International Spillovers and Bailouts*

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Abstract

We study how cross-country macroeconomic spillovers caused by sovereign default affect equilibrium bailouts. Because of portfolio diversification, the default of one country causes a macroeconomic contraction in other countries, which motivates a bailout. But why do creditor countries choose to bail out debtor countries instead of their own private sector? We show that this is because an external bailout could be cheaper than a domestic bailout. We also show that, although anticipated bailouts lead to higher borrowing, they can be Pareto improving not only ex-post (after a country has defaulted) but also ex-ante (before the country chooses its debt).

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1 Introduction

This paper is motivated by two observations related to sovereign debt markets. The first observation is that portfolio gross holdings are internationally diversified. Figure 1 illustrates this fact for Germany and for five European countries: Greece, Ireland, Italy, Portugal and Spain (GIIPS). The left panel plots the shares of public debt issued by Germany (solid line) and by the GIIPS countries (dashed line), held by non-resident banks. The right panel plots the shares of public debt issued by other European Union countries held by resident banks in Germany (solid line) and GIIPS countries (dashed line).

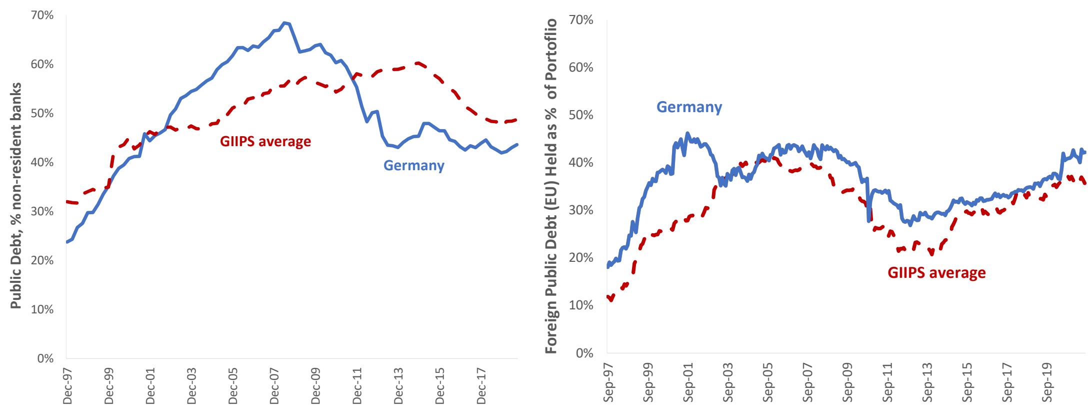

Figure 1: Sovereign debt and portfolio diversification. *First panel:* The solid line is the share of public debt issued by Germany that is held by banks that are non-resident of Germany. The dashed line is the share of public debt issued by GIIPS countries held by banks that are non-resident of the GIIPS countries. Data is from Merler and Pisani-Ferry (2012) and the debt is measured as general government debt. *Second panel:* The solid line is the holdings of German banks of public debt issued by other EU countries over the total EU public debt held by German banks. The dashed line is the holdings of GIIPS banks of public debt issued by other EU countries over the total EU public debt held by GIIPS banks. Data is from Statistical Data Warehouse of the Bank of International Settlements, Balance Sheet Items and the variable in the numerator is "Holdings of debt securities issued by other EU Member States General Government reported by MFI (stock)". For GIIPS countries, we first compute a series for each country and then we calculate the GDP-weighted average.

The figure shows that countries have, at the same time, large shares of foreign liabilities (first panel) and hold large shares of foreign assets (second panel). The international diversification is not limited to public debt but also to private debt, portfolio investments, and FDI (Lane and Milesi-Ferreti (2007, 2017)). In this paper, however, we focus on sovereign debt because of its special role in providing liquidity.

The second observation is that many episodes of sovereign debt crises are resolved with restructuring programs negotiated with creditor countries (bailouts). Figure

2, reconstructed from Mitchener and Trebesch (2021), shows that 75% of modern-era debt crises have been resolved without formal default, although they typically involve some form of financial assistance (bailout). For example, bailout episodes that succeeded in preventing default include those experienced by periphery countries in Europe (excluding Greece) during the 2011-2012 crisis. According to Gourinchas, Martin and Messer (2020), these transfers amounted to 3.6% of GDP for Cyprus, 1.29% for Ireland, 2.93% for Portugal, and 0.49% for Spain.

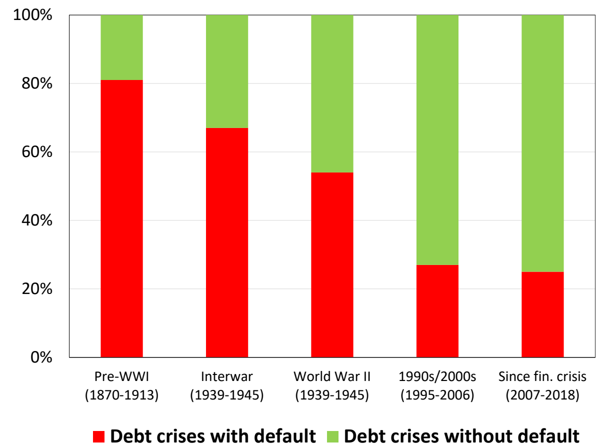

Figure 2: Shares of sovereign debt crises with and without default. *Source*: Mitchener and Trebesch (2021)

The fact that sovereign debt crises tend to be resolved with bailout programs raises two questions addressed by this paper. First, why do creditor countries choose to bail out defaulting countries? Second, although bailouts could be welfare improving ex-post, are they welfare improving also ex-ante—that is, before countries choose how much to borrow?

The first result of the paper is to show that external bailouts could be cheaper than domestic bailouts, which explains why creditor countries may prefer an external bailout. Because of international financial diversification (Figure 1), default generates negative macroeconomic spillovers in creditor countries. The spillovers could be mitigated, or even averted, with either an external bailout (by transferring resources to the foreign government) or with a domestic bailout (by helping domestic investors). Because an external bailout brings macroeconomic benefits also to the defaulting country, the same macroeconomic outcome can be achieved with a smaller external bailout than a domestic bailout.

The second result of the paper is that external bailouts could be welfare improving not only ex-post, but also ex-ante. This may appear surprising since the anticipation of bailouts creates, typically, a moral hazard problem leading to higher indebtedness. However, when public debt provides liquidity (which is a feature of our model) and part of the debt is held abroad (also a feature of our model), countries might issue too little debt. The anticipation of bailouts, then, could bring the equilibrium debt closer to the efficient level.

The third result of the paper is to show that a debt crisis in a country could be triggered or amplified by higher borrowing in other countries. This is because, keeping the debt of the country fixed, the higher borrowing (supply of debt) by other countries allows, in equilibrium, greater diversification of portfolios for the countries that do not increase their debt. But higher diversification of portfolios increases the incentive to default. In that sense, a debt crisis could be externally induced by a global financial expansion.

We show these results in a two-country model where both types of sovereign debt crises—with and without default—could arise in equilibrium. The model features cross-country holdings of debt issued by the two countries. Although international diversification may improve production efficiency and allow for greater risk-sharing, it also increases macroeconomic interdependence across countries. This implies that the macroeconomic impact of a crisis in one country spills to the other country.

According to Das, Papaioannou, and Trebesch (2013), cross-border feedback channels arise because banks and financial institutions are exposed to the sovereign default risks of other countries. For example, Rieffel (2003) shows that banks in the US were heavily exposed to emerging country debt during the 1980s. A 30% write-off of emerging country debt would have depleted the capital of most large US banks, creating unpredictable consequences for financial markets and the whole US economy. We capture the international interdependence by formalizing a consolidated bank/business sector that chooses optimally the holdings of domestic and foreign debt.

Public debt is a liquid asset that facilitates production. Default reduces the stock of debt and, therefore, generates negative macroeconomic consequences for all countries holding the debt. A bailout is modelled as a Nash bargaining game between the governments of the creditor and defaulting countries. The game determines a transfer made by the creditor country to the defaulting country in exchange for a higher repayment of the debt. This is similar to the debt restructuring mechanism studied in Yue (2010) and Bai and Zhang (2012), but with an endogenous threat value. Effectively, with a bailout program, creditor countries encourage debt repayment. By doing so, they reduce the financial losses experienced by domestic banks/businesses and alleviate the negative macroeconomic consequences caused by default. We refer to the transfer resulting from the bargaining game as 'external bailout.'

Creditor countries could also alleviate the negative macroeconomic consequences of default by directly bailing out its own banks/businesses rather than the defaulting country. This takes the form of a transfer to domestic entrepreneurs to cover, partially or in full, the financial losses incurred on their holdings of foreign debt. We refer to this transfer as 'domestic' bailout. We show that, under some conditions, creditor countries prefer to bail out foreign governments rather than their own banks/businesses.

To understand why, we have to consider that an external bailout reduces the macroeconomic losses in both countries. A domestic bailout, instead, reduces the losses only in the creditor country. Because the defaulting country also benefits from the higher repayment facilitated by the external bailout, the creditor country can extract part of that benefit in the bargaining game. Another way to say this is that the external transfer needed to induce the same repayment to domestic entrepreneurs is smaller than the domestic transfer needed with a domestic bailout.

In a dynamic setting, the anticipation of bailouts increases the incentive to borrow which, ex-post, would require larger bailouts. This is the typical moral hazard problem in international borrowing. In spite of this, however, external bailouts could be Pareto improving also ex-ante, that is, before the issuance of debt. This is because, with financial integration, part of the debt is purchased by foreign residents and, therefore, the benefits are shared with other countries. This implies that the issuing country does not fully internalize the benefits of its debt and, as a result, borrows less than globally desirable. Anticipated bailouts could act as a counterbalancing force for this externality because they increase the incentive to issue more debt. Cross-country diversification of portfolios is crucial for this result: in absence of diversification the benefits of public debt as well as the macroeconomic consequences of default would be fully internalized. We show these results quantitatively with the infinite horizon version of the model calibrated to European countries.

We also simulate the model over the period 1999-2013 and show that its dynamics capture some of the salient features of the 2011-2012 European debt crisis. An interesting aspect of the European experience was the rising debt issued by Germany prior to the crisis. The simulation shows that the increase in German debt, a safe asset, exacerbated the later crisis. As the safe country (Germany) issues more debt, part of the debt is purchased by the residents of the risky country (GIIPS). As a result, the risky country becomes more diversified. But higher diversification reduces the macroeconomic cost of default for the risky country, which increases the incentive to default. The higher incentive to default arises even if the debt issued by the risky country and the fraction held by foreigner residents does not change. This shows that a debt crisis in risky countries could be triggered, or amplified, by the financial expansion in creditor countries.

The rest of the paper is organized as follows. After a discussion of the related literature, we describe the model in Section 2. We then define the competitive equilibrium in Sections 3 and the political equilibrium 4. Section 5 provides analytical intuitions by focusing on a one-period version of the model. Sections 6 through 8 conduct the quantitative analysis with the infinite-horizon model calibrated to European countries. Section 9 concludes.

Related literature

Following the seminal work by Eaton and Gersovitz (1981), several contributions in the literature have studied the dynamics of sovereign default in the context of small open economy models. In those models, default does not generate macroeconomic costs for the creditor countries besides the lower repayment.[1] Moreover, the macroeconomic cost is often exogenous. In our model, default creates endogenous macroeconomic costs in both, the debtor and creditor countries, because government debt provides liquidity to the private sector of both countries.

The sovereign-bank nexus is studied in Gennaioli, Martin and Rossi (2014), Perez (2015), and Sosa-Padilla (2018). These papers also consider small open economies with an endogenous cost of default. However, their goal is not to study the international spillovers of sovereign defaults, which justifies the use of the small economy paradigm.[2] An important contribution of our study, instead, is to show the relevance of macroeconomic spillovers generated by sovereign default using a model with 'large' open economies, in the spirit of Azzimonti, de Fracisco and Quadrini (2014).[3] The consideration of large open economies with endogenous spillovers are important for understanding the optimality of bailouts.

Cross-country spillovers play an important role for the desirability of bailouts in Tirole (2015) and Gourinchas, Martin, and Messer (2020). Our work differs in two dimensions. First, we provide a micro-foundation for international spillovers which derive from 'endogenous' cross-country holdings of financial assets and the role that these assets play in production. Second, we show that ex-post bailouts could generate Pareto improvements ex-ante (i.e. before debt is issued) *because* they lead to more, not less borrowing. In our setup, the debt issued by one country is beneficial for the production of both countries, and this generates a positive externality in the issuance of the debt. In absence of policy coordination or anticipated bailouts, public debt is inefficiently low. In Tirole (2015), the benefits from borrowing are always internalized but the cost of default is not (because spillovers are exogenously given). This leads to over-issuance of debt in absence of bailouts.

Bolton and Jeanne (2011) also study a model with a safe and risky country. With financial integration, the issuance of public debt from the safe country is too low while the issuance from the risky country is too high. In our model, instead, also the risky country issues too little debt. Importantly, they find that the safe country always loses, ex ante, from fiscal integration (bailout), which is different from our finding. The different finding derives from the assumption that in our model the debt of the risky country also provides liquidity. Because the risky country does not internalize

[1] Examples include Aguiar and Gopinath (2006), Aguiar and Amador (2016), Arellano (2008), Cuadra, Sanchez, and Sapriza (2010), and Yue (2010). Aguiar and Amador (2014) and Tomz and Wright (2013) provide earlier reviews of this literature, whereas the handbook chapters by Aguiar, Chatterjee, Cole and Stangebye (2016) and D'Erasmo, Mendoza, and Zhang (2016) provide a more recent discussion of the literature on sovereign default and sustainable public debt.

[2] See also Guembel and Sussman (2009), Broner, Martin, and Ventura (2010), Broner and Ventura (2011), Brutti (2011), and Di Casola and Sichlimiris (2017). More recently, Bocola (2016) and Farhi and Tirole (2018) study the interaction between sovereign debt and domestic financial institutions.

[3] Arellano and Bai (2013) also consider an environment in which sovereign default affects other countries. Their mechanism is based on the interest rate channel along the lines of Borri and Verdelhan (2009), Park (2014), Lizarazo (2013), and Pouzo and Presno (2016). Our channel of transmission, instead, relies on the destruction of financial assets.

the liquidity benefit of its debt for the other country, the debt issuance is too low. The anticipation of bailouts corrects for this inefficiency.

Moreover, differently from Bolton and Jeanne (2011), Tirole (2015) and Gourinchas et al. (2020), we show that our main theoretical results are quantitatively relevant by calibrating an infinite-horizon version of the model.

Our results also differ from those in the doom-loop literature (Farhi and Tirole (2012, 2018), Cooper and Nikolov (2018), and Hur, Sosa-Padilla, and Yom (2021)). One of the findings of this literature is that bailouts generate excessive financial fragility and, hence, reduce welfare. The contribution of our paper is complementary, not alternative, to the doom-loop literature because we focus on the positive effects of anticipated bailouts (external and domestic) funded by creditor countries.[4]

A special feature of our analysis is the emphasis on portfolio diversification in affecting default decisions. Importantly, the mechanism differs from the one emphasized in the literature where, typically, public debt is used for private consumption smoothing (e.g. Eaton and Gersovitz (1981) or Arellano (2008)). We show that international diversification increases the incentive of a country to default because the domestic macroeconomic cost is smaller when portfolios are diversified. This is different from (and additional to) the cross-country redistribution from foreign to domestic residents, which is common to many sovereign default models such as Amador (2003), Aguiar, Amador, Farhi, and Gopinath (2013), Hatchondo, Martinez, and Sapriza (2009), and Mendoza and Yue (2012). Through this channel we are also able to study the importance of financial booms in creditor countries for the incentives and consequences of default in debtor countries. Our model also features internal redistribution of default from domestic agents that hold the debt to other domestic agents. In this respect our paper relates to D'Erasmo and Mendoza (2016, 2021) who consider domestic default in an economy with heterogeneous agents.

2 The model

Time is discrete and the economy lasts for N periods (with N potentially being infinity). There are two large countries. The first country is characterized by a strong political system that guarantees commitment to repay debt obligations. We refer to the first country as 'safe' and denote it with superscript S. The second country, instead, is characterized by weaker institutions which are less effective at disciplining politicians. As a result, the country may default on part or all of its debt obligations. We refer to the second country as 'risky' and denote it with superscript

[4]Bianchi (2016) finds that bailouts may be welfare-improving ex-ante in an environment where the risky government bails out its domestic private sector but does not consider default. Our work, instead, studies cross-country bailouts subject to sovereign risk. The optimality of central government bailouts of sub-national units is studied in Chari and Kehoe (2007), Cooper, Kempf, and Peled (2008), and Dovis and Kirpalani (2020). Dovis (2019), Fink and Scholl (2016), and Roch and Uhlig (2018) consider instead bailouts provided by international financial institutions.

R. In each country $i \in \{S, R\}$ there are two types of atomistic agents: a measure μ^i of workers and a measure μ^i of entrepreneurs. The population size is $2\mu^i$.

2.1 Workers

Workers value consumption and leisure with lifetime utility

$$\mathbb{E} \sum_{t=1}^{N} \beta^{t-1} \left\{ c_t^i - \frac{\nu}{1+\nu} \left(\ell_t^i\right)^{\frac{1+\nu}{\nu}} \right\}.$$

The variable c_t^i is consumption, ℓ_t^i is the supply of labor, $0 < \beta < 1$ is the intertemporal discount factor, and $\nu > 0$ is the elasticity of labor supply. Workers do not participate in financial markets, so their budget constraint is

$$c_t^i = w_t^i \ell_t^i - \frac{T_t^i}{\mu^i}.$$

The variable T_t^i/μ^i denotes government lump-sum taxes paid by each worker (or transfers if negative), and w_t^i is the wage rate. The assumption that workers cannot borrow simplifies the exposition but it is not essential.[5] The solution to the worker's problem provides the individual supply of labor as a function of the wage rate,

$$\ell_t^i = \left(w_t^i\right)^{\nu}. \tag{1}$$

2.2 Entrepreneurs

There is a consolidated banking/business sector, where each bank/firm is run by an entrepreneur. The lifetime utility of entrepreneurs in country i is

$$\mathbb{E} \sum_{t=1}^{N} \beta^{t-1} \ln(d_t^i),$$

where d_t^i denotes consumption.

There is no market for contingent claims and the only assets traded by entrepreneurs are one-period government bonds. With internationally integrated financial markets, entrepreneurs can hold bonds issued by both countries. We denote by b_t^{ji} the bonds issued by country $j \in \{S, R\}$ held by an entrepreneur residing in country $i \in \{S, R\}$. Therefore, the first superscript indicates the issuing country while the second superscript indicates the residency of the holder. The price for bonds issued by country j is q_t^j. Because the debt repayment of country S could differ from that of country R, their prices could differ, that is, $q_t^S \neq q_t^R$.

[5] We could allow workers to borrow up to a limit. As long as the interest rate is lower than the intertemporal discount rate—which will be the case in the general equilibrium of the calibrated model—workers will borrow up to the limit and the model would have similar properties.

Entrepreneurs in country i enter period t with assets b_t^{Si} and b_t^{Ri}. After default from country R, which arises at the beginning of the period before any other decision is made, the financial wealth of entrepreneurs could be smaller than its pre-default value. Denoting by m_t^i the after-default wealth, default could make $m_t^i < b_t^{Si} + b_t^{Ri}$. The precise definition of m_t^i will be provided below. For the moment, we just need to remember that a default reduces entrepreneurs' wealth.

A central feature of the model is that financial wealth affects production decisions. We formalize this by assuming that entrepreneurs operate the production function

$$y_t^i = z_t^i (m_t^i)^\alpha (l_t^i)^{1-\alpha}, \tag{2}$$

where m_t^i is their financial wealth after default, l_t^i is the input of labor, and $z_t^i \in \{z_{Low}, z_{High}\}$ is an aggregate country-specific productivity shock. Productivity in each country follows a stationary Markov process. Its realization becomes known at the beginning of the period before entrepreneurs choose the input of labor l_t^i.

The production function captures, in reduced form, the inter-dependence between the financial and production sectors based on the idea that financial wealth provides working capital complementary to labor. Production also carries a cost ϕm_t^i. The cost increases with the production scale, which is captured by the production input m_t^i. We interpret this cost as depreciation of fixed capital.[6]

Given the value of financial assets, m_t^i, and the wage rate, w_t^i, entrepreneurs choose the input of labor l_t^i to maximize gross profits,

$$\max_{l_t^i} \Big\{ z_t^i (m_t^i)^\alpha (l_t^i)^{1-\alpha} - w_t^i l_t^i \Big\}.$$

The solution to the maximization problem returns the demand for labor,

$$l_t^i = \left[\frac{(1-\alpha) z_t^i}{w_t^i}\right]^{\frac{1}{\alpha}} m_t^i, \tag{3}$$

which we can use to derive the entrepreneur's gross profits and end-of period wealth,

$$\pi_t^i = \alpha z_t^i \left[\frac{(1-\alpha) z_t^i}{w_t^i}\right]^{\frac{1-\alpha}{\alpha}} m_t^i, \tag{4}$$

$$a_t^i = (1-\phi) m_t^i + \pi_t^i. \tag{5}$$

[6]Suppose that each entrepreneur needs to use a fixed amount of physical capital $\bar{k}$ in production. When production increases, the fixed capital is utilized more intensively, which leads to higher depreciation. We could use an alternative specification in which the input of production is not financial wealth but it is physical capital accumulated by firms endogenously. By further assuming that the utilization of capital requires working capital, we introduce a motive to hold financial wealth similar to the current specification. This alternative model would increase the complexity of the analytical exposition but would not change the basic properties characterized in the paper. We would also like to point out that the cost ϕm_t^i is only relevant for the calibration of the model but it is irrelevant for the qualitative properties.

End-of-period wealth is in part consumed and in part saved in domestic and foreign bonds. The entrepreneur's budget constraint is

$$a_t^i = d_t^i + q_t^S b_{t+1}^{Si} + q_t^R b_{t+1}^{Ri}.$$

Denote by δ_{t+1} the fraction of debt repaid by country R next period. When $\delta_{t+1} = 1$ the government fully repays its debt obligations. A value $\delta_{t+1} < 1$ indicates a partial default. This is an endogenous variable that will be derived in equilibrium. We refer to $1 - \delta_{t+1}$ as the *haircut*.[7]

Entrepreneurs take δ_{t+1} as given when solving their optimization problem. The portfolio decisions are characterized by the following lemma.

Lemma 1 *Entrepreneurs' optimal consumption and portfolio decisions satisfy*

$$\begin{aligned} d_t^i &= (1-\bar{\beta}_t)a_t^i, \\ q_t^S b_{t+1}^{Si} &= \theta_t \bar{\beta}_t a_t^i, \\ q_t^R b_{t+1}^{Ri} &= (1-\theta_t)\bar{\beta}_t a_t^i, \end{aligned}$$

where $\bar{\beta}_t = \frac{\beta - \beta^{N-t+1}}{1-\beta^{N-t+1}}$ *and* θ_t *solves the first order condition*

$$1 = \mathbb{E}_t \left\{ \frac{\delta_{t+1}}{(1-\theta_t)\delta_{t+1} + \theta_t q_t^R / q_t^S} \right\}. \tag{6}$$

Proof. See Appendix A.

Entrepreneurs consume a fraction $1 - \bar{\beta}_t$ of their wealth a_t^i and save the remaining fraction $\bar{\beta}_t$.[8] A fraction θ_t of savings is then invested in bonds issued by the safe country and the remaining fraction $1 - \theta_t$ is invested in bonds issued by the risky country. Notice that θ_t does not have the country subscript. This means that entrepreneurs in both countries choose the same composition of portfolios, that is, they allocate the same fraction of wealth to bonds issued by the two countries.[9]

2.3 Governments and policies

The government of country $j \in \{S, R\}$ enters the period with debt B_t^j. This debt is held by entrepreneurs in countries S and R. Thus,

$$B_t^j = B_t^{jS} + B_t^{jR}.$$

[7] We allow for partial default which is a feature of the data. Although full default is more common in the literature, there are some exceptions like Arellano, Mateos-Planas and Rios-Rull (2019).

[8] The fraction $\bar{\beta}_t$ is age-specific because of the finite life span. With an infinite horizon $\bar{\beta}_t = \beta$.

[9] The portfolio symmetry is not consistent with empirical evidence characterized by 'home bias': residents hold larger shares of domestic bonds. In the online appendix we consider an extension of the model with home bias and show that, once re-calibrated, the model generates similar results.

The variable $B_t^{ji} = b_t^{ji}\mu^i$ is the total debt issued by country j held by entrepreneurs in country i. Capital letters denote aggregate values. By summing the holdings of both countries, B_t^{jS} and B_t^{jR}, we obtain the total debt of country j.

Country S always repay its debt obligations. In country R, instead, there is a probability ρ^R that its government does not have the commitment to repay and could attempt to restructure the debt. Let $\varrho^R = 1$ be the state of the world in which country R has *commitment* (repays the debt in full) and $\varrho^R = 0$ the state in which it has *flexibility* to default. This variable, which could be the result of political turnover, is realized at the beginning of the period. In the event of no commitment, the risky country can approach the creditor country to renegotiate its debt obligations, triggering a restructuring episode. If it chooses to do so, we say that country R is in a "sovereign debt crisis."

To capture the fact that some debt crises end up in straight default while others are resolved with some rescue package (see Figure 2), we assume that the safe country is able to renegotiate the debt and offer a bailout only with probability ρ^S. With probability $1 - \rho^S$, instead, offering an external bailout is not politically feasible. In this case, the debt crisis can only be resolved with an outright default.

We denote with $\varrho^S = 1$ the state of the world in which country S has the flexibility to offer a bailout to the risky country, and $\varrho^S = 0$ the state in which it *cannot* bail out the risky country. As for the commitment of country R, we interpret the ability of country S to negotiate as reflecting the political environment that prevails in that country. Both ϱ^R and ϱ^S are revealed before country R decides to trigger a restructuring episode.

The sequence of play is summarized in Figure 3 and the associated payoffs will be defined in Section 4. At the beginning of the period the states ϱ^R and ϱ^S become known and they determine the main branches of the tree drawn in the figure.

1. **First branch**: This is the case in which $\varrho^R = 1$. Country R has full commitment to repay and, therefore, the realization of ϱ^S is irrelevant.

2. **Second branch**: This is the case with $\varrho^R = 0$ and $\varrho^S = 0$. Country R has two options: it can repay the debt or default without receiving a bailout (since country S is unable to negotiate). If it chooses to default, the repayment δ_t is chosen unilaterally to maximize the welfare of country R. Country S, however, can bail out its own entrepreneurs (domestic bailout).

   With a domestic bailout, country S makes transfers to domestic entrepreneurs by taxing domestic workers. The transfer received by an entrepreneur is proportional to the 'individual' wealth of the entrepreneur, that is, $\tilde{\tau}_t^d = \chi_t\left(b_t^{SS} + \delta_t b_t^{RS}\right)$. The average per-entrepreneur transfer is then

$$\tau_t^d = \chi_t\left(\frac{B_t^{SS} + \delta_t B_t^{RS}}{\mu^S}\right).$$

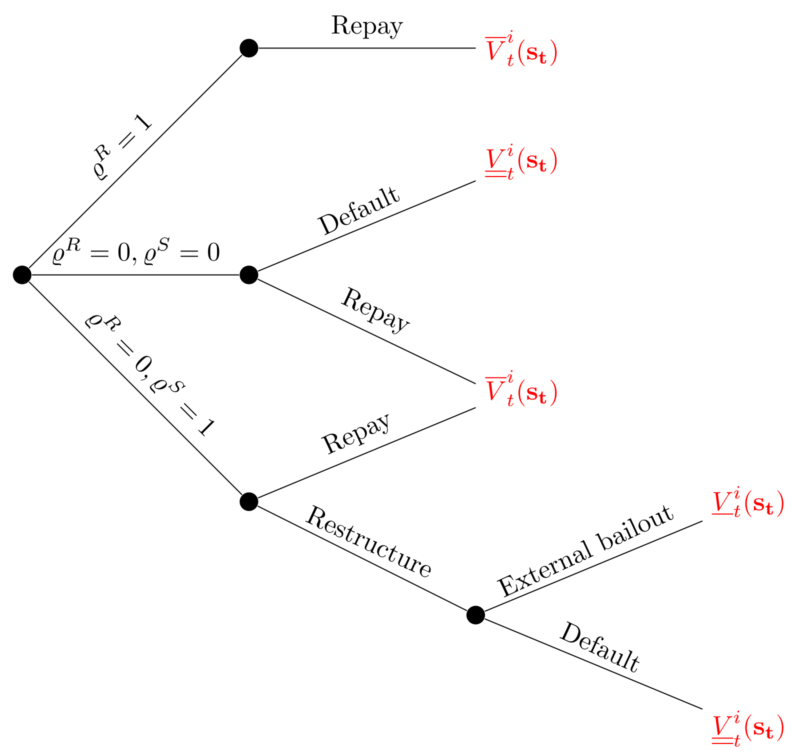

Figure 3: Sequence of events in debt restructuring, bailout, and default.

Since in equilibrium there is a representative entrepreneur, each will get the same transfer τ_t^d. Still, the assumption that transfers are proportional to individual wealth, rather than the actual losses incurred by each entrepreneur, simplifies the portfolio decision.[10]

The domestic transfer made by country S cannot exceed the average losses,

$$\tau_t^d \leq \frac{(1-\delta_t)B_t^{RS}}{\mu^S}.$$

3. **Third branch**: This is the case with $\varrho^R = 0$ and $\varrho^S = 1$. Country R can ask for restructuring and country S has the ability to negotiate a bailout. Negotiation

[10]The alternative assumption would make the transfer equal to the actual loss incurred by the entrepreneur on the defaulted bond, that is, $\tilde{\tau}_t^d = (1-\delta_t)b_t^{RS}$. Since in equilibrium entrepreneurs are homogeneous, the received transfers will be the same. However, the two assumptions imply different optimality conditions for the portfolio choice. In particular, under the alternative assumption, the portfolio composition will be different in the two countries. Specifically, entrepreneurs in country S would hold a larger fraction of R bonds while entrepreneurs in country R would hold a larger fraction of S bonds. This implies the counterfactual property that entrepreneurs hold most of their wealth in foreign assets, the opposite of the home bias.

is over the repayment fraction δ_t and total transfers $\tau_t^e \mu^S$ from country S to country R. We refer to the negotiated transfers as an "external bailout." If the negotiated repayment fraction δ_t is smaller than 1, country S can also make a transfer $\tau^d \geq 0$ to its own entrepreneurs as described above (domestic bailout).

Governments in both countries balance their budget with taxes T_t^i (or transfers if negative) paid by workers. Their budget constraints are

$$
\begin{aligned}
T_t^S + q_t^S B_{t+1}^S &= B_t^S + (\tau_t^d + \tau_t^e)\mu^S, \\
T_t^R + q_t^R B_{t+1}^R &= \delta_t B_t^R - \tau_t^e \mu^S.
\end{aligned}
\tag{7}
$$

The two budget constraints encompass all possible cases of 'no restructuring', 'restructuring without bailout', and 'restructuring with bailout'. In the case of no restructuring, country R fully repays its debt obligations. Thus, $\delta_t = 1$, $\tau_t^d = 0$ and $\tau_t^e = 0$. In the case of restructuring without bailout, there is a sovereign debt crisis with default where $\delta_t < 1$, $\tau_t^d \geq 0$ and $\tau_t^e = 0$. Recall that the safe country can always bail out its own entrepreneurs ('no bailout' means absence of 'external' bailout). In the case of restructuring with bailout, we have $\delta_t \leq 1$, $\tau_t^d \geq 0$ and $\tau_t^e > 0$.

Given the restructuring outcome, which is captured by the variables δ_t, τ_t^d and τ_t^e, individual entrepreneurs' wealth becomes

$$
\begin{aligned}
m_t^S &= b_t^{SS} + \delta_t b_t^{RS} + \tau_t^d, \\
m_t^R &= b_t^{SR} + \delta_t b_t^{RR}.
\end{aligned}
\tag{8}
$$

These expressions make clear how default—with or without external bailout—affects entrepreneurs wealth, which in turn affects aggregate macroeconomic activity. A lower repayment fraction δ_t causes a reduction in entrepreneurs' wealth. Country S, however, can alleviate the wealth losses of its own entrepreneurs with a domestic bailout that transfers τ_t^d from workers to entrepreneurs.

In the event of default, country R is excluded from the market for only one period and $B_{t+1}^R = \delta_t B_t^R$. Starting in the next period, however, the country re-enters the financial market and chooses the new debt optimally.

To simplify the analysis, we abstract from the optimal choice of debt in country S. Instead, we assume that the supply of safe assets is exogenous and changes stochastically according to a first order Markov process where B_t^S can take two values, B_{Low} and B_{High}. A transition from B_{Low} to B_{High} acts as a financial shock that increases the supply of safe assets.

2.4 Timing within a period

The sequence of events within each period t is illustrated in Figure 4. For illustrative purposes, we can think of a period as divided in three subperiods:

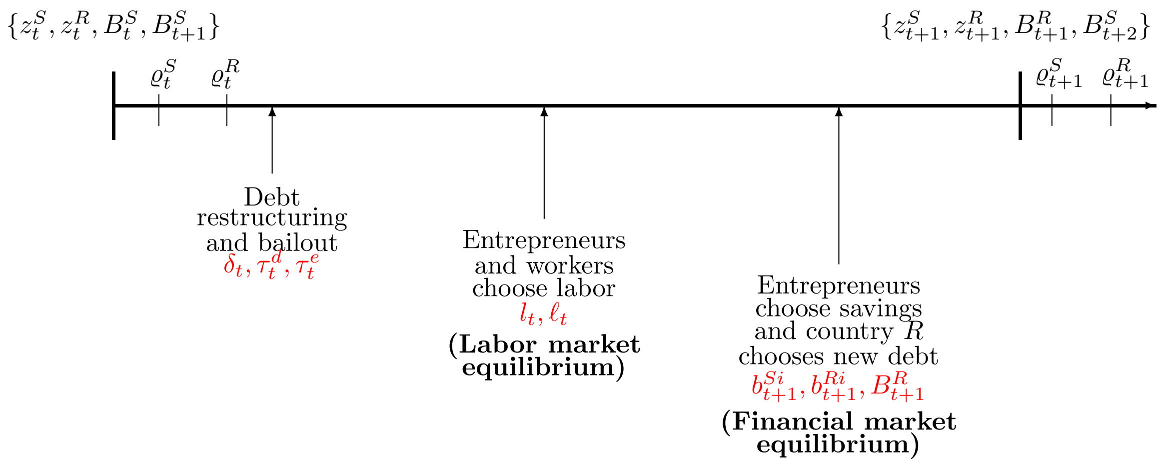

Figure 4: Timing within a period.

Subperiod 1: Debt restructuring with or without bailout takes place at the beginning of the period as illustrated in Figure 3. The exogenous states include the aggregate productivity $z_t^i \in \{z_{Low}, z_{High}\}$, current and new bonds issued by the safe country, $B_t^S \in \{B_{Low}, B_{High}\}$ and $B_{t+1}^S \in \{B_{Low}, B_{High}\}$, and the commitment variables ϱ_t^R and ϱ_t^S. For the later derivation of the spreads, it would be convenient to assume that ϱ^R is observed before ϱ^S. Debt restructuring, however, takes place only after they both become known. The endogenous states (not shown in the figure) are the bonds issued by the risky country in the previous period, B_t^R, and the portfolio holdings of entrepreneurs, B_t^{ij}.

Subperiod 2: Given the post-default wealth, entrepreneurs choose the input of labor and workers choose the supply of labor. Clearing in the labor market determines the wage w_t^i and employment L_t^i in each country $i \in \{S, R\}$.

Subperiod 3: Entrepreneurs choose the portfolio allocation, b_{t+1}^{Si} and b_{t+1}^{Ri}, which determines the demands for safe and risky debt. The government of country R chooses the supply of debt B_{t+1}^R. In the event of default $B_{t+1}^R = \delta_t B_t^R$. Clearing in the bond market determines the prices for bonds, q_t^S and q_t^R.

3 Competitive equilibrium

In this section, we characterize some of the properties of the competitive equilibrium for given government policies (restructuring, bailout, and debt issuance). Given the particular timing structure, we analyze separately the competitive equilibrium in the labor market and the competitive equilibrium in the financial market. We start with the labor market equilibrium, which we can characterize analytically.

3.1 Labor market equilibrium

Figure 5 shows the supply and demand for labor derived from the aggregation of individual supplies—eq. (1)—and individual demands—eq. (3).

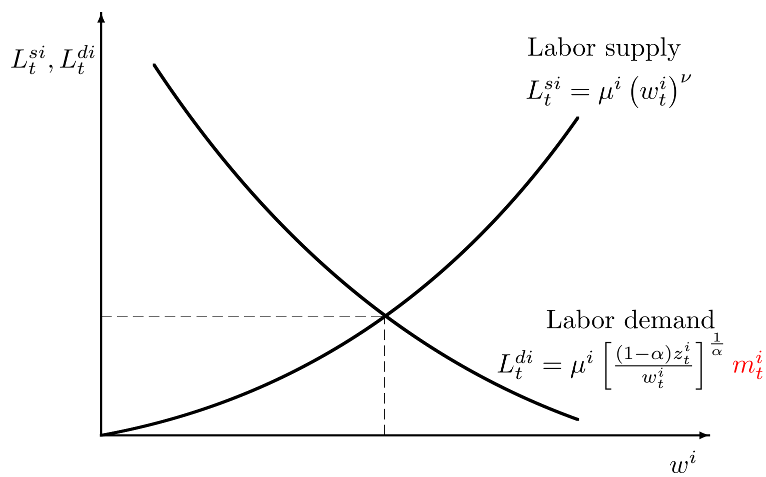

Figure 5: Labor Market Equilibrium.

The demand for labor is a function of entrepreneurs' wealth m_t^i. An increase in m_t^i shifts the demand for labor to the right and leads to higher employment and wage. Equivalently, a decrease in m_t^i shifts the demand for labor to the left, reducing equilibrium employment and wage. Through the impact on m_t^i, the default of country R has real macroeconomic consequences in both countries.

The equilibrium wage is given by

$$w_t^i = \left[(1-\alpha)z_t^i\right]^{\frac{1}{1+\alpha\nu}} (m_t^i)^{\frac{\alpha}{1+\alpha\nu}}. \tag{9}$$

The wage rate is increasing in productivity z_t^i and financial wealth m_t^i. Once we find the equilibrium wage, we can determine the individual supply of labor from workers ℓ_t^i (eq. 1), the individual input of labor used in production by entrepreneurs l_t^i (eq. 3), individual profits π_t^i (eq. 4), and end-of-period net worth a_t^i (eq. 5).

Aggregate output is derived from the aggregation of individual productions, that is, $Y_t^i = y_t^i \mu^i$. Using the production function, eq. (2), aggregate output is

$$Y_t^i = (1-\alpha)^{\frac{\nu(1-\alpha)}{1+\alpha\nu}} (z_t^i)^{\frac{1+\nu}{1+\alpha\nu}} (m_t^i)^{\frac{\alpha(1+\nu)}{1+\alpha\nu}} \mu^i. \tag{10}$$

Eq. (10) shows that a drop in m_t^i caused by default from country R has a negative impact on the production of both countries. Thus, the model generates endogenous macroeconomic spillovers from default. In other models such as Tirole (2015), spillovers are exogenous.

Another feature of the model is that the contraction induced by default is larger during economic booms. This is stated formally in the following proposition.

Proposition 2 *Default causes a larger output drop in country $i \in \{S, R\}$ when the productivity of the country is higher, that is,*

$$\frac{\partial Y^i(z_t^i = z_{High})}{\partial \delta_t} > \frac{\partial Y^i(z_t^i = z_{Low})}{\partial \delta_t}. \tag{11}$$

Proof. We obtain the derivative of Y_t^i with respect to δ_t using eq. (10) and the definition of m_t^i provided in eq. (8). The obtained derivative is increasing in z_t^i. ■

This property has two implications: (i) the incentive of the risky country to default increases when the country is in recession; (ii) the bailout incentive of the safe country increases when the creditor country is in an economic boom. While the first implication (higher default incentive in recession) is a feature of many sovereign default models, the second (bailout incentive) is a novel feature of our model. It points out that the business cycle is important not only for the incentive to default, but also for bailouts.

3.2 Financial market equilibrium

The financial market's equilibrium requires that the demand of bonds issued by each country equals its supply,

$$B_{t+1}^{SS} + B_{t+1}^{SR} = B_{t+1}^{S}, \tag{12}$$
$$B_{t+1}^{RS} + B_{t+1}^{RR} = B_{t+1}^{R}. \tag{13}$$

The aggregation of individual portfolio decisions characterized in Lemma 1 provides the entrepreneurs' demands for safe bonds,

$$B_{t+1}^{SS} = \frac{\theta_t \bar{\beta}_t a_t^S \mu^S}{q_t^S}, \tag{14}$$
$$B_{t+1}^{SR} = \frac{\theta_t \bar{\beta}_t a_t^R \mu^R}{q_t^S}, \tag{15}$$

and entrepreneurs' demands for risky bonds,

$$B_{t+1}^{RS} = \frac{(1-\theta_t) \bar{\beta}_t a_t^S \mu^S}{q_t^R}, \tag{16}$$
$$B_{t+1}^{RR} = \frac{(1-\theta_t) \bar{\beta}_t a_t^R \mu^R}{q_t^R}. \tag{17}$$

Entrepreneurs consume a fraction $1-\bar{\beta}_t$ of their wealth a_t^i and save the remaining fraction $\bar{\beta}_t$. A fraction θ_t of savings are then invested in safe bonds, and the remaining fraction $1-\theta_t$ in risky bonds. The fraction θ_t, determined by condition (6), is independent of i because entrepreneurs in both countries choose the same portfolio composition. The variable $a_t^i = (1-\phi)m_t^i + \pi_t^i$ is the individual end-of-period net worth of entrepreneurs in country i. The aggregate net worth is $a_t^i \mu^i$.

Given the end-of-period net worth, a_t^S and a_t^R, and the aggregate supplies of bonds, B_{t+1}^S and B_{t+1}^R, we can use eq. (6) together with eqs. (12)-(17) to find the equilibrium values of B_{t+1}^{SS}, B_{t+1}^{SR}, B_{t+1}^{RS}, B_{t+1}^{RR}, q_t^S, q_t^R, θ_t. Because θ_t depends on the repayment fraction δ_{t+1}, which depends non-linearly on the next period states, we cannot derive an analytical solution for δ_{t+1}. We can only characterize it numerically.

4 Political equilibrium

To use a compact notation, we denote by $\mathbf{p}_t = (\delta_t, \tau_t^d, \tau_t^e)$ the outcome of debt restructuring. The vector $\mathbf{p}_t$, together with the new debt chosen by the risky country, B_{t+1}^R, represent the policy variables that are determined in the political equilibrium.

We consider only government policies with a Markov structure, that is, they are functions of the aggregate states $\mathbf{s}_t = (z_t^S, z_t^R, B_t^{SS}, B_t^{SR}, B_t^{RS}, B_t^{RR}, B_{t+1}^S, \varrho_t^S, \varrho_t^R)$. The state vector includes B_{t+1}^S since the debt issued by the safe country follows a Markov process and new borrowing is revealed at the beginning of the period. The policy variables can be expressed as

$$\begin{aligned} \mathbf{p}_t &= \Upsilon_t(\mathbf{s}_t), \\ B_{t+1}^R &= \Phi_t(\mathbf{s}_t, \mathbf{p}_t). \end{aligned}$$

Given the policy functions, the government's value in country i can be written as

$$V_t^i(\mathbf{s}_t) = U^i(\mathbf{s}_t, \mathbf{p}_t, B_{t+1}^R) + \beta\, \mathbb{E}_t V_{t+1}^i(\mathbf{s}_{t+1}), \tag{18}$$

where the period utility is given by

$$\begin{aligned} U^i(\mathbf{s}_t, \mathbf{p}_t, B_{t+1}^R) &= \Psi \cdot \ln\left[(1-\phi)m_t^i + \pi_t^i - q_t^S b_{t+1}^{Si} - q_t^R b_{t+1}^{Ri}\right] + \\ &\quad (1-\Psi)\cdot\left[w^i \ell_t^i - \frac{T_t^i}{\mu^i} - \frac{\nu}{1+\nu}\left(\ell_t^i\right)^{\frac{1+\nu}{\nu}}\right] - \lambda \frac{B_{t+1}^i}{\mu^i}. \end{aligned}$$

This is the sum of utilities of entrepreneurs and workers, weighted by Ψ and $1-\Psi$. Note that the expression is written in 'per-entrepreneur' terms (thus, aggregate variables such as taxes and government debt are divided by the mass μ^i of entrepreneurs, which is also the mass of workers). We have added a further assumption: the stock of debt B_{t+1}^i has a negative impact on government's welfare. This cost, which in reduced form reflects political risks associated with excessive borrowing, does not play any significant role for the qualitative properties of the model. We included it only for calibration purposes.[11]

[11] As we will see, the calibration of λ allows us to match the targeted debt-to-GDP ratio together with the targeted risk-free interest rate.

Given states $\mathbf{s}_t$, and policies $\mathbf{p}_t$ and B^R_{t+1}, the period utility $U^i(\mathbf{s}_t, \mathbf{p}_t, B^R_{t+1})$ can be derived analytically. However, the continuation value can only be computed numerically. We should point out that the value function $V^i_t(\mathbf{s}_t)$ has the time subscript t because the horizon could be finite. The subscript can be omitted when $N = \infty$.

When the governments of the two countries solve their optimization problems at time t, they take as given the continuation value $V^i_{t+1}(\mathbf{s}_{t+1})$, in addition to the policy responses of the other government. The future impact of current policies is fully captured by their impact on next period states $\mathbf{s}_{t+1}$. Given the timing structure depicted in Figure 4, we describe first the optimal borrowing of country R (last sub-period), given the debt restructuring outcome. We will then describe the debt restructuring problem (first sub-period), given the anticipated borrowing policy.

4.1 Borrowing policy

The government of country R chooses its borrowing by solving the following problem

$$\mathcal{V}^R_t(\mathbf{s}_t, \mathbf{p}_t) = \max_{B^R_{t+1}} \left\{ U^R(\mathbf{s}_t, \mathbf{p}_t, B^R_{t+1}) + \beta\, \mathbb{E} V^R_{t+1}(\mathbf{s}_{t+1}) \right\} \tag{19}$$

subject to:

$$B^R_{t+1} = \delta_t B^R_t \qquad \text{if} \qquad \mathbf{p}_t \neq (1, 0, 0)$$

where $V^R_{t+1}(\mathbf{s}_{t+1})$ is the continuation value defined in (18). The government takes as given the default and renegotiation outcome $\mathbf{p}_t$ since this is determined in subperiod 1. If the country restructured the debt, that is, $\mathbf{p}_t \neq (1, 0, 0)$, it will not be able to re-optimize and the new debt is $B^R_{t+1} = \delta_t B^R_t$. The difference between $\mathcal{V}^R_t(\mathbf{s}_t, \mathbf{p}_t)$ and $V^R_{t+1}(\mathbf{s}_t)$ is that the former is conditional on a given restructuring policy $\mathbf{p}_t$, while in the latter the equilibrium $\mathbf{p}_t$ has already been substituted out.

The solution to Problem (19) returns the borrowing policy $B^R_{t+1} = \Phi_t(\mathbf{s}_t, \mathbf{p}_t)$. The government's value in country S is

$$\mathcal{V}^S_t(\mathbf{s}_t, \mathbf{p}_t) = U^S\Big(\mathbf{s}_t, \mathbf{p}_t, \Phi_t(\mathbf{s}_t, \mathbf{p}_t)\Big) + \beta\, \mathbb{E} V^S_{t+1}(\mathbf{s}_{t+1}). \tag{20}$$

4.2 Restructuring policies

We now have all the ingredients to characterize default, renegotiation, and bailout policies, $\mathbf{p}_t = \Upsilon_t(\mathbf{s}_t)$. To do so we need to consider three possible scenarios: a) Full repayment; b) Debt restructuring without external bailout (default); c) Debt restructuring with external bailout.

a) Full repayment

Full repayment arises in two cases. The first is when country R has full commitment to repay ($\varrho^R_t = 1$). This is the first branch of the tree depicted in Figure 3. The second is when country R does not have the commitment to repay ($\varrho_t^R = 0$) but it does not find optimal to restructure the debt. This is captured by the second and third branches in Figure 3.

With full repayment, we have $\mathbf{p}_t = (1, 0, 0)$: the repayment fraction is $\delta_t = 1$, and there are not bailout transfers, $\tau_t^d = \tau_t^e = 0$. The country's welfare is

$$\overline{V}_t^i(\mathbf{s}) = \mathcal{V}_t^i\Big(\mathbf{s}_t, (1, 0, 0)\Big), \tag{21}$$

with $\mathcal{V}_t^i(\mathbf{s}_t, \mathbf{p}_t)$ defined in eqs. (19) and (20).

b) Debt restructuring without bailout (default)

In this scenario, country R finds it optimal to restructure the debt. However, either country S is unable to negotiate ($\varrho_t^S = 0$) or, even if the country is able to negotiate ($\varrho^S = 1$), it does not find it optimal. The first case could arise in the second branch of Figure 3. The second case could arise in the third branch. In both cases $\tau^e = 0$. However, country S can still implement a domestic bailout, in which case $\tau^d > 0$.

The two governments make their choices sequentially: Country R chooses the repayment δ_t first and country S chooses τ_t^d knowing the value of δ_t. They play a Stackelberg game where country R is the leader and country S is the follower.

We solve the strategic game starting with the optimization problem solved by country S (the follower). After country R has chosen the repayment fraction δ_t, the optimal domestic transfer chosen by the safe country is

$$g_t^\tau(\mathbf{s}_t, \delta_t) \;\;=\;\; \arg \max_{\tau_t^d \in [0, (1-\delta_t) B^{RS}/\mu^i]} \mathcal{V}_t^S\Big(\mathbf{s}_t, (\delta_t, \tau_t^d, 0)\Big)$$

The constraint imposes that the domestic transfer is non-negative and does not exceed the average losses incurred by each entrepreneur.

Country R chooses the optimal repayment fraction δ_t taking as given the response function $g^\tau(\mathbf{s}_t, \delta_t)$ of the safe country. The optimal δ_t solves

$$g_t^\delta(\mathbf{s}_t) \;\;=\;\; \arg \max_{\delta_t \in [0\,,\,1]} \mathcal{V}_t^R\bigg(\mathbf{s}_t, \Big(\delta_t, g_t^\tau(\mathbf{s}_t, \delta_t), 0\Big)\bigg). \tag{22}$$

Using the optimal policies chosen by the two countries, the value for country i is

$$\underline{V}_t^i(\mathbf{s}_t) = \mathcal{V}_t^i\left(\mathbf{s}_t, \bigg(g_t^\delta(\mathbf{s}_t), g_t^\tau\Big(\mathbf{s}_t, g_t^\delta(\mathbf{s}_t)\Big), 0\bigg)\right). \tag{23}$$

c) Debt restructuring with bailout

If country R chooses to restructure the debt and country S has the ability to negotiate a bailout ($\varrho^S = 1$), the restructuring outcome $\mathbf{p}_t = (\delta_t, \tau_t^d, \tau_t^e)$ is determined with

Nash bargaining. If bargaining fails, the countries revert to the equilibrium with restructuring but without bailout (default). Therefore, the threat values are $\underline{\underline{V}}_t^S(\mathbf{s}_t)$ and $\underline{\underline{V}}_t^R(\mathbf{s}_t)$ defined above.

Given η, the bargaining power of country S, the restructuring outcome solves[12,13]

$$g_t^{\mathbf{P}}(\mathbf{s}_t) \quad = \quad \arg\max_{\delta_t,\tau_t^d,\tau_t^e} \left[\mathcal{V}_t^S\Big(\mathbf{s}_t,(\delta_t,\tau_t^d,\tau_t^e)\Big) - \underline{\underline{V}}_t^S(\mathbf{s}_t)\right]^{\eta} \left[\mathcal{V}_t^R\Big(\mathbf{s}_t,(\delta_t,\tau_t^d,\tau_t^e)\Big) - \underline{\underline{V}}_t^R(\mathbf{s}_t)\right]^{1-\eta}, \tag{24}$$

subject to:

$$\delta_t \in [0\,,\,1]$$
$$\tau_t^d \in \left[0\,,\,\frac{(1-\delta_t)B_t^{RS}}{\mu^S}\right]$$
$$\tau_t^e \geq 0.$$

Using the negotiated policy $g_t^{\mathbf{P}}(\mathbf{s}_t)$, we define the value for country i as

$$\underline{V}_t^i(\mathbf{s}_t) = \mathcal{V}_t^i\Big(\mathbf{s}_t, g_t^{\mathbf{P}}(\mathbf{s}_t)\Big) \tag{25}$$

The solution obtained in the case of debt restructuring without bailout (default) is also feasible in the bargaining problem considered here. Therefore, an external bailout always (weakly) dominates a debt restructuring without bailout. This implies that $\underline{V}_t^i(\mathbf{s}_t) \geq \underline{\underline{V}}_t^i(\mathbf{s}_t)$. Thus, a domestic bailout without renegotiation never strictly dominates an external bailout.

After deriving the value functions associated with debt restructuring (with and without bailout), we can determine whether country R prefers to repay or restructure the debt. This is the decision made when $\varrho_t^R = 0$, which arises in the second and third branches of Figure 3. The optimal decision—denoted by $\xi_t(\mathbf{s}_t)$—solves

$$\xi_t(\mathbf{s}_t) = \arg\max_{x\in\{0,1\}} \begin{cases} x \cdot \underline{\underline{V}}_t^R(\mathbf{s}_t) + (1-x)\cdot \overline{V}_t^R(\mathbf{s}_t), & \text{if} \quad \varrho_t^S = 0, \\ \\ x \cdot \underline{V}_t^R(\mathbf{s}_t) + (1-x)\cdot \overline{V}_t^R(\mathbf{s}_t), & \text{if} \quad \varrho_t^S = 1. \end{cases}$$

Given the restructuring policy $\xi_t(\mathbf{s}_t)$, we can compute the associated welfare as

$$V_t^i(\mathbf{s}_t) = \begin{cases} \xi_t(\mathbf{s}_t)\cdot \underline{\underline{V}}_t^i(\mathbf{s}_t) + (1-\xi_t(\mathbf{s}_t))\cdot \overline{V}_t^i(\mathbf{s}_t), & \text{if} \quad \varrho_t^S = 0, \\ \\ \xi_t(\mathbf{s}_t)\cdot \underline{V}_t^i(\mathbf{s}_t) + (1-\xi_t(\mathbf{s}_t))\cdot \overline{V}_t^i(\mathbf{s}_t), & \text{if} \quad \varrho_t^S = 1. \end{cases}$$

This is the value function for the government welfare used in eq. (18).

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[12]Even though the two countries have different populations $2\mu^i$, we can write the Nash bargaining problem using 'per-entrepreneur' surpluses. Aggregate surpluses would give exactly the same result

[13]Alternatively, we could assume a different timing where country S proposes a contingent bailout transfer, and country R chooses whether to accept or reject it. The online appendix shows that this case is equivalent to assuming $\eta = 1$ and the quantitative results do not change significantly.

5 One-period economy ($N = 1$)

We start the characterization of the political equilibrium focusing on the special version of the model with only one period, that is, $N = 1$. This is also the equilibrium in the terminal period when $N > 1$.

With only one period, new borrowing is zero and the optimal policies are limited to default and renegotiation. Since the equilibrium with commitment to repay is trivial in the one-period model, we concentrate on the case in which $\varrho^R = 0$. To simplify notation, we eliminate time subscripts.

Given the states $\mathbf{s} = (z^S, z^R, B^{SS}, B^{SR}, B^{RS}, B^{RR})$, the governments' policies are $\mathbf{p} = (\delta, \tau^d, \tau^e)$, which are determined by the sequential game depicted in Figure 3. The government's objectives are defined by eqs. (19) and (20) but with the next period debts set to zero, $B^S_{N+1} = B^R_{N+1} = 0$. Since the parameter ϕ does not affect the qualitative properties of the model, for the analysis of this section we assume that $\phi = 0$. The objective function for country i simplifies to

$$\mathcal{V}^i(\mathbf{s}, \mathbf{p}) = \Psi \cdot \ln\Big[m^i + y^i - w^i l^i\Big] + (1-\Psi) \cdot \left[w^i \ell^i - \frac{T^i}{\mu^i} - \frac{\nu}{1+\nu}\left(\ell^i\right)^{\frac{1+\nu}{\nu}}\right]. \qquad (26)$$

This is the (per-capita) weighted sum of utilities for entrepreneurs and workers. Entrepreneurs' utility is increasing in individual consumption, which in the one-period model is equal to the end-of-period wealth $a^i = m^i + \pi^i$. The welfare of workers is equal to consumption net of the dis-utility from working. Appendix B derives the analytical expression for the government's welfare as a function of δ, τ^d and τ^e.

5.1 Debt restructuring without bailout (default)

When the safe country cannot negotiate an external bailout, the transfer τ^e is zero. Since the repayment decision of country R does not depend on τ^d, we characterize the repayment fraction δ ignoring the domestic bailout chosen by country S.[14]

a) Repayment decision by country R

Country R chooses $\delta \in [0, 1]$ in order to maximize objective (26), subject to $m^R = (B^{SR} + \delta B^{RR})/\mu^R$ and $T^R = \delta B^R$. To characterize the solution, we consider first the unconstrained problem, whose first order condition can be written as

[14] The independence of the repayment from the domestic bailout of country S does not apply to the multi-period model because τ^d affects savings and, therefore, the equilibrium interest rates.

$$\underbrace{-\Psi\left(\frac{1}{d^R}\right)\left(\frac{\partial y^R}{\partial m^R}-w^R\frac{\partial l^R}{\partial m^R}\right)\left(\frac{B^{RR}}{B^R}\right)-(1-\Psi)\left(w^R-\ell^{R\frac{1}{\nu}}\right)\frac{\partial \ell^R}{\partial m^R}\left(\frac{B^{RR}}{B^R}\right)}_{\text{1. Macroeconomic effect}} \quad +$$

$$\underbrace{\Psi\left(\frac{1}{d^R}\right)l^R\frac{\partial w^R}{\partial m^R}\left(\frac{B^{RR}}{B^R}\right)-(1-\Psi)\ell^R\frac{\partial w^R}{\partial m^R}\left(\frac{B^{RR}}{B^R}\right)}_{\text{2. Price effect}} \quad - \quad \underbrace{\Psi\left(\frac{1}{d^R}\right)\left(\frac{B^{RR}}{B^R}\right)+1-\Psi}_{\text{3. Redistribution effect}} \quad = \quad 0. \tag{27}$$

This left hand side shows the marginal welfare effect for country R when its debt is reduced by one unit per entrepreneur/worker. This requires δ to drop by μ^R/B^R. The welfare effect is the sum of three terms, each capturing the impact of a particular channel. We can also see the impact of each channel separately on entrepreneurs (terms multiplied by Ψ) and workers (terms multiplied by $1-\Psi$).

1. *Macroeconomic effect*: A lower repayment decreases the wealth of entrepreneurs, m^R, which in turn decreases the demand for labor, l^R. Since both m^R and l^R are inputs of production, output y^R drops. This affects entrepreneurs' welfare through the drop in profits (converted in utility units through the marginal utility of consumption, $1/d^R$). The welfare effect on workers comes from the decrease in labor income and the lower dis-utility of working.

2. *Price effect*: Lower entrepreneurs' wealth reduces the demand for labor and causes a reduction in wages. This is captured by the term $\partial w^R/\partial m^R$, which is positive for entrepreneurs (since it decreases the cost of labor) and negative for workers (since it reduces their compensation).

3. *Redistribution effect*: By repaying less, the government redistributes wealth from creditors (entrepreneurs) to tax payers (workers).[15] Lowering B^R by one unit per entrepreneur/worker, decreases the wealth of entrepreneurs in country R by B^{RR}/B^R. The welfare effect on workers is the reduction in taxes, which is equal to 1. As long as some of the bonds issued by country R are held by foreign entrepreneurs, the wealth losses incurred by entrepreneurs are smaller than the reduction in taxes for workers, that is, $\frac{B^{RR}}{B^R}<1$.

Although the redistribution effect of a lower repayment has positive welfare effects for workers, the macroeconomic and price effects have negative consequences for them. Thus, from the perspective of workers, default implies a trade-off: reducing taxes at the expense of lower pre-tax income. For entrepreneurs, default implies only a cost:

[15] The assumption that only workers pay taxes is not essential. This would be true even if taxes were equally paid by workers and entrepreneurs. What matters is that taxes are not proportional to the holding of the debt so that agents who hold the debt (entrepreneurs) experience a net gain from taxes while agents who do not hold the debt (workers) experience a net loss.

in addition to the direct loss of wealth, they also earn lower incomes (profits decline even if entrepreneurs pay lower wages). A key insight is that the macroeconomic cost is lower when domestic portfolios are diversified, that is, B^{RR}/B^R is smaller. This is captured by the fact that most terms in eq. (27) are multiplied by B^{RR}/B^R.

The next step is to establish existence and uniqueness of the equilibrium and to characterize its properties. Let's define σ^S and σ^R as the shares of debt issued by country S and country R, respectively, held by entrepreneurs in country R,

$$\begin{aligned} B^{SR} &= \sigma^S B^S, \\ B^{RR} &= \sigma^R B^R. \end{aligned}$$

Proposition 3 *There exists a unique solution to the unconstrained problem with the optimal repayment rate* $\underline{\underline{\delta}}^u(\mathbf{s})$ *strictly increasing in productivity* z^R*. Keeping the portfolio shares* σ^S *and* σ^R *constant,* $\underline{\underline{\delta}}^u(\mathbf{s})$ *is strictly decreasing in* B^S *and* B^R*.*

Proof. Appendix C.

The first part of the proposition establishes that it is optimal to repay more when productivity is high. This is because the macroeconomic impact of default rises with productivity (see previous Proposition 2).

The second part of the proposition establishes that the incentive to repay declines when the stocks of domestic and/or foreign debt are higher. This property relies on the assumption that the portfolio shares σ^S and σ^R do not change. This would be the case if entrepreneurs in both countries choose the same composition of portfolio, that is, $B^{SS}/B^{RS} = B^{SR}/B^{RR}$. This is consistent with the optimal portfolio choices characterized in Lemma 1 for the the multi-period model.

That an increase in B^R leads to a lower repayment ratio may not be surprising: A lower repayment generates a larger redistribution from foreign workers to domestic workers. Less obvious is the dependence on B^S: When B^S is higher, entrepreneurs in country R hold more safe assets which reduces the macroeconomic cost of default.[16]

Proposition 3 characterizes the unconstrained problem and the optimal repayment $\underline{\underline{\delta}}^u(\mathbf{s})$ could be negative or bigger than 1. This implies that the constrained solution could be at a corner. The constrained repayment policy is

$$g^\delta(\mathbf{s}) = \min\left\{ \max\left\{\underline{\underline{\delta}}^u(\mathbf{s}) \ , \ 0\right\} \ , \ 1\right\}. \tag{28}$$

Figure 6 illustrates the properties of the optimal repayment, $g^\delta(\mathbf{s})$ as a function of B^R, the stock of debt issued by country R. The level of productivity z^R and the shares σ^S and σ^R are kept constant. When safe debt is low, B^S_{Low} (continuous line), country R fully repays for values of debt that are smaller than the threshold $\hat{B}^R_{Low}$. However, the country partially defaults (so the debt haircut is positive) when the stock of B^R exceeds $\hat{B}^R_{Low}$.

[16]It is worth noting that the repayment rate is also decreasing in σ^S and σ^R until it reaches zero.

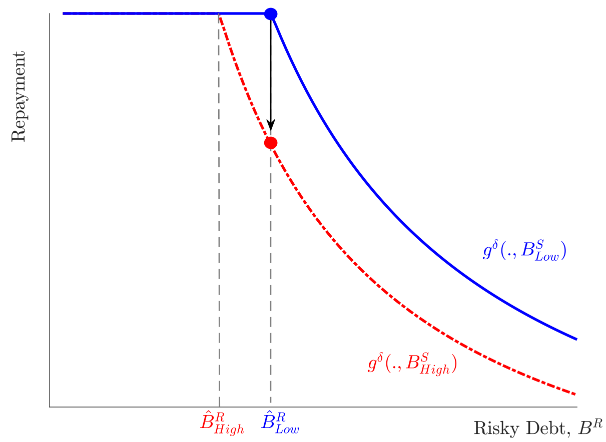

Figure 6: Optimal repayment rate δ as a function of risky debt, B^R, for a given productivity z^R and constant shares σ^S and σ^R. Continuous line: low value safe debt (B^S_{Low}). Dashed line: high safe debt (B^S_{High}).

Portfolio diversification and externally induced default. Figure 6 also illustrates the importance of portfolio diversification in country R. When country S increases its debt to B^S_{High}, entrepreneurs in country R hold more safe debt (their portfolio becomes less risky). This increases the incentive of country R to default as illustrated by the change of the repayment function from the continuous line to the dashed line. The threshold above which the repayment is partial ($\delta < 1$) decreased from $\hat{B}^R_{Low}$ to $\hat{B}^R_{High}$. Furthermore, for any value of the debt above $\hat{B}^R_{High}$, the repayment ratio becomes smaller, before reaching zero.

The comparison of the continuous line with the dashed line, for a given B^R, isolates the portfolio channel emphasized in this paper from the 'standard' channel widely studied in the literature: if more of the risky debt is held by entrepreneurs in country S, default becomes more attractive for country R because it generates a larger redistribution in favor of country R. The mechanism discussed here, instead, is different because it keeps the debt of country R and its cross-country holdings fixed.

The importance of the diversification channel is to highlight that a country's default could be driven or amplified by the increase in debt issued by other countries. The size of the haircut (for a given level of risky debt) increases with the level of safe assets held by residents of country R. This will also be highlighted in the calibration exercise, where we show that the expansion of the financial sector in safe countries (like Germany) impacts the incentives to default of risky countries (like the GIIPS).

Although in the model we only allow for financial assets issued by governments, the effect described here could also be driven by the expansion of private borrowing. For example, a financial boom in advanced economies, either private or public, could induce the default of emerging countries even if financial and real conditions have not changed in these countries. It could also be the result of an increase in global assets that are not controlled domestically, such as foreign currency or gold. It is in this sense that default could be 'externally' driven.

b) Domestic bailout decision by country S

Default by the risky country also destroys entrepreneurial wealth held by entrepreneurs in country S and, therefore, it affects adversely employment and output in the safe country. To alleviate the negative macroeconomic spillover, the government of the safe country could bail out its own entrepreneurs.

With domestic bailout, country S chooses $\tau^d \in [0, (1-\delta)B^{RS}/\mu^S]$ to maximize (26) subject to $m^S = (B^{SS} + \delta B^{RS})/\mu^S + \tau^d$ and $T^S = B^S + \tau^d \mu^S$. The domestic transfer τ^d cannot be negative or bigger than the financial losses incurred by entrepreneurs.

The first order condition with respect to τ^d for the unconstrained problem is

$$\underbrace{\Psi\left(\frac{1}{d^S}\right)\left(\frac{\partial y^S}{\partial m^S} - w^S\frac{\partial l^S}{\partial m^S}\right) + (1-\Psi)\left(w^S - (\ell^S)^{\frac{1}{\nu}}\right)\frac{\partial \ell^S}{\partial m^S}}_{\textit{1. Macroeconomic effect}}$$
$$\underbrace{-\Psi\left(\frac{1}{d^S}\right)l^S\frac{\partial w^S}{\partial m^S} + (1-\Psi)\ell^S\frac{\partial w^S}{\partial m^S}}_{\textit{2. Price effect}} + \underbrace{\Psi\left(\frac{1}{d^S}\right) - (1-\Psi)}_{\textit{3. Redistribution effect}} = 0 \tag{29}$$

The welfare consequences of a domestic bailout are also decomposed into (1) the macroeconomic effect (reduces the size of the recession induced by default), (2) price effect (higher wages), (3) redistribution effect (from workers to entrepreneurs).

The optimality condition for domestic bailout—eq. (29)—is similar to the optimality condition for default—eq. (27). However, it has an inverted sign. Effectively, the domestic bailout 'counteracts' the negative impact of default on country S. Also notice that the various effects in eq. (27) are multiplied by B^{RR}/B^R, with the exception of the second term in the 'redistribution effect'. The term B^{RR}/B^R captures the importance of portfolio diversification which is not relevant for a domestic bailout.

Proposition 4 *Given δ chosen by country R, there exists a unique solution to the unconstrained domestic bailout problem with the optimal transfer $\underline{\underline{\tau}}^d(\mathbf{s};\delta)$ strictly increasing in productivity z^S. Furthermore, keeping the portfolio shares σ^S and σ^R constant, the transfer $\underline{\underline{\tau}}^d(\mathbf{s};\delta)$ strictly decreases in B^S, B^R and δ.*

Proof. Appendix D.

Also the constrained solution to the domestic bailout could be at a corner. We can then write it as

$$g^\tau(\mathbf{s},\delta) = \min\left\{\max\left\{\underline{\underline{\tau}}^d(\mathbf{s};\delta)\ ,\ 0\right\}\ ,\ \frac{(1-\delta)B^{RS}}{\mu^S}\right\}. \tag{30}$$

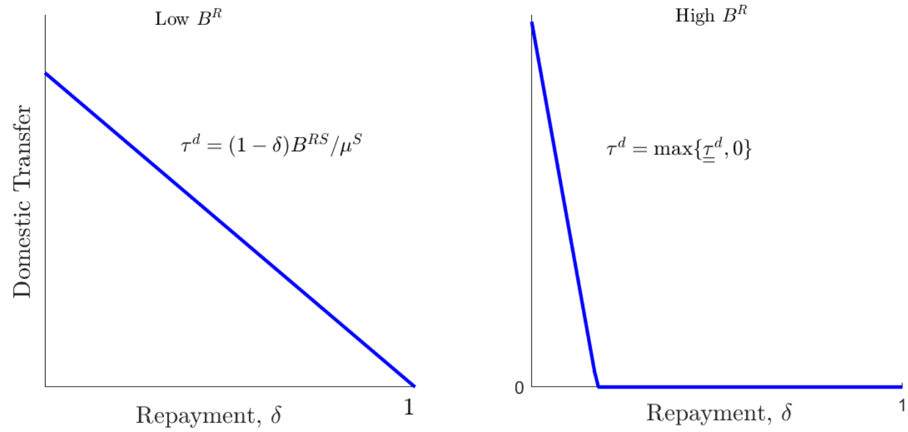

Figure 7: Domestic bailout transfer τ^d as function of δ. The plots are constructed keeping σ^S, σ^R and B^S constant.

Figure 7 illustrates the optimal domestic transfer $g^\tau(\mathbf{s}, \delta)$ as a function of δ. The left panel considers a value of B^R that is relatively low. Because of the scarcity of financial assets, the government of country S fully compensates the losses incurred by domestic entrepreneurs with bailout transfers. Thus, the lower is the repayment ratio δ, the higher the domestic transfer is.

The right panel considers a higher value of B^R. Provided that the debt of country R is fully repaid, the financial wealth held by domestic entrepreneurs exceeds what is considered optimal by the S government. This implies that, if country R repays only part of its debt, the government of country S does not fully bail out domestic entrepreneurs. The second panel of Figure 7 shows that the domestic transfer declines faster with the repayment ratio and becomes zero before δ reaches 1.

5.2 Debt restructuring with bailout

Without a rescue package from the safe country—which is the case analyzed in the previous subsection—the government of country R ignores the macroeconomic cost of default incurred by country S. This introduces an externality that could make the equilibrium socially inefficient and both countries could gain from renegotiating.

Renegotiation arises only if country R triggers a debt crisis, that is, if it asks for a debt restructuring and country S is able to renegotiate ($\varrho^S = 1$). In this case, the governments of the two countries solve the Nash bargaining problem defined in eq. (24). The choice variables are the repayment rate $\delta \in [0, 1]$, the domestic transfer $\tau^d \in [0, (1-\delta)B^{RS}/\mu^S]$, and the external transfer $\tau^e \geq 0$. If the two countries do not reach an agreement, they revert to the equilibrium without external bailout. The threat value is $\underline{\underline{V}}^i(\mathbf{s})$ defined in Section 5.1.

We start with the unconstrained case. The first order condition with respect to τ^d

is identical to the one derived in Section 5.1, eq. (29). This is because the domestic bailout only affects the welfare of residents in country S. Thus, given the repayment δ, the optimal domestic transfer is identical to the one without bailout.

The first order condition with respect to the external transfer τ^e is

$$\frac{\eta}{1-\eta} = \left[\frac{\mathcal{V}^S(\mathbf{s},\mathbf{p}) - \underline{\underline{V}}^S(\mathbf{s})}{\mathcal{V}^R(\mathbf{s},\mathbf{p}) - \underline{\underline{V}}^R(\mathbf{s})}\right] \frac{\mu^S}{\mu^R}. \tag{31}$$

The term in parenthesis is the ratio of the net bargaining surpluses for the two countries: for country S in the numerator and for country R in the denominator. This ratio, re-scaled by the relative size of the two countries, is equalized to the ratio of the relative bargaining powers.

The first order condition with respect to δ is

$$\frac{\partial \mathcal{V}^R(\mathbf{s};\mathbf{p})}{\partial \delta}\mu^R + \frac{\partial\ \mathcal{V}^S(\mathbf{s};\mathbf{p})}{\partial \delta}\mu^S = 0. \tag{32}$$

This condition has a simple intuition: the weighted sum of the (net) marginal benefits from repaying an extra unit of debt must be equal to zero. Equivalently, the marginal cost for the R country must be equal to the marginal benefit for the S country. This result is independent of the bargaining weights but depends on the relative size of the two countries. Denoting with $\underline{\delta}^u(\mathbf{s})$ the unconstrained optimal repayment ratio with renegotiation, we have the following proposition.

Proposition 5 *The repayment ratio is strictly bigger with an external bailout,* $\underline{\delta}^u(\mathbf{s}) > \underline{\underline{\delta}}^u(\mathbf{s})$. *The domestic bailout transfer* τ^d *is strictly smaller with an external bailout.*

Proof. Appendix E.

Through negotiation, the choice of the repayment ratio internalizes the macroeconomic spillovers associated with a lower repayment (see eq. (32)). This leads to a higher repayment, which in turn allows the safe country to reduce the domestic bailout needed to sustain the same level of entrepreneurs' wealth m^S.

As for the case of debt restructuring without bailout, the solution to the bargaining problem could be at the corner. We can then write the general solution as

$$g^\delta(\mathbf{s}) = \min\left\{\max\left\{\underline{\delta}^u(\mathbf{s})\ ,\ 0\right\}\ ,\ 1\right\}. \tag{33}$$

Notice that the solution under renegotiation is constrained efficient as it is equivalent to the allocation chosen by a benevolent planner that faces the same instruments ($\tau^e \geq 0$, $\tau^d \geq 0$, and $0 \leq \delta \leq 1$) and the same utility weight ($0 < \Psi < 1$ for entrepreneurs). The proof is provided in Appendix F.

5.3 Domestic bailout vs. external bailout

Provided that $\varrho^S = 1$, the safe country can choose whether to bail out the risky country. This raises the question of whether the external bailout is preferred to a domestic bailout: Rather than making a transfer to the risky country in exchange for a higher repayment, the safe country could limit the macroeconomic consequences of default by making a higher transfer directly to its own entrepreneurs. The answer is provided by the following proposition.

Proposition 6 *Country S is never worse off with an external bailout compared to only a domestic bailout.*

Proof. Consider the debt restructuring problem without bailout. The equilibrium repayment is $g^\delta(\mathbf{s})$ and the domestic transfer is $g^\tau(\mathbf{s})$. These are also feasible solutions in the bargaining problem. With this solution the two countries get their reservation values. Therefore, country S cannot do worse than with an external bailout. ■

Although an external bailout cannot be worse than a domestic bailout, it might not be strictly better. What are the conditions for which an external bailout brings higher welfare to both countries? This depends on many variables including productivity, the stocks of debt, and portfolios' composition. A full characterization conditional on all these variables is very complex. Here, we focus on the special case in which countries have the same productivity and the same portfolio holdings.

Proposition 7 *Let $z^S = z^R = z$ and $\sigma^S = \sigma^R \equiv \sigma = \frac{1}{2}$. Given $B^R > 0$, there exist $\bar{B}^S > \underline{B}^S \geq 0$ such the renegotiation outcome is characterized by $\tau^d = 0$ and $\tau^e > 0$ and both countries experience higher welfare if and only if $B^S \in (\underline{B}^S, \bar{B}^S)$.*

Proof. See Appendix G.

The proposition states that, for given debt issued by country R, renegotiation takes place only if the debt of country S is within a non-empty interval. When B^S is large, even if country R defaults and repays zero, entrepreneurs in country S have still plenty of liquidity so that a bailout (domestic or external) would not increase welfare. When B^S is very small, country R does not have an incentive to default and will repay the debt in full. Notice that, if B^R is large, the lower bound for B^S could be negative. Although the proposition is stated under the condition that the two countries have the same productivity and portfolio holdings, we show in the next section that this is also true in the calibrated model without symmetry.

Another property stated in the proposition is that, under renegotiation, the domestic transfer τ^d is zero while the external transfer τ^e is positive. This means that country S prefers an external bailout to a domestic bailout. Why is that? The reason is that an external bailout brings macroeconomic benefits also to the debtor country. Because of this, country S is able to convince country R to repay an extra dollar by giving a subsidy that is smaller than the part of the dollar received in repayment by domestic entrepreneurs. In other words, the external bailout is cheaper than a domestic bailout. This is shown in Figure 8 with a numerical example.[17]

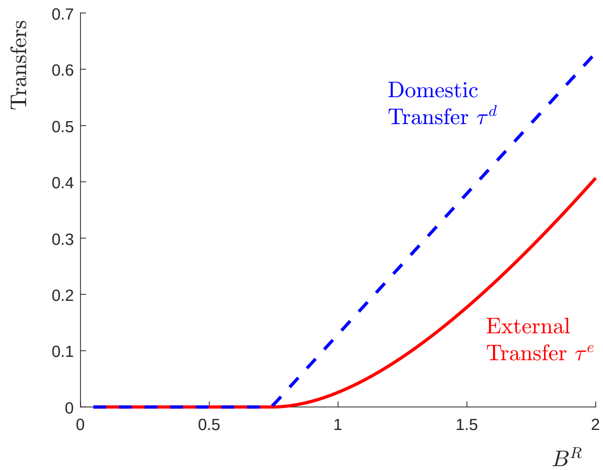

Figure 8: Domestic or external bailout transfers as functions of B^R needed to keep m^S unchanged. For a given B^R, τ^d is the domestic transfer (without external transfer) needed to keep m^S to the pre-default value; τ^e is the external transfer (without domestic transfer) needed also to keep m^S to the pre-default value. Parameters values are: $\Psi = 0.5$, $\nu = 2$, $\mu^R = \mu^S = 1$, $\alpha = 1/3$, $z^S = z^R = 1.5$, $B^S = 1.02$, and $\sigma^R = 0.5$.

The dashed line represents, in absence of external bailout, the domestic transfer made to entrepreneurs in country S needed to cover the losses incurred with default. If the repayment ratio is $\delta < 1$, then $\tau^d = (1-\delta)B^{RS}$. This ensures that the wealth of domestic entrepreneurs remains $m^S = b^{SS} + b^{RS}$. If B^R is small, country R repays the debt in full ($\delta = 1$) and the domestic transfer is zero. As B^R rises, however, the repayment ratio δ starts to decrease and the transfer $\tau^d = (1-\delta)B^{RS}$ increases.

The solid line is the external transfer τ^e required to induce country R to fully repay its debt. With the plotted transfer country R is indifferent between repaying

[17] An external bailout increases the surplus primarily because it allows for higher production (and, therefore, consumption). Another way to say this is that an external bailout has a higher multiplier for global output than a domestic bailout. Of course, bailouts could increase the surplus through other endogenous channels that are not fully captured by our model, For example, it could improve the distribution of consumption between entrepreneurs and workers in a way that increases government's welfare. A similar result could be obtained in a model where the cost of default is exogenous in both countries and it could be avoided with a bailout. In this environment, the safe country would still prefer an external bailout to a domestic bailout, provided that its bargaining power is positive. Although this alternative model could deliver similar results, modeling the cost of default endogenously provides a more intuitive explanation of why a bailout could reduce the cost of default. Notice that the assumption that country R cannot use a domestic bailout is crucial for the result. If country R can also use a domestic bailout, effectively, it would be able to default only on foreign creditors. It is then not possible to increase the surplus with an external bailout.

the debt in full ($\delta = 1$) or defaulting ($\delta < 1$ but $\tau^e = 0$). As B^R increases, country R has an incentive to repay less and must be compensated with a higher τ^e.

The message conveyed by Figure 8 is that, provided that country R has an incentive to default, τ^e is always smaller than τ^d. Thus, country S can achieve the same outcome with a smaller external transfer than with a domestic transfer. This, together with Proposition 7, is one of the most important findings of this section.

6 Calibration

The analysis of the one-period model has shown that debt restructuring depends on the initial debt of the two countries. But what determines the initial debt? To answer this question we need a multi-period version of the model where the debt of country R is endogenous. With more than one period, however, analytical solutions are not available. We will then characterize the equilibrium numerically.

For the quantitative application we consider the infinite horizon version of the model and focus on the European countries more directly involved in the 2011-2012 sovereign debt crisis. We think of the risky country R as representative of Greece, Ireland, Italy, Portugal and Spain (GIIPS), the five countries that experienced the largest increase in interest rate spreads.

Figure 9 plots the debt-to-GDP ratios of the five European countries (left panel) together with the corresponding interest spreads over the period 1999-2019. Although the height of the European debt crisis took place in 2011-2012, borrowing and interest rate spreads started to rise with the global financial crisis. Initially, the patterns of borrowing and spreads were similar among the five European countries. At the height of the crisis, however, they diverged significantly. While the debt-to-GDP ratio of Greece declined sharply, this was not the case for the other GIIPS countries. Also, the increase in the interest rate spread for Greece was much bigger than for the other GIIPS countries, reaching a peak of over 30 percent. In the language of Mitchener and Trebesch (2021), Greece suffered a sovereign debt crisis *with* default while the other countries experienced a sovereign debt crisis *without* default.

The experience of the five European countries provides a case study for the application of our model. In that spirit, we use averages for these countries to calibrate the parameters that are specific to country R. For the parameters that are specific to country S, instead, we use moments for Germany because it is the largest country in Europe, widely regarded as safe. Moments are computed over the sample period 1999-2019. The starting year coincides with the introduction of the EURO in 1999 while the ending year is motivated by data availability.

We divide the parameters in four groups. The first includes parameters that are standard in the macro literature. In the the second group there are parameters that are more directly related to the financial structure of the economy. The third group contains parameters that determine the size of the two countries. The fourth group gathers the residual parameters that need to be calibrated jointly to match several

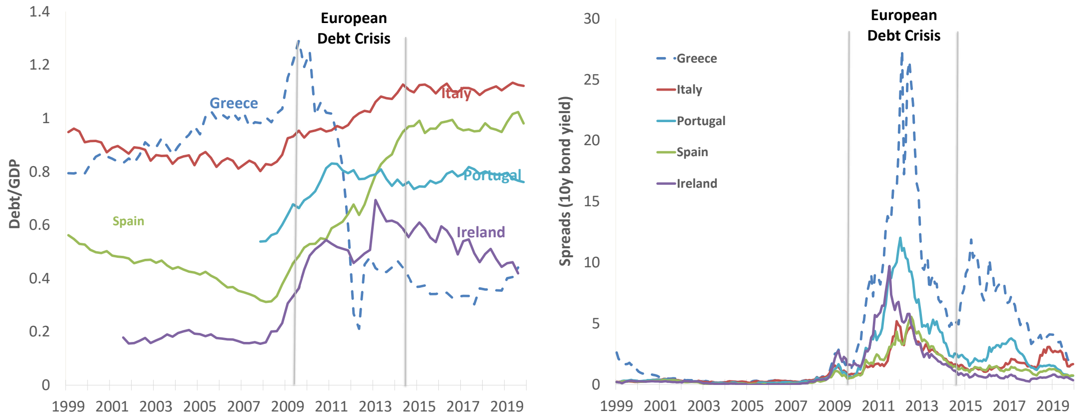

Figure 9: Debt to GDP (left) and Spreads (right) during the European Debt Crisis. Quarterly data with annualized GDP.

moments. The full list of parameters with their calibrated values are in Table 1.

Table 1: Calibrated parameter values.

Parameters	Description	Values
β	Entrepreneur's discount factor	0.930
ν	Elasticity of labor supply	1.000
α	Capital income share in production	0.333
ϕ	Production cost (depreciation)	0.390
ρ^S	Probability bailout in crisis	0.750
ρ_R	Probability commitment to repay	0.903
η	Bargaining power share	0.500
Ψ	Government weight on entrepreneurs	0.111
λ	Government dis-utility from debt	0.141
μ^R	Relative size country R ($\mu^S = 1$)	1.255
z_L, z_H, ρ_z	Distribution productivity	(0.975, 1.025, 0.9)
$B^S_{Low}, B^S_{High}, \rho_B$	Distribution debt country S	(0.295, 0.369, 0.9)

Group 1 (Standard macro parameters). We start with the inter-temporal discount factor that we set to $\beta = 0.93$. This implies a return on equity for entrepreneurs of about 7%. The elasticity of labor supply is set to $\nu = 1$, which is common in macro.

The production function is $y = zm^\alpha l^{1-\alpha}$, where m is financial wealth and l is the input of labor. The capital income share is set to $\alpha = 1/3$. Productivity z follows a symmetric two-state Markov chain with persistent probability of 0.9. The average z is normalized to 1 and the standard deviation is set to 2.5 percent. The process for productivity is within the range of estimations for linearly detrended TFP.

Production also carries the cost ϕm. The parameter ϕ is important for determining the equilibrium interest rate, which we use as a calibration target. However, since the interest rate depends also on other parameters, we calibrate ϕ jointly with the other parameters included in the residual Group 4.

Group 2 (Parameters related to debt and default). Debt issued by the safe country follows a symmetric two state-Markov chain, that is, $B_t^S \in \{B_{Low}^S, B_{High}^S\}$. To calibrate the parameters that characterize this process we use the public debt of Germany. Our measure of public debt is given by general government debt (internal and external), including the debt held by the Central Bank and other public institutions. We include the latter because, indirectly, it also provides liquidity.

The debt-to-GDP ratio of Germany displays relatively slow movements over the sample period 1999-2019. This suggests a high persistence probability for B_t^S, which we set to 0.9. The average ratio over this period is 67% and the standard deviation is 7%. These moments provide us with two targets to calibrate B_{Low}^S and B_{High}^S. Since in the model the average and standard deviation of the debt-to-output ratio depend on other parameters (due to the endogeneity of output), the values of B_{Low}^S and B_{High}^S are determined jointly with other parameters included in Group 4.

The probability ρ^S that country S is able to renegotiate ($\varrho^S = 1$) is set to 0.75. This is consistent with Figure 2 showing that, in more recent times, the frequency of sovereign debt crises without default is about 75 percent. The probability that country R commits to repay—the parameter ρ^R—is calibrated to target a frequency of debt crises of 5%. Although not based on precise empirical observations, the 5% target is within the range of values used in the literature.

In our model, a crisis emerges when country R asks for debt restructuring. Some times this leads to renegotiation (with probability $\rho^S = 0.75$) and some times to straight default (with probability $1 - \rho^S = 0.25$). However, country R does not necessarily ask for restructuring every time it has the option to do so. This implies that the frequency of debt crises depends non-trivially on other parameters. Thus, we need to calibrate ρ^R together with the residual parameters included in Group 4.

For the bargaining parameter η we lack direct information allowing us to pin down its value and we set it to the mid value of 0.5. The sensitivity analysis conducted in Appendix H shows that this parameter is not important for the quantitative results.

The welfare weight assigned to entrepreneurs, Ψ, is important for determining the haircuts and interest rate spreads. In general, the repayment ratio δ increases with the weight assigned to entrepreneurs. Based on this, we set the value of this parameter to target an average spread in crises of 5.46%. This is the GDP-weighted average spread for the GIIPS countries over the period 2009-2013. We use 2009-2013 because the spreads for these countries started to rise with the financial crisis and began to normalized after 2013. Since the spread in the model depends also on other parameters, we calibrate Ψ jointly with the other parameters included in Group 4.

Group 3 (Country size). The relative size of the two countries plays an important role because it determines the contribution of each country to the general equilibrium. This is especially relevant because the choices of borrowing, default, and bailout are made by governments who internalize the general equilibrium effects of their decisions.

If we interpret country S as capturing a perfectly integrated rest of the world, the impact of country R's decisions on the world interest rate and on international macroeconomic spillovers would be negligible. In reality, however, financial integration is not perfect. Even in today's highly globalized world, national markets continue to be characterized by strong regional components. An example is the European banking system. In principle, there is perfect capital mobility within the EURO countries. This should induce banks in different EURO countries to hold similar portfolios investments across member states. The 2012 crisis, however, showed that the foreign holding of Greek sovereign debt was concentrated in few European countries, making their economies more exposed to the debt crisis in Greece.

Whether a bailout is led by the more exposed countries or through a cooperative effort carried out by informal institutions (such as the Paris Club or the London Club) or via international organizations (such as the IMF, ECB, or the European Commission) is secondary. What is important is that countries or regions more directly exposed to the sovereign default have the political interest to influence negotiations.

Based on these considerations, we should interpret country S as representing the regions that are more exposed to the default of the risky country. But this is endogenous in the model, as the level of exposure is determined by portfolio decisions, which in turn depend on the relative size of the two countries.[18] In particular, our model predicts that, given the debt issued by the two countries, the fraction of country R's debt held by foreign residents increases with the relative size of country S. Because of this, we calibrate the relative size μ^R/μ^S to target the share of country R's public debt held by non-resident banks, which we set to 44.5%. This is the GDP-weighted average for the five GIIPS countries over the period 1999-2019. Since the share of foreign ownership also depends on other parameters, we calibrate μ^R/μ^S jointly with the parameters included in Group 4.[19]

Group 4 (Residual parameters calibrated jointly). At this point we are left with seven parameters: (i) the cost per unit of financial wealth, ϕ; (ii) low value of safe debt, B^S_{Low}; (iii) high value of safe debt, B^S_{High}; (iv) probability of commitment to repay, ρ^R; (v) government weight on entrepreneurs, Ψ; (vi) relative size of risky country, μ^R/μ^S; (vii) government dis-utility from public debt, λ. We calibrate these parameters by targeting seven moments: (i) an average interest rate in country S (risk-free rate) of 1.28%; (ii) an average debt-to-output ratio in country S of 67%; (iii) a standard deviation of debt-to-output ratio in country S of 7%; (iv) an average

[18] Notice that only the relative size matters.

[19] In the online appendix we consider an extension of the model that allows for home bias. We then re-calibrate the model to target the portfolio shares of **both** countries and obtain similar results.

frequency of debt crises of 5%; (v) an average interest rate spread conditional on a crisis of 5.46%; (vi) an average share of country R's debt held by foreign residents of 44.5%; (vii) an average debt-to-output ratio in country R of 80%. The empirical and simulated moments are reported in Table 5. The empirical moments are computed using data from Merler and Pisani-Ferry (2012) and the ECB data Warehouse.

Table 2: Calibration moments: data and model.

	Data	**Model**
Real risk-free rate	1.3%	1.3%
Public debt-to-output ratio Country S	67.0%	67.2%
Standard deviation public debt-to-output ratio Country S	7.0%	7.0%
Frequency of crises	5.0%	4.9%
Spread conditional on crisis	5.5%	5.4%
Share of Country R's debt held by Country S	44.5%	44.5%
Public debt-to-output ratio Country R	80.0%	79.8%

Notes: Data on public debt and GDP are "Bruegel database of sovereign bond holdings developed in Merler and Pisani-Ferry (2012)." We use general government debt held domestically and abroad, including debt held by the Central Bank and other public institutions. Data is available for the period 1999-2019. The share of public debt held abroad is computed as 'Non-residents' debt in percentage of total general debt. The real risk-free interest rate is the long-term nominal interest rate in Germany, 10 years maturity (obtained from the ECB Warehouse) minus the inflation rate computed from changes in the German CPI (obtained from FRED). The spread for the risky countries is the GDP-weighted average difference between the interest rates on 10-year bonds for Greece, Ireland, Italy, Portugal and Spain, and the 10-year bonds interest rate for Germany. The 'Spread conditional on crisis' is computed as an average over the 2009-2013 period. To compute the moments in the model, we draw a long sequence of random shocks and simulate the model over 110,000 periods. We discard the simulated variables for the first 10,000 periods and calculate the moments as averages of the remaining 100,000 periods. We compute the spread at the beginning of the period conditional on (i) $\varrho^R = 0$, and (ii) country R chooses to trigger a restructuring episode, either with bailout ($\varrho^S = 1$) or without bailout ($\varrho^S = 0$).

7 Quantitative properties of the model

This section presents the quantitative properties of the calibrated model starting with the optimal policies.

7.1 Optimal policies

Country R's decision to borrow at the end of the period, $B^R_{t+1} = \Phi(\mathbf{s}_t, \mathbf{p}_t)$, is a function of the aggregate states, $\mathbf{s}_t$, and governments' policies $\mathbf{p}_t = \Upsilon(\mathbf{s}_t)$. In the infinite horizon the policy function is stationary so we can omit the time subscript.

The top panels of Figure 10 plot B^R_{t+1} as a function of the current debt, B^R_t, when country R has commitment to repay ($\varrho^R_t = 1$). The commitment to repay affects

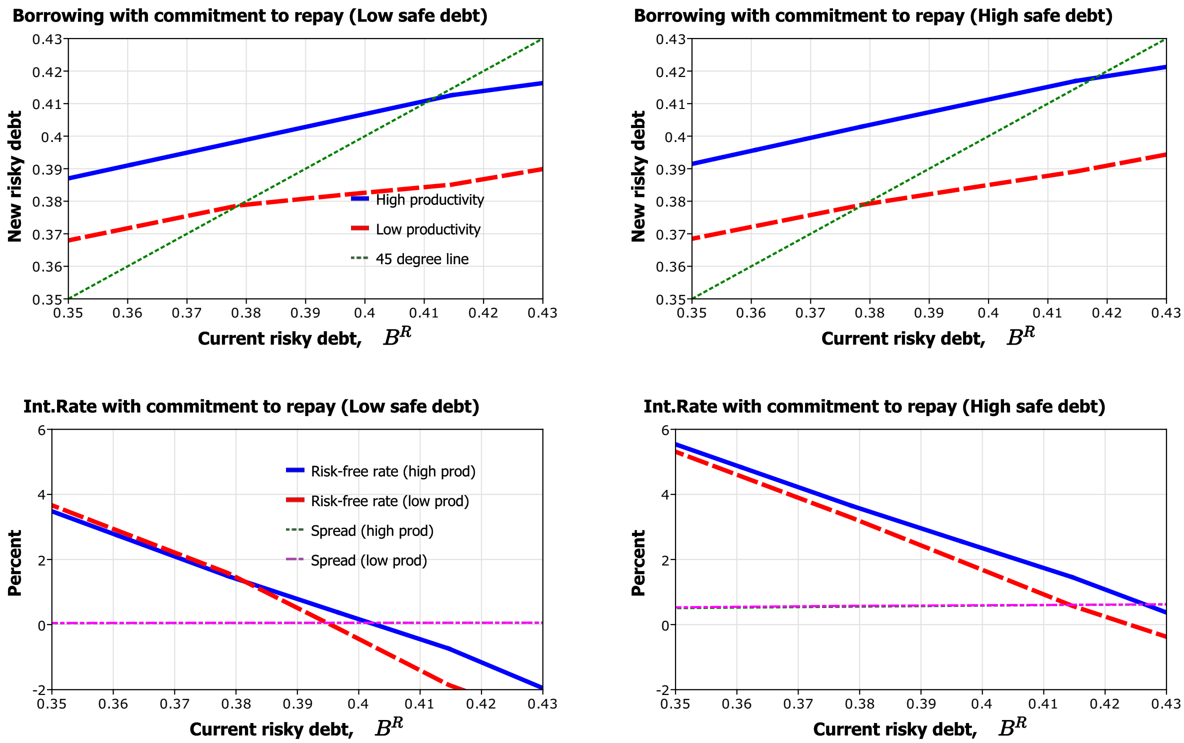

Figure 10: Optimal borrowing, risk-free interest rate and spread when country R has commitment to repay.

borrowing since in the event of a debt crisis country R cannot re-optimize its debt. The figure is plotted for the range of B^R that contains in the ergodic distribution.

Borrowing policies also depend on the productivity of the two countries, z_t^S and z_t^R, and on the debt issued by the safe country, B_t^S and B_{t+1}^S. The optimal borrowing can be plotted for different combinations of these states. Since productivity and the debt of country S can each take two values, we have 16 possibilities. For readability, however, we only plot the states with $z_t^S = z_t^R$ and $B_t^S = B_{t+1}^S$. The left panel is for the case with low safe debt, B_{Low}, whereas the right panel is for B_{High}. Each panel shows the optimal borrowing for low and high productivity, z_{Low} and z_{High}. The 45 degree line visualizes whether next period debt is higher or lower than current debt.

There are two properties worth emphasizing. First, next period debt increases with current debt. At some point, the borrowing curve crosses the 45 degree line. Thus, country R raises its debt when the current value is low and reduces it when it is high. Second, optimal debt increases with productivity. This is intuitive since, indirectly, the debt acts as an accumulated input of production. Because productivity is persistent, higher productivity in the current period signals higher productivity in the future, which increases the incentive to invest. Note that the cyclical dynamics of debt in our model differs from the cyclical properties of a standard sovereign default model such as Arellano (2008). The main role of borrowing in the standard model is to smooth consumption and, therefore, countries borrow more in recessions. In our model, instead, countries borrow more in booms.

The bottom panels of Figure 10 plot the risk-free interest rate, corresponding to the interest rate on the debt of country S, as a function of B_t^R (under commitment to repay). For any level of productivity, the risk-free rate is decreasing in B_t^R. When B_t^R is low, country R chooses $B_{t+1}^R > B_t^R$ (see top panels). To induce entrepreneurs to hold more bonds, the interest rate must rise. On the other hand, when B_t^R is high, country R chooses $B_{t+1}^R < B_t^R$. To induce entrepreneurs to hold less bonds, the interest rate must drop. The dependence of the interest rate on productivity (continuous versus dashed lines) can go in either direction.[20]

The bottom panels of Figure 10 also plot the interest rate spread, given by the difference between the interest rate on B^R (which is defaultable) and the interest rate on B^S (which is always repaid). When B_t^S is low (left panel), the spread is zero because government R finds optimal to repay debt next period even if it has the option to default. When B_t^S is high (right panel), country R will default in the next period if it does have commitment to repay and country S cannot offer a bailout. However, the probability that country R has no commitment to repay, coupled with no bailout, is small (less than 2%). This explains why the spread is positive but small.

The variables plotted in Figure 10 are conditional on the commitment to repay ($\varrho_t^R = 1$). In the calibrated model this happens with a probability of 90.3%. Figure 11, instead, plots the actual and effective debt haircuts when country R can default ($\varrho_t^R = 0$). The actual haircut is $1 - \delta_t$, while the effective haircut is calculated as

$$\text{Effective haircut} = 1 - \delta_t + \tau_t^e / B_t^R.$$

The first four panels of Figure 11 are for the case in which country S has flexibility to negotiate a bailout (which happens with probability $\rho^S = 0.75$) while the remaining panels are for the case it doesnot (with probability $1 - \rho^S = 0.25$).

When the B^S is low, the actual haircut is always zero. Without bailout, country R chooses to fully repay the debt and next period borrowing is chosen as shown in the top panels of Figure 10. If country R has the option to restructure, the effective haircut becomes positive for large values of B^R. When the effective haircut is positive, next period debt equals $B_{t+1}^R = \delta_t B_t^R$.

When B^S is high, debt restructuring becomes the preferred option for country R. As we can see from the panels in the second row, the effective haircut is positive and increasing in B^R. The haircut is positive because the two countries reach an agreement for the full repayment of debt ($\delta_t = 1$) but in exchange for a bailout

[20] This is because productivity affects both the demand and supply of bonds. On the one hand, higher productivity raises the entrepreneurs' demand for bonds since financial wealth becomes more productive. This lowers the interest rate. On the other, country R has more incentive to issue debt for the same reason (financial wealth becomes more productive), which raises the interest rate. These contrasting effects explain why in the figure the interest rate curves cross each other.

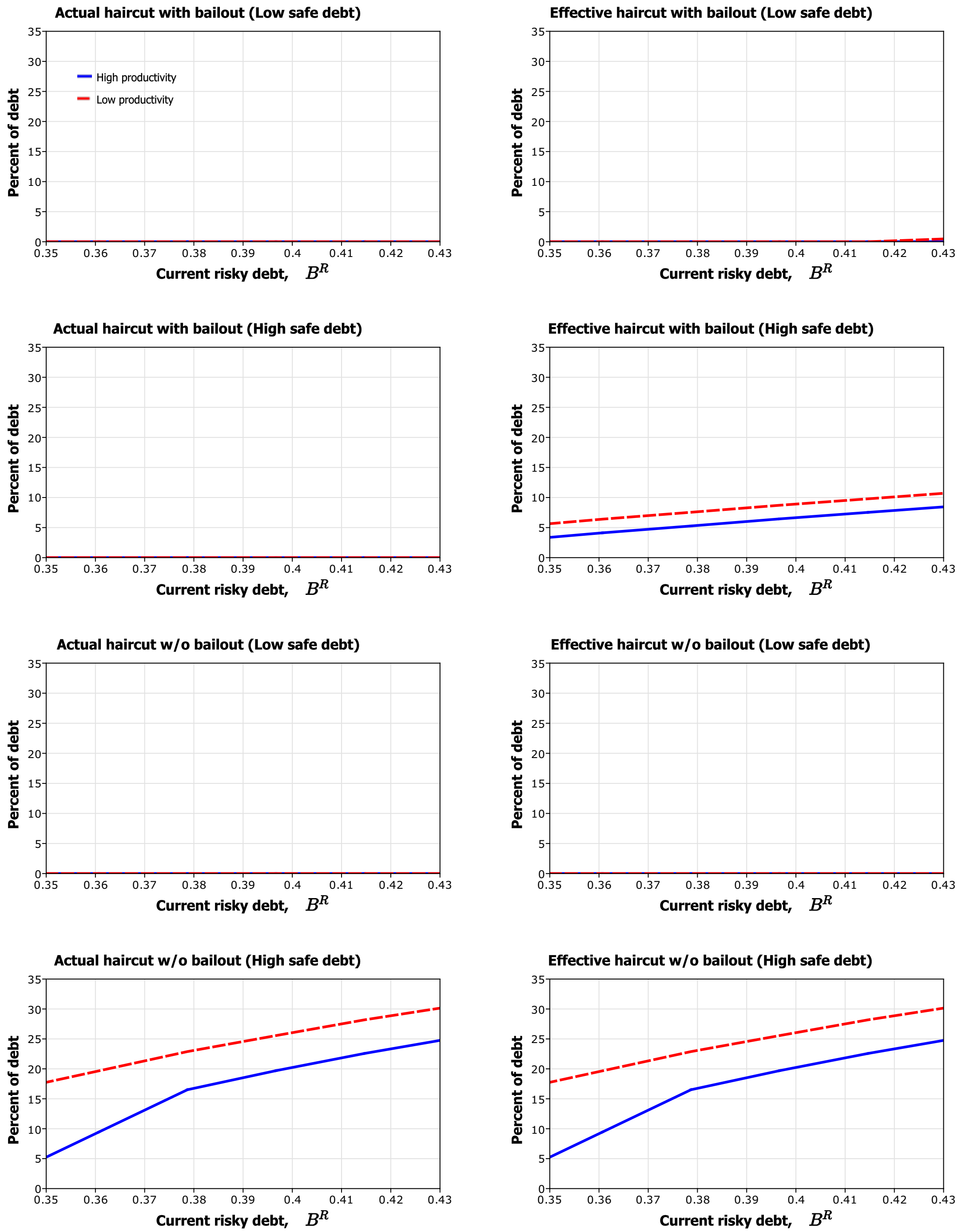

Figure 11: Effective debt haircut when country R has flexibility.

transfer τ_t^e. Although the repayment ratio δ_t negotiated by the two countries could be smaller than 1, it is always 1 (the actual haircut is zero) in the calibrated model.

Next period's debt follows $B^R_{t+1} = \delta_t B^R_t$.

In the event in which country S cannot negotiate a bailout, the effective haircut derives from a lower repayment: $\delta_t < 1$ (and the effective haircut is equal to the actual haircut). As can be seen in the last panels of Figure 11, the haircut increases with B^R. We also observe that the haircut is substantially bigger without a bailout (second and fourth rows).

Why does country R choose to trigger a restructuring episode (inducing a debt crisis) only when the safe debt is high? The intuition is similar to the one described in the one-period model (see Figure 6). When B^S is high, entrepreneurs' wealth is also high. Therefore, its marginal value in production is lower, making default less costly. In the eventuality of renegotiation, the debt crisis materializes in a positive bailout transfer from country S to country R, while creditors receive full repayment $\delta_t = 1$. Without bailout, the debt crisis leads to outright default with repayment $\delta_t < 1$. A similar logic explains why a debt crisis arises for high values of B^R, in addition to the high value of B^S.

Figure 12 plots three variables: (i) the probability of a debt crisis; (ii) the interest rate spread when country R does not have commitment to repay ($\varrho^R_t = 0$), but before knowing whether country S is able to renegotiate a bailout (ϱ^S_t is still unknown); (iii) the interest rate spread when a debt crisis is resolved without bailout ($\varrho^S_t = 0$).

Recall that the commitment to repay, ϱ^R, is observed first and the ability to bail out, ϱ^S, is observed after (see Figure 4). A "debt crisis" arises when country R does not have commitment to repay ($\varrho^R_t = 0$) and chooses to restructure its debt. The probability of a debt crisis is computed before observing ϱ^R_t and ϱ^S_t. The "spread with debt crisis" is the interest rate spread conditional on a debt crisis. This is calculated by solving a hypothetical portfolio problem in which entrepreneurs trade the bonds issued by both countries after observing ϱ^R_t but before observing ϱ^S_t. Denoting by $\tilde{q}^R_t$ and $\tilde{q}^S_t$ the prices for the bonds of country R and country S under this hypothetical scenario, the first order conditions give

$$\frac{\tilde{q}^S_t}{\tilde{q}^R_t} = \frac{\mathbb{E}_{\varrho^S}\left(\frac{1}{m^i_t} \mid \mathbf{s}_t\right)}{\mathbb{E}_{\varrho^S}\left(\frac{\delta_t}{m^i_t} \mid \mathbf{s}_t\right)}.$$

This provides an expression for the gross spread.

Finally, the "spread with default" is computed conditional on a debt crisis after observing $\varrho^S_t = 0$. At this point there is no longer uncertainty about repayment. Therefore, the gross spread is just $1/\delta_t$ (the inverse of the haircut).

Consistently with the previous discussion based on Figure 11, the probability of a debt crisis becomes positive for high values of debt in both countries. The spreads also rise with the debt of country R because the repayment ratio declines. Importantly, debt crises emerge only when the safe debt is high (right panels), that is, when there is high worldwide liquidity. We also observe that the spread is higher in recessions, that is, when productivity is low. When the safe debt is low (left panels), the probability

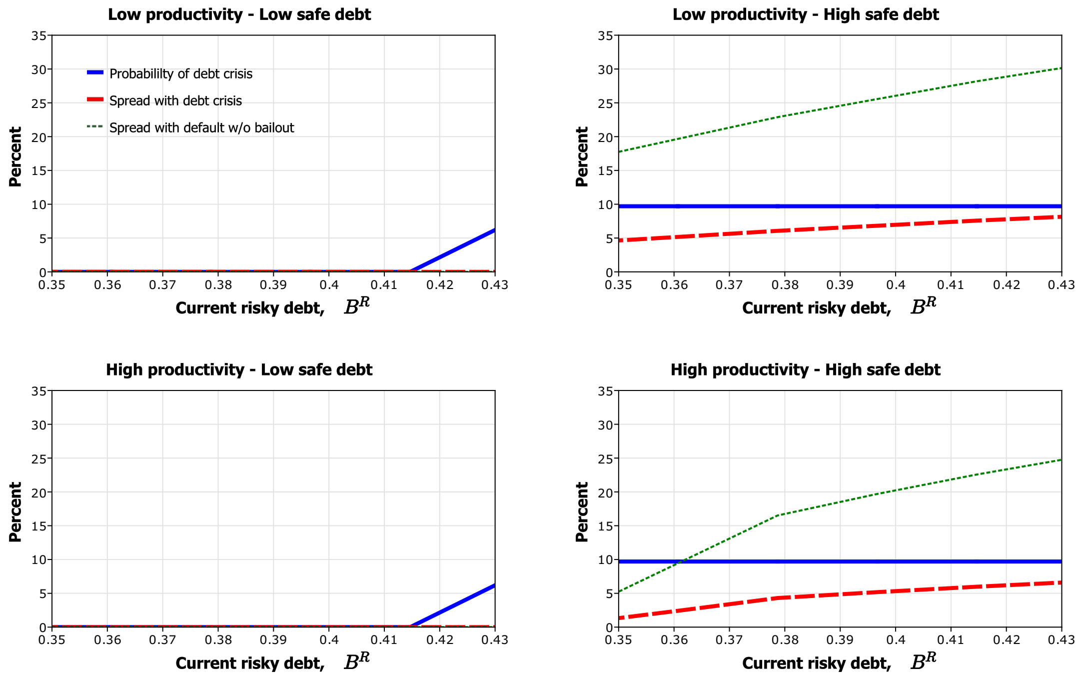

Figure 12: Probability of debt crises and interest rate spreads.

of a debt crisis is basically zero. When the safe debt is high (right-hand-side panels), it becomes optimal for country R to restructure the debt even if country S is unable to offer a bailout. It is possible then for creditors to receive a lower repayment (in absence of a bailout). This explains why the spread becomes positive. The reason the spread increases with B_t^R is because the repayment fraction δ_t declines with B_t^R.

7.2 Anticipated bailouts and borrowing

To show the importance of debt renegotiation for equilibrium borrowing, Figure 13 plots the borrowing policies for the baseline model (top panels) along with the borrowing policies for an alternative economy where there are no bailouts (bottom panels). This is obtained by imposing $\rho^S = 0$ and all debt crises end up in default. Country S, though, could still implement a domestic bailout in the event of default.

Comparing the top panels of Figure 13 (baseline model with $\rho^S = 0.75$) with the bottom panels (alternative model with $\rho^S = 0$), we see that the optimal borrowing without bailout is always smaller than with external bailout. The prospect of renegotiation, which would allow country R to extract transfers from country S, increases the incentive to borrow. We can then infer that the average debt in the environment with bailout will be higher than in the environment without bailout. In addition, since in absence of bailouts debt crises are resolved with default, the repayment δ_t is

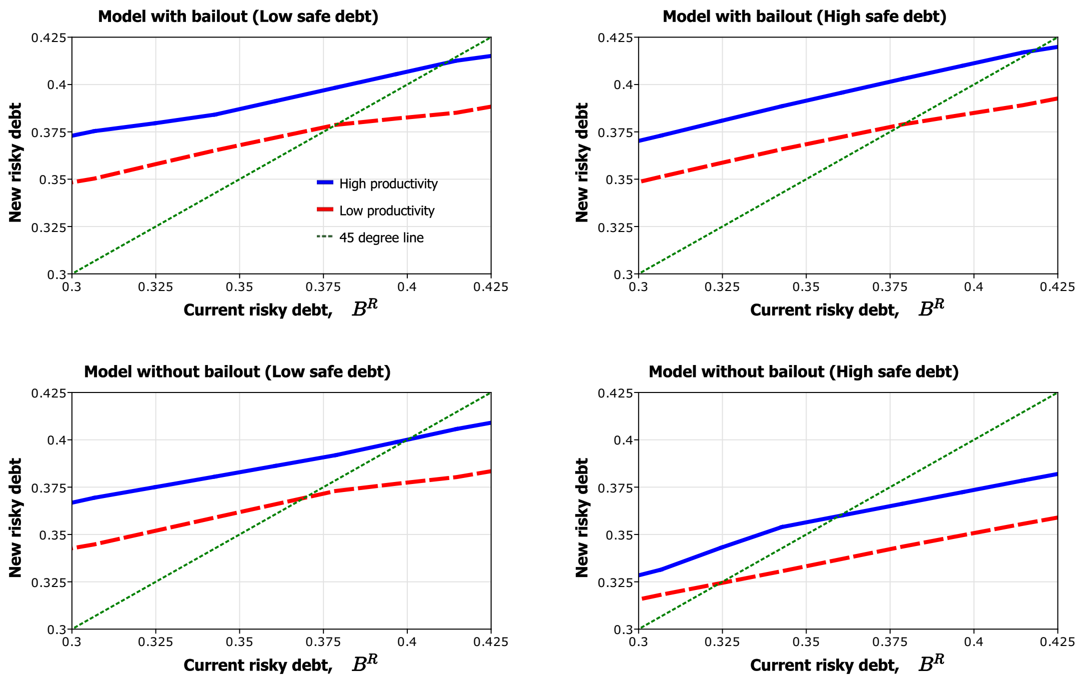

Figure 13: Optimal borrowing in the regime with and without renegotiation.

smaller than 1. The higher frequency of $\delta_t < 1$ in the environment without bailouts also contributes to reduce the average debt in country R.

Table 8 reports the average values of key variables generated by simulating the model with external bailout ($\rho^S = 0.75$) and no bailout ($\rho^S = 0$). As anticipated, the average debt is higher in the baseline model with bailout. The average debt-to-output ratio issued by country R is 79.78% in the economy with bailout and 74.93% in the economy without bailout.

The table also shows that there is a significant difference between the average interest rate on the debt of the safe and risky countries. Conditional on a debt crisis, the spread between the two becomes 19.61 percent. In contrast, it was only 5.44 percent when bailouts were possible (with 75% probability). The difference arises because all crises are resolved with default when $\rho^S = 0$, whereas in the baseline model default can be averted in 75% of debt crises.

The higher average debt in the economy with bailout facilitates higher production. The last two rows of Table 3 show that the average output in the economy with bailout is about 2 percent higher than in the economy without bailout. As we will see in the next subsection, this could explain why the commitment to not to bail out could reduce welfare in both countries.

Table 3: Average statistics over ergodic distribution.

	With bailout ($\rho^S = 0.75$)	**Without bailout** ($\rho^S = 0$)
Debt-to-output ratio country S (in %)	67.24	68.86
Debt-to-output ratio country R (in %)	79.78	74.93
Interest rate country S (in %)	1.34	-0.06
Interest rate country R (in %)	1.64	0.66
Unconditional repayment ratio (in %)	99.75	99.20
External transfers τ^e (% of Y^S conditional on bailout)	5.34	NA
Domestic transfers τ^d (% of Y^S conditional on default)	9.44	6.74
Probability debt crisis (in %)	4.88	5.01
Spread conditional on debt crisis (in %)	5.44	19.61
Spread conditional on default (no bailout) (in %)	26.57	19.61
Output country S	0.494	0.483
Output country R	0.493	0.478

7.3 Ex-ante optimality of bailouts

The equilibrium properties discussed so far show that the expectation of bailouts leads to higher borrowing from country R. The higher borrowing is encouraged by the expectation of transfers from country S. After a debt crisis, that is, after country R asks for the restructuring of the debt, external bailouts are always optimal. Are they optimal also ex-ante?

Suppose that country S commits permanently, at time $t = 0$, not to bail out country R. This is equivalent to the environment with $\rho^S = 0$. Is the welfare of country S higher with commitment? What is the welfare impact on country R?

Denote by $V^i(\mathbf{s}_0; NoBail)$ country i's welfare at time $t = 0$ when country S commits to never bail out country R. Similarly, denote by $V^i(\mathbf{s}_0; Bail)$ country i's welfare when the government of country S can bail out country R with probability $\rho^S = 0.75$ (the baseline model). The welfare gain from committing to never bail out is calculated as the proportional increase in consumption in the allocation with bailout that makes government welfare equal to welfare in the environment without bailout.

Let $\hat{V}^i(\mathbf{s}_0; Bail, \alpha)$ be the welfare of country i when consumption allocation with bailout is increased proportionally by α. The proportional increase is for both entrepreneurs and workers, and in all periods and contingencies. The welfare gain from committing not to bail out is then the value of α that satisfies $V^i(\mathbf{s}_0; NoBail) = \hat{V}^i(\mathbf{s}_0; Bail, \alpha)$. If $\alpha > 0$, the welfare of country i is higher when country S commits to not bail out. If $\alpha < 0$, the commitment of country S is welfare reducing.[21]

Figure 14 plots the welfare gains as functions of the initial debt issued by country R, for different initial states. The plotted range for the initial debt B_0^R combines the ergodic sets of the two environments (with and without bailout). Therefore, independently of the environment, if the initial debt is within the plotted range, it will remain in that range. In the top panels country R is initially committed to repay ($\varrho_1^R = 0$), while in the bottom panels country R does not have initial commitment.

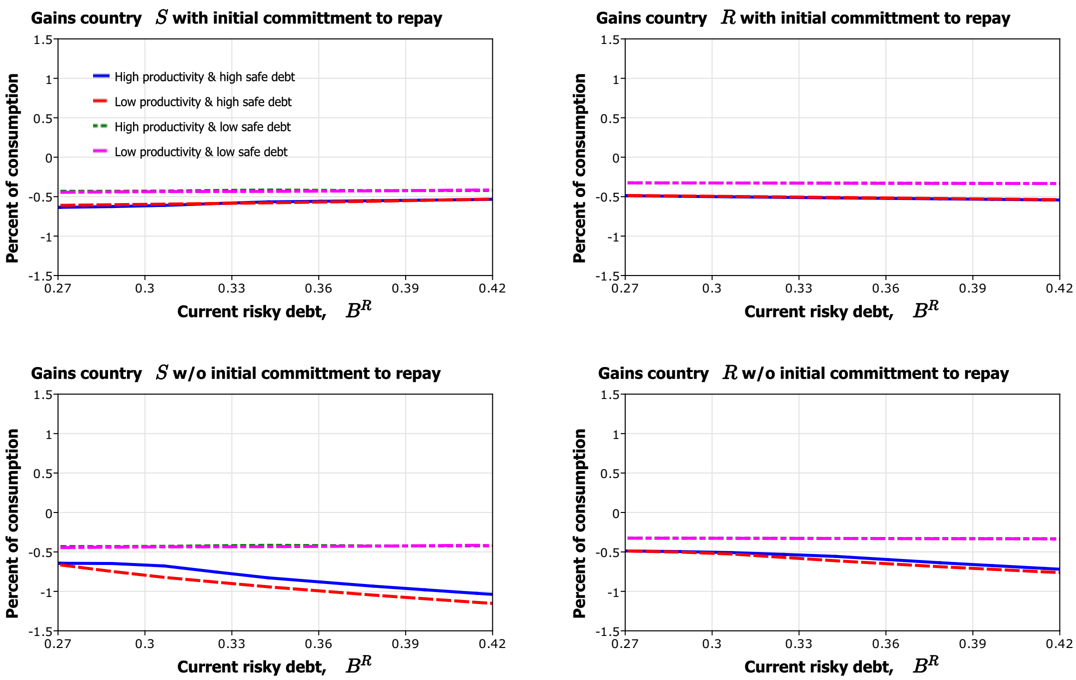

Figure 14: Welfare gains when country S commits to never bail out R.

The welfare gains are always negative independently of the initial states. Therefore, the commitment to not bail out has negative welfare consequences for both countries. The welfare losses change with the initial states when the commitment is decided (time zero). On average, however, they are about 0.5 percent of consumption.

Although the negative welfare effects for country R were to be expected, the finding that country S also incurs welfare losses is more subtle. It derives from the fact that, without external bailouts, the debt issued by country R is lower (see Table

[21]The welfare gain for country $i \in \{S, R\}$ is equal to

$$\alpha = \frac{(1-\beta)\Big[V^i(\mathbf{s}_0; NoBail) - V^i(\mathbf{s}_0; Bail)\Big]}{\Psi + (1-\Psi)(1-\beta)\mathbb{E}_0 \sum_{t=0}^{\infty} c_t^i},$$

where c_t^i is the consumption of workers in the equilibrium allocation with bailout.

3). There are two reasons for this. First, when country R chooses its borrowing, it ignores the benefits that higher debt brings to the other country (since part of the debt will be purchased by entrepreneurs in country S). The second reason is that, in states in which country R has the ability to default, debt repayment is lower. This destroys financial wealth and takes time to rebuilt. Even if country S can avoid the macroeconomic cost of default with a domestic bailout, the future wealth of its own entrepreneurs will be lower in general equilibrium. Also, for country S, the domestic bailout is less beneficial than an external bailout. The anticipation of bailouts, then, increases the incentive to issue debt and could have positive welfare consequences also for country S. The second effect was absent in the one-period model.

The sign and magnitude of the welfare effects depend on parameter values. In Appendix H, we conduct a sensitivity analysis and show that the signs are quite robust to moderate variations in parameter values.

8 Application to the EURO crisis

In this section, we simulate the model over the period 1999-2013 to assess how the European debt crisis was affected by the pre-crisis dynamics. We pay special attention to the dynamics of the safe debt. We choose 2013 as the ending period because, by then, the interest rate spreads for the four European countries that did not default (Ireland, Italy, Portugal and Spain) had declined significantly.

The exogenous sources of fluctuation are the productivity shocks, z_t^S and z_t^R, the debt issued by the safe country, B_t^S, the commitment to repay for the risky country, ϱ_t^R, and the bailout capability of the safe country, ϱ_t^S. The simulation is based on a particular sequence of the exogenous states that reflect the dynamics observed over the period 1999-2013.

- **Productivity, z_t^i:** European countries experienced a decline in productivity starting with the great recession in 2009. Since productivity takes only two values, for both countries we assign z_{High} during the pre-crisis period 1999-2008, and z_{Low} during the crisis period 2009-2013.

- **Commitment to repay, ϱ_t^R:** Until the financial crisis, there were not significant signs of a debt crisis in Europe. Therefore, we set $\varrho_t^R = 1$ for 1999-2008. Around the great recession, however, the spreads for the GIIPS countries started to increase, suggesting that market participants started to worry about repayment. Based on this, we set $\varrho_t^R = 0$ in 2009-2013.

- **Bailout ability, ϱ_t^S:** We consider two cases. In the first case country S is always able to offer a bailout, $\varrho_t^S = 1$. We interpret this case as capturing the experience of Ireland, Italy, Portugal and Spain. These countries experienced debt difficulties with significant increases in interest rate spreads. However, they received financial support and, ultimately, they did not default.

In the second case, country S has the ability to offer a bailout ($\varrho_t^S = 1$) until 2010 but it is unable to offer a bailout during 2011-2013 ($\varrho_t^S = 0$). This aims at capturing the experience of Greece.[22]

- **Safe debt, B_t^S:** Given the central role of Germany as supplier of safe bonds in Europe, we use the German public debt to set the exogenous sequence of B_t^S.

  Figure 15 plots the debt-to-GDP ratio of Germany during 1999-2013 (continuous line) and shows that it increased from slightly below 60% to about 70% during the pre-crisis period. Based on this, we assume that B_t^S increases gradually during 1999-2008 and stays at a high level for the remaining simulation period.[23]

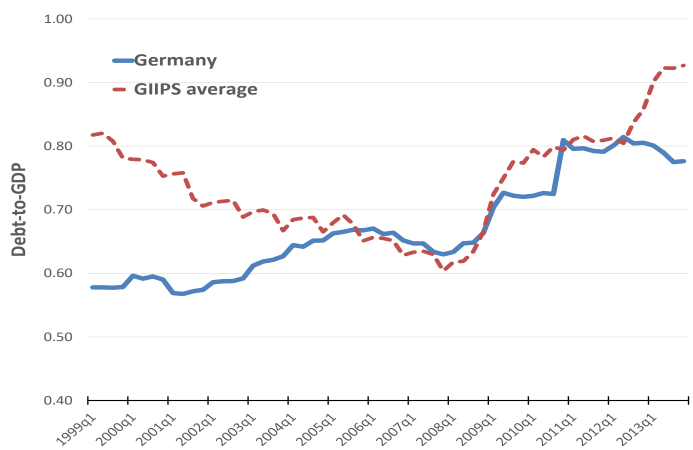

Figure 15: Debt-to-GDP ratio in Germany and weighted average for GIIPS countries.

- **Initial risky debt, B_{1999}^R:** Public debt in the risky country is endogenous and responds to shocks. However, we need to initialize its value in the first simulation year, 1999. We set it to the limiting value of B_t^R when the shocks remain at the 1999 levels.

The goal of the engineered sequence of shocks is to assess whether the model replicates the overall macroeconomic and financial experience of the periphery countries in Europe. The simulation results are shown in Figures 16 and 17. The first figure is for the environment with bailout, while the second figure is for the environment where S cannot bail out R during 2011-2013. These two environments are meant to differentiate the experience of Ireland, Italy, Portugal and Spain (who did receive a

[22]Although the debt of Greece experienced a significant haircut, it also received some financial support which is absent in our particular simulation.

[23]Since in the calibrated model B_t^S can take only two values, we generate a gradual increase by repeating the simulation many times. In each simulation we draw $B_t^S = B_{Low}^S$ with probability p_t, and $B_t^S = B_{High}^S$ with probability $1-p_t$. The expected value of the safe debt is $p_t B_{Low}^S + (1-p_t) B_{High}^S$. We then choose the probability p_t in each year to generate the desired increase in average debt.

bailout and did not default at the end) from the experience of Greece (which instead defaulted). The top panels of each figure illustrate the exogenous variables, while the middle panels plot the responses of the endogenous variables.

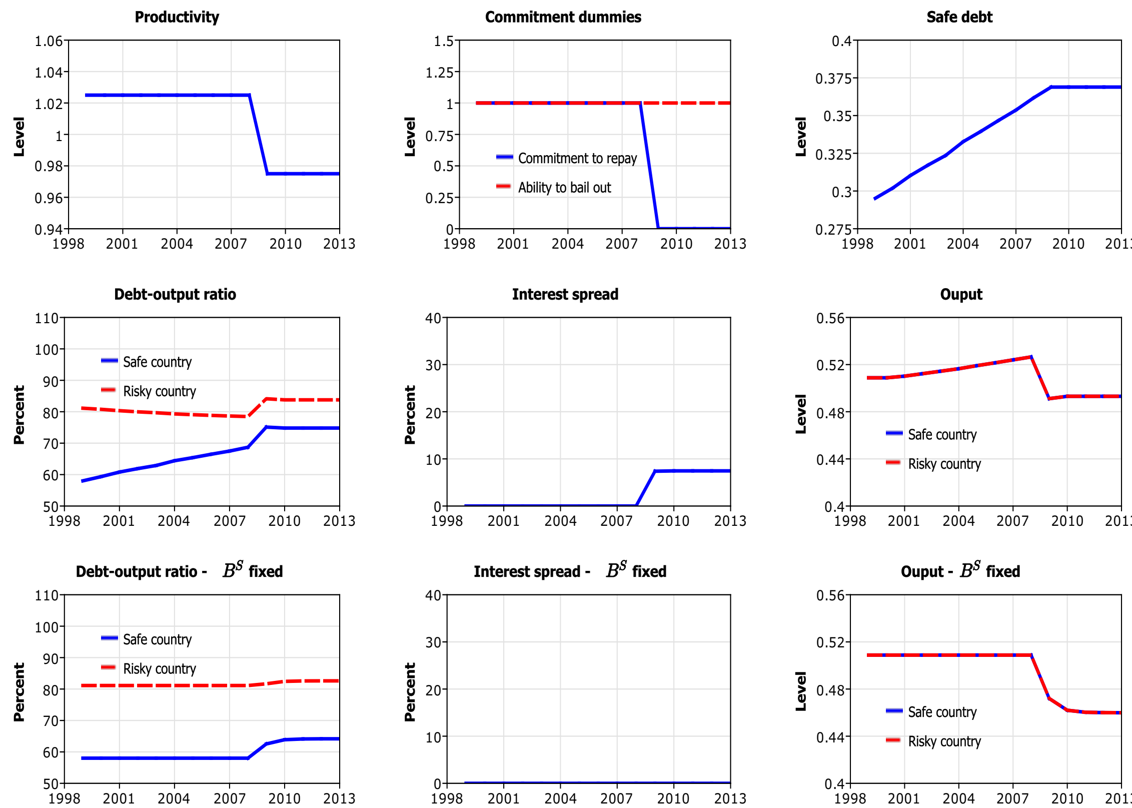

Figure 16: European crisis without default: Simulation responses over 1999-2013.

Debt Crisis without Default (Italy, Ireland, Portugal and Spain): Looking at the panels in the second row of Figure 16, we see that the debt-to-output ratio of country R decreases endogenously in the pre-crisis period. This follows from the optimal response of country R's borrowing to the exogenous increase in the debt of country S: since an increase in B_t^S provides more financial assets for country R entrepreneurs, there is less need to create liquidity by issuing B_t^R. The same pattern is observed in the data for GIIPS countries (dashed line in Figure 15).[24]

[24] Since the debt of country S increases while the debt of country R decreases, entrepreneurs in the risky country hold a larger share of foreign debt relatively to the global debt. This is consistent with the dashed line in the second panel of Figure 1 before the crisis. Entrepreneurs in country S, instead, hold a smaller share of foreign debt relatively to the global debt. This is also consistent with the continuous line in the second panel of Figure 1 before the crisis. The model, however, does not replicate the dynamics shown in the second panel of Figure 1 after the financial crisis.

The arrival of the crisis in 2009 is associated with an increase in the debt-to-output ratio. This is mostly due to the output decline caused by the drop in productivity (the denominator of the ratio). Overall, this captures the dynamics of debt observed in the data for the average of the GIIPS countries as shown in Figure 15. The assumption that the GIIPS countries lost the commitment to repay in 2009 is important for two reasons. First, debt restructuring without default implies that debt remains high in subsequent years. The drop in productivity then increases the debt-to-output ratio. Second, the fact that there is not commitment to repay implies that the market anticipates the possibility of default. This increases the interest rate spread. Even though we simulate the model with a deterministic sequence of shocks, they are not anticipated by the market. As we can see in the middle panel of Figure 16, the interest rate spread raises quickly starting in 2009 (this happens more gradually in the data).

The last panel in the second row of Figure 16 shows that output drops considerably. This is driven almost exclusively by the drop in productivity. Since the debt crisis is resolved with a bailout, there is no drop in the financial wealth held by entrepreneurs.

External Liquidity and the European Debt Crisis: Consider a counterfactual simulation in which we keep the safe debt B_t^S fixed for the entire simulation at its initial value. The corresponding responses are shown in the bottom panels of Figure 16. We can see that the debt-to-output ratio stays constant in both countries before the crisis. With the crisis, the debt-to-output ratio increases, but this is driven by the drop in output (the denominator of the ratio). What is interesting is the response of the interest rate spread. The increase in spread is now much smaller to the point that we would barely talk about a sovereign debt crisis.

The counterfactual simulation highlights the novel mechanism of this paper: when global liquidity increases, the incentive to default also increases. This is especially noteworthy because in the first simulation the debt of country R is actually smaller before the crisis compared to the counterfactual simulation. Despite the fact that country R has a smaller debt, the increase in the interest rate spread is much bigger. Also notice that the increase in safe debt causes an output boom, something that does not happen when B_t^S is constant. In both cases, however, the drop in output during the crisis is essentially driven by the exogenous drop in productivity since the debt crisis is resolved with a bailout and creditors are fully repaid.

Debt Crisis with Default (Greece): We now consider the simulation underlying Figure 17. The only difference between this simulation and the one in Figure 16 is that country S loses the ability to bail out country R in 2011-2013. The simulation tries to capture what happens when a country faces a debt crisis and there is no bailout that prevents default like the experience of Greece.

The dynamics that precedes the crisis is the same as in the previous simulation. However, as the crisis evolves, default leads to a large drop in the debt of country R (see first panel in the second row of the figure). The interest rate spread first increases moderately (since at first country R receives a bailout). But then, in the last three years, the spread jumps close to 40%. This is very similar to the spread dynamics we observed for Greece. In addition, the drop in output for country R is much bigger with default than in the previous case without default (Figure 16). This extra drop in production is fully endogenous. It derives from the fact that default destroys financial assets which then causes a reduction in production. Notice that the drop in output in country R is larger than in country S. This is because country S is able to smooth out the drop in entrepreneurial wealth with a domestic bailout.

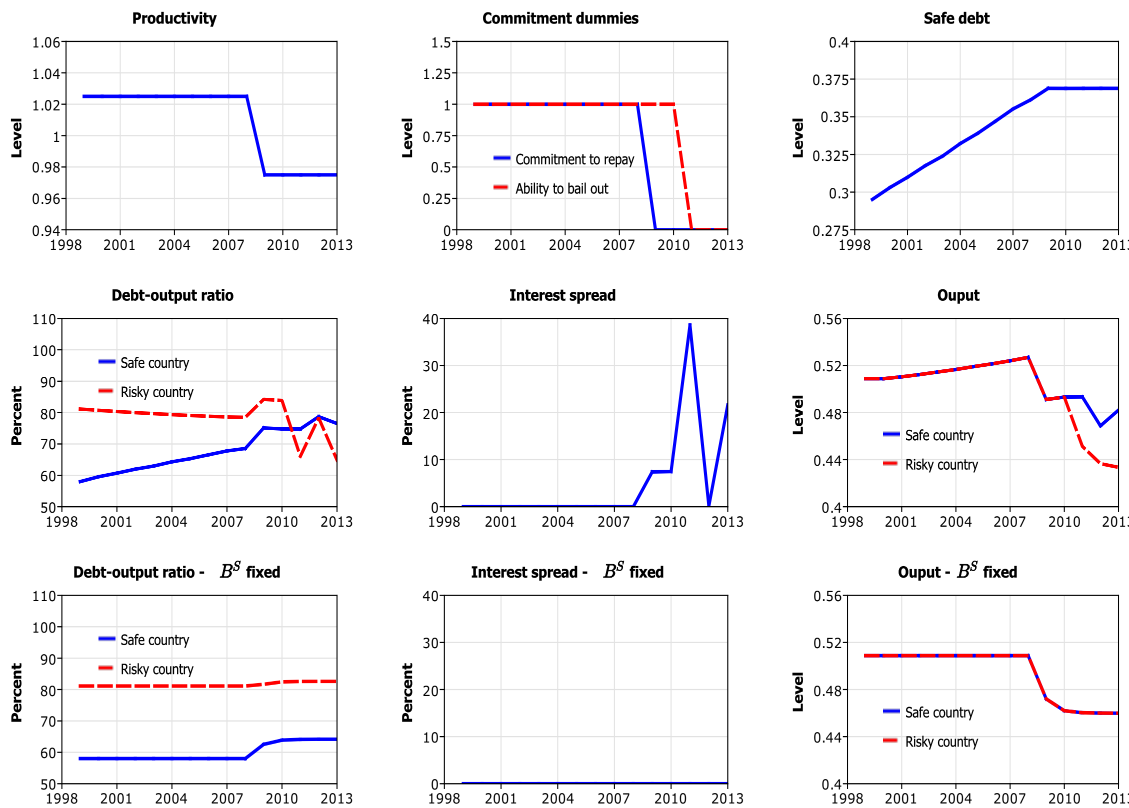

Figure 17: European crisis with default: Simulation responses over 1999-2013.

How would the responses of the economy change if the debt of country S had not increased in the pre-crisis period? The bottom row of Figure 17 shows the responses for the counterfactual simulation in which we keep B_t^S fixed. First, the interest rate spread increases only slightly despite the fact that country R defaults. This is because, upon default, country R finds it optimal to repay a higher share when the safe debt is low. Remarkably, this is the case even though the debt of country R is actually higher in the counterfactual simulation. Second, the drop in output upon default is smaller, especially for the defaulting country R. This also shows the importance of global liquidity for the severity of a financial crisis. Even if in our model a debt crisis can emerge only when $\varrho_t^R = 0$, which is exogenous, the magnitude of the crisis (interest rate spreads and aggregate output) is fully endogenous.[25]

9 Conclusion

Many episodes of sovereign debt crisis are not resolved with outright default. Instead, they involve bailout programs negotiated with creditor countries. With these programs, creditor countries incur direct or indirect costs with the purpose of sustaining higher repayments. But why do creditor countries bail out defaulting countries when they could achieve a similar outcome at home by bailing out domestic banks/businesses? The paper shows that this is because external bailouts could be cheaper than domestic bailouts.

Although external bailouts alleviate the macroeconomic costs of default and are efficient ex-post, are they efficient also ex-ante? Would creditor countries benefit by committing not to bailing out defaulting countries? The paper provides a negative answer to the second question. This is because the anticipation of bailouts leads to higher issuance of debt, which could be beneficial because it corrects for an externality: in absence of international policy coordination, countries issue too little debt as they fail to internalize the benefits of their debt for other countries. Although we showed this result only with a calibrated model, the finding challenges the view that anticipated bailouts have negative welfare consequences because they create moral hazard problems that lead to excessive borrowing.

The paper could be extended in many interesting dimensions. In our baseline model, debt issued by the safe country follows an exogenous process. This captures a situation in which the borrowing decision of the safe country does not react to the debt issued by the risky country. A possible extension is to endogenize the borrowing decisions of the safe country. We conjecture that country S would still choose a level of debt that is lower than what is globally optimal. This is because the government of country S would also ignore the liquidity benefits of issuing its debt for country R and in equilibrium there will be too little debt. Anticipated bailouts, then, could be welfare improving because they would correct for the under-issuance.

Another extension would be to allow both countries to default. This would allow us to analyze the benefits of creating a supra-national institution with the ability to provide bailouts to defaulting countries. Finally, we could add a third country that is risky and analyze cascade defaults in absence of bailouts.

[25] In both simulations illustrated in Figures 16 and 17, the possibility of bailout is fully anticipated (with probability 75%). As we have seen, the anticipation of bailout leads country R to issue more debt. Based on the simulation results shown in Figure 14, we conjecture that the higher debt induced by the anticipation of bailout is welfare improving.

A Proof of Lemma 1

The entrepreneurs' portfolio choice, given policy, solves

$$\max_{\{d_t^i, b_{t+1}^{Ri}, b_{t+1}^{Si}\}} \mathbb{E}_{\mathbf{s}} \left[\sum_{t=1}^{N} \beta^{t-1} \ln(d_t^i) \right]$$

subject to

$$\begin{aligned} m_t^i &= b_t^{Si} + \delta_t b_t^{Ri}, \\ a_t^i &= m_t^i(1-\phi) + \pi_t^i(m_t^i), \\ d_t^i &= a_t^i - q_t^S b_{t+1}^{Si} - q_t^R b_{t+1}^{Ri}. \end{aligned}$$

Profits are defined in eq. (4) and can be rewritten more compactly as $\pi_t^i(m_t^i) = A_t^i m_t^i$, where $A_t^i = \alpha z_t^i \left[\frac{(1-\alpha)z_t^i}{w_t^i}\right]^{\frac{1-\alpha}{\alpha}}$. Using the profit function $\pi_t^i(m_t^i) = A_t^i m_t^i$ updated one period forward, we can write the next period net worth of the entrepreneur as

$$a_{t+1}^i = (1 + A_{t+1}^i - \phi)(b_{t+1}^{Si} + \delta_{t+1} b_{t+1}^{Ri}).$$

The first order conditions with respect to b_{t+1}^{Si} and b_{t+1}^{Ri} are

$$\frac{q_t^S}{d_t^i} = \beta \mathbb{E}_t \left[\frac{(1 + A_{t+1}^i - \phi)}{d_{t+1}^i} \right], \tag{34}$$

$$\frac{q_t^R}{d_t^i} = \beta \mathbb{E}_t \left[\frac{(1 + A_{t+1}^i - \phi)\delta_{t+1}}{d_{t+1}^i} \right]. \tag{35}$$

Let's guess that optimal consumption for the entrepreneur takes the form

$$d_t^i = (1 - \bar{\beta}_t) a_t^i,$$

where $\bar{\beta}_t$ is only a function of time. We also guess the optimal portfolio policies,

$$q_t^S b_{t+1}^{Si} = \theta_t^i \bar{\beta}_t a_t^i \qquad \text{and} \qquad q_t^R b_{t+1}^{Ri} = (1 - \theta_t^i) \bar{\beta}_t a_t^i, \tag{36}$$

where θ_t^i is the fraction of saved wealth, $\bar{\beta}_t a_t^i$, that is used to purchase safe bonds. The remaining fraction $1 - \theta_t^i$ is used to purchase risky bonds.

Multiplying eq. (34) by b_{t+1}^{Si} and eq. (35) by b_{t+1}^{Ri}, adding the resulting expressions, and using the equations that define dividends and next period assets, we obtain

$$\frac{q_t^S b_{t+1}^{Si} + q_t^R b_{t+1}^{Ri}}{(1 - \bar{\beta}_t) a_t^i} = \frac{\beta}{1 - \bar{\beta}_{t+1}}.$$

Replacing our guesses for the portfolio policies in eq. (36) yields

$$\frac{\bar{\beta}_t (1 - \bar{\beta}_{t+1})}{(1 - \bar{\beta}_t)\beta} = 1. \tag{37}$$

Since $\bar{\beta}_N = 0$, we can use eq. (37) iteratively to solve for

$$\bar{\beta}_t = \frac{\beta(1-\beta^{N-t})}{1-\beta^{N-t+1}}.$$

This shows that the saving rate $\bar{\beta}_t$ depends only on time, confirming our guess.

We now replace the guess for the consumption policy into eq. (35), to obtain

$$1 = \mathbb{E}_t \left[\frac{\delta_{t+1}}{\theta_t^i \frac{q_t^R}{q_t^S} + (1-\theta_t^i)\delta_{t+1}} \right]. \tag{38}$$

This condition, together with eq. (37), determines the share of savings invested in safe bonds, the variable θ_t^i. Since entrepreneurs in both countries receives the same repayment ratio on risky bonds (the variable δ_{t+1}) and they pay the same prices q_t^S and q_t^R (due to free mobility of capital), condition (38) implies that θ_t^i must be the same for both countries. In other words, entrepreneurs in both countries choose the same portfolio composition, that is, $\theta_t^S = \theta_t^R = \theta_t$. ■

B Government welfare with $N = 1$

Policies are denoted by the triplet $\mathbf{p} = (\delta, \tau^d, \tau^e)$ and the state of the economy by $\mathbf{s} = (z^S, z^R, B^{SS}, B^{SR}, B^{RS}, B^{RR}, \varrho^R, \varrho^S)$. The objective of government i is

$$\mathcal{V}^i(\mathbf{s}, \mathbf{p}) = \Psi \ln \left(m^i + \pi^i \right) + (1-\Psi) \left\{ w^i \ell^i - \frac{T^i}{\mu^i} - \frac{\nu}{1+\nu} \left(\ell^i \right)^{\frac{1+\nu}{\nu}} \right\}.$$

Replacing equilibrium wages w^i from eq. (9) into eqs. (3) and (4), we obtain closed-form expressions for labor ℓ^i and profits π^i. Substituting these expressions into the equation above and simplifying, we obtain

$$\mathcal{V}^i(\mathbf{s}, \mathbf{p}) = \Psi \ln \left(m^i + \gamma(z^i) \left(m^i \right)^{\tilde{\nu}} \right) + (1-\Psi) \left(\zeta(z^i) \left(m^i \right)^{\tilde{\nu}} - \frac{T^i}{\mu^i} \right), \tag{39}$$

with

$$\begin{aligned} \tilde{\nu} &= \frac{\alpha(1+\nu)}{1+\alpha\nu} \\ \zeta(z^i) &= \frac{1}{1+\nu} \left[(1-\alpha) z^i \right]^{\frac{1+\nu}{1+\alpha\nu}} \\ \gamma(z^i) &= \alpha z^i \left[(1-\alpha) z^i \right]^{\frac{\nu(1-\alpha)}{1+\alpha\nu}}. \end{aligned}$$

Eq. (39) defines $\mathcal{V}^i(\mathbf{s}, \mathbf{p})$ as a function of m^i and T^i. Eqs. (7) and (8) show that m^i and T^i are functions of δ, τ^d and τ^d. Therefore, we can express $\mathcal{V}^i(\mathbf{s}, \mathbf{p})$ analytically as a function of these three policy variables. ■

C Proof of Proposition 3

Replace $T^R = \delta B^R$ and $m^R = b^{SR} + \delta b^{RR}$ into the objective function of the government, eq. (39). The resulting function is strictly concave in δ, which we can show by taking the second derivative of the function,

$$\frac{\partial \mathcal{V}^R(\mathbf{s},\mathbf{p})^2}{\partial^2 \delta} = \Psi (b^{RR})^2 \left[\frac{\gamma(z^R)\tilde{\nu}(\tilde{\nu}-1)(m^R)^{\tilde{\nu}-2} d^R - \left(1+\gamma(z^R)\tilde{\nu}(m^R)^{\tilde{\nu}-1}\right)^2}{(d^R)^2} \right] + (1-\Psi)(b^{RR})^2 \zeta(z^R)\tilde{\nu}(\tilde{\nu}-1)(m^R)^{\tilde{\nu}-2},$$

where $d^i = m^i + \gamma(z^i)(m^i)^{\tilde{\nu}}$. The negativity of the second derivative follows from the assumption that $\nu > 1$, which in turn implies $\tilde{\nu} < 1$.

Because the objective function is strictly concave, the first order condition is necessary and sufficient to characterize the unconstrained optimum. The FOC is

$$\frac{1+\gamma(z^R)\tilde{\nu}\left(m^R\right)^{\tilde{\nu}-1}}{m^R + \gamma(z^R)\left(m^R\right)^{\tilde{\nu}}} + \tilde{\Psi}\zeta(z^R)\tilde{\nu}(m^R)^{\tilde{\nu}-1} = \tilde{\Psi}\frac{B^R}{B^{RR}}, \tag{40}$$

where $\tilde{\Psi} = \frac{1-\Psi}{\Psi}$.

For analytical convenience we use $m^R = b^{SR} + \delta b^{RR}$ as the optimizing variable. Once we have found the optimal value of m^R we go back to δ.

Assuming that the share σ^R is constant, the RHS of (40) becomes

$$RHS = \tilde{\Psi}\frac{1}{\sigma^R},$$

which is independent of m^R. The LHS of (40) is strictly decreasing in m^R, which we can show by differentiation,

$$\frac{\partial LHS}{\partial m^R} = \frac{\gamma(z^R)\tilde{\nu}(\tilde{\nu}-1)(m^R)^{\tilde{\nu}-2}}{d^R} - \frac{\left(1+\gamma(z^R)\tilde{\nu}(m^R)^{\tilde{\nu}-1}\right)^2}{(d^R)^2} + \tilde{\Psi}\zeta(z^R)\tilde{\nu}(\tilde{\nu}-1)(m^R)^{\tilde{\nu}-2}.$$

Using L'Hopital's rule, we can show that $\lim_{m^R\to 0} = \infty$ and $\lim_{m^R\to\infty} = 0$. Hence, the unconstrained solution exists and it is unique. The preferred financial wealth depends only on z^R, and we denote it by $\underline{\underline{m}}^{Ru}(z^R)$. Using $m^R = b^{SR} + \delta b^{RR}$, we get

$$\underline{\underline{\delta}}^u(\mathbf{s}) = \left[\underline{\underline{m}}^{Ru}(z^R) - \frac{\sigma^S}{\mu^R}B^S\right]\frac{\mu^R}{\sigma^R B^R},$$

which is a decreasing function of B^S and B^R until it reaches zero. ∎

D Proof of Proposition 4

Replace $T^S = \delta B^S + \tau^d \mu^S$ and $m^S = b^{SS} + \delta b^{RS} + \tau^d$ into the objective function of the government, eq. (39). The second derivative of the objective function with respect to τ^d is

$$\frac{\partial \mathcal{V}^S(\mathbf{s},\mathbf{p})^2}{\partial^2 \tau^d} = \Psi \left[\frac{\gamma(z^S)\tilde{\nu}(\tilde{\nu}-1)(m^S)^{\tilde{\nu}-2} d^S - \Big(1+\gamma(z^S)\tilde{\nu}(m^S)^{\tilde{\nu}-1}\Big)^2}{(d^S)^2} \right] + (1-\Psi)\zeta(z^S)\tilde{\nu}(\tilde{\nu}-1)(m^S)^{\tilde{\nu}-2}.$$

Because $\tilde{\nu} < 1$, the second derivative is negative and, therefore, the objective function is strictly concave in τ^d.

Because the objective function is strictly concave, the first order condition is necessary and sufficient to characterize the unconstrained optimum. The FOC is,

$$\frac{1+\gamma(z^S)\tilde{\nu}\big(m^S\big)^{\tilde{\nu}-1}}{m^S + \gamma(z^S)\big(m^S\big)^{\tilde{\nu}}} + \tilde{\Psi}\zeta(z^S)\tilde{\nu}(m^S)^{\tilde{\nu}-1} = \tilde{\Psi}. \tag{41}$$

The RHS of (41) is independent of m^S. The LHS is strictly decreasing in m^S, which we can show by differentiating

$$\frac{\partial LHS}{\partial m^S} = \frac{\gamma(z^S)\tilde{\nu}(\tilde{\nu}-1)(m^S)^{\tilde{\nu}-2}}{d^S} - \frac{\Big(1+\gamma(z^S)\tilde{\nu}(m^S)^{\tilde{\nu}-1}\Big)^2}{(d^S)^2} + \tilde{\Psi}\zeta(z^S)\tilde{\nu}(\tilde{\nu}-1)(m^S)^{\tilde{\nu}-2}.$$

Using L'Hopital's rule, $\lim_{m^S \to 0} = \infty$ and $\lim_{m^S \to \infty} = 0$. Hence, the unconstrained solution exists and is unique. The preferred level of financial wealth, denoted by $\underline{\underline{m}}^{Su}(z^S)$, depends only on z^S. Using $m^S = b^{SS} + \delta b^{RS} + \tau^d$, and our assumptions about portfolio composition, we obtain

$$\underline{\underline{\tau}}^d(\mathbf{s}) = \underline{\underline{m}}^{Su}(z^S) - \frac{1}{\mu^S}\Big[(1-\sigma^S)B^S + \delta(1-\sigma^R)B^R\Big].$$

This shows that the optimal domestic transfer decreases in B^R, B^S, and δ. ■

E Proof of Proposition 5

The Nash bargaining problem is

$$\max_{\delta,\tau^d,\tau^e} \Big[\mathcal{V}^S\Big(\mathbf{s},\mathbf{p}\Big) - \underline{\underline{V}}^S(\mathbf{s})\Big]^{\eta} \Big[\mathcal{V}^R\Big(\mathbf{s},\mathbf{p}\Big) - \underline{\underline{V}}^R(\mathbf{s})\Big]^{1-\eta}, \tag{42}$$

subject to $\delta \in [0\,,\,1]$, $\tau^d \in [0\,,\,(1-\delta)B^{RS}/\mu^S]$, and $\tau^e \geq 0$. The function $\mathcal{V}^i$ is defined in eq. (39).

Financial wealth is equal to

$$m^S = b^{SS} + \delta b^{RS} + \tau^d \qquad \text{and} \qquad m^R = b^{SR} + \delta b^{RR},$$

and taxes are

$$T^S = B^S + (\tau^d + \tau^e)\mu^S \qquad \text{and} \qquad T^R = \delta B^R - \tau^e \mu^S.$$

The first order condition with respect to τ^d is

$$\frac{\partial \mathcal{V}^S(\mathbf{s},\mathbf{p})}{\partial m^S} = 1 - \Psi. \tag{43}$$

After some algebraic manipulation, this becomes identical to eq. (41). Hence, the unconstrained interior solution for financial wealth in country S is the same as in the no renegotiation case. Thus, $\underline{m}^{Su}(z^S) = \underline{\underline{m}}^{Su}(z^S)$.

The first order conditions with respect to τ^e and δ are given by eqs. (31) and (32). Eq. (32) can be written as

$$\frac{\partial \mathcal{V}^S(\mathbf{s},\mathbf{p})}{\partial m^S} B^{RS} + \frac{\partial \mathcal{V}^R(\mathbf{s},\mathbf{p})}{\partial m^R} B^{RR} - (1-\Psi) B^R = 0. \tag{44}$$

Substituting eq. (43) and noting that $B^R = B^{RR} + B^{RS}$, we get

$$\frac{\partial \mathcal{V}^R(\mathbf{s},\mathbf{p})}{\partial m^R} = 1 - \Psi.$$

Using the functional form for $\mathcal{V}^R$ and simplifying, we obtain

$$\frac{1 + \gamma(z^R)\tilde{\nu}\big(m^R\big)^{\tilde{\nu}-1}}{m^R + \gamma(z^R)\big(m^R\big)^{\tilde{\nu}}} + \tilde{\Psi}\zeta(z^R)\tilde{\nu}(m^R)^{\tilde{\nu}-1} = \tilde{\Psi}. \tag{45}$$

The LHS is the same as in eq. (45), whereas the the RHS is smaller. Since the LHS is decreasing in m^R and the RHS is independent of it, the solution with renegotiation implies a higher level of preferred financial wealth than in the case without renegotiation. Therefore, $\underline{m}^{Ru}(z^R) > \underline{\underline{m}}^{Ru}(z^R)$.

Using the definitions of m^R and m^S, we derive

$$\begin{aligned} \underline{\delta}^u(\mathbf{s}) &= \left[\underline{m}^{Ru}(z^R) - \frac{\sigma^R}{\mu^R} B^S\right] \frac{\mu^R}{\sigma^R B^R}, \\ \underline{\tau}^d(\mathbf{s}) &= \underline{m}^{Su}(z^S) - \frac{1}{\mu^S}\Big[(1-\sigma^S)B^S + \underline{\delta}^u(\mathbf{s})(1-\sigma^R)B^R\Big]. \end{aligned} \tag{46}$$

Since we have already shown that $\underline{m}^{Su}(z^S) = \underline{\underline{m}}^{Su}(z^S)$ and $\underline{m}^{Ru}(z^R) > \underline{\underline{m}}^{Ru}(z^R)$, the above equations allow us to establish that $\underline{\delta}^u(\mathbf{s}) > \underline{\underline{\delta}}^u(\mathbf{s})$ and $\underline{\tau}^d(\mathbf{s}) < \underline{\underline{\tau}}^d(\mathbf{s})$. ■

F Efficiency of bailouts

We show here that the negotiated repayment ratio δ and domestic transfer τ^d are equal to those chosen by a planner with utility weights Ψ and μ. The planner faces the same participation constraints of the two countries, that is, their welfare cannot be smaller than the welfare they can get under default.

The planner's problem can be written as

$$\max_{\delta,\tau^d,\tau^e} \quad \eta\mathcal{V}^S\Big(\mathbf{s},\mathbf{p}\Big) + (1-\eta)\mathcal{V}^R\Big(\mathbf{s},\mathbf{p}\Big) \tag{47}$$

$$\text{subject to:}$$

$$\mathcal{V}^S\Big(\mathbf{s},\mathbf{p}\Big) \geq \underline{\underline{V}}^S(\mathbf{s}),$$

$$\mathcal{V}^R\Big(\mathbf{s},\mathbf{p}\Big) \geq \underline{\underline{V}}^R(\mathbf{s}),$$

with the function $\mathcal{V}^i$ defined in eq. (39). The problem is also subject to $\delta \in [0\,,\,1]$, $\tau^d \in [0\,,\,(1-\delta)B^{RS}]$, and $\tau^e \geq 0$. However, we first characterize the unconstrained problem which ignores these constraints.

Define $\eta\lambda^S$ the Lagrange multiplier associated with the participation constraint of country S (first constraint) and $(1-\eta)\lambda^R$ the Lagrange multiplier associated with the participation constraint of country R (second constraint). The first order condition with respect to δ is

$$\eta(1+\lambda^S)\frac{\partial\mathcal{V}^S(\mathbf{s},\mathbf{p})}{\partial m^S}B^{RS} + (1-\eta)(1+\lambda^R)\frac{\partial\mathcal{V}^R(\mathbf{s},\mathbf{p})}{\partial m^R}B^{RR} = (1-\eta)(1+\lambda^R)(1-\Psi)B^R.$$

The first order condition with respect to τ^e returns

$$\eta(1+\lambda^S) = (1-\eta)(1+\lambda^R).$$

Substituting in the first order condition for δ we obtain

$$\frac{\partial\mathcal{V}^S(\mathbf{s},\mathbf{p})}{\partial m^S}B^{RS} + \frac{\partial\mathcal{V}^R(\mathbf{s},\mathbf{p})}{\partial m^R}B^{RR} = (1-\Psi)B^R. \tag{48}$$

Eq. (48) is exactly the same as condition (44) for the bargaining problem.

We show next that the first order condition for τ^d is also the same. In the planner's problem (47) the first order condition with respect to τ^d is

$$\frac{\partial\mathcal{V}^S(\mathbf{s},\mathbf{p})}{\partial m^S} = 1-\Psi. \tag{49}$$

The same condition can be derived by differentiating the bargaining problem (42).

Notice that the first order conditions (48) and (49) do not depend on τ^e. Since these conditions are identical and they do not depend on τ^e, the unconstrained solutions will have the same δ and τ^d. The external transfer τ^e could differ in the bargaining solution and in the planner's solution. However, the transfer does not affect the total surplus but only its distribution between the two countries.

So far we have shown that the unconstrained solutions are equivalent for the bargaining problem (42) and for the planner's problem (47). Since both problems are subject to the same constraints, also the constrained solutions will be equivalent, except in the distribution of the surplus through τ^e. ■

G Proof of Proposition 7

When $\sigma^S = \sigma^R = 1/2$, entrepreneurs' wealth in the two countries is

$$m^S = \frac{1}{2}(B^S + \delta B^R) + \tau^d, \tag{50}$$

$$m^R = \frac{1}{2}(B^S + \delta B^R). \tag{51}$$

The proofs of Propositions 3, 4, 5 in Appendices C, D, E, provide the conditions for the optimal entrepreneurs' wealth m in the environments with default and with renegotiation. In the environment with default, the desired values of m for the two countries are given by conditions (40) and (41). With $\sigma^S = \sigma^R = 1/2$ and $z^S = z^R = z$, these conditions can be written as

$$\begin{aligned} F(m^S) &= 1, \\ F(m^R) &= 2, \end{aligned}$$

where the function $F(.)$ is strictly decreasing in m^i. We denote by $\underline{\underline{m}}^S$ and $\underline{\underline{m}}^R$ the values of entrepreneurs' wealth that solve these two conditions. The decreasing property of the function $F(.)$ implies that $\underline{\underline{m}}^S > \underline{\underline{m}}^R$.

In the environment with renegotiation, the optimal values of entrepreneurs' wealth for the two countries are both determined by condition (45). With $\sigma^S = \sigma^R = 1/2$ and $z^S = z^R = z$, the conditions for the two countries can be written as

$$\begin{aligned} F(m^S) &= 1, \\ F(m^R) &= 1, \end{aligned}$$

and the solutions are denoted by $\underline{m}^S$ and $\underline{m}^R$. Since the conditions are identical for the two countries, we have that $\underline{m}^S = \underline{m}^R$. We also notice that the optimal conditions for country S in the environments with default and with renegotiation are also identical. Therefore, $\underline{\underline{m}}^S = \underline{m}^S$. In summary, we have that

$$\underline{\underline{m}}^R < \underline{\underline{m}}^S = \underline{m}^S = \underline{m}^R$$

In words, under default, country R prefers a lower m than country S. The optimal m for country S is the same under default and under renegotiation. Finally, with renegotiation the optimal m is the same for the two countries. Once we have the optimal values of m^i, the definitions of entrepreneurs' wealth provided in eqs. (50) and (51) allow us to determine δ and τ^d.

We are now ready to prove that with renegotiation $\tau^d = 0$. This follows from the fact that $\underline{m}^R = \underline{m}^S$. Eqs. (50) and (51) then imply that τ^d must be zero.

The next step is to prove that, for given $B^R > 0$, there exists $\underline{B}^S < \bar{B}^S$ such that renegotiation takes place if and only if $B^S \in (\underline{B}^S, \bar{B}^S)$.

Define $\bar{B}^S = 2\underline{\underline{m}}^S$. For $B^S \geq \bar{B}^S$, we have that $m^S \geq \underline{\underline{m}}^S$ even if the repayment ratio and the domestic transfer are both zero. In fact, we can see from (50) that, if $\delta = \tau^d = 0$, then $m^S = (1/2)B^S$. If $B^S \geq \bar{B}^S$, then $m^S \geq \underline{\underline{m}}^S$. Country S would like to reduce entrepreneurs' wealth but that would requires either $\delta < 0$ and/or $\tau^d < 0$, which is not feasible. Therefore, renegotiation can only take place if $B^S < \bar{B}^S = 2\underline{\underline{m}}^S$.

Now define $\underline{B}^S = 2\underline{\underline{m}}^R - B^R$. As long as $B^R > 0$, we have that $\underline{B}^S$ is smaller than $\bar{B}^S = 2\underline{\underline{m}}^S$. If $B^S \leq \underline{B}^S$, the (constrained) optimal repayment ratio under default is $\underline{\underline{\delta}}^u = 1$. To show this, consider eq. (51). With $\delta = 1$ we have that $m^R = (1/2)(B^S + B^R)$. If $B^S \leq \underline{B}^S$ then $m^R \leq \underline{\underline{m}}^R$. This means that country R would like to increase m but to do this it has to choose a repayment $\delta > 1$ which is not feasible. Therefore, the constrained repayment ratio under default is $\underline{\underline{\delta}} = 1$. Then, given full repayment, there cannot be renegotiation.

So far we have shown that renegotiation can take place only if $B^S \in (\underline{B}^S, \bar{B}^S)$, where $\underline{B}^S = 2\underline{\underline{m}}^R - B^R$ and $\bar{B}^S = 2\underline{\underline{m}}^S$. However, we also need to show that, if $B^S \in (\underline{B}^S, \bar{B}^S)$, then renegotiation will actually take place.

If $B^S > \underline{B}^S$, we have that $m^R > \underline{\underline{m}}^R$ when $\delta = 1$. This implies that, under default, country R would like to repay a fraction of the debt smaller than 1, that is, $\underline{\underline{\delta}} < 1$. Since $\underline{\underline{m}}^R < \underline{\underline{m}}^S$ and $\underline{\underline{\delta}} < 1$, renegotiation will take place. With renegotiation, then $\underline{m}^R = \underline{m}^S$, which requires a higher repayment ratio.

To complete the proof we have to show that, upon renegotiation, the external transfer is positive, that is, $\tau^e > 0$, and both countries gain. Since $\underline{m}^R \neq \underline{\underline{m}}^R$, the first order condition $F(m^R) = 2$ is no longer satisfied for country R. In absence of a transfer, then, the welfare of country R would be lower than with default. To convince country R to cooperate, country S has to give a positive transfer τ^e.

The final step of the proof is to show that both countries gain from renegotiating. Let's notice first that, when $B^S \in (\underline{B}^S, \bar{B}^S)$, the net surplus from renegotiation is positive. The transfer τ^e will then determine how the net surplus would be split between the two countries. In fact, after fixing $\underline{\delta}$, τ^e is a pure transfer from workers in country S to workers in country R. Since workers have linear utility and the governments of the two countries assign the same weights $1 - \Psi$ to the utility of workers, τ^e does not affect the total surplus. We can then find a value of τ^e that makes the welfare with renegotiation bigger for both countries. The unique value τ^e is such that country S gets a fraction η of the net surplus and country R gets the remaining fraction $1 - \eta$. ■

H Sensitivity analysis

In Section 5, we characterized the properties of the one-period model analytically. However, to establish the ex-ante welfare implications of bailouts we need more than one period so that the debt issued by the risky country is endogenous. Unfortunately, even the simplest extension with only two periods makes the analytical characterization unfeasible. Furthermore, in a two-period model we still have that the initial states, which are important for the welfare results, are somewhat arbitrary since they are not determined within the model. For these reasons, we characterized the dynamic properties quantitatively in the infinite horizon model. With an infinite horizon we can derive the ergodic distribution of the states and compute the welfare gains starting from any of the ergodic states. This is what we did in Subsection 7.3.

One limitation of the quantitative analysis is that the results are specific to the particular calibration. Even if the calibrated model predicts that the commitment to not bail out reduces welfare for both countries, we cannot claim that this is true for any choice of parameters. Therefore, in this section we conduct a sensitivity analysis to shed some light on the robustness of our results.

Table 4 reports average statistics for some of the variables of interest, after changing one of the parameters. For each parameter we consider two values, one below and one above the baseline calibration. The baseline parameter, reported in Table 1, is around the middle of the two values considered for the sensitivity. The corresponding baseline statistics for the variables of interest are in Table 8.

The first parameter we change is the relative size of the risky country, μ^R (with μ^S normalized to 1). A larger size of country R makes commitment not to renegotiate less costly for both countries. This is because the incentive to default is lower and, therefore, the gains from bailout become smaller.

The dis-utility λ decreases the value of bailouts for both countries. To understand why, we should first notice that a higher λ lowers the average debt B^R. When the risky debt is smaller, country R has less incentive to default and there is less need of bailouts. Thus, the welfare losses from committing not to bail out are smaller.

The third parameter we change is the weight Ψ that the government assigns to entrepreneurs. With a higher Ψ, the government gives more value to the utility of entrepreneurs. Since default is costly for entrepreneurs (they get repaid less) while it could be beneficial for workers (they pay less taxes), a higher Ψ reduces the incentive of country R to default. This raises the average debt and makes the economy with and without bailout more similar, which explains why the losses of commitment not to bailout become smaller for country R. Country S still loses by committing not to bail out. However, if we keep increasing Ψ, we reach a point in which country R never defaults. In that case bailouts become irrelevant for welfare.

When Ψ is small, default is not very costly since governments assign low weight to those who lose from default (entrepreneurs). This implies that the renegotiation surplus is not large. Therefore, country S does not benefit much from renegotiation. The debt of country R drops in absence of bailouts, which reduces welfare. However,

Table 4: Sensitivity analysis - Averages over ergodic distribution.

	$\mu^R = 1.0$		$\mu^R = 1.5$		$\lambda = 0.12$		$\lambda = 0.16$	
	Bail	**NoBail**	**Bail**	**NoBail**	**Bail**	**NoBail**	**Bail**	**NoBail**
B^S/Y^S	67.81	69.89	66.69	68.11	65.22	66.90	68.91	70.98
B^R/Y^R	79.36	72.22	80.38	76.62	85.25	80.34	75.10	69.83
B^{RS}/B^R	50.17	50.64	40.06	40.23	44.56	44.87	44.45	44.67
r^S	0.90	-0.94	1.78	0.69	2.54	1.12	0.25	-1.10
r^R	1.37	0.61	1.97	1.15	2.87	2.15	0.53	-0.40
Prob. crisis	4.96	4.98	5.03	4.97	4.94	4.90	4.96	5.14
Spread in crises	8.40	38.12	3.55	9.59	5.91	22.57	5.16	15.25
Spread in default	45.75	38.12	15.97	9.59	28.98	22.57	24.62	15.25
Gain country S	-0.57%		-0.52%		-0.51%		-0.48%	
Gain country R	-0.75%		-0.24%		-0.48%		-0.39%	

	$\Psi = 0.06$		$\Psi = 0.16$		$\eta = 0.4$		$\eta = 0.6$	
	Bail	**NoBail**	**Bail**	**NoBail**	**Bail**	**NoBail**	**Bail**	**NoBail**
B^S/Y^S	71.45	74.15	63.01	63.91	67.02	68.85	67.31	68.85
B^R/Y^R	68.12	60.67	91.93	89.59	80.03	74.97	79.63	74.97
B^{RS}/B^R	44.65	45.27	44.38	44.36	44.51	44.75	44.51	44.75
r^S	-1.58	-3.96	4.17	3.67	1.36	-0.04	1.30	-0.04
r^R	-0.86	-1.41	4.21	3.73	1.66	0.86	1.61	0.86
Prob. crisis	5.03	5.01	4.23	3.77	4.97	4.96	5.08	4.96
Spread in crises	12.11	75.78	0.82	1.44	5.51	19.79	5.47	19.79
Spread in default	80.36	75.29	3.33	1.44	26.85	19.79	26.54	19.79
Gain country S	0.56%		-0.53%		-0.50%		-0.50%	
Gain country R	-1.08%		-0.05%		-0.45%		-0.41%	

	$\rho^R = 0.85$		$\rho^R = 0.95$		$B^S = 0.299$		$B^S = 0.365$	
	Bail	**NoBail**	**Bail**	**NoBail**	**Bail**	**NoBail**	**Bail**	**NoBail**
B^S/Y^S	66.93	69.85	67.36	68.15	61.40	62.84	73.00	75.14
B^R/Y^R	80.36	72.21	79.30	77.00	82.73	78.25	76.77	71.11
B^{RS}/B^R	44.56	44.83	44.44	44.60	44.49	44.69	44.53	44.81
r^S	1.37	-0.73	1.27	0.58	0.80	-0.47	1.81	0.32
r^R	1.85	0.28	1.42	1.11	1.04	0.16	2.19	1.44
Prob. crisis	7.64	7.69	2.54	2.60	5.19	5.14	4.87	5.15
Spread in crises	5.58	14.23	5.35	22.75	4.39	13.27	6.62	25.50
Spread in default	27.05	14.23	26.03	22.75	20.28	13.27	33.20	25.50
Gain country S	-0.88%		-0.23%		-0.62%		-0.40%	
Gain country R	-0.68%		-0.22%		-0.34%		-0.53%	

it also lowers the interest rate. Since country S has more debt than country R when Ψ is small, the lower interest rate implies that country S has a lower net interest burden on its debt (difference between the interests paid to country R minus the interests that domestic entrepreneurs earn on country R's debt). This explains why country S now benefits from committing not to bail out. The opposite is true for country R.

The fourth parameter we change is η, the bargaining power of the safe country. Comparing the case with $\eta = 0.4$ and $\eta = 0.6$, we can see that this parameter is not important for the results. Although η determines the split of the renegotiation surplus, it does not affect significantly the equilibrium debt and the interest spreads.

As expected, country R looses a little when η is higher but not that much.

Next we change the probability of commitment to repay for country R, the parameter ρ^R. When the commitment probability to repay is bigger, debt crises arise less frequently. Therefore, there is less need of bailouts. This explains why the welfare losses from commitment are lower for both countries. In the limiting case with $\varrho^R = 1$, commitment not to bailout is irrelevant because debt crises never arise.

The debt issued by the safe country, B^S, plays an important role in the model. Since B^S is exogenous, we can conduct a sensitivity analysis with respect to the average value of the safe debt. We consider 10 percent deviations from the average value in the baseline calibration. Higher values of B^S lower the benefit of bailouts for country S but increases the benefits for country R (making the losses from commitment not to bail out smaller). As B^S increases, country R reduces the issuance of its debt and, therefore, its portfolio becomes safer. This implies that it is not very costly for country R to default and, therefore, it repays less in a debt crisis. Bailouts then, allow for more debt stability which is ex-ante beneficial. At the same time, however, since it is less costly to default for country R, the renegotiation surplus is smaller which implies lower gains for country S.

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ONLINE APPENDIX

A Description of numerical solution

The model has N periods and the timing within a period is shown in Figure 1.

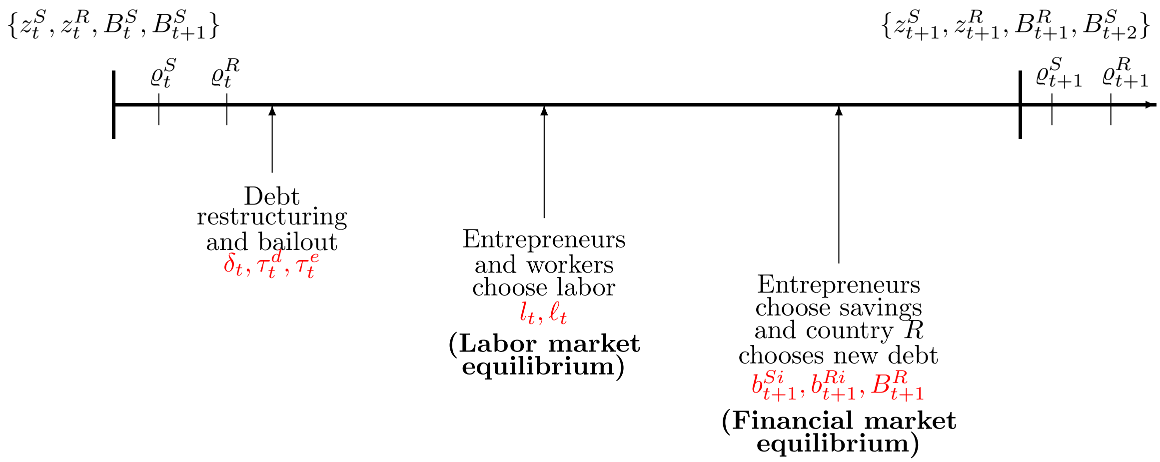

Figure 1: Timing within a period.

The goal of the numerical procedure is to find the policies

$$\begin{aligned} \mathbf{p}_t &= \Upsilon_t(\mathbf{s}_t), \\ B_{t+1}^R &= \Phi_t(\mathbf{s}_t, \mathbf{p}_t), \end{aligned}$$

where $\mathbf{p}_t = (\delta_t, \tau_t^d, \tau_t^e)$ and $\mathbf{s}_t = (z_t^S, z_t^R, B_t^{SS}, B_t^{SR}, B_t^{RS}, B_t^{RR}, B_{t+1}^S, \varrho_t^S, \varrho_t^R)$ and $t = 1, .., N$.

We first use the property of the model stated in Lemma 1 to reduce the sufficient set of state variables. Since entrepreneurs in the two countries choose the same portfolio composition, we replace the four states $B_t^{SS}, B_t^{SR}, B_t^{RS}, B_t^{RR}$ with ω_t, B_t^S, B_t^R, where ω_t is the share of entrepreneurs wealth in country S, that is, $\omega_t = (B_t^{SS} + B_t^{RS})/(B_t^S + B_t^R)$. Once we know B_t^S, B_t^R, ω_t, we can calculate $B_t^{SS}, B_t^{SR}, B_t^{RS}, B_t^{RR}$.

Second, since ϱ^S is irrelevant when $\varrho^R = 1$, we define the combined state

$$\varrho = \begin{cases} I, & \text{if } \varrho^R = 1 \quad \text{(Repayment)}, \\ II, & \text{if } \varrho^R = 0 \text{ and } \varrho^S = 0 \quad \text{(Repayment or default)}, \\ III, & \text{if } \varrho^R = 0 \text{ and } \varrho^S = 1 \quad \text{(Repayment or external bailout)}. \end{cases}$$

In the first state, the debt is always repaid in full. In the second state, the risky country could restructure the debt and, if it chooses to do so, country S cannot renegotiate, in which country R defaults. In the third state, the risky country could restructure the debt and, if it chooses to do so, country S has the ability to renegotiate (through an external bailout). We can then reduce the set of state variables to

$$\mathbf{s}_t = (z_t^S, z_t^R, \omega_t, B_t^S, B_t^R, B_{t+1}^S, \varrho_t).$$

The government's value in country S and country R are given, respectively, by

$$\begin{aligned} V_t^S(\mathbf{s}_t) &= \Psi \ln\left((1-\bar{\beta}_t)a_t^S\right) \\ &+ (1-\Psi)\left[w_t^S \ell_t^S - \frac{\nu}{1+\nu}\left(\ell_t^S\right)^{\frac{1+\nu}{\nu}} - \frac{B_t^S}{\mu^S} - \tau_t^d - \tau_t^e + \frac{q_t^S B_{t+1}^S}{\mu^S}\right] - \lambda \frac{B_{t+1}^S}{\mu^S} \\ &+ \beta\, \mathbb{E}_t V_{t+1}^S(\mathbf{s}_{t+1}), \end{aligned}$$

$$\begin{aligned} V_t^R(\mathbf{s}_t) &= \Psi \ln\left((1-\bar{\beta}_t)a_t^R\right) \\ &+ (1-\Psi)\left[w_t^R \ell_t^R - \frac{\nu}{1+\nu}\left(\ell_t^R\right)^{\frac{1+\nu}{\nu}} - \frac{\delta_t B_t^R}{\mu^R} + \frac{\tau_t^e \mu^S}{\mu^R} + \frac{q_t^R B_{t+1}^R}{\mu^R}\right] - \lambda \frac{B_{t+1}^R}{\mu^R} \\ &+ \beta\, \mathbb{E}_t V_{t+1}^R(\mathbf{s}_{t+1}), \end{aligned}$$

with all variables at their equilibrium values. Notice that we have used Lemma 1 to substitute entrepreneurs' consumption $d_t^i = (1-\bar{\beta}_t)a_t^i$.

For computational purposes, we rewrite the two value functions as

$$\begin{aligned} V_t^S(\mathbf{s}_t) &= (1-\Psi)\left[w^S \ell_t^S - \frac{\nu}{1+\nu}\left(\ell_t^S\right)^{\frac{1+\nu}{\nu}} - \frac{B_t^S}{\mu^S} - \tau_t^d - \tau_t^e\right] + \tilde{V}_t^S(\tilde{\mathbf{s}}_t), \\ V_t^R(\mathbf{s}_t) &= (1-\Psi)\left[w^R \ell_t^R - \frac{\nu}{1+\nu}\left(\ell_t^R\right)^{\frac{1+\nu}{\nu}} - \frac{\delta_t B_t^R}{\mu^R} + \frac{\tau_t^e \mu^S}{\mu^R}\right] + \tilde{V}_t^R(\tilde{\mathbf{s}}_t), \end{aligned}$$

where

$$\tilde{V}_t^i(\tilde{\mathbf{s}}_t) = \Psi \ln\left((1-\bar{\beta}_t)a_t^i\right) + (1-\Psi)\frac{q_t^i B_{t+1}^i}{\mu^i} - \lambda \frac{B_{t+1}^i}{\mu^i} + \beta\, \mathbb{E}_t V_{t+1}^i(\mathbf{s}_{t+1}),$$

and $\tilde{\mathbf{s}}_t = (z_t^S, z_t^R, a_t^S, a_t^R, B_{t+1}^S, B_{t+1}^R)$.

The decomposition splits the value function in two components. The function $\tilde{V}_t^i(\tilde{\mathbf{s}}_t)$ is the value after the determination of $\mathbf{p}_t$. This function depends on the states $\tilde{\mathbf{s}}_t$. The states ω_t, B_t^S, B_t^R, ϱ_t and the policies $\mathbf{p}_t$ are incorporated in the new states a_t^S and a_t^R. Therefore, a_t^S and a_t^R become the new states and we no longer need ω_t, B_t^S, B_t^R, ϱ_t. This is why the new set of sufficient states is $\tilde{\mathbf{s}}_t$ instead of $\mathbf{s}_t$. The reason we make the split is because it allows us to reduce the computational time significantly as we will remark below.

Discrete state space To obtain a numerical representation of the above functions, we create a discrete grid for the states B^R, a^S and a^R. All other states take finite discrete values by assumption. Therefore, from now on, $\mathbf{s}$ and $\tilde{s}$ should be thought as taking values in finite (multidimensional) grids. We will then find values for V_t^i and for the policy variables $\mathbf{p}_t$ and B_{t+1}^R in the grid for $\mathbf{s}$. Similarly, we will find values of $\tilde{V}_t^i$ in the grid for $\tilde{\mathbf{s}}_t$.

A.1 Backward iteration

Since we have N periods, we solve the model backward starting with the terminal period $t = N$. When solving at a particular time t, we use the solutions for $V_{t+1}^i(\mathbf{s})$ and $\mathbf{p}_t = \Upsilon_{t+1}(\mathbf{s})$ we computed in the previous step, except in the terminal period where we have

$$\begin{aligned} B_{N+1}^S &= 0, \\ V_{N+1}^i &= 0, \\ \mathbf{p}_{N+1} &= (0,0,0). \end{aligned}$$

We now describe the computational procedure at each time $t = N, N-1, N-2, .., 1$. We first describe the procedure to solve for the financial market equilibrium given the new debt chosen by the risky country, B_{t+1}^R. This allows us to derive the function $\tilde{V}_t^i(\tilde{s})$. Then, given the numerical solution for $\tilde{V}_t^i(\tilde{\mathbf{s}})$, we describe the procedure to solve for the debt repayment, labor market equilibrium and new borrowing chosen by the risky country. This allows us to find the equilibrium policies $\mathbf{p}_t$ and B_{t+1}^R, and the associated value function $V_t^i(\mathbf{s})$.

A.2 Solving for financial market equilibrium given B_{t+1}^R

For each grid point of the states $\tilde{\mathbf{s}}_t = (z_t^S, z_t^R, a_t^S, a_t^R, B_{t+1}^S, B_{t+1}^R)$, we want to find the prices of bonds, q_t^S and q_t^R, and the portfolio allocation B_{t+1}^{SS}, B_{t+1}^{SR}, B_{t+1}^{RS}, B_{t+1}^{RR}. This is done by solving the following nonlinear system of equations:

$$\begin{aligned} q_t^S B_{t+1}^{SS} &= \theta_t \bar{\beta}_t a_t^S, \\ q_t^R B_{t+1}^{RS} &= (1-\theta_t)\bar{\beta}_t a_t^S, \\ q_t^S B_{t+1}^{SR} &= \theta_t \bar{\beta}_t a_t^R, \\ q_t^R B_{t+1}^{RR} &= (1-\theta_t)\bar{\beta}_t a_t^R, \\ 1 &= \mathbb{E}_t \left\{ \frac{\delta_{t+1}}{(1-\theta_t)\delta_{t+1} + \theta_t q_t^R / q_t^S} \right\}, \\ B_{t+1}^S &= B_{t+1}^{SS} + B_{t+1}^{SR}, \\ B_{t+1}^R &= B_{t+1}^{RS} + B_{t+1}^{RR}. \end{aligned}$$

where $\bar{\beta}_t = \frac{\beta - \beta^{N-t+1}}{1-\beta^{N-t+1}}$.

The first five conditions derive from the optimal portfolio choice of entrepreneurs established in Lemma 1. The last two are the market clearing conditions. The seven equations allow us to solve for the seven unknowns $q_t^S, q_t^R, B_{t+1}^{SS}, B_{t+1}^{SR}, B_{t+1}^{RS}, B_{t+1}^{RR}, \theta_t$, at each grid point for $\tilde{s}_t$. We can then compute the wealth share of entrepreneurs in country S as $\omega_{t+1} = (B_{t+1}^{SS} + B_{t+1}^{RS})/(B_{t+1}^S + B_{t+1}^R)$. This is one of the endogenous states in the next period. Notice that the fifth condition depends on the next period repayment ratio δ_{t+1}. Therefore, in order to find a solution we need to use the policy function $\mathbf{p}_{t+1} = \Upsilon_{t+1}(\mathbf{s}_{t+1})$ we derived in the previous step. Also, since the next period states determined by the above conditions, in particular x_{t+1}, are not necessarily on the grid point, we use linear interpolations to find intermediate values.

We now have all the elements to compute

$$\tilde{V}_t^i(\tilde{\mathbf{s}}) = \Psi \ln\left((1-\bar{\beta}_t)a_t^i\right) + (1-\Psi)\frac{q_t^i B_{t+1}^i}{\mu^i} - \lambda \frac{B_{t+1}^i}{\mu^i} + \beta\, \mathbb{E}_t V_{t+1}^i(\mathbf{s}'),$$

for any grid point of $\tilde{\mathbf{s}}_t = (z^S_t, z^R_t, a^S_t, a^R_t, B^S_{t+1}, B^R_{t+1})$. Notice that, to find the current value of this function, we need to use the next period value function $V_{t+1}(\mathbf{s}_{t+1})$ that we derived in the previous step. Also in this case we use linear interpolation to find intermediate values.

Solving the above system of nonlinear equations numerically is the most computational expensive step. By solving the system only on the grid for $\tilde{\mathbf{s}}$, allows us to speed the whole computational procedure. Instead of solving the system every time we change $\mathbf{p}_t$ and B^R_{t+1} to find the equilibrium values in the next step, we simply interpolate the function $\tilde{V}^i_t(\tilde{\mathbf{s}}_t)$.

A.3 Solving for repayment policy $\mathbf{p}_t$ and new borrowing B^R_{t+1}

We first notice that, given the states $\mathbf{s}$, once we know the repayment and bailout policies $\mathbf{p} = (\delta, \tau^d, \tau^e)$, we can determine the residual wealth of entrepreneurs,

$$
\begin{aligned}
m^S &= \frac{B^{SS}}{\mu^S} + \delta\frac{B^{RS}}{\mu^S} + \tau^d. \\
m^R &= \frac{B^{SR}}{\mu^R} + \delta\frac{B^{RR}}{\mu^R}.
\end{aligned}
$$

This, in turn, allows us to solve for the labor market equilibrium in both countries so that we can determine

$$
\begin{aligned}
l^i &= \left[\frac{(1-\alpha)z^i}{w^i}\right]^{\frac{1}{\alpha}} m^i, \\
w^i &= (l^i)^{\frac{1}{\nu}}, \\
\pi^i &= \alpha z^i \left[\frac{(1-\alpha)z^i}{w^i}\right]^{\frac{1-\alpha}{\alpha}} m^i, \\
a^i &= (1-\phi)m^i + \pi^i.
\end{aligned}
$$

The values of a^S and a^R is what we need to solve for the optimal borrowing of country R given the repayment policy $\mathbf{p}_t$.

Provided that the debt is fully repaid without bailout, that is, $\mathbf{p} = (1, 0, 0)$, the new borrowing of country R is determined by the following maximization problem:

$$
\max_{B^{R'}} \tilde{V}^R_t(\tilde{\mathbf{s}}),
$$

where the function $\tilde{V}^R_t(\tilde{\mathbf{s}})$ has been derived above for grid points of the states $\tilde{\mathbf{s}}$. Since we may need to solve this problem for values of a^S and a^R that are not necessarily in the grid points for $\tilde{s}$, we use linear interpolation over these two variables.

The optimal $B^{R'}$ is found with a grid search. However, in order to increase accuracy, the number of grid points we search for the optimal borrowing is bigger than the number of grid points for the current state B^R. This implies that we need to interpolate the function $\tilde{V}^R_t(\tilde{\mathbf{s}})$ also in the $B^{R'}$ dimension.

We can now solve for the repayment and bailout policy $\mathbf{p}$. In determining this policy, the various problems described below will take into account the impact on country R's borrowing, the function $B^{R'} = \Phi_t(\mathbf{s}, \mathbf{p})$.

To derive the repayment policy we have to distinguish three cases associated with the realization of the exogenous state $\varrho \in \{I, II, III\}$.

a) $\varrho = I$: Full repayment.

In this case the solution is trivially given by $\mathbf{p}_t = (1, 0, 0)$ and the value function is

$$V^i\Big(\mathbf{s} \,|\, \varrho = I\Big) = (1-\Psi)\left[w^i \ell^i - \frac{\nu}{1+\nu}\left(\ell^i\right)^{\frac{1+\nu}{\nu}} - \frac{B^i}{\mu^i}\right] + \tilde{V}_t^i\Big(\tilde{\mathbf{s}} \,|\, B^{R'} = \Phi_t(\mathbf{s}, (1,0,0))\Big).$$

b) $\varrho = II$: Repayment or default

For any state $\mathbf{s}$ in the grid, we need to find a solution if country R chooses to restructure the debt. In this case we need to solve the problem

$$\max_{\delta \in [0\,,\,1]} \qquad (1-\Psi)\left[w^R \ell^R - \frac{\nu}{1+\nu}\left(\ell^R\right)^{\frac{1+\nu}{\nu}} - \frac{\delta B^R}{\mu^R}\right] + \tilde{V}_t^R\Big(\tilde{\mathbf{s}} \,|\, B^{R'} = \delta B^R\Big),$$

subject to:

$$\max_{\tau^d \in \left[0, \frac{(1-\delta)B^{RS}}{\mu^i}\right]} (1-\Psi)\left[w^S \ell^S - \frac{\nu}{1+\nu}\left(\ell^S\right)^{\frac{1+\nu}{\nu}} - \frac{B^S}{\mu^S} - \tau^d\right] + \tilde{V}_t^S\Big(\tilde{\mathbf{s}} \,|\, B^{R'} = \delta B^R\Big).$$

Here δ and τ^d are solved sequentially and taking as given the response of next period debt which, in this case, is equal to $\mathbf{B}^{R'} = \delta B^R$. The two optimization problems are solved using first order conditions together with the Kuhn-Tucker conditions for possible corner solutions. We derive the first order conditions with numerical differentiation. Using the solutions for δ and τ^d, we can derive the value for country R when $\varrho = II$ and the country chooses to default,

$$\hat{V}_t^R\Big(\mathbf{s} \,|\, \varrho = II\Big) = (1-\Psi)\left[w^R \ell^R - \frac{\nu}{1+\nu}\left(\ell^R\right)^{\frac{1+\nu}{\nu}} - \frac{\delta B^R}{\mu^R}\right] + \tilde{V}_t^R\Big(\tilde{\mathbf{s}} \,|\, B^{R'} = \delta B^R\Big).$$

Finally, we consider the decision of country R to repay or default,

$$\xi = \arg\max_{x \in \{0,1\}} \left\{x \cdot V_t^R\Big(\mathbf{s} \,|\, \varrho = I\Big) + (1-x) \cdot V_t^R\Big(\mathbf{s} \,|\, \varrho = II\Big)\right\},$$

which allows us to determine the value for the two countries

$$V_t^i\Big(\mathbf{s} \,|\, \varrho = II\Big) = \xi \cdot V_t^i\Big(\mathbf{s} \,|\, \varrho = I\Big) + (1-\xi) \cdot V_t^i\Big(\mathbf{s} \,|\, \varrho = II\Big).$$

c) $\varrho = III$: Repayment or bailout

For any state $\mathbf{s}$ in the grid, we need to find a solution if country R chooses to restructure the debt and country R agrees to renegotiate. Since the two countries solve a bargaining problem, we first need to determine the threat values, which are given by $V_t^i\Big(\mathbf{s} \,|\, \varrho = II\Big)$, computed in the previous point b).

Define the net renegotiation surpluses of the two countries as

$$\begin{aligned}\Delta_t^S\Big(\mathbf{s};\mathbf{p}\Big) &= (1-\Psi)\left[w^S\ell^S - \frac{\nu}{1+\nu}\left(\ell^S\right)^{\frac{1+\nu}{\nu}} - \frac{B^S}{\mu^S} - \tau^d - \tau^e\right] + \tilde{V}_t^S\Big(\tilde{\mathbf{s}} \,|\, B^{R'} = \delta B^R\Big) \\ &- V^S\Big(\mathbf{s}|\varrho = II\Big), \\ \Delta_t^R\Big(\mathbf{s};\mathbf{p}\Big) &= (1-\Psi)\left[w^R\ell_t^R - \frac{\nu}{1+\nu}\left(\ell^R\right)^{\frac{1+\nu}{\nu}} - \frac{\delta B^R}{\mu^R} + \frac{\tau^e\mu^S}{\mu^R}\right] + \tilde{V}_t^R\Big(\tilde{\mathbf{s}} \,|\, B^{R'} = \delta B^R\Big) \\ &- V^R\Big(\mathbf{s}|\varrho = II\Big)\end{aligned}$$

To solve for the bargaining outcome we need to solve the problem

$$\arg\max_{\delta,\tau^d,\tau^e} \quad \left[\Delta_t^S\Big(\mathbf{s};\mathbf{p}\Big)\right]^\eta \left[\Delta_t^R\Big(\mathbf{s};\mathbf{p}\Big)\right]^{1-\eta}.$$

Since the optimal solution for δ and τ^d does not depend on τ^e, we first solve for the repayment ratio and domestic transfer using first order conditions together with the Kuhn-Tucker conditions for possible corner solutions. We derive the first order conditions with numerical differentiation. Given the optimal solution for δ and τ^d, we find the value of τ^e so that country S gets a share η of the net total surplus and country R get the remaining share $1-\eta$. Specifically, given the optimal δ and τ^d, we find τ^e so that

$$\Delta_t^S\Big(\mathbf{s};(\delta,\tau^d,\tau^e)\Big) = \eta\left[\Delta_t^S\Big(\mathbf{s};(\delta,\tau^d,\tau^e)\Big) + \Delta_t^R\Big(\mathbf{s};(\delta,\tau^d,\tau^e)\Big)\right].$$

The value when $\varrho = III$ and country R chooses to restructure the debt is,

$$\hat{V}_t^R\Big(\mathbf{s}|\varrho = III\Big) = (1-\Psi)\left[w^R\ell^R - \frac{\nu}{1+\nu}\left(\ell^R\right)^{\frac{1+\nu}{\nu}} - \frac{\delta B^R}{\mu^R} + \frac{\tau^e\mu^S}{\mu^R}\right] + \tilde{V}_t^R\Big(\tilde{\mathbf{s}}|B^{R'} = \delta B^R\Big).$$

Finally, the decision of country R to repay or default is determined by,

$$\xi = \arg\max_{x\in\{0,1\}}\left\{x\cdot V_t^R\Big(\mathbf{s}|\varrho = I\Big) + (1-x)\cdot V_t^R\Big(\mathbf{s}|\varrho = III\Big)\right\},$$

which allows us to determine the value for the two countries

$$V_t^i\Big(\mathbf{s}|\varrho = III\Big) = \xi\cdot V_t^i\Big(\mathbf{s}|\varrho = I\Big) + (1-\xi)\cdot V_t^i\Big(\mathbf{s}|\varrho = III\Big).$$

We have then determined the value of $V_t^i\Big(\mathbf{s}\Big)$ at any grid point for $\mathbf{s}$, as well as the policy function $\mathbf{p} = \Upsilon_t(\mathbf{s})$. We can then move to the earlier period $t-1$ and continue until we have solved for the initial period $t = 1$. When N is sufficiently large so that the solution at $t = 1$ is approximately equal to the solution at $t = 2$ for all grid points, the approximate solution for the infinite horizon model is that at $t = 1$. We will then use the value function $V^i(\mathbf{s}) = V_1^i(\mathbf{s})$ and policy functions $\Upsilon(\mathbf{s}) = \Upsilon_1(\mathbf{s})$ and $\Phi(\mathbf{s};\mathbf{p}) = \Phi_1(\mathbf{s};\mathbf{p})$ to simulate the infinite horizon model.

B Extended model with home bias

In this appendix, we extend the model so that portfolios feature home bias. We recalibrate the model to target the observed portfolio compositions of the two countries and compare the main quantitative properties with the model without home bias presented in the paper.

To generate portfolio home bias we assume that a constant fraction κ of bonds issued by country j are purchased, every period, by domestic entrepreneurs who do not have access to foreign bonds. Given B^j_{t+1}, the bonds issued by country j, the quantity acquired by domestic entrepreneurs without access to foreign markets is then κB^j_{t+1}. The quantity acquired by entrepreneurs with access to foreign bonds (both resident and non-residents) is $(1-\kappa)B^j_{t+1}$.

Entrepreneurs who have access to external financial markets are selected by a random variable realized before the investing decisions are made. We denote by α^j_t the fraction of entrepreneurs without access to foreign markets. This fraction (which is also the probability that an entrepreneur does not have access to foreign markets) changes every period to ensure that the constant fraction κ of domestic bonds are purchased by constrained entrepreneurs. It must satisfy

$$\alpha^j_t = \frac{q^j_t \kappa B^j_{t+1}}{\beta a^j_t \mu^j}.$$

This follows from the fact that, with logarithmic utility, the saving of an individual entrepreneur is $\beta a_{j,t}$ regardless of his type (with or without access to foreign financial markets).

The remaining fraction of entrepreneurs with access to foreign bonds, $1-\alpha^j_t$, solve a portfolio choice problem where they invest a share θ_t of their savings in bonds issued by country S and a fraction $1-\theta_t$ in bonds issued by country R. The condition determining θ_t is the same as in the model without home bias (see Condition (6) in Lemma 1).

The market equilibrium for bonds issued by country S is

$$q^S_t B^S_{t+1} = (1-\alpha^S_t)\theta_t \beta a^S_t \mu^S + \alpha^S_t \beta a^S_t \mu^S + (1-\alpha^R_t)\theta_t \beta a^R_t \mu^R.$$

The left-hand side is the supply of bonds while the right-hand-side is the demand. The demand has three components. The first is the demand coming from domestic entrepreneurs with access to foreign investments. Because they can diversify, they invest only a fraction θ_t of their savings in bonds issued by country S. The second component is the demand from domestic entrepreneurs without access to foreign investments. Since they cannot purchase foreign bonds, they invest all their savings in bonds issued by country S. The third component is the demand from foreign entrepreneurs with access to foreign investments. Since they can invest in both domestic and foreign bonds, they allocate only a fraction θ_t of savings to bonds issued by country S.

The equilibrium for bonds issued by country R is

$$q^R_{t+1} B^R_t = (1-\alpha^R_t)(1-\theta_t)\beta a^R_t \mu^R + \alpha^R_t \beta a^R_t \mu^R + (1-\alpha^S_t)(1-\theta_t)\beta a^S_t \mu^S.$$

The left-hand side is the supply of bonds while the right-hand-side is the demand. The first component of the right-hand-side is the demand coming from domestic entrepreneurs with access to foreign investments. Because they can diversify, they invest only a fraction $1-\theta_t$ of their savings in bonds issued by country R. The second component is the demand from domestic entrepreneurs with no access to foreign investments. As in the previous case, all their savings are invested in bonds issued by country R (recall that they cannot purchase foreign bonds by construction). The third component is the demand from foreign entrepreneurs with access to foreign investments. Since they can invest in both domestic and foreign bonds, they allocate only a fraction $1-\theta_t$ of their savings to bonds issued by country R.

Proposition 8 *Define $\mu = \mu^R/\mu^S$ the size of country R relative to the size of country S. We have that*

$$B^{SS}_{t+1} = \Big[(1-\kappa)x_{t+1} + \kappa\Big] B^S_{t+1}, \tag{1}$$

$$B^{SR}_{t+1} = \Big[(1-\kappa)(1-x_{t+1})\Big] B^S_{t+1}, \tag{2}$$

$$B^{RS}_{t+1} = \Big[(1-\kappa)x_{t+1}\Big] B^R_{t+1}, \tag{3}$$

$$B^{RR}_{t+1} = \Big[(1-\kappa)(1-x_{t+1}) + \kappa\Big] B^R_{t+1}, \tag{4}$$

where

$$x_{t+1} = \frac{(1-\alpha^S_t)a^S_t}{(1-\alpha^S_t)a^S_t + (1-\alpha^R_t)a^R_t \mu}.$$

Proof. Define $\hat{B}^j_{t+1}$ the debt issued by country j purchased by non-constrained entrepreneurs (residents and non-residents) and $\hat{\hat{B}}^j_{t+1}$ the debt issued by country j purchased by constrained (resident) entrepreneurs. Therefore, $B^j_{t+1} = \hat{B}^j_{t+1} + \hat{\hat{B}}^j_{t+1}$.

The fraction of $\hat{B}^S_{t+1}$ purchased by unconstrained entrepreneurs in country S is

$$\begin{aligned}
\frac{(1-\alpha^S_t)\theta_t \beta a^S_t \mu^S / q^S_t}{(1-\alpha^S_t)\theta_t \beta a^S_t \mu^S / q^S_t + (1-\alpha^R_t)\theta_t \beta a^R_t \mu^R / q^S_t} &= \\
\frac{(1-\alpha^S_t)a^S_t \mu^S}{(1-\alpha^S_t)a^S_t \mu^S + (1-\alpha^R_t)a^R_t \mu^R} &= x_{t+1}.
\end{aligned}$$

The fraction of $\hat{B}^S_{t+1}$ purchased by unconstrained entrepreneurs in country R is then $1-x_{t+1}$.

The fraction of $\hat{B}^R_{t+1}$ purchased by unconstrained entrepreneurs in country S is

$$\begin{aligned}\frac{(1-\alpha^S_t)(1-\theta_t)\beta a^S_t \mu^S/q^R_t}{(1-\alpha^S_t)(1-\theta_t)\beta a^S_t \mu^S/q^R_t + (1-\alpha^R_t)(1-\theta_t)\beta a^R_t \mu^R/q^R_t} &= \\ \frac{(1-\alpha^S_t)a^S_t \mu^S}{(1-\alpha^S_t)a^S_t \mu^S + (1-\alpha^R_t)a^R_t \mu^R} &= x_{t+1}.\end{aligned}$$

The fraction of $\hat{B}^R_{t+1}$ purchased by unconstrained entrepreneurs in country R is then $1 - x_{t+1}$.

We observe next that

$$\begin{aligned}B^{SS}_{t+1} &= \hat{B}^S_{t+1} x_{t+1} + \hat{\hat{B}}^S_{t+1} \\ B^{SR}_{t+1} &= \hat{B}^S_{t+1}(1 - x_{t+1}) \\ B^{RS}_{t+1} &= \hat{B}^R_{t+1} x_{t+1} \\ B^{RR}_{t+1} &= \hat{B}^R_{t+1}(1 - x_{t+1}) + \hat{\hat{B}}^R_{t+1}\end{aligned}$$

We use now the assumption that a fraction κ of bonds issued by each country is purchased by constrained domestic entrepreneurs. This implies that $\hat{B}^j_{t+1} = (1-\kappa)B^j_{t+1}$ and $\hat{\hat{B}}^j_{t+1} = \kappa B^j_{t+1}$. Substituting we obtain eqs. (1)-(4). ■

Knowing B^S_{t+1}, B^R_{t+1} and x_{t+1}, we can reconstruct the whole allocation of bonds, that is, B^{SS}_{t+1}, B^{SR}_{t+1}, B^{RS}_{t+1}, B^{RR}_{t+1}. We will then be able to solve the model with the same number of state variables as in the model without home bias. Notice that when $\kappa = 0$, we have that $\alpha^i_t = 0$ and the model collapses to the model without home bias characterized in the paper.

B.1 Re-calibration

The additional parameter κ allows us to differentiate the portfolio composition of the two countries. In the model without home bias, we were able to target the portfolio composition of only one country. In particular, we targeted the share of country R's debt held abroad (country S). With the additional parameter κ, we can target the portfolio composition of both countries. Specifically, we now target the following moments:

1. The long-run average share of German holdings of non-German EU debt over all German holdings of EU debt (including German debt) between 1997 and 2019. This is the average of the solid-blue line in the right panel of Figure 1 presented in the paper, and it is equal to 35.5%. In the model it corresponds to

$$\frac{B^{RS}}{B^{RS} + B^{SS}} = 0.355.$$

2. The long-run average share of GIIPS holdings of non-GIIPS EU debt over all GIIPS holdings of EU debt (including GIIPS debt) between 1997 and 2019. This is the average of the dashed-red line in the right panel of Figure 1 presented in the paper, and it is equal to 30.1%. In the model this corresponds to

$$\frac{B^{SR}}{B^{SR} + B^{RR}} = 0.301.$$

The averages are computed using GDP weights. Data details are provided at the bottom of Figure 1. The model has been re-calibrated to match the original targets together with the two additional portfolio targets described above.[1]

Table 5: Calibration moments: data and model.

	Data	Baseline model	Extended model
Real risk-free rate	1.3%	1.3%	1.2%
Debt-to-output ratio Country S	67.0%	67.2%	67.7%
Standard deviation debt-to-output ratio Country S	7.0%	7.0%	6.8%
Frequency of crises	5.0%	4.9%	4.9%
Spread conditional on crisis	5.5%	5.4%	5.5%
Share of Country R's debt held by Country S	44.5%	44.5%	**36.2%**
Share of foreign debt held by Country S	35.5%	57.4%	**34.9%**
Share of foreign debt held by Country R	30.1%	42.5%	**30.1%**
Public debt-to-output ratio Country R	80.0%	79.8%	80.8%

Notes: Data on public debt and GDP are "Bruegel database of sovereign bond holdings developed in Merler and Pisani-Ferry (2012)." We use general government debt held domestically and abroad, including debt held by the Central Bank and other public institutions. Data is available for the period 1999-2019. The share of public debt held abroad is computed as 'Non-residents' debt in percentage of total general debt. The real risk-free interest rate is the long-term nominal interest rate in Germany, 10 years maturity (obtained from the ECB Warehouse) minus the inflation rate computed from changes in the German CPI (obtained from FRED). The spread for the risky countries is the GDP-weighted average difference between the interest rates on 10-year bonds for Greece, Ireland, Italy, Portugal and Spain, and the 10-year bonds interest rate for Germany. The 'Spread conditional on crisis' is computed as an average over the 2009-2013 period. To compute the moments in the model, we draw a long sequence of random shocks and simulate the model over 110,000 periods. We discard the simulated variables for the first 10,000 periods and calculate the moments as averages of the remaining 100,000 periods. We compute the spread at the beginning of the period conditional on (i) $\varrho^R = 0$, and (ii) country R chooses to trigger a restructuring episode, either with bailout ($\varrho^S = 1$) or without bailout ($\varrho^S = 0$).

Table 5 reports the targeted moments in the data and the values generated by: (i) the model without home bias (corresponding to the baseline case presented in the paper) and (ii) the extended model with home bias. The table lists three portfolio

[1] We have replaced the previous portfolio target B^{RS}/B^R with the two targets $B^{RS}/(B^{RS}+B^{SS})$ and $B^{SR}/(B^{SR} + B^{RR})$. In the previous calibration, the portfolio target was matched mainly by changing the relative size μ. In the new calibration, the portfolio targets are matched mainly by changing the relative size μ and the parameter κ.

moments. The first moment—'Share of country R's debt held by country S'—is a target for the original model without home bias, but not for the extended model with home bias. That is why this moment is matched only by the baseline model. The other two moments—'Share of foreign debt held by country S' and 'Share of foreign debt held by country R'—are targeted by the extended model with home bias, but not in the baseline model without home bias. Hence, they are only matched in the extended model.

Table 6: Parameter values.

Parameters	Description	Baseline model	Extended model
β	Entrepreneur's discount factor	0.930	0.930
ν	Elasticity of labor supply	1.000	1.000
α	Capital income share in production	0.333	0.333
ϕ	Production cost (depreciation)	0.390	0.390
ρ^S	Probability bailout in crisis	0.750	0.750
ρ_R	Probability commitment to repay	0.903	0.905
η	Bargaining power share	0.500	0.500
Ψ	Government weight on entrepreneurs	0.111	**0.072**
λ	Government dis-utility from debt	0.141	**0.095**
μ	Relative size country R	1.255	**0.880**
κ	Share of external debt in S's portfolio	n.a.	**0.350**
z_L, z_H, ρ_z	Distribution productivity	(0.975, 1.025, 0.9)	(0.975, 1.025, 0.9)
$B^S_{Low}, B^S_{High}, \rho_B$	Distribution debt country S	(0.295, 0.369, 0.9)	(0.295, 0.369, 0.9)

Table 6 reports the parameter values for the model without home bias and the extended model with home bias. The important point to observe is that, in the extended model, the weight Ψ assigned to entrepreneurs is smaller. This is necessary to generate the same target for the interest rate spread. Keeping the same weight Ψ but having less diversified portfolios would reduce the incentive of country R to default. We would then have very small spreads. A lower value of Ψ increases the incentive to default and allows us to generate the targeted spread.

B.2 Properties

Table 8 displays the average statistics for the ergodic distribution of the baseline model and the extended model with home bias. For each version, the statistics are computed for the environments with and without bailout. As can be seen, the differences between the baseline model (without home bias) and the extended model (with bailout) are not large.

Figures 2 and 3 plot the welfare gains of committing never to bailout the risky country in the two versions of the model. Although the magnitudes are slightly

Table 7: Average statistics for ergodic distribution of baseline and extended model. All parameters of the extended model are re-calibrated to match all targeted moments.

	Baseline model		Extended model	
	With bailout ($\rho^S = 0.75$)	Without bailout ($\rho^S = 0$)	With bailout ($\rho^S = 0.75$)	Without bailout ($\rho^S = 0$)
Debt-to-output ratio country S (in %)	67.24	68.86	67.73	69.23
Debt-to-output ratio country R (in %)	79.78	74.93	80.77	75.70
Interest rate country S (in %)	1.34	-0.06	1.14	0.03
Interest rate country R (in %)	1.64	0.66	1.43	0.94
Unconditional repayment ratio (in %)	99.75	99.20	99.75	99.16
External transfers τ^e (% Y^S cond. on bailout)	5.34	NA	1.93	NA
Domestic transfers τ^d (% Y^S cond. on default)	9.44	6.74	5.31	3.87
Probability debt crisis (in %)	4.88	5.01	4.91	4.98
Spread conditional on debt crisis (in %)	5.44	19.61	5.50	20.86
Spread conditional on default (in %)	26.57	19.61	26.84	20.86
Output country S	0.494	0.483	0.491	0.481
Output country R	0.493	0.478	0.491	0.478

different, the welfare gains are always negative. This means that in both versions of the model, eliminating the ability to bailout the risky country is welfare reducing for both S and R.

B.3 Home bias without changing other parameters

After introducing home bias, we have re-calibrated all parameters so that the model matches not only the portfolio compositions of the two countries but also all other moments targeted in the calibration of the baseline model. After doing that, we have shown that the model delivers very similar results. In this subsection we show that this finding depends on the re-calibration of all other parameters. In fact, if we do not re-calibrate all other parameters, the quantitative properties of the model change substantially.

Suppose that we calibrate κ and the relative size μ to match the portfolio compositions $B^{RS}/(B^{SS} + B^{RS}) = 0.355$ and $B^{SR}/(B^{SR} + B^{RR}) = 0.301$. All other parameters are kept at the calibrated values for the baseline model. The average statistics for this case are shown in Table 8.

We can now see that the probability of crises in the extended model is basically zero. This also implies that the interest rate spread is also very close to zero. The intuition is that, when portfolios are not very diversified, it becomes much more costly

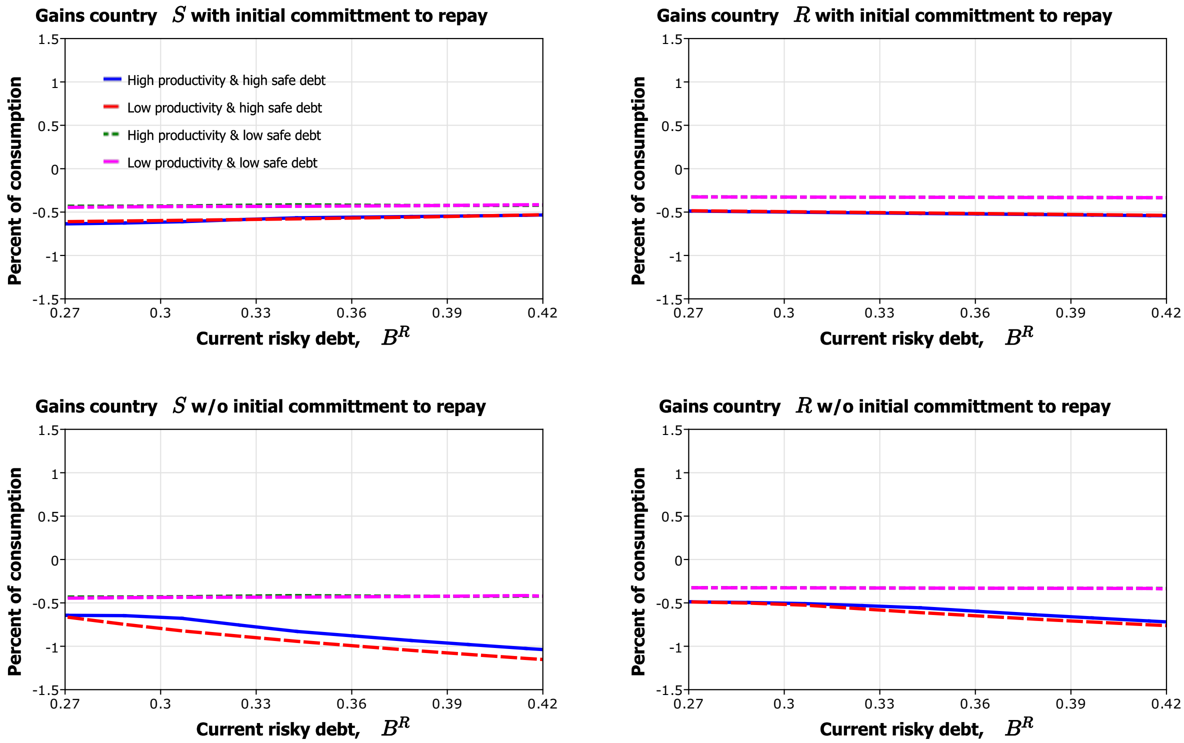

Figure 2: Welfare gains when country S commits to never bail out country R. Baseline model without home bias.

for country R to default. In equilibrium, then, the repayment ratio is always 1 or very close to 1. Bailout, then, becomes irrelevant: the equilibrium with bailout is very similar to the equilibrium without bailout and the welfare gains from commitment to not bail out are basically zero.

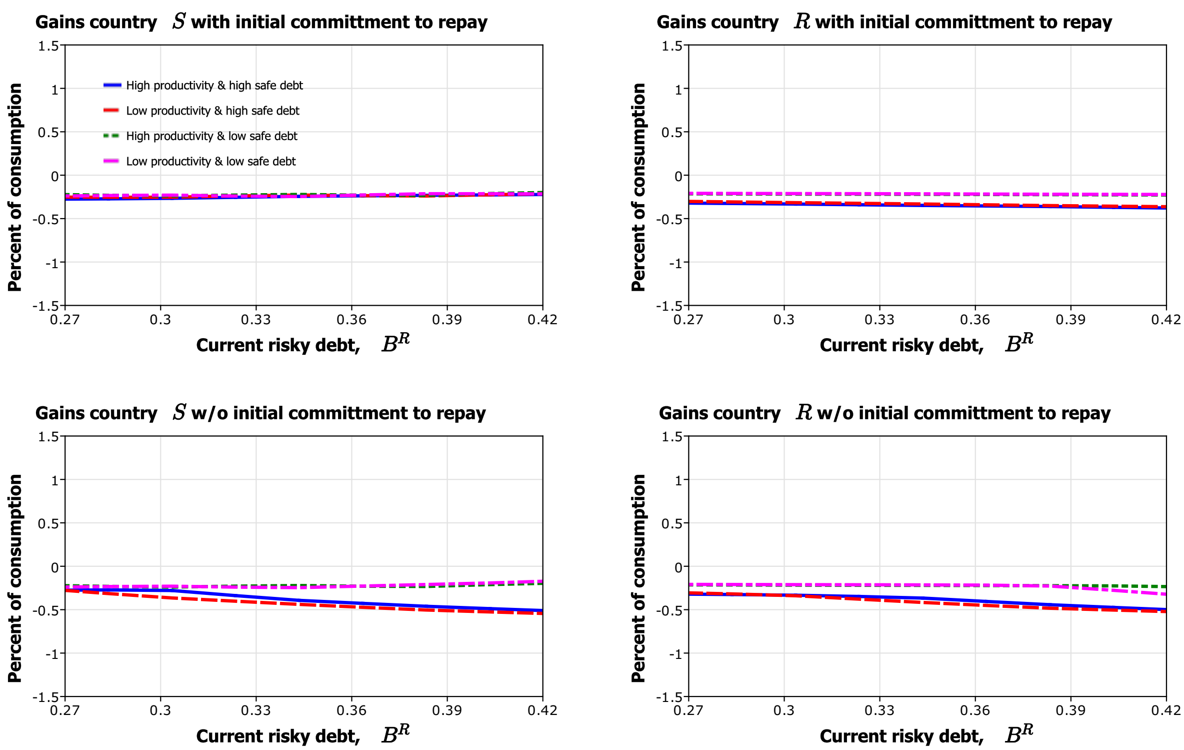

Figure 3: Welfare gains when country S commits to never bail out country R. Extended model with home bias.

Table 8: Average statistics for ergodic distribution of baseline and extended model. Only μ and κ are calibrated to match portfolio compositions of both countries.

	Baseline model		Extended model	
	With bailout ($\rho^S = 0.75$)	Without bailout ($\rho^S = 0$)	With bailout ($\rho^S = 0.75$)	Without bailout ($\rho^S = 0$)
Debt-to-output ratio country S (in %)	67.24	68.86	69.13	69.20
Debt-to-output ratio country R (in %)	79.78	74.93	75.72	75.67
Interest rate country S (in %)	1.34	-0.06	0.27	0.28
Interest rate country R (in %)	1.64	0.66	0.27	0.28
Unconditional repayment ratio (in %)	99.75	99.20	100.00	100.00
External transfers τ^e (% Y^S cond. on bailout)	5.34	NA	0.04	NA
Domestic transfers τ^d (% Y^S cond. on default)	9.44	6.74	0.07	0.11
Probability debt crisis (in %)	4.88	5.01	0.47	0.43
Spread conditional on debt crisis (in %)	5.44	19.61	0.02	0.09
Spread conditional on default (in %)	26.57	19.61	0.06	0.09
Output country S	0.494	0.483	0.481	0.481
Output country R	0.493	0.478	0.481	0.481

C Sensitivity to bailout timing

In the baseline model, we assumed that countries R and S Nash-bargain bailouts and repayment rates after a restructuring episode is triggered. Alternatively, we could assume a different timing where country S makes a proposal and country R decides whether to accept or reject this proposal. If creditors choose the external bailout first, it must be that they commit to a bailout strategy that depends on the repayment chosen by the debtor country in the second stage of the game (that is, after the creditors have chosen the bailout strategy).

Consider the following problem solved by the creditor country:

$$\max_{\delta_t, \tau_t^d, \tau_t^e} \mathcal{V}_t^S\Big(\mathbf{s}_t, (\delta_t, \tau_t^d, \tau_t^e)\Big) \tag{5}$$

subject to:

$$\mathcal{V}_t^S\Big(\mathbf{s}_t, (\delta_t, \tau_t^d, \tau_t^e)\Big) \geq \underline{\underline{V}}_t^S(\mathbf{s}_t),$$

$$\mathcal{V}_t^R\Big(\mathbf{s}_t, (\delta_t, \tau_t^d, \tau_t^e)\Big) \geq \underline{\underline{V}}_t^R(\mathbf{s}_t),$$

$$\tau_t^d \in \left[0\,,\,\frac{(1-\delta_t)B_t^{RS}}{\mu^S}\right] \quad \text{and} \quad \tau_t^e \geq 0$$

In this problem, the creditor country (country S) chooses the triplet $\{\delta_t, \tau_t^d, \tau_t^e\}$ that maximizes its own welfare, subject to the participation of the debtor country (country R). Denoting by $\{\hat{\delta}_t, \hat{\tau}_t^d, \hat{\tau}_t^e\}$ the solution to this problem, the creditor country commits to pay the external transfer $\tau_t^e = \hat{\tau}_t^e$ and the domestic transfer $\tau_t^d = \hat{\tau}_t^d$ if country R repays $\delta_t = \hat{\delta}_t$. Otherwise it pays $\tau_t^e = 0$ and $\underline{\underline{\tau}}_t^d$ if the repayment is different from $\hat{\delta}_t$. Obviously, with this bailout strategy, country R does not have an incentive to choose a repayment $\delta_t \neq \hat{\delta}_t$.

We can now observe that the above problem is equivalent to the Nash bargaining problem considered in the paper when country S has the whole bargaining power, that is, $\eta = 1$. This implies that the ability of country S to move first would make the external bailout even more attractive for this country (first mover advantage). Table 9 compares the statistics of the baseline model with $\eta = 0.5$ and the model with $\eta = 1$. As can be seen, the average statistics do not change in important ways.

Table 9: Average statistics for ergodic distribution of baseline model with $\eta = 0.5$ and model with $\eta = 1.0$. With the exception of η all parameters are at the baseline values.

	Baseline model		Extended model	
	With bailout ($\eta = 0.5$)	Without bailout ($\eta = 0.5$)	With bailout ($\eta = 1.0$)	Without bailout ($\eta = 1.0$)
Debt-to-output ratio country S (in %)	67.24	68.86	67.53	68.92
Debt-to-output ratio country R (in %)	79.78	74.93	78.86	74.93
Interest rate country S (in %)	1.34	-0.06	1.11	-0.04
Interest rate country R (in %)	1.64	0.66	1.41	0.87
Unconditional repayment ratio (in %)	99.75	99.20	99.76	99.18
External transfers τ^e (% Y^S cond. on bailout)	5.34	NA	1.74	NA
Domestic transfers τ^d (% Y^S cond. on default)	9.44	6.74	8.94	6.72
Probability debt crisis (in %)	4.88	5.01	5.07	5.07
Spread conditional on debt crisis (in %)	5.44	19.61	5.29	19.73
Spread conditional on default (in %)	26.57	19.61	25.54	19.73
Output country S	0.494	0.483	0.492	0.483
Output country R	0.493	0.478	0.491	0.478