Dual Approach for One Goal: The Internationalization of Renminbi during 2010-2021*

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This Version: March 2023

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Abstract

The last decade has witnessed China's policy reform toward building a global currency and the accelerated process of renminbi internationalization. We provide a quantitative assessment of the effects of these policies using a dynamic general equilibrium model. The model incorporates key functions of an international currency serving both as a store of value and a medium of exchange. Due to the complementarity between different functions, combined policies that simultaneously address frictions in the international asset market and trade market are more effective than policies with a single focus, so the dual-approach reform adopted by Chinese government has been largely optimal. Quantification of the model reveals that (1) China's reform in the past decade has lowered the return wedge in renminbi bond by 4.51%, and lowered the revenue wedge in renminbi-settled trade by 1.40%; (2) had the reform been targeting the renminbi payment share by only addressing frictions in international trade market, the cost of the reform would be 1.55 times larger.

*For their helpful comments we thank....

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1 Introduction

Building a global currency is a keystone in a country's internationalization agenda. An aspiring country achieves this goal usually by adopting a combination of policies. For example, as shown in Figure 1, during the past decade, Chinese government has initiated a series of policy reforms, which can be roughly categorized into those facilitating transactions in renminbi asset market or those facilitating international trade settled in renminbi. During the same time, it has witnessed an accelerated path for renminbi internationalization: the share of renminbi bonds held by foreign investors has increased from 1.06% to 3.92%, and the share of international transactions settled in renminbi has increased from 0.30% to 2.22%.

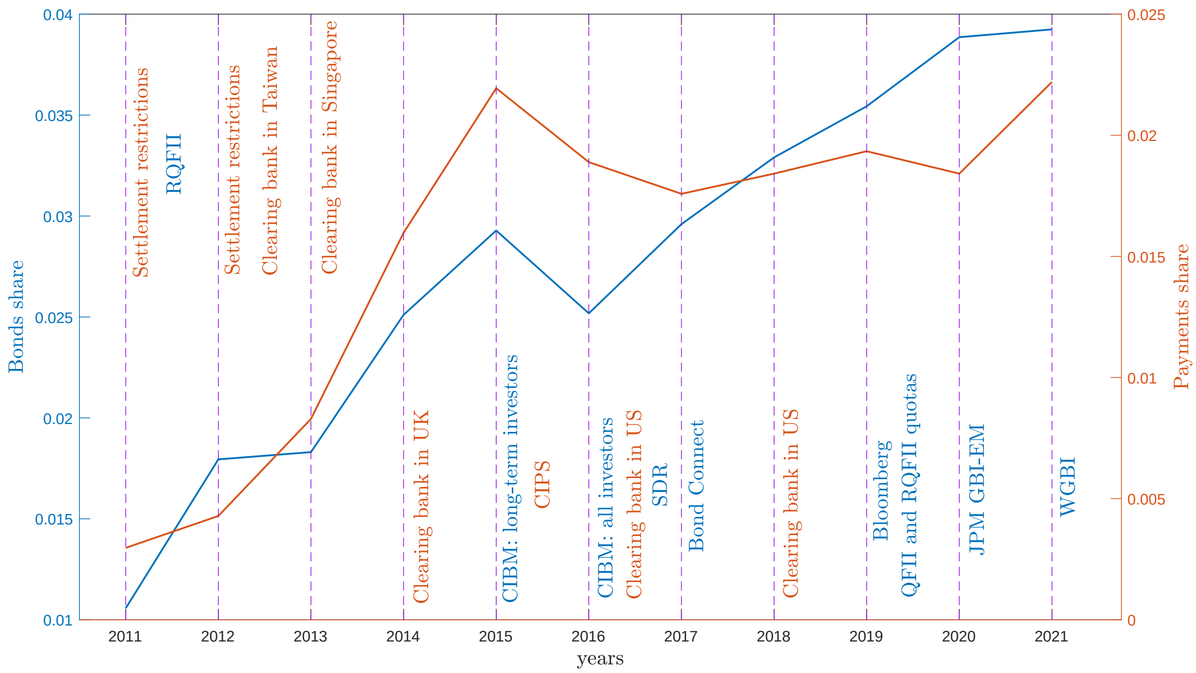

Figure 1: Bonds share and payments share during 2011-2021

Note: The data source for foreign holdings of renminbi bonds is International Investment Position Table. The data sources for volume of renminbi bonds are PBoC and China bond. The data source for renminbi payment share is RMB Tracker document, which is published by SWIFT.

One may then be tempted to attribute the observed accelerated internationalization of renminbi to the observed reforms, but such a correlation is not yet causal as it is confounded by other factors such as the change in the relative size of countries and evolving international trade patterns. The contribution of the current paper is to provide a quantitative assessment of the impact of these policies on renminbi internationalization, controlling for potential confounding factors that can be reflected by other macro variables.

We also assess the cost-effectiveness of the observed policy compared to alternative ones that aim at achieving the same goals. We argue that the dual approach adopted by Chinese government, by simultaneously addressing frictions in the asset market and the international trade market, are largely optimal.

Toward this end, we build a multi-country dynamic general equilibrium model with a typical core-periphery structure that has been used in the recent studies on endogenous currency choices and emergence of global currency. The key model structure is similar to Chahrour and Valchev (2021) except that the model is extended to capture asymmetries between a leading economy and a catch-up follower, and, importantly, to incorporate a series of policy instruments that can be inferred from the data.

In more detail, the model consists of two large economies, labelled the US and China, and a continuum of small economies that represent the rest of the world (RoW). There are two types of bonds, each denominated in one of the global currencies labelled dollar and renminbi, respectively. Households in each economy engage in inter-temporal transactions by holding a portfolio of bonds. There are no aggregate shocks, so bond returns are equalized in equilibrium, except that they earn different liquidity premia for serving as collaterals that enable international trade.

Firms in a country engage in profitable international trade by searching trading firms from other countries. Due to contractual frictions in cross-border trade, both parties in a transaction need to put up bonds as collateral, which can be acquired from households in the form of intra-period loans (trade finance). The trade finance markets are segmented within a period: firms make a currency choice and are paired with domestic households holding bonds in that currency via random search. Firms that successfully find trade finance pay a usage fee to households, after which they are paired with trading partners in the origin-destination specific international trade market. A pair of matched exporter and importer then conducts trade and splits their joint profits, from which a currency mismatch cost is deducted if the two firms' currency choices are different.

Two layers of strategic complementarity in currency choices arise in the model. The first is among the currency choices across firms: since a cost is incurred on trading profits upon currency mismatch, firms will choose the currency that they expect their partners to choose most often, and thus coordinate on the currency choice. The second is between firms and households within in a country: since households and firms are paired by random search in the trade finance market, firms tend to choose the currency in which the bonds are in greatest supply, and households tend to choose the currency of bond for which trade finance is in greatest demand. As shown in Chahrour and Valchev (2021), there exist multiple stable equilibria with different dominant currency if the strategic complementarities are sufficiently strong, under empirically plausible parameterization.

We model policies on currency internationalization in the asset market as reducing existing wedges on bond returns faced by international investors, and in the trade market as reducing wedges on the revenue of international trade settled in the currency. We show that the same strategic complementarity which gives rise to a dominant currency also implies that combined policies bolstering different functions of an international currency are more cost effective than any single policy. In a simplified version of the model, we characterize such a policy complementarity analytically. Locally, we show that starting from any steady state, a reduction in the wedge of bond return or firm revenue associated with a currency increases the choice of trade finance in that currency, and the marginal effect of reduction in revenue wedge is larger if the initial bond holding in that currency is higher. Globally, we show that starting from an economy in which renminbi cannot be a dominant currency due to existing frictions, achieving the status of a global currency by reducing two frictions simultaneously is more cost effective than doing so by reducing any single friction alone.

We gauge the quantitative importance of the policy complementarity and assess the optimality of observed policies by taking the model to China's experience during 2010-2021. Our quantitative exercise follows a structural accounting approach: by choosing time-varying model parameters and policy parameters, the model's *transition dynamics* exactly matches salient macroeconomic times series, including international investors' bond portfolio compositions, international firms' currency choices, and other macro variables such as relative country sizes and volumes of government bonds. The backed out wedges in renminbi bond return and wedges in firms' revenue settled in renminbi are thus the inferred effects of policies once other macro conditions are controlled for.

The calibration reveals the dual endeavors of Chinese government in promoting renminbi internationalization in the past decade. On average, the inferred wedge on renminbi bond return to international investors has been reduced by 4.51%, and the inferred wedge on firms' revenue trading in renminbi has been reduced by 1.40%. By projecting the inferred wedges over extracted keywords from related policy documents over the period, we find that the reduction in bond return wedge is more likely to be associated with policy keywords in addressing asset-market frictions, whereas the reduction in firms' revenue wedge is more likely to be associated with keywords in addressing trade-market frictions. These correlations do not only serve to validate our structural accounting approach, but also construct a mapping from policy keywords to reduced-form wedges that can used in predicting the impact of future policy reforms from their documents.

By interpreting the reductions in wedges as real transfers to foreign households or firms, we calculate the costs of implementing these policies imposed on the government budget. We find the costs of achieving the internationalization are about 0.1% of GDP, which implies the policy is costly. Note that part of the reductions in wedges are due to removals of actual frictions, which raise efficiencies and should not show up as the government's burden. Nevertheless, these calculations highlight the difficult tradeoffs when a country like China aspires to start the journey toward internationalizing its own currency.

Despite the considerable costs in the internationalization process, we find that the policy combinations enacted by Chinese government are largely optimal. We reach this conclusion by considering alternative internationalization process by either using single policy to address the frictions in bond market, or the trade market, alone. We show that, for the purpose of raising firms' use of renminbi by the same magnitude via reducing bond return wedge alone, the costs would be 1.26 times larger than those required by the observed policy. Similarly, the costs for a policy that only changes the revenue wedge would be 1.55 times larger. These exercises demonstrate that combined policies are usually more cost effective than any single policy.

Finally, we plan to project the future of renminbi internationalization based on the calibrated model, taking into account the projected future changes in relative country sizes, bond volumes, and policies.

Literature. The paper is related to three strands of literature. First, our paper is related to the literature that explores the emergence of a dominant currency. Several literature documents empirical evidence that dominant currency dominates in many aspects, which includes currency of pricing in international trade (Gopinath, 2015; Gopinath et al., 2020), structure of external balance sheets (Gourinchas and Rey, 2007a; Gourinchas and Rey, 2007b), denomination of debt contracts (Eren and Malamud, 2022), and currency composition of private portfolios (Maggiori et al., 2020). Theoretically, literature provides explanations for the emergence of a dominant currency based on the three roles of international currency, which includes a unit of account (Engel, 2006; Gopinath et al., 2010; Doepke and Schneider, 2017; Mukhin, 2022), a medium of exchange (Matsuyama et al., 1993; Devereux and Shi, 2013; Liu et al., 2019), and a store of value (Maggiori, 2017; He et al., 2019; Bocola and Lorenzoni, 2020; Eren and Malamud, 2022).

Instead of focusing single role of international currency, Chahrour and Valchev (2021), Gopinath and Stein (2021) explain the emergence of dominant currency based on complementarity between different functions of international currency. In Chahrour and

Valchev (2021), firms in different countries tend to use bond denominated in same currency as collateral to avoid mismatch cost in international trade. Households and firms within the same country also tend to choose bond denominated in same currency. This ensures that household can hold bonds with higher liquidity premia, and firms can find bonds as collateral with higher probability. Gopinath and Stein (2021) focus on complementarity between a unit of account and a store of value. In their model, household choose to hold dollar-dominated safe asset to avoid exchange rate risk, and the high demand of dollar deposits lowers the interest rate of dollar-denominated loans, which makes more firms choose to price goods in dollar and verifies the expectation of household. [1] **?** show that dominant currency can arise due to complementarity between debt issuance choices of private firms. Firms tend to issue debt in the unit of asset with greatest liquidity, which further increases liquidity of this kind of asset. There also exists complementarity between unit of account and store of value in their model, but they focus on how dominance in finance generates dominance in trade. Our contribution relative to them is that we focus on policy complementarity and provide quantitative accounting along the transition path.

Our paper is also related to literature that studies openness policies and international capital flows in China. Song et al. (2011) build a two-sector model with financial friction to explain the observed patterns of capital flows in China. Song et al. (2014) analyze the effects of exchange rate policy, interest rate policy, and deposit rate policy based on the model in Song et al. (2011) with capital controls. Liu et al. (2021) explore optimal capital account liberalization based on a model with financial repression. Jermann et al. (2022) study exchange rate policy in China since 2015. They show empirically that the policy aims to balance exchange rate flexibility and RMB index stability based on market pillar and the basket pillar. Compared with their work, we highlight the impact of openness policies on renminbi internationalization, a topic that draws many recent policy and academic attentions.

The analysis is closely related to recent work on the policy to internationalize domestic currency. Bahaj and Reis (2022) analyze how the swap line can help the rising international currency to become dominant currency theoretically. Empirically, they find that the creation of swap lines by People's Bank of China is significantly associated with the increase in the use of the renminbi in international payment. Georgiadis et al. (2021) contrast the effects of fundamentals and policies on share of trade invoicing empirically. They find that higher export and import exposure to China strengthen the role of dollar.

[1] The high demand of dollar- denominated loans, as shown in Gopinath and Stein (2018), also increases the dollar reserves of central bank.

Even though the higher trade share also leads to increase in renminbi share, the effects are not significant. For the role of policies, they find that creation of swap lines by PBoC is associated with increase in renminbi invoicing. Clayton et al. (2022) focus on how China selectively opens up domestic bond market. They build a dynamic reputation model to explain why China first allow stable investor to enter domestic bond market and then gradually lift the restrictions to flightier investors to enter. We contribute to this literature by considering policy combination and complementarity between different policies. We also provide quantitative structural decomposition for effects of the policies.

Outline. Section 2 introduces the policy background. Section 3 presents the model and shows analytical analysis. Sections 4 shows the quantitative results. Section 5 concludes.

2 Policy background

We provide a brief overview about the policies to promote renminbi internationalization in this section. As is mentioned before, Chinese government internationalized the renminbi by initiating policies in different aspects. We roughly categorize them into those facilitating transactions in renminbi asset market or those facilitating international trade settled in renminbi. For renminbi assets, we focus on renminbi bonds, so we mainly introduce policy reforms related to renminbi bonds market.

2.1 Policy to facilitate transactions in renminbi bonds market

In 2002, the Qualified Foreign Institutional Investor (QFII) was introduced and qualified foreign investors were allowed to enter the stock and exchange-traded bond markets in China, but they were not allowed to enter the China Interbank Bond Market (CIBM), which consists a very large share of renminbi bonds. Since 2010, foreign central banks, renminbi clearing banks in Hongkong and Macao, and overseas participating banks were allowed to use renminbi to invest China Interbank Bond Market. In 2011, the Renminbi Qualified Foreign Institutional Investor (RQFII) was introduced, which allowed foreign investors to use renminbi to invest securities in China. The restrictions on the access to China's bond markets were further lifted in 2013, when the QFII participants could enter the China Interbank Bond Market. The quotas on QFII and RQFII were removed in 2019, and the two programs were merged in 2020. In 2015, foreign central banks, international financial institutions and sovereign wealth funds had easier access to the interbank market, and the investment quota limit on them was removed. In 2016, restrictions on investing in the China Interbank Bond Market were relaxed for almost all foreign investors. In the same year, renminbi was included in the special drawing rights (SDR) basket.

Bond Connect was launched in 2017, which provided a new way for foreign investors to invest in the China Interbank Bond Market. Oversea investors could invest in the China Interbank Bond Market through connection between financial infrastructure institutions of Mainland China and Hongkong. Bond Connect provided much convenience for the foreign investors, and it was seen as an important reform to help renminbi bonds to be included in global bond indices (Clayton et al., 2022). Renminbi bonds were included in Bloomberg Barclays Global Aggregate Index (BBGA), JP Morgan Government Bond Index - Emerging Market (GBI-EM), and FTSE World Government Bond Index (WGBI) during year 2019-2021. In July 2022, Swap Connect was announced to be launched six months later. Swap Connect allows oversea investors to trade interest rate swap products in the Mainland China through the connection between financial infrastructure institutions of Mainland China and Hongkong. The launch of Swap Connect can facilitate the opening of derivatives market in China and provide more access to manage interest risk for the oversea investors.

2.2 Policy to facilitate international trade settled in renminbi

Before year 2009, only a very small fraction of trade was allowed to be settled in renminbi. In year 2009, *Administrative Rules on the Pilot Program of RMB Settlement of Cross-border Trade Transactions* was issued and renminbi settlement was allowed for trade between qualified firms in five cities (Shanghai, Guangzhou, Shenzhen, Dongguan, and Zhuhai) and Association of Southeast Asian Nations (ASEAN) member states as well as Hong Kong and Macao. In 2010, coverage of the pilot scheme was expanded to 20 provinces, and qualified firms were allowed to use renminbi to trade with firms in all countries. Restrictions on renminbi settlement for Chinese importing firms and exporting firms were lifted in year 2011 and 2012, respectively.

During the past decades, China has established several renminbi clearing banks in different regions. For example, clearing banks in Taiwan, Singapore, and the United Kingdom were established during 2012-2014. At the same time, China signed a lot of swap lines with different countries, and most of them were firstly signed before year 2016. Swap lines can reduce the volatility of borrowing costs for foreign firms in renminbi, which increases the incentives for foreign firms to use renminbi (Bahaj and Reis, 2022). To facilitate renminbi settlement in international trade, the Cross-border Interbank

Payment System (CIPS) was launched in year 2015. CIPS was designed to be built in two phases, and CIPS Phase Two was launched in year 2018. The CIPS Phase One adopted real time gross settlement mechanism, and CIPS Phase Two also introduced deferred net settlement mode. The launch of CIPS provided longer operation hours and enhanced the efficiency for the renminbi settlement. [2]

To summarize, the policies introduced above show the multipronged strategy of Chinese government in promoting renminbi internationalization during the past decade, which motivates us to analyze the rationale for the combined policies.

3 The Model

We consider a multi-country model with three regions: the U.S., China, and the rest of the world which contains a continuum of small countries. Countries are indexed by $j \in \{us, cn, [0, \mu_{rw}]\}$. Time is discrete and infinite. There are two types of bonds in the world: dollar bonds issued by U.S. government and renminbi bonds issued by Chinese government. Households hold these two types of bonds. They make consumption and saving decisions in each period. Due to contractual frictions in cross-border trade, firms in each country choose to search and borrow one of the two types of bonds from the households as collateral in each period.

The U.S. is leading economy in the model, whose currency is the dominant currency in initial state. As a catch-up follower, China has smaller country size and lower bond supply than the U.S.. To internationalize the renminbi, China can use policies to reduce return wedges in renminbi bond to increase the incentives of households to hold renminbi bonds, and to reduce revenue wedge in renminbi-settled trade to encourage more firms to use renminbi bonds as collateral. We next describe detailed settings about each part of the model.

[2] Before the launch of CIPS, there were three ways for oversea agents to settle in renminbi. The first is Non-Resident Account (NRA) mode. Foreign firms can directly open accounts with banks in China for RMB settlement with the approval of the PBoC. The second is correspondent banks mode. Oversea banks open account for renminbi settlement in correspondent banks, and the correspondent banks use renminbi for settlement through domestic settlement system. The third is oversea clearing banks mode. The oversea clearing banks open account for renminbi settlement in PBoC, and the renminbi settlement is done through the clearing banks.

3.1 Household

Representative household in country j chooses consumption and bond holdings to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{C_{jt}^{1-\sigma}}{1-\sigma} \tag{3.1}$$

where C_{jt} is Cobb-Douglas aggregate of domestic and foreign goods. The share of domestic goods is home bias parameter $a_{h,j}$, and share of foreign goods is proportional to size of the exporting country. Taking consumption basket in the U.S. for example:

$$C_{us,t} = \left(C_{us,t}^{us}\right)^{a_{h,us}} \left(C_{us,t}^{cn}\right)^{\frac{\left(1-a_{h,us}\right)\widetilde{\mu}_{cn}}{\widetilde{\mu}_{cn}+\widetilde{\mu}_{rw}}} \left(C_{us,t}^{rw}\right)^{\frac{\left(1-a_{h,us}\right)\widetilde{\mu}_{rw}}{\widetilde{\mu}_{cn}+\widetilde{\mu}_{rw}}} \tag{3.2}$$

where the subscript denotes the country and the superscript denotes the goods. μ_j is population share of country j and $\widetilde{\mu}_j$ is share of size of country j. We have $\widetilde{\mu}_j = \frac{\mu_j Y_j}{\mu_{us}Y_{us}+\mu_{cn}Y_{cn}+\mu_{rw}Y_{rw}}$, where Y_j is endowment in country j, which is exogenous in the model. $C_{jt}^{rw} \equiv (\int_0^{\mu_{rw}} (C_{jt}^i)^{(\eta-1)/\eta} di)^{\eta/(\eta-1)}$ is index of consumption of goods in rest of the world. Consumption baskets in China and the rest of the world are similar to (3.2), which are given in appendix A.

Compared with standard household problem, one new feature in the model is that holding bond can bring the household additional endogenous liquidity premia, which is the fees paid by firms when firms borrow bonds from the households. The budget constraint of household is:

$$\begin{aligned} P_{jt}C_{jt} + \left(1-\Delta_{jt}^{USD}\right) P_{us,t}^{us} Q_t^{USD} B_{jt}^{USD} + \left(1-\Delta_{jt}^{RMB}\right) P_{cn,t}^{cn} Q_t^{RMB} B_{jt}^{RMB} + \psi_{jt} \\ = P_{us,t}^{us} B_{jt-1}^{USD} + P_{cn,t}^{cn} B_{jt-1}^{RMB} + P_{jt}^{j} Y_{jt} + \Pi_{jt}^{T} + T_{jt} \end{aligned} \tag{3.3}$$

where Δ_{jt}^{USD} and Δ_{jt}^{RMB} denote liquidity premia of the dollar bond and renminbi bond, respectively. The liquidity premia is determined endogenously in equilibrium. P_{jt} is price index of consumption basket C_{jt}. P_{jt}^j is price of country j's goods. Q_t^{USD} and Q_t^{RMB} are price of dollar bond and renminbi bond, respectively. Π_{jt}^T is total profit of firms in the country j and T_{jt} is lump-sum taxes. ψ_{jt} is bond adjustment cost, which takes the form:

$$\psi_{jt} \equiv P_{us,t}^{us} Q_t^{USD} \tau(B_{jt}^{USD}, \underline{B}_{jt}^{USD}) + P_{cn,t}^{cn} Q_t^{RMB} \tau(B_{jt}^{RMB}, \underline{B}_{jt}^{RMB}) \tag{3.4}$$

where $\tau(B_{jt}^i, \underline{B}_{jt}^i) \equiv \frac{\bar{\tau}}{2} \left(\frac{B_{jt}^i - \underline{B}_{jt}^i}{\underline{B}_{jt}^i}\right)^2 \underline{B}_{jt}^i$. $\bar{\tau}$ is parameter related to bond adjustment cost. $\underline{B}_{j,t}^i = B_{j,t}^i$ for country-currency pair (j,i) that does not see bond adjustment cost: (rw, USD),

(us, USD), (cn, RMB). The bond adjustment cost mainly captures the frictions in bond market, and $\underline{B}_{j,t}^{RMB}$ reflects country-j households' willingness to hold renminbi bonds. Increase in $\underline{B}_{j,t}^{RMB}$ implies country-j households are more willing to hold renminbi bonds.

The Euler equations are given by

$$1 = \beta E_t \left[\left(\frac{C_{j,t+1}}{C_{j,t}} \right)^{-\sigma} \frac{P_{j,t}}{P_{j,t+1}} \frac{P_{us,t+1}^{us}}{P_{us,t}^{us}} \frac{1}{Q_t^{USD} \left(1 - \Delta_{j,t}^{USD} + \tau' \left(B_{j,t}^{USD}, \underline{B}_{j,t}^{USD} \right) \right)} \right] \tag{3.5}$$

$$1 = \beta E_t \left[\left(\frac{C_{j,t+1}}{C_{j,t}} \right)^{-\sigma} \frac{P_{j,t}}{P_{j,t+1}} \frac{P_{cn,t+1}^{cn}}{P_{cn,t}^{cn}} \frac{1}{Q_t^{RMB} \left(1 - \Delta_{j,t}^{RMB} + \tau' \left(B_{j,t}^{RMB}, \underline{B}_{j,t}^{RMB} \right) \right)} \right] \tag{3.6}$$

where

$$\tau' \left(B_{j,t}^{RMB}, \underline{B}_{j,t}^{RMB} \right) = \bar{\tau} \left(\frac{B_{j,t}^{RMB}}{\underline{B}_{j,t}^{RMB}} - 1 \right), \quad j \in \{us, rw\} \tag{3.7}$$

is bond return wedge in renminbi bond. Chinese government can use policy to reduce existing frictions in bond market, which increases $\underline{B}_{j,t}^{RMB}$ and then country-j households are more willing to hold renminbi bonds.

3.2 Firms

In our endowment economy, firms in each country search for trading partners to exchange goods. Due to contractual frictions in cross-border trade, both parties in a transaction need to borrow bonds from households as collateral. In each period, firm goes through three stages, where the timeline of the firm's decisions is shown in figure 2. First, firms need to decide whether to operate, and choose ex-ante probability for exporting to and importing from each country if they decide to operate. In equilibrium, the probability is the fraction of firms with different trading statuses. After deciding to enter the market, firms need to make funding decisions and search for bonds. For firms that find the bonds successfully, they go to international trade market to search for trading partners to exchange goods. We next describe firm's decision in each stage, where we trace decision of country-j dollar-funded exporter. Decisions of firms with other trading statuses are similar.

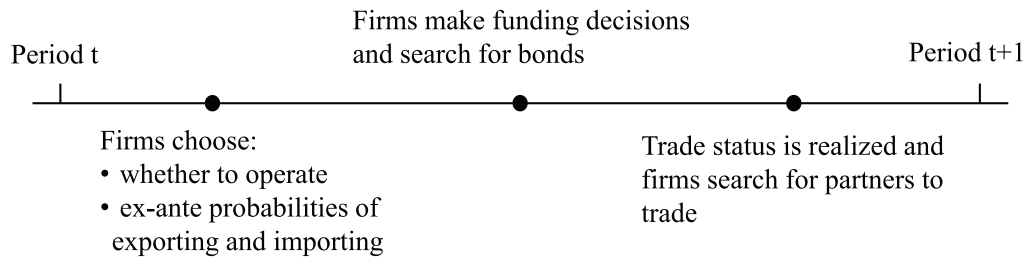

Figure 2: Timeline of the firms' decisions

3.2.1 Exchange of Goods

We first introduce the final stage in the timeline, when the firms have decided to operate and found the bonds as collateral. We follow den Haan et al. (2000) to use the matching function with the following form:

$$M^T(u,v) = \frac{uv}{\left(u^{\frac{1}{\varepsilon_T}} + v^{\frac{1}{\varepsilon_T}}\right)^{\varepsilon_T}} \tag{3.8}$$

where ε_T is matching elasticity. For country-j exporter that searches for country-i importer, the probability of successful matching is

$$p^{ei}_{jit} = \frac{M^T\left(\tilde{m}^{ex}_{jit}, \tilde{m}^{im}_{ijt}\right)}{\tilde{m}^{ex}_{jit}} = \left(1 + \left(\frac{\mu_j p^{ex}_{jit} m_{jt}\left(p^{USD}_{jt} X_{jt} + p^{RMB}_{jt}\left(1 - X_{jt}\right)\right)}{\mu_i p^{im}_{ijt} m_{it}\left(p^{USD}_{it} X_{it} + p^{RMB}_{it}\left(1 - X_{it}\right)\right)}\right)^{1/\varepsilon_T}\right)^{-\varepsilon_T} \tag{3.9}$$

where $\tilde{m}^{ex}_{jit}$ and $\tilde{m}^{im}_{ijt}$ are mass of the country-j exporter with destination i and country-i importer with destination j, respectively. m_{jt} is mass of country-j firms that choose to operate. p^{ex}_{jit} is probability of choosing to export to country i. $p^{USD}_{jt} X_{jt} + p^{RMB}_{jt}\left(1 - X_{jt}\right)$ is fraction of funded country-j firms, where p^{USD}_{jt} and p^{RMB}_{jt} are probabilities of finding dollar bonds and renminbi bonds, respectively. X_{jt} is fraction of country-j firms that choose to search for dollar bonds.

For firms that successfully match with their trading partners, the exporters sell goods to importers. The country-j exporter and country-i importer split total profit $P^j_{it} - P^j_{jt}$. The trading price is determined by Nash bargaining, and we denote Nash bargaining power of exporter as α, then the effective "wholesale" price for trading between country-j exporter and country-i importer is $P^{whol}_{jit} = P^j_{jt} + \alpha(P^j_{it} - P^j_{jt})$. Since we assume firms borrow one numeraire unit of bonds, the trading volume between country-j exporter and country-i importer is $1/P_{jit}^{whol}$. Finally, firms face mismatch cost when the two matching firms use collateral with bonds in different currencies. Based on these settings, the expected profit of country-j dollar-funded exporter with destination i is

$$\pi_{jit}^{USD,ex} = p_{jit}^{ei} \frac{\alpha}{P_{jit}^{whol}} \left[P_{it}^{j} - P_{jt}^{j} - \kappa P_{jit}^{whol} \left(1 - \frac{p_{it}^{USD} X_{it}}{p_{it}^{USD} X_{it} + p_{it}^{RMB} (1 - X_{it})} \right) \right] \tag{3.10}$$

where κ is parameter that measures magnitude of mismatch cost, and the term in the bracket is the fraction of country-i renminbi-funded firms among country-i funded firms.

3.2.2 Searching for Bonds

Now we come to second stage in the timeline, when the firms have chosen to operate, and they need to decide to search for dollar bonds or renminbi bonds. The match is between firms and households in the same country. The form of matching function is similar to that in international trade market, which is

$$M^F(u, v) = \frac{uv}{\left(u^{\frac{1}{\varepsilon_F}} + v^{\frac{1}{\varepsilon_F}} \right)^{\varepsilon_F}} \tag{3.11}$$

where ε_F is matching elasticity. The probability of finding U.S. bonds for country-j firm is

$$p_{jt}^{USD} = \frac{M^F \left(m_{jt} X_{jt}, \nu P_{us,t}^{us} B_{jt}^{USD} Q_t^{USD} \right)}{m_{jt} X_{jt}} = \left(1 + \left(\frac{m_{jt} X_{jt}}{\nu P_{us,t}^{us} B_{jt}^{USD} Q_t^{USD}} \right)^{1/\varepsilon_F} \right)^{-\varepsilon_F} \tag{3.12}$$

where $m_{jt} X_{jt}$ is the mass of country-j firms that search for U.S. bonds, and $m_{jt}(1 - X_{jt})$ is mass of country-j firms that search for renminbi bonds. Moreover, the calibration is based on annual data, and ν captures the fact that typical trade finance arrangement is shorter than one year. When firms successfully find the bonds, the profit with dollar bonds and renminbi bonds are

$$v_{jt}^{USD,(l)} = \Pi_{jt}^{USD} + \theta_{jt}^{USD,(l)}, \quad v_{jt}^{RMB,(l)} = \Pi_{jt}^{RMB} + \theta_{jt}^{RMB,(l)}$$

Note that besides the expected profits obtained directly from the trade, there also exists preference shock that affects firm's decision. $\theta_{jt}^{USD,(l)}$ and $\theta_{jt}^{RMB,(l)}$ are preference shocks that are Gumbel with scale parameter σ_θ and country-specific location parameter. Expected profit obtained directly from the trade with dollar funding is given by

$$\Pi_{jt}^{USD} = p_{jt}^{USD}(\sum_{i \neq j} \left(p_{jit}^{ex} \pi_{jit}^{USD,ex} + p_{jit}^{im} \pi_{jit}^{USD,im} \right) - r) \tag{3.13}$$

where $\pi_{jit}^{USD,ex}$ and $\pi_{jit}^{USD,im}$ are trading profit, which are determined in the final stage. p_{jit}^{ex} and p_{jit}^{im} are probabilities for choices of different trading statuses, which are determined in the first stage. r is fees paid by firms. The expression of Π_{jt}^{RMB} is similar, except that Chinese government can use policy to reduce revenue wedge in renminbi-settled trade in the rest of the world:

$$\Pi_{jt}^{RMB} = p_{jt}^{RMB}(\sum_{i \neq j} \left(p_{jit}^{ex} \pi_{jit}^{RMB,ex} + p_{jit}^{im} \pi_{jit}^{RMB,im} \right) - r + \mathbb{1}_{\{j \in \{rw\}\}} \tau_{f,t}) \tag{3.14}$$

where $\tau_{f,t}$ is amount of reduction in revenue wedge, so the increase in $\tau_{f,t}$ corresponds to reduction in revenue wedge. $\mathbb{1}_{\{j \in \{rw\}\}} = 1$ if $j \in \{rw\}$, $\mathbb{1}_{\{j \in \{rw\}\}} = 0$ if $j \in \{us, cn\}$, which means China only implements policy to reduce frictions related to renminbi-settled trade in the rest of the world . When deciding which bond to search for, firms choose the bond that can bring higher profit, so fraction of country-j firms that search for dollar bonds is

$$X_{jt} = \int \mathbb{1} \left(v_{jt}^{USD,(l)} \geq v_{jt}^{RMB,(l)} \right) F \left(d\theta_{jt}^{l} \right) = \frac{1}{\exp \left(\sigma_{\theta} \left(\Pi_{jt}^{RMB} - \Pi_{jt}^{USD} + \theta_{j}^{diff} \right) \right)} \tag{3.15}$$

where θ_{j}^{diff} is the country-specific difference in the location parameter of $\theta_{jt}^{RMB,(l)}$ and $\theta_{jt}^{USD,(l)}$.

3.2.3 Entry Decision and Trading Status Choice

For the first stage, we analyze how firms decide whether to operate and how they choose the probabilities of exporting, importing and trading partner's country. When deciding whether to pay the fixed cost to operate, firms face the problem below

$$\begin{aligned} \max_{\left\{ p_{jit}^{im}, p_{jit}^{ex} \right\}} \quad & X_{jt} \Pi_{jt}^{USD} + \left(1 - X_{jt} \right) \Pi_{jt}^{RMB} - \phi_j P_{jt} \\ \text{s.t.} \quad & \sum_{i \neq j} p_{jit}^{im} + \sum_{i \neq j} p_{jit}^{ex} = 1 \end{aligned} \tag{3.16}$$

where firms pay fixed cost ϕ_j in units of domestic composite goods. In equilibrium, the expected profits of choosing operating should be equal to fixed entry cost. Moreover, firms are indifferent to export and import, and they are indifferent to export to which country or import from each country. We can use these conditions to pin down mass of operating firms and probabilities of each trading status.

3.3 The Rest of the Model

Government in large country issues bonds in each period. We denote the quantity of bonds as $\bar{B}_t^j$, which is exogenous and we calibrate it to match corresponding statistics in data. The U.S. government's budget is

$$P_{us,t}^{us}\bar{B}_{t-1}^{USD} + T_{us,t} = Q_t^{USD}P_{us,t}^{us}\bar{B}_t^{USD} \tag{3.17}$$

The government budget in China is similar. Since government of country in the rest of world doesn't issue bond, we have $T_{rw,t} = 0$.

To close the model, we need to characterize the liquidity premia and market clearing conditions. The liquidity premia comes from the fees paid by firms, which is product of probability of finding firms, velocity ν, and fees paid by firms r:

$$\Delta_{jt}^{USD} = \left(1 + \left(\frac{m_{jt}X_{jt}}{\nu P_{us,t}^{us}Q_t^{USD}B_{jt}^{USD}}\right)^{1/\varepsilon_F}\right)^{-\varepsilon_F}\nu r \tag{3.18}$$

Finally, bonds market and goods market need to clear and the detailed conditions are given in appendix A.

3.4 Analytical Analysis

Before taking the model to data, we analyze the policy complementarity analytically in a simplified version of the full model. To simplify the analysis, we remove the mismatch cost and set the matching elasticity to 1. We also assume trading profit and currency choices of firms in the U.S. and China are exogenous. The detailed setting of the simplified model is given in appendix B. In the simplified model, we can characterize the steady state renminbi bond holdings of households and currency choices of firms in the rest of the world by the following two equations:

$$\underbrace{\frac{1}{1+\bar{B}^{RMB}-\frac{\mu_{rw}}{\mu_{cn}}B_{rw}^{RMB}}r}_{\Delta_{cn}^{RMB}} = \underbrace{\frac{1-X_{rw}}{1-X_{rw}+B_{rw}^{RMB}}r}_{\Delta_{rw}^{RMB}} - \underbrace{\bar{\tau}(\frac{B_{rw}^{RMB}}{\underline{B}_{rw}^{RMB}}-1)}_{\tau'(B_{rw}^{RMB},\underline{B}_{rw}^{RMB})} \tag{3.19}$$

$$X_{rw} = (1+\exp(\sigma_\theta(\underbrace{p_{rw}^{RMB}(\pi - r + \tau_f) - p_{rw}^{USD}(\pi - r)}_{\Pi_{rw}^{RMB}\text{-}\Pi_{rw}^{USD}})))^{-1} \tag{3.20}$$

where π is exogenous trading profit in simplified model. Other notations in above equations are same with that in the full model. Equation (3.19) is obtained by combining steady steady euler equations in China and the rest of the world. It shows that RoW households tend to hold more renminbi bonds when renminbi bond return wedge decreases. Equation (3.20) characterizes currency choice of firms in the rest of the world, which depends on difference between profit with renminbi bonds and dollar bonds, and the reduction in revenue wedge increases incentive for RoW firms to use renminbi.

Similar to the full model, Chinese government can implement policy to reduce return wedges and revenue wedges to internationalize renminbi, which is reflected in changes in $\underline{B}_{rw}^{RMB}$ and τ_f. We first analyze the effects of $\underline{B}_{rw}^{RMB}$ and τ_f on X_{rw} and the complementarity between the two policies locally in a linearized system, where variables with hat represent deviation from steady state value and variables with subscript 0 represent steady state variables. The related result is summarized in the proposition 1 and the proof is shown in appendix C.1.

Proposition 1. *Starting from a dollar-dominant steady state with* $(\underline{B}_{rw}^{RMB}, \tau_f) = (\underline{B}_{rw,0}^{RMB}, \tau_{f,0})$*, an increase in* $\underline{B}_{rw}^{RMB}$ *or* τ_f *decreases* X_{rw}*, and the marginal effect of* τ_f *could be written as a function of* $(B_{rw,0}^{RMB}, X_{rw,0}, \tau_{f,0})$*, which increases in* $B_{rw,0}^{RMB}$*.*

Proposition 1 formally proves that an increase in $\underline{B}_{rw}^{RMB}$ or τ_f leads to increase in share of firms that use renminbi in trade finance market, so government has at least two choices to raise firms' use of renminbi: using policy to reduce frictions in bond market or using policy to reduce frictions related to renminbi settlement. The second half of proposition 1 shows that besides single effect of the policy, there exists complementarity between the two policies: when government uses policy to reduce bond return wedge, demand for renminbi bonds increase, then policy to reduce revenue wedge has larger effect on renminbi share. The complementarity between the two policies implies the third choice may be the best: using combined policies to achieve the targeted share. [3]

[3] Note that the marginal effect in proposition 1 is marginal effect in partial equilibrium. We show policy

To further compare the combined policies and single policy, we compare the costs of the policies given that they achieve the same target, where we interpret the reductions in wedges as real transfers to foreign households or firms and calculate the costs of implementing these policies imposed on the government budget. Specifically, the government spending on reducing bond return wedge is

$$G^B = \frac{\mu_{rw}}{\mu_{cn}} \left(\Delta\tau'(B_{rw}^{RMB}, \underline{B}_{rw}^{RMB}) B_{rw}^{RMB} \right)$$

where

$$\Delta\tau'(B_{rw}^{RMB}, \underline{B}_{rw}^{RMB}) = \bar{\tau} \left(\frac{B_{rw,0}^{RMB}}{\underline{B}_{rw,0}^{RMB}} - \frac{B_{rw}^{RMB}}{\underline{B}_{rw}^{RMB}} \right)$$

is reduction in return wedge. The government spending on reducing firms' revenue wedge is

$$G^F = \frac{\mu_{rw}}{\mu_{cn}} \left(p_{rw}^{RMB} (1 - X_{rw}) \tau_f \right)$$

where we consider the case that the economy stays at state with $(\underline{B}_{rw}^{RMB}, \tau_f) = (\underline{B}_{rw,0}^{RMB}, 0)$ before the implementation of policy reforms. Now we compare the costs of combined policies and single policy globally. To simplify the analysis, we consider the case that starting from a initial dollar-dominant steady state with $(\underline{B}_{rw}^{RMB}, \tau_f) = (\underline{B}_{rw,0}^{RMB}, 0)$, government uses policy to achieve symmetric steady state with $X_{rw} = 0.5$. [4] The related result is summarized in the proposition 2 and the proof is shown in appendix C.2.

Proposition 2. *If $\pi - r > \underline{\pi}(\underline{B}_{rw,0}^{RMB}, r)$, combined policies can achieve the symmetric steady state with a lower government spending than the single policy to reduce revenue wedge, where $\underline{\pi}(\underline{B}_{rw,0}^{RMB}, r)$ is increasing in $\underline{B}_{rw,0}^{RMB}$ and r.*

Proposition 2 shows that achieving the symmetric steady state by using combined policies is more cost effective than doing so by using single policy to reduce revenue wedge alone. Intuitively, when there exists policy complementarity, combined policies can achieve the target with lower degree of policy, so the cost of the combined policies may be lower. Proposition 2 also characterizes the condition when combined policies is more likely to be efficient. Intuitively, when $\underline{B}_{rw,0}^{RMB}$ is lower, households in the rest of the world are less willing to hold renminbi bonds and there exist more frictions in bond market in initial state, so addressing the frictions in bond market is more likely to be

complementarity in general equilibrium numerically in section 4.

[4]Ideally, we need to compare government spending on combined policies and single policy when achieving steady state with $X_{rw} < 0.5$, but achieving symmetric steady state can simplify the algebra and allow us to do some analytical analysis. After achieving symmetric steady state, the government can reduce the wedge a little bit to achieve steady state with $X_{rw} < 0.5$

cost effective. Same amount of increase in renminbi bond holdings in RoW have more effects on firm's profit when net trading profit $\pi - r$ is higher, so reducing return wedge is more effective. Finally, when financing cost r is lower, households earn lower liquidity premia, and we need less reduction in return wedge when increasing same amount of renminbi bond holdings in RoW, so cost of the policy to reduce return wedge is lower when r is lower.

4 Quantification and Results

4.1 Calibration

In this section, we take our model to China's experience during 2010-2021 to gauge the quantitative importance of the policy complementarity and assess the optimality of observed policies. Starting from initial year 2010 with large frictions in bond market and trade market, China initiates policy reforms to reduce frictions during 2011-2021 and the economy begins to transit with increasing renminbi bonds share and renminbi payments share. Since we allow endowment to grow and the economy eventually converges to a balanced growth path, we solve a detrended system, where we detrend the country-j variables by country-j endowment Y_{jt}. The detailed detrended equilibrium conditions are given in appendix A. After detrending the system, we solve the full transition path, which calibrates parameters and solve the transition path simultaneously. Note that starting from a given state, we can solve the transition path given the parameters, so we can calibrate parameters to match moments on transition path with their data counterparts.

We first report externally calibrated parameters, which are shown in table 1. We set $\mu_{cn} = 0.1946$ and $\mu_{us} = 0.0449$ to match population share of China and the U.S. in data, respectively. One period in the model corresponds to one year, so β is set to 0.96. ε_T is set to 0.01 to minimize the search friction in international trade market. Mismatch cost κ is set to 0.01, which implies the total mismatch cost for the two parties is 1% of the goods value. We set $\alpha = 0.5$, which means exporters and importers have equal bargaining power. The velocity ν is set to 8 to match the fact that the observed maturity of a typical letter of credit contract is 45 days in data, and borrowing interest rate r is set to 0.0050 to match the typical cost of letter of credit (Chahrour and Valchev,2021). We assign a large value to σ_θ such that the effect of preference shock on currency share in trade finance market as small as possible. For coefficient on bond adjustment cost, we follow Chahrour and Valchev (2021) to set $\bar{\tau} = 0.04$. [5]

We calibrate remaining parameters to match model moments with their data counterparts on the transition path. The initial bond holdings are solved in a steady state, where the parameters are calibrated to match statistics in the year 2009. Note that bond shares in the steady state are same with their data counterparts in data, and we solve the transition path starting from the bond holdings solved in this steady state. To solve the transition path, we assume time-varying parameters $\{\underline{B}_t^{RMB}, \tau_{f,t}, \bar{B}_t^{RMB}, \bar{B}_t^{USD}, \underline{B}_{cn,t}^{USD}\}$ after 2021 are same with that in 2021, and the growth rates of the three regions on balanced growth path are 3%. Along the transition path, we calibrate parameters to match share of foreign holdings of renminbi bonds, share of China's holdings of foreign bonds, renminbi payments share, government bonds over GDP in China and the U.S.. Growth rates of endowments are externally calibrated to match growth rates of real GDP per capita in data, and the initial endowment ratio between two countries in the model is externally calibrated to match real GDP per capita ratio. We also calibrate parameters to match some moments in initial year 2010. We report time-invariant parameters and targeted moments in initial year in table 2 and 3, respectively. For the targeted moments in initial year, we match trade volume over GDP in the rest of the world, which is 0.5424. The import markup is 10% according to micro-level evidence in Coşar et al. (2018). The share of foreign currency usage of Chinese firm is 0.85, which is share of foreign currency settlement for trade in goods in China. The share of dollar usage of the U.S. firm is 0.9989, and the share of dollar usage of the RoW firm in initial year is 0.9988. [6]

4.2 Transition Dynamics

The time series of inferred $\hat{\underline{B}}_t^{RMB}$, $\tau_{f,t}$ and reductions in the two wedges are shown in figure 3. The reductions are relative to wedges in the year 2010. The choice of the initial year are motivated by the time line of policy reforms and data. As is described in section 2, most policy reforms were initiated since 2011. In particular, RMB settlement pilot program for cross-border trade was expanded to the whole country in 2011. We can also find that foreign holdings of renminbi bonds and renminbi payment share increase significantly since 2011. Note that we normalize $\tau_{f,t}$ to 0 in 2010, so values of $\tau_{f,t}$ during 2011-2021 are also the reductions in revenue wedges. During this period, on

[5] We also consider other values of $\bar{\tau}$ and find our results are robust to the choice of $\bar{\tau}$.

[6] These two shares are calculated based on dollar share in trade finance market in the U.S. and RoW, and the renminbi payment share in the world. According to Chahrour and Valchev (2021), dollar share in trade finance market in the U.S. and RoW are 90% and 80%, respectively. Since we only have two currencies in model, we re-scale the share of dollar and renminbi such that their relative ratio is same with that in data and the sum of the shares of the two currencies are equal to 1.

Table 1: Predetermined parameters

Parameters	Description	Value
β	Discount factor	0.9600
μ_{us}	Population share of the U.S.	0.1946
μ_{cn}	Population share of China	0.0449
κ	Mismatch cost	0.0100
σ	Risk aversion coefficient	1.0000
r	Funding fee	0.0050
ν	Velocity	8.0000
α	Exporters bargaining parameter	0.5000
σ_θ	Scale parameter of Gumbel distribution	2000
ε_T	Matching elasticity for trade	0.0100
$\bar{\tau}$	Bond adjustment costs	0.0400

Table 2: Calibrated Parameters

Parameters	Description	Target	Value
a_h	Home bias	RoW trade/GDP	0.7210
ϕ_{us}	Fixed entry costs for U.S. firms	Import markup for U.S. goods	0.0006
ϕ_{cn}	Fixed entry costs for Chinese firms	Import markup for Chinese goods	0.0038
ϕ_{rw}	Fixed entry costs for RoW firms	Import markup for RoW goods	0.0146
ε_F	Matching elasticity for bonds	USD usage	0.6647
$\theta_{us}^{\text{diff}}$	shifted mean for U.S. firms	U.S. USD usage	-0.0031
$\theta_{cn}^{\text{diff}}$	shifted mean for Chinese firms	CN USD usage	-0.0342

Table 3: Targeted Moments

Moment	Data	Model
RoW trade/GDP	0.5424	0.5424
Import markup for U.S. goods	1.1000	1.1000
Import markup for Chinese goods	1.1000	1.1000
Import markup for RoW goods	1.1000	1.1000
US USD usage	0.9989	0.9989
CN USD usage	0.8500	0.8500
RoW USD usage in initial year	0.9988	0.9988

average, the inferred bond return wedge has been reduced by 4.51%, and the inferred firms' revenue wedge has been reduced by 1.40%. The simultaneous reductions in the two wedges reveals the dual endeavors of Chinese government in promoting renminbi internationalization in the past decade.

Moreover, we can find that patterns of reductions in the two wedges are different. The amount of reduction in revenue wedge increases before 2015 but decreases after 2015, while reduction in return wedge doesn't exhibit this pattern. This result is consistent with the time line of the policy reforms. As is described in section 2, most policy reforms related to reducing frictions of renminbi settlement were implemented before 2016. For example, most of swap lines were firstly signed before year 2016. Policy reforms related to reducing frictions in bond market were active during the whole period, such as launch of RQFII (2011), CIBM Direct (2015), and launch of Bond Connect (2017). All of them have increased the the willingness of international investors to hold renminbi bonds, and we can find corresponding reduction in bond return wedge in the model. One one hand, this implies the changes of the wedges in our model can capture the effects of related policy reforms, which validates our structural accounting approach; on the other hand, the result shows that we can predict the impact of future policy reforms by constructing a mapping from policy keywords to reduced-form wedges.

Figure 3: Inferred reductions in wedges along the transition path

We next decompose the effects of policies on renminbi payment share. To decompose the effects, we first fix the value of $\tau_{f,t}$ to its value in the initial year and set values of other parameters to the calibrated values. The change in $X_{rw,t}$ along this transition path reflects effect of reducing return wedge alone. Similarly, we next fix the value of $\underline{\hat{B}}_{rw,t}^{RMB}$ to its value in the initial year and set values of other parameters to the calibrated values. The changes of $X_{rw,t}$ along this transition path reflects effect of reducing revenue wedge alone. Due to complementarity between the two policies, the sum of these two single effects are less than the effect of combined policies, and we regard the difference between effect of combined policies and sum of these two single effects as effect of complmentarity between the two policies. [7] As is shown in figure 4, complementarity between the policies is quantitatively important. The effect of reducing revenue wedge first increases and then decreases, which is consistent with the pattern of reductions in revenue wedges in figure 3. We can find that the effect of reducing revenue wedge is relatively small, which implies that when offshore liquidity is low, reducing revenue wedge alone has little effect on firm's currency choice in the rest of the world. This also reflects the importance of the policy complementarity.

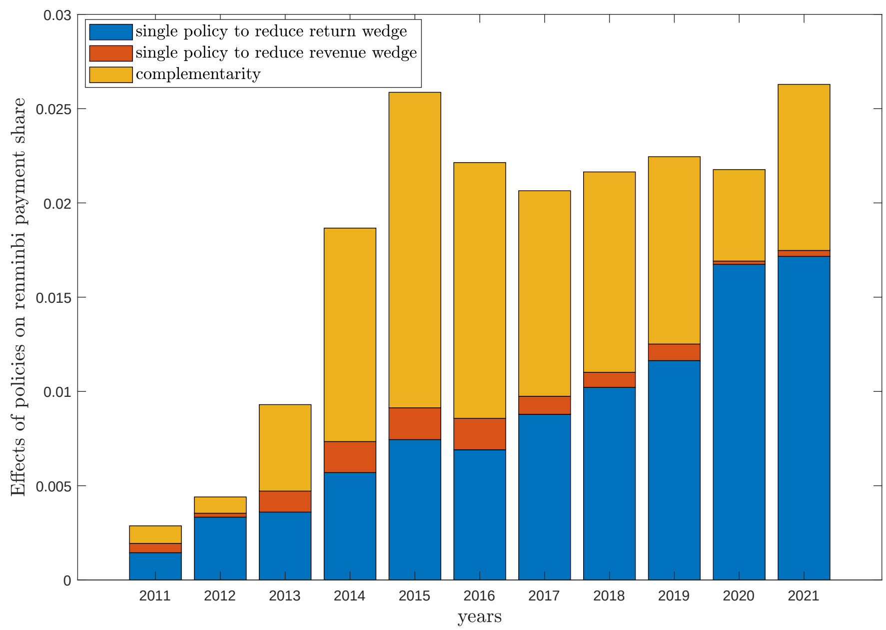

Figure 4: Decomposition for effects of the policies on renminbi payment share

Above exercise reveals that Chinese government used combined policies to internationalize the renminbi, then how effective are the observed policies? To answer this question, we consider alternative internationalization process by using single policy to address the frictions in bond market, or the trade market, alone. To compare combined policies and single policy, we consider the counterfactual that using single policy to target renminbi payment share in data. The dashed line in figure 5 shows the levels of $\underline{\hat{B}}_{rw}^{RMB}$

[7] When calculating effects of different policies on $X_{rw,t}$, we actually calculate the change in $X_{rw,t}$ relative to that along the transition path without policy reforms to control the effects of changes in fundamentals

and $\tau_{f,t}$ in single policy case. For comparison, we also plot the levels of $\underline{\hat{B}}_{rw}^{RMB}$ and $\tau_{f,t}$ in combined policies case. We can find that levels of $\underline{\hat{B}}_{rw}^{RMB}$ and $\tau_{f,t}$ in single policy case are significantly higher than that in combined policies case. This result shows the quantitative importance of the policy complementarity. Due to complementarity between the two policies, we can achieve the same goal with lower levels of $\underline{\hat{B}}_{rw}^{RMB}$ and $\tau_{f,t}$ by using combined policies.

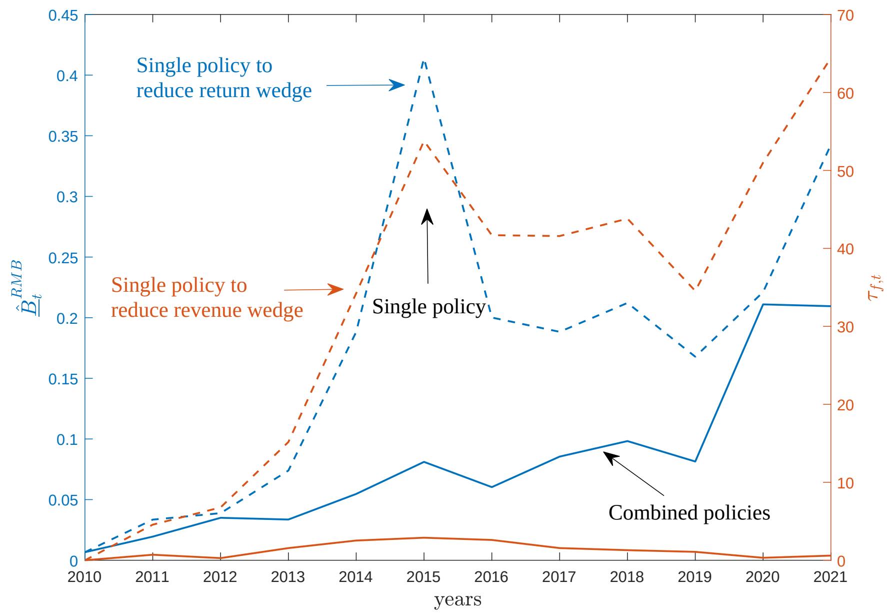

Figure 5: $\underline{\hat{B}}_t^{RMB}$ and $\tau_{f,t}$ along the transition path: single policy case

To further assess the optimality of the combined policies, we interpret the reductions in wedges as real transfers to foreign households or firms and calculate the costs of implementing these policies imposed on the government budget. Left panel in figure 6 plots government spending over GDP in each year during 2010-2021. On average, government spending on combined policies is about 0.1% of GDP, which implies the policy is formidably costly. Note that part of the reductions in wedges are due to removals of actual frictions, which raise efficiencies and should not show up as the government's burden, so this method tends to overestimate the cost. However, this calculation can illustrate the relative cost between combined policies and single policy. Right panel in figure 6 plots cumulative government spending during 2010-2011. The result shows that had the reform been targeting the renminbi payment share by using single policy to address frictions in bond market or trade market alone, the costs of the reforms would be 1.55 times larger and 1.27 times larger, respectively. Therefore, combined policies are usually more cost effective than any single policy.

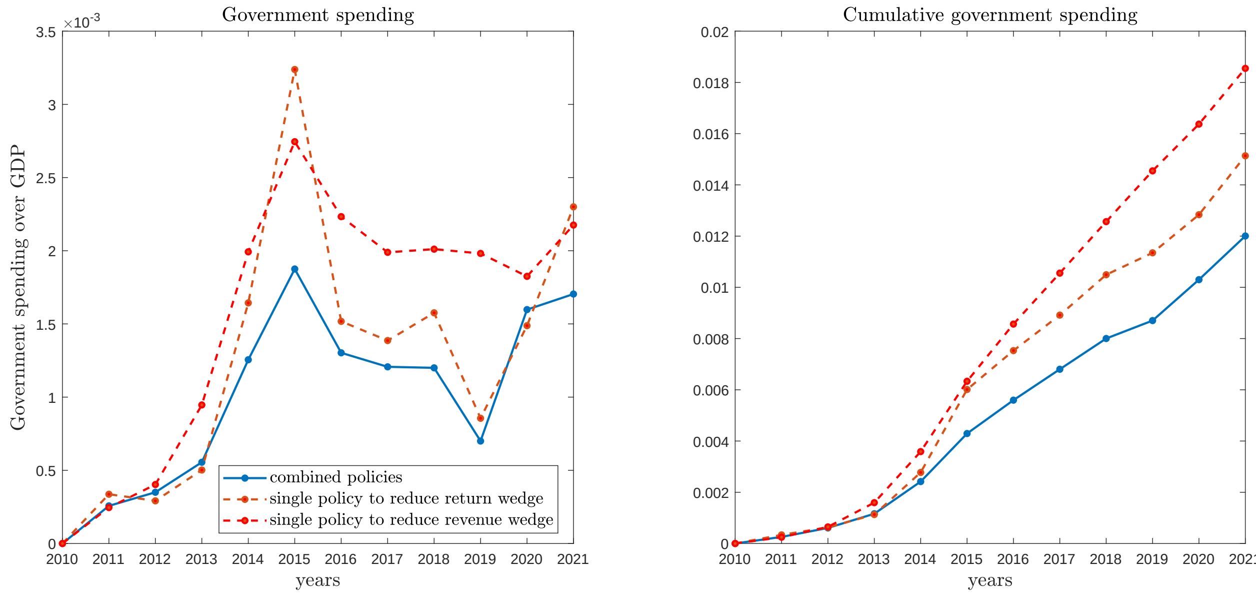

Figure 6: Government spending on policies

During the past decade, China experienced fast growth, and renminbi bond supply also increased a lot, so it is possible that the increase of renminbi payment share was mainly drived by the changes in fundamentals instead of the policy reforms. To verify whether this argument is correct, we consider the counterfactual that there were no policy reforms during 2011-2021. The dashed red line in figure 7 shows pattern of renminbi payment share without policy reforms. We also plot renminbi payment share in combined policies case for comparison. Surprisingly, even though relative country size and renminbi bond supply increase a lot, renminbi payment share nearly remains unchanged. Possible reason is that there exist large frictions in asset market and in before the initiation of policy reforms, and the changes in fundamentals have little effects on payment share due to existing frictions. Moreover, most Chinese firms choose dollar as settlement currency. When size of China increases, firms in the rest of the world are more likely to trade with Chinese firms, and they are more willing to choose to use renminbi as settlement currency to reduce mismatch cost if most Chinese firms use renminbi as settlement currency. However, given the fact most that Chinese firms choose dollar as settlement currency, this channel is very weak.

This result is consistent with the empirical result in Georgiadis et al. (2021). They find that increase in trade share of China strengthens the dominant status of dollar and has positive insignificant effect on invoicing share of renminbi, while creation of swap lines by PBoC is associated with increase in renminbi invoicing. Note that our results don't mean the changes in fundamentals have no effects on international status of the currency. The conclusion depends on domestic condition and current status of the currency. The implication of the result is that policy reforms are important at the initial stage of internationalization process. Effects of changes in fundamentals are limited by existing frictions in initial stage, which needs policy reforms to reduce these frictions to internationalize the domestic currency.

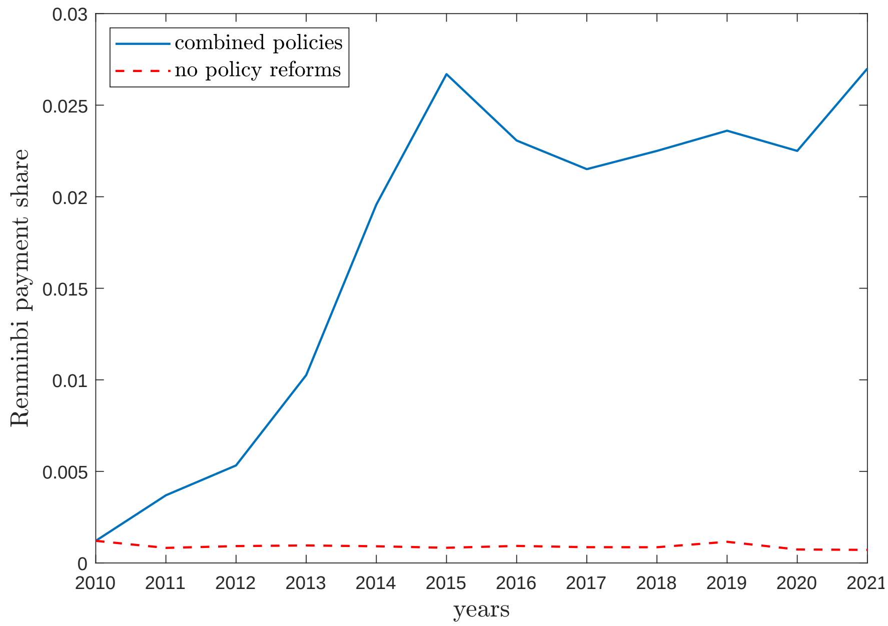

Figure 7: Transition path without the policy reforms

5 Conclusions

China has initiated a series of policies to promote RMB internationalization in the last decades. To quantitative evaluate the effects of these polices, this paper build a multi-country dynamic general equilibrium model that incorporates key functions of an international currency serving both as a store of value and a medium of exchange. We demonstrate there are strong policy complementarities existing in China's case, and thus the combined policies that simultaneously address frictions in the international asset market and trade market are more effective than policies with a single focus. Our quantitative results show that China's reform in the past decade has lowered the return wedge in RMB bond by 4.51%, and lowered the revenue wedge in RMB-settled trade by 1.40%. Moreover, compared to the combined policies, if targeting the observed payment share, it would cost 1.55 times higher for the policies only reducing bonds market frictions, and 1.26 times higher for the policies only reducing RMB-settled trade frictions. Either case, the cost of achieving the internationalization is non-trivial, highlighting the difficult tradeoffs when a country like China aspires to start an extensive process to internationalize its currency.

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Appendix

A Detrended Equilibrium Conditions

A.1 Household

We write down detrended equilibrium conditions in this section. Denote $G_{jt} \equiv \frac{Y_{jt}}{Y_{j,0}}$ and $g_{jt} \equiv \frac{G_{jt}}{G_{jt-1}}$. The detrended budget constraints in the three regions read

$$P_{jt}\hat{C}_{jt} + \left(1-\Delta_{jt}^{USD}\right) P_{us,t}^{us} Q_t^{USD} \hat{B}_{jt}^{USD} + \hat{\psi}_{jt}^{USD} + \left(1-\Delta_{jt}^{RMB}\right) P_{cn,t}^{cn} Q_t^{RMB} \hat{B}_{jt}^{RMB} + \hat{\psi}_{jt}^{RMB}$$
$$= P_{us,t}^{us} \frac{\hat{B}_{jt-1}^{USD}}{g_{j,t}} + P_{cn,t}^{cn} \frac{\hat{B}_{jt-1}^{RMB}}{g_{j,t}} + P_{jt}^{j} + \hat{\Pi}_{jt}^{T} + \hat{T}_{jt}, \quad j \in \{us, cn, rw\} \tag{A.1}$$

where

$$\hat{\psi}_{j,t}^{USD} = P_{us,t}^{us} Q_t^{USD} \tau\left(\hat{B}_{j,t}^{USD}, \underline{\hat{B}}_{j,t}^{USD}\right), \quad \hat{\psi}_{j,t}^{RMB} = P_{cn,t}^{cn} Q_t^{RMB} \tau\left(\hat{B}_{j,t}^{RMB}, \underline{\hat{B}}_{j,t}^{RMB}\right)$$

with $\hat{X}_{jt} \equiv \frac{X_{jt}}{Y_{jt}}$ for variable X, and adjustment cost function takes the form

$$\tau\left(\hat{B}_{j,t}^{i}, \underline{\hat{B}}_{j,t}^{i}\right) \equiv \bar{\tau}\left(\left(\hat{B}_{j,t}^{i} - \underline{\hat{B}}_{j}^{i}\right) / \underline{\hat{B}}_{j}^{i}\right)^2 \underline{\hat{B}}_{j}^{i}/2$$

We have $\underline{\hat{B}}_{j,t}^{i} = \hat{B}_{j,t}^{i}$ for the country-currency pair (us, USD), (rw, USD), and (cn, RMB).

Euler equations:

$$1 = \beta E_t \left[\left(g_{j,t+1} \frac{\hat{C}_{j,t+1}}{\hat{C}_{j,t}} \right)^{-\sigma} \frac{P_{j,t}}{P_{j,t+1}} \frac{P_{us,t+1}^{us}}{P_{us,t}^{us}} \frac{1}{Q_t^{USD}\left(1-\Delta_{j,t}^{USD} + \tau'\left(\hat{B}_{j,t}^{USD}, \underline{\hat{B}}_{j,t}^{USD}\right)\right)} \right], \quad j \in \{us, cn, rw\} \tag{A.2}$$

$$1 = \beta E_t \left[\left(g_{j,t+1} \frac{\hat{C}_{j,t+1}}{\hat{C}_{j,t}} \right)^{-\sigma} \frac{P_{j,t}}{P_{j,t+1}} \frac{P_{cn,t+1}^{cn}}{P_{cn,t}^{cn}} \frac{1}{Q_t^{RMB}\left(1-\Delta_{j,t}^{RMB} + \tau'\left(\hat{B}_{j,t}^{RMB}, \underline{\hat{B}}_{j,t}^{RMB}\right)\right)} \right], \quad j \in \{us, cn, rw\} \tag{A.3}$$

Conditions related to allocation among different goods:

Before writing down first order conditions, we first write down consumption basket in the three regions:

$$\hat{C}_{us,t} = \left(\hat{C}^{us}_{us,t}\right)^{a_{h,us}} \left(\hat{C}^{cn}_{us,t}\right)^{\frac{\left(1-a_{h,us}\right)\widetilde{\mu}_{cn}}{\widetilde{\mu}_{cn}+\widetilde{\mu}_{mw}}} \left(\hat{C}^{rw}_{us,t}\right)^{\frac{\left(1-a_{h,us}\right)\widetilde{\mu}_{rw}}{\widetilde{\mu}_{cn}+\widetilde{\mu}_{rw}}}$$

$$\hat{C}_{cn,t} = \left(\hat{C}^{cn}_{cn,t}\right)^{a_{h,cn}} \left(\hat{C}^{us}_{cn,t}\right)^{\frac{\left(1-a_{h,cn}\right)\widetilde{\mu}_{us}}{\widetilde{\mu}_{us}+\widetilde{\mu}_{rw}}} \left(\hat{C}^{rw}_{cn,t}\right)^{\frac{\left(1-a_{h,cn}\right)\widetilde{\mu}_{rw}}{\widetilde{\mu}_{us}+\widetilde{\mu}_{rw}}}$$

$$\hat{C}_{rw,t} = \left(\hat{C}^{rw}_{rw,t}\right)^{a_{h,rw}} \left(\left(\hat{C}^{us}_{rw,t}\right)^{\widetilde{\mu}_{us}} \left(\hat{C}^{cn}_{rw,t}\right)^{\widetilde{\mu}_{cn}} \left(\hat{C}^{rw}_{rwj,t}\right)^{\widetilde{\mu}_{rw}}\right)^{1-a_{h,rw}}$$

First order conditions:

$$\hat{C}^{us}_{us,t} = a_{h,us}\frac{P_{us,t}\hat{C}_{us,t}}{P^{us}_{us,t}} \tag{A.4}$$

$$\hat{C}^{cn}_{us,t} = (1-a_{h,us})\omega^{cn,rw}_{cn}\frac{P_{us,t}\hat{C}_{us,t}}{P^{cn}_{us,t}} \tag{A.5}$$

$$\hat{C}^{rw}_{us,t} = (1-a_{h,us})\omega^{cn,rw}_{rw}\frac{P_{us,t}\hat{C}_{us,t}}{P^{rw}_{us,t}} \tag{A.6}$$

$$\hat{C}^{cn}_{cn,t} = a_{h,cn}\frac{P_{cn,t}\hat{C}_{cn,t}}{P^{cn}_{cn,t}} \tag{A.7}$$

$$\hat{C}^{us}_{cn,t} = (1-a_{h,cn})\omega^{us,rw}_{us}\frac{P_{cn,t}\hat{C}_{cn,t}}{P^{us}_{cn,t}} \tag{A.8}$$

$$\hat{C}^{rw}_{cn,t} = (1-a_{h,cn})\omega^{us,rw}_{rw}\frac{P_{cn,t}\hat{C}_{cn,t}}{P^{rw}_{cn,t}} \tag{A.9}$$

$$\hat{C}^{rw}_{rw,t} = a_{h,rw}\frac{P_{rw,t}\hat{C}_{rw,t}}{P^{rw}_{rw,t}} \tag{A.10}$$

$$\hat{C}^{us}_{rw,t} = (1-a_{h,rw})\tilde{\mu}_{us}\frac{P_{rw,t}\hat{C}_{rw,t}}{P^{us}_{rw,t}} \tag{A.11}$$

$$\hat{C}^{cn}_{rw,t} = (1-a_{h,rw})\tilde{\mu}_{cn}\frac{P_{rw,t}\hat{C}_{rw,t}}{P^{cn}_{rw,t}} \tag{A.12}$$

$$\hat{C}^{rw}_{rwj,t} = (1-a_{h,rw})\tilde{\mu}_{rw}\frac{P_{rw,t}\hat{C}_{rw,t}}{P^{rw}_{rwj,t}} \tag{A.13}$$

$$P_{us,t} = \frac{1}{K_{us}}\left(P^{us}_{us,t}\right)^{a_{h,us}}\left(\left(P^{cn}_{us,t}\right)^{\omega^{cn,rw}_{cn}}\left(P^{rw}_{us,t}\right)^{\omega^{cn,rw}_{rw}}\right)^{1-a_{h,us}} \tag{A.14}$$

$$P_{cn,t} = \frac{1}{K_{cn}}\left(P^{cn}_{cn,t}\right)^{a_{h,cn}}\left(\left(P^{us}_{cn,t}\right)^{\omega^{us,rw}_{us}}\left(P^{rw}_{cn,t}\right)^{\omega^{us,rw}_{rw}}\right)^{1-a_{h,cn}} \tag{A.15}$$

$$P_{rw,t} = \frac{1}{K_{rw}} \left(P^{rw}_{rw,t}\right)^{a_{h,rw}} \left(\left(P^{us}_{rw,t}\right)^{\tilde{\mu}_{us}} \left(P^{cn}_{rw,t}\right)^{\tilde{\mu}_{cn}} \left(P^{rw}_{rwj,t}\right)^{\tilde{\mu}_{rw}} \right)^{1-a_{h,rw}} \tag{A.16}$$

where

$$K_{us} \equiv a_{h,us}^{a_{h,us}} \left(1 - a_{h,us}\right)^{1-a_{h,us}} \left(\omega^{cn,rw}_{cn}\right)^{\left(1-a_{h,us}\right)\omega^{cn,rw}_{cn}} \left(\omega^{cn,rw}_{rw}\right)^{\left(1-a_{h,us}\right)\omega^{cn,rw}_{rw}} \tag{A.17}$$

$$K_{cn} \equiv a_{h,cn}^{a_{h,cn}} \left(1 - a_{h,cn}\right)^{1-a_{h,cn}} \left(\omega^{us,rw}_{us}\right)^{\left(1-a_{h,cn}\right)\omega^{us,rw}_{us}} \left(\omega^{us,rw}_{rw}\right)^{\left(1-a_{h,cn}\right)\omega^{us,rw}_{rw}} \tag{A.18}$$

$$K_{rw} \equiv a_{h,rw}^{a_{h,rw}} \left(1 - a_{h,rw}\right)^{1-a_{h,rw}} \left(\tilde{\mu}_{us}\right)^{\left(1-a_{h,rw}\right)\tilde{\mu}_{us}} \left(\tilde{\mu}_{cn}\right)^{\left(1-a_{h,rw}\right)\tilde{\mu}_{cn}} \left(\tilde{\mu}_{rw}\right)^{\left(1-a_{h,rw}\right)\tilde{\mu}_{rw}} \tag{A.19}$$

$$\omega^{i,s}_{j} \equiv \frac{\tilde{\mu}_j}{\tilde{\mu}_i + \tilde{\mu}_s} \tag{A.20}$$

$$\tilde{\mu}_j \equiv \frac{\mu_j Y_j}{\mu_j Y_j + \mu_i Y_i + \mu_s Y_s} = \frac{\mu_j}{\mu_j + \mu_i \frac{Y_i}{Y_j} + \mu_s \frac{Y_s}{Y_j}} \tag{A.21}$$

A.2 Firms

1. Firms' trading profit π_{jit} (Stage 3):

$$\pi^{USD,ex}_{jit} = p^{ei}_{jit} \frac{\alpha}{P^{whol}_{jit}} \left[P^j_{i,t} - P^j_{j,t} - \kappa P^{whol}_{jit} \left(1 - \xi^{USD}_{i,t}\right) \right] \tag{A.22}$$

$$\pi^{USD,im}_{jit} = p^{ie}_{jit} \frac{1-\alpha}{P^{whol}_{ijt}} \left[P^i_{j,t} - P^i_{i,t} - \kappa P^{whol}_{ijt} \left(1 - \xi^{USD}_{i,t}\right) \right] \tag{A.23}$$

$$\pi^{RMB,ex}_{jit} = p^{ei}_{jit} \frac{\alpha}{P^{whol}_{jit}} \left[P^j_{i,t} - P^j_{j,t} - \kappa P^{whol}_{jit} \xi^{USD}_{i,t} \right] \tag{A.24}$$

$$\pi^{RMB,im}_{jit} = p^{ie}_{jit} \frac{1-\alpha}{P^{whol}_{ijt}} \left[P^i_{j,t} - P^i_{i,t} - \kappa P^{whol}_{ijt} \xi^{USD}_{i,t} \right] \tag{A.25}$$

where

$$P^{whol}_{jit} = P^j_{j,t} + \alpha \left(P^j_{i,t} - P^j_{j,t}\right) \tag{A.26}$$

$$\xi^{USD}_{i,t} = \frac{p^{USD}_{i,t} X_{i,t}}{p^{USD}_{i,t} X_{i,t} + p^{RMB}_{i,t} \left(1 - X_{i,t}\right)} \tag{A.27}$$

$$p^{ei}_{jit} = \left(1 + \left(\frac{Y_{jt}}{Y_{it}} \frac{\mu_j p^{ex}_{jit} \hat{m}_{jt} \left(p^{USD}_{jt} X_{jt} + p^{RMB}_{jt} \left(1 - X_{jt}\right)\right)}{\mu_i p^{im}_{ijt} \hat{m}_{it} \left(p^{USD}_{it} X_{it} + p^{RMB}_{it} \left(1 - X_{it}\right)\right)} \right)^{1/\varepsilon_T} \right)^{-\varepsilon_T} \tag{A.28}$$

$$p_{jit}^{ie} = \left(1 + \left(\frac{Y_{jt}}{Y_{it}} \frac{\mu_j p_{jit}^{im} \hat{m}_{jt} \left(p_{jt}^{USD} X_{jt} + p_{jt}^{RMB}\left(1 - X_{jt}\right)\right)}{\mu_i p_{ijt}^{ex} \hat{m}_{it}\left(p_{it}^{USD} X_{it} + p_{it}^{RMB}\left(1 - X_{it}\right)\right)}\right)^{1/\varepsilon_T}\right)^{-\varepsilon_T} \tag{A.29}$$

2. Funding decisions (Stage 2):

Probabilities of finding funds:

$$p_{jt}^{USD} = \left(1 + \left(\frac{\hat{m}_{jt} X_{jt}}{\nu P_{us,t}^{us} Q_t^{USD} \hat{B}_{jt}^{USD}}\right)^{1/\varepsilon_F}\right)^{-\varepsilon_F}$$

$$p_{jt}^{RMB} = \left(1 + \left(\frac{\hat{m}_{jt}(1 - X_{jt})}{\nu P_{cn,t}^{cn} Q_t^{RMB} \hat{B}_{jt}^{RMB}}\right)^{1/\varepsilon_F}\right)^{-\varepsilon_F}$$

Firms' profit $\tilde{\Pi}_{jit}$ and Π_{jit}:

$$\tilde{\Pi}_{jt}^{USD} = \sum_{i \neq j} p_{jit}^{im} \pi_{jit}^{USD,im} + \sum_{i \neq j} p_{jit}^{ex} \pi_{jit}^{USD,ex} \tag{A.30}$$

$$\tilde{\Pi}_{jt}^{RMB} = \sum_{i \neq j} p_{jit}^{im} \pi_{jit}^{RMB,im} + \sum_{i \neq j} p_{jit}^{ex} \pi_{jit}^{RMB,ex} \tag{A.31}$$

$$\Pi_{jt}^{USD} = p_{jt}^{USD}(\tilde{\Pi}_{jt}^{USD} - r) \tag{A.32}$$

$$\Pi_{jt}^{RMB} = p_{jt}^{RMB}(\tilde{\Pi}_{jt}^{RMB} - r) \tag{A.33}$$

Currency choices:

$$X_{jt} = \int \mathbb{1}\left(v_{jt}^{USD,(l)} \geq v_{jt}^{RMB,(l)}\right) F\left(d\theta_{jt}^{l}\right) = \frac{1}{1 + \exp\left(\sigma_\theta(\Pi_{jt}^{RMB,(l)} - \Pi_{jt}^{USD,(l)} + \theta_j^{diff})\right)}, \quad j \in \{us, cn, rw \tag{A.34}$$

where

$$v_{jt}^{USD,(l)} = \Pi_{jt}^{USD,(l)} + \theta_{jt}^{USD,(l)}, \quad v_{jt}^{RMB,(l)} = \Pi_{jt}^{RMB,(l)} + \theta_{jt}^{RMB,(l)}$$

$\theta_{jt}^{USD,(l)}$ and $\theta_{jt}^{RMB,(l)}$ are preference shocks that follow Gumbel distribution. θ_j^{diff} is the country specific difference in the location parameter of $\theta_{jt}^{RMB,(l)}$ and $\theta_{jt}^{USD,(l)}$. We assume $\theta_{jt}^{USD,(l)}$ and $\theta_{jt}^{RMB,(l)}$ have same scale parameters σ_θ.

3. Indifference conditions and zero-profit conditions (Stage 1):

Probabilities of choosing to be importers and exporters:

$$\sum_{i \neq j} p_{jit}^{im} + \sum_{i \neq j} p_{jit}^{ex} = 1 \tag{A.35}$$

$$X_j \pi_{jit}^{USD,im} + \left(1 - X_j\right) \pi_{jit}^{RMB,im} = X_j \pi_{ji't}^{USD,im} + \left(1 - X_j\right) \pi_{ji't}^{RMB,im} \tag{A.36}$$

$$X_j \pi_{jit}^{USD,ex} + \left(1 - X_j\right) \pi_{jit}^{RMB,ex} = X_j \pi_{ji't}^{USD,ex} + \left(1 - X_j\right) \pi_{ji't}^{RMB,ex} \tag{A.37}$$

$$X_j \pi_{jit}^{\mathrm{USD},im} + (1 - X_{us}) \pi_{jit}^{RMB,im} = X_{us} \pi_{jit}^{USD,ex} + (1 - X_{us}) \pi_{jit}^{RMB,ex} \tag{A.38}$$

Zero-profit conditions:

$$X_{jt} \Pi_{jt}^{USD} + \left(1 - X_{jt}\right) \Pi_{jt}^{RMB} - \phi_j P_{jt} = 0 \tag{A.39}$$

A.3 Market Clearing Conditions

1. Bonds market clearing conditions:

$$\mu_{us} \hat{B}_{us,t}^{USD} + \mu_{cn} \hat{B}_{cn,t}^{USD} \frac{Y_{cn,t}}{Y_{us,t}} + \mu_{rw} \hat{B}_{rw,t}^{USD} \frac{Y_{rw,t}}{Y_{us,t}} = \mu_{us} \hat{\bar{B}}_{us} \tag{A.40}$$

$$\mu_{us} \hat{B}_{us,t}^{RMB} \frac{Y_{us,t}}{Y_{cn,t}} + \mu_{cn} \hat{B}_{cn,t}^{RMB} + \mu_{rw} \hat{B}_{rw,t}^{RMB} \frac{Y_{rw,t}}{Y_{cn,t}} = \mu_{cn} \hat{\bar{B}}_{cn} \tag{A.41}$$

2. Goods market clearing conditions:

$$\mu_{us} \hat{C}_{us,t}^{us} + \mu_{cn} \hat{C}_{cn,t}^{us} \frac{Y_{cn,t}}{Y_{us,t}} + \mu_{rw} \hat{C}_{rw,t}^{us} \frac{Y_{rw,t}}{Y_{us,t}} = \mu_{us} \tag{A.42}$$

$$\mu_{us} \hat{C}_{us,t}^{cn} \frac{Y_{us,t}}{Y_{cn,t}} + \mu_{cn} \hat{C}_{cn,t}^{cn} + \mu_{rw} \hat{C}_{rw,t}^{cn} \frac{Y_{rw,t}}{Y_{cn,t}} = \mu_{cn} \tag{A.43}$$

$$\mu_{us} \hat{C}_{us,t}^{rw} \frac{Y_{us,t}}{Y_{rw,t}} + \mu_{cn} \hat{C}_{cn,t}^{rw} \frac{Y_{cn,t}}{Y_{rw,t}} + \mu_{rw} \hat{C}_{rwj,t}^{rw} + \mu_{rw} \hat{C}_{rw,t}^{rw} = \mu_{rw} \tag{A.44}$$

$$\hat{C}_{j,t}^{i} = \frac{p_{jit}^{ie} p_{jit}^{im} \hat{m}_{j,t} \left(p_{j,t}^{USD} X_{j,t} + p_{j,t}^{RMB} \left(1 - X_{j,t}\right)\right)}{P_{jit}^{whol}}, \quad j \in \{us, cn\}, i \in \{us, cn, rw\} / j \tag{A.45}$$

$$\mu_{rw} \hat{C}_{rw,t}^{us} = \frac{p_{us,rw,t}^{ei} p_{us,rw,t}^{ex} \mu_{us} \hat{m}_{us,t} \frac{Y_{us,t}}{Y_{rw,t}} \left(p_{us,t}^{USD} X_{us,t} + p_{us,t}^{RMB} \left(1 - X_{us,t}\right)\right)}{P_{us,rw,t}^{whol}} \tag{A.46}$$

$$\mu_{rw} \hat{C}_{rw,t}^{cn} = \frac{p_{cn,rw,t}^{ei} p_{cn,rw,t}^{ex} \mu_{cn} \hat{m}_{cn,t} \frac{Y_{cn,t}}{Y_{rw,t}} \left(p_{cn,t}^{USD} X_{cn,t} + p_{cn,t}^{RMB} \left(1 - X_{cn,t}\right)\right)}{P_{cn,rw,t}^{whol}} \tag{A.47}$$

$$\hat{C}^{rw}_{rwj,t} = \frac{p^{ie}_{rw,rw,t} p^{im}_{rw,rw,t} \hat{m}_{rw,t} \left(p^{USD}_{rw,t} X_{rw,t} + p^{RMB}_{rw,t} \left(1 - X_{rw,t}\right)\right)}{P^{whol}_{rw,rw,t}} \tag{A.48}$$

A.4 Other conditions

1. Liquidity premia:

$$\Delta^{USD}_{jt} = \frac{\nu \hat{m}_{jt} X_{jt}}{\left[\left(\hat{m}_{jt} X_{jt}\right)^{1/\varepsilon_F} + \left(\nu P^{us}_{us,t} \hat{B}^{USD}_{jt} Q^{USD}_t\right)^{1/\varepsilon_F} \right]^{\varepsilon_F}} r \tag{A.49}$$

$$\Delta^{RMB}_{jt} = \frac{\nu \hat{m}_{jt} \left(1 - X_{jt}\right)}{\left[\left(\hat{m}_{jt} \left(1 - X_{jt}\right)\right)^{1/\varepsilon_F} + \left(\nu P^{cn}_{cn,t} \hat{B}^{RMB}_{jt} Q^{RMB}_t\right)^{1/\varepsilon_F} \right]^{\varepsilon_F}} r \tag{A.50}$$

2. Government:

$$\hat{T}_{us,t} = P^{us}_{us,t} (Q^{USD}_t \hat{\bar{B}}_{us,t} - \frac{\hat{\bar{B}}_{us,t-1}}{g_{us,t}}) \tag{A.51}$$

$$\hat{T}_{cn,t} = P^{cn}_{cn,t} (Q^{RMB}_t \hat{\bar{B}}_{cn,t} - \frac{\hat{\bar{B}}_{cn,t-1}}{g_{cn,t}}) \tag{A.52}$$

$$\hat{T}_{rw,t} = 0 \tag{A.53}$$

3. Total profit of country $j's$ firms

$$\hat{\Pi}^T_{jt} = \hat{m}_{jt} (X_{jt} \Pi^{USD}_{jt} + \left(1 - X_{jt}\right) \Pi^{RMB}_{jt}) + \hat{\Gamma}_{jt} \tag{A.54}$$

where

$$\begin{aligned} \hat{\Gamma}_{jt} =& \kappa \hat{m}_{jt} \sum_{i \neq j} X_{jt} p^{USD}_{jt} (\alpha p^{ex}_{jit} p^{ei}_{jit} + (1-\alpha) p^{im}_{jit} p^{ie}_{jit})(1 - \xi^{USD}_{i,t}) \\ &+ \kappa \hat{m}_{jt} \sum_{i \neq j} (1 - X_{jt}) p^{RMB}_{jt} (\alpha p^{ex}_{jit} p^{ei}_{jit} + (1-\alpha) p^{im}_{jit} p^{ie}_{jit}) \xi^{USD}_{i,t} \end{aligned} \tag{A.55}$$

Γ_{jt} is total mismatch cost in country j.

4. Determination of relative endowment

Given $\left\{g_{j,t+1}\right\}^{\infty}_{j,t=0}$, can solve a detrended equilibrium system and then add back the time-varying trend to construct the original equilibrium. This is noted by constructing

$$\frac{Y_{i,t}}{Y_{j,t}} = \frac{Y_{i,0}}{Y_{j,0}} \frac{\prod^t_{s=1} g_{i,s}}{\prod^t_{s=1} g_{j,s}}, \text{ for } t > 0, \tag{A.56}$$

where we calibrate $\frac{Y_{i,0}}{Y_{i0}}$ to match relative GDP per capita in initial year.

According to Walras law, one market clearing condition is redundant. We set $P^{rw}_{rw,t} \equiv 1$.

B Simplified Model

B.1 Household

Households in China and the U.S. face the following maximization problem:

$$\max_{C_{jt}, B^{USD}_{jt}, B^{RMB}_{jt}} E_0 \sum_{t=0}^{\infty} \beta^t u(C_{jt})$$

$$\text{s.t. } C_{jt} + Q^{USD}_t(1-\Delta^{USD}_{jt})B^{USD}_{jt} + Q^{RMB}_t(1-\Delta^{RMB}_{jt})B^{RMB}_{jt} = B^{USD}_{jt-1} + B^{RMB}_{jt-1} + Y_{jt} + T_{jt} + \Pi^T_{jt}, \tag{B.1}$$

In simplified model, we assume $X_{us} = 1$ and $X_{cn} = 0$. Both of them are exogenous. The assumption that $X_{cn} = 0$ is inconsistent with data, but we make this assumption to simplify the algebra and focus on the complementarity between policies. With this assumption, it is easy to see that in steady state, $B^{RMB}_{us} = B^{USD}_{cn} = 0$.[8]

Households in RoW face the maximization problem below:

$$\max_{C_{j,t}, B^{USD}_{j,t}, B^{RMB}_{j,t}} E_0 \sum_{t=0}^{\infty} \beta^t u(C_{j,t}) \tag{B.2}$$

$$\text{s.t. } C_{j,t} + Q^{USD}_t(1-\Delta^{USD}_{j,t})B^{USD}_{j,t} + Q^{RMB}_t(1-\Delta^{RMB}_{j,t})B^{RMB}_{j,t} + \psi_{j,t} = B^{USD}_{j,t-1} + B^{RMB}_{j,t-1} + Y_{j,t} + T_{j,t} + \Pi^T_{j,t}$$

where $\psi_{j,t} = Q^{RMB}_t \frac{\tau}{2}(\frac{B^{RMB}_{j,t} - \underline{B}^{RMB}_{j,t}}{\underline{B}^{RMB}_{j,t}})^2 \underline{B}^{RMB}_j$, which is same with that in the full model expect that we only have one goods in the simplified model and there is no relative price.

We consider simple form of matching function in simplified model:

$$M(u, v) = \frac{uv}{u+v}$$

We also assume bonds enter the matching function with quantity instead of value,

[8] We don't model the bond adjustment cost in China and the U.S. because we still have $B^{RMB}_{us} = B^{USD}_{cn} = 0$ when there exists adjustment costs, so whether there exists adjustment costs has no effect on the results.

then probability of finding dollar bonds and renminbi bonds in country j are given by

$$p_{j,t}^{USD} = \frac{B_{j,t}^{USD}}{B_{j,t}^{USD} + X_{j,t}}, \quad p_{j,t}^{RMB} = \frac{B_{j,t}^{RMB}}{B_{j,t}^{RMB} + 1 - X_{j,t}}$$

and the liquidity premia is

$$\Delta_j^{USD} = \frac{X_{j,t}}{B_{j,t}^{USD} + X_{j,t}} r, \quad \Delta_j^{RMB} = \frac{1 - X_{j,t}}{B_{j,t}^{RMB} + 1 - X_{j,t}} r$$

We consider symmetric equilibrium that $X_{jt} = X_{rw,t}$ for all $j \in [0, \mu_{rw}]$. The steady state euler equations imply that

$$\Delta_{rw}^{USD} = \Delta_{us}^{USD}, \quad \bar{\tau}\left(\frac{B_{rw}^{RMB}}{\underline{B}_{rw}^{RMB}} - 1\right) = \Delta_{rw}^{RMB} - \Delta_{cn}^{RMB} \tag{B.3}$$

Since $\Delta_{rw}^{USD} = \Delta_{us}^{USD}$, we have

$$\frac{B_{rw}^{USD}}{X_{rw}} = B_{us}^{USD} \tag{B.4}$$

Bond market clearing conditions are given by

$$\mu_{rw} B_{rw}^{USD} + \mu_{us} B_{us}^{USD} = \mu_{us} \bar{B}^{USD}$$

$$\mu_{rw} B_{rw}^{RMB} + \mu_{cn} B_{cn}^{RMB} = \mu_{cn} \bar{B}^{RMB}$$

Combining equations (B.3), (B.4) with bond market clearing conditions, we have

$$B_{rw}^{USD} = \frac{\mu_{us} \bar{B}^{USD} X_{rw}}{\mu_{us} + \mu_{rw} X_{rw}} \tag{B.5}$$

$$\bar{\tau}\left(\frac{B_{rw}^{RMB}}{\underline{B}_{rw}^{RMB}} - 1\right) = \frac{1 - X_{rw}}{1 - X_{rw} + B_{rw}^{RMB}} r - \frac{1}{1 + \bar{B}^{RMB} - \frac{\mu_{rw}}{\mu_{cn}} B_{rw}^{RMB}} r \tag{B.6}$$

B.2 Firm

Compared with the full model, we largely simplify the settings related to firms. We assume the trading profit π is exogenous and there is no mismatch cost. Firms just make funding decisions. After they make funding choices and search for bonds successfully, they can certainly match with trading partners and earn net profit $\pi - r$. Same with that in full model, firms face preference shocks when making decisions, where we assume preference shocks are Gumbel with scale parameter σ_θ. Then share of firms with dollar funding in RoW is

$$X_{rw} = (1 + \exp(\sigma_\theta(\Pi_{rw}^{RMB} - \Pi_{rw}^{USD})))^{-1} \tag{B.7}$$

where

$$\Pi_{rw}^{USD} = p_{rw}^{USD}(\pi - r)$$

$$\Pi_{rw}^{RMB} = p_{rw}^{RMB}(\pi - r + \tau_f)$$

Plugging expression of profits into equation (B.7), we get

$$X_{rw} = (1 + \exp(\sigma_\theta(\frac{B_{rw}^{RMB}}{1 - X_{rw} + B_{rw}^{RMB}}(\pi - r + \tau_f) - \frac{\bar{B}^{USD}}{\bar{B}^{USD} + \mu_{us} + \mu_{rw}X_{rw}}(\pi - r))))^{-1} \tag{B.8}$$

Equations (B.6) and (B.8) characterize steady state renminbi bond holdings of households and currency choices of firms in the rest of the world. We conduct analytical analysis based on these two equations.

C Proof of Proposition

C.1 Proof of Proposition 1

Proof: We linearize the system of equations around steady state with $(\underline{B}_{rw}^{RMB}, \tau_f) = (\underline{B}_{rw,0}^{RMB}, \tau_{f,0})$. Note that since we focus on change of level of X_{rw} in this version, we linearize the equations instead of log-linearizing them. We first linearize equation (B.6):

$$-\bar{\tau}\frac{B_{rw,0}^{RMB}}{(\underline{B}_{rw,0}^{RMB})^2}\hat{\underline{B}}_{rw}^{RMB} + \frac{\bar{\tau}}{\underline{B}_{rw,0}^{RMB}}\hat{B}_{rw}^{RMB} = -\frac{B_{rw,0}^{RMB}}{\left(1 - X_{rw,0} + B_{rw,0}^{RMB}\right)^2}r\hat{X}_{rw}$$
$$- \left(\frac{1 - X_{rw,0}}{\left(1 - X_{rw,0} + B_{rw,0}^{RMB}\right)^2}r + \frac{\mu_{rw}r}{\mu_{cn}\left(1 + B_{cn,0}^{RMB}\right)^2}\right)\hat{B}_{rw}^{RMB}$$

Rearranging above equation, we have

$$\hat{B}_{rw}^{RMB} = G_1\hat{\underline{B}}_{rw}^{RMB} - G_2\hat{X}_{rw} \tag{C.1}$$

where

$$G_1 = \frac{\bar{\tau} B^{RMB}_{rw,0}}{H(\underline{B}^{RMB}_{rw,0})^2}$$

$$G_2 = \frac{B^{RMB}_{rw,0} r}{H(1 - X_{rw,0} + B^{RMB}_{rw,0})^2}$$

$$H = \frac{1 - X_{rw,0}}{(1 - X_{rw,0} + B^{RMB}_{rw,0})^2} r + \frac{\bar{\tau}}{\underline{B}^{RMB}_{rw,0}} + \frac{\mu_{rw} r}{\mu_{cn}(1 + B^{RMB}_{cn,0})^2}$$

We next linearize equation (B.8):

$$-\frac{1}{X^2_{rw,0}} \hat{X}_{rw} = (\frac{1}{X_{rw,0}} - 1)\sigma_\theta (P^{RMB}_{rw,0} \hat{\tau}_f + \frac{(1 - X_{rw,0})(\pi - r + \tau_{f,0})}{(1 - X_{rw,0} + B^{RMB}_{rw,0})^2} \hat{B}^{RMB}_{rw} + \frac{B^{RMB}_{rw,0}(\pi - r + \tau_f)}{(1 - X_{rw,0} + B^{RMB}_{rw,0})^2} \hat{X}_{rw} + \frac{\mu_{us} \bar{B}^{USD}(\pi - r)}{(\mu_{us} \bar{B}^{USD} + \mu_{us} + \mu_{rw} X_{rw,0})^2} \hat{X}_{rw})$$

Rearranging above equation, we have

$$\hat{X}_{rw} = -A_1 \hat{\tau}_f - A_2 \hat{B}^{RMB}_{rw} \tag{C.2}$$

where

$$A_1 = \frac{X_{rw,0}(1 - X_{rw,0})\sigma_\theta B^{RMB}_{rw,0}}{F(1 - X_{rw,0} + B^{RMB}_{rw,0})}$$

$$A_2 = \frac{X_{rw,0}(1 - X_{rw,0})^2 \sigma_\theta (\pi - r + \tau_{f,0})}{F(1 - X_{rw,0} + B^{RMB}_{rw,0})^2}$$

$$F = 1 + X_{rw,0}(1 - X_{rw,0})\sigma_\theta (\frac{B^{RMB}_{rw,0}(\pi - r + \tau_{f,0})}{(1 - X_{rw,0} + B^{RMB}_{rw,0})^2} + \frac{\mu_{us} \bar{B}^{USD}(\pi - r)}{(\mu_{us} \bar{B}^{USD} + \mu_{us} + \mu_{rw} X_{rw,0})^2})$$

Combining the two linearized equations, we have

$$\hat{X}_{rw} = -\frac{A_1}{1 - A_2 G_2} \hat{\tau}_f - \frac{A_2 G_1}{1 - A_2 G_2} \hat{\underline{B}}^{RMB}_{rw} \tag{C.3}$$

$$\hat{B}^{RMB}_{rw} = \frac{A_1 G_2}{1 - A_2 G_2} \hat{\tau}_f + \frac{G_1}{1 - A_2 G_2} \hat{\underline{B}}^{RMB}_{rw} \tag{C.4}$$

We next show $0 < 1 - A_2 G_2 < 1$. To see this, $A_2 > 0, G_2 > 0$, and

$$G_2 < \frac{B^{RMB}_{rw,0}}{1 - X_{rw,0}} \Rightarrow A_2 G_2 < \frac{X_{rw,0}(1 - X_{rw,0}) B^{RMB}_{rw,0} \sigma_\theta (\pi - r + \tau_f)}{F(1 - X_{rw,0} + B^{RMB}_{rw,0})^2} < 1$$

Therefore, $\frac{A_1}{1-A_2G_2} > 0$, $\frac{A_2G_1}{1-A_2G_2} > 0$, and an increase in $\underline{B}_{rw}^{RMB}$ or τ_f decreases X_{rw}. In partial equilibrium, the marginal effect of τ_f on X_{rw} is A_1, where

$$A_1 = \frac{X_{rw,0}(1-X_{rw,0})\sigma_\theta \frac{B_{rw,0}^{RMB}}{1-X_{rw,0}+B_{rw,0}^{RMB}}}{1+X_{rw,0}(1-X_{rw,0})\sigma_\theta(\frac{B_{rw,0}^{RMB}(\pi-r+\tau_{f,0})}{(1-X_{rw,0}+B_{rw,0}^{RMB})^2}+\frac{\mu_{us}\bar{B}^{USD}(\pi-r)}{(\mu_{us}\bar{B}^{USD}+\mu_{us}+\mu_{rw}X_{rw,0})^2})}$$
$$= \frac{1}{(\frac{1}{X_{rw,0}(1-X_{rw,0})\sigma_\theta}+\frac{\mu_{us}\bar{B}^{USD}(\pi-r)}{(\mu_{us}\bar{B}^{USD}+\mu_{us}+\mu_{rw}X_{rw,0})^2})(\frac{1-X_{rw,0}}{B_{rw,0}^{RMB}}+1)+\frac{\pi-r+\tau_{f,0}}{1-X_{rw,0}+B_{rw,0}^{RMB}}}$$

We can find that the marginal effect A_1 is a function of $(B_{rw,0}^{RMB}, X_{rw,0}, \tau_{f,0})$, which increases in $B_{rw,0}^{RMB}$.

C.2 Proof of Proposition 2

Proof: In symmetric steady state, we have $X_{rw} = 0.5$, and the system of equations become

$$\bar{\tau}(\frac{B_{rw}^{RMB}}{\underline{B}_{rw}^{RMB}}-1)-\frac{1}{1+2B_{rw}^{RMB}}r = -\frac{1}{1+\bar{B}^{RMB}-\frac{\mu_{rw}}{\mu_{cn}}B_{rw}^{RMB}}r \tag{C.5}$$

$$p_{rw}^{USD}(\pi-r) = p_{rw}^{RMB}(\pi-r+\tau_f) \tag{C.6}$$

Given $\underline{B}_{rw}^{RMB}$, we can solve corresponding (B_{rw}^{RMB}, τ_f). Therefore, we can express variables in symmetric steady state as function of $\underline{B}_{rw}^{RMB}$.

Suppose $\underline{B}_{rw}^{RMB} = \underline{B}_{rw,0}^{RMB}$ at initial dollar-dominant steady state, and government decreases revenue wedge alone to achieve symmetric steady state. We denote $\tau_{f,0}$ as the amount of reduction in wedge that we need to achieve symmetric steady state. When government uses single policy to reduce revenue wedge, the government spending is

$$G(\underline{B}_{rw,0}^{RMB}) = \frac{\mu_{rw}}{\mu_{cn}}\frac{1}{2}p_{rw,0}^{RMB}\tau_{f,0}$$

where $p_{rw,0}^{RMB} = \frac{B_{rw,0}^{RMB}}{1-X_{rw,0}+B_{rw,0}^{RMB}}$ is probability of finding renminbi bonds in RoW in symmetric steady state with $\underline{B}_{rw}^{RMB} = \underline{B}_{rw,0}^{RMB}$. Now suppose government also uses policy to reduce return wedge, the government spending becomes

$$G(\underline{B}_{rw}^{RMB}) = \frac{\mu_{rw}}{\mu_{cn}}(\frac{1}{2}p_{rw}^{RMB}\tau_f+\bar{\tau}\left(\frac{B_{rw,0}^{RMB}}{\underline{B}_{rw,0}^{RMB}}-\frac{B_{rw}^{RMB}}{\underline{B}_{rw}^{RMB}}\right)B_{rw}^{RMB})$$

Plugging equation (C.6) into it, we have

$$G(\underline{B}_{rw}^{RMB}) = \frac{\mu_{rw}}{\mu_{cn}}(\frac{1}{2}(\pi - r)(p_{rw}^{USD} - p_{rw}^{RMB}) + \bar{\tau}\left(\frac{B_{rw,0}^{RMB}}{\underline{B}_{rw,0}^{RMB}} - \frac{B_{rw}^{RMB}}{\underline{B}_{rw}^{RMB}}\right)B_{rw}^{RMB})$$

We next show compared with only using policy to reduce revenue wedge, we can lower the government spending by using combined policies. To show this, we aim to prove $\frac{dG(\underline{B}_{rw}^{RMB})}{d\underline{B}_{rw}^{RMB}}\Big|_{\underline{B}_{rw}^{RMB}=\underline{B}_{rw,0}^{RMB}} < 0$.

Write down the expression of $\frac{dG(\underline{B}_{rw}^{RMB})}{d\underline{B}_{rw}^{RMB}}$:

$$\frac{dG(\underline{B}_{rw}^{RMB})}{d\underline{B}_{rw}^{RMB}} = \frac{\mu_{rw}}{\mu_{cn}}\left(-\frac{\pi - r}{2}\frac{dp_{rw}^{RMB}}{dB_{rw}^{RMB}}\frac{dB_{rw}^{RMB}}{d\underline{B}_{rw}^{RMB}} + \bar{\tau}\left(\frac{B_{rw,0}^{RMB}}{\underline{B}_{rw,0}^{RMB}} - \frac{2B_{rw}^{RMB}}{\underline{B}_{rw}^{RMB}}\right)\frac{dB_{rw}^{RMB}}{d\underline{B}_{rw}^{RMB}} + \bar{\tau}\left(\frac{B_{rw}^{RMB}}{\underline{B}_{rw}^{RMB}}\right)^2\right) \tag{C.7}$$

It is easy to see that

$$\frac{dG(\underline{B}_{rw}^{RMB})}{d\underline{B}_{rw}^{RMB}}\Big|_{\underline{B}_{rw}^{RMB}=\underline{B}_{rw,0}^{RMB}} = \frac{\mu_{rw}}{\mu_{cn}}\left(\left(-\frac{\pi - r}{(1+2B_{rw,0}^{RMB})^2} - \frac{\bar{\tau}B_{rw,0}^{RMB}}{\underline{B}_{rw,0}^{RMB}}\right)\frac{dB_{rw}^{RMB}}{d\underline{B}_{rw}^{RMB}}\Big|_{\underline{B}_{rw}^{RMB}=\underline{B}_{rw,0}^{RMB}} + \bar{\tau}\left(\frac{B_{rw,0}^{RMB}}{\underline{B}_{rw,0}^{RMB}}\right)^2\right)$$

where we plug $\frac{dp_{rw}^{RMB}}{dB_{rw}^{RMB}} = \frac{2}{(1+2B_{rw}^{RMB})^2}$ into equation (C.7).

To prove $\frac{dG(\underline{B}_{rw}^{RMB})}{d\underline{B}_{rw}^{RMB}}\Big|_{\underline{B}_{rw}^{RMB}=\underline{B}_{rw,0}^{RMB}} < 0$, we only need to prove

$$\left(\frac{\pi - r}{(1+2B_{rw,0}^{RMB})^2} + \frac{\bar{\tau}B_{rw,0}^{RMB}}{\underline{B}_{rw,0}^{RMB}}\right)\frac{dB_{rw}^{RMB}}{d\underline{B}_{rw}^{RMB}}\Big|_{\underline{B}_{rw}^{RMB}=\underline{B}_{rw,0}^{RMB}} > \bar{\tau}\left(\frac{B_{rw,0}^{RMB}}{\underline{B}_{rw,0}^{RMB}}\right)^2 \tag{C.8}$$

To prove this, We first find un upper bound for $B_{rw,0}^{RMB}$ based on equation (C.5). When $\tau_f = \tau_{f,0}$ we have

$$\frac{1}{1+\bar{B}^{RMB} - \frac{\mu_{rw}}{\mu_{cn}}B_{rw,0}^{RMB}}r = \frac{1}{1+2B_{rw,0}^{RMB}}r - \bar{\tau}(\frac{B_{rw,0}^{RMB}}{\underline{B}_{rw,0}^{RMB}} - 1)$$

Note that LHS of above equation increases in $B_{rw,0}^{RMB}$ (notice that $\mu_{rw}B_{rw,0}^{RMB} \le \mu_{cn}\bar{B}^{RMB}$), and RHS of above equation decreases in $B_{rw,0}^{RMB}$. When $B_{rw,0}^{RMB} = 0$,

$$\frac{1}{1+\bar{B}^{RMB}}r = LHS < RHS = r + \bar{\tau}$$

and when $B^{RMB}_{rw,0} = \frac{\mu_{cn}}{\mu_{rw}}\bar{B}^{RMB}$,

$$r = LHS > RHS = \frac{1}{1 + \frac{2\mu_{cn}}{\mu_{rw}}\bar{B}^{RMB}}r - \bar{\tau}(\frac{\mu_{cn}\bar{B}^{RMB}}{\mu_{rw}\underline{B}^{RMB}_{rw,0}} - 1)$$

Therefore, there exists a unique solution for above equation. When $B^{RMB}_{rw,0} = \bar{B}^{RMB}_{rw,0} \equiv (1 + \frac{r}{\tau})\underline{B}^{RMB}_{rw,0}$[9],

$$\frac{1}{1 + \frac{1}{\mu_{cn}}(\bar{B}^{RMB} - \mu_{rw}\bar{B}^{RMB}_{rw,0})}r = LHS > RHS = \frac{1}{1 + 2\bar{B}^{RMB}_{rw,0}}r - r$$

Therefore, $\bar{B}^{RMB}_{rw,0} \equiv (1 + \frac{r}{\tau})\underline{B}^{RMB}_{rw,0}$ is an upper bound for $B^{RMB}_{rw,0}$.

We next derive the expression of $\frac{dB^{RMB}_{rw}}{d\underline{B}^{RMB}_{rw}}\Big|_{\underline{B}^{RMB}_{rw} = \underline{B}^{RMB}_{rw,0}}$. According to equation (C.5), we have

$$\left(\frac{2}{(1 + 2B^{RMB}_{rw,})^2}r + \frac{\bar{\tau}}{\underline{B}^{RMB}_{rw}}\right)\frac{dB^{RMB}_{rw}}{d\underline{B}^{RMB}_{rw}} - \frac{\bar{\tau}B^{RMB}_{rw}}{(\underline{B}^{RMB}_{rw})^2} = -\frac{\mu_{rw}r}{\mu_{cn}(1 + B^{RMB}_{cn})^2}\frac{dB^{RMB}_{rw}}{d\underline{B}^{RMB}_{rw}}$$

Rearranging above equation, we get

$$\frac{dB^{RMB}_{rw}}{d\underline{B}^{RMB}_{rw}}\Big|_{\underline{B}^{RMB}_{rw} = \underline{B}^{RMB}_{rw,0}} = \frac{\bar{\tau}\frac{B^{RMB}_{rw,0}}{(\underline{B}^{RMB}_{rw,0})^2}}{\frac{2}{(1+2B^{RMB}_{rw,0})^2}r + \frac{\bar{\tau}}{\underline{B}^{RMB}_{rw,0}} + \frac{\mu_{rw}r}{\mu_{cn}(1+B^{RMB}_{cn,0})^2}} \tag{C.9}$$

Plugging (C.9) into inequality (C.8), it is sufficient to prove

$$\frac{\pi - r}{(1 + 2B^{RMB}_{rw,0})^2} > \left(\frac{2}{(1 + 2B^{RMB}_{rw,0})^2}r + \frac{\bar{\tau}}{\underline{B}^{RMB}_{rw,0}} + \frac{\mu_{rw}r}{\mu_{cn}(1 + B^{RMB}_{cn,0})^2}\right)B^{RMB}_{rw,0}$$

and it is sufficient to find condition such that

$$\frac{\pi - r}{(1 + 2B^{RMB}_{rw,0})^2} - (2 + \frac{\mu_{rw}}{\mu_{cn}})rB^{RMB}_{rw,0} - \frac{\bar{\tau}}{\underline{B}^{RMB}_{rw,0}}B^{RMB}_{rw,0} > 0$$

Notice that LHS of above inequality decreases in $B^{RMB}_{rw,0}$, so above inequality holds if

$$\frac{\pi - r}{(1 + 2\bar{B}^{RMB}_{rw,0})^2} - (2 + \frac{\mu_{rw}}{\mu_{cn}})r\bar{B}^{RMB}_{rw,0} - \frac{\bar{\tau}}{\underline{B}^{RMB}_{rw,0}}\bar{B}^{RMB}_{rw,0} > 0$$

[9] Here we consider the case that $\bar{B}^{RMB}_{rw,0} < \frac{\mu_{cn}}{\mu_{rw}}\bar{B}^{RMB}$. If $\bar{B}^{RMB}_{rw,0} \geq \frac{\mu_{cn}}{\mu_{rw}}\bar{B}^{RMB}$, $\bar{B}^{RMB}_{rw,0}$ automatically serves as un upper bound for $\bar{B}^{RMB}_{rw}$.

where $\bar{B}^{RMB}_{rw,0}$ is upper bound for B^{RMB}_{rw}, which we have derived its expression before. Plugging $\bar{B}^{RMB}_{rw,0} \equiv (1+\frac{r}{\bar{\tau}})\underline{B}^{RMB}_{rw,0}$ into above inequality and rearranging it, we can show that above inequality holds when [10]

$$\pi - r > \underline{\pi}(\underline{B}^{RMB}_{rw,0}, r) \equiv (1 + 2(1+\frac{r}{\bar{\tau}})\underline{B}^{RMB}_{rw,0})^2((2+\frac{\mu_{rw}}{\mu_{cn}})(r+\frac{r^2}{\bar{\tau}})\underline{B}^{RMB}_{rw,0} + r + \bar{\tau})$$

[10]Note that this is sufficient condition, but not the necessary condition.