RISK TAKING UNDER ASSIMILATION AND CONTRAST: THEORY, EXPERIMENTS, AND APPLICATIONS

GIORGIO SAPONARO[†]

Abstract. Field evidence suggests that gradual changes are often not obvious to agents, but large and sudden changes frequently result in overreactions. I develop, apply, test, and structurally estimate a portable model of history-dependent decision making under risk that produces this phenomenon (known as the "boiling-frog effect") as the interaction between memory and attention. If a risky prospect looks similar to the past, it does not catch attention. If it is very dissimilar, it is contrasted away from the past and receives too much attention. I provide an experimental test of the model and display evidence of four novel effects of history-dependent risk preferences: (i) assimilation, (ii) contrast, (iii) the boiling-frog and (iv) a recency effect. The model parsimoniously reconciles some pricing anomalies in asset and housing markets and provides a novel comparative static.

1. Introduction

Gradual changes attract less attention than drastic ones. For example, a person might fail to appreciate the cumulative worsening of his health conditions when this happens gradually, like when aging. In contrast, he might be shocked by a drastic, even if transient, health deterioration and overreact because of it. Studies from financial markets show that news is appreciated less when it is received gradually, compared to when it is received drastically: Da et al. (2014), and Grinblatt and Moskowitz (2004) find that past gradual price increases predict higher future returns than drastic increases of the same cumulative magnitude. Pricing instability of changes in fundamentals is also found in housing markets. Prices overreact to the opening of industrial plants that pose serious health risks (Currie et al. 2015, Sanders 2012).[1] On the other hand, prices do not react when hazardous pollution changes in a gradual fashion (Greenstone and Gallagher 2008).

I develop, test and apply a model of decision making that shows how these and other instabilities in evaluation of risky prospects arise as the byproduct of the interaction between memory and attention. When a decision maker faces a risky prospect she spontaneously thinks of past similar prospects she faced, and thinks more easily about the ones faced closer in time. The past is an anchor for her

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[1]Relatedly, Davis (2004) shows a negligible house price reaction when evidence of a cancer cluster is dispersed across multiple years, and a very significant price change when a number of cases appears within the same year.

evaluation. If the currently faced prospect looks like the past, she assimilates towards it and does not pay much attention to the change. If the current prospect is very different from the past, she contrasts away from it, and pays too much attention to the change.[2] This leads the decision maker to underreact to a cumulatively sizeable gradual change: at each period she is biased by recent past remembered expectations and does not notice enough the small bit of additional news because of assimilation. In the alternative case where the news comes in one big chunk, the remembered past prospects look very different from the updated prospect, and the decision maker overreacts to the news. Assimilation is different from one of the premises of Prospect Theory (Kahneman and Tversky, 1979) that people are sensitive to small changes. The model can be seen as the most natural analogue of Bordalo et al. (2020) in the domain of choice under risk.

1.1. **Model and applications.** To fix ideas, suppose an individual evaluates the health risk from air pollution of her neighbourhood, that is a random damage D. When she considers D she spontaneously retrieves past air pollution risks that she faced, and she collapses them in an average D^m. More recent memories are easier to retrieve, hence D^m relies more on the recent past. She anchors the evaluation on the expected past damage $\mathbb{E}[D^m]$ and pays attention to how D differs from D^m. I model how much she notices the change from D^m to D with the attention weight $g(\mathbb{E}[D] - \mathbb{E}[D^m]) \geq 0$. Her evaluation is anchored to the past expected damage and adjusted to the change from the past, with attention weight g

$$\underbrace{\mathbb{E}[D^m]}_{\text{anchor}} + \overbrace{g(\mathbb{E}[D] - \mathbb{E}[D^m])}^{\text{attention weight}} \left(\underbrace{\mathbb{E}[D] - \mathbb{E}[D^m]}_{\text{adjustment}} \right).$$

If D looks similar to D^m then $0 < g(\mathbb{E}[D] - \mathbb{E}[D^m]) < 1$: in this case the decision maker does not pay much attention to the change in health risk $\mathbb{E}[D] - \mathbb{E}[D^m]$ and underreacts to it. If D contrasts away from D^m then $g(\mathbb{E}[D] - \mathbb{E}[D^m]) > 1$: in this case she pays too much attention to the change in expectation and overreacts to it.

Take for example three individuals, *Ann*, *Bob*, and *Carl* that live in different places, evaluate the yearly health risk from pollution in their residential area, and decide at which price they are willing to sell their house. In the place where *Ann* lives, the *yearly* health risk is 2% in week 1, 2.1% in week 2, 2.2% in week 3, 2.3% in week 4, and it jumps to 5% in week 5. Instead *Bob* receives news in a more gradual fashion: the health risk is 2% in week 1, 2.8% in week 2, 3.6% in week 3, 4.2% in week 4, and it is 5% in week 5. Instead, *Carl* does not receive news and faces a health risk equal to 5%. When evaluating the risk in week 5, *Ann* retrieves much lower past risks: 5% contrasts away from them and seems large ($g > 1$). This makes *Ann* willing to sell at a lower price than *Carl*. Instead, *Bob* retrieves

[2]This is a manifestation of a natural tendency, widely detected in decades of research in cognitive psychology (measuring perception of physical quantities, Helson and Rohles, 1959), to assimilate an item to a reference that is close to it, and to contrast an item that is largely different from the reference. The presence of these two effects is also documented in studies in social psychology, pioneered by Sherif, M., & Hovland, C. I. (1961), where assimilation and contrast is found in assessment of people, groups and objects (Herr, Sherman, and Fazio 1983, Strack, Schwarz, and Gschneidinger 1985, Herr 1986, Lombardi, Higgins, and Bargh 1987, Manis, Nelson, and Shedler 1988, Wilder and Thompson 1988, Shimp, Stuart, and Engle 1991).

more similar risks from the past and assimiliates the 5% risk to them, so that the 5% risk seems smaller ($g < 1$). So *Bob* is willing to sell at a higher price than *Carl*.

The model yields predictions that are testable in the lab through binary choices between gambles. (i) An individual is less likely to choose a gamble that has improved gradually over time than one that has improved sizeably all of the sudden. I call this property, following folk wisdom, the boiling-frog effect in choice under risk. This term comes from the (empirically disputed) conjecture that a frog would jump out of a pan whose water is being heated up quickly, while it would stay in it if water were heated up gradually, and eventually die. The model also yields two predictions that are symmetric with respect to each other. (ii) An individual is more likely to choose a gamble if it comes as a large improvement from the past, rather than as a large worsening from the past. This is the contrast effect, because the gamble is contrasted against the past and looks much better or worse than it. The assimilation effect is the opposite phenomenon: a gamble that departs from the past by a small amount looks similar to it: (iii) an individual is more likely to accept a gamble after a mild worsening rather than a mild improvement.[3]

I show in Appendix E that the model can coherently and parsimoniously reconcile asset pricing anomalies in light of assimilation or contrast, which depend on the size of the change in fundamentals. In the logic of prediction (i), when positive news about a stock is given in small bits, investors' evaluations are assimilated to what they remember. Instead, contrast occurs when the news is given drastically. After a big chunk of positive news, the price goes up by more than when the cumulatively equal amount of news is released gradually. In the long run investors correct the price towards the rational benchmark, so that returns after a stream of small positive news are larger than after a big chunk of good news, as shown in Grinblatt and Moskowitz (2004) and Da et al. (2014). In a housing market application, the market notices (neglects) drastic (gradual) changes in pollution because the updated expectations are contrasted against (assimilated to) the past. For this mechanism, pollution premia measured via time variation within location in pollution are larger than premia measured in the crossection, if the change in pollution is drastic (as found for air quality by Chay and Greenstone, 2005), and smaller than crossectional premia if the change is gradual (as found by Greenstone and Gallagher, 2008 for hazardous waste).[4][5] Moreover, the contrast effect (ii) is consistent with findings in Currie et al. (2015): a drastic

[3] More formally, a gamble is chosen more frequently if it is close but dominated (in first-order stochastic sense) by a past recent one, than if it is close and dominant.

[4] Assimilation explains evidence in financial markets presented by Giglio and Shue (2014).They find that the passage of time after a merger announcement predicts changes in future probability of success, but such changes are not fully priced by the market. The model says that the gradual nature of these changes in expectations produces underreaction, hence predicting future returns.
Assimilation also can help make sense of the positive average returns of stocks experiencing predictable long run demographic shifts that gradually affect the customer base of their industry, as in Della Vigna and Pollet (2007).

[5] Murfin and Spiegel (2020), indeed finds no discount in houses exposed to sea level rise even controlling for altitude. Harrison, Smersh, and Schwartz (2001), relatedly, finds that the price discount in exposed areas is less than the present discounted value of insurance premia. Further evidence of underreaction of pricing of long term sea level rise is given by Keys (2020) that finds no premia in Florida between 2013 and 2018, and some risk premia emerging between 2018 and 2020.

increase in health risk, caused by the opening of a polluting plant, creates contrast of the present against the past. Price overshoots on impact and partly reverts as people get accustomed to the plant.[6]

A comparative static of the model can be tested with US stock market returns. When there is on average small news (hence volatility is small), investors are inattentive: inattention generates positive autocorrelation of returns. On the other hand when investors receive big news (hence volatility is high), they pay too much attention to big news and overreact on average: this generates negative autocorrelation. I test a weaker version of this prediction in the US stock market and find that the volatility of returns negatively predicts subsequent return autocorrelation both at the market and at the individual stock level.

1.2. **Experiments.** I test predictions (i), (ii) and (iii) with between-subjects experiments administered to an online sample of 2010 people. Subjects face sequences of either 4 or 5 choice sets that comprise two binary lotteries. The risky lottery pays a higher payoff with a smaller chance than the safe lottery, and both lotteries pay \$0 with some chance. The safer lottery is the same across all treatments and stages of the choice sequences. One choice set is the same in all treatments. Subjects differ in the history of choice sets faced before the common choice set. In the first (second) "contrast" treatment, the history of choice sets displays a riskier option that has a much higher (lower) probability of upside, compared to the one in the common final choice set. In the first (second) "assimilation" treatment, choice sets are faced such that the probability of upside of the riskier option declines (increases) across stages by a small percentage in each stage. As an example, in the "assimilation" treatments one subject sees that the risky option improves gradually across stages, by paying \$80 with 2% in Stage 1, 5% in Stage 2, 6% in Stage 3, 7% in Stage 4, and 9% in Stage 5. Another subject, instead, faces the same sequence of risky options in reverse order. As a result, when judging the same gamble, the two subjects differ in that one has just faced a similar slightly worse option, while the other has seen a similar slightly better one.[7] As another example, the test of prediction (i) is obtained by comparing the second "contrast" treatment against the the second "assimilation" treatment. In both treatments the target risky option at the final stage pays \$80 with 9% chance and \$0 otherwise (the alternative pays \$9 with 60% chance in all stages). In the "contrast" treatment the final stage is preceded by three stages where the risky option pays \$80, respectively, with $2\%, 4\%$ and 3% chance. Subjects in the "assimilation" treatment face risky options' probabilities in this order: 2%, 5%, 6%, 7%. I run analogous treatments where, instead of the upside's probability, the upside's payoff changes in a fashion similar to the mentioned four treatments.

Results show evidence of all of the three predictions, and make the case for the proposition that individual decisions depend on the recent past, with biases of contrast and assimilation. (i) I find higher take-up (32 pp vs. 23 pp) of the riskier option by subjects who faced a history of largely less-paying risky options compared to a history of less-paying risky options whose upside probability mildly increased in each stage. This suggests that the drastic change in a risky gamble is noticed more than a gradual one. (ii) I find higher take-up (32 pp vs. 11 pp) of the risky gamble by subjects who previously faced

[6]Relatedly, Bin and Landry (2013) estimate a discount between 6% and 20% of houses in a flood zone after a hurricane, but such discount diminishes with time from the event, suggesting overreaction.

[7]With better and worse I mean, respectively, a first-order stochastically dominant gamble and a dominated one.

a history of significantly higher-paying risky options than a history of significantly lower-paying ones. This suggests that a drastic upward change in a gamble boosts its evaluation up, or that a drastic downward change depresses it. (iii) I find higher take-up (21 pp vs. 16 pp) of the risky gamble by subjects who previously faced a history of mildly higher-paying risky options than a history of mildly lower-paying ones. This suggests that a small downward change in a gamble distorts its evaluation up, while a small upward change distorts it down. The first two results also clearly hold in treatments where one lottery changes in the payoff value as opposed to its probability, while a weaker version of the third result holds.

I estimate the preference parameters of my model and provide indirect evidence of a shape in valuation akin to the one theorized by Bordalo et al (2020) in consumer choice. To my knowledge, this work is the first one to estimate a reference dependent valuation function of this family. Valuation shrinks towards the reference point for options similar to it, and is exaggerated away for options different from it.[8]

The model predicts also (iv) a recency effect in choice: a given history of past gambles is more likely to generate contrast if the most recent gambles are far away from the one under evaluation, than if they are close. I design two experimental treatments where the same choice sets before a final choice set are faced in different order across subjects, and find strong empirical support for the recency effect.

1.3. **Related Literature.**

1.3.1. *Experiments.* To the best of my knowledge, this work is the first to provide experimental evidence of sequential contrast in risk taking and the simplest to provide evidence of sequential assimilation in risk taking and the boiling frog effect. Two experimental works are primarily related to this paper. Schram and Sonneman (2011) finds that in a virtual experiment of insuring against a health risk with some switching costs, subjects stick longer with an undercovering plan if the simulated health conditions decline gradually, as opposed to when they decline drastically. My experiment detects the Boiling Frog effect in a neutral setting without any explicit switching cost. Frydman and Jin (2022) finds that risk taking is smaller when the risky lottery is within a sequence of lotteries with relatively lower upsides: I show that the model can recast such evidence as assimilation in risk taking.

Previous literature asking subjects a sequence of choices has focused on other phenomena than this paper, such as the endowment effect (Sprenger 2015) and compromise effect in choice under risk, typically elicited via multiple price lists (Miller, Meyer and Lanzetta, 1969; Birnbaum 1992; Binswanger, 1981; Murnighan, Roth and Schoumaker, 1987; Harrison, Lau, Rutstrom, and Sullivan 2005; Freeman, Halevy, and Kneeland, 2019; Beauchamp, Benjamin, Laibson, Chabris, 2020). The design of the present paper involves sequential choices without subjects knowing how many of the total choices they will have to make, nor they being reminded what they chose, nor they being able to change their previous choices. Moreover, as shown in the Appendix B, when subjects face all questions in the same page, the assimilation effect disappears.

[8]I also study the counterfactual scenario where decision maker face a high payoff reference point. The estimates predict that contrast in probability changes is stronger when stakes are higher. This suggests that in real world high stakes problems overreaction is more likely to occur.

1.3.2. *Theories.* The joint results of the experiment I design in this paper are hard to account with existing models of risky choice. Under the assumption of plausible expectations, expectations-based reference-dependence (Koszegi and Rabin 2006), can only capture assimilation, and not the boiling frog and the contrast effect. Theories of underreaction (including rational inattention Sims, 2003, Caplin and Dean, 2015, sparsity as in Gabaix, 2014 and noisy perception of numbers as in Woodford 2012, 2014) can explain assimilation as the product of costly attention (the first two theories), or of estimation of noisy numbers using the previously observed numbers as a prior (the latter theory). They predict that as shocks get larger (either if the signal is stronger like in rational inattention or the variance gets larger as in sparsity) the decision maker should act more rationally, while my theory, as tested experimentally, also features overreaction, that is not rational. Bushong et al. (2021) can capture the boiling frog effect but not the evidence of contrast. The contrast effect cannot easily be explained by Expectations Based Reference Dependence (Koszegi and Rabin, 2006, 2009). In the simplest case of binary lotteries, the theory predicts that, if people expect the past chosen lottery, they prefer to take the risky lottery most similar to it rather than a safer one.[9]

The financial application of the model can feature dynamics of medium term momentum (Jegadeesh and Titman, 1993) and delayed overreaction (De Bondt and Thaler 1985) found empirically in financial markets, which is modelled by Barberis et al. (1998), Daniel, Hirshleifer and Subrahmaniam (1998), Hong and Stein (1999), and Mullainathan (2002). The boiling-frog effect is a distinguishing prediction of my model from that literature.

Theories of beliefs such as diagnostic expectations (Bordalo et al., 2018) do not predict a switch from under to over reaction depending on the size of the shock.[10] Rabin and Vayanos (2010) can imply both under and overreaction in risk taking: differently from that work, my model predicts overreaction to large shocks from steady state, and does not predict the occurrence of underreaction to arbitrarily long sequences.

1.4. **Structure of the paper.** The structure of the paper proceeds as follows. Section 2 presents the model, Section 3 discusses the main predictions, Section 4 presents the experimental test of the predictions of Section 3 and discusses the results, including the estimation, Section 7 presents applications and one test with field data, Section 5 presents secondary predictions, Section 6 concludes.

2. A model of decision making

2.1. **Evaluation compared to the past.** A decision maker faces choices in periods $1, ..., T$ between risky prospects that realize at T. At time t, she considers the risky prospect A_t and retrieves a combination of recently encountered risky prospects, A_t^m,which will be defined formally later. Throughout the paper, I shall use the term risky prospect, lottery or gamble intercheangebly. Lotteries are elements of the space $\Delta(\mathbb{R})$ which is the set of probability distributions over consumption units. Uncertainty is

[9] I show this in Appendix C.

[10] It can predict underreaction under other distributional assumptions

realized at time T.[11] A stylized example of a decision in such a domain is the problem of investing into a risky asset that pays at a final period and on which the investor receives information at each $t \leq T$

Example 1. *(Learning about an asset)* Suppose an investor can either pick a stock with a random payment X at time T or a safe asset paying S. Suppose the investor receives information on X in each period t. In the model this is as if, at each period t, the choice set is

$$C_t = \{A_t, \delta_S\}$$

where $A_t = f_{X|t}$, that is, A_t is the marginal distribution of the stock X conditional on information received up to time t, while δ_S is the lottery giving S w.p.1, to be delivered at T as well. This domain will be used in Subsection 7. ▲

For every outcome x, the evaluation of the probability $A_t(x)$ is anchored to $A_t^m(x)$ and adjusted to $A_t(x)$. This is according to the following expression:

$$\pi\left(A_t(x)\,|A_t^m\right) := \underbrace{A_t^m(x)}_{\text{anchor}} + \overbrace{g\left(F_{A_t}, F_{A_t^m}\right)}^{\text{attention}} \left(\underbrace{A_t(x) - A_t^m(x)}_{\text{adjustment}}\right) \tag{2.1}$$

where $F_{A_t}, F_{A_t^m}$ are the cumulative distribution functions of A_t and A_t^m. The extent to which the adjustment catches attention is modulated by the function $g \geq 0$. The adjustment of A_t from A_t^m catches a lot of attention when g is large, while little attention when g is small. Crucially, g is monotone, meaning that the adjustment from A_t^m to A_t gets more attention the further away A_t is from A_t^m. This representation resembles Bordalo et al. (2020) which posits a model of evaluation of attributes, where a change from the reference point is weighted by the salience function which satisfies the ordering property, to which monotonicity is an analogue in this case.

Definition 1. (attention weighting function)

Let F, G be cumulative distribution functions.

$$F, G \longmapsto g(F, G) \in \mathbb{R}_+$$

The function g has the following properties

(i) Monotonicity

$$F >_{FOSD} G >_{FOSD} H \implies g(F, H) > g(G, H) \tag{2.2}$$

and $\forall H$ there exists $\underline{H} <_{FOSD} H <_{FOSD} \bar{H}$, such that

$$g\left(\bar{H}, H\right) = g\left(\underline{H}, H\right) = 1 \tag{2.3}$$

while

$$g(H, H) = 0 \tag{2.4}$$

[11] The slight abuse of notation makes the discussion simpler without any ambiguity. The non abusive notation specifies a set of temporal lotteries defined recursively as $H_t = \Delta\left(\{0\} \times H_{t+1}\right)$ with $H_T = \Delta(\mathbb{R})$ .and the space of the lotteries being such that, for any lottery A_t, the marginal distribution at t of consumption in T is the same as the marginal at T.

(ii) Continuity

$$g(\cdot,\cdot)$$

is continuous in both arguments in the L^2 norm.

(iii) Symmetry

$$g(F,H) = g(H,F) \tag{2.5}$$

The monotonicity property determines how the strength of similarity produces anchoring to the anchor (g small) or overreaction to the adjustment (g large). I say that A_t is **assimilated** to A_t^m when

$$0 < g\left(F_{A_t}, F_{A_t^m}\right) < 1,$$

which happens if L^2 distance between F_{A_t} and $F_{A_t^m}$ is small enough, i.e. when the two lotteries are close. If this is the case, the DM **underreacts** to the change between what he remembers, A_t^m, and what he faces, A_t.

We say that A_t is **contrasted** away from A_t^m, when

$$g\left(F_{A_t}, F_{A_t^m}\right) > 1,$$

which happens only if $d\left(F_{A_t}, F_{A_t^m}\right)$ is large enough, i.e. when the two lotteries are far away. If this is the case, the DM **overreacts** to the change between A_t^m and A_t.

The utility of the decision maker is

$$U(A_t|A_t^m) := \int u(x)\,\mathrm{d}\pi\left(A_t(x)\,|A_t^m\right)$$

which becomes

$$U(A_t|A_t^m) = \underbrace{\mathbb{E}_{A_t^m}[u]}_{\text{anchor}} + \overbrace{g\left(F_{A_t}, F_{A_t^m}\right)}^{\text{attention weight}} \left\{\underbrace{\mathbb{E}_{A_t}[u] - \mathbb{E}_{A_t^m}[u]}_{\text{adjustment}}\right\} \tag{2.6}$$

where u is a strictly increasing function which represents consumption utility. The distorted utility of A_t is anchored to the expected utility of the reference A_t^m, and adjusted to the rational expected utility of A_t, by a degree commanded by the similarity between A_t and A_t^m. If A_t is very far from the remembered lottery A_t^m, $g > 1$ and $U(A_t|A_t^m) > \mathbb{E}_{A_t}[u]$ and the DM overreacts. When $g < 1$, she underreacts to the change.

Example 2. One convenient example of g is

$$g(F,G) = \theta\left\{\left|\mathbb{E}_F[v] - \mathbb{E}_G[v]\right|\right\}^{\beta} \tag{2.7}$$

where v is a strictly increasing function of x, $\theta > 0$, and $\beta > 0$. ▲

Assimilation and contrast of evaluation of goods' attributes such as price and quality is modelled by Bordalo et al. (2020) (henceforth BGS), who suggest that the evaluation of a price of a good is anchored to remembered prices in similar contexts and adjusted to the observed price, with larger adjustment the larger the surprise. The logic of my model is analogous. A lottery A_t cues the retrieval of past similar lotteries summarized by A_t^m which anchors evaluation. The final evaluation of A_t depends on how A_t differs from what the DM remembers: significantly large differences from A_t^m command large adjustments, that is overreaction, and small differences command underreaction. The monotonicity property of the function g can be seen as the counterpart in the risky domain of the ordering property of the salience function which is proposed in BGS.

This representation mirrors two widely detected features of human perception: assimilation occurs when some element is judged as tending toward a reference element, and contrast occurs when some element is judged as being more opposed to a reference one (Leeuwenberg 1982): for example, in a task of lifting two weights sequentially, experimental subjects report estimates biased toward the first weight if it is moderate, and biased away from it, if it is extreme.[12] These phenomena are detected in domains including the perception of brightness and size (Helson and Rohles, 1959, Helson and Joy, 1962, Helson, 1963).[13]

Example 2 shows one special case of attention functon g that is useful for applications. Example 3 shows how the similarity between A_t and its reference A_t^m leads to upward and downward bias in the evaluation, in the simplest possible case of binary lotteries. For notational ease, (K, p) means a lottery that pays K with probability p and 0 with probability $1-p$.

Example 3. *(binary lotteries)* Suppose the decision maker evaluates a binary lottery A_t and has linear utility u.

$$A_t(x) = \begin{cases} p & x = K \\ 1-p & x = 0 \end{cases}$$

and remembers a binary lottery A_t^m

$$A_t^m(x) = \begin{cases} p^m & x = K \\ 1-p^m & x = 0 \end{cases}$$

To save space I sometimes describe A_t as (K, p) and A_t^m as (K, p^m). The evaluation of A_t is biased by the distance between p and p^m. Just for expositional ease, assume a quadratic function for g :

$$g\left((K, p), (K, p^m)\right) = \theta \left(Kp - Kp^m\right)^2$$

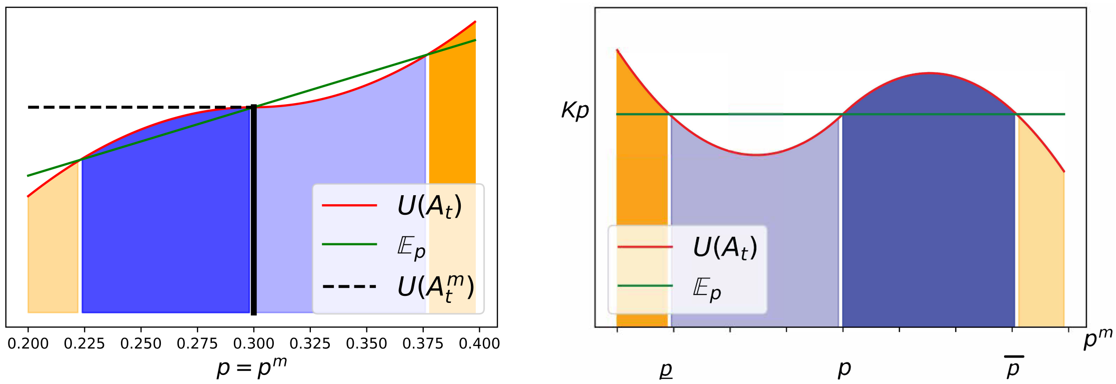
with $\theta > 0$. The utility, anchored to the expected utility of A_t^m

$$U\left(A_t | A_t^m\right) = \underbrace{Kp^m}_{\text{anchor}} + \underbrace{\theta \left\{K\left(p - p^m\right)\right\}^3}_{\text{attention}\times\text{adjustment}}$$

[12] The notion of contrast in optics appears in Helmholtz (1866).

[13] Such empirical regularities have provided further motivation to adaptation level theory (Helson, 1964) whose basic assumptions are that stimuli are judged based on past recent experiences. An individual whose perceptual system has adapted to an environment, contrasts the stimulus away. Instead, if the stimulus is adjacent to the environment, the perception of the stimulus adapts to it. An implication of this view is that small differences in stimuli are attenuated, and large ones are exaggerated.

Figure 2.1. Assimilation and contrast of a binary lottery (K,p) when remembered lottery is (K,p^m)



Left graph: the red curve plots the utility $U((K,p)\,|\,(K,p^m))$ for fixed $p^m = 0.3$ and p on the x axis and linear consumption utility. The green line plots the average $\mathbb{E}_p = Kp$ as a function of p. The dark and light blue areas identify the assimilation region, where p is close to p^m, while the dark and light orange areas identify the contrast region, where p is far from p^m. Darker areas identify values for p such that the decision maker is risk loving, that is where $U(A_t|A_t^m) > \mathbb{E}_p$, while lighter areas identify values for p where the decision maker is risk averse, $U(A_t|A_t^m) < \mathbb{E}_p$.
Right graph: the red curve plots $U((K,p)\,|\,(K,p^m))$ for fixed p, while p^m varies on the x axis. The green horizontal line is the expected value Kp. $\underline{p}$ and $\bar{p}$ identify the switching points between assimilation and contrast. Area colors define the same behavior as in the left graph.

Figure 2.1 shows, for fixed memory p^m, the evaluation $U(A_t|A_t^m)$ as a function of p. The key lesson is that there is an assimilation region $\left[\underline{p},\bar{p}\right]$ with $\bar{p} > p^m > \underline{p}$ where, the anchoring of the evaluation is such to bias the evaluation towards it, that is if $\bar{p} > p > p^m$ utility is downward biased towards the anchor, that is

$$Kp^m \underbrace{< U(A_t|A_t^m) <}_{A_t \text{similar to} A_t^m} Kp$$

while, if $\underline{p} < p < p^m$, utility is upward biased towards the anchor, that is

$$Kp^m \underbrace{> U(A_t|A_t^m) >}_{A_t \text{ similar to} A_t^m} Kp$$

the opposite occurs for probabilities in the contrast region $p \notin \left[\underline{p},\bar{p}\right]$. ▲

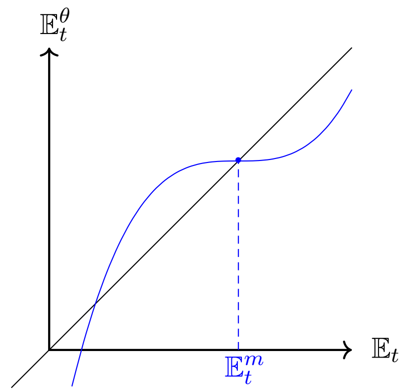
Example 4. *(Average distorted by assimilation and contrast)*

Suppose the similarity weight is $g(F,G) = \theta\left|\mathbb{E}_t - \mathbb{E}_t^m\right|$ where $\mathbb{E}_t = \int x \mathrm{d}A_t$ and $\mathbb{E}_t^m = \int x \mathrm{d}A_t^m$. If the decision maker has linear consumption utility u we can represent the distorted utility as

$$U(A_t|A_t^m) =: \mathbb{E}_t^\theta = \mathbb{E}_t^m + \theta \begin{cases} (\mathbb{E}_t - \mathbb{E}_t^m)^2 & if\, \mathbb{E}_t > \mathbb{E}_t^m \\ -(\mathbb{E}_t - \mathbb{E}_t^m)^2 & if\, \mathbb{E}_t < \mathbb{E}_t^m \end{cases}$$

For expected value $\mathbb{E}_t$ close and above $\mathbb{E}_t^m$, $\mathbb{E}_t^\theta$ is below $\mathbb{E}_t$, implying underreaction of the distorted expected value to the change from $\mathbb{E}_t^m$ to $\mathbb{E}_t$. For expected value $\mathbb{E}_t$ far above remembered $\mathbb{E}_t^m$, $\mathbb{E}_t^\theta$ is

Figure 2.2. Distorted Average



The blue curve plots $\mathbb{E}_t^\theta$ as a function of$\mathbb{E}_t$ on the x axis, for fixed $\mathbb{E}_t^m$.

above $\mathbb{E}_t$, implying overreaction to the change from $\mathbb{E}_t^m$ to $\mathbb{E}_t$. Figure 2.2 graphically represents the verbal description. ▲

2.2. **Past history shapes the reference point.** I specify A_t^m, in the spirit of BGS, by resorting to norm theory (Kahneman and Miller, 1986) which suggests that the reference point is shaped by *recency* and *similarity*.[14]

At each t, the DM faces choice set

$$C_t \subseteq \Delta(\mathbb{R})$$

each element of C_t specifies lotteries that pay at a final date T, with uncertainty resolved at T. For a given $A_t \in C_t$, the decision maker retrieves the most similar past lotteries encountered and discounts them by recency.[15] One case of a sequence of choice sets of options that pay at a final period is an environment where there is learning as in the Example 1 below.

We are now ready to define how the remembered lottery A_t^m is constructed.

Definition 2. Let A_t^m be the remembered lottery cued by A_t

$$A_t^m = (1-\rho)\sum_{j=1}^{t}\rho^{j-1}\hat{A}_{t-j} + \rho^t A_t$$

where $\rho \in \left[0,1\right)$ is the recency discount factor, and $\hat{A}_{t-j}$ is the most similar lottery to A_t from those encountered at time $t-j$. That is:

$$\hat{A}_{t-j} = \begin{cases} \arg\min_{\nu \in C_{t-j}} d(\nu, A_t) & if\, A_t\, non\, deterministic \\ A_t & if\, A_t\, deterministic \end{cases}$$

[14]As the authors themselves write, these ideas are imported from Avant and Helson (1973), Ward (1979) and Lockhead and King (1983).

[15]Even if I use this restriction to highlight the key predictions, the domain can be easily generalized to multi period lotteries.

where

$$d(\nu, A_t) = \begin{cases} \int (F_\nu(x) - F_{A_t}(x))^2 \,\mathrm{d}x & if\, \nu\, non\, deterministic \\ +\infty & if\, \nu\, deterministic \end{cases}$$

In words, when the DM faces a new distribution A_t she recalls the past distributions $\hat{A}_{t-j}$ which are closest to it, and discount those further away by factor ρ.

Example 5. *(gradual vs drastic changes in past history lead to different reference points)* Suppose there 5 periods and two decision makers, G and D. Between period 1 and 4, one risky prospect that they face changes in time (say because they receive information about it and update). The payment is realized at time 5.

$$C_s^i = \left\{ \left(\$80, p_s^i\right), (\$9, 60\%) \right\} \; i = D, G, \; s = 1, .., 4$$

where

$$p_s^D = 0.02, 0.025, 0.03, 0.09$$
$$p_s^G = 0.02, 0.04, 0.065, 0.09$$

that is, D faces a drastic change between period 3 and 4, while D faces gradual changes throughout. When they evaluate the lottery

$$(\$80, 9\%)$$

they retrieve two different remembered lotteries $A_4^{m,D}$ and $A_4^{m,G}$

$$A_4^{m,D} = \left(\$80, (1-\rho)\left\{0.03 + \rho 0.025 + \rho^2 0.02\right\} + \rho^3 0.09\right)$$

while

$$A_4^{m,D} = \left(\$80, (1-\rho)\left\{0.065 + \rho 0.04 + \rho^2 0.02\right\} + \rho^3 0.09\right)$$

since G has faced a path of better lotteries, in first-order stochastic dominance sense, G retrieves a better lottery than D, that is $A_4^{m,G} >_{FOSD} A_4^{m,D}$.▲

Example. 1 (continued) Take the investor in Example 1. The expected value of the remembered stock payment A_t^m is

$$\mathbb{E}_t^m[X] := \mathbb{E}_{A_t^m}[x] = (1-\rho)\sum_{j=1}^{t}\left(\rho^{j-1}\mathbb{E}_{t-j}[X]\right) + \rho^t \mathbb{E}_t[X]$$

that is, the decision maker remembers an average of prior expectations and gives more weight to the more recent ones.

Suppose the investor has linear utility $u(x) = x$ and g is as in Example 2, with v linear.

with $\beta > 0$. Suppose, additionally, that the decision maker only retrieved the most recent past, which is $\rho = 0$, then

$$\mathbb{E}_t^\theta[X] := U(A_t | A_t^m) = \mathbb{E}_{t-1}[X] + \theta \begin{cases} (\mathbb{E}_t[X] - \mathbb{E}_{t-1}[X])^{\beta+1} & if\, \mathbb{E}_t[X] \geq \mathbb{E}_{t-1}[X] \\ -(\mathbb{E}_t[X] - \mathbb{E}_{t-1}[X])^{\beta+1} & if\, \mathbb{E}_t[X] < \mathbb{E}_{t-1}[X] \end{cases}$$

▲

3. Main predictions: history dependent risk taking

The first key result from analyzing the model is that when a risky prospect looks similar to past ones, its evaluation is distorted towards them. Instead, when the prospect looks very different from the past remembered, its evaluation is contrasted away. A natural consequence of these two forces is the second result: the effect on evaluation produced by a change in a risky prospect from t_0 to t_1 is attenuated by the addition of intermediate steps populated with intermediate (in first-order stochastic sense) lotteries. I call this the boiling frog effect in choice under risk: small intermediate steps attenuate the reaction to a change in risky prospects. In subsection 7 I show that we can interpret some phenomena in asset prices as the manifestation of this force.

Proposition 1 discusses assimilation and contrast. Suppose there are two identical people, *Ann* and *Bob*, evaluating the same risky prospect (K, p), that means K with probability p and 0 otherwise. *Bob* has seen risky prospects that slightly first-order stochastically dominate (K, p), like $\left(K, p_s^{Bob}\right)$ with $p_s^{Bob} \gtrsim p$, *Ann* has seen prospects that are slightly dominated, like $\left(K, p_s^{Ann}\right)$ with $p_s^{Ann} \lesssim p$. Then *Bob* assimilates the risky prospect to the slightly better (in FOSD sense) ones and has a higher evaluation than *Ann* who assimilates the prospect to a slightly worse one (in FOSD sense). Suppose instead, the former was exposed to risky prospects largely better (in FOSD sense), that is $p_s^{Bob} >> p$, while the latter was exposed to largely worse risky prospects, $p_s^{Ann} << p$. The first one considers (K, p) as a drastic deterioration from the past, while the second will view it as a drastic improvement. Hence, assimilation (contrast) to better options biases choice in favor (against) the current risky prospect A_t. The next proposition states this more generally. Let $A_t^{i,m}$ be the memory anchor for $i = Ann, Bob$.

Proposition 1. *Suppose, at time t, $C_t = \{A_t, B\}$. Consider two alternative histories of choice sets for $s < t$,*

$$C_s^i = \left\{A_s^i, B\right\} \; i = Ann, Bob$$

such that $d(A_s, B) > d(A_s, A_t)$ for every s. Assume either ρ is small or t is large. There are threshold lotteries

$$\underline{\mu}^{A_t} <_{FOSD} A_t <_{FOSD} \bar{\mu}^{A_t}$$

such that:

(Assimilation) If

$$\underline{\mu}^{A_t} <_{FOSD} A_s^{Ann} <_{FOSD} A_t <_{FOSD} A_s^{Bob} <_{FOSD} \bar{\mu}^{A_t}$$

then

$$U\left(A_t|A_t^{Ann,m}\right) \geq U\left(B|B^{Ann,m}\right) \implies U\left(A_t|A_t^{Bob,m}\right) > U\left(B|B^{Bob,m}\right).$$

(Contrast) If

$$A_s^{Ann} <_{FOSD} \underline{\mu}^{A_t} <_{FOSD} \bar{\mu}^{A_t} <_{FOSD} A_s^{Bob}$$

$$U\left(A_t|A_t^{Bob,m}\right) \geq U\left(B|B^{Bob,m}\right) \implies U\left(A_t|A_t^{Ann,m}\right) > U\left(B|B^{Ann,m}\right).$$

Proposition 1 says that we can find thresholds $\underline{\mu}^{A_t}$ and $\bar{\mu}^{A_t}$ that identify a cutoff between the assimilation (where $U\left(A_t|A_t^{Ann,m}\right) < U\left(A_t|A_t^{Bob,m}\right)$) and the contrast (where $U\left(A_t|A_t^{Ann,m}\right) >$

$U\left(A_t|A_t^{Bob,m}\right))$ regions. In the assimilation region the value of A_t is attracted to the value of past histories. The contrast region is where the value of A_t is exaggerated away from the past. Think of the example of an insurance against an adverse event with time-varying likelihood p_s,$s = 0, 1$. Suppose a decision maker with linear utility u decided her willingness to pay for the insurance in period 1. The assimilation region determines the maximum change in probability from period 0 to 1, so that the DM still wants to pay a price below fair value (or, alternatively, does not insure, if the price is fixed at fair value). If the change from 0 to 1 is above the cutoff that identifies the assimilation region, the DM exaggerates the change and is willing to pay an insurance above fair value.The cutoffs identified in the proposition are such that

$$U\left(A_t|\underline{\mu}^{A_t}\right) = U\left(A_t|\bar{\mu}^{At}\right) = U\left(A_t|A_t\right) = \mathbb{E}_{A_t}\left[u.\right]$$

In the example of linear utility, are they correspond to the past historical distributions that make the agent risk neutral when evaluating A_t. Figure 2.1 plots the expected utility of a lottery $(K, p; 0, 1-p)$, which pays K with probability p and 0 otherwise, as a function of the probability of past remembered lotteries of the form (K, p_s) where $p^m = (1-\rho)\sum \rho^{s-1}p_s + \rho^t p$. The past p^m distorts the evaluation. Past slightly better expectations, so that $p^m \in [p, \bar{p}]$, make the lottery look better, as in the dark blue region. Past significantly better expectations, so that $p^m > \bar{p}$, make the lottery look worse, as in the light orange region. Past significantly (slightly) worse expectations, so that $p^m < \underline{p}$, $\left(p^m \in \left[\underline{p}, p\right]\right)$ make the lottery look better (worse), as in the dark orange (light blue) region. Corollary 2 states the result of Proposition 1 for binary lotteries, which is tested experimentally in Section 4.

Consider again, the case of a decision maker that decides how much to pay for insurance against a time-varying probability in the adverse event. We just discussed that, if a change is above a cutoff, she will exaggerate her willingness to pay for insurance above fair value, because she will contrast the new risk against what she has in memory. This fact yields an interesting additional implication, which is that, if she faces a gradual change in risk, her memory gets populated with moderately higher risks. As a result, a given probability p_t will look less strikingly different from the past p_t^m relative to the case where the DM experienced a discrete jump. This hypothetical behavior is sometimes referred to as a Boiling Frog effect, which I state in Proposition 2, and on which I provide experimental evidence in Section 4.

Proposition 2. *(Boiling Frog) Under the same assumptions of Proposition 1, suppose* $A_1^{Ann} = A_1^{Bob} = A_1 \leq_{FOSD} \underline{\mu}^{A_t}$ *suppose*

$$A_s^{Ann} <_{FOSD} \underline{\mu}^{A_t}$$

for every s and

$$A_s^{Ann} <_{FOSD} A_s^{Bob}$$

for all s between 1 and t. Then

$$U\left(A_t|A_t^{Bob,m}\right) \geq U\left(B|B^{Bob,m}\right) \implies U\left(A_t|A_t^{Ann,m}\right) > U\left(B|B^{Ann,m}\right).$$

In words Ann values A_t more than Bob if she saw lotteries dominated by some seen by b

Proposition 2 says that when evaluating lottery A_t, if, before t, Ann has seen lotteries below the contrast threshold $\underline{\mu}^{A_t}$, while Bob has seen the same starting lotteries than Ann, but relatively better (in first-order stochastic sense) lotteries between 1 and t, then Ann has a higher evaluation of A_t relative to Bob. This is because Ann contrasts A_t upwards more than *Bob* does.

To get the simple meaning of Proposition 2, think about a probability of adverse event that evolves in three periods, starting from $p_0 = \underline{p}$ and being equal to p_2 in the last period. If the change from p_0 to p_2 is large enough, Ann faced $p_1^{Ann} = \underline{p}$, that is a drastic change from $\underline{p}$ to p_2, and evaluates the risk to be larger than Bob who faced $p_1^{Bob} \in \left(\underline{p}, p_2\right)$, that is a more gradual path leading to p_2.

The predictions so far rely on the memory anchor A_t^m being a backward looking average of past lotteries, and not much on the exponential discounting of past lotteries. Relevant other implications descend from the assumption of discounting: the reaction to the present depends on the order of past experiences. To fix ideas, suppose Ann and Bob has seen past lotteries

$$A_s^i = \left(K, p_s^i; 0, 1 - p_s^i\right) \quad i = Ann, Bob$$

such that $p_s^i < p$, $s < t$. Suppose they saw the same lotteries but not in the same order: in particular *Ann* has seen them ordered in a declining fashion: $p_s^{Ann} < p_{s-1}^{Ann}$. Suppose at time t they evaluate the same lottery$A_t = (K, p; 0, 1 - p)$. Then, if the recent lotteries end up being different enough, *Ann* chooses A_t more than *Bob*. This has an application in any setup where a principal wants to manage surprises to boost the perception of an outcome, as I exemplify in Subsection 1: a given set of surprises produces larger reaction if it is observed in increasing order: a convex trend produces overreaction. I test for the history-order dependence of risk taking in Subsection 4.6. The following corollary states the prediction more generally.

Corollary 1. *(Recency effect) Under the same assumptions of Proposition 1. Then there exists* $0 < \bar{\rho} < 1$ *such that, if*

$$0 < \rho < \bar{\rho}$$

and $\bigcup_{s<t} \left\{A_s^{Ann}\right\} = \bigcup_{s<t} \left\{A_s^{Bob}\right\}$ *and*

$$A_{t-1}^{Ann} \leq_{FOSD} \underline{\mu}^{A_t}$$

and

$$A_s^{Ann} <_{FOSD} A_{s-1}^{Ann} <_{FOSD} A_t$$

for $s < t$*, then*

$$U\left(A_t | A_t^{Bob,m}\right) \geq U\left(B | B^{Bob,m}\right) \implies U\left(A_t | A_t^{Ann,m}\right) > U\left(B | B^{Ann,m}\right).$$

The Corollary says that *Ann* values A_t more than *Bob* if *Ann* saw a declining order of lotteries before A_t, while *Bob* saw the same lotteries but in different order. This prediction will be tested in Subsection 4.6.

4. Experimental Evidence

In this section I experimentally test the main novel predictions of the model.

(1) The contrast prediction says that if lottery pays *much* more (less) in first-order stochastic sense than the most similar ones remembered, it is chosen more (less) often.

(2) The assimilation prediction consists in the notion that if a lottery pays *slightly* more (less) in first-order stochastic sense than the most similar ones remembered, it will be chosen less (more) often.

(3) Assimilation and contrast interact, implying that gradual upward (in stochastic dominance sense) changes in a lottery make the resulting lottery chosen less often than drastic upward changes.

I test these predictions through randomly assigned cross subject experimental treatments where each subject faces a sequence of binary choices between binary lotteries, and only one lottery changes across choice sets. In the first set of treatments the probability of the outcome of a lottery changes across stages (Subsection 4.1). In the second set of treatments the outcome of one of the lotteries changes across stages (Subsection 4.7).

The experiment was pre registered at the American Economics Association's RCT registry, with ID AEARCTR-0009601 on June 18th 2022. The experiments have taken place in July and August 2022 on the online platform Prolific. The task takes about 2 minutes. Subjects were instructed that they were paid a fixed amount of \$0.40 (an hourly rate of \$12) plus, if they got selected, the random payment of one of the lotteries they chose. The uncertainty regarding the payoff of the lottery drawn is resolved at the end of the experiment.

4.1. **Experimental paradigm 1: changes in probability.** In each treatment subjects make a sequence of choices between two lotteries. In stage t, subject in treatment j must choose between either \$80 with probability q_t^{Tj}, and \$0 with probability $1 - q_t^{Tj}$, or \$9 with probability 0.6, and \$0 with probability 0.4. This means that the choice sets are as follows:

$$C_t^j = \left\{ \left(\$80, q_t^{Tj}; \$0, 1 - q_t^{Tj} \right), ((\$9, 0.6; \$0, 0.4)) \right\}$$

with j indexing the treatment $j = 1, 2, 3, 4$.

When subjects face a choice set at stage t, they neither see their previous choice sets, nor they are reminded what they chose. They are told in the instructions that one choice will be implemented for one subject in their experimental wave.

I refer to a lottery that pays A with probability q and 0 otherwise as (A, q).

Treatment 1.

$$\left(q_t^{T1}\right)_{t=1}^{5} = (0.09,\ 0.07,\ 0.06,\ 0.05,\ 0.02)$$

Subjects choose between an alternative that varies across choice sets and a safer lottery that is invariant. The option paying \$80 *decreases* in first-order stochastic dominance across choice sets.

Treatment 2.

$$\left(q_t^{T2}\right)_{t=1}^{5} = (0.02,\, 0.05,\, 0.06,\, 0.07,\, 0.09)$$

The option paying \$80 *increases* in first-order stochastic dominance across choice sets.

Treatment 3. The sequence of choice sets is

$$\left(q_t^{T3}\right)_{t=1}^{4} = (0.21,\, 0.23,\, 0.22,\, 0.09)$$

The option paying \$80 *decreases* in first-order stochastic dominance across choice sets, with a large jump between $t = 3$ and $t = 4$.

Treatment 4. The sequence of choice sets is

$$\left(q_t^{T4}\right)_{t=1}^{4} = (0.02,\, 0.04,\, 0.03,\, 0.09)$$

The option paying \$80 *increases* in first-order stochastic dominance across choice sets, with a large jump between $t = 3$ and $t = 4$.

One wave of subjects (N=1210) was randomly assigned either to Treatment 1 or 2. Another simultaneous wave of subjects (N=400) was randomly assigned either to Treatment 3 or 4.

Table 3 summarizes the different experimental treatments.

4.2. **Model's derived hypotheses .** The following predictions come from Corollary 2. Let us call

$$P\left(A, q|TJ\right)$$

the frequency of choice of option (A, q) in Treatment J.

Prediction 1 - Contrast

$$P\left((\$80, 9\%)|T3\right) < P\left((\$80, 9\%)|T4\right)$$

Prediction 2 - Weak assimilation

In Treatment 1 the option paying \$80 is chosen more often than in Treatment 2 on average

$$\frac{\sum_{t=1}^{T} P\left((\$80, q_t)|T1\right)}{T} > \frac{\sum_{t=1}^{T} P\left((\$80, q_t)|T2\right)}{T}$$

The strong assimilation prediction is a stronger test of the model and says the following.

Prediction 3 - Strong Assimilation

If $P\left((\$80, 9\%)|T4\right) \leq P\left((\$80, 9\%)|T1\right)$ *then*

$$P\left((\$80, q_t)|T1\right) > P\left((\$80, q_t)|T2\right)$$

for each

$$t = 1, ..., T$$

The last prediction connects contrast and assimilation and says that, for fixed starting and ending choice sets, (A, q_T) is chosen less often if it comes after a sequence of choice sets that entails a gradual positive change in the probability of A than if it comes after a drastic change from q_{T-1} to q_T. The intuition is that, after a drastic change, contrast is at play and the lottery gets overvalued. While after a gradual change either assimilation or weaker contrast make the lottery (A, q_T) stand out less noticeably.

Prediction 4 - Boiling Frog

In Treatment 4 the option paying \$80 is chosen more often than in Treatment 2 at stage 4

$$P\left((\$80, 9\%)|T2\right) < P\left((\$80, 9\%)|T4\right)$$

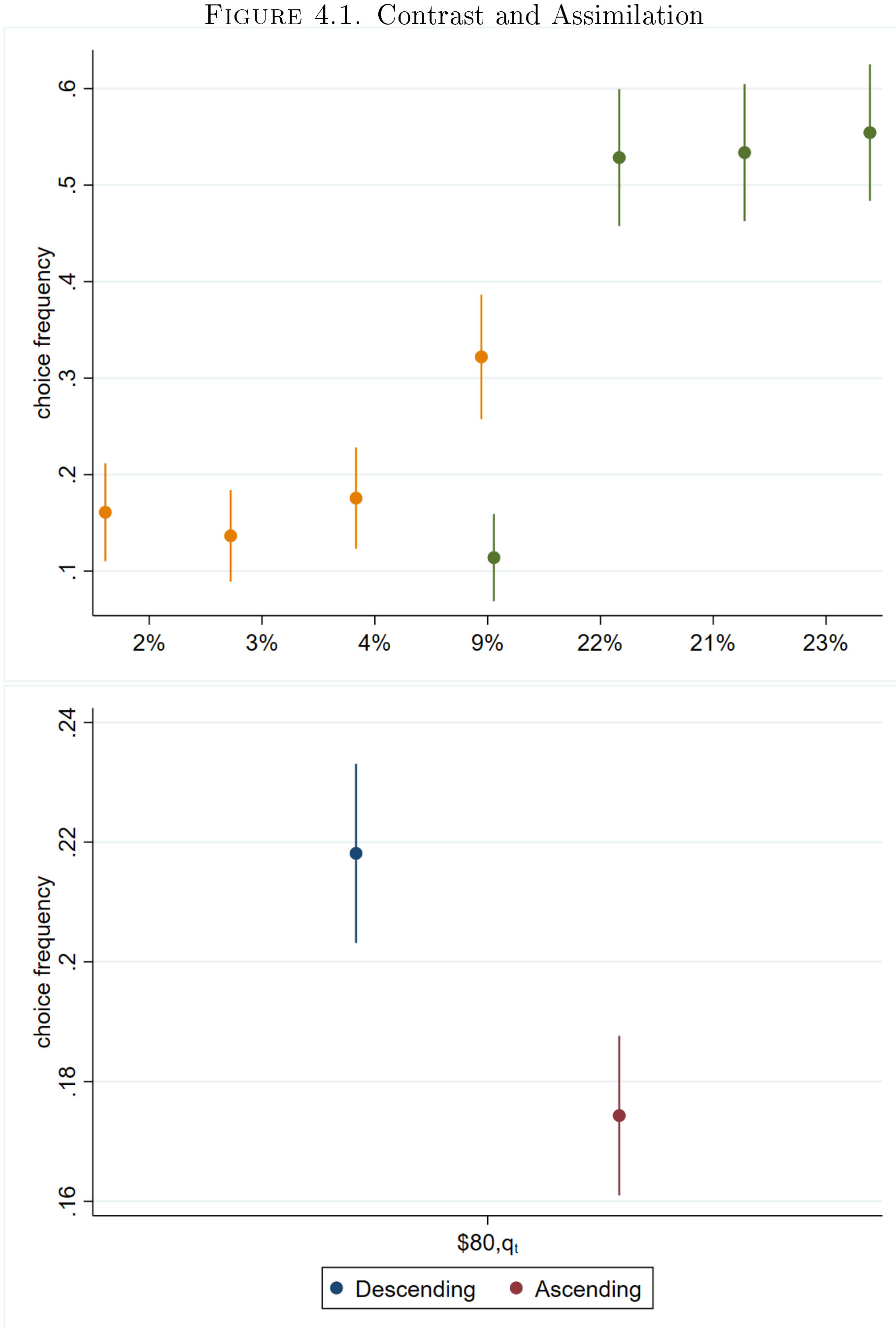
4.3. **Results.**

4.3.1. *Evidence of Contrast.* The top panel in Figure 4.1 below compares the choice frequency of the risky option $(\$80, 9\%)$ across Treatment 3 (where it is shown after subjects have seen $(\$80, 21\%)$, $(\$80, 23\%)$ and $(\$80, 22\%)$), and Treatment 4 (where it is shown after subjects have seen $(\$80, 2\%)$, $(\$80, 4\%)$ and $(\$80, 3\%)$). The risky option is chosen by 32% of the subjects when it appears larger than the preceding options in Treatment 4, and less than half of the times (12%) when it comes after larger options. Column (1) in the top panel of Table 1 shows that the treatment difference of about -20% is statistically significant at every meaningful level, and also economically significant: a decision maker that sees the option after a much better one is more than 50% less likely to choose it compared to a decision maker that has seen worse choices first.

Another relevant comparison is between the choice likelihood of a decision maker who observed better options (T3) with a decision maker who sees the option without having observed anything before (T1). The top panel in Figure 4.2 shows that the choice frequency of $(\$80, 9\%)$ when it is encountered first (Treatment 1) is approximately 36% which is not statistically distinguishable from the 32% of treatment 4. Also in this case, the treatment difference between T3 and T1 is large and statistically significant, implying that a decision maker that observed largely first-order stochastically dominating options before is less than half as likely to choose the risky option as a decision maker who has seen nothing before. The insignificant difference between T1 and T3 choice frequency of $(\$80, 9\%)$, moreover, suggests that $(\$80, 9\%)$ is approximately in the crossing point between the assimilation and the contrast region, relative to the reference lottery which is a combination of $(\$80, 4\%)$,$(\$80, 3\%)$, $(\$80, 2\%)$. This is only a heuristic argument: the rigorous inference of the crossing point can be found in subsection 6.4.1, where I perform a structural estimation of the contrast parameter θ , the recency parameter ρ,and the coefficient of relative risk aversion $1 - \alpha$.

4.3.2. *Evidence of Assimilation .* The bottom panel of Figure 4.1 compares the cross period average choice frequency of the risky options $(\$80, q_t$ between Treatment 1, where the options are showed sequentially in a decreasing order of stochastic dominance (q_t goes smoothly from 9% to 2%), and Treatment 2, where the options are showed in increasing order of stochastic dominance (q_t goes gradually from 2% to 9%). This way the same option in Treatment 1 is preceded by slightly better ones, while in Treatment 2 is preceded by slightly worse ones. The table shows that subjects choose $(\$80, q_t$ around 22% of the time if they are preceded by slightly FOS dominant ones (in Treatment 1), and around 17% of the time when the options are preceded by slightly FOS dominated ones in Treatment 2. As shown

FIGURE 4.1. Contrast and Assimilation



Choice frequencies of option \$80, q are shown, with q on the x axis and segments indicating 95% confidence intervals. Green dots indicate Treatment 3, while orange dots indicate Treatment 4.
The blue dot is the cross period average frequency of Treatment 1, while the red dot is of Treatment 2.

in the bottom panel of Table 1, the treatment effect of 4% is statistically different from 0. The size of the effect is economically relevant: on average subjects can decrease their propensity to take risk by 20% if they are shown worse risky options that look similar to the target.

A more nuanced picture is revealed if we estimate treatment effects on choice for every option separately: the choice frequency of ($\$80, 9\%$) is larger by 10% when it is seen as a first option rather than when it is preceded by the increasing sequence of FOS dominated options. Subjects observing a gradual decline in the probability of $80 are also significantly more likely to choose ($\$80, 9\%$), ($\$80, 7\%$) and ($\$80, 6\%$) compared to subjects that, before facing it, have seen the sequence of worse options $(\$80, 2\%), (\$80, 5\%), (\$80, 6\%)$. The effect weakens and disappears for values below 6%, and actually reverses for the option ($\$80, 2\%$): this means that ($\$80, 2\%$) is contrasted away from the previous options in Treatment 3. The decline of the size of the point estimate of the effect is suggestive that the assimilation acts from below. That is, subjects might be primed more effectively by low probability options, and thus assimilate options that slightly FOS dominate them, while they might not be assimilating an option preceded by options that slightly FOS dominate it, or they might even contrast it. In order to test whether this mechanism is at play, choice frequencies of the intermediate options $(\$80, 2\%), (\$80, 5\%), (\$80, 6\%), (\$80, 7\%)$ should be estimated in an experiment where they are not preceded by any other option. In absence of that, it can be asserted that the difference in risk taking in the choice set containing the relatively better option ($\$80, 9\%$) all comes from assimilation to the worse options in treatment 2, since in treatment 1 the option ($\$80, 9\%$) is observed in isolation.

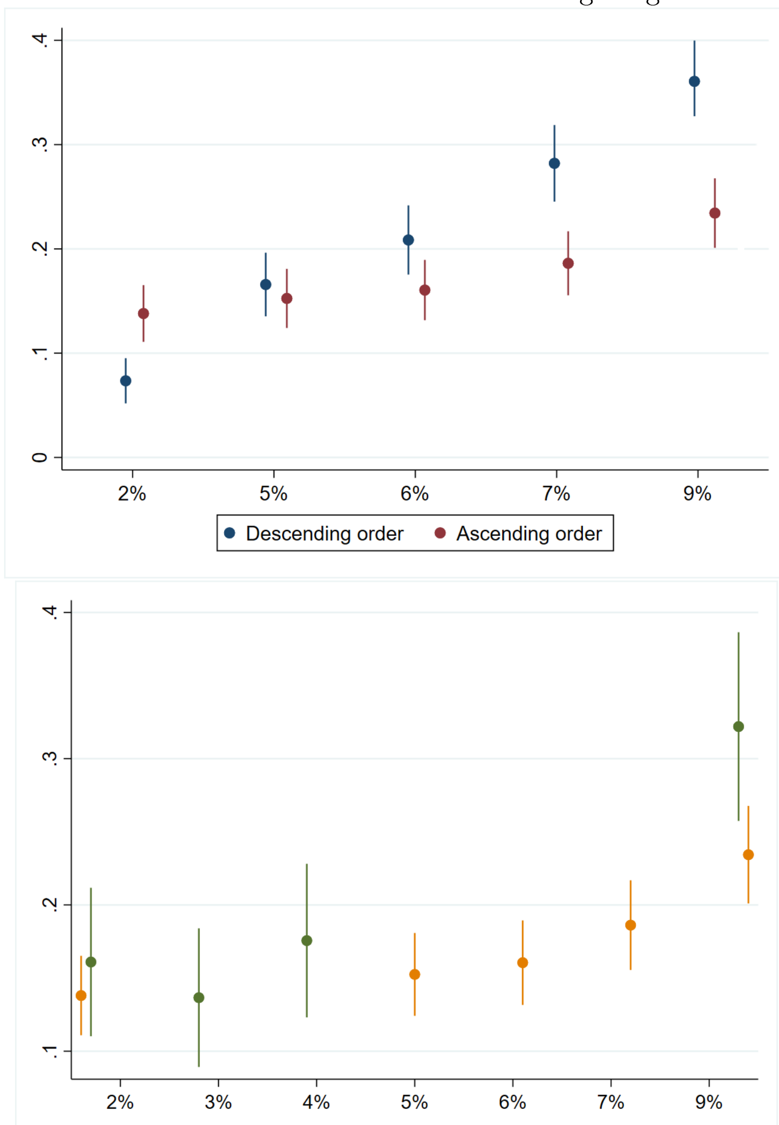
4.3.3. *Boiling Frog effect in choice under risk.* As shown in the bottom panel of Figure 4.2, the choice frequency of the option ($\$80, 9\%$), which is encountered last in both Treatments 2 and 4, is larger by 8 in Treatment 4, where the probability of $80 rises more drastically across choice sets than in Treatment 2. Such difference is statistically significant at 5% level, as shown by Column (2) of the top panel of Table 1. I conclude that Prediction 4 is confirmed.

4.4. **Discussion.** The results overall reveal sizable contrast and assimilation effects. The choice of an option is less likely if the option looks like a drastic improvement from the past, relative to when it looks like a drastic deterioration from the past, and also relative to when it looks like a gradual improvement from the past.

The results from the comparison between T3 and T4 can be consistent with inertia: subjects in T4 choose the safer option at the beginning and keep on choosing it because they are either stuck to the status quo or do not even pay attention. Inertia is ruled out by the comparison between $T4$ and T2, which reveals the boiling frog effect: if subjects are stuck to the status quo or are not paying attention to features of the option that they are not choosing, then observing ($\$80, 9\%$) after a smooth sequence of improving options as in T2 or after a drastic change as in T4 should not make a difference, because the starting and ending choice sets are the same for both treatments. Instead, as we have discussed, there is an economically and statistically significant difference of about 8% in choice frequencies of ($\$80, 9\%$) between the two treatments.

We have seen in subsection 4.3.2 evidence consistent with an hypothesis that subjects assimilate only from "below" that is, they are not sensitive to slight improvements in options. This brings subjects to be less willing to take an option is preceded by FOS dominated ones. In order for subjects in treatment T1 to be more likely to take ($\$80, 9\%$) than in T2, it is not necessary for them to be biased by assimilation in T1. The pattern observed implies that there is assimilation at least in one of the two treatments,

FIGURE 4.2. Assimilation and the Boiling Frog effect



Average choice frequencies of option $80, q are shown. q is reported on the x axis. Segments show 95% confidence intervals. Blue dots represent the choice frequencies in Treatment 1, red and orange dots represent those of Treatment 2, green dots represent choice frequencies of Treatment 4

but not necessarily in both. The comparison of $P(\$80, 9\%)$ across T1 and T2 shows that there is assimilation in T2, because subject are less risk taking after an ascending path of probabilities than if considering the option in isolation. Another question is whether subjects in T1 are also assimilating, that is the choice frequency of the risky options are larger than in the case where they see them in isolation.[16] Given that we only get to see choices in isolation of only one option, we cannot estimate the assimilation or contrast effect of T1 only. Subjects in T1 may be less likely to choose the risky option in T1 than if they saw each risky option in isolation, implying that they are actually contrasting

[16]With "in isolation" I mean facing a choice set without having seen anything else similar before

Table 1. Assimilation, Contrast and the Boiling Frog effect

	(1) ($80,9%)	(2) ($80,9%)
D Drastic effect	-0.208***	
	(0.0404)	
A Drastic	0.322***	
	(0.0281)	
A Drastic effect		0.0876**
		(0.0351)
A Gradual		0.234***
		(0.0174)
Observations	398	828
R-squared	0.063	0.008

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

	(1) 9%	(2) 7%	(3) 6%	(4) 5%	(5) 2%
D Drastic effect	0.126***	0.0959***	0.0480**	0.0133	-0.0645***
	(0.0260)	(0.0242)	(0.0223)	(0.0211)	(0.0177)
A Gradual	0.234***	0.186***	0.161***	0.152***	0.138***
	(0.0181)	(0.0168)	(0.0155)	(0.0147)	(0.0123)
Observations	1,208	1,208	1,208	1,208	1,208
R-squared	0.019	0.013	0.004	0.000	0.011

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

The top table shows the regressions of a dummy(=1 if ($80, 9%) is chosen) on a constant ("A Drastic" in Column (1) and "A Gradual" in Column (2)) and a Treatment dummy. The dummy "D Drastic effect" is =1 if subject is in Treatment 3, the dummy "A Drastic effect" is =1 if subject is in Treatment 4. In Column (1) the sample includes Treatments 3 and 4, while in Column (2) it includes Treatments 2 and 4.
The bottom table shows, for each column, the regression coefficients of a dummy = 1 if ($80, q) is chosen and 0 otherwise, with q varying across columns. The regressor "D Gradual effect" is a dummy =1 if the observation is in Treatment 1 and =0 if it is in Treatment 2. The regressor "A Gradual" is the constant. The sample includesTreatment 1 and 2.

the options against the preceding ones. An answer to this question can be drawn from an additional treatment, called T1*, that is discussed in Appendix B, where subjects observe one more choice set $\{(\$80, 11\%), (\$9, 60\%)\}$before the choice sets of T1, in their same order. Results displayed in the top left table of Table 6 show that

$$P((\$80, 9\%) | T1^*) \approxeq 27\% < 35\% \approxeq P((\$80, 9\%) | T1)$$

suggesting that subjects contrast ($80, 9%) in treatment T1*. The model, at least under functional forms that I later use in this paper, predicts

$$P((\$80, 9\%) | T2) < P((\$80, 9\%) | T1) < P((\$80, 9\%) | T1^*)$$

that is, if ($80, 9%) is assimilated in $T2$ to lower probability options, than it should be assimilated in $T1^{*}$to the higher probability option ($80, 11%). This inconsistency between the model and the data suggests that other forces can be additionally at play, whose investigation is beyond the scope of this work.

4.5. **Switching point between assimilation and contrast.** In Subsection 4.3.1 I have argued qualitatively where might be the switching point between assimilation and contrast in the probability of receiving $80, conditional on a history of options observed. The purpose of this subsection is twofold: first I provide a rigorous answer to the question of where is the elicited switching point between assimilation and contrast in the current experiments. Secondly, I answer the question of how does a DM with preferences estimated in my experiment reacts to risks of much larger magnitude, which are not feasible in an experimental setting. In order to answer these questions, I estimate preference parameters under the assumption of a functional form equal to

$$g(F,G) = \theta \left| \mathbb{E}_F[u] - \mathbb{E}_G[u] \right|$$

Estimation details are in Appendix D. The key parameters are θ, ρ and the curvature of utility α.

Suppose a subject has seen many times ($\$80, \bar{p}$) and now evaluates ($\$80, \bar{p}+\delta$). The switching point is defined as the value δ such that

$$U(80, \bar{p}+\delta | 80, \bar{p}) = \mathbb{E}_{(80,\bar{p}+\delta)}[u].$$

Such value depends not only on the structural parameters, but also on the values at stake:

$$\delta = \frac{1}{\theta}\frac{1}{80^{\alpha}}$$

where α comes from a functional form assumption over consumption utility:

$$u(x) = x^{\alpha}.$$

Appendix D shows that the estimate of θ and α is

$$\hat{\theta}_1 = 1.29$$

and

$$\hat{\alpha}_1 = 0.52$$

That yield the switching point

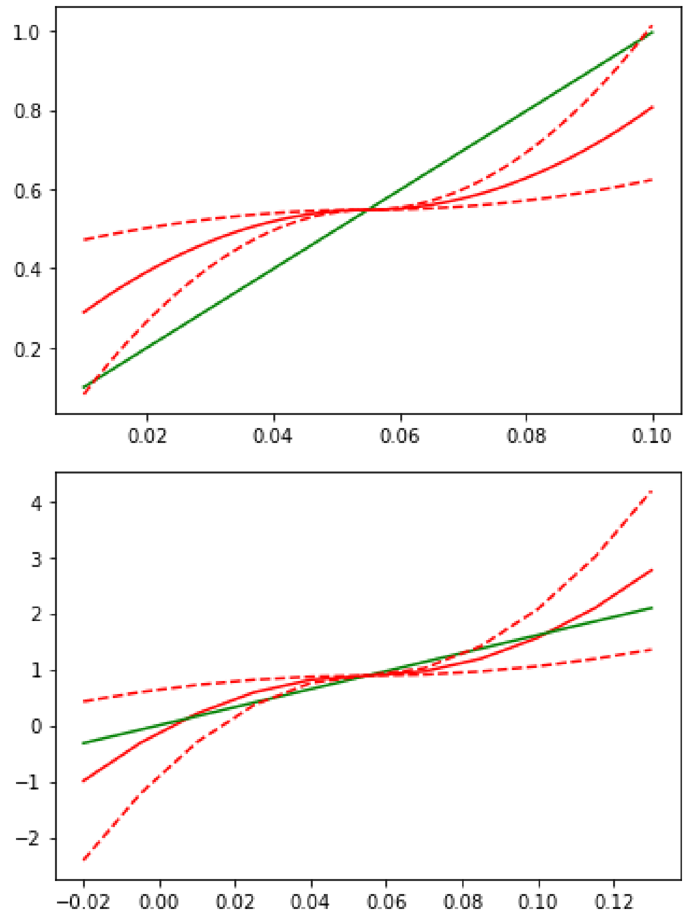
$$\hat{\delta}_1 = 0.079$$

Suppose $\bar{p}$ is equal to 5.5%, like in Figure 4.3. After having faced the lottery ($80, 5.5%),the decision maker will like the lottery

$$(\$80, 12.4\%)$$

just as much as if she saw such a lottery without being primed with anything. In other words, the estimated parameters predict the choice frequency of ($80, 12.6%) to be the same both when seen in

FIGURE 4.3. Estimated preferences



Both figures plot in red the distorted expected utility of a lottery ($\$K, p, \$0, 1-p$) conditional on past lotteries being ($\$K, 5.5\%, \$0, 94.5\%$). p is reported on the x axis. In the top panel $K = 80$, while in the bottom panel $K = 200$. The figure is constructed using $\hat{\theta}_1$ and $\hat{\alpha}_1$, which are estimates from the sample including T1, T2, T3 and T4. The the undistorted expected utility is plotted in green. Dashed lines represent 95% confidence intervals, obtained via block bootstrap.

isolation and when seen after a long sequence of $(\$80, 5.5\%)$.[17] This calculation immediately yields the assimilation region that we would obtain in an experiment we have not run. Suppose we wanted a decision maker not to buy a risky lottery paying $(\$200, 12\%)$ sold at fair value. We may naively think that, since we estimated an assimilation region of $5.5\% \pm 12.4\%$ in our experiment, $(\$200, 12\%)$ would be assimilated downwards to a priming lottery of $(\$200, 5.5\%)$, so that the priming would succeed in reducing risk taking of a subject. The model instead says that the assimilation region is stake dependent, and shrinks as stakes scale up: Figure 4.3, bottom panel, shows that $(\$200, 12\%)$ lies in the contrast region of $(\$200, 5.5\%)$. This means that the priming would *increase* risk taking, backfiring the original purpose.

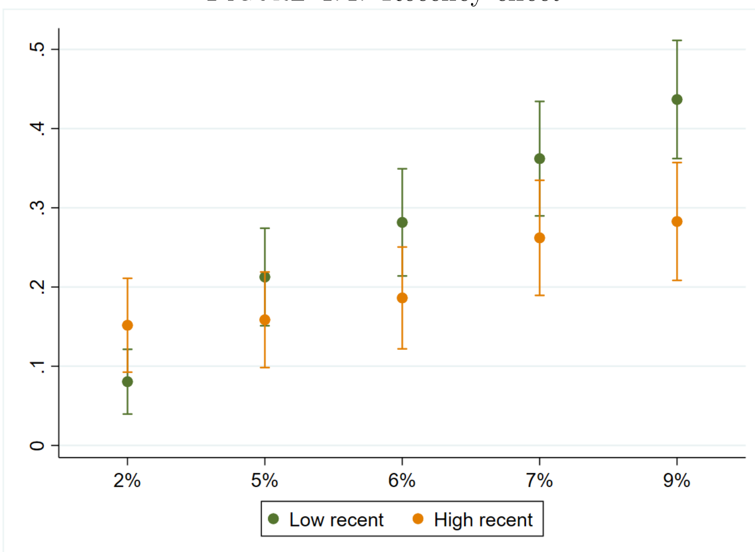
[17]In other words, the certainty equivalent of $(\$80, 12.6\%)$ is the same whether the option comes after $(\$80, 5.5\%)$, or when it is seen in isolation.

TABLE 2. Recency effect. Treatments

	R1	R2
Stage 1	\$80, 7% vs \$9, 60%	\$80, 2% vs \$9, 60%
Stage 2	\$80, 6% vs \$9, 60%	\$80, 5% vs \$9, 60%
Stage 3	\$80, 5% vs \$9, 60%	\$80, 6% vs \$9, 60%
Stage 4	\$80, 2% vs \$9, 60%	\$80, 7% vs \$9, 60%
Stage 5	\$80, 9% vs \$9, 60%	\$80, 9% vs \$9, 60%

Each column displays the sequence of choice sets faced by subjects in Treatment R1 and R2, respectively.

FIGURE 4.4. Recency effect



Choice frequencies of (\$80, q%), q *is on the* x *axis. Green dots: Treatment R1. Yellow dot: Treatment R2.*

4.6. **Recency effect: estimation and one additional experimental test.** The structural estimation described in the appendix allows me to recover an estimate of the recency parameter ρ which is equal to $\hat{\rho}_1 = 0.02$. This essentially means that subjects acts as if the remembered lottery was equal to the last lottery they saw. This suggests that changing the order of the history, holding the elements of the past the same, should have dramatic effects on choice. I run the additional treatments on 320 subjects on Prolific, summarized in Table 2, to test for this. In R1 subjects see a declining path of probability of the upside \$80 to 2%, right before a jump to 9%. In R2, subjects see the same choice sets as in R1, but in a gradual increase from 2% to 9%. The recency parameter just estimated says that subjects in stage 5 of R1 perceive a jump from the remembered lottery (\$80, 2%) to (\$80, 9%) that makes them evaluate the lottery approximately on the cutoff between assimilation and contrast. Instead, subject in stage 5 in R2 should assimilate (\$80, 9%) to (\$80, 7%) and choose it less often than subjects in R1

The results are in Figure 4.4. The choice frequency of (\$80, 9%) is significantly larger (by about 15 percentage points) in subjects in R1 than in R2, confirming the prediction of the model.

4.7. **Experimental paradigm 2: changes in payoffs.** The model predicts that the same forces of contrast and assimilation should be at play also when the lottery under evaluation does not share the same support of its reference lottery. In other words, if a lottery (A, q) is assimilated to a lottery $(A, q + \delta)$, for δ small, then the model says that it is also assimilated to $(A + \epsilon, q)$ with ϵ small. Symmetrically, for ϵ large, it gets contrasted away from $(A + \epsilon, q)$. It depends on the contrast parameter θ how large (small) must ϵ be to deliver contrast (assimilation). In the spirit of the main treatments explained in Subsection 4.1, I design four additional cross subject treatments where a sequence of choice sets is shown, with the following structure

$$C_t^j = \left\{ (\$80, 0.09) \, , \left(\left(\$B_t^{Tj}, 0.6 \right) \right) \right\}$$

where the treatments are indexed by $j = 5, 6, 7, 8$.

Treatment 5:

$$\left(B_t^{T5}\right)_{t=1}^{4} = (5,\ 4,\ 2,\ 1)$$

B_t^{T2} gradually decreases across choice sets.

Treatment 6:

$$\left(B_t^{T6}\right)_{t=1}^{4} = (1,\ 2,\ 4,\ 5)$$

B_t^{T2} gradually increases across choice sets.

Treatment 7:

$$\left(B_t^{T7}\right)_{t=1}^{4} = (9,\ 8,\ 8.5,\ 5)$$

B_t^{T2} drastically decreases from stage $t = 3$ to $t = 4$.

Treatment 8:

$$\left(B_t^{T8}\right)_{t=1}^{4} = (1,\ 2,\ 1.5,\ 5)$$

B_t^{T2} drastically increases from stage $t = 3$ to $t = 4$.

One wave of subjects (N=400) was randomly assigned either to Treatment 5 or 6. Another simultaneous wave of subjects (N=400) was randomly assigned either to Treatment 7 or 8.

Table 3 gives a summary of the Treatments.

TABLE 3. Treatments

	T1	T2	T3	T4
Stage 1	\$80, 9% vs \$9, 60%	\$80, 2% vs \$9, 60%	\$80, 21% vs \$9, 60%	\$80, 2% vs \$9, 60%
Stage 2	\$80, 7% vs \$9, 60%	\$80, 5% vs \$9, 60%	\$80, 23% vs \$9, 60%	\$80, 4% vs \$9, 60%
Stage 3	\$80, 6% vs \$9, 60%	\$80, 6% vs \$9, 60%	\$80, 22% vs \$9, 60%	\$80, 3% vs \$9, 60%
Stage 4	\$80, 5% vs \$9, 60%	\$80, 7% vs \$9, 60%	\$80, 9% vs \$9, 60%	\$80, 9% vs \$9, 60%
Stage 5	\$80, 2% vs \$9, 60%	\$80, 9% vs \$9, 60%		
	T5	**T6**	**T7**	**T8**
Stage 1	\$80, 9% vs \$5, 60%	\$80, 9% vs \$1, 60%	\$80, 9% vs \$9, 60%	\$80, 9% vs \$1, 60%
Stage 2	\$80, 9% vs \$4, 60%	\$80, 9% vs \$2, 60%	\$80, 9% vs \$8, 60%	\$80, 9% vs \$2, 60%
Stage 3	\$80, 9% vs \$2, 60%	\$80, 9% vs \$4, 60%	\$80, 9% vs \$8.5, 60%	\$80, 9% vs \$1.5, 60%
Stage 4	\$80, 9% vs \$1, 60%	\$80, 9% vs \$5, 60%	\$80, 9% vs \$5, 60%	\$80, 9% vs \$5, 60%

Predictions:

The model predicts assimilation in Treatment 5 and 6, contrast in Treatments 7 and 8, and the boiling frog effect in Treatments 6 and 8.

(1) *(Contrast)*

$$C\left(\$5, p|T8\right) > C\left(\$5, p|T7\right)$$

(2) *(Weak Assimilation)*

$$\frac{\sum_{t=1}^{T} C\left((\$B_t, p)|T5\right)}{T} > \frac{\sum_{t=1}^{T} C\left((\$B_t, p)|T6\right)}{T}$$

(3) *(Strong Assimilation)*

$$C\left((\$B_t, p)|T5\right) > C\left((\$B_t, p)|T6\right)$$

at every period t

(4) *(Boiling Frog)*

$$C\left(\$5, p|T6\right) > C\left(\$5, p|T8\right)$$

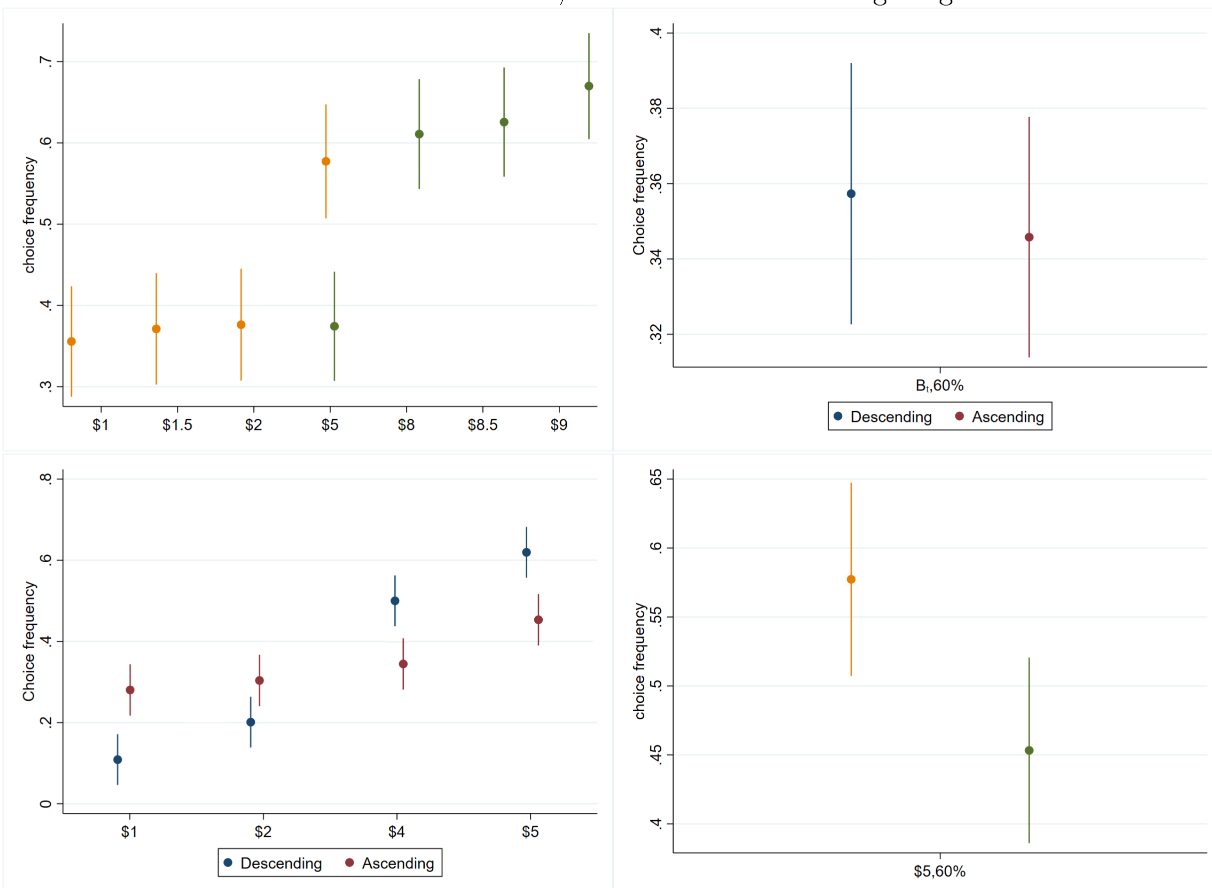
4.7.1. *Results.* The data show a strong contrast effect. The top left panel in Figure 4.5 shows that the option paying \$5 with probability 60%, in Treatment 8, is chosen 58% of the time, while only 37% of the times in Treatment 7. As shown in Column (1) in the top panel of Table 7, the difference in choice frequencies is economically sizable and statistically significant at any meaningful confidence level. Hence evidence confirms that the same lottery is contrasted away from the previous lotteries in at least one of the two Treatments.

The comparison between treatments yields less clear results when I test for assimilation (Treatment 5 vs Treatment 6). Here the hypothesis is that options paying B_t in Treatment 5 should be chosen more frequently than in Treatment 6. The top right panel of Figure 4.5 shows that subject do not seem to do so, since we cannot reject that the cross period choice frequencies are the same across treatments.

The analysis of the heterogenoeus treatment effects yields a more nuanced picture, represented in the bottom left panel of Figure 4.5: contrast occurs for some options and assimilation for others. In particular, the \$5 and \$4 dollar options are chosen less likely in Treatment 6, where where they are preceded by $(\$1, 0.6)$ and $(\$2, 0.6)$, than in Treatment 5, where the order is decreasing. This suggests that the \$5 option in Treatment 6 is assimilated to the previous lower paying options, while in Treatment 6 it is evaluated in isolation, since it is seen first. The opposite occurs for the low paying options \$1 and \$2: the two options are chosen less likely when they are presented in decreasing order of payoffs, suggesting that they are contrasted away from the previous options which pay more. Such Treatment differences are statistically significant at 5% or lower levels, as shown in the bottom panel of Table 7.

When comparing Treatments 8 and 6 in the bottom right panel of Figure 4.5 we detect a statistically significant difference in the choice frequency of the option in the last stage, $(\$5, 0.6)$, which is chosen 12% more when presented after a drastic upward change in B rather than after a gradual change (Column (2) top panel of Table 7). Hence the data confirm predictions 1. and 4. that are contrast and the boiling frog.

FIGURE 4.5. Contrast, Assimilation and Boiling Frog



The top right panel shows the cross period average frequency of $(B, 60\%)$ in Treatment 5 (blue dot) and Treatment 6 (red dot). The panels on the left show the choice frequencies of $(\$B, 0.6)$, with B on the x axis, across Treatment 7 (orange dots) and Treatment 8 (green dots), Treatment 5 (blue dots) and Treatment 6 (red dots). The bottom right panel shows the choice frequency of $(\$5, 0.6)$ in Treatment 8 (orange dot) and Treatment 5 (green dot). Bars indicate 95% confidence intervals.

4.8. **Expectations Based Reference Dependence, contrast, assimilation and the Boiling Frog.** The Expectations Based Reference Dependent (EBRD) utility model proposed by Koszegi and Rabin (2006) predicts that choice is indirectly distorted by past choice sets through the endogenous formation of expectations. The formal analysis conducted in the Appendix C shows that the expectations required to rationalize the contrast effect and the boiling frog effect are not reasonable, for the same reason why the ones required to rationalize the assimilation effect can be reasonable.

I first show the predictions of EBRD in Treatments detecting contrast and assimilation through variation of the probability. Compare two EBRD agents with same preference parameters, each making two choices. In the first stage, one faces $\left(\$80, p^{high}\right) vs\,(\$9, q)$ and the other $\left(\$80, p^{low}\right) vs\,(\$9, q)$, with $p^{high} > p > p^{low}$. In the second stage both face $(\$80, p)\, vs\,(\$9, q)$. Suppose we observe that the agent who faced p^{high} picks$(\$9, q)$ in the second stage, while the agent who faced p^{low} picks $(\$80, p)$ in the second stage. EBRD predicts that the agent who faced p^{high} expects to receive \$80 (\$9) with a *lower* (*higher*) chance than the agent who faced p^{low}. This is not reasonable, because subjects in Treatment 3 choose $\left(\$80, p^{high}\right)$, with higher frequency than the one in which subject in Treatment 4 choose $(\$80, p^{low})$, so they sould be expecting the opposite of what EBRD says. An alternative way to view this is that an EBRD agent who was accustomed to p^{high} is used to more risk than an agent accustomed to p^{low}: hence she should pick $(\$80, p)$. This does not happen in the contrast effect, but it happens in assimilation: suppose we observe, in stage 2, that the EBRD agent who faced p^{high} picks $(\$80, p)$ while the agent who observed p^{low} does not. EBRD predicts that it must be that the agent who faced p^{high} expects to receive \$80 with *higher* chance than the agent who faced p^{low}. These expectations are consistent with what subjects see and choose.

The predictions of EBRD in the Treatments that vary the payoff of a lottery, as opposed to the probability, follow a similar logic. An EBRD agent in the low payoff condition should expect the alternative lottery more than the subject in the high payoff condition, hence should choose the target option $(\$5, q)$ less often, which is not what subjects do in Treatments 7 and 8.

4.9. **Summary of experimental results and last remarks.** I find evidence both of overreaction of choice to big changes and underreaction to small changes in risk when the support of the lotteries is held fixed. I find evidence of overreaction to big changes in payoffs of lotteries, while mixed evidence of underreaction and overreaction depending on the direction of the change in payoffs. The bottomline of the results is condensed in Table 4.

The monotonic treatment differences between Treatment 5 and 6 highlighted in the top right panel of Figure 4.5 resonate the monotonic treatment differences found in Treatments 1 and 2, where assimilation seems to be stronger at higher levels of probability, suggesting the insensitivity is stronger when an option goes up in probability (or payoff) than when it goes down. Such findings suggest that a better fit might be an attention function g which is asymmetric, which I rule out for tractability purposes.

Table 4. Contrast, Assimilation and the Boiling Frog effect in probability and payoffs

	Probability	Payoff
Contrast	Yes	Yes
Assimilation	Yes	Mixed
Boiling Frog	Yes	Yes
Recency	Yes	*

* Left to future research.

5. Other predictions

5.1. **An endowment effect for risk.** In hypothetical decisions, subjects are more risk averse when trading a sure amount for a gamble than vice versa (Hershey, Kunreuther, and Schoemaker 1982, McCord and de Neufville 1985, Schoemaker 1990, although such methodologies have been criticized by Holt and Laury 2002, Plott and Zeiler 2005). Sprenger (2015) shows that subjects are more risk averse when asked to state probability equivalents to certain payments than when asked certain equivalents to risky payments. According to my model, a similar effect can be accounted for if decision makers are very sensitive to contrast: a DM that remembers a safe lottery and later faces a risky lottery with same expected value, contrasts the smaller expected utility against the one induced by the safe payment and is more risk averse than one who remembers the risky lottery.

Remark 1. Assume $\rho = 0$ (that is, the DM remembers only the past lottery) and consider two decision makers a and b with same preferences but the following choice sets $C_1^a = \{(Kp, 1)\}$, $C_1^b = \{(K, p; 0, 1-p)\}$ and $C_2^a = C_2^b = \{(K, p; 0, 1-p), (x, 1)\}$ for fixed p and K. Let g be as in Example 2, with $\nu = u$ concave. Then, if θ large,

$$U\left((K, p; 0, 1-p) \middle| (Kp, 1)\right) < U\left((K, p; 0, 1-p) \middle| (K, p; 0, 1-p)\right)$$

that is, a will accept a lower certainty equivalent than b in period 2.

The negative of this result is that, if this behavior is observed also for small stakes lotteries, the inferred shape of the weight g implies a very small assimilation region, which is in tension with experimental results yielding assimilation that I lay out later in the paper. Whether these facts can be quantitatively jointly matched is a matter af structural parameter estimation. Even if I do not estimate parameters based on samples of the papers mentioned, in Appendix D I compare estimates obtained from my sample and from the one by Frydman and Jin (2022), and do not reject the null that the preference parameters are the same across samples. However, the testing for the endowment effect as in Remark 1, and an estimation that takes into account such evidence, is left to future work.

5.2. **Assimilation and efficient coding.** Experimental literature on numerical cognition and choice has studied how the dispersion of options perceived by the decision maker during several decisions impacts taste for risk. In particular, Frydman and Jin (2022) (FJ) shows in binary choices between a risky and riskless option that more risk taking is associated with distributions of past upsides that have a higher mean. While the authors interpret fact as the effect of a change in the prior in the simulation of numbers used as a proxy of value, my model offers a different explanation for the same fact. Namely that assimilation makes the perception of the lottery closer to the mean distribution of the past lotteries if they are sufficiently close to it. I consider here a setting that adherently represents FJ's experimental paradigm.

Consider two decision makers a and b choose between $\left(X_t^i, p; 0, 1-p\right)$ and $\left(C_t^i, 1\right)$. Over many decision stages, the frequencies of upsides are $X_t^a \sim f^{dec}$ $t \in \{1, ..., T\}$ where f^{dec} is an decreasing distribution over the support $[k, K]$. Another group of decision makers observes upsides $X_t^b \sim f^{inc}, t \in \{1, ..., T\}$ with f^{inc} increasing on the support $[k, K]$.

Proposition 3. *Assume u is linear and g is as in Example 2 with $\beta = 1$. If $[k, K]$ is sufficiently small, a decision maker that has observed random sequence X_t^a will be, unconditionally, more likely to be risk averse than the DM who observed random sequence X_t^b. That is, for any $\forall x_t \in [k, K]$]*

$$Pr\left(U\left((x_t, p; 0, 1-p) \middle| \{X_s^a\}_{s<t} \sim f^{dec}\right) < U\left((x_t p, 1) \middle| \{X_s^a\}_{s<t} \sim f^{dec}\right)\right) >$$
$$Pr\left(U\left((x_t, p; 0, 1-p) \middle| \left\{X_s^b\right\}_{s<t} \sim f^{inc}\right) < U\left(x_t p, 1\right) \left\{X_s^b\right\}_{s<t} \sim f^{inc}\right)$$

if and only if

$$K \in \left[k, k + \frac{1}{2\theta p}\right]$$

5.3. **Stakes affect assimilation and contrast.** Suppose a DM is relatively insensitive to a change by 2% in the probability of his car breaking down. Does the model say that she will be insensitive of a 2% change in any risk? One might find appealing for the model to take into account some stake dependent sensitivity to changes. That is, even if the DM underreacts to a 2% change of a relatively modest consequence, she might overreact to a 2% change of the chance of a high stake. The model can easily take this into account, by allowing the attention weight g to increase if lotteries scale up. That is, suppose we scale up the payoffs paid by a distribution F by α: the decision maker has increasing sensitivity in stakes if

$$g\left(F\left(\frac{x}{\alpha}\right),G\left(\frac{x}{\alpha}\right)\right) > g\left(F\left(x\right),G\left(x\right)\right)\ \forall\alpha > 1. \tag{5.1}$$

Such property is satisfied by the functional form in Example 2, if v is a power function.

Under this restriction, the assimilation region decreases in stakes. Hence we might neglect a 2% change in car breakdown risk, but overreact to a 2% change in health risk. The following Remark states it formally.

Remark 2. Let $u\left(x\right) = kx^{\zeta}$. Let us call, for a generic lottery L, $L^{\alpha}\left(x\right) = L\left(\frac{x}{\alpha}\right)$. $A_t^{\alpha}\left(x\right) = A_t\left(\frac{x}{\alpha}\right)$. Suppose

$$C_s = \left\{\underline{\mu}^{\alpha}, B^{\alpha}\right\}$$

and

$$C_t = \left\{A_t, B^{\alpha}\right\}$$

Let B be such that

$$U\left(A_t|\underline{\mu}\right) = U\left(B|B\right)$$

where $\underline{\mu} = \underline{\mu}^{A_t}$ as defined in Proposition 1. Then

$$\alpha > 1 \implies U\left(A_t^{\alpha}|\underline{\mu}^{\alpha}\right) \quad > \quad U\left(B^{\alpha}|B^{\alpha}\right)$$

The statement simply says that the value of the lottery, if seen after dominated lotteries $\underline{\mu}^{\alpha}$, increases in α.

6. Conclusion

I develop, test and apply a new portable theory of choice under risk that builds on Bordalo et al. (2020) that is based on two biases studied for decades in social and cognitive psychology.

Anomalies in markets, often discussed in separate terms of underreaction and overreaction, can be unified in this work. I show that my model can explain them as the result of decision makers assimilating environments which look similar to what they are used to, and contrasting environments that starkly differ from them, and generates new testable predictions. One prediction is that periods when large news come in the market are more likely to see overreaction (and successive corrections in the opposite direction of the news), while periods when small news come are more likely to see underreaction (and positive correction after the news). This implies a relationship between market volatility and autocorrelation which I find to be true for US stock market data. Moreover, assimilation and contrast are found in the experiments which I have run and discussed in this work. Experimental, theoretical and field work might evaluate the implications of such theory and findings to policy making. A version of the Lucas' Critique applies: which gradual policies are indeed underreacted does not only depend on the perceptual biases I focus on in this paper, but also on the ability of agents to infer from intentionally manipulated changes. Agents that are sophisticated enough about the policy maker's intention to exploit the naives' bias, might undo the effect of intentionally gradual or drastic policies. Understanding real world contexts where this is more prevalent is the goal of future work.

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APPENDIX A

Proof. of Proposition 1

Note by monotonicity of g there must be distributions $\hat{H} >_{FOSD} F_{A_t} >_{FOSD} \dot{H}$ such that $g\left(F_{A_t}, \dot{H}\right) > 1$. By continuity, we can find $\hat{\rho}$, $\dot{\rho}$such that

$$g\left(\left(1-\hat{\rho}^t\right)\hat{H}+\hat{\rho}^t F_{A_t}, F_{A_t}\right)=1 \tag{6.1}$$

and

$$g\left(\left(1-\dot{\rho}^t\right)\dot{H}+\dot{\rho}^t F_{A_t}, F_{A_t}\right)=1$$

pick $\rho^* = \min\{\hat{\rho}, \dot{\rho}\}$. WLOG suppose $\rho = \rho^* = \hat{\rho}$. Then, by continuity and monotonicity of g we can find $F_{A_t} >_{FOSD} \tilde{H} >_{FOSD} \dot{H}$ such that

$$g\left(\left(1-\hat{\rho}^t\right)\tilde{H}+\hat{\rho}^t F_{A_t}, F_{A_t}\right)=1 \tag{6.2}$$

Let us call $\underline{\mu}$ and $\bar{\mu}$ the lotteries with cdf equal to, respectively, $\tilde{H}$ and $\hat{H}$.

Note that $d\left(A_s^i, A_t\right) < d\left(B, A_t\right)$ implies

$$A_t^{i,m} = (1-\rho)\sum \rho^{j-1} A_{t-j}^i + \rho^t A_t \;\; i = a, b$$

(assimilation) By monotonicity $\underline{\mu} <_{FOSD} A_s^a <_{FOSD} A_t <_{FOSD} A_s^b <_{FOSD} \bar{\mu}$ implies

$$g\left((1-\rho)\sum \rho^{j-1} F_{A_{t-j}^a} + \rho^t F_{A_t}, F_{A_t}\right) < 1 \tag{6.3}$$

and

$$g\left((1-\rho)\sum \rho^{j-1} F_{A_{t-j}^b} + \rho^t F_{A_t}, F_{A_t}\right) < 1 \tag{6.4}$$

This implies that

$$U\left(A_t | A_t^{a,m}\right) = \mathbb{E}_{A_t^{a,m}}[u] + g\left(F_{A_t^{a,m}}, F_{A_t}\right)\left\{\mathbb{E}_{A_t}[u] - \mathbb{E}_{A_t^{a,m}}[u]\right\} < \mathbb{E}_{A_t}[u] \tag{6.5}$$

and

$$U\left(A_t | A_t^{b,m}\right) = \mathbb{E}_{A_t^{a,m}}[u] + g\left(F_{A_t^{b,m}}, F_{A_t}\right)\left\{\mathbb{E}_{A_t}[u] - \mathbb{E}_{A_t^{b,m}}[u]\right\} > \mathbb{E}_{A_t}[u] \tag{6.6}$$

Where (6.5) comes from (6.3) and (6.6) comes from (6.4). The result follows from (6.5) , (6.6) and the fact that

$$U\left(B | B_t^{i,m}\right) = \mathbb{E}_B[u] \tag{6.7}$$

(contrast) By monotonicity $\underline{\mu} >_{FOSD} A_s^a \;\; A_s^b >_{FOSD} \bar{\mu}$ implies

$$g\left((1-\rho)\sum \rho^{j-1} F_{A_{t-j}^a} + \rho^t F_{A_t}, F_{A_t}\right) > 1 \tag{6.8}$$

and

$$g\left((1-\rho)\sum \rho^{j-1} F_{A_{t-j}^b} + \rho^t F_{A_t}, F_{A_t}\right) > 1 \tag{6.9}$$

This implies that

$$U\left(A_t | A_t^{a,m}\right) = \mathbb{E}_{A_t^{a,m}}[u] + g\left(F_{A_t^{a,m}}, F_{A_t}\right)\left\{\mathbb{E}_{A_t}[u] - \mathbb{E}_{A_t^{a,m}}[u]\right\} > \mathbb{E}_{A_t}[u] \tag{6.10}$$

and

$$U\left(A_t|A_t^{b,m}\right) = \mathbb{E}_{A_t^{a,m}}[u] + g\left(F_{A_t^{b,m}}, F_{A_t}\right)\left\{\mathbb{E}_{A_t}[u] - \mathbb{E}_{A_t^{b,m}}[u]\right\} < \mathbb{E}_{A_t}[u] \tag{6.11}$$

Where (6.10) comes from (6.8) and (6.11) comes from (6.9). The result follows from (6.10) , (6.11) and (6.7) □

Proof. of Proposition 2

$A_s^a <_{FOSD} A_s^b$ implies $A_t^{a,m} <_{FOSD} A_t^{b,m}$ which implies

$$g\left(F_{A_t^{a,m}}, F_{A_t^a}\right) > g\left(F_{A_t^{b,m}}, F_{A_t^b}\right) \tag{6.12}$$

by monotonicity. Moreover, by inequality 6.8,

$$g\left(F_{A_t^{a,m}}, F_{A_t^a}\right) > 1. \tag{6.13}$$

As a result

$$U\left(A_t|A_t^{a,m}\right) - U\left(A_t|A_t^{b,m}\right) = \mathbb{E}_{A_t^{a,m}}[u] - \mathbb{E}_{A_t^{a,m}}[u] + g\left(F_{A_t^{a,m}}, F_{A_t}\right)\left\{\mathbb{E}_{A_t}[u] - \mathbb{E}_{A_t^{a,m}}[u]\right\} +$$

$$-g\left(F_{A_t^{b,m}}, F_{A_t}\right)\left\{\mathbb{E}_{A_t}[u] - \mathbb{E}_{A_t^{b,m}}[u]\right\} >$$

$$> \mathbb{E}_{A_t^{a,m}}[u] - \mathbb{E}_{A_t^{a,m}}[u] + g\left(F_{A_t^{a,m}}, F_{A_t}\right)\left\{\mathbb{E}_{A_t^{b,m}}[u] - \mathbb{E}_{A_t^{a,m}}[u]\right\} > 0 \tag{6.14}$$

Where the first inequality is implied by 6.12 and the last inequality is implied by 6.13. This completes the proof. □

Proof. of Proposition 3

First notice that

$$U\left((x_t, p; 0, 1-p)\,\middle|\,\left\{X_s^i\right\}_{s<t}\right) = pX_t^{m,i} + \theta \begin{cases} \left(px_t - pX_t^{m,i}\right)^2 & x_t > X_t^m \\ -\left(px_t - pX_t^{m,i}\right)^2 & x_t < X_t^m \end{cases}$$

where $i = a, b$ and

$$X_t^{m,i} = (1-\rho)\sum_{j=1}^{t}\rho^{j-1}pX_{t-j}^i + \rho^t p x_t$$

Where $X_s^i \sim f^i$. Note that f^bincreasing and f^a decreasing implies $F^b < F^a$ over the support $\left[k, K\right]$ which implies $X_s^b >_{FOSD} X_s^a$ which implies $X^{m,b} >_{FOSD} X^{m,b}$. Given that p is fixed, rewrite for notational ease

$$U\left((x_t, p; 0, 1-p)\,\middle|\,\left\{X_s^i\right\}_{s<t}\right) = V\left(x_t|X_t^{m,i}\right)$$

Note that, for fixed x_t

$$\frac{\partial}{\partial X_t^{m,i}}V\left(x_t|X_t^{m,i}\right) > 0 \iff X_t^{m,i} \in \left[x_t - \frac{1}{2\theta p}, x_t + \frac{1}{2\theta p}\right]$$

By assumption,

$$\mathrm{supp} f^b = \mathrm{supp} f^a = [k, K].$$

By the definition of support,

$$
\begin{aligned}
Pr\left(\frac{\partial}{\partial X_t^{m,i}}V\left(x_t|X_t^{m,i}\right)>0\right)=Pr\left(X_t^{m,i}\in\left[x_t-\frac{1}{2\theta p},x_t+\frac{1}{2\theta p}\right]\right)=1\ \forall x_t\in[k,K]\iff \\
\iff \left[x_t-\frac{1}{2\theta p},x_t+\frac{1}{2\theta p}\right]\supseteq[k,K]\ \ \forall x_t\in[k,K]\iff \\
\iff x_t-\frac{1}{2\theta p}\le k\wedge x_t+\frac{1}{2\theta p}\ge K\ \forall x_t\in[k,K]\iff \\
\iff K-\frac{1}{2\theta p}\le k\wedge k+\frac{1}{2\theta p}\ge K\iff \\
\iff K-k\le\frac{1}{2\theta p}
\end{aligned}
\tag{6.15}
$$

(if part) If condition 6.15 is satisfied, then $V\left(x_t\middle|X^{m,i}\right)$ is strictly increasing in $X^{m,i}$ in its support. This implies that $X^{m,b}>_{FOSD}X^{m,a}$ implies

$$
V\left(x_t\middle|X^{m,b}\right)>_{FOSD}V\left(x_t\middle|X^{m,a}\right). \tag{6.16}
$$

By definition of first-order stochastic dominance, inequality (6.16) implies that $\forall c\in \text{Supp}V$

$$
Pr\left(V\left(x_t\middle|X^{m,b}\right)>c\right)>Pr\left(V\left(x_t\middle|X^{m,a}\right)>c\right)
$$

and

$$
Pr\left(V\left(x_t\middle|X^{m,b}\right)>c\right)=Pr\left(V\left(x_t\middle|X^{m,a}\right)>c\right)
$$

for $\forall c\notin \text{Supp}V$.

This, and the fact that $U\left((c_t,1)\middle|\left\{X_s^i\right\}_{s<t}\right)=c_t$ implies the result.

(only if part) suppose condition 6.15 is not satisfied. WLOG, take $k=0$. Then $K=\frac{1}{2\theta p}+\delta$ for some $\delta>0$. Assume $\rho=0$. Take $x_t=0$, and $c=p\left(\frac{1}{2\theta p}+\frac{\delta}{2}\right)-\theta\left(\frac{1}{2\theta p}+\frac{\delta}{2}\right)^2$. Take f^a continuous. Then there exists $\epsilon>0$ such that

$$
P\left(V\left(0\middle|X^{m,a}\right)>c\right)=\epsilon
$$

Take f^b such that

$$
f^b(x)=\begin{cases} l & x\in\left[k,\frac{1}{2\theta p}+\frac{\delta}{2}\right] \\ L & x\in\left(\frac{1}{2\theta p}+\frac{\delta}{2},K\right] \end{cases}
$$

where

$$
L=\frac{2}{\delta}\chi
$$

with $\chi>1-\epsilon$ and

$$
l=\frac{1-\chi}{\frac{1}{2\theta p}+\frac{\delta}{2}}.
$$

It follows by construction that

$$
P\left(V\left(0\middle|X^{m,b}\right)>c\right)<\epsilon
$$

$\square$

Proof. of Proposition 7

$$\rho = 0 \implies P_t^\theta - P_{t-1}^\theta = u_t + \theta\left[\left|u_t\right|u_t - \left|u_{t-1}\right|u_{t-1}\right]$$

$$Cov\left(\Delta P_{t+1}^\theta, \Delta P_t^\theta\right) = Cov\left(-\left|u_t\right|u_t, u_t - \theta\left|u_t\right|u_t\right) =$$

$$= \frac{1}{\sigma}\left[A\int_0^\infty x e^{\frac{1}{2\sigma^2}x^2}\mathrm{d}x - B\int_0^\infty x^4 e^{\frac{1}{2\sigma^2}x^2}\mathrm{d}x\right] =$$

$$= K_1\sigma^3 - K_2\sigma^4 = \sigma^3\left(K_1 - K_2\sigma\right)$$

where $K_1 > 0$ and $K_2 > 0$. This implies that

$$Cov\left(\Delta P_{t+1}^\theta, \Delta P_t^\theta\right) > 0 \iff \sigma < \frac{K_1}{K_2}$$

□

The following Corollaries exemplify Proposition 1

Corollary 2. *Suppose, at time t, $C_t = \{A_t, B\}$, where $A_t = (K, p; 0, 1-p)$. Consider two alternative histories of choice sets for $s < t$,*

$$C_s^i = \left\{A_s^i, B\right\}\ i = a, b$$

such that $d(A_s, B) > d(A_s, A_t)$ for every s and $A_s^i = \left(K, p_s^i\right)$ for $i = a, b$. Assume either ρ is small or t is large. Then there are threshold lotteries

$$\underline{p} < p < \bar{p}$$

such that:

(Assimilation) If

$$\underline{p} < p_s^a < p < p_s^b < \bar{p}$$

then

$$U\left(A_t|A_t^{a,m}\right) \geq U\left(B|B^{a,m}\right) \implies U\left(A_t|A_t^{b,m}\right) > U\left(B|B^{b,m}\right).$$

(Contrast) If

$$p_s^a < \underline{p} < \bar{p} < p_s^b$$

$$U\left(A_t|A_t^{b,m}\right) \geq U\left(B|B^{b,m}\right) \implies U\left(A_t|A_t^{a,m}\right) > U\left(B|B^{a,m}\right).$$

Corollary 3. *Suppose, at time t, $C_t = \{A_t, B\}$, where $A_t = (K, p; 0, 1-p)$. Consider two alternative histories of choice sets for $s < t$,*

$$C_s^i = \left\{A_s^i, B\right\}\ i = a, b$$

such that $d(A_s, B) > d(A_s, A_t)$ for every s and $A_s^i = \left(K_s^i, p; 0, 1-p\right)$ for $i = a, b$. Assume either ρ is small or t is large. Then there are threshold lotteries

$$\underline{K} < K < \bar{K}$$

such that:

(Assimilation) If

$$\underline{K} < K_s^a < K < K_s^b < \bar{K}$$

then

$$U\left(A_t|A_t^{a,m}\right) \geq U\left(B|B^{a,m}\right) \implies U\left(A_t|A_t^{b,m}\right) > U\left(B|B^{b,m}\right).$$

(Contrast) If

$$K_s^a < \underline{K} < \bar{K} < K_s^b$$

$$U\left(A_t|A_t^{b,m}\right) \geq U\left(B|B^{b,m}\right) \implies U\left(A_t|A_t^{a,m}\right) > U\left(B|B^{a,m}\right).$$

The following Corollary restates the result about contrast in a formulation that closely tracks the experimental paradigm that I will use to test the model, and compares two individuals who have the same utility parameters, but experienced different histories of choice sets. The individuals are called T3 and T4 to represent the treatment conditions that will be used in the experiment.

Appendix B: Additional experimental treatments

In this section I describe the experimental treatments that have been run in addition to the one presented in the main paper.

Treatments 1*, 2*,$\tilde{1}$,$\tilde{2}$, 9, 10

Description. In all of the treatments, subjects are asked to make binary choices between lotteries and are told that one of their choices will be implemented. Table 5 displays the sequences of choice sets across the different treatments. Treatments 1*and 2* are administered on Cloud Research to a total of 380 subjects, of which 362 pass basic comprehension checks. The other treatments are administered on Prolific, with Treatments $\tilde{1}$ and $\tilde{2}$ being administered to a total of 500 subjects, and 9 and 10 to a total of 298 subjects, all of which pass the basic checks. Treatments 1* and 2* consist of, respectively, the same choice sets of Treatment 1 with the addition of the choice set

$$C_1^{1^*} = \{(\$80, 11\%), (\$9, 60\%)\},$$

and Treatment 2 with the addition of the choice set

$$C_6^{2^*} = C_1^{1^*}$$

This means that Treatment 2* shows to subjects the same choice sets as Treatment 1*, in reverse order.

Treatments $\tilde{1}$and $\tilde{2}$ face the subjects with the same exact questions as Treatments 1 and 2, but show all of them on the screen, rather than presenting the choices sequentially on different screens.

Treatments 9 and 10 show choice sets of the form

$$C_t^j = \left\{\left(\$K_t^j, 9\%\right), (\$5, 60\%)\right\}$$

with

$$\left(K_t^j\right)_{t=1}^4 = \begin{cases} (90, 86, 88, 50) & if\, j = 9 \\ (20, 24, 22, 50) & if\, j = 10 \end{cases}$$

That is, Treatments 9 and 10 show the same final choice set

$$C_4^j = \{(\$50, 9\%), (\$5, 60\%)\}$$

but the riskier option $(\$50, 9\%)$ is preceded, in Treatment 9, by options paying much a larger upside, while, in Treatment 10, it is preceded by options paying a much smaller upside.

Predictions. The model predicts that we should observe assimilation in Treatments 1* and 2*, because we have detected assimilation in Treatments 1 and 2: the addition of one option which is marginally different from the riskier option of choice set C_1^1 (C_5^2), that is $(\$80, 9\%)$, should not change dramatically the results. That is, $\left(\$80, q_t^j\right)$ should be chosen more frequently for $j = 1$ than for $j = 2$

The model also predicts that we should *not* observe assimilation in Treatments $\tilde{1}$ and $\tilde{2}$, because choice should be distorted by *remembered* lotteries, and not by choice sets faced simultaneously.

Finally, the model predicts that we should observe contrast in Treatments 9 and 10. The riskier lotteries in stages $t = 1, 2, 3$ are constructed to be different, in expected value terms, from the lottery in stage 4 as much as the lotteries in Treatments 3 and 4 are from the riskier lottery in the last stage.

Results. The results confirm the predctions. The top left table in Table 6 reports results from the comparison between Treatments 1* and 2*. Each column reports the regression of the indicator variable which is equal to 1 when the riskier option is chosen, and 0 otherwise, on the indicator for Treatment 1* (called Treatment 1* effect) and the constant. Each column reports the coefficients estimated for each choice set separately, going from Column (1) that refers to the choice set $\{(\$80, 11\%), (\$9, 60\%)\}$, to Column (6) which refers to $\{(\$80, 2\%), (\$9, 60\%)\}$. The regressions reveal that the riskier options $(\$80, 11\%)$ and $(\$80, 9\%)$ are about 10% more likely to be chosen in Treatment 1* than in 2*, while results are economically smaller and statistically insignificant for the other options. This is consistent with assimilation: $(\$80, 11\%)$ and $(\$80, 9\%)$ in Treatment 2* look similar to previously faced options that pay \$80 with a lower chance, and are chosen less often than in Treatment 1*. The top panel of Figure 6.1 displays the average choice frequencies of the riskier option across treatments and choice sets.

The comparison of the choice frequencies of $(\$50, 9\%)$ across Treatments 9 and 10 reveals the contrast effect. As both the central panel of Figure 6.1 and the bottom left table of Table 6 show the riskier option in stage 4 is chosen 27% less likely in Treatment 9 than in Treatment 10. This suggests that either $(\$50, 9\%)$ looks exaggerately larger than $(\$22, 9\%)$ or exaggerately smaller than $(\$88, 9\%)$, or both.

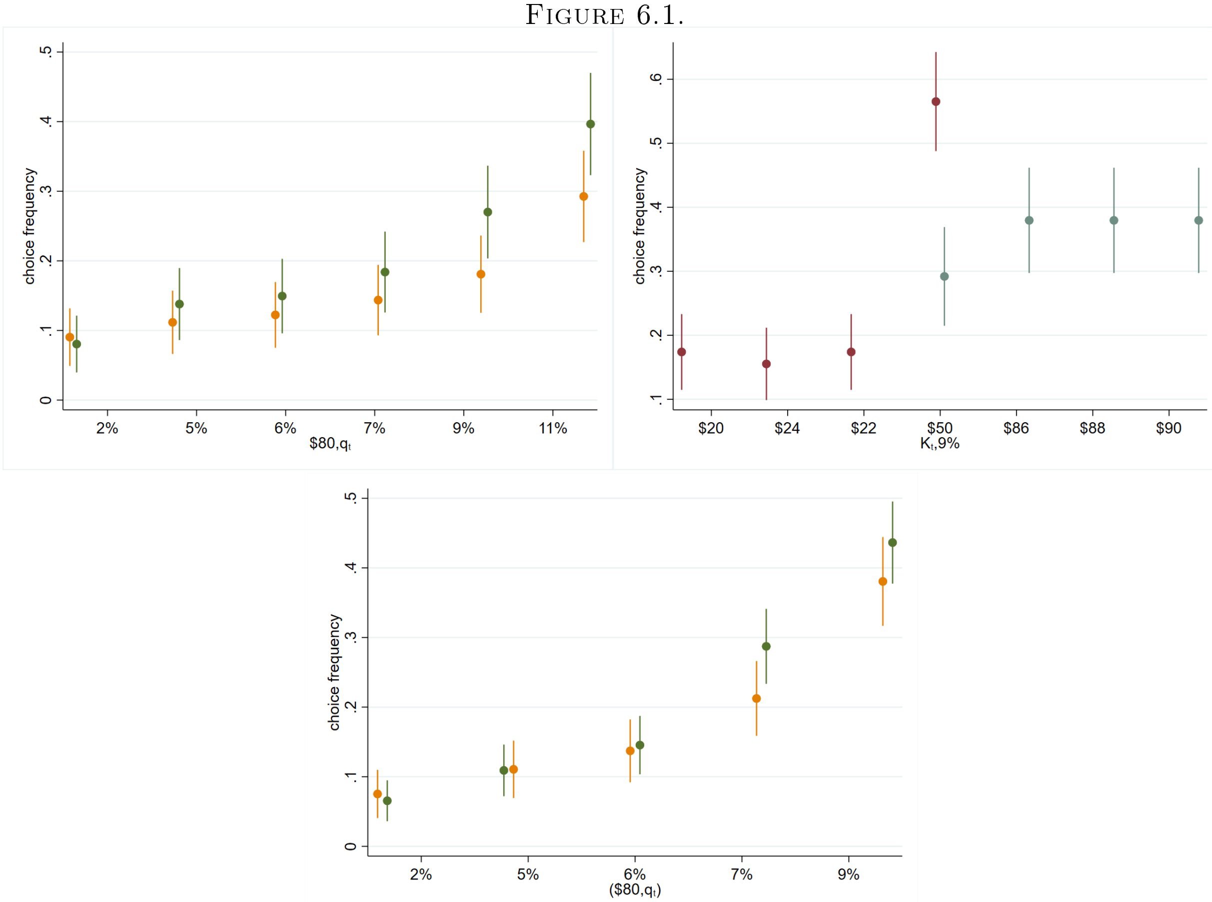
The comparison of treatment effects between Treatments 1, 2, and Treatments $\tilde{1}$, $\tilde{2}$, shows that the assimilation effect is weakened when choice sets are shown all in the same screen. The top right Table in Table 6 displays the Treatment differences between 1 and 2 as regression coefficients. As discussed in subsection 4.3.2 the Treatment difference is statistically significantly positive when the probability of \$80 is $6\%, 7\%$ and 9% (implying assimilation of the corresponding options), and negative for 2% (contrast). The analogous Treatment differences between $\tilde{1}$ and $\tilde{2}$ reveal much smaller (in absolute

Table 5. Additional treatments

	T1*	T2*	T9	T10
Stage 1	\$80, 11% vs \$9, 60%	\$80, 2% vs \$9, 60%	\$80, 9% vs \$5, 60%	\$20, 9% vs \$5, 60%
Stage 2	\$80, 9% vs \$9, 60%	\$80, 5% vs \$9, 60%	\$76, 9% vs \$5, 60%	\$24, 9% vs \$5, 60%
Stage 3	\$80, 7% vs \$9, 60%	\$80, 6% vs \$9, 60%	\$78, 9% vs \$5, 60%	\$22, 9% vs \$5, 60%
Stage 4	\$80, 6% vs \$9, 60%	\$80, 7% vs \$9, 60%	\$50, 9% vs \$5, 60%	\$50, 9% vs \$5, 60%
Stage 5	\$80, 5% vs \$9, 60%	\$80, 9% vs \$9, 60%		
Stage 6	\$80, 2% vs \$9, 60%	\$80, 11% vs \$9, 60%		

value) treatment effects. I conclude that the observation of choice sets in the same screen dampens the extent to which subjects are biased by order effects consistently with my theory.

FIGURE 6.1.



The top panel shows the choice frequencies of ($80,q) in Treatment 1 (green dots) and 2* (orange dots),with q on the x axis. The central panel shows choice frequencies of ($B,9%) in Treatment 9 (grey dots) and 10 (brown dots), with B on the x axis. The lower panel shows choice frequencies of ($80,q) in Treatment $\tilde{1}$ (green dots) and $\tilde{2}$ (orange dots), with q on the x axis. Segments represent 95% confidence intervals.*

Appendix C: Expectations based reference dependence and the Experiments

I hereby relate my experimental findings to Expectations based reference dependent gain loss utility, as in Koszegi and Rabin (2006). The decision maker decides according to a reference dependent utiliry defined as follows.

Definition 3. The utility of lottery μ with reference lottery ν is

$$U(\pi|\nu) = \int u(x)\, d\pi(x) + \int\int \mu(u(x) - u(y))\, \mathrm{d}\pi(x)\, \mathrm{d}\nu(y)$$

where

$$\mu(z) = \eta \{\mathbb{I}(z > 0) + \lambda \mathbb{I}(z \leq 0)\} z$$

with $\eta > 0$ and $\lambda > 1$.

TABLE 6. Assimilation and contrast

	(1) 11%	(2) 9%	(3) 7%	(4) 6%	(5) 5%	(6) 2%
Treatment 1* effect	0.104**	0.0893**	0.0403	0.0271	0.0262	-0.00997
	(0.0498)	(0.0437)	(0.0389)	(0.0361)	(0.0348)	(0.0295)
Treatment 2*	0.293***	0.181***	0.144***	0.122***	0.112***	0.0904***
	(0.0345)	(0.0303)	(0.0270)	(0.0250)	(0.0241)	(0.0205)
Observations	362	362	362	362	362	362
R-squared	0.012	0.011	0.003	0.002	0.002	0.000

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

	(1) 9%	(2) 7%	(3) 6%	(4) 5%	(5) 2%
Treatment 1 effect	0.126***	0.0959***	0.0480**	0.0133	-0.0645***
	(0.0260)	(0.0242)	(0.0223)	(0.0211)	(0.0177)
Treatment 2	0.234***	0.186***	0.161***	0.152***	0.138***
	(0.0181)	(0.0168)	(0.0155)	(0.0147)	(0.0123)
Observations	1,208	1,208	1,208	1,208	1,208
R-squared	0.019	0.013	0.004	0.000	0.011

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

	(1) ($50,9%)
Treatment 9 effect	-0.273***
	(0.0557)
Treatment 10	0.565***
	(0.0377)
Observations	298
R-squared	0.075

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

	(1) 9%	(2) 7%	(3) 6%	(4) 5%	(5) 2%
Treatment $\tilde{1}$ effect	0.0558	0.0749*	0.00829	-0.00153	-0.00977
	(0.0442)	(0.0390)	(0.0314)	(0.0281)	(0.0229)
Treatment $\tilde{2}$	0.381***	0.212***	0.137***	0.111***	0.0752***
	(0.0327)	(0.0289)	(0.0232)	(0.0208)	(0.0170)
Observations	501	501	501	501	501
R-squared	0.003	0.007	0.000	0.000	0.000

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

The top left table shows, for each column, the regression coefficients of a dummy = 1 if the riskier option is chosen and 0 otherwise. The regressor Treatment 1^* effect is a dummy =1 if the observation is in Treatment 1^* and =0 if it is in Treatment 2^*. The regressor Treatment 2^* is the constant.
The top right table shows, for each column, the regression coefficients of a dummy = 1 if the riskier option is chosen and 0 otherwise. The regressor Treatment 1 effect is a dummy =1 if the observation is in Treatment 1 and =0 if it is in Treatment 2. The regressor Treatment 2 is the constant.
The bottom left table shows, for each column, the regression coefficients of a dummy = 1 if the riskier option is chosen and 0 otherwise. The regressor Treatment 9 effect is a dummy =1 if the observation is in Treatment 9 and =0 if it is in Treatment 10. The regressor Treatment 10 is the constant.
The bottom right table shows, for each column, the regression coefficients of a dummy = 1 if the riskier option is chosen and 0 otherwise. The regressor Treatment $\tilde{1}$ effect is a dummy =1 if the observation is in Treatment 1 and =0 if it is in Treatment 2. The regressor Treatment $\tilde{2}$ is the constant.

TREATMENTS 1, 2 ,3 4

Throughout I consider the evaluation of the same lottery

$$\pi = (K, p)$$

by two decision makers a and d, (a for ascending order and d for descending order) that represent subjects in two different treatment groups. Let us call ν^j the reference lottery based on which decision maker j evaluates gains and losses. The DM evaluates π at time T after having faced

$$S_t^j = \left\{ \left(K, r_t^j \right), (k, q) \right\} \quad for \ \ j = a, d \ and \ t = 1, 2, T-1$$

where $K > k$. At time T, j chooses π over (k, q) if and only if

$$U\left(\pi|\nu^j\right) - U\left((k,q)\,|\nu^j\right) > 0.$$

Contrast Effect. If a is in Treatment 4 and d is in Treatment 3, we are interested in knowing the conditions for contrast effect, that is

$$U\left(\pi|\nu^a\right) - U\left((k,q)\,|\nu^a\right) > 0.$$

and

$$U\left(\pi|\nu^d\right) - U\left((k,q)\,|\nu^d\right) < 0.$$

TABLE 7. Contrast, Assimilation and the Boiling Frog (2)

	(1) ($5,60%)	(2) ($5,60%)
D Drastic effect	-0.203***	
	(0.0492)	
A Drastic	0.577***	
	(0.0352)	
A Drastic effect		0.124**
		(0.0493)
A Gradual		0.453***
		(0.0340)
Observations	397	408
R-squared	0.041	0.015

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

	(1) $5	(2) $4	(3) $2	(4) $1
D Gradual effect	0.166***	0.154***	-0.103**	-0.172***
	(0.0496)	(0.0491)	(0.0437)	(0.0395)
A Gradual	0.453***	0.346***	0.304***	0.280***
	(0.0337)	(0.0334)	(0.0297)	(0.0268)
Observations	398	398	398	398
R-squared	0.028	0.024	0.014	0.046

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

The top table shows the regressions of a dummy(=1 if ($5, 60%) is chosen) on a constant ("A Drastic" in Column (1) and "A Gradual" in Column (2)) and a Treatment dummy. The dummy "D Drastic effect" is =1 if subject is in Treatment 7, the dummy "A Drastic effect" is =1 if subject is in Treatment 8. In Column (1) the sample includes Treatments 7 and 8, while in Column (2) it includes Treatments 6 and 8.
The bottom table shows, for each column, the regression coefficients of a dummy = 1 if $\$B, 60\%$ is chosen and 0 otherwise, with B varying across columns. The regressor "D Gradual effect" is a dummy =1 if the observation is in Treatment 5 and =0 if it is in Treatment 6. The regressor A Gradual is the constant. The sample includes Treatment 5 and 6.

Case 1: each decision maker expects a linear combination of the lotteries previously chosen. Suppose that decision makers, just before seeing the choice set S_T^j, believe they will face one of the past ones. This is plausible given that there is not any trend in the probability of any of the two options in the periods before T in either of the two treatments. We examine, cases where each j has chosen either always $\left(K, r_t^i\right)$ or always (k, q) at any

$$t < T$$

this is consistent with the evidence of the experiment, where almost all subjects do not switch between the risky and the safer lottery between period 1 and 3. Such assumption implies that the reference lottery of decision maker j is

$$\nu^j = \begin{cases} \left(K, \sum_t \alpha_t^a r_t^a\right) & if\, j = a\, and\, picked\, \left(K, r_t^a\right) \\ (k, q) & if\, j = a, d\, and\, picked\, (k, q) \\ \left(K, \sum_t \alpha_t^d r_t^d\right) & if\, j = d\, and\, picked\, \left(K, r_t^d\right) \end{cases} \tag{6.17}$$

where

$$\sum_{t<T} \alpha_t^j = 1 \;\; j = a, d$$

We call

$$r^j := \sum_{t<T} \alpha_t^j r_t^j$$

Before describing all the possible choice combinations at time T that can be observed from a and d , I introduce a useful result that states that if a decision maker, accustomed to (k, q), picks (K, p) over (k, q), then she would also pick (K, p) if she was accustomed to $(K, \bar{r})$ with $\bar{r}$ arbitrary.

Proposition 4. *For every $r_0 \in (0, 1)$,if*

$$U(\pi|k, q) - U(k, q|k, q) > 0 \tag{6.18}$$

then

$$U(\pi|K, \bar{r}) - U(k, q|K, \bar{r}) > 0. \tag{6.19}$$

Moreover a decision maker expecting $(K, \bar{r})$ chooses π over (k, q) if and only if

$$\mathbb{E}_\pi [u] > \mathbb{E}_{(k,q)} [u]. \tag{6.20}$$

That is

$$U(\pi|K, \bar{r}) - U(k, q|K, \bar{r}) > 0 \tag{6.21}$$

if and only if

$$u(K)p - u(k)q > 0. \tag{6.22}$$

Proof. We first prove the second result. Assuming $K > k$ and $p > q$, simple algebra yields

$$U(\pi|K, \bar{r}) - U(k, q|K, \bar{r}) = (u(K)p - u(k)q)\{1 + \eta + \eta(\lambda - 1)\bar{r}\} > 0 \iff$$

$$\iff u(K)p > u(k)q$$

Next we prove the first result.

$$U(\pi|k, q) - U(k, q|k, q) =$$

$$= (u(K)p - u(k)q)\{1 + \eta\} - u(k)q\eta(\lambda - 1)[p - q] <$$

$$< (u(K)p - u(k)q)\{1 + \eta\}.$$

As a result if

$$0 < U(\pi|k, q) - U(k, q|k, q)$$

it has to be that

$$\left(u\left(K\right)p-u\left(k\right)q\right)\left\{1+\eta\right\}>0$$

which implies

$$u\left(K\right)p>u\left(k\right)q$$

that yields the desired result. □

We want to know the possible combinations of period T choices that can be observed simultaneously in a and d. There are 4 possible combinations of past experiences, two for each decision maker j, that make four possible combinations of ν^d and ν^a that determine four different relations between their choices.

(1) d has always chosen $\left(K,r_t^d\right)$ while a has always chosen $\left(k,q\right)$. Since $\left(K,r_t^d\right)$ is riskier than $\left(k,q\right)$, d is accustomed to taking more risk than a. That is

$$\nu^d=\left(K,r^d\right)$$
$$\nu^a=\left(k,q\right)$$

This makes d more willing to take risk than a. Hence it has to be that if a picks π then also d picks π.

(2) d has always chosen $\left(K,r_t^d\right)$ while a has always chosen $\left(K,r_t^a\right)$. Then

$$\nu^d=\left(K,r^d\right)$$

and

$$\nu^a=\left(K,r^a\right)$$

where $r^d>r^a$. The gain loss utility that agents feel in proportion of the rational expected utility is the same for both options for both agents, and a picks π if and only if d does. This is the Corollary of the second result of Proposition 4 that states that, conditional on reference lottery $(K,\bar{r})$, risk taking is independent of $\bar{r}$.

(3) Both d and a have always chosen $\left(k,q\right)$,that is

$$\nu^d=\nu^a=\left(k,q\right)$$

They have identical preferences: hence a picks π if and only if d does.

(4) d has always chosen $\left(k,q\right)$ and a has always chosen $\left(K,r_t^a\right)$. This case is symmetric to case 1. Hence if d picks π it means that $u\left(K\right)p>u\left(k\right)q$: thus also a picks π. But if η or λ are high and $u\left(K\right)p>u\left(k\right)q$, d picks $\left(k,q\right)$ and a picks π, which is the contrast effect.

Hence, there can be contrast on average only if the majority of the subjects is like in point 4. But we observe in the data that decision maker a usually picks $\left(k,p\right)$ i periods $t<T$. Thus cases 1. and 3. apply, ruling out the contrast effect. This statement can be made formal. We impose that there is individual specific randomness in the decision making, that allows us to evaluate the predictions of Expectations Based reference dependence in terms of choice frequencies. The following Proposition states that if, in periods $t<T$, individuals of the a type choose $\left(k,q\right)$ more often than individuals of the d type, then it

must be that they choose (k,q) more often also at time T. In other words, individuals in the ascending treatment are accustomed to take less risk than individuals in the descending treatment: thus, at time T, individuals in the ascending treatment choose the risky option less frequently than individuals in the descending treatment. This is the opposite of what we observe in the treatment comparison between T3 and T4.

Proposition 5. *Assume each subject's utility has an iid shock* ϵ_i^j

$$U_i\left(\pi|\nu_i^j\right) - U_i\left(k,q|\nu_i^j\right) = U\left(\pi|\nu_i^j\right) - U\left(k,q|\nu_i^j\right) + \epsilon_i^j. \tag{6.23}$$

where

$$j = a, d$$

and a is the ascending treatment while d is the descending treatment

Assume that the distribution of ϵ_i^j is identical across populations a and d

Assume that, for every i, the reference lottery at time T, ν_i^j, is a combination of the past chosen lotteries:

$$\nu_i^j = \sum_{t=1}^{T-1} \alpha_t^j \phi_{t,i}^j \tag{6.24}$$

where $\phi_{t,it}^j$ is the lottery chosen at time t by subject i in population j and $\sum_{t=1}^{T-1} \alpha_t^j = 1$.

Assume K and p are such that

$$u(K)\, p > u(k)\, q \tag{6.25}$$

then

$$Pr\left(\left(K, r_{T-1}^j\right) \text{ } is\ chosen|j = d\right) > Pr\left(\left(K, r_{T-1}^j\right) \text{ } is\ chosen|j = a\right) \tag{6.26}$$

implies that

$$Pr\left(\pi\, is\, chosen|j = d\right) > Pr\left(\pi\, is\, chosen\, |j = a\right) \tag{6.27}$$

Proof. Notice that, depending on the choices at periods $t < T$, under the restrictions that individual i in population j either chooses always $\left(K, r_t^j\right)$ or $(k,q)$the reference lottery is

$$\nu_i^j = \begin{cases} (K, \sum_t \alpha_t^a r_t^a) & if\ j = a\, and\, i\, picked\ (K, r_t^a) \\ (k,q) & if\ j = a\, and\, i\, picked\ (k,q) \\ \left(K, \sum_t \alpha_t^d r_t^d\right) & if\ j = d\, and\, i\, picked\ \left(K, r_t^d\right) \\ (k,q) & if\ j = d\, and\, i\, picked\ (k,q) \end{cases} \tag{6.28}$$

For brevity let us call

$$r^j := \sum_t \alpha_t^j r_t^j \tag{6.29}$$

$$U\left(\pi|K, r^j\right) - U\left(k, q|K, r^j\right) =: \Delta U^j \tag{6.30}$$

let us also call

$$U\left(\pi|k,q\right) - U\left(k,q|k,q\right) =: \Delta\tilde{U} \tag{6.31}$$

Notice that inequality 6.25 implies

$$\Delta U^j > \Delta \tilde{U} \tag{6.32}$$

Moreover $r_t^d > r_s^a \; \forall t, s < T$ implies

$$\Delta U^d > \Delta U^a \tag{6.33}$$

Let us call

$$\delta^j := Pr\left(\left(K, r_t^j\right) \, was \, chosen \, | j\right)$$

$$Pr\left(\pi \, is \, chosen | j\right) =$$

$$= Pr\left(\pi \, is \, chosen | j, \left(K, r_t^j\right) \, was \, chosen\right) \delta^j + Pr\left(\pi \, is \, chosen | j, (k, q) \, was \, chosen\right) \left(1 - \delta^j\right) \tag{6.34}$$

Let us call F the cdf of ϵ_i^j

$$Pr\left(\pi \, is \, chosen | j\right) = \left[1 - F\left(-\Delta U^j\right)\right] \delta^j + \left[1 - F\left(-\Delta \tilde{U}\right)\right] \left(1 - \delta^j\right) \tag{6.35}$$

$$Pr\left(\pi \, is \, chosen | d\right) > \left[1 - F\left(-\Delta U^d\right)\right] \delta^a + \left[1 - F\left(-\Delta \tilde{U}\right)\right] \left(1 - \delta^a\right) >$$

$$> \left[1 - F\left(-\Delta U^a\right)\right] \delta^a + \left[1 - F\left(-\Delta \tilde{U}\right)\right] \left(1 - \delta^a\right) = Pr\left(\pi \, is \, chosen | a\right)$$

Where the first inequality follows from inequality (6.27) and (6.32), and the second inequality follows from (6.33). □

Case 2: decision maker's expectations only restricted to the support of the past options. Suppose that, instead of expecting to choose between sets that they already encountered, they expect to choose among sets that contain options that have the same support of the options they faced, but need not be a linear combination of them.

That is, a decision maker has seen $\left\{\left(K, r_t^j\right), (k, q)\right\}$ and has a distribution over possible choice sets $\{(K, \tilde{r}), (k, q)\}$. That is, she attaches to each $\{(K, \tilde{r}), (k, q)\}$ a probability $Q^j(\tilde{r})$. Her reference lottery is

$$\nu^j = \int \phi(\tilde{r}, q) \, \mathrm{d}Q^j(\tilde{r}) \tag{6.36}$$

where

$$\phi(\tilde{r}, q) := \arg \max_{X \in \{(K, \tilde{r}), (k, q)\}} U(X) \tag{6.37}$$

Given this structure, we allow for beliefs $Q^j(\tilde{r})$ to be arbitrary. This implies that ν^j can be written as

$$\nu^j = \left(K, r^j; k, q^j; 0, 1 - r^j - q^j\right) \tag{6.38}$$

with

$$0 \leq r^j + q^j \leq 0$$

we omit the 0 outcome of the lottery for brevity and rewrite

$$\nu^j = \left(K, r^j; k, q^j\right) \tag{6.39}$$

Unlike in the previous subsection, in this case it is not possible to determine choice at time T as a function of past observable choices, because belief Q^jneeds not depend on them. On the other hand, it is possible to characterize the unobservable ν^j as a function of the choice at time T. The reader is then left to decide whether the ν^j that delivers the contrast effect is reasonable or not.

Proposition 6. *(Contrast and reference beliefs) Let*

$$\nu^j = \left(K, r^j; k, q^j\right) \tag{6.40}$$

If, at T, a picks π and d picks (k, q) it must be either that a expected to choose (k, q) less often than d did, or that a expects to receive K with higher probability than d does.

Proof. Simple algebra yields

$$U\left(\pi|\nu^j\right) - U\left((k,q)\,|\nu^j\right) =$$
$$= \left[u\left(K\right)p - u\left(k\right)q\right]\left\{1 + \eta + r^j \eta\left(\lambda - 1\right)\right\} - u\left(x\right)q^j \eta\left[p - q + \lambda\left(1-p\right)\right] \tag{6.41}$$

Note that

$$U\left(\pi|\nu^a\right) - U\left((k,q)\,|\nu^a\right) > 0 \tag{6.42}$$

if and only if

$$r^a > \frac{Aq^a - B}{C} \tag{6.43}$$

where

$$A := u\left(x\right)\eta\left[p - q + \lambda\left(1-p\right)\right] > 0 \tag{6.44}$$

where the inequality follows from $\lambda > 1$,

$$B = \left[u\left(K\right)p - u\left(k\right)q\right]\left\{1+\eta\right\} > 0 \tag{6.45}$$

where the inequality follows from (6.42) and (6.44)

$$C = \left[u\left(K\right)p - u\left(k\right)q\right]\left\{\eta\left(\lambda - 1\right)\right\} > 0 \tag{6.46}$$

and

$$U\left(\pi|\nu^d\right) - U\left((k,q)\,|\nu^d\right) < 0 \tag{6.47}$$

is true if and only if

$$r^d < \frac{Aq^d - B}{C} \tag{6.48}$$

Subtracting (6.44) from (6.48) we get

$$r^d - r^a < \frac{A}{C}\left[q^d - q^a\right]$$

which implies $r^d < r^a$ or $q^d > q^a$, that yields the result. $\square$

The Proposition means that, under the assumption of arbitrary beliefs on the support of observed distributions, for subjects in the ascending treatment $T4$ to choose (K, p) more often than subjects in the descending treatments, it must be that they expected, before observing the choice set at time T, either to K to be more likely than subjects in $T3$, or that k was less likely than subjects in $T3$. This is puzzling, because, in $t < T$, subjects in $T4$ face a probability of K that is much smaller than subjects in $T3$, and choose the option (k, q) much more likely than subjects in $T3$.

I conclude that Expectations Based reference dependence as in Koszegi and Rabin (2006) does not capture the contrast effect, if we tie expectations to previously chosen options, and it can capture if we allow beliefs to take a counterintuitive shape.

Boiling Frog. Consider now the comparison between subjects in Treatment $T4$ and $T2$. Both groups face an ascending order of the probability of K. As before we take two representative agents, a and a^*, the former observing the choice sets of $T4$, that is a drastic change after $T-1$, and the latter $T2$, which is a gradual change. Just for notational convenience, I denote T as the last period choice set, and I assume that a had observed $S_0^a = \emptyset$ while

$$S_0^{a^*} = \left\{ \left(K, r_0^{a^*} \right), (k, q) \right\}$$

This is motivated by the fact that in $T4$ subjects make one choice less than in $T2$.

I assume that the beliefs before observing choice set T, that is ν^j, are as assumed in Proposition 6, that is

$$\nu^j = \left(K, r^j; k, q^j \right) \quad j = a, a^* \tag{6.49}$$

I want to characterize beliefs ν^j , in period T, that are implied by the fact that a chooses (K, p) and a^* chooses (k, q), that is the stylized fact which I call gradualism. If a picks $\pi = (K, p)$, it must be that she was expecting to receive K (k) more (less) likely than a^* was. The following Corollary formalizes the point.

Corollary 4. *(Boiling frog and reference beliefs) Let*

$$\nu^j = \left(K, r^j; k, q^j \right) \quad j = a, a^* \tag{6.50}$$

If, at T, a chooses π and a^* chooses (k, q), then at least one of the two conditions is true: (i) $r^a > r^{a^*}$ (ii) $q^a < q^{a^*}$

Proof. The proof of Proposition 6 applies, replacing d with a^*. □

Let us compare the result of Corollary 4 with our experimental evidence. In periods $t < T$, subjects represented by a^*, that is in $T2$, choose the option paying K more often than subjects represented by a, in $T4$. Hence, their history up to T suggests that a should hold higher (lower) expectations to receive K (or k) relative to a^*, in contradiction with Corollary 4.

Assimilation. Consider now the comparison between subjects in $T1$ and $T2$. The former group faces a gradual descending order in the probability of K, and the latter faces an ascending order. Let us call d^* the representative agent of $T1$. For brevity of exposition, let us restrict our analysis to the first 3 choices of each individual. Both individuals face the same choice set at time $T = 3$,

$$S_T = \{(K, p), (k, q)\}$$

and at $t < T$ they face choice sets of the following shape

$$S_t^j = \left\{ \left(K, r_t^j \right), (k,q) \right\} \quad for \;\; j = a^*, d^* \; and \; t = 1, 2$$

where

$$r_1^{a^*} < r_2^{a^*} < p < r_2^{d^*} < r_1^{d^*}$$

I assume that the beliefs before observing choice set T, that is ν^j, are as assumed in Proposition 6, that is

$$\nu^j = \left(K, r^j; k, q^j\right) \quad j = a^*, d^* \tag{6.51}$$

I want to characterize beliefs ν^j , in period T, that are implied from the fact that d^* chooses (K,p) and a^* chooses (k,q). The model says that if d^* chooses (K,p), while she must be expecting to the receive K (k) more (less) likely than a^*. The following Corollary formalizes the point.

Corollary 5. *(Assimilation and reference beliefs) Let*

$$\nu^j = \left(K, r^j; k, q^j\right) \quad j = a, a^* \tag{6.52}$$

If at T d^* chooses $\pi = (K,p)$ and a^* chooses (k,q), then either $r^{d^*} > r^{a^*}$ or $q^{d^*} < q^{a^*}$.

Proof. The proof of Proposition 6 applies, replacing d with a^* and a with d^* □

Let us now relate Corollary 5 to the experiment. The properties on ν^{a^*} and ν^{d^*}revealed by the Corollary are consistent with what experimental subjects observe and choose before T, on average. Indeed, subjects in $T1$ (represented by d^*in our results) face and choose,K (k) with larger (lower) likelihood than those in $T2$ (represented by a^*) do. This suggest that it is not implausible for d^*, relative to a^*, to expect a higher (lower) likelihood of K (k) going forward. On the other hand, one could counter argue that decision makers form expectations extrapolating from a trend: if this happens, having faced $q_2^{d^*} < q_1^{d^*}$can lead d^* to believe $r^{d^*} < r^{a^*}$ thus believing that the likelihood of choosing k might be $q^{a^*} > q^{d^*}$. Both arguments are plausible. Thus, I conclude that Expectations Based can predict Assimilation with reference expectations that are plausible in the experimental setting, but there exist equally plausible expectations that would predict the opposite of assimilation in this setting.

6.1. **Contrast and the Rabin's critique.** One of the most well known experimental challenges to concave Expected Utility is the fact that subjects in experiments reject small stakes binary lotteries with equally likely negative and positive outcomes and expected value close to zero (that is, they are loss averse). The elicited risk aversion such experiments implies that subjects should prefer an unreasonably small payment in compensation to a lottery with positive support and very large expected value (Rabin, (2000)). I show that this implication needs not to hold in my model.

Let us assume that there are two periods, 0 and 1. The choice sets are $C_0 = \{(0,1)\}$ and $C_1 = \{(k,1), \pi_1\}$, that is, no risk is faced in the first period while in the second period the subject chooses between a risky π_1 and a riskless k. Let us also assume $\rho = 0$, that is the memory reference point in period 1 is equal to the previous period lottery, that is 0 with probability 1. Furthermore let us assume

$u(x) = v(\bar{c} + x) - v(\bar{c})$ where

$$v(c) = \frac{1}{1-\alpha} y^{1-\alpha}$$

Two cases mimic Rabin's point. The first case is when $k = 0$ and

$$\pi_1 = \left(A(1+\epsilon), \frac{1}{2}; -A, \frac{1}{2} \right)$$

that is, in period 1 the agent chooses whether to accept or reject the lottery π_1 with$A + \epsilon > A > 0$. We mirror the standard finding by supposing that the agent rejects $\pi_1$even if $A \to 0_+$This is possible in the model (as in EU), by concavity

$$U(0,1) = 0$$

and

$$U(\pi_1) = \theta g(\pi_1, (0,1)) \frac{1}{2(1-\alpha)} \left[(\bar{c} + A(1+\epsilon))^{1-\alpha} + (\bar{c} - A)^{1-\alpha} - 2\bar{c}^{1-\alpha} \right]$$

Just like in expected utility, for A fixed and very small, we can find an arbitrarily large α such that

$$U(\pi_1) < 0$$

that is a very risk avers subject rejects an extremely small stakes lottery of that kind. We now ask the Rabin's critique question: will such a risk averse subject require a modest certainty equivalent for a very large positive lottery? The answer is not necessarily: a very nice lottery contrasts away his memory reference point by looking even nicer, while the certainty equivalent does not. Suppose

$$\pi_1 = (A, q; 0, (1-q))$$

and

$$k = qA$$

The utility from the safe prospect is

$$U((qA, 1)) = \frac{1}{1-\alpha} \left[(\bar{c} + qA)^{1-\alpha} - \bar{c}^{1-\alpha} \right]$$

while the utility from the risky prospect is

$$U(\pi_1) = \frac{q\theta}{1-\alpha} g(\pi_1, (0,1)) \left[(\bar{c} + qA)^{1-\alpha} - \bar{c}^{1-\alpha} \right]$$

which is arbitrarily large if A is large and g is unbounded, or if θ is large

Hence, decision makers accustomed to safe payments can be both loss averse when deciding over small stakes and risk loving when deciding over large stakes.

APPENDIX D: Structural estimation

In Subsection 4.3.1 I have argued qualitatively where is the switching point between assimilation and contrast in the probability of receiving \$80, conditional on a history of options observed. The rigorous answer requires the estimation of the structural parameters that include the contrast parameter θ and the curvature of the instantaneous consumption utility. The estimate allow us (i) to test the stability of preference parameters across domains and (ii) predict counterfactual scenarios.

In order to address (i) I estimate the parameters of a member of the class of preferences defined in this paper through experimental data collected by Frydman and Jin (2022). Their experiment is an appealing comparison term because the experimental paradigm differs from the one used in this paper mainly by the number of choices made by each subject (600 vs 5). This allows me to investigate whether the extent to which hundreds of fast decisions and just a few equally fast decisions get remembered similarly, yielding comparable recency parameter estimates ρ. I thus perform the same estimation through my experimental data presented in Section (4). In order to address (ii) I determine the size of the assimilation region combining the parameter estimates.

6.2. **Brief description of Frydman and Jin (2021) experiment II.** In the experiment II of FJ, $N = 200$ subjects face a sequence of $T = 600$ binary choice sets of the form

$$S_{i,t} = \left\{ \left(x_{i,t}, \frac{1}{2}; 0, \frac{1}{2} \right), (c_{i,t}, 1) \right\}$$

where for subject i $x_{i,t}$ is drawn at each t from a distribution of support $[\$2, \$8]$, and $x_{i,t}$ is drawn from a distribution of support $[\$1, \$4]$.[18]

6.3. **Random utility formulation.** First we recall the representation of (**??**). We assume $g(\cdot, \cdot)$ has the same shape used in example 2 and in the applications in Sections 9 and 10. Call

$$\mu_{i,t} = \left(x_{i,t}, \frac{1}{2}; 0, \frac{1}{2} \right)$$

and

$$\mu_{i,t}^{m}$$

being the associated remembered lottery which we will shortly derive. The utility is

$$U\left(\mu_{i,t} | \mu_{i,t}^{m}\right) = \mathbb{E}_{\mu_{i,t}^{m}}[u] + \begin{cases} \theta \left\{ \mathbb{E}_{\mu_{i,t}}[u] - \mathbb{E}_{\mu_{i,t}^{m}}[u] \right\}^2 & if\, \mathbb{E}_{\mu_{i,t}}[u] > \mathbb{E}_{\mu_{i,t}^{m}}[u] \\ -\theta \left\{ \mathbb{E}_{\mu_{i,t}}[u] - \mathbb{E}_{\mu_{i,t}^{m}}[u] \right\}^2 & if\, \mathbb{E}_{\mu_{i,t}}[u] < \mathbb{E}_{\mu_{i,t}^{m}}[u] \end{cases} \tag{6.53}$$

For a given $\left(x_{i,t}, \frac{1}{2}; 0, \frac{1}{2}\right)$, the closest lottery in l^2 norm from choice set $S_{i,t-j}$ is $\left(x_{i,t-j}, \frac{1}{2}; 0, \frac{1}{2}\right)$. Hence

$$U\left(\mu_{i,t} | \mu_{i,t}^{m}\right) = \left(\frac{1-\rho}{2}\right) \sum_{j=1}^{t-1} \rho^{j-1} \left(u\left(x_{i,t-j}\right)\right) + \left(\frac{\rho^{t-1}}{2}\right) u\left(x_{i,t}\right) + \tag{6.54}$$

$$+ \begin{cases} \theta \frac{1}{4} \left\{ \left(1-\rho^{t-1}\right) u\left(x_{i,t}\right) - (1-\rho) \sum_{j=1}^{t-1} \rho^{j-1} \left(u\left(x_{i,t-j}\right)\right) \right\}^2 & if\, \left(\frac{1-\rho^{t-1}}{1-\rho}\right) u\left(x_{i,t}\right) > \sum_{j=1}^{t-1} \rho^{j-1} \left(u\left(x_{i,t-j}\right)\right) \\ -\theta \frac{1}{4} \left\{ \left(1-\rho^{t-1}\right) u\left(x_{i,t}\right) - (1-\rho) \sum_{j=1}^{t-1} \rho^{j-1} \left(u\left(x_{i,t-j}\right)\right) \right\}^2 & if\, \left(\frac{1-\rho^{t-1}}{1-\rho}\right) u\left(x_{i,t}\right) < \sum_{j=1}^{t-1} \rho^{j-1} \left(u\left(x_{i,t-j}\right)\right) \end{cases}$$

The utility of lottery $(c_{i,t}, 1)$ is instead

$$U\left(\left(c_{i,t}, 1\right)\right) = u\left(c_{i,t}\right)$$

[18]For half of the sample the distribution of $x_{i,t}$ and $c_{i,t}$ first-order stochastically dominates the one of the other half of the sample.

We now impose the assumption that u takes the form of a power function

Assumption 1

$u(x) = x^{\alpha}$.

Now I impose that there is some stochasticity in choice, modelled as an additive random normally distributed noise:

Assumption 2

$(x_{i,t}, p_{it},)$ is chosen over $(c_{i,t}, q_{it})$ if and only if

$$U(x_{i,t}, p_{it}, 0, 1 - p_{it}) - U(c_{i,t}, q_{it}) + \epsilon_{i,t} > 0$$

where

$$\epsilon_{i,t} \sim_{i.i.d.} N\left(0, \sigma^2\right)$$

The model yields a likelihood as follows

$$L(\boldsymbol{y}, \boldsymbol{x}, \boldsymbol{c}, \rho, \theta, \alpha) = \prod_{i=1}^{N} \prod_{t=1}^{T} \Phi\left(\frac{U(x_{i,t}) - U(c_{i,t})}{\sigma}\right)^{y_{t,i}} \left(1 - \Phi\left(\frac{U(x_{i,t}) - U(c_{i,t})}{\sigma}\right)\right)^{1 - y_{t,i}}$$

where $y_{i,t} = \mathbb{I}((x_{i,t}, p_{it})\ is\, chosen)$

Thus I provide an estimate of $\rho, \theta, \alpha, \sigma$ via maximum likelihood:

$$\hat{\boldsymbol{\beta}} = \begin{pmatrix} \hat{\rho} \\ \hat{\theta} \\ \hat{\alpha} \\ \hat{\sigma} \end{pmatrix} = \arg \max_{\rho \in [0,1], \theta, \alpha, \sigma} L(\boldsymbol{y}, \boldsymbol{x}, \boldsymbol{c}, \rho, \theta, \alpha, \sigma)$$

I call the estimate obtained from the sample of our own experiment $\hat{\boldsymbol{\beta}}_1$ while I call the estimate obtained from the sample of FJ $\hat{\boldsymbol{\beta}}_2$

6.4. **Estimation result.** I use the full sample of both experiment, that is 200 subjects times 600 repetitions in FJ, and, for my experimental sample, 1,810 subjects times 5 repetitions plus 400 subjects times 4 repetition, who correspond to Treatment 1, 2, 3 and 4. The other Treatments will soon be included in the analysis.

$$\hat{\beta}_1 = \begin{pmatrix} \hat{\rho}_1 \\ \hat{\theta}_1 \\ \hat{\alpha}_1 \\ \hat{\sigma}_1 \end{pmatrix} = \begin{pmatrix} 0.02 \\ 1.28 \\ 0.28 \\ 1.53 \end{pmatrix}$$

$$\hat{\beta}_2 = \begin{pmatrix} \hat{\rho}_2 \\ \hat{\theta}_2 \\ \hat{\alpha}_2 \\ \hat{\sigma}_2 \end{pmatrix} = \begin{pmatrix} 0.40 \\ 1.34 \\ 0.72 \\ 1.20 \end{pmatrix}$$

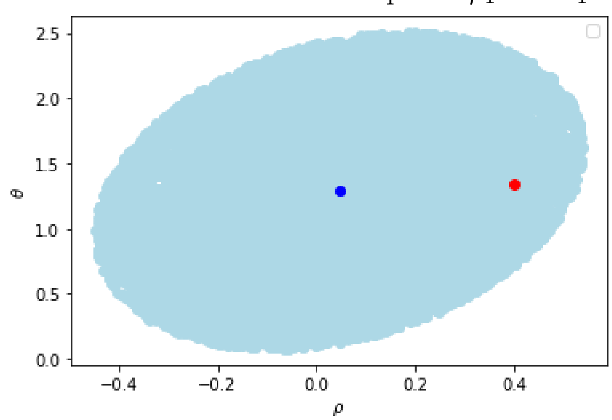
the comparison between the two estimate yields two main insights. Subjects the sample of our experiments, by having a $\hat{\rho}_1$ close to 0 , act as if the memory based lottery was approximately only the lottery encountered in the previous period. Indeed if $\rho = 0$

$$\mu_{i,t}^m = (x_{i,t-1}, p_{it}, 0, 1 - p_{it})$$

Instead, subjects in the sample of FJ exhibit a discounting $\hat{\rho}_2$ of about 40%. This means that the memory based lottery attaches weight of about 60% to the lottery of $t-1$, and 40% to the discounted average of the past lotteries earlier than $t-1$. We conclude that subjects in FJ behave as if they had a longer experimental memory compared to my subjects.

The second interesting comparison is between the contrast parameters estimated across the two samples, $\hat{\theta}_1$ and $\hat{\theta}_2$. The two estimates are very close to each other. Overall, we cannot reject the null hypothesis that the two pairs of estimate of structural parameters (ρ, θ) are the same. Figure 6.2 plots the 95% confidence ellipse of the memory and contrast estimators pair $(\hat{\rho}_1, \hat{\theta}_1)$ based on my sample. The fact that the estimate based on FJ $(\hat{\rho}_2, \hat{\theta}_2)$ lies within the ellipse implies that we cannot reject the null hypothesis that the parameters estimated are equal across the two samples. This confirms my hypothesis that such parameters are structural and thus stable across samples.

FIGURE 6.2. Confidence ellipse of $\hat{\rho}_1$ and $\hat{\theta}_1$



The light blue area plots the 95 confidence ellipse for $\hat{\rho}_1$ and $\hat{\theta}_1$. ρ is on the x axis, and θ is on the y axis. The blue dot corresponds to $(\hat{\rho}_1, \hat{\theta}_1)$, while the red dot corresponds to $(\hat{\rho}_2, \hat{\theta}_2)$.

6.4.1. *Estimation of the assimilation region.* The third point that I have alluded to in subsection 4.3.1 is: for given past history of lotteries $(\$80, \bar{p})$, what is the crossing probability $\bar{p} \pm \delta$ which divides the assimilation region from the contrast region? That is δ such that

$$U(80, \bar{p} + \delta | 80, \bar{p}) = \mathbb{E}_{(80, \bar{p}+\delta)}[u]$$

Which is identified by

$$80^\alpha \bar{p} + \theta 80^{2\alpha} \delta^2 = 80^\alpha (\bar{p} + \delta)$$

that yields

$$\delta = \frac{1}{\theta 80^{\alpha}}$$

Such a point is a function of θ and α. The larger θ or α, the smaller the assimilation region, i.e.

$$\frac{\partial}{\partial \alpha}\delta < 0 \, and \, \frac{\partial}{\partial \theta}\delta < 0$$

Figure 6.3 plots how the distorted expected utility of lottery $(\$80, p, \$0, 1-p)$ varies with p, conditioning on memory lottery $(\$80, 5.5\%, \$0, 94.5\%)$. This setup is close to the assimilation treatment of our experiment, where the risky option pays \$80 with $p \in \{2\%, 5\%, 6\%, 7\%, 9\%\}$. The top figure shows the estimated distorted utility based on the sample of FJ, that is based on estimates $\left(\hat{\rho}_2, \hat{\theta}_2, \hat{\alpha}_2\right)$. The figure reveals

$$\hat{\delta}_2 = \frac{1}{\hat{\theta}_2 80^{\hat{\alpha}_2}} = 2.0\%,$$

which means that an option paying \$80 with 7% is assimilated to $(\$80, 5.5\%)$, while, instead, an option paying $(\$80, 9\%)$ is contrasted away from it.

The bottom figure shows the distorted utility based on estimates from my sample $\left(\hat{\rho}_1, \hat{\theta}_1, \hat{\alpha}_1\right)$. The figure reveals a much larger assimilation region, due to the fact that estimates for θ and α are smaller in my sample than in FJ. The assimilation region consists of a

$$\hat{\delta}_1 = 7.5\%$$

implies, for example, that $(\$80, 9\%)$ is assimilated to $(\$80, 5.5\%)$, and $(\$80, 14\%)$ is contrasted away from it.

The estimate of $\hat{\delta}_1$ is aligned to the intuition I gave in subsection 4.3.1. Let us take the comparison between the choice frequency of $\$80, 9\%$ in Treatment 4 as opposed to Treatment 1. Since $\hat{\rho}_1 \approx 0$

$$\mu_5^m \approx (\$80, 3\%)$$

in Treatment 4. While

$$\mu_1^m = (\$80, 9\%)$$

in Treatment 1. It follows that in Treatment 4

$$\bar{p} = 3\%$$

Now, since

$$9\% < \bar{p} + \hat{\delta}_1$$

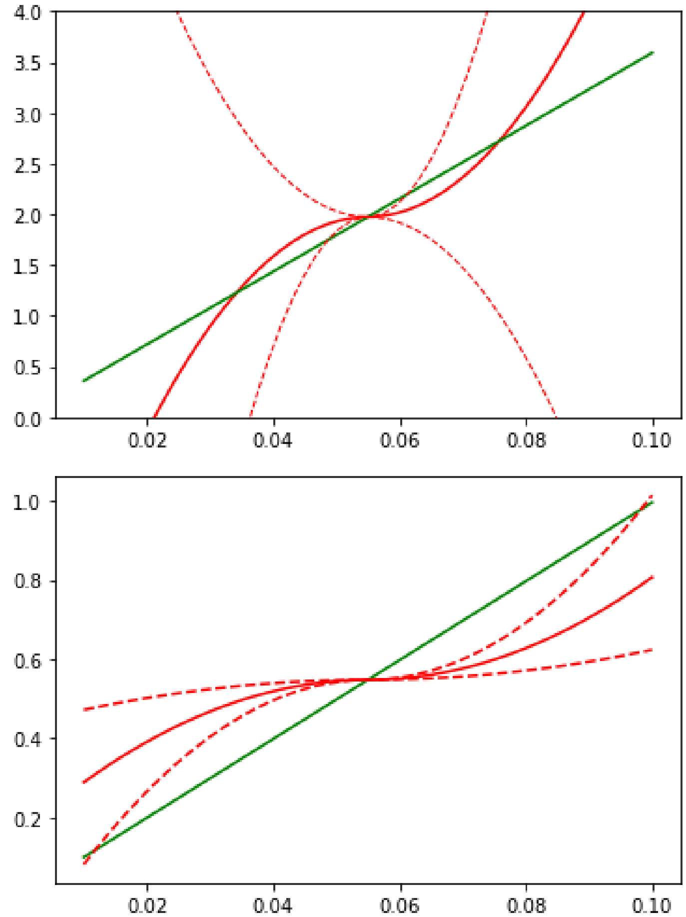
it has to be that $(\$80, 9\%)$ in treatment 4 falls in the assimilation region. This is confirmed by the experimental fact that

$$C\left(\$80, 9\% | T4\right) \leq C\left(\$80, 9\% | T1\right)$$

7. Intermezzo: a more general domain useful for applications

One can easily extend the model to the domain of choice of lotteries that can pay in every period. In section (2) choice set at time t lies in the space of lotteries that pay only at time T, that is, it belongs

FIGURE 6.3. Estimated assimilation and contrast regions from FJ and my sample



Both figures plot in red the distorted expected utility of a lottery ($\$80, p, \$0, 1-p$) conditional on past lotteries being ($\$80, 5.5\%, \$0, 94.5\%$). p is reported on the x axis. The estimates of the first figure are performed on the FJ sample: $\hat{\theta}_2$ and $\hat{\alpha}_2$. The estimates of the second figure are $\hat{\theta}_1$ and $\hat{\alpha}_1$, and have been performed on the sample which comprises T1, T2, T3 and T4. The the undistorted expected utility is plotted in green. Dashed lines represent 95% confidence intervals, obtained via block bootstrap.

to

$$H_t = \Delta\left(\{0\} \times H_{t+1}\right)$$

where

$$H_T = \Delta\left(\mathbb{R}_+\right)$$

The choice set at each t is now

$$C_t \subset H_t^* = \Delta\left(\mathbb{R}_+ \times {H^*}_{t+1}\right)$$

I assume a time separable utility function so that the decision maker is indifferent with respect to autocorrelation and temporal resolution of uncertainty.[19] Given this, it is sufficient to represent a lottery

$$\boldsymbol{\mu^t} \in C_t$$

[19] Stanca (2022) clarifies that the two properties are distinct and correspond to two different representations of recursive utility preferences.

as the sequence of lotteries over future periods that it induces

$$\boldsymbol{\mu^t} = \left\{\mu_{t+h}^t\right\}_{h=0}^{+\infty}$$

As before, μ^t cues a sequence of past lotteries $\left\{\hat{\boldsymbol{\mu}}^{t-j}\right\}_{j=1}^{\infty}$. The remembered $t-j$ lottery cued by $\boldsymbol{\mu}^t$ is

$$\hat{\boldsymbol{\mu}}^{t-j} = \arg\min_{\boldsymbol{\nu}\in C_{t-j}} \sum_{h=0}^{\infty} \gamma^h d\left(\nu_{t+h}, \mu_{t+h}^t\right)$$

that is, the decision maker, when thinking of lottery $\boldsymbol{\mu}^t$, retrieves one lottery faced at $t-j$ that induces the sequence of lotteries, from t onwards, that is the closest in the discounted sum of euclidean norms, where the discounting ensures that such a measure is finite.

Once the past lotteries have been retrieved $\left\{\hat{\boldsymbol{\mu}}^{t-j}\right\}_{j=1}^{\infty}$ the decision maker aggregates them to form the remembered average lottery

$$\boldsymbol{\mu}^{m,t} = (1-\rho)\sum \rho^{j-1}\hat{\boldsymbol{\mu}}^{t-j}$$

$\boldsymbol{\mu}^{m,t}$ specifies a marginal cdf for every future period $t+h$:

$$\left\{F_{t+h}^{m,t}\right\}_{h=0}^{\infty}$$

Example 6. Consider a variant of Example 5, assuming a finite horizon T.

$$C_0 = \left\{ \begin{pmatrix} \$0, 100\% \\ \$10, 5\% \\ \$10, 5\% \end{pmatrix}, \begin{pmatrix} \$0, 100\% \\ \$3, 100\% \\ \$3, 100\% \end{pmatrix} \right\}$$

$$C_1 = \left\{ \begin{pmatrix} \$10, 5\% \\ \$10, 30\% \end{pmatrix}, \begin{pmatrix} \$3, 100\% \\ \$3, 100\% \end{pmatrix} \right\}$$

$$C_2 = \left\{ (\$10, 30\%), (\$3, 100\%) \right\}$$

Let us focus on time 3. When considering lottery $\mu^1 = \begin{pmatrix} \$10, 5\% \\ \$10, 5\% \end{pmatrix}$ the agent retrieves the lotteries

$$\hat{\mu}^0 = \begin{pmatrix} \$0, 100\% \\ \$10, 5\% \\ \$10, 5\% \end{pmatrix}$$

$$\hat{\mu}^{-1} = \begin{pmatrix} \$10, 5\% \\ \$10, 30\% \end{pmatrix}$$

So the memory lottery at time 1 specifies the lottery that pays at time 2 and the lottery that pays at time 3

$$\boldsymbol{\mu}^{m,1} = \begin{pmatrix} \$10, 5\% \\ \$10, (1-\rho)\,5\% + \rho 30\% \end{pmatrix}$$

Now that we have specified the memory lottery vector we can produce the vector of distorted lotteries. Each component $\boldsymbol{\mu_{t+h}^t}$ of the vector of lotteries $\boldsymbol{\mu^t}$ is distorted by a function that represents how much the whole vector $\boldsymbol{\mu}^t$ departs from $\boldsymbol{\mu}^{m,t}$

$$\pi\left(\boldsymbol{\mu}^t\right) = \boldsymbol{\mu}^{m,t} + \theta \sum_{h=0}^{\infty} g\left(F_{\mu^t_{t+h}}, F^m_{\mu_t}\right)\left(\boldsymbol{\mu^t} - \boldsymbol{\mu}^{m,t}\right)$$

where g is the same as in Section 2.

Notice that the setup in section 1 is a particular case of this one. Indeed for a lottery that only has a final payment

$$\boldsymbol{\mu}^t = \begin{pmatrix} 0, 100\% \\ 0, 100\% \\ \vdots \\ \mu_T \end{pmatrix}$$

it holds the following

$$\pi\left(\boldsymbol{\mu}^t\right) = \boldsymbol{\mu}^{m,t} + \theta \sum_{h=0}^{T} g\left(F_{\mu^t_{t+h}}, F_{\mu^{m,t}_t}\right)\left(\boldsymbol{\mu^t} - \boldsymbol{\mu}^{m,t}\right) =$$

$$= \begin{pmatrix} 0, 100\% \\ 0, 100\% \\ \vdots \\ \mu^{m,t}_{T-1} \end{pmatrix} + \theta g\left(F_{\mu_{T-1}}, F^m_{\mu_t}\right) \begin{pmatrix} 0, 100\% \\ 0, 100\% \\ \vdots \\ \mu_{T-1} - \mu^{m,t}_{T-1} \end{pmatrix}$$

hence, the distorted lottery pays 0 in every period except in the last period, where it is equal to the lottery

$$\mu^{m,t}_T + \theta g\left(F_{\mu_T}, F^m_{\mu_t}\right)\left(\mu_T - \mu^{m,t}_T\right)$$

Let us call $\pi_{t+h}\left(\boldsymbol{\mu^t}\right)$ the distorted distribution at time $t+h$ induced by $\pi\left(\boldsymbol{\mu^t}\right)$, that is the $t+h$ component of $\pi\left(\boldsymbol{\mu}^t\right)$. We are now ready to state the representation

Definition 4. The discounted expected utility of $\boldsymbol{\mu}^t \in C_t$ is equal to

$$V_t = U\left(\pi_t\left(\boldsymbol{\mu^t}\right)\right) + \beta V_{t,t+1} \tag{7.1}$$

$$V_{t,t+j} = U\left(\pi_{t+j}\left(\boldsymbol{\mu^t}\right)\right) + \beta V_{t,t+j+1}$$

With $\beta \in (0, 1)$

APPENDIX E

8. Applications: sequences of news in markets

8.1. **A simple asset pricing illustration and an empirical test.** As an example on how to apply my theory of risk preferences, I derive its predictions in the context of the evaluation of risky dividend streams. The minimal setup so far introduced is sufficient to produce a rich dynamics of asset prices as a function of public news received in the economy. Throughout this subsection I assume that the function g is specified as in Example 2. Suppose there is risky asset that pays a dividend D_T at a final date T, in finite supply equal to 1, and a price of P^{θ}_t. Suppose an investor with linear consumption utility has wealth equal to 1 to be invested either in the risky asset or in a safe asset. Both assets pay at a final period T. The investor at each t is naive about the possibility to sell an asset at intermediate periods before T.[20] Thus, the investor has the following utility[21] at period t

$$U\left(A_t|A_t^m\right)=\begin{cases}\frac{\mathbb{E}_t^{\theta}[D_T]}{P_t^{\theta}} & if\, she\, picks\, the\, risky\, asset\\ 1 & else\end{cases}$$

In equilibrium, the price must be such that she is indifferent between picking the risky asset and holding cash, that is

$$P_t^{\theta}=\mathbb{E}_t^{\theta}\left[D_T\right]$$

Suppose the final dividend D_T is the sum of interim dividends, that is $D_T=D_0+\sum_{j=1}^{T}u_j$where

$$u_j\sim_{iid}N\left(0,\sigma^2\right)$$

and u_j realized at time j. This means that the investor receives news about the stock at each period so that the rational expectation of D_T is a random walk, $\mathbb{E}_t\left[D_T\right]=\mathbb{E}_{t-1}\left[D_T\right]+u_t$. The investor biases such expected value: if the news is small, she assimilate $\mathbb{E}_t\left[D_T\right]$ to $\mathbb{E}_{t-1}\left[D_T\right]$ and underreacts. If the news is large she contrasts $\mathbb{E}_t\left[D_T\right]$ away from $\mathbb{E}_{t-1}\left[D_T\right]$ and overreacts. To see this, recall the construction of $\mathbb{E}_t^{\theta}\left[D_T\right]$ from Example 1 (continued). For t large,

$$\mathbb{E}_t^{\theta}\left[D_T\right]\approx\underbrace{(1-\rho)\sum_{j=1}^{t}\rho^j\mathbb{E}_{t-j}\left[D_T\right]}_{\text{memory anchor}}+\theta\begin{cases}\left(\mathbb{E}_t\left[D_T\right]-(1-\rho)\sum_{j=1}^{t}\rho^j\mathbb{E}_{t-j}\left[D_T\right]\right)^2 & for\, \mathbb{E}_t>\mathbb{E}_t^m\\ \underbrace{-\left(\mathbb{E}_t\left[D_T\right]-(1-\rho)\sum_{j=1}^{t}\rho^j\mathbb{E}_{t-j}\left[D_T\right]\right)^2}_{\text{attentionweight}\times\text{adjustment}} & for\, \mathbb{E}_t<\mathbb{E}_t^m\end{cases}$$

When positive news is received about the final payoff, the expected value of $\mathbb{E}_t\left[D_T\right]$ increases. If the news is not big enough, the investor thinks not much has changed, and the price moves too little. Over time he gets accustomed to the news, as the remembered expectation $\mathbb{E}_t^m=(1-\rho)\sum_{j=1}^{t}\rho^j\mathbb{E}_{t-j}\left[D_T\right]$ incorporates the recent news, and slowly reacts to the news.

8.1.1. *Small sequences of news and momentum in asset prices (Grinblatt and Moskowitz 2004, Da et al. 2014).* The investor under appreciates information which does not move expectations much from her memory and over appreciates the information that moves it by a lot. This suggests that a surprise released in small bits over time generates smaller contemporaneous price reaction than one concentrated in one period, and predicts higher future returns, as the memory anchor gradually adjusts to the new information set. A sequence of small surprises makes the stock look less dissimilar from the past, compared to a one-shot big piece of news. This is analytically clearer from the following rewriting of

[20] This is a common assumption in asset pricing literature, there called myopic portfolio rule.

[21] In appendix I show how to keep the spirit of the model while assuming mean variance utility. Predictions are analogous.

$\mathbb{E}_t^{\theta}[D_T]$

$$P_t^{\theta} = \mathbb{E}_t^{\theta}[D_T] = \mathbb{E}_t[D_T] + \left(u_t + \sum_{j=1}^{t-t_0-1} \rho^j u_{t-j}\right)\left\{\underbrace{\left|u_t + \sum_{j=1}^{t-t_0-1} \rho^j u_{t-j}\right|}_{\text{attention weight}} \theta - 1\right\} \tag{8.1}$$

Imagine past surprises are equal to $u_j = u > 0$ with $j = 1, .., t$. The attention given to the change in the expectation of the stock at time t from what the investor retrieves depends on the recency-discounted past surprises, and amounts to

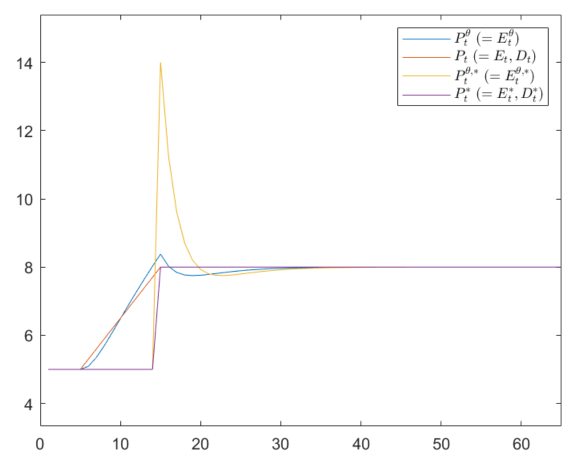
$$g\left(F_{D_T|t}, F_{D_T^m|t}\right) = u \sum_{j=0}^{t-t_0-1} \rho^j \theta$$

Think about an alternative history when no surprise occur until t, and all the surprises come at t in one chunk equal to $u^* = (t - t_0)\, u$. The attention to the change of the stock from the past is

$$g\left(F^*_{D_T|t}, F^*_{D_T^m|t}\right) = (t_1 - t_0)\, u > g\left(F_{D_T|t}, F_{D_T^m|t}\right)$$

Figure 8.1 plots these two scenarios. P_t^{θ} is the price of the stock in the first scenario, under a sequence of small surprises from period 5 to 15, while $P_t^{\theta,*}$ is the price when the same quantity of surprise occurs all at the same period 15. P_t and P_t^* are the price paths if investors were rational under, respectively, the first and second scenario. While $P_t = P_t^*$ for $t = 15$, since the total amount of news is the same, $P_t^{\theta,*} > P_t^{\theta}$, that is the price of the drastic history scenario reacts more than the gradual scenario, implying lower future returns. The dynamics mirror what Da et al. (2014) label as the “Boiling Frog”

FIGURE 8.1. Continuous vs Discrete surprise



Time on the x axis. P_t, P_t^* are, respectively, the rational price after gradual news and drastic news. P_t^{θ} and $P_t^{\theta,*}$ are the behavioral price under gradual and drastic news.

effect that they document for individual stock returns (and is documented with a similar measure by Grinblatt and Moskowitz 2004), and Huang et al. (2021) finds for cross stock returns: a given past price increase is more associated with momentum if it is gradual. The following Remark states this formally, as a prediction of the model.

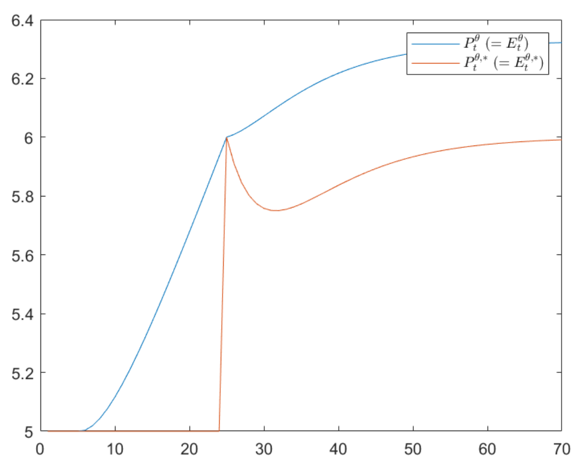
Remark 3. Consider two different price paths such that $P^{\theta}_{t_0} = P^{\theta,*}_{t_0}$ and $P^{\theta}_{t_1} = P^{\theta,*}_{t_1}$, where $\left\{P^{\theta}_{j}\right\}_{j=t_0}^{t_1}$ is generated by a stream of positive news $\{u_j\}_j$with $u_j > 0$, while $\left\{P^{\theta,*}_{j}\right\}_{j=t_0}^{t_1}$ is generated by $\left\{u^*_j\right\}_j$with $u^*_j = 0$ for $j < t_1$ and $u_t > 0$. If $\Delta P^{\theta*}_{t_1}$ is not too small, then

-
$$\mathbb{E}_t\left[\Delta P^{\theta,*}_{t_1,t_1+h}\right] < \mathbb{E}_t\left[\Delta P^{\theta}_{t_1,t_1+h}\right]$$

for h above some constant k

- $\lim_{h\to+\infty}\mathbb{E}_{t_1}\left[P^{\theta}_{t_1+h}\right] > \lim_{h\to+\infty}\mathbb{E}_{t_1}\left[P^{\theta,*}_{t_1+h}\right]$

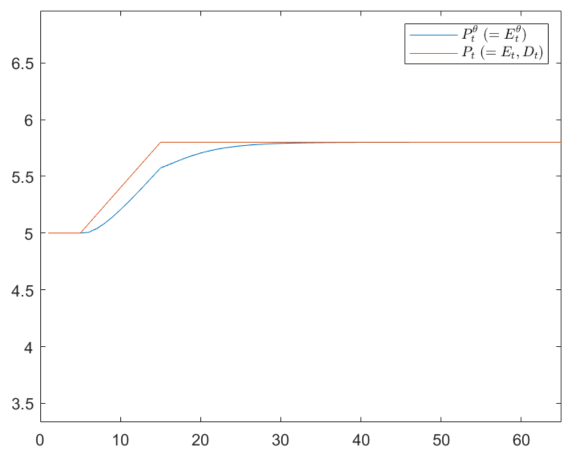
FIGURE 8.2. Gradual price increases predict higher returns



Time on the x axis. P^{θ}_t *and* $P^{\theta,*}_t$ *defined as in Proposition 3.* P^{θ}_t *:price under gradual news.* $P^{\theta,*}_t$ *: price under drastic news.*

8.1.2. *Underreaction to the passage of time (Giglio and Shue 2014). Delayed overreaction to long sequences of news.* The assimilation effect, interacted with the gradual updating of the memory anchor naturally yields the prediction that a sequence of sufficiently small bits of positive (negative) news predicts positive (negative) returns after the news flow has stopped, as the investor fails to appreciate the change in the stock from the stock she remembers. This logic explains why in some settings the mere passage of time, predicts future positive (negative) returns, when such passage is a good (bad) news. For example, Giglio and Shue (2014) shows that, when the passage of one week should mean positive (negative) revision on the hazard rate of the success of a merger, higher (lower) future returns *after* the week has passed, suggesting investors underappreciate the informational content of one week passing. While this fact can be explained with absence of explicit news being less attention grabbing than explicit news, my theory explains it as a product of assimilation: the passage of time is in itself gradual, and so is the change in the probability of success: the future prospect looks more similar to the past than it actually is, leading to future price corrections (Figure 8.3). One way to distinguish the two explanations would be selecting episodes when the passage of one unit of time is associated with a drastic informational content (say because a strict deadline is reached): in such episodes, the mentioned theory predicts the same degree of return predictability, on the contrary my theory would predict less positive, or even negative predictability.

FIGURE 8.3. Price response to sequence of very small surprises



Time on the x axis. A sequence of small shocks $u_t = u > 0$ for $t = 5, ..., 15$. P_t: rational price. P_t^θ : price in the behavioral economy.

While a sequence of small positive news leads to positive future returns because of assimilation, a very long sequence is associated with initial under and delayed overreaction. This is because, as the news cumulates, the investor still remembers some of the features of the stock from far back in time, since she retrieves a backward looking discounted average of past expectations. As the stock gets far enough from the remembered one, the investor contrasts it and overreact it, pricing above the rational investor and drastically reverting below rational when the news flow stops.

Remark 4. If there is sequence of surprises $u_t = u > 0$ for $t \in \{t_0, .., t_1\}$, there exist constants $\underline{u}, \bar{u}$ such that

- If $u < \underline{u}$, the price is below rational and $\mathbb{E}_{t_1}\left[\Delta P^\theta_{t_1,t_1+h}\right] > 0$ for any h above some constant.
- If $u \in [\underline{u}, \bar{u}]$, the price is below rational for some $t < t_1$ and above rational afterwards, so that $\mathbb{E}_{t_1}\left[\Delta P^\theta_{t_1,t_1+h}\right] < 0$ for any h above some constant.
- If $u > \bar{u}$, the price is above rational and $\mathbb{E}_{t_1}\left[\Delta P^\theta_{t_1,t_1+h}\right] < 0$ for any h above some constant.

8.1.3. *Volatility and under/overreaction: an empirical test.* Suppose one lives in an environment with frequent big positive and negative news (large σ). In such environment the investor allocates too much attention to these news because they make the asset look quite different from the past, hence the investor overreacts. In more normal times, instead, when there is on average small news, the investor fails to detect changes, assimilates the updated risky prospects, anchors to the past, and underreacts.

Table 8. Volatility and under/overreaction. Market returns.

	$autocorr_t$(monthly)		$autocorr_t$(yearly)	
	1926-2022	1990-2022	1926-2022	1990-2022
vol_{t-1}	-1.29^{***}	-1.14^{***}		
vol_{t-1}			-0.59^{***}	-0.38
Obs.	1056	352	96	32

The table shows in the first (last) two columns the coefficient of a linear regression of within-month (year) autocorrelation of daily (monthly) value weighted market returns on volatility of returns in the previous month (year). Results are presented for the whole sample (1926-2022) and the restricted sample (1990-2022).

This yields the prediction that when the volatility of the market is high, there should be on average overreaction to information, while when the volatility is low, there is underreaction, which leads to the following final result.

Proposition 7. *Assume $\rho = 0$, that is investors anchor to $t-1$ memories. There is some $\bar{z}$ such that $\sigma^2 > \bar{z} \implies Cov\left(\Delta P_t^\theta, \Delta P_{t+1}^\theta\right) < 0$ and $\sigma^2 < \bar{z} \implies Cov\left(\Delta P_t^\theta, \Delta P_{t+1}^\theta\right) > 0$.*

The weaker version of this prediction is that market volatility and market return autocorrelation are negatively related. I test this prediction in the time series of US stock market returns from 1926 to 2022 following Campbell et al. (2022) that extends Chiah et al. (2020) and Campbell et al. (2001). The return volatility of period t is defined as in Campbell et al. (2022), and is given by equation (8.2).

$$vol_t = \sqrt{\sum_{s \in t} \left(R_{s,m} - \bar{R}_{m,t}\right)^2} \tag{8.2}$$

I conduct the analysis both looking at within year volatility ($t =$ year) and within-month volatility ($t =$ month) of daily ($s =$ day) returns, and consider analogous return autocorrelations within the same period. $R_{s,m}$ is the value weighted market return of period s, and $\bar{R}_{m,t}$ is the average of market returns of period s comprised in unit of time t. Market returns are taken from CRSP. Moreover, I further test the prediction for individual stock returns.[22] Table 8 shows that there is a statistically and economically significant negative relation between return autocorrelation and volatility.[23] The relation is robust to restricting to the more recent part of the sample, after the occurrence of the downward trend in market autocorrelation documented by Froot and Perold (1995). The analysis at the stock level confirms that monthly volatilty of daily stock returns negatively predicts autocorrelation in the next month, even when controlling for stock fixed effects and market level volatility.

The sign of the predictability of autocorrelation with volatility is consistent with the model's prediction that there is larger sensitivity to larger surprises. The results cannot be produced by persistent stochastic volatility of returns: simulating a GARCH model yields a zero correlation between return autocorrelation and lagged return volatility. While the predictability at the market level can be explained by time-varying market liquidity generating reversal in daily returns as in Nagel (2012), liquidity is

[22]The use of lagged volatility avoids the negative mechanical relationship that arises when comparing the two simulaneous series. In case the reader is nonetheless interested, statistics are reported in Table 12 of Appendix E.

[23]The correlation coefficients are shown in Table 13 of Appendix E.

Table 9. Volatility and under/overreaction. Individual stock returns.

	$autocorr_t$(monthly) (1)	(2)	(3)	(4)
vol_{t-1}	-0.41***	-0.46***	-0.40***	-0.47***
Obs.	2448630	2448630	2448630	2112520

The table shows the coefficient of a linear regression of within-month autocorrelation of daily returns on volatility of returns in the previous month for individual stocks. Univariate regression results with stock fixed effects are presented in column (1), while column (2) controls for market level monthly volatilty, column (3) adds year fixed effects, and column (4) controls for the interaction between market volatility and the absolute value of rolling betas. The sample covers years from 1990 to 2018.

less likely to explain stock level predictability of autocorrelation when controlling for market volatility. Behavioral or rational theories of inattention, such as, respectively, Gabaix (2014) and Sims (2006), do not predict negative autocorrelation for large enough volatility, as I find in the later portion of the data, consistent with my theory. Theories of beliefs distortion such as Diagnostic Expectations (Bordalo et al. 2018) or the gambler's fallacy (Rabin and Vayanos, 2010) can feature underreaction and delayed overreaction, but do not predict a that the sensitivity to a shock depends on its size. Theories of categorization as Mullainathan (2002) predict size-dependent sensitivity to news, but not the boiling-frog effect documented in the financial literature.[24]

9. Asset Pricing under Assimilation and Contrast

9.1. **Setup.** Let us assume that there is a riskless asset (cash) in inelastic supply and a risky asset in fixed supply of 1, which pays a dividend at a terminal date. The dividend is the sum of a random iid shocks that realize at each period. With probability β the asset survives and gets to the next period, while with probability $(1-\beta)$ it dies and pays off the sum of the realized shocks until that period. That is the asset pays, if it reaches $t+h$, $\bar{D}+\sum_{j=1}^{t+h}u_j$where $u_j \sim_{iid} N\left(0,\sigma_u^2\right)$. Conditional on reaching time t, the dividend paid at time $t+h$ is thus

$$D_{t+h}=\begin{cases}\bar{D}+\sum_{j=1}^{t+h}u_j & w.p.\,\beta^h\left(1-\beta\right)\\ 0 & w.p.\,1-\beta^h\left(1-\beta\right)\end{cases}$$

We call $D_T$the total random payment of the asset, that is the sum of the dividends. Conditional on reaching time t, it is

$$D_T=\sum_{h=0}^{\infty}D_{t+h}$$

this setup is convenient since the expectation of D_T is a random walk

$$\mathbb{E}_t\left[D_T\right]=\mathbb{E}_{t-1}\left[D_T\right]+u_t$$

and the variance is constant, avoiding horizon effects of asset prices.

[24]The reason of this is that in Mullainathan (2002) the category to which a posterior belongs is independent of history.

The agent, endowed with a stock of financial wealth W_0 at time 0 has mean variance preferences over final wealth. That is, she selects demand of the risky asset X_t to solve

$$\max_{X_t}\left\{X_t\left(\mathbb{E}_t^{\theta}\left[D_T\right]-P_t\right)-\frac{\gamma}{2}X_t^2\mathbb{V}_t\left[D_T\right]\right\}$$

this problem is standard except for one difference, which is that the expectation $\mathbb{E}^{\theta}$ is taken over the distorted distribution rather than the rational one.[25] The demand of the risky asset is thus

$$X_t=\frac{\mathbb{E}_t^{\theta}\left[D_T\right]-P_t}{\gamma\mathbb{V}\left[D_T\right]}$$

where

$$\mathbb{V}\left[D_T\right]=\mathbb{V}_t\left[D_T\right]$$

which is constant over t. The equilibrium price is thus

$$P_t=\mathbb{E}_t^{\theta}\left[D_T\right]-\gamma\mathbb{V}\left[D_T\right]$$

9.2. **Distorted expectation.** As described in previous sections the decision maker exaggerates the probability of events that have become largely more likely, while she attenuates the probability of events that have only become slightly more likely. The distorted expected value of D_T conditional on information at time t is the sum of the the distorted expectations of the stochastic dividend of each period D_{t+h}, where each of them is distorted by a function of the total departure of the process $\{D_{t+h}\}_h$ from the one retrieved from memory.

$$\mathbb{E}_t^{\theta}\left[D_T\right]=\underbrace{(1-\rho)\sum_{j=1}^{\infty}\rho^{j-1}\mathbb{E}_{t-j}\left[D_T\right]+}_{\text{expectations norm}}\underbrace{\theta\left\{\sum_{h=1}^{\infty}g\left(F_t^{D_{t+h}},(1-\rho)\sum_{j=1}^{\infty}\rho^{j-1}F_{t-j}^{D_{t+h}}\right)\right\}}_{\text{salience}}\underbrace{\left\{\mathbb{E}_t\left[D_T\right]-(1-\rho)\sum_{j=1}^{\infty}\rho^{j-1}\mathbb{E}_{t-j}\left[D_T\right]\right\}}_{\text{deviation from norm}}$$

The distorted expectation is anchored to a weighted average of all the previous expectations with a larger weight on the most recent ones, plus a deviation term which is exaggerated if the distribution of dividends $F_t^{D_{t+h}}$ is significantly different from the weighted average of the past distributions. Large positive deviations make $\mathbb{E}_t^{\theta}\left[D_T\right]>\mathbb{E}_t\left[D_T\right]$ while

Lemma 1. *If*

$$g\left(F,G\right)=\left|\mathbb{E}_F-\mathbb{E}_G\right|$$

the distorted expectation $\mathbb{E}^{\theta}$ *can be rewritten as*

$$\mathbb{E}_t^{\theta}\left[D_T\right]=(1-\rho)\sum_{j=1}^{\infty}\rho^{j-1}D_{t-j}+\begin{cases}\theta\left\{D_t-(1-\rho)\sum_{j=1}^{\infty}\rho^{j-1}D_{t-j}\right\}^2 & if\ D_t>(1-\rho)\sum_{j=1}^{\infty}\rho^{j-1}D_{t-j}\\ -\theta\left\{D_t-(1-\rho)\sum_{j=1}^{\infty}\rho^{j-1}D_{t-j}\right\}^2 & if\ D_t<(1-\rho)\sum_{j=1}^{\infty}\rho^{j-1}D_{t-j}\end{cases}\tag{9.1}$$

[25]The reader might ask why just the expectation is assumed to be distorted and not also the variance. One reason is that the purpose of the application is to highlight the additional explanatory power from a standard model with only one additional change. Numerical simulations of the model, with distorted variance, show results similar to the ones discussed. The second reason is that an undistorted variance makes the problem more analytically tractable without the need to resort to numerical simulations.

A shock in the dividend at time t which brings it above the historical average dividend $(1-\rho)\sum_{j=1}^{\infty}\rho^{j-1}D_{t-j}$ is overestimated if

$$D_t >> (1-\rho)\sum_{j=1}^{\infty}\rho^{j-1}D_{t-j}$$

because the innovation is salient and is over appreciated by the investor, while it can be underestimated if

$$D_t \approxeq (1-\rho)\sum_{j=1}^{\infty}\rho^{j-1}D_{t-j}$$

as the dividend gets assimilated to the past historical average. To see this more clearly, we can simplify further expression (9.1) as

$$\mathbb{E}_t^{\theta}\left[D_T\right] = \mathbb{E}_t\left[D_T\right] + \left(u_t + \sum_{j=1}^{t-t_0-1}\rho^j u_{t-j}\right)\left\{|u_t + \sum_{j=1}^{t-t_0-1}\rho^j u_{t-j}|\theta - 1\right\}$$

if the innovation in dividend is large, so that

$$|u_t + \sum_{j=1}^{t-t_0-1}\rho^j u_{t-j}| > \frac{1}{\theta}$$

the distorted expectation overshoots the rational expectation: the dividend at time t suprises the investor who had in memory a combination of past dividends $(1-\rho)\sum_{j=1}^{\infty}\rho^{j-1}D_{t-j}$ that contrast the realization D_t, hence the agent overshoots her expectation for the terminal dividend D_T.

On the contrary, suppose there has been a sequence of small positive innovations to dividends$\{u_j\}_{t_0}^{t}$, if these innovations are small enough the distorted expectation undershoots the rational expectation: small innovations are not appreciated because dividend D_j is assimilated to the past dividends in memory.

9.3. **Predictions.** The price is a linear function of investors' expectations

$$P_t = \mathbb{E}_t^{\theta}\left[D_T\right] - \gamma\mathbb{V}\left[D_T\right] =$$

$$= \mathbb{E}_t\left[D_T\right] + \gamma\mathbb{V}\left[D_T\right] + \left(u_t + \sum_{j=1}^{t-t_0-1}\rho^j u_{t-j}\right)\left\{|u_t + \sum_{j=1}^{t-t_0-1}\rho^j u_{t-j}|\theta - 1\right\}$$

The last term of the expression summarizes the effect of the distortion on the processing of the news: if the news is very large and positive, the term is positive meaning that the expectation overshoots the rational and so the price is above rational. θ, the contrast parameter, determines what size the shocks to the dividend must take for the investors to overreact to such shocks. If the news, or the sequence of past news, is small, then the term is negative, since the rational expectations are close to the memory norm, and the price is below rational. The opposite is true for negative news: the price is below rational for large negative news, and above rational for small negative ones.

$$\Delta P_t = u_t + \left(\sum_{j=0}^{t-t_0-1}\rho^j u_{t-j}\right)\left\{|\sum_{j=0}^{t-t_0-1}\rho^j u_{t-j}|\theta - 1\right\} - \left(\sum_{j=0}^{t-t_0-1}\rho^j u_{t-j-1}\right)\left\{|\sum_{j=0}^{t-t_0-1}\rho^j u_{t-j-1}|\theta - 1\right\}$$

where the first term u_t is the dividend shock at time t, which in an economy populated by rational investors should be equal to the price change. In this economy, there are two additional terms. the second term is the bias in price at time t which depends on the shocks realized until t.

The third term is the bias in price at time $t-1$ which depends on dividend shocks realized until $t-1$

9.3.1. *Underreaction and overreaction to one shot surprise.* Consider the simple case of a shock from the steady state. That is, dividends have remained constant until a positive shock occurs at time t. The memory norm is equal to the dividend D_{t-1} realized before the shock, since all shocks $\{u_{t-j}\}_{j=1}^{t-t_0-1}$are 0

$$(1-\rho)\sum_{j=1}^{\infty}\rho^{j-1}\mathbb{E}_{t-j}\left[D_T\right]=(1-\rho)\sum_{j=1}^{\infty}\rho^{j-1}D_{t-j}=D_{t-1}$$

. As an innovation occurs to D_t the investor updates her expected value of D_T

$$\mathbb{E}_t^{\theta}\left[D_T\right]=\mathbb{E}_t\left[D_T\right]+u_t\left\{|u_t|\theta-1\right\} \tag{9.2}$$

where this simplification follows from the fact that the past shocks are 0. From this expression it is clear that the distorted expected value $\mathbb{E}_t^{\theta}\left[D_T\right]$ overreacts to the rational expectation if $u_t$is large, underreacts if u_t is small. The intuition is that if $u_t$is small the posterior distribution on D_T is very close to the remembered prior, so it is assimilated and the distorted expectation undershoots. An coincidental intuition is that if u_t is small D_t looks close to D_{t-1} which is remembered by the investor since it is the past expectations on D_T. As D_t is assimilated to D_{t-1}, the investor does not fully appreciate the change in dividend.

The underreaction in expected value is reflected into underreaction in the price: after a small positive shock u_t at time t the price adjusts less than it would if the investor was rational, predicting average positive return after t.

A large shock, on the other hand, has the opposite effect on future average returns. If D_t is much larger than D_{t-1} (that is, if u_t is large enough), the long run expectation of D_T is contrasted against the remembered one, hence it looks large and overshoots the rational expectation. This makes the price go above the rational price, causing overreaction at time t, which is followed by negative average returns. The following proposition summarizes the above description.

Proposition 8. *Assume the steady state at $t-1$*

(Assimilation $\Longrightarrow$ underreaction) If $|u_t|<\frac{1}{\theta}$
- *If $R_{t-1,t}>0$ then $\mathbb{E}_t\left[R_{t,t+h}\right]>0$ for h large*
- *If $R_{t-1,t}<0$ then $\mathbb{E}_t\left[R_{t,t+h}\right]<0$ for h large*

(Contrast $\Longrightarrow$ overreaction) If $|u_t|>\frac{1}{\theta}$
- *If $R_{t-1,t}>0$ then $\mathbb{E}_t\left[R_{t,t+h}\right]<0$ for h large*
- *If $R_{t-1,t}<0$ then $\mathbb{E}_t\left[R_{t,t+h}\right]>0$ for h large*

Such results help compare the theory with other models that capture over and under reaction to one shot news. The most related one is Mullainathan (2000): a model with homogeneous investors who update picking the posterior from the same finite set of categories generates an extreme version of underreaction: if the news is small enough for investors not to change category, the price does not

FIGURE 9.1. Assimilation: momentum after a small shock

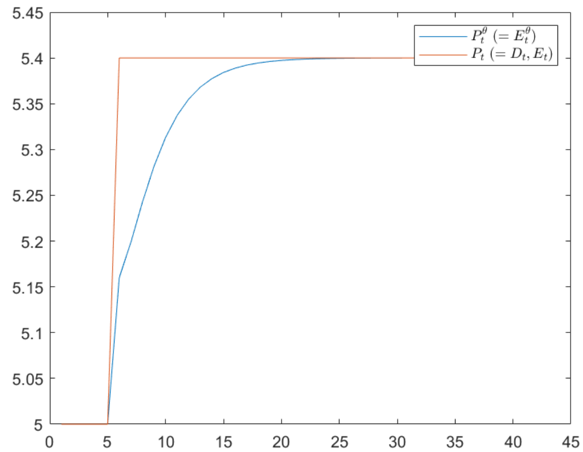
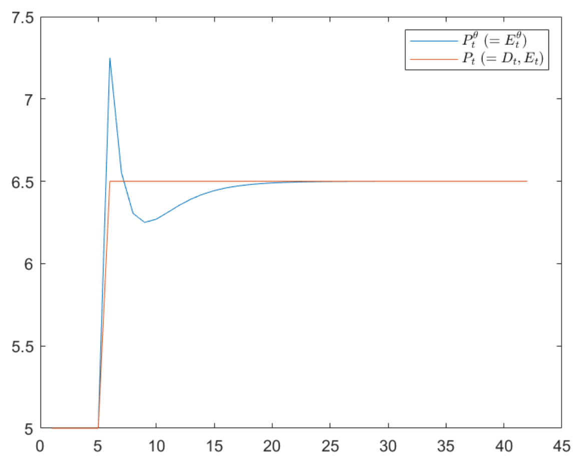


FIGURE 9.2. contrast: reversal after a large shock



move at all. Investor heterogeneity is necessary to generate price movements. In my model, price movements occur due to the gradual adjustment of the memory norm to the rational expectations. Rabin and Vayanos (2009) shows how an individuall mistakenly believing in mean reversion of signals overestimates changes in the underyling state after a long sequence of positive news and thus overreacts, while underreacts to short sequence of news. The model predicts that the effect on the next period forecast of a one off positive shock from steady state does not depend on the shock's size: the decision maker believes the signal mean reverts in the next period while updates on the state correctly, hence her expectation underreacts, no matter the size of the shock[26]. Diagnostic expectations (Bordalo et al, 2018), on the contrary, predicts overreaction to every shock, under the assumption of normally distributed dividends, which is the most frequently applied case.

[26]Hence, after a one off shock from steady state, the model predicts positive expected returns

9.3.2. *Sequence of surprises.* So far we have seen that it is the size of the surprise that determines whether price goes above or below rational, generating subsequent momentum or reversal. The dynamics of price are more nuanced when more than one surprise occurs. Small positive surprises to dividends, if they are small enough, do not change the posterior distribution from the memory average very much, so that the investors consistently underreact to the surprise, generating momentum after the flow of surprises has stopped coming. The effect is due to assimilation, just like in Proposition (8), where all the action comes from the distorted expected value $\mathbb{E}_t^\theta$

$$\mathbb{E}_t^\theta = \underset{\text{memory}}{(1-\rho)\sum \rho^{j-1}\mathbb{E}_{t-j}+} \quad \underset{\text{distortion } \mathbf{<1}}{|\mathbb{E}_t - (1-\rho)\sum \rho^{j-1}\mathbb{E}_{t-j}|} \quad \underset{\text{deviation from memory}}{\left(\mathbb{E}_t - (1-\rho)\sum \rho^{j-1}\mathbb{E}_{t-j}\right)}$$

The surprises of the sequence can be so small that $|\mathbb{E}_t - (1-\rho)\sum \rho^{j-1}\mathbb{E}_{t-j}| < 1$ for the whole length of the sequence. Since the price is linear in the expected value, we conclude that sequences of small surprises in dividends generate momentum both during their occurrence and after they stop.

More insteresting is the case of sequences of surprise a relatively larger magnitude: as the news flow starts, the posterior distribution over D_T might still be assimilated to the remembered one, and so its expectation. Hence the price reacts less than rational. As the surprises cumulate, the posterior gets sufficiently far from memory so that

$$|\mathbb{E}_t - (1-\rho)\sum \rho^{j-1}\mathbb{E}_{t-j}| > 1$$

At this point the present information looks enough different from the one in memory, hence the investor overreacts, bringing price above rational. As soon as the news stops flowing, the remembered expectations get accustomed to the recent expectations, so the effect of the surprises gradually weakens. The bubble bursts and price reverts back to the rational level in a few periods. Crucially, when the bubble bursts the price falls below rational before reverting to it: the intuition is that the remembered expectations get closer to the rational expectations from below: hence the rational expectation is assimilated to memory, the distorted expectation is thus biased below the rational, and the price too.

If the size of the suprise, instead, is large enough, the price goes above rational on impact because the distorted expectation overshoots above the rational and starts reverting downward after the news flow stops, following analogous dynamics. The next proposition formally expresses the behavior of the price depending on the different sequences of surprises.

FIGURE 9.3. Price response to sequence of medium small surprises

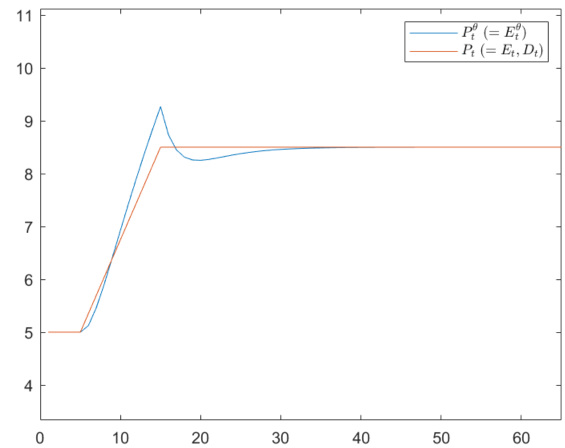
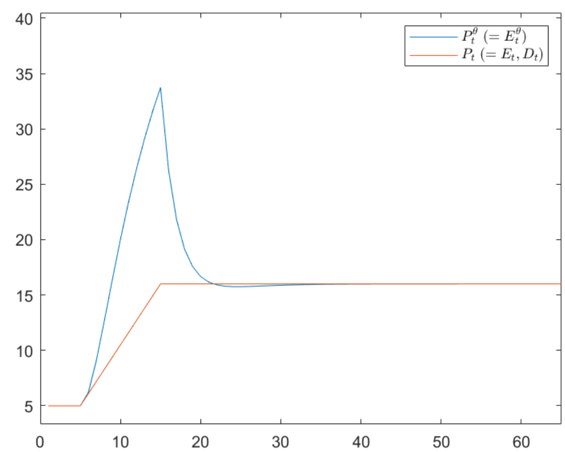


FIGURE 9.4. Price response to sequence of very small surprises



9.3.3. *Continuous vs discrete news: the Frog in the pan effect.* In this economy, as shown in Proposition (8), i

9.3.4. *Consistent vs alternating news.* In this economy populated with investors biased by remembered expectations the current period realization of dividends D_t does not contain all the information predictive of future returns. The dividend D_t in combination with past realizations of the dividends can generate over or undervaluation the stock. The history of surprises received until date t matters in determining valuation: if D_t is realized after a period of consistent and increasing surprises, past dividends are contrastingly lower than D_t (because they have been increasing over time), so past expectations are lower than current rational ones. As a result the investor contrasts the present expected value from the past ones (or alternatively, contrasts $D_t$from the past dividends) and overestimates it, pricing the asset above its rational price. Since the price P_t^θ is above rational, it will decline on average. As a result, a recent history of subsequent surprises in expected dividends predicts lower returns than a history of alternated surprises. Symmetrically, a recent history of subsequent disappointments produces overshooting of distorted expecations below rational, predicting future higher returns. The following figures compare three price and information histories, all of which share the same total amount of information released, that is

$$D_{t_1} = \bar{D} + \sum_{t_0}^{t_1} u_t$$

for all three cases ($t_1 = 35$ in the graphs). The difference across the three histories is the order in which surprises have occurred: in the first one, surpises alternate, in the second one bad surprises occur at the beginning of the history and better ones later on. In the third history, good surprises occur early and bad ones later on. Given that the total information is the same at time t_1, a rational investor at time t_1 shoud price the asset the same in all three cases (as it is the case in the figures). Instead, for a behavioral investor, contrasting the t_1 distribution against the history of past expectations distorts prices. In the second case, $\mathbb{E}_{t_1}$ is contrasted away from the average past expectations, which is low because of low initial realizations of dividends. Hence, $\mathbb{E}^{\theta}{}_{t_1}$overshoots relative to $\mathbb{E}_{t_1}$driving up price. In the first case, the alternation of news is such that the investor does not notice salient deviations of the rational distribution from the remembered average, hence does not significantly inflate nor deflate prices. The third case is symmetrical to the second one: at t_1 the investor remembers the high past expectations induced by the past good news: when the bad news arrive and cumulates she contrast the expectation against the high remembered expectation, and overshoots the price downwards. As a result, returns after t_1 will be on average higher in case 3, where the expectation is contrasted downwards from a high remembered expectation, relative to case 1 which has average returns higher than case 2 where the expectation is contrasted upwards.

FIGURE 9.5. Mixed surprises

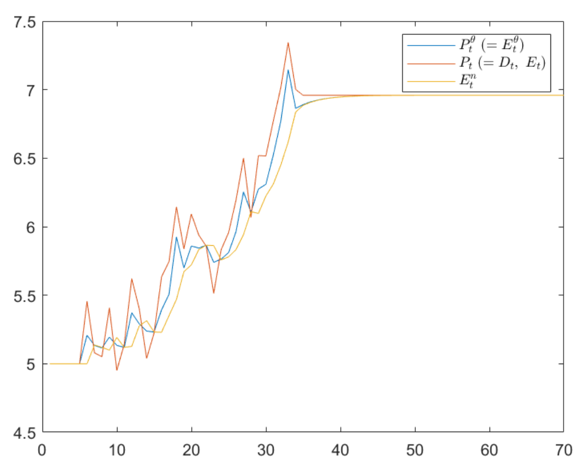


FIGURE 9.6. Ordered surprises (negative to positive)

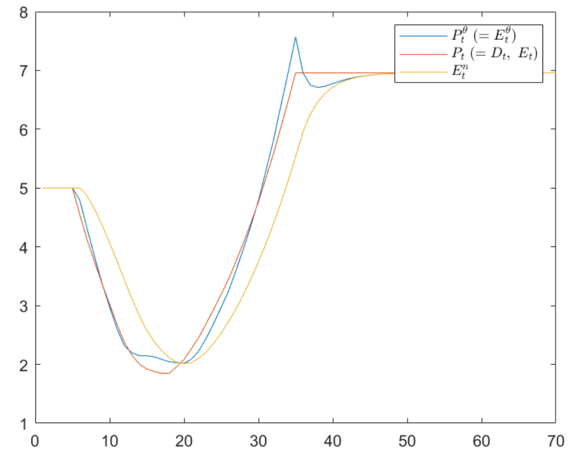
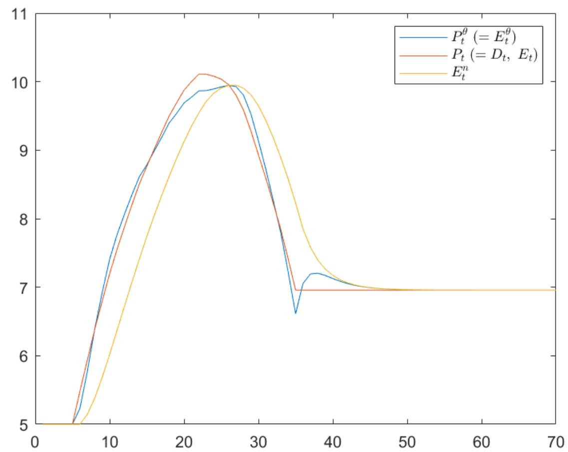


FIGURE 9.7. Ordered surprises (positive to negative)



The following proposition states formally the result, which holds for price histories where the total surprise received $\sum_{t_0}^{t_1} u_t$ is large enough.

Proposition 9. *Assume the market at t_0 is in steady state. Consider a stream of shocks $\{u_t\}_{t_0}^{t_1}$ such that $u_t \geq u_{t-1}$ $\forall t$ (at least one of the inequalities being strict), and such that $\sum_t u_t > \kappa$. Consider another stream of news $\{u_t^*\}_{t_0}^{t_1}$ which is a permutation of $\{u_t\}_{t_0}^{t_1}$. Then,*

- *the expected price change at t_1 after $\{u_t\}_{t_0}^{t_1}$ will be lower than after $\{u_t^*\}_{t_0}^{t_1}$, that is*

$$\mathbb{E}_t\left[\Delta P_{t+1}\right] < \mathbb{E}_t\left[\Delta P_{t+1}^*\right]$$

- *the expected $t+h$ return under $\{u_t\}_{t_0}^{t_1}$ is lower than under $\{u_t^*\}_{t_0}^{t_1}$ for h large*
- $\lim_{h\to+\infty} \mathbb{E}_t\left[P_{t+h}\right] = \lim_{h\to+\infty} \mathbb{E}_t\left[P_{t+h}^*\right] = P_t = P_t^*$

TABLE 10. Return under and overreaction by percentiles of market volatility

	$autocorr_t$			
	Daily		Monthly	
	1926-2022	1990-2022	1926-2022	1990-2022
$constant$	0.098^{***}	-0.017	0.150	0.025
	(0.009)	(0.015)	(0.017)	(0.024)
$vol_t \geq 50^{th}$	-0.049^{***}	-0.016	-0.078^{***}	0.020
	(0.014)	(0.015)	(0.026)	(0.036)
$vol_t \geq 90^{th}$	-0.079^{***}	-0.091^{**}	-0.0744^{*}	-0.148^{**}
	(0.024)	(0.038)	(0.042)	(0.047)
Obs.	1056	352	96	32

The table shows the results of the regression of market autocorrelation (daily or monthly) on the constant, one dummy =1 if simultaneous volatility is above the 50^{th} percentile, and one dummy=1 if simultaneous volatility is above the 90^{th} percentile. The regressions are perrformed both on the whole sample (1926-2022) and on the recent sample (1990-2022).

TABLE 11. Return under and overreaction by percentiles of market volatility

	$autocorr_t$			
	Daily		Monthly	
	1926-2022	1990-2022	1926-2022	1990-2022
$constant$	0.094^{***}	-0.001	0.146^{***}	0.001
	(0.010)	(0.015)	(0.017)	(0.027)
$vol_{t-1} \geq 50^{th}$	-0.040^{***}	-0.060^{**}	-0.078^{***}	-0.003
	(0.014)	(0.023)	(0.026)	(0.040)
$vol_{t-1} \geq 90^{th}$	-0.075^{***}	-0.027	-0.058	-0.074
	(0.024)	(0.038)	(0.043)	(0.059)
Obs.	1056	352	96	32

The table shows the results of the regression of value weighted market return autocorrelation (daily or monthly) within period t (t = month for daily returns, t = year if monthly returns) on the constant, one dummy =1 if the return volatility at $t-1$ is above the 50^{th} percentile, and one dummy=1 if volatility at $t-1$ is above the 90^{th} percentile. The regressions are perrformed both on the whole sample (1926-2022) and on the recent sample (1990-2022).

TABLE 12. Correlation between volatility and under/overreaction

	acorr_tdaily returns		acorr_t monthly returns	
	1926-2022	1990-2022	1926-2022	1990-2022
vol_t	-0.18^{***}	-0.16^{**}		
vol_t			-0.38^{***}	-0.52^{**}
Obs.	1056	352	96	32

The table shows the correlation coefficient between within-month autocorrelation of daily returns and simultaneous volatility (first and second columns), and the correlation coefficient between within year autocorrelation of monthly returns and volatility (third and fourth columns). Results are presented for the whole sample (1926-2022) and the restricted sample (1990-2022). Standard errors are computed via bootstrap.

Empirical test, additional tables.

10. Pricing amenities under Assimilation and Contrast

The following section applies my model to a standard housing market where rents are pinned down by a geographic indifference condition between amenities and prices reflect future rents. I show that

Table 13. Correlation between volatility and under/overreaction

	acorr_tdaily returns		acorr_t monthly returns	
	1926-2022	1990-2022	1926-2022	1990-2022
vol_{t-1}	-0.17^{***}	-0.15^{**}		
vol_{t-1}			-0.38^{***}	-0.28^{*}
Obs.	1056	352	96	32

The table shows the correlation coefficient between within-month autocorrelation of daily returns and lagged volatility (first and second columns), and the correlation coefficient between within year autocorrelation of monthly returns and volatility (third and fourth columns). Results are presented for the whole sample (1926-2022) and the restricted sample (1990-2022). Standard errors are computed via bootstrap.

when agents hold distorted perception of the future equilibrium prices, changes in amenities are underappreciated when occurring slowly and overappreciated when occurring drastically. This application helps make sense of the evidence of negligble effect of gradual government policies, of the overreaction of prices to news about the opening of polluting plants and of the delayed effect of sea level rise change in expectations on prices.

Consider two cities A and B. If a worker, at time t, chooses to live in city A, she earns wage w_t, perceives amenity a_t, and pay rent R_t. If she chooses city B,she earns $\bar{w}$, gets amenity $\bar{a}$ and pays $\bar{R}$. Workers' utility is linear in consumption and amenity. Consumption is equal to

$$w_t - R_t$$

$$U_t^A\left(a_t, w_t - R_t\right) = a_t + w_t - R_t$$

city B is the reservation city, that is, it has deterministic rent, wage and amenities,[27] so that the utility derived from living in city B is

$$U_t^B = \bar{a} + \bar{w} - \bar{R}$$

While workers cannot buy a house, landlords can own one. Landlords have linear utility in consumption, discount factor β and have no wealth, but can borrow at interest rate equal to r for one period and purchase one, and only one, unit of housing at price P_t. If she buys a house, in period t she gets consumption

$$R_t^A$$

and, in period $t+1$, she repays the loan by selling it, incurring consumption

$$P_{t+1} - P_t$$

If she does not buy the house at time t she gets consumption 0. Let us assume that (i) landlords' utility is linear in consumption and (ii) random payments are evaluated as separate lotteries (i.e. landlords distort random sale prices as separate from random rents). At each period they choose whether to own or not to own a house. Rent at time t, R_t^A, is known before they make the decision.

$$y_t \in \{own,\, not\, own\}$$

[27]This is a convenient expositional assumption often made in the urban economics literature (Rosen, 1979, Roback 1982)

$$V_t(y_t) = \begin{cases} R_t + \beta\mathbb{E}_t^\theta[P_{t+1}] - P_t + \beta E_t V_{t+1}^\theta(y_{t+1}) & if\, y_t = own \\ 0 + E_t V_{t+1}^\theta(y_{t+1}) & if\, y_t = not\, own \end{cases}$$

$$E_t V_{t+j}^\theta(y_{t+1}) = \begin{cases} \mathbb{E}_t^\theta[R_{t+j}] + \beta\mathbb{E}_t^\theta[P_{t+j+1}] - \beta\mathbb{E}_t^\theta[P_{t+j}(1+r)] + \beta E_t V_{t+j+1}^\theta(y_{t+j+1}) & y_{t+1} = own \\ 0 + \beta E_t V_{t+j+1}^\theta(y_{t+j+1}) & y_{t+1} = not\, own \end{cases}$$

Lemma 2. *(Landlords' indifference)*

In equilibrium

$$R_t = a_t - \bar{a} + w_t - \bar{w} + \bar{R}$$

Moreover, if (i) landlords' utility is linear in consumption, and (ii) random payments are evaluated as separate lotteries.

$$P_t = R_t + \beta\mathbb{E}_t^\theta[P_{t+1}]$$

the housing premium for location A amenities and wages is

$$P_t - P^B = a_t - \bar{a} + w_t - \bar{w} + \beta\mathbb{E}_t^\theta[X_{t+1}] \tag{10.1}$$

where the price of location B housing is

$$P^B = \bar{R}\frac{1-\beta^{T+1}}{1-\beta}$$

and

$$X_{t+1} = a_t - \bar{a} + w_t - \bar{w} + \beta\mathbb{E}_{t+1}^\theta[X_{t+2}]$$

The lemma above restates two trivial facts. The first one says that rent, pinned down by indifference of workers between the two cities, positively depends on amenities and wage. The second one shows that, under appropriate restrictions, the price of housing follows a familiar recursive formulation and is equal to the rent plus the discounted expected future price. Expression (10.1) shows that the price differential depends on the distorted expectations of fundamentals of the cities, that is the amenity differential $a_t - \bar{a}$ and the wage differential $w_t - \bar{w}$.

To illustrate the main insight of this application, assume $w_t = \bar{w}$ for all t. That is

$$P_t - P^B = a_t - \bar{a} + \beta\mathbb{E}_t^\theta[X_{t+1}]$$

where

$$X_{t+1} = a_t - \bar{a} + \beta\mathbb{E}_{t+1}^\theta[X_{t+2}]$$

Let us now highlight the key implication of the model by further simplifying. Suppose $\beta = 1 - \rho = 1$, and amenities in city A are constant in period 1 and 2, that is $a_t = \hat{a}$ for all $t < T$. Like in GIn period 1, the city government decides to improve amenities by undertaking a policy whose purpose is to reduce hazardous waste at period T. That is,

$$a_T = \hat{a} + \sum_{j=1}^{T}\epsilon_t$$

where ϵ_t are i.i.d shocks which realize at t and have symmetric pdf. These shocks capture the action that the government undertakes in the periods which gets converted into amenity a_T in period 3. [28]

The price differential between the two cities thus depends on the updated values of amenity in city A in period 3. Assume that $\rho = 1$, that is that agents in the economy evaluate random payments by comparing them with what they remember from the previous period. The amenity premium π_t depends on the expected improvements that payoff at time T, which are distorted.

$$P_t - P^B = (T - t)(\hat{a} - \bar{a}) + \mathbb{E}_t^\theta [a_T]$$

Lemma 3. *(Slow vs fast changes in amenities) Assume* $1 - \rho = \beta = 1$ *and*

$$g(F, G) = (|\mathbb{E}_F[X] - \mathbb{E}_G[X]|)^\alpha$$

with $\alpha > 0$*. If* T *is large, gradual increases (declines) in long term amenity* a_T *are priced less than sharp ones. That is,*

$$P_{T-1} - P_0 > 0$$

is smaller if

$$\mathbb{E}_t^\theta [a_T] - \mathbb{E}_{t-1}^\theta [a_T] = \delta$$

for $t = 1, ..., T - 1$*than if*

$$\mathbb{E}_t^\theta [a_T] - \mathbb{E}_{t-1}^\theta = \begin{cases} \delta (T-1) & t = t_1 \\ 0 & t \neq t_1 \end{cases}$$

The following result is another manifestation of contrast and assimilation. When shocks are large (high variance) price changes revert back (the covariance of price changes is negative). When shocks are small (small variance) the price underreacts and exhibits momentum.

Lemma 4. *Let* $\epsilon_t \sim_{i.i.d.} U[-k, k]$*. Let* $\hat{a} = \bar{a}$*. Then there is momentum in house prices if* $k < \bar{K}$*,while there is overreaction in the premium if* $k > \bar{K}$*. That is*

$$Cov_0 [\Delta P_t, \Delta P_{t-1}] > 0$$

if $k < \bar{K}$ *and*

$$Cov_0 [\Delta P_t, \Delta P_{t-1}] < 0$$

if $k > \bar{K}$

10.1. **Application 2: gradual and drastic changes in pollution. Between vs within comparisons.** Suppose there are two cities, A, and B. A has a fixed level of pollution characterized by amenity $\bar{a}$ and B has time-varying pollution, modelled by mean reverting amenities

$$a_{t+1} = (1 - \delta)\bar{\mu} + \delta a_t + \epsilon_{t+1} \tag{10.2}$$

with $\epsilon_t \sim N(0, \sigma^2)$ with $\delta \in (0, 1]$. Suppose agents in the economy had discount rate β and, in particular, landlords could own one unit of property in either of the two cities. Housing in city A has

[28] Notice that in this application the law of iterated expectations holds, that is $\mathbb{E}_t^\theta \left[\mathbb{E}_{t+k}^\theta [a_T]\right] = \mathbb{E}_t^\theta [a_T]$. This is not true in general.

price $\bar{P}$, while price P_t^θ in city B is determined in equilibrium. In the appendix, I show that the price P_t^θ is a linear function of the distorted expected value of the sum of future amenities which boils down to a function of the current amenity level a_t and the past level a_{t-1}.

$$\mathbb{E}_t^\theta\left[\sum_{j=1}^{\infty}\beta^j a_{t+j}\right] = \underbrace{\frac{\beta}{1-\beta}\bar{\mu} + \frac{\beta\delta}{1-\beta\delta}\left(a_{t-1}-\bar{\mu}\right)}_{\text{memory anchor}} + \theta\left(\frac{\beta\delta}{1-\beta\delta}\right)^2 \cdot \begin{cases} \left[a_t - a_{t-1}\right]^2 & if\; a_t > a_{t-1} \\ \underbrace{-\left[a_t - a_{t-1}\right]^2}_{\text{adjustment}\times\text{dissimilarity}} & if\; a_t < a_{t-1} \end{cases} \tag{10.3}$$

The price depends on the difference in the long run means of amenities between the two cities and on the present and past shocks

$$P_t^\theta = \bar{P} + \frac{\beta}{1-\beta}\underbrace{\left(\bar{\mu}-\bar{a}\right)}_{\text{crossectional difference}} + \frac{\beta\delta}{1-\beta\delta}\left(a_{t-1}-\bar{\mu}\right) + \kappa\begin{cases} \left[a_t - a_{t-1}\right]^2 & if\; a_t > a_{t-1} \\ -\left[a_t - a_{t-1}\right]^2 & if\; a_t < a_{t-1} \end{cases} \tag{10.4}$$

Suppose we had a long panel of observations of pollution and house prices. If we estimated the premium with a between estimator, like in crossectional studies, and regressed linearly price on amenities, we would obtain a coefficient of $\frac{\beta}{1-\beta}$ which is the effect of a higher long run $\bar{\mu}$ on P^θ. If, instead, we wanted to estimate a linear regression using a within estimator, the size of the coefficient would depend on how the distorted expectation in city B varies. For pollution that varies by a small amount, the price reacts too little, as the current pollution is assimilated to the past one, so the effect of amenities on prices will be small than $\frac{\beta}{1-\beta}$. When pollution varies by a large amount, instead, it contrasts against what agents remember, so the price is oversensitive to variation in pollution, and the estimated coefficient will be higher than $\frac{\beta}{1-\beta}$. This can help explain why the estimated air pollution premia from within city variation by Chay and Greenstone (2005) are larger than crossectional estimates, as the authors show that the change in air pollution is very drastic. Also this explains why the hazardous waste premia estimated within cities by Greenstone and Gallagher 2008 are much smaller than crossectional estimates. This is because the shock to pollution whose effect they estimate is distributed in several small shocks over a long period of time: the environment is assimilated to what people remember, and so they take much longer to realize the total change.