Trust at Scale: The Economic Limits of Cryptocurrencies and Blockchains*†

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October 24, 2023

Abstract

A recent innovation from computer science suggests the possibility for an economic system that does not need the support of government or rule of law (Nakamoto, 2008). Trust and security instead arise from a combination of cryptography and economic incentives. This paper presents a three equation argument that this new form of trust, while undeniably ingenious, is prohibitively expensive: the recurring, "flow" cost of maintaining the trust must be large relative to the one-off, "stock" benefits of attacking the system. This result also implies that the cost of securing the trust scales linearly with the potential value of attack — which means that if Bitcoin and other cryptocurrencies were to become significantly more economically useful than they have been to date, then the cost of maintaining their trust must grow to absurd levels. The key contrast with traditional trust grounded in rule of law, and complementary sources such as reputations, relationships and collateral, is the economies of scale that arise from credible deterrence as implicit in Hayek (1960) and Becker (1968): Society or a firm pays a fixed cost to enjoy trust over a large quantity of economic activity at low or zero marginal cost.

*This paper originally circulated in June 2018 in a shorter form as Budish (2018).

†Acknowledgments: I am grateful to the co-editor Andrei Shleifer and five anonymous referees for advice that greatly improved the paper. Thanks are also due to Susan Athey, Vitalik Buterin, Glenn Ellison, Alex Frankel, Joshua Gans, Edward Glaeser, Austan Goolsbee, Hanna Halaburda, Zhiguo He, Joi Ito, Steve Kaplan, Anil Kashyap, Judd Kessler, Scott Kominers, Randall Kroszner, Robin Lee, Jacob Leshno, Neale Mahoney, Gregor Matvos, Sendhil Mullainathan, Vipin Narang, Neha Narula, David Parkes, Tim Roughgarden, John Shim, Adi Sunderam, Scott Stornetta, Alex Tabarrok, Rakesh Vohra, Aviv Zohar, and seminar participants at Chicago Booth, the MIT Digital Currency Initiative, NBER Monetary Economics, Harvard, Carnegie Mellon, UPenn, Virtual Market Design, UIC, University of Tokyo, Northwestern, Iowa State Market Design Conference, Columbia and Stanford. Ethan Che, Natalia Drozdoff, Matthew O'Keefe, Anand Shah, Peyman Shahidi, Jia Wan and Tianyi Zhang have provided excellent research assistance. Disclosure: the author is a technical advisor to a project pursuing frequent batch auctions for decentralized finance. The author does not have any other financial interests that relate to this research.

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1 Introduction

Economists have long widely agreed that the market system requires some form of government and rule of law for support. This is uncontroversial among even the most free-market oriented thinkers. Adam Smith (1776) mostly argues for reducing government interference in markets, but he does not go all the way to zero, finding a critical role for the government in the enforcement of contracts and property rights and the provision of public security (as well as certain other public goods). Hayek (1960) grapples at length with the paradox that to maximize freedom—which he defines as the absence of coercion—it is necessary to have a government that has the power to coerce. Friedman (1962) famously described the government's role establishing the "rules of the game" for the market system and acting as its "umpire." The debate within modern economics is about what else government should do beyond these basic supports for the market system (e.g., social insurance, correcting externalities, provision of public goods). That there is some role for government and rule of law is essentially taken for granted.

A recent innovation from computer science suggests the possibility for an economic system that does not need the support of government and rule of law (Nakamoto, 2008).[1] Trust and security instead arise from a combination of cryptography and economic incentives. The details are complex and will be described below in Section 2, but at a high level, Nakamoto (2008) invented a way to achieve what computer scientists call "permissionless consensus" a large, anonymous, decentralized, freely-entering and -exiting mass of computational power around the world is incentivized to pay attention to and collectively maintain a common data set, enabling trust in this data set without the need for rule of law or any specific trusted party. The parties involved do not even need to know the identity or number of other parties involved. This invention enabled cryptocurrencies, including Nakamoto's own creation Bitcoin. The specific data structure maintained by the large mass of computational power is called a blockchain.[2]

It is no understatement to say that Nakamoto's invention captured the world's attention. One oft-cited figure is the $3 trillion of market capitalization of Bitcoin and other crypto assets at their 2021 peak, but even this figure seems to understate the amount of cultural, political,

[1] The anti-government views of early Bitcoin enthusiasts have been widely documented. See Popper (2015) for one early account and satoshi.nakamotoinstitute.org as a primary source. A few example quotes from these sources give a sense: "It's completely decentralized with no server or central authority" (Nakamoto email, 1/8/2009); "a new territory of freedom" (Nakamoto email, 11/6/2008); "outside the reach of any government" (Popper, pg 48).

[2] Not widely appreciated is that the blockchain data structure, *without* the novel method of trust, significantly predates Nakamoto, at least in terms of the core scientific ingredients if not popular and commercial appreciation of its usefulness (Haber and Stornetta, 1991; Bayer, Haber and Stornetta, 1993). This form of blockchain is sometimes called a permissioned or private blockchain, and is in essence a well-architected database that is append-only, has clear rules about what parties can add what data, and uses cryptography to prove that past data has not been deleted or tampered with. This form of blockchain, with trust grounded in rule of law and other traditional sources, is discussed in Budish and Sunderam (2023).

and commercial attention that has been paid to blockchains and cryptocurrencies. This level of attention is understandable given the nature of the idea—the premise of an economic system without government or rule of law is radical. Yet, at the same time, the economic usefulness of Nakamoto's invention remains an open question. To date, the majority of cryptocurrency volume appears to be speculative, with the other most widely documented use case being black-market transactions (Makarov and Schoar, 2021; Gensler, 2021; Buterin, 2022).[3] Moreover, the majority of this speculative volume has been through cryptocurrency exchanges—which are, at least in principle, centralized, trusted financial intermediaries. That is, the largest use of cryptocurrencies to date does not even take advantage of the novel form of trust.[4]

So which view is correct? Can trust and security be engineered from cryptography and incentives alone? Or is rule of law essential for the market system?

This paper will show that Nakamoto's novel form of trust—while undeniably ingenious—is economically implausible, at least in its literal and pure form without any implicit support from government or rule of law. It is too expensive in absolute terms relative to the stakes involved, and its expense *scales linearly* with the stakes involved. Put differently, if Nakamoto trust were to become more economically useful, then the costs of securing its trust would become preposterous. The analysis serves both as an explanation for why cryptocurrencies and blockchains have not been very economically useful to date, and as a reason to be skeptical that Nakamoto's anonymous, decentralized trust will play a major role in the global economy and financial system in the future—again, at least in its pure form without implicit support from rule of law. In so doing, this paper will also sharpen our conceptual understanding of what is special about traditional forms of

[3]Makarov and Schoar (2021) find that about 75% of Bitcoin transaction volume since 2015 involves cryptocurrency exchanges or exchange-like entitites, once the data are cleaned to account for spurious volume (such as a user moving their own funds from one address to another). They conclude that "the vast majority of Bitcoin transactions between real entities are for trading and speculative purposes." In a dataset from an earlier time period and using a different data cleaning and classification methodology, Foley, Karlsen and Putniņš (2019) find that 46% of Bitcoin transactions that do not involve cryptocurrency exchanges relate to illegal activity. Many credible public observers have also described cryptocurrency activity to date as mostly speculative or black-market. For example, Treasury Secretary Janet Yellen said in Feb 2021 "I don't think that bitcoin ... is widely used as a transaction mechanism ...To the extent it is used I fear it's often for illicit finance. ... It is a highly speculative asset." (Cox, 2021). SEC Chair Gary Gensler said in Aug 2021 "Primarily, crypto assets provide digital, scarce vehicles for speculative investment. ... These assets haven't been used much as a unit of account. We also haven't seen crypto used much as a medium of exchange. To the extent that it is used as such, it's often to skirt our laws ..." (Gensler, 2021). Ethereum founder Vitalik Buterin wrote in a Dec 2022 essay that he is most excited about applications still to come in the future, not the ones that already exist which he describes as "hyperfinancialized" (Buterin, 2022).

[4]Some of the trust in cryptocurrency exchanges has clearly been misplaced. Most famously, FTX abruptly collapsed and filed for bankruptcy in November 2022 and is reported to have misused on the order of $9 billion of customer funds. See Levine (2022) for a good account. Additionally, cryptocurrency exchanges' relationship to rule of law has been tenuous at best. FTX domiciled most of its business in the Bahamas in part to circumvent U.S. laws and regulations. Coinbase and Binance are currently under investigation by the U.S. Securities and Exchange Commission for operating as unregistered securities exchanges, and Binance is under investigation by the U.S. Department of Justice over money laundering concerns (including for terrorist groups, see Berwick and Talley, 2023).

trust that are grounded in rule of law and other complementary sources such as reputations, relationships and collateral. The key distinction will prove to be *economies of scale in the production of trust*.

The core of the paper's argument is just three simple equations. The first equation is a zero-profits condition that says that the amount of computing power devoted to maintaining Nakamoto trust will directly reflect the compensation paid to this computing power. For a sense of magnitudes, in 2021/2022 the compensation to Bitcoin computing power (known as "miners") averaged about $250,000 per block of data, or about $36M per day, and Bitcoin miners performed an average of about 200 million trillion calculations per second as an equilibrium response to this compensation. The computer science details behind this process are complicated, and vary to some extent by blockchain protocol, but the economics is standard free-entry logic. Variations of this first equation have appeared in numerous other prior papers.

The second equation is an incentive compatibility condition: how much trust does a given level of computational power produce? The Achilles' heel of the form of trust invented by Nakamoto (i.e., permissionless consensus) is that it is vulnerable to what is known as a "majority attack." Nakamoto's method for creating an anonymous, decentralized consensus about the state of a dataset relies on a majority of the computing power devoted to maintaining the data to behave honestly. This is not an obscure point; it is in the *abstract* of the famous Nakamoto (2008) paper. Intuitively, permissionless consensus, whether in the form invented by Nakamoto or other subsequent variations such as proof-of-stake, always relies on some implicit version of majority or super-majority voting to adjudicate what the state is in case there is a dispute. The second equation captures that it must not be economically profitable for a potential attacker to acquire a 51% majority (or greater) of the total computational power to manipulate the state. Prior economic analysis of cryptocurrencies had mostly abstracted away the possibility of attack, focusing instead on other economic issues.

Equation (3) connects equations (1) and (2), i.e., connects the free-entry/zero-profits condition to the incentive compatibility condition. The reason these two equations can be linked is that the amount of honest computing power appears in both. In equation (1), the amount of honest computing power reflects the recurring payments to this computing power. In equation (2), the amount of honest computing power determines the cost of attack. Equation (3) then tells us that the payments to the honest computing power in the zero-profit equilibrium must be large relative to the value of attacking the system.[5]

[5]Under idealized attack circumstances, because the attacker earns the compensation that would have been paid to honest participants, we can obtain an even stronger result—which is that the net cost of attack is zero. Like with other zero theorems in economics, the import of this result is not its conclusion (which clearly cannot be right) but that it helps us think through what frictions might obtain that would make the conclusion false.

This is a very expensive form of trust! The recurring payments to miners are a "flow", whereas the value of attacking the system is more like a "stock." So equation (3) tells us that the flow-like costs of maintaining the trust must exceed the stock-like value of breaking the trust.

The essential difference between the Nakamoto trust model and the traditional model grounded in rule of law is depicted in Figure 1. A criminal is thinking of robbing a bank. In the traditional model, the criminal must first consider how many security guards he will need to overcome. Then he will have to take into account that the bank will call in reinforcements from police, and that, if caught, he will go to jail. Similarly, consider a country thinking of whether to invade another country. They will have to consider the soldiers at the border (analog of security guards), but also that the invaded country will call in military reinforcements (analog of police), and that the invaded country may launch a counter-attack in retaliation (analog of Becker-ian deterrence from courts).[6] In contrast, the Nakamoto model is just to have a very large number of security guards at the bank, or soldiers at the border. This works, but it is very expensive and scales terribly with the stakes.

Notice two sources of scale economies in the traditional model. First, the police do not have to be present at the particular bank to provide security—they can provide security to a large number of locations at once as long as they are not all attacked simultaneously. Second, the courts can deter crime with just the credible threat to prosecute and imprison—a fixed cost investment in court capacity can deter a large quantity of potential criminal activity. This is the essence of Becker's (1968) model of optimal deterrence, and is central to Hayek's (1960) resolution of the paradox noted above that freedom requires a government with the power to coerce.[7]

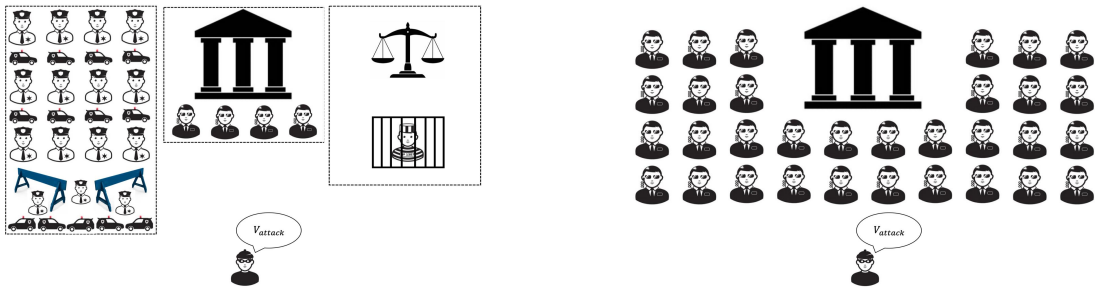
Notice as well a subtle additional source of inefficiency in the Nakamoto trust model—the full scale of the computational power is present for all transactions, whether for large sums or small. This is like having the same number of security guards outside the local bank branch as outside Fort Knox or the Federal Reserve.

There are many other sources of trust that are familiar to economists besides rule of law,

[6] I thank Edward Glaeser for drawing this connection to military strategy and Vipin Narang for a helpful discussion about the topic.

[7] Hayek's resolution is that a government's credible threat of coercion, in response to violations of clear, predictable, and symmetrically enforced laws, is both (i) not a violation of freedom, and (ii) sufficient to secure freedom. "The *threat* of coercion has a very different effect from that of *actual and unavoidable* coercion ... The great majority of the threats to coercion that a free society must employ are of this avoidable kind ... The sanctions of the law are designed only to prevent a person from doing certain things or to make him perform obligations that he has voluntarily incurred. ... Provided that I know beforehand ... I need never be coerced." (Pgs 209-210, emphasis added). "It is the cases that never come before the courts, not those that do, that are the measure of the certainty of the rule of law." (Pg. 316). "There is little difference between the knowledge that if he builds a bonfire on the floor of his living room his house will burn down, and the knowledge that if he sets his neighbor's house on fire he will find himself in jail. Like the laws of nature the laws of the state provide fixed features in the environment in which he has to move." (Pg. 221)

Figure 1: Comparison of Security Models: Bitcoin versus Traditional



Traditional Security	Bitcoin Security
cost of overcoming guards + cost of overcoming police reinforcements + risk × punishment if caught $> V_{attack}$	cost of overcoming guards $> V_{attack}$

such as reputations, brands, relationships, collateral, and cultural or organizational norms.[8] The contrast versus Nakamoto trust is similar: in each of these cases the trust is more secure than the flow level of investment in maintaining the trust. It also bears emphasis that these sources of trust often work in conjunction with rule of law, sometimes implicitly. For example, a customer trusts Starbucks to provide good coffee because of its brand, but also because it is illegal for a different entity to impersonate Starbucks' name and imagery. In a Levin (2003) relational contract, the employee trusts that if they put in high effort they will get paid a performance bonus, but in the background, it is also the case that the employee knows the employer will pay at least the promised minimum because of rule of law, and the employer trusts the employee not to rob the company because of the rule of law.[9]

There are also many alternative models for creating data security that are familiar to computer scientists and related experts. This includes traditional cryptography. To attack the Bitcoin blockchain requires that the attacker has more computing power than the honest miners; to attack

[8]Foundational work on trust from rule of law includes Schelling (1960), Becker (1968), Hart (1995), La Porta et al. (1998). Important work on other sources of trust includes Nelson (1974), Kreps et al. (1982), Fudenberg, Levine and Maskin (1994), Tadelis (1999) on brands and reputations; Baker, Gibbons and Murphy (2002), Levin (2003) on relationships; Kandori (1992), Holmstrom and Milgrom (1994), La Porta et al. (1997), Guiso, Sapienza and Zingales (2006) on norms. Also closely related is work on trust specifically in the context of financial markets, including La Porta et al. (1998), Sapienza and Zingales (2012), Gennaioli, Shleifer and Vishny (2015) and Zingales (2015).

[9]Formally, in Levin (2003)'s model, the employer pays the employee at least the fixed salary w_t no matter what, and the employee's lowest action, denoted $e_t = 0$, harms the firm only through poor effort, not theft. If the firm could pay the worker nothing or the worker could rob the firm, the scope for cooperation in the relational contract would be far worse (formally, each party's "renege" option would be much more attractive, undermining the relational contract's ability to be self-enforcing.)

data that is secured by traditional cryptography requires more computing power than a trillion Amazon Web Services, run for more time than the age of the entire universe.[10]

There is a way out of my flow-stock argument that may explain why Bitcoin has not already been attacked given this paper's analysis. If both (i) the technology used to maintain the blockchain is specialized (as opposed to repurposable), and (ii) an attack not only allows the attacker to steal money but also causes a collapse in the value of the cryptocurrency, then the attacker's cost becomes a stock, not a flow, because their specialized capital will collapse in value too.[11] However, vulnerability to collapse is itself a serious problem, and raises the possibility of an attack motivated by this collapse per se ("sabotage"). This part of the analysis thus suggests a "pick your poison" critique of Nakamoto trust: it is either extremely expensive relative to its economic usefulness, or it is vulnerable to sabotage and collapse. The model also identifies specific collapse scenarios.

The remainder of this paper is organized as follows. Section 2 provides a description of Bitcoin and the Nakamoto (2008) blockchain. Section 3 presents the heart of the economic critique of Nakamoto, equations (1)-(3). Section 4 uses the model to quantify the cost of keeping Nakamoto trust secure against double-spending attacks. Section 5 considers the possibility of a "sabotage" attack and derives a pick your poison result. Section 6 considers collapse scenarios implied by the paper's analysis. Section 7 contrasts Nakamoto trust with traditional trust grounded in rule of law. Section 8 concludes. Appendix A discusses responses to this paper's argument since it first circulated in 2018. Appendix B provides technical results in support of the double-spending attack analysis.

[10]Bitcoin's current level of computing power is about 3×10^{20} hashes per second. Hence, to attack the Bitcoin network requires an amount of computing power greater than 3×10^{20} hashes per second. As I will discuss below, this level of computing power has a flow cost of about $40 million per day and would require access to about $10 billion of specialized capital. To break an SHA-256 encrypted data set through brute force would require $2^{256} \approx 10^{77}$ calculations. I estimate that if you had a trillion Amazon Web Services' worth of compute power (about $65 billion trillion of capital), running for 14 billion years, that would get you to about 10^{45} hashes.

[11]Ethereum's recent adoption of proof-of-stake consensus with "slashing", which is the confiscation of a participant's capital under pre-defined conditions, is an attempt to make a double-spending attack cost a stock not a flow without needing the whole system to a collapse (see Buterin, 2016). Unfortunately, this approach too bumps up against serious economic limits. Most significantly, an impossibility theorem of Tas et al. (2023) shows that it is impossible to guarantee that the protocol can confiscate the attacker's capital before they spend it elsewhere, which defeats the purpose. Recent work of Lewis-Pye, Roughgarden and Budish (2023) derives a possibility result for this kind of security under stronger assumptions but only if the attacker is smaller than 5/9 of the total; if the attacker is larger than that then rule of law or some other external source of trust is needed for security. See further discussion in Section 2.5.3 and Appendix A.1.

2 Overview of the Nakamoto Blockchain

Sections 2.1-2.4 provide an overview of Bitcoin and the Nakamoto (2008) blockchain. The goal is to provide an overview that is self-contained and at a sufficient level of detail to justify the economics analysis in the rest of the paper.[12] Section 2.5 clarifies the relationship between the Nakamoto blockchain and three other ideas: permissioned blockchains, smart contracts, and proof-of-stake consensus.

Readers already familiar with the relevant background may skip this section without much loss.

2.1 Transactions

The first step in describing Bitcoin and the Nakamoto blockchain is to describe transactions, and the limitations of other methods of keeping track of transactions.

Elements of a Bitcoin Transaction. The key elements of a Bitcoin transaction are the sender of funds, the receiver of funds, the transaction amount, and a cryptographic signature. The sender and receiver are represented as alphanumeric strings called addresses; addresses are somewhat analogous to account numbers. The cryptographic signature uses standard ideas from public-key cryptography to prove that the transaction was initiated by the sender; that is, the signature could only be created by someone who knows the sender's private key for that address. The cryptographic signature also encodes the other transaction details, including the receiver and the transaction amount; it is like not only signing a check but also signing the seal of the envelope that contains the check, so the recipient and amount cannot be subsequently altered.

There are two additional details regarding transactions to note, both of which will make more sense later after additional terms are defined. First, transactions also include a fee amount, payable to the miner who adds the transaction to the blockchain. Second, transactions indicate not just the amount of funds (e.g., 10 Bitcoins) but which specific Bitcoins the sender wishes to send (e.g., these specific 10 Bitcoins). The sender does this by referencing a previous transaction or transactions in which they received those specific Bitcoins, where previous means in an earlier block in the blockchain.

[12] Readers interested in additional computer science detail should consult sources such as the textbook treatment of Narayanan et al. (2016), the website Bitcoin.Org (especially its Bitcoin Developer Guide), Tim Roughgarden's (2023) online course, and the original Nakamoto (2008) paper. Lewis-Pye and Roughgarden (2023) survey the computer science literature on permissionless consensus more broadly. There are several surveys with additional detail aimed specifically at economists as well, including Halaburda et al. (2022) and Böhme et al. (2015).

Limitations of a Shared Public Spreadsheet of Transactions. Imagine keeping track of such transactions on a shared public spreadsheet, such as a Google Doc. The cryptographic signature provides a certain level of trust in the data, in that only Alice, or someone in possession of Alice's private key, can add correctly-signed transactions in which Alice is the sender of funds. However, there are three vulnerabilities:

1. Alice could add a transaction in which she sends money she does not have.
2. Alice could add multiple transactions at the same or similar time, in which she sends money she does have but to multiple parties at the same time.
3. Alice could delete previous transactions from the shared public spreadsheet; either her own or others'.

Thus, while a shared public spreadsheet of transactions could be utilized among parties that trust each other — e.g., a modern version of the babysitting co-op parable in Krugman (1998) — this system is not suitable for tracking transactions among parties that do not have such a level of trust.

Limitations of a Trusted Party. Imagine keeping track of transactions through a widely trusted party that keeps track of balances, such as a central bank. This approach addresses the three vulnerabilities described above with respect to the shared public spreadsheet: the trusted party can ensure that only valid transactions are added to the ledger and that previous transactions are not deleted. However, the limitation is that it requires such a trusted party. A central goal of Nakamoto (2008) is to develop a trusted ledger of transactions that does not require a trusted party.

2.2 What is the Nakamoto Blockchain?

This section will describe the Nakamoto blockchain, in four steps.

I: Pending Transactions List. Users submit transactions to a pending transactions list, called the mempool. One can think of the mempool as in essence the shared public spreadsheet discussed above. However, transactions in the mempool are not considered official yet.

II: Valid Blocks. Any computer around the world can compete for the right to add transactions from the mempool to a data structure called the *blockchain*. The computational competition will be described in the next step.

The phrase blockchain references that transactions are added in blocks (for Bitcoin, consisting of about 1000-2000 transactions), and each block of transactions "chains" to the previous block by including a hash of the data in the previous block. See Figure 2. This use of hashes to chain together a sequence of blocks of data was invented by Haber and Stornetta (1991) and Bayer, Haber and Stornetta (1993). Since the hash of the current block depends on the data in the previous block, which in turn includes its hash of the block before that, etc., any change to any element in the history of transactions affects the value of the hash of the current block.

For a block of transactions to be valid, the following three criteria must all be true:

1. Each individual transaction must be properly signed: the cryptographic signature could only be generated by a user in possession of the sender's private key.
2. Each individual transaction must be properly funded: given all transactions in previous blocks in the chain, the sender must be in possession of the Bitcoins she or he is sending.
3. The transactions in a block must not contradict each other: there cannot be two or more transactions in a block in which a common sender sends the same Bitcoins to multiple receivers.

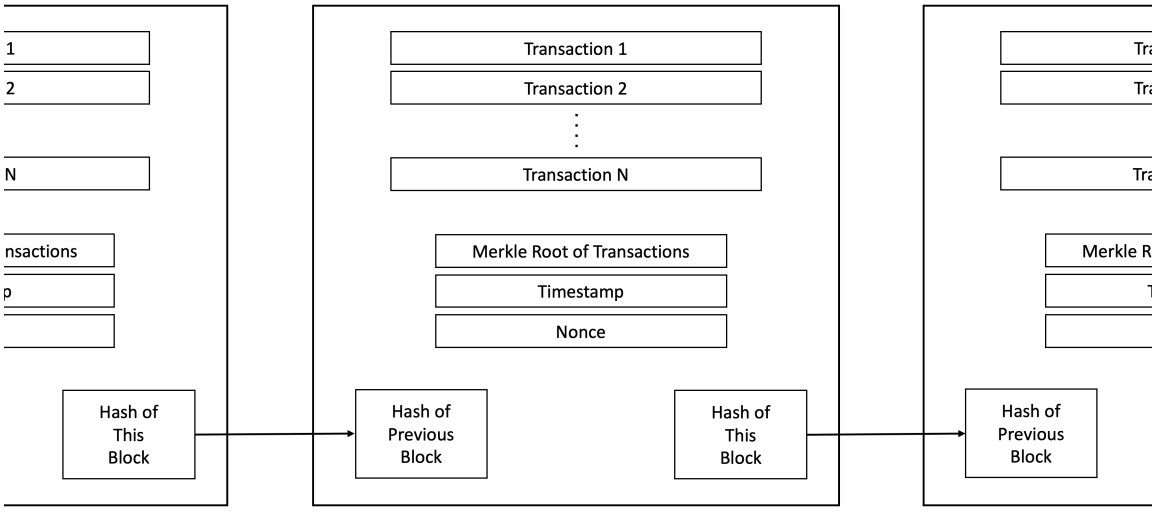
III: Bitcoin Mining Computational Tournament. The competition to add new blocks boils down to a massive, brute-force search for a lucky random alphanumeric string. More precisely, Bitcoin miners — where "miners" is just the terminology for computational power that attempts to add new blocks of transactions to the Bitcoin blockchain — choose a valid block of Bitcoin transactions from the mempool that they wish to chain to the previous block of transactions, and search for an alphanumeric string (called a nonce) such that when that alphanumeric string, in combination with all of the data in the new block of transactions they are adding (summarized by its Merkle Root), and the hash of the previous block of transactions that they are chaining to, is all hashed together using the hash function SHA-256, the result has a very large number of leading zeros.

For readers unfamiliar with hash functions, it is highly recommended to go to a website like https://www.movable-type.co.uk/scripts/sha256.html to get a feel for how they work. For example, the hash of the title of this paper is 09b23bf1eb4b7cda... which has one leading zero. A block added to the Bitcoin blockchain in April 2022, block 729,999 has the hash

0000000000000000008b6f6fb83f8d74512ef1e0af29e642dd20daddd7d318f

which has 19 leading zeros. Since each digit in the hash can take on values 0-9 and a-f, and the SHA-256 hash function is pseudorandom, the likelihood of finding an alphanumeric string that produces a hash with 19 leading zeros is 1 out of 16^{19}, which is about 1 out of 75 billion trillion.

Figure 2: Illustration of the Blockchain Data Structure



Notes: See the text of Section 2.1 for a description of transactions and the text of Section 2.2 for a description of the overall blockchain data structure and the other elements in the diagram.

The number of leading zeros required is calibrated by the Bitcoin system every roughly two weeks, based on the current amount of computational power devoted to Bitcoin mining, to ensure that blocks are successfully mined on average every 10 minutes. (This calibration can be finer than is possible using just zeros; for instance the hash might have to have 19 leading zeros and a 20th digit weakly less than 9.) As of this writing, the amount of computational power devoted to Bitcoin mining is about 375 million trillion hashes per second.

When a miner finds a lucky alphanumeric string, they publicly broadcast their block — consisting of the transactions, the hash of the previous block, their lucky alphanumeric string, and their block's hash — to all of the other Bitcoin miners. Other Bitcoin miners can quickly check whether the block is valid; that is, does the set of transactions in the block meet the criteria listed above in Step II, and does the alphanumeric string indeed produce a valid hash with enough leading zeros. Note, critically, that while finding a lucky alphanumeric string is extremely computationally intensive, checking the validity of a given block is computationally trivial. For this reason, a valid block is "proof-of-work" — proof that the miner who found the block did a large amount of computational work in expectation.

The lucky miner who broadcast the valid block gets compensated in two ways. First, the miner is compensated with new Bitcoins. This is called the "block reward", which was originally 50 Bitcoins per block, and halves every roughly four years, most recently in May 2020 to 6.25 Bitcoins per block. Second, the miner earns any transactions fees associated with the transactions they included in their block. The economics of these transactions fees are considered in depth in

Huberman, Leshno and Moallemi (2021); users who place a high value on getting their transaction added to the blockchain quickly can ensure faster service by offering a larger transaction fee, so there is an auction-theoretic flavor to the fees, as well as queueing and congestion issues.[13]

IV: Longest-Chain Convention. Once a valid block is broadcast and the other miners have checked its validity, miners are supposed to move on to mining the next block. To induce this behavior, Nakamoto proposed the *longest-chain convention* — the convention that, if there are multiple chains of blocks, the longest chain, as measured by the amount of computational work, is the official consensus record of transactions.

Intuitively, Nakamoto's longest-chain convention provides a decentralized way to coordinate miners' efforts. If miners focus their attention on the current longest chain, and they find a lucky alphanumeric string and mine a block, then their new block will be part of the new longest chain, and hence new official record, and the miner will earn the block reward. Section 11 of Nakamoto (2008) shows formally that as long as a majority of computational power follows the longest-chain convention, then the longest chain will outpace attackers with probability that converges to one exponentially in the honest-majority's share and the deficit the attacker must overcome.[14]

A related intuition is that the longest-chain convention provides a decentralized way to adjudicate disputes — computational power "votes" on the true state, and the majority rules.

The game-theoretic validity of longest-chain consensus has received considerable academic attention. The most general treatment to date is Biais et al. (2019), who show that honest mining on the longest chain is indeed a Nash equilibrium, though there can be other equilibria as well. Carlsten et al. (2016) show that longest-chain mining is an equilibrium only if the block reward component of miner compensation is large enough. Kroll, Davey and Felten (2013) provide credible intuition for why longest-chain mining is a Nash equilibrium, though without a formal game-theoretic model.

However, all of these prior works explicitly assume that all miners are "small" — that is, they assume away the possibility of majority attack. Majority attack, discussed next, will be at the heart of this paper's analysis.

2.3 Vulnerability to Majority Attack

Nakamoto's blockchain is vulnerable to attack by an adversary with 51% or more of the computational power. This is because the adversary, whenever they like, can create an alternative chain

[13] Using transaction fees to bid for processing priority has led to forms of front-running in decentralized finance applications. See Daian et al. (2019) and Gans and Holden (2022).

[14] Indeed, this analysis comprises over 2 pages of Nakamoto's 8-page paper.

of blocks that will outpace the honest chain of blocks with probability one, and hence become the new consensus. This vulnerability of Nakamoto consensus is widely understood — it is even in the abstract of the Nakamoto (2008) paper (excerpted below). Moreover, that Nakamoto consensus is vulnerable to attack is not surprising to computer scientists in the sense that previous approaches to distributed consensus were also vulnerable to attacks by a too-large adversary. For example, the Byzantine Fault Tolerance (BFT) paradigm for consensus has been known to be vulnerable to attack by an adversary with $> \frac{1}{3}$ of the power since the 1980's, except under very restrictive assumptions (Pease, Shostak and Lamport 1980; Lamport, Shostak and Pease 1982; Dolev and Strong 1983; Fischer, Lynch and Paterson 1985; Dwork, Lynch and Stockmeyer 1988).[15]

The canonical attack Nakamoto (2008) worried about is called a double-spend: the attacker sends Bitcoins in transactions on the original honest chain, and then deletes those transactions from the consensus record with their alternative chain, allowing them to spend the same currency twice. Section 4 will describe double-spending attacks in detail and analyze their economic implications.

2.4 Nakamoto Blockchain: Summary

The abstract of Nakamoto (2008) succinctly summarizes the accomplishment and its vulnerability:

> "A purely peer-to-peer version of electronic cash would allow online payments to be sent directly from one party to another without going through a financial institution. Digital signatures provide part of the solution, but the main benefits are lost if a trusted third party is still required to prevent double-spending. We propose a solution to the double-spending problem using a peer-to-peer network. The network timestamps transactions by hashing them into an ongoing chain of hash-based proof-of-work, forming a record that cannot be changed without redoing the proof-of-work. The longest chain serves not only as proof of the sequence of events witnessed, but proof that it came from the largest pool of CPU power. *As long as a majority of CPU power is controlled by nodes that are not cooperating to attack the network, they'll generate the longest chain and outpace attackers.* The network itself requires minimal structure. Messages are broadcast on a best effort basis and nodes can leave and rejoin the network at will,

[15]The exception in which communication among honest parties is secure even in the presence of an unbounded adversary requires that the honest parties have access to a communication network that never experiences delays longer than a known bound (the "synchronous model") and have access in advance of communication to all honest parties' cryptographic public keys (the "public key infrastructure" assumption). These assumptions are frequently satisfied in practical applications with pre-existing trust (e.g., secure military communications) but are widely viewed to be incompatible with the kind of communication Nakamoto (2008) is trying to facilitate over the internet among parties without pre-existing trust. See Lecture 3 of Roughgarden (2023) for an accessible treatment and the papers cited in the text for some key historical results on BFT consensus.

> accepting the longest proof-of-work chain as proof of what happened while they were gone." (Emphasis added)

The accomplishment is a "purely peer-to-peer version of electronic cash" without the use of a "trusted third party." Trust in the integrity of the data emerges from the hash-based proof-of-work, conducted by an unstructured network with free entry and exit. The longest chain is the official record of "what happened" — i.e., is the (permissionless) consensus.

The vulnerability is majority attack — the construction relies on the assumption that "a majority of CPU power is controlled by nodes that are not cooperating to attack the network."

The economic limits that majority attack places on Nakamoto's novel form of trust are at the heart of this paper's analysis.

2.5 Clarifications and Discussion

2.5.1 Permissioned Blockchains

As interest in Bitcoin and Nakamoto's blockchain surged, many started to use the phrase "blockchain" to describe similarly-architected databases maintained by *known, trusted parties* — that is, *without* the central innovation of Nakamoto (2008). This concept is sometimes known as a permissioned or private blockchain, or sometimes as distributed ledger technology (see, e.g., Bakos and Halaburda, 2021). An IBM marketing campaign called it "Blockchain for Business." Goldman Sachs called such blockchains "The New Technology of Trust." (Goldman Sachs, 2018)

Many researchers and observers view this use of the phrase "blockchain" as hype for what is in essence just an append-only distributed database with well-defined permissions. The financial columnist Matt Levine memorably wrote:

> "If you announce that you are updating the database software used by a consortium of banks to track derivatives trades, the New York Times will not write an article about it. If you say that you are blockchaining the blockchain software used by a blockchain of blockchains to blockchain blockchain blockchains, the New York Times will blockchain a blockchain about it." (Levine, 2017)

As should be clear, this paper's analysis and concern are about blockchain in the sense of Nakamoto (2008). It should be uncontroversial that well-architected databases are economically useful, even if there is a heated debate about what to call them. Indeed, what this paper highlights is that it is exactly the aspect of Bitcoin and Nakamoto (2008) that is innovative relative to traditional distributed databases — the anonymous, decentralized trust that emerges from proof-of-work — that is the source of its economic limits.

2.5.2 Smart Contracts

Notice that Nakamoto's novel form of trust is not specific to currency transactions. One can replace "Alice sends Bob 10 Bitcoins, signed by Alice" with any executable computer instruction signed by Alice. This idea is often called a "smart contract" (see Buterin, 2014*a*).

The analysis framework of this paper applies analogously to blockchains that allow smart contracts though the attack possibilities may differ.

2.5.3 Proof-of-Stake

The computational work Bitcoin miners must perform to add a new block serves the role of sybil resistance, i.e., making it expensive to add new identities to the permissionless system. Without sybil resistance an attacker could create infinitely many identities.

Since Nakamoto (2008) there have been several other approaches taken to sybil resistance for permissionless consensus, the most prominent of which is proof-of-stake. Roughly, instead of voting for the correct chain with computational work, participants vote for the correct chain with stake in the cryptocurrency. Ethereum, the second-most valuable cryptocurrency project after Bitcoin, switched from proof-of-work to proof-of-stake in Fall 2022, and its founder has been discussing the potential benefits of proof-of-stake since as early as 2014 (Buterin, 2014*b*, 2016).

One motivation for proof-of-stake over proof-of-work is to reduce environmental externalities. The computational work that powers Bitcoin consumes on the order of 0.3-0.8% of all global electricity, which is a fairly astonishing figure.[16] As will become clear, however, the environmental issue is orthogonal to the concerns raised in this paper about Nakamoto (2008) — just replace the opportunity cost of mining with the opportunity cost of holding stake and the arguments go through relatively unchanged.

What is interesting about proof-of-stake for the purpose of this paper's argument is that stakes are not memory-less like computational work: stakes are locked up on chain, like collateral, and observably persist over time, like reputation. This opens up the possibility of punishing attackers by confiscating their stakes, which makes attacks more expensive and hence the blockchain more secure. Intuitively, this is an attempt to algorithmically mimic the traditional trust that is created by rule of law in combination with financial collateral (Buterin, 2014*b*, 2016; Buterin and Griffith, 2019).

Recent research suggests that this approach to security, while intuitively compelling, may not work. Tas et al. (2023) show as their Theorem 1 that it is impossible to guarantee that the

[16]De Vries (2018); Digiconomist (2022). The 0.8% figure is Digiconomist (2022)'s main estimate, whereas the 0.3% figure is based on its best-case analysis under the assumption that all Bitcoin mining equipment is maximally energy efficient.

attacker's stake can be successfully confiscated before the attacker withdraws their stake, unless one imposes external trust assumptions such as rule of law. Under slightly stronger assumptions about the nature of the networking environment, Lewis-Pye, Roughgarden and Budish (2023) show that slashing the attacker's stake works if and only if the attacker's majority is bounded by 5/9 of the total locked-up stake.[17] An interpretation is that a proof-of-stake blockchain can successfully mimic traditional trust to deter small attacks, but that rule of law must step in in the case of a large-enough attacker. See further discussion in Appendix A.1.

3 Nakamoto Blockchain: A Critique in 3 Equations

Sections 3.1-3.3 present the three equation critique of Nakamoto (2008). Section 3.4 presents a result that shows that the net cost of attack may be *zero* under strong assumptions. Section 3.5 presents a one-shot game version of the analysis that captures the essence of the critique while abstracting from many details. This one-shot version will be helpful for discussion later in the paper and may be simpler to teach.

3.1 Zero-Profit Condition (Honest Play)

Our conceptual question here is: how much computational power will maintain Nakamoto's anonymous, decentralized trust, if we restrict all participants to behave honestly?

Let there be a large finite number I of honest participants, who follow the Nakamoto longest-chain protocol automatically. We may think of I as representing all people who could potentially provide part of the decentralized support for Nakamoto trust. For example, I is the number of people connected to the internet around the world.

Each player i chooses a quantity of "trust support" $x_i \in \mathbb{R}^+$, which we may think of as their quantity of computational work in Nakamoto (2008)'s proof-of-work blockchain, or their quantity of some other costly action in another blockchain (stake, storage space, memory, etc.). Let $N = \sum_{i=1}^{I} x_i$ denote the total quantity of trust support. A player can choose a quantity of zero if they like, which is how we can think about people not participating in the decentralized trust. Our equilibrium concept for N will be a zero-profit condition. This is meant to capture the

[17] The stronger assumption in Lewis-Pye, Roughgarden and Budish (2023) is that there has to be a substantial delay period between when a user (including honest users) asks to unlock their stake and when they can use it in a transaction, and this delay period must be longer than any feasible attack. Otherwise the attacker could withdraw their stake and spend it before their attack is detected and punished, as in Tas et al. (2023). This implies that, if a proof-of-stake cryptocurrency were to become economically useful (to date, this remains an open question, see Buterin, 2022), then only a fraction of the total stake can be locked up for trust support, with the rest unlocked for actual use.

permissionless, free-entry / free-exit nature of Nakamoto trust. Nash equilibrium is studied in the one-shot game analysis of Section 3.5 and is very similar.

Let c denote the cost per unit time to supply one unit of trust support. For example, for proof-of-work, this is the cost per unit time to run one unit of computational power, including variable costs such as electricity and a rental cost of capital for capital equipment. We will sometimes use the notation $c = rC + \eta$, where rC is the rental cost of capital and η is the variable cost of electricity.

Let p_{block} denote the economic reward paid to a participant who successfully mines a new block of transactions, i.e., wins a computational tournament. For the purpose of this paper, we will consider the compensation to the lucky miner in aggregate, without distinguishing between whether this compensation is in the form of newly issued Bitcoins (which are a form of seignorage tax on holders of the currency) or transaction fees. We will treat p_{block} as exogenous and derive constraints on it below. Participants' probability of winning the next reward p_{block} is equal to their share of trust support. Specifically, player i wins the next block with probability $\frac{x_i}{N}$.

Let D denote the block difficulty level, defined as the number of units of trust-support-time needed, in expectation, to mine one block. Assume blocks arrive Poisson. That is, if there are N units of trust support, blocks are solved according to a Poisson point process with mean $\frac{D}{N}$.

Note a potential source of confusion is that costs c are incurred per unit time whereas rewards p_{block} are earned per block. The next two concepts will help map between objects that are per unit time and objects that are per block. First, we can define profits per unit of trust support per unit time as

$$\frac{1}{N}\frac{D}{N}p_{block} - c,$$

because some unit of trust support solves a block every $\frac{D}{N}$ time in expectation and each of the N units is equally likely to be the winner.

Second, define honest equilibrium as follows:

Definition 1. A *zero-profit honest mining equilibrium* consists of quantities $\{x_i^*\}_{i=1}^I$ and a difficulty level D^* such that participants (i) solve one block per unit time (as a normalization), and (ii) earn zero economic profits in expectation.

Proposition 1. *Let $N^* = \sum_{i=1}^I x_i^*$. In any zero-profit honest mining equilibrium,*

$$N^* c = p_{block}. \tag{1}$$

and $D^* = N^*$.

Proof. For participants to solve one block per unit time (condition (i)) requires $D^* = N^*$. This in

turn implies that profits per unit of trust support per unit time are $\frac{1}{N}\frac{D}{N}p_{block} - c = \frac{1}{N^*}p_{block} - c$. For these profits to be zero (condition (ii)) in turn implies $N^*c = p_{block}$. □

Proposition 1 is widely known and is the standard characterization of a rent-seeking tournament: the prize in the tournament, p_{block}, is dissipated by expenditures aimed at winning the prize, N^*c.[18] Prat and Walter (2021) provide empirical support that equation (1) describes actual equilibrium behavior in the Bitcoin mining market, with some additional nuances related to capital adjustment costs. There are also numerous websites that compare current block rewards to current mining costs, which lends further empirical support to the free-entry / zero-profits logic.

The Bitcoin Wiki acknowledges the rent-seeking competition among miners and the logic of equation (1) in detail, under the heading "Weaknesses –> Energy Consumption":

> "... the economic equilibrium for the mining rate is reached when global electricity costs for mining approximate the value of mining reward plus transaction fees. So the higher the value of one bitcoin, the higher the value of mining rewards and transaction fees, the higher the energy consumption of the bitcoin network in the long run. More efficient mining gear does not reduce energy use of the bitcoin network. ... cheaper energy linearly increases mining energy use ... the same conclusions apply to all proof-of-work based currencies." (Bitcoin Wiki, 2020*b*)

For a sense of magnitudes, in 2021 Bitcoin's block reward averaged roughly $318,000 per 10 minute block, which corresponds to about $2 million per hour, $46 million per day, and about $17 billion per year. Ethereum's block reward in 2021 averaged about $8,000 per block, but since Ethereum's block interval is just 13 seconds on average this corresponds to a similar overall magnitude, of about $50 million per day and about $19 billion per year.

3.2 Incentive Compatibilty Condition (Majority Attack)

Our conceptual question here is: how much security is generated by the amount of honest trust-support characterized in equation (1)? As discussed in Section 2.3, it is widely understood that an agent with a majority of computational power could successfully attack Nakamoto's novel form of trust. Specifically, such a player could double spend with probability one.

[18] See, for example: Kroll, Davey and Felten (2013) pg. 8; Huberman, Leshno and Moallemi (2021) Theorem 1; Easley, O'Hara and Basu (2019) equation (2); Chiu and Koeppl (2022) Lemma 1; Ma, Gans and Tourky (2018) equation (7); and Halaburda et al. (2022) equation (4). It is also straightforward to allow for heterogeneous mining costs. Let $c(\cdot)$ denote a continuous weakly increasing function where $c(n)$ gives the per-block cost of the nth unit of computational power. Then (1) becomes $N^*c(N^*) = p_{block}$. The marginal unit of computational power earns zero economic profits.

If there are N^* units of honest trust support, an outsider gains a majority with $N^* + \epsilon$ units, at cost $(N^* + \epsilon)c$ per unit time. An insider gains a majority with as little as $\frac{N^*}{2} + \epsilon$ units of computational power. The analysis will mostly focus on the costs of outside attacks, both to be conservative and because that case most cleanly expresses the conceptual critique of Nakamoto's novel form of trust. That said, readers thinking about the logistics of majority attacks in practice may find insider attacks more worrisome; they are also cheaper.

Consider an additional player, the attacker, not restricted to honest play. This player can attack by choosing AN^* units of trust support, $A > 1$, for an $\frac{A}{A+1}$ majority at cost AN^*c per unit time. Denote the expected duration of the attack by $t(A)$; we will derive a closed form for $t(A)$ under some assumptions in Section 4. Call $AN^*c \cdot t(A)$ the gross cost of attack. The attacker can choose A to minimize $A \cdot t(A)$. Call this optimum $A^* \cdot t(A^*)$, which we will study as well in Section 4.

Let V_{attack} denote the value of attack. For now, let us think about this value of attack in the abstract, but have in mind that the value of an attack will grow if the blockchain's economic usefulness and importance grow.

Definition 2. The Nakamoto blockchain is *incentive compatible against an outsider attack, on a gross-cost basis*, if the gross cost of attack exceeds the benefits of attack:

$$A^*N^*c \cdot t(A^*) > V_{attack}. \tag{2}$$

Economically, the key thing to note about (2) is that the cost of attack on the left-hand-side is related to the *flow* cost of maintaining the blockchain, i.e., to N^*c. In contrast, consider, e.g., mutually-beneficial cooperation in a relationship and the associated temptation to cheat, or a trusted brand that is tempted to shirk on quality. In such cases, the cost of cheating to the cheating party is related to the *stock* value of the relationship or brand they are destroying, not the flow cost of its maintenance.[19] Another contrast is trust that is supported by rule-of-law. In such cases, the cost of cheating to the cheating party is related not to the direct costs of conducting the crime, but to the costs of potentially getting caught and punished (Becker, 1968). As emphasized, the ingenious aspect of the Nakamoto (2008) form of trust is that it is completely anonymous and decentralized, without any reliance on rule-of-law, relationships or other traditional sources. But, this aspect also makes the Nakamoto (2008) form of trust economically cheaper to violate.

[19] An early version of this insight is due to Schelling (1956): "What makes many agreements enforceable is only the recognition of future opportunities for agreement that will be eliminated if mutual trust is not created and maintained, and whose value outweighs the momentary gain from cheating in the present instance." Aumann (1959) is perhaps the earliest mathematical formalization (see Nobel Prize Committee, 2005). For additional references and literature connections please see the introduction.

From a computer security perspective, the key thing to note about (2) is that the security of the blockchain is *linear* in the amount of expenditure on trust support, i.e., linear in N^*c in the left-hand-side of (2). In contrast, in many other contexts investments in computer security yield convex returns (e.g., traditional uses of cryptography) — analogously to how a lock on a door increases the security of a house by more than the cost of the lock. It is much more expensive to break modern cryptography than it is to implement it! Imagine if the cost of attacking Visa was that you had to have as much computational power as Visa for a few hours.

Two additional remarks are in order. First, as noted, an attack could also come from an insider, i.e., part of the current honest trust support. The outside attacker IC condition (2) seems more attactive conceptually because it treats the honest participants as "small", which is the Nakamoto ideal, and in equilibrium with the level of compensation p_{block}. That said, an inside attacker migh be more realistic in practice, since mining is often concentrated (Makarov and Schoar, 2021; Cong, He and Li, 2021). Second, the left-hand-side of (2) is the gross cost of attack. In the Nakamoto blockchain, the attacker would earn rewards for the blocks in their attacker chain, which subsidizes the attacker. We will come back to the distinction between gross and net costs of attack in Section 3.4.

3.3 Flow-Stock Problem

In the hoped-for equilibrium in which participants are honest, the amount of trust support devoted to maintaining the blockchain is characterized by the zero-profit equilibrium (1).[20] The incentive-compatibility condition then relates this amount of trust support to the level of security generated. Since N^*c appears in both the zero-profit condition (1) and the incentive-compatibility condition (2), we can combine the two equations:

Proposition 2. *The zero-profit condition (1) and the incentive-compatibility condition (2) together imply the equilibrium constraint:*

$$p_{block} > \frac{V_{attack}}{A^* \cdot t(A^*)} \tag{3}$$

In words: the equilibrium per-block payment to participants for maintaining the trust on the blockchain must be large relative to the benefits of attacking it.

Proof. (3) follows directly from combining (1) and (2). □

Equation (3) places potentially serious economic constraints on the applicability of the Nakamoto (2008) blockchain. The blockchain can only be used in economic contexts where users are willing

[20] If miners earn positive economic profits in equilibrium then the quantity N^* characterized by the zero-profit condition in (1) is an upper bound, and (3) obtains as is.

to pay a per-block transactions cost, p_{block}, that is large relative to the value of attacking the system, V_{attack}.

Again, consider the contrast versus traditional sources of trust such as brands and rule of law. Imagine if a brand were only as trustworthy as its flow investment in advertising, or users of the Visa network had to pay fees to Visa, every ten minutes, that were large relative to the value of a successful attack on the Visa network. Or, imagine that a country were only as secure as its flow expenditure on soldiers at the border.

Another crucial economic issue with (3) is that the flow expenditure on trust p_{block} must scale linearly with the value of attacking the system V_{attack}. For instance, if attacking the system grows 1000 times more attractive, then the cost of securing the system must grow 1000 times as well for the system to remain secure. The genius of Becker-ian (1968) deterrence, also emphasized by Hayek (1960) and Schelling (1960), is that a government able to credibly impose large punishments (the parameter f in Becker's model) can deter large attacks or crimes at comparatively low cost — e.g., with the threat of nuclear retaliation or with the threat of imprisonment or large fines.

3.4 Zero Net Attack Cost Theorem

What we may call the *net* cost of attack can differ from the gross cost of attack, modeled above, for three potential reasons: block rewards, attacker cost frictions, and effects of the attack on the value of Bitcoin itself.

First, the attacker earns block rewards from their attack. That is, after the attacker's alternative chain replaces the honest chain, the attacker earns the block rewards associated with the blocks in the new longest chain. These block rewards in effect subsidize the attack. An A attacker who attacks for t time units performs $At \cdot N^*$ units of trust support. If the difficulty stays constant at $D' = D^* = N^*$, then this corresponds to At block rewards in expectation. If the difficulty on the attacker chain adjusts upwards, i.e., $D' > D^*$, then the attacker will earn $At \cdot \frac{N^*}{D'} < At$ block rewards.

Second, the attacker may face frictions relative to the cost of honest mining. For example, if the attacker's compute power is less energy efficient than the honest miners' compute power, or because there are costs of starting and stopping the attack. Let $\kappa \geq 0$ parameterize the attacker's cost inefficiency relative to honest mining, such that their total cost of attack is $(1 + \kappa)At \cdot N^*c$.

Third, the attack may harm the value of Bitcoin. This reduces the value of the attacker's block rewards and reduces the value of the Bitcoin the attacker is left with after double spending. If we let $\Delta_{attack} \geq 0$ parameterize this decline, this reduces the value of the attacker's block rewards by $\Delta_{attack}At \cdot N^*c$ and reduces the benefit of a double spending attack originally worth V_{attack} by $\Delta_{attack}V_{attack}$. If the capital equipment is specific to the attacked cryptocurrency, then the attack

would reduce the value of the capital equipment as well; we will return to this issue in Section 5.

In the ideal case for the attacker with respect to these three sources of cost difference, we have the following remarkable conclusion:

Proposition 3. *If the attacker does not face any cost frictions relative to the costs of honest participants ($\kappa = 0$), the attack concludes without any difficulty adjustment ($D' = D^*$), and the attack does not cause the value of Bitcoin to fall ($\Delta_{attack} = 0$), then the net cost of attack is zero.*

Proof. The attackers' trust-support cost of attack is $(1+\kappa)At \cdot N^*c$. The net value of the attacker's block rewards is $At \cdot \frac{N^*}{D'} p_{block}(1 - \Delta_{attack})$. The reduction in the value of the Bitcoin the attacker is left with after double spending is $\Delta_{attack} V_{attack}$. If $\kappa = \Delta_{attack} = 0$ and $D' = N^*$, then substituting $p_{block} = N^*c$ from equation (1) yields a net cost of 0. □

The intuition behind this result is that the attacker is fully compensated for their computational costs for the same reason that honest miners are fully compensated for their costs under honest play. In effect, Nakamoto (2008) consensus treats the attacker "as if" they are an honest participant, because the longest chain is the true state.

Moroz et al. (2020) and Auer (2019) derive similar results to Proposition 3 building off of Budish (2018). To my knowledge, Jacob Leshno (in a verbal communication with the author), Alex Tabarrok (in a January 2019 blog post), Auer (2019) and Moroz et al. (2020) all discovered this consequence of the Budish (2018) model roughly contemporaneously.[21] Bonneau (2016)'s analysis of "bribery" attacks deserves credit for the intuition that the net cost of attacking Bitcoin might be very small because of the block rewards subsidy. Recent work of Gans and Halaburda (2023) generalizes the zero net attack cost result and, by incorporating Bitcoin transaction fees into their model, shows that it is possible for an inside attacker to have a slighty negative net attack cost.

To be clear, zero attack frictions seems unrealistic. But, zero friction is often useful as a benchmark case, and the result does reinforce that Nakamoto trust is economically implausible when taken literally.

3.5 One-Shot Game Analysis

The analysis above uses a price-theoretic zero-profit equilibrium concept for honest mining, and contains several details that are specific to aspects of Nakamoto (2008). As a complement to that

[21] The analysis in the June 2018 draft artificially constrained the attacker to earn at most t block rewards. The June 2018 draft also did not have explicit cost frictions. Rather, the assumption that an attacker earns at most t block rewards is like an implicit cost friction, related to starting and stopping the attack, of $(A-1)t \cdot N^*c$. As a result, the June 2018 draft had slightly different simulated net costs than here, and that draft did not have Proposition 3.

approach consider the following stylized one-shot game, which yields a Nash equilibrium solution and abstracts from some complexities of the above.[22]

There are I players. Each player i chooses a quantity x_i of trust support (work, stake, etc.) and a "posture" $a_i \in \{Honest, Attack\}$. Costs are c per unit of trust support. Define $N = \sum_{i=1}^{I} x_i$.

Payoffs are as follows. If there is a player i with $x_i > \frac{N}{2}$ and $a_i = Attack$, this player gets a payoff of V_{attack}, gross of their costs. All other players get zero. Else, each player gets a payoff of $\frac{x_i}{N}p$.

Our question is: under what conditions does there exist a Nash equilibrium, denoted $\{(a_i^*, x_i^*)\}_{i=I}^{I}$, in which all players choose $a_i^* = Honest$? Call such a profile, if one exists, an honest equilibrium.

Lemma 1. *If there is an honest equilibrium, then $N^*c \leq p$.*

Proof. Towards a contradiction, assume there is an honest equilibrium with $N^*c > p$. Choose any player i with $x_i^* > 0$. Player i's net payoff is $\frac{x_i^*}{N^*}p - x_i^*c < x_i^*c - x_i^*c = 0$. So the player has a profitable deviation by choosing $x_i' = 0$ instead. Contradiction. □

In words, this lemma tells us that the amount spent on trust support N^*c will be no greater than the compensation paid for this trust support p, analogously to equation (1) above.

Proposition 4. *A necessary condition for an honest equilibrium is $p \geq \frac{V_{attack}}{1+\frac{1}{I}}$.*

Proof. Conjecture an honest equilibrium. Player i's payoff in honest equilibrium is $\frac{x_i^*}{N^*}p - x_i^*c$. Consider a deviation by i in which they attack by choosing $a_i' = Attack$ and $N_{j\neq i}^* = \sum_{j\neq i} x_i^* + \epsilon$ for some $\epsilon > 0$. For this to be worse for player i requires:

$$V_{attack} - N^*{}_{j\neq i}c - \epsilon \leq \frac{x_i^*}{N^*}p - x_i^*c$$

Note that if $x_i^* = 0$ this simplifies within an epsilon to $N^*c \geq V_{attack}$, analogously to (2) above. Rearranging, using the Lemma, and noting that $\min(x_i^*) \leq \frac{1}{I}N^*$ yields

$$V_{attack} \leq p(1 + \frac{1}{I}).$$

□

As the number of players I grows large, the necessary condition for honest play becomes $p \geq V_{attack}$, analogously to (3) above. An interpretation of the timing of this game is that p and c now represent, respectively, blockchain compensation and trust-support-costs for an amount of time commensurate with the duration of an attack, i.e., the analog of $A^* \cdot t(A^*)$ in (3). The

[22] I thank Rakesh Vohra for suggesting this modeling idea.

analysis tells us that the cost of running the blockchain, for an attack-duration amount of time, must exceed the value of attacking it.

4 Analysis of Double-Spending Attacks

The canonical attack Nakamoto (2008) worries about is called a "double spend." In a double spend, an attacker sends Bitcoins (or some other cryptocurrency) to another party in exchange for goods or assets, and then uses their computational majority to effectively delete the transaction in which they sent their Bitcoins. This leaves the attacker with both the goods or assets they bought and their Bitcoins, which they can spend again (hence "double").

This section analyzes double-spending attacks under the assumption that the attack does not cause a post-attack decline in the value of the cryptocurrency, i.e., $\Delta_{attack} = 0$. The case where the attack does cause such a decline will be considered in the next section. The analysis will use the gross cost of attack (i.e., equations (2)-(3)), under various assumptions about escrow periods. This is equivalent to using the net cost of attack considered in Section 3.4 under the assumption that attack frictions cancel out block rewards, i.e., $\kappa = 1$.

4.1 Mechanics of a Double-Spending Attack

What a majority attacker can and cannot do. Before discussing double-spending attacks it is useful to clarify what, technologically, a majority attacker can and cannot do. Because a majority attacker can find lucky hashes faster in expectation than the honest minority, the attacker can create an alternative longest chain of transactions, and replace the honest chain with their alternative chain at a strategically opportune moment. This allows the attacker to control what transactions get added to the blockchain, and allows the attacker, within computational limits, to remove recent transactions from the blockchain — by creating an alternative chain starting from the recent past. The attacker even earns the block rewards for each period of their alternative chain after they make it the new longest public chain.[23]

[23]Block rewards do not vest for 100 block intervals (see Bitcoin Protocol Rules, section ""tx" messages", item 11; and Bitcoin Developer Guide, section "Transaction Data" (Bitcoin Wiki, 2020*a*; Bitcoin.org, 2022)). Thus, as long as the attacker replaces the honest chain within 100 blocks, there is no ambiguity as to who gets the block rewards. If the attacker replaces the honest chain after more than 100 periods have elapsed things get more complicated. In this scenario, the bitcoins that vested to miners on the honest chain would become unusable because they do not exist on the new longest chain. It seems likely that, in practice, such a long-horizon attack (and possibly even shorter-horizon attacks) would generate an attempt to counter-attack or hard-fork to restore the original honest chain (see Moroz et al., 2020), and that in general there would be a period of chaos and uncertainty in this scenario. I will return to this issue in Section 5 under the discussion of Sabotage attacks and in Appendix A.4 under the discussion of counter-attacks.

What the attacker *cannot* do is create new transactions that spend other participants' Bitcoins. Creating new transactions that spend other participants' coins would require not just a majority of computational power, but enough computational power to break modern cryptography: creating a transaction that spends another participant's coins requires learning their private key. A majority attacker cannot simply "steal all the Bitcoins."[24]

Description of double spending. Figure 3 illustrates a double-spending attack. The attacker engages in the following actions in sequence:

(i) The attacker spends Bitcoins. That is, the attacker signs one or more transactions in which they send Bitcoins to other parties in exchange for other goods or assets.

(ii) The attacker allows those transactions to be added to the blockchain. That is, the transactions are added to the longest chain as parts of mined blocks in the usual way as described in Section 2.2.

(iii) The attacker works in secret to create an alternative longest chain. In this alternative chain, the Bitcoins that were sent to other parties in (i) are instead sent to other addresses controlled by the attacker.

(iv) The attacker waits for any escrow periods to elapse, so they receive the goods or assets they transacted for in (i). In Bitcoin a common escrow period is 6 blocks, or about 1 hour.

(v) The attacker then releases their alternative longest chain. The attacker now has the goods or assets they received in (iv) but also has the Bitcoins which they have sent to themselves in the chain in (iii).

4.2 Analysis Framework

Equation (3) tells us that the possibility of a double-spending attack places economic constraints on Nakamoto's anonymous, decentralized trust. To understand these constraints we need to

[24]Here is a detailed excerpt on what majority attackers can and cannot do from the Bitcoin Wiki, under "Attacker Has a Lot of Computing Power":

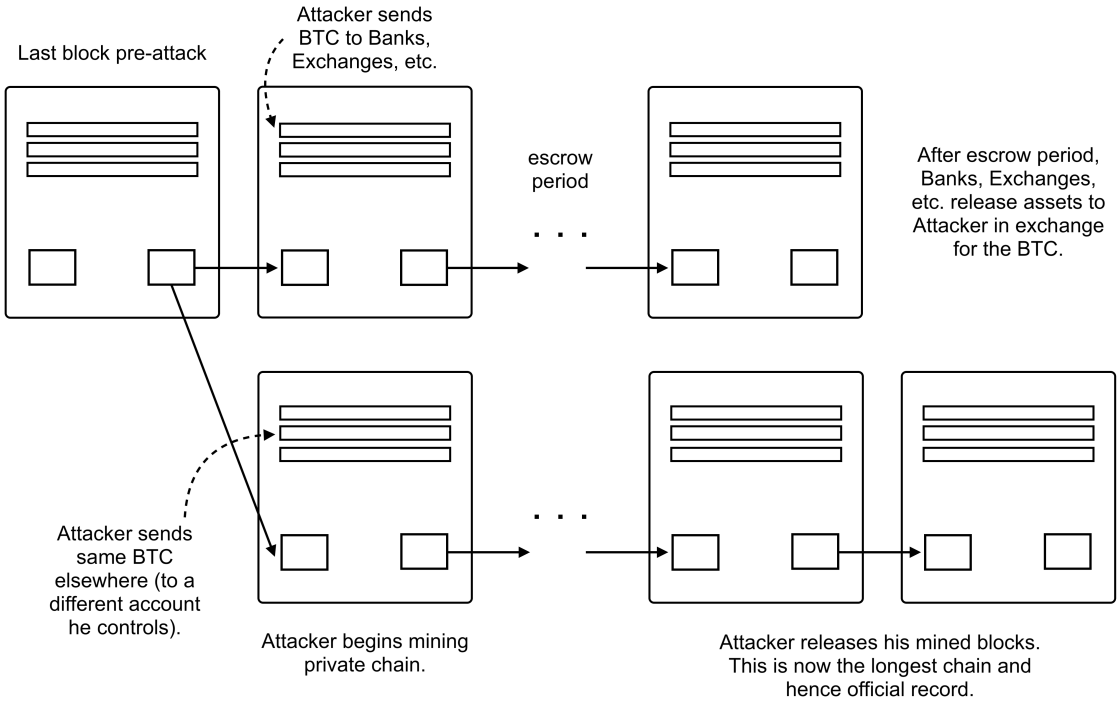
"An attacker that controls more than 50% of the network's computing power can, for the time that he is in control, exclude and modify the ordering of transactions. This allows him to:

- Reverse transactions that he sends while he's in control. This has the potential to double-spend transactions that previously had already been seen in the block chain.
- Prevent some or all transactions from gaining any confirmations.
- Prevent some or all other miners from mining any valid blocks.

The attacker *can't*:

- Reverse other people's transactions without their cooperation.
- Prevent transactions from being sent at all (they'll show as 0/unconfirmed).
- Change the number of coins generated per block.
- Create coins out of thin air.
- Send coins that never belonged to him." (Bitcoin Wiki, 2020*c*)

Figure 3: Illustration of Double-Spending Attack



Notes: See the text for description.

analyze the benefits of a double-spending attack (the V_{attack} term) and the expected cost of a double-spending attack in block-compute-cost units (the $A^* \cdot t(A^*)$ term).

Before proceeding with the analysis, I would like to reiterate the "if-then" nature of this paper's argument. This paper is trying to take seriously the "if" possibility in which cryptocurrencies and Nakamoto's anonymous, decentralized trust become a more important and useful part of the global economic and financial system. Some responses to the first draft of this paper's double-spending analysis were about why double-spending attacks would be hard to execute at the scale imagined in this section at *present* — not in the hypothesized future in which cryptocurrencies and Nakamoto trust are much more integrated with the global economy and financial system.

4.2.1 Benefits of Double Spending: V_{attack}

A majority attacker will not use their majority to double spend for a cappucino at Starbucks. They will use their majority to conduct transactions that are as large as possible given the current uses of the Nakamoto blockchain. Furthermore, they might engage in many such transactions using multiple addresses.

V_{attack}, therefore, should be understood as a statistic on the amount of transaction volume that large *honest* users of Bitcoin can conduct in a short period of time. The more value that honest

users can transact using Bitcoin, the more value an attacker can double spend.[25]

Therefore, I will consider a wide range of values for V_{attack}. I use \$1,000 as the low-end of this range, representing Bitcoin's early days when even buying a pizza was remarkable. I use \$100 billion as the high-end of this range. While arbitrary, this seems a reasonable order of magnitude for a large-scale attack on the global financial system. This figure also represents about 10% of Bitcoin's peak market capitalization.

4.2.2 Cost of Attack in Block-Compute-Cost Units: $A^* \cdot t(A^*)$

It is possible to obtain a closed-form expression for the expected duration t of a double-spending attack. Let A denote the attacker's majority and e denote the escrow period. For simplicity, assume that if the attacker engages in multiple transactions, they are all added to the honest chain at the same time. Motivated by the description of the mining process above, assume that honest miners mine new blocks as a Poisson process with arrival rate 1 and the attacker mines new blocks as a Poisson process with arrival rate A. As a reminder, we interpret one unit of time as the amount of time it takes in expectation to mine a single block in honest equilibrium.

In the Appendix I show the following:

Proposition 5. *The expected duration t of the double-spending attack, as a function of the attacker majority A and escrow period e, is given by:*

$$t(A,e) = (1+e) + \left[\sum_{i=0}^{1+e} \left(\frac{i+1}{A-1}\right) \cdot \frac{(1+2e-i)!}{(1+e-i)!e!} \left(\frac{A}{1+A}\right)^{1+e-i} \left(\frac{1}{1+A}\right)^{1+e}\right]. \tag{4}$$

As the attacker majority grows large ($A \to \infty$), $t(A,e)$ converges to $1+e$. In the limit as $A \to_+ 1$, we have $t(A,e) \to \infty$.

Proof. See Appendix B. □

Expression (4) can be understood as follows. In the attacker's best case, their attack takes $1+e$ time. That is, as soon as their assets are released from escrow, the attacker releases their alternative longest chain. This best case occurs if the attacker mines $1+e+1$ blocks before the honest miners mine $1+e$ blocks. Suppose, on the other hand, that the attacker is behind the honest chain by $i \geq 0$ blocks at the time the honest miners mine their $1+e$ block. Given the

[25] This point likely seems obvious, but it was missed in past academic literature on double-spending attacks. The computer science literature did not explicitly model the economic benefits of attack, and therefore missed how they would scale with Bitcoin's usefulness (Rosenfeld, 2014; Eyal and Sirer, 2014; Bonneau, 2016). Within economics, a model of Chiu and Koeppl (2022) assumes that an attack involves just a single transaction and holds this transaction size fixed. The authors conclude that the system becomes more secure as its economic value grows relative to this fixed transaction size. This is like noting that it is less attractive to engage in a double-spending attack for a cappucino in 2023 than it was in 2009.

Poisson arrival processes, it will take the attacker $\frac{i+1}{A-1}$ of time in expectation to strictly surpass the honest chain. The last part of the expression gives the probability that the attacker's deficit is i blocks, as a function of the escrow period e and attacker majority A.

Table 1 provides example calculations of duration t and the gross block-compute-cost term At for a wide variety of escrow periods and attacker majorities. For example, if the escrow period is $e = 6$, which is a fairly common escrow period for Bitcoin, then attacker majorities ranging from $A = 1.2$ to $A = 1.5$ result in attack durations t ranging from 8.77 to 14.37, and gross block-compute-costs At ranging from 13.15 to 17.24. Notably, smaller majorities lead to significantly longer attack durations and costs. If $A = 1.05$, which corresponds to a 51.2% attacker majority, the duration t is 45.06 blocks and the cost term At is 47.31.

If the escrow period is significantly longer than common practice, say $e = 100$ blocks (roughly 16 hours), then attack durations range from 101.0 to 105.1 for attackers with majorities ranging from $A = 1.2$ to $A = 1.5$, and gross block-compute costs At range from 126.2 to 151.5 for attackers with majorities in this range. Notice that as the escrow period grows longer, the average attack duration t gets proportionally closer to the escrow period. The intuition is simple law-of-large numbers.

Also note that, even at very long escrow periods, the gross-cost-minimizing attacker majority is larger than 51%. For escrow periods ranging between $e = 6$ to $e = 1000$, the cost-minimizing attacker majority ranges from about $A = 1.5$ (60%) to about $A = 1.1$ (52%). It is true that a 51% majority is enough to ensure statistically that the attack will eventually succeed, but a cost-minimizing attacker will choose a somewhat larger majority. This is speculative, but it seems possible that the widespread use of the phrase "51% attack" generated a false sense of security about how long a successful attack would take, and hence how expensive attacks would be on a gross-cost basis.[26]

Appendix B provides numerical analysis of the cost-minimizing attacker majority A^*, and hence minimum block-compute-costs $A^* \cdot t(A^*)$, as a function of the escrow period e.

4.2.3 Cases

Since $A^* \cdot t(A^*)$ depends on the escrow period, it will be helpful in our analysis to consider several cases:

- A base case in which the escrow period is the standard $e = 6$ blocks (60 minutes). In this case, the cost-minimizing attacker majority is $A = 1.53$ (or 60%), an average attack

[26] Indeed, as shown in Proposition 5, in the limit as the attacker majority converges to 50% from above, the attack duration t goes to infinity, hence so too do the attacker's gross costs.

Table 1: Expected Duration and Gross Cost of Attack

A. Expected Duration of Attack (t)

	$e = 0$	$e = 1$	$e = 6$	$e = 10$	$e = 100$	$e = 1000$
$A = 1.05$	25.51	29.77	45.06	54.44	181.32	1,067.82
$A = 1.1$	13.02	15.42	24.48	30.35	125.81	1,004.04
$A = 1.2$	6.79	8.28	14.37	18.65	105.13	1,001.0
$A = 1.25$	5.54	6.86	12.41	16.44	102.79	1,001.0
$A = 1.33$	4.34	5.49	10.57	14.40	101.47	1,001.0
$A = 1.5$	3.08	4.07	8.77	12.49	101.03	1,001.0
$A = 2$	1.89	2.78	7.39	11.23	101.0	1,001.0
$A = 5$	1.12	2.06	7.00	11.00	101.0	1,001.0

B. Gross Block-Compute Costs (At)

	$e = 0$	$e = 1$	$e = 6$	$e = 10$	$e = 100$	$e = 1000$
$A = 1.05$	26.78	31.26	47.31	57.17	190.38	1,121.22
$A = 1.1$	14.32	16.96	26.92	33.39	138.39	1104.35
$A = 1.2$	8.14	9.93	17.24	22.38	126.15	1,201.20
$A = 1.25$	6.93	8.57	15.51	20.55	128.49	1,251.25
$A = 1.33$	5.78	7.31	14.06	19.15	134.96	1,331.33
$A = 1.5$	4.62	6.11	13.15	18.73	151.54	1,501.5
$A = 2$	3.78	5.56	14.78	22.45	202.0	2,002.0
$A = 5$	5.59	10.29	35.01	55.00	505.0	5,005.0

Notes: Expected duration t, as a function of attacker majority A and escrow period e, is computed using formula (4) in the text and double-checked using a computational simulation.

takes about $t = 8.6$ time (86 minutes), and the attacker's average block-compute-costs are $A^* \cdot t(A^*) = 13.14$ (See Appendix Table 4 for all calculations).

- An expensive attack case in which the escrow period is a full day, or $e = 144$ blocks. In this case, the cost-minimizing attacker majority is $A = 1.16$ (or 54%), and the attacker's average block-compute costs $A^* \cdot t(A^*)$ are 176.

- A very expensive attack case in which the escrow period is a full week, or $e = 1008$ blocks. In this case, the cost-minimizing attacker majority is $A = 1.07$ (or 52%), and the attacker's average block-compute-costs are $A^* \cdot t(A^*) = 1100$.

Please note that while the expensive and very-expensive cases are defined in terms of longer escrow periods, they can equivalently be interpreted in terms of assumptions about higher attacker frictions. For example, if the escrow period is the standard $e = 6$, the expensive case can be

interpreted as attacker frictions κ increasing attack costs by a factor of ≈ 10, and the very expensive case can be interpreted as attacker frictions increasing attack costs by a factor of ≈ 100.

4.3 Results

Base Case Results for the base case are presented in Table 2. To keep the Nakamoto blockchain secure in the base case requires a per-block cost that is about 7.6% of the value secured against a double-spending attack. This follows directly from equation (3), rewritten as $\frac{p_{block}}{V_{attack}} \geq \frac{1}{A^* \cdot t(A^*)}$. Per transaction, assuming 2000 transactions per block, the cost is 0.004% of the value secured.

These costs likely sound economically plausible. But consider how they scale with time and with the amount of value secured. A cost of 7.6% per block amounts to over 1,000% of the value secured per day, and about 400,000% of the value secured per year. For example, to secure the system against a $1bn attack requires $4trn of annual security expense. To secure the system against a $100bn attack requires $400trn of annual security expense—or more than 4 times global GDP.

The per-transaction fee of 0.004% likely sounds very small, but this is a percentage of the value secured against attack, not the size of the transaction. For example, if an attack could be worth $1bn, then each transaction must pay 0.004% of $1bn which is $40,000 of security costs. The intuition is that every transaction has to implicitly pay for the costs of the large standing army — as if a transaction for a cappucino has the security required by Fort Knox.[27]

Sensitivity Analysis Table 3 provides the results of the sensitivity analysis. The expensive and very expensive cases improve the picture by one or two orders of magnitude, but the costs are still extremely high. In the expensive case, with a one-day escrow period, to secure against a $1 billion attack requires per-transaction costs of $3k and annual security costs of $300 billion. In the very-expensive case, with a one-week escrow period, to secure against a $1 billion attack requires per-transaction costs of $450 and annual security costs of about $50 billion. Even in the very-expensive case, to secure the system against a $100 billion attack requires a per-year security cost of about $5 trillion, which is more than 5% of global GDP.

[27] This observation explains the motivation for what are known as "level two" blockchain applications which batch small transactions off-chain to economize on fees. For example, the local coffee shop might only reconcile cappucino purchases on the blockchain once per year. The conceptual tension with such ideas, at least to date, is that they rely on off-chain sources of trust, which begs the question of the use of Nakamoto trust in the first place. See further discussion in the Conclusion and Appendix A.5.

Table 2: Cost Per (3) to Secure Against Attack: Base Case Analysis

	Per-Block	Per-Day	Per-Year	Per-Transaction
Security Costs as % of Value Secured	7.61%	1,096%	400,129%	0.004%
To Secure:				
$1 thousand	$76.1 dollars	$11.0 thousand	$4.0 million	3.8 cents
$1 million	$76.1 thousand	$11.0 million	$4.0 billion	$38.1 dollars
$1 billion	$76.1 million	$11.0 billion	$4.0 trillion	$38.1 thousand
$100 billion	$7.6 billion	$109.6 billion	$400.1 trillion	$3.8 million

Notes: See equation (3) and the text of Section 4.3 for description. The Base Case scenario assumes an escrow period of $e = 6$ blocks (one hour).

Table 3: Cost Per (3) to Secure Against Attack: Sensitivity Analysis

A. Security Costs as % of Value Secured

Attack Scenarios	Per-Block	Per-Day	Per-Year	Per-Transaction
Base Case	7.61%	1,096%	400,129%	0.004%
Expensive	0.57%	82.0%	29,939%	0.0003%
Very Expensive	0.09%	13.1%	4,777.9%	0.00005%

B. Cost to Secure Against $1 Billion Attack

Attack Scenarios	Per-Block	Per-Day	Per-Year	Per-Transaction
Base Case	$76 million	$11 billion	$4 trillion	$38 thousand
Expensive	$6 million	$820 million	$299 billion	$3 thousand
Very Expensive	$1 million	$131 million	$48 billion	$455 dollars

Notes: See equation (3) and the text of Section 4.3 for description. The Base Case scenario assumes an escrow period of $e = 6$ blocks (one hour), the Expensive scenario assumes an escrow period of $e = 144$ blocks (one day), and the Very Expensive scenario assumes an escrow period of $e = 1008$ blocks (one week).

4.4 Discussion

The double-spending analysis is consistent with the modest early use cases of Bitcoin, in which Bitcoin was primarily used by hobbyists and for small-scale black market activity (e.g., online gambling, Silk Road). In these early days, the amount that could be gained in a double-spending attack was not very high, because there were not high-value transaction opportunities. If a double-spending attack could gain at most $1,000, then the implicit cost per transaction in the base case

necessary to secure the trust is just $0.04.

The double-spending analysis is also consistent with larger-scale black-market uses of cryptocurrencies, especially as black-market users may be most willing to pay the high implicit costs. For example, if a double-spending attack could gain at most $10 million, then the implicit cost per transaction in the base case needs to be about $380. This is modest relative to the costs of transporting large amounts of cash (Rogoff, 2017).

Where the analysis suggests greater skepticism is the use of cryptocurrencies and Nakamoto trust as a major component of the mainstream global financial system. If cryptocurrencies and Nakamoto trust were to become more integrated with the mainstream global financial system, then it would be possible to move amounts of value that are ordinary in the scheme of global finance, and hence it would be possible to double spend for amounts of value that are ordinary in the scheme of global finance. The analysis suggests that this scenario is unrealistic because of the way the trust model scales. To secure the system against attacks of $1 billion — which is less than 0.2% of daily trading volume in the U.S. Treasury market alone — requires a per-transaction security cost of $38,000, and an annual security cost of $4 trillion. To secure against attacks of $100 billion requires an annual security cost of four times global GDP. While market power and fees in traditional finance are clearly an important economic issue (Greenwood and Scharfstein, 2013; Philippon, 2015), and Huberman, Leshno and Moallemi (2021) are careful to remind us to compare the costs of the Nakamoto trust model against the costs of market power in traditional finance, it is clear from these calculations that Nakamoto trust is absurdly expensive relative to traditional trust. We will return to this comparison between Nakamoto trust and traditional trust in Section 7.

5 Analysis of Sabotage Attacks

A reasonable response to the analysis in the previous section is that a large-value double-spending attack would be widely noticed, which would cause a decline in the value of Bitcoin (or the relevant cryptocurrency), which reduces the value of engaging in the double-spending attack in the first place. While the attacker still has the falsely-obtained goods or assets, the Bitcoins the attacker is left with to double spend are now worth less than before. Moreover, the attack would reduce the value of any capital equipment the attacker holds that is Bitcoin-specific. The Bitcoin Wiki classifies the majority attack into its "Probably Not a Problem" category for this reason (Bitcoin Wiki, 2022).

In this section I will show that this argument is logically correct. However, this line of reasoning raises the possibility of an attacker motivated by harming the cryptocurrency per se. I call this

possibility Sabotage, and derive a "pick your poison" result.

5.1 Pick Your Poison

Assume that the double-spending attack analyzed in Section 4 causes a proportional decline in the value of Bitcoin of Δ_{attack}. Assume that the attacker holds the minimum amount of Bitcoin necessary to conduct the attack, namely V_{attack} worth, the amount they will double spend. For this section, maintain the assumption from Section 3.1 that the attacker's cost of mining is c per unit of compute power per unit time. In Section 5.3 I will explicitly model the stock cost of capital, C, and assume that the attack harms the value of the attacker's capital stock too.

The Δ_{attack} decline in Bitcoin's value modifies the attacker's incentive compatibility condition in two ways. First, the attacker double spends for V_{attack} of value (e.g., traditional financial assets from a traditional financial institution), but to realize this benefit has to hold Bitcoins worth this amount, which decline in value $\Delta_{attack}V_{attack}$. Hence the net benefit of the attack is $(1-\Delta_{attack})V_{attack}$. Second, the attacker's net cost of attack has to be adjusted for the decline in the value of the block rewards they earn. An A attacker who attacks for t time still earns At block rewards in expectation, but each block reward declines in value by Δ_{attack}. The net incentive compatibility condition thus becomes:

$$(\kappa+\Delta_{attack})At\cdot N^{*}c>(1-\Delta_{attack})V_{attack}. \tag{5}$$

Substituting in equation (1) yields the following modification of (3):

$$p_{block}>\frac{(1-\Delta_{attack})}{At(\kappa+\Delta_{attack})}V_{attack} \tag{6}$$

The larger is Δ_{attack}, the smaller is the per-block cost necessary to deter the double-spending attack. This is easiest to see by considering the extreme case of $\Delta_{attack}=1$, i.e., if the attack causes a total collapse in the value of Bitcoin. In this case, the attacker loses exactly as much in Bitcoin value as they gain from the double spending, so the numerator on the right-hand-side of (6) is zero — in effect, there is no chance to "double spend" at all. More generally, we have the following simple result:

Proposition 6. *For any potential value of a double-spending attack $V_{attack}>0$, and any level of block reward $p_{block}>0$, the Nakamoto blockchain is secure against the double-spending attack if the post-attack decline in Bitcoin's value, Δ_{attack}, is sufficiently high.*

Proof. Follows directly from (6), noting that the numerator of the right-hand-side goes to zero as Δ_{attack} goes to one. □

Proposition 6 may sound reassuring about Bitcoin's security against double spending, but it raises another more troubling possibility: an attacker motivated by the harm to Bitcoin's value per se. Δ_{attack} is thus a "pick your poison" parameter: If Δ_{attack} is small, then the system is vulnerable to the double-spending attack analyzed in Section 4, and the implicit tax on economic activity using the blockchain has to be high. If Δ_{attack} is high, then the system is indeed secure against double-spending attacks — but this concedes that an attack would significantly harm Bitcoin, which in turn raises the possibility of an attacker movitated by this harm per se.[28]

5.2 Sabotage Value

What is the value of a sabotage attack on Bitcoin (or another significant cryptocurrency)? It is hard to say of course, but easy to imagine that the magnitudes are already large, and would be larger still if cryptocurrencies become more significantly integrated into the global financial system. Open interest in CME Bitcoin futures as of Sept 2023 is about 15,000 contracts, each tracking 5 Bitcoins, worth about $2 billion at current prices. According to data from The Block, open interest in Bitcoin futures aggregated across the major crypto exchanges has exceeded $20 billion (and is about $10 billion as of Sept 2023), and open interest in Ethereum futures has been as high as $10 billion (about $5 billion as of Sept 2023).[29] These figures give a sense of magnitudes for what could be made from a short-selling attack.

The market capitalization of cryptocurrencies gives another sense of magnitudes for the amount of economic harm a bad actor could cause. Bitcoin's market capitalization ranged from about $600 billion to $1 trillion in 2022. Across all crypto assets tracked by CoinMarketCap, market capitalization peaked at about $3 trillion in Nov 2021 and ranged from about $1.2 trillion to $2.2 trillion in 2022. Paypal co-founder Peter Thiel (2022) recently predicted that Bitcoin will be worth more than $100 trillion.

Last, Ethereum founder Vitalik Buterin described a future in which it is "just considered normal for there to be trillion dollar assets that are managed on Ethereum." (Klein, 2022) If

[28] You can't have your cake and eat it too. If your view is that Bitcoin's value would fall in the immediate aftermath of the double-spending attack, but then would recover and this is all predictable, then the attacker can just hold on to their Bitcoins until the value recovers, and their cost of attack becomes what it was as originally analyzed in Section 4. A more elaborate version of this argument involves the Bitcoin community coodinating on a hard fork after the attack, to both nullify the attack and enable a recovery in the value of Bitcoin. This "community response" is a valid argument but is inconsistent with Nakamoto's anonymous, decentralized trust. See Appendix A.2 for further discussion.

[29] CME open interest data is available via its website. I found open interest data from crypto exchanges at https://www.theblock.co/data/crypto-markets/futures/. I believe this to be a credible source but am less confident in it than I am the CME figures. For what it's worth, when I wrote the June 2018 draft of this paper, CME + CBOE open interest was about $160 million, and crypto exchange futures did not, to my knowledge, exist at the time. That is, futures market open interest has grown by two orders of magnitude in the past few years.

indeed assets of that magnitude are managed on Ethereum or other blockchains, without implicit or explicit protections from rule of law, then the value and risk of sabotage would be large.

5.3 Sabotage and Blockchain-Specific Capital

Nakamoto (2008) envisioned that blockchain mining would be performed by ordinary computers: "one-CPU-one-vote." Since 2013, however, Bitcoin mining has been dominated by computational equipment that is extremely specialized to Bitcoin mining. These machines have a large number of specialized chips called ASICs (application specific integrated circuits) which have the SHA-256 hash function programmed directly into their hardware — making them extremely fast at Bitcoin mining, and useless for any task that does not involve computing a large number of SHA-256 hashes. Such chips are reportedly on the order of 10,000 times more efficient at Bitcoin mining than re-purposable alternatives such as GPUs or FPGAs.[30]

As emphasized in the introduction, if the capital used to maintain Nakamoto's anonymous, decentralized trust is non-repurposable, and the attack causes a collapse of the trust, then the attacker cost model needs to be modified. In addition to charging the attacker the flow cost of attack, the attacker must also be charged for the decline in the value of their specialized capital. This makes the attacker's cost more like a stock than a flow, and thus makes the blockchain more secure — but this security rests on the fragile precipice of specific capital and vulnerability to sabotage.

Let $c = rC + \eta$ denote the per-unit-time compute cost from above, where C denotes the capital cost of one unit of computational power (e.g., 1 ASIC machine), r denotes the rental cost of capital per unit time, inclusive of risk and depreciation, and η denotes the electricity cost per unit time. The honest-mining equilibrium (1) tells us that

$$N^*(rC + \eta) = p_{block}. \tag{7}$$

An outside attacker would need at least N^*C worth of capital to conduct the attack, while an inside attacker would need at least $\frac{N^*C}{2}$ of capital.

Consider the extreme case in which the attack causes a total collapse of the economic value of the blockchain, including the specialized equipment; this is the case for which the incentive constraint against attack is least constraining. Given how small the flow costs of attack are, as

[30]Remarkably, even if one controlled all of the (re-purposable) computational power owned by Amazon Web Services, one would only have <0.1% of Bitcoin's hash rate. This calculation is based on AWS owning $65 billion of technology capital per its 2021 10-K filing, the calculations below that the Bitcoin capital stock is about $10 billion, and an assumption that specialized ASIC chips are at least 10,000 times more economically efficient at SHA-256 hashing than general-purpose computers.

analyzed in Section 4, ignore these and focus only on the stock cost of the specialized capital. This yields an incentive compatibility constraint for an outside sabotage attack of

$$N^*C > V_{attack} \tag{8}$$

and

$$\frac{N^*C}{2} > V_{attack}$$

for an inside sabotage attack. We can compute N^*C as a function of p_{block}. Let $\mu = \frac{rC}{rC+\eta}$ denote the capital share of mining. The honest-mining equilibrium (7) can be rewritten as

$$N^*C = \frac{\mu p_{block}}{r} \tag{9}$$

Hence we can derive a modified version of (3):

$$p_{block} > \frac{r}{\mu} V_{attack}. \tag{10}$$

This is several orders of magnitude more secure than before because r is the interest rate per block. Here is an example calculation. Assume the capital share of mining is $\mu = 0.4$ (De Vries, 2018; Digiconomist, 2022), and the annual discount rate for ASICs is 50% (ASICs depreciate quickly and mining is risky), which implies that the per-unit-time discount rate is $r \approx 0.001\%$. Now compare $\frac{r}{\mu}$ on the right-hand-side of (10) to the $\frac{1}{A^* \cdot t(A^*)}$ factor on the right-hand-side of (3). If we use the base case value of $\frac{1}{A^* \cdot t(A^*)} = 7.6\%$, we have an over 2000-fold improvement in security. If we use these same values for μ and r and use p_{block} of \$250,000, then (9) implies a capital stock of \$10 billion, which about matches what is implied by current prices for state-of-the-art ASIC machines.[31] This suggests these magnitudes are reasonable.

Thus, if one concedes that a majority attack on Bitcoin would effectively be a sabotage that would cause the entire trust model to collapse, then, given the specialized computational equipment currently used for Bitcoin mining, Bitcoin is significantly more secure than is implied by the analysis in Section 4 of double-spending attacks.[32] Equations (9)-(10) suggest that Bitcoin is currently secure against sabotages worth on the order of \$10 billion if the attacker comes from the outside, and for sabotages worth on the order of \$5 billion from inside attackers.

[31] Specifically, a Bitmain Antminer S19j XP has a current retail price of \$4,983 (https://www.bitmain.com/, accessed August 31, 2023) and it would take about 2.3 million of these machines to match Bitcoin's current hash rate, for a total capital cost of about \$11.5 billion at retail prices. I do not have any information on how retail prices relate to the prices paid by large-scale miners.

[32] A blog post of Joseph Bonneau (2014) is the earliest written version I am aware of of the argument that ASICs might make Bitcoin more secure. I thank Arvind Naranayan for calling my attention to it after I circulated the earlier version of this paper.

6 Collapse Scenarios

Section 5 gives a candidate answer to what we can colloquially call the "Chicago Lunch Table" question: if the analysis of Section 3 is right, why has Bitcoin not been attacked already? Our answer is the combination of (i) specialized equipment; (ii) an attack would not just steal money, but would in effect be a sabotage that causes collapse of the trust; and (iii) at present, the sabotage possibilities do not merit the cost.

Suppose, for the purpose of a speculative discussion, that this answer is correct. That is, the Bitcoin blockchain currently does satisfy the IC constraint (8) that is based on a stock cost of attack,

$$N^*C > V_{attack},$$

but does not satisfy the IC constraint (2) that is based on a flow cost of attack:

$$A^*N^*c \cdot t(A^*) < V_{attack}.$$

For a sense of magnitudes, at current levels of p_{block}, these assumptions place the value of a successful attack on Bitcoin at less than $10 billion (a rough estimate for N^*C) but greater than anywhere from about $4 million (base case) to about $250 million (very expensive case). While subjective, this seems plausible.[33]

This paper's analysis framework then suggests three possible scenarios that could precipitate a successful attack.

6.1 Attack Scenario I: Cheap-enough specialized chips

Suppose there are previous-generation ASIC chips that are not economically efficient for mining, but are powerful enough for the purpose of an attack, and exist in large quantity. Formally, let $c = rC + \eta$ denote the cost per unit time of one unit of computational power for the most efficient specialized chips, and assume that the previous-generation chips have an electricity cost per unit of computational power that on its own makes the chips cost-prohibitive for mining even if the

[33] To date, the largest 51% attack on a proof-of-work cryptocurrency has been the one on Bitcoin Gold in June 2018, for $18 million. In April 2022 there was a $182 million 51% attack on Beanstalk, an algorithmic stablecoin project. This attack was not a double-spending attack, but rather the attacker temporarily obtained >51% of the stake in the project, and then voted to adopt a change to the project's code that siphoned off all of its reserve funds to the attacker. This is not exactly a proof-of-stake consensus protocol, but has some similarities when thinking about attack vulnerability. There have been several other blockchain attacks worth >$100 million that are based on attacking faulty code, as opposed to a 51% attack. See Vigna, 2022 on the Beanstalk attack; Lovejoy, 2020 for a list of 51% attacks (including the Bitcoin Gold attack); and *rekt.news* for general exposition on a variety of recent attacks.

capital is free: $\eta' > c$.[34]

In honest-mining equilibrium, therefore, these previous-generation chips will have a market value of zero. This implies, however, that these chips could be used by an attacker if they exist in sufficient quantity: if there are at least N^* computational-units of previous generation chips with $\eta' > c$ and hence zero capital cost, then an attacker could attack at a flow cost of $N^*\eta'$.

At present, there is no reason to think that there are enough previous-generation chips for the purpose of attack. Both the quantity and efficiency of ASICs in the market have been growing dramatically.[35]

However, as the Bitcoin ASIC market matures, it seems plausible that this could change. More generally, if the security of Nakamoto trust depends on specialized chips, then Nakamoto trust is vulnerable to other kinds of changes in the chips market. For example, it is conceptually plausible that specialized ASIC chips used for SHA-256 hashing become used in a much wider variety of applications than Bitcoin. At present, large-scale use of hash power is primarily for proof-of-work cryptocurrencies, and most large cryptocurrencies have their own hash function.[36] If the use of Nakamoto blockchain becomes much more widespread than it is at present, it is at least plausible that Bitcoin's share of the world's SHA-256 hash power is much smaller than it is today. In this case, even though the chips themselves are specialized to SHA-256, they would be usable for other purposes even if Bitcoin itself were to collapse.

6.2 Attack Scenario II: Sufficient Fall in Honest Mining Rewards

Suppose there is a large decline in Bitcoin's price for reasons unrelated to this paper's analysis. Since at present most of the block reward, p_{block}, consists of newly issued Bitcoins, such a fall in Bitcoin's price would directly cause a fall in p_{block}. This, in turn, could lead to a glut of ASICs relative to the amount needed to maintain equilibrium in the honest mining market, (1).

Here is an example calculation. Suppose that we are presently in equilibrium as defined by (1), with $N^*(rC + \eta) = p_{block}$. As above, define the capital share $\mu = \frac{rC}{rC+\eta}$. Suppose the block reward declines from p_{block} to αp_{block}, where $\alpha < (1 - \mu)$. Then $N^*\eta > \alpha p_{block}$, meaning that some capital will be "mothballed" (i.e., turned off) because if all N^* units of capital are utilized then

[34] It may help to conceptualize a unit of computational power as some fixed amount of hashes per second, like 1TH/sec.

[35] On an energy-efficiency basis, Bitmain Antminer ASIC machines have improved by a factor of nearly 100x from 2013 to 2022, and by a factor of 3.5x since 2018, when the most recent generation of ASIC chips came online. On a quantity basis, Bitcoin's hash rate is up by a factor of over 10,000x since end-2013, and up by a factor of 4 since 2018.

[36] I thank Glenn Ellison for this observation and for connecting it to the theory in this paper. If cryptocurrencies A and B each use the same hash function X, and A has more hash power than B, then hash power from A can be temporarily diverted to attack B, making the cost of attacking B a flow not a stock.

mining loses money on the basis of electricity costs alone. If the decline is such that $\alpha < \frac{1-\mu}{2}$, then more than 50% of capital will be mothballed. Economically, the opportunity cost of using otherwise-mothballed equipment to attack is very low. Logistically, large amounts of mothballed equipment might make an attack easier to execute.

Additionally, the number of Bitcoins issued per block reward halves every four years. The next such halving will occur in around March 2024, to 3.125 Bitcoins. In 2032 the reward will halve to less than 1 Bitcoin, and by 2044 the reward will be less than 0.1 Bitcoin. Unless the dollar value per Bitcoin grows significantly, or transaction fees increase significantly, these halvings will cause significant drops over time in p_{block}, and hence could also cause significant amounts of mining capital to be mothballed.

In effect, a large enough fall in the reward to honest mining, whether due to a decline in the value of Bitcoin or a decline in the number of new Bitcoins issued per block or both, could cause the "cheap-enough specialized chips" envisioned in Attack Scenario I.

6.3 Attack Scenario III: Bitcoin Grows in Economic Importance (Relative to Cost)

The first two scenarios identify conditions under which the cost of attack changes from a stock cost to a flow cost, moving us from the incentive constraint (8) to the incentive constraint (3).

The other possible scenario is that Bitcoin grows in economic importance, relative to its level of compensation to miners and capital stock, to the point where the stock-cost incentive constraint (8) itself no longer holds. That is, $V_{attack} > N^*C$.

Speculatively, this seems most likely to occur if Bitcoin were to become more fully integrated into the global financial system. While $10 billion is certainly a lot of money, it is small in the scheme of global finance.

7 Comparison of Nakamoto Trust and Traditional Trust

In this section we return to the contrast discussed in the introduction between Nakamoto trust and traditional trust supported by rule of law and complementary sources, such as reputations and relationships. The essential difference is economies of scale.

7.1 Beckerian Deterrence as an Economy of Scale

For concreteness, consider a financial transaction between two parties of size V, where one of the parties has an opportunity to cheat and steal the other party's assets. Specifically, Party 1

chooses an action from the set $\{Engage,\ Don't\ Engage\}$; Party 2 chooses an action from the set $\{Honest,\ Cheat\}$; if the players choose $Engage$ and $Honest$ then both parties get a payoff of $b > 0$, representing the net benefit of transacting; but if Party 1 chooses $Engage$ and Party 2 chooses $Cheat$, then Party 2 gets a payoff of V and Party 1 gets a payoff of $-V$, representing that Party 2 has stolen Party 1's assets. If Party 1 chooses $Don't\ Engage$ then both parties get a payoff of zero. Clearly, in the game as described so far, the only equilibrium is for Party 1 to choose $Don't\ Engage$, so the parties will forego the benefit of transacting.

Now add a legal system with the power to enforce contracts. Specifically, if Party 2 plays $Cheat$, then Party 1 can pay a cost c_l to adjudicate the transaction in court, and the court can perfectly observe whether Party 2 played $Cheat$ or $Honest$. If the court observes that Party 2 played $Cheat$ it can compel Party 2 to return Party 1's assets and punish Party 2 with a large fine f. In this scenario, Party 1's payoff is $-c_l$, the cost of bringing the matter to court, and Party 2's payoff is $-f$, the cost of the fine.

Clearly, the legal system makes it an equilibrium for the parties to transact honestly; the credible threat of a large fine deters Player 2 from cheating. And, since the players will transact honestly on the equilibrium path, the court need not even involve itself with most transactions in the first place. This is the point I emphasized in the introduction about the economies of scale in traditional trust, and that seems implicit in Hayek (1960) and Becker (1968). The key insight is: *A society that pays a fixed cost of operating a court system can facilitate honest transactions that have zero marginal cost of security because of the deterrence effect. This is a scale economy for traditional trust.*

Similar scale economies of trust arise in the private sector from fixed-cost investments in brands, reputation, relationships or collateral. Often such investments work in conjunction with rule of law. For example, a firm's brand, reputation or relationships can serve as a credible commitment to provide high quality versus low quality on those dimensions of quality that are not legally contractible (Nelson, 1974; Fudenberg, Levine and Maskin, 1994; Tadelis, 1999; Baker, Gibbons and Murphy, 2002; Levin, 2003).

Collateral is a particularly important example to discuss in our context. Imagine that a bank intermediates the transaction above between Party 1 and Party 2 and has at least V of general-purpose collateral on its balance sheet. The bank can be trusted not to abscond with the parties' assets, without any appeal to reputation or brand, if a court can compel it to compensate the parties out of of its general-purpose collateral if it cheats. Moreover, the cost of general-purpose collateral as a source of trust support is low because collateral earns a market rate of return. Under the assumptions of the Modigliani-Miller theorem the cost of collateral as a source of trust support is *zero*, because collateral earns exactly the risk-adjusted market rate of return (e.g., the risk-free

rate for treasury collateral) and a firm's value is independent of its capital structure (Modigliani and Miller, 1958). Estimates from the empirical literature on the magnitudes of violations of the Modigliani-Miller theorem find that the cost of collateral is not literally zero, but is less than 1% per year of the collateral amount, which likely translates to less than 0.01% of transaction volume.[37]

7.2 Simple Mathematical Comparision

For a simple mathematical comparison of Nakamoto trust and traditional trust, consider the stripped-down model of Section 3.5 augmented as follows. First, add a parameter V_{honest} that represents the average volume transacted per period by honest users of the system if trust is secured. So, under Nakamoto trust, if the per-period payment for security p exceeds the value of attacking the system V_{attack}, then honest participants transact V_{honest} volume. Second, model traditional trust as costing a fixed cost F plus a variable cost per unit transacted of c, such that society's cost of traditional trust is $F + cV_{honest}$ per period. In Figure 1, the c can be interpreted as the cost of the security guards and the F can be interpreted as society's cost of police and courts.

The two trust models' cost per unit volume are thus:

$$Traditional\ Trust : \frac{F}{V_{honest}} + c \tag{11}$$

$$Nakamoto\ Trust : \frac{V_{attack}}{V_{honest}}$$

Traditional trust has scale economies to the extent that fixed costs that support trust can scale over a large quantity of transaction volume. Several examples have been discussed already, such as the cost of credible Becker-ian deterrence, the cost of a brand or reputation, or the cost of general-purpose financial collateral. Traditional trust also often has variable costs that do not have economies of scale per se, but, by working in complement with fixed-cost assets, can be much smaller than variable costs under Nakamoto. The security guards in Figure 1 are a conceptual illustration. I will give specific magnitudes for finance in the next subsection.

Nakamoto trust, in contrast, only enjoys economies of scale with transaction volume if the scope for attacking the system V_{attack} does not grow with the system's usefulness for honest participants V_{honest}, which seems unlikely without support from rule of law. It is also worth emphasizing that the ratio $\frac{V_{attack}}{V_{honest}}$ could easily exceed 1, as a majority attacker will engage in large transactions

[37] See Section 3.1.2 of Budish and Sunderam (2023), which estimates that, if one uses a conservative interpretation of the literature on violations of the Modigliani-Miller theorem, the magnitude of the cost of collateral as a source of trust support is about 0.5-1.0 basis points of transaction volume, i.e., 0.005-0.01% of transaction volume. So for a transaction of size V, the bank would incur a marginal cost of 0.01% times V.

whereas V_{honest} measures the size of average transactions. For example, if transaction sizes have a Pareto distribution with shape parameter 2, then the ratio of the size of 99th percentile transactions to mean transactions is 5.

7.3 Sense of Magnitudes

Total annual spending on police, prisons and courts, at the state, local and federal level, is about $300 billion per year in the United States (Urban Institute, 2023). Real value added in the U.S. financial industry is about $800 billion per year.[38] So we could use $1 trillion per year as a conservative upper bound for the cost of trust in the U.S. financial sector, since the former figure includes spending that is unrelated to finance and the latter figure includes spending that is unrelated to trust.[39] We could use $100 billion per year as a less conservative ballpark magnitude. We can use $1 quadrillion per year as a lower bound for transaction volume in the U.S. financial sector.[40] We can thus upper bound the cost of trust support $\frac{F}{V_{honest}} + c$ in traditional finance by 0.1% of transaction volume and use 0.01% as a less conservative measure. Clearly this is just a rough ballpark, but it seems credible as an order of magnitude given the figures just described. Many fees in traditional finance, especially for large transactions, are on the order of 0.01% or less of transaction volume.[41]

[38]This figure is taken from the U.S. Bureau of Economic Analysis "Real Value Added by Industry" data, lines 56-57 ("Federal Reserve banks, credit intermediation, and related activities" and "Securities, commodity contracts, and investments.") Real value added measures payments to both capital and labor. See Philippon (2015) on why it is a useful measure for the cost of the financial sector.

[39]Gennaioli, Shleifer and Vishny (2015) distinguish between financial sector trust in the sense of security from expropriation or theft and trust in the sense of confidence to take risks. They argue that high fees and market power in the financial sector, especially as relates to investment management (see Greenwood and Scharfstein, 2013), can often be intrepreted as demand for the latter kind of trust, confidence to take risks. The former kind of trust, security from theft, is the aspect I have in mind here as relevant for the comparison to Nakamoto trust.

[40]According to data from the Securities Industry and Financial Markets Association, total transaction volume in U.S. Treasury bonds is about $750bn per trading day, other fixed income securities is about $300bn per day, and equity markets is about $500bn per trading day. Together this comes to over $400 trillion per year. CME Group trading volume for its top 10 futures contracts (interest rates, equity indices and oil) is over $500 trillion per year. A New York Fed report indicates that foreign exchange volume in North America (of which over 90% involves USD) is over $300 trillion per year. ACH cleared nearly $80 trillion of cash transactions in 2022. The Fedwire funds service clears over $1 quadrillion per year. There are numerous other assets with high trading volume, such as other futures contracts, commodities, and options. So $1 quadrillion seems a reasonable magnitude for a lower bound. These figures are from Budish and Sunderam (2023), sifma.org, and cmegroup.com.

[41]For example, Hu, Pan and Wang (2021) report that the median fee charged in the treasury repo market is 2 basis points (0.02%) on an annual basis which translates to about 0.00005% per day. Interactive Brokers' fees for large foreign exchange transactions are about 0.001%. Budish, Lee and Shim (forthcoming) find that the average exchange trading fee in the U.S. stock market is $0.0001 per-share-per-side, or about 0.0001% on a $100 share of stock. Fees are higher for retail transactions but still often relatively small. Visa's annual operating expenses are less than 0.1% of their annual transaction volume and their revenue is about 0.25% of volume (Visa Annual Report, 2022). Asset management fees for hundreds of Vanguard index funds are less than 0.10% (https://investor.vanguard.com/investment-products/list/all). There are of course numerous other fees in finance that are much higher, but these tend not to be fees for trust (in the sense of security from theft, see previous

Nakamoto trust costs as studied in Section 4, while expressed in a different metric, are dramatically higher. To secure the system against attacks worth $1 billion — a drop in the bucket for traditional finance — costs over $3 trillion per year, or more than three times our upper bound for the cost of trust in traditional finance. To secure against attacks worth $100 billion — which is clearly large but still less than 3% of daily transaction volume in the U.S. financial system — costs over 4 times global GDP.

To reconcile these two sets of numbers, consider that $100 billion per year on trust in traditional finance amounts to about $50 million per hour (assuming 2000 trading hours per year), or about $100-$150 million for the amount of time it takes to conduct a double-spending attack against Nakamoto per the analysis in Table 1. Whereas, traditional finance has volume of over $500 billion per trading hour. For $50 million spent on trust per hour to support $500 billion of volume per hour under Nakamoto trust, one would need that a majority attacker can only double-spend for 0.01% of average volume, i.e., $\frac{V_{attack}}{V_{honest}} = 0.0001$, which seems highly unlikely.[42]

This discussion is not meant to be an apology for traditional finance. Fees in traditional finance are certainly large in dollar terms and as a fraction of the overall economy (see Greenwood and Scharfstein, 2013 and Philippon, 2015), and the financial sector performs poorly on many measures of public trust (Sapienza and Zingales, 2012 and Zingales, 2015). Still, the comparison versus the cost of Nakamoto trust is night and day, because traditional finance enjoys economies of scale in trust that arise from rule of law in conjunction with fixed-cost assets like collateral and reputation.

8 Conclusion

The anonymous, decentralized trust enabled by the Nakamoto (2008) blockchain, while ingenious, is *expensive*. Equation (3) says that for the trust to be meaningful requires that the flow cost of running the blockchain is large relative to the one-shot value of attacking it. In the double-spending attack considered in Section 4, the implication is that the transaction costs of the blockchain must be large in relation to the highest-value economic uses of the blockchain, which can be interpreted as a very large implicit tax — e.g., from $500 to $63,000 per transaction if the system is to be secured against $1 billion attacks, all growing linearly with the value of attack. If the system is

fn.) but fees that reflect market power over consumers (Campbell, 2006, Greenwood and Scharfstein, 2013) or consumers' willingness to pay for financial advice (Gennaioli, Shleifer and Vishny, 2015). A famous paper of Philippon (2015) estimates the cost of the financial sector as a percentage of the value of real intermediation (as opposed to transaction volume) and finds that this is about 1.5-2%.

[42]For example, in the May 2018 double-spending attack against Bitcoin Gold, the attacker double-spent $18 million against average trading volume per hour of less than $1 million that month per CoinMarketCap data. That is, $\frac{V_{attack}}{V_{honest}} = 18$.

to be secured against $100 billion attacks the cost can exceed global GDP. The argument that an attack is more expensive than this flow cost, considered in Sections 5-6, requires one to concede both (i) that the security of the blockchain relies on its use of scarce, non-repurposable technology (contra to the Nakamoto (2008) vision of "one-CPU-one-vote"), and (ii) that the blockchain is vulnerable to a sabotage attack that causes its novel form of trust to collapse. These concessions, in combination with the analysis framework of this paper, in turn imply specific collapse scenarios: if conditions change in the specialized chip market, if a fall in Bitcoin's value or mining rewards leads to enough capital being mothballed, or if Bitcoin grows economically important enough to tempt a saboteur. Overall, the results place serious economic constraints on the use of the Nakamoto (2008) blockchain.

It bears emphasis that the paper's analysis is consistent with the continued use of cryptocurrencies and the Nakamoto blockchain for black-market purposes, and more generally in use cases where users are willing to pay the high implicit costs of anonymous, decentralized trust. Rather, this paper suggests skepticism and caution about large-scale uses of cryptocurrencies and the Nakamoto blockchain by traditional global businesses or the traditional global financial system. Such entities have access to cheaper forms of trust.

Relatedly, this paper's analysis is also consistent with the usefulness of the blockchain data structure *without* Nakamoto (2008)'s novel form of trust. This is often called distributed ledger technology or a permissioned blockchain (see fn. 2 and Section 2.5.1). Indeed, what this paper highlights is that it is exactly the aspect of Bitcoin and Nakamoto (2008) that is so innovative relative to these kinds of distributed databases — the anonymous, decentralized trust that emerges from proof-of-work — that is the source of its economic limits. As one specific example, it is completely consistent with this paper's analysis that Central Bank Digital Currencies (CBDCs) could be of high economic value. CBDCs take some technical inspiration from cryptocurrencies but are anchored in traditional trust from rule of law and the reputation of central banks, and thus do not face the scaling problem of Nakamoto trust highlighted in this paper.

At a broader level, this paper builds on the view tracing all the way back to Adam Smith that government and laws are essential ingredients for the market system. A central point this paper has tried to emphasize is a fairly simple one implicit in Hayek (1960) and Becker (1968), though I have not seen it stated explicitly in this language, which is that traditional trust supported by rule of law enjoys economies of scale. Society pays the fixed cost of the apparatus of rule of law, or firms pay the fixed cost of building a brand or reputation or holding collateral (each of which works in conjunction with laws), and these fixed cost assets can provide trust over a large number of economic activities at low or zero marginal cost.

I want to close with two directions for future research given the message of this paper. First,

and most directly, is there a "solution" to this paper's critique of Bitcoin and Nakamoto trust? Informally, is there a way to generate trust in a public dataset (or, specifically, a cryptocurrency) that has some of the anonymity and decentralization aspects of Nakamoto while being signficantly less economically constrained by the arguments in this paper?[43] Appendix A describes several of the responses this paper has received since it first circulated in 2018. The most promising responses combine blockchain-based trust with traditional trust in some way. For example, Ethereum's new protocol algorithmically mimics the combination of collateral plus rule of law to punish small attackers but must rely on some external source of trust support to punish large attackers. If large sums of money were in dispute, it seems obvious that this external source of trust support would involve rule of law.[44] Another interesting response is to concede that blockchain trust is intrinsically very expensive, per this paper's argument, but to only use it for occasional large transactions with long escrow periods ("Layer 1"), while most transactions are conducted off chain ("Layer 2"), supported by external sources of trust. An open conceptual question about these responses is what the permissionless consensus part adds given the external sources of trust support.

The second direction for future research is a broader conceptual question: How should economists model trust that comes from a combination of technology and rule of law? More generally, how should economists understand trust when it comes from multiple sources in the same transaction, that work in complement with each other?[45] This is often the case in practice, with trust arising from some combination of rule of law, reputations, relationships, brands, collateral, norms, technology, etc., often implicitly and without drawing notice. Consider the completely ordinary transaction of buying a cup of coffee at the local coffee shop. The consumer trusts the coffee shop to provide quality coffee because of reputational incentives, and perhaps implicitly food-safety laws. The coffee shop trusts the consumer's payment if cash because counterfeiting is technologically complex and illegal, and if electronic because of traditional cryptography and because

[43] One formalization of this question is in recent work of Lewis-Pye, Roughgarden and Budish (2023). They define "economically secure permissionless consensus" as a permissionless consensus protocol that makes it expensive to attack in the sense of costing a stock not a flow (the attacker loses some of their capital like in (8) as opposed to (3)) without honest participants suffering impairment of their capital in any way (as occurs if the system collapses). See Section 2.5.3 and Appendix A.1 for further discussion.

[44] See the discussion in Section 2.5.3 and Appendix A.1 for details. This external source of trust support is sometimes called "social consensus," but it seems unlikely that the process can be purely social if large sums are under dispute. Would Goldman Sachs defer to the outcome of a Vitalik Buterin Twitter poll?

[45] One simple preliminary exploration of this question is in Section 4.1 of Budish and Sunderam (2023) who conceptualize trust as getting to cooperate-cooperate in a prisoner's dilemma (as discussed in La Porta et al., 1997), and model (i) technology as eliminating some actions from the possibility set, (ii) law as changing the payoffs to some actions via punishment, and (iii) reputation as the differential incentive to cooperate if play is repeated versus one-shot (as in the traditional folk theorem arguments of Aumann (1959), Fudenberg, Levine and Maskin, 1994 and others). This approach seems a useful first step but does not seem to capture the richness of what we might call "multi-layered trust."

the financial intermediary has reputational, relational, and legal reason to follow through. The employee trusts their employer to follow through with promised compensation because of laws and the implicit relational contract. Both the customer and the employee trust the other not to rob them because of laws and social norms. All this trust for a simple cup of coffee!

As one appreciates how many different sources of trust work together for even the most ordinary of economic transactions, it is hard not to regard the traditional market system with a sense of wonder. I hope that this paper has persuaded the reader that a blockchain-based economy without rule of law is not a viable alternative.

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Appendix

A Discussion of Responses to this Paper's Argument

This paper first circulated in shorter form in June 2018. I received a lot of comments and counter-arguments in response to the paper's main line of argument.

I have tried to handle the central line of counter-argument throughout the main text of this updated draft. This is the point made by Huberman, Leshno and Moallemi (2021) and many practitioners that we should compare Bitcoin's costs to the costs of market power in traditional finance, which are also high.[46] I hope the present draft of the text makes more clear the conditional nature of the paper's argument: if Bitcoin becomes more economically useful, then it will have to get even more expensive, linearly, or it will be vulnerable to attack. I hope as well that the more explicit computational simulations, for varying levels of V_{attack} all the way up to $100 billion, make clear that the way Bitcoin's security cost model scales is importantly different from how costs scale for traditional finance protected by rule-of-law.

In this appendix I discuss several of the other most common comments and counter-arguments I have received about this paper since it was first circulated.

A.1 Ethereum Proof-of-Stake with Slashing as an Attempt to Mimic Collateral Plus Rule of Law

This sub-section expands on the discussion of proof-of-stake in the main text in Section 2.5.3.

In its simplest form, proof-of-stake is vulnerable to the exact same critique as proof-of-work. Just conceptualize c as the per-block opportunity cost of locking up stake for validation. Equations (1)-(3) go through virtually unchanged.

However, the use of stakes rather than computational work opens up possibilities for punishing attackers that do not exist in Nakamoto (2008). This is for two reasons. First, stakes are locked on chain, somewhat analogously to collateral, whereas the computers used for proof-of-work blockchains exist off chain in the physical world. Second, the use of stakes opens up the use of a different paradigm for permissionless consensus called Byzantine Fault Tolerance (BFT). Very roughly speaking, all locked-up stake explicitly signs all transactions, and for a transaction to be confirmed, 2/3 of locked-up stake must sign it. Therefore, in the event of a double-spending attack in which two blocks sending the same funds to alternative destinations are confirmed, at least 1/3

[46] See Philippon (2015) and Greenwood and Scharfstein (2013) on high costs of traditional finance, and see Cochrane (2013) for a counterpoint.

of the total stake (i.e., 2/3 + 2/3 - 1) must have signed conflicting transactions. This creates algorithmically observable proof that an attacker has misbehaved, which can be the grounds for the algorithmic confiscation of the attacker's capital — analogously to the use of rule of law. This is a key reason why Ethereum, the second largest cryptocurrency after Bitcoin, adopted proof-of-stake starting in September 2022 (Buterin, 2014*b*; Buterin, 2016; Buterin, 2020; Buterin and Griffith, 2019).

This approach would be a compelling response to the issues raised in this paper if it works, because it would make the cost of a double-spending attack a stock not a flow without needing the whole system of trust to collapse. Mathematically, if we denote C the value of a unit of stake, c the opportunity cost of stake per unit time, and N^* the equilibrium level of honest stake (i.e., $N^*c = p_{block}$ as in (1)), then an attacker from the outside will have at least $\frac{N^*C}{2}$ of capital confiscated and an attacker from the inside will have at least $\frac{N^*C}{3}$ of capital confiscated. At recent levels of N^*C for Ethereum, this is about \$10-\$15 billion. So a double-spending attack against Ethereum is similarly expensive to the sabotage attacks against Bitcoin studied in Section 5, but *without the premise that an attack would collapse the system.* This is clearly an important improvement if it works.

Unfortunately, as previewed in the main text in Section 2.5.3, recent research suggests that this approach may not work — at least on its own without external support from rule of law. The subtle issue is how do you ensure that the attacker's stake is confiscated before they can withdraw and spend it — keeping in mind that the attacker is large, so can control the blockchain including potentially its adjudication of the attacker's own crime. First, Tas et al. (2023) show an impossibility result for what they call "slashable safety", which requires that the double-spend attacker's stake is slashed before the attacker can withdraw it from the system. Their result holds for any size attacker and any strictly-positive amount of slashing. Second, Lewis-Pye, Roughgarden and Budish (2023) make a stronger assumption about the nature of possible network outages and latencies, and specifically assume that it is possible to define a finite amount of time that is long enough to ensure that any network outage repairs by then. Under this assumption, they show that slashing an attacker's stake can work if and only if the attacker's majority is bounded by 5/9 of the total stake.[47] If the attacker can have more than 5/9 of the total capital then the attacker can both double spend and prevent the protocol's version of a legal system from punishing him. An interpretation of this result is that the Ethereum blockchain can mimic the combination of

[47]The intuition for where the 5/9 bound comes from is as follows. As described above, at least 1/3 of the total stake must have signed conflicting transactions to double spend. So an attacker with 5/9 would be left with 5/9 - 1/3 = 2/9 of the total stake post-slashing, while honest participants would have the other 4/9. So, post-slashing, the attacker would have weakly less than 1/3 of the post-slashing stake, which is the critical threshold for the usual positive results for BFT consensus to come into play (see citations in Section 2.3).

collateral plus rule of law to deter small attackers, but must rely either on a collapse argument or traditional rule of law external to the blockchain to punish large-enough attackers.

A separate issue is that the use of the BFT consensus paradigm creates a vulnerability to what are known as "liveness" attacks, as distinct from "safety" attacks like double spending. Tradeoffs between safety and liveness properties are commonplace in the literature on distributed consensus protocols. The issue is that since in BFT consensus 2/3 of all stake must sign a transaction for it to be finalized, an attacker can halt consensus for a long period of time by simply refusing to sign transactions. Importantly, in the case of a liveness attack, unlike a double-spending attack, the attacker has not observably done anything wrong like sign conflicting transactions. They could just be having a network outage. For this reason, confiscating stake for being silent is controversial and Ethereum has chosen to do so only very slowly. Using Ethereum's slashing formula as of its Fall 2022 switch to proof-of-stake, I compute that if the total amount of stake on Ethereum is $25 billion, an attacker could silence Ethereum for an hour for $13,000, a day for $7 million, or a full week for $388 million. If Ethereum became a significant component of the global financial system, these costs of shutting it down seem trivial to a motivated saboteur.

I conclude that Ethereum's new approach to permissionless consensus does achieve an important security improvement relative to Nakamoto (2008) and this improvement can be understood through the lens of this paper. Under plausible assumptions about network latency, the combination of proof-of-stake and slashing can make double-spending attacks by a sub-5/9 attacker economically expensive — i.e., cost a stock not a flow — without reliance on a collapse argument. However, given the vulnerability to larger attackers and to liveness attacks, it remains difficult to see how Ethereum or others like it can be secure on their own without rule of law as an external source of support. As noted in the conclusion, a very interesting question for future research is how conceptually to think about combinations of trust from technology and trust from rule of law. What do combinations of the two make economically possible that is not feasible with either alone?

A.2 Community

As noted above in Section 5, a majority attack on Bitcoin, or any other major cryptocurrency, would be widely noticed. A line of argument I heard frequently in response to the June 2018 draft is that the Bitcoin community would organize a response to the attack. For example, the community could organize a "hard fork" off of the state of the blockchain just prior to the attack, which would include all transactions perceived to be valid, void any perceived-as-invalid transactions, possibly confiscate or void the attacker's other Bitcoin holdings if these are traceable, and possibly change the hash function or find some other way to ignore or circumvent the attacker's majority of compute

power.[48]

The community response argument seems valid as an argument that attacks might be more expensive or difficult to execute than is modeled here, but it raises two important issues.

First, and most obviously, the argument contradicts the notion of anonymous, decentralized trust. It relies on a specific set of trusted individuals in the Bitcoin community.

Second, consider the community response argument from the perspective of a traditional financial institution. In the event of a large-scale attack that involves billions of dollars, the traditional financial institution would, in this telling, be left in the hands of the Bitcoin community. At present, reliance on a tight-knit community of those most invested in Bitcoin (whether financially, intellectually, etc.), may sound reassuring — those with the most to lose would rally together to save it. But now imagine the hypothetical future in which Bitcoin becomes a more integral part of the global financial system, and imagine there is a fight over whether an entity like a Goldman Sachs is entitled to billions of dollars worth of Bitcoin that it believes was stolen — but the longest chain says otherwise. Will the "vampire squid" be made whole by the "Bitcoin community?" Quite possibly, but one can hopefully see the potential weakness of relying on an amorphous community as a source of trust for the global economic and financial system.

A.3 Rule of Law

A related line of argument is that, in the event of a large-scale attack specifically on a financial institution such as a bank or exchange, rule of law would step in. For example, the financial institutions depicted as the victims of a double-spend in Figure 3, once they realize they no longer have the Bitcoins paid to them because of the attack, would obtain help from rule-of-law tracing down the attacker and recovering the stolen funds.

This response, too, seems internally valid while contradicting the idea of anonymous, decentralized trust. It also seems particularly guilty of wanting to "have your cake and eat it too." In this view, cryptocurrencies are mostly based on anonymous, decentralized trust — hence evading most forms of scrutiny by regulators and law enforcement — but, if there is a large attack, then rule-of-law will come to the rescue.

A.4 Counterattacks

Moroz et al. (2020) extend the analysis in Budish (2018) to enable the victim of a double-spending

[48]The phrase "hard fork" means that in addition to coordinating on a particular fork of a blockchain if there are multiple — in this case, the attacker's chain, which is the longest, and the chain the community is urging be coordinated on in response — the code used by miners is updated as well. This could include hard-coded state information such as the new chain or information about voided Bitcoins held by the attacker, code updates such as a new hash function, etc.

attack to attack back. They consider a game in which there is an Attacker and a Defender. If the Attacker double spends against the Defender for v dollars, the Defender can then retaliate, themselves organizing a 51% or more majority, to attack back so that the original honest chain becomes the longest chain again. This allows the Defender to recover their property.

For example, suppose the escrow period is 6, denote the initial double-spend transaction as taking place in block 1, and suppose the attacker chain replaces the honest chain as soon as the escrow period elapses, as in Figure 3. Notationally, suppose the honest chain consists of blocks {1, 2, ..., 7} at the time the honest chain is replaced, and the attacker chain that replaces it is {1', 2', ..., 7', 8'}. If the Defender can quickly organize a majority of their own, then they can build off of the {1, 2, ..., 7} chain, and eventually surpass the attacker chain, recovering their property. For example, maybe the honest chain reaches block 10 before the Attacker chain reaches block 10', so then {1, 2, ..., 10} is the new longest chain and the Defender has their property back from the correct transaction in block 1.

This argument is game theoretically valid, and indeed there are theoretical subtleties to the argument that the reader can appreciate for themselves in the paper. That said, it relies on every large-scale participant in the Bitcoin system being able and willing to conduct a 51% attack on a moment's notice. This is kind of like requiring every major financial institution to have not just security guards, but access to a standing army.

A.5 Modification to Nakamoto I: Increase Throughput

Bitcoin processes about 2000 transactions per block, which is about 288,000 per day or 105 million per year. In contrast, Visa processes about 165 billion transactions per year (Visa, 2021).

The reader will notice that the logic in equations (1)-(3) does not depend directly on the number of transactions in a block. If the number of transactions in a Bitcoin block were to increase by 1000x (to roughly Visa's level), then the required p_{block} to keep Bitcoin secure against a given scale of attack V_{attack}, per equation (3) would not change. Thus, the required cost *per transaction* to keep Bitcoin secure against a given scale of attack would decline by a factor of 1000.

In this scenario of a 1000x throughput increase, Bitcoin's security costs per transaction are still large, but less astonishingly so. In the base case, to secure Bitcoin against a $1 billion attack would require costs per transaction of $31 instead of $31,000. To secure against a $100 billion attack would require costs per transaction of $3,100 instead of $3.1 million.

A subtlety is that as the number of transactions per block grows, so too might the scope for attack. That is, V_{attack} might grow as well.

Still, this seems a promising response to the logic of this paper. A particularly interesting variation on this idea is the paradigm called "Level 2." In this paradigm, the Bitcoin blockchain

("Level 1") would be used for relatively large transactions, but smaller transactions would be conducted off-chain, possibly supported with traditional forms of trust, with just occasional netting on the main Bitcoin blockchain. In this paradigm, as well, the large transactions on chain could also have a long escrow period, making attacks more expensive.[49]

A.6 Modification to Nakamoto II: Tweak Longest-Chain Convention

The discussion above in A.2 expressed skepticism about the "community" response to the logic of this paper. However, what about modifying the longest-chain convention to try to encode what the community would *want* to do in the event of an attack.

The modification to the longest-chain convention could take advantage of two specific features of double-spending attacks:

1. The Attacker has to sign transactions both to the victim of the double-spending attack — call this the Bank — and to another account they control — call this the Cousin account. The fact that there are multiple-signed transactions for the same funds is an initial proof that something suspicious has happened.
2. The Attacker has to make the signed transaction to the Bank public significantly before — in "real-world clock time" — the signed transaction to their Cousin account.

The difficulty with just using facts #1 and #2 to void the transaction to the Cousin is alluded to with the phrase "real-world clock time." Part of what the Nakamoto (2008) blockchain innovation accomplishes is a sequencing of data that does not rely on an external, trusted, time-stamping device.

Relatedly, the difficulty with just using fact #1 and having the policy "if there are multiple correctly signed transactions sending the same funds, destroy the funds" is that the victim of the double-spending attack, the Bank, will by now have sent real-world financial assets to the Attacker — and this transaction, in the real world (off the blockchain), cannot be voided no matter how we modify the blockchain protocol. A different way to put the concern is that such a policy would allow any party that sends funds on the blockchain in exchange for goods or financial assets off the blockchain, to then void the counterparty's received funds after the fact. This seems a recipe for sabotage of the traditional financial sector.

The open question, then, is whether the protocol can be modified so that in the event of fact #1, multiple signed transactions, there is some way to appeal to fact #2, grounded in the sequencing of events in real-world clock time, not adjudicated by the longest-chain convention's determination of the sequence of events.

[49] I thank Neha Narula for several helpful conversations about this approach.

One pursuit along these lines is Leshno, Pass and Shi (in preparation). Their approach, which they call "Stubborn Nakamoto", is fully secure against double-spending attacks but, instead, has to permanently halt in response to observing conflicting transactions. In consensus terminology, it trades a security problem for a liveness problem. In conjunction with a source of external trust support, such as rule of law, to restart the system in case of such an outage, this could work. The open conceptual question then becomes what the permissionless consensus part adds given the source of external trust support (i.e., the same question asked in the Conclusion and A.1).

B Double-Spending Attack Technical Appendix

B.1 Proof of Proposition 5 (Closed-Form Expression for Duration of Double-Spending Attack)

Let $s = 0$ denote the time of the last block prior to the attack. As a reminder, time is normalized so that one unit of time is the amount it takes on average for honest miners to mine one block, e.g., 10 minutes for Bitcoin.

The attacker spends Bitcoins in exchange for other goods or assets in the honest miners' first block after time 0. In parallel, the attacker mines an alternative chain starting from the last block prior to the attack.

Honest miners mine blocks as a Poisson process with rate 1, and the attacker mines at rate $A > 1$. Both the honest miners' and the attacker's chains are time-independent Poisson processes, with:

$$B_H(s) \coloneqq \text{Number of blocks on honest chain at time } s,$$
$$B_A(s) \coloneqq \text{Number of blocks on attacker chain at time } s.$$

The attack is completed when both (i) the honest chain has mined at least $1+e$ blocks, therefore passing the attacker transaction's escrow period, and (ii) the attacker chain has mined strictly more blocks than the honest chain. Therefore, the expected duration of the double-spending attack, as a function of the attacker majority A and escrow period e, is given by the stopping time formula:

$$t(A, e) = E[\inf\{s : B_H(s) \geq 1 + e,\, B_A(s) > B_H(s)\}].$$

It will be useful to define a random variable that denotes the time at which the honest chain

completes the escrow period. Call this S_H^{1+e}:

$$S_H^{1+e} := \inf\{s : B_H(s) \geq 1 + e\}.$$

Similarly, it will be useful to define the difference in length between the honest chain and the attacker chain at the random time at which the honest chain completes the escrow period. Call this D^{1+e}:

$$\begin{aligned} D^{1+e} &:= B_H(S_H^{1+e}) - B_A(S_H^{1+e}) \\ &= (1 + e) - B_A(S_H^{1+e}). \end{aligned}$$

If the realization of $D^{1+e} < 0$, the attacker chain is strictly longer than the honest chain at the conclusion of the escrow period, and the attacker immediately completes the double-spending attack. The total duration of attack is simply the time elapsed in completing the escrow period.

Else, if the realization of $D^{1+e} \geq 0$, the attacker faces a deficit and must continue the attack after the conclusion of the escrow period. In this case, the total duration of attack is the length of the escrow period plus the time it takes for the attacker to overcome the deficit. Note, if the attacker deficit is i blocks, to overcome the deficit the attacker must mine $i + 1$ more blocks than the honest miners, as the attacker chain must be strictly longer than the honest chain to complete the attack.

Hence, we can partition $t(A, e)$ based on the sign of D^{1+e} for a tractable expression for $t(A, e)$:

$$\begin{aligned} t(A, e) &= E[S_H^{1+e} | D^{1+e} < 0)] \times P(D^{1+e} < 0) \\ &\quad + \sum_{i=0}^{1+e} \Big(E[S_H^{1+e} | D^{1+e} = i] + E[\text{Time for attacker to overcome deficit} = i] \Big) \times P(D^{1+e} = i) \\ &= E[S_H^{1+e}] + \sum_{i=0}^{1+e} E[\text{Time for attacker to overcome deficit} = i] \times P(D^{1+e} = i). \end{aligned}$$

The second equality follows from the law of total probability, $\sum_{k=-\infty}^{1+e} E[S_H^{1+e} | D^{1+e} = k] \times P(D^{1+e} = k) = E[S_H^{1+e}]$. Now, there are three terms left to simplify: $E[S_H^{1+e}]$, $E[\text{Time for attacker to overcome deficit} = i]$, and $P(D^{1+e} = i)$.

Consider the first term, $E[S_H^{1+e}]$. A well-known property of Poisson processes is that arrivals are distributed according to the Gamma distribution, $S_H^{1+e} \sim Gamma(1 + e, 1)$. This Gamma distribution has a simple expression for its mean:

$$E[S_H^{1+e}] = 1 + e.$$

Now consider the second term, $E[\text{Time for attacker to overcome deficit} = i]$. Via the Markov property, we know this random variable does not depend on *when* the honest chain finishes the escrow period, only the deficit itself. So, consider the stochastic process:

$$\begin{aligned} D_{i+1}(s) &:= \overline{B_H}(s) - \overline{B_A}(s) \\ &= \text{Difference between (auxiliary)} \\ &\quad \text{honest and attacker chains at } s. \\ \overline{B_H}(0) &= i+1 \\ \overline{B_A}(0) &= 0 \end{aligned}$$

That is, start two auxiliary honest and attacker chains at $s = 0$, but initialize the difference between the length of the two chains to be $i + 1$, as the attacker must overcome a deficit of i. The stochastic movement of this difference process can be thought of as an $M/M/1$ queue, where 'arrivals' are blocks on the honest chain, and 'departures' are blocks on the attacker's chain. We want the time it takes the difference process $D_{i+1}(s)$ to reach 0 – i.e., how long it takes the attacker to overcome the deficit i. In the queueing literature, this is known as the "first passage time" of a queue, $\text{FPT}(i+1) \coloneqq \inf\{s : D_{i+1}(s) = 0\}$. The mean of the first passage time of the $M/M/1$ queue is $E[\text{FPT}(i+1)] = \frac{i+1}{A-1}$ (equation 41 in Bailey, 1957). Hence,

$$E[\text{Time for attacker to overcome deficit} = i] = \frac{i+1}{A-1}.$$

Finally, consider the term $P(D^{1+e} = i)$. Recall D^{1+e} is the difference between the honest and attacker's chains' length at the time the honest chain completes the escrow period. Hence, we can write:

$$\begin{aligned} \{D^{1+e} = i\} &= \left\{B_H(S_H^{1+e}) - B_A(S_H^{1+e}) = i\right\} \\ &= \left\{(1+e) - B_A(S_H^{1+e}) = i\right\} \\ &= \left\{B_A(S_H^{1+e}) = 1 + e - i\right\}. \end{aligned}$$

Thus, we want to find $P\left(B_A(S_H^{1+e}) = 1 + e - i\right)$. To proceed, we first find the probability $P(B_A(r) = k)$ for any realization r of the random escrow length S_H^{1+e} and any possible value of the attacker chain length k as of the time the honest chain completes the escrow period. Then, we will integrate over all possible realizations of r according to the probability distribution of S_H^{1+e}. The attacker's

chain is $Poisson(A)$ and S_H^{1+e} is distributed $Gamma(1+e,1)$, so that:

$$\begin{aligned} P(B_A(S_H^{1+e})=k) &= \int_0^\infty P(B_A(r)=k \mid S_H^{1+e}=r)\cdot P(S_H^{1+e}=r)\,dr \\ &= \int_0^\infty \frac{(Ar)^k\cdot\exp(-Ar)}{k!}\cdot\frac{r^e\cdot\exp(-\mathrm{r})}{\Gamma(1+e)}\,dr \\ &= \frac{A^k}{k!\,e!}\cdot\frac{\Gamma(k+e)}{(1+A)^{k+e}}\int_0^\infty r\cdot\frac{(1+A)^{k+e}\cdot r^{k+e-1}\cdot\exp(-(1+A)r)}{\Gamma(k+e)}\,dr \\ &= \frac{(k+e)!}{k!\,e!}\left(\frac{A}{1+A}\right)^k\left(\frac{1}{1+A}\right)^{1+e}. \end{aligned}$$

The second equality exploits the independence of $B_A(s)$ and S_H^{1+e} (inherited from the independence of B_A and B_H) and substitutes the expressions for the respective Poisson and Gamma densities. The third equality moves terms out of the integral and multiplies and divides by $\frac{\Gamma(k+e)}{(1+A)^{k+e}}$, so that the integrand is exactly the first moment of $Gamma(k+e,1+A)$. The expression for the mean is well known: $\frac{k+e}{1+A}$. The fourth equality substitutes this expression and simplifies. Hence, plugging in $k=1+e-i$, the probability of an attacker deficit i at the time the honest chain completes the escrow period is:

$$P(D^{1+e}=i)=\frac{(1+2e-i)!}{(1+e-i)!\,e!}\left(\frac{A}{1+A}\right)^{1+e-i}\left(\frac{1}{1+A}\right)^{1+e}.$$

Substituting these three expressions into that of $t(A,e)$, we have

$$t(A,e)=(1+e)+\left[\sum_{i=0}^{1+e}\left(\frac{i+1}{A-1}\right)\cdot\frac{(1+2e-i)!}{(1+e-i)!\,e!}\left(\frac{A}{1+A}\right)^{1+e-i}\left(\frac{1}{1+A}\right)^{1+e}\right]$$

obtaining expression (4) in the text as required.

To complete the proof let us consider the limits as $A\to_+ 1$ and $A\to\infty$. Define $f(A,e)$ as the bracketed expression above,

$$f(A,e)\equiv\left[\sum_{i=0}^{1+e}\left(\frac{i+1}{A-1}\right)\cdot\frac{(1+2e-i)!}{(1+e-i)!\,e!}\left(\frac{A}{1+A}\right)^{1+e-i}\left(\frac{1}{1+A}\right)^{1+e}\right],$$

such that $t(A,e)$ takes the form:

$$t(A,e)\equiv(1+e)+f(A,e).$$

First, consider the limit $\lim_{A\to\infty} t(A,e)$. Observe that each term in $f(A,e)$ either goes to 0 or is bounded by a constant. The first and fourth terms go to 0 in the limit: $0\le\lim_{A\to\infty}\left(\frac{i+1}{A-1}\right)\le$

$\lim_{A\to\infty}\left(\frac{2+e}{A-1}\right)=0$ and $\lim_{A\to\infty}\left(\frac{1}{1+A}\right)^{1+e}=0$. The second and third terms are bounded by a constant: $\frac{(1+2e-i)!}{(1+e-i)!\,e!}$ is constant in A and $\lim_{A\to\infty}\left(\frac{A}{1+A}\right)^{1+e-i}\leq 1$. Hence, the product of these terms is 0 in the limit, so $\lim_{A\to\infty} t(A,e)=(1+e)+0=1+e$ as desired.

Second, consider the limit $\lim_{A\to_+1} t(A,e)$. The first term in $f(A,e)$ goes to ∞ in the limit while all other terms are strictly positive and bounded below. Formally, for the first term, $\lim_{A\to_+1}\left(\frac{i+1}{A-1}\right)\geq \lim_{A\to_+1}\left(\frac{1}{A-1}\right)=\infty$. For the other terms: $\frac{(1+2e-i)!}{(1+e-i)!\,e!}>0$ is constant in A; $\lim_{A\to_+1}\left(\frac{A}{1+A}\right)^{1+e-i}=\left(\frac{1}{2}\right)^{1+e-i}>0$; and $\lim_{A\to_+1}\left(\frac{1}{1+A}\right)^{1+e}=\left(\frac{1}{2}\right)^{1+e}>0$. Hence, the product of these terms goes to infinity in the limit, so $\lim_{A\to_+1} t(A,e)=\infty$ as desired.

B.2 Numerical Analysis of Cost-Minimizing Attacker Majority

The gross cost of attack, for an attacker with majority $A>1$ and an attack that takes t time in expectation, is defined as $At\cdot N^*c$. Proposition 5 provides an explicit formula for $t(A,e)$, the expected duration of a double-spending attack as a function of the attacker majority A and the escrow period e.

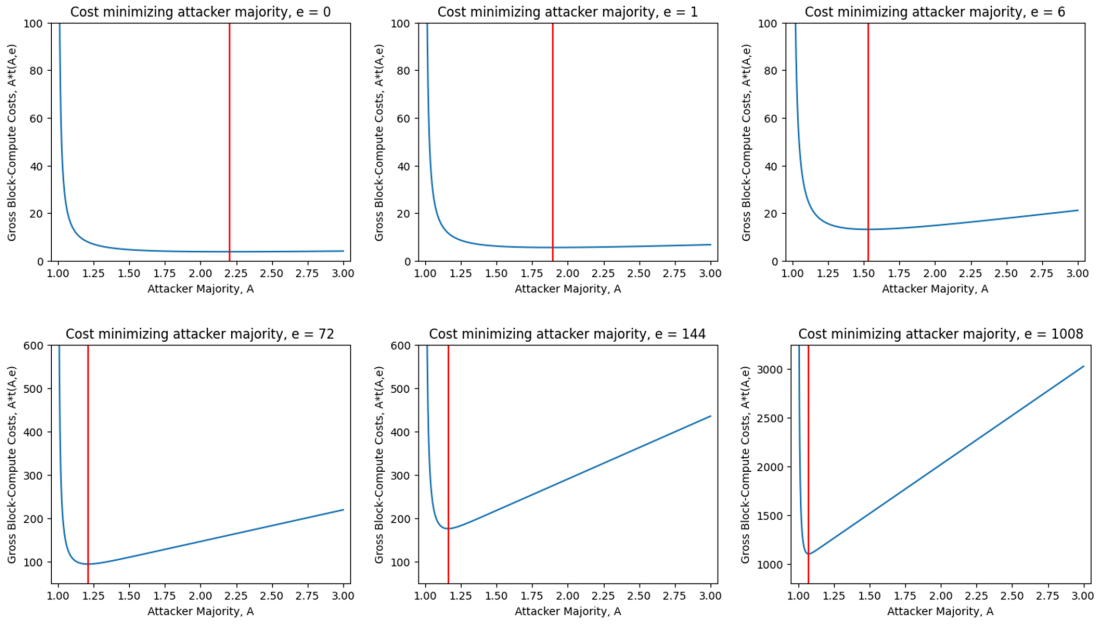
In this section of the Appendix, we use this definition and formula to numerically study the cost-minimizing attacker majority A as a function of the escrow period e.

Formally, the gross-cost-minimization problem is given by:

$$\begin{aligned}A^*(e)&:=\arg\min_A A\cdot t(e,A)\\&=\arg\min_A A\cdot(1+e)+A\cdot\left[\sum_{i=0}^{1+e}\left(\frac{i+1}{A-1}\right)\cdot\frac{(1+2e-i)!}{(1+e-i)!\,e!}\left(\frac{A}{1+A}\right)^{1+e-i}\left(\frac{1}{1+A}\right)^{1+e}\right].\end{aligned}$$

While this minimization problem is not analytically tractable, it is straightforward to solve numerically. Figure 4 plots the cost of attack for a variety of escrow periods, as well as the cost-minimizing $A^*(e)$.

Figure 4: Attacker Gross Cost Minimization



Notes: The gross cost of attack as a function of majority A is in blue, plotted as $A \cdot t(A, e)$. As discussed in the main text, this quantity needs to be multiplied by equilibrium per-block compute costs N^*c to obtain gross costs in dollars. The gross-cost-minimizing attacker majority $A^*(e)$ is denoted in red, and is obtained via scipy.optimize.minimize_scalar, a numerical solver in Python.

Intuitively, an attacker majority that is too large will mine more blocks than is necessary for the attack to succeed, whereas an attacker majority that is too close to $A \approx 1$ will, as shown in Proposition 5, have an attack duration that converges to infinity, and hence also be more expensive than is optimal. Because the double-spending attack must be at least as long as the escrow length, the cost-minimizing choice of $A^*(e)$ decreases as the escrow length increases. The longer the escrow length is, the more sure a large majority is to mine more blocks than is necessary, by simple law-of-large numbers reasoning.

Table 4 provides the cost-minimizing majority $A^*(e)$, the duration of attack at this attacker majority $t(A^*(e), e)$, and the total gross cost of attack at this attacker majority $A^*(e) \cdot t(A^*(e), e)$ for a variety of escrow periods.

Table 4: Optimal Attacker Majority, Duration and Compute Costs

	$e = 0$	$e = 1$	$e = 6$	$e = 72$	$e = 144$	$e = 1008$
$A^*(e)$	2.21	1.89	1.53	1.21	1.16	1.07
$t(A^*(e), e)$	1.70	2.92	8.57	77.53	151.2	1,024.4
$A^*(e) \cdot t(A^*(e), e)$	3.74	5.53	13.14	93.88	175.6	1,100.1

Notes: $A^*(e)$ is solved numerically as described in the text. The expected duration of attack then follows from Proposition 5. Gross compute costs are in units of equilibrium per-block compute costs N^*c.

As before, the duration of attack is given in block units of the honest miner, and the total gross cost of attack is given in compute units (N^*c). Note that even very large escrow periods induce a cost-minimizing majority larger than 51% — for example, an escrow period of 1000 blocks induces an optimal attacker majority of $A = 1.07$, or 51.7%.