Maximizing the Effect of Altruism

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Motivation

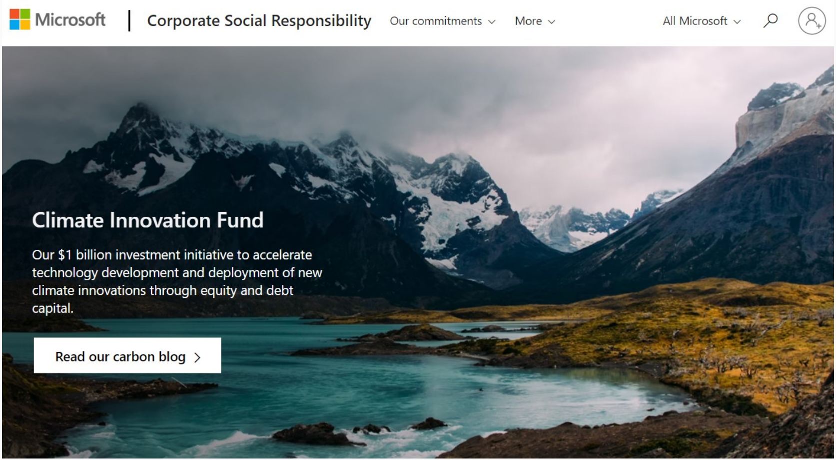
Microsoft will be carbon negative by 2030

Jan 16, 2020 | Brad Smith - President & Vice Chair

Motivation

"Reforestation in the style of Picasso," created with DALL-E

Motivation



"We will primarily deploy this capital in two areas: (1) to accelerate ongoing technology development by investing in project and debt finance; and (2) to invest in new innovations through equity and debt capital."

Motivation

CLIMATE

Opinion | Advance Market Commitments Worked for Vaccines. They Could Work for Carbon Removal, Too.

Through "advance market commitments," we can incentivize the development of transformative carbon removal approaches.

Politico Opinion piece by Athey, Glennerster, Ransohoff, Snyder (2022)

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Model: principal wishes to maximize production of agent subject to

- Heterogeneous private production costs
- Ex-post budget constraint
- No value for leftover budget
- Extension with outside options

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Characterization which implies that

- Optimal schedule always pools most efficient types
- For separable costs, pool size independent of budget size

Practical Takeaways

- Principal should always pool most efficient firms.
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- If efficient firms are sufficiently rare, offer bang-bang schedule.
  $\Longrightarrow$ A fund with a single take-it-or-leave-it offer of a fixed investment amount for a fixed production amount.
- With (better) outside option, have higher expectations of efficient types and lower expectations of inefficient types.
  $\Longrightarrow$ If there are cheap carbon credits on the open market, the fund should offer steeper schedules.

Related Work

- Monopoly Regulation: Baron and Myerson (1983), Laffont and Tirole (1986, 1993), Amador and Bagwell (2022)
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- Delegation: Alonso and Matouschek (2008), Amador and Bagwell (2013), Amador, Bagwell and Frankel (2018), Amador and Bagwell (2022) $\implies$ allow transfers
- Similar features: Gomes and Pavan (2016), Kominers, Dworczak, and Akbarpour (2021), Kang (2023) $\implies$ different environments

Outline

- Model
- Optimal mechanism
- Qualitative features
- Separable types

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Principal-agent design problem where the principal would like an agent to produce something costly.

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  Normalize $\Psi(0, \theta) = 0$ for all θ.

Assumptions

Assumptions on Ψ:

- Ψ is strictly increasing in both arguments, twice continuously differentiable, and supermodular.
- Ψ and Ψ_θ are convex in x.
- μ admits a density f, CDF F

Interpretation: costs are convex in x, and θ is a scalability parameter, so higher θ means costs grow faster.

Example: $\Psi = \theta x^2$.

Optimization Problem

By revelation principle, sufficient to find production levels $x : \Theta \to X$ and transfers $t : \Theta \to \mathbb{R}_+$ that maximize:

$$
\begin{aligned}
\max_{x,t} \quad & \int_{\theta\in\Theta} x(\theta) f(\theta)\, d\theta \\
\text{s.t.} \quad & t(\theta) - \Psi(x(\theta), \theta) \geq t(\theta') - \Psi(x(\theta'), \theta) \quad && \forall \theta, \theta' \in \Theta \quad && \text{(IC)} \\
& t(\theta) - \Psi(x(\theta), \theta) \geq 0 && \forall \theta \in \Theta && \text{(IR)} \\
& t(\theta) \leq T && \forall \theta \in \Theta && \text{(B)}
\end{aligned}
$$

Note: Any feasible solution can be equivalently represented as a transfer schedule $\hat{t} : X \to \mathbb{R}_+$.

Optimal Mechanism

Monotonicity

Lemma

$x : \Theta \to X$ is a feasible schedule iff following two conditions hold:

1. *x is nonincreasing.*
2. $\Psi(x(\underline{\theta}), \underline{\theta}) + \int_{\underline{\theta}}^{\bar{\theta}} \Psi_\theta(x(s), s)\, ds \leq T$

Further, given a feasible schedule x, a transfer function that supports the schedule is given by

$$t(\theta) = \Psi(x(\theta), \theta) + \int_{\theta}^{\bar{\theta}} \Psi_\theta(x(s), s)\, ds$$

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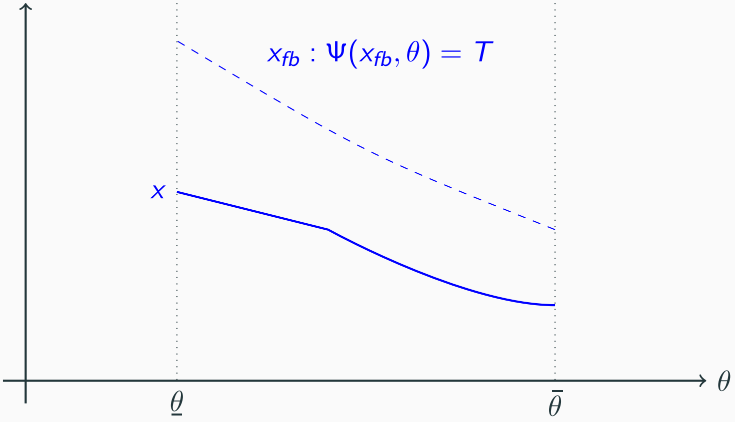
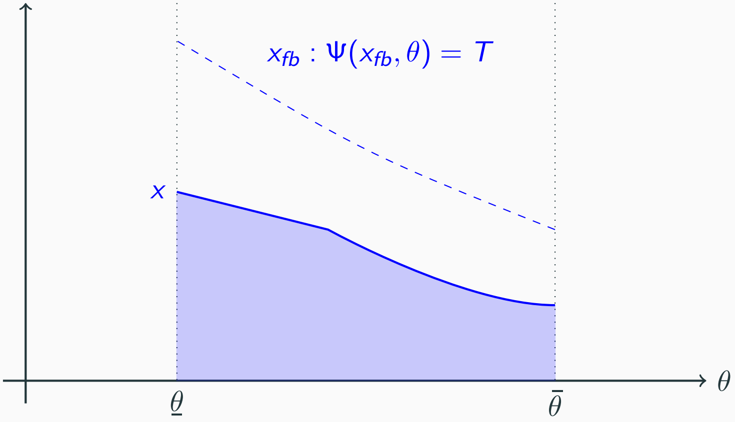
Implication: a feasible mechanism is implementable as a subsidy schedule

Illustration



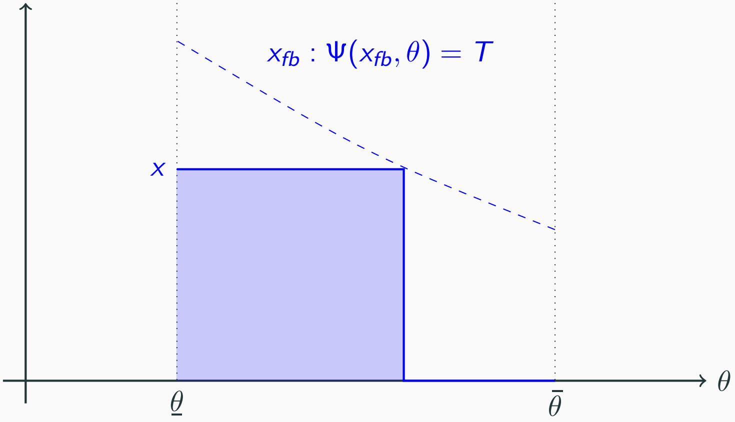
Any decreasing x will do...

Illustration



... as long as the weighted area is less than T.

Illustration



Example: weighted area is exactly T.

Optimal Control Problem

Rewrite as optimal control problem where "time" is type θ.

$$\begin{aligned}
\max_{x} \quad & \int_{\Theta} x(\theta) f(\theta)\, d\theta \\
\text{s.t.} \quad & \dot{x}(\theta) = u(\theta) \leq 0 && \forall \theta \in \Theta \\
& \Psi(x(\underline{\theta}), \underline{\theta}) + \int_{\underline{\theta}}^{\bar{\theta}} \Psi_{\theta}(x(s), s)\, ds \leq T
\end{aligned}$$

Let $\rho(\theta) \geq 0$ be costate variable of first constraint; $\lambda \geq 0$ be Lagrangian of second constraint.

Interpretation: λ is shadow value of money

Characterization Result

Theorem

An optimal mechanism (x, t) exists, is unique, and, together with Lagrange multiplier $\lambda \geq 0$ and costate function $\rho(\theta) \geq 0$, satisfies:

$$\rho(\theta) > 0 \implies \dot{x}(\theta) = 0 \tag{1}$$

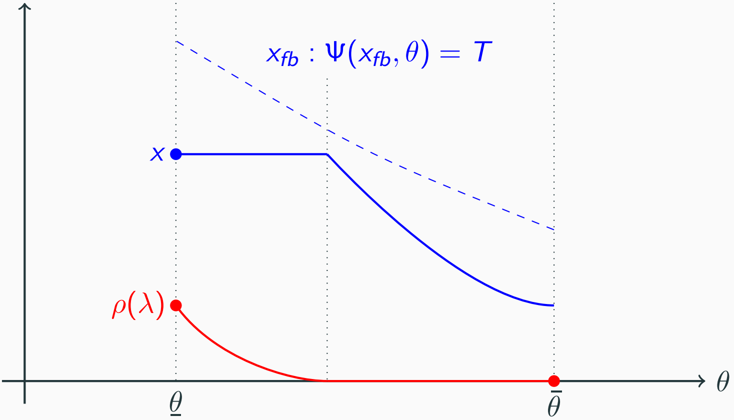
$$\dot{\rho}(\theta) = \lambda \Psi_{x\theta}(x(\theta), \theta) - f(\theta) \tag{2}$$

$$\rho(\bar{\theta}) = 0 \tag{3}$$

$$\rho(\underline{\theta}) = \lambda \Psi_x(x(\underline{\theta}), \underline{\theta}) \tag{4}$$

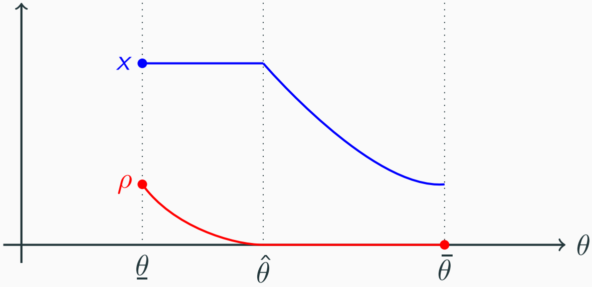
$$t(\underline{\theta}) = \Psi(x(\underline{\theta}), \underline{\theta}) + \int_{\underline{\theta}}^{\bar{\theta}} \Psi_\theta(x(s), s)\, ds = T \tag{5}$$

Illustration



Qualitative Features

Pooling at the Top



- if $\rho > 0$, then x is constant by complementary slackness
- pooling at the top: $\rho(\underline{\theta}) = \lambda \Psi_x(x(\underline{\theta}), \underline{\theta}) > 0$

Pooling Threshold

Proposition

Suppose optimal x^* is not constant on $[\underline{\theta}, \bar{\theta}]$. Let $\underline{x} \equiv x^*(\underline{\theta})$. Then largest type $\hat{\theta}$ that receives full transfer has:

$$\Psi_x(\underline{x}, \hat{\theta}) f(\hat{\theta}) = \Psi_{x\theta}(\underline{x}, \hat{\theta}) F(\hat{\theta})$$

Furthermore, there is always pooling at the top, i.e., $\hat{\theta} > \bar{\theta}$.

Rationale: Principle faces trade off between utilizing more budget on inefficient types with providing info rents to efficient types.

- LHS is marginal direct cost paid to threshold type
- RHS is marginal information rent paid to more efficient types

Shadow Value of Money

The threshold equation is:

$$\Psi_x(\underline{x}, \hat{\theta}) f(\hat{\theta}) = \Psi_{x\theta}(\underline{x}, \hat{\theta}) F(\hat{\theta})$$

Shadow Value of Money

The threshold equation is:

$$\lambda^* \Psi_x(\underline{x}, \hat{\theta}) = F(\hat{\theta})$$

The marginal value of extra budget is how much the principal can get out of the types that are more efficient than the threshold.

Shadow Value of Money

We should have complete pooling if for all θ:

$$\lambda^* \Psi_x(\underline{x}, \theta) \geq F(\theta)$$

Complete Pooling

Let $\bar{x}$ be such that $T = \Psi\left(\bar{x}, \bar{\theta}\right)$. Then

Proposition

The optimal mechanism pools all types if and only if $\forall\theta$,

$$F(\theta) \leq \Psi_x\left(\bar{x}, \theta\right) / \Psi_x(\bar{x}, \bar{\theta})$$

Interpretation: If agents of efficient types are sufficiently rare, then it is optimal to offer a single menu item $(\bar{x}, T)$.

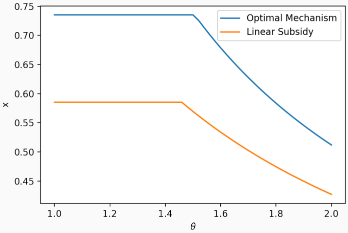
Constant-Rate Subsidy

Proposition

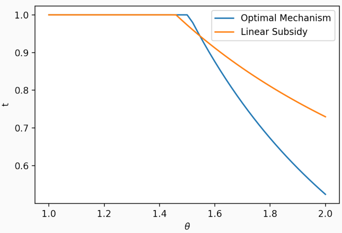
Let x_r be the outcome from offering a constant-rate subsidy at rate r up to the budget. Then there exists a feasible schedule x^* that Pareto dominates it.

Intuition: under a linear subsidy, agent chooses marginal cost equal to subsidy rate. A mechanism can set transfers to target average cost equal to subsidy rate.

Constant-Rate Subsidy: Illustration

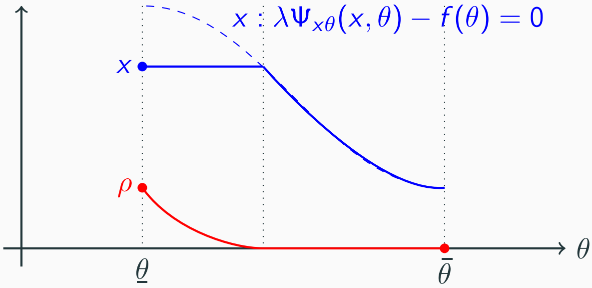


(a) Production schedule x



(b) Transfer schedule t

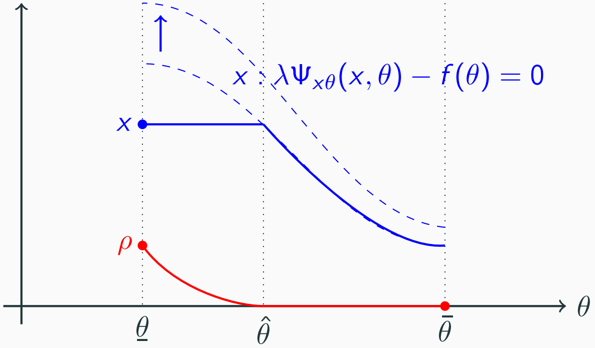
The Shadow Value of Money



Recall λ is shadow value of money.

- if $\dot{x} < 0$, then $\rho = 0$ by complementary slackness
- hence x defined by costate evolution constraint, $\lambda\Psi_{x\theta}(x, \theta) - f(\theta) = \dot{\rho} = 0$

Pool Size



If budget increases, shadow value λ decreases,

- increasing dotted-blue curve pointwise

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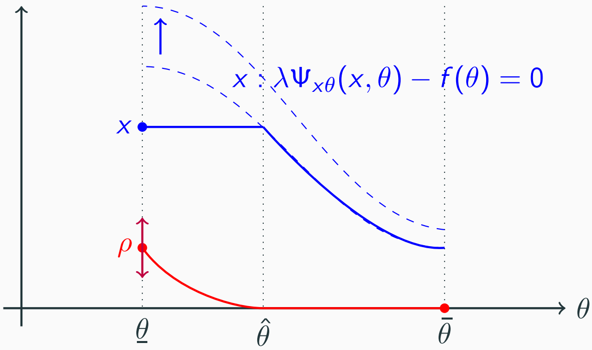

If budget increases, shadow value λ decreases,

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- whether ρ increases or decreases depends: $\rho(\underline{\theta}) = \lambda \Psi_x(x(\underline{\theta}), \underline{\theta})$ and $\dot{\rho} = \lambda \Psi_{x\theta}(x, \theta) - f(\theta)$

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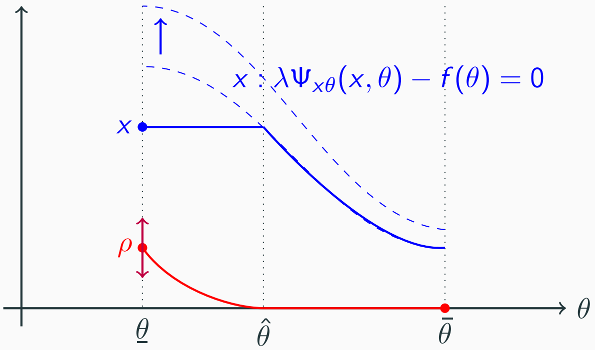

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  $\rho(\underline{\theta}) = \lambda\Psi_x(x(\underline{\theta}), \underline{\theta})$ and $\dot{\rho} = \lambda\Psi_{x\theta}(x,\theta) - f(\theta)$
- so $\hat{\theta}$ depends on how fast Ψ_x and $\Psi_{x\theta}$ grow in x

Separable Costs

Characterization

Special case: multiplicatively separable Ψ, e.g., $\Psi = \theta x^2$.

Theorem

Suppose that $\Psi(x, \theta) = \theta \Gamma(x)$. *Then the optimal mechanism* (x^*, t^*) *induces a production schedule*

$$x^*(\theta) = (\Gamma')^{-1} \left(\frac{\tilde{f}(\theta)}{\lambda^*} \right)$$

where $\tilde{f} = \frac{d}{d\theta} (\text{cav}^* F)$ *and* λ^* *is the Lagrange multiplier chosen so that* $\int_\Theta \Gamma(x^*(\theta))\, d\theta = T$.

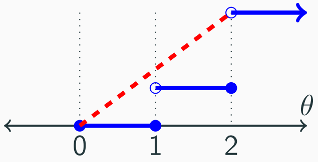
Example: $\Psi = \theta x^2$; $\Gamma = x^2$, so

$$x^*(\theta) = \frac{\tilde{f}(\theta)}{2\lambda^*} \implies x^* \text{ is linearly proportional to } \tilde{f}$$

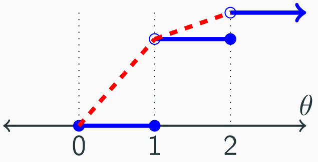
Pooling

Agents are pooled according to linear segments in the concave hull of the type CDF F.

Example: $\Psi(x,\theta) = \theta x^2$, $\theta \in \{1,2\}$, $\mu = \Pr[\theta = 1]$



(a) complete pooling: $\mu \le 1/2$



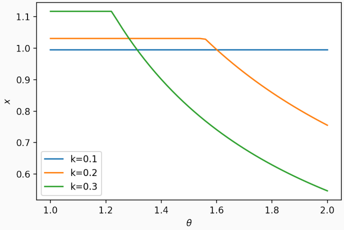
(b) separating: $\mu > 1/2$

Note: Pool size is invariant to changes in budget T!

Outside Value: Pooling

Suppose principal can obtain x in outside market at per-unit cost of k, i.e., objective changes to $\int_{\theta \in \Theta} [x(\theta) - kt(\theta)] f(\theta)\ d\theta$.

Example: Opt mechs for $\Psi(x, \theta) = \theta x^2,\ T = 2,\ \theta \sim U[1, 2]$

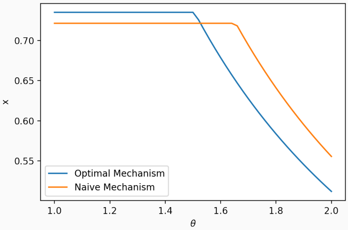


The pooling size shrinks and the optimal mechanism demands more of most efficient types, and less of least efficient types, as k grows.

Outside Value: Constant-Rate Subsidy

Naive Mechanism: ignoring the budget constraint

Example: $\Psi(x, \theta) = \theta x^2$, $T = 1$, $\theta \sim U[1, 2]$, $k = 0.3$



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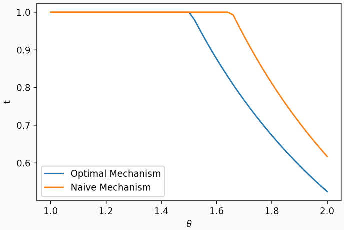

(b) Transfer schedule t

Naive mechanism uses more budget in expectation, obtains less output from more efficient types, and separates fewer types.

Conclusion

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- Buying at a price: A common form of advance market commitment is a constant-rate subsidy: buy quantity at fixed rate r up to the budget T. This is always strictly suboptimal.

Conclusion

- Pooling at the Top: The principal always pools a positive measure of the most efficient types.
- Buying at a price: A common form of advance market commitment is a constant-rate subsidy: buy quantity at fixed rate r up to the budget T. This is always strictly suboptimal.
- Scalability: As the distribution over agent production technologies becomes less efficient (scalable), more types are pooled in the optimal contract.

  If efficient firms are sufficiently rare, the principal should only offer a single target and pay entire budget on meeting target.