# What's in a Question? Using Item Response Data to Better Represent Learning 

Jesse Bruhn<br>joint with Mike Gilraine, Jens Ludwig, and Sendhil Mullainathan

Disclaimer: The conclusions of this research do not necessarily reflect the opinions or official positions of the Texas Education Research Center, the Texas Education Agency, the Texas Higher Education Coordinating Board, the Texas Workforce Commission, or the State of Texas

## Testing is a major part of education



Ex: standardized testing and prep occupy as much as $18 \%$ of instructional time.

Tests are used to make high-stakes decisions

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## Tests are used to make high-stakes decisions



## Test results


T. Chris Riley-Tillman | Matthew K. Burns | Stephen P. Kilgus


Which equation is best reoresented bv this araoh?
A $y=-\frac{7}{4} x+4 \quad \mathbf{1}$ Which comparison is true?
B $y=-\frac{7}{4} x+7$
A $68>649$
C $y=-\frac{4}{7} x+4$
D $y=-\frac{4}{7} x+7$
B $571>582$

C $730<806$

D $709<692$

## 12 Janet has 2 new game

- Each game has 3 packs of cards.
- Each pack has 10 cards.
to find the total number of cards Jane has for these 2 games?

F \begin{tabular}{|l|l|l|l|l|l|}
\hline 10 \& 10 \& 10 <br>
\hline

$\quad$

\hline 10 \& 10 \& 10 <br>
\hline
\end{tabular}

G | 3 | 3 | 3 |
| :--- | :--- | :--- |

5 What is the solution to this system of equations?

$$
\begin{gathered}
2 x+y=40 \\
x-2 y=-20
\end{gathered}
$$

H | 3 | 10 |
| :--- | :--- |

10


A $(12,16)$
B $(15,17.5)$
C There is no solution.
D There are an infinite number of solutions.
22 A person dives into a pool from its edge to swim to the other side.
The table shows the depth in feet of the person from the sur
of the water after $x$
quadratic function.

| Pool |  |
| :---: | :---: |
| Time, $x$ (seconds) | Depth of Person from <br> Surface of Water, $d(x)$ (feet) |
| 1 | -2.85 |
| 4 | -8.28 |
| 6 | -9.3 |
| 8.5 | -7.65 |
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| 11.5 | -1.38 |

Which function best models the data?
F $d(x)=0.05 x^{2}+0.74 x$
G $d(x)=0.05 x^{2}+0.74 x+9.17$
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## Aggregator

Performance Measure

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## Statistics behind aggregation (IRT, MIRT)

- Under some assumptions, this will be optimal.

$$
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## What do we lose?

How we evaluate teachers

How we evaluate students

How we evaluate interventions

## What do we lose?

## Teachers:

## Districts use test results to evaluate teachers.

Item data reveals variability obscured by average growth.
As much as 30\% of teachers in bottom decile value-add land in the top decile of item performance.
"Good versus bad teachers" is a less accurate model than "different teachers are differentially good at promoting different aspects of achievement."

In total, aggregation destroys $\sim 60-70 \%$ of the predictable variation in student performance

## What do we lose?

Teachers: $\sim 60-70 \%$ of the
predictable variation in $\longrightarrow$
student performance

## What do we lose?

## Students: We use test scores as proxies for later life outcomes.

In total, aggregation destroys as much as $55 \%$ of predictable variation in graduation, college attendance, and earnings.

Less than 50\% agreement re: "ineffective" educators using predicted student outcomes versus typical aggregates.

Summary statistics using alternative weights lead to different policies and priorities.

## What do we lose?

Teachers:


Students: As much as $55 \%$ of
 long-run outcomes

Due to comparative
advantage across items

Different priorities from different averages

## What do we lose?

## Interventions: The impact of pre-K, small class size, and quality teachers "fades-out" on test scores only to reemerge later in life. <br> Fadeout is heterogeneous item-by-item.

Fade-out is partly an illusion due to changing composition of items across tests

Even very crude alternative weighted averages based on item difficulty can double persistence

Can even find weighted averages that "fade-in"

## What do we lose?

## Teachers:



Due to comparative advantage across items

## Students: <br> As much as $55 \%$ of



Different priorities from different averages

Fadeout: At least 50\% of persistence

## What do we lose?

Teachers: $\sim 60-70 \%$ of the


Due to comparative advantage across items student performance

Students: As much as $55 \%$ of predictable variation in


Different priorities from different averages long-run outcomes

Fadeout: At least $50 \%$ of persistence

## Contribution

## Educational measurement:

- e.g. Anaya et al. (2022) Bond \& Lang (2018), Cascio \& Staiger (2012), Cunha et al. (2008, 2010), Jacob \& Rothstein (2016), Kaur et al. (2023), Lang (2010), Nielsen (2019, 2023), Reyes (2023).
- Explore implications of item aggregation for measuring educational performance.


## Teacher value-add:

- e.g. Chetty et al. (2014a, 2014b), Gilraine \& Pope (2022), Jackson (2018), Mulhern \& Opper (2022), Papay (2011), Rose et al (2022), and many others
- Highlight potential for item data to generate new / nuanced TVA measures.


## Fadeout:

- e.g. Bailey et al (2017), Cascio \& Staiger (2012), Chetty et al. (2011), Currie \& Thomas (1995), Ludwig \& Miller (2007), Deming (2009), Heckman et al (2013), Puma et al (2010), Gray-Lobe et al (2022).
- New explanation based on the changing composition of item content.


## Universe of Texas

K-12 students:

- 4.5 million students
- 14 million student-years
- 1.24 billion student-yeartest items

Linked to:

- Test scores
- Item responses
- Teachers
- Graduation, college attendance, earnings.

|  | Full <br> Sample <br> $(1)$ | Teacher-Student <br> Matched Sample <br> $(2)$ |
| :--- | :---: | :---: |
| Panel A: Standardized Tests <br> \# of items on Math Test <br> \% Correct on Math Test | 52.0 | 49.0 |
| \# of items on English Test | 57.3 | 56.8 |
| \% Correct on English Test | 65.1 | 45.2 |
| Panel B: Demographics |  | 65.9 |
| \% Hispanic | 51.5 | 51.3 |
| \% Black | 12.7 | 13.0 |
| \% Free Lunch Eligible | 51.1 | 51.9 |
| Class Size | - | 22.0 |
| \# of Students | $4,495,344$ | $3,644,164$ |
| \# of Teachers | - | 81,628 |
| Observations | $14,014,753$ | $9,073,848$ |
| (student-year) | $1,240,841,152$ | $855,056,544$ |
| Observations |  |  |

## What do we lose?

Teachers: $\sim 60-70 \%$ of the

student performance

## Students: As much as $55 \%$ of



Fadeout:


Due to comparative advantage across items

## How much information do we lose about teachers?

$$
D_{i q t}=\alpha_{q t}+\Gamma X_{i t}+\eta_{i q t}
$$

$D_{i q t} \Rightarrow$ Takes a value of one if student $i$ correctly answered item $\boldsymbol{q}$ in year $t$.
$\alpha_{q t} \Rightarrow$ Question fixed effect.
$X_{i t} \Rightarrow$ Standard Chetty et al. (2014a,b) vector of teacher value-added covariates, including lagged average score.

## How much information do we lose about teachers?

$D_{i q t}=\alpha_{q t}+\Gamma X_{i t}+\eta_{i q t}$
$\operatorname{var}(\eta) \longrightarrow$ Unexplained student performance
$D_{i q t}=\alpha_{q t}+\delta_{q t(j, t)}+\Gamma X_{i t}+u_{i q t}$
$\delta_{q t i(i, t)} \Rightarrow$ Teacher $j(i, t)$ by item $q$ in year $t$ fixed effect

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$D_{i q t}=\alpha_{q t}+\delta_{j(i, t)}+\Gamma X_{i t}+\epsilon_{i q t}$
$\delta_{t j(i, t)} \Rightarrow$ Teacher $j(i, t)$ by year $t$ fixed effect.

- Up to a scaling, equivalent to "standard" TVA for average scores.

How much information do we lose about teachers?
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$\operatorname{var}(\eta)-\operatorname{var}(u) \longrightarrow$ Explained by teachers
$D_{i q t}=\alpha_{q t}+\delta_{j(i, t)}+\Gamma X_{i t}+\epsilon_{i q t}$
$\operatorname{var}(\epsilon)-\operatorname{var}(u) \longrightarrow$ Lost by averaging.

## How much information do we lose about teachers?



## What kind of info? Comparative advantage.

Teacher Rank

## What do we lose?

## Teachers:



Due to comparative advantage across items

Students: As much as $55 \%$ of

long-run outcomes

Fadeout:


Statistical artifact of
Different priorities from different averages
test composition

How much do we lose about student outcomes?

$$
\begin{aligned}
& Y_{i}=F\left(X_{i}, W_{i}\right)+\eta_{i} \\
& Y_{i}=G\left(\bar{X}_{i}, \bar{W}_{i}\right)+\epsilon_{i}
\end{aligned}
$$

Where:

$$
X_{i}=\left\{x_{i}\right\}_{a \in M} \longrightarrow
$$

Indicator variables denoting exact answers ( $\sim 160$ per grade-year) to math items.
$W_{i}=\left\{w_{i}\right\}_{a \in E} \longrightarrow$
Indicator variables denoting exact answers ( $\sim 160$ per grade-year) to ELA items.

Learned from data using a Gradient
$F()$ and $G()$ Boosted Tree algorithm
(Chen \& Guestrin, 2016)

# How much do we lose about student outcomes? 

## Explanatory Power Loss (\%)

0.4
0.2
0.0

## "Outcome" value-add versus test score value-add



View of "ineffective" varies with individual item weighting

## What do we lose?

## Teachers:



Due to comparative advantage across items

## Students: <br> As much as $55 \%$ of



Different priorities from different averages

Fadeout: At least 50\% of persistence

## Fadeout

Impact of a 1-SD Teacher


But tests aren't like wages...

## Potential explanations:

## Real skill depreciation, similar to fadeout of job training on wages (e.g. Crépon et al., 2013) <br> Non-cognitive skills (Heckman et al., 2013) <br> Artifact of normalization (Cascio and Staiger, 2012)

## Different tests measure different concepts.

## $4^{\text {th }}$ Grade Math item $\longrightarrow$ ? $\longrightarrow \longrightarrow 5^{\text {th }}$ Grade Math item

8 Which equation shows a decimal and a fraction that are equivalent?
F $23.5=23 \frac{5}{100}$
G $23.55=23 \frac{55}{10}$
H $23.05=23 \frac{5}{10}$
J $23.5=23 \frac{50}{100}$

2 A worker is building toys at a factory. The relationship between the
 Which graph represents this relationship?



Is fadeout uniform across items?



Fadeout is not uniform across items.


## Crude reweighting schemes can double persistence.



## Even find fade-in for certain weighted averages.



## What do we lose?

Teachers: $\sim 60-70 \%$ of the
predictable variation in student performance

Students: As much as $55 \%$ of predictable variation in long-run outcomes

Fadeout: At least 50\% of persistence

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Different priorities from different averages

Statistical artifact of test composition

Thank you!

