Endowment Spending Rules*

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I. Introduction

We study historical endowment spending by colleges in the U.S., focusing on how those colleges respond to sharp fluctuations in the size of the endowment. Using data reported by private, non-profit colleges and universities (IRS 990 tax filings, including Schedule D) we explore their spending from endowments and the extent to which they follow specific rules or exercise greater discretion. Our data allow us to estimate the spending rules followed by institutions and examine deviations from these spending rules and what might be explaining them. Colleges and universities can deviate from their spending rules both by distributing an amount from the endowment that differs from the amount called for by their rule but can also do so by distributing resources from the endowment to units on campus who then do not actually spend them. Actual spending from the endowment depends on this as well as the amount distributed, and the IRS 990 data allows us to gain some insight into the importance of this behavior.

Our sample period begins in 2008-2009, which is the first year that colleges filed Schedule D, as part of their 990 tax returns and continues to 2021-2022, the most recent fiscal year for which colleges have filed returns. Schedule D is essential to our analysis because it reports endowment levels along with revenues (1. investment returns; 2. contributions) and costs (3. grants and scholarships; 4. administrative costs; 5. other costs) in a cleaner fashion than in previous 990 returns. This sample period is of particular interest because it corresponds to a prolonged positive period from the end of the Great Recession to the start of the pandemic as well as the unexpectedly positive year for stock market investments during the first full fiscal year of the pandemic in 2020-21. At the same time, this sample period also has some limitations, and in particular, we are unable to study the reaction of colleges to downturns (the focus of J. Brown and Weisbenner (2014)), as the two primary negative years for endowment returns during this time occurred during the first (2008-09) and last (2021-22) years of this period.

Our paper is distinguished from previous studies of endowment spending both because of the nature and time period of our analysis. Previous studies (such as K. Brown and Tiu (2013), J. Brown et al. (2014), Goetzmann and Oster (2014), and Bulman (2022)) primarily utilized a combination of IPEDS and NACUBO (“National Association of College and University Business Officers”) survey data. The NACUBO data are self-reported and do not include all
institutions with endowments, although both private and public institutions participate. By rule, our data is comprehensive and since it consists of official tax filings, it is presumably more accurate than the survey data compiled by NACUBO. Recent papers by Dehiya and Yermack (2021) and Lo, Matveyev, and Zeume (2022) use 990 filings to analyze returns and spending for a broader set of non-profit organizations but do not focus on colleges or other subgroups of those organizations.

Our paper is distinct from two other active strains of the literature in objective as our primary goal is to use tax filings to reconstruct the spending rules used by universities. One other set of papers studies university asset allocation with an eye towards assessing whether they are choosing optimal risk levels in their investments (see for example Dimmock (2012), Blanchett (2014), Gilbert and Hrdlicka (2015), Ang, Ayala, and Goetzmann (2018), Campbell and Sigalov (2021), Cejnek, Franz, and Stoughton (2023)). Similarly, Filosa (2023) considers endowment levels and spending relative to university debt levels. A second set of papers considers the implications of sustainability on optimal spending from an endowment (D. Brown and Scholz (2019), Dybvig and Qin (2021), Campbell and Martin (2023)). Our paper is mostly closely related to Halem, Lo, Matveyev, and Quraishi (2022), who assess the interaction between stated spending rules and current asset allocations for Harvard, MIT, Stanford, and Yale and project the trajectories of their endowment levels across a variety of different scenarios.¹

An understanding of how institutions use their endowments is of interest for more than purely academic reasons. The so-called “demographic cliff” may reduce overall college enrollment by as much as fifteen percent in coming years (Grawe, 2018). While this is unlikely to impact the most highly selective institutions (e.g. Ivy Plus), many tuition-dependent private non-profits may need to rely more on endowment spending in order to navigate the new enrollment landscape. Second, while we are not able to study the endowment behavior of most public institutions due to data limitations related to IRS reporting requirements, these institutions may

¹ Cejnek et al. (2014) provides a broader survey of past research on university endowments, while Chambers, Dimson, and Kaffe (2020) studies endowment spending of 12 institutions over a longer period of 75 years.
also need to rely more on their endowments as a result of past (and likely future) declines in state support for higher education (Webber, 2018).

Our paper makes the following contributions to the literature. First, we provide a detailed theoretical discussion of how to think about the tradeoffs inherent in different spending rules. Anecdotally, many faculty appear to have a poor understanding of both what is possible (e.g. restricted vs. unrestricted spending) and what the long-term drawbacks are to increased endowment spending. On the other hand, institutions can often lack transparency in the manner in which they spend their endowment. Money is fungible, and endowment spending nominally directed toward financial aid may not functionally be spent on that category if money from a different institutional source is reduced. In order to shed some light on the above questions, we first empirically estimate the spending rules for each school in our sample. Using a small set of assumptions and a simple empirical model, we are able to closely (within 10%) approximate the spending rules of 80 percent of the sample.

The paper proceeds as follows. Section 2 describes the data and the sample of institutions that we study. Section 3 provides descriptive statistics. Section 4 analyzes the theoretical properties that result from the application of accounting identities to the one-step spending rules described by many colleges. Section 5 presents the results of simulations designed to reverse engineer the specific accounting rules utilized by each of the colleges. Section 6 presents regression analysis of spending as reported in Schedule D filings and IPEDS reports. Section 7 concludes.
II. Data and the Sample

Our sample of four-year institutions includes the 187 private, non-profit colleges and universities with highest endowments per full-time equivalent (FTE) student in 2021. We excluded public institutions because they are not required to file 990 tax returns. We also exclude three institutions that would otherwise qualify for our sample: (1) Salem College because its returns combine the financial information for a college and an associated high school; (2) Cedarville University and Earlham college, whose 990 form includes one year with a large negative value for “Other expenditures”, as that entry disrupts our ability to assess their spending rules.

This is a snapshot of the relevant portion of the Amherst College 2021-2022 Schedule D form. The entries in lines 1b and 1c represent inflows or endowment revenues. The entries in lines 1d, 1e, and 1f represents or endowment payouts. By accounting rule, the balance at the end of the year is determined by initial balance adjusted for the difference of yearly revenues and costs:

Ending Balance (1g) = Starting Balance (1a) + Revenues (1b + 1c) – Costs (1d + 1e + 1f)

Complete if the organization answered “Yes” on Form 990, Part IV, line 10.

The entries in lines 2a, 2b, and 2c are also of interest. In particular, Line 2a reports the current percentage of the endowment designated as “Funds Functioning as Endowment”, which is sometimes described as “quasi-endowment”. This is essentially another version of savings; increases in the corresponding value from year to year in this category may be indicative of hoarding, whereby the college payout or from other sources of revenue, such as a surplus in the operating budget. We also make use of reported values of (1) “Cash-not-interest-balance” and (2) “Savings and temporary cash investments” from Section X (“Balance Sheet”) of the 990.
College 990 filings are public information and are collected and made available by ProPublica and other organizations. We used existing files of 990 data compiled by the Federal Reserve Bank for nine of the fourteen years in the sample period and filled out the data set by manual entry of data found on the ProPublica website. We checked the data for all anomalies and corrected a number of errors in either initial data entry or in the original 990 files. Our data includes 14 years of 990 filings for each of the colleges in the sample with the single exception of the filing for one year for Illinois Wesleyan where the publicly available version of the 990 file is incomplete.

We divide the sample of colleges into five groups based on their initial endowment values at the start of the sample period, July 1, 2008. Table 1 provides the criteria for each of these groups as well as a selection of colleges from each group.

### Table 1: Subgroups of Colleges by Initial Endowment Levels

<table>
<thead>
<tr>
<th>GROUP 1</th>
<th>GROUP 2</th>
<th>GROUP 3</th>
<th>GROUP 4</th>
<th>GROUP 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ivy League</td>
<td>Amherst</td>
<td>Bowdoin</td>
<td>American</td>
<td>Belmont</td>
</tr>
<tr>
<td>Chicago</td>
<td>Baylor</td>
<td>Brandeis</td>
<td>Babson</td>
<td>Bennington</td>
</tr>
<tr>
<td>Duke</td>
<td>Boston College</td>
<td>Carleton</td>
<td>Bates</td>
<td>Univ. of Dallas</td>
</tr>
<tr>
<td>Emory</td>
<td>Boston U</td>
<td>Davidson</td>
<td>Fordham</td>
<td>Elon</td>
</tr>
<tr>
<td>Johns Hopkins</td>
<td>Cal Tech</td>
<td>Haverford</td>
<td>Franklin Marshall</td>
<td>Emerson</td>
</tr>
<tr>
<td>MIT</td>
<td>Carnegie Mellon</td>
<td>Macalester</td>
<td>Gonzaga</td>
<td>Endicott</td>
</tr>
<tr>
<td>Northwestern</td>
<td>Case Western</td>
<td>Middlebury</td>
<td>Lewis and Clark</td>
<td>Evansville</td>
</tr>
<tr>
<td>Notre Dame</td>
<td>George Wash.</td>
<td>Northwestern</td>
<td>Loyola Marymt.</td>
<td>Hampshire</td>
</tr>
<tr>
<td>NYU</td>
<td>Smith</td>
<td>Oberlin</td>
<td>Marquette</td>
<td>Knox</td>
</tr>
<tr>
<td>Pittsburgh</td>
<td>Swarthmore</td>
<td>RPI</td>
<td>Providence</td>
<td>Lake Forest</td>
</tr>
<tr>
<td>Rice</td>
<td>TCU</td>
<td>Syracuse</td>
<td>Quinnipiac</td>
<td>Millsaps</td>
</tr>
<tr>
<td>Stanford</td>
<td>Tufts</td>
<td>Vanderbilt</td>
<td>Reed</td>
<td>Mt. St. Mary’s</td>
</tr>
<tr>
<td>USC</td>
<td>Tulane</td>
<td>Vassar</td>
<td>St. Lawrence</td>
<td>Sarah Lawrence</td>
</tr>
<tr>
<td>Vanderbilt</td>
<td>Wellesley</td>
<td>Wake Forest</td>
<td>Tulsa</td>
<td>Thomas Aquinas</td>
</tr>
<tr>
<td>Washington U</td>
<td>Williams</td>
<td>Wesleyan</td>
<td>Villanova</td>
<td>Westmont</td>
</tr>
<tr>
<td><strong>22 colleges</strong></td>
<td><strong>24 colleges</strong></td>
<td><strong>33 colleges</strong></td>
<td><strong>78 colleges</strong></td>
<td><strong>29 colleges</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>START BALANCE</th>
<th>START BALANCE</th>
<th>START BALANCE</th>
<th>START BALANCE</th>
<th>START BALANCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 x 10^9 or more</td>
<td>1 x 10^9 to 2 x 10^9</td>
<td>5 x 10^8 to 1 x 10^9</td>
<td>1 x 10^8 to 5 x 10^8</td>
<td>0 to 10^8</td>
</tr>
</tbody>
</table>

2 In many cases, an error in the initial data entry by a college was straightforward to correct because the college fixed the error in reports provided in subsequent years. As shown in the Schedule D provided for Amherst, each filing provides information from the current year and four most recent years. Except in cases of obvious mistakes, we use the report for a given year for each year’s filing rather than the information provided for that year in subsequent 990 filings, primarily because the Federal Reserve Bank files only contain entries for the current year.
III. Descriptive Statistics for Endowment Returns

The fifteen or so years since the end of the Great Recession have provided an unusual period of nearly consistently positive stock returns and minimal inflation. As suggested by Ehrenberg (2000), Figure 1 compares yearly endowment returns to the percentage change in the Higher Education Price Index (HEPI) compiled by the Commoufud Institute during the sample period.

Figure 1: Endowment Returns and Inflation: 2008-09 to 2021-22

As shown in Table 2, the universities in our sample accrued average endowment revenues at a yearly rate of 10.5%, which was well more than their yearly average endowment spending of 5.1%. There was relatively little difference in nominal yearly endowment growth by size of initial endowment: increases in endowment for colleges in each group outpaced increases in the HEPI by at least an average of 2.35 percentage points per year over the sample period.

Table 2: Average Yearly Revenues and Costs as % of Endowment

<table>
<thead>
<tr>
<th></th>
<th>All</th>
<th>Group 1</th>
<th>Group 2</th>
<th>Group 3</th>
<th>Group 4</th>
<th>Group 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Invest Return</td>
<td>7.28%</td>
<td>8.70%</td>
<td>7.77%</td>
<td>7.43%</td>
<td>7.14%</td>
<td>6.23%</td>
</tr>
<tr>
<td>Contributions</td>
<td>3.27%</td>
<td>3.00%</td>
<td>2.32%</td>
<td>2.46%</td>
<td>3.14%</td>
<td>5.54%</td>
</tr>
<tr>
<td>Grants</td>
<td>1.60%</td>
<td>1.03%</td>
<td>1.35%</td>
<td>1.79%</td>
<td>1.79%</td>
<td>1.54%</td>
</tr>
<tr>
<td>Other Cost</td>
<td>3.16%</td>
<td>3.82%</td>
<td>3.50%</td>
<td>3.02%</td>
<td>2.87%</td>
<td>3.35%</td>
</tr>
<tr>
<td>Admin Cost</td>
<td>0.30%</td>
<td>0.27%</td>
<td>0.35%</td>
<td>0.32%</td>
<td>0.35%</td>
<td>0.15%</td>
</tr>
<tr>
<td>Revenues</td>
<td>10.54%</td>
<td>11.74%</td>
<td>10.09%</td>
<td>9.89%</td>
<td>10.18%</td>
<td>11.77%</td>
</tr>
<tr>
<td>Total Cost</td>
<td>5.07%</td>
<td>5.12%</td>
<td>5.20%</td>
<td>5.13%</td>
<td>5.00%</td>
<td>5.04%</td>
</tr>
<tr>
<td>Change in HEPI</td>
<td>2.48%</td>
<td>2.48%</td>
<td>2.48%</td>
<td>2.48%</td>
<td>2.48%</td>
<td>2.48%</td>
</tr>
</tbody>
</table>
Table 3 divides the sample period into four subperiods. During the first four years from 2008-09 to 2011-12, these colleges were still recovering from the Great Recession; with most using the initial period of stock market recovery to recoup losses from 2008-09 their endowment levels were fairly flat for this period as a whole. In the next two sets of four years, colleges in each group expanded their endowments despite somewhat lower return years in 2015-16 and 2019-20. Finally, colleges in each group benefitted from the extraordinary positive stock market year of 2020-21 though this was followed by average losses in 2021-22.

Table 3: Cumulative Change in Endowment by Time Period

<table>
<thead>
<tr>
<th>Time Period</th>
<th>Group 1</th>
<th>Group 2</th>
<th>Group 3</th>
<th>Group 4</th>
<th>Group 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>2008-09 to 2011-12</td>
<td>-1.18%</td>
<td>-4.72%</td>
<td>-3.91%</td>
<td>-2.13%</td>
<td>11.03%</td>
</tr>
<tr>
<td>2012-13 to 2015-16</td>
<td>25.74%</td>
<td>18.12%</td>
<td>20.73%</td>
<td>25.07%</td>
<td>43.81%</td>
</tr>
<tr>
<td>2016-17 to 2019-20</td>
<td>32.67%</td>
<td>22.14%</td>
<td>20.18%</td>
<td>20.43%</td>
<td>28.98%</td>
</tr>
<tr>
<td>2020-21 to 2021-22</td>
<td>31.20%</td>
<td>29.70%</td>
<td>24.92%</td>
<td>22.22%</td>
<td>12.48%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Time Period</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>July 2008 to June 2022</td>
<td>116.12%</td>
<td>82.39%</td>
<td>75.98%</td>
<td>84.31%</td>
<td>134.25%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Balance</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Balance 2009</td>
<td>$8.17 B (8.09 B)</td>
<td>$1.38 B (303 M)</td>
<td>$676 M (101 M)</td>
<td>$248 M (94.4 M)</td>
<td>$51.4 M (26.9 M)</td>
</tr>
<tr>
<td>End Balance 2022</td>
<td>$16.2 B (12.9 B)</td>
<td>$2.51 B (781 M)</td>
<td>$1.19 B (339 M)</td>
<td>$451 M (224 M)</td>
<td>$105 M (81.1 M)</td>
</tr>
<tr>
<td># of Institutions</td>
<td>22</td>
<td>24</td>
<td>34</td>
<td>78</td>
<td>29</td>
</tr>
</tbody>
</table>

In all, on a nominal basis, the colleges in each group doubled or nearly doubled their average endowment levels during the sample period.

Figure 2 graphs this information in slightly different form, highlighting the extraordinary nature of the conditions in 2021-22. The colleges in Group 1 had increased their endowment levels by an average of 60% from the start of 2008-09 to the end of 2019-2020 and then were able to increase their endowments by nearly 40% in 2020-2021. There was somewhat more positive correlation between endowment size and investment return in 2021-2022, as the colleges in Group 1 had investment gains of more than 40% of endowment levels as the start of that fiscal year, while the colleges in Groups 2 to 4 had investment returns between 33 and 38%, and the colleges in Group 5 had investment returns of 25% for that fiscal year..
Figure 2: Endowment Changes by Group and Year

Figure 3 compares the change in costs reported in Schedule D, once again highlighting the last two years of the sample period. On a percentage basis, the institutions in the sample increased their endowment expenditures at a faster rate than the For 2008-09 to 2019-20. With the exception of colleges in Group 5, the colleges in the sample increased expenditures per year by an average of nearly 50% during the first twelve years of the sample from 2008-09 to 2019-20.

Figure 3: Changes in Reported Costs (Schedule D) by Group and Year

The colleges in Group 5 increased expenditures by the greatest proportional amount in 2020-21, the unusually positive year for investments, then had a much lower rate of increase the following year, which was an unusually negative year for investments. By contrast, colleges in Group 1 had little change in 2020-21 but a substantial increase of 7.1% in Schedule D costs in 2021-22. These findings suggest that the colleges in Group 5 respond quickly whereas colleges in Group 1 may also respond more slowly but fairly decisively to changes in economic conditions.

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3 We exclude Williamette University (in group 4) from the computations for Figure 3 because it reported substantial grants for most years and 0 expenditures on grants for 2020-2021.
IV. Theoretical Properties of Spending Rules

Endowment spending rules provide specific guidance about the dispersion of revenues over time. Sedlacek and Jarvis (2010) identify four categories of these rules based on institutional responses to the NACUBO survey: about 75% report a Moving Average Rule that applies a target annual spending rate to the average of past endowment levels (usually on a quarterly basis for a three-year period). Sedlacek and Jarvis further observe 12 to 15% of institutions with largest endowment levels use a Hybrid Rule based on a weighted sum of recent endowment level and previous year’s endowment spending. K. Brown and Tiu (2013) tracked the policies reported by institutions in their 2003 to 2011 survey responses. Surprisingly, only half of the institutions maintained a single rule through those nine years and on average, 25% of respondents reported a change in approach or target spending rate in each given year. In this section, we provide a brief accounting-based overview of the properties of the Moving Average and Hybrid rules as precursor to using them to simulate the spending patterns that we observe in the tax data.

A. Basic Framework and Spending Rules

We define $X_t$ as the endowment level, $R_t$ as revenue, and $S_t$ as spending in year (or period) $t$ and assume initially that there is no inflation and no return on endowment investments. The endowment changes over time according to the accounting rule (i.e. “The Law of Motion”)

$$X_{t+1} = X_t + R_t - S_t.$$  \hfill (1)

**Simple Proportional Spending Rule**

The simplest Moving Average rule sets a yearly endowment payout as a fixed proportion of current endowment based on payout rate $\alpha$, which is often .05 for non-profit organizations. We begin discussion with this rule – which is simpler than the rules used by most institutions in our sample – because its properties are so clear.

$$S_t = \alpha X_t.$$  \hfill (2)

Looking forward one period, then

$$X_{t+1} = X_t + R_t - S_t = (1 - \alpha) X_t + R_t$$  \hfill (3)

$$S_{t+1} = \alpha X_{t+1} = \alpha(1 - \alpha) X_t + R_t$$  \hfill (4)

As this pair of equations suggests, revenue in period $t$ first influences endowment spending at period $t+1$. In each period, proportion $\alpha$ of the remaining value of revenue $R_t$ is designated for
payout but that remaining value declines over time. Specifically, each additional dollar of revenue in period \( t \) yields additional payout \( a(1-\alpha)^{k-1} \) in period \( k \).  

**Multiperiod Moving Average Spending Rule**

While the most common practice is to base endowment payouts on an equally weighted Moving Average of endowment levels for the past 3 to 5 years (meaning that this average includes 12 to 20 quarterly values), for the purpose of this theoretical discussion, we consider the simplest form of moving average with just two terms and allow the weights on the years to be different.  

\[
S_t = W_1 X_t + W_2 X_{t-1}
\]  

(5)

Since the endowment levels \( X_t \) and \( X_{t-1} \) are linked by the one-step accounting rule, we can rewrite \( S_t \) as a function of endowment level, revenue, and spending in period \( t-1 \).  

\[
S_t = W_1 [X_{t-1} + R_{t-1} - S_{t-1}] + W_2 X_{t-1}
\]

OR  

\[
S_t = (W_1 + W_2) X_{t-1} + W_1 R_{t-1} - S_{t-1}
\]

(6)

Once again, with this rule, revenue from period \( t \) first influences spending in period \( t+1 \) but now the influence of that revenue continues to increase until it affects each of the terms, \( X_t \) and \( X_{t-1} \), in the spending rule.

Figure 4 shows the effect of a change in the number of terms in the moving average on the distribution of new revenue accrued in period 0 with a target spending rate of 5%. With a one period average, the Moving Average becomes a Proportional Spending Rule, so this spending peaks immediately at 5% in period 1 and then declines gradually from there. Increasing the time frame to five or ten periods for the weighted average lengthens the time until the peak spending level is reached and also smooths the spending curve around the peak level to some degree. In addition, spending never reaches the target level in any period for any choice of time frame longer than one period. Beyond the case of a Proportional Spending Rule, endowment spending is based on an average past endowment level less than total revenue from time 0 since the average includes periods where some of that revenue had already been spent.

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4 That is, the dispersion of revenue \( R_t \) follows a geometric series forward from period \( t+1 \) with initial value \( aR_t \) and multiplier \((1-\alpha)\) so by the geometric series rule, the eventual sum of revenue payouts over time is \( \frac{aR_t}{1-(1-\alpha)} = R_t \).  

5 The institution can implement a target spending rate by setting \( W_1 + W_2 = \alpha \) and then can represent this spending rule as \( S_t = \alpha [W_1 X_t + (1-W_1) X_{t-1}] \) so that choice of weight \( W_1 \) and target \( \alpha \) fully describe the spending rule.
Sedlacek and Jarvis (2010) explain that hybrid spending was introduced by Stanford and is known interchangeably as the Stanford and Yale rules depending on the weights. Yale University currently describes this policy on its website as follows:

*The payout under the Spending Policy is equal to 80% of the prior year’s spending plus 20% of the long-term spending rate applied to the previous year’s beginning endowment market value, with the sum adjusted for inflation.*

https://your.yale.edu/policies-procedures/policies/2202-endowment-spending-and-distributions

One important element of Yale’s policy is that spending in a given year is a function of lagged values of both endowment level and spending, so that revenues in year $t$ influence the endowment level in year $t+1$ but do not affect endowment spending until year $t+2$. With this in mind, an endowment spending rule of this form is defined by parameters $\alpha$ and $\beta$ where $\alpha$ represents the targeted long-term spending rate (once again, this is commonly set to .05) and $\beta$ is the proportional weight on the endowment level. That is,

$$S_t = \beta (\alpha X_{t-1}) + (1- \beta) S_{t-1}. \quad (7)$$

To track the effect of revenues $R_t$ on future spending, we apply this definition to period $t+2$ since that is the first time that those revenues affect spending

$$S_{t+2} = \beta (\alpha X_{t+1}) + (1- \beta) S_{t+1}. \quad (8)$$
Applying accounting rule (1) $X_{t+1} = X_t + R_t - S_t$ and using the definition of $S_t$ to substitute for $S_{t+2}$ gives $S_{t+2}$ as a function of values from period $t$.

$$S_{t+2} = \beta \alpha (X_t + R_t - S_t) + (1- \beta) \left[ \beta (\alpha X_t) + (1- \beta) S_t \right]$$

OR

$$S_{t+2} = \beta \alpha R_t + \beta (1-\beta)^2 \alpha S_t + \alpha \beta (2-\beta) X_t.$$

In period $t+1$, revenue $R_t$ in period $t$ affects the endowment value but not the spending level. Starting in period $t+2$, those revenues $R_t$ in period $t$ influence both the spending level and the endowment level. Assuming that $\alpha$ is relatively small, revenue $R_t$ has a greater influence in period $t+3$ when it flows into spending from two channels where it only affects spending through the endowment level in period $t+1$. Figure 5 compares payout rates over time for different values of $\beta$, the weight for the endowment level, holding fixed the target payout rate $\alpha = .05$.

**Figure 5: Spending of Endowment Revenue Over Time with a Hybrid Rule**

We highlight several features of the spending patterns that are evident in Figure 5.

- Increasing $\beta$ shifts the dispersion of revenue from year 0 earlier in time because new revenues have an immediate effect on endowment level and only a gradual effect on spending levels.
- The payout of revenues from year 0 are always less than the target payout rate $\alpha = .05$ with maximum payout decreasing in $\beta$.
- Yearly payouts initially increase and reach a maximum value between years 5 and 10.
- Yearly payouts for lower values of $\beta$ eventually catch up and surpass payouts for higher values of $\beta$.  

13
Amherst uses a variant of the hybrid rule that relies on a moving average of past endowments.

the … spending rule … combines a three-year smoothed historical endowment value (30%) with a modest increase (inflation) in last year’s spending value (70%).

Naturally, by incorporating this moving average in its rule, Amherst combines properties of the Moving Average and Hybrid spending rules. Specially, this choice reduces the influence of very recent changes in the endowment with the result that Amherst spends a lower proportion of new revenues in the first years than for the corresponding weighted average rule in Figure 6. As Ehrenberg (2009) summarizes,

wide fluctuation in endowment values may suggest the need to base spending on a longer period of endowment values to provide less variability in the flow of spending. ...The downside of basing spending rules on longer historical periods is that during prolonged upswings in market valuations (such as that experienced during most of the past 20 years), spending as a share of the current value of the endowment will fall below the target percentage share.

Figure 6 compares the cumulative payout rates over time associated with these spending patterns and suggests that the value of $\beta$ has limited effect on long-term cumulative spending. About half of initial revenues are spent in 15 years for any of these rules and cumulative spending patterns essentially overlap for every value of $\beta$ for year 20 and beyond.

**Figure 6: Cumulative Spending of New Revenue with a Hybrid Rule**
Figure 7 shows how the distribution of revenues is affected by investment returns. In this case, we assume a 4% annual return on investments; since this is less than the target payout rate, the endowment does not grow forever and new revenues are still dispersed over time.

- As in Figure 1, spending increases over time and is initially larger for larger values of $\beta$.
- The endowment is initially growing because the investment return is initially higher than the effective payout rate for the new revenue.
- The peak payout rate goes above the target rate of .05.
- Dispersion of new revenues is quite elongated. After 100 years, 35% of the initial revenue still remains in the endowment for each of these three weighing schemes.

**Figure 7: Spending of New Revenue with Hybrid Rule and Investment Returns**

---

**B. Relationship of the Hybrid and Moving Average Spending Rules**

Iterative application of the Hybrid and Moving Average rules yield long-term patterns for the dispersion of the inflow of revenue in (say) period 0. It is natural to wonder if the two rules overlap to some degree in the spending patterns they produce. To make this comparison, we use generalized versions of the spending rules with weights ($W_{1M}$, $W_{2M}$) for the Moving Average rule and ($W_{1H}$, $W_{2H}$) for the Hybrid rule with no mention of the typical spending rate $\alpha$.

**MOVING AVERAGE:** $S_t = W_{1M} X_t + W_{2M} X_{t-1}$.

**HYBRID RULE:** $S_t = W_{1H} X_{t-1} + W_{2H} S_{t-1}$. 

---
Proposition 1: There is a direct translation of any Hybrid Rule to or from a Moving Average rule, but if the weights for Moving Average Rule are both positive, then the translation requires one of the weights for the Hybrid rule to be negative.

Proof: This follows from direct application of the accounting rule (1) to the definitions of the spending rules. When all revenue is accrued at time 0, (1) simplifies to \(X_t = X_{t-1} - S_{t-1}\). Thus, with the Moving Average rule,

\[
S_t = W_{1M} X_t + W_{2M} X_{t-1}
\]

(9)

OR

\[
S_t = W_{1M} (X_{t-1} - S_{t-1}) + W_{2M} X_{t-1}
\]

(10)

OR

\[
S_t = (W_{1M} + W_{2M}) X_{t-1} - W_{1M} S_{t-1}
\]

(11)

The key point is that we started with a Moving Average rule in (9) and produced an equivalent Hybrid rule in (11). That is, a Moving Average rule with weights \((W_{1M}, W_{2M})\) yields exactly the same spending rule as a Hybrid rule with weights \((W_{1H} = W_{1M} + W_{2M}, W_{2H} = -W_{1M})\), where we emphasize the negative sign in \(W_{2H} = -W_{1M}\) so \(W_{2H}\) is negative if \(W_{1M}\) is positive.

We can apply the same steps in reverse to convert a Hybrid rule to a Moving Average rule.

\[
S_t = W_{1H} X_{t-1} + W_{2H} S_{t-1}
\]

(12)

OR

\[
S_t = W_{1H} X_{t-1} + W_{2H} (X_{t-1} - X_t)
\]

(13)

OR

\[
S_t = (W_{1H} + W_{2H}) X_{t-1} - W_{2H} X_t
\]

(14)

Once again, the negative coefficient on \(X_t\) in (14) indicates that if the weights are both positive for a Hybrid Rule, then the corresponding weight on current endowment must be negative in the equivalent Moving Average rule.

Examples: The Yale rule sets 80% weight on previous spending and 20% on current endowment which yields \(W_{1H} = 0.04, W_{2H} = 0.8\) with target spending rate \(\alpha = .05\). This corresponds to a Moving Average rule with weights \(W_{1M} = W_{1H} + W_{2H} = 0.84\) and \(W_{2M} = -W_{2H} = -0.8\).

A equal weighted Moving Average rule with \(W_{1M} = W_{2M} = 0.025\) (for target spending \(\alpha = .05\)) corresponds to a Hybrid rule with weights \(W_{1H} = W_{1M} + W_{2M} = 0.05, W_{2H} = -W_{1M} = -.025\).
We note that Proposition 1 relies on two important assumptions that simplify the derivation. First, although we assume that the Hybrid Rule includes a lag so that current spending is a function of both lag-1 endowment and lag-1 spending, whereas we allowed the Moving Average Rule to include a term for current endowment level. In the Appendix, we extend the proof of Proposition 1 to show that the result holds when both rules are based entirely on lagged values.

Second, and more substantively, Proposition 1 limits the Moving Average rule to be a weighted sum of two endowment levels. While there is a natural mapping of Moving Average to and from Hybrid Rule when each incorporates two terms and two weight coefficients, that property does not hold when there are different numbers of terms in the two rules. In practice, Moving Average rules have more than two terms and two coefficients. In essence, the addition of more terms yields more degrees of freedom for a Moving Average rule, which makes it impossible to mimic every Moving Average Rule with (say) three terms with a two term Hybrid Rule.

C. Implications of the Theoretical Analysis for Simulations

Section V conducts simulations using Moving Average and Hybrid rules to attempt to recreate the observed endowment spending pattern for each of the institutions in our sample. In these simulations, we explicitly restrict the coefficients used in the rules to be positive, since the reported spending rules for these institutions always use positive coefficients. Thus, although there is a theoretical equivalence between the Moving Average and Hybrid rules, this equivalence is broken when it is not possible for any weighting coefficient to take a negative value. In fact, the reported results for the two rules diverge with neither rule appearing to dominate the other, which provides suggestive evidence that each rule has theoretical advantages and disadvantages given the restriction that none of the weights can be negative.
V. Spending Rule Simulations

We conducted simulation exercises to attempt to reverse engineer the spending rules of the institutions in the sample. In each case, we estimate distinct weights for individual institutions to simulate the effect of Hybrid and Moving Average spending rules for them. The definition of Hybrid Rules leaves little room for adjustment, but we have to decide how many lagged terms to include in the Moving Average simulations.

Estimating Weights for Each College’s Spending Rules

Table 4 reports the results of preliminary descriptive OLS regressions for Moving Average spending rules with observed spending as the dependent variable and lagged values of the endowment level as independent variables. Equation (15) depicts the form of the Lagged model where $S_{j,t}$ represents endowment spending for institution $j$ in period $t$ and $X_{j,t-k}$ represents the lagged $k$ value of endowment for institution $j$.

$$S_{j,t} = \beta_0 X_{j,t} + \beta_1 X_{j,t-1} + \beta_2 X_{j,t-2} + \beta_3 X_{j,t-3} + \beta_4 X_{j,t-4} + \epsilon_{j,t}$$  \hspace{1cm} (15)

The results suggest that there are significant results for up to 3 lagged terms but not necessarily for more than that because the coefficient on the 2nd lagged value of endowment is not significant in the Lag 3 model. The coefficients in the last two columns are similar but with larger standard errors when we restrict the sample to the institutions with the largest endowments.

Table 4: Regression Results for Moving Average Spending Rules

<table>
<thead>
<tr>
<th>Lag 0</th>
<th>Lag 1</th>
<th>Lag 2</th>
<th>Lag 3</th>
<th>Lag 4</th>
<th>Lag 4 Group 1-2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Current Endowment</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Current Endowment</td>
<td>.0452**</td>
<td>.0035**</td>
<td>.0048**</td>
<td>.0039**</td>
<td>.0047**</td>
</tr>
<tr>
<td></td>
<td>(.0002)</td>
<td>(.0009)</td>
<td>(.0011)</td>
<td>(.0012)</td>
<td>(.0013)</td>
</tr>
<tr>
<td><strong>Endowment Lag 1</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Endowment Lag 1</td>
<td>.0482**</td>
<td>.0341**</td>
<td>.0346**</td>
<td>.0257**</td>
<td>.0260**</td>
</tr>
<tr>
<td></td>
<td>(.0010)</td>
<td>(.0017)</td>
<td>(.0027)</td>
<td>(.0037)</td>
<td>(.0073)</td>
</tr>
<tr>
<td><strong>Endowment Lag 2</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Endowment Lag 2</td>
<td>.0129**</td>
<td>-.0009</td>
<td>.0137**</td>
<td>.0135</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.0016)</td>
<td>(.0030)</td>
<td>(.0043)</td>
<td>(.0087)</td>
<td></td>
</tr>
<tr>
<td><strong>Endowment Lag 3</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Endowment Lag 3</td>
<td>.0149**</td>
<td>.0020</td>
<td>.0018</td>
<td>.0064</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.0015)</td>
<td>(.0031)</td>
<td>(.0063)</td>
<td>(.0039)</td>
<td></td>
</tr>
<tr>
<td><strong>Endowment Lag 4</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Endowment Lag 4</td>
<td>.0065**</td>
<td>.0064</td>
<td>.0064</td>
<td>.0064</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.0019)</td>
<td>(.0039)</td>
<td>(.0039)</td>
<td>(.0039)</td>
<td></td>
</tr>
<tr>
<td><strong>R^2</strong></td>
<td>.9586</td>
<td>.9805</td>
<td>.9825</td>
<td>.9834</td>
<td>.9834</td>
</tr>
<tr>
<td><strong>Observations</strong></td>
<td>2,618</td>
<td>2,431</td>
<td>2,244</td>
<td>2,057</td>
<td>1,870</td>
</tr>
</tbody>
</table>

* = significant at .05, ** = significant at .01 level.

---

6 We exclude the constant term in each model to match the description of Moving Average spending rules.
Based on consideration of the results in Table 4, we chose to apply a Lag 2 rule for simulating Moving Average spending, with separate terms for current, lag 1, and lag 2 endowment levels. We include current endowment and two lags (rather than using Lag 1 to Lag 3 values) to allow for the inclusion of as many years of data from the sample as possible in the simulation. That is, we use spending rules of the following form for the simulations:

**Spending Model Equations for Simulations**

**MOVING AVERAGE:**

\[ S_{jt} = W_0X_{jt} + W_1X_{j,t-1} + W_2X_{j,t-2} \]  

(16)

**HYBRID:**

\[ S_{jt} = W_3X_{t-1} + W_5S_{t-1} \]  

(17)

As an initial background step for our simulations, we applied separate OLS regressions for Hybrid and Moving Average models for each college (13 observations for the Hybrid, 12 for the Moving Average model) with spending \( S_t \) as the dependent variable in (18) and (19).

**Regression Equations to Estimate Coefficients for the Simulations**

**MOVING AVERAGE:**

\[ S_{jt} = \hat{\beta}_0X_{jt} + \hat{\beta}_1X_{j,t-1} + \hat{\beta}_2X_{j,t-2} + \epsilon_{jt} \]  

(18)

**HYBRID:**

\[ S_{jt} = \hat{\beta}_E X_{t-1} + \hat{\beta}_S S_{t-1} + \epsilon_{jt} \]  

(19)

We excluded a constant term in these regression specifications, so these OLS models do not incorporate a target spending level or restrict the values of the coefficients or their sums, thereby allowing for average per-year spending levels other than 5% of endowment level.

Next, we assessed the plausibility of the regression coefficients for use as spending rule weights in our simulations according to the following criteria:

- For the Moving Average model, we required regression coefficients on the Lag 1 and Lag 2 variables to satisfy \( 0 \leq \hat{\beta}_1 \leq 0.06 \) and \( 0 \leq \hat{\beta}_2 \leq 0.06 \), so that each lagged endowment value contributes a positive amount less than 6 percent of its nominal value to projected spending.

- For the Hybrid model, we required regression coefficients on the two independent variables to satisfy \( 0 \leq \hat{\beta}_E \leq 0.06 \) and \( 0 \leq \hat{\beta}_S \leq 1.2 \), so that each component contributes a positive amount, but not too large an amount to estimated spending.

When these conditions held, which occurred for 59.4% of cases with the Moving Average and 80.8% of cases with the Hybrid rule, we apply the regression coefficients directly as weights so
that \( W_0 = \hat{\beta}_0, W_1 = \hat{\beta}_1, W_2 = \hat{\beta}_2 \) for Moving Average and \( W_E = \hat{\beta}_E, W_S = \hat{\beta}_S \) for the Hybrid rule in the simulations. If other cases where the regression coefficients did not meet these conditions, we reverted to a default choice of parameters \( W_0 = 0, W_1 = 0.025, W_2 = 0.025 \) for Moving Average and \( W_E = 0.03, W_S = 0.4 \) for the Hybrid rule.\(^7\)

**B. Detailed Rules for the Simulations**

With the general form of the one-step spending rule equations and the weights for the spending rules in place, we made the following further assumptions to carry out the simulations.

- We assume that each college generates the same proportion \( \gamma_{j,t} \) of revenues for college in \( j \) in year \( t \) in the simulations as in practice. That is, the ratio of the sum of (1) investment earnings and (2) contribution to endowment value at the start of the fiscal year as reported in Scheduled D is the same in the simulations as in practice.
- We work in terms of nominal values with no adjustments for inflation.
- We take the actual endowment and spending values as given for one year (2009) for Hybrid and two years (2009, 2010) for Moving Average simulations since (16) includes one lagged observation and (17) includes two lagged observations.

To carry out the simulations for a given college, we then apply equation (16) for the Moving Average and (17) for the Hybrid rule to compute spending \( \hat{S}_{j,t} \), compute revenue in year \( t \) as proportion \( \gamma_{j,t} \) of the starting endowment in that year \( \hat{R}_{j,t} = \gamma_{j,t} \hat{X}_{j,t} \) and apply accounting rule (1) to compute simulated endowment for the next year according to

\[
\hat{X}_{j,t+1} = \hat{X}_{j,t} + \hat{R}_{j,t} - \hat{S}_{j,t}.
\]

By iterating this series of steps, applying (20) once for each year, we fill out a complete time series of estimated endowment, revenue, and spending values \( \hat{X}_{j,t}, \hat{R}_{j,t}, \hat{S}_{j,t} \) for each college \( j \) and years \( t \) from 2010 to 2022 for the Hybrid and 2011 to 2022 for the Weighted Spending rules.

---

\(^7\) These default parameters correspond to a 5% target rule for contributions. For instance, weights of 60% on past endowment and 40% on past cost with a 5% target rule yield a weight of \( 0.6 \times 0.05 = 0.03 \) on past endowment.
C. Results of the Simulations

Once the simulated spending patterns were generated, we compared the simulated and actual
endowment values at the end of the sample period in July 2022 according to the absolute value
of the percentage difference between simulated $\hat{X}_{2022}$ and actual final endowment level $X_{2022}$.

\[
\text{Error Rate} = \left| \frac{\hat{X}_{2022} - X_{2022}}{X_{2022}} \right|
\]

We classify the simulation results into four categories: (1) Error Rate < 5%; (2) Error Rate
between 5 and 10%; (3) Error Rate greater than 10 %. Table 5 compares the results for Hybrid
(Row) and Moving Average (Column) simulations and shows that the Hybrid Model
outperforms the Moving Average Model with projected final endowment within 10% of the true
value for 82.9% of the institutions whereas the Moving Average rule achieves that level of
accuracy for 71.7% of institutions.8

Table 5: Accuracy of Simulated Spending Patterns

<table>
<thead>
<tr>
<th>Hybrid Model</th>
<th>0 to 5% Absolute Error</th>
<th>5.1 to 10% Absolute Error</th>
<th>10.1 or more Absolute Error</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 to 5% Absolute Error</td>
<td>105 (56.2%)</td>
<td>10 (5.4%)</td>
<td>17 (9.1%)</td>
<td>132 (70.6%)</td>
</tr>
<tr>
<td>5.1 to 10% Absolute Error</td>
<td>4 (3.7%)</td>
<td>10 (5.4%)</td>
<td>6 (3.2%)</td>
<td>23 (12.3%)</td>
</tr>
<tr>
<td>10.1 or more Absolute Error</td>
<td>5 (2.7%)</td>
<td>4 (2.1%)</td>
<td>23 (12.3%)</td>
<td>29 (17.1%)</td>
</tr>
<tr>
<td>TOTAL</td>
<td>103 (62.6%)</td>
<td>24 (12.8%)</td>
<td>46 (24.6%)</td>
<td>184 (100%)</td>
</tr>
</tbody>
</table>

To be clear, we do not interpret this comparison as suggesting that a larger percentage of
institutions use Hybrid and smaller percentage of them use Moving Average rules than they
report in the NACUBO survey, for there are a number of reasons that our simulations might
produce distorted projections for Moving Average rules.

8 This comparison also holds for institutions with the highest initial endowments though the gap narrows in that
case: restricting attention to institutions in Groups 1 and 2, the Hybrid Rule yields projected final endowment within
10% of the true value for 89.1% of the institutions and the Moving Average Rule achieves that level of accuracy for
84.8% of them.
• The current sample may not provide sufficient data to accurately estimate the weights for the Moving Average rule; in the cases where our regression method yields valid weights for this rule, the simulated final endowment falls within 5% of the true value in 80.5% of the cases and within 10% of the true value in 95.5% of the cases.

• Moving Average rules are typically applied to lagged quarterly endowment levels but our data only includes yearly endowment levels;

• Institutions that use Moving Average rules use a varying number of lagged endowment values while we always use a weighted average of three years of endowment values.

Figure 8a depicts the simulated and actual costs for Yale University where these relationships are fairly typical of cases where both simulations yield final endowment levels fairly close to the actual value (here the final simulated endowment for the Hybrid model was 1.2% away and the final simulated endowment level for the Moving Average rule 1.0% away from the true value.) Both rules produce results close to but clearly distinct from actual spending, with both periods of underestimating (from about 2011 to 2015) and overestimating (about 2015 through 2019) actual reported costs.

**Figure 8a: Simulated and Actual Spending for Yale University**
Figures 8b and 8c graph the projected costs for simulations where one of the two methods more closely approximates the final endowment level. Figure 8b shows the results for Washington and Lee University, where the Moving Average simulation initially tracks the downturn in costs after the end of the Great Recession, but then systematically underestimates the subsequent rise in endowment spending from 2012 on.

**Figure 8a: Simulated and Actual Spending for Washington and Lee University**

Figure 8c shows the results for Brown University, where the Hybrid Rule simulation substantially overestimates endowment spending at both the beginning and end of the sample period, whereas the Moving Average simulation rule provides a more even balance of periods of time where it overestimates and underestimates costs relative by comparison to actual spending.

**Figure 8c: Simulated and Actual Spending for Brown University**
As these examples suggest, institutional spending tends to be less predictable than these spending rules suggest. The projected costs in the simulations are most inaccurate when the Hybrid and Moving Average rules cannot reproduce relatively complicated spending patterns of individual colleges. While our method is clearly ad hoc and we are only judging the accuracy of the simulation based on the end result in terms of endowment level, these results suggest that the spending patterns of the colleges are at least reasonably described by a weighted average spending rule. For this reason, we focus attention on the projected weights for the 85% of cases where the Hybrid Rule produces a simulated final endowment within 10% of the true value.

Equation (17) \( S_{j,t} = W_E X_{t-1} + W_S S_{t-1} \) for simulations of the Hybrid Spending Rule corresponds to a target spending rate \( \alpha \) when \( \frac{W_E}{\alpha} + W_S = 1 \) which means \( \alpha = \frac{W_E}{1-W_S} \). Table 6 reports the average weights for simulations of the Hybrid Rule for the 155 institutions with final projected endowment within 10% of the actual value. The given average weights in each category correspond to target spending rates between 5.3 and 5.7%, which seems broadly plausible. Institutions in Groups 1 to 4 are estimated to use average weights of 50 to 65% on previous spending, which imposes considerable smoothness on the spending pattern, though not as much as with the 80% weight on prior spending from the Yale Rule.

### Table 6: Average Weights for Successful Hybrid Simulations

<table>
<thead>
<tr>
<th></th>
<th>All</th>
<th>Group 1</th>
<th>Group 2</th>
<th>Group 3</th>
<th>Group 4</th>
<th>Group 5</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Endowment Weight</strong></td>
<td>.025</td>
<td>.029</td>
<td>.020</td>
<td>.020</td>
<td>.026</td>
<td>.033</td>
</tr>
<tr>
<td><strong>Lagged Cost Weight</strong></td>
<td>.531</td>
<td>.492</td>
<td>.636</td>
<td>.637</td>
<td>.516</td>
<td>.358</td>
</tr>
<tr>
<td><strong>Target Spending</strong></td>
<td>5.33%</td>
<td>5.71%</td>
<td>5.50%</td>
<td>5.51%</td>
<td>5.37%</td>
<td>5.14%</td>
</tr>
<tr>
<td><strong>Colleges</strong></td>
<td>155</td>
<td>20</td>
<td>21</td>
<td>29</td>
<td>65</td>
<td>20</td>
</tr>
</tbody>
</table>

* The standard deviation for each reported value is reported in parentheses. The target spending level is computed at the average weights listed in each column.
It is notable that the implied target spending rates listed in Table 6 are somewhat greater than 5% even though the descriptive statistics in Table 2 showed that average spending was close to 5% of current endowment levels for each group of colleges. These results are actually consistent with each other in a period of rising endowments since then a spending rule designed for target spending equal to proportion $\alpha$ of recent past endowment levels will produce endowment payout of less than that proportion $\alpha$ of the current endowment level.

There are several possible explanations for the common finding in Table 6 of target spending levels higher than the reported standard level of 5%.

- Some institutions may use higher target spending levels. For example, Yale reports a target level of 5.25% as part of the policy described in Section IV.
- Our computations don’t include inflation and a 5% real target spending rate corresponds to a higher nominal rate.
- Our analysis in Section VI below finds that contributions are spent more rapidly than other portions of endowment funds, which may result in greater spending than would be suggested by a 5% target rate.

The Hybrid Rule is conducive to simulations of future endowment paths because it requires so little information and is straightforward to apply. Figure x projects the results of simulations of four scenarios for the trajectory of the endowment path for Yale University through 2050 using the estimated Hybrid Rule weights for Yale as described above. Scenarios 1 through 3 assume that Yale receives revenue each year (starting in 2022-23) equal to a fixed proportion of its current endowment, where that proportion is equal to the yearly payout rate in Scenario 1, 6% in Scenario 2 (approximately one percentage point higher than the yearly payout rate) in Scenario 2, and 2.5% in Scenario 3 (approximately half the yearly payout rate). Scenario 4 makes one adjustment to Scenario 1, assuming a one-time negative event that causes a 25% loss of the endowment in investment returns in 2024-25.
As shown in Figure 9, total costs stabilize relatively quickly in Scenarios 1 and 4, indicating that the endowment level also stabilizes in those cases, though at a much lower level in Scenario 4 than in Scenario 1. By contrast, we project steady increases in Scenario 2 and steady reductions in Scenario 3 for both total costs and endowment level. These results indicate that the Hybrid Rule is an especially good match for relatively stable conditions but that it will result in consistent under- or over-spending if the target payout rate is different than the rate of revenue generation.
D. Descriptive Regression Analysis of Spending

Our simulations implicitly give equal weight to contributions and investment returns. To distinguish between these two sources of revenue, we conduct descriptive regressions to predict current spending levels as a function of past investment returns and contributions and a prior endowment level. As shown in Table 8, investment gains and contributions have estimated positive and generally significant effects on future endowment spending, but there are apparent distinctions in the effects of these two separate sources of funds.

- Contributions have largest effect in the first year after realization and estimated substantially diminishing effect beyond that to the point where they have no discernible effect in year 3 for the institutions in Groups 1 and 2 (those with highest endowments).
- Investment gains have relatively small initial effect and then growing effect over time, consistent with the properties of one-step spending rules.

Table 8: Regression Results for Spending as Function of Investment Returns and Contributions

<table>
<thead>
<tr>
<th></th>
<th>All</th>
<th>All</th>
<th>Group 1-2</th>
<th>Group 1-2</th>
<th>Group 3-5</th>
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<td>Balance Lag 3</td>
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<td>.046**</td>
<td>.043**</td>
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<td>.053**</td>
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<tr>
<td></td>
<td>(.0004)</td>
<td>(.0008)</td>
<td>(.0008)</td>
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<td>.008**</td>
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<td>-.005</td>
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<tr>
<td></td>
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<td>(.001)</td>
<td>(.0025)</td>
<td>(.002)</td>
<td>(.003)</td>
<td>(.0025)</td>
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<tr>
<td></td>
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<tr>
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<td>(.016)</td>
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<td>(0.16)</td>
<td>(0.12)</td>
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<td>(0.66 M)</td>
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<td>Fixed Effects By Institution</td>
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<td>NO</td>
<td>YES</td>
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<td>YES</td>
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<td>.9866</td>
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<td>506</td>
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</table>

* = significant at .05, ** = significant at .01 level.
VI. Categories within the Endowment

As part of their 990 Schedule D filings, colleges and universities report the division of the endowment into three categories defined by the Financial Accounting Standards Board (FASB): (1) quasi-endowment, often known as “Funds Functioning as Endowment”; (2) permanent endowment; (3) term or restricted endowment. New donations are classified in the second category as “permanent endowment” but investment earnings on funds in the permanent endowment are typically moved into term endowment.

**Figure 10: Average Division of Endowments by Category**

Figure 10 shows that the average proportion of endowments in the term endowment category approximately doubled from 2008-09 to 2013-14 and then remained fairly steady after that. This increase in the proportion held as term endowments appears to be explained almost entirely by a corresponding decline in the proportion of funds held in the permanent endowment. These changes may be explained at least partly by an initial period of limited donations and solid investment returns in the aftermath of the Great Recession. It is also interesting to see that the proportion held as quasi-endowment remained relatively close to 30% throughout the sample period, which suggests that the spending power of these institutions grew in proportion to the increase in real endowment level.
Table 10 presents the results of descriptive regressions with the percentage of the endowment in a given category (in a given fiscal year) as the dependent variable. For Columns (1) and (2), the dependent variable is the percentage of the endowment held as “quasi-endowment”; for Columns (1) and (2), the dependent variable is the percentage of the endowment held as “Permanent Endowment”; for Columns (1) and (2), the dependent variable is the percentage of the endowment held as “Term Endowment”. All variables in these specifications are included as percentages of the endowment at the beginning of the relevant fiscal year.

### Table 8: Regression Results for Spending as Function of Investment Returns and Contributions

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
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<td>Perm Endow</td>
<td>Perm Endow</td>
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<td>Invest Gains Lag 2</td>
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<td>.787** (.012)</td>
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<td>.929** (.007)</td>
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<tr>
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<td>-.173** (.027)</td>
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<td>.051 (.070)</td>
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<td>-.233** (.066)</td>
<td>-.183 (.124)</td>
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<td><strong>Fixed Effects By Institution</strong></td>
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<td>YES</td>
<td>NO</td>
<td>YES</td>
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<td>.7745</td>
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<td>2,237</td>
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</table>

* = significant at .05, ** = significant at .01 level.
We report separate regression results for each dependent variable without and with institution-level fixed effects. Contributions appear to have more robust effects than investment returns for these endowment categories and an increase in contributions in a given year predicts an increase in the percentage held in quasi-endowment and a decline in the percentage held in permanent endowment. While the increase in term endowment percentages during the first part of the sample period stands out in Figure 10, current total cost (endowment expenditure) is the only significant independent variable as a predictor of term endowment percentage in Column (6) when we include institution-level fixed effects.

VII. IPEDS Spending
We utilize IPEDS data to look at specific changes in expenditures by institutions, focusing on four categories (Academic Support, Institutional Support, Instruction, Student Support) that were consistently reported in our sample. Webber and Ehrenberg (2010) and Deming and Walters (2017) establish a particular connection between funding for Student Support and student success in terms of persistence and degree completion, so we focus on that category in our analysis.9 Figure 11a shows overall increases of 40 to 50% in total IPEDS costs (the sum of expenses reported in the four categories listed above) from 2008-09 to 2019-20, with modest decline followed by subsequent increases in the next two years.

Figure 11a: % Increases in IPEDS Reported Costs

9 “Student services” includes admissions, registrar activities, and activities for which the primary purpose is to contribute to students’ emotional and physical well-being and to their intellectual, cultural, and social development outside the context of the formal instructional program. Intercollegiate athletics and student health services may also be included except when operated as self-supporting auxiliary enterprises.  
https://nces.ed.gov/programs/coe/indicator/cue/postsecondary-institution-expense
Figure 11b shows larger proportional increases in spending on student support both during the first 12 years of the sample period and in 2021-22 where spending increased by an average between 12% and 18% in each of the five groups of colleges. Investment in health and specifically mental health services for students likely contribute to the cost increases observed in student services for 2021-22; a total of 77% of institutions reported expansions of mental health service during 2022 in a separate survey.¹⁰

Figure 11b: % Change in Yearly Spending on Student Support

VIII. Discussion and Conclusion

Previous papers by Tobin (1974), Hansmann (1990), Merton (1993), and Hoxby (2013), among others, discussed the conceptual basis for university endowments. Hoxby conceptualizes universities as organizations that promote increases in intellectual capital through research and teaching, using endowments to fund both current and future productive activity. From this perspective, the optimal payout rate for endowments should equate the marginal value of present and expected future activities at the university that create intellectual capital. Consistent with this view, Ehrenberg (2009) suggests a rule of thumb for spending that maintains the university’s endowment level in real terms over time:

*To provide future generations with protection against inflation, the endowment for a specific funded activity must grow over time by the average rate of inflation faced by the university.*

This prescription is at odds with the consistent growth of the university endowments that we study during the sample period. In fact, 85% of the institutions in our sample and almost all of the initially wealthiest institutions produced endowment growth greater than the cumulative inflation rate of 33% from 2008-09 to 2021-22. Similarly, Lerner, Schoar, and Wang (2008) found a clear positive correlation between endowment size and endowment growth for the period from 1992 to 2005, primarily because institutions with largest endowments had superior average returns on investments during that time period.

Table 9: Endowment Growth Relative to Inflation

<table>
<thead>
<tr>
<th>Group</th>
<th>Endowment Growth &gt; Inflation</th>
<th>Endowment Growth &lt; Inflation</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>22 (100%)</td>
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<td>22</td>
</tr>
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<td>2</td>
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<td>3</td>
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<td>4</td>
<td>63 (80.8%)</td>
<td>15 (19.2%)</td>
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<td>5</td>
<td>22 (75.9%)</td>
<td>7 (24.1%)</td>
<td>29</td>
</tr>
<tr>
<td>TOTAL</td>
<td>159 (85.0%)</td>
<td>28 (15.0%)</td>
<td>187</td>
</tr>
</tbody>
</table>

Endowment growth also consistently outpaced inflation prior to our sample period. Lerner, Shoar, and Wang observed median annual endowment growth rates (among 1,300 institutions that provided survey responses to NACUBO) of 7.4% during the thirteen-year period from 1992 to 2005 when the average annual inflation rate was 2.6%. Further, Piketty (2014) summarized results from the longer period from 1980 to 2010 and found in his analysis of 850 institutions
that “U.S. universities earned an average real return of 8.2% on their capital endowments, and all the more so for higher endowments.” Hoxby (2013) argues that long-term real growth of university endowments can be justified by a particular combination of underlying circumstances: 

*the financial side should only grow persistently as a share of the total portfolio if (i) the returns on future intellectual capital projects are substantially higher than those of today's intellectual capital projects and (ii) adjustment costs are such that when those future days arrive, the cost of suddenly needing to provide infrastructure and expertise to them will not be exorbitant.*

Hoxby suggests that long periods of real endowment growth suggest suboptimal spending rules: 

*Since circumstances (i) and (ii) probably do not often arise in conjunction, a university should examine itself if its financial market portfolio's share of its total portfolio rises very persistently. The likely answer is that the university is not solving its investment problem correctly.*

As Stein (2023) observes, endowment spending rules that adjust payouts to reduce short- and medium-term accounting deficits and surpluses are flawed because they do not consider cost-benefit tradeoffs and in particular do not use net present value as a criterion the present vs. future comparisons highlighted by Ehrenberg and Hoxby. Assuming that once the endowment payout is determined for a given year, universities fund the most promising projects in terms of benefit-cost ratio, the threshold for present day expenditures should fall both because (1) if present-day expenditures are held constant, the threshold for future marginal investments in terms of benefit-cost ratio falls on average; (2) this increase in funding could open new possibilities for larger investments that have outsize benefit-cost ratios but that were not previously feasible.

One particular possible use of endowment funds would be to provide additional financial aid to promote economic mobility. Baum and Lee (2019) suggest that it is paradoxical that some institutions with outsize endowments practice need-aware admissions, meaning that they have a systematic policy of rejecting some well-qualified students solely on the basis of financial need. Bulman (2022) studies the effects of investment returns on future spending and concludes that plausibly exogenous increases in endowments lead to more spending, but do not appear to increase diversity or the number of low-income students who enroll. Similarly, Baum, Hill, and Schwartz (2018) find that “Institutions with high endowments per student do use these resources
to lower net prices for students, but not necessarily to enroll greater shares of students with financial need compared to other institutions.” These authors emphasize that since most institutions with very large endowments are need-blind and offer generous financial aid programs to students who are admitted, it would only make sense to direct additional funds to financial in tandem with substantive changes in recruiting and admission practices.

A broader issue is that universities receive many donations with stipulations attached, so much so that it is common for them to report that 70% or more of their endowment funds are restricted in their use. If these restrictions are especially strict, a university could experience long-lasting endowment growth on paper and yet still face ongoing financial stress. To take a possibly extreme case, if half of an institution’s endowment is so restricted that it can’t be used for any practical purpose then annual costs equal to 5% of the listed endowment level a represent a true 10% payout rate on the effective endowment level. On the other hand, many restrictions on the endowment direct funds towards uses, such as financial aid, that the university would choose on its own. From the outside, however, it is not easy to determine the degree to which restrictions on endowment spending do or don’t align with university priorities. As Baum, Hill, and Schwartz (2018) conclude, “The available data do not make it possible to determine to what extent these restrictions in fact constrain the spending decisions of colleges and universities.”

The cliche “A rising tide lifts all boats” applies both to restricted and unrestricted endowment funds; a period of unusually large investment returns would necessarily open new possibilities for unrestricted spending for any institution that was not initially dramatically underwater. The last two years of our sample period and the nearly two years since the end of our sample period may meet this definition. Our sample data indicated nominal endowment increases averaging 30% in 2020-21 in a year with essentially no inflation. Though 2021-22 brought a combination of endowment declines and outsize inflation, both 2022-23 and the first three quarters of 2023-24 have been fertile periods for investments. For example, the S&P 500 increased 17.6% in 2022-23 (from 3785 on 6/30/22 to 4450 on 6/23) and another 17.6% so far in 2023-24, a year of declining inflation. Based on these figures, it could be common for an institution to increase its endowment by 50% in real terms during these four years. A change in endowment status of that size would likely provide a variety of new opportunities and suggest substantive adjustments to existing endowment spending rules.
References


IRS Form 990 data, including Schedule D.


