# Entry and Exit in Treasury Auctions 

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#### Abstract

Many markets, including those for Treasury securities, are organized around a dealership structure, where dealers commit to participate while entry by non-dealers is irregular. This introduces a trade-off between competition and volatility, which we study using data on twenty-three years of Canadian Treasury auctions. We document a consistent exit trend by dealer banks, and increasing, but volatile participation by non-dealer hedge funds. Leveraging a structural model, we evaluate the impact of dealer bank exit on hedge fund participation and its consequences on market competition and volatility. The results reveal that dealer bank exit was a main driver of hedge fund entry. Using our structural model, we quantify the trade-off between competition and volatility, and find that rules that commit a subset of bidders to participate are necessary to ensure market stability. These findings have broader implications for auction and non-auction markets with regular and irregular participants.


JEL: D44, D47, G12, G28
Keywords: Treasury market, auction, intermediaries, hedge funds, regulation

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## 1 Introduction

Governments worldwide have traditionally relied on regulated banks, known as primary dealers, to consistently purchase government debt and facilitate trade between investors, such as firms, public entities, and individuals, effectively "making markets." More recently, other institutions have assumed an increasingly important role. For instance, hedge funds doubled their (gross) exposure to U.S. Treasuries between early 2018 and February 2020, reaching $\$ 2.39$ trillion (Banegas et al. (2021)). Given that hedge funds and other non-dealer institutions, referred to as customers, are subject to less stringent regulations than dealers and have no obligation to participate in market-making activities, the implications of this observed trend on the functioning of Treasury markets remain uncertain. ${ }^{1}$

The goal of this paper is to better understand the tradeoff between having committed dealers vs. increased participation by non-dealer customers in Treasury markets and evaluate the consequences for market functioning. Answering these questions is challenging because there is limited data that allows us to study how customers, in particular hedge funds, trade Treasuries. We overcome this challenge by focusing on the Canadian primary market in which the government issues bonds via regularly held auctions. ${ }^{2}$ We document dealer exit and rising, yet irregular, customer participation. We then introduce and estimate a structural model to assess the role of dealer exit in explaining customer entry, and to quantify the benefits of greater customer competition against the costs of higher volatility.

Our data combines bidding information of all Canadian Treasury auctions from 1999 until 2022 with price information from the secondary market, the futures market and the repo market. Two types of bidders participate in the auctions: dealers and customers. Only dealers can submit bids directly to the auctioneer; customers must bid via a dealer. We observe bidder types, unique anonymized bidder identifiers, and all submitted bids. We also know via which dealer a customer submits bids, which security is issued, and where the market cleared.

With these data, we document a series of facts. The first set of facts is about entry and

[^1]exit. We show that dealers have systematically exited the primary market since the very first auctions took place. In contrast, customers, in particular hedge funds, have entered the market. However, unlike dealers, who have an obligation to regularly attend auctions and buy sufficient amounts of debt, hedge fund auction participation is irregular. This suggests that hedge funds select specific auctions, depending on market conditions. In line with this idea, we show that customers participate at auction when bid-ask spreads in the days leading up to the auction are high. The second set of facts help us model bidding behavior, conditional on auction participation. We show that dealers who observe an aggressive bid by a customer systematically adjust their own bids in anticipation of greater competition in the auction. Sophisticated customers, such as hedge funds, should take this adjustment into account when bidding.

Motivated by the empirical evidence and institutional features of the market, we construct a model that mimics a fiscal year. At the start of each year, when the government announces its debt issuance plan, each dealer decides, at a cost, whether to commit to bidding in all auctions of the upcoming year, and the number of participating dealers is announced publicly. Then, before each auction, customers observe the market conditions, and decide whether or not to enter that specific auction, at a cost. Conditional on participation, all customers and dealers draw private signals about how much they value the bond-representing how much profit they expect to generate post-auction-, and place their bids. The auction clears at the price at which aggregate demand meets supply and each bidder pays their offered prices for all units won.

To solve for equilibrium conditions of this game and estimate dealer and customer values and entry costs, we build on the empirical literature on multi-unit auctions, in particular Hortaçsu and Kastl (2012), Guerre et al. (2000), Hortaçsu (2002) and Kastl (2011). Unlike existing studies, we ( $i$ ) allow customers to behave strategically in that they anticipate dealer updating, and (ii) endogenize bidder participation. This contributes to the empirical auction literature that allows for endogenous entry, but has so far focused on single-object auctions (see Hortacsu and Perrigne (2021) for an overview). Estimation approaches for entry in single object auctions require knowledge of how the equilibrium bid function behaves. In multiunit auctions this is a complicated object and so these tools are not directly transferable. Instead, we leverage estimated bounds on bidder-specific surpluses together with a matching
procedure to estimate entry costs in multi-unit auctions. This approach could be used to endogenize bidder participation in other settings, including electricity, renewable energy, or carbon allowances.

We estimate high participation costs for both dealers and customers, and find that customers are willing to pay more for bonds than dealers. This suggests that (participating) customers expect to execute more profitable trading strategies with the bond post-auction than (participating) dealers. One reason for this may be that customers face less stringent regulation than dealers, especially post global financial crisis. Consistent with this, we find that dealer values are not significantly different from customer values prior to 2007-2009, but differ significantly afterwards.

To disentangle whether the surge in customer engagement resulted from dealers leaving the market, potentially influenced by regulatory shifts, or from broader alterations in market conditions favoring customer bond purchases, we employ our structural model and conduct a counterfactual analysis. In this process, we calculate counterfactual bids without relying on the conventional assumption of truthful bidding, as typically employed in multi-unit auction studies. Instead, we utilize the empirical guess-and-verify method introduced by Richert (2021). By reintroducing the dealers who withdrew from the market since 2014 and simulating customer participation in these counterfactual auctions, we observe that an average customer would have been approximately $44 \%$ less likely to partake in an average auction if dealers had not exited. This substantial reduction in participation indicates that dealer exit is a very significant economic driver of customer entry.

We next use our model to evaluate market consequences from the rise in customer participation. On the one hand, stronger participation may increase competition in the spirit of Bulow and Klemperer (1996). This would reduce debt funding costs and price distortions due to bid-shading. On the other hand, irregular bidder participation may increase volatility in market outcomes, such as the market price. We start with a simplified environment with one type of bidder (dealers) that bids directly to the auctioneer. We find that the expected price drops by $11 \%$ when removing one bidder, because of stronger shading, and because there is a non-zero risk of auction failure due to insufficient demand. When the expected number of participating customers drops by one, there is no risk of auction failure. However, the expected price decreases by $0.7 \%$, and bid-shading increases by $7.9 \%$ due to weaker
competition.
To compare the competition and volatility effect from rising, yet irregular customer participation, we compare the revenue gain from attracting one extra customer in expectation with the expected revenue loss coming from across auction variation in customer participation probabilities. Volatility in participation results in revenue losses relative to consistent participation at the average level, since revenues decrease more sharply when participation falls below its average level than they increase when it is above. Random participation therefore generates lower revenues than consistent participation at the average level, as revenue improvements when participation is high are more than offset by losses under low participation. With our model estimates, the revenue effects from competition and volatility are roughly similar ( $\mathrm{C} \$ 2.9 \mathrm{M}$ or 9 bps ).

In light of the competition-volatility trade-off, we propose a simple policy rule that aims at increasing competition while decreasing volatility at the same time. Our idea is to strategically shift supply from auctions in which we predict strong customer participation to auctions with low predicted customer participation. The hope is that by doing so, we can stabilize customer participating and attract sufficiently many market participants to guarantee a high level of competition. Indeed, we find that the median revenue increases by about $\mathrm{C} \$ 16$ million per auction (or roughly 48 bps ) when implementing our proposed rule.

The competition-volatility trade-off we highlight might be present in other settings. It is common in auctions for financial products to have a set of regular and irregular bidders (e.g., Hendricks et al. (2023) for mortgage securities; Richert (2022) for credit event auctions). Our framework can be easily adjusted to fit these applications. Furthermore, the economic insights generalize to non-auction markets which are populated by regular and irregular participants. Examples include market makers versus opportunistic traders in financial markets, global versus local firms in production markets, loyal versus non-loyal customers in consumption markets, and irregular versus stable energy generation in electricity markets (Petersen et al. (2022)).

## 2 Institutional details and data

In most countries, government bonds are issued in primary auctions to a small set of regulated banks, often called primary dealers, and to customers (see Appendix Figure A1).

Canadian Treasury market. Canada adopted a tiered market structure that consisted of primary dealers and customers to distribute it's debt in November 1998. Since then, Treasury auctions are held regularly according to an annual auction schedule. For instance, between 1999 and 2022 there have been about 28 government bond auctions per year, with an average issuance size of $\mathrm{C} \$ 3.24$ billion.

Anyone can participate in Treasury auctions, but only dealers can bid directly. ${ }^{3}$ Other bidders, called customers, can only participate indirectly by placing their bids via one of the dealers. This gives dealers access to more information that customers (and other dealers). ${ }^{4}$

Customers include different types of institutions, such as pension, mutual, and ETF funds, insurance companies, sovereigns, or bank treasuries. For over a decade, the biggest customer category are "alternative investments companies" (AIC), which includes hedge funds. For simplicity, we use the term hedge fund to describe AICs throughout the paper.

A bidder may submit and update two types of bids from the time the tender call opens until the auction closing. The first type is a competitive bid. This is a step-function with at most 7 steps, which specifies how much a bidder offers to pay for specific amounts of the asset for sale. Bids must be stated in multiples of $\mathrm{C} \$ 1,000$; the minimum bid is $\mathrm{C} \$ 100,000$; the maximal bid of a primary dealer is $25 \%$ of supply for its own account, and for customers during regular times. In 2020, the bidding limit was temporarily increased to $40 \%$. The second type of bidding is a non-competitive bid, which is a quantity order that the bidder will win for sure at the average price of all accepted competitive bid prices. However, since non-competitive bids cannot be larger than $\mathrm{C} \$ 10$ million for dealers and $\mathrm{C} \$ 3-\mathrm{C} \$ 5$ million for customers, their size is trivial relative to competitive bids.

Market players and regulations. In Canada, as elsewhere, dealers have an obligation to actively buy bonds in the primary market, and to act as market-makers in the secondary

[^2]cash and repurchase (repo) markets where they provide liquidity to investors who seek to exchange government bonds for cash. In exchange, dealers enjoy benefits. For instance, dealers have privileged access to liquidity facilities and overnight repurchase operations, and extract auction rents from observing customer bids.

Given the important role dealers play in the market, they are heavily regulated. In the aftermath of the 2007-2009 financial crisis, regulation tightened for dealers (and large banks more broadly). For instance, Banks faced heightened capital requirements. A notable illustration of this is the Basel III leverage ratio, which was enforced at the close of 2014, and represents a significant limitation for bond trading (CGFS (2016); Allen and Wittwer (2022); Favara et al. (2022)).

Traditionally, customers, such as hedge funds, played a negligible role in Treasury markets. ${ }^{5}$ We know that this has changed in recent years, but we have a limited understanding of what hedge funds are doing, given that there are only a handful of empirical studies. For example, Sandhu and Vala (2023) argue that hedge funds can act as market makers, engaging in trades that counter the positions of other investors. During times of distress, such as March 2020, hedge funds can contribute to market imbalances, reduced liquidity, and increased price volatility (e.g., Barth and Kahn (2020); Vissing-Jorgensen (2021)). Increased hedge fund trading may also have implications for systemic risk in the market, given that hedge funds are more likely to employ riskier trading strategies (e.g., Dixon et al. (2012)) —an effect that we do not consider in our analysis.

Data. Understanding the activities and impact of hedge funds on market functioning poses challenges due to limited data availability. For one, comprehensive long panels of trade-level data with unique identifiers for all traders are not readily accessible. For instance, the U.S. started collecting trade-level data through TRACE in mid-2017, but customer reporting with unique IDs is not mandatory. Similarly, while the Bank of England and Bank of Canada provide firm IDs, identifying all customers remains difficult (e.g., Kondor and Pintér (2022); Allen and Wittwer (2021); Pintér and Semih (2022); Barth et al. (2022)). Additionally, customers, unlike dealers, are not obliged to report their trades. Consequently, studies analyzing hedge fund trading behavior in the past years face limitations in examining trades

[^3]Table 1: Data Summary of Bond Auctions

|  | Mean | SD | Min | Max |
| :--- | ---: | ---: | ---: | ---: |
| Issued amount (face value C\$ 100, in C\$M) | 32.42 | 10.45 | 10.00 | 70.00 |
| Revenue (in C\$B) | 3.25 | 1.04 | 0.88 | 7.00 |
| Number of Dealers | 14.46 | 2.61 | 11 | 23 |
| Number of Customers | 6.74 | 2.56 | 1 | 15 |
| Comp demand of a dealer (as \% of supply) | 14.80 | 7.51 | 0.00 | 40 |
| Comp demand of a customer (as \% of supply) | 5.83 | 4.70 | 0.01 | 25 |
| Non-comp demand of a dealer (as \% of supply) | 0.09 | 0.04 | 0.00 | 0.30 |
| Non-comp demand of a customer (as \% of supply) | 0.19 | 0.16 | 0.00 | 0.76 |
| Number of submitted steps of a dealer | 4.34 | 1.71 | 1 | 7 |
| Number of submitted steps of a customer | 1.86 | 1.02 | 1 | 7 |
| Amount won by a dealer (as \% of supply) | 4.79 | 5.85 | 0 | 35 |
| Amount won by a customer (as \% of supply) | 4.02 | 5.90 | 0 | 25 |

Table 1 displays summary statistics of our sample, which goes from 10 February, 1999 until January 27, 2022. There are 645 auctions. The total number of competitive bidding functions (including updates) is 62,813 . The total number of non-competitive bids is 10,552 .
between hedge funds and dealers, which could represent a biased sample of hedge fund trades. ${ }^{6}$

We overcome the data challenge by focusing on the primary market, where we can trace market participants over a long time horizon, thanks to time-persistent identifiers. ${ }^{7}$ We observe all winning and losing bids in regular government bond auctions from the beginning of 1999 until the end of January, 2022. This represents the entire auction history with the exception of three auctions held in December of 1998. Table 1 provides key summary statistics for our auction sample.

We augment the auction data with daily average prices of each security from the secondary market, the futures market, and the repo-market from the Canadian Depository for Securities (CDS). These data range from the beginning of 2014 until the end of 2021.

[^4]
## 3 Empirical evidence: Exit, entry and bid updating

We document a series of stylized facts to motivate our structural model.
Exit and entry. We observe two striking time trends regarding entry and exit in the Canadian primary market since the adoption of the primary dealer model. Appendix Figure A3 visualizes similar trends using public data of U.S. Treasury auctions to illustrate that these trends are not Canadian-specific.

On the one hand, dealers have exited the market. The total number of dealers declined from 24 in 1999 to 15 in 2021 (see Figure 1A). After an early round of exits, the number of dealers remained stable until 2014, when global players - such as Deutsche Bank and Morgan Stanley - exited the primary market for Canadian debt. Smaller broker-dealers-PI Financial Corporation and Ocean Securities-followed in 2015. ${ }^{8}$ The exit of global banks from the Canadian bond market around 2013-2015, during a period of tighter banking regulations and monitoring, suggests that stricter regulations may have pushed some banks out of the market. ${ }^{9}$ More recently, two dealers sought buyers, which would mean exiting via acquisition: RBC (which is also a dealer) has, subject to government approval, purchased HSBC, and Laurentian Bank (parent company of Laurentian Securities Inc.) failed to find a buyer.

On the other hand, customers (in particular hedge funds) have become more active, in particular since around 2014 (see Figure 1B). The number of hedge funds has increased from zero to ten in 2021. Furthermore, they have been buying an increasingly larger share of the auction allotments relative to dealers and other customer groups (see Appendix Figures A4, and A5). However, since customers have no obligation to participate regularly, like dealers, their auction participation is highly volatile.

To better understand what drives customer participation, we regress the number of participating customers on a set of explanatory variables, using data from 2014 onward (when customers are almost exclusively hedge funds). The first variable we include indicates the auction dates for which we estimate that buying a bond and shorting the future, could be

[^5]Figure 1: Dealer exit and hedge fund entry from 1999-2022


Figure 1A shows how many primary dealers and government security distributors have participated in primary auctions since the first auction in 1999 until 2022. Figure 1B shows how many hedge funds participate in each auction from 1999 until 2022.
profitable (inspired by Barth and Kahn (2020); Banegas et al. (2021)). ${ }^{10}$ The second indicator variable tells us whether the bond-to-be-issued has benchmark status, which is the Canadian equivalent of being on-the-run (Berger-Soucy et al. (2018)). It could either be that hedge funds buy more liquid on-the-run bonds because they are easier to sell, or that they buy cheaper off-the-run bonds (while shorting more expensive on-the-run-bonds, as suggested by Banegas et al. (2021)).

The third and fourth indicator variables capture the importance of monetary policy committee meetings (MPC) and quantitative easing. Including an indicator for MPC meetings is inspired by the findings of Lou et al. (2023), who demonstrate that hedge funds tend to purchase bonds outside of the pre-MPC window to avoid interest rate uncertainty. Including an indicator for auction-days on which the central bank conducts a bond issuance in the morning and engages in quantitative easing by purchasing bonds in the afternoon, where hedge funds can sell the bonds they just bought (similar to An and Song (2018, 2023)).

[^6]The fifth and sixth variables measure whether the rate at which Canadian dollars are exchanged to U.S. dollars on auction day, or the spread at which a to-be-issued bond is traded prior to the auction influence customer participation. We approximate the spread by the average difference between the highest and lowest price within a day at which a bond-to-be-issued is traded in the secondary market three days prior to the auction. The seventh variable counts the number of dealers who participate in the auction. ${ }^{11}$

Our estimation findings, reported in Table 2, indicate that customers are more likely to participate in auctions when the secondary-market spread is high, which suggests that they might be trading bonds post-auction. In addition, we find some support for Lou et al. (2023)'s idea that hedge funds avoid buying bonds prior to monetary policy announcements. The other explanatory variables are statistically insignificant, when including year-fixed effects. For the number of dealers, this is because there is little variation within a year-a feature we will incorporate in our model. For the other explanatory variables, this might be because of low statistical power. For example, between 2014 and 2021 there are only five cases in which we find it profitable to buy at auction and simultaneously short futures.

Low statistical power, in addition to a moderately sized $R^{2}$ (in the regression without year fixed-effects), indicates that there may be unobservable factors that play a significant role in driving customer participation-a feature our model will incorporate.

Dealer updating. Our auction model, presented in Section 4, builds on Hortaçsu and Kastl (2012), who model the bidding process of Canadian Treasury auctions, in which customers must bid via dealers.

Intuitively, dealers who observe an aggressive customer bid should aggressively update their own bid, as they face more competition than expected. ${ }^{12}$ Supporting this conjecture, the correlation between the change in the dealer's (quantity-weighted) average bid, conditional on updating, and the customer's (quantity-weighted) average bid is positive (see Appendix Figure A6; Hortaçsu and Kastl (2012)). The (quantity-weighted) average bid is the price a

[^7]Table 2: Drivers of Customer Participation

|  | (OLS1) |  | (OLS2) |  | (Year-FE) |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\beta_{1}:$ Basis trade | +1.637 | $(0.931)$ | +1.134 | $(0.924)$ | +0.955 | $(0.852)$ |
| $\beta_{2}:$ Benchmark status | +0.304 | $(0.353)$ | +0.122 | $(0.356)$ | -0.063 | $(0.329)$ |
| $\beta_{3}:$ MPC | $-1.849^{*}$ | $(0.859)$ | -0.614 | $(0.900)$ | $-1.722^{*}$ | $(0.846)$ |
| $\beta_{4}:$ QE | $+0.945^{*}$ | $(0.392)$ | +0.574 | $(0.430)$ | -0.418 | $(0.436)$ |
| $\beta_{5}:$ Exchange rate | -1.008 | $(1.983)$ | -0.178 | $(2.064)$ | -4.535 | $(3.311)$ |
| $\beta_{6}:$ Spread | $+0.366^{* * *}$ | $(0.056)$ | $+0.388^{* * *}$ | $(0.059)$ | $+0.361^{* * *}$ | $(0.055)$ |
| $\beta_{7}:$ Number of dealers | $-0.467^{* * *}$ | $(0.116)$ | $-0.325^{*}$ | $(0.128)$ | +0.066 | $(0.140)$ |
| Extra controls | - |  | $\checkmark$ |  | $\checkmark$ |  |
| Adjusted $R^{2}$ | 0.211 |  | 0.251 |  | 0.369 |  |
| Observations | 327 |  | 327 |  | 327 |  |

Table 2 shows the estimation results of regressing the observed number of participating customers in an auction on a series of explanatory variables using data from the beginning of 2014 until the end of 2021 in column (OLS1). In column (OLS2) we add additional controls that capture the interest rate environment and expectations about the stance of monetary policy, and therefore future bond prices. We construct an OIS curve which represents the market expectations of the Bank of Canada's interest rate target for the overnight lending rate over 12 months. We also include 1- and 3-month Canadian Dollar Offered Rates (CDOR). CDOR is used as the main interest rate benchmark for calculating the floating-rate component of both over-the-counter and exchange-traded Canadian-dollar derivative products (McRae and Auger (2018)). In column (Year-FE) we include year fixed effects, in addition. Standard errors are in parenthesis.
bidder is willing to pay per-unit of the bond. Naturally, there might be a selection problemwe only observe how a dealer updates their bid if there is an actual update.

To overcome the selection concern, we need an instrument that exogenously affects the probability that a dealer updates (conditional on observing a customer's bid) but not the bid itself. Our idea is to use the time until the auction closes as an instrument-dealers are less likely to be able to update their bids the less time they have left in the auction-and estimate a Heckman-Selection model. ${ }^{13}$ One complication when analyzing in which direction a dealer updates her bid in response to observing a customer's bid arises from the fact that bids are step functions, and it is not obvious how to capture the shape and movement of that function. To get a sense of what matters most, we consider three moments of the customer's bidding step function: the (quantity-weighted) average bid, the number of steps, and the highest amount demanded.

In line with our expectations, we find in Table 3 that a dealer is more likely to update

[^8]Table 3: Heckman-Selection model of bid updating

|  |  |  |
| :--- | :--- | :--- |
| $\Delta$ bid $^{d}$ |  |  |
| $\quad$ qw-bid | $0.000800^{* * *}$ | $(0.000215)$ |
| $\quad$ number of steps | 0.0393 | $(0.0314)$ |
| total demand | 0.000264 | $(0.000163)$ |
| constant | $-0.905^{* * *}$ | $(0.166)$ |
| $\mathbb{I}$ (Update\| customer bid) |  |  |
| $\quad$ time_left | $0.000318^{* * *}$ | $(0.0000169)$ |
| $\quad$ qw-bid | $-0.000422^{* * *}$ | $(0.0000744)$ |
| $\quad$ number of steps | $0.0981^{* * *}$ | $(0.0133)$ |
| $\quad$ total demand | $0.000346^{* * *}$ | $(0.0000695)$ |
| $\quad$ constant | $0.102^{* * *}$ | $(0.0294)$ |
| Inverse mills | $1.198^{* * *}$ | $(0.280)$ |
| $\rho$ | 0.44449 |  |
| $\sigma$ | 2.69551 |  |
| Observations | 16156 |  |

Table 3 shows the results from estimating Heckman-Selection model, using bidding data from all regular Treasury auctions from 1999 until 2022. The model tests whether, and in which direction a dealer changes her quantity-weighted bid, $\Delta$ bid $^{d}$, (in bps) after observing a customer's bidwhich we summarize by its quantity-weighted bid (in bps), the number of steps and the total demand (in million $\mathrm{C} \$$ )—, using the seconds left until auction closure as an exogenous driver of selection into updating. Standard errors are in parentheses.
her bid conditional on observing a customer bid when she has more time until the auction closes. A dealer bids more aggressively when observing a more aggressive customer bid. Of the moments in the customer's step function that we consider, only the quantity-weighted average is statistically significant-a detail that we will leverage in our estimation presented in Section 5.

## 4 Model of the Canadian primary market

Motivated by the empirical evidence, we construct a model with two main features, which are both novel relative to the existing literature. First, we allow customers to be strategic in that they take into account that dealers will systematically update their bids when observing their bid. We show in Appendix A. 2 that this feature is empirically relevant, in that model estimates are biased if we assume that customers do not anticipate dealer updating.

Second, we endogenize the participation decision. Here we distinguish between the
dealer's decision to exit the market (at an annual frequency) and the customer's decision to enter specific auctions. This allows us to highlight the benefit of greater competition versus the cost of higher market volatility when an increasing share of bidders participates irregularly. Random variables will be highlighted in bold.

### 4.1 Players, timing and preferences

There are two groups ( $g$ ) of market participants: dealers ( $d$ ) and customers ( $h$ ). The number of dealers who consider remaining as dealers, $\bar{N}^{d}$, and the number customers who are interested in bidding, $N^{h}$, are commonly known. ${ }^{14}$ Similarly, all distributions and functional forms specified in Assumptions 1-6 are commonly known.

At the beginning of a year, when the debt issuance plan is announced, according to which $T$ auctions will be held. At this point, dealers decide whether they wish to continue being a dealer, which means committing to participate in all auctions in the upcoming year. Whether this is profitable depends on the (private) annual cost each dealer faces, $\gamma_{i}^{d}$. One may think of this as an opportunity cost that an institution that acts as a (primary) dealer suffers because it cannot do other things during the time it fulfills dealer-activities, such as bidding at auction and making markets.

Assumption 1. At the beginning of the year, dealers' private commitment costs for all $T$ auctions of the year, $\gamma_{\boldsymbol{i}}^{\boldsymbol{d}}$, are drawn independently from a common atomless distribution $G^{d}$. After each dealer makes their decision, the market is informed about the number of bidders who will act as dealers in the upcoming year, $N^{d}$. In reality, this information is posted on the website of the debt management office.

Before each auction $t$, customers observe how costly it is for them to enter the auction. Similar to dealers, one may think of these entry costs as opportunity costs, since it takes time monitor the market and compute competitive bids.

Assumption 2. Before each auction $t$, customers' entry costs for auction $t$, $\gamma_{\boldsymbol{t i}}^{\boldsymbol{h}}$, are drawn independently from a common atomless distribution $G^{h}$.

[^9]In addition, customers observe the (commonly known) distribution from which they will draw private signal that affect their willingness to pay if they choose to bid at auction. This distribution is specified in Assumption 3. It captures current market conditions, which can be unobserved to the econometrician. For example, the expected willingness to pay may be high when bid-ask spreads are high (as suggested by the evidence in Table 2), or when interest rates are expected to fall.

With this knowledge, each customer decides whether to enter an auction. Note that the entry decision is taken before the customer learns their private signal, which captures institution-specific knowledge, including information about the balance sheet or outstanding client orders, on auction day. This timing reflects the idea that most customers are part of large institutions, who tend to allocate tasks (such as bidding at auction or trading other assets) some time in advance, before the bidding process starts.

Conditional on participation in auction $t$, each customer and dealer draws a private (multi-dimensional) signal $\boldsymbol{s}_{\boldsymbol{t i}}^{\boldsymbol{g}}$. The signals may be drawn from two different distributions for dealers and customers.

Assumption 3. Dealers' and customers' private signals $s_{\boldsymbol{t i}}^{\boldsymbol{d}}$ and $\boldsymbol{s}_{\boldsymbol{t i}}^{\boldsymbol{h}}$ are for all bidders $i$ independently drawn from common atomless distribution functions $F_{t}^{d}$ and $F_{t}^{h}$ with support $[0,1]$ and strictly positive densities $f_{t}^{d}$ and $f_{t}^{h}$.

Within an auction, a bidder's signal must be independent from all other signals, conditional on everything that bidders know when bidding, which includes a reference price-range provided by the auctioneer. To support this assumption, we follow Hortaçsu and Kastl (2012) and test whether dealers who observe customer's bid's only learn about the degree of competition in the auction (and not about the fundamental value of the bond). Our findings, reported in Appendix Table A2, support the conditional independence assumption.

Assumptions 1-3 rule out that bidders have incentive to adopt strategies that connect multiple auctions. Thus, technically, our game consists of dealers' exit decisions plus $T$ ex-ante identical, and separate auction sub-games. We think that this is reasonable in our setting, given that the typical bidder sells their auction-purchased bonds quickly on days around the auction, before the next auction takes place. Therefore, the bidder is less likely to implement bidding strategies that span across auctions. In other settings, it can be
important to take these types of strategic considerations into account. For example, Rüdiger et al. (2023) analyze inter-temporal bidding behavior in Argentinian Treasury auctions; Allen et al. (2020) consider interconnections between auctions that take place in parallel.

How bidders bid in a specific auction $t$ depends on how much they value the bond at that time. This, in turn, is driven by their signals.

Assumption 4. A bidder $i$ of group $g \in\{d, h\}$ with signal $s_{t i}^{g}$ values amount $q$ of the bond by $v_{t}^{g}\left(q, s_{t i}^{g}\right)$. This value function is non-negative, measurable, bounded strictly increasing in $s_{t i}^{g}$ for all $q$, weakly decreasing in $q$ for all $s_{t i}^{g}$.

Given their values before auction $t$, bidders place bids. Each bid is a step function that characterizes the price the bidder would like to pay for each amount. For simplicity, we fix the number of steps, $K_{t i}$, by assuming that $K_{t i}$ is part of the bidder's private signal. Alternatively, we could follow Kastl (2011) and introduce a (private) cost of computing and submitting steps which would pin down the number of steps endogenously.

Assumption 5. Bidder $i$ has the following action set to place a bid in auction $t$ :

$$
A_{t i}=\left\{\begin{array}{l}
(b, q): \operatorname{dim}(b)=\operatorname{dim}(q)=K_{t i} \\
b_{k} \in[0, \infty) \text { and } q_{k} \in[0,1] \\
b_{k}>b_{k+1} \text { and } q_{k}>q_{k+1} \forall k<K_{t i}
\end{array}\right.
$$

Following Hortaçsu and Kastl (2012), bidding evolves in three rounds. First, dealers can place early bids directly to the auctioneer. To rationalize early bidding, one dimension of dealer $i$ 's signal $\boldsymbol{s}_{\boldsymbol{t} \boldsymbol{d}}^{\boldsymbol{d}}$ is a random variable $\Psi_{i} \in[0,1]$ that corresponds to the mean of another Bernoulli random variable, $\boldsymbol{\Phi}_{\boldsymbol{i}}$, which determines whether the dealer's later bids will make it in time to be accepted. Second, each participating customer is randomly matched to a dealer and places her bid with this dealer. ${ }^{15}$ Third, each dealer observes these customer bids (if any), and may update their own bids. In the last stage, the dealer observes her own signal, $s_{t i}^{d}$, in addition to $Z_{t i}^{d}$, which includes the bids of the customer(s) that was (were) matched

[^10]to this dealer or the fact that no customer bid arrived, and the realization $\omega_{i} \in\{0,1\}$ of $\Phi_{i}$, where $\omega_{i}=1$ means that the late bid will make it in time.

A pure bidding strategy is a mapping from the information set of a bidder to the action space at each stage of the game. To capture everything that a bidder knows, we introduce a bidder's type, labeled $\theta_{i \tau}^{g}$, where time $\tau$ summarizes the auction date and the bidding stage. The type equals to the private signal of the bidder in the first and second stage of the game, and it may include $Z_{t i}^{d}$ at the final stage. With this, bidding strategies can be represented by bidding functions, labeled $b_{i \tau}^{g}\left(\cdot, \theta_{i \tau}^{g}\right)$, for bidder $i$ of group $g$ with type $\theta_{i \tau}^{g}$ at time $\tau$.

When choosing the bidding function, a customer anticipates that their dealer can update their own bid-here we differ from Hortaçsu and Kastl (2012) who focus on dealers. However, since the customer doesn't know the dealers type, $\theta_{i \tau}^{d}$, she doesn't know whether and how the dealer will update their bid. As a result, a customer cannot be sure if the market-clearing price will increase (or decrease) when she marginally increases her own demand, $q_{k}$, at step $k$-anything can happen. ${ }^{16}$ This makes it complex for the customer to determine her optimal bid.

To render the customers' optimization problem solvable, we assume that dealers only pay attention to finite sets of moments of the customers' bidding function when updating their own bid - motivated by the empirical evidence presented in Table 3. Alternatively, we could assume that customers only think that this is the case, even though the dealer responds to the full curve. Formally, a moment is a mapping $\mu_{t}^{l}$ that transforms the bidding function, $b_{i \tau}^{h}\left(\cdot, \theta_{i \tau}^{h}\right)$ for type $\theta_{i \tau}^{h}$, into a real number $\mathbb{R}$. We restrict attention to moments that are differentiable w.r.t. $q$ at each price. This includes, for example, the intercept with the price or quantity axis, some smooth approximation of the slope, or the quantity-weight bid (qw-bid): ${ }^{17}$

$$
\begin{equation*}
\mu_{t}^{l}\left(b_{i \tau}^{h}\left(\cdot, \theta_{i \tau}^{h}\right)\right)=\frac{\sum_{k}^{K_{t i}} b_{k} q_{k}}{\sum_{k} q_{k}} \text { when }\left\{b_{k}, q_{k}\right\}_{k=1}^{K_{t i}} \text { constitute bidding function } b_{i \tau}^{h}\left(\cdot, \theta_{i \tau}^{h}\right) \text {. } \tag{1}
\end{equation*}
$$

[^11]Once all bidders submit their step function, the market clears at the lowest price, $P_{t}^{*}$, at which the aggregated submitted demand satisfies the total supply. The supply is unknown to bidders when they place bids because a significant fraction of the total allotment goes to non-competitive bidders, i.e., to the Bank of Canada who is the largest non-competitive bidder.

Assumption 6. Supply $\boldsymbol{Q}_{\boldsymbol{t}}$ is a random variable distributed on $\left[\underline{Q_{t}}, \bar{Q}_{t}\right]$ with strictly positive marginal density conditional on $s_{t i}^{g} \forall i, g=h, d$.

Given all bidding functions, $b_{i \tau}^{g}\left(\cdot, \theta_{i \tau}^{g}\right)$, bidder $i$ of group $g$ wins amount $q_{t i}^{g *}$ at market clearing. She pays the amount she offered to win for each unit won. In case there is excess demand at the market clearing price, each bidder is rationed pro-rata on-the-margin (see Kastl (2011) for details).

### 4.2 Equilibrium conditions

We first characterize the equilibrium in auction $t$ conditional on customer and dealer participation. Then, we determine the entry and exit decisions by customers, and dealers, respectively.

Auction. To find the optimal bidding strategy, a dealer maximizes her expected total surplus, taking the behavior of other bidders as given. For bidder $i$ in group $g$ of type $\theta_{i \tau}^{g}$ the expected total surplus is

$$
\begin{equation*}
T S_{t i}^{g}=\mathbb{E}_{t}\left[\int_{0}^{q_{t i}^{g *}}\left[v_{t}^{g}\left(x, s_{t i}^{g}\right)-b_{i \tau}^{g}\left(x, \theta_{i \tau}^{g}\right)\right] d x\right] \tag{2}
\end{equation*}
$$

The expectation is taken over the amount the bidder will win at market clearing, $\boldsymbol{q}_{\boldsymbol{t} \boldsymbol{i}}^{\boldsymbol{g} \boldsymbol{*}}$, which depends on the strategies and types of all bidders, as well as the unknown supply.

We focus on type-symmetric Bayesian Nash equilibrium (BNE) in which all dealers and customers play the same bidding strategy if they have the same type. Formally, a pure strategy BNE of auction $t$ is a collection of bidding functions $b_{i \tau}^{g}\left(\cdot, \theta_{i \tau}^{g}\right)$ that each bidder $i$ and almost every type $\theta_{i \tau}^{g}, b_{i \tau}^{g}\left(\cdot, \theta_{i \tau}^{g}\right)$ maximizes the bidder's expected total surplus.

Proposition 1. Fix a set of $L_{t}$ moment functions that map a customer's bid function into a real number, $\mu_{t}^{l}: b_{i \tau}^{h}\left(\cdot, \theta_{i \tau}^{h}\right) \rightarrow \mathbb{R}$, and consider a type-symmetric BNE.
(i) Every step $k$ but the last step in the dealer's bid function $b_{i \tau}^{d}\left(\cdot, \theta_{i \tau}^{d}\right)$ has to satisfy

$$
\begin{equation*}
\operatorname{Pr}\left(b_{k}>\boldsymbol{P}_{\boldsymbol{t}}^{\boldsymbol{*}}>b_{k+1} \mid \theta_{i \tau}^{d}\right)\left[v_{t}^{d}\left(q, s_{t i}^{d}\right)-b_{k}\right]=\operatorname{Pr}\left(b_{k+1} \geq \boldsymbol{P}_{\boldsymbol{t}}^{*} \mid \theta_{i \tau}^{d}\right)\left(b_{k}-b_{k+1}\right) . \tag{3}
\end{equation*}
$$

At the last step, $b_{K_{t i}}=v_{t}^{d}\left(\bar{q}\left(\theta_{i \tau}^{d}\right), s_{t i}^{d}\right)$, where $\bar{q}\left(\theta_{i \tau}^{d}\right)$ is the maximal amount the dealer may be allocated in the auction equilibrium.
(ii) Every step $k$ in a customer's bid function $b_{i \tau}^{h}\left(\cdot, \theta_{i \tau}^{h}\right)$ that generates moments $m_{t}^{l}=$ $\mu_{t}^{l}\left(b_{i \tau}^{h}\left(\cdot, \theta_{i \tau}^{h}\right)\right)$ for all $l$ has to satisfy:

$$
\begin{align*}
& \operatorname{Pr}\left(b_{k}>\boldsymbol{P}_{\boldsymbol{t}}^{*}>b_{k+1} \mid \theta_{i \tau}^{h}, m_{t}\right)\left[v_{t}^{h}\left(q, s_{t i}^{h}\right)-b_{k}\right]= \\
& \operatorname{Pr}\left(b_{k+1} \geq \boldsymbol{P}_{\boldsymbol{t}}^{*} \mid \theta_{i \tau}^{h}, m_{t}\right)\left(b_{k}-b_{k+1}\right)-\sum_{l=1}^{L_{t}} \lambda_{t}^{l} \frac{\partial \mu_{t}^{l}\left(b_{i \tau}^{h}\left(\cdot, \theta_{i \tau}^{h}\right)\right)}{\partial q}+\operatorname{Ties}\left(b_{i \tau}^{h}\left(\cdot, \theta_{i \tau}^{h}\right)\right), \tag{4}
\end{align*}
$$

with Ties $\left(b_{i \tau}^{h}\left(\cdot, \theta_{i \tau}^{h}\right)\right)=\operatorname{Pr}\left(b_{k}=\boldsymbol{P}_{\boldsymbol{t}}^{*}\right) \mathbb{E}\left[\left.v_{t}^{h}\left(\boldsymbol{q}_{\boldsymbol{t i}}^{\boldsymbol{h} *}, s_{t i}^{h}\right) \frac{\partial \boldsymbol{q}_{t i}^{\boldsymbol{h} *}}{\partial q_{k}} \right\rvert\, b_{k}=\boldsymbol{P}_{\boldsymbol{t}}^{*}\right]-\operatorname{Pr}\left(b_{k+1} \geq \boldsymbol{P}_{\boldsymbol{t}}^{*}\right) \mathbb{E}\left[\left.v_{t}^{h}\left(\boldsymbol{q}_{\boldsymbol{t i}}^{\boldsymbol{h} \boldsymbol{*}}, s_{t i}^{h}\right) \frac{\partial \boldsymbol{q}_{\boldsymbol{q}}^{\boldsymbol{h} \boldsymbol{*}}}{\partial q_{k}} \right\rvert\, b_{k+1} \geq\right.$ $\left.\boldsymbol{P}_{\boldsymbol{t}}^{*}\right]+\operatorname{Pr}\left(b_{k}=\boldsymbol{P}_{\boldsymbol{t}}^{*}\right) \mathbb{E}\left[\left.\frac{\partial \boldsymbol{q}_{\boldsymbol{t}_{i t}^{*}}}{\partial q_{k}} \right\rvert\, b_{k}=\boldsymbol{P}_{\boldsymbol{t}}^{*}\right]+\operatorname{Pr}\left(b_{k+1}=\boldsymbol{P}_{\boldsymbol{t}}^{*}\right) \mathbb{E}\left[\left.\frac{\partial \boldsymbol{q}_{t i_{i}^{*}}^{\boldsymbol{h}}}{\partial q_{k}} \right\rvert\, b_{k+1}=\boldsymbol{P}_{\boldsymbol{t}}^{*}\right]+\operatorname{Pr}\left(b_{k+1}<\boldsymbol{P}_{\boldsymbol{t}}^{*}\right) \mathbb{E}\left[\left.\frac{\partial \boldsymbol{q}_{\boldsymbol{t}}^{\boldsymbol{h}}}{\partial q_{k}} \right\rvert\, b_{k+1}<\right.$ $\left.P_{\boldsymbol{t}}^{*}\right]$, where we have omitted the dependence on $\theta_{i \tau}^{h}$ and $\left\{m^{l}\right\}_{l=1}^{L}$, and

$$
\begin{equation*}
\lambda_{t}^{l}\left[m_{t}^{l}-\mu_{t}^{l}\left(b_{i \tau}^{h}\left(\cdot, \theta_{i \tau}^{h}\right)\right)\right]=0 \text { for all } l \text { in auction } t . \tag{5}
\end{equation*}
$$

(iii) The moments, $\left\{m^{l}\right\}_{l=1}^{L}$, are such that expected total surplus (2) is maximized, and $m^{l}=\mu^{l}\left(b_{i \tau}^{h}\left(\cdot, \theta_{i \tau}^{h}\right)\right)$ for all $l$ and all customers.

From the existing literature we know how the dealer determines her equilibrium bid. Essentially, she chooses her bid to maximize total surplus, $T S_{t i}^{d}$, defined in (2), subject to market clearing. She trades off the expected surplus on the marginal infinitesimal unit versus the probability of winning it (see Kastl (2017), p. 237 for more details). Given that it is never optimal for a dealer to submit a bid above her true value, dealer demand is never rationed in equilibrium, except for the last step. At the last step the dealer submits her true value, because it is not possible to increase the winning probability of (non-existing) subsequent steps by shading the bid.

Our innovation is to characterize the equilibrium bidding of a customer. The key difference between the dealer and the customer comes from the fact that customers take into account the dealer's response to observing their bid. The customer faces a complicated trade-off when placing bids. To illustrate this, assume that only one moment - the quantity-
weighted bid-matters, and consider a bidding function with $K_{t i}$ steps that would be be optimal if dealers didn't update (e.g., Figure 2). As for dealers, at the last step of this (hypothetical) function the customer submits her true value. The customer could deviate from this function, for example, by extending the last step out. Note that she cannot add another step, because the bidder's type specifies the number of steps, $K_{t i}$. This would reduce the quantity-weighted bid, and might induce less aggressive dealer bidding in response. This benefit is evaluated against the cost of surplus lost from winning additional units and having to pay a price above your value for those units. A similar trade-off arises for all other possible deviations.

To derive a deeper understanding for how the customer chooses her bid, take the perspective of a customer and let all other bidders play the equilibrium strategy. Assume that the customer considers changing her own bidding function $b_{i \tau}^{h}\left(\cdot, \theta_{i \tau}^{h}\right)$. She thinks that the dealer (who observes the perturbed bidding function) changes their bid only if the customer's perturbed bidding function generates different moments, $\left\{m_{t}^{l}\right\}_{l=1}^{L}$, than the original bid. With this information, the customer determines her best response by considering perturbation of the bidding function that leave the moments unchanged. Fixing some $\left\{m_{t}^{l}\right\}_{l=1}^{L}$, the customer finds the bidding function that maximizes her expected total surplus such that $\mu_{t}^{l}\left(b_{i \tau}^{c}\left(\cdot, \theta_{i \tau}^{h}\right)\right)=m_{t}^{l}$ for all $l$. This is summarized in Proposition 1 (ii). Here $\lambda_{t}^{l} \in \mathbb{R}$ denote Lagrange multipliers of the moments' constraints. The term $\operatorname{Ties}\left(b_{i \tau}^{h}\left(\cdot, \theta_{i \tau}^{h}\right) \mid \theta_{i \tau}^{h}, m_{t}^{l}\right)$ includes all the cases in which the customer's bid ties with some other bid and must be rationed. This happens with non-zero probability at any step, because it can be optimal for a customer to place a bid above her true value to favorably affect dealer updating. In addition, the customer chooses moments $\left\{m_{t}^{l}\right\}_{l=1}^{L}$ so that the total surplus is actually maximized—Proposition 1 (iii).

Entry and exit decisions. A customer $i$ enters an auction $t$ if her entry cost for auction $t, \gamma_{t i}^{h}$, is smaller than the total surplus she expects to earn from participating in the auction before she observes her private signal, but after observing the signal distributions, $F_{t}^{g}$, and the shape of the value functions, $v_{t}^{g}(\cdot, \cdot)$, for $g=\{h, d\}$. These two elements capture current market conditions.

Proposition 2. Customer $i$ with entry cost $\gamma_{t i}^{h}$ enters auction $t$ if

$$
\begin{equation*}
\gamma_{t i}^{h} \leq \mathbb{E}_{t}\left[\boldsymbol{T} S_{\boldsymbol{t i}}^{\boldsymbol{h}} \mid N^{d}\right] \text { with } T S_{t i}^{h} \text { given by (2). } \tag{6}
\end{equation*}
$$

The expectation is taken over the customer's private signal $s_{t i}^{h}$, and conditional on $N^{d}$ dealers bidding in the auction.

Anticipating all auctions, $t=1, \ldots, T$ of the upcoming year, dealer $i$ exits the market if her commitment cost is higher than the surplus she expects to earn from bidding in all $T$ auctions of the upcoming year.

Proposition 3. At the beginning of the year, dealer $i$ with commitment cost $\gamma_{i}^{d}$ exits the market if

$$
\begin{equation*}
\gamma_{i}^{d} \geq \sum_{N^{d}=1}^{\bar{N}^{d}}\left(\sum_{t=1}^{T} \mathbb{E}_{t}\left[\boldsymbol{T} \boldsymbol{S}_{\boldsymbol{t i}}^{\boldsymbol{d}} \mid N^{d}\right]\right) \operatorname{Pr}\left(\boldsymbol{N}^{\boldsymbol{d}}=N^{d}\right) \text { with } T S_{t i}^{d} \text { given by (2). } \tag{7}
\end{equation*}
$$

The dealer cares about the aggregate surplus she expects to earn over the entire year, and when taking her decision she doesn't know how many other dealer's will compete in the auctions, $N^{d}$. Therefore, the dealer considers all possible realizations of $N^{d}$ and weights each by how likely it is to occur, $\operatorname{Pr}\left(N^{d}=N^{d}\right)$.

## 5 Identification and estimation

The goal is to learn about the unobserved bidder values, $v_{t}^{g}\left(\cdot, s_{t i}^{g}\right)$, and the entry and exit cost distributions, $G^{h}$ and $G^{d}$. For this, we fix the maximal number of dealers to what we observe in our sample, i.e., $\bar{N}^{d}=24$. We define the number of potential customers in a year, $N^{h}$, as the maximal number of customers we observe bidding in any auction of that year.

Identifying and estimating values. The idea behind identifying and estimating bidder values from bidding data is to infer how much each bidder is truly willing to pay from the equilibrium conditions under the assumption that everyone plays this equilibrium, here specified in Proposition 1. Note that we observe all elements that enter these conditions, but the values (we seek to identify), and the probabilities of where the market will clear. We estimate these probabilities, by extending Hortaçsu and Kastl (2012)'s resampling procedure and allowing information of a bidder to be correlated over the course of the auction (details in Appendix B). With this, we can point-identify dealer values at all submitted steps, $q_{k}$

Figure 2: Illustration of what we can infer about values from bids


Figure 2 illustrates what can infer about values of a dealer who submits a bid function with 4 steps (shown in blue). We can point-identify her values at $q_{1}, q_{2}, q_{3}$ and $q_{4}$-the black circles. Further, since the value function is decreasing in quantity and bounded, we know that the dealer's values for quantities in between must lie in the dashed boxes.
(Kastl (2012)). This implies that we can construct an upper and a lower bound for the entire value curve, as illustrated in Appendix Figure 2.

Our contribution is to learn about customer values. This is difficult because the customer's equilibrium condition (4) involves ties, which implies that it must be evaluated at all quantity points on the bidding curve, and not only at submitted steps. Therefore, customer values cannot be point-identified. However, we can construct sets of informative bounds on customer values that are consistent with the observed bids. For this, we assume that dealers only pay attention to the qw-bid when updating their own bid-motivated by the empirical evidence in Table 3. Condition (4) then simplifies to an equation with a single moment, the qw-bid (1). For simplicity, we drop the super- and subscript $l=1$.

To identify customer value bounds, we proceed in three steps. First, we guess a Lagrange multiplier, $\lambda_{t} \in \mathbb{R}$, and determine $K_{t i}$ upper and $K_{t i}$ lower values for $v_{t}^{h}\left(q_{k}, s_{t i}^{h}\right)$ for a customer $i$ who submits $K_{t i}$ steps in a fixed auction $t$, by relying on Proposition 1 (ii). For this, we utilize boundedness and monotonicity of $v_{t}^{h}\left(\cdot, s_{t i}^{h}\right)$, which implies, for example, that the upper bound value in case of a tie and thus rationing at step $k$ is $\bar{v}_{t}^{h}\left(q_{k}, s_{t i}^{h}\right)$, and the lower bound value is $\underline{v}_{t}\left(q_{k+1}, s_{t i}^{h}\right)$.

Second, we simplify this system of $2 K_{t i}$ equations, by showing that at a subset of steps rationing never occurs in equilibrium, which implies that we can eliminate the terms involving rationing at these steps in the equilibrium condition. We do this by constructing profitable deviations at these steps (see Lemma 2). For instance, consider a step $k<K_{t i}$ for the case
in which $\lambda_{t}<0$. Slightly increasing the bid at step $k$ can break the tie and either generate a positive surplus or decrease the loss from placing a bid to favorably influence dealer updating.

Third, we check that the guessed Lagrange multiplier, $\lambda_{t}$, is valid in equilibrium. For this, we rely on Proposition 1 (iii) according to which the equilibrium bidding function must generate a moment $m_{t}^{l}$ that is optimal. For illustration, assume for an instant that we could point-identify the customer's entire value curve. Then, increasing or decreasing $m_{t}^{l}$ by a small amount, $\epsilon>0$, must decrease the customer's expected total surplus. We obtain two inequality conditions: $T S_{t i}^{h}\left(m_{t}^{l}\right)-T S_{t i}^{h}\left(m_{t}^{l}+\epsilon\right) \geq \lambda_{t} \epsilon$ and $-T S_{t i}^{h}\left(m_{t}^{l}\right)+T S_{t i}^{h}\left(m_{t}^{l}-\epsilon\right) \leq \lambda_{t} \epsilon$. Here we highlight the dependence of $T S_{t}^{h}$, defined in expression (2), on a chosen moment, $m_{t}^{l}$, for clarity. These inequality conditions imply a set of Lagrange multipliers that are valid in equilibrium. If our guessed Lagrange multiplier is in this set, we are done. Otherwise, we make a different guess and start over. The same logic applies when using bounds on value functions, just that the RHS of the inequality conditions are derived from bounds; see Appendix D. 2 for details.

Identifying and estimating cost distributions. To learn about the cost distributions, $G^{h}$ and $G^{d}$, we rely on Propositions 2 and 3 , respectively. For this, we compute bounds on how much customers and dealers expect to gain from participating in the game, i.e., the RHS of (7) and (6), respectively.

For customers, we construct upper and lower bounds of the surplus (2) a customer (who submitted bid function $i$ ) expects to gain after entering auction $t, \overline{T S}_{t i}^{h}$, and $\underline{T S}_{t i}^{h}$. Visually, $\overline{T S}_{t i}^{h}$ is the area between the upper bound on the value function and the bid function in Figure (2), where each quantity is weighted by the probability that it is won at market clearance. To approximate bounds on the surplus customers expect to gain before entering the auction, $\mathbb{E}_{t}\left[\overline{\boldsymbol{T S}}{ }_{t i}^{h} \mid N^{d}\right]$ and $\mathbb{E}_{t}\left[\underline{T S}_{t i}^{h} \mid N^{d}\right]$, we take the averages of $\overline{T S_{t i}^{h}}$ and $\underline{T S_{t i}^{h}}$ across customers $i$ in auction $t$. Here we rely on the assumption that customers signals are drawn iid from the same distribution.

For dealers, constructing bounds on the expected annual auction surplus, given in (6), is more difficult, since dealers make their entry decision before knowing how many other dealers will participate in the upcoming year. This implies that we need to compute two additional objects besides the expected total surplus given the observed number of dealers
(which we obtain as for customers). First, we get the expected total surpluses when different numbers of dealers participate. We could exactly compute these counterfactual surpluses using a similar approach presented in our counterfactual exercises. However, this is computationally intensive. Therefore, we approximate the counterfactual surpluses by finding auctions that are similar to an auction in question in that bidders expect a similar per-unit surplus but with a different number of participating dealers. Within this set of auctions, we compute $\overline{T S}_{t i}^{d}$ for all dealers and take an average in order to obtain the expected surplus for a fixed counterfactual $N^{d}, \mathbb{E}_{t}\left[\overline{T S}_{\boldsymbol{t i}}^{\boldsymbol{d}} \mid N^{d}\right]$. Second, we compute how likely it is that $N^{d}$ dealers participate when $\bar{N}^{d}$ is the maximal number of dealers from the empirical probability that a dealer participates in a fixed year: $\operatorname{Pr}\left(N^{d}=N^{d}\right)=\binom{\bar{N}^{d}}{N^{d}}\left(N^{d} / \bar{N}^{d}\right)^{N^{d}}\left(1-\left(N^{d} / \bar{N}^{d}\right)^{\bar{N}^{d}-N^{d}}\right.$.

With the bounds on how much dealers and customers expect to gain from auction participation, we identify bounds on the cost distributions by matching the predicted participation probability of a customer and a dealer according to Propositions 2 and 3 to what we observe in the data. For example, the predicted probability for a customer of entering an auction, $\operatorname{Pr}\left(\gamma_{\boldsymbol{t} \boldsymbol{i}}^{\boldsymbol{h}} \leq \mathbb{E}_{t}\left[\boldsymbol{T} \boldsymbol{S}_{\boldsymbol{t} \boldsymbol{h}}^{\boldsymbol{h}} \mid N^{d}\right]\right)$, must equal the observed entry probability (the number of customers who bid in an auction, $N_{t}^{h}$, relative to the number of customers who consider bidding, $N^{h}$ ). Relying on the fact that

$$
\begin{equation*}
\operatorname{Pr}\left(\gamma_{t i}^{\boldsymbol{h}} \leq \mathbb{E}_{t}\left[\underline{\boldsymbol{T}}_{\boldsymbol{t} \boldsymbol{i}}^{\boldsymbol{h}} \mid N^{d}\right]\right) \leq \frac{N_{t}^{h}}{N^{h}} \leq \operatorname{Pr}\left(\gamma_{t i}^{\boldsymbol{h}} \leq \mathbb{E}_{t}\left[\overline{\boldsymbol{T S}}_{\boldsymbol{t} \boldsymbol{i}}^{\boldsymbol{h}} \mid N^{d}\right]\right) \tag{8}
\end{equation*}
$$

we could identify a lower and upper bound for the cost distribution non-parametrically as long as there are sufficiently many different surpluses, $\left.\mathbb{E}_{t}\left[\overline{\boldsymbol{T S}}_{\boldsymbol{t} \boldsymbol{i}}^{\boldsymbol{h}} \mid N^{d}\right]\right)$ and $\mathbb{E}_{t}\left[\underline{\boldsymbol{T}}_{\boldsymbol{t} \boldsymbol{i}}^{\boldsymbol{h}} \mid N^{d}\right]$ ), from different auctions, to cover the full support of the cost distribution. With our data, we $\underline{\left.\text { impose an exponential distribution with parameter } \beta^{h} \text { for customers, and } \beta^{d} \text { for dealers. }{ }^{18}{ }^{18}\right) .}$
${ }^{18}$ Concretely, for customers we estimate a set of $\beta^{h}$, using the following criterion function: $Q^{\prime}\left(\beta^{h}\right)=$ $Q\left(\beta^{h}\right)-\inf _{\beta^{\prime}} Q\left(\beta^{\prime}\right)$ with $\left.\left.Q\left(\beta^{h}\right)=\left(\frac{N_{t}^{h}}{N^{h}}-H\left(\mathbb{E}_{t}\left[\underline{\boldsymbol{T}} \underline{\boldsymbol{S}}_{t i}^{h} \mid N^{d}\right] ; \beta^{h}\right)\right)_{+}\right)^{2}+\left(\frac{N_{t}^{h}}{N^{h}}-H\left(\mathbb{E}_{t}\left[\overline{\boldsymbol{T}} \overline{\boldsymbol{S}}_{t i}^{h} \mid N^{d}\right] ; \beta^{h}\right)\right)_{-}\right)^{2}$, where $H$ is the CDF of an exponential distribution with parameter $\beta^{h}$. As the sample size grows, all points in the identified set produce criterion values of zero. To account for finite sample errors, we define a contour set of level $c_{n}$, and estimate the parameter set $\left\{\beta^{h} \mid Q^{\prime}\left(\beta^{h}\right) \leq c_{n}\right\}$. We choose the cutoff $c_{n}$ proportionally to the number of auctions in our sample $c_{n}=\log (645) / 645$, inspired by Chernozhukov et al. (2007).

## 6 Estimated values and costs

For each auction $t$, we estimate dealer values, $\hat{v}_{i t k}$, and bounds for customer values, $\underline{v}_{i t k}$ and $\bar{v}_{i t k}$, at each submitted quantity step $k$. In addition, we obtain upper and lower bounds for the (exponential) cost distributions of both bidder groups, $G^{h}, G^{d}$.

Dealer and customer values. We find that customers who participate in the auctions tend to have higher values than dealers, per average unit of a bond (see Figure 3). A customer is willing to pay about 6.9-7.7 bps more than a dealer in the median. This is true when using customer values at the lower bound and upper bound, given that bounds are relatively tight. For the rest of the paper, we rely on the lower-bounded values to be conservative.

A 6.9 bps difference in willingness to pay is economically meaningful compared to the median (average) market yield-to-maturity of the bonds, which is 161 bps ( 214 bps ). We show in Appendix A. 3 that the difference is statistically significant at the $5 \%$ level, where standard errors are computed via bootstrap.

Since values reflect how much an auction participant expects to earn from trading bonds in the secondary market post-auction, this finding suggests that (participating) customers anticipate larger average returns (per unit) from buying bonds than dealers. This pattern cannot be driven by customer selection into more profitable auctions given that we are considering value differences conditional on auction participation. It is in line with evidence documented by Sandhu and Vala (2023), who argue that hedge funds are able to obtain higher than average returns in the secondary market for Canadian government bonds, because they have more flexibility to employ complex and risky trading strategies (Ontario Securities Commission (2007)).

Different to customers, dealers have been facing increasingly stringent regulations since the global financial crisis in 2007-2009. In line with this fact, Figure 3 shows that customer values have increased over time relative to dealer values, suggesting that the average customer has become more profitable over time relative to the average dealer. This trend could come from the same customers becoming more profitable over time. Alternatively, if different customer types (e.g., hedge funds vs. pension funds) have systematically different value distributions, it could come from a change in the composition of customer types over time.

We verify that higher customer values correlate with the observable factors that predict

Figure 3: Difference between customers and dealers values


Figure 3A shows the distribution of the difference in quantity-weighted average values between participating customers and dealers, using customer lower bound values on the LHS, and upper bound values on the RHS. We remove outliers, defined as a value that is more than three scaled median absolute deviations from the median. Figure 3B plots the point estimates and $95 \%$ confidence intervals from regressing the difference between the average customer value and the average dealer value in each auction on a set of time indicator variables. The first indicator is 1 for 1999-2000, the second indicator is 1 for 2001-2002, and so forth, until 2021-2022. The confidence intervals are computed using the point-estimates of dealer and customer values at the lower bounds, and therefore do not account for noise in the estimation of these values. Prices are in C $\$$ with a face value of 100 .
stronger customer participation to validate our value estimates (shown in Table 2). For this we regress the average quantity-weighted customer value per auction on the same explanatory variables that we used in the regression of Table (2). Our findings, reported in Appendix Table A3, are consistent with the descriptive evidence, according to which the auctionsecondary market spread is the main driver of customer participation. ${ }^{19}$

Participation costs. Participation costs capture the opportunity cost of profits that customers and dealers could generate outside the Treasury market. These costs differ from values in that they are independent of how much an institution wins at auction; they must be payed when an institution spends time bidding at auction (and in the case of dealers, conduct other market making-activities), even if the institution doesn't buy any bonds.

[^12]On average, we estimate an annual cost of being a dealer between $\mathrm{C} \$ 3.198$ and $\mathrm{C} \$ 4.310$ million with (bootstrapped) confidence interval $[\mathrm{C} \$ 2.837 \mathrm{M}, \mathrm{C} \$ 4.551 \mathrm{M}]$. This is sizable compared to the average annual profit a bank generates from its market-making activities in all financial markets combined, which is roughly C $\$ 413$ million (Allen and Wittwer (2021)). Our cost estimate for dealers appear larger than Rüdiger et al. (2023)'s estimate for how valuable it is to be a primary dealer in Argentinian Treasury auctions.

The average entry cost of a customer is between $\mathrm{C} \$ 429,500$ and $\mathrm{C} \$ 459,300$ per auction with a (bootstrapped) confidence interval [C\$402,000, C $\$ 490,710$ ]. Given that there are about 30 auctions per year, the customer cost is larger than the dealer's cost (which is about C $\$ 106,600$ - $\mathrm{C} \$ 143,670$ per auction). This is in line with the idea that customers are better at executing profitable trading strategies-here outside of the Treasury market, driving up the opportunity participation cost.

Summarizing, our findings highlight systematic differences between dealers and customers, both in terms of values and entry costs.

## 7 Drivers and consequences of customer participation

With the estimated model, we conduct counterfactuals to understand why hedge funds entered the market in 2014-2015, and evaluate the consequences for market functioning. For this, we assume that all model primitives, such as the distributions of values and costs, remain fixed when changing market rules. We use final bids only and the lower bound estimates of values and costs. Ideally, we would compute counterfactual outcomes for all auctions in our sample. However, this is computationally intense. We therefore only consider every third auction since 2014, i.e., the period when hedge funds became the dominating customer group (recall Figure A4).

Computing counterfactuals Conducting counterfactuals for multi-unit auctions in which bidders have multi-unit demand is challenging because it is impossible to solve for an equilibrium analytically. Since we can only characterize necessary equilibrium conditions, it is generally difficult to compute counterfactual bids. Another complication arises from the fact that we want to allow bidders to demand more when they are competing against fewer
competitors, without ignoring (unobservable) constraints that limit the total amount bidders are able to buy at once. To overcome these challenges we proceed in two steps (explained in more details in Appendix C.1).

First, we construct an empirical distribution of maximal demand for dealers and customers, respectively. One observation in the distribution for dealers (customers), is the maximal percentage of supply that one of the dealers (customers) ever demanded. If this percentage is above $25 \%$, we replace it by $25 \%$ to incorporate the feature that, during regular times, bidders face a bidding limit of $25 \%$ of supply. Only during the COVID pandemic the limit was increased temporarily.

Second, we build on Richert (2021)'s empirical guess-and-verify approach, and solve for the counterfactual bid distributions such that two conditions are satisfied: (i) the distribution of values implied by these bids, and optimal bidding, are indistinguishable from the distribution of the estimated values; and (ii) the counterfactual distribution of maximal demand in an auction is first-order stochastically dominated by the empirical distribution of maximal demand across all auctions. As robustness, we shift the empirical distribution of maximal demand by 5 percentage point to the right.

Our approach might not fully capture the size of demand in counterfactual auctions that are far from any observed auction. We therefore focus our discussion on local changes where the approach provides more reliable predictions. To avoid focusing on any particular draw from the cost or value distribution, we take the ex ante perspective, and compare expected market outcomes. For instance, we analyze the expected price at which an auction clears under current market rules, relative to counterfactual market rules.

Why did customers enter? We first aim at understanding whether customers entered the market because of dealer exit or because of changing market conditions. For this we compare the status quo, in which two dealers left in 2014, with a counterfactual with two additional dealers (fixing dealer participation but allowing customers to select into auctions).

Our findings suggest that customer participation was partially driven by dealer exit and partially due to changes in the market that made it more profitable for customers to buy bonds (as the time-trend in Figure 3B suggests). Adding two dealers reduces a customer's participation probability by roughly 17.5 percentage points, or $44.3 \%$, on average.

Figure 4: Why did customers enter?


Figure 4 shows the probability for a customer to participate in every 3rd auction from 2014 onward in the status quo (on the x-axis) and the counterfactual in which we add back the two dealers who left (on the y-axis). Probability is in percentage points.

Furthermore, Figure 4 highlights that the probability of participating in some auctions drops to almost zero. ${ }^{20}$

Competition-volatility trade-off. Next, we illustrate the competition-volatility trade off that arises when adding market participants who don't participate with regularity.

For this, we first consider a hypothetical auction environment with only dealers. This eliminates effects coming from changes in bidder composition or from dealer bid-updating. In this simplified auction environment, we ask by how much the expected auction price and auction coverage varies in the number of competing dealers.

Consider a typical auction, shown in Figure 5: If 14 dealers compete, the market clears at a competitive price, which is similar to the observed one. If only 13 or 12 dealers compete, the expected price drops by close to $10 \%$, and $20 \%$ respectively. ${ }^{21}$ The expected price and revenue drop for two reasons. First, with fewer bidders, the auction is less competitive, so that bidders increase the extent to which they shade their values. Second, auction failure,

[^13]Figure 5: Typical auction: Varying number of dealers


Figure 5 shows how the range of expected price (in C\$) at which an auction clears varies as the number of dealers increases in an auction which issues the average supply with medium customer participation, i.e., 3 customers. In theory, there is one expected price for each fixed number of dealers. In practice, we determine a range of prices (marked in black) given that our numerical procedure to determine counterfactual bids allows for small differences between the value distribution that is implied by the counterfactual bids and the true value distribution (see Richert (2021)). The blue horizontal line shows the average observed bid, which is close to the observed market clearing price.
because total demand is insufficient, becomes more likely (as shown in Figure 5 (B)). To disentangle these two effects, we can compare these price drops to the price change that would result if bids were unchanged but maintaining the same share of failed auctions. This price change is roughly equal to the share of auctions that fail to clear (since values are close to 100 ). For example, if only 13 dealers compete, the failure probability is $5 \%$, which means that the total price drop of $10 \%$ is evenly split between the competition-effect and the auction-failure-effect.

Next, we analyze the effect of varying the probability with which customers participate in a typical auction, fixing the number of dealers-see Figures 6 and A10. When the expected number of customers falls by one, there is no risk of auction failure, the expected price decreases by $0.7 \%$, expected revenue decreases by $0.08 \%$ (or 2.7 million) and bid-shading increases by $7.9 \%$.

To compare the competition effect to the volatility effect, we compute the expected revenue loss from a reduction in expected customer participation of one competitor on average,
and the expected revenue loss from across auction variation in customer entry rates. ${ }^{22}$ For the median auction, the competition effect $(\mathrm{C} \$ 2.9 \mathrm{M})$ is slightly larger than the loss from irregular participation $(\mathrm{C} \$ 2.85 \mathrm{M})$. However, when the distribution of customer entry probabilities gives more weight to auctions with fewer customers than in our data, the volatility loss can out-weight the competition effect. This is because low auction revenues on bad days, when few customers enter, dominates the smaller increases in auction revenues on good days.

Taken together, these findings highlight (i) a significant impact on market competition when bidders are removed, and (ii) the potential risk of introducing volatility in auction coverage and clearing prices when bidder participation is irregular. Our results also indicate that losing additional dealers could have detrimental effects unless an adequate number of new customers enter the market. This is not a hypothetical concern, as one of the dealers (HSBC) has been acquired by another dealer (RBC) and a second dealer is for sale.

Alternative policies. In the final part of the paper, we aim at determining a simple policy that both reduces volatility and increases competition relative to the status quo.

As a starting point, we consider modifications to commitment requirements. We first eliminate dealer commitment, meaning that we allow dealers to freely decide whether to enter each auction like customers in the status quo in the hope that this increases competition without harming volatility. Then, we require customers to commit to participating in the same way as dealers in the hope that this decreases volatility without harming competition. Due to endogenous bidder participation, in both cases, it is theoretically possible to increase competition and decrease volatility relative to the status quo. Empirically, we find that neither of these two alternative policy regimes achieves that goal (see Appendix C. 2 for details).

Rather than changing bidder commitment, we propose to strategically reshuffle supply across auctions to incentivize and stabilize customer participation in the hope to increase competition and decrease volatility at the same time. The idea is that we can predict (with

[^14]Figure 6: Typical auction: Varying customer participation probability


Figure 6 shows how the range of expected price (in $\mathrm{C} \$$ ) at which an auction clears varies as the participation probability of (all) customers varies between 0 and 1 computed at grid points ( $0, .05, .1, .2, .5,1$ ) in an auction which issues the average supply with medium customer participation, i.e., 3 customers. In theory, there is one expected price for each counterfactual. In practice, we determine a range of prices (marked in black) given that our numerical procedure to determine counterfactual bids allows for small differences between the value distribution that is implied by the counterfactual bids and the true value distribution (see Richert (2021)). The blue horizontal line shows the average observed bid, which is close to the observed market clearing price.
some noise) how many customers each auction would attract under the current supply schedule based on observable market conditions to then shift some of the supply from unattractive auctions to attractive auctions.

Concretely, we rely on the OLS regression from Table 2 to predict how many customers will want to participate under the current supply schedule, given observable market conditions using auctions from 2014 until the end of 2019. We exclude 2020 onward when debt issuance spiked in responds to the COVID pandemic to focus on normal times. We rank the predicted number of participating customers, $\hat{N}_{t}^{c}$, from smallest to highest, and obtain a quantile ranking, ranging from 0 (smallest $\hat{N}_{t}^{c}$ ) until 1 (highest $\hat{N}_{t}^{c}$ ) from the empirical distribution of $\hat{N}_{t}^{c}$. The quantile ranking is a single dimensional index, $s \in[0,1]$, which captures the intensity of predicted customer participation.

Based on this participation index, we increase supply by $10 \%$ in the auction with lowest participation $(s=0)$ and decrease supply by $10 \%$ in the auction with highest participation $(s=1)$. We adjust supply linearly for all auctions in between. Formally, the new supply

Figure 7: Reshuffling supply to incentivize customer participation


Figure 7A shows the distribution of customer participation probabilities across auctions in the status quo and the counterfactual in which we strategically reshuffle supply to incentivize stable customer participation (in pp). Figure 7B displays the corresponding distributions of expected auction prices (in $\mathrm{C} \$$ ).
in an auction with index $s$ is $Q_{t}+0.2(0.5-s)$ given the observed supply is $Q_{t}$. This rule implies that the supply in the auction with median participation $(s=0.5)$ is left unchanged. To avoid issuing too extreme amounts of debt in any counterfactual auction relative to the status quo, we normalize all initial supplies to one, i.e., $Q_{t}=1$ for all $t{ }^{23}$

We find that implementing such supply adjustments successfully increases competition while decreasing volatility (see Figure 7). Both, the median expected price and the median number of participating customers increase by 0.09 pp and 50.71 pp respectively, suggesting enhanced competition, relative to the status quo. In addition, the customer participation probability per auction stabilizes around a median of $36 \%$, and price volatility diminishes. Median revenue per auction increases by about $\mathrm{C} \$ 16 \mathrm{M}$ (or about 48 bps ).

This simple rule does not account for all factors that influence the complicated decision on how to issue government debt. For example, it abstracts from the term-structure of bonds, and therefore ignores complications that arise from rolling over debt in the future.

[^15]However, the rule has at least three attractive features. First, it is easy to implement and politically feasible, given that the central bank already changes the supply issued to bidders by placing non-negligible non-competitive bids. Second, the rule is supply-neutral in relative terms in that we add the same percentages of supply in one auction that we subtract in another. Since the observed supplied quantity (in dollars) is uncorrelated with the quantity changes we make (in dollars) the rule is also supply-neutral in absolute terms, as long as we repeat our exercise with sufficiently many auctions. Third, the rule is essentially revenueneutral. This is because that there is no statistically significant correlation between the average (quantity-weighted) value for the auctioned bond of participating bidders and the predicted number of participating customers. ${ }^{24}$ Therefore, our rule does not systematically shift supply from low-value auctions, that clear at low prices, to high-value auctions, or vice versa.

## 8 Conclusion

We study dealer exit and customer entry in the primary market for Canadian government debt, and analyze consequences for the functioning of the market. We show that hedge fund participation has increased starting around 2014, but remains highly volatile. We introduce and estimate a structural model to trade-off the benefits of higher competition from customer entry with the costs of higher market volatility. Our framework could be used in other settings with regular and irregular market participants.

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## ONLINE APPENDIX

## Entry and Exit in Treasury Auctions

By Jason Allen, Ali Hortaçsu, Eric Richert, and Milena Wittwer

Appendix A presents additional empirical test.
Appendix B explains how we estimate dealer and customer values.
Appendix C. 1 provides details about how to compute counterfactuals.
Appendix D presents mathematical details, including formal proofs.

## A Empirical tests

## A. 1 Testing independent private values

We perform the test introduced in Hortaçsu and Kastl (2012) for the auctions of bonds in our sample and obtain a p-value of 0.384 , which does not provide evidence of common values. The test checks for equality of the estimated marginal values of dealers before and after observing a customer bid. In a common values environment, a customers bid would provide the dealer with information that changes their expected marginal values for acquiring the bond being sold. Under independent private values, this bid reveals information about the expected level of competition, but should not affect their marginal value. Similar to the findings of Hortaçsu and Kastl (2012) for bills, we fail to find evidence that dealer marginal values are shifted by the information learned through the customers bids in the bond market.

## A. 2 Testing bounds

Here we test whether customers take dealer updating into account when placing a bid. Formally, we want to know if $\lambda=0$. To do this, we construct measures $T_{i}=\mid v_{i}\left(q, s_{i} ; \lambda=\right.$ $0)-v_{i}\left(q, s_{i}\right) \mid$ and construct a test statistic analagous to $S S Q_{T}$ from Hortaçsu and Kastl (2012). The test rejects the null hypothesis with p-value of 0.00 , where the p-value is computed via bootstrap.

## A. 3 Testing value differences

Here we test whether customer values are significantly above dealer values. The null hypothesis is that customer values at the lower bound are weakly smaller than dealer values at the upper bound. We consider four different tests. First, we compute three aggregate test statistics following Hortaçsu and Kastl (2012), but because we are interested in a one sided null hypothesis (are customer values larger than dealer values), we drop the absolute value. The first is in the spirit of a Chi-squared test, the second based on the 95 th percentile of across auction differences, and the third based on the max difference in values across auctions. In all cases we omit the subset of auctions where no customers showed up. We compute these test statistics for the differences in average quantity-weighted value, the average max value, and the average min value of dealers/customers. We also provide confidence intervals for a set estimate of the mean difference. Results are reported in Table A2. For all measures the customer value appears to be above the dealer value, however the differences in the average max value are less precise, with some of the test statistics insignificant.

## B Resampling procedure to estimate values

To back out dealer values and bounds on customer values from the equilibrium conditions of Proposition 1, we need to estimate the probabilities that enter these conditions, which are determined by the distribution of the market clearing price, $\boldsymbol{P}_{\boldsymbol{t}}^{*}$.

To build an intuition for how to estimate the price distribution, consider a simpler setting with $N$ ex-ante symmetric bidders. Here, we fix a bidder in an auction, and draw $N-1$ bid functions with replacement from all observed bids in that auction. ${ }^{25}$ This simulates one possible market outcome for our fixed bidder. Repeating this many times, we obtain the distribution of the market clearing price $P_{\boldsymbol{t}}^{*}$ for this bidder.

Our setting is more complicated because there are dealers and customers, and customers must bid via dealers. Hortaçsu and Kastl (2012) introduce a resampling procedure to back out dealer values for this setting. We extend their method in two ways. First, we allow

[^17]signals within a bidder to be correlated over the course of an auction to avoid estimation bias coming from the fact that we observe some bidders updating their bids without observing a bid of a customer. For simplicity, our model does not rationalize such updates, but an extended model based on Hortaçsu and Kastl (2012) could. Second, we obtain all elements in the customer's FOC needed to bound customer values.

To simulate a market outcome in our setting, we first construct the residual supply curve that a bidder $i$ faces in auction $t$. For this, we start by randomly drawing a customer bid from the set of $N^{h}$ potential customer bids. If the customer did not participate in the auction, her bid is 0 ; if she updated her bid, we randomly select one of her bids. Next, we find a dealer that saw a similar bid to the bid of that customer. ${ }^{26}$ If the selected dealer made multiple bids, we select at random one of the bids from the set of bids submitted by that dealer after they observed a bid similar to the current customer's bid. If the dealer did not update her bid after learning the customer's bid, we choose the last bid before learning it. Once a bid is selected, we drop all other bids from that dealer. We repeat this procedure $N^{h}$ times if $i$ is a dealer and $N^{h}-1$ times if $i$ is a customer.

Next, we resample the uninformed dealers. Starting with a list of uninformed dealers, draw one such dealer. ${ }^{27}$ If they submitted more than one bid, randomly select one bid and drop the others. Continue drawing from the set of uninformed dealers so that there are $N^{h}+N^{d}-1$ included bidding curves. This is one realization of the residual supply curve that $i$ expects to face. Repeat the process many times to estimate the distribution of the clearing price, each time starting with the full set of bids made in the auction.

As in Kastl (2011), consistency of the estimator requires that the probability of the market clearing at each step is strictly bounded away from zero. However, in our finite sample this event may occur, leading the estimated marginal value to explode. For steps with win probabilities close to or equal to zero we therefore mix the estimated clearing price distribution with a uniform distribution over the range of the bids made in order to give all the bids a small win probability. In addition to reducing the sensitivity of the analysis

[^18]to these small probabilities, we truncate the estimated marginal values by assuming that marginal values are below the maximum bid ever made in the year of the auction for a bond with the same maturity as the bond being sold plus $\mathrm{C} \$ 1,000$ /months to maturity, which is roughly equivalent to 10 bps .

With the estimated price distributions, we can solve for the value that rationalizes a dealer's bid at each step, using condition (3). To obtain customer value bounds, we implement an estimation procedure that follows our identification argument; for details see Appendix D.2.

## C Details regarding counterfactuals

In Appendix C. 1 we explain how we compute counterfactual equilibria. In Appendix C. 2 we present what happens when we make changes to the rules of bidder commitment.

## C. 1 Computational details

We construct a criterion function which allows us to compare the implied and true value distributions along a number of dimensions. We are interested in the set of bid distributions for which the value of this criterion is close enough to zero such that remaining violations are likely due to errors from the number of simulations used in estimation of the implied value distribution. The criterion involves three main components.

First, we evaluate the marginal distributions of values at quantiles of the quantity-bid distribution, corresponding to orders for $1.1,1.7,2.3,3.4,3.6,4.5,5.6$ and $25 \% \mathrm{~s}$ of the total supply. At each of these quantity levels we construct bounds on the marginal distribution of values in auction $j$ for bidder type $g$ along a grid of evenly spaced points running from the value at the $5^{t h}$ percentile in auction $j$ for a bidder of type $g$ to the value at the $95^{t h}$ percentile. At each point we compare the bounds on the implied values from the guess of the bid distribution to the true values and add to the criterion function $\max \left(F_{L}^{I M}-F_{U}, 0\right)^{2}$ and $\min \left(F_{U}^{I M}-F_{L}, 0\right)^{2}$, where $F_{L}^{I M}$ denotes the implied marginal distribution of marginal values at each of the quantity levels evaluated on the grid points and $F_{U}$ denotes the corresponding upper bound on the marginal distribution known from the data.

The second component in the criterion measures the distribution of slopes across the
same fixed grid of quantity levels. Given the bounds on bidders' values across these points we calculate the largest and smallest slope that fit within these bounds (or minimize the violations of the bounds). We then compare the marginal distribution of the slopes, along an evenly spaced grid of slopes, again adding to the criterion function $\max \left(F_{L}^{I M}-F_{U}, 0\right)^{2}$ and $\min \left(F_{U}^{I M}-F_{L}, 0\right)^{2}$, where $F$ now represents the marginal distribution of the slopes (and the notation for bounds, and implied values is as above).

Third, we want the distribution across bidders of type $g$ of the largest quantity bid on in auction $j$ to be smaller than the distribution of the largest quantities (within a bidder) ever purchased. The restriction is designed to capture the fact that not all bidders have the capacity to purchase up to the $25 \%$ limit and even in a counterfactual with lower prices may not be interested in purchasing up to this limit. Therefore, we require the distribution of largest quantity bid by each bidder of type $g$ in auction $j$ to be first-order stochastically dominated by the distribution of max (across auctions) quantities ever bid. Evaluating this along a set of grid points (evenly spaced in quantity-space from 0 to $25 \%$ ), this results in an additional contribution to the criterion function of 0 if the counterfactual distribution of max quantities at a given grid point is above the max-quantity distribution in the data and the squared difference of the probabilities of quantities less than that grid point when the counterfactual distribution is below the distribution from the data.

In an alternative specification we increase the maximum quantities by 5pp (of supply). Our findings are similar with slightly smaller price effects and lower likelihood of auction failure (see Appendix Figures A11, and A12). If we allow each bidder to demand up to the bidding limit ( $25 \%$ ) , no auction would fail. However, assuming that all bidders have the capacity to buy Treasuries worth more than $\mathrm{C} \$ 81$ million in each auction is unrealistic. This would be possible only if a bidder receives an extraordinary amount of many client orders, or has sufficient balance sheet space despite stringent regulatory constraints.

Finally, recall that we specified the customers' optimization problem using a two-step optimization problem with the exterior optimization over moments represented in the bidders problem via a constraint with multiplier $\lambda$. Since these $\lambda$ will change in the counterfactual world, we don't have a primitive distribution to match for this parameter and it appears as though this introduces additional degree of freedom which might make informative counterfactuals impossible. However, fixing any proposal distribution the $\lambda$ are not free parameters,
but are pinned down by the FOC (in particular, by the condition for optimality of the moment chosen). Therefore, given a bid distribution, the $\lambda$ cannot be changed to map into different value distributions. In practice, both the $\lambda$ and the marginal values of a bidder are set-identified, which will lead to some information loss. This loss will result in wider counterfactual bounds in the empirical analysis.

## C. 2 Evaluating the importance of commitment

In light of the competition-volatility trade-off presented in the main text, we evaluate two alternative policy regimes regarding bidder commitment. First, we make changes to assess the extent to which primary auctions run smoothly without forcing regular participation by dealers. Then, we attempt to minimize volatility by requiring customers to commit to participating in the same way as dealers. Due to endogenous bidder participation, in both cases, it is theoretically ambiguous whether competition and volatility increase or decrease relative to the status quo.

To analyze the importance of dealer commitment, we compare two settings-the first has dealers commit as in the status quo but allows customers to place bids directly with the auctioneer; the second has both types bid directly with the auctioneer but doesn't impose obligatory participation on dealers. To compute the counterfactual without dealer commitment, we assume that a dealer's cost to enter one auction equals her estimated annual cost divided by the average number of auctions in a year.

We find that most auctions attract sufficiently many bidders to guarantee full auction coverage, even without obligating dealers to regularly participate (see Appendix Figure A13). However, both dealer and customer participation is highly volatile. Moreover, three out of one-hundred and one auctions risk failure without dealer commitment (where we define an auction to be at risk if it's chance of not clearing is above $5 \%$ ). The expected price of these at-risk auctions drops by more than $5 \%$. Further, even in fully covered auctions, expected revenue decreases by $0.04 \%$ in the median without dealer commitment.

To assess the impact of customer commitment, we force customers to decide at the beginning of each year whether or not to commit to participating in all auctions of the upcoming year. They enter the market if their annual participation cost (approximated by
the estimated auction-specific cost scaled by the average number of auctions in a year) is larger than the total surplus they expect from participating in all auctions of that year. Dealer participation is fixed.

Appendix Table A4 presents the results. We find that between one and two fewer customers would participate if they had to commit. Nevertheless, auctions would remain relatively competitive since sufficiently many bidders would remain in the market. Expected revenue drops by $\mathrm{C} \$ 3.6 \mathrm{M}$, or about $0.11 \%$ on average.

## D Mathematical appendix

## D. 1 Auxiliary statements and proofs

Lemma 1. The moment function, which computes the quantity-weighted bid,

$$
\begin{equation*}
\mu^{l}\left(b_{i \tau}^{h}\left(q, \theta_{i \tau}^{h}\right)\right)=\frac{\sum_{k}^{K} b_{k} q_{k}}{\sum_{k} q_{k}} \text { with }\left\{b_{k}, q_{k}\right\}_{k=1}^{K} \text { constituting bidding function } b_{i \tau}^{h}\left(\cdot, \theta_{i \tau}^{h}\right) \tag{1}
\end{equation*}
$$

is differentiable w.r.t. $q$.

Proof of Lemma 1. We want to show that $\mu^{l}\left(b_{i \tau}^{h}\left(q, \theta_{i \tau}^{h}\right)\right)$ is differentiable at any $q$, meaning that $\lim _{h \rightarrow 0} \frac{\mu^{l}\left(b_{\tau}^{h}\left(q+h, \theta_{i \tau}^{h}\right)\right)-\mu^{l}\left(b_{i \tau}^{h}\left(q, \theta_{i \tau}^{h}\right)\right)}{h}$ exists. Given (1) there are $K$ different cases, depending on which $q$ we consider. The first case is:

$$
\lim _{h \rightarrow 0} \frac{\frac{b_{1}\left(q_{1}+h\right)+\sum_{k=2}^{K} b_{k} q_{k}}{q_{1}+h+\sum_{k=2}^{K} q_{k}}-\frac{\sum_{k}^{K} b_{k} q_{k}}{\sum_{k} q_{k}}}{h}=\frac{b_{1} \sum_{k=2}^{K} q_{k}-\sum_{k=2}^{K} b_{k} q_{k}}{\left(\sum_{k=1}^{K} q_{k}\right)^{2}} \geq 0 \text { since } b_{1} \geq b_{2} \geq b_{3} .
$$

The limit exists. An analogous argument applies to all other cases, only that the limit becomes negative for steps towards the last step. At the last step, the limit is non-positive. Therefore, the moment function is differentiable w.r.t. $q$.

Proof of Proposition 1. The proof of statement (i) is analogous to the proof of Kastl (2011)'s Proposition 1. To show statement (ii), take the perspective of customer $i$ and assume all other bidders play an equilibrium. Fix $L$ moment functions $\mu^{l}$. Given that the customer thinks that the dealer only updates its own function when the customer submits
a function with at least one different moment $\left\{m^{l}\right\}_{l=1}^{L}$, we can solve a two-step problem to characterize the necessary condition for the customer's best reply - which, by assumption, is an equilibrium bid function given that all other bidders play an equilibrium strategy.

First, we fix some set of moments $\left\{m^{l}\right\}_{l=1}^{L}$, and find $b_{i \tau}^{c}\left(\cdot, \theta_{i \tau}^{h}\right)$ that maximizes her expected total surplus $T S^{h}\left(b_{i \tau}^{c}\left(\cdot, \theta_{i \tau}^{h}\right),\left\{m^{l}\right\}_{l=1}^{L}\right)$ such that $\mu^{l}\left(b_{i \tau}^{c}\left(\cdot, \theta_{i \tau}^{h}\right)\right)=m^{l}$ for all $l$. The objective function is:

$$
\begin{equation*}
T S^{h}\left(b_{i \tau}^{h}\left(\cdot, \theta_{i \tau}^{h}\right),\left\{m^{l}\right\}_{l=1}^{L}\right)-\sum_{l=1}^{L} \lambda_{l}\left(m^{l}-\mu^{l}\left(b_{i \tau}^{h}\left(\cdot, \theta_{i \tau}^{h}\right)\right),\right. \tag{9}
\end{equation*}
$$

with $\left\{b_{k}, q_{k}\right\}_{k=1}^{K}$ making up bidding function $b_{i \tau}^{h}\left(\cdot, \theta_{i \tau}^{h}\right)$. Given this objective function, we can follow Kastl (2011)'s perturbation argument in the original proof step by step (shown in his Appendix A.2.). There are two differences. First, there is an additional term that comes from the constraints that all moments $l$ of chosen function, $\mu^{l}\left(b_{i \tau}^{h}\left(\cdot, \theta_{i \tau}^{h}\right)\right.$ must equal the fixed moments $m^{l}$. However, since by assumption all moment functions are differentiable w.r.t. $q$, no complication arises from this extra term. Second, with dealer updating it can be more difficult to predict where the market will clear when the customer demands a different amount than $q$. However, with the assumption that the customer thinks that dealers only update their own bids when a moment of the customer's bidding function changes, Kastl (2011)'s original arguments go through and we obtain equation A2, A3 and the additional term, $\sum-\lambda \frac{\partial m u^{l}\left(b_{i \tau}^{h}\left(\cdot \theta_{i \tau}^{h}\right)\right.}{\partial q_{k}}$. Note that unlike in Kastl (2011), these expressions do not simplify because it may be optimal for a bidder to tie. A tie allows the bidder to request additional quantity in order to move their moments, without necessarily winning all the quantity bid. A necessary condition for optimality is that the sum of these three components is zero.

The above characterizes an equilibrium bidding function for a fixed set of moments $\left\{m^{l}\right\}_{l=1}^{L}$. For our estimation these conditions are sufficient. The reason is that we assume that the observed bids are equilibrium bids and that we observe not only the bids but also the moments that these bid functions generate (for fixed moment functions).

However, for completeness of the argument, in equilibrium, the customer chooses the
moments $\left\{m^{l}\right\}_{l=1}^{L}$ so that $b_{i \tau}^{c}\left(\cdot, \theta_{i \tau}^{h}\right)$ actually maximizes the total surplus:

$$
\begin{equation*}
\max _{\left\{m^{l}\right\}_{l=1}^{L}} T S^{h}\left(b_{i \tau}^{h}\left(\cdot, \theta_{i \tau}^{h}\right),\left\{m^{l}\right\}_{l=1}^{L}\right) \text { subject to: } \mu^{l}\left(b_{i \tau}^{h}\left(\cdot, \theta_{i \tau}^{h}\right)\right)=m^{l} \text { for all } l . \tag{10}
\end{equation*}
$$

Solving this maximization is challenging (even when we restrict attention to moments that are real numbers) because the objective function is not differentiable. To see this consider a change in moment $m^{l}$. The dealer who observes the corresponding bid function updates her own bid because a change in $m^{l}$ (weakly) changes the dealer's information set and with that its type, $\theta_{i \tau}^{d}$. Therefore, the dealer submits a different bid function, i.e., a step function with steps at different points. This changes the customer's beliefs about the price at which the auction will clear. Formally, the distribution of the clearing price, when fixing the customer's own bid function, changes, and since biding functions are step functions, a change in such a function easily leads to non-continuous jumps that render the objective function $T S^{h}$ non-differentiable.

Lemma 2. (i) For a customer $s_{i}^{h}$ : Ties occur with zero probability for a.e. $s_{i}^{h}$ in any equilibrium, for either all steps except the last step, or at the last step. (ii) For a dealer $s_{i}^{d}$, Kastl (2011) Lemma 1 applies.

Proof of Lemma 2. Case 1: $\lambda>0$. Consider a step $k=K_{i}$. Suppose bidder $i$ ties on a step $k=K_{i}$. Take some $q_{m}=\sup \left\{q \mid v\left(q, s_{i}\right) \leq b_{k}\right\}$ and let $\bar{q}=\max \left\{q_{k}-\delta, q_{m}\right\}$ with $\delta$ some strictly positive step size bounded above by $q_{k}-q_{k-1}$, i.e., you step either to where you get positive surplus from the amount you purchase if possible or if not at least you buy less negative. Take the deviation to bid $b_{i k}^{\prime}=b_{i k}+\epsilon$, where $\epsilon$ is sufficiently small, at $\bar{q}$ (and the associated step). Bidder $i$ gets less units than they would in a tie and each one of these they no longer have to pay for. There is no loss to them from not winning these units.

Then the constraint part gives $-\lambda\left(m^{l}-\mu(\bar{q})\right)$. Because $\mu\left(\bar{q}, b^{\prime}\right)>\mu\left(q_{k}, b\right)=m^{l}$, the bracketed term is negative, $\lambda$ is positive (by assumption) and there is one more negative sign so this term is positive overall. This represents an additional strictly positive profit from the deviation.

Case 2: $\lambda<0$. Suppose bidder $i$ ties on a step $k<K_{i}$. Take some $q_{m}=\sup \left\{q \mid v\left(q, s_{i}\right) \leq\right.$ $\left.b_{k}\right\}$ and let $\bar{q}=\max \left\{q_{k}-\delta, q_{m}\right\}$ with $\delta$ some strictly positive step size bounded above by
$q_{k}-q_{k-1}$, i.e., you step either to where you get positive surplus from the amount you purchase if possible or if not at least you buy less negative. Take the deviation to bid $b_{i k}^{\prime}=b_{i k}+\epsilon$, where $\epsilon$ is sufficiently small, at $\bar{q}$ (and the associated step). Bidder $i$ gets less units than they would in a tie and each one of these they longer have to pay for. There is no loss to them from not winning these units.

Then the constraint part gives $-\lambda\left(m^{l}-\mu(\bar{q})\right)$. Because $\mu\left(\bar{q}, b^{\prime}\right)<\mu\left(q_{k}, b\right)=m^{l}$, the bracketed term is positive, $\lambda$ is negative (by assumption) and there is one more negative sign so this term is positive overall. This represents an additional strictly positive profit from the deviation.

Case 3: $\lambda=0$. This is the world of Kastl (2011). Follows from that proof.

## D. 2 Mathematical details on bounding customer values

In this section, we provide formal details on how to identify bounds on the customer's value distribution from equilibrium condition (4) of Proposition 1. For this, we fix an auction $t$ and a customer $i$, and drop the auction $t$, time $\tau$, customer $h$, bidder $i$ subscripts for simplicity. Further, we assume that only one moment, $m^{l}$, matters, for instance, the quantity-weighted bid, consistent with our estimation, and drop the $l$-subscript.

The identification argument is complicated by the fact that condition (4)—here slightly rearranged-not only contains marginal values at submitted steps, $v\left(q_{k}, s\right)$, but also values
at some intermediate quantities between submitted steps, $v\left(\boldsymbol{q}^{*}, s\right)$ :

$$
\begin{align*}
0= & \operatorname{Pr}\left(b_{k}>\boldsymbol{P}^{*}>b_{k+1} \mid \theta, m\right) v(q, s)+ \\
& \operatorname{Pr}\left(b_{k}=\boldsymbol{P}^{*} \mid \theta, m\right) \mathbb{E}\left[\left.v\left(\boldsymbol{q}^{*}, s\right) \frac{\left.\partial \boldsymbol{q}^{*}\right)}{\partial q_{k}} \right\rvert\, b_{k}=\boldsymbol{P}^{*}, \theta, m\right]+ \\
& \operatorname{Pr}\left(b_{k+1} \geq \boldsymbol{P}^{*} \mid \theta, m\right) \mathbb{E}\left[\left.v\left(\boldsymbol{q}^{*}, s\right) \frac{\partial \boldsymbol{q}^{*}}{\partial q_{k}} \right\rvert\, b_{k+1} \geq \boldsymbol{P}^{*}, \theta, m\right]- \\
& \operatorname{Pr}\left(b_{k}>\boldsymbol{P}^{*}>b_{k+1} \mid \theta, m\right)\left(b_{k}\right)- \\
& \operatorname{Pr}\left(b_{k+1} \geq \boldsymbol{P}^{*} \mid \theta, m\right)\left(b_{k}-b_{k+1}\right)- \\
& \operatorname{Pr}\left(b_{k}=\boldsymbol{P}^{*} \mid \theta, m\right) \mathbb{E}\left[\left.\frac{\partial \boldsymbol{q}^{*}}{\partial q_{k}} \right\rvert\, b_{k}=\boldsymbol{P}^{*}, \theta, m\right]- \\
& \left.\operatorname{Pr}\left(b_{k+1}=\boldsymbol{P}^{*} \mid \theta, m\right) \mathbb{E}\left[\left.\frac{\partial \boldsymbol{q}^{*}}{\partial q_{k}} \right\rvert\, b_{k+1}=\boldsymbol{P}^{*}, \theta, m\right]\right]- \\
& \operatorname{Pr}\left(b_{k+1}<\boldsymbol{P}^{\boldsymbol{*}} \mid \theta, m\right) \mathbb{E}\left[\left.\frac{\partial \boldsymbol{q}^{*}}{\partial q_{k}} \right\rvert\, b_{k+1}<\boldsymbol{P}^{*}, \theta, m\right]+\lambda \frac{\partial \mu(b(\cdot, \theta))}{\partial q_{k}} . \tag{11}
\end{align*}
$$

However, we can still obtain set-identification of these marginal values, by using the monotonicity and boundedness of the step function. That is, for all $q_{k-1} \leq q \leq q_{k}$ we know that $v\left(q_{k-1}, s\right) \geq v(q, s) \geq v\left(q_{k}, s\right)$. We also are able to sign the derivatives of the rationed quantity: increasing the bid at step $k$ increases the quantity rationed in the event of a tie at step $k$ and decreases the quantity rationed in the event of a tie at step $k+1$.

We obtain upper and lower bounds on the marginal values at each $q_{k}$ that are satisfied with non-zero probability. Bids of $q_{k}$ with zero win probability, are bounded in the same way as the intermediate $q$. We obtain an upper bound on the marginal value at $q_{k}$ by making the terms involving the marginal value and the derivative of the rationed quantity in the event of a tie as small as possible (and making them as big as possible for the lower bound). To do this, we plug in $\bar{v}\left(q_{k}, s\right)$, the max possible value at intermediate quantities along the next step for terms involving rationed quantities at the next step (e.g. line 3 of equation (11)) and $\underline{v}\left(q_{k}, s\right)$ the smallest possible value at intermediate quantities along the current step for terms involving ties at the current step (e.g. line 2. of equation (11)). To obtain a lower bound, we instead substitute the maximum value at the current step, $\bar{v}\left(q_{k-1}, s\right)$ (in line 2 . of equation (11)) and the minimum value at the next step, $\underline{v}\left(q_{k+1}, s\right)$ (in line 3 of equation (11)). By substituting these values into the first order conditions from each step, we are left
with a system of $2 K$ linear equations with $2 K$ unknowns to solve.
We can make use of some additional information to simplify the resulting system, by adding zeros into the matrix of coefficients for the linear system. In particular, at the last step, all terms involving $b_{K}$ drop out. When the current $\lambda$ is negative, at steps before the last step ties cannot be optimal, and the terms involving the derivative of the rationed quantity drop out except for at the second-to-last step. When the current $\lambda$ is positive, the only simplification occurs at the last step, where the rationing terms all drop out. To see why, suppose that a bidder wanted to tie with positive probability. If the step is above the last, raising the quantity increases the quantity-weighted average bid while at the last step, it decreases it. Lemma 2 formalizes this idea and summarizes all possible simplifications.

With these simplifications, we obtain the following system of equations for two cases $\lambda>0$, and $\lambda<0$. Below, we express each equation to guarantee readability. It can help to transform the system of equation into a single matrix, one for each case, to get a better sense for whether the system is identified.
(i) When $\lambda<0$ :

$$
\begin{aligned}
& A_{1}=\operatorname{Pr}\left(b_{1}>P^{*}>b_{2} \mid \theta, m\right) \bar{v}\left(q_{1}, s\right) \\
& A_{1}=\operatorname{Pr}\left(b_{1}>P^{*}>b_{2} \mid \theta, m\right) \underline{v}\left(q_{1}, s\right)
\end{aligned}
$$

$$
A_{K-1}=\operatorname{Pr}\left(b_{K-1}>\boldsymbol{P}^{*}>b_{K} \mid \theta, m\right) \bar{v}\left(q_{K-1}, s\right)+\operatorname{Pr}\left(b_{K} \geq \boldsymbol{P}^{*} \mid \theta, m\right) \mathbb{E}\left[\left.\frac{\partial \boldsymbol{q}^{*}}{\partial q_{K-1}} \right\rvert\, b_{K} \geq \boldsymbol{P}^{*}, \theta, m\right] \bar{v}\left(q_{K-1}, s\right)
$$

$$
A_{K-1}=\operatorname{Pr}\left(b_{K-1}>\boldsymbol{P}^{*}>b_{K} \mid \theta, m\right) \underline{v}\left(q_{K-1}, s\right)+\operatorname{Pr}\left(b_{K} \geq \boldsymbol{P}^{*} \mid \theta, m\right) \mathbb{E}\left[\left.\frac{\partial \boldsymbol{q}^{*}}{\partial q_{K-1}} \right\rvert\, b_{K} \geq \boldsymbol{P}^{*}, \theta, m\right] \underline{v}\left(q_{K}, s\right)
$$

$$
A_{K}=\operatorname{Pr}\left(b_{K}>\boldsymbol{P}^{*}>0 \mid \theta, m\right) \bar{v}\left(q_{K}, s\right)+\operatorname{Pr}\left(b_{K}=P^{*} \mid \theta, m\right) \mathbb{E}\left[\left.\frac{\partial \boldsymbol{q}^{*}}{\partial q_{K}} \right\rvert\, b_{K}=P^{*}, \theta, m\right] \underline{v}\left(q_{K}, s\right)
$$

$$
A_{K}=\operatorname{Pr}\left(b_{K}=\boldsymbol{P}^{*} \mid \theta, m\right) \mathbb{E}\left[\left.\frac{\partial \boldsymbol{q}^{*}}{\partial q_{K}} \right\rvert\, b_{K}=P^{*}, \theta, m\right] \bar{v}\left(q_{K-1}, s\right)+\operatorname{Pr}\left(b_{K}>\boldsymbol{P}^{*}>0 \mid \theta, m\right) \underline{v}\left(q_{K}, s\right)
$$

where

$$
\begin{aligned}
A_{k} & =\operatorname{Pr}\left(b_{k}>\boldsymbol{P}^{*}>b_{k+1} \mid \theta, m\right)\left(b_{k}\right)+\operatorname{Pr}\left(b_{k+1} \geq \boldsymbol{P}^{*} \mid \theta, m\right)\left(b_{k}-b_{k+1}\right) \\
& \left.+\operatorname{Pr}\left(b_{k}=\boldsymbol{P}^{*} \mid \theta, m\right) \mathbb{E}\left[\left.\frac{\partial \boldsymbol{q}^{*}}{\partial q_{k}} \right\rvert\, b_{k}=\boldsymbol{P}^{*}, \theta, m\right]+\operatorname{Pr}\left(b_{k+1}=\boldsymbol{P}^{*} \mid \theta, m\right) \mathbb{E}\left[\left.\frac{\partial \boldsymbol{q}^{*}}{\partial q_{k}} \right\rvert\, b_{k+1}=\boldsymbol{P}^{*}, \theta, m\right]\right] \\
& +\operatorname{Pr}\left(b_{k+1}<\boldsymbol{P}^{*} \mid \theta, m\right) \mathbb{E}\left[\left.\frac{\partial \boldsymbol{q}^{*}}{\partial q_{k}} \right\rvert\, b_{k+1}<\boldsymbol{P}^{*}, \theta, m\right]-\lambda \frac{\partial \mu(b(\cdot, \theta))}{\partial q_{k}}
\end{aligned}
$$

for all $k=1, \ldots, K_{1}$, and

$$
A_{k}=\operatorname{Pr}\left(\boldsymbol{P}^{*}>b_{K} \mid \theta, m\right)\left(b_{K}\right)+\operatorname{Pr}\left(b_{K}=\boldsymbol{P}^{*} \mid \theta, m\right) \mathbb{E}\left[\left.\frac{\partial \boldsymbol{q}^{*}}{\partial q_{K}} \right\rvert\, b_{K}=\boldsymbol{P}^{*}, \theta, m\right]-\lambda \frac{\partial \mu(b(\cdot, \theta))}{\partial q_{K}}
$$

at the last step K.
(ii) When $\lambda>0$ :

$$
\begin{aligned}
B_{1} & =\operatorname{Pr}\left(b_{1}>\boldsymbol{P}^{*}>b_{2} \mid \theta, m\right) \bar{v}\left(q_{1}, s\right)+\operatorname{Pr}\left(b_{2} \geq \boldsymbol{P}^{*} \mid \theta, m\right) \mathbb{E}\left[\left.\frac{\partial \boldsymbol{q}^{*}}{\partial q_{1}} \right\rvert\, b_{1}=\boldsymbol{P}^{*}, \theta, m\right] \underline{v}\left(q_{1}, s\right) \\
B_{1} & =\operatorname{Pr}\left(b_{1}>\boldsymbol{P}^{*}>b_{2} \mid \theta, m\right) \underline{v}\left(q_{1}, s\right)+\operatorname{Pr}\left(b_{2} \geq \boldsymbol{P} \mid \theta, m\right) \mathbb{E}\left[\left.\frac{\partial \boldsymbol{q}^{*}}{\partial q_{1}} \right\rvert\, b_{2} \geq \boldsymbol{P}^{*}, \theta, m\right] \underline{v}\left(q_{2}, s\right) \\
B_{2} & =\operatorname{Pr}\left(b_{2}>\boldsymbol{P}^{*}>b_{3} \mid \theta, m\right) \bar{v}\left(q_{2}, s\right)+\operatorname{Pr}\left(b_{3} \geq \boldsymbol{P}^{*} \mid \theta, m\right) \mathbb{E}\left[\left.\frac{\partial \boldsymbol{q}^{*}}{\partial q_{2}} \right\rvert\, b_{3} \geq \boldsymbol{P}^{*}, \theta, m\right] \bar{v}\left(q_{2}, s\right) \\
B_{2} & =\operatorname{Pr}\left(b_{3}=\boldsymbol{P}^{*} \mid \theta, m\right) \mathbb{E}\left[\left.\frac{\partial \boldsymbol{q}^{*}}{\partial q_{2}} \right\rvert\, b_{2}=\boldsymbol{P}^{*}, \theta, m\right] \bar{v}\left(q_{1}, s\right)+\operatorname{Pr}\left(b_{2}>\boldsymbol{P}^{*}>b_{3} \mid \theta, m\right) \underline{v}\left(q_{2}, s\right) \\
& +\operatorname{Pr}\left(b_{3} \geq \boldsymbol{P} \mid \theta, m\right) \mathbb{E}\left[\left.\frac{\partial \boldsymbol{q}^{*}}{\partial q_{2}} \right\rvert\, b_{3} \geq \boldsymbol{P}^{*}, \theta, m\right] \bar{v}\left(q_{4}, s\right)
\end{aligned}
$$

$$
\begin{aligned}
B_{K-1} & =\operatorname{Pr}\left(b_{K-1}>\boldsymbol{P}^{*}>b_{K}\right) \bar{v}\left(q_{K-1}, s\right)+\operatorname{Pr}\left(b_{K-1}=\boldsymbol{P}^{*}\right) \underline{v}\left(q_{K-1}, s\right) \mathbb{E}\left[\left.\frac{\partial \boldsymbol{q}^{*}}{\partial q_{K-1}} \right\rvert\, b_{K-1}=\boldsymbol{P}^{*}, \theta, m\right] \\
B_{K-1} & =\operatorname{Pr}\left(b_{K-1}=\boldsymbol{P}^{*}\right) \bar{v}\left(q_{K-1}, s\right) \mathbb{E}\left[\left.\frac{\partial \boldsymbol{q}^{*}}{\partial q_{K-1}} \right\rvert\, b_{K-1}=\boldsymbol{P}^{*}, \theta, m\right]+\operatorname{Pr}\left(b_{K-1}>\boldsymbol{P}^{*}>b_{K}\right) \underline{v}\left(q_{K-1}, s\right) \\
B_{K} & =\operatorname{Pr}\left(b_{K}>\boldsymbol{P}^{*}>0\right) \bar{v}\left(q_{K}, s\right) \\
B_{K} & =\operatorname{Pr}\left(b_{K}>\boldsymbol{P}^{*}>0\right) \underline{v}\left(q_{K}, s\right)
\end{aligned}
$$

were

$$
\begin{aligned}
& \left.B_{1}=A_{1}-\bar{v}\left(q_{1}, s\right) \operatorname{Pr}\left(b_{1}=P^{*} \mid \theta, m\right)\right) \mathbb{E}\left[\left.\frac{\partial q^{*}}{\partial q_{1}} \right\rvert\, b_{1}=P^{*}, \theta, m\right] \\
& B_{k}=A_{k} \text { for } k=2, \ldots K
\end{aligned}
$$

In either case the system of equations is still underidentified as $\lambda$, which appears in the terms $A_{k}$ and $B_{k}$ and determines which set of equations apply, is unknown. However, in addition to this system of equations we know that any perturbation in $m$ must not be optimal: we consider two such possible perturbations to $m+\epsilon$ and $m-\epsilon$ respectively and argue this can
be used to construct bounds on $\lambda$.

$$
\begin{align*}
& T S(b(\cdot, \theta), m)-T S(b(\cdot, \theta), m+\epsilon) \geq \lambda \epsilon  \tag{12}\\
& T S(b(\cdot, \theta), m-\epsilon)-T S(b(\cdot, \theta), m) \leq \lambda \epsilon \tag{13}
\end{align*}
$$

where

$$
\begin{aligned}
T S(b(\cdot, \theta), m) & =\sum_{k=1}^{K}\left[\operatorname{Pr}\left(b_{k}>\boldsymbol{P}^{*}>b_{k+1} \mid \theta, m\right) V\left(q_{k}, s\right)-\operatorname{Pr}\left(b_{k}>\boldsymbol{P}^{*} \mid \theta, m\right) b_{k}\left(q_{k}-q_{k-1}\right)\right] \\
& +\sum_{k=1}^{K} \operatorname{Pr}\left(b_{k}=\boldsymbol{P}^{*} \mid \theta, m\right) \mathbb{E}\left[V\left(\boldsymbol{q}^{*}, s\right)-b_{k}\left(\boldsymbol{q}^{*}-q_{k-1}\right) \mid b_{k}=\boldsymbol{P}^{*}, \theta, m\right]
\end{aligned}
$$

with $q_{0}=b_{K+1}=0 . T S(b(\cdot, \theta), m+\epsilon)$ and $T S(b(\cdot, \theta), m-\epsilon)$ are defined analogously.
We can use (15) to find bounds for (12) and (13). Consider (12) first, and omit conditioning on $\theta$ for simplicity. We obtain:

$$
\begin{aligned}
& \sum_{k=1}^{K}\left[\operatorname { m a x } \left(0, \Delta \operatorname{Pr}\left(b_{k} \geq \boldsymbol{P}^{*} \geq b_{k+1}\right)\left(\sum_{j=1}^{k} \bar{v}\left(q_{j-1}, s\right)\left(q_{j}-q_{j-1}\right)\right)+\right.\right. \\
& \min \left(0, \Delta \operatorname{Pr}\left(b_{k} \geq \boldsymbol{P}^{*} \geq b_{k+1}\right)\left(\sum_{j=1}^{k} \underline{v}\left(q_{j}, s\right)\left(q_{j}-q_{j-1}\right)\right)+\Delta \operatorname{Pr}\left(b_{k}>\boldsymbol{P}^{*}\right) b_{k}\left(q_{k}-q_{k-1}\right)\right]+ \\
& \sum_{k=1}^{K}\left[\max \left(0, \Delta \operatorname{Pr}\left(b_{k}=\boldsymbol{P}^{*}\right)\right)\left(\sum_{j=1}^{k-1}\left(\bar{v}\left(q_{j-1}, s\right)\left(q_{j}-q_{j-1}\right)\right)\right)+\min \left(0, \Delta \operatorname{Pr}\left(b_{k}=\boldsymbol{P}^{*}\right)\right)\left(\sum_{j=1}^{k-1}\left(\underline{v}\left(q_{j}, s\right)\left(q_{j}-q_{j-1}\right)\right)\right)\right. \\
& +\left[\operatorname{Pr}\left(b_{k}=\boldsymbol{P}^{*} \mid m\right) \bar{v}\left(q_{j-1}, s\right)\left(\mathbb{E}\left[\boldsymbol{q}^{*} \mid b_{k}=\boldsymbol{P}^{*}, m\right]-q_{k-1}\right)-\right. \\
& \left.\operatorname{Pr}\left(b_{k}=\boldsymbol{P}^{*} \mid m+\epsilon\right) \bar{v}\left(q_{j-1}, s\right)\left(\mathbb{E}\left[\boldsymbol{q}^{*} \mid b_{k}=\boldsymbol{P}^{*}, m+\epsilon\right]-q_{k-1}\right)\right] \\
& \left.1\left(\operatorname{Pr}\left(b_{k}=\boldsymbol{P}^{*} \mid m\right)\left(\mathbb{E}\left[\boldsymbol{q}^{*} \mid b_{k}=\boldsymbol{P}^{*}, m\right]-q_{k-1}\right)-\operatorname{Pr}\left(b_{k}=\boldsymbol{P}^{*} \mid m+\epsilon\right) \mathbb{E}\left[\boldsymbol{q}^{*} \mid b_{k}=\boldsymbol{P}^{*}, m+\epsilon\right]-q_{k-1}\right)>0\right) \\
& +\left[\operatorname{Pr}\left(b_{k}=\boldsymbol{P}^{*} \mid m\right) \underline{v}\left(q_{j}, s\right)\left(\mathbb{E}\left[\boldsymbol{q}^{*} \mid b_{k}=\boldsymbol{P}^{*}, m\right]-q_{k-1}\right)-\right. \\
& \left.\operatorname{Pr}\left(b_{k}=\boldsymbol{P}^{*} \mid m+\epsilon\right) \underline{v}\left(q_{j}, s\right)\left(\mathbb{E}\left[\boldsymbol{q}^{*} \mid b_{k}=\boldsymbol{P}^{*}, m+\epsilon\right]-q_{k-1}\right)\right] \\
& \left.1\left(\operatorname{Pr}\left(b_{k}=\boldsymbol{P}^{*} \mid m\right)\left(\mathbb{E}\left[\boldsymbol{q}^{*} \mid b_{k}=\boldsymbol{P}^{*}, m\right]-q_{k-1}\right)-\operatorname{Pr}\left(b_{k}=\boldsymbol{P}^{*} \mid m+\epsilon\right) \mathbb{E}\left[\boldsymbol{q}^{*} \mid b_{k}=\boldsymbol{P}^{*}, m+\epsilon\right]-q_{k-1}\right)<0\right) \\
& \left.-b_{k}\left(\mathbb{E}\left[\boldsymbol{q}^{*}-q_{k} \mid b_{k}=\boldsymbol{P}^{*}, m\right]-\mathbb{E}\left[\boldsymbol{q}^{*}-q_{k} \mid b_{k}=\boldsymbol{P}^{*}, m+\epsilon\right]\right)\right] \geq \lambda \epsilon,
\end{aligned}
$$

where $\Delta \operatorname{Pr}(\cdot)$ indicates taking a difference between $\operatorname{Pr}(\cdot \mid . ., m)$ and $\operatorname{Pr}(\cdot \mid . ., m+\epsilon)$ and $v\left(q_{k-1}, s\right)=$ $\bar{v}\left(q_{1}, s\right)$ for $k=1$.

Similarly for (15):

$$
\begin{aligned}
& \sum_{k=1}^{K}\left[\operatorname { m a x } \left(0, \Delta \operatorname{Pr}\left(b_{k} \geq \boldsymbol{P}^{*} \geq b_{k+1}\right)\left(\sum_{j=1}^{k} \underline{v}\left(q_{j}, s\right)\left(q_{j}-q_{j-1}\right)\right)+\right.\right. \\
& \min \left(0, \Delta \operatorname{Pr}\left(b_{k} \geq \boldsymbol{P}^{*} \geq b_{k+1}\right)\left(\sum_{j=1}^{k} \bar{v}\left(q_{j-1}, s\right)\left(q_{j}-q_{j-1}\right)\right)+\Delta \operatorname{Pr}\left(b_{k}>\boldsymbol{P}^{*}\right) b_{k}\left(q_{k}-q_{k-1}\right)\right]+ \\
& \sum_{k=1}^{K}\left[\max \left(0, \Delta \operatorname{Pr}\left(b_{k}=\boldsymbol{P}^{*}\right)\right)\left(\sum_{j=1}^{k-1}\left(\underline{v}\left(q_{j}, s\right)\left(q_{j}-q_{j-1}\right)\right)\right)+\min \left(0, \Delta \operatorname{Pr}\left(b_{k}=\boldsymbol{P}^{*}\right)\right)\left(\sum_{j=1}^{k-1}\left(\bar{v}\left(q_{j-1}, s\right)\left(q_{j}-q_{j-1}\right)\right)\right)\right. \\
& +\left[\operatorname{Pr}\left(b_{k}=\boldsymbol{P}^{*} \mid m-\epsilon\right) \underline{v}\left(q_{j}\right)\left(\mathbb{E}\left[\boldsymbol{q}^{*} \mid b_{k}=\boldsymbol{P}^{*}, m-\epsilon\right]-q_{k-1}\right)-\right. \\
& \left.\operatorname{Pr}\left(b_{k}=\boldsymbol{P}^{*} \mid m\right) \underline{v}\left(q_{j}\right)\left(\mathbb{E}\left[\boldsymbol{q}^{*} \mid b_{k}=\boldsymbol{P}^{*}, m\right]-q_{k-1}\right)\right] \\
& \left.1\left(\operatorname{Pr}\left(b_{k}=\boldsymbol{P}^{*} \mid m-\epsilon\right)\left(\mathbb{E}\left[\boldsymbol{q}^{*} \mid b_{k}=\boldsymbol{P}^{*}, m-\epsilon\right]-q_{k-1}\right)-\operatorname{Pr}\left(b_{k}=\boldsymbol{P}^{*} \mid m\right) \mathbb{E}\left[\boldsymbol{q}^{*} \mid b_{k}=\boldsymbol{P}^{*}, m\right]-q_{k-1}\right)>0\right) \\
& +\left[\operatorname{Pr}\left(b_{k}=\boldsymbol{P}^{*} \mid m-\epsilon\right) \bar{v}\left(q_{j-1}\right)\left(\mathbb{E}\left[\boldsymbol{q}^{*} \mid b_{k}=\boldsymbol{P}^{*}, m-\epsilon\right]-q_{k-1}\right)-\right. \\
& \left.\operatorname{Pr}\left(b_{k}=\boldsymbol{P}^{*} \mid m\right) \bar{v}\left(q_{j-1}\right)\left(\mathbb{E}\left[\boldsymbol{q}^{*} \mid b_{k}=\boldsymbol{P}^{*}, m\right]-q_{k-1}\right)\right] \\
& \left.1\left(\operatorname{Pr}\left(b_{k}=\boldsymbol{P}^{*} \mid m\right)\left(\mathbb{E}\left[\boldsymbol{q}^{*} \mid b_{k}=\boldsymbol{P}^{*}, m-\epsilon\right]-q_{k-1}\right)-\operatorname{Pr}\left(b_{k}=\boldsymbol{P}^{*} \mid m+\epsilon\right) \mathbb{E}\left[\boldsymbol{q}^{*} \mid b_{k}=\boldsymbol{P}^{*}, m\right]-q_{k-1}\right)<0\right) \\
& \left.-b_{k}\left(\mathbb{E}\left[\boldsymbol{q}^{*}-q_{k} \mid b_{k}=\boldsymbol{P}^{*}, m-\epsilon\right]-\mathbb{E}\left[\boldsymbol{q}^{*}-q_{k} \mid b_{k}=\boldsymbol{P}^{*}, m\right]\right)\right] \leq \lambda \epsilon
\end{aligned}
$$

where $\Delta \operatorname{Pr}(\cdot)$ instead now indicates taking a difference between $\operatorname{Pr}(\cdot \mid . ., m-\epsilon)$ and $\operatorname{Pr}(\cdot \mid . ., m)$.
Summarizing, we now have a system of $2 K$ linear equations, $2 K+1$ unknowns, and 2 inequalities from the optimality of the $K$ steps submitted by bidder $i$.

We implement these ideas via the following procedure to construct the identified set of $v(q, s)$. To begin, we search over the (one-dimensional) set of $\lambda$. For each feasible $\lambda$, there is a unique set of lower and upper bounds for $v$ at each $q_{k}$ where that customer submitted a step that satisfy equation (4). Using these implied $v$, together with the definition of the total surplus function allows us to obtain upper and lower bounds on the $\lambda$ based on (15). This range of $\left(\lambda_{L}, \lambda_{U}\right)$ are the set of feasible $\lambda$ which are consistent with the observed choices and values. To obtain the maximum and minimum we find the marginal value at each point that maximizes (minimizes) the value of the change in the total surplus function. Whether this is the upper or lower envelope of the set of marginal values consistent with the observed bids, depends only on the sign of the change in clearing probabilities under the increased $m$. If the initial $\lambda$ is in the range $\left(\lambda_{L}, \lambda_{U}\right)$, than the associated marginal value curve is part of the identified set of values that can rationalize the behavior of the given customer.

When it is outside the range, the bid is not consistent with equilibrium behavior for that set of values. To trace out the identified set, we repeat this exercise along a grid of possible $\lambda$.

Appendix Table A1: List of dealers that exited and entered auctions, and when

| Dealer name | Bill auctions | Bond auctions |
| :--- | :--- | :--- |
| CT Securities | Entry 05jan1999, exit 18jan2000 | Entry 24feb1999, exit 24nov1999 |
| Salomon | Entry 13apr1999 exit 19dec2000 | Entry 10feb1999, exit 18apr2001 |
| Goldman Sachs | Entry 05jan1999, exit 13feb2001 | Entry 10feb1999, exit 29aug2002 |
| SG Valeurs | Entry 13apr1999, exit 02nov2004 | Entry 10feb1999, exit 10mar2004 |
| JP Morgan | Entry 05jan1999, exit 21dec2007 | Entry 10feb1999, exit 28nov2007 |
| Deutsche Bank | Entry 05jan1999, exit 02jul2014 | Entry 10feb1999, exit 09jul2014 |
| Morgan Stanley | Entry 05jan1999, exit 10apr2001 | Entry 10feb1999, exit 28may2014 |
| PI Financial Corp | Entry 12may2009, exit 10feb2015 | Entry 04feb2009, exit 18feb2015 |
| Ocean Securities | Entry 21feb2006, exit 04mar2008 | Entry 24feb1999, exit 23sep2015 |
| Sherbrooke SSC | Entry Apr2020 | Entry Apr2020 |

Appendix Table A1 lists all entries and exits of primary dealers in bill and bond auctions from 1999 until 2022.

Appendix Table A2: Differences in Customer and Dealer Values

|  | P95 | Sum | Max | CI |
| :--- | :---: | :---: | :---: | :---: |
| QWA-Value | 0.00 | 0.00 | 0.00 | $[825,2906]$ |
| Max-Value | 0.06 | 0.00 | 0.93 | $[-836,2406]$ |
| Min-Value | 0.00 | 0.00 | 0.00 | $[615,1976]$ |

Appendix Table A2 shows the results from testing whether customer values are above dealer values. Columns P95, Sum, and Max present p-values for the test-statistics which take the 95th percentile, the sum of squared standardized differences, and the maximum difference across auctions of the average values of dealers less the lower bound of customer values. P-values are computed using the bootstrap. The confidence intervals are for interval estimates of the mean difference. The QWA-value is the average (within customers and dealers) of the individual participants quantity-weighted average values. The Max-Value row compares the within group average values of the individual bidders' maximum value (at their first submitted step). The Min-Value row compares the within group average values of the individual bidders' minimum value (at their last submitted step).

Appendix Table A3: Drivers of Customer Participation and Customer Values

|  | (OLS1) |  | (OLS2) |  | (Year-FE) |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\beta_{1}:$ Basis trade | -1.414 | $(2.688)$ | -1.807 | $(2.716)$ | -1.127 | $(2.718)$ |
| $\beta_{2}:$ Benchmark status | +0.573 | $(1.019)$ | +0.464 | $(1.047)$ | +0.803 | $(1.049)$ |
| $\beta_{3}:$ MPC | -0.562 | $(2.478)$ | -1.575 | $(2.646)$ | -0.605 | $(2.698)$ |
| $\beta_{4}:$ QE | $+3.421^{* *}$ | $(1.132)$ | $+2.679^{*}$ | $(1.265)$ | +2.587 | $(1.391)$ |
| $\beta_{5}:$ Exchange rate | +8.936 | $(5.724)$ | +4.855 | $(6.070)$ | -0.122 | $(10.56)$ |
| $\beta_{6}:$ Spread | $+0.803^{* * *}$ | $(0.172)$ | $+0.802^{* * *}$ | $(0.174)$ | $+0.856^{* * *}$ | $(0.176)$ |
| $\beta_{7}:$ Number of dealers | $+0.924^{* *}$ | $(0.335)$ | +0.620 | $(0.375)$ | +0.253 | $(0.446)$ |
| Extra controls | - |  | $\checkmark$ |  | $\checkmark$ |  |
| Adjusted $R^{2}$ | 0.114 |  | 0.127 |  | 0.136 |  |
| Observations | 327 |  | 327 |  | 327 |  |

Appendix Table A3 is analogous to Table 2, but regresses the estimated average of the quantityweighted values of customers (instead of the observed number of participating customers) on the observable variables listed in the table in column (OLS1). Using observed quantity-weighted bids instead of estimated values results in similar estimation findings. The data ranges from the beginning of 2014 until the end of 2021. In column (OLS2) we add the same controls that proxy for the interest rate environment and monetary policy expectations as before; in column (Year-FE) we include year fixed effects. Standard errors are in parenthesis.

Appendix Table A4: Customer commitment

| Year | No. of customers |  | Clearing Price |  | Revenue |  | Std of Revenue |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
|  | Com | SQ | Com | SQ | Com | SQ | Com | SQ |
| 2015 | 7 | 6.68 | 99.34 | 99.74 | 3.52 | 3.53 | 0.41 | 0.28 |
| 2016 | 7 | 8.94 | 100.18 | 100.30 | 3.46 | 3.47 | 0.28 | 0.26 |
| 2017 | 5 | 6.10 | 99.65 | 99.76 | 3.59 | 3.59 | 0.26 | 0.44 |
| 2018 | 6 | 6.44 | 98.59 | 98.56 | 3.82 | 3.82 | 0.43 | 0.50 |
| 2019 | 7 | 9.18 | 98.43 | 98.27 | 3.91 | 3.91 | 0.49 | 0.62 |
| 2020 | 7 | 11.52 | 98.03 | 97.95 | 4.25 | 4.25 | 0.61 | 0.60 |
| 2021 | 7 | 8.84 | 98.19 | 98.15 | 4.09 | 4.09 | 0.59 | 0.68 |
| 2022 | 7 | 9.39 | 98.51 | 98.60 | 4.14 | 4.15 | 0.68 | 0.68 |

Appendix Table A4 compares the counterfactual with customer commitment (Com) to the status quo (SQ), where customers make per-auction entry decisions. Dealer participation is fixed. Note that the equilibrium number of customer entrants depends on auction-specific profits for each of the 30 auctions per year across 8 years. To avoid computing the equilibrium in all of these auctions for each possible number of customers, we utilize a selected sample of five auctions. These auctions are strategically chosen to align the number of customers with percentiles (5th, 25th, 50th, 75th, and 95th) of the customer participation distribution since 2014, while the quantity sold approximates the average amount. When calculating profits, surpluses, and prices for each year, we re-weight the predictions from these five auctions to match the composition of auctions in that specific year. Expected revenues are in C $\$$ billions.

Appendix Figure A1: Primary auctions in different countries


Appendix Figure A1, taken from Muller (2019), shows an overview of how different countries issue debt. Towards the left are countries like Canada who heavily rely on dealers to make markets. Towards the right are countries like the U.S. who let anyone participate in primary auctions.

Appendix Figure A2: What a dealer sees when bidding


Appendix Figure A2 shows a screenshot of what a dealer sees when placing its bids, either for its own account or on behalf of a customer.

Appendix Figure A3: Auction allotment by investor class for U.S. government bond auctions


Appendix Figure A3 shows the auction allotment in percentage of supply in U.S. government bond auctions from beginning of 2010 until end of January 2022 for broker/dealers (plus) and for investment funds (circle). Broker/dealers include Includes primary dealers, other commercial bank dealer departments, and other non-bank dealers and brokers; Investment funds include mutual funds, money market funds, hedge funds, money managers, and investment advisors. To create this graph we use public data from TreasuryDirect.org, available at https://home.treasury.gov/ data/investor-class-auction-allotments, accessed on July 19, 2023.

Appendix Figure A4: Purchased amount by dealers, customers, and hedge funds


Appendix Figure A4 shows the distribution of how much dealers, customers, and hedge funds win (as a group) in percentage of the total amount issued across all bond auctions in our sample for each year from 1999 until 2022.

Appendix Figure A5: Purchased amount by investor groups


Appendix Figure A5 shows a binned scatter plot of how much each investor group wins in percentage of the total supply bought by non-dealers from 1999 until 2022.

Appendix Figure A6: Dealer adjustment and customer bid


Appendix Figure A6 shows the correlation between the change in the dealer's quantity-weighted bid and the quantity-weighted bid of the customer, conditional on the dealer updating her bid. A quantity-weighted bid is the total amount a bidder bid divided by the total amount she demanded. A bid is measured in yields to maturity and expressed in bps.

Appendix Figure A7: Random Matching of Customers to Dealers


Appendix Figure A7A shows the distribution of how many dealers a customer places a bid through within an auction. The median is 1 . Figure A7B plots the distribution of the number of unique dealers used by a customer in all auctions in the data and the number of unique dealers that would be predicted for each customer in our model where customers are randomly matched to dealers. The model prediction fixes the maximum number of dealers at the median number of dealers across years (12). The distributions are broadly similar, but the model predicted distribution somewhat overestimates the probability that a customer sometimes uses all of the possible dealers.

Appendix Figure A8: Revenue and Price Effect-Adding Back Dealers


Appendix Figure A8A shows the probability for a hedge fund (HF) to participate in every 15th auction from 2014 onward in the status quo (on the $x$-axis) and the counterfactual in which we add back the two dealers who left (on the y-axis). Probability is in percentage points. Figure A8B shows the expected number of dealers and hedge funds that participate in each auction in the status quo and the counterfactual. Figure A8C shows the distribution of the expected auction revenues in million $\mathrm{C} \$$. Figure A8E a time series of the percentage change in the expected price when going from the status quo to the counterfactual. Prices are in C $\$$ with a face value of 100 .

Appendix Figure A9: Expected Price and Number of Dealers for auction p5, and p95


Appendix Figure A9 is analogous to Figure 5. It shows how the range of expected price (in C $\$$ ) at which an auction clears varies as the number of dealers increases from 7 to 14 in two representative auctions, which issue the average supply with 1 participating customer in (A), and with 4 participating customers in (B). Here, 1 and 4 are the 5th and 95 th percentile of the observed distribution of the number of participating customers. In (C) and (D) we show the auction failure for these two auctions, respectively. In theory, there is one counterfactual equilibrium for each fixed number of dealers. In practice, we determine a range of prices (marked in black) given that our numerical procedure to determine counterfactual bids allows for small differences between the value distribution that is implied by the counterfactual bids and the true value distribution (see Richert (2021) for details). The blue horizontal line shows the average observed bid, which is close to the observed market clearing price.

Appendix Figure A10: Varying Customer Participation Probability


Appendix Figure A10 shows how the range of expected price (in $\mathrm{C} \$$ ) at which an auction clears varies as the customer participation probability varies between 0 and 1 in an auction in two representative auctions, which issue the average supply with 1 participating customer in (A), and with 4 participating customers in (B). Here, 1 and 4 are the 5 th and 95 th percentile of the observed distribution of the number of participating customers. In theory, there is one expected price for each counterfactual. In practice, we determine a range of prices (marked in black) given that our numerical procedure to determine counterfactual bids allows for small differences between the value distribution that is implied by the counterfactual bids and the true value distribution (see Richert (2021) for details). The blue horizontal line shows the average observed bid, which is close to the observed market clearing price.

Appendix Figure A11: Varying Number of Dealers Under the Alternative Distribution of Maximal Quantities


Appendix Figure A11 is analogous to Figure 5, but using the alternative distribution of maximal quantities of Appendix C.1. It shows how the range of expected price (in $\mathrm{C} \$$ ) at which an auction clears varies as the number of dealers increases from 7 to 14 in an auction which issues the average supply with medium customer participation-by which we mean 3 customers participate. In theory, there is one expected price for each fixed number of dealers. In practice, we determine a range of prices (marked in black) given that our numerical procedure to determine counterfactual bids allows for small differences between the value distribution that is implied by the counterfactual bids and the true value distribution (see Richert (2021) for details). The blue horizontal line shows the average observed bid, which is close to the observed market clearing price.

Appendix Figure A12: Varying Customer Participation Probability Under the Alternative Distribution of Maximal Quantities


Appendix Figure A12 is analogous to Figure 6, but using the alternative distribution of maximal quantities of Appendix C.1. It shows how the range of expected price (in $\mathrm{C} \$$ ) at which an auction clears varies as the customer participation probability varies between 0 and 1 in an auction which issues the average supply with medium customer participation-by which we mean 3 customers participate. In theory, there is one expected price for each counterfactual. In practice, we determine a range of prices (marked in black) given that our numerical procedure to determine counterfactual bids allows for small differences between the value distribution that is implied by the counterfactual bids and the true value distribution (see Richert (2021) for details). The blue horizontal line shows the average observed bid, which is close to the observed market clearing price.

Appendix Figure A13: No dealer commitment


Figure A13A shows the probability a customer participates in every 3rd auction from 2014 onward in the status quo (on the x-axis) and the counterfactual in which we add back the two dealers who left (on the y-axis). Probability is in percentage points. Figure A13B shows the expected number of dealers and customers that participate in each auction in the status quo and the counterfactual. Figure A13C shows the distribution of the percentage change in the expected auction revenue. Figure A13D a time series of the percentage change in the expected price when going from the status quo to the counterfactual. Prices are in $\mathrm{C} \$$ with a face value of 100 .


[^0]:    *The presented views are those of the authors, not necessarily of the Bank of Canada. All errors are our own. We thank Walter Muiruri for excellent research assistance. We are especially grateful to Philippe Besnier, Mark de Guzman, Faizan Maqsood, Jabir Sandhu, Matthieu Truno, and Rishi Vala for sharing their expertise on Canadian Treasuries. In addition, we thank Daniel Barth, Jay Kahn, Paulo Somani, and all participants at seminars at Boston College, Boston University, Microsoft Research in New England, the NY Federal Reserve, and the Federal Reserve Board, as well as SITE Market Design, and David K. Levine's conference at SAET. Jason Allen - Bank of Canada, Email: jallen@bankofcanada.ca, Ali Hortaçsu: University of Chicago, Email: Hortacsu@gmail.com; Eric Richert, University of Chicago, Email: richert.eric@gmail.com, Milena Wittwer, Carroll School of Management, Email: wittwer@bc.edu.

[^1]:    ${ }^{1}$ Unprecedented market turmoil in March 2020 triggered a policy debate on whether to reform Treasury market rules and capital requirements (e.g., Logan (2020), Ackerman and Hilsenrath (2022), Grossman and Goldfarb (2022)).
    ${ }^{2}$ In most of the paper, we focus on bonds rather than bills because bonds are more profitable and have attracted hedge funds.

[^2]:    ${ }^{3}$ Strictly speaking, there are two types of dealers in Canada: primary dealers and government security distributors, which are smaller dealers. We do not distinguish between these groups.
    ${ }^{4}$ Appendix Figure A2 shows what a dealer sees when bidding.

[^3]:    ${ }^{5}$ In stock markets, hedge funds have long been active, and their role in these markets has been discussed in the academic literature (e.g., Stein (2009)).

[^4]:    ${ }^{6}$ For instance, Banegas et al. (2021) acknowledge challenges in assessing the extent of hedge funds' Treasury selling during March 2020 due to a lack of detailed data on hedge funds' cash and derivatives positions.
    ${ }^{7}$ The bidder identifiers were created by Bank of Canada staff who observe bidder names, and therefore are able to account for mergers, acquisitions, and name changes.

[^5]:    ${ }^{8}$ Appendix Table A1 lists all dealer entry/exits.
    ${ }^{9}$ This would align with anecdotal evidence. For instance, according to research by Greenwich Associatesa leading financial consultancy-regulations implemented after the 2008 global financial crises caused general retreat from Canadian debt markets in 2014 (Altstedter (2014)).

[^6]:    ${ }^{10}$ To determine profitability of buying bonds at auction and shorting the corresponding futures contract, we approximate the bond's value as the quantity-weighted average price of winning bids (by customers) plus accrued interest between the auction date and the futures' expiration date. If this price is below the price of the future multiplied by a conversion rate that is determined by the Bank of Canada, we say that a cash/future trade is profitable. The conversion rates are published here: https://www.m-x.ca/en/markets/ interest-rate-derivatives/bond-futures-conversion-factor, accessed on 08/23/2023.

[^7]:    ${ }^{11}$ In addition, even though transactions data only starts in 2016, we did experiment with including interdealer repo rates and spreads to capture the cost of overnight borrowing, but the coefficients are not statistically significant.
    ${ }^{12}$ In theory, it could also be that after observing an aggressive customer bid, a dealer re-evaluates their own valuation for the bond upwards. In our empirical application we reject the hypothesis that dealers learn about fundamentals (see Appendix A.1). This is consistent with Hortaçsu and Kastl (2012).

[^8]:    ${ }^{13}$ Recall that a dealer has to electronically submit bids using the platform presented in Appendix Figure A2. Bid-updating, therefore, is not instantaneous.

[^9]:    ${ }^{14}$ Alternatively, we could distinguish between different types of customers, for instance hedge funds versus other customers. Theoretically, such an extension would be straightforward. However, empirically, non-hedge fund customers play such a small role that the cost of complicating the model and increasing measurement error (due to more bidder groups in the resampling procedure described below) outweighs the benefit of separating non-hedge fund customers from other customers.

[^10]:    ${ }^{15}$ The assumption of random matches simplifies the equilibrium conditions and estimation procedure. In Appendix Figure A7 we provides some evidence that random matching is a reasonable approximation of reality.

[^11]:    ${ }^{16}$ As a comparison, when dealers don't update bids, the market clearing price will weakly increase in all states of the world if the customer increases her quantity $q_{k}$ at price $b_{k}$ by a little bit, assuming that all other participants play as in the equilibrium. This monotonicity helps the dealer to find the best reply.
    ${ }^{17}$ Lemma 1 in Appendix D. 1 formally proves that the qw-bid function, which we focus on later, is differentiable w.r.t. $q$.

[^12]:    ${ }^{19}$ When including all observable variables, all coefficients that are statistically significant at a $10 \%$ level in one of the regressions of Table 2 or Appendix Table A3 share the same sign. There is one exception, which is the number of participating dealers. This observable factor is negatively correlated in Table 2 and insignificantly positive in Appendix Table A3. Including the year-fixed effect makes both coefficients positive and statistically insignificant.

[^13]:    ${ }^{20}$ Appendix Figure A8 shows by how much the expected revenue and expected price changes in the counterfactual relative to the status quo. Given that all auctions are relatively competitive given the observed number of bidders, as we explain next, these effects are relatively small.
    ${ }^{21}$ Appendix Figure A9 shows the analogous results for other auctions, specially the p 5 and p 95.

[^14]:    ${ }^{22}$ Formally, let $p r$ denote the probability that a customer participates, and pr_data $=\frac{1}{T} \sum_{t} N_{t}^{h} / \bar{N}^{h}$. The competition effect $=\mathbb{E}\left[\boldsymbol{r e v e n u e} \mid p r=p r_{-} d a t a\right]-\mathbb{E}\left[\boldsymbol{r e v e n u e} \left\lvert\, p r=p r_{-} d a t a-\frac{1}{\frac{1}{T} \sum_{t} N_{t}^{h}}\right.\right]$ is the difference between the expected auction revenue with observed customer entry probabilities and the expected revenue when we remove one customer, in expectation. The volatility effect $=\mathbb{E}\left[\right.$ revenue $\left.\mid p r=p r \_d a t a\right]-$ $\mathbb{E}[\mathbb{E}[$ revenue $\mid p r]]$ is the expected revenue with observed entry probabilities minus the expectation of expected revenues over the distribution of customer entry probabilities.

[^15]:    ${ }^{23}$ To illustrate why normalizing supply reduces the supply changes we propose, consider one auction that supplies 4 billion and one auction that supplies 1 billion in the status quo. Assume that our rule would suggest shifting $10 \%$ of $\mathrm{C} \$ 4$ billion from the first to the second auction. This would mean increasing supply by C $\$ 400$ million - a massive percentage increase of $40 \%$.

[^16]:    ${ }^{24}$ The point estimate from regressing the average quantity-weighted value of all bidders in an auction that issues a bond with $\mathrm{C} \$ 100$ face value on the predicted number of participating customers is -0.03 . The confidence interval is $[-0.14,+0.07]$.

[^17]:    ${ }^{25}$ In order to allow the auctions to differ in some characteristics which are unobserved to the econometrician but are observed by bidders, we focus on resampling schemes that use the empirical distribution of bids within a given auction.

[^18]:    ${ }^{26}$ Ideally, we would choose a dealer that saw the exact same bid, however given the limited sample size this event is extremely unlikely. To reflect the uncertainty that the customer faces about the value of the dealer observing their bid, we set a bandwidth and define similar bids using the quantity-weighted average bids.
    ${ }^{27}$ This includes sampling bids from dealers that later become informed and place a later bid but who were not selected in the simulated residual supply curve in the customer resampling step.

