US PCE Inflation

Source: Bureau of Economic Analysis & authors’ calculations.
Real Consumer Expenditure

Source: Bureau of Economic Analysis & authors’ calculations.
Production and Imports

(a) Real Gross Output

Real Quantity Index (2017Q1=1)


Goods Services

(b) Real Imports

Real Quantity Index (2017Q1=1)


Consumer Goods (ex. autos) Industrial Materials & Supplies

Source: Bureau of Economic Analysis & authors’ calculations.
Import Prices

Source: Bureau of Economic Analysis & authors’ calculations.
Constraints in the Supply Chain

1. Did supply chain constraints trigger the inflation surge?
   - What is the nature of the constraints?
     - Potentially-binding capacity constraints on firm output.
     - Binding constraints produce non-linear outcomes.
   - Are constraints domestic or international in scope?
     - Much attention on import constraints.

2. What role for shocks to demand (including monetary policy) vs. supply in explaining inflation?
   - Did high demand exhaust existing capacity?
     - Or, did negative supply-side shocks reduce capacity?
   - Did binding constraints amplify the impacts of other shocks?
Framework Overview

Multisector, New Keynesian, small open economy.

- Continuum of firms under monopolistic competition in each sector.
- Standard CES demand and production structure.
  - Representative consumer; separable consumption/leisure preferences. Nested CES preferences across sectors and home/foreign goods.
  - Firms use labor, home inputs, and foreign inputs to produce. And there are input-output linkages across sectors.

- Pricing assumptions:
  - Dollar invoicing for imports and exports.
  - Rotemberg adjustment costs for output prices.
  - Flexible wages [extension with wage rigidity to come].

- Complete international financial market.
- Taylor-type rule with inertia and policy shocks. [ZLB details later.]

The Twist: potentially binding constraints for foreign & domestic firms.
Pricing Problem for Home Firms

Suppress sector & end use notation for clarity.

Firm $\omega$ sets $P_t(\omega)$ to solve:

$$\max_{\{P_t(\omega)\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \frac{S_{0,t}}{P_t} \left[ (P_t(\omega) - MC_t(\omega)) Y_t(\omega) - \Phi(P_{t-1}(\omega), P_t(\omega)) \right]$$

s.t. $Y_t(\omega) = \left( \frac{P_t(\omega)}{P_{Ht}} \right)^{-\varepsilon} Y_t$

and $Y_t(\omega) \leq \bar{Y}_t$

with $\Phi(P_{t-1}(\omega), P_t(\omega)) \equiv \frac{\phi}{2} \left( \frac{P_t(\omega)}{P_{t-1}(\omega)} - 1 \right)^2 P_{Ht} Y_t$. 
Pricing in Symmetric Equilibrium

Optimal Pricing:

\[ 0 = 1 - \varepsilon \left(1 - \frac{MC_t + \mu_t}{P_{Ht}}\right) - \phi (\Pi_{Ht} - 1) \Pi_{Ht} \]

\[ + E_t \left[ \frac{S_{t,t+1}}{\Pi_{t+1}} \phi (\Pi_{Ht+1} - 1) \Pi_{Ht+1}^2 \frac{Y_{t+1}}{Y_t} \right] \]

with \( \Pi_{Ht} \equiv P_{Ht}/P_{H,t-1} \).

Complementary Slackness Condition:

\[ \mu_t \left[ \bar{Y}_t - Y_t \right] = 0 \]

plus \( \mu_t \geq 0 \) and \( Y_t \leq \bar{Y}_t \).

Slack constraint \( \Rightarrow \mu_t = 0 \Rightarrow \) usual domestic price Phillips Curve holds.

Binding constraint \( \Rightarrow Y_t = \bar{Y}_t \Rightarrow \) price determined by demand.
Phillips Curves

Adding notation: \( s \in \{1, \ldots, S\} \) and \( u \in \{C, M\} \):

\[
\pi_{Ht}(s) = \frac{\varepsilon - 1}{\phi(s)} [ \hat{rmc}_t(s) - \hat{rp}_{Ht}(s) ] + \frac{\varepsilon}{\phi(s)} \hat{\mu}_t(s) + \beta \mathbb{E}_t [\pi_{Ht+1}(s)]
\]

\[
\pi_{uFt}(s) = \frac{\varepsilon - 1}{\phi(s)} [ \hat{rmc}_t^*(s) + \hat{q}_t - \hat{rp}_{uFt}(s) ] + \frac{\varepsilon}{\phi(s)} \hat{\mu}_{ut}(s) + \beta \mathbb{E}_t [\pi_{uFt+1}(s)]
\]

1. Binding constraints \( \sim \) markup (cost-push) shocks.
   - Stable market structure (elasticity), change in pricing conduct.
   - Distinct from “capital utilization” approach to capacity. [Details]
   - Bernanke and Blanchard (2023) & Del Negro et al. (2022): cost-push shocks account for US inflation.
   - Markup channel is also consistent with resilience of profits. [Profits]

2. Prices tell us whether constraints bind, not why they bind.
   - Positive demand shocks vs. negative capacity shocks.
   - Both manifest as supply-side “markup shocks.”
   - We need data on prices & quantities to pin down shocks. [IRFs]
Two sectors: goods and services. Labor is homogeneous and mobile across sectors. CES export demand for each sector’s output.

Foreign consumption goods and inputs are distinct goods, but are subject to the same cost shocks: $\widehat{rmc}_t^*(s)$.

Two potentially binding constraints:
1. Foreign input goods production capacity.
2. Domestic goods production capacity.

Monetary policy:

$$1 + i_t = (1 + i_{t-1})^{\rho_i} \omega_t^{(1-\rho_i)} (Y_t/Y_0)^{(1-\rho_i)\rho_y} \Psi_t$$
Shocks

1. Demand shocks:
   - Time discount shock: \( E_0 \sum_{t=0}^{\infty} \beta^t \Theta_t \left[ \frac{C_t^{1-\rho}}{1-\rho} - \lambda \frac{L_t^{1+\psi}}{1+\psi} \right] \).
   - Goods-biased demand shock: \( C_t(g) = \zeta_t(g) \left( \frac{P_t(g)}{P_t} \right)^{-\vartheta} C_t \), with \( \zeta_t(g) + \zeta_t(s) = 1 \).

2. Monetary policy shocks: \( \Psi_t \).

3. Shocks to domestic and foreign capacity: \( \bar{Y}_t(g), \bar{Y}_{Mt}^*(g) \).

4. Cost shocks:
   - Sector-level TFP: \( Z_t(s) \).
   - Foreign real marginal cost: \( \widehat{rmc}_t^*(s) \).

Exogenous variables follow AR(1) process.

Note: no labor market shocks; more to come in an extension.
Solution Method

Non-linearities due to occasionally binding constraints
→ construct piece-wise linear solution [Guerrieri and Iacoviello (2015)].

General Solution:

\[ X_t = J (X_{t-1}, \varepsilon_t; \theta) + Q (X_{t-1}, \varepsilon_t; \theta) X_{t-1} + G (X_{t-1}, \varepsilon_t; \theta) \varepsilon_t \]

Policy function depends on whether constraints bind today, and how long they are expected to bind into the future.

Looking forward to estimation, re-write the solution:

\[ X_t = J(D_t, \theta) + Q(D_t, \theta)X_{t-1} + G(D_t, \theta)\varepsilon_t \]

- \( D_t = [d_t, d_{Mt}] \) is \# of periods each constraint binds from date \( t \).
- Given \( D_t \), can solve for time-varying coefficients in policy matrices. Then verify path of \( X_t \) is consistent with the guess.
Estimation

- Calibrate subset of parameters, estimate the remainder.
  - Excess steady-state capacity for domestic goods (5%) and foreign goods inputs (10%).
  - Sufficiently high so that constraints are slack prior to 2020.
  - Calibrated level isn’t important; magnitude of capacity shocks adjusts.

- Structural parameters $\theta$ to be estimated:
  - Stochastic process for exogenous variables: shock variance, AR coeffs.
  - Elasticities of substitution between home and foreign goods, separately for consumption and inputs.
  - Parameters of monetary policy rule.

- Treat durations of binding constraints ($D_t$) as estimable parameters.
  - We extend this work by imposing equilibrium constraints on durations.
  - Allow constraints to potentially bind from 2020:Q2 onward.

- Likelihood a function of parameters and durations $\mathcal{L}(\theta, D|Y^{obs})$
Observables

Data for 1990:Q1 to 2022:Q4

- Consumer inflation and expenditure by sector.
- Industrial production and aggregate nominal GDP.
- Value-added per worker by sector.
- Inflation and expenditure for imported goods inputs (ex. fuels).
- Inflation and expenditure for imported consumer goods.
- Shadow Fed Funds rate:
  - We use updated Wu and Xia (2016) shadow rate as the policy rate.
  - We have also explored explicit ZLB constraint in our model, as in Kulish et al. (2017) and Jones et al. (2022).
Model Fit: Inflation

(a) Aggregate Consumer Inflation

(b) Consumer Services Inflation

(c) Consumer Goods Inflation

(d) Inflation for Imported Inputs
We plot reduced-form markup shocks in the domestic and import price Phillips Curves:

\[
\left( \frac{\varepsilon \phi(s)}{P_0 P_{H0}(s)} \right) \hat{\mu}_t(s) \quad \text{and} \quad \left( \frac{\varepsilon \phi(s)}{P_0 P_{uF0}(s)} \right) \hat{\mu}^*_u(s).
\]
Counterfactual: Slack Capacity Constraints
Aggregate Consumer Price Inflation

Note: Simulated values include measurement error, for comparability to data.
Counterfactual: Slack Capacity Constraints

(a) Goods Inflation

(b) Services Inflation

Note: Simulated values include measurement error, for comparability to data.
Counterfactual: Slack Capacity Constraints

(a) Goods Output

(b) Goods Profits Per Unit
# Decomposition Consumer Price Inflation

## Individual Shocks

<table>
<thead>
<tr>
<th>Year</th>
<th>Percentage Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>2020q1</td>
<td></td>
</tr>
<tr>
<td>2020q3</td>
<td></td>
</tr>
<tr>
<td>2021q1</td>
<td></td>
</tr>
<tr>
<td>2021q3</td>
<td></td>
</tr>
<tr>
<td>2022q1</td>
<td></td>
</tr>
<tr>
<td>2022q3</td>
<td></td>
</tr>
<tr>
<td>2023q1</td>
<td></td>
</tr>
</tbody>
</table>

- **All Shocks**
- **Demand Shocks**
- **Monetary Policy Shocks**
- **Capacity Shocks**
- **Cost Shocks**

**Draw parameters & filter data for smoothed shocks.**

**Introduce shocks one-by-one and solve model with potentially binding constraints.**

**Repeat 1000 times and plot median across simulations.**
Repeat the same exercise, now combining capacity and non-capacity shocks. Tight capacity amplifies the impact of monetary policy shocks in 2021-2022.
Other Extensions

- Remove energy from the domestic price indexes
- Labor supply shocks and constraints
Labor Supply Shocks and Constraints

Add three new features to enrich labor market:

1. **Wage rigidity** → Phillips Curve for wages.

   \[
   \pi_{Wt} = \left( \frac{\epsilon_L - 1}{\phi_W} \right) [\widehat{mrs}_t - \widehat{rw}_t] + \left( \frac{\epsilon_L}{\phi_W} \frac{P_0}{W_0} \right) \hat{\mu}_{Lt} + \beta E_t (\pi_{Wt+1})
   \]

   with \( \widehat{mrs}_t = \hat{\lambda}_t + \psi \hat{I}_t - \rho \hat{C}_t \).

2. **Labor disutility shocks** (\( \hat{\lambda}_t \)) → raise cost of labor supply, moving up the wage Phillips Curve.

3. **Labor supply constraints**: \( L_t \leq \bar{L}_t \)

   → when constraint binds (\( \hat{\mu}_{Lt} > 0 \)), wage Phillips Curve shifts up.

Re-estimate model, adding data on real wages and hours worked.
Multipliers on the Labor Constraint

Binding labor constraint helps explain lack of deflation in 2020.
Counterfactuals

C1: Goods constraints are slack.
C2: Labor constraint is slack.
C3: Goods and labor constraints are slack.
Counterfactual: Policy & Capacity Shocks

Median values across 1000 simulations.
Concluding Remarks

- We have developed a quantitative framework to study inflation that places capacity constraints at center stage.

- Binding constraints introduce a wedge in the Phillips Curve relationship between inflation and real marginal costs.

- Quantitatively, we find that binding capacity constraints explain about half of the rise in US inflation during 2021-2022.

- Why do constraints bind? Increases in demand, triggered by loose monetary policy, plus negative capacity shocks.

- Next: optimal policy & mistakes with capacity constraints.
Contrast with Capacity (Capital) Utilization

Recall Greenwood, Hercowitz and Huffman (1988):

\[ Y_t = Z_t(U_tK_t)^\alpha L_t^{1-\alpha} \]
\[ K_t = I_t + (1 - \delta(U_t))K_{t-1} \]
\[ \hat{rmc}_t = \hat{z}_t + \alpha(\varepsilon \delta \ast \hat{u}_t + \hat{r}q_t) + (1 - \alpha)\hat{r}w_t \]
\[ \pi_{Ht}(s) = \frac{\varepsilon - 1}{\phi(s)} [\hat{rmc}_t(s) - \hat{r}p_{Ht}(s)] + \frac{\varepsilon}{\phi(s)} + \beta \mathbb{E}_t [\pi_{Ht+1}(s)] \]

Capital utilization \((\hat{u}_t) \rightarrow \hat{rmc}_t(s) \rightarrow \pi_{Ht}(s)\).

Our approach to capacity works through markups, conditional on \(rmc\). It changes the structural relation between \(\pi\) and \(rmc\).
Demand vs. Capacity Shocks

(a) Demand Shock: Inflation

(b) Capacity Shock: Inflation

(c) Demand Shock: Output

(d) Capacity Shock: Output

Positive demand shock causes domestic constraint to bind in (a) & (c).
Negative domestic capacity shock causes constraint to bind in (b) and (d).
(a) Domestic Markup Shock

(b) Foreign Markup Shock
Profits per Unit of Output

Nominal corporate profits (NIPA Table 6.16) per unit of gross output.

Source: Bureau of Economic Analysis & authors’ calculations.
Estimation Details

- The likelihood, $L(\theta, D|Y^{obs})$ is a function of both the structural parameters ($\theta$) and the sequence of durations ($D = \{D_t\}_{t=1}^T$).

- We set priors over structural parameters and independent priors over durations to construct the posterior.

- For each proposed draw, we check that durations are consistent with rational expectations equilibrium.
  - Draw proposed durations and parameters.
  - Construct time-varying policy matrices for those parameters.
  - Kalman-filter data and construct smoothed shocks.
  - For each date $\tau$, project endogenous variables forward given duration ($d_\tau$) and smoothed shock ($\tilde{\varepsilon}_\tau$), assuming no future shocks.
  - Reject the draw if constraints are violated.
    Otherwise, accept it and evaluate the likelihood.
  - We accept about 25% of parameter/duration draws.
Energy Shocks

Energy prices rise (late 2021), then fall (late 2022).
We removed oil/fuels from the import price index.
Now, remove from domestic price indexes, and re-estimate.
Estimation on Simulated Data

- Simulate a 70 period path using OccBin
- Engineer monetary expansion in last 10 quarters of simulation
- OccBin gives duration of endogenously binding constraints
Simulation

(a) Inflation

(b) Interest Rate

(c) Domestic Duration

(d) Foreign Duration
Likelihood for Foreign Duration in Period 65

![Graph showing the likelihood for foreign duration in period 65 with a downward trend as duration increases]
Smoothed Multipliers

(a) Domestic Multiplier

(b) Foreign Multiplier
Consumers

$$U \left( \{ C_t, L_t \}_{t=0}^{\infty} \right) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Theta_t \left[ \frac{C_t^{1-\rho}}{1-\rho} - \chi \frac{L_t^{1+\psi}}{1+\psi} \right]$$

with

$$C_t = \left( \sum_s \zeta_t(s)^{1/\vartheta} C_t(s)^{(\vartheta-1)/\vartheta} \right)^{\vartheta/(\vartheta-1)}$$

$$C_t(s) = \left( \sum_s \gamma(s)^{1/\epsilon(s)} C_{Ht}(s)^{(\epsilon(s)-1)/\epsilon(s)} + (1 - \gamma(s))^{1/\epsilon(s)} C_{Ft}(s)^{(\epsilon(s)-1)/\epsilon(s)} \right)^{\epsilon(s)/(\epsilon(s)-1)}$$
Domestic Firms

\[ Y_t(s, \omega) = Z_t(s, \omega) A(s) (L_t(s, \omega))^{1-\alpha(s)} (M_t(s, \omega))^{\alpha(s)} \]

\[ M_t(s, \omega) = \left( \sum_{s'} \left( \frac{\alpha(s', s)}{\alpha(s)} \right)^{1/\kappa} M_t(s', s, \omega)^{(\kappa-1)/\kappa} \right)^{\kappa/(\kappa-1)} \]

\[ M_t(s', s, \omega) = \left[ \xi(s', s) \frac{1}{\eta(s')} M_{Ht}(s', s, \omega)^{\eta(s')-1} \eta(s') \right] + 

\left( 1 - \xi(s', s) \right) \frac{1}{\eta(s')} M_{Ft}(s', s, \omega)^{\eta(s')-1} \eta(s') \right] \frac{\eta(s')}{\eta(s')-1}, \]
Market Clearing

\[ Y_t(s) = C_{Ht}(s) + \sum_{s'} M_{Ht}(s, s') + X_t(s) + \frac{\phi(s)}{2} \left( \frac{P_t(s)}{P_{t-1}(s)} - 1 \right)^2 Y_t(s) \]

\[ Y^*_{Ct}(s) = C_{Ft}(s) + \frac{\phi(s)}{2} \left( \prod_{CFt}(s) - 1 \right)^2 Y^*_{Ct}(s) \]

\[ Y^*_{Mt}(s) = \sum_{s'} M_{Ft}(s, s') + \frac{\phi(s)}{2} \left( \prod_{MFt}(s) - 1 \right)^2 Y^*_{Mt}(s) \]
<table>
<thead>
<tr>
<th>Param</th>
<th>Value</th>
<th>Reference/Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi$</td>
<td>2</td>
<td>Labor supply elasticity of 0.5</td>
</tr>
<tr>
<td>$\rho$</td>
<td>2</td>
<td>Intertemporal elasticity of substitution of 0.5</td>
</tr>
<tr>
<td>$\beta$</td>
<td>.995</td>
<td>Annual risk-free real rate of 2%</td>
</tr>
<tr>
<td>$\vartheta$</td>
<td>0.5</td>
<td>Elasticity of substitution across sectors in cons.</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>4</td>
<td>Elasticity of substitution between varieties</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>0.3</td>
<td>Elasticity of substitution for inputs across sectors</td>
</tr>
<tr>
<td>$\sigma(s)$</td>
<td>1.5</td>
<td>Export demand elasticity</td>
</tr>
<tr>
<td>$\phi$</td>
<td>35.468</td>
<td>To yield first order equivalence to Calvo pricing, with average price duration of 4 quarters</td>
</tr>
</tbody>
</table>
We use data on labor productivity growth in manufacturing and total (private sector) labor productivity growth from the Bureau of Labor Statistics. We assume that labor productivity growth in manufacturing coincides with goods labor productivity (growth in real value added per worker) in the model, while also matching aggregate (economy-wide) labor productivity growth in the model.

We use data on import price inflation for consumption goods, and we proxy input price inflation in the model using data on import price inflation for imported industrial materials (excluding fuels).

We use data for consumer goods (except food and automotive) to proxy for consumption imports, and we construct proxies for imported inputs (excluding fuels) by removing the subcategory of petroleum and products from industrial materials and supplies using standard chain index formulas and auxiliary NIPA data on the sub-categories of imports.


