What’s in a Question? Using Item Response Data to Better Represent Learning

Jesse Bruhn

joint with Mike Gilraine, Jens Ludwig, and Sendhil Mullainathan

Disclaimer: The conclusions of this research do not necessarily reflect the opinions or official positions of the Texas Education Research Center, the Texas Education Agency, the Texas Higher Education Coordinating Board, the Texas Workforce Commission, or the State of Texas
Testing is a major part of education

Ex: standardized testing and prep occupy as much as 18% of instructional time.
(Nelson et al, 2013)
Tests are used to make high-stakes decisions
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Test results
15. The graph of a linear function is shown on the grid.

Which equation is best represented by this graph?

A  \( y = -\frac{7}{4}x + 4 \)  
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C  \( y = \frac{4}{7}x + 4 \)  
D  \( y = \frac{4}{7}x + 7 \)

1 Which comparison is true?

A  68 > 649  
B  571 > 582  
C  730 < 806  
D  709 < 692

12. Janet has 2 new games.

- Each game has 3 packs of cards.
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Which model can be used to find the total number of cards Janet has for these 2 games?

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5. What is the solution to this system of equations?

\[
2x + y = 40 \\
x - 2y = -20
\]

A  (12, 16)  
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\begin{array}{ccc}
10 & 10 & 10 \\
10 & 10 & 10 \\
\end{array}
\]

G
\[
\begin{array}{ccc}
3 & 3 & 3 \\
3 & 3 & 3 \\
\end{array}
\]

H
\[
\begin{array}{ccc}
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- Under some assumptions, this will be optimal.
What information do we lose?

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\[ p_i(\theta) = c_i + \frac{1 - c_i}{1 + e^{-a_i(\theta - b_i)}} \]

Average of items

What information do we lose?
What do we lose?

How we evaluate teachers

How we evaluate students

How we evaluate interventions
What do we lose?

Teachers: Districts use test results to evaluate teachers. Item data reveals variability obscured by average growth. As much as 30% of teachers in bottom decile value-add land in the top decile of item performance.

“Good versus bad teachers” is a less accurate model than “different teachers are differentially good at promoting different aspects of achievement.”

In total, aggregation destroys ~60-70% of the predictable variation in student performance.
What do we lose?

Teachers: ~60-70% of the predictable variation in student performance

Due to teacher comparative advantage
What do we lose?

**Students:** We use test scores as proxies for later life outcomes.

In total, aggregation destroys as much as 55% of predictable variation in graduation, college attendance, and earnings.

Less than 50% agreement re: “ineffective” educators using predicted student outcomes versus typical aggregates.

Summary statistics using alternative weights lead to different policies and priorities.
What do we lose?

**Teachers:** ~60-70% of the predictable variation in student performance

**Due to comparative advantage across items**

**Students:** As much as 55% of predictable variation in long-run outcomes

**Different priorities from different averages**
What do we lose?

**Interventions:** The impact of pre-K, small class size, and quality teachers “fades-out” on test scores only to reemerge later in life.

Fadeout is heterogeneous item-by-item.

**Fade-out is partly an illusion due to changing composition of items across tests**

Even very crude alternative weighted averages based on item difficulty can double persistence.

Can even find weighted averages that “fade-in”
What do we lose?

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Different priorities from different averages

Fadeout: At least 50% of persistence

Statistical artifact of test composition
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Statistical artifact of test composition
Contribution

Educational measurement:


• Explore implications of item aggregation for measuring educational performance.

Teacher value-add:

• e.g. Chetty et al. (2014a, 2014b), Gilraine & Pope (2022), Jackson (2018), Mulhern & Opper (2022), Papay (2011), Rose et al (2022), and many others

• Highlight potential for item data to generate new / nuanced TVA measures.

Fadeout:


• New explanation based on the changing composition of item content.
Universe of Texas K-12 students:

- 4.5 million students
- 14 million student-years
- 1.24 billion student-year-test items

Linked to:

- Test scores
- Item responses
- Teachers
- Graduation, college attendance, earnings.

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<tr>
<td># of items on Math Test</td>
<td>52.0</td>
<td>49.0</td>
</tr>
<tr>
<td>% Correct on Math Test</td>
<td>57.3</td>
<td>56.8</td>
</tr>
<tr>
<td># of items on English Test</td>
<td>44.0</td>
<td>45.2</td>
</tr>
<tr>
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<td>65.1</td>
<td>65.9</td>
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<tr>
<td>% Hispanic</td>
<td>51.5</td>
<td>51.3</td>
</tr>
<tr>
<td>% Black</td>
<td>12.7</td>
<td>13.0</td>
</tr>
<tr>
<td>% Free Lunch Eligible</td>
<td>51.1</td>
<td>51.9</td>
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<tr>
<td>Class Size</td>
<td>-</td>
<td>22.0</td>
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<td># of Students</td>
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<tr>
<td># of Teachers</td>
<td>-</td>
<td>81,628</td>
</tr>
<tr>
<td>Observations (student-year)</td>
<td>14,014,753</td>
<td>9,073,848</td>
</tr>
<tr>
<td>Observations (student-item-year)</td>
<td>1,240,841,152</td>
<td>855,056,544</td>
</tr>
</tbody>
</table>
What do we lose?

Teachers: ~60-70% of the predictable variation in student performance

Students: As much as 55% of predictable variation in long-run outcomes

Fadeout: At least 50% of persistence

Due to comparative advantage across items

Different priorities from different averages

Statistical artifact of test composition
How much information do we lose about teachers?

\[ D_{iqt} = \alpha_{qt} + \Gamma X_{it} + \eta_{iqt} \]

\( D_{iqt} \Rightarrow \) Takes a value of one if student \( i \) correctly answered item \( q \) in year \( t \).

\( \alpha_{qt} \Rightarrow \) Question fixed effect.

\( X_{it} \Rightarrow \) Standard Chetty et al. (2014a,b) vector of teacher value-added covariates, including lagged average score.
How much information do we lose about teachers?

\[ D_{iqt} = \alpha_{qt} + \Gamma X_{it} + \eta_{iqt} \]

\[ \text{var}(\eta) \quad \rightarrow \quad \text{Unexplained student performance} \]

\[ D_{iqt} = \alpha_{qt} + \delta_{qtj(i,t)} + \Gamma X_{it} + u_{iqt} \]

\[ \delta_{qtj(i,t)} \quad \Rightarrow \quad \text{Teacher } j(i, t) \text{ by item } q \text{ in year } t \text{ fixed effect} \]
How much information do we lose about teachers?

\[ D_{iqt} = \alpha_{qt} + \Gamma X_{it} + \eta_{iqt} \]

\[ \text{var}(\eta) \rightarrow \text{Unexplained student performance} \]

\[ D_{iqt} = \alpha_{qt} + \delta_{qtj(i,t)} + \Gamma X_{it} + u_{iqt} \]

\[ \text{var}(\eta) - \text{var}(u) \rightarrow \text{Explained by teachers} \]

\[ D_{iqt} = \alpha_{qt} + \delta_{j(i,t)} + \Gamma X_{it} + \epsilon_{iqt} \]

\[ \delta_{tj(i,t)} \Rightarrow \text{Teacher } j(i, t) \text{ by year } t \text{ fixed effect.} \]

- Up to a scaling, equivalent to “standard” TVA
  for average scores.
How much information do we lose about teachers?

\[ D_{iqt} = \alpha_{qt} + \Gamma X_{it} + \eta_{iqt} \]

\[ \text{var}(\eta) \quad \rightarrow \quad \text{Unexplained student performance} \]

\[ D_{iqt} = \alpha_{qt} + \delta_{qt(i,t)} + \Gamma X_{it} + u_{iqt} \]

\[ \text{var}(\eta) - \text{var}(u) \quad \rightarrow \quad \text{Explained by teachers} \]

\[ D_{iqt} = \alpha_{qt} + \delta_{j(i,t)} + \Gamma X_{it} + \epsilon_{iqt} \]

\[ \text{var}(\epsilon) - \text{var}(u) \quad \rightarrow \quad \text{Lost by averaging.} \]
How much information do we lose about teachers?

Predictable Variation Loss (%)
What kind of info? Comparative advantage.

<table>
<thead>
<tr>
<th>Teacher Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Best</td>
</tr>
<tr>
<td>Worst</td>
</tr>
<tr>
<td>Individual Item Performance</td>
</tr>
<tr>
<td>Best</td>
</tr>
<tr>
<td>Average Performance</td>
</tr>
<tr>
<td>Worst</td>
</tr>
</tbody>
</table>
What do we lose?

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Different priorities from different averages

**Fadeout:** At least 50% of persistence

Statistical artifact of test composition
How much do we lose about student outcomes?

\[
Y_i = F(X_i, W_i) + \eta_i \\
Y_i = G(\bar{X}_i, \bar{W}_i) + \epsilon_i
\]

Where:

\(X_i = \{x_i\}_{a \in M} \rightarrow\) Indicator variables denoting exact answers (~160 per grade-year) to math items.

\(W_i = \{w_i\}_{a \in E} \rightarrow\) Indicator variables denoting exact answers (~160 per grade-year) to ELA items.

\(F()\) and \(G()\) \rightarrow\) Learned from data using a Gradient Boosted Tree algorithm (Chen & Guestrin, 2016)
How much do we lose about student outcomes?

<table>
<thead>
<tr>
<th>Explanatory Power Loss (%)</th>
<th>Test Scores</th>
<th>Graduation</th>
<th>College</th>
<th>Earnings</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
“Outcome” value-add versus test score value-add

View of “ineffective” varies with individual item weighting
What do we lose?

**Teachers:** ~60-70% of the predictable variation in student performance  ⇒  Due to comparative advantage across items

**Students:** As much as 55% of predictable variation in long-run outcomes  ⇒  Different priorities from different averages

**Fadeout:** At least 50% of persistence  ⇒  Statistical artifact of test composition
Real skill depreciation, similar to fadeout of job training on wages (e.g. Crépon et al., 2013)

Potential explanations:
- Real skill depreciation, similar to fadeout of job training on wages (e.g. Crépon et al., 2013)
- Non-cognitive skills (Heckman et al., 2013)
- Artifact of normalization (Cascio and Staiger, 2012)

But tests aren’t like wages…
Different tests measure different concepts.

4th Grade Math item

8 Which equation shows a decimal and a fraction that are equivalent?

F 23.5 = 23\(\frac{5}{100}\)

G 23.55 = 23\(\frac{55}{10}\)

H 23.05 = 23\(\frac{5}{10}\)

J 23.5 = 23\(\frac{50}{100}\)

5th Grade Math item

2 A worker is building toys at a factory. The relationship between the number of hours the employee works, \(x\), and the number of toys the employee builds, \(y\), is represented by the equation \(y = 9x\).

Which graph represents this relationship?

Is fadeout uniform across items?
Fadeout is not uniform across items.
Crude reweighting schemes can double persistence.
Even find **fade-in** for certain weighted averages.
What do we lose?

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Due to comparative advantage across items

Students: As much as 55% of predictable variation in long-run outcomes

Different priorities from different averages

Fadeout: At least 50% of persistence

Statistical artifact of test composition

Thank you!