Robust Real Rate Rules

Tom D. Holden, Deutsche Bundesbank* 28/09/2023

Abstract: Central banks wish to avoid self-fulfilling fluctuations. Interest rate rules with a unit response to real rates achieve this under the weakest possible assumptions about the behaviour of households and firms. They are robust to household heterogeneity, hand-to-mouth consumers, non-rational household or firm expectations, active fiscal policy, missing transversality conditions and to any form of intertemporal or nominal-real links. They are easy to employ in practice, using inflation-protected bonds to infer real rates. With a time-varying short-term inflation target, they can implement arbitrary inflation dynamics, including optimal policy. This provides a way to translate policy makers' desired path for inflation into one for nominal rates. US Federal Reserve behaviour is remarkably close to that predicted by a real rate rule, given US monetary policymakers' desired inflation path. Real rate rules work thanks to the key role played by the Fisher equation in monetary transmission.

Keywords: robust monetary rules, determinacy, Taylor principle, inflation dynamics, monetary transmission mechanism. **JEL codes:** E52, E43, E31

* Address: Mainzer Landstraße 46, 60325, Frankfurt am Main, Germany.
E-mail: thomas.holden@gmail.com. Website: https://www.tholden.org/.
The views expressed in this paper are those of the author and do not represent the views of the Deutsche Bundesbank, the Eurosystem or its staff.
The author would like to thank the editor, Charles Jones, four anonymous referees, Klaus Adam, Parantap Basu, Christiane Baumeister, Florin Bilbiie, Mark Bils, Nicola Borri, R. Anton Braun, Xiaoshan Chen, John Cochrane, Pablo Cuba-Borda, Davide Debortoli, Martin Eichenbaum, George Evans, Andrea Ferrero, Rafael Gerke, Robert Hall, Cosmin Ilut, Daniel Kienzler, Seunghyeon Lee, Eric Leeper, Campbell Leith, Vivien Lewis, Felix Geiger, François Gourio, Refet Gürkaynak, Marcus Hagedorn, Karl Harmenberg, Michael Hatcher, Seppo Honkapohja, Guido Imbens, Peter Ireland, Malte Knueppel, Robert Kollmann, Philipp Lieberknecht, Bartosz Maćkowiak, Jochen Mankart, Christoph Meinerding, Nikolaos Melissinos, Elmar Mertens, Stéphane Moyen, Matthias Rottner, Hikaru Saijo, Sebastian Schmidt, Guido Schultefrankenfeld, Lars Svensson, Harald Uhlig, Mu-Chun (Tomi) Wang, Iván Werning and Alexander Wolman for helpful comments.
Ancestors of this paper were circulated under the titles "The One Equation New Keynesian Model" and "A Robust Monetary Rule".

1 Introduction

Today you start work as president of the Fictian Central Bank (FCB). As FCB president, you have a clear mandate to stabilize inflation, even if that results in unemployment or output losses. How should you act? Having studied New Keynesian macro, you are inclined to follow some variant of the Taylor rule. You recall the Taylor principle: the response of nominal rates to inflation should be greater than one to ensure determinacy—the existence of a unique stable solution, without self-fulfilling fluctuations. But you also remember reading other papers which talked of the Taylor principle being insufficient if there are hand-to-mouth households (Galí, Lopez-Salido & Valles 2004), firm-specific capital (Sveen & Weinke 2005), high government spending (Natvik 2009), or if the inflation target is positive (Ascari & Ropele 2009), particularly in the presence of trend growth and sticky wages (Khan, Phaneuf & Victor 2019). Indeed, you recollect that the Taylor principle inverts if there are sufficiently many hand-to-mouth households (Bilbiie 2008), financial frictions (Lewis & Roth 2018; Manea 2019), or non-rational expectations (Branch & McGough 2010; 2018). You also recall that if real government surpluses do not respond to government debt levels, then following the Taylor principle can lead to explosive inflation (Leeper 1991; Leeper & Leith 2016; Cochrane 2023). Is there a way you could act to ensure determinacy and stable inflation, even if one or more of these circumstances is true? This paper provides a family of "robust real rate rules" that manage to do this. We then reassess classic questions of monetary economics through the lens of these rules.

For a central bank to use a real rate rule, both nominal and real bonds must be traded in the economy. If a unit of the former is purchased at t, it returns the principal plus a nominal yield of i_t in period $t+1$. If a unit of the latter is purchased at t, it returns the principal plus a nominal yield of $r_t + \pi_{t+1}$ in period $t+1$, where π_{t+1} is realized inflation between t and $t+1$. US Treasury Inflation Protected Securities (TIPS) are one example of a real bond.

Arbitrage between the nominal and real bond markets implies that the Fisher equation must hold, i.e.:

$$i_t = r_t + \mathbb{E}_t \pi_{t+1}, \tag{1}$$

where $\mathbb{E}_t \pi_{t+1}$ is the full information rational expectation of period $t+1$'s inflation rate, given period t's information. We are abstracting for the moment from inflation risk premia, term premia and liquidity premia, all of which can generate endogenous wedges in the Fisher equation. However, all our results are robust to such wedges, as we show in Section 3.

We suppose that the central bank observes the nominal and real bond markets, and that it can intervene in the former. Then the central bank can choose to set nominal interest rates according to the simple "real rate rule":

$$i_t = r_t + \phi \pi_t, \tag{2}$$

where $\phi > 1$ (the Taylor principle). Combining these two equations gives that:

$$\mathbb{E}_t \pi_{t+1} = \phi \pi_t,$$

which has a unique non-explosive solution of $\pi_t = 0$. Determinate inflation!

Here we have ignored the zero lower bound (ZLB), as the focus of this paper is on determinacy away from the ZLB. Nonetheless, in Section 4 we show that real rate rules continue to perform well even with the ZLB. We are also sidestepping the equilibrium selection issues raised by Cochrane (2011) and following the standard New Keynesian literature in assuming agents select non-explosive paths for inflation. The limited memory arguments of Angeletos & Lian (2023) give one justification for this. Alternatively, the escape clause rules of Christiano & Takahashi (2018; 2020) give central banks a way to ensure coordination on expectations consistent with non-explosive inflation.

Why are real rate rules so robust? Firstly, they do not require an aggregate Euler equation to hold, even approximately. For the Fisher equation (1) to hold (still ignoring risk/term/liquidity premia for now), there only need to be two deep pocketed, fully informed, rational agents. Arbitrage takes care of the rest. Even full information is not necessary. In a large market, the Fisher equation can come to hold even when information about future inflation is dispersed amongst market participants (Hellwig 1980; Lou et al. 2019). If there is an aggregate Euler equation, its role is only to determine equilibrium real interest rates, given the values of other variables.

Given that the rule does not require an aggregate Euler equation to hold, it is automatically robust to household heterogeneity, hand-to-mouth agents, and

non-rational consumer expectations. The only expectations that matter are the expectations of participants in the markets for nominal and real bonds. It is much more reasonable to assume that financial markets lead to pricing consistent with rational expectations than to assume rationality of households more generally. In fact, even financial market participants do not need to be fully rational. Real rate rules continue to work if financial market participants are learning, or if they are not fully rational.

Real rate rules also have a second source of robustness: they do not require an aggregate Phillips curve to hold. The slope of the Phillips curve can have no impact on the dynamics of inflation. If a central bank is unconcerned with output, they do not even need to know if the Phillips curve holds, let alone its slope. Nor does it matter how firms form inflation expectations. The Fisher equation and the monetary rule pin down inflation, so while non-rational firm expectations could affect output fluctuations, they will not alter inflation dynamics. The only requirement is that at least some prices are updated each period using current information. If there is a Phillips curve, it determines the output gap from the level of inflation consistent with the Fisher equation and the monetary rule.

The possibility of decoupling inflation from the rest of the economy has wide ranging implications. For example, there is a tradition in monetary economics of examining model features producing amplification or dampening of monetary shocks. Under a real rate rule, if the Fisher equation holds then no change to the model can ever produce amplification or dampening, except a change to the monetary rule itself. Thus, such amplification/dampening results were always highly dependent on the particular monetary rule being used. With a greater than unit response to real rates, amplification can be flipped to dampening, and vice versa.

Likewise, a persistent question in monetary economics has been "which shocks drive inflation?". Here too, the answer must be crucially sensitive to the monetary rule being used. Under a real rate rule, only monetary policy shocks or shocks to the Fisher equation could move inflation. Central banks have the power to control inflation almost perfectly, so ultimate responsibility for

inflation must rest with them.

An even more fundamental question of monetary economics is "how does monetary policy work?". The traditional answer involves movements in nominal rates leading to movements in real rates, due to sticky prices. But this cannot be the transmission mechanism under flexible prices, as then real rates are exogenous. Nor too can it be the transmission mechanism under a real rate rule, as then real rate movements are irrelevant. In these cases, monetary policy works exclusively through the Fisher equation's link between nominal rates and expected inflation. Since outcomes under a real rate rule are qualitatively so similar to those under a traditional rule, our results suggest that Fisher equation transmission is the norm. Rupert & Šustek (2019) draw the same conclusion based on the observation that contractionary (positive) monetary shocks can lower real rates in New Keynesian models with capital.

The rest of this paper further examines real rate rules, along with the classic questions of monetary economics they help answer. The next section shows how real rate rules can ensure inflation hits an arbitrary time varying short-term inflation target. Hence, real rate rules can implement optimal policy, attaining high welfare, and can also explain observed inflation dynamics. We also discuss the benefits of smoothing real rate rules.

Section 3 looks at the impact of monetary shocks and Fisher equation wedges. We also examine the implications of following a real rate rule in a simple New Keynesian model. Next, Section 4 presents a modified real rate rule designed to ensure determinacy even in the presence of the zero lower bound.

Section 5 discusses how a real rate rule could be implemented in practice. We show that it is easy to adapt real rate rules to work with longer bonds, and that neither information nor indexation lags present a challenge to the performance of these rules. In Section 6 we show that the practical real rate rule of Section 5 provides an excellent fit to actual US Federal Reserve behaviour, even when the short-term inflation target is disciplined by data from the Summary of Economic Projections.

Section 7 examines some potential challenges to the performance of real rate rules. We show they work in fully non-linear models and that they are robust

to bounded rationality and learning. We also show that, generically, real rate rules continue to work even when inflation is determined by something other than monetary policy, as under the fiscal theory of the price level.

Prior literature. Rules like equation (2) have appeared in Adão, Correia & Teles (2011), Lubik, Matthes & Mertens (2019) and Holden (2021) amongst other places. However, in the prior literature they have chiefly been introduced for analytic convenience, rather than as serious proposals. One exception is the work of Cochrane (2017; 2023), who briefly discusses rules of this form within the context of a wider discussion of rules that hold $i_t - r_t$ constant (i.e. rules with $\phi = 0$). Cochrane (2018) further explores rules holding $i_t - r_t$ constant.

The "indexed payment on reserve" rules of Hall & Reis (2016) also rely on observable real rates, but use a different mechanism to achieve determinacy. They propose that the central bank issues an asset ("reserves") with nominal return from \$1 of $\$(1 + r_t)\frac{p_{t+1}}{p_t^*}$ or $\$(1 + i_t)\frac{p_t}{p_t^*}$. Additionally, in older work, Hetzel (1990) proposes using the spread between nominal and real bonds to guide monetary policy, and Dowd (1994) proposes targeting the price of futures contracts on the price level. This has a similar flavour to a real rate rule, as these rules effectively use expected inflation as the instrument of monetary policy.

Forecast targeting has also been proposed by Hall & Mankiw (1994) and Svensson (1997), amongst others. Bernanke & Woodford (1997) examine the desirability of responding to private sector inflation forecasts, finding that this can lead to indeterminacy. The difference is that whereas the Bernanke & Woodford (1997) rules set nominal rates to a multiple of expected inflation, real rate rules set expected inflation to a multiple of current inflation. Rules of the form $i_t = \mathbb{E}_t \pi_{t+1} + \cdots$, are a special case of the Bernanke & Woodford (1997) class, and have been used by Bilbiie (2008; 2011) for analytic convenience. By the Fisher equation, following such a rule is equivalent to directly setting the real rate, which has led some authors to refer to these rules as "real rate rules" (Beaudry, Preston & Portier 2022), although they are quite different to the "real rate rules" studied in this paper. However, rules setting real rates break under flexible prices and wages (as then real rates are exogenous), leading to lower robustness. Our real rate rules do not have this problem.

There is also an established literature looking at rules tracking the efficient or "natural" real interest rate (Woodford 2003). This uses rules of the form $i_t = n_t + \phi\pi_t$, where n_t is the natural real rate. While this looks very similar to our equation (2) (swap r_t for n_t), it is a very different idea. By the Fisher equation, the previous natural rate rule implies that $\pi_t = \phi^{-1}(r_t - n_t) + \phi^{-1}\mathbb{E}_t\pi_{t+1} = \mathbb{E}_t \sum_{k=0}^{\infty} \phi^{-k-1}(r_{t+k} - n_{t+k})$, assuming that $\phi > 1$ and that inflation is non-explosive. However, since r_t is endogenous, this "solution" for π_t does not automatically imply uniqueness, unlike under a real rate rule. Nor does it imply zero inflation in equilibrium, except under restrictive assumptions on the rest of the model, such as the absence of cost-push shocks. Furthermore, while real rates are observable through inflation-protected securities, natural real rates must be inferred from estimates of shocks under a particular model. For example, the natural rate usually depends on shocks to technology and discount factors, neither of which are directly observable.

Money growth rules also generally deliver determinacy (Carlstrom & Fuerst 2003). However, they translate fluctuations in money demand or velocity into fluctuations in inflation, so they do not give the degree of inflation control provided by real rate rules. Despite this, even simple money growth rules can perform comparably to traditional rules, and even outperform them when the ZLB binds frequently (Belongia & Ireland 2022; Billi, Söderström & Walsh 2023). We show that real rate rules keep their robust performance even in the presence of the ZLB.

2 Time-varying short-term inflation targets

The robust real rate rule of equation (2) ensures zero inflation in all periods.[1] But this is not always desirable. For example, with sticky prices, it is optimal for the central bank to tolerate higher inflation in response to an unexpected increase in mark-ups. The traditional solution is for the central bank to respond to output gaps in their monetary rule, but any response to other variables risks reducing robustness.[2] A better solution is for the central bank to

[1] See Online Appendix A for a discussion of outcomes off the equilibrium path.

[2] Online Appendix B shows that real rate rules keep some robustness even with a response to other endogenous variables.

adopt a time-varying short-term inflation target. With this target responding to other endogenous variables or shocks, the central bank can produce desirable movements in inflation without compromising robustness. With such a target, real rate rules can determinately implement any target path for inflation, no matter the rest of the model. This implies they can also implement optimal policy, maximizing welfare. It also means that any observed inflation and interest rate dynamics are consistent with a real rate rule.

Time-varying short-term inflation targets also solve one of the greatest challenges to the real-world uptake of monetary rules. No central bank governor wants to give up their ability to respond to unusual circumstances in unusual ways. A time-varying short-term inflation target splits the monetary decision in two. The governor and board announce the level of inflation they would like to hit over the next month(s), while the trading desk can mechanically follow a real rate rule to achieve that target. This combines the flexibility benefits of discretionary reactions to current circumstances, via changes to the target path of inflation, with the determinacy benefits of rigid commitment to a rule.[3]

How do these time-varying short-term targets work? To start, let π_t^* be the central bank's period t inflation target for period t inflation. This can be a function of any of the model's endogenous variables and exogenous shocks.[4] For example, in order to dampen the output response to mark-up shocks, the central bank could set π_t^* either as a decreasing function of the output gap, or as an increasing function of the mark-up shock. The central bank should publish this target each period, else the limited information of market participants could lead to additional volatility.

With a time-varying inflation target, the real rate rule becomes:

---

[3] The two-part approach to implementation also helps resolve the concern implicitly raised by Afrouzi et al. (2023). Within a period, the central bank would like to move after firms have set prices, so it can punish their deviations. But it would also like to move before firms so that it can influence their price setting that period. This conflict is resolved by announcing the current period inflation target at the start of the period but setting nominal rates via a real rate rule at the end of the period, responding to observed prices.

[4] Ireland (2007) also allows the central bank's inflation target to respond to other structural shocks. He presents evidence that the US Federal Reserve has reacted to mark-up shocks.

$$i_t = r_t + \mathbb{E}_t \pi^*_{t+1} + \phi(\pi_t - \pi^*_t). \tag{3}$$

From the Fisher equation (1), this implies, $\mathbb{E}_t(\pi_{t+1} - \pi^*_{t+1}) = \phi(\pi_t - \pi^*_t)$. Again with $\phi > 1$, there is a unique non-explosive solution for $\pi_t - \pi^*_t$, now with $\pi_t = \pi^*_t$ for all t. I.e., at all periods of time, and in all states of the world, realised inflation is equal to π^*_t. The central bank can choose an arbitrary path for inflation as the unique, determinate equilibrium outcome.

There are only two constraints on the short-term inflation target. The first is that the central bank must be capable of calculating a reasonable approximation to $\mathbb{E}_t \pi^*_{t+1}$. One way to ensure this is to make π^*_t only a function of $t-1$ dated variables. Alternatively, the central bank could respond to variables for which there are liquid futures or option markets, or the central bank could form these expectations using a forecasting model. Errors in these forecasts will show up as monetary policy shocks, increasing the variance of $\pi_t - \pi^*_t$, but we will see that this can be dampened with a large ϕ.

The second constraint on the inflation target is that if the monetary rule is replaced with the equation $\pi_t = \pi^*_t$, then inflation should still be stable and determinate. For example, we cannot set $\pi^*_t := 2\pi_{t-1} + \varepsilon_{*,t}$, for some target shock $\varepsilon_{*,t}$, as then with $\pi_t = \pi^*_t$, $\pi_t = 2\pi_{t-1} + \varepsilon_{*,t}$, which is an explosive process. One way to ensure this condition is for π^*_t to be only a function of exogenous variables, but this is far from necessary. However, responding to exogenous variables is enough to mimic the outcome of any other monetary policy regime, as in a stationary equilibrium, endogenous variables must have a representation as a function of the infinite history of the economy's shocks.[5]

This has two important implications. Firstly, it means that appropriately designed real rate rules can implement (timeless/unconditional/etc.) optimal policy, and thus attain the highest possible level of welfare.[6] Rules with time varying targets can also mimic outcomes under rules responding to additional endogenous variables. Secondly, it means that without direct evidence on π^*_t, it

[5] We show this formally in Supplemental Appendix J.6.

[6] Other papers have examined the implementation of optimal policy using instrument rate rules (see e.g. Svensson & Woodford 2003; Dotsey & Hornstein 2006; Evans & Honkapohja 2006; Evans & McGough 2010). Ours is unique in enabling the implementation of a certain inflation path robustly across models.

is impossible to test empirically if a central bank is using a general real rate rule. Any dynamics of inflation and interest rates are consistent with a real rate rule like (3), for an appropriately chosen π_t^*. Thus, real rate rules are observationally equivalent to any other specification for central bank behaviour.

To see how implementing optimal monetary policy with a real rate rule works, suppose that the model contains the New Keynesian Phillips curve:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa x_t + \kappa \omega_t, \tag{4}$$

where x_t is the output gap and ω_t is a mark-up/cost-push shock.[7] And suppose that the central bank wants to minimise a discounted weighted combination of the variances of inflation and the output gap, $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [\pi_t^2 + \lambda x_t^2]$, where $\lambda > 0$. Then, under the timelessly optimal perspective of Woodford (1999), the central bank wants to ensure $\pi_t = -\kappa^{-1}\lambda(x_t - x_{t-1})$ in all periods. They can do this by setting $\pi_t^* := -\kappa^{-1}\lambda(x_t - x_{t-1})$ and using the rule of equation (3). This is determinate as long as $\beta > 0$.[8] It ensures that $\pi_t = -\kappa^{-1}\lambda(x_t - x_{t-1})$ even if the central bank's model of the economy is misspecified, and the true link between inflation and the output gap is not given by equation (4). This contrasts with other proposals for policy implementation,[6] which depend on all the parameters of the Euler and Phillips curves, and hence may produce strange outcomes under misspecification.

Alternatively, the central bank may prefer to announce a target for next period's inflation, not this period's. The optimal path for one period ahead inflation, $\pi_{t+1|t}$, minimizes $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\pi_{t|t-1}^2 + \lambda x_t^2\right]$ subject to $\pi_{t|t-1} = \beta \pi_{t+1|t} + \kappa x_t + \kappa \omega_t$. This is a purely backwards-looking constraint, so there is no longer any difference between discretion, commitment, or timeless policy. The solution is $\pi_{t+1|t} = -\kappa^{-1}\lambda \mathbb{E}_t(x_{t+1} - x_t)$, which can be determinately implemented with $\beta > 0$ by setting the period t target for period $t+1$ inflation, $\pi_{t+1|t}^*$, to

[7] Throughout this paper, we multiply the mark-up shock by κ as the ratio of the response to x_t and the response to ω_t is not a function of either the (Calvo) price adjustment probability or the (Rotemberg) price adjustment cost. See Khan (2005) for derivations.

[8] From substituting $\pi_t = -\kappa^{-1}\lambda(x_t - x_{t-1})$ into the Phillips curve, we see that determinacy requires the quadratic $q(A) = \beta\lambda A^2 - [\kappa^2 + \lambda(1+\beta)]A + \lambda$ to have one root inside the unit circle and one root outside. With $\beta > 0$, determinacy then follows from the facts that $q''(0) = 2\beta\lambda > 0$, $q'(0) = -[\kappa^2 + \lambda(1+\beta)] < 0$, $q(0) = \lambda > 0$ and $q(1) = -\kappa^2 < 0$.

$-\kappa^{-1}\lambda\mathbb{E}_t(x_{t+1}-x_t)$,[9] and using the rule $i_t = r_t + \pi^*_{t+1|t} + \phi\big(\pi_t - \pi^*_{t|t-1}\big)$.[10]

We have seen that time-varying short-term inflation targets make real rate rules into effective tools for implementing optimal policy. One final note is in order though: It is important to distinguish the time-varying short-term inflation target from the central bank's constant long-term target (of 2% say). Just as central banks currently need to communicate carefully when choosing not to raise rates despite inflation being above target, so with a time-varying short-term target, a central bank would want to carefully justify their choice to allow temporarily higher inflation. But it need pose no greater risk to central bank credibility. Indeed, the US Federal Reserve already effectively announces a path for π^*_t through the Summary of Economic Projections. This gives monetary policy makers' forecasts for inflation conditional on their beliefs about "appropriate monetary policy".[11]

2.1 Adding interest rate smoothing

High degrees of interest rate smoothing are often thought to be a good description of actual central bank behaviour given the rarity of large interest rate changes. However, since the rule (3) can generate arbitrary inflation dynamics (and hence arbitrary nominal rate dynamics), we cannot conclude based on observed nominal rates that the central bank is actually smoothing rates. Nonetheless, interest rate smoothing is worth investigating, as it can be a source of added robustness.

With a fully smoothed real rate rule, the central bank sets interest rates so:[12]

$$i_t - r_t = (i_{t-1} - r_{t-1}) + (\mathbb{E}_t\pi^*_{t+1} - \mathbb{E}_{t-1}\pi^*_t) + \theta(\pi_t - \pi^*_t), \tag{5}$$

where $\theta > 0$ and where π^*_t is the short-term inflation target, as before. Note: the central bank smooths $i_t - r_t$, not just i_t. This ensures real rates can still be substituted out from the Fisher equation.

One advantage of full smoothing is that it removes the impact of permanent

[9] Determinacy follows from substituting $x_t = \kappa^{-1}(\pi_{t|t-1} - \beta\pi_{t+1|t})$ into $\pi_{t+1|t} = -\kappa^{-1}\lambda\mathbb{E}_t(x_{t+1}-x_t)$, leading to the same characteristic quadratic as in the previous footnote.

[10] It is important to note that it is π_t not $\pi_{t|t-1}$ that enters the rule, as although inflation will be predetermined in equilibrium, out of equilibrium it might not be.

[11] See Supplemental Appendix I.1 for more information on the Summary of Economic Projections.

[12] We examine partially smoothed real rate rules in Supplemental Appendix J.7.

wedges in the Fisher equation. Suppose that due to static convenience yields, risk, or liquidity premia (say), the Fisher equation took the form $i_t = r_t + \mathbb{E}_t\pi_{t+1} + \nu$, for some constant wedge ν. Now let $\hat{\pi}_t := \pi_t - \pi_t^*$ and $e_t := \mathbb{E}_t\hat{\pi}_{t+1}$, then the Fisher equation and monetary rule imply $\hat{\pi}_t = \theta^{-1}(e_t - e_{t-1})$. Substituting this back into the definition of e_t then implies $\mathbb{E}_t e_{t+1} = (1+\theta)e_t$, which has the unique non-explosive solution $e_t = 0$ as $\theta > 0$. Thus, in equilibrium, $\pi_t = \pi_t^*$.

This establishes that when $\nu = 0$, our smoothed real rate rule produces the same equilibrium inflation (and hence the same nominal rates) as our unsmoothed real rate rule, equation (3). Moreover, the smoothed rule ensures inflation stays at target even when $\nu \neq 0$, unlike the unsmoothed rule. We will see in Section 5 that smoothing also ensures inflation remains stationary even when there is a non-stationary Fisher equation wedge or monetary shock.

It is also more robust in one further important respect. Whereas the rule in equation (3) required a response to current inflation of $\phi > 1$, the fully smoothed real rate rule just needs a response to current inflation of $\theta > 0$. In practice, it may be hard for central banks to commit to responding more than one for one to inflation. Even if they manage this, it will be hard for them to convince other economic agents they really will be so aggressive. Since inflation and nominal rates are identical for any $\phi > 1$, there is no way for these agents to observe ϕ. Even with $\phi < 1$, there are equilibria which are observationally equivalent to the equilibria with $\phi > 1$. It would be far easier for central banks to convince economic agents that they at least respond positively to inflation. This is all that is needed for a fully smoothed real rate rule.

This gives a compelling argument for the preferability of smoothing real rate rules. In Section 4 we will see another benefit of such rules: smoothing prevents the existence of sunspot equilibria in the presence of the zero lower bound.

3 Monetary shocks and Fisher equation wedges

With the Fisher equation (1) and the monetary rule of equations (2), (3) or (5), inflation is always at its target. For inflation to move from this target, there must be a shock to either the Fisher equation or the monetary rule. In this section, we examine the consequences of these shocks, including their

implications for real variables in the three equation New Keynesian (NK) model. We also examine what happens if the Fisher equation contains an endogenous wedge, coming from time varying risk premia, for example.

3.1 Monetary policy shocks

We can add a monetary policy shock, ζ_t, to the rule of equation (2), giving:

$$i_t = r_t + \phi\pi_t + \zeta_t. \tag{6}$$

One source of monetary policy shocks could be the central bank's limited information. If the central bank does not perfectly observe current inflation, and sets interest rates to $i_t = r_t + \phi\tilde{\pi}_t$, where $\tilde{\pi}_t$ is its signal about inflation, then it will end up setting a slightly different level for nominal rates than that dictated by the rule $i_t = r_t + \phi\pi_t$, effectively generating monetary policy shocks.[13,14]

From combining (6) with the Fisher equation (1) we have:

$$\mathbb{E}_t\pi_{t+1} = \phi\pi_t + \zeta_t, \tag{7}$$

which has the unique solution $\pi_t = -\frac{1}{\phi-\rho}\zeta_t$, if $\phi > 1$ and ζ_t follows an AR(1) process with persistence $\rho \in (-1,1)$.

A contractionary (positive) monetary policy shock results in a fall in inflation, as expected. One way to understand this is to note that the rule of equation (6) is actually a special case of the rule with a time-varying target in equation (3). Implicitly, equation (6) targets $\pi_t^* := -\frac{1}{\phi-\rho}\zeta_t$, as substituting this into equation (3) gives equation (6). Contractionary monetary shocks are equivalent to temporary reductions in the inflation target.

We also see that if the central bank is more aggressive, so ϕ is larger, then inflation is less volatile. We will see that this result extends to shocks to the Fisher equation, and many other departures from the simple setup of equations

[13] In fact, this kind of limited information is inconsistent with our simple model's assumptions. Real bonds bought at $t-1$ give a return in period t which is a function of π_t. Hence, π_t must be available to all parties in period t. (It is not "true" inflation that matters, but whatever inflation measure is used in the real bond contract.) Of course, in reality inflation is released with a lag, and real bonds have additional indexation lag. We explicitly model these lags in Section 5, and our conclusions remain the same.

[14] Schmitt-Grohé & Uribe (2007) look at monetary rules responding to lagged information and show that they perform as well as rules responding to current information. Lubik, Matthes & Mertens (2019) look at the determinacy consequences of a central bank that filters inflation signals in order to retrieve the optimal estimate. The determinacy problems they highlight all disappear if the central bank directly responds to its signal.

(1) and (2). Large ϕ squashes shocks, bringing inflation nearer to target.

In this model, only monetary policy shocks affect inflation. Of course, if there is a nominal rigidity in the model, monetary shocks may have an impact on real variables. But as long as the central bank follows a rule like this, these real disruptions have no feedback to inflation. Causation runs from inflation to real variables, not the other way round. We can understand inflation without worrying about the rest of the economy.

This result may be surprising, but in fact an extensive body of empirical evidence finds no role for the Phillips curve in forecasting inflation (see e.g. Atkeson & Ohanian 2001; Ang, Bekaert & Wei 2007; Stock & Watson 2009; Dotsey, Fujita & Stark 2018). For example, Dotsey, Fujita & Stark (2018) find that post-1984, Phillips curve based forecasts perform worse than those of a simple IMA(1,1) model, both unconditionally and conditional on various measures of the state of the economy. This is consistent with causation only running from inflation to the output gap, not in the opposite direction.[15] Likewise, Miranda-Agrippino & Ricco (2021) find that a contractionary monetary policy shock causes an immediate fall in the price level, while impacts on unemployment materialise much more slowly. Again, this suggests that causation runs from inflation to unemployment, not the other way round.

3.2 Robust real rate rules in the three equation NK world

What is the role of the Phillips curve and Euler equation under a real rate rule? Suppose in the setup of the previous subsection that ζ_t is independent of other structural shocks, and we have the Phillips curve of equation (4) and the discounted/compounded Euler equation:

$$x_t = \delta \mathbb{E}_t x_{t+1} - \varsigma(r_t - n_t), \tag{8}$$

where n_t is the exogenous natural real rate of interest and x_t is the output gap (as before). This form of discounted/compounded Euler equation appears in Bilbiie (2019) and (under discounting) in McKay, Nakamura & Steinsson (2017). The latter paper shows it provides a good approximation to a heterogeneous agent model with incomplete markets. We recover the standard

[15] McLeay & Tenreyro (2019) provide an alternative explanation: optimal policy prescribes a negative correlation between inflation and output, making difficult empirical identification of the Phillips curve.

Euler equation if $\delta = 1$ and ς is the elasticity of intertemporal substitution. This specification also nests the two agent, limited asset market participation ("TANK") model of Bilbiie (2008) when $\delta = 1$, but ς is allowed to be negative. And, it nests the behavioural NK model of Gabaix (2020) if β and δ are reduced from their values under full rationality by cognitive discounting.

Since $\pi_t = -\frac{1}{\phi-\rho}\zeta_t$, and ζ_t is AR(1) with persistence $\rho \in (-1,1)$, the Phillips curve (4) implies that $x_t = -\frac{1}{\kappa}\frac{1-\beta\rho}{\phi-\rho}\zeta_t - \omega_t$. The Phillips curve is determining the output gap, given the already determined level of inflation. This is consistent with the evidence of Dotsey, Fujita & Stark (2018), as x_t is no help in forecasting π_t here. $\mathbb{E}_t\pi_{t+1} = -\frac{\rho}{\phi-\rho}\zeta_t = \rho\pi_t$, so once you know π_t, you already have all the information you need to form the optimal forecast of π_{t+1}. The correlation in π_t and x_t provides no extra information.[16]

This model also enables us to show the robustness of our rule's determinacy in practice. Note that with x_t expressed as a linear combination of exogenous variables, there is no need to solve the Euler equation (8) forward, so the degree of discounting (δ) cannot have an effect on determinacy. Not needing to solve the Euler equation forward also gives robustness to a missing transversality constraint on household assets, as under an overlapping generations structure. The only role of the Euler equation is to pin down real rates, given inflation and the output gap. For example, if ω_t is independent across time, then the Euler equation implies $r_t = n_t + \frac{1}{\varsigma}\left[\frac{1}{\kappa}\frac{(1-\beta\rho)(1-\delta\rho)}{\phi-\rho}\zeta_t + \omega_t\right]$. Via the Fisher equation, this in turn implies that $i_t = n_t + \frac{1}{\varsigma}\left[\frac{1}{\kappa}\frac{(1-\beta\rho)(1-\delta\rho)-\kappa\varsigma\rho}{\phi-\rho}\zeta_t + \omega_t\right]$ in equilibrium.[17]

The irrelevance of Euler equation parameters for determinacy contrasts with the prior literature on determinacy under standard monetary rules. For example, with a standard monetary rule, Bilbiie (2019) finds that when $\varsigma > 0$ and $\beta \leq 1$, the Taylor principle ($\phi > 1$) is only sufficient for determinacy in the

[16] This result is robust to generalizing to an ARMA(1,1) process for ζ_t. See Supplemental Appendix J.2.

[17] For high values of ρ, $\frac{(1-\beta\rho)(1-\delta\rho)-\kappa\varsigma\rho}{\kappa\varsigma}$ is likely to be negative, so positive monetary policy shocks actually lower the nominal rate in equilibrium. (The author thanks a referee for this observation.) This is relatively common in NK models (Holden 2021; Bilbiie 2022), and should be unsurprising given the equivalence between positive monetary shocks and temporary reductions in the inflation target. The arguments of Bilbiie (2022) suggest that sunspot driven liquidity traps are more likely when positive monetary shocks have negative effects. We look for such sunspot equilibria under real rate rules in Section 4.

discounting case ($\delta \leq 1$),[18] and Bilbiie (2008) finds that when $\delta = 1$ and $\varsigma < 0$, the Taylor principle ($\phi > 1$) is neither necessary nor sufficient for determinacy.[19,20] Under our rule (6), the Taylor principle is necessary and sufficient for determinacy whether there is discounting or compounding, and whether ς is positive or negative (at least given $\phi \geq 0$).[21]

Note that even though a nominal rate peg is determinate when $\delta = 0$ (say), inflation is not "over-determined" under a real rate rule with $\delta = 0$. As ever, the monetary rule and the Fisher equation pin down inflation, the Phillips curve then pins down the output gap, and the Euler equation gives the level of real rates that is consistent with these values.

The rule is also robust to the presence of lags in the Euler or Phillips curve. For example, suppose the Phillips curve and Euler equation are given by:

$$\begin{aligned} \pi_t &= \tilde{\beta}(1-\varrho_\pi)\mathbb{E}_t\pi_{t+1} + \tilde{\beta}\varrho_\pi\pi_{t-1} + \kappa x_t + \kappa\omega_t, \\ x_t &= \tilde{\delta}(1-\varrho_x)\mathbb{E}_t x_{t+1} + \tilde{\delta}\varrho_x x_{t-1} - \varsigma(r_t - n_t), \end{aligned} \tag{9}$$

where $\tilde{\beta}$ and $\tilde{\delta}$ may not have the same structural interpretation as β and δ (depending on the precise micro-foundation). These equations have no impact on the solution for inflation, which remains $\pi_t = -\frac{1}{\phi-\rho}\zeta_t$. Instead, the lag in the Euler equation changes the dynamics of the real interest rate, with no impact on inflation or output gaps, while the lag in the Phillips curve affects both output gap and real rate dynamics, with no impact on inflation. In particular, the output gap is given by $x_t = \frac{1}{\kappa(\phi-\rho)}\Big[\Big(\tilde{\beta}\varrho_\pi - \rho\big(1-\tilde{\beta}(1-\varrho_\pi)\rho\big)\Big)\zeta_{t-1} - \big(1-\tilde{\beta}(1-\varrho_\pi)\rho\big)\varepsilon_{\zeta,t}\Big] - \omega_t$. As before, the output gap has a closed form solution in terms of the monetary policy and cost push shocks. Monetary policy shocks are still always contractionary, but they only have a short-lived impact on the output gap if ϱ_π is around $\frac{\rho(1-\tilde{\beta}\rho)}{\tilde{\beta}(1-\rho^2)}$.

[18] See equation (40) of Appendix C.1 of Bilbiie (2019).

[19] See Proposition 7 of Appendix B.1 of Bilbiie (2008).

[20] One might wonder whether including a response to the price level improves the robustness of standard monetary rules. In Supplemental Appendix J.3 we show this is not the case. Under mild parameter restrictions, a small positive response to the price level only produces determinacy if $\kappa\varsigma > 0$.

[21] This is robust to monetary responses to the real rate which are not exactly equal to 1. This is a corollary of the more general result given in Online Appendix B. We give an alternative direct proof in Supplemental Appendix J.4.

3.3 Wedges in the Fisher equation

How do real rate rules perform if the Fisher equation does not hold exactly? In this subsection, we show that even endogenous "wedges" in the Fisher equation do not challenge the robustness of real rate rules.

Risk premia are one source of such a wedge in the Fisher equation, but certainly not the only one. For example, nominal bonds may supply greater liquidity services or convenience yield than real bonds, and so nominal bonds may command a premium. Such a premium is documented by Fleckenstein, Longstaff & Lustig (2014), based on comparing synthetic treasury bonds constructed from TIPS and inflation swaps to actual treasury bonds. On the other hand, TIPS provide deflation protection, which instead increases the value of TIPS. A Fisher equation wedge could also come from bounded rationality of market participants, distorting expectations. We look in more detail at risk premia and bounded rationality in Section 7, but for now we present general results that apply independently of the source of the wedge.

Suppose then that the linearized Fisher equation takes the form:

$$i_t = r_t + \mathbb{E}_t \pi_{t+1} + \nu_t,$$

where ν_t is a potentially endogenous wedge term. We assume though that ν_t is stationary, and that there exists some $\overline{\mu}_0, \overline{\mu}_1, \overline{\mu}_2, \overline{\gamma}_0, \overline{\gamma}_1, \overline{\gamma}_2 \geq 0$ such that for any stationary solution for π_t, $|\mathbb{E}\nu_t| \leq \overline{\mu}_0 + \overline{\mu}_1 |\mathbb{E}\pi_t| + \overline{\mu}_2 \operatorname{Var} \pi_t$ and $\operatorname{Var} \nu_t \leq \overline{\gamma}_0 + \overline{\gamma}_1 |\mathbb{E}\pi_t| + \overline{\gamma}_2 \operatorname{Var} \pi_t$, for all $t \in \mathbb{Z}$. This assumption is extremely mild, as all these coefficients may be arbitrarily large. For example, if ν_t were to come purely from an inflation risk premium, we would expect $\overline{\mu}_2 > 0$ and $\overline{\gamma}_0 > 0$ but all other coefficients to be zero. Alternatively, if ν_t were to come purely from the liquidity services provided by nominal bonds, we would expect $\overline{\mu}_0$, $\overline{\gamma}_0$ and $\overline{\mu}_1$ to be positive (the latter as the value of liquidity services might vary over the cycle), but all other coefficients to be zero.

Combining the modified Fisher equation with the simple rule in (2) gives:

$$\mathbb{E}_t \pi_{t+1} + \nu_t = \phi \pi_t.$$

Comparing this to the equilibrium condition with a monetary policy shock, (7), reveals that exogenous shocks to the Fisher equation act just like monetary shocks, only with the opposite sign. In the general case in which ν_t is

endogenous, we still have that:

$$\pi_t = \mathbb{E}_t \sum_{k=0}^{\infty} \phi^{-k-1} \nu_{t+k} + \lim_{k\to\infty} \left[\phi^{-k} \mathbb{E}_t \pi_{t+k}\right] = \mathbb{E}_t \sum_{k=0}^{\infty} \phi^{-k-1} \nu_{t+k},$$

assuming as ever that we select the stationary equilibrium for inflation.[22] Thus, with $\phi > 1$:

$$|\mathbb{E}\pi_t| = \frac{|\mathbb{E}\nu_t|}{\phi - 1} \leq \frac{\overline{\mu}_0 + \overline{\mu}_1 |\mathbb{E}\pi_t| + \overline{\mu}_2 \operatorname{Var} \pi_t}{\phi - 1},$$

and:[23]

$$\operatorname{Var} \pi_t = \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \phi^{-j-1} \phi^{-k-1} \operatorname{Cov}\left(\mathbb{E}_t \nu_{t+j}, \mathbb{E}_t \nu_{t+k}\right) \leq \frac{\overline{\gamma}_0 + \overline{\gamma}_1 |\mathbb{E}\pi_t| + \overline{\gamma}_2 \operatorname{Var} \pi_t}{(\phi - 1)^2}.$$

So, for sufficiently large ϕ:[24]

$$|\mathbb{E}\pi_t| \leq \frac{[(\phi-1)^2 - \overline{\gamma}_2]\overline{\mu}_0 + \overline{\mu}_2 \overline{\gamma}_0}{(\phi - 1 - \overline{\mu}_1)[(\phi-1)^2 - \overline{\gamma}_2] - \overline{\mu}_2 \overline{\gamma}_1} = \mathrm{O}\left(\frac{1}{\phi}\right) \text{ as } \phi \to \infty,$$

$$\operatorname{Var} \pi_t \leq \frac{(\phi - 1 - \overline{\mu}_1)\overline{\gamma}_0 + \overline{\mu}_0 \overline{\gamma}_1}{(\phi - 1 - \overline{\mu}_1)[(\phi-1)^2 - \overline{\gamma}_2] - \overline{\mu}_2 \overline{\gamma}_1} = \mathrm{O}\left(\frac{1}{\phi^2}\right) \text{ as } \phi \to \infty.$$

Hence, as $\phi \to \infty$, $\mathbb{E}\pi_t \to 0$ and $\operatorname{Var} \pi_t \to 0$. While the central bank can no longer guarantee precisely zero inflation in the presence of an endogenous wedge, if they are aggressive enough, they can ensure the mean and variance of inflation are arbitrarily close to zero. And, as we already saw, using smoothed rules further limits the impact of Fisher equation wedges. Thus, wedges in the Fisher equation do not present a substantial challenge to the performance of real rate rules.

Inflation swap real rate rules. If the pricing of nominal bonds is highly distorted by the liquidity services they provide (for example), then the central bank may reach lower inflation bias and variance for a given ϕ by intervening in inflation swap markets rather than nominal bond ones. In our notation, an inflation swap is a contract agreed in period t between two parties, A and B, in which A promises to make a net payment of $\Pi_{t+1} - K_t$ to B in period $t+1$, where K_t is the negotiated contract rate. Writing Ξ_{t+1} for the real stochastic discount factor between periods t and $t+1$, this contract rate must solve:

---

[22] Ireland (2015) finds a role for risk premia in explaining US inflation fluctuations, so it is empirically plausible that the Fisher equation wedge should appear in the solution for inflation.

[23] As by the Cauchy-Schwarz inequality, the law of total variance and stationarity: $\operatorname{Cov}\left(\mathbb{E}_t \nu_{t+j}, \mathbb{E}_t \nu_{t+k}\right) \leq \sqrt{(\operatorname{Var} \mathbb{E}_t \nu_{t+j})(\operatorname{Var} \mathbb{E}_t \nu_{t+k})} = \sqrt{(\operatorname{Var} \nu_{t+j} - \mathbb{E} \operatorname{Var}_t \nu_{t+j})(\operatorname{Var} \nu_{t+k} - \mathbb{E} \operatorname{Var}_t \nu_{t+k})} \leq \operatorname{Var} \nu_t$.

[24] In particular, we need $\phi - 1 > \overline{\mu}_1$, $(\phi-1)^2 > \overline{\gamma}_2$ and $(\phi - 1 - \overline{\mu}_1)[(\phi-1)^2 - \overline{\gamma}_2] > \overline{\mu}_2 \overline{\gamma}_1$.

$$\mathbb{E}_t \frac{\Xi_{t+1}}{\Pi_{t+1}} (\Pi_{t+1} - K_t) = 0.$$

So, from log-linearizing, $k_t = \log K_t = \mathbb{E}_t \pi_{t+1}$, to first order.

The central bank can then use the inflation swap real rate rule, $k_t = \phi \pi_t$. Combined with the inflation swap pricing equation, this gives $\mathbb{E}_t \pi_{t+1} = \phi \pi_t$, just like when the central bank intervenes in nominal bond markets. The advantage of directly targeting inflation swap contract rates is that inflation swaps are unlikely to supply liquidity services, unlike nominal bonds, meaning the inflation swap pricing equation may be less distorted than the Fisher equation. One final benefit of directly targeting inflation swap contract rates is that inflation swaps do not include the deflation protection given by TIPS. This removes an added source of distortion in the $i_t - r_t$ gap.

4 The zero lower bound

All our examples so far have ignored the zero lower bound (ZLB) on nominal interest rates. The ZLB is problematic for real rate rules as it prevents the central bank from fixing $i_t - r_t$ when $i_t = 0$. This means that at the ZLB, the Euler equation again becomes relevant for outcomes, reducing robustness. This section presents a simple solution to restore robustness in the presence of the ZLB. In Supplemental Appendix G we give two other potential solutions: price level real rate rules, and perpetuity real rate rules. We also show there that when households hold perpetuities, appropriately constructed real rate rules can rule out both permanent ZLB traps as well as explosive paths for inflation, answering Cochrane (2011).

4.1 The problems caused by the ZLB for real rate rules

We can see the problems caused by the ZLB even in the simple set-up used in this paper's introduction. In the presence of the zero lower bound, under the introduction's set-up, we have that:

$$\max\{0, r_t + \phi \pi_t\} = i_t = r_t + \mathbb{E}_t \pi_{t+1}.$$

While without the ZLB, we can cancel out the r_t in the monetary rule with the r_t from the Fisher equation, now this is no longer possible. Instead, we have that $\max\{-r_t, \phi \pi_t\} = \mathbb{E}_t \pi_{t+1}$. Thus, real rates (and hence the Euler equation)

potentially matter for inflation dynamics and determinacy. Holden (2021) points out that even if r_t is exogenous, with $r_t = 0$ for $t \neq 1$, and even if we assume that $\pi_t \to 0$ as $t \to \infty$, still there are multiple solutions for a value of r_1 ($r_1 = 0$), and no solution for other values of r_1 ($r_1 < 0$).

Holden (2021) shows this multiplicity and non-existence of perfect foresight solutions is the rule for NK models with a ZLB, even with a terminal condition on inflation ensuring an eventual escape from the ZLB. Additionally, there are further solutions converging to a deflationary steady state with interest rates at zero (Benhabib, Schmitt-Grohé & Uribe 2001). Furthermore, under rational expectations there are always at least as many solutions as under perfect foresight, as well as a continuum of further solutions which switch based on a sunspot (Holden 2021).

For example, forward looking NK models without fundamental shocks often have absorbing sunspot solutions of the following form (Mertens & Ravn 2014; Schmidt 2016; Bilbiie 2022). The economy starts at the ZLB in period 1. While at the ZLB, there is a constant probability of $q \in [0,1]$ of remaining there in the next period. With probability $1 - q$ though, the economy returns to the intended steady state and stays there for ever. As an example, suppose that the model is given by the following four equations:[25]

$$\pi_t - \pi^* = \beta \mathbb{E}_t(\pi_{t+1} - \pi^*) + \kappa x_t, \tag{10}$$

$$x_t = \delta \mathbb{E}_t x_{t+1} - \varsigma(r_t - n), \tag{11}$$

$$\max\{0, r_t + \pi^* + \phi(\pi_t - \pi^*)\} = i_t = r_t + \mathbb{E}_t \pi_{t+1},$$

with $\kappa\varsigma \neq 0$, $\phi > 1$ and $n + \pi^* > 0$ (so there is a steady state with positive nominal rates). Then an equilibrium of the form described exists if and only if $\frac{(1-\beta q)(1-\delta q)-q\kappa\varsigma}{\kappa\varsigma} \leq 0$.[26,27] This holds for q sufficiently large when $\kappa\varsigma > 0$ and either $\beta \leq 1$ and $\delta \geq 1$, or $\beta \geq 1$ and $\delta \leq 1$, or $\delta \geq 0$ and $\beta \in [1 - \kappa\varsigma, 1]$, or $\beta \geq 0$

[25] This is the model of Subsection 3.2, but without shocks, and allowing for a non-zero long-run inflation target π^* with full indexation of non-resetting firms to this target.

[26] This is proven in Supplemental Appendix J.8. We also examine two-state sunspot solutions without an absorbing state in that appendix. Nakata & Schmidt (2022) look at such equilibria under optimal policy.

[27] In line with the results of Bilbiie (2022), this is identical to the condition for positive monetary shocks to lower the nominal rate when $q = \rho$, in the notation of footnote 17. (The author again thanks a referee for this observation.)

and $\delta \in [1-\kappa\varsigma, 1]$. And it holds for q sufficiently small when $\kappa\varsigma < 0$. Hence for most reasonable calibrations of the model (including those aimed at capturing the impact of heterogeneity) it holds for at least some q, implying that the existence of sunspot equilibria is common even under real rate rules.

4.2 Modified inflation targets

One of the sources of equilibrium non-existence is that the monetary rule is implicitly targeting an infeasible level for inflation when real rates are low. If inflation is at target, nominal interest rates should be positive. A modified inflation target can ensure this. Furthermore, by introducing some history dependence we can rule out sunspot equilibria of the type previously considered; interest rate smoothing will do.

Building on the smoothed rule of equation (5), suppose that from period 1 onwards, the central bank uses the rule:

$$i_t = \max\{0, r_t + (i_{t-1} - r_{t-1}) + (\mathbb{E}_t \breve{\pi}^*_{t+1} - \mathbb{E}_{t-1}\breve{\pi}^*_t) + \theta(\pi_t - \breve{\pi}^*_t)\}, \tag{12}$$

where $\theta > 0$ and where the modified inflation target, $\breve{\pi}^*_t$, is given by:

$$\breve{\pi}^*_t := \max\{\pi^*_t, \epsilon - r_{t-1}\}, \tag{13}$$

with π^*_t the original inflation target, and $\epsilon > 0$ some small constant (10 annualized bps say). Note that if inflation is at the modified target, then nominal rates must be positive, by the Fisher equation.[28] We assume that the central bank announces the modified inflation target at the start of each period, so agents do not necessarily need to understand the rule that produces it. Since it is hard to give sense to expectations of the modified target before the rule is introduced, we will sometimes also assume that $\mathbb{E}_0\breve{\pi}^*_1 = \mathbb{E}_0\pi_1$, or at least that the central bank acts in period 1 as if that were true.

Under this modified rule, $\pi_t = \breve{\pi}^*_t - \theta^{-1}\mathbb{E}_{t-1}(\pi_t - \breve{\pi}^*_t)$ for all $t \geq 1$ is an equilibrium.[29] This means that $\pi_t = \breve{\pi}^*_t$ for all $t > 1$,[30] and this will also hold for

[28] This modified target is higher (in expectation) than necessary to ensure positive nominal rates. It would be enough to set $\breve{\pi}^*_t := \pi^*_t + \max\{0, \epsilon - r_{t-1} - \mathbb{E}_{t-1}\pi^*_t\}$, which has a lower $t-1$ dated conditional expectation than $\max\{\pi^*_t, \epsilon - r_{t-1}\}$, by Jensen's inequality. However, the target of equation (13) is likely to be easier to communicate, and easier to learn, as it is a constraint on observables, not expectations.

[29] That this is the unique equilibrium in the absence of the ZLB follows from the results of Subsection 2.1. Thus, it suffices to establish that $i_t > 0$ for all $t \geq 1$ under this equilibrium. This follows as $\mathbb{E}_t\pi_{t+1} = \mathbb{E}_t\breve{\pi}^*_{t+1} - \theta^{-1}\mathbb{E}_t(\pi_{t+1} - \breve{\pi}^*_{t-1})$, i.e., $(1+\theta^{-1})\mathbb{E}_t\pi_{t+1} = (1+\theta^{-1})\mathbb{E}_t\breve{\pi}^*_{t+1}$, so $i_t - r_t = \mathbb{E}_t\pi_{t+1} = \mathbb{E}_t\breve{\pi}^*_{t+1} \geq$

$t = 1$ under the initial condition $\mathbb{E}_0 \check{\pi}_1^* = \mathbb{E}_0 \pi_1$. Hence, under this monetary rule, no matter the form of the rest of the model, there is a closed form solution for inflation in terms of observables. The existence of a closed form solution is particularly desirable as it is likely to be easier for agents to coordinate on simple solutions. Even the existence of a solution is notable, as under the simple rule of the previous subsection there could be no solution at all.

Additionally, under the $\pi_t = \check{\pi}_t^* - \theta^{-1}\mathbb{E}_{t-1}(\pi_t - \check{\pi}_t^*)$ solution, π_t is bounded below by $-r_{t-1}$.[31] This prevents the severe deflations that can accompany shocks taking the economy to the ZLB under standard monetary rules. It also removes all of the deflationary bias that usually accompanies the ZLB (Hills, Nakata & Schmidt 2019). Instead, the definition of $\check{\pi}_t^*$ implies that $\mathbb{E}\pi_t \geq \mathbb{E}\pi_t^*$,[31] so there is a mild inflationary bias.

Moreover, at least in the absence of uncertainty, the $\pi_t \equiv \check{\pi}_t^* - \theta^{-1}\mathbb{E}_{t-1}(\pi_t - \check{\pi}_t^*)$ solution is unique, assuming that π_t is bounded, and that the economy eventually escapes the ZLB for good.[32] This is another improvement on the performance of the naïve real rate rule of the previous subsection, where there were multiple perfect-foresight paths even given these assumptions.

4.3 Ruling out sunspot equilibria

The rule of equation (12) also helps rule out sunspot equilibria. The usual explanation for the benefit of history dependence when there is a ZLB is as follows: history dependence leads to higher inflation after exiting the ZLB, raising inflation expectations even while at the ZLB. However, this channel cannot help rule out the sunspot equilibria that exist when the ZLB state is sufficiently persistent, as in the fully persistent limit, inflation in the non-ZLB

$\epsilon - r_t > -r_t$, implying $i_t > 0$.

[30] Let $t > 1$. By the previous footnote, $\mathbb{E}_{t-1}\pi_t = \mathbb{E}_{t-1}\check{\pi}_t^*$, so from the solution for inflation, $\pi_t = \check{\pi}_t^*$.

[31] At least for $t > 1$, but also for $t = 1$ under the initial condition $\mathbb{E}_0 \check{\pi}_1^* = \mathbb{E}_0 \pi_1$.

[32] Strictly, without further assumptions we only have uniqueness conditional on the path of $\check{\pi}_t^*$. This is proven in Supplemental Appendix J.9. For example, an endogenous π_t^* could produce multiple solutions for $\check{\pi}_t^*$. However, at least when π_t^* is exogenous, there is unconditional uniqueness for standard models. For example, with the rest of the model given by equations (10) and (11), with π_t^* exogenous and $\frac{\beta+\delta}{\kappa\varsigma} > -\frac{1}{2}$, there is a unique perfect foresight solution under the additional terminal condition $\check{\pi}_t^* - \pi_t^* \to 0$ as $t \to \infty$ (again proved in Supplemental Appendix J.9). Moreover, with $\frac{\beta+\delta}{\kappa\varsigma} \geq 0$, this uniqueness is robust, in the sense that no small, continuous change to the model or its parameters could overturn it.

state(s) has no impact on inflation in the ZLB state.[33] Instead, history dependence helps rule out sunspot equilibria by preventing any transition into the ZLB state. History dependence allows the central bank to have a weak contemporaneous response to inflation. With the monetary rule flatter than the Fisher equation, uniqueness is restored.

We want to know when it is possible for the economy to make a sunspot driven jump to the ZLB. So, we need to look at two or more state sunspot solutions without an absorbing "good" state. Supplemental Appendix J.8 looks at two-state sunspot solutions to the model of Subsection 4.1 with the naïve real rate rule. It shows that given mild parameter restrictions,[34] a sunspot solution only exists when both the "bad" (ZLB) state and the "good" (non-ZLB) state are sufficiently persistent. Thus, to examine whether the rule of equation (12) prevents similar sunspot solutions, it suffices to consider the extreme case in which the economy remains in its current state with probability one. This avoids the technical challenges of solving for sunspot solutions with endogenous state variables.

Consider then the model of equations (10), (11), (12) and (13), with $\pi_t^* = \pi^*$ for all t, $n + \pi^* > \epsilon > 0$, $\theta > 0$, $\kappa\varsigma > 0$ and $(1-\beta)(1-\delta) - \kappa\varsigma < 0$.[35] Suppose that in period 0, the economy was away from the ZLB, and was expected to stay there with probability one. Thus, $i_0 - r_0 = \mathbb{E}_0\pi_1 = \mathbb{E}_0\check{\pi}_1^* = \check{\pi}_1^* = \pi^*$. However, in period 1, a "zero probability sunspot shock" hits, so that with probability one, for all $t \geq 1$, $0 = i_t = r_t + \pi_{t+1}$. (The expectation drops out of the Fisher equation as there is no other uncertainty.) Thus for $t \geq 1$, the Phillips curve and Euler equation imply that $\pi_t = \pi_Z$ and $x_t = x_Z$ where $(1-\beta)(\pi_Z - \pi^*) = \kappa x_t$ and $(1-\delta)x_Z = \varsigma(\pi_Z + n)$, so:

$$\pi_Z - \pi^* = \frac{\kappa\varsigma(n + \pi^*)}{(1-\beta)(1-\delta) - \kappa\varsigma} < 0.$$

[33] Nie & Roulleau-Pasdeloup (2022) show that the higher post-ZLB inflation channel can help rule out sunspots given a fixed persistence of the ZLB state. But with policy fixed, sunspots always exist in their set-up if the ZLB state is allowed to be sufficiently persistent.

[34] $\kappa\varsigma > 0$, $(1-\beta)(1-\delta) - \kappa\varsigma < 0$, $\beta\delta \geq 0$, ϕ sufficiently large. Note: if $\kappa\varsigma > 0$, then $(1-\beta)(1-\delta) - \kappa\varsigma < 0$ when either $\beta \leq 1$ and $\delta \geq 1$, or $\beta \geq 1$ and $\delta \leq 1$, or $\delta > 0$ and $\beta \in [1 - \kappa\varsigma, 1]$, or $\beta > 0$ and $\delta \in [1 - \kappa\varsigma, 1]$.

[35] The final two assumptions ensure that with the naïve real rate rule, a sunspot solution only exists with sufficiently persistent states. The previous footnote gives sufficient conditions for $(1-\beta)(1-\delta) - \kappa\varsigma < 0$.

This is consistent with equilibrium if and only if the interest rate would be non-positive for $t \geq 1$ were it not for the ZLB. In period 1, this requires:

$$\begin{aligned} 0 &\geq r_1 + (i_0 - r_0) + (\mathbb{E}_1 \check{\pi}_2^* - \mathbb{E}_0 \check{\pi}_1^*) + \theta(\pi_1 - \check{\pi}_1^*) \\ &= \max\{0, \epsilon + \pi_Z - \pi^*\} + (\theta - 1)(\pi_Z - \pi^*). \end{aligned}$$

However, if $\theta < 1$, then $(\theta - 1)(\pi_Z - \pi^*) > 0$, so the condition cannot hold. Thus, as long as the central bank does not respond too aggressively to inflation, there cannot be sunspot solutions of the kind previously described. Furthermore, it follows that as long as the economy is currently sufficiently close to the "good" steady state, there is no way for the economy to ever jump to the ZLB. Crucial to this result is the fact that interest rate smoothing means we only need θ to be positive. It does not need to be greater than 1, as the state variables ensure the response to anticipated inflation deviations is $1 + \theta > 1$, as required by the Taylor principle. Thus, the modified inflation target rule delivers robust uniqueness, even in the presence of the ZLB.

5 Practical implementation of real rate rules

Until recently, central banks concentrated their monetary interventions in overnight debt markets. However, with the rise of quantitative easing, many central banks have been buying substantial quantities of longer maturity sovereign debt. There is no reason then that central banks could not conduct open market operations to fix the interest rate on longer maturity bonds. Since long maturity bonds are less likely to hit the ZLB, this lessens the ZLB's impact. Furthermore, using longer maturity bonds is convenient under a real rate rule, as in most countries, inflation-protected securities are only issued a few times per year, and at long maturities, e.g., five years. As a result, markets in shorter maturity inflation-protected securities may be illiquid or even unavailable, and it can be difficult to reconstruct the short end of the real yield curve.

Inflation indexation lags further complicate the use of short maturity inflation-protected securities (see e.g. Gürkaynak, Sack & Wright (2010)). For example, with time measured in quarters, 3-month maturity US TIPS have a period $t + 1$ realized yield of $r_t + \pi_t$, not $r_t + \pi_{t+1}$ as one might have expected. Additionally, there is an information lag as inflation is not observed contemporaneously. By using longer maturity bonds, the impact of these

indexation and information lags are reduced. This section examines the performance of real rate rules when the central bank implements them using multiperiod debt in the presence of indexation and information lags.

5.1 Set-up

We aim to describe a set-up with many of the frictions that would be problematic for a naïve implementation of a real rate rule. The central bank's trading desk would be tasked with maintaining a particular level of the gap between nominal and real rates, according to the market for bonds of a certain maturity. We let $T \geq 1$ be the time to maturity of these bonds, measured in periods. The units of time do not need to coincide with the maturity of the bond. For example, T may be 60 if periods are months and five-year bonds are used.

We allow for the possibility that inflation is not observed contemporaneously. For example, US CPI is released with a one-month lag. To capture this, while keeping to the convention that $\mathbb{E}_t v_t = v_t$ for all t-dated endogenous variables v_t, we assume that market participants and the central bank use the $t - S$ information set in period t (i.e. they know the values of all $t - S$ and earlier dated variables), for some $S \geq 0$. Since the central bank does not know π_t at t, we assume that they respond to deviations of π_{t-S} from target, rather than to deviations of π_t.

We write $i_{t|t-S}$ for the nominal yield per-period on a T-period nominal bond at t, and $r_{t|t-S}$ for the real yield per-period on a T-period inflation-protected bond at t. This notation captures the fact that period t nominal and real yields must be fixed in period $t - S$: market participants and the central bank only have access to the period $t - S$ information set at t, and these agents must know period t nominal and real rates.

We allow for a wedge in the Fisher equation to capture inflation risk premia, liquidity premia, asymmetric term premia and even departures from full information rational expectations amongst market participants. Since only $t - S$ dated variables are known in period t, we denote the period t value of this shock by $\nu_{t|t-S}$. I.e., risk premia (etc.) will be determined S periods in advance, though market participants and the central bank will not act on this, as they use S period old data.

We also allow for the possibility of an indexation lag in the return of the real bond. We assume that the lag is L periods, where $L \geq S$. If periods are months, then L would be 3 for the US (Gürkaynak, Sack & Wright 2010).

5.2 The generalized Fisher equation and monetary rule

Given all this, the Fisher equation coming from arbitrage between nominal and real bonds states that:

$$i_{t|t-S} = r_{t|t-S} + \nu_{t|t-S} + \mathbb{E}_{t-S} \frac{1}{T} \sum_{k=1}^{T} \pi_{t+k-L}.$$

We assume that $T - L \geq -S$, so that inflation dated $t - S$ or later enters this equation. Otherwise, the Fisher equation becomes backward looking and determinacy conditions may be quite different. So, for the US, the central bank would have to use bonds with maturity of at least two months.

Supplemental Appendix H presents empirical evidence that this five-year Fisher equation holds. There, we review some prior evidence, and show both that professional forecasts predict breakeven rates, and that breakeven rates forecast realised inflation.

Based upon our previous rule (12), we suppose that from period $1 - S$ onwards, the central bank intervenes in T-period nominal bond markets to ensure that:

$$\begin{aligned} i_{t|t-S} = \max \Big\{ & 0, r_{t|t-S} + \bar{\nu}_{t|t-S} + (i_{t-1|t-1-S} - r_{t-1|t-1-S} - \bar{\nu}_{t-1|t-1-S}) \\ & + \mathbb{E}_{t-S} \frac{1}{T} \sum_{k=1}^{T} \check{\pi}^*_{t+k-L} - \mathbb{E}_{t-1-S} \frac{1}{T} \sum_{k=1}^{T} \check{\pi}^*_{t-1+k-L} + \theta(\pi_{t-S} - \check{\pi}^*_{t-S}) \\ & + \frac{1}{T} [(\pi_{t-S} - \check{\pi}^*_{t-S}) - (\pi_{t-L} - \check{\pi}^*_{t-L})] \Big\}, \end{aligned}$$

where $\check{\pi}^*_t$ is the modified inflation target (to be defined), $\bar{\nu}_{t|t-S}$ is the central bank's period t belief about the level of $\nu_{t|t-S}$, and $\theta > 0$. $\bar{\nu}_{t|t-S}$ could also include a monetary policy shock component. We stress that the $t|t-S$ index here does not mean that the private sector knows monetary policy shocks S periods in advance, as the private sector (and the central bank) uses the $t - S$ information set at t. The final term here is a response to the change in relative inflation from period $t - L$ to $t - S$. This ensures that $\theta > 0$ is sufficient for determinacy even when $S < L$.[36]

Note that while under conventional monetary policy, nominal interest rates are approximately constant between monetary policy committee meetings, this may not be the case here. The rule effectively specifies a period t level for $i_{t|t-S} - r_{t|t-S}$, not for $i_{t|t-S}$. The level of $r_{t|t-S}$ may fluctuate (perhaps in part due to unexpected changes in $i_{t|t-S}$), so the central bank's trading desk could have to continuously tweak the level of $i_{t|t-S}$ to hold $i_{t|t-S} - r_{t|t-S}$ at its desired level. While this is a departure from current operating procedures, there is no reason why holding $i_{t|t-S} - r_{t|t-S}$ approximately constant should be any harder than holding $i_{t|t-S}$ approximately constant. This is thanks to the real-time observability of $r_{t|t-S}$ via inflation-protected bonds.

The central bank could also directly control $i_{t|t-S} - r_{t|t-S}$ by promising to freely exchange \$1 face value of real debt for $\$\exp[T(i_{t|t-S} - r_{t|t-S})]$ face value of nominal debt, as suggested by Cochrane (2017; 2018). Alternatively, the central bank could buy or sell a long-short portfolio containing \$1 face value of nominal debt, and −\$1 face value of real debt to hold the portfolio's per-period return fixed at $\$(i_{t|t-S} - r_{t|t-S})$.[37] Or, the central bank could directly pin down the contract rate on inflation swaps, as suggested in Subsection 3.3.

We define the modified inflation target to ensure that $i_{t|t-S} \geq \epsilon$, where $\epsilon > 0$ is a small constant (10 annualized bps say). As in the one period case, the central bank should announce the modified inflation target each period, so firms and households do not need to understand the precise law of motion of $\check{\pi}_t^*$. In particular, we define:

$$\check{\pi}_t^* := \max\{\check{\pi}_t^{(j)} | j \in \{1, \dots, T\}\},$$

where for $j \in \{1, \dots, T\}$:

$$\check{\pi}_t^{(j)} := \pi_t^* + \max\left\{0, \frac{T}{j}\left(\epsilon - r_{t+j-1-T+L|t+j-1-T+L-S} - \bar{\nu}_{t+j-1-T+L|t+j-1-T+L-S}\right) - \frac{1}{j}\sum_{k=1}^{T-j}\check{\pi}_{t-k}^* - \mathbb{E}_t\frac{1}{j}\sum_{k=0}^{j-1}\pi_{t+k}^*\right\}.$$

If $T = 1$ and $L = S = 0$, then much as in Subsection 4.2, $\check{\pi}_t^* = \max\{\pi_t^*, \epsilon - r_{t-1|t-1} - \bar{\nu}_{t-1|t-1}\}$. More generally, the $\check{\pi}_t^{(1)}$ component is enough to ensure that

[36] We examine determinacy without this term in Online Appendix E, and show that there is still determinacy as long as $\theta > \frac{2}{T}$, so in the continuous time limit, $\theta > 0$ is again sufficient.

[37] The author thanks Peter Ireland for this suggestion.

$r_{t|t-S} + \bar{\nu}_{t|t-S} + \frac{1}{T}\sum_{k=1}^{T} \breve{\pi}^*_{t+k-L} \geq \epsilon$ for all t.[38] The $\breve{\pi}^{(j)}_t$ components for $j > 1$ help to smooth the inflation increases over time. Rather than increasing the inflation target just in the final period of a bond that would otherwise violate the constraint, instead we smooth this increase over the life of the bond. The particular structure here is designed to minimise the risk of self-fulfilling dynamics from the various bounds. Note that we do not attempt to ensure that shorter maturity bonds are away from the ZLB, thus the higher is T, the closer $\breve{\pi}^*_t$ should be to π^*_t.

5.3 Solution and robustness

Define $\Delta_t := (\nu_{t+S|t} - \bar{\nu}_{t+S|t}) - (\nu_{t-1+S|t-1} - \bar{\nu}_{t-1+S|t-1})$ and:

$$e_t := \mathbb{E}_t \frac{1}{T} \sum_{k=1}^{T-L+S} (\pi_{t+k} - \breve{\pi}^*_{t+k}).$$

Then combining the multi-period Fisher equation and the monetary rule implies that if the ZLB never binds (as our modified target should ensure), then $e_t + \Delta_t = e_{t-1} + \theta(\pi_t - \breve{\pi}^*_t)$. (The final term in the monetary rule has dropped out due to cancellation with the first $L - S$ terms of the sums.) Substituting this back into the definition of e_t gives the purely forward-looking recurrence:

$$\theta T e_t = \mathbb{E}_t \sum_{k=1}^{T-L+S} (e_{t+k} - e_{t+k-1} + \Delta_{t+k}). \tag{14}$$

Any solution for e_t gives a corresponding solution for π_t as $\pi_t = \breve{\pi}^*_t + \theta^{-1}(e_t - e_{t-1} + \Delta_t)$. Note that since we assume the rule is introduced in period $1 - S$, equation (14) will hold for all $t \geq 1$.

We start by proving determinacy when Δ_t is exogenous. We separately consider the two cases, $T = L - S$ and $T > L - S$ (recall that we assume $T \geq L - S$). In the first case, equation (14) states that $\theta T e_t = 0$, giving a unique solution. In the latter case, there is a unique solution if and only if it has a unique solution when $\Delta_t = 0$ for all t. In this case, via the substitution $e_t = c\lambda^t$ we have the characteristic polynomial, $\theta T = \lambda^{T-L+S} - 1$, meaning $|\lambda| = (1 + \theta T)^{\frac{1}{T-L+S}} > 1$. Therefore, all the polynomial's roots are outside the unit circle, which implies

[38] Strictly, we want to ensure that $0 < i_{t|t-S} = r_{t|t-S} + \nu_{t|t-S} + \mathbb{E}_{t-S}\frac{1}{T}\sum_{k=1}^{T} \breve{\pi}^*_{t+k-L}$. If $\bar{\nu}_{t|t-S} \neq \nu_{t|t-S}$, then this is not guaranteed. However, if the tracking error $|\bar{\nu}_{t|t-S} - \nu_{t|t-S}|$ can be bounded with probability one, then we can set ϵ to that bound and ensure $0 < i_{t|t-S}$. If the tracking error is not bounded with probability one, then we can still make the probability of hitting the ZLB negligible by setting a high enough ϵ.

determinacy as equation (14) is purely forward looking. Thus, at least when Δ_t is exogenous, there is a unique solution for inflation.[39] In the special case in which the central bank observes ν_t so $\bar{\nu}_t = \nu_t$, then $\pi_t = \pi_t^*$ for all $t > 1$.

In the general case in which Δ_t is potentially endogenous, as long as it is stationary, the solution must take the form $e_t = \mathbb{E}_t \sum_{j=1}^{\infty} A_j \Delta_{t+j}$. Substituting this into (14) then matching terms gives that for $j \leq T - L + S$, $A_j = (1 + \theta T)^{-1}$, while for $j > T - L + S$, $A_j = (1 + \theta T)^{-1} A_{j-T+L-S}$. Thus:

$$e_t = \mathbb{E}_t \sum_{j=1}^{\infty} (1 + \theta T)^{-\left\lceil \frac{j}{T-L+S} \right\rceil} \Delta_{t+j},$$

where for all z, $\lceil z \rceil$ denotes the smallest integer greater or equal to z. For example, in the simple case in which Δ_t is independent across time (meaning that $\nu_{t+S|t} - \bar{\nu}_{t+S|t}$ follows a random walk), $e_t = 0$, so for $t > 1$, $\pi_t = \breve{\pi}_t^* + \theta^{-1} \Delta_t$. Or if Δ_t follows an AR(1) process with persistence ρ, then $e_t = E\Delta_t$ (also AR(1)), where $E := \rho\big(1 - \rho^{T-L+S}\big)(1-\rho)^{-1}\big(1 - \rho^{T-L+S} + \theta T\big)^{-1}$, so $x_t := \pi_t - \breve{\pi}_t^* = \theta^{-1}\big((1+E)\Delta_t - E\Delta_{t-1}\big)$.

Furthermore, under even weaker conditions than those used in Subsection 3.3, we have that $\pi_t \approx \pi_t^*$ for large θ, even when Δ_t is endogenous (proven in Supplemental Appendix J.10). These conditions are very mild, as already argued in Subsection 3.3. Thus, with θ large, even if the central bank imperfectly tracks the risk (etc.) premium ν_t, and even if their error is endogenous and non-stationary ($I(1)$), it will still be the case that $\pi_t \approx \pi_t^*$ in all periods. I.e., even in the presence of unobservable endogenous, non-stationary wedges in the Fisher equation, the central bank can still determinately implement an arbitrary path for inflation. The presence of information or indexation lags makes no fundamental difference to this.

6 Empirical test

Would the behaviour of the US Federal Reserve have been drastically different if it were following a real rate rule? Or have nominal rates in the US closely tracked what they would have been under a real rate rule? We previously argued that real rate rules could explain any observed outcomes,

[39] We do not have the indeterminacy issues for rules setting long-rates that were noted by McGough, Rudebusch & Williams (2005), due to the presence of the real rate in our rule.

given an appropriate path of π_t^*. Thus, to design a reasonable empirical test of the explanatory power of real rate rules, we need additional data to discipline π_t^*. Luckily, for the US this is available, via the Summary of Economic Projections. Each quarter, Federal Reserve board members and regional bank presidents are asked for their projections for the economy, conditional on the Fed following what they believe to be "appropriate monetary policy".[11] Their projections for inflation thus represent what they believe to be the ideal outcome for inflation, given the economy's state, i.e. they are measures of π_t^*. In Supplemental Appendix I we document how we recover a quarterly time series for π_t^* from data on these projections at different horizons.

Armed with estimates of π_t^*, we can then estimate the practical real rate rule introduced in Section 5 on actual quarterly US CPI and TIPS breakeven inflation data (Q4 2008 to Q4 2022). We work with five-year US treasuries and TIPS, so $T = 20$. Since annual yields on five-year US treasuries never dropped below 0.19 over our sample,[40] we ignore the ZLB. We take $L = 1$ (i.e., three months) and $S = 0$, since the true CPI release delay of below one month is less than half of the length of a period (three months). For simplicity we write (e.g.) ν_t rather than $\nu_{t|t}$. Thus, we wish to estimate:

$$y_t := \mathbb{E}_t \frac{1}{T}\sum_{k=0}^{T-1}(\pi_{t+k} - \pi^*_{t+k}) - \mathbb{E}_{t-1}\frac{1}{T}\sum_{k=0}^{T-1}(\pi_{t-1+k} - \pi^*_{t-1+k}) + \nu_t - \nu_{t-1}$$
$$- \frac{1}{T}\left[(\pi_t - \pi_t^*) - (\pi_{t-1} - \pi_{t-1}^*)\right] = \theta x_t + \varepsilon_{\bar{\nu},t},$$

where $x_t := \pi_t - \pi_t^*$ and $\varepsilon_{\bar{\nu},t} := \bar{\nu}_t - \bar{\nu}_{t-1}$. We describe the empirical counterpart of y_t in Supplemental Appendix I.5.

We begin by estimating $y_t = \theta x_t + \varepsilon_{\bar{\nu},t}$ by OLS. This gives $\theta \approx 0.033$ with a heteroskedasticity and autocorrelation (HAC) robust p-value of 0.002 for a test of $\theta = 0$. Thus, the OLS slope is significantly different from 0 at 1%. We plot the data and the OLS fit in Figure 8.[41]

However, OLS is likely to be biased for two reasons. Firstly, the presence of measurement error in π_t^* leads to attenuation bias. Secondly, aside from any measurement error, x_t is still likely to be correlated with the error term, and

[40] Data from https://fred.stlouisfed.org/series/DGS5.

[41] See Supplemental Appendix I for replication instructions for all these results.

hence endogenous. According to the model, x_t is linear in $\Delta_t = (\nu_t - \nu_{t-1}) - (\bar{\nu}_t - \bar{\nu}_{t-1})$ and its lags, and the error term is $\varepsilon_{\bar{\nu},t} = \bar{\nu}_t - \bar{\nu}_{t-1}$. If, as is plausible, Δ_t and $\bar{\nu}_t - \bar{\nu}_{t-1}$ are correlated, then x_t will be endogenous.

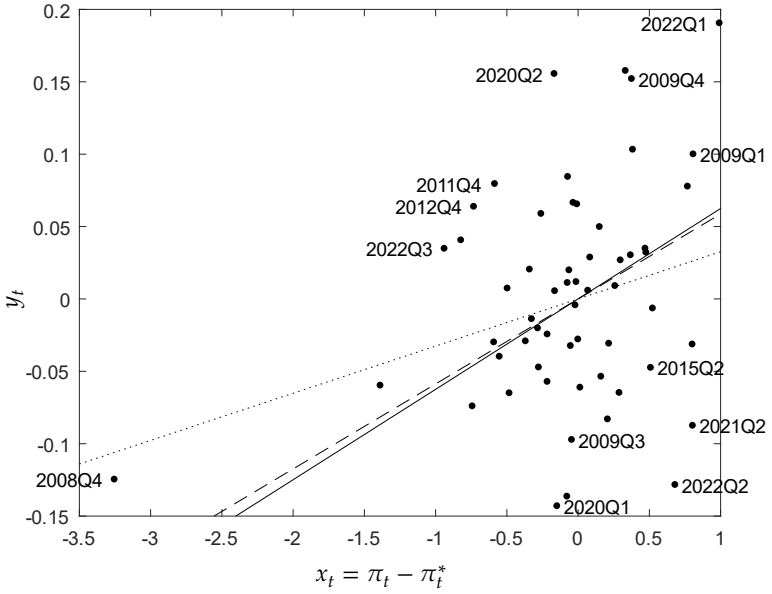

Figure 1: Data and linear fits. The dotted line is the OLS estimate. The dashed line is the monetary shock-based estimate. The solid line is the IV estimate.

To reduce these biases, we instrument x_t with the oil supply news shocks of Känzig (2021).[42] The oil price news shocks have an immediate impact on inflation, as shown by Känzig (2021). They ought not to be driven by the monetary shock ($\varepsilon_{\bar{\nu},t}$) or changes in π_t^*, as they are constructed using a tight window around OPEC announcements, during which the US monetary stance should not have changed. As evidence, Känzig documents that the shocks are uncorrelated with standard measures of monetary policy surprises, and do not lead to immediate movements in the Fed funds rate.

So, the oil price news shocks should be correlated with x_t, but not with $\varepsilon_{\bar{\nu},t}$ (the monetary shock), as is required to be a valid instrument. This means they must be correlated with $\nu_t - \nu_{t-1}$, the change in the Fisher wedge. If Δ_t is IID, then $y_t = \nu_t - \nu_{t-1}$, so y_t should be correlated with these shocks. We observe a correlation of 36%, significantly different from 0 at a 1% level.

A potential challenge to this instrument is that were the central bank really

[42] Känzig provides these shocks to download here: https://github.com/dkaenzig/oilsupplynews. We aggregate them to quarterly.

following a real rate rule, the Fed's trading desk might detect the movement in $\nu_t - \nu_{t-1}$ and respond with a similar change in $\bar{\nu}_t - \bar{\nu}_{t-1}$, even within the tight window used in constructing the shocks. However, given the observed smooth path of Fed Treasury holdings, this seems unlikely in practice.[43]

The IV estimates give $\theta \approx 0.062$ with a HAC p-value of 0.003 for a test of $\theta = 0$.[44] Thus, our preferred estimate of θ is significantly different from 0 even at a 1% level.[45] The residuals from this regression are measures of the monetary policy shock, $\varepsilon_{\bar{\nu},t}$. As such, they should be correlated with the monetary shocks identified by Bauer & Swanson (2023), aggregated to quarterly. This correlation is 14% over the available sample (Q4 2008 to Q4 2019). That this number is not higher is perhaps unsurprising given that nominal rates were near zero for most of the sample, making it hard for the Bauer & Swanson high frequency identification method to pick up much of a signal.

The correlation of the residuals with the Bauer Swanson shocks suggests an alternative estimate of θ as the value that maximizes the correlation between the residuals and these shocks.[46] This gives an almost identical value of $\theta \approx 0.059$, albeit with far larger standard errors. That this estimate is so close nonetheless supplies further evidence in support of our IV estimate.

Given our estimate of θ, we can also examine how much of the variance of various rates can be explained by terms other than the direct effect of the monetary policy shock $\varepsilon_{\bar{\nu},t}$. In particular, we calculate $\text{RSS} := (y_t - \theta x_t)^2$ and then evaluate $1 - \frac{\text{RSS}}{\text{TSS}}$, where TSS is the total sum of squares from a rate of interest. The IV estimates explain 49.1% of the variance of changes in five-year breakeven inflation expectations, 53.3% of the variance of changes in five-year

[43] See https://fred.stlouisfed.org/series/WSHOTSL.

[44] We also run the OLS and IV regressions without the $\frac{1}{T}[(\pi_t - \pi_t^*) - (\pi_{t-1} - \pi_{t-1}^*)]$ term on the left-hand side. This gives $\theta \approx 0.089$ (OLS) and $\theta \approx 0.130$ (IV) with both HAC p-values for a test of $\theta = 0$ below 0.0001. Reassuringly, the critical value for determinacy without the extra term is $\frac{2}{T} = 0.1$, so the IV estimate is high enough for determinacy, even without the extra term. (The p-value on a test of $\theta = 0.1$ is 0.26.)

[45] The first stage F statistic is 42.17 which is generally considered high enough for reliable inference.

[46] Straightforward algebra shows that this is equivalent to running the regression $m_t = ax_t + by_t + o_t$, where m_t is the monetary shock series and o_t captures noise in the monetary shock series, and then estimating θ by $-\frac{a}{b}$. Standard errors can then be estimated via the delta-method. This estimator is consistent if $m_t = \chi\varepsilon_{\bar{\nu},t} + o_t$, where the noise o_t is uncorrelated with x_t and y_t.

treasury yields, 97.4% of the variance of levels of five-year breakeven inflation expectations, and 97.5% of the variance of levels of five-year treasury yields. Thus, our monetary rule explains almost all the variance in five-year yields. A real rate rule is a good model of actual Federal Reserve behaviour.

7 Challenges to real rate rules

Of course, the world is more complicated than the simple linear models we have presented in this paper. While the robustness to the presence of endogenous wedges ought to reassure us that real rate rules continue to work under many departures from our base assumptions, still we may worry about how real rate rules work in non-linear economies, or under bounded rationality. We address these particular concerns in the first two subsections below. We then go on to show that there is generically a stable equilibrium under a real rate rule even with active fiscal policy.

7.1 Risk premia and non-linear models

We have focussed on linearized models in the rest of this paper. In this subsection we verify that real rate rules still work in fully non-linear models. While we have already shown that real rate rules continue to work in the presence of endogenous risk premia under mild conditions (in Subsection 3.3), it is still reassuring to check things work in the non-linear case.

Suppose that Ξ_{t+1} is the real stochastic discount factor (SDF) between period t and period $t+1$, that I_t is the gross nominal interest rate (so $i_t = \log I_t$) and that R_t is the gross real interest rate (so $r_t = \log R_t$). Then the pricing equations for one-period nominal and real bonds imply:

$$I_t \mathbb{E}_t \frac{\Xi_{t+1}}{\Pi_{t+1}} = 1, \qquad R_t \mathbb{E}_t \Xi_{t+1} = 1.$$

We suppose that the central bank's target for period $t+1$ gross inflation is $\Pi^*_{t+1|t}$, which they announce in period t. With this target, the nonlinear version of equation (3) is the following rule:

$$I_t = R_t \Pi^*_{t+1|t} \left(\frac{\Pi_t}{\Pi^*_{t|t-1}} \right)^{\phi}.$$

Combining this rule with the bond pricing equations implies that:

$$\mathbb{E}_t \frac{\Xi_{t+1}}{\mathbb{E}_t \Xi_{t+1}} \frac{\Pi^*_{t+1|t}}{\Pi_{t+1}} = \left(\frac{\Pi^*_{t|t-1}}{\Pi_t} \right)^{\phi}.$$

It is easy to see that $\Pi_t = \Pi^*_{t|t-1}$ is always one solution of this equation, as $\mathbb{E}_t \frac{\Xi_{t+1}}{\mathbb{E}_t \Xi_{t+1}} = 1$. Thus, robust real rate rules are always consistent with stable inflation, even in fully non-linear models.

Furthermore, under mild assumptions,[47] there exists a constant $\overline{Z} \geq 1$ such that for all sufficiently high ϕ, $1 \leq \frac{\Pi^*_{t|t-1}}{\Pi_t} \leq \overline{Z}^{\frac{1}{\phi-1}}$. This upper bound tends to 1 as ϕ goes to ∞, thus for large ϕ, any solution must have $\Pi_t \approx \Pi^*_{t|t-1}$. This holds even if the SDF, Ξ_t, is a complicated function of inflation and its history. Under slightly stronger assumptions on the SDF,[48] we can even guarantee that $\Pi_t = \Pi^*_{t|t-1}$ is the unique solution for all sufficiently large ϕ. We prove these results in Online Appendix D.

7.2 Learning and bounded rationality

Our general results on Fisher equation wedges also imply substantial robustness to departures from full rationality (non-rational expectations equal rational expectations plus a wedge). But as before, it is reassuring to see how this works in practice. We summarise results here for several prominent models of bounded rationality. In all cases, we suppose that the central bank follows the monetary rule of equation (6), $i_t = r_t + \phi \pi_t + \zeta_t$, where ζ_t is an AR(1) process with persistence $\rho \in (-1,1)$, and where $\phi > 0$ at least. Full details and proofs are given in Online Appendix C. Evidence on departures from full rationality is surveyed in Coibion, Gorodnichenko & Kamdar (2018) and in the handbook edited by Bachmann, Topa & Klaauw (2023).

Adaptive, naïve, and extrapolative expectations. Branch & McGough (2009) suppose that aggregate inflation expectations are a linear combination of rational expectations and an additional term capturing adaptive, naïve or extrapolative expectations. In particular, agents' period t expectation of period $t+1$ inflation is given by $\alpha \mathbb{E}_t \pi_{t+1} + (1-\alpha)\theta \pi_{t-1}$. Here, $\alpha \in [0,1]$ gives the

[47] Π_t must be bounded above, as it is in the non-linear NK model. See Online Appendix D.1 for further discussion of this. Furthermore, the SDF must have bounded moments of some (positive) order.

[48] This requires the SDF to have a finite upper bound (almost surely), and for the gap between the realized SDF and the SDF if $\Pi_t = \Pi^*_{t|t-1}$ to be bounded by a linear function of $\frac{\Pi^*_{t|t-1}}{\Pi_t}$.

weight on rational expectations, and $\theta \geq 0$ controls whether the non-rational part is adaptive ($\theta < 1$), naïve ($\theta = 1$) or extrapolative ($\theta > 1$). This leads to the behavioural Fisher equation $i_t = r_t + \alpha \mathbb{E}_t \pi_{t+1} + (1-\alpha)\theta \pi_{t-1}$. We show that the solution is unique and stable as long as $\phi > \alpha + (1-\alpha)\theta$. (This may be stronger than $\phi > 1$ if $\theta > 1$.) Furthermore, as $\phi \to \infty$, $\operatorname{var} \pi_t \to 0$. This means that sufficiently aggressive monetary policy can squash the variance of inflation, even in the presence of adaptive, naïve, or extrapolative expectations.

Diagnostic expectations. Under diagnostic expectations (Bordalo, Gennaioli & Shleifer 2018; L'Huillier, Singh & Yoo 2021; Bianchi, Ilut & Saijo 2023), agents' expectations overreact to new information, as measured by changes in rational forecasts. As in the previous case, the solution under diagnostic expectations is unique and stable as long as ϕ is sufficiently large ($\phi > 2.40$ would do according to the estimates of Bianchi, Ilut & Saijo (2023)). And again, as $\phi \to \infty$, $\operatorname{var} \pi_t \to 0$. Furthermore, as $\phi \to \infty$, $\operatorname{var}\big((\phi - \rho)\pi_t + \zeta_t\big) \to 0$, which means that with even moderately high ϕ, inflation's dynamics will be very close to its dynamics under rational expectations.

Finite horizon planning. Woodford (2019) gives a model of limited planning horizons. Agents are assumed to optimize over decisions in finitely many future periods, using a learned value function to evaluate outcomes at their planning horizon. We find that $\phi > 1$ is stronger than necessary, and, as before, as $\phi \to \infty$, $\operatorname{var} \pi_t \to 0$, and $\operatorname{var}\big((\phi - \rho)\pi_t + \zeta_t\big) \to 0$ as well. It is particularly reassuring that with finite horizon planning, determinacy conditions are weaker than under rational expectations. Given a mix of finite horizon expectations and diagnostic or extrapolative ones, it is likely that ϕ not much larger than one would be sufficient.

Least squares learning. Under least squares learning (Marcet & Sargent 1989; Evans & Honkapohja 2001), agents update their beliefs about the laws of motion of endogenous variables via recursive least squares. For simplicity, we assume agents can directly observe the monetary shock ζ_t. We suppose that in period t, agents believe that for all s, $\pi_s = a_t + b_t \zeta_s + \varepsilon_s$, where they believe $\mathbb{E}_{s-1} \varepsilon_s = 0$. Allowing for a constant seems natural, as they may not know the inflation target (assumed to be zero), or the size of the static Fisher

equation wedge (also assumed to be zero). They estimate the coefficients a_t and b_t by recursive least squares, given some initial prior beliefs. We show that if $\phi > 1$, then under recursive least squares learning, with probability one, a_t converges to 0 and b_t converges to $-\frac{1}{\phi-\rho}$. Furthermore, $\pi_t + \frac{\zeta_t}{\phi-\rho}$ converges in probability to zero. Thus, agents succeed in learning the rational expectations solution, no matter what the initial conditions are. This guarantee of global stability under least squares learning is an improvement over the situation with standard monetary rules, for which at best local stability can be proven (see e.g. Bullard & Mitra (2002)).

Constant gain learning. If agents believe parameters may be non-stationary, then it is no longer reasonable to perform least squares learning. Instead, it is natural to assume that they learn with a constant gain coefficient on new observations (Evans & Honkapohja 2001). This replaces the decreasing gain of recursive least squares with some constant, $\gamma > 0$. We prove that with $\rho = 0$, $\phi > 1$ and γ sufficiently low, a_t and b_t converge in probability to the truth. Thus, even though agents are using a constant gain, they still manage to exactly learn the true parameters, whatever the initial conditions. It is easy for agents to learn the rational expectations equilibrium under a real rate rule!

7.3 The fiscal theory of the price level and over determinacy

As long as the linear Fisher equation holds, robust real rate rules can never fail to rule out sunspots. However, in an economy in which the price level is determinate independent of monetary policy, they may still produce explosive inflation.[49] This is true of any monetary rule respecting the Taylor principle, not just the real rate rules we examine in this paper. Inflation becomes "over determined", and an explosive solution is all that remains.

For example, suppose that government debt is all one period and nominal, and that real government surpluses are not responsive to government debt levels, meaning fiscal policy is "active" in the sense of Leeper (1991). Then the

[49] Note: it is certainly not the case though that in any model in which an interest rate peg is determinate, a real rate rule would produce explosive inflation. For example, in the New Keynesian model with a discounted Euler equation, from Subsection 3.2, if $\delta \in \left(-\frac{1+\beta+\kappa\varsigma}{1+\beta}, \frac{1-\beta-\kappa\varsigma}{1-\beta}\right)$ then an interest rate peg is determinate. We saw that the real rate rule is also determinate (and non-explosive) in this model.

price level is pinned down by the government debt valuation equation (see e.g. Cochrane (2023)), in line with the fiscal theory of the price level (FTPL). With flexible prices and constant real interest rates, to a first order approximation:

$$\pi_t - \mathbb{E}_{t-1}\pi_t = -\varepsilon_{s,t}, \tag{15}$$

where $\varepsilon_{s,t}$ is a shock to the present value of real primary government surpluses, scaled by the value of outstanding real government debt, with $\mathbb{E}_{t-1}\varepsilon_{s,t} = 0$.[50] Suppose in this world that the central bank did follow the basic real rate rule $i_t = r_t + \phi\pi_t + \varepsilon_{\zeta,t}$, where $\phi > 1$ and $\mathbb{E}_{t-1}\varepsilon_{\zeta,t} = 0$. Then, from the Fisher equation, $\mathbb{E}_{t-1}\pi_t = \phi\pi_{t-1} + \varepsilon_{\zeta,t-1}$, implying from (15) that $\pi_t = \phi\pi_{t-1} + \varepsilon_{\zeta,t-1} - \varepsilon_{s,t}$. With $\phi > 1$, this is an explosive process. We know from Subsection 3.1 that if there were to be a stationary solution for π_t, it must have $\pi_t = -\frac{1}{\phi}\varepsilon_{\zeta,t}$. But this is inconsistent with equation (15) as long as $\varepsilon_{\zeta,t} \neq \phi\varepsilon_{s,t}$, so only the non-stationary solution is left.

However, this is a knife edge result. For example, suppose that the government issues multi-period (geometric coupon) debt, and that both monetary and fiscal policy are active (i.e., real primary government surpluses do not respond to debt, and the monetary rule satisfies the Taylor principle). Based on results with one period debt, researchers have tended to assume that this "active-active" combination will inevitably produce explosive inflation. This is incorrect.

In Online Appendix F.1 we examine the equilibria of a non-linear model with multi-period debt under flexible prices. We show that under active fiscal policy, there is at least one equilibrium in which real variables and inflation are stable and independent of surpluses, whether or not monetary policy is active. These equilibria feature a growing bubble in the price of government debt which is balanced by declining debt quantities. The initial debt price jumps to ensure the transversality condition is still satisfied, giving a "Fiscal Theory of the Debt Price". These equilibria exist as long as the geometric decay factor for the bond coupons is not precisely equal to zero (the one-period debt case). Furthermore, under passive monetary policy, we find a continuum of equilibria, contrary to the usual claim that the active fiscal, passive monetary,

[50] See Cochrane (2023), Subsection 2.5 and following.

combination ensures unique outcomes (which is again only true with one period debt). These equilibria feature arbitrarily high inflation.

These results are not specific to the particular model set-up we use in Online Appendix F.1. Firstly, in Online Appendix F.2 we show that these results also hold in a linearised model with sticky prices. Then, in Online Appendix F.3 we show that generically, any model achieving determinacy via an FTPL-type mechanism must admit a stable solution under a real rate rule. There are only two main restrictions for this result. Firstly, the potentially explosive variables such as bond prices must not feed back to the real economy. Secondly, the equations determining the potentially explosive variables must not be too forward looking. Both assumptions are satisfied by standard FTPL models under geometric coupon debt.[51] Therefore, only in knife edge cases will following the Taylor principle guarantee explosive inflation. Real rate rules are robust to the risk of active fiscal policy, or other sources of over determination.

8 Conclusion

This paper's implications are stark. Under a real rate rule: the central bank can always achieve its inflation target, no matter the rest of the economy; any movement in inflation must be due to insufficient central bank aggression, or a central bank choice to so move inflation; monetary policy works in spite of, not because of, real rate movements; causation runs exclusively from inflation to the output gap, not the other way round; household and firm decisions, constraints and inflation expectations are irrelevant for inflation dynamics; and nothing can amplify or dampen the impact of shocks on inflation, except changes in the central bank's own behaviour. With a time-varying short-term inflation target, real rate rules can determinately implement optimal monetary policy, or match observed dynamics. They continue to work in the presence of the ZLB, endogenous wedges in the Fisher equation, or active fiscal policy.

To a policy maker, these conclusions may be shocking. However, for readers familiar with New Keynesian models, perhaps they are not completely

[51] Note that the geometric coupon bond first order condition $Q_t = \mathbb{E}_t \frac{\Xi_{t+1}}{\Pi_{t+1}} (1 + \omega Q_{t+1})$ can be rewritten as the two equations $E_t = \frac{1+\omega Q_t}{Q_{t-1}}$, and $1 = \mathbb{E}_t \frac{\Xi_{t+1}}{\Pi_{t+1}} E_{t+1}$. Here Q_t is potentially explosive, but is determined by a backward-looking equation, while E_t is asymptotically stable.

surprising. In models in which an aggressive response to inflation produces determinacy, with an extremely aggressive response, the variance of inflation can be pushed down to near zero. And Rupert & Šustek (2019) argue that even in New Keynesian models with a standard monetary rule, monetary policy does not operate via real rates. Rather, real rate rules just crystallise the monetary policy transmission mechanism that is at work in all New Keynesian models. Monetary policy acts via the Fisher equation, and via the Taylor principle's promise to induce explosive inflation should inflation deviate from target. Plausible arguments ruling out the explosive and deflationary equilibria include those of Angeletos & Lian (2023), of Christiano & Takahashi (2018; 2020) and of our Supplemental Appendix G.1.

Real rate rules are not just a mere theoretical curiosity though. We have presented a positive vision for the practical implementation of a real rate rule with a time-varying short-term inflation target. Under this proposal, central bank boards keep the crucial role of choosing the desired path of inflation. Only the technical decision of how to set rates to hit that path is delegated to the rule. The rule embeds no politically sensitive views about the slope of the Phillips curve or the costs of inflation. And the rule can be implemented using assets for which there is already a liquid market: either nominal and real long-maturity bonds, or inflation swaps. Current Federal Reserve behaviour is remarkably close to this practical rule: the rule explains 97.5% of the variance of five-year US treasury yields.

References

Adão, Bernardino, Isabel Correia & Pedro Teles. 2011. 'Unique Monetary Equilibria with Interest Rate Rules'. *Review of Economic Dynamics* 14 (3) (July): 432–442.

Afrouzi, Hassan, Marina Halac, Kenneth Rogoff & Pierre Yared. 2023. 'Monetary Policy without Commitment'.

Ang, Andrew, Geert Bekaert & Min Wei. 2007. 'Do Macro Variables, Asset Markets, or Surveys Forecast Inflation Better?' *Journal of Monetary Economics* 54 (4) (May 1): 1163–1212.

Angeletos, George-Marios & Chen Lian. 2023. 'Determinacy without the Taylor

Principle'. *Journal of Political Economy* 131 (8) (August): 2125–2164.

Ascari, Guido & Tiziano Ropele. 2009. 'Trend Inflation, Taylor Principle, and Indeterminacy'. *Journal of Money, Credit and Banking* 41 (8): 1557–1584.

Atkeson, Andrew & Lee E Ohanian. 2001. 'Are Phillips Curves Useful for Forecasting Inflation?' Edited by Edward J Green & Richard M Todd. *Federal Reserve Bank of Minneapolis Quarterly Review* (Winter 2001): 12.

Bachmann, Ruediger, Giorgio Topa & Wilbert van der Klaauw, ed. 2023. *Handbook of Economic Expectations*. London: Academic Press, Elsevier.

Bauer, Michael D. & Eric T. Swanson. 2023. 'A Reassessment of Monetary Policy Surprises and High-Frequency Identification'. *NBER Macroeconomics Annual* 37 (May): 87–155.

Beaudry, Paul, Andrew Preston & Franck Portier. 2022. 'Does It Matter to Assume That U.S. Monetary Authorities Follow a Taylor Rule?' Tilburg University.

Belongia, Michael T. & Peter N. Ireland. 2022. 'A Reconsideration of Money Growth Rules'. *Journal of Economic Dynamics and Control* 135 (February 1): 104312.

Benhabib, Jess, Stephanie Schmitt-Grohé & Martín Uribe. 2001. 'The Perils of Taylor Rules'. *Journal of Economic Theory* 96 (1–2): 40–69.

Bernanke, Ben S. & Michael Woodford. 1997. 'Inflation Forecasts and Monetary Policy'. *Journal of Money, Credit and Banking* 29 (4): 653–684.

Bianchi, Francesco, Cosmin Ilut & Hikaru Saijo. 2023. 'Diagnostic Business Cycles'. *The Review of Economic Studies* (February 21): rdad024.

Bilbiie, Florin O. 2008. 'Limited Asset Markets Participation, Monetary Policy and (Inverted) Aggregate Demand Logic'. *Journal of Economic Theory* 140 (1) (May 1): 162–196.

———. 2011. 'Nonseparable Preferences, Frisch Labor Supply, and the Consumption Multiplier of Government Spending: One Solution to a Fiscal Policy Puzzle'. *Journal of Money, Credit and Banking* 43 (1): 221–251.

———. 2019. *Monetary Policy and Heterogeneity: An Analytical Framework*. 2019 Meeting Papers. Society for Economic Dynamics.

———. 2022. 'Neo-Fisherian Policies and Liquidity Traps'. *American Economic*

Journal: Macroeconomics 14 (4) (October): 378–403.

Billi, Roberto M., Ulf Söderström & Carl E. Walsh. 2023. 'The Role of Money in Monetary Policy at the Lower Bound'. *Journal of Money, Credit and Banking* 55 (4): 681–716.

Bordalo, Pedro, Nicola Gennaioli & Andrei Shleifer. 2018. 'Diagnostic Expectations and Credit Cycles'. *The Journal of Finance* 73 (1): 199–227.

Branch, William A. & Bruce McGough. 2009. 'A New Keynesian Model with Heterogeneous Expectations'. *Journal of Economic Dynamics and Control* 33 (5). Complexity in Economics and Finance (May 1): 1036–1051.

———. 2010. 'Dynamic Predictor Selection in a New Keynesian Model with Heterogeneous Expectations'. *Journal of Economic Dynamics and Control* 34 (8) (August 1): 1492–1508.

———. 2018. 'Chapter 1 - Heterogeneous Expectations and Micro-Foundations in Macroeconomics'. In *Handbook of Computational Economics*, edited by Cars Hommes & Blake LeBaron, 4:3–62. Handbook of Computational Economics. Elsevier.

Bullard, James & Kaushik Mitra. 2002. 'Learning about Monetary Policy Rules'. *Journal of Monetary Economics* 49 (6) (September): 1105–1129.

Carlstrom, Charles T & Timothy S Fuerst. 2003. 'Money Growth Rules and Price Level Determinacy'. *Review of Economic Dynamics* 6 (2) (April 1): 263–275.

Christiano, Lawrence J. & Yuta Takahashi. 2018. 'Discouraging Deviant Behavior in Monetary Economics'. *NBER Working Paper Series* No. 24949.

———. 2020. 'Anchoring Inflation Expectations'.

Cochrane, John H. 2011. 'Determinacy and Identification with Taylor Rules'. *Journal of Political Economy* 119 (3): 565–615.

———. 2017. 'The Grumpy Economist: Target the Spread'. *The Grumpy Economist*.

———. 2018. 'The Zero Bound, Negative Rates, and Better Rules' (March 2): 27.

———. 2023. *The Fiscal Theory of the Price Level*. Princeton University Press.

Coibion, Olivier, Yuriy Gorodnichenko & Rupal Kamdar. 2018. 'The Formation of Expectations, Inflation, and the Phillips Curve'. *Journal of Economic Literature* 56 (4) (December): 1447–1491.

Dotsey, Michael, Shigeru Fujita & Tom Stark. 2018. 'Do Phillips Curves Conditionally Help to Forecast Inflation?' *International Journal of Central Banking*: 50.

Dotsey, Michael & Andreas Hornstein. 2006. 'Implementation of Optimal Monetary Policy'. *Economic Quarterly*: 113–133.

Dowd, Kevin. 1994. 'A Proposal to End Inflation'. *The Economic Journal* 104 (425): 828–840.

Evans, George W. & Seppo Honkapohja. 2001. *Learning and Expectations in Macroeconomics*. Frontiers of Economic Research. Princeton and Oxford: Princeton University Press.

———. 2006. 'Monetary Policy, Expectations and Commitment'. *The Scandinavian Journal of Economics* 108 (1): 15–38.

Evans, George W. & Bruce McGough. 2010. 'Implementing Optimal Monetary Policy in New-Keynesian Models with Inertia'. *The B.E. Journal of Macroeconomics* 10 (1).

Fleckenstein, Matthias, Francis A. Longstaff & Hanno Lustig. 2014. 'The TIPS-Treasury Bond Puzzle'. *The Journal of Finance* 69 (5): 2151–2197.

Gabaix, Xavier. 2020. 'A Behavioral New Keynesian Model'. *American Economic Review* 110 (8) (August): 2271–2327.

Galí, Jordi, J. David Lopez-Salido & Javier Valles. 2004. 'Rule-of-Thumb Consumers and the Design of Interest Rate Rules'. *Journal of Money, Credit, and Banking* 36 (4) (July 28): 739–763.

Gürkaynak, Refet S., Brian Sack & Jonathan H. Wright. 2010. 'The TIPS Yield Curve and Inflation Compensation'. *American Economic Journal: Macroeconomics* 2 (1): 70–92.

Hall, Robert E. & N. Gregory Mankiw. 1994. 'Nominal Income Targeting'. *NBER Chapters*: 71–94.

Hall, Robert E. & Ricardo Reis. 2016. *Achieving Price Stability by Manipulating the Central Bank's Payment on Reserves*. Working Paper. National Bureau of Economic Research.

Hellwig, Martin F. 1980. 'On the Aggregation of Information in Competitive Markets'. *Journal of Economic Theory* 22 (3) (June 1): 477–498.

Hetzel, Robert L. 1990. 'Maintaining Price Stability: A Proposal'. *Economic Review of the Federal Reserve Bank of Richmond* (March): 3.

Hills, Timothy S., Taisuke Nakata & Sebastian Schmidt. 2019. 'Effective Lower Bound Risk'. *European Economic Review* 120 (November 1): 103321.

Holden, Tom D. 2021. 'Existence and Uniqueness of Solutions to Dynamic Models with Occasionally Binding Constraints'. *The Review of Economics and Statistics* (October 15): 1–45.

Ireland, Peter N. 2007. 'Changes in the Federal Reserve's Inflation Target: Causes and Consequences'. *Journal of Money, Credit and Banking* 39 (8): 1851–1882.

———. 2015. 'Monetary Policy, Bond Risk Premia, and the Economy'. *Journal of Monetary Economics* 76 (November 1): 124–140.

Känzig, Diego R. 2021. 'The Macroeconomic Effects of Oil Supply News: Evidence from OPEC Announcements'. *American Economic Review* 111 (4) (April): 1092–1125.

Khan, Hashmat. 2005. 'Price-Setting Behaviour, Competition, and Markup Shocks in the New Keynesian Model'. *Economics Letters* 87 (3) (June 1): 329–335.

Khan, Hashmat, Louis Phaneuf & Jean Gardy Victor. 2019. 'Rules-Based Monetary Policy and the Threat of Indeterminacy When Trend Inflation Is Low'. *Journal of Monetary Economics* (March): S0304393219300479.

Leeper, Eric M. 1991. 'Equilibria under "Active" and "Passive" Monetary and Fiscal Policies'. *Journal of Monetary Economics* 27 (1) (February 1): 129–147.

Leeper, Eric M. & Campbell Leith. 2016. 'Chapter 30 - Understanding Inflation as a Joint Monetary–Fiscal Phenomenon'. In *Handbook of Macroeconomics*, edited by John B. Taylor & Harald Uhlig, 2:2305–2415. Elsevier.

Lewis, Vivien & Markus Roth. 2018. 'Interest Rate Rules under Financial Dominance'. *Journal of Economic Dynamics and Control* 95 (October 1): 70–88.

L'Huillier, Jean-Paul, Sanjay R. Singh & Donghoon Yoo. 2021. 'Incorporating Diagnostic Expectations into the New Keynesian Framework'. *Working Papers*. Working Papers (February 9).

Lou, Youcheng, Sahar Parsa, Debraj Ray, Duan Li & Shouyang Wang. 2019.

'Information Aggregation in a Financial Market with General Signal Structure'. *Journal of Economic Theory* 183 (September 1): 594–624.

Lubik, Thomas A., Christian Matthes & Elmar Mertens. 2019. *Indeterminacy and Imperfect Information*. Federal Reserve Bank of Richmond Working Papers.

Manea, Cristina. 2019. 'Collateral-Constrained Firms and Monetary Policy': 66.

Marcet, Albert & Thomas J Sargent. 1989. 'Convergence of Least Squares Learning Mechanisms in Self-Referential Linear Stochastic Models'. *Journal of Economic Theory* 48 (2) (August 1): 337–368.

McGough, Bruce, Glenn D. Rudebusch & John C. Williams. 2005. 'Using a Long-Term Interest Rate as the Monetary Policy Instrument'. *Journal of Monetary Economics* 52 (5) (July 1): 855–879.

McKay, Alisdair, Emi Nakamura & Jón Steinsson. 2017. 'The Discounted Euler Equation: A Note'. *Economica* 84 (336): 820–831.

McLeay, Michael & Silvana Tenreyro. 2019. 'Optimal Inflation and the Identification of the Phillips Curve'. In . NBER Chapters. National Bureau of Economic Research, Inc.

Mertens, Karel RSM & Morten O. Ravn. 2014. 'Fiscal Policy in an Expectations-Driven Liquidity Trap'. *The Review of Economic Studies*: rdu016.

Miranda-Agrippino, Silvia & Giovanni Ricco. 2021. 'The Transmission of Monetary Policy Shocks'. *American Economic Journal: Macroeconomics* 13 (3) (July): 74–107.

Nakata, Taisuke & Sebastian Schmidt. 2022. 'Expectations-Driven Liquidity Traps: Implications for Monetary and Fiscal Policy'. *American Economic Journal: Macroeconomics* 14 (4) (October): 68–103.

Natvik, Gisle James. 2009. 'Government Spending and the Taylor Principle'. *Journal of Money, Credit and Banking* 41 (1): 57–77.

Nie, He & Jordan Roulleau-Pasdeloup. 2022. 'The Promises (and Perils) of Control-Contingent Forward Guidance'. *Review of Economic Dynamics* (August 2).

Rupert, Peter & Roman Šustek. 2019. 'On the Mechanics of New-Keynesian Models'. *Journal of Monetary Economics* 102. Carnegie-Rochester-NYU Conference Series on Public Policy. "A Conference Honoring the

Contributions of Charles Plosser to Economics" Held at the University of Rochester Simon Business School, April 20-21, 2018 (April 1): 53–69.

Schmidt, Sebastian. 2016. 'Lack of Confidence, the Zero Lower Bound, and the Virtue of Fiscal Rules'. *Journal of Economic Dynamics and Control* 70 (September 1): 36–53.

Schmitt-Grohé, Stephanie & Martín Uribe. 2007. 'Optimal Simple and Implementable Monetary and Fiscal Rules'. *Journal of Monetary Economics* 54 (6): 1702–1725.

Stock, James & Mark W. Watson. 2009. 'Phillips Curve Inflation Forecasts'. In *Understanding Inflation and the Implications for Monetary Policy*, edited by Jeffrey Fuhrer, Yolanda Kodrzycki, Jane Little & Giovanni Olivei, 99–202. Cambridge: MIT Press.

Sveen, Tommy & Lutz Weinke. 2005. 'New Perspectives on Capital, Sticky Prices, and the Taylor Principle'. *Journal of Economic Theory* 123 (1). Monetary Policy and Capital Accumulation (July 1): 21–39.

Svensson, Lars E. O. 1997. 'Inflation Forecast Targeting: Implementing and Monitoring Inflation Targets'. *European Economic Review* 41 (6) (June 1): 1111–1146.

Svensson, Lars E. O. & Michael Woodford. 2003. 'Implementing Optimal Policy through Inflation-Forecast Targeting' (June).

Woodford, Michael. 1999. 'Commentary: How Should Monetary Policy Be Conducted in an Era of Price Stability?' In *New Challenges for Monetary Policy*. Jackson Hole, Wyoming: Federal Reserve Bank of Kansas City.

———. 2003. *Interest and Prices. Foundations of a Theory of Monetary Policy*. Princeton University Press.

———. 2019. 'Monetary Policy Analysis When Planning Horizons Are Finite'. *NBER Macroeconomics Annual* 33 (January): 1–50.