Optimal Gradualism*

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Abstract

This paper studies how gradualism affects the welfare gains from trade, technology, and reforms. When workers face adjustment frictions, gradual shocks create less adverse distributional effects in the short run. We show that there are welfare gains from inducing a more gradual transition via temporary taxes on trade and technology and provide formulas for the optimal path for taxes. Our formulas account for the possibility that reallocation effort responds to policy and for the existence of income taxes and assistance programs. Using these formulas, we compute the optimal temporary taxes needed to mitigate the distributional consequences of rising import competition from China and the deployment of automation technologies substituting for routine jobs. Our formulas can also be used to compute the optimal timing of economic reforms or trade liberalizations. We study Colombia's trade liberalization in 1990 and conclude that optimal policy called for a more gradual reform.

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Technological progress, trade, and economic reforms can generate periods of adjustment during which some workers fall behind, lose their jobs, experience wage declines, and see their livelihoods disrupted.[1] Even if technology and trade are positive developments in the long run, dealing with these short-run disruptions remains an important policy concern, especially in the wake of rapid changes in the economy.[2]

Existing evidence points to large disruptions. Autor et al. (2014) document that an average US worker in an industry exposed to Chinese import competition experienced a cumulative income loss equal to half their annual earnings in 1990 over the 1992–2007 period relative to unexposed workers. Cortes (2016) shows that US workers who in 1985 held routine jobs—those that can be more easily automated—experienced a subsequent decline in wages of 16% by 2007 relative to similar workers in other occupations.

How should policy respond during these adjustment periods? Do short-run disruptions imply that more gradual advances in technology and trade are preferable?

This paper shows that short-run disruptions create potential gains from gradualism and justify temporary taxes on new technologies and trade or embracing gradual reforms. Our main contribution is to provide formulas for the optimal path for taxes on new technologies and trade that capture the gains from gradualism. We evaluate these formulas in a calibrated version of our model that matches the empirical estimates of Autor et al. (2014) for trade and Cortes (2016) for the automation of routine jobs. Our formulas call for temporary taxes on trade and automation technologies of 10%, phased out over time. We also use our formulas to study Colombia's trade liberalization in 1990 and show that optimal policy called for a more gradual reform.

We derive these formulas in a model where workers are displaced by technology or trade. Ex-ante identical workers are allocated across islands à la Lucas and Prescott (1974). Some islands represent jobs automated by new technologies (e.g., welding or data-entry clerks) or segments of industries disrupted by international trade (e.g., low-cost apparel or household electronics). At time t_0, a new technology arrives, capable of replacing workers in these

[1]For evidence in the context of trade, see Autor and Dorn (2013); Autor et al. (2014). For evidence in the context of automation technologies, see Cortes (2016); Adão et al. (2021); Acemoglu and Restrepo (2020, 2022). Finally, see Goldberg and Pavcnik (2005) for evidence of how dismantling trade protection reduces the relative wages of workers in exposed industries.

[2]In the US, industrial robots installations and imports from China tripled in a few years (see Autor et al., 2013; Acemoglu and Restrepo, 2020, respectively), and the share of e-commerce in retail went from 0.6% to 10% from 1999 to 2019 (see US Census, 2022). As Erik Brynjolfsson and Andrew McAfee put it in *The Second Machine Age*, "People are falling behind because technology is advancing so fast and our skills and organizations aren't keeping up" (Brynjolfsson and McAfee, 2014). Managing short-run disruptions is also a key policy concern when it comes to policy reforms (see, for example, Rodrik, 1995).

islands by producing the same output at lower costs. These costs decline over time as technology improves exogenously, capturing advances in automation (as in Acemoglu and Restrepo, 2022) or improvements in Chinese exporters' productivity (as in Caliendo et al., 2019; Galle et al., 2022). Real wages at disrupted islands fall over time, and real wages at other islands increase. As in Alvarez and Shimer (2011), workers reallocate at a rate $\alpha > 0$, which represents the time it takes to find new jobs or acquire skills required in other jobs. The transition features a temporary decline in real wages and consumption for disrupted workers and higher real wages for all in the long run.[3]

Using this model, we provide analytical answers to the two questions above:

Given a path for technological progress, should the government induce a more gradual adjustment via temporary taxes on new technologies?

Using a variational approach, we derive formulas for the optimal tax path on new technologies and show that the optimum involves a temporary increase in taxes that is phased out over time. Temporary taxes are optimal even if technology, trade, and reforms make everyone better off in the long run. Taxes on new technologies should be higher when disrupted workers experience a large drop in consumption during the transition, which is linked to the decline in income documented by Autor et al. (2014) for the China Shock and Cortes (2016) for the automation of routine jobs.

Our formulas account for the possibility that taxing new technologies might dampen incentives for reallocation, creating a motive for a faster phase-out of the initial tax. Our formulas also extend to a scenario in which the planner can reform the income tax system or expand the generosity of assistance programs and the safety net to ease the transition for disrupted households. We characterize the optimal increase in the generosity of these programs during the transition and show that taxes on new technologies are still optimal in the short run.[4] The logic follows Naito (1999) and Costinot and Werning (2022): taxing new technologies is valuable because it assists workers affected by technological disruptions

[3]Our model is designed to capture the effects of technologies that work by substituting workers at some of their existing roles. We see trade, offshoring, and automation technologies working in this way. These technologies have the potential to reduce the wages of displaced workers despite increasing aggregate output. Other developments, such as factor-neutral improvements in technology or technologies that directly complement skilled workers, do not fit our description.

[4]The key assumption here is that these programs cannot be conditioned on island. This is certainly true for income taxes and many broad-based assistance programs, such as unemployment and disability insurance or universal basic income. If island-specific assistance programs or wage subsidies were available, redistribution could be done without distorting production—an implication of the Second Welfare Theorem and Diamond and Mirrlees (1971). In practice, these additional (and more desirable policy tools) might be limited since identifying workers whose livelihoods were disrupted by technology and trade as opposed to regular economic churn and sectoral fluctuations might be costly.

or globalization and not those who reduce their work effort to exploit the tax system.

Conditional on government policy, does the economy benefit from more gradual technological advances along the transition?

The finding that it is optimal to tax automation and trade during a transition does not imply that slower technological progress is desirable. For example, we show that more rapid technological progress is always welcomed when optimal taxes on new technologies are in place. More generally, faster technological progress can increase or reduce welfare when automation technologies and trade go untaxed, depending on parameters. This points to an important distinction. Both taxes and slower technological progress deliver first-order distributional gains. However, taxing technologies has a negative second-order effect on production, while slower technological progress has a negative first-order effect on the production frontier, creating an ambiguous effect on welfare.

Applications: We apply our framework to study the automation of routine jobs, the China Shock, and Colombia's 1990 trade liberalization. We calibrate the model to match the evidence in Cortes (2016) on the automation of routine jobs and in Autor et al. (2014) on the China Shock. The evidence points to limited opportunities for reallocation and implies low values for the transition rate out of disrupted islands α of 2.7% per year for workers in routine jobs and 1.8% per year for the China Shock. In addition, we back out the underlying path for technology from data on occupational wages or import shares.

We find that the automation of routine jobs had a negative welfare effect of 6%–8% on workers in disrupted islands, depending on assumptions about the ability to share the risk of transitioning late and capacity to borrow. Welfare losses are driven by a short-run income decline of 12% from 1985 to 2000, which recovers by 2025. From a utilitarian perspective, these short-run disruptions justify an optimal tax on automation technologies of 10%–12.5% over 1985–1995. The tax is subsequently phased out, reaching a level of 4% by 2020 in the least gradual scenario. This policy limits welfare losses for workers in disrupted islands to less than 2%.

The China Shock had a negative welfare effect of 15%–19% on workers in disrupted islands, though these account for only 1.6% of the US workforce. From a utilitarian perspective, these disruptions justify an optimal tax on Chinese imports of 10%–15% over 1991–2000. The tax is subsequently phased out, reaching a level of 3% by 2020 in the least gradual scenario. This policy limits welfare losses for workers in disrupted islands to less than 12.5%.

In both applications, we estimate no gains (from a utilitarian point of view) of slower technological progress, even absent taxes on technologies or trade.

In a final application, we compute the optimal trade liberalization policy for Colombia. In 1990, Colombia embarked on a rapid and ambitious program of trade liberalization, reducing effective tariffs by 37.5% in two years. We calibrate our model to match the immediate increase in imports following the reform and the drop in wages in previously protected industries estimated by Goldberg and Pavcnik (2005). The evidence points to a slow reallocation rate α of 3% per year, similar to our estimates in the other applications. Optimal policy calls for a more gradual reform, with tariffs remaining at a fourth of their initial level by 2000—10 years after the reform started. Reallocation rates of 20% per year—one order of magnitude larger than our estimate—are required to justify Colombia's swift drop in tariffs.

Related literature: Our optimal tax formulas relate to the tariff formula in Grossman and Helpman (1994) and the formula for optimal taxes on new technologies in Propositions 1 and 3 of Costinot and Werning (2022). Grossman and Helpman (1994) and Helpman (1997) focus on redistribution via tariffs across workers specialized in different industries. Our optimal tax formula shares the same structure as theirs and generalizes it to a dynamic environment where workers reallocate over time.[5]

Our characterization of optimal taxes builds on Costinot and Werning (2022) and extends their variational arguments to a dynamic setting.[6] Despite methodological similarities, the problem solved by Costinot and Werning differs from ours. They are interested in *pre-distribution*: taxing technology to reduce permanent inequality between ex-ante different people. Technologies that reduce wages at the bottom of the income distribution relative to the top have "tagging" value. Taxing these technologies achieves a better distribution of income than using income taxes alone, an insight that goes back to Naito (1999). The problem we study is complementary. In our model, winners and losers are ex-ante identical, and new technologies are taxed to ease the transition. This is why our formulas for optimal taxes are linked to the short-run decline in income for exposed workers (i.e., the regressions in Autor et al., 2014; Cortes, 2016, used here), and do not depend on how robots or trade affect wages at different points of the income distribution (i.e., the quantile regressions in Acemoglu and Restrepo, 2020; Chetverikov et al., 2016, used by Costinot and Werning). The formulas in Costinot and Werning prescribe long-run taxes on the basis of

[5]The formulas also differ in that Grossman and Helpman assume a quasi-linear aggregator across islands, and their weights emerge from lobbying and not necessarily from welfare considerations.

[6]Variational arguments have been used extensively to characterize properties of optimal income tax schedules (Saez, 2001; Tsyvinski and Werquin, 2017; Saez and Stantcheva, 2016).

pre-distribution, and our formulas prescribe short-run taxes to ease transitions.

Our paper also contributes to recent literature on the optimal taxation of automation motivated by distributional considerations (Thuemmel, 2018; Guerreiro et al., 2021; Donald, 2022) or inefficiencies (Acemoglu et al., 2020; Beraja and Zorzi, 2022).[7] Thuemmel (2018); Guerreiro et al. (2021) show that non-zero taxes on robots are justified even when income taxes are available as an additional tool for redistribution, in line with Naito (1999). Like us, Guerreiro et al. (2021) emphasize that optimal taxes on robots are positive along the transition and zero in the long run when affected cohorts of workers retire from the labor market. Beraja and Zorzi (2022) also argue for temporary taxes on automation technologies, though in their case, taxation is motivated by efficiency considerations: firms make investment decisions on automation with the wrong discount factor, which generates excessive automation. We contribute to this literature by providing intuitive and general formulas for optimal taxes on automation technologies that provide a tight link between theory and empirical evidence and identify the key features of the data that inform optimal taxes. We also show that the formulas can be applied to study how trade competition should be handled during a transition period and how economic reforms should be conducted.

Finally, we contribute to the literature on the optimal timing of reforms and trade liberalization, going back to Mussa (1984) and with contributions by Edwards and van Wijnbergen (1989); Rodrik (1995); Bond and Park (2002); Chisik (2003). Mussa argued that "a general case for gradualism in trade liberalization can be based on a desire to limit the income and wealth losses sustained by owners of resources initially employed in protected industries," which is the rationale for gradualism studied in this paper.

Roadmap: Section 1 introduces our model and characterizes the transitional dynamics following advances in trade or automation technologies. Section 2 derives formulas for optimal taxes and the gains from technological gradualism. Sections 3, 4, and 5 apply our framework to the automation of routine jobs in the US, the China Shock, and Colombia's trade liberalization. Proofs and derivations are in the Appendix.

[7] A complementary line of work studies the compensation of displaced workers via changes in income taxes (see Antràs et al., 2017; Tsyvinski and Werquin, 2017) and derives formulas for welfare that account for distributional considerations when these optimal taxes are in place.

1 A Model of Economic Disruptions

Status quo: Consider an economy with a mass 1 of households and a set of islands $x \in \mathcal{X}$. Each island produces a good $y_{x,t}$, which combines with the output of other islands into a final numeraire good y_t according to a constant returns to scale production function

$$y_t = f\left(\{y_{x,t}\}_{x \in \mathcal{X}}\right).$$

Initially, islands produce these goods with labor so that $y_{x,t} = \ell_{x,t}$, where $\ell_{x,t}$ denotes the mass of workers in island x. We assume that the initial allocation of workers $\ell_{x,0}$ across islands before the shock equates wages to a common level $\bar{w}$. This can be thought of as the steady state of the reallocation process introduced below.[8]

The disruption: At time $t = 0$ a new technology arrives. For a subset of *disrupted islands* $x \in \mathcal{D} \subset \mathcal{X}$, it becomes possible to produce their goods using a new technology embodied in capital $k_{x,t}$. New capital can be produced from $1/A_{x,t}$ units of the final good and fully depreciates after use. The new technology's productivity $A_{x,t}$ increases over time and converges to A_x.[9]

Following the arrival of new technologies, the production of $y_{x,t}$ becomes

$$y_{x,t} = \begin{cases} \ell_{x,t} + k_{x,t} & \text{if } x \in \mathcal{D} \\ \ell_{x,t} & \text{if } x \notin \mathcal{D}. \end{cases}$$

The disruption leads to permanent changes in island wages $w_{x,t}$, which prompt workers to reallocate with Poisson probability $\alpha_x > 0$ to an island of their choice.

Taxes: The government sets taxes $\tau_{x,t}$ on new technologies, raising revenue

$$T_t = \sum_{x \in \mathcal{D}} \tau_{x,t} \cdot \frac{k_{x,t}}{A_{x,t}},$$

[8] Our derivations apply to an economy with no ex-ante inequality between or within islands. This helps us isolate the incentives to induce gradual transitions to mitigate the cost of the disruption for affected households. Appendix D extends some of our results to an economy with ex-ante inequality across households within and between islands. Under mild assumptions, inequality within islands does not affect our tax formulas. Inequality across islands creates incentives for higher taxes on technology if disrupted islands had lower consumption (in line with the logic of pre-distribution in Costinot and Werning (2022)).

[9] We assume an exogenous path for advances in technology. This formulation abstracts from external learning by doing or other externalities that could generate a technological transition that is inefficiently slow to begin with. For this reason, our formulas for optimal taxes must be interpreted as a tax on top of any subsidy needed to reach the first best level of technological deployment.

which gets redistributed equally to households in a lump sum way. We first consider a baseline version of our model where the government has no other tools for redistribution or assistance and study these tools in our extensions.

Households: Households are ex-ante identical in their labor productivity and are endowed with 1 unit of labor. Before the shock, households in island x hold assets $a_{x,0}$. We assume that households are not insured against the initial disruption.

After the shock, households make consumption and saving decisions to maximize

$$U_{x,0} = \mathbb{E}\left[\int_0^\infty e^{-\rho t} \cdot u(c_{x,t}) \cdot dt\right] - \kappa(\alpha_x)$$

subject to some budget constraints that are left unspecified but that could capture various scenarios, ranging from hand-to-mouth to perfect risk sharing within islands. In all cases, we assume this is a small open economy and the interest rate r is fixed.

The term $\kappa_x(\alpha_x)$ captures reallocation costs. We consider two cases. With *exogenous reallocation,* $\alpha_x > 0$ is fixed and we set $\kappa_x(\alpha_x) = 0$. With *endogenous reallocation,* α_x is chosen by households to maximize indirect utility $\mathcal{U}\left(\{w_{x',t} + T_t\}_{x' \in \mathcal{X}}, a_{x,0}; \alpha_x\right) - \kappa_x(\alpha_x)$.

1.1 Transitional Dynamics and Equilibrium

We impose two assumptions on f, which are satisfied by commonly used aggregators.

Assumption 1 (Symmetry) *For all islands $x', x'' \notin \mathcal{D}$ and any island $x \in \mathcal{D}$, we have*

$$\frac{\partial^2 f}{\partial y_x \partial y_{x'}} = \frac{\partial^2 f}{\partial y_x \partial y_{x''}}.$$

This assumption ensures that technology benefits non-disrupted islands equally and implies a common wage w_t at non-disrupted islands along the transition. We thus focus on redistribution between non-disrupted and disrupted islands—the winners and losers of trade, technological progress, or reforms—and abstract from redistribution between winners (i.e., software engineers benefiting more than economists from the automation of sales jobs).

Let $c^f(p)$ denote the unit cost function associated with the aggregator f. With some abuse of notation, we denote by $c^f(\{w_x\}_{x \in \mathcal{D}}, w)$ the price of the final good when the price of the island x output is w_x for $x \in \mathcal{D}$ and w for other islands. Also, we denote by c^f_x and c^f_w the partial derivatives of this cost function with respect to w_x and w.

Assumption 2 (Adoption) *For any vector of wages with $w_x < \bar{w}$ for $x \in \mathcal{D}$ and a wage $w > \bar{w}$ in non-disrupted islands such that $c^f(\{w_x\}_{x\in\mathcal{D}}, w) = 1$, we have*

$$\frac{c_x^f(\{w_x\}_{x\in\mathcal{D}}, w)}{c_w^f(\{w_x\}_{x\in\mathcal{D}}, w)} > \frac{c_x^f(\{\bar{w}\}_{x\in\mathcal{D}}, \bar{w})}{c_w^f(\{\bar{w}\}_{x\in\mathcal{D}}, \bar{w})}$$

This assumption ensures that new technologies are adopted in all disrupted islands (so long as the after-tax cost of the new technology is below the initial market wage). It prevents adoption in one island from reducing demand in others.

Assumptions 1 and 2 hold when there is a single disrupted and a single non-disrupted island, but also when there are many islands whose outputs are combined via a constant-elasticity of substitution aggregator, f.

The following propositions characterize the transitional dynamics of the economy. Throughout, we maintain Assumptions 1 and 2.

Proposition 1 *Suppose that $\bar{w} > (1+\tau_{x,t})/A_{x,t}$, so that the new technologies are adopted. For a given path for taxes $\{\tau_{x,t}\}$ we have a transition with wages given by*

$$w_{x,t} = \begin{cases} (1+\tau_{x,t})/A_{x,t} & \text{if } x \in \mathcal{D} \\ w_t & \text{if } x \notin \mathcal{D}, \end{cases}$$

where the wage w_t in non-disrupted islands satisfies $w_t > w_{x,t}$ and can be computed as

$$1 = c^f(\{w_{x,t}\}_{x\in\mathcal{D}}, w_t).$$

Island employment is given by $\ell_{x,t} = e^{-\alpha_x t} \cdot \ell_{x,0}$ for $x \in \mathcal{D}$ and $\ell_t = 1 - \sum_{x\in\mathcal{D}} e^{-\alpha_x t} \cdot \ell_{x,0}$ for undisrupted islands. Finally, the quantity of goods produced with the new technology at island $x \in \mathcal{D}$ is given by

$$k_{x,t} = \ell_t \cdot \frac{c_x^f(\{w_{x,t}\}_{x\in\mathcal{D}}, w_t)}{c_w^f(\{w_{x,t}\}_{x\in\mathcal{D}}, w_t)} - \ell_{x,t} > 0.$$

The disruption has the following implications for households' consumption. A fraction $P_{x,t} = e^{-\alpha_x \cdot t}$ of households from island $x \in \mathcal{D}$ still works in the disrupted island at time t and consumes $c_{x,t}$ at time t. The remaining $1 - P_{x,t}$ households have reallocated by that time, with a fraction $\alpha_x \cdot e^{-\alpha_x \cdot t_n}$ reallocating to a non-disrupted island at time $t_n \in [0,t]$. We denote their consumption at time t by $c_{x,t_n,t}$. Households at non-disrupted islands do not reallocate, face no uncertainty, and consume c_t at t.

The paths for $c_{x,t}$, $c_{x,t_n,t}$ and c_t depend on the set of tools to share risk and smooth consumption available to households. In our applications, we compute the welfare gains from this transition and implement our formulas for optimal taxes under four scenarios:

I. Hand-to-mouth (transition risk and no borrowing): This scenario assumes households are hand-to-mouth. This implies $c_{x,t} = w_{x,t} + T_t$, $c_{x,t_n,t} = c_t = w_t + T_t$. In this scenario, households cannot borrow to smooth their consumption along the transition and face the risk of transitioning late.

II. No borrowing and no transition risk: This scenario assumes no borrowing from other islands or foreigners. However, we allow households to share the risk of transitioning late within their island. One can also think of this as a case where each household owns a mass 1 of units of labor, which it retools at a rate α to be used in other islands so that it faces no uncertainty. In both cases, $c_{x,t} = c_{x,t_n,t} = P_{x,t} \cdot w_{x,t} + (1 - P_{x,t}) \cdot w_t + T_t$, and $c_t = w_t + T_t$.

III. Borrowing with transition risk: This scenario assumes that households can borrow at an exogenous interest rate r but face the risk of transitioning late. In disrupted islands, households' problem can be summarized by the following system of HJB equations

$$\rho v_x(a,t) - \dot{v}_x(a,t) = \max_c u(c) + \frac{\partial v_x(a,t)}{\partial a} \cdot (ra + w_{x,t} - c) + \alpha_x \cdot (v(a,t) - v_x(a,t)),$$
$$\rho v(a,t) - \dot{v}(a,t) = \max_c u(c) + \frac{\partial v(a,t)}{\partial a} \cdot (ra + w_t - c).$$

Here, $v_x(a,t)$ is the value function of households in disrupted islands at time t with assets a (and taking α_x as given), and $v(a,t)$ is the value function of households in non-disrupted islands with assets a. Appendix E shows how to solve this problem numerically using the tools from Achdou et al. (2021).

IV. Borrowing with no transition risk: This scenario assumes ex-post complete markets. That is, households can freely save and borrow at an exogenous interest rate r and share transition risks within their islands. Households' problem becomes

$$\max \int_0^\infty e^{-\rho t} \cdot u(c_{x,t}) \cdot dt \quad \text{s.t.: } 0 \le \int_0^\infty e^{-rt} \cdot \left[P_{x,t} \cdot w_{x,t} + (1 - P_{x,t}) \cdot w_t + T_t - c_{x,t}\right] \cdot dt + a_{x,0}.$$

which implies $c_{x,t} = c_{x,t_n,t} = c_{x,0}$ and $c_t = c_0$ with $c_0 > c_{x,0}$.

2 Optimal Policy and the Gains from Gradualism

We evaluate policies using a symmetric welfare function $W_0 = \sum_{x \in \mathcal{X}} \int_{h \in x} \mathcal{W}\left(U_{x,0}^h\right) \cdot dh$, where $U_{x,0}^h = U_{x,0}$ is the expected lifetime utility of household h in island x after the shock and $\mathcal{W}$ is an increasing and concave function.

We provide formulas for the change in welfare and optimal taxes in terms of the per-capita social value of increasing income in island x at time t. This can be computed as

$$\chi_{x,t} = \begin{cases} g_x \cdot e^{-\rho t} \cdot u'(c_{x,t}) & \text{if } x \in \mathcal{D} \\ \dfrac{1}{\ell_t} \cdot \left(\sum_{x \in \mathcal{D}} \ell_{x,0} \cdot (1 - P_{x,t}) \cdot g_x \cdot e^{-\rho t} \cdot \mathbb{E}[u'(c_{x,t_n,t}) | t_n \leq t] + \ell_0 \cdot g \cdot e^{-\rho t} \cdot u'(c_t) \right) & \text{otherwise,} \end{cases}$$

where $g_x = \mathcal{W}'(U_{x,0}) \geq 0$ is the Pareto weight for households from disrupted island x and $g = \mathcal{W}'(U_0) \geq 0$ is the Pareto weight for households from undisrupted islands.[10] Note that χ_t (i.e., the common value for $\chi_{x,t}$ in disrupted islands) accounts for the changing composition of these islands due to reallocation. The χ's summarize the relevant information regarding households' marginal utilities of consumption during the transition, which in turn depend on their income paths and their ability to smooth consumption and share risks.

We first provide a general lemma that characterizes the change in welfare resulting from an arbitrary variation in taxes and technology utilization. This lemma relates to Lemma 1 in Costinot and Werning (2022) but differs in that it accounts for the fact that variations in taxes affect households' incomes in all future states. It also builds on variational arguments from Saez (2001); Saez and Stantcheva (2016); Tsyvinski and Werquin (2017).

Lemma 1 (Variations Lemma) *A variation in taxes that induces a change in wages dw_t, $dw_{x,t}$, technology $dk_{x,t}$, tax revenue dT_t, and reallocation effort $d\alpha_x$ changes welfare by*

$$dW_0^{reform} = \int_0^\infty \bar{\chi}_t \cdot \left[\underbrace{\sum_{x \in \mathcal{D}} \tau_{x,t} \cdot \frac{dk_{x,t}}{A_{x,t}}}_{aggregate\ efficiency} + \underbrace{\sum_{x \in \mathcal{X}} \ell_{x,t} \cdot \left(\frac{\chi_{x,t}}{\bar{\chi}_t} - 1 \right) \cdot dw_{x,t}}_{distributional\ considerations} \right] \cdot dt, \tag{1}$$

where $\bar{\chi}_t$ denotes the population-weighted average of $\chi_{x,t}$ across islands at t.

Equation (1) explains how distorting technology affects welfare. Any tax reform affects welfare by changing the aggregate efficiency of the economy (the first term in (1)) and via

[10] Our formulas for optimal taxes apply more generally when using *generalized social marginal welfare weights* that depend on broader ethical and political considerations (as in Saez and Stantcheva, 2016). Both features can be captured by having g_x and g depend on additional arguments.

distributional considerations (the second term in (1)).

The change in aggregate efficiency captures the total increase in resources available to households for consumption. Because there are no inefficiencies in our economy, this term is equal to the fiscal externality $\sum_{x \in \mathcal{D}} \tau_x \cdot dk_{x,t}/A_{x,t}$.

The distributional consideration term assesses the extent to which the reform reallocates resources to households that need them the most, as summarized by the social value of income $\chi_{x,t}$. A reform that curbs the use of automation technologies or trade during the transition brings positive distributional gains by increasing the wages of households stuck in disrupted islands, who have the highest marginal utilities of consumption.

2.1 Optimal Policy with Exogenous Reallocation

The next Proposition provides our first formula for optimal taxes. In this and the following sections, we use the partial derivative $\partial \ln w_x / \partial \ln z$ to denote the effect of a log change in quantity z on the log of the wage $w_x = \partial f / \partial y_x$ holding all other quantities constant.

PROPOSITION 2 *Let $m_{x',t} = k_{x',t}/A_{x',t}$ denote expenditure on $k_{x',t}$. With exogenous reallocation effort, a necessary condition for an optimal tax sequence is that*

$$\tau_{x',t} = \sum_{x \in \mathcal{X}} \frac{\ell_{x,t} \cdot w_{x,t}}{m_{x',t}} \cdot \left(\frac{\chi_{x,t}}{\bar{\chi}_t} - 1 \right) \cdot \left(-\frac{\partial \ln w_{x,t}}{\partial \ln k_{x',t}} \right). \tag{2}$$

Moreover, the formula in equation (2) implies zero taxes in the long run $\lim_{t \to \infty} \tau_{x',t} = 0$.

The derivation follows Costinot and Werning (2022). The idea is that, at an optimum, a variation that changes $k_{x',t}$ should not change welfare. Using Lemma 1 to evaluate this variation yields the optimal tax formula in equation (2).

Equation (2) shows that the optimal tax on automation or trade equals the distributional gains from curbing automation or trade—the right-hand side. Intuitively, optimal policy equates the marginal fiscal externality from reducing $k_{x',t}$—the foregone tax revenue on the left-hand side—to the distributional gains from distorting production.

The distributional gains (and the optimal tax on automation and trade) depend on:

1. **The influence of technology on wages:** Taxes should be higher when technology has a sizable negative impact on the wage of disrupted islands, as captured by the elasticities $-\partial \ln w_{x,t} / \partial \ln k_{x',t}$. In this case, distorting technology use is an effective tool to redistribute income toward households in disrupted islands.

2. **The gap in marginal social values, $\frac{\chi_{x,t}}{\bar{\chi}_t} - 1$:** Taxes depend on the gap in marginal social values $\chi_{x,t}$ between disrupted and non-disrupted islands, which capture the gains from redistribution. This gap is large when disrupted households experience a sizable drop in consumption during the transition.

   The gap and gains from redistribution depend on how big the income drop experienced by disrupted households is but also on households' ability to smooth consumption and share risks during the transition. The gap is larger when households from disrupted islands do not share the risk of transitioning late (so that $u'(c_{x,t}) > \mathbb{E}[u'(c_{x,t_n,t})|t_n \leq t]$), and when households from disrupted islands become constrained and cannot borrow against their future income.[11]

3. **The share of workers that have not been able to reallocate, $\ell_{x,t}$:** Taxes should be higher in the short run, when more disrupted workers will benefit from an increase in wages at disrupted islands. Over time, curbing automation or trade loses its tagging value since most of the disrupted workers have left disrupted islands and no longer benefit from the increase in wages. In the long run, taxes are zero because all workers reallocate away from disrupted islands.

2.2 Endogenous Reallocation

We extend Proposition 2 to allow for endogenous reallocation. In this case, any policy reform changes reallocation rates by $d\alpha_x$. The effect of $d\alpha_x$ through households' transition probabilities is second order because households internalize this benefit. However, changes in reallocation bring general equilibrium effects on wages and tax revenue at all points in time (for a given level of technology utilization) that also impact welfare.

To account for these GE effects, we need additional notation. Let $\mathcal{U}_{x,\alpha} = \partial \mathcal{U}_x / \partial \alpha_x$ denote the utility gains of changing the reallocation rate for a household in island x. Households'

[11] This can be seen by decomposing the gap as in Dávila and Schaab (2022):

$$\frac{\chi_{x,t}}{\bar{\chi}_t} - 1 = \underbrace{\frac{\chi_{x,t}}{\bar{\chi}_t} - \frac{\bar{\chi}_{x,t}}{\bar{\chi}_t}}_{\text{transition insurance}} + \underbrace{\frac{\bar{\chi}_{x,t}}{\bar{\chi}_t} - \frac{\bar{\chi}_x}{\bar{\chi}}}_{\text{consumption smoothing}} + \underbrace{\frac{\bar{\chi}_x}{\bar{\chi}} - 1}_{\text{pure redistribution}},$$

where $\bar{\chi}_{x,t} = P_{x,t} \cdot g_x \cdot e^{-\rho t} \cdot u'(c_{x,t}) + (1 - P_{x,t}) \cdot g_x \cdot e^{-\rho t} \cdot \mathbb{E}[u'(c_{x,t_n,t})|t_n \leq t]$ is the average social value of income at time t for households initially at island x, $\bar{\chi}_x = \int_0^\infty \bar{\chi}_{x,s} \cdot ds$ is the average social value of income for households initially at island x, and $\bar{\chi}$ is the average social value of income for all households. The decomposition shows that the distributional gains from distorting technology arise due to (i) improved risk sharing among households from disrupted islands, (ii) improved consumption smoothing for disrupted households, and (iii) pure redistribution of income towards households from disrupted islands.

choice of α_x satisfies the first-order condition $\mathcal{U}_{x,\alpha} = \kappa'_x(\alpha_x)$. Let $\mathcal{U}_{x,\alpha,d,t} \cdot dt$ denote the marginal effect of changes in income at time t in island x on $\mathcal{U}_{x,\alpha}$ and $\mathcal{U}_{x,\alpha,n,t} \cdot dt$ denote the marginal effect of changes in income at time t in non-disrupted islands on $\mathcal{U}_{x,\alpha}$. In general, $\mathcal{U}_{x,\alpha,d,t}$ is negative and $\mathcal{U}_{x,\alpha,n,t}$ positive, reflecting the disincentives of policies that tax new technology on reallocation efforts. Define $\varepsilon_{x'',x} \cdot \delta$ as the rate of change in $\alpha_{x''}$ when $\mathcal{U}_{x,\alpha}$ changes by δ. The cross partials $\varepsilon_{x'',x}$ depend on the curvature of the cost function $\kappa_x(\alpha_x)$ and the way in which reallocation away from island x affects factor prices and tax revenue, shaping incentives for reallocating away from island x''. When $\varepsilon_{x'',x} = 0$ for all x'', x we are back in the exogenous reallocation case.

Proposition 3 provides formulas for optimal taxes in terms of the disincentive effects $\mathcal{U}_{x,\alpha,d,t}$, $\mathcal{U}_{x,\alpha,n,t}$ and the cross partials $\varepsilon_{x,x''}$. The Appendix provides formulas for these objects in terms of primitives.

Proposition 3 *When effort is endogenous, a necessary condition for an optimal tax sequence is that*

$$\tau_{x',t} = \sum_{x \in \mathcal{X}} \frac{\ell_{x,t} \cdot w_{x,t}}{m_{x',t}} \cdot \left(\frac{\chi^{end}_{x,t}}{\bar{\chi}^{end}_t} - 1 \right) \cdot \left(- \frac{\partial \ln w_{x,t}}{\partial \ln k_{x',t}} \right), \tag{3}$$

where the χ^{end}'s are now given by

$$\chi^{end}_{x,t} = \begin{cases} \chi_{x,t} + \dfrac{1}{\ell_{x,t}} \cdot \displaystyle\sum_{x'' \in \mathcal{D}} \ell_{x'',0} \cdot \mu_{x''} \cdot \varepsilon_{x'',x} \cdot \mathcal{U}_{x,\alpha,d,t} & \text{if } x \in \mathcal{D} \\ \chi_{x,t} + \dfrac{1}{\ell_t} \cdot \displaystyle\sum_{x, x'' \in \mathcal{D}} \ell_{x'',0} \cdot \mu_{x''} \cdot \varepsilon_{x'',x} \cdot \mathcal{U}_{x,\alpha,n,t} & \text{otherwise,} \end{cases}$$

and μ_x is the social value per displaced worker of increasing the reallocation rate α_x:

$$\mu_x = \int_0^\infty (-s \cdot e^{-\alpha_x s}) \cdot \left[\sum_{x'' \in \mathcal{X}} \ell_{x'',s} \cdot (\chi_{x'',s} - \bar{\chi}_s) \cdot \frac{\partial w_{x'',s}}{\partial \ell_{x,s}} \right] \cdot ds. \tag{4}$$

The formula for optimal taxes shares the same structure as before. All that is needed is redefining the χ's, so that the social marginal value of increasing future income at different islands accounts for its effect on reallocation rates and the social benefit of reallocation.

To illustrate this result, consider the scenario with hand-to-mouth households and a single disrupted island. We have $\mathcal{U}_{x,\alpha} = \int_0^\infty e^{-\rho t} \cdot (t \cdot P_{x,t}) \cdot [u(w_t + T_t) - u(w_{x,t} + T_t)] \cdot dt$, $\mathcal{U}_{x,\alpha,d,t} = -(t \cdot P_{x,t}) \cdot \lambda_{x,d,t}$, and $\mathcal{U}_{x,\alpha,n,t} = (t \cdot P_{x,t}) \cdot \lambda_{x,n,t}$. The social marginal values of increasing incomes at disrupted and non-disrupted islands become

$$\chi_{x,t}^{\text{end}} = \begin{cases} \chi_{x,t} - \dfrac{\ell_{x,0}}{\ell_{x,t}} \cdot \mu_x \cdot \varepsilon_{x,x} \cdot (t \cdot P_{x,t}) \cdot \lambda_{x,d,t} & \text{if } x \in \mathcal{D} \\ \chi_t + \dfrac{\ell_{x,0}}{\ell_t} \cdot \mu_x \cdot \varepsilon_{x,x} \cdot (t \cdot P_{x,t}) \cdot \lambda_{x,n,t} & \text{otherwise,} \end{cases}$$

We can see from these formulas that $\chi_{x,t}$ has an extra negative term that accounts for the disincentive effects of taxing new technologies on reallocation efforts. These disincentives vanish in the very short run since few workers expect to be able to reallocate early on but become more important later in the transition since more workers expect to have reallocated by then. As a result, the optimal policy still involves a positive tax on automation and trade in the short run but now features a more rapid phase-out designed to provide backloaded incentives for reallocation. When reallocation is very responsive to incentives (i.e., the elasticity $\varepsilon_{x,x}$ is high), the disincentive effects of taxing automation and trade justify *subsidizing* these technologies in the medium run to provide stronger backloaded incentives for reallocation.

2.3 Income Tax Reforms and Assistance Programs

The design of optimal taxes on trade and technology depends on the availability of alternative policy instruments. Propositions 2 and 3 characterize optimal taxes when there are no other tools for assisting disrupted workers. At the other extreme, if *island-specific* transfers or wage subsidies are available, there is no rationale for distorting technology—a consequence of the Second Welfare Theorem and Diamond and Mirrlees (1971). In practice, these alternative (and more desirable) instruments might be limited since identifying and targeting losers from trade and technology is costly.

This section focuses on an intermediate and more realistic case where governments can ease transitions by adjusting the progressivity of the income tax system or the generosity of the broader safety net and existing assistance programs. The defining feature of many of these alternative tools is that they can only be conditioned on workers' income (or employment) and not on their identity or initial island. This is certainly true for income taxes and many broad-based assistance programs, such as unemployment and disability insurance, universal basic income, or wage insurance systems. To make our formulas tractable, we simplify things and assume that income taxes and the safety net act as a linear income tax with a marginal tax rate of $\mathcal{R}_t$ on households' taxable income. The marginal tax rate $\mathcal{R}_t$ summarizes the progressivity of the income tax system and the generosity of the safety net.

To capture the distortions generated by a more progressive income tax or a more generous safety net, we allow households to affect their taxable income by choosing their work effort, $n_{x,t}$, so that their taxable income becomes $n_{x,t} \cdot w_{x,t}$. These unobserved actions limit the use of income taxes and assistance programs since they cannot distinguish between workers with a low income because their job was disrupted by technology or workers with a low income due to their lack of work effort.[12] Effort is costly, and reduces households flow utility to $u(c_{x,t} - \psi(n_{x,t}))$, where ψ is a convex power function. These quasi-linear preferences imply that effort is not affected by income effects and responds to wages with a constant elasticity $\varepsilon_\ell \geq 0$.

The next proposition characterizes optimal policy when governments optimize jointly over paths for taxes on new technologies and the generosity of the safety net $\mathcal{R}_t$.

Proposition 4 *When reallocation effort is exogenous, optimal taxes on new technologies* $\{\tau_{x,t}\}$ *and marginal income tax rates* $\{\mathcal{R}_t\}$ *satisfy the necessary conditions*

$$\tau_{x',t} = \sum_{x \in \mathcal{X}} \frac{\ell_{x,t} \cdot n_{x,t} \cdot w_{x,t}}{m_{x',t}} \cdot \left(\frac{\chi_{x,t}}{\bar{\chi}_t} - 1\right) \cdot (1 - \mathcal{R}_t) \cdot \left(-\frac{\partial \ln w_{x,t}}{\partial \ln k_{x',t}}\right) - \varepsilon_\ell \cdot \mathcal{R}_t \cdot \sum_{x \in \mathcal{X}} \frac{\ell_{x,t} \cdot n_{x,t} \cdot w_{x,t}}{m_{x',t}} \cdot \frac{\partial \ln w_{x,t}}{\partial \ln k_{x',t}}, \tag{5}$$

$$\frac{\mathcal{R}_t}{1 - \mathcal{R}_t} = \frac{1}{\varepsilon_\ell} \cdot \sum_{x \in \mathcal{X}} \frac{\ell_{x,t} \cdot n_{x,t} \cdot w_{x,t}}{m_{\ell,t}} \cdot \left(\frac{\chi_{x,t}}{\bar{\chi}_t} - 1\right) \cdot \left[(1 - \mathcal{R}_t) \cdot \frac{\partial \ln w_{x,t}}{\partial \mathcal{R}_t} - 1\right] + \mathcal{R}_t \cdot \sum_{x \in \mathcal{X}} \frac{\ell_{x,t} \cdot n_{x,t} \cdot w_{x,t}}{m_{\ell,t}} \cdot \frac{\partial \ln w_{x,t}}{\partial \mathcal{R}_t}, \tag{6}$$

where $m_{\ell,t}$ *denotes total labor income in the economy.*

The degree of insurance provided by the tax system and the safety net affect optimal taxes on new technologies in three ways. First, a higher $\mathcal{R}_t$ leads to less dispersion in marginal utilities of consumption between disrupted and undisrupted households. Second, with a safety net and income taxes in the back, distorting technology to manipulate wages becomes a less powerful tool. For every dollar of higher wages at disrupted islands, households receive $1 - \mathcal{R}_t$ dollars of after-tax income. Third, taxing technology creates a fiscal externality (captured in the last line of the formula) since it affects the level of work effort.

[12] One can also think of $n_{x,t}$ as costly actions that reduce the probability of losing your job or experiencing a large income drop. For example, households might reduce their work effort and get fired because of shirking and not because of technological disruptions. Or households may decide to stop investing in their skills, causing their income to drop, not because of technology but due to their choices.

The proposition also gives a formula for the optimal level of insurance provided by the tax system and the safety net in response to technological disruptions. The formula in equation (6) shows that the optimal level of insurance trades off disincentives for work effort (the left-hand side) with the direct benefits from redistribution and the pecuniary and fiscal externalities induced by general equilibrium effects.[13] Equation (6) characterizes the optimal marginal income tax in an economy where the only role of income taxes is to assist disrupted households. In practice, the design of optimal income taxes also depends on the degree of inequality in permanent income. Likewise, the optimal generosity of the safety net also depends on the prevalence of idiosyncratic shocks unrelated to technological change. Both considerations are absent from our model. For this reason, we interpret the formula in equation (6) as characterizing the *additional* provision of insurance via income taxes and assistance programs justified in response to a technological disruption.[14]

The proposition shows that, so long as $\varepsilon_\ell > 0$, it will be optimal to respond to disruptions with a combination of temporary taxes on automation technologies or trade in the short run and an expansion of the generosity of the safety net. Taxing technology continues to be optimal because of its tagging value: reducing its use assists workers affected by disruptions and not those who reduced their work effort to exploit the generosity of the tax system and assistance programs. A complementary intuition is that distorting technology is helpful because it directly manipulates the wage distribution in favor of disrupted workers, whereas income taxes or assistance programs are a blunt tool since they can only manipulate the after-tax income of all households independently of their circumstances.[15]

[13]Shutting down general equilibrium effects by setting $\frac{\partial \ln w_{x,t}}{\partial \mathcal{R}_t} = 0$ yields the usual formula for an optimal linear income tax $\frac{\mathcal{R}_t}{1-\mathcal{R}_t} = \frac{1}{\varepsilon_\ell} \cdot \sum_{x \in \mathcal{X}} \frac{\ell_{x,t} \cdot n_{x,t} \cdot w_{x,t}}{m_{\ell,t}} \cdot \left(1 - \frac{\chi_{x,t}}{\bar{\chi}_t}\right)$.

[14]While our formulas apply to broad-based assistance programs available to all workers, targeted assistance programs face similar limitations in practice. For example, the US Trade Adjustment Assistance Program (TAA) includes extended unemployment insurance, retraining subsidies, and a wage insurance program for workers displaced by trade. However, the process for determining and certifying which workers were displaced by trade is costly (involving the creation of its own bureaucracy), noisy, and subjective (see Hyman, 2018, for an overview of the process). In line with this description, Autor et al. (2013) estimate that for every dollar of labor income loss due to trade disruptions, TAA transfers increase by less than one cent, suggesting a limited targeting capacity of this program. A broader set of income-based programs generate the bulk of transfers received by workers in regions experiencing Chinese import competition.

[15]This is the same rationale for why it is optimal to distort technology in Naito (1999), Guerreiro et al. (2021), and Costinot and Werning (2022). For example, in Costinot and Werning (2022), distorting technology is desirable because it redistributes towards workers whose wages are low because of technological reasons and not towards those whose income is low due to lack of work effort. Costinot and Werning (2022) also show that this result extends to a case with general non-linear income taxes.

2.4 Optimal Reforms and Trade Liberalizations

We can apply the formulas in Propositions 2, 3, and 4 to the question of how to conduct economic reforms or trade liberalizations.

Consider a variant of our model where the new technology already exists at time 0 and has constant productivity A_x but is not in use because of a distortionary tax $\bar{\tau}_x > 0$ in place preventing this, so that $\frac{1+\bar{\tau}_x}{A_{x,0}} \geq \bar{w}$ for some protected islands $x \in \mathcal{D}$. One could think of these as industries shielded from competition via trade tariffs or barriers to entry or as industries and firms that have been subsidized at the expense of others.

Our formulas characterize the optimal path for a reform that removes these distortions. Depending on parameters, the optimal reform path could involve an instantaneous jump to $\tau_{x,0} \in (0, \bar{\tau}_x)$ and a gradual decline to $\tau_{x,t} = 0$. Or it might involve keeping the tax at $\bar{\tau}_x$ for some time and phasing it out gradually in the future. This second scenario is equivalent to announcing the reform in advance, allowing people to save and reallocate in preparation.

2.5 Do we want slower technological progress?

The previous section showed that it is optimal to tax technology in the short run to assist disrupted households. That does not mean that the economy benefits from moving to a counterfactual world where technological progress advances more slowly. This is a different thought experiment.

To illustrate the difference, the following proposition considers the welfare change of shifting the path for $A_{x,t}$ to a more gradual one. To simplify the exposition, we consider a case with no income-based safety net.

Proposition 5 *Consider a new technology path that changes $A_{x,t}$ by $dA_{x,t}$. On this new path, technology use changes by $dk_{x,t}$, wages by $dw_{x,t}$, and welfare by*

$$(7) \quad dW_0^{tech} = \int_0^\infty \bar{\chi}_t \cdot \left[\underbrace{\sum_{x \in \mathcal{D}} \frac{k_{x,t}}{A_{x,t}} \cdot (\tau_x \cdot d \ln k_{x,t} + d \ln A_{x,t})}_{\textit{aggregate efficiency}} + \underbrace{\sum_{x \in \mathcal{X}} \ell_{x,t} \cdot \left(\frac{\chi_{x,t}}{\bar{\chi}_t} - 1 \right) \cdot dw_{x,t}}_{\textit{distributional considerations}} \right] \cdot dt.$$

The comparison between equations (1) and (7) reveals a key asymmetry between taxing the use of technology and having slower technological progress. Taxing the use of automation technologies or trade brings a first-order distributional gain, but only creates a second-order distortion in production (the term $\tau_{x,t} \cdot dk_{x,t}$ in equation 1, which is zero starting from an equilibrium with no distortions). Slower technological progress brings the exact same distributional gains, but now comes at the expense of a first-order reduction in production efficiency (the term $\frac{k_{x,t}}{A_{x,t}} \cdot d\ln A_{x,t}$ in equation 1). This asymmetry explains why it is optimal to tax the new technology by at least some positive amount in the short run while slower technological progress might reduce welfare. A complementary intuition is that taxing automation brings distributional benefits and generates revenue that can be redistributed to households. Instead, slower technological progress generates distributional benefits but no revenue.

One corollary of Proposition 5 is that, when optimal taxes on automation technologies and trade are in place, we have

$$dW_0^{\text{tech}} = \int_0^\infty \bar{\chi}_t \cdot \left[\sum_{x \in \mathcal{D}} \frac{k_{x,t}}{A_{x,t}} \cdot d\ln A_{x,t} \right] \cdot dt > 0,$$

and faster technological progress always increases welfare. This holds even if the government has no other tools for redistribution or assistance programs. This result follows from a simple envelop logic: a government can always mimic the less gradual path for technology by taxing its use.

More generally, more rapid technological progress can increase welfare even if governments do not intervene. This is the case when: i. differences in consumption between disrupted and non-disrupted households are small; ii. workers reallocate rapidly; and iii. technological advances are back-loaded, so that technology increases slowly initially.

3 Application I: The Automation of Routine Jobs

3.1 Description, Empirical Evidence, and Calibration

Description: There are 5 islands. Islands 2–5 are in $\mathcal{D}$ and represent segments of routine occupations $o(x)$ (where $o(x)$ denotes the occupation associated with island x) disrupted by technological progress: i. clerical and administrative occupations (10% of employment in 1985); ii. sales occupations (5% of employment); iii. production occupations (18.5% of employment); iv. transportation and material handling occupations (4% of employment). These four occupational groups are identified as routine in Acemoglu and Autor (2011). The first island represents segments of these occupations unaffected by the automation of routine jobs plus non-routine occupations.

Not all jobs that are part of an occupation are replaced by technology. A fraction $s_{o(x)}$ of all jobs in occupation $o(x)$ are disrupted and belong to island x. This implies that island x accounts for a fraction $s_{o(x)} \cdot \Omega_{o(x)}$ of initial employment, where $\Omega_{o(x)}$ is the initial share of employment in occupation $o(x)$.[16] We treat $s_{o(x)} \in [0, 1]$ as an unobservable to be calibrated in order to match the scope of the technological disruption.

The evidence in Cortes (2016): This paper estimates trends in occupational wages and shows that workers employed in routine jobs experienced lower wage growth. Cortes uses the Panel Study of Income Dynamics to estimate a variant of the model:

$$\log \text{hourly wage}_{j,o,t} = \delta_t + \text{Routine}_o \cdot \theta_t + X'_{j,t} \cdot \zeta + X'_j \cdot \zeta_t + \gamma_{j,o} + u_{j,o,t}. \tag{8}$$

The model explains hourly wages for person j employed in occupation o at time t as a function of: i. time trends, δ_t; ii. a differential time-path for routine occupations, $\text{Routine}_o \cdot \theta_t$, where Routine_o takes the value of 1 for routine occupations and θ_t captures differential wage trends in these jobs; iii. time varying individual covariates $X'_{j,t} \cdot \zeta$; iv. differential time trends by individual fixed characteristics $X'_j \cdot \zeta_t$; and v. permanent differences in the productivity of individual j in occupation o, $\gamma_{j,o}$. The last term accounts for selection in persistent attributes that make some individuals more productive at some occupations.[17]

Panels A and B in Figure 1 provide estimates of equation (8). Panel A reports estimates of θ_t from Cortes' data for different specifications: 1. controlling for permanent wage differences by individuals across occupations and demographics; 2. allowing for differential trends by region of residence and whether the person resides in urban or suburban areas; 3. controlling for union membership; and 4. allowing for differential trends over time by education level. These specifications show a permanent decline of 22–30% in relative wages paid in routine jobs since 1985. The more demanding specification that controls for trends in wages by educational levels dates the start of the decline to 1986. Panel B reports separate estimates $\theta_{o(x),t}$ for routine occupational groups using the most demanding specification. All routine occupational groups exhibit a similar pattern of declining wages, though the speed and extent of the decline varies across groups.

Cortes (2016) estimates of occupation wages over time are informative of the behavior

[16] One may consider a mass 1 of islands. Each island represents a differentiated job within an occupation, with island x belonging to occupation $o(x)$, and a mass $\Omega_{o(x)}$ of islands in each occupation. The automation of routine jobs corresponds to an improvement in the productivity of specialized equipment and software that substitutes for labor in a share $s_{o(x)}$ of the jobs that are part of occupation $o(x)$.

[17] Cortes (2016) also estimates the price associated with cognitive occupations, but for our purposes, the relevant object is the price of routine occupations relative to all others.

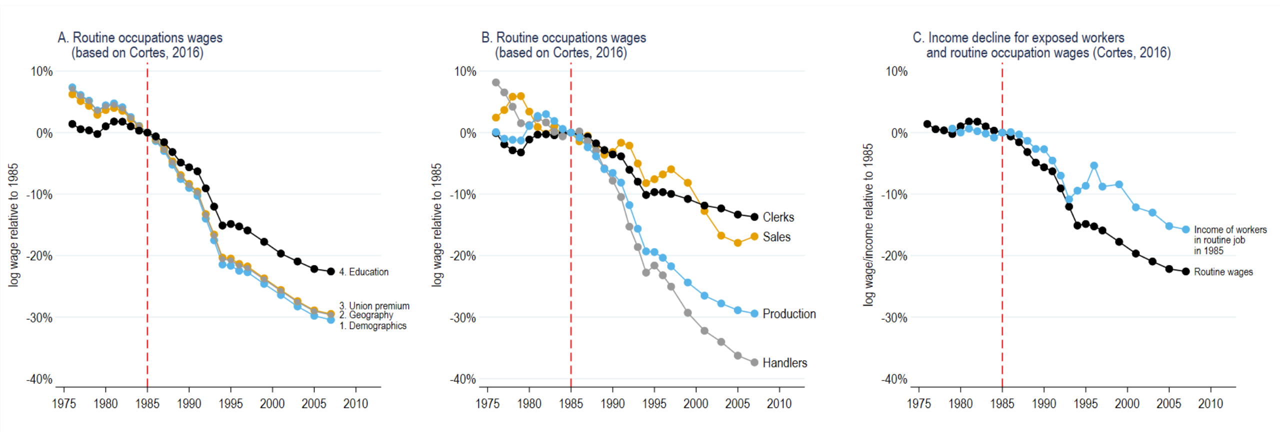

FIGURE 1: ESTIMATES OF WAGE TRENDS FOR ROUTINE OCCUPATIONS. Panel A reports estimates of θ_t in equation (8) using the data and specifications from Cortes (2016). Panel B reports separate estimates of $\theta_{o(x),t}$ for occupational groups. Panel C reports estimates of the incidence of these shocks on workers employed in routine occupations in 1985. All series smoothed using a 3-year moving average.

of $w_{x,t}$ in our model, which in turn provides information on the path for technology $A_{x,t}$. For example, if all jobs within an occupation are disrupted, then $\ln(w_{o(x),t}/w_t) = \theta_{o(x),t}$, which one can use to invert for advances in technology $A_{x,t}$.

Cortes (2016) also provides estimates of future wage growth for workers employed in routine occupations at time t_0. In particular, Cortes estimates the model:

$$(9) \qquad \Delta \log \text{wage income}_{j,t} = \delta_t + \beta \cdot \text{Routine}_{j,t_0} + X'_{j,t_0} \cdot \zeta + u_{j,t}.$$

This model explains wage growth between t_0 and t as a function of individual characteristics at time t_0 and a dummy for whether the individual worked at a routine occupation at time t_0.[18] We use Cortes data an estimate this regression for $t = 1976, \ldots, 2007$, controlling for age, demographics, union membership and education in 1985. Individuals employed in routine jobs by 1985 experienced 16% less wage growth during 1985–2007 than comparable workers, which aligns with the 20-year growth estimates from Table 2 in Cortes (2016).

Panel C in Figure 1 report our preferred estimates for occupational wage changes $\theta_{o(x),t}$ from 1985 to 2007 and the incidence of this shock on workers employed in these occupations in 1985 (from equation 9). There is a large incidence of the shock, with these workers experiencing 70% of the wage decline implied by the full shock.[19] Through the lens of our

[18] We focus on a variant of the specification used in Cortes (2016) that looks at total income accounting for hours worked. This is because reduced work hours might be an important margin of adjustment for workers. This strategy does not account for non-employment, though we did not find evidence of sizable employment effects in the PSID.

[19] These estimates of incidence account for non-employment and changes in hours worked. The 16%

model, the high incidence estimated by Cortes (2016) points to small reallocation rates α_x.

Calibration: Aggregate output y_t is given by a CES aggregator

$$y_t = \left(\nu^{\frac{1}{\sigma}} \cdot \ell_t^{\frac{\sigma-1}{\sigma}} + \sum_{x \in \mathcal{D}} \nu_x^{\frac{1}{\sigma}} \cdot y_{x,t}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}. \tag{10}$$

We take a value of $\sigma = 0.85$ from the literature on occupational polarization (see Goos et al., 2014). We normalize status-quo wages to $\bar{w} = 1$, which implies $\nu + \sum_{x \in \mathcal{D}} \nu_x = 1$.

Technology evolves according to

$$\ln A_{x,t} = \mathcal{S}(t; \pi_x, h_x, \kappa_x) = \pi_x \cdot \frac{1 + h_x \cdot (t_f - t_0)^{-\kappa_x}}{1 + h_x \cdot (t - t_0)^{-\kappa_x}} \text{ for } t \in [t_0, t_f] \tag{11}$$

where $\mathcal{S}$ is a parametric S-curve. $A_{x,t}$ starts at 1 and converges to $\exp(\pi_x)$—the long-run level of cost-saving gains due to the technology—at time t_f. From there on, we set $A_{x,t} = \pi_x$. The parameters h_x and κ_x govern the shape of the S-curve: a higher κ_x implies an S-curve with a steeper inflection; a higher h_x implies a faster adjustment. Building on the findings in Cortes (2016), we assume that the automation of routine jobs starts in $t_0 = 1986$. Acemoglu and Restrepo (2020) estimate cost-saving gains of 30% for the automation of production jobs via industrial robots. We set $\pi_x = 30\%$ and $t_f = 2007$, which assumes similar cost-saving gains of automating other routine occupations by 2007.

We calibrate $s_{o(x)}$ (or equivalently $\nu_x = \Omega_{o(x)} \cdot s_{o(x)}$), $A_{x,t}$, and $\alpha_{0,x} = \alpha_0$ to match the occupational wages $\theta_{o(x),t}$ over time and the incidence of the shock by 2007. These parameters are jointly calibrated, but their values are tightly linked to these moments:

- The choice of $\pi_x = 30\%$ pins down the level of A_{x,t_f}.
- The occupation-level wage decline by 2007, $\theta_{o(x),t_f}$, pins down $s_{o(x)}$—the exposure of each occupation to technological progress. In particular, $s_{o(x)} \to 1$ implies $\theta_{o(x),t_f} \to \ln(w_{x,t_f}/w_{t_f})$, where $w_{x,t_f}/w_{t_f}$ depends only on π_x, while $s_{o(x)} \to 0$ implies $\theta_{o(x),t_f} \to 0$.
- Occupational wages $\theta_{o(x),t}$ pin down the path for $A_{x,t}$ between $t_0 = 1986$ and $t_f = 2007$.
- The incidence of the shock pins down α_0. A higher incidence implies a lower reallocation rate. We calibrate a common $\alpha_0 = 2.7\%$ per year to match the average incidence

income decline is due to both lower wages and a greater likelihood of non-employment. We also estimated incidence separately for workers in each of the four routine occupational groups. The estimates range from 18% for sales, to 71–96% for the remaining groups. We use the average incidence as a target in our calibration because the group-specific estimates are noisy.

of 70% on exposed workers.

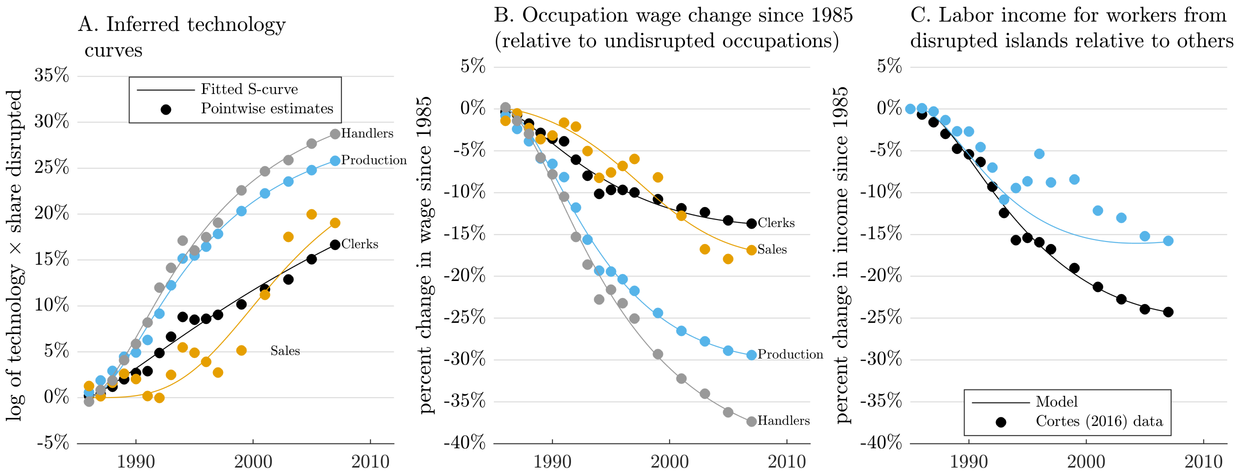

Figure 2: Calibrated paths for technologies replacing for routine jobs. Panel A reports yearly estimates of $A_{x,t}$ and their corresponding S-curve separately for each routine occupation. Panel B reports the model-implied occupational wage decline since 1985 and compares this to the estimates of $\theta_{o(x),t}$ by occupational group. Panel C reports estimates of the implied incidence of these shocks on workers employed in routine occupations by 1985.

This procedure yields yearly estimates for $\hat{A}_{x,t}$ from 1986 to 2007. Using these values, we fit the S-curve in equation (11) via non-linear least squares. Panel A in Figure 2 reports the yearly estimates of $\hat{A}_{x,t}$ and the fitted $S-$curves for the disrupted segments of routine occupations. To facilitate the interpretation, these estimates are scaled by $s_{o(x)}$, so that the figure is informative of the scope and the timing of the shock across routine occupations. Panels B and C show that our model matches the empirical evidence in Cortes (2016). Panel B reports the model-implied decline in relative wages by occupation since 1986 and compares it to the empirical estimates for $\theta_{o(x),t}$ from Cortes (2016). Panel C reports the model-implied incidence of the shock on workers initially employed in routine occupations. Our calibration generates a relative income decline of 16% for these workers, which matches the 70% incidence of the shock.

Table 1 reports the remaining parameters. We let $u(c) = c^{1-\gamma}/(1-\gamma)$, set the inverse elasticity of intertemporal substitution $\gamma = 2$, and set the discount and interest rate to 5% per year. For the versions of our model in which households are not hand-to-mouth, we assume zero initial assets for disrupted households. This aligns with the evidence in Kaplan et al. (2014) which points to median liquid wealth for US households of $1,714 in 2010.

3.2 Optimal policy in response to the automation of routine jobs

Using the formula in Proposition 2, we compute the optimal path for taxes on automation technologies since 1986. We first focus on the case with exogenous reallocation effort and report optimal taxes for the four scenarios described in section 2: i. hand-to-mouth; ii. no borrowing and no transition risk; iii. borrowing but transition risk; iv. ex-post complete markets. These four scenarios determine the mapping between the observed decline in income documented by Cortes (2016) and matched by our model, and the unobserved marginal utilities of consumption that are relevant for policy. We report optimal taxes obtained for an utilitarian welfare function, which implies $g_x = g = 1$.

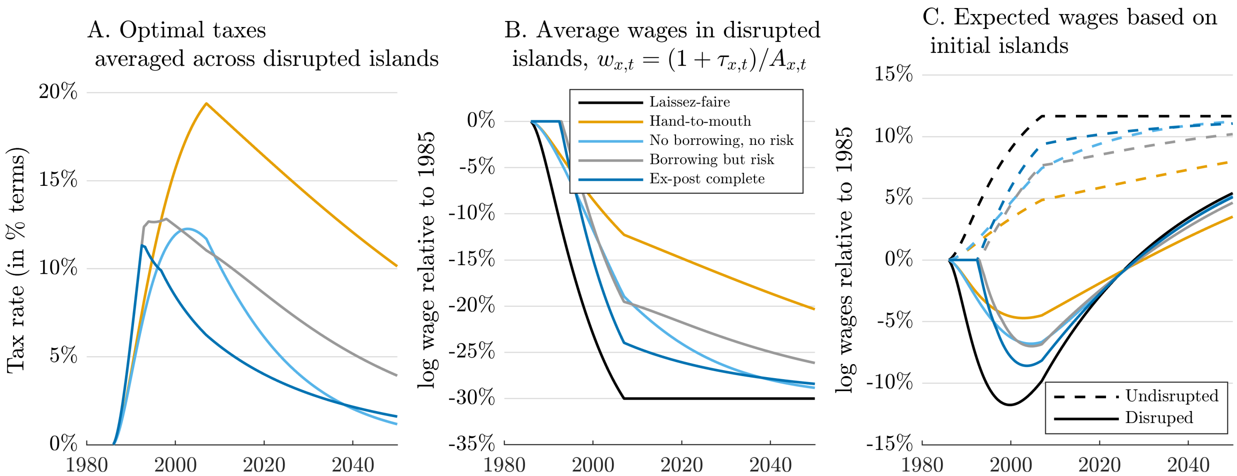

FIGURE 3: OPTIMAL TAXES AND PATH FOR WAGES ASSOCIATED WITH THE AUTOMATION OF ROUTINE JOBS. Panel A reports optimal taxes obtained under the four scenarios introduced in Section 1. Panel B reports the average wage in disrupted islands relative to its 1985 level. This panel also includes the laissez-faire path where automation goes untaxed for comparison. Panel C reports the expected wage of workers initially employed in disrupted islands (solid lines) and non-disrupted islands (dashed lines) relative to their 1985 levels.

Panel A in Figure 3 plots optimal tax paths for these scenarios. Optimal policy calls for a 10–18% tax on automation technologies which is then phased out over time. In the most conservative scenario with ex-post complete markets, optimal policy calls for a short-run increase in taxes on automation technologies of 11%, phased out to 4% by 2020.

Panel B plots the average wage among disrupted occupations and compares it to a laissez-faire scenario with no taxes. Optimal policy induces a more gradual wage decline in affected islands. By 2020, wages in disrupted islands are 4–15% higher than in laissez-faire.

Panel C plots expected wage paths for workers who were initially in disrupted islands. Relative to the previous panel, this one accounts for the role of reallocation. The solid black line shows a large income decline of 12% from 1985 to 2000, which aligns with the high estimated incidence in Cortes (2016). Optimal policy induces a more shallow and less persistent income drop of 7% for households in disrupted islands over 1985–2020. This panel also plots income for households in non-disrupted island. With no taxes on automation technologies, wages for unaffected workers grow gradually by 11%. Optimal policy makes wage growth more gradual for unaffected occupations.

The comparisons across scenarios illustrates the different margins through which taxing automation can generate distributional gains. The yellow line provides the optimal tax for hand-to-mouth households. The light blue line plots optimal taxes when households share transition risk but cannot save or borrow outside their initial island. This comparison shows that the presence of transition risk calls for higher and more persistent taxes on automation technologies, which redistribute towards unlucky workers stuck in disrupted islands for long periods.[20]

Going from the light blue line to the dark blue shuts down the role of consumption smoothing. When households cannot smooth their consumption, it is optimal to have taxes on automation technologies that trace their income path (from Panel C). When households can save and borrow, it is optimal to have higher taxes on automation technologies early on instead and phase them out more quickly. Early taxes are less distortive since the fiscal externality scales with expenditure in the new technology, and initial expenditure on automation is low. They also allow households to build assets during the early phase of the transition to prepare for the large income drop experienced in 2000. In our calibration, optimal policy fully shields workers from automation technologies for 6 years to allow them to build their savings. This is equivalent to announcing the arrival of the technology and committing to deploy it gradually 6 years from now.

Figure 4 turns to welfare implications. Panel A reports the change in welfare in consumption-equivalent terms for households initially located in unaffected islands. Panel B reports the average change in welfare for households from disrupted islands. With no taxes, the automation of routine jobs leads to a welfare gain of 7.5% for workers who are not adversely affected and a 6–8% welfare loss for workers disrupted by this technological change. In all scenarios, optimal policy leads to a sizable improvement in welfare for disrupted workers of 4–7 pp; while the cost for non-disrupted workers is small (1–2 pp).

We now consider the role of endogenous reallocation effort. We focus on scenarios i, ii, and iv, for which we have tractable formulas. Figure 5 plots optimal taxes for these

[20]Building on Dávila and Schaab (2022) and the decomposition in footnote 11, Appendix E.4 reports a decomposition of optimal taxes into components motivated by providing transition insurance, improving consumption smoothing, and pure redistribution.

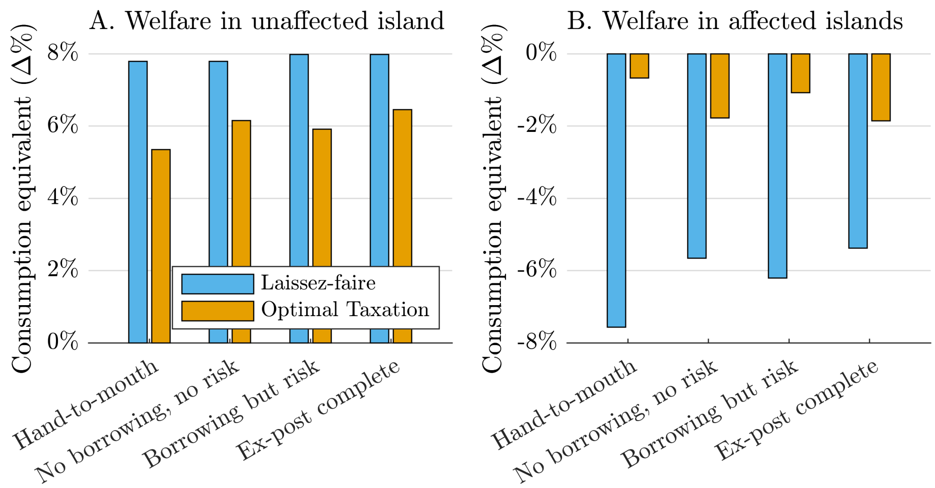

FIGURE 4: WELFARE CHANGES IN RESPONSE TO THE DECLINE IN ROUTINE JOBS. Panel A reports consumption-equivalent welfare changes for workers initially employed in non-disrupted occupations under laissez-faire and under optimal policy. Panel B reports average welfare changes for workers initially employed in disrupted occupations.

scenarios. We consider a simple specification of the effort elasticities with $\varepsilon_{x'',x} = 0$ for $x'' \neq x$ and $\varepsilon_{x,x} = \varepsilon > 0$ is calibrated so that moving to the (previous) optimal plan with exogenous effort results in an offsetting reduction of reallocation effort by 10%, 20%, and 50%. A greater offset implies that moving to the (previous) optimal policy becomes more costly. The figure provides the optimal tax in each case once we account for the induced reduction in reallocation effort using the formula in Proposition 3. Endogenous effort leads to a more rapid phase out of taxes, and in some cases, justifies a subsidy to the new technology by 2020 to provide backloaded incentives for reallocation.

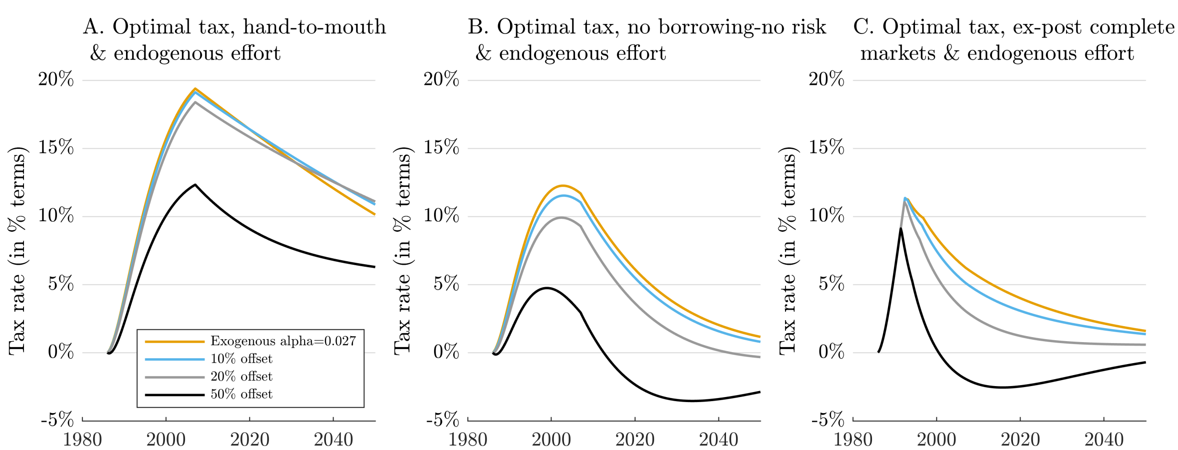

FIGURE 5: OPTIMAL TAX ON AUTOMATION WITH ENDOGENOUS REALLOCATION EFFORT. Panel A considers hand-to-mouth households. Panel B considers households that share reallocation risk but cannot borrow or save outside their islands. Panel C considers ex-post complete markets.

We conclude by exploring the role of insurance provided by income taxes and assistance programs. Figure 6 plots optimal taxes when the government can use these alternative tools. The panels consider the same scenarios for households used above but focus on the case with exogenous reallocation effort. The blue line plots the optimal tax when work effort is endogenous and responds to wages with an elasticity $\varepsilon_\ell = 0.3$, which matches estimates of the Hicksian elasticity of labor supply (in hours) in Chetty et al. (2011), but the government does not set any assistance program or income taxes. This differs slightly from previous estimates because of endogenous changes in hours worked.

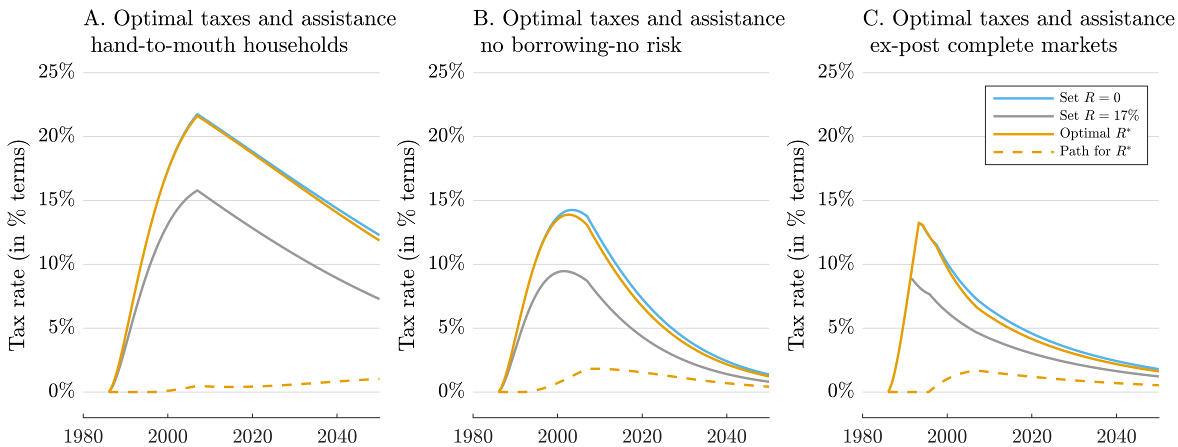

FIGURE 6: OPTIMAL TAX ON AUTOMATION TECHNOLOGIES AND ASSISTANCE PROGRAMS. Panel A considers hand-to-mouth households. Panel B considers households that share reallocation risk but cannot borrow or save outside their islands. Panel C considers ex-post complete markets.

The solid and dashed orange lines plot optimal paths for taxes on automation and the marginal tax rate on income $\mathcal{R}_t$ from Proposition 4. Optimal policy calls for a small increase in marginal income taxes of 1–2 pp, while optimal taxes on automation are unaffected by this additional policy lever. The reason why assistance programs are not used more intensively as part of the optimal policy bundle is that they are a blunt tool. The automation of routine jobs affects a small group of workers. Because assistance programs cannot be targeted to these households, they will be exploited by the large majority of workers who are not disrupted by technology, generating costly reductions in effort.

As explained in Section 2, our formula for the optimal level of insurance provided by taxes and assistance programs only accounts for the role of these tools in easing the transition for disrupted households. Other factors not modeled here, such as ex-ante inequality between workers or idiosyncratic income shocks might justify a more generous tax system and safety net, which provides support for disrupted workers. To account for the role of existing income taxes and assistance programs, we provide estimates (in gray) for the optimal tax on automation technologies obtained by fixing $\mathcal{R}_t = 17\%$, which matches estimates of the insurance provided by the US safety net and tax system.[21]

Our interpretation from the exercises in Figure 6 is that the insurance provided by the existing income tax system and safety net lead to smaller and more short-lived optimal taxes on automation. However, our results also show that short-run taxes on automation are a better way to ease the adjustment for disrupted workers than reforming the existing safety net or raising income taxes during a transition period due to their tagging value.

3.3 Slower technological progress of automation technologies

Our calibrated model also allows us to explore if the welfare gains from the automation of routine jobs would have been higher if the technology had advanced more slowly. To explore this, we compute the welfare gains from moving to a new path for technology governed by the S-curve in equation (11) but with advances taking place at a 50% slower rate over time. This implies that the economy still converges to the same level, but the transition is slower, not because of taxes but due to technological reasons. We estimate that this shift in the path for technology would *reduce* welfare by 0.2% in consumption-equivalent terms even with no automation taxes in place.

4 Application II: The China Shock

4.1 Description, Empirical Evidence, and Calibration

Description: there are 21 islands. Islands 2–21 are in $\mathcal{D}$ and represent segments of 2-digit manufacturing industries $i(x)$ disrupted by import competition from China. As before, we assume that a fraction $s_{i(x)}$ of the products or varieties in industry $i(x)$ are exposed to Chinese competition and calibrate $s_{i(x)}$ to match the scope of the disruption brought by the China Shock in each industry. This implies that the disrupted island x associated with industry $i(x)$ accounts for a fraction $\nu_x = s_{i(x)} \cdot \Omega_{i(x)}$ of initial value added, where $\Omega_{i(x)}$ is the industry share of value added.[22] Table 2 lists all 2-digit manufacturing industries, their

[21] Available estimates suggest that marginal income taxes for US workers below the median are of 7% (see Guner et al., 2014). These are the workers that have been more exposed to technological and trade disruptions in recent years. On top of this, assistance programs, such as disability and unemployment insurance, replace an extra 6.5–10% of income losses, depending on the shock being analyzed. For example, the estimates in Tables 8 and 9 of Autor et al. (2013) and Tables A17 and A19 of Acemoglu and Restrepo (2020) show that, for every dollar of labor income loss due to trade disruptions or automation via industrial robots, government transfers increase by 10 cents.

[22] As before, one may consider a mass 1 of islands partitioned into industries. Island x belong to industry $i(x)$, and there is a mass $\Omega_{i(x)}$ of islands in each industry. Each island produces a differentiated variety. We

SIC codes, and their shares of value added in 1991. The first island represents segments of manufacturing industries that were not exposed to Chinese competition plus all non-manufacturing industries.

The empirical evidence in Autor et al. (2013) and Autor et al. (2014): These papers provide two key moments. Autor et al. (2013) measure Chinese import penetration by industry using data from Comtrade for 1991 to 2007. They document that the increase in Chinese imports within industries is highly correlated across advanced countries, which suggest that the China Shock is driven by improvements in Chinese exporters productivity.

Panel A in Figure 7 summarizes their data and plots the *change in normalized import shares* by manufacturing industries. This is computed as

$$\text{Change in normalized import share}_i = \frac{1}{\Omega_i} \cdot \left(m_{i,t}/y_t - m_{i,t_0}/y_{t_0} \right),$$

where $m_{i,t}$ denotes the value of Chinese imports in industry i. Normalizing by Ω_i makes these estimates comparable across industries. Normalizing imports by GDP at time t ensures that this measure does not capture a mechanical increase in imports driven by US economic growth. While Chinese imports started to increase for some industries since the early 90s, there is a more pronounced and pervasive increase in 2000–2007.[23]

Panel B reports the increase in normalized import shares over 1991–2007 for 2-digit manufacturing industries. On average, Chinese imports rose by 11 pp of manufacturing value added, though there is sizable heterogeneity, with industries such as apparel, leather products, and miscellaneous manufacturing products experiencing 40–110 pp increases in import competition. The increase in normalized import shares is informative of the share of segments disrupted $s_{i(x)}$ and the path of Chinese productivity $A_{x,t}$ in these islands.

Autor et al. (2014) provide evidence that workers initially employed in industries facing more import competition from China experienced lower income growth after 1991. They

can model the rise of Chinese imports as resulting from improvements in Chinese exporters productivity at a share $s_{i(x)}$ of the islands associated with industry $i(x)$. The remaining varieties are shielded from Chinese competition. For example, Holmes and Stevens (2014) show that import competition affected primarily large firms engaged in the production of standardized goods in exposed industries. This interpretation also aligns with the fact that there are sizable differences in Chinese import penetration across detailed products, even within the 2-digit manufacturing industries used in our analysis (see Autor et al., 2013).

[23] Part of the acceleration can be attributed to China's accession to the World Trade Organization (WTO) at the end of 2001, which resulted on the US granting Permanent Normal Trade Relations to China (see Pierce and Schott, 2016). This created an incentive for US firms to open plants abroad, which can be thought of as an increase in $A_{x,t}$ in our model.

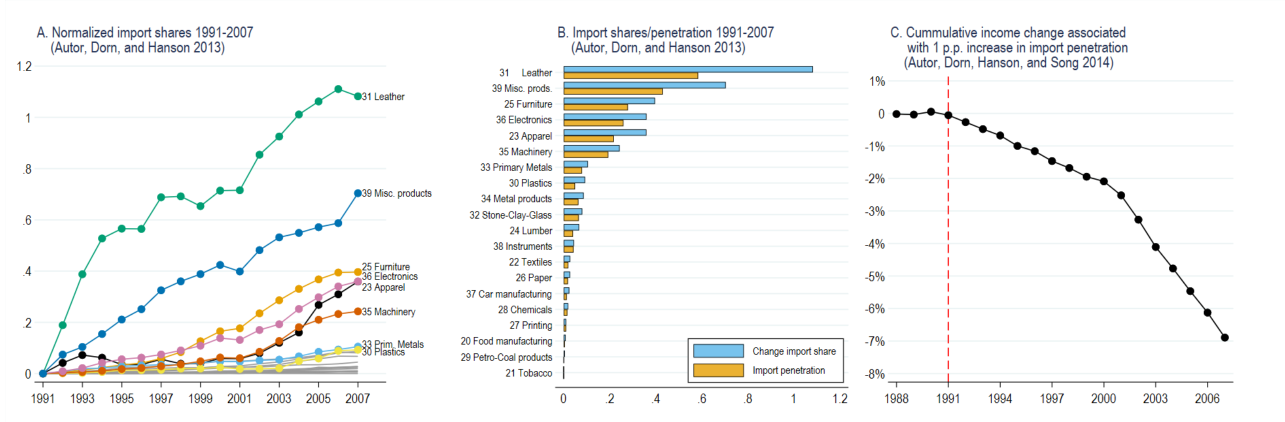

FIGURE 7: MEASURES OF IMPORT COMPETITION FROM CHINA AND THE EFFECT OF THE CHINA SHOCK ON WAGE GROWTH OF EXPOSED WORKERS. Panel A reports estimates of normalized import shares over 1991–2007 using the data from Autor et al. (2013). Panel B reports the increase over 1991–2007 in normalized import shares and compares this to Autor et al. measure of import penetration. Panel C reports estimates from Autor et al. (2014) of the effects of a 1 pp increase in import penetration on cumulative future income growth of exposed workers.

use data from the Social Security Administration to estimate the model:

$$\text{cumulative earnings}_{j,t} = \beta_{\mathrm{w},t} \cdot \Delta \mathrm{IP}\ 91\text{–}07_j + \theta \cdot \mathrm{X}_j + u_{j,t}, \tag{12}$$

which explains cumulative earnings for person j from 1991 to year t as a function of import penetration in their industry of employment in 1991 IP 91–07$_j$, individual characteristics X_j, and an error term.[24] Cumulative earnings are measured relative to a baseline average

$$\text{cumulative earnings}_{j,t} = \frac{\sum_{t'=1992}^{t} \text{labor income}_{j,t'}}{(1/4) \cdot \sum_{t'=1988}^{1991} \text{labor income}_{j,t'}}$$

and account for years of non-employment or zero labor income. Their import penetration measure is similar to the normalized share of imports introduced above, but differs in that it normalizes import growth by total US consumption of industry i output in 1991. For comparison, Panel B plots their import penetration measure. Both measures are highly correlated (correlation of 0.99), but normalized import shares are convenient for our model.

Panel C in Figure 7 plots the estimates of $\beta_{w,t}$ in equation (12), obtained from Figure III in Autor et al. (2014). The estimates show no pre-trends in labor income prior to 1991. Labor income then declines in relative terms, and by 2007, workers who were employed

[24] Autor et al. (2014) report IV estimates of equation (12). They instrument Chinese import penetration in the US using Chinese import penetration in the same industry in other high-income countries. This strategy isolates the variation in Chinese imports coming from changes in supply. Their first-stage results suggest that 80% of the variation in import penetration is due to rising exporters productivity in China.

in industries with a 7.5 pp higher import penetration (the average level in manufacturing industries) saw their cumulative income from 1992 to 2007 decline by 50% of their baseline annual income (7.5× 6.8) relative to workers not exposed to Chinese competition. This sizable effects suggests that the China Shock had considerable incidence on workers employed at disrupted industries, which points to limited opportunities for workers to reallocate.[25]

Calibration: Aggregate output y_t is given by the CES in (10). We set $\sigma = 2$, which matches the median elasticity of substitution across varieties at the level of aggregation in our analysis from Broda and Weinstein (2006).[26] As before, we normalize $\bar{w} = 1$.

Technology evolves according to the S-curve in (11). Building on the findings in Autor et al. (2013) and Autor et al. (2014), we assume that the China Shock starts in $t_0 = 1991$.[27] We also set a common value of $\pi_x = \pi$ across industries representing the cost-saving gains from trading with China by $t_f = 2007$. We calibrate π to match empirical estimates of price declines generated by Chinese import competition in US markets. Bai and Stumpner (2019) document that a 1% decrease in the share of goods produced domestically in an industry is associated with a 0.36–0.5% decline in consumer prices. In our model, the relationship between industry prices and domestic production shares satisfies

$$\Delta \ln P_i \approx \text{constant} + \pi \cdot \Delta \ln \text{share domestic production}_i.$$

Intuitively, substituting a variety produced domestically for a Chinese variety generates a cost-saving gain of π per variety substituted. We set $\pi = 50\%$, to match the upper end of the estimates in Table 1 of Bai and Stumpner (2019).

We calibrate $s_{i(x)}$ (or equivalently $\nu_x = \Omega_{i(x)} \cdot s_{i(x)}$), $A_{x,t}$, and $\alpha_{0,x} = \alpha_0$ to match the increase in the share of Chinese imports over 1991–2007 and the incidence of this shock on workers employed in disrupted industries. These parameters are jointly calibrated, but their values are tightly linked to these moments:

[25] Autor et al. (2014) also show that within a 2-digit industry, all of the incidence of the China Shock falls on workers that specialized in the detailed industries experiencing the biggest increase in import penetration. This points to limited opportunities for reallocation even within an industry.

[26] Disrupted islands correspond to specific products or varieties within each 2-digit SIC industry. The closest level of aggregation considered in Broda and Weinstein (2006) is the elasticity of substitution among products within a 3-digit SIC level. It is also important to note that $\sigma - 1$ is not equivalent to the *trade elasticity* that features in various trade models. In our model, the equivalent of a trade elasticity for island x is trade elasticity $= \sigma \cdot \frac{y_{x,t}}{k_{x,t}} - 1 > \sigma - 1$. This elasticity exceeds $\sigma - 1$, because our model features an extensive margin of trade (infinite elasticity) and an intensive margin (elasticity $\sigma - 1$). Our calibration produces an *average* trade elasticity for disrupted islands of 4.8.

[27] There was a small level of pre-existing trade with China before 1991. Appendix E explains how we extend our model to capture pre-existing trade and how we deal with it in our calibration.

- The choice of $\pi_x = 50\%$ pins down the level of A_{x,t_f}.
- Normalized import shares by 2007 are proportional to $s_{i(x)}$—the exposure of each industry to Chinese import competition.
- The time path for normalized imports pins down $A_{x,t}$ for $t = 1991, \ldots, 2007$.
- The incidence of the shock pins down α_0. A higher incidence implies a lower reallocation rate. We calibrate a common $\alpha_0 = 1.8\%$ per year to match the estimates of cumulative wage growth from Autor et al. (2014) by 2007 for affected workers.

This procedure yields estimates for $\hat{A}_{x,t}$ for all years since 1991. Using these values, we fit the S-curve in equation (11) via non-linear least squares. Panel A in Figure 8 reports yearly estimates of $\hat{A}_{x,t}$ and the fitted $S-$curves for the disrupted segments of manufacturing industries. To facilitate the interpretation, these estimates are scaled by $s_{i(x)}$, so that the figure is informative of the scope and the timing of the shock across industries. We report these for the top 8 industries with highest exposure to import competition. Panels B and C show that our model matches the empirical evidence in Autor et al. (2013) and Autor et al. (2014). Panel B reports the model-implied rise in imports since 1991 and compares it to the data. Panel C reports the incidence of the shock on workers associated with a 1 pp increase in Chinese import penetration. Our model matches the cumulative income decline for exposed workers by 2007. Though not targeted, our model matches the time path of income for exposed workers for all years.

Table 2 reports the remaining parameters used in our calibration, which are the same used in the application of our model to the decline in routine jobs.

The calibration for the decline in routine jobs and the China Shock vary in details but exploit similar information. In both cases, we calibrate the rate of reallocation to match empirical estimates of future wage growth for workers employed at disrupted industries or occupations. The high incidence of these shocks on these workers points to limited opportunities to reallocate. We then show that one can use trends in occupational wages or imports by industry to recover the time path for $A_{x,t}$.

4.2 Optimal policy for the China Shock

Using the formula in Proposition 2, we compute the optimal path for taxes on Chinese imports starting in 1991. We do so for the same four scenarios for households introduced before. These four scenarios determine the mapping between the observed decline in income

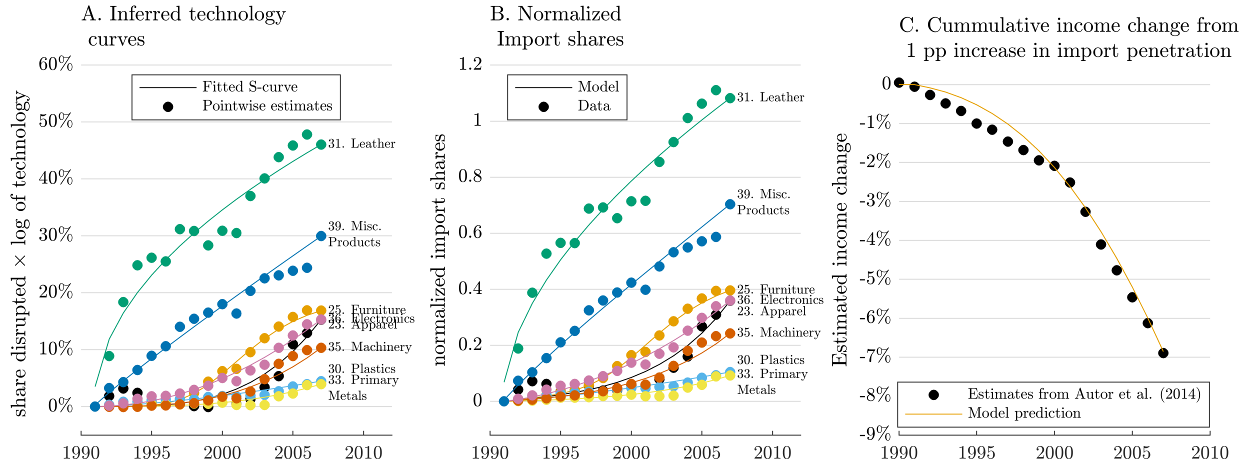

FIGURE 8: CALIBRATED PATHS FOR ADVANCES IN CHINESE EXPORTERS PRODUCTIVITY AND TARGETED MOMENTS. Panel A reports estimates for $A_{x,t}$ and their S-curve for the top 8 industries with the highest exposure to Chinese competition. Panel B reports the model-implied import shares since 1991 and compares this to the estimates in Autor et al. (2013). Panel C reports estimates of the implied incidence of these shocks on workers in industries with a 1 pp higher exposure to import penetration and compares it to the estimates in Autor et al. (2014).

documented by Autor et al. (2014) and matched by our model, and the unobserved marginal utility of consumption of disrupted households over time. As before, we report results for an utilitarian welfare function.

Panel A in Figure 3 plots optimal tax paths for these scenarios. The dark blue line provides the most conservative scenario, obtained when households can borrow and insure against the risk of transitioning late. Optimal policy calls for a short-run increase in taxes on Chinese imports of 12%, phased out over time and reaching a level of 4% by 2020.

Panel B plots the resulting wages in disrupted islands and compares it to their level in a laissez-faire world. Optimal policy induces a more gradual reduction in wages at affected islands. By 2020, wages in disrupted islands are 5–15% higher than in a world with no taxes on Chinese imports. When workers can save, optimal policy fully delays the China Shock by 5 years, which allows workers to build their savings and reallocate in advance.

Panel C plots expected wage paths for workers initially employed in disrupted islands. Relative to the previous figure, this one accounts for the role of reallocation. The solid black line shows a large income decline of 30% from 1985 to 2000, which aligns with the high estimated incidence in Autor et al. (2014). Optimal policy induces a more modest income drop for households in disrupted islands over 1985–2040. This panel also plots income for households in non-disrupted islands over time. With no taxes on Chinese imports, wages for unaffected workers grow gradually by 1% thanks to the gains from trade.

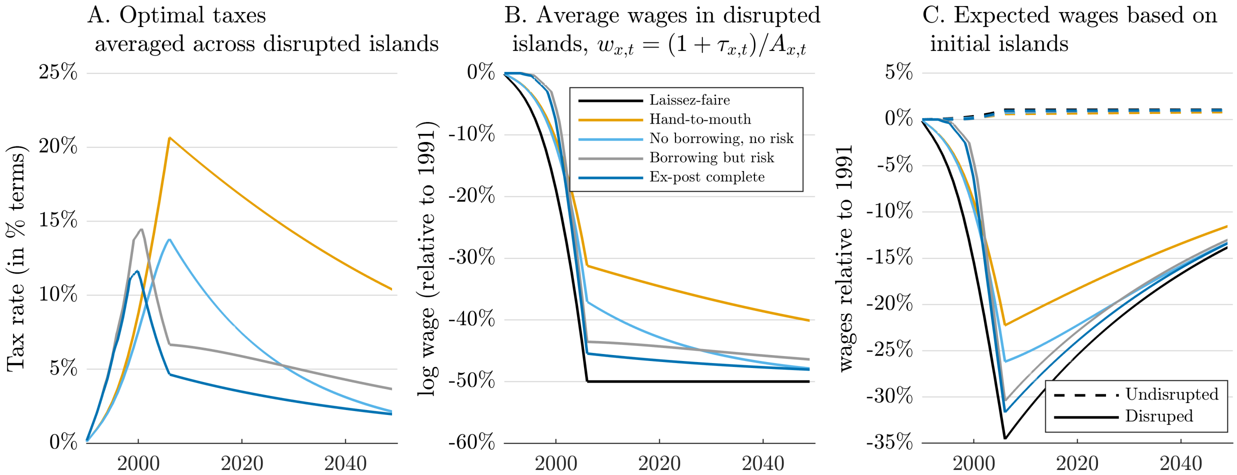

FIGURE 9: OPTIMAL TAXES AND PATH FOR WAGES ASSOCIATED WITH THE CHINA SHOCK. Panel A reports optimal taxes obtained under the four scenarios introduced in Section 1. Panel B reports the average wage in disrupted islands relative to its 1991 level. This panel also includes the laissez-faire path where Chinese imports go untaxed for comparison. Panel C reports the expected wage of workers initially employed in disrupted islands (solid lines) and non-disrupted islands (dashed lines) relative to their 1991 levels.

As in the case of automation, we find that the incentive to reduce transition risks calls for higher and more persistent taxes (comparing scenarios I and II). When households cannot save (I and II), optimal tariffs peak around 2002 when income losses are maximized, which helps them smooth consumption. In Scenario IV, when this incentive is removed, we see that optimal policy involves early taxes that fully shield workers for about 4 years to help them build assets to be consumed later in the transition.

Figure 10 turns to welfare. Panel A and B report welfare changes in consumption-equivalent terms for households initially employed in unaffected and disrupted islands, respectively. Without taxes on imports, the China Shock improves welfare by 0.6% for workers who are not exposed to international competition and reduces welfare by 15% for those who are (though these workers represent only 1.6% of the US workforce). The small gains from trade align with the trade literature and reflect the low aggregate levels of Chinese import penetration in the US. For example, Galle et al. (2022) estimate gains from trade of 0.3% from trade with China. Optimal policy leads to sizable welfare gains for disrupted workers of 3–6 pp at a small cost for non-disrupted workers (0.05–0.1 pp).

We now consider the role of endogenous reallocation effort. As before, we focus on scenarios for which we have tractable formulas (I, II, IV) and report estimates for optimal taxes for different assumed levels of offset (the percent reduction in effort implied by moving to the optimal policy that assumes exogenous effort). Figure 11 reports our estimates.

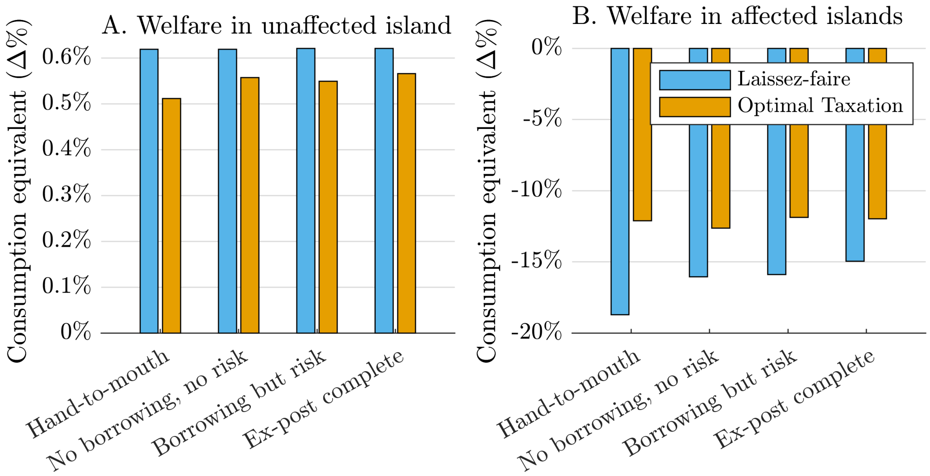

Figure 10: Welfare changes in response to the China Shock. Panel A reports consumption-equivalent welfare changes for workers initially employed in non-disrupted industries under laissez-faire and under optimal policy. Panel B reports average welfare changes for workers initially employed in disrupted industries.

Endogenous effort leads to a more rapid phase-out of taxes on trade, and in the last scenario to a small import subsidy by 2040 to provide back-loaded incentives for reallocation.

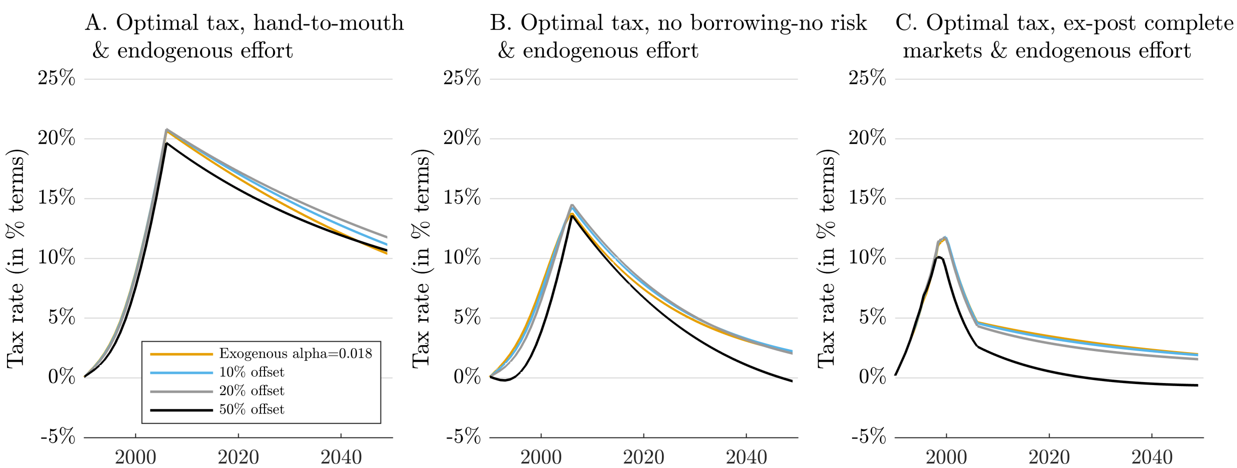

Figure 11: Optimal tax on Chinese imports when reallocation effort is endogenous. Panel A considers hand-to-mouth households. Panel B considers households that share reallocation risk but cannot borrow or save outside their islands. Panel C considers the case of ex-post complete markets.

Finally, we explore the role of income taxes and assistance programs in Figure 11. The panels consider the same household scenarios as above but focus on the case with exogenous reallocation effort. The blue line plots optimal taxes when work effort is endogenous and responds to wages with an elasticity $\varepsilon_\ell = 0.3$ but we set $\mathcal{R}_t = 0$. The solid and dashed orange lines plot the optimal tax and the optimal marginal tax rate on income from Proposition 4. Optimal policy calls for a small increase in marginal income taxes of 1 pp, while optimal taxes on Chinese imports are unaffected by this additional policy lever. As before, we provide estimates (in gray) for the optimal tax on trade obtained by fixing $\mathcal{R}_t = 17\%$, which captures the existence of a safety net and income taxes that justified by considerations outside of our model. These lead to a quicker phase-out of taxes on Chinese imports.

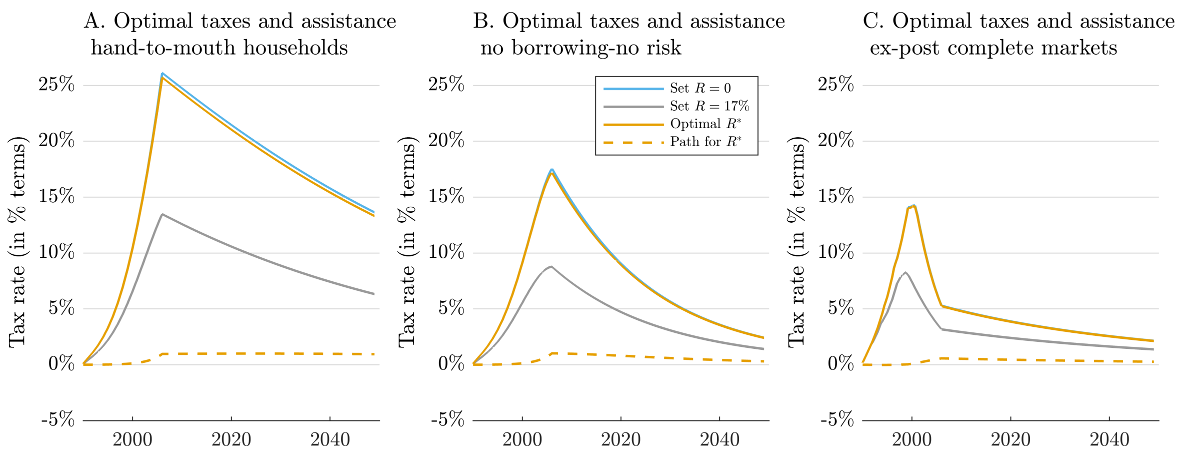

FIGURE 12: OPTIMAL TAX ON CHINESE IMPORTS AND ASSISTANCE PROGRAMS. Panel A considers hand-to-mouth households. Panel B considers households that share reallocation risk but cannot borrow or save. Panel C considers the case of ex-post complete markets.

Our conclusion from the estimates in Figure 12 is that the existing safety net and income tax system call for less pronounced taxes on trade. Still, it is optimal to set short-run taxes of the order of 7–13% on Chinese imports to ease the transition for disrupted households. Import taxes are better than reforming income taxes or expanding the safety net because these tools cannot be targeted to the small share of workers disrupted by trade.

4.3 Slower technological progress of Chinese exporters

As before, we compute the welfare gains from moving to a new path for Chinese exporters' productivity governed by the S-curve in equation (11) but with advances taking place at a 50% slower rate over time. We estimate that this shift in the path for productivity in China would *reduce* welfare by 0.03% in consumption-equivalent terms even with no optimal tariffs in place.

5 APPLICATION III: TRADE LIBERALIZATION IN COLOMBIA

5.1 Description, Empirical Evidence, and Calibration

Our final application explores Colombia's trade liberalization in 1990. Before the reform, Colombia had arresting levels of trade protection, with average nominal tariffs on manufacturing imports of 40%, and effective tariffs—which account for other barriers and surcharges—reaching levels of 75% (see Goldberg and Pavcnik, 2005; Eslava et al., 2013). Pre-reform levels of trade protection also featured vast dispersion both within and across industries, with apparel and shoes enjoying effective tariffs of close to 120%, and intermediate-goods imports being subsidized or enjoying no protection. With the government of President César Gaviria in 1990, Colombia embarked in an ambitious program of economic reforms that included liberalizing labor markets and opening up to trade.[28] The initial plan was for trade liberalization to be implemented gradually. But concerns about the credibility of the reform process and the potential for the emergence of political roadblocks led to a swift implementation (see Edwards and Steiner, 2008).[29] By 1992, Colombia reduced all nominal tariffs to common international levels of close to 13% and removed almost all additional trade barriers, leading to a new trade structure with uniform effective taxes of 25% across most manufacturing industries, with the exception of imported food products.

Figure 13 depicts the large and rapid decline in effective tariffs starting in 1990 for 3-digit manufacturing industries in Colombia. Average effective tariffs declined by 45 pp from 1990 to 1992. The middle panel shows that imports increased immediately, with their share in GDP rising from 10% in 1989 to 15% in 1993. The right panel summarizes the cumulative decline in effective tariffs over 1989–2002 and the increase in import shares over this period for manufacturing industries.

To map the theory to the data, we must deal with two aspects of Colombia's trade liberalization. First, tariffs were not lowered to zero—the optimal level in our model. The reform lowered tariffs to "internationally acceptable levels" of $\tau_{x,t_f} = 13\%$ for nominal tariffs and $\tau_{x,t_f} = 25\%$ for effective tariffs across most industries and products. To avoid confounding the gains from gradualism with issues related to the optimal long-run level of tariffs, we assume that there is another distortion in the economy that makes the post 1990 level of protection optimal and report series for net tariffs, defined as $1 + \tau^{net}_{x,t} = \frac{1+\tau_{x,t}}{1+\tau_{x,t_f}}$.[30]

Second, there were large trade volumes in some industries before the reform, especially

[28] These reforms were part of a broader regional movement away from decades of protectionism under the auspices of *import substitution programs*. Chile was at the forefront of the reform movement, and had a gradual liberalization process starting in 1975. Subsequent reformers, such as Argentina, Colombia, Costa Rica, and Nicaragua embraced more rapid reforms. For example, Nicaragua reduced nominal tariffs from 110% to 12% from 1990 to 1992. See Edwards (1994) for more on the reform movement in Latin America.

[29] The first concern was that a gradual reform would not do enough to convince Colombian firms to restructure their operations. The second concern was that a gradual reform could be stopped before achieving the desired level of liberalization if the political climate changed.

[30] Formally, we assume that imported goods in island x have an exogenous subsidy of $1+\tau_{x,t_f}$, where τ_{x,t_f} denotes the effective tax rate in 2002. In this case, the optimal tax is $1+\tau_{x,t} = (1+\tau^{net}_{x,t})\cdot(1+\tau_{x,t_f})$, where $\tau^{net}_{x,t}$ is the optimal tax characterized in Proposition 2 after redefining technology to $A^{net}_{x,t} = A_{x,t}/(1+\tau_{x,t_f})$.

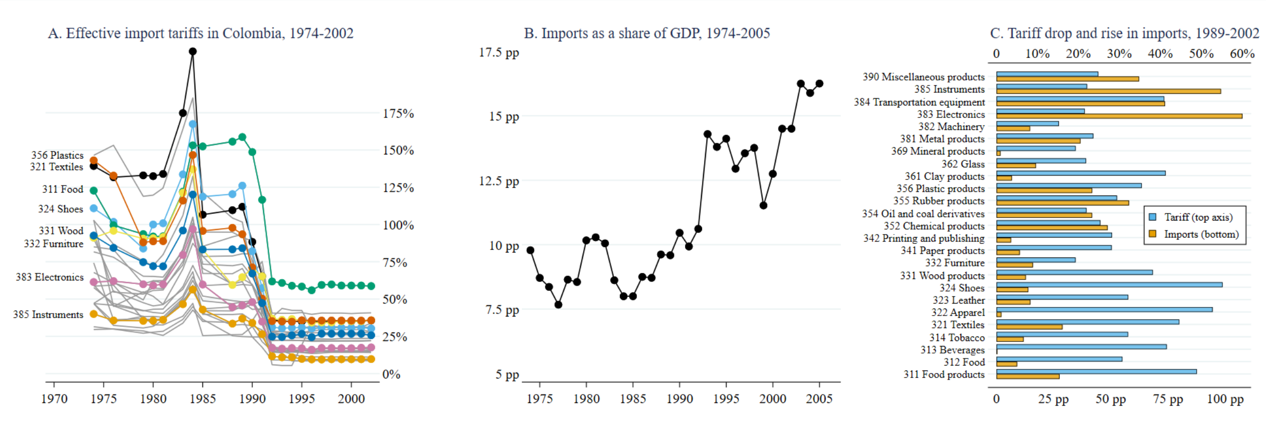

FIGURE 13: TRADE LIBERALIZATION IN COLOMBIA. Panel A reports the time series for effective tariffs for 3-digit manufacturing industries over 1974–2002. Panel B reports imports as a share of GDP. Panel C reports the percent decline in tariffs, defined as the percent change in $1 + \tau_{x,t}$ from 1989 to 2002 and the associated increase in normalized import shares for each industry. Data on tariffs comes from Colombia's *Ministerio del Comercio* and data on imports comes from the *Departamento Nacional de Planeación*, DNP.

for industries producing intermediate and capital goods, many of which had been liberalized prior to 1989. We account for this feature of Colombia's tariff and trade structure by assuming that some goods and services were already produced abroad by 1989 and these goods experienced no change in tariffs during Colombia's trade liberalization. Instead, Colombia's trade liberalization worked at the extensive margin: by making it profitable to import a widening range of final goods that used to be produced domestically.[31]

Calibration: We consider an economy with 26 islands. Islands 2–26 represent segments of manufacturing industry $i(x)$ that survived trade competition because of the protection granted by the high tariffs in 1989, but were out-competed following the trade liberalization.[32] These segments account for a share $s_{i(x)}$ of industry i. We assume that the initial level of protection in industry $i(x)$ is set at the minimum level required to ensure that imports did not disrupt island x. This allow us to recover the pre-tax productivity of imports as $A_x = (1 + \bar{\tau}_{x,t_0}) \cdot \bar{w}$ for all disrupted islands.

As in the China-Shock application, we set $\sigma = 2$ and calibrate the shares $s_{i(x)}$ to match the observed increase in normalized import shares by industry following the trade liberalization. Conditional on the decline in effective tariffs, industries with a larger share of exposed

[31]Most of the increase in trade after the liberalization took place at industries producing final goods. These industries had high levels of protection and low levels of trade by 1989.

[32]We exclude the intermediate-goods industries 351 "industrial chemicals", 353 "oil refined products", 371 "steel and iron" and 372 "primary metals" from our exercise because they were already liberalized prior to 1990 and experienced no increase in import penetration since then.

segments, $s_{i(x)}$, should see a more pronounced increase in normalized import shares.[33]

For this application, we do not have data on the incidence of trade liberalization on workers previously employed in exposed industries. However, Goldberg and Pavcnik (2005) provide a related piece of evidence. Exploiting variation in changes in protection over time across Colombian industries, they show that a 10 percentage point decrease in tariffs is associated with a decline in industry wage premiums of 1%. Their estimate of the decline in industry wage premia contains information on α. In the limit with $\alpha = \infty$, workers earn the same wage in all industries and trade does not affect industry wage premia. A value of $\alpha = 3\%$ matches Goldberg and Pavcnik (2005) estimates.

We set $t_0 = 1989$—the year before Gaviria's reforms—and feed the observed path for effective tariffs to obtain the path for wages, imports, and aggregates under the reform. Table 3 reports the remaining parameters used in our calibration, which are the same used in the application of our model to the decline in routine jobs and the China Shock.

5.2 Optimal trade liberalization

We use Proposition 2 to compute the optimal reform path for effective tariffs under a utilitarian welfare function, focusing on the four scenarios described in Section 1. Figure 14 reports our findings. Panel A depicts the observed tariffs following the 1990 trade liberalization and the optimal path implied by Proposition 2. The comparison suggests that Colombia's trade liberalization was too rapid. Optimal policy called for an immediate drop in net tariffs to 12–15% and a gradual tariff decline reaching a level of 5–10% by 2010.

Despite the fact that some industries enjoyed more protection, optimal policy calls for a proportional reduction in tariffs across industries, retaining the dispersion in tariffs during the transition. This is the opposite of what one would get on pure efficiency grounds, which call for more aggressive dismantling of tariffs in heavily protected sectors (i.e., Mussa, 1984).

Panel B shows the observed path for imports and the counterfactual path under the optimal policy. We see that both in the model and data, imports rose rapidly after the 1990 trade liberalization. Optimal policy induces a more gradual increase in imports.

Panel C reports optimal reform paths for different values of α under the conservative

[33]The assumption behind this procedure is that the observed increase in import penetration in the 90s was entirely due to the large reduction in effective tariffs. This is reasonable, especially when considering the vast drop in tariffs, and taking into account the fact that, as shown in Figure 13, the decline in tariffs was met by an immediate rise in imports. Our procedure matches the rise in imports for all industries from 1989 to 2002, except for 385 (scientific and medical instruments) and 383 (electronics). For these two industries, the restriction that $s_{i(x)} \leq 1$ binds and our model understates the increase in imports.

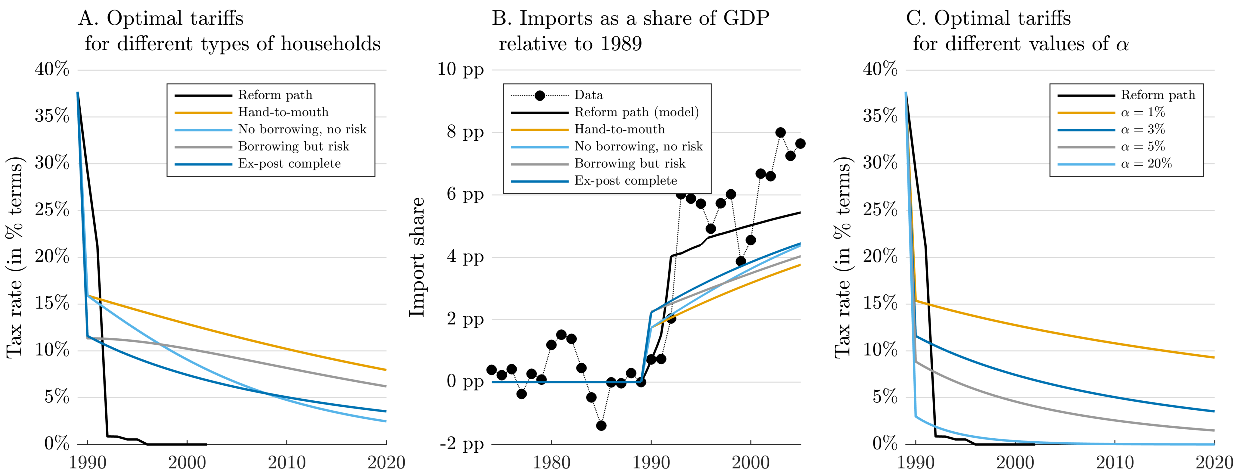

FIGURE 14: OPTIMAL TARIFFS AND OBSERVED TARIFFS FOR COLOMBIA'S TRADE LIBERALIZATION. Panel A reports optimal taxes obtained under the four scenarios introduced in Section 1. Panel B reports imports as a share of GDP relative to 1989 under the observed and the optimal policy. Panel C reports optimal taxes obtained for different values of the reallocation rate α when households face ex-post complete markets.

assumption that households can borrow and share transition risks. The swift reform conducted in Colombia (and in much of Latin America during that period) is justified for reallocation rates of 20% per year—an order of magnitude larger than our estimate.

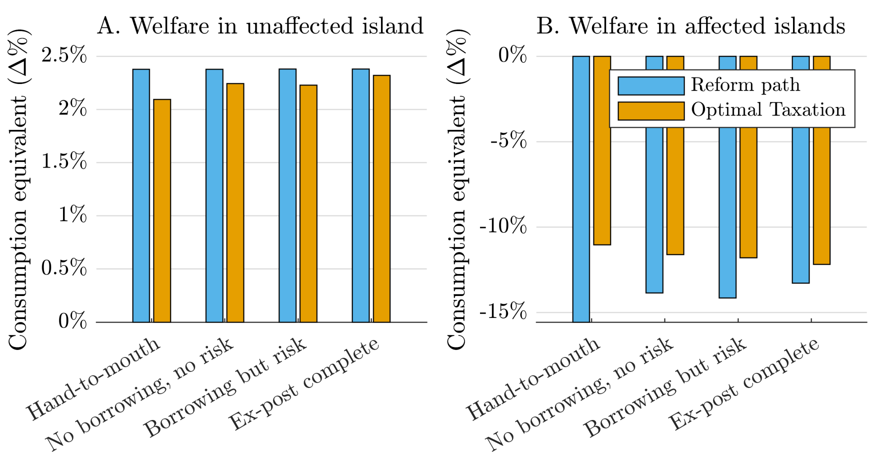

FIGURE 15: WELFARE CHANGES IN CONSUMPTION-EQUIVALENT TERMS, COLOMBIA'S TRADE LIBERALIZATION. Panel A reports consumption-equivalent welfare changes for workers initially employed in non-disrupted industries under the actual reform and under optimal policy. Panel B reports average welfare changes for workers initially employed in protected industries.

Figure 15 reports welfare gains and costs from trade liberalization under different scenarios and paths for tariffs. Colombia's trade liberalization brought welfare gains of 2.2% for unaffected workers and welfare losses of 12–14% for disrupted workers (3.4% of the workforce). A more gradual reform would mitigate losses by 1–4 pp and come at a small welfare cost for unaffected workers of 0.05-0.3 pp.

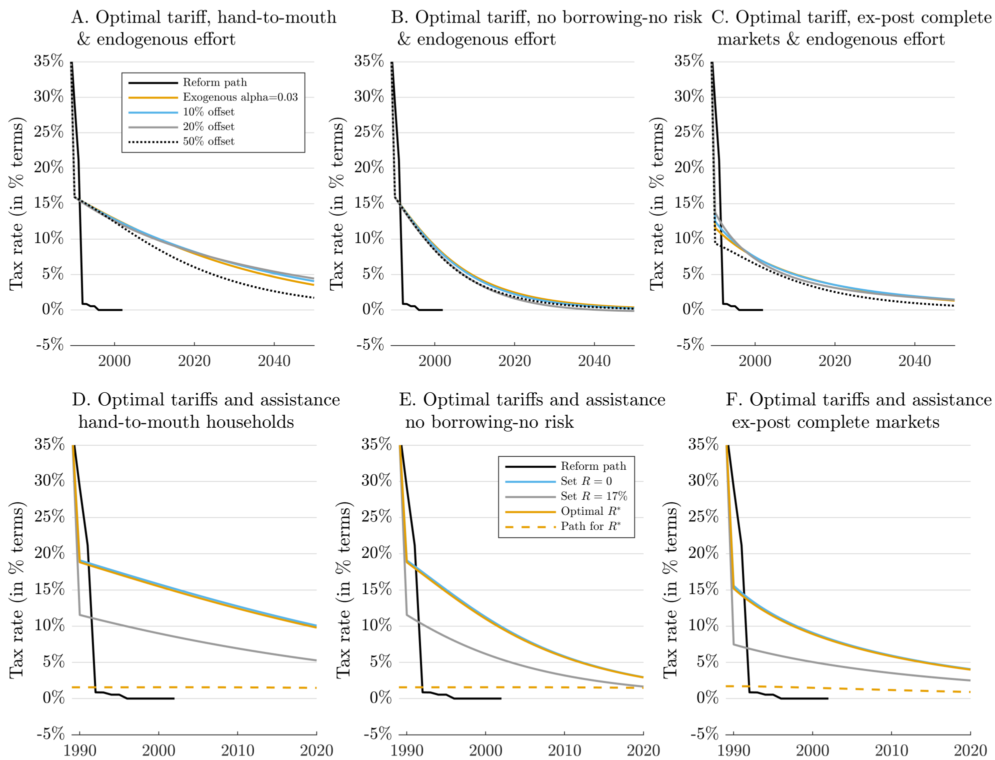

FIGURE 16: OPTIMAL TARIFF PATH FOR COLOMBIA'S TRADE LIBERALIZATION WHEN REALLOCATION EFFORT IS ENDOGENOUS (TOP PANELS) OR IT CAN BE COMPLEMENTED BY ASSISTANCE PROGRAMS. Panels A, D consider hand-to-mouth households. Panels B, E consider households that share reallocation risk but cannot borrow or save outside their islands. Panels C, F considers the case of ex-post complete markets.

The conclusion that optimal policy calls for a more gradual trade liberalization holds when we account for endogenous reallocation effort or the availability of income taxes and assistance programs. These scenarios are summarized in Figure 16. Endogenous reallocation effort has a small effect on the optimal policy path. As before, assistance programs are a blunt tool to deal with the adverse distributional effects of rapid reforms. A marginal income tax rate of 17% (as the one induced by the US system) calls for a slightly more rapid reform, but nowhere as rapid as in practice.

6 Concluding remarks

This paper explores how gradualism mediates the gains from trade, technological change, and reforms. We argue that gradual changes have less adverse distributional effects in the short run and justify the use of temporary taxes or gradual reforms. We provide formulas for the optimal path for taxes in response to technological change, trade, or during a reform.

We apply our theory to study the decline in routine jobs, the rise in Chinese import competition in the US, and Colombia's trade liberalization. Our formulas suggest that optimal policy calls for temporary taxes on the order of 10% when we calibrate our model to match the short-run income declines experienced by some workers due to the automation of routine jobs or rising import competition from China. They also suggest that the swift trade liberalization of 1990 in Colombia can only be justified when workers can reallocate at a rate of 20% per year—an order of magnitude larger than our estimates for the US.

These conclusions remain valid when we consider the possibility of dealing with disruptions by reforming the income tax system or increasing the marginal tax rate of assistance programs. We show that these programs are too blunt to deal with technological disruptions that only affect some segments of the workforce, and these disruptions do not justify reforming the existing tax system or safety net. Instead, taxing new technologies or trade in the short run offers a more direct way of easing the transition for disrupted workers.

The fact that short-run taxes on automation and trade are desirable does not mean that the US economy did not benefit from rapid advances in Chinese exporting productivity or the development of automation technologies. In both scenarios, our calibrated model points to welfare losses from moving to a world with more gradual advances.

Our formulas show that the desirability of taxes and the gains from technological gradualism depend on the extent to which disrupted households cut their consumption during a period of adjustment. Most of the existing literature focuses on estimating the impact of trade and technological disruptions on income. From a policy perspective, understanding how these disruptions affect consumption seems even more important, and a natural question for future research.

One interesting extension of our theory involves a case with congestion in reallocation; for example, because retraining large numbers of people in a single period might be subject to aggregate diminishing returns. Though we believe this offers an important rationale for gradualism, we did not explore the implications of congestion in our empirical applications.

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Table 1: Calibration for the decline in routine jobs over 1985–2007.

Panel I. Islands and technology

Occupation	*Data & moments, Cortes (2016)*			*Estimated objects*		
	Employment share 1985	Wage decline 85–07	Incidence	Size disrupted islands, $\nu_{o(x)}$	Share disrupted, $s_{o(x)}$	$S-$curve parameters $\{\pi_x, \kappa_x\}$
Clerical jobs	10%	-14.4%	86.1%	5.6%	56.3%	$\{0.3, 1.0615\}$
Production jobs	18.5%	-30.1%	71.1%	16.0%	86.6%	$\{0.3, 1.7104\}$
Sales jobs	5%	-11.9%	18%	2.46%	49.1%	$\{0.3, 11.325\}$
Handling jobs	4%	-40.8%	96.4%	3.98%	99.4%	$\{0.3, 1.3335\}$

Panel II. Elasticities, reallocation rate, and households

Elasticity of substitution	$\sigma = 0.85$	From literature on polarization (see Goos et al., 2014)
Reallocation rate per year	$\alpha_0 = 2.7\%$	Calibrated to match average incidence of 71%
Inverse elasticity of intertemporal substitution	$\gamma = 2$	Standard macro calibration.
Discount rate and interest rate	$r = \rho = 5\%$	Standard macro calibration.
Initial assets	0	Low median liquid assets in US Survey of Consumer Finances

Notes: The table summarizes the data used to calibrate the model to match the wage decline in routine jobs and the resulting parameters. The employment shares of routine occupations come from Acemoglu and Autor (2011); their wage decline from 1985–2007 from Cortes (2016); and the incidence of the wage decline also from Cortes (2016). The scale parameter of the S-curve h_x in equation (11) is not reported because it has no clear interpretation. Section 3 describes the calibration approach and data in detail.

Table 2: Calibration for the China Shock 1991–2007.

Panel I. Islands and technology

SIC code and industry	Data & moments, Autor et al. (2013), Autor et al. (2014)			Estimated objects		
	Value-added share 1991	Normalized import share 91–07 (pp)	Import Penetration 91–07 (pp)	Size disrupted islands, $\nu_{i(x)}$	Share disrupted, $s_{i(x)}$	S−curve parameters $\{\pi_x, \kappa_x\}$
20 Food & Kindred Products	1.77%	0.87	0.48	0.01%	0.74%	{0.5,3.9768}
21 Tobacco Products	0.29%	0.02	0.02	0.00%	0.01%	{0.5,14.217}
22 Textile Mill Products	0.32%	2.8	1.99	0.01%	2.38%	{0.5,4.3931}
23 Apparel	0.43%	35.97	21.76	0.13%	30.57%	{0.5,3.4398}
24 Lumber & Wood Products	0.36%	6.74	4.05	0.02%	5.72%	{0.5,2.3346}
25 Furniture & Fixtures	0.28%	39.69	27.88	0.09%	33.73%	{0.5,4.5851}
26 Paper & Allied Products	0.76%	2.75	1.83	0.02%	2.34%	{0.5,2.1375}
27 Printing & Publishing	1.31%	1.07	1.03	0.01%	0.91%	{0.5,1.6949}
28 Chemical & Allied Products	1.95%	1.94	1.58	0.03%	1.65%	{0.5,2.8831}
29 Petroleum & Coal Products	0.35%	0.54	0.13	0.00%	0.46%	{0.5,3.4844}
30 Rubber & Plastics Products	0.63%	10.53	7.95	0.06%	8.95%	{0.5,1.4856}
31 Leather & Leather Products	0.06%	108.38	58.44	0.05%	92.11%	{0.5,0.5088}
32 Stone, Clay, & Glass Products	0.43%	8.06	6.53	0.03%	6.85%	{0.5,1.0715}
33 Primary Metal Industries	0.66%	9.29	4.95	0.05%	7.90%	{0.5,3.0595}
34 Fabricated Metal Products	1.03%	8.69	6.37	0.07%	7.38%	{0.5,2.5377}
35 Industrial Machinery	1.67%	24.32	19.33	0.34%	20.67%	{0.5,2.9514}
36 Electronic Equipment	1.36%	36.04	25.96	0.41%	30.63%	{0.5,1.9978}
37 Transportation Equipment	1.88%	2.44	1.32	0.04%	2.07%	{0.5,2.5975}
38 Instruments & Related	1.04%	4.46	4.26	0.04%	3.79%	{0.5,0.9968}
39 Miscellaneous Manufacturing	0.26%	70.49	43.05	0.15%	59.91%	{0.5,0.9271}

Panel II. Elasticities, reallocation rate, and households

Elasticity of substitution	$\sigma = 2$	From Broda and Weinstein (2006)
Reallocation rate per year	$\alpha_0 = 1.8\%$	Calibrated to match incidence regressions in Autor et al. (2014)
Inverse elasticity of intertemporal substitution	$\gamma = 2$	Standard macro calibration.
Discount rate and interest rate	$r = \rho = 5\%$	Standard macro calibration.
Initial assets	0	Low median liquid assets in US Survey of Consumer Finances

Notes: The table summarizes the data used to calibrate the model to match the China Shock and the resulting parameters. Industry value added shares come from the NBER-CES, and are adjusted using aggregate data from the BEA-BLS integrated industry accounts to recognize the fact that the NBER-CES does not remove intermediate services from value added. Normalized import shares and import penetration measures come from Autor et al. (2013) and Autor et al. (2014). The scale parameter of the S-curve h_x in equation (11) is not reported because it has no clear interpretation. Section 4 describes the calibration approach and data in detail.

TABLE 3: Calibration for Colombia's trade liberalization.

PANEL I. ISLANDS AND TECHNOLOGY

SIC code and industry	*Data & moments, Eslava et al. (2013), DNP*				*Estimated objects*	
	Value-added share 1989	Effective tariff 1989	Percent decline in effective tariff	Change normalized import shares 89–02	Size disrupted islands, $\nu_{i(x)}$	Share disrupted, $s_{i(x)}$
311 Food products	1.59%	48.86%	158.71%	27.6 pp	0.40%	22.19%
312 Food	1.59%	30.72%	91.34%	9.1 pp	0.18%	10.07%
313 Beverages	2.42%	41.53%	95.03%	0.4 pp	0.01%	0.37%
314 Tobacco	0.43%	32.10%	80.42%	11.9 pp	0.06%	12.92%
321 Textiles	2.01%	44.61%	111.89%	28.9 pp	0.58%	24.92%
322 Apparel	0.58%	52.73%	116.54%	2.2 pp	0.01%	1.66%
323 Leather products	0.14%	32.13%	70.53%	14.9 pp	0.03%	16.11%
324 Shoes	0.23%	55.14%	126.10%	13.9 pp	0.03%	10.15%
331 Wood products	0.15%	38.15%	84.11%	12.9 pp	0.02%	12.44%
332 Furniture	0.10%	19.30%	64.67%	16.0 pp	0.03%	22.93%
341 Paper products	0.72%	28.10%	61.72%	10.2 pp	0.10%	12.03%
342 Printing and publishing	0.59%	28.19%	73.49%	6.4 pp	0.05%	7.53%
352 Chemical products	1.40%	25.37%	48.22%	48.6 pp	0.98%	60.58%
354 Oil and coal derivatives	0.11%	21.97%	43.35%	41.8 pp	0.07%	56.32%
355 Rubber	0.31%	29.41%	65.44%	58.0 pp	0.24%	66.29%
356 Plastic products	0.55%	35.43%	93.49%	41.9 pp	0.27%	42.49%
361 Clay products	0.15%	41.27%	93.38%	6.8 pp	0.01%	6.16%
362 Glass	0.25%	21.86%	45.24%	17.3 pp	0.07%	23.34%
369 Mineral products	0.89%	19.32%	48.11%	1.8 pp	0.03%	2.63%
381 Metal products	0.65%	23.63%	59.76%	36.7 pp	0.35%	47.61%
382 Machinery (exc. electric)	0.34%	15.25%	32.95%	14.7 pp	0.09%	23.48%
383 Electronics	0.73%	21.56%	45.87%	107.6 pp	0.84%	100.00%
384 Transportation equipment	1.09%	40.92%	101.65%	73.7 pp	0.84%	67.67%
385 Instruments	0.17%	22.11%	36.90%	98.2 pp	0.20%	100.00%
390 Miscellaneous products	0.22%	24.84%	63.68%	62.4 pp	0.20%	78.62%

PANEL II. ELASTICITIES, REALLOCATION RATE, AND HOUSEHOLDS

Elasticity of substitution	$\sigma = 2$	Imputed from China-Shock application and Broda and Weinstein (2006)
Reallocation rate per year	$\alpha_0 = 3\%$	Matches decline in industry premium in Goldberg and Pavcnik (2005)
Inverse elasticity of intertemporal substitution	$\gamma = 2$	Standard macro calibration.
Discount rate and interest rate	$r = \rho = 5\%$	Standard macro calibration.
Initial assets	0	Imputed from China-Shock application

Notes: The table summarizes the data used to calibrate the model to match Colombia's trade liberalization and the resulting parameters. Industry value added shares come from the *Departamento Nacional De Planeación*, DNP. Effective tariffs come from the *Ministerio de Comercio*, and are described in detail in Eslava et al. (2013). The change in import shares before and after the reform come from the *Departamento Nacional De Planeación*, DNP. We compute the 1989 level of imports as an average over 1985–1989 and the post reform level as an average over 1998–2002. We exclude industries 351 "industrial chemicals", 353 "oil refined products", 371 "steel and iron" and 372 "primary metals" from the analysis because they was already liberalized prior to 1990 and experienced no increase in import penetration since then. Section 5 describes the calibration approach and data in detail.

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Appendix to "Optimal Gradualism"

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A Proofs for Section 1

Proof of Proposition 1. Suppose that $k_{x,t} > 0$. We verify this is the case at the end of the proof. Firms in island x must be indifferent between producing with workers or producing using the new technology. This implies $w_{x,t} = (1 + \tau_{x,t})/A_{x,t}$.

We now show that wages across non-disrupted islands are equalized. Assumption 1 ensures that this holds at $t = 0$. Each period, a flow $\alpha \cdot (1 - \ell_t)$ of workers joins these islands. Directed search implies that these workers allocate in a way that preserves wage equality.

The expression pinning down the common wage w_t comes from the fact that we have normalized the price of the final good to 1, which implies that island wages lie along the iso-cost curve $1 = c^f\left(\{w_{x,t}\}_{x\in\mathcal{D}}, w_t\right)$. This equation implies that w_t is implicitly a function of the vector of after tax productivities $\{(1 + \tau_{x,t})/A_{x,t}\}$.

To derive an expression for output, we use Shepard's lemma, which implies $y_{x,t} = y_t \cdot c^f_x$. Adding across non-disrupted islands and using market clearing, yields

$$\ell_t = y_t \cdot \sum_{x\notin\mathcal{D}} c^f_x = y_t \cdot c^f_w\left(\{w_{x,t}\}_{x\in\mathcal{D}}, w_t\right) \quad \Rightarrow \quad y_t = \ell_t \cdot \frac{1}{c^f_w\left(\{w_{x,t}\}_{x\in\mathcal{D}}, w_t\right)}.$$

To derive an expression for $k_{x,t}$ for $x \in \mathcal{D}$, we use market clearing at disrupted islands:

$$k_{x,t} = y_t \cdot c^f_x\left(\{w_{x,t}\}_{x\in\mathcal{D}}, w_t\right) - \ell_{x,t}.$$

Substituting the expression for output yields the expression for $k_{x,t}$ in the proposition.

To conclude, we show that $k_{x,t} > 0$. Assumption 2 implies

$$\frac{c^f_x\left(\{w_{x,t}\}_{x\in\mathcal{D}}, w_t\right)}{c^f_w\left(\{w_{x,t}\}_{x\in\mathcal{D}}, w_t\right)} > \frac{c^f_x\left(\{\bar{w}\}_{x\in\mathcal{D}}, \bar{w}\right)}{c^f_w\left(\{\bar{w}\}_{x\in\mathcal{D}}, \bar{w}\right)} = \frac{\ell_{x,0}}{\ell_0} \geq \frac{\ell_{x,t}}{\ell_t}.$$

Rearranging this inequality yields $k_{x,t} > 0$. ■

B Proofs for Section 2

This section provides proofs for Lemma 1 and Propositions 2, 3, and 4.

Proof of Lemma 1. We first derive an expression for the change in households' income. We work with a slightly more general case that only assumes that output y is produced using a constant returns to scale function of different capital, k_m, and labor types, ℓ_j. To simplify the notation, we ignore time subscripts.

Only capital is taxed, so that tax revenue and the change in tax revenue are given by

$$T = \sum_m \tau_m \cdot \frac{k_m}{A_m} \quad \Rightarrow \quad dT = \sum_m \tau_m \cdot \frac{dk_m}{A_m} + \sum_m \frac{k_m}{A_m} \cdot d\tau_m.$$

Producers of the final good maximize profits but make zero profits in equilibrium due to constant returns to scale. The equilibrium allocation therefore satisfies

$$0 = \max_{y,\{k_m\},\{\ell_j\}} y - \sum_m \frac{k_m}{A_m} \cdot (1+\tau_m) - \sum_j \ell_j \cdot w_j.$$

The envelope theorem implies that the change in profits in response to the reform is

$$0 = -\sum_m \frac{k_m}{A_m} \cdot d\tau_m - \sum_j \ell_j \cdot dw_j.$$

Adding this equation and the equation for dT yields:

$$\sum_j \ell_j \cdot dw_j + dT = \sum_m \tau_m \cdot \frac{dk_m}{A_m}, \tag{A1}$$

which shows that total household income changes by $\sum_m \tau_m \cdot \frac{dk_m}{A_m}$—the aggregate efficiency term in equation (1).

We now turn to the change in welfare. The welfare function is given by

$$W_0 = \sum_{x\in\mathcal{D}} \ell_{x,0} \cdot \mathcal{W}\left(\max_{\alpha_x} \mathcal{U}_x(\{w_{x,t} + T_t, w_t + T_t\}_{t=0}^{\infty}; \alpha_x)\right) + \ell_0 \cdot \mathcal{W}\left(\mathcal{U}(\{w_t + T_t\}_{t=0}^{\infty})\right),$$

where $\mathcal{U}_x(\{w_{x,t} + T_t, w_t + T_t\}_{t=0}^{\infty}; \alpha_x)$ is the indirect utility function for disrupted households (net of reallocation effort costs, if endogenous) and $\mathcal{U}(\{w_t + T_t\}_{t=0}^{\infty}$ is the indirect utility function for households from non-disrupted islands. The envelope theorem implies that an infinitesimal change in $w_{x,t} + T_t$ changes $\mathcal{U}_x(\{w_{x,t} + T_t, w_t + T_t\}_{t=0}^{\infty}; \alpha_x)$ by

$$d\mathcal{U}_x(\{w_{x,t} + T_t, w_t + T_t\}_{t=0}^{\infty}; \alpha_x) = P_{x,t} \cdot e^{-\rho t} \cdot u'(c_{x,t}) \cdot (dw_{x,t} + dT_t),$$

where the right-hand side gives the expected marginal utility of consuming the extra income.

Likewise, an infinitesimal change in $w_t + T_t$ changes $\mathcal{U}_x(\{w_{x,t} + T_t, w_t + T_t\}_{t=0}^{\infty}; \alpha_x)$ by

$$\begin{aligned} d\mathcal{U}_x(\{w_{x,t} + T_t, w_t + T_t\}_{t=0}^{\infty}; \alpha_x) = & \int_0^t e^{-\rho t} \cdot u'(c_{x,t_n,t}) \cdot (dw_t + dT_t) \cdot \alpha_x \cdot e^{-\alpha_x t_n} \cdot dt_n \\ = & (1 - P_{x,t}) \cdot e^{-\rho t} \cdot \mathbb{E}[u'(c_{x,t_n,t}) | t_n \leq t] \cdot (dw_t + dT_t), \end{aligned}$$

where this expression integrates over all potential histories in which the household is at the non-disrupted island at time t and can consume this extra income.

Finally, an infinitesimal change in $w_t + T_t$ changes $\mathcal{U}(\{w_t + T_t\}_{t=0}^{\infty})$ by

$$d\mathcal{U}(\{w_t + T_t\}_{t=0}^{\infty}) = e^{-\rho t} \cdot u'(c_t) \cdot (dw_t + dT_t).$$

Combining these observations, we can express the change in welfare as

$$\begin{aligned} dW_0 = \int_0^{\infty} \Bigg[& \sum_{x \in \mathcal{D}} \ell_{x,0} \cdot \Big(P_{x,t} \cdot g_x \cdot e^{-\rho t} \cdot u'(c_{x,t}) \cdot (dw_{x,t} + dT_t) \\ & + (1 - P_{x,t}) \cdot g_x \cdot e^{-\rho t} \cdot \mathbb{E}[u'(c_{x,t_n,t}) | t_n \leq t] \cdot (dw_t + dT_t) \Big) \\ & + \ell_0 \cdot g \cdot e^{-\rho t} \cdot u'(c_t) \cdot (dw_t + dT_t) \Bigg] \cdot dt. \end{aligned}$$

Using the definition of $\chi_{x,t}$ in the text, we can rewrite this as

$$\text{(A2)} \qquad dW_0 = \int_0^{\infty} \left[\bar{\chi}_t \cdot \left(\sum_{x \in \mathcal{X}} \ell_{x,t} \cdot dw_{x,t} + dT_t \right) + \sum_{x \in \mathcal{X}} \ell_{x,t} \cdot (\chi_{x,t} - \bar{\chi}_t) \cdot dw_{x,t} \right] \cdot dt,$$

which separates the welfare change into a component due to increased household income and a component due to distributional considerations. Plugging the expression for the change in household income from equation (A1), yields the change in welfare in (1). ■

Remark: The lemma also applies when there is endogenous reallocation effort. This is because changes in reallocation effort have a second order effect on $U_{x,0}$ (households are optimizing with respect to α_x).

Proof of Proposition 2. The proof follows Costinot and Werning (2022). Consider a tax reform that changes $k_{x',t}$ by $dk_{x',t}$ but keeps the utilization of all other types of capital unchanged. At a social optimum, this variation cannot affect welfare. Using equation (1) to evaluate the change in welfare from this variation, we obtain

$$\tau_{x',t} \cdot \frac{dk_{x',t}}{A_{x',t}} = \sum_{x \in \mathcal{X}} \ell_{x,t} \cdot \left(\frac{\chi_{x,t}}{\bar{\chi}_t} - 1 \right) \cdot (-dw_{x,t}).$$

Dividing by $\frac{dk_{x',t}}{A_{x',t}}$ and rearranging terms yields the formula in (2). The derivation clarifies that $\frac{\partial \ln w_{x,t}}{\partial \ln k_{x',t}}$ refers to a partial derivative (i.e., the percent change in wages across islands resulting from a change in $k_{x',t}$ holding all other $k_{x,t}$ as well as the allocation of workers across islands constant). ■

Proof of Proposition 3. We use Lemma 1, which continues to be valid when reallocation effort is endogenous. Suppose we are at an optimum. Consider a reform that changes $k_{x',t}$ by $dk_{x',t}$ but leaves all other $k_{x,s}$ unchanged. This reform changes α_x by $d\alpha_x$ and, because the reform kept $k_{x,s}$ fixed for all other x, s, it also changes wages and tax revenue at all points in time and islands.

Define the direct effect of the reform as the effect on welfare through wages and tax revenue holding all α_x constant. For an outcome $h_{x,s}$, denote by $d_k h_{x,s}$ the direct effect of the reform—i.e., the change induced by $k_{x',t}$—and by $d_\alpha h_{x,s}$ the indirect effect—i.e., due to changes in α_x.

Using this notation and following the steps from Lemma 1, we can compute the welfare gains from a reform that changes $k_{x',t}$ by $dk_{x',t}$ and leaves all other $k_{x,s}$ unchanged as

$$dW_0 = \underbrace{\sum_{x \in \mathcal{X}} \ell_{x,t} \cdot \chi_{x,t} \cdot (d_k w_{x,t} + d_k T_t) \cdot dt}_{\text{direct effects of reform}} + \underbrace{\sum_{x'' \in \mathcal{D}} \ell_{x'',0} \cdot \mu_{x''} \cdot d\alpha_{x''}}_{\text{indirect effects via } \alpha_{x''}}, \tag{A3}$$

where μ_x denotes the per-worker welfare gains from increasing the reallocation rate out of island x. Note that there is a dt multiplying the aggregate efficiency and distributional considerations terms, since these only accrue in an instant of time. This implies that both the direct and indirect effects are "of the order of" dt.

To compute the indirect effects, we turn to the determination of $d\alpha_x$. The first-order condition for α_x is $\kappa_x'(\alpha_x) = \mathcal{U}_{x,\alpha}$. Totally differentiating this equation we get

(A4)
$$\kappa_x''(\alpha_x)\cdot d\alpha_x = \sum_{x''\in\mathcal{D}} \frac{\partial \mathcal{U}_{x,\alpha}}{\partial \alpha_{x''}}\cdot d\alpha_{x''} + \mathcal{U}_{x,\alpha,d,t}\cdot(d_k w_{x,t} + d_k T_t)\cdot dt + \mathcal{U}_{x,\alpha,n,t}\cdot(d_k w_t + d_k T_t)\cdot dt.$$

In this equation, $\frac{\partial \mathcal{U}_{x,\alpha}}{\partial \alpha_{x''}}$ gives the effect of changing $d\alpha_{x''}$ on $\mathcal{U}_{x,\alpha}$ via changes in wages and tax revenue over time (this object has to be computed holding $k_{x,s}$ constant for all x,s). The equation shows that the direct effects of the reform alter $\mathcal{U}_{x,\alpha}$, but these effects are "of the order of" dt, since the direct effect only changes wages and tax revenue at a point in time t.

Stacking equation (A4) for all $x\in\mathcal{D}$ we get the system of linear equations

$$\Psi\cdot \text{stack}(d\alpha_x) = \text{stack}(\mathcal{U}_{x,\alpha,d,t}\cdot(d_k w_{x,t} + d_k T_t)\cdot dt + \mathcal{U}_{x,\alpha,n,t}\cdot(d_k w_t + d_k T_t)\cdot dt),$$

where $\text{stack}(h_x)$ is the column vector of h_x across disrupted islands and the matrix Ψ has diagonal entries given by $\kappa''(\alpha_x) - \frac{\partial \mathcal{U}_{x,\alpha}}{\partial \alpha_x}$ and off-diagonal entries $-\frac{\partial \mathcal{U}_{x,\alpha}}{\partial \alpha_{x''}}$. We assume this matrix is invertible, which is equivalent to saying that the equilibrium exists and is unique.

The system can be solved as

$$\text{(A5)}\qquad d\alpha_x'' = \sum_{x\in\mathcal{D}} \varepsilon_{x'',x}\cdot(\mathcal{U}_{x,\alpha,d,t}\cdot(d_k w_{x,t} + d_k T_t) + \mathcal{U}_{x,\alpha,n,t}\cdot(d_k w_t + d_k T_t))\cdot dt,$$

where the $\varepsilon_{x'',x}$ are the entries of Ψ^{-1}. The $\varepsilon_{x'',x}$ tell us how changes in the direct incentives to reallocate in island x affect reallocation rates from island x''.

Plugging the formula for $d\alpha_{x''}$ in equation (A3), we obtain

$$\begin{aligned} dW_0 = &\sum_{x\in\mathcal{X}} \ell_{x,t}\cdot\chi_{x,t}\cdot(d_k w_{x,t} + d_k T_t)\cdot dt \\ &+ \sum_{x\in\mathcal{D}}\sum_{x''\in\mathcal{D}} \ell_{x'',0}\cdot\mu_{x''}\cdot\varepsilon_{x'',x}\cdot(\mathcal{U}_{x,\alpha,d,t}\cdot(d_k w_{x,t} + d_k T_t) + \mathcal{U}_{x,\alpha,n,t}\cdot(d_k w_t + d_k T_t))\cdot dt.\end{aligned}$$

Using equation (A1) for the change in households' income and the definition of the $\chi_{x,t}^{\text{end}}$ in the proposition, we can rewrite the change in welfare from this reform as

$$dW_0 = \bar{\chi}_t^{\text{end}}\cdot\tau_{x',t}\cdot\frac{dk_{x',t}}{A_{x',t}}\cdot dt + \sum_{x\in\mathcal{X}} \ell_{x,t}\cdot(\chi_{x,t}^{\text{end}} - \bar{\chi}_t^{\text{end}})\cdot d_k w_{x,t}\cdot dt.$$

At an optimum, we must have that this variation cannot increase welfare. Following the same steps as in the proof of Proposition 2, we get

$$\tau_{x',t} = \sum_{x\in\mathcal{X}} \frac{\ell_{x,t}\cdot w_{x,t}}{m_{x',t}}\cdot\left(\frac{\chi^{\text{end}}_{x,t}}{\bar{\chi}^{\text{end}}_t}-1\right)\cdot\left(-\frac{d_k \ln w_{x,t}}{d_k \ln k_{x',t}}\right)+\frac{\ell_t\cdot w_t}{m_{x',t}}\cdot\left(\frac{\chi^{\text{end}}_{t}}{\bar{\chi}^{\text{end}}_t}-1\right)\cdot\left(-\frac{d_k \ln w_{t}}{d_k \ln k_{x',t}}\right)$$

This derivation also clarifies that $\frac{d_k \ln w_{x,t}}{d_k \ln k_{x',t}}$ and $\frac{d_k \ln w_{t}}{d_k \ln k_{x',t}}$ are partial derivatives: they correspond to the change in wages given a change in capital holding reallocation rates constant.

To complete the proof of the proposition, we compute μ_x. Lemma 1 implies that the welfare gains from a change in reallocation rates holding all $k_{x',s}$ constant are

$$\mu_x\cdot\ell_{x,0}\cdot d\alpha_x = \int_0^\infty\left[\sum_{x''\in\mathcal{X}}\ell_{x'',s}\cdot(\chi_{x'',s}-\bar{\chi}_s)\cdot d_\alpha w_{x'',s}\right]\cdot ds.$$

The formula in (4) follows from the fact that

$$d_\alpha w_{x'',s} = \frac{\partial w_{x'',s}}{\partial \ell_{x,s}}\cdot(-s\cdot e^{-\alpha_x s})\cdot\ell_{x,0}\cdot d\alpha_x.$$

Note that these are partial derivatives since we are interested on the effect of $\alpha_{x'}$ on wages and tax revenues holding all other α_x and $k_{x,s}$ constant. ■

B.1 Deriving Formulas for $\mathcal{U}_{x,\alpha}$, $\mathcal{U}_{x,\alpha,d,t}$, $\mathcal{U}_{x,\alpha,n,t}$.

Hand-to-mouth: In this case, we have

$$\mathcal{U}_x = \int_0^\infty e^{-\rho t}\cdot\left[P_{x,t}\cdot u(w_{x,t}+T_t)+(1-P_{x,t})\cdot u(w_t+T_t)\right]\cdot dt.$$

Differentiating this with respect to α, and then with respect to wages at time t, we obtain:

$$\begin{aligned}\mathcal{U}_{x,\alpha} &= \int_0^\infty e^{-\rho t}\cdot(t\cdot P_{x,t})\cdot\left[u(w_t+T_t)-u(w_{x,t}+T_t)\right]\cdot dt,\\ \mathcal{U}_{x,\alpha,d,t} &= -(t\cdot P_{x,t})\cdot\lambda_{x,d,t},\\ \mathcal{U}_{x,\alpha,n,t} &= (t\cdot P_{x,t})\cdot\lambda_{x,n,t}.\end{aligned}$$

No borrowing and no transition risk: Let

$$c_{x,t} = P_{x,t}\cdot w_{x,t}+(1-P_{x,t})\cdot w_t+T_t$$

In this case, we have

$$\mathcal{U}_x = \int_0^\infty e^{-\rho t} \cdot u\left(c_{x,t}\right) \cdot dt.$$

Differentiating this with respect to α, and then with respect to wages at time t, we obtain:

$$\begin{aligned}
\mathcal{U}_{x,\alpha} &= \int_0^\infty e^{-\rho t} \cdot \left(t \cdot P_{x,t}\right) \cdot \left(w_t - w_{x,t}\right) \cdot u'\left(c_{x,t}\right) dt, \\
\mathcal{U}_{x,\alpha,d,t} &= -\left(t \cdot P_{x,t}\right) \cdot \lambda_{x,d,t} + e^{-\rho t} \cdot \left(t \cdot P_{x,t}\right) \cdot \left(w_t - w_{x,t}\right) \cdot u''\left(c_{x,t}\right) \cdot P_{x,t}, \\
\mathcal{U}_{x,\alpha,n,t} &= \left(t \cdot P_{x,t}\right) \cdot \lambda_{x,n,t} + e^{-\rho t} \cdot \left(t \cdot P_{x,t}\right) \cdot \left(w_t - w_{x,t}\right) \cdot u''\left(c_{x,t}\right) \cdot \left(1 - P_{x,t}\right).
\end{aligned}$$

Borrowing with transition risk: In this case there are no simple analytical expressions for $\mathcal{U}_{x,\alpha}$, $\mathcal{U}_{x,\alpha,d,t}$, $\mathcal{U}_{x,\alpha,n,t}$, nor a simple way of computing these objects numerically. For this reason, we do not analyze this scenario with endogenous reallocation effort.

Ex-post complete markets: Assume that $u(c) = c^{1-\gamma}/(1-\gamma)$ and let

$$h_{x,0} = a_{x,0} + \int_0^\infty e^{-rt} \cdot \left[P_{x,t} \cdot w_{x,t} + \left(1 - P_{x,t}\right) \cdot w_t + T_t\right] \cdot dt$$

denote the effective wealth of households in disrupted islands at time 0. We can solve analytically for $\mathcal{U}_x$ as

$$\mathcal{U}_x = \left[r - \frac{1}{\gamma}(r - \rho)\right]^{-\gamma} \cdot h_{x,0}^{1-\gamma}/(1-\gamma).$$

Differentiating this with respect to α, and then with respect to wages at time t, we obtain:

$$\begin{aligned}
\mathcal{U}_{x,\alpha} &= \left[r - \frac{1}{\gamma}(r - \rho)\right]^{-\gamma} \cdot h_{x,0}^{-\gamma} \cdot \int_0^\infty e^{-rt} \cdot \left(t \cdot P_{x,t}\right) \cdot \left(w_t - w_{x,t}\right) \cdot dt, \\
\mathcal{U}_{x,\alpha,d,t} &= -\left[r - \frac{1}{\gamma}(r - \rho)\right]^{-\gamma} \cdot h_{x,0}^{-\gamma} \cdot e^{-rt} \cdot \left(t \cdot P_{x,t}\right) - \gamma \cdot \frac{P_{x,t}}{h_{x,0}} \cdot \mathcal{U}_{x,\alpha}, \\
\mathcal{U}_{x,\alpha,n,t} &= \left[r - \frac{1}{\gamma}(r - \rho)\right]^{-\gamma} \cdot h_{x,0}^{-\gamma} \cdot e^{-rt} \cdot \left(t \cdot P_{x,t}\right) - \gamma \cdot \frac{1 - P_{x,t}}{h_{x,0}} \cdot \mathcal{U}_{x,\alpha}.
\end{aligned}$$

B.2 Optimal Assistance Programs

We now study optimal policy design for a government that can tax technology and adjust income taxes or the generosity of the safety net. Households utility is now given by

$$U_{x,0} = \mathbb{E}\left[\int_0^\infty e^{-\rho t} \cdot u(c_{x,t} - \psi(n_{x,t})) \cdot dt\right] - \kappa(\alpha_x)$$

Moreover, households' income in island x at time t is

$$n_{x,t} \cdot (1 - \mathcal{R}_t) \cdot w_{x,t} + T_t.$$

Without loss of generality we normalize initial effort to 1 at all islands, so that work effort is given by

$$n_{x,t} = \left(\frac{(1 - \mathcal{R}_t) \cdot w_{x,t}}{\bar{w}}\right)^{\varepsilon_\ell}.$$

We also denote optimal work effort at undisrupted islands by

$$n_t = \left(\frac{(1 - \mathcal{R}_t) \cdot w_t}{\bar{w}}\right)^{\varepsilon_\ell}.$$

Finally, government tax revenue is given by

$$T_t = \sum_{x \in \mathcal{D}} \tau_{x,t} \cdot \frac{k_{x,t}}{A_{x,t}} + \sum_{x \in \mathcal{X}} \ell_{x,t} \cdot n_{x,t} \cdot \mathcal{R}_t \cdot w_{x,t}.$$

We first extend Lemma 1 to this environment.

Lemma A1 (Joint variations Lemma) *Consider a variation in taxes on technology and the marginal tax rate that induces a change in wages $dw_t, dw_{x,t}$, technology $dk_{x,t}$, tax revenue $d\tilde{T}_t$, and reallocation effort $d\alpha_x$. This variation changes welfare by*

$$dW_0^{joint} = \int_0^\infty \Bigg[\bar{\chi}_t \cdot \left(\sum_{x \in \mathcal{X}} \ell_{x,t} \cdot \mathcal{R}_t \cdot w_{x,t} \cdot dn_{x,t} + \sum_{x \in \mathcal{D}} \tau_{x,t} \cdot \frac{dk_{x,t}}{A_{x,t}}\right) + \sum_{x \in \mathcal{X}} \ell_{x,t} \cdot n_{x,t} \cdot (\chi_{x,t} - \bar{\chi}_t) \cdot ((1 - \mathcal{R}_t) \cdot dw_{x,t} - w_{x,t} \cdot d\mathcal{R}_t)\Bigg] \cdot dt, \tag{A6}$$

Proof. We first derive an expression for the change in households' income (net of work effort). We work with a slightly more general case that only assumes that output y is produced using a constant returns to scale function of different capital, k_m, and labor types, ℓ_j. To simplify the notation, we ignore time subscripts in this step.

The change in tax revenue is given by

$$dT = \sum_m \tau_m \cdot \frac{dk_m}{A_m} + \sum_m d\tau_m \cdot \frac{k_m}{A_m} + \sum_j \ell_j \cdot dn_j \cdot \mathcal{R} \cdot w_j + \sum_j \ell_j \cdot n_j \cdot d\mathcal{R} \cdot w_j + \sum_j \ell_j \cdot n_j \cdot \mathcal{R} \cdot dw_j$$

Producers of the final good maximize profits but make zero profits in equilibrium due to constant returns to scale. The equilibrium allocation therefore satisfies

$$0 = \max_{y,\{k_m\},\{\ell_j \cdot n_j\}} y - \sum_m \frac{k_m}{A_m} \cdot (1 + \tau_m) - \sum_j \ell_j \cdot n_j \cdot w_j.$$

The envelope theorem implies that the change in profits in response to the reform is

$$0 = -\sum_m \frac{k_m}{A_m} \cdot d\tau_m - \sum_j \ell_j \cdot n_j \cdot dw_j.$$

Adding this equation and the equation for dT yields:

$$\text{(A7)} \qquad \sum_j \ell_j \cdot n_j \cdot (1 - \mathcal{R}) \cdot dw_j - \sum_j \ell_j \cdot n_j \cdot w_j \cdot d\mathcal{R} + dT = \sum_m \tau_m \cdot \frac{dk_m}{A_m} + \sum_j \mathcal{R} \cdot \ell_j \cdot w_j \cdot dn_j,$$

which shows that total household income changes by $\sum_m \tau_m \cdot \frac{dk_m}{A_m} + \sum_j \mathcal{R} \cdot \ell_j \cdot w_j \cdot dn_j$—the aggregate efficiency term in equation (A6).

We now turn to the change in welfare. The envelope theorem implies that an infinitesimal change in after tax wages $(1 - \mathcal{R}_t) \cdot w_{x,t}$ and transfers T_t changes $U_{x,0}$ by

$$dU_{x,0} = P_{x,t} \cdot e^{-\rho t} \cdot u'(c_{x,t} - \psi(n_{x,t})) \cdot (n_{x,t} \cdot (1 - \mathcal{R}_t) \cdot dw_{x,t} - n_{x,t} \cdot w_{x,t} \cdot d\mathcal{R}_t + dT_t),$$

where the right-hand side gives the expected marginal utility of consuming the extra income. Note that changes in $n_{x,t}$ induced by the policy have a second-order effect on $U_{x,0}$ since households where supplying labor optimally. Likewise, an infinitesimal change in after tax wages $(1 - \mathcal{R}_t) \cdot w_t$ and transfers T_t changes $U_{x,0}$ by

$$\begin{aligned} dU_{x,0} &= \int_0^t e^{-\rho t} \cdot u'(c_{x,t_n,t} - \psi(n_t)) \cdot (n_t \cdot (1 - \mathcal{R}_t) \cdot dw_t - n_t \cdot w_t \cdot d\mathcal{R}_t + dT_t) \cdot \alpha_x \cdot e^{-\alpha_x t_n} \cdot dt_n \\ &= (1 - P_{x,t}) \cdot e^{-\rho t} \cdot \mathbb{E}[u'(c_{x,t_n,t} - \psi(n_t)) | t_n \leq t] \cdot (n_t \cdot (1 - \mathcal{R}_t) \cdot dw_t - n_t \cdot w_t \cdot d\mathcal{R}_t + dT_t), \end{aligned}$$

where this expression integrates over all potential histories in which the household is at the non-disrupted island at time t and can consume this extra income. Finally, an infinitesimal change in $w_t + T_t$ changes U_0—utility for households from undisrupted islands—by

$$dU_0 = e^{-\rho t} \cdot u'(c_t) \cdot (n_t \cdot (1 - \mathcal{R}_t) \cdot dw_t - n_t \cdot w_t \cdot d\mathcal{R}_t + dT_t).$$

Combining these observations, we can express the change in welfare as

$$dW_0 = \int_0^\infty \left[\sum_{x \in \mathcal{X}} \ell_{x,t} \cdot \chi_{x,t} \cdot (n_{x,t} \cdot (1 - \mathcal{R}_t) \cdot dw_{x,t} - n_{x,t} \cdot w_{x,t} \cdot d\mathcal{R}_t + dT_t) \right] \cdot dt,$$

where the per-capita social value of increasing income in island x at time t is now

$$\chi_{x,t} = \begin{cases} g_x \cdot e^{-\rho t} \cdot u'(c_{x,t} - \psi(n_{x,t})) & \text{if } x \in \mathcal{D} \\[2em] \dfrac{1}{\ell_t} \cdot \left(\displaystyle\sum_{x \in \mathcal{D}} \ell_{x,0} \cdot (1 - P_{x,t}) \cdot g_x \cdot e^{-\rho t} \cdot \mathbb{E}[u'(c_{x,t_n,t} - \psi(n_t)) | t_n \leq t] \right. \\ \qquad \left. + \ell_0 \cdot g \cdot e^{-\rho t} \cdot u'(c_t - \psi(n_t)) \right) & \text{otherwise,} \end{cases}$$

This can be rearranged as

$$dW_0 = \int_0^\infty \Bigg[\bar{\chi}_t \cdot \left(\sum_{x \in \mathcal{X}} (\ell_{x,t} \cdot n_{x,t} \cdot (1 - \mathcal{R}_t) \cdot dw_{x,t} - \ell_{x,t} \cdot n_{x,t} \cdot w_{x,t} \cdot d\mathcal{R}_t) + dT_t \right)$$
$$+ \sum_{x \in \mathcal{X}} \ell_{x,t} \cdot n_{x,t} \cdot (\chi_{x,t} - \bar{\chi}_t) \cdot ((1 - \mathcal{R}_t) \cdot dw_{x,t} - w_{x,t} \cdot d\mathcal{R}_t) \Bigg] \cdot dt,$$

which separates the welfare change into a component due to increased household income (first line) and a component due to distributional considerations (Second line). Plugging the expression for the change in household income from equation (A7), yields the change in welfare in (A6). ■

We now use Lemma A1 to prove Proposition 4.

Proof of Proposition 4. Consider a variation that changes $k_{x',t}$ by $dk_{x',t}$ but keeps the utilization of all other types of capital and the marginal tax rate of assistance programs at all periods unchanged. At a social optimum, this variation cannot affect welfare. Using Lemma A1 to evaluate the welfare change from this variation we obtain

$$0 = \bar{\chi}_t \cdot \left(\sum_{x \in \mathcal{X}} \ell_{x,t} \cdot \mathcal{R}_t \cdot w_{x,t} \cdot dn_{x,t} + \tau_{x',t} \cdot \frac{dk_{x',t}}{A_{x',t}} \right) + \sum_{x \in \mathcal{X}} \ell_{x,t} \cdot n_{x,t} \cdot (\chi_{x,t} - \bar{\chi}_t) \cdot (1 - \mathcal{R}_t) \cdot dw_{x,t}.$$

Exploiting the fact that $dn_{x,t} = \varepsilon_\ell \cdot n_{x,t} \cdot d\ln w_{x,t}$ and rearranging terms, we obtain

$$\tau_{x',t} \cdot \frac{dk_{x',t}}{A_{x',t}} + \varepsilon_\ell \cdot \mathcal{R}_t \cdot \sum_{x\in\mathcal{X}} \ell_{x,t} \cdot n_{x,t} \cdot w_{x,t} \cdot d\ln w_{x,t} = -\sum_{x\in\mathcal{X}} \ell_{x,t} \cdot n_{x,t} \cdot \left(\frac{\chi_{x,t}}{\bar{\chi}_t} - 1\right) \cdot (1 - \mathcal{R}_t) \cdot (-dw_{x,t}).$$

Dividing by $dk_{x',t}$ and solving for $\tau_{x',t}$ yields the optimal-tax formula in (5). This derivation shows that $\frac{\partial \ln w_{x,t}}{\partial \ln k_{x',t}}$ corresponds to the change in wages given a change in capital holding other forms of capital constant but incorporating the effects from changes in work effort.

We now turn to the formula for optimal marginal tax rates. Consider a variation that changes $d\mathcal{R}_t$ but keeps technology utilization constant at all times and islands. At a social optimum, this variation cannot affect welfare. Using Lemma A1 to evaluate the welfare change from this variation we obtain

$$0 = \bar{\chi}_t \cdot \sum_{x\in\mathcal{X}} \ell_{x,t} \cdot \mathcal{R}_t \cdot w_{x,t} \cdot dn_{x,t} + \sum_{x\in\mathcal{X}} \ell_{x,t} \cdot n_{x,t} \cdot (\chi_{x,t} - \bar{\chi}_t) \cdot ((1 - \mathcal{R}_t) \cdot dw_{x,t} - w_{x,t} \cdot d\mathcal{R}_t).$$

Exploiting the fact that $dn_{x,t} = -\varepsilon_\ell \cdot n_{x,t} \cdot \frac{d\mathcal{R}_t}{1-\mathcal{R}_t} + \varepsilon_\ell \cdot n_{x,t} \cdot d\ln w_{x,t}$ and rearranging terms yields

$$\begin{aligned}\bar{\chi}_t \cdot \varepsilon_\ell \cdot \frac{\mathcal{R}_t}{1 - \mathcal{R}_t} \cdot m_{\ell,t} \cdot d\mathcal{R}_t = &\sum_{x\in\mathcal{X}} \ell_{x,t} \cdot n_{x,t} \cdot (\chi_{x,t} - \bar{\chi}_t) \cdot ((1 - \mathcal{R}_t) \cdot dw_{x,t} - w_{x,t} \cdot d\mathcal{R}_t) \\ &+ \bar{\chi}_t \cdot \varepsilon_\ell \cdot \mathcal{R}_t \cdot \sum_{x\in\mathcal{X}} \ell_{x,t} \cdot n_{x,t} \cdot w_{x,t} \cdot d\ln w_{x,t}\end{aligned}$$

where $m_{\ell,t} = \sum_{x\in\mathcal{X}} \ell_{x,t} \cdot n_{x,t} \cdot w_{x,t}$. Dividing by $d\mathcal{R}_t$ and solving for $\frac{\mathcal{R}_t}{1-\mathcal{R}_t}$ yields the optimal-tax formula in (5).

When implementing the formula in equation (6), we compute the general equilibrium derivatives $\frac{\partial \ln w_t}{\partial \mathcal{R}_t}$ and $\frac{\partial \ln w_{x,t}}{\partial \mathcal{R}_t}$ as $(1/w_{x,t}) \cdot dw_{x,t}/d\mathcal{R}_t$ and $(1/w_t) \cdot dw_t/d\mathcal{R}_t$, where these objects are given by the solution to the system of equations

$$\begin{aligned} dw_{x,t} =& \varepsilon_\ell \cdot \sum_{x'\in\mathcal{D}} f_{y_x,y_{x'}} \cdot \ell_{x',t} \cdot n_{x',t} \left(\frac{dw_{x',t}}{w_{x',t}} - \frac{d\mathcal{R}_t}{1 - \mathcal{R}_t}\right) + \varepsilon_\ell \cdot f_{y_x,y} \cdot \ell_t \cdot n_t \cdot \left(\frac{dw_t}{w_t} - \frac{d\mathcal{R}_t}{1 - \mathcal{R}_t}\right) \\ dw_t =& \varepsilon_\ell \cdot \sum_{x'\in\mathcal{D}} f_{y,y_{x'}} \cdot \ell_{x',t} \cdot n_{x',t} \left(\frac{dw_{x',t}}{w_{x',t}} - \frac{d\mathcal{R}_t}{1 - \mathcal{R}_t}\right) + \varepsilon_\ell \cdot f_{y,y} \cdot \ell_t \cdot n_t \cdot \left(\frac{dw_t}{w_t} - \frac{d\mathcal{R}_t}{1 - \mathcal{R}_t}\right). \end{aligned}$$

■

C Proofs for Section 2.5

Proof of Proposition 5. The proof follows the steps used in Lemma 1. We first derive an expression for the change in households' income. We work with a slightly more general case that only assumes that output y is produced using a constant returns to scale function of different capital, k_m, and labor types, ℓ_j. To simplify the notation, we ignore time subscripts.

Only capital is taxed, so that tax revenue and the change in tax revenue are given by

$$T = \sum_m \tau_m \cdot \frac{k_m}{A_m} \quad \Rightarrow \quad dT = \sum_m \tau_m \cdot \frac{dk_m}{A_m} + \sum_m \frac{k_m}{A_m} \cdot d\tau_m - \sum_m \frac{k_m}{A_m} \cdot \tau_m \cdot d\ln A_m.$$

Producers of the final good maximize profits but make zero profits in equilibrium due to constant returns to scale. The equilibrium allocation therefore satisfies

$$0 = \max_{y,\{k_m\},\{\ell_j\}} y - \sum_m \frac{k_m}{A_m} \cdot (1+\tau_m) - \sum_j \ell_j \cdot w_j.$$

The envelope theorem implies that the change in profits in response to the reform is

$$0 = \sum_m \frac{k_m}{A_m} \cdot (1+\tau_m) \cdot d\ln A_m - \sum_m \frac{k_m}{A_m} \cdot d\tau_m - \sum_j \ell_j \cdot dw_j.$$

Adding this equation and the equation for dT yields:

$$\sum_j \ell_j \cdot dw_j + dT = \sum_m \tau_m \cdot \frac{dk_m}{A_m} + \sum_m \frac{k_m}{A_m} \cdot d\ln A_m, \tag{A8}$$

which shows that total household income changes by $\sum_m \tau_m \cdot \frac{dk_m}{A_m} + \sum_m \frac{k_m}{A_m} \cdot d\ln A_m$—the aggregate efficiency term in equation (7).

We now turn to the change in welfare. Following the same steps from Lemma 1, we can show that the change in welfare is given by (A2). Plugging the expression for the change in household income from equation (A8), yields the change in welfare in (7). ∎

D Theoretical Extensions

This section provides theoretical extensions of our baseline model.

D.1 Inequality Between and Within Islands

This section extends our results to an economy with ex-ante differences in labor productivity within and between islands. We also discuss a rationale for ignoring pre-existing income differences across islands when deciding how to compensate the losers of globalization and technological progress.

We consider a model where workers differ in absolute advantage. Households are endowed with $\xi > 0$ units of labor. We refer to ξ as the type of the household. The distribution of ξ in island x has cdf $\Phi_x(\xi)$, and the distribution of ξ in non-disrupted islands has cdf $\Phi(\xi)$. This definition implies $\int_\xi \xi \cdot d\Phi_x(\xi) = \ell_{x,0}$ and $\int_\xi \xi \cdot d\Phi(\xi) = \ell_0$.

We make the following assumptions:

- The utility function is of the form $u(c) = \frac{1}{1-\gamma} c^{1-\gamma}$ for some $\gamma > 0$.
- A household of type ξ receives labor income $\xi \cdot w_{x,t}$ in island x and a proportional lump-sum rebate of $\xi \cdot T_t$. It also faces a cost of reallocation $\xi^{1-\gamma} \cdot \kappa_x(\alpha)$ and is endowed with initial assets $\xi \cdot a_{0,x}$.
- The budget restrictions faced by households imply that a consumption plan $\xi \cdot c$ is feasible for a household of type ξ from island x if and only if $\xi' \cdot c$ is feasible for a household of type ξ' from island x.

These assumptions imply that all households in island x choose paths for consumption and savings that are proportional to each other. In particular, let $\{c_{x,t_n,t}, c_{x,t}, \alpha_x\}$ denote the optimal consumption plan for a household with $\xi = 1$ from island x, and let $\{c^\xi_{x,t_n,t}, c^\xi_{x,t}, \alpha^\xi_x\}$ denote the optimal consumption plan for a household of type ξ from island x. The assumptions imply $c^\xi_{x,t_n,t} = \xi \cdot c_{x,t_n,t}$, $c^\xi_{x,t} = \xi \cdot c_{x,t}$, and $\alpha^\xi_x = \alpha_x$. In addition, households' utility is $U^\xi_{x,0} = U_{x,0} \cdot \xi^{1-\gamma}$. The same applies to households from non-disrupted islands.

Consider a welfare function of the form

$$W_0 = \sum_{x \in \mathcal{D}} \int_\xi \mathcal{W}\left(((1-\gamma) \cdot U^\xi_{x,0})^{\frac{1}{1-\gamma}}; \xi\right) \cdot d\Phi_x(\xi) + \int_\xi \mathcal{W}\left(((1-\gamma) \cdot U^\xi_0)^{\frac{1}{1-\gamma}}; \xi\right) \cdot d\Phi(\xi).$$

This welfare function accounts for heterogeneity in ξ. We do not require the welfare function to be symmetric, and in particular, we let $\mathcal{W}$ depend on ξ to capture societal preferences for redistribution across households with different types. We also wrote the welfare function in terms of consumption equivalent terms $((1-\gamma) \cdot U^\xi_{x,0})^{\frac{1}{1-\gamma}}$ and $((1-\gamma) \cdot U^\xi_0)^{\frac{1}{1-\gamma}}$, but this is done for tractability only.

Proposition A1 *The results in Propositions 2, 3 and 4 apply to this general economy with Pareto weights re-defined as*

$$g_x = \int_\xi \mathcal{W}'\left(\xi \cdot ((1-\gamma)\cdot \mathcal{U}_{x,0})^{\frac{1}{1-\gamma}};\xi\right) \cdot ((1-\gamma)\cdot \mathcal{U}_{x,0})^{\frac{\gamma}{1-\gamma}} \cdot \frac{\xi \cdot d\Phi_x(\xi)}{\ell_{x,0}}$$
$$g = \int_\xi \mathcal{W}'\left(\xi \cdot ((1-\gamma)\cdot \mathcal{U}_{0})^{\frac{1}{1-\gamma}};\xi\right) \cdot ((1-\gamma)\cdot \mathcal{U}_{0})^{\frac{\gamma}{1-\gamma}} \cdot \frac{\xi \cdot d\Phi(\xi)}{\ell_{0}}.$$

Proof:. Using the fact that $U^\xi_{x,0} = U_{x,0}\cdot\xi^{1-\gamma}$ and $U^\xi_0 = U_0\cdot\xi^{1-\gamma}$, we can rewrite the welfare function as

$$W_0 = \sum_{x\in\mathcal{D}} \int_\xi \mathcal{W}\left(\xi\cdot((1-\gamma)\cdot U_{x,0})^{\frac{1}{1-\gamma}};\xi\right)\cdot d\Phi_x(\xi) + \int_\xi \mathcal{W}\left(\xi\cdot((1-\gamma)\cdot U_{0})^{\frac{1}{1-\gamma}};\xi\right)\cdot d\Phi(\xi).$$

Changes in welfare are then given by

$$dW_0 = \ell_{x,0}\cdot g_x \cdot dU_{x,0} + \ell_0 \cdot g \cdot dU_0,$$

which coincides with the change in welfare in the main text. As explained in the main text (see footnote 10) all the results in the paper hold for arbitrary Pareto weights, g_x, g, and so in particular, they also hold after redefining g_x and g. ∎

The proposition illustrates how ex-ante inequality affects optimal policy. Suppose that $\mathcal{W}'(c;n) = c^{-\eta}$ for $\eta \geq \gamma$, so that the welfare function is scale free, symmetric, and concave in individual utilities. Then:

$$g_x = \left(\int_\xi \xi^{-\eta}\cdot\frac{\xi\cdot d\Phi_x(\xi)}{\ell_{x,0}}\right)\cdot\left(((1-\gamma)\cdot\mathcal{U}_{x,0})^{\frac{1}{1-\gamma}}\right)^{-\eta}\cdot((1-\gamma)\cdot\mathcal{U}_{x,0})^{\frac{\gamma}{1-\gamma}}$$
$$g = \left(\int_\xi \xi^{-\eta}\cdot\frac{\xi\cdot d\Phi(\xi)}{\ell_{0}}\right)\cdot\left(((1-\gamma)\cdot\mathcal{U}_{0})^{\frac{1}{1-\gamma}}\right)^{-\eta}\cdot((1-\gamma)\cdot\mathcal{U}_{0})^{\frac{\gamma}{1-\gamma}}.$$

Ex-ante inequality across households only matters via the terms $\int_\xi \xi^{-\eta}\cdot\frac{\xi\cdot d\Phi_x(\xi)}{\ell_{x,0}}$ and $\int_\xi \xi^{-\eta}\cdot\frac{\xi\cdot d\Phi(\xi)}{\ell_0}$. These terms are larger for islands x with households that have fewer units of labor on average, introducing a motive for taxing $k_{x,t}$ more aggressively due to its tagging value. On the other hand, within island inequality does not affect optimal taxes conditional on these tagging terms. Optimal taxes are also zero in the long run, since distorting $k_{x,t}$ loses its tagging value as people reallocate away from island x.

The proposition also identifies conditions under which ex-ante inequalities do not interact with the problem of protecting losers. Suppose that $\mathcal{W}$ satisfies

$$\mathcal{W}'(\xi \cdot c; \xi) = \mathcal{W}'(c).$$

This captures a situation where the public considers it fair for households of type ξ to enjoy a higher consumption, proportional to their higher labor endowment. In this case

$$\begin{aligned} g_x =& \mathcal{W}'\left(((1-\gamma)\cdot \mathcal{U}_{x,0})^{\frac{1}{1-\gamma}}\right)\cdot((1-\gamma)\cdot \mathcal{U}_{x,0})^{\frac{\gamma}{1-\gamma}} \\ g =& \mathcal{W}'\left(((1-\gamma)\cdot \mathcal{U}_{0})^{\frac{1}{1-\gamma}}\right)\cdot((1-\gamma)\cdot \mathcal{U}_{0})^{\frac{\gamma}{1-\gamma}} \end{aligned}$$

and inequality of labor endowments between and within islands is irrelevant for the problem of compensating winners and losers. This offers a rationale for ignoring ex-ante inequalities across (and within) islands when selecting optimal taxes on technologies or trade motivated exclusively by compensating the losers. In particular, we can ignore ex-ante inequalities across islands if they are considered fair.

D.2 Retraining Subsidies and Active Labor Market Policies

As our second extension, we consider the problem of taxing technologies when the government has access to other tools that allow it to select the socially optimal level of reallocation. These tools might include retraining subsidies or active labor market policies.

PROPOSITION A2 *Suppose the planner has other policy tools that implement the optimal social level of reallocation. A necessary condition for an optimal tax sequence is that:*

$$\tau_{x',t} = \sum_{x\in\mathcal{X}} \frac{\ell_{x,t}\cdot w_{x,t}}{m_{x',t}} \cdot \left(\frac{\chi_{x,t}}{\bar{\chi}_t} - 1\right)\cdot\left(-\frac{\partial \ln w_{x,t}}{\partial \ln k_{x',t}}\right), \tag{A9}$$

where the multipliers on the right-hand side are now evaluated along an equilibrium with the socially optimal level of α_x.

Proof:. Optimal reallocation effort maximizes social welfare. The envelope theorem implies that the effect of any reform on welfare is equal to the direct effect holding α_x constant, which leads to the same optimality condition as in Proposition 2. ■

E Calibration and Details of Numerical Algorithms

E.1 Calibration Details, China Shock

Calibrating π: Industry prices are initially given by $P_i = 1$. Following the disruption, we get a price index

$$P_{i,t_f} = c_i(W_t, \exp(-\pi)),$$

for some cost function c_i with $c_i(1,1) = 1$. Assuming that π is small, we can log-linearize this equation around $(1,1)$ as

$$\ln P_{i,t_f} \approx \text{share domestic production}_{i,t_f} \cdot \ln W_t - \text{share Chinese production}_{i,t_f} \cdot \ln \pi.$$

This implies

$$\ln P_{i,t_f} =\approx \text{share domestic production}_{i,t_f} \cdot (\ln W_t + \ln \pi) - \ln \pi.$$

Let $s = \max\{\text{share Chinese production}_{i,t_f}\}$ and suppose that s is small, as is the case in the data. Then

$$\ln P_{i,t_f} =\approx \ln \text{share domestic production}_{i,t_f} \cdot \ln \pi - \ln \pi.$$

This shows that the regression in Bai and Stumpner (2019) across industries identifies $\ln \pi$.

Pre-existing trade: In the applications of our framework to the China Shock and Colombia's trade liberalization, we have to deal with the fact that there was some pre-existing trade.

For the China Shock, we handle pre-existing trade by assuming that there is a mass $\nu_{p(i)}$ of islands associated with industry i that were already replaced by Chinese imports and hosted no workers by 1991. We normalize the cost of Chinese imports in these islands to 1, which implies that the cost function associated with (10) becomes

$$c_u^f(\{w_x\}, w) = \left(\nu_p + \nu \cdot w^{1-\sigma} + \sum_{x \in \mathcal{D}} \nu_x \cdot w_x^{1-\sigma}\right)^{\frac{1}{1-\sigma}},$$

where $\nu_p = \sum_i \nu_{p(i)}$. The normalization $\bar{w} = 1$ in status quo then requires $\nu_p + \nu + \sum_{x \in \mathcal{D}} \nu_x = 1$. In our calibration, we set $\nu_p = 2.5\%$—the share of imports in GDP before the China Shock.

We assume that the China Shock is driven by advances in the productivity of Chinese imports at other islands, and not by cost reductions of established products. These assumptions imply that the status-quo level of imports in industry i is

$$\frac{m_{i,t_0}}{y_{i,t_0}} = \nu_{p(i)};$$

while imports in industry i at time t after the China Shock are given by

$$\frac{m_{i,t}}{y_{i,t}} = \nu_{p(i)} + \frac{1}{y_t \cdot A_{x,t}} \cdot \left(\ell_t \cdot \frac{c_x^f}{c_w^f} - \ell_{x,t} \right),$$

where x is defined as the island associated with industry i (i.e., the one for which $i(x) = i$). In this expression, the first term accounts for imports at islands with pre-existing trade and the second term accounting for imports in new islands. The change in normalized import shares at time t is then equal to

$$\text{(A10)} \quad \text{Change in normalized import share}_{i(x),t} = \frac{1}{\Omega_i} \cdot \frac{1}{y_t \cdot A_{x,t}} \cdot \left(\ell_t \cdot \frac{c_x^f}{c_w^f} - \ell_{x,t} \right) \text{ for } x \in \mathcal{D}.$$

Equation (A10) provides a system of equation across industries that we use to calibrate $\nu_{i(x)}$ and $s_{i(x)}$ in a first step to match the change in normalized imports by 2007 (recall that $A_{x,t_f} = \exp(\pi)$ at this point), and then to calibrate a path for $A_{x,t}$ in a second step, as described in the main text.

For Colombia's trade liberalization, we assume that a mass $\nu_{p(i)}$ of segments were already produced internationally and hosted no workers by 1989. In addition, we assume this segments were not protected by 1989, and experienced no subsequent decline in tariffs after the 1990 trade liberalization. Under these assumptions, we have that the status-quo level of imports in industry i is

$$\frac{m_{i,t_0}}{y_{i,t_0}} = \nu_{p(i)};$$

while imports in industry i at time t after the liberalization are

$$\frac{m_{i,t}}{y_{i,t}} = \nu_{p(i)} + \frac{1 + \tau_{x,t}}{y_t \cdot A_{x,t}} \cdot \left(\ell_t \cdot \frac{c_x^f}{c_w^f} - \ell_{x,t} \right),$$

where x is defined as the island associated with industry i (i.e., the one for which $i(x) = i$).

The change in normalized import shares at time t is then equal to

$$\text{(A11)} \quad \text{Change in normalized import share}_{i(x),t} = \frac{1}{\Omega_i} \cdot \frac{1+\tau_{x,t}}{y_t \cdot A_{x,t}} \cdot \left(\ell_t \cdot \frac{c_x^f}{c_w^f} - \ell_{x,t} \right) \text{ for } x \in \mathcal{D}.$$

Equation (A11) provides a system of equation across industries that we use to calibrate $\nu_{i(x)}$ and $s_{i(x)}$ to match the increase in normalized import shares between 1989 and 2002.

E.2 Implementing the Formulas in Equations (2) and (3):

This subsection describes the numerical procedure used to compute optimal taxes.

Exogenous reallocation effort. We compute optimal taxes with exogenous reallocation effort as follows:

1. Start with $\tau_{x,t}^{(0)} = 0$ (laissez-faire).
2. Compute equilibrium objects for $\tau_{x,t}^{(n)}$, identified with the superscript (n) below.
3. Use equation (2) to update optimal taxes as

$$\tau_{x',t}^{(n+1)} = \sum_{x \in \mathcal{D}} \frac{\ell_{x,t} \cdot w_{x,t}^{(n)}}{m_{x',t}^{(n)}} \cdot \left(\frac{\chi_{x,t}^{(n)}}{\bar{\chi}_t^{(n)}} - 1 \right) \cdot \left(-\frac{\partial \ln w_{x,t}^{(n)}}{\partial \ln k_{x',t}^{(n)}} \right) + \frac{\ell_t \cdot w_t^{(n)}}{m_{x',t}^{(n)}} \cdot \left(\frac{\chi_t^{(n)}}{\bar{\chi}_t^{(n)}} - 1 \right) \cdot \left(-\frac{\partial \ln w_t^{(n)}}{\partial \ln k_{x',t}^{(n)}} \right),$$

4. Repeat steps 2–3 until convergence.

Endogenous reallocation effort. We compute optimal taxes with exogenous reallocation effort as follows:

1. Start with $\tau_{x,t}^{(0)} = 0$ (laissez-faire) and the observed rate of reallocation $\alpha_x^{(0)}$.
2. Compute equilibrium objects for $\tau_{x,t}^{(n)}$ and $\alpha_x^{(0)}$, identified with the superscript (n) below.
3. Use equation (2) to update optimal taxes as

$$\tau_{x',t}^{(n+1)} = \sum_{x \in \mathcal{D}} \frac{\ell_{x,t} \cdot w_{x,t}^{(n)}}{m_{x',t}^{(n)}} \cdot \left(\frac{\chi_{x,t}^{\text{end},(n)}}{\bar{\chi}_t^{\text{end},(n)}} - 1 \right) \cdot \left(-\frac{\partial \ln w_{x,t}^{(n)}}{\partial \ln k_{x',t}^{(n)}} \right) + \frac{\ell_t \cdot w_t^{(n)}}{m_{x',t}^{(n)}} \cdot \left(\frac{\chi_t^{\text{end},(n)}}{\bar{\chi}_t^{\text{end},(n)}} - 1 \right) \cdot \left(-\frac{\partial \ln w_t^{(n)}}{\partial \ln k_{x',t}^{(n)}} \right),$$

4. Update the reallocation rate using

$$\alpha_x^{(n+1)} = \alpha_x^{(n)} + \Delta\alpha_x^{(n)},$$

where $\Delta\alpha_x^{(n)}$ is given by

$$\Delta\alpha_x^{(n)} = \sum_{x''} \varepsilon_{x,x''} \cdot \int_0^\infty \left(\mathcal{U}_{x'',\alpha,d,t}^{(n)} \cdot (\Delta^{(n)} w_{x'',t} + \Delta^{(n)} T_t) + \mathcal{U}_{x'',\alpha,n,t}^{(n)} \cdot (\Delta^{(N)} w_t + \Delta^{(n)} T_t) \right) \cdot dt,$$

and $\Delta^{(n)} w_{x'',t}$, $\Delta^{(n)} w_t$, $\Delta^{(n)} T_t$ is the change in wages and tax revenue generated by the update in taxes from iteration n to $n+1$.

5. Repeat steps 2–4 until convergence.

This procedure only requires us to specify values for $\varepsilon_{x,x''}$ and solve for the optimal tax and the equilibrium path without having to specify the κ function. This comes at the cost of assuming that the elasticities $\varepsilon_{x,x''}$ remain roughly unchanged for the variations in taxes considered. It also ignores the effect of changes in household utility on the multipliers g_x, which is second order due to the envelope theorem, but could be non-negligible for large changes in reallocation effort α_x.

As an alternative, we experimented with the following procedure, which requires parameterizing the κ_x function:

1. Start with $\tau_{x,t}^{(0)} = 0$ (laissez-faire) and the observed rate of reallocation $\alpha_x^{(0)}$.

2. Compute equilibrium objects for $\tau_{x,t}^{(n)}$ and $\alpha_x^{(0)}$, identified with the superscript (n) below.

3. Compute $\varepsilon_{x,x''}^{(n)}$ by solving the system of equations in (A4).

4. Use equation (2) to update optimal taxes as

$$\tau_{x',t}^{(n+1)} = \sum_{x \in \mathcal{D}} \frac{\ell_{x,t} \cdot w_{x,t}^{(n)}}{m_{x',t}^{(n)}} \cdot \left(\frac{\chi_{x,t}^{\mathrm{end},(n)}}{\bar{\chi}_t^{\mathrm{end},(n)}} - 1 \right) \cdot \left(- \frac{\partial \ln w_{x,t}^{(n)}}{\partial \ln k_{x',t}^{(n)}} \right) + \frac{\ell_t \cdot w_t^{(n)}}{m_{x',t}^{(n)}} \cdot \left(\frac{\chi_t^{\mathrm{end},(n)}}{\bar{\chi}_t^{\mathrm{end},(n)}} - 1 \right) \cdot \left(- \frac{\partial \ln w_t^{(n)}}{\partial \ln k_{x',t}^{(n)}} \right),$$

using the values of $\varepsilon_{x,x''}^{(n)}$ to compute the χ's.

5. Update the reallocation rate using

$$\kappa'(\alpha_x^{(n+1)}) = \mathcal{U}_{x,\alpha}^{(n)}.$$

6. Repeat steps 2–5 until convergence.

E.3 Details for the Savings Problem with no Risk Sharing:

As explained in the text, households problem can be summarized by the following system of HJB equations

$$\rho v_x(a,t) - \dot{v}_x(a,t) = \max_c u(c) + \frac{\partial v_x(a,t)}{\partial a} \cdot (ra + w_{x,t} - c) + \alpha_x \cdot (v(a,t) - v_x(a,t)),$$
$$\rho v(a,t) - \dot{v}(a,t) = \max_c u(c) + \frac{\partial v(a,t)}{\partial a} \cdot (ra + w_t - c).$$

Here, $v_x(a,t)$ is the value function of households in disrupted islands at time t with assets a when they exert reallocation effort α_x (kept as an implicit argument to simplify notation), and $v(a,t)$ is the value function of households in non-disrupted islands with assets a.

Let $h_{x,t} = \int_t^\infty e^{-(s-t)r} \cdot w_{x,s} ds$ and $h_t = a_t + \int_t^\infty e^{-(s-t)r} \cdot w_s ds$. We can rewrite these HJB equations using $z = a + h$—effective wealth—as the state variable:

$$\rho v_x(z,t) - \dot{v}_x(z,t) = \max_c u(c) + \frac{\partial v_x(z,t)}{\partial z} \cdot (rz - c) + \alpha_x \cdot (v(z + h_t - h_{x,t}) - v_x(z,t)),$$
$$\rho v(z) = \max_c u(c) + \frac{\partial v(z)}{\partial z} \cdot (rz - c).$$

Note that the HJB equation for $v(z)$ is now stationary, since interest rates are constant. For $u(c) = c^{1-\gamma}/(1-\gamma)$, we can solve analytically for $v(z)$ as

$$v(z) = \left[r - \frac{1}{\gamma}(r-\rho)\right]^{-\gamma} \cdot \frac{z^{1-\gamma}}{1-\gamma}.$$

Moreover, policy functions in the non-disrupted island are given by

$$c_t = \left[r - \frac{1}{\gamma}(r-\rho)\right] \cdot z_t, \qquad \dot{z}_t = \frac{1}{\gamma}(r-\rho) \cdot z_t.$$

This implies

$$\lambda_{x,n,t} = \frac{1}{1 - P_{x,t}} \cdot \int_0^t e^{-\rho t} \cdot \alpha_x \cdot e^{-\alpha_x t_n} \cdot \left(\left[r - \frac{1}{\gamma}(r-\rho)\right] \cdot (z_{x,t_n} + h_{t_n} - h_{x,t_n}) \cdot e^{\frac{1}{\gamma}(r-\rho)(t-t_n)}\right)^{-\gamma} \cdot dt_n,$$

where $z_{x,t}$ denotes the effective wealth held by households in disrupted islands at time t.

This expression uses the fact that

$$c_{x,t_n,t} = \left[r - \frac{1}{\gamma}(r - \rho)\right] \cdot \left(z_{x,t_n} + h_{t_n} - h_{x,t_n}\right) \cdot e^{\frac{1}{\gamma}(r-\rho)(t-t_n)}.$$

To characterize $z_{x,t}$ we solve the HJB equation for $v_x(z,t)$ numerically using the finite-differences method described in Achdou et al. (2021). This method characterizes the common path of consumption $c_{x,t}$ and assets $z_{x,t}$ for households in disrupted islands starting from $z_{x,0} = a_{x,0} + h_{x,0}$. From this method, we also obtain

$$\lambda_{x,d,t} = e^{-\rho t} \cdot c_{x,t}^{-\gamma}.$$

Figure A1 plots typical path for consumption $c_{x,t}$ and assets $z_{x,t}$ starting from $z_{x,0} = h_{x,0} = 1$ in an economy where $h_t - h_{x,t}$ is positive and rises from 0.3 to 0.5 over time. For this examples, we consider a baseline scenario with $\alpha_x = 5\%$, $r = \rho = 5\%$, and $\gamma = 2$ and report variants.

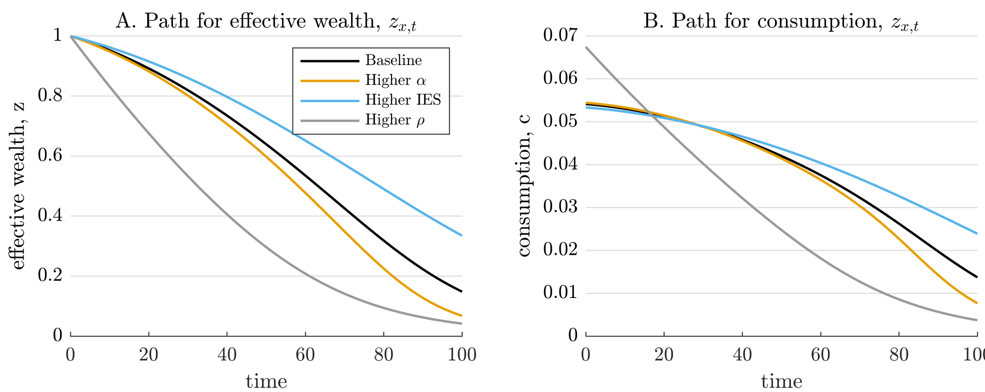

FIGURE A1: CONSUMPTION AND WEALTH PATH IN DISRUPTED ISLANDS. The figure reports examples of the optimal path for effective wealth and consumption in disrupted islands when households can borrow but face uncertainty regarding when they will reallocate. These paths are obtained numerically using the finite-differences method described in Achdou et al. (2021).

E.4 Decomposing the sources of welfare gains from the policy

This section uses the decomposition in Dávila and Schaab (2022) to decompose our formula for optimal taxes into the distributional benefits from: (i) improved sharing of transition risks; (ii) improved consumption smoothing time; (iii) pure redistribution across islands.

Using the notation from footnote 11, we can write optimal taxes from Proposition 2 as

(A12)

$$\tau_{x',t} = \sum_{x\in\mathcal{X}} \frac{\ell_{x,t}\cdot w_{x,t}}{m_{x',t}} \cdot \left(\underbrace{\frac{\chi_{x,t}}{\bar{\chi}_t} - \frac{\bar{\chi}_{x,t}}{\bar{\chi}_t}}_{\text{transition insurance}} + \underbrace{\frac{\bar{\chi}_{x,t}}{\bar{\chi}_t} - \frac{\bar{\chi}_x}{\bar{\chi}}}_{\text{consumption smoothing}} + \underbrace{\frac{\bar{\chi}_x}{\bar{\chi}} - 1}_{\text{pure redistribution}} \right) \cdot \left(-\frac{\partial \ln w_{x,t}}{\partial \ln k_{x',t}} \right).$$

Figures A2 and A3 decompose the optimal taxes on automation technologies and Chinese imports into these components. We provide the decomposition for scenarios I–III. By construction, the optimal tax in scenario IV is driven only by pure redistribution.

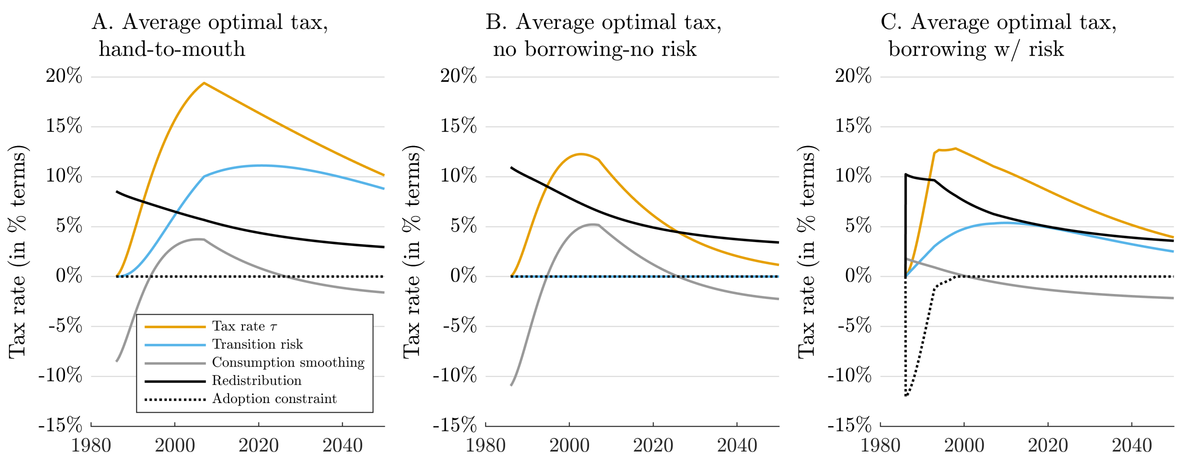

FIGURE A2: DECOMPOSITION OF OPTIMAL TAX FOR ROUTINE AUTOMATION. The figure reports population-weighted path for optimal tax in disrupted islands and its decomposition following (A12). Panel A reports figures for hand-to-mouth consumers, Panel B considers the no borrowing - no risk scenario, and Panel C covers the scenario with borrowing and transition risk.

In line with the discussion in the text, we find that in scenario I and III, reducing transition risk creates a motive for having more persistent taxes on automation and trade. In scenarios I and II, improving consumption smoothing calls for a more concentrated tax around the 2000s when income losses by exposed workers are at a maximum. Finally, the pure redistribution component is stable across scenarios and declines gradually over time. In the third scenario, the decomposition is not exact because the constraint $\frac{1+\tau_{x,t}}{A_{x,t}} \le \bar{w}$ binds early on in the transition.

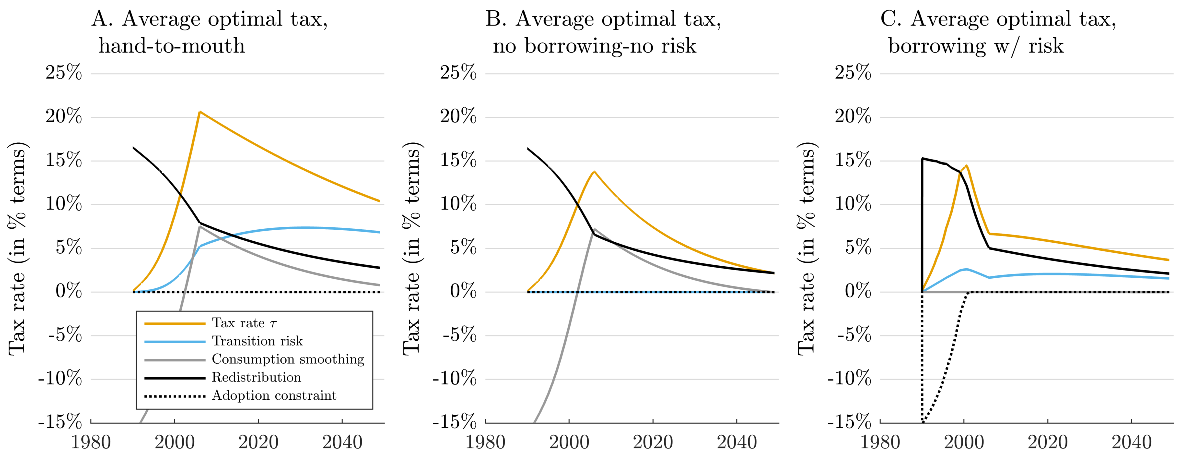

Figure A3: Decomposition of optimal tax for China Shock. The figure reports population-weighted path for optimal tax in disrupted islands and its decomposition following (A12). Panel A reports figures for hand-to-mouth consumers, Panel B considers the no borrowing - no risk scenario, and Panel C covers the scenario with borrowing and transition risk.

Figure A4 decompose the optimal tax on tariffs for Colombia's trade liberalization. The decomposition shows that the pure redistributional motive and the desire to reduce transition risk are the main factors that generate a more gradual optimal liberalization.

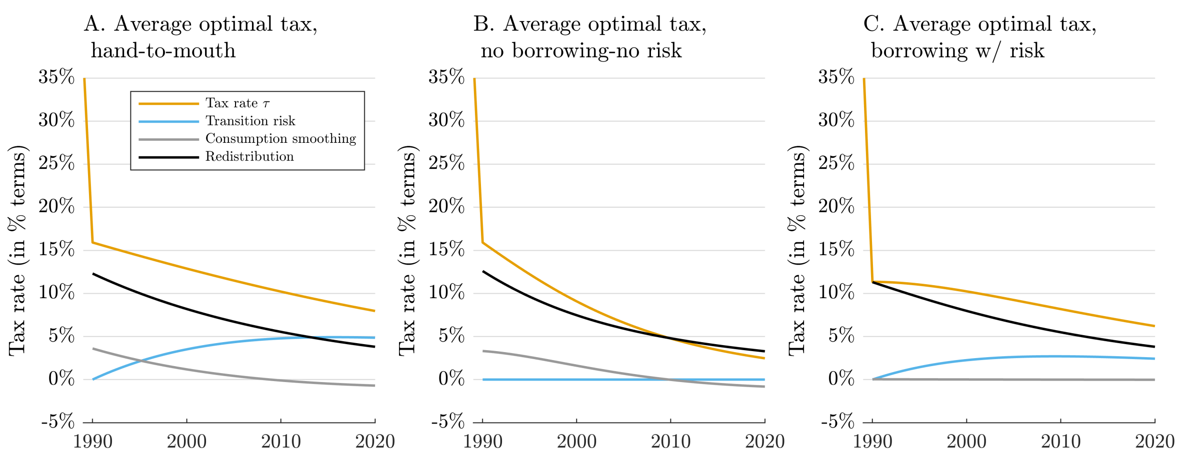

Figure A4: Decomposition of optimal tax for Colombia's trade liberalization. The figure reports population-weighted path for optimal tax in disrupted islands and its decomposition following (A12). Panel A reports figures for hand-to-mouth consumers, Panel B considers the no borrowing - no risk scenario, and Panel C covers the scenario with borrowing and transition risk.