Automation and Rent Dissipation: Implications for Inequality, Productivity, and Welfare*

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Abstract

This paper studies the effects of automation technologies in economies with labor market distortions where workers earn rents on their jobs. We show that automation is targeted at high-rent jobs. This creates rent dissipation, reducing wages for displaced workers and aggregate TFP beyond their competitive effects. It also implies that automation reduces wage dispersion among equal workers. Using data for the US from 1980 to 2016, we provide empirical evidence consistent with the rent dissipation mechanism. We also show how the general equilibrium effects of automation accounting for rent dissipation can be estimated. Our results suggest that the baseline ("competitive") effects of automation account for 44% of the increase in between-group inequality in the US since 1980, while rent dissipation adds another 16% to this number. We estimate that automation brought a small reduction in TFP and (utilitarian) welfare gains on net since 1980.

Keywords: tasks, automation, productivity, technology, inequality, wages, distortions, rents

JEL Classification: J23, J31, O33.

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1 Introduction

US labor markets have experienced epochal changes since 1980: not only did inequality increase greatly, but the real wages of workers without a college degree declined or stagnated.[1] While there is no consensus on the causes of this development, the automation of tasks performed by low-education workers appears to have played an important role (see Acemoglu and Restrepo, 2022).

This paper contributes to our understanding of these trends by studying the implications of automation for wages, productivity, and welfare in economies where labor markets have noncompetitive elements. The distinguishing feature of our framework is that workers receive rents for certain tasks—meaning they are paid above their outside option.[2] Rents reflect forces or barriers that prevent firms from bidding down wages and expanding employment. For example, rents can arise in jobs where workers cannot be monitored and must be paid efficiency wages, jobs where wages are artificially bid up by unions or licenses, or jobs where workers hold up firms. Rents contribute to within-group inequality, as identical workers are paid differently across tasks. Rents also distort firms' hiring decisions, reducing employment at high-rent jobs below what would be socially optimal.

Our first contribution is to show theoretically that the impacts of automation differ from what we would see in a competitive labor market. This is because of a novel *rent dissipation* mechanism: all else equal, high-rent tasks are more likely to be automated. Rent dissipation has important implications for within-group wage dispersion, between-group wage differences, productivity, and welfare:

1. *Within-group:* By reallocating workers away from high-rent jobs, automation reduces wage dispersion within groups of equal workers.

2. *Between-group:* In competitive models, automation affects the relative wage paid to a group of workers by reducing the share of tasks they perform—the *displacement effect*. In our model, the rent dissipation mechanism negatively affects exposed groups' wages by pushing displaced workers to lower-rent jobs.

3. *Productivity and welfare:* In a competitive market, automation always increases TFP by reducing the cost of producing automated tasks. In our model, automation has an

[1]See Goldin and Katz (2008), Acemoglu and Autor (2011), and Autor (2019) for overviews.

[2]We adopt a reduced-form modeling of rents via wedges for ease of exposition and to highlight that similar insights apply, in general, across a range of noncompetitive models. In a companion paper, Acemoglu and Restrepo (2023), we build a search and matching model that micro-founds this reduced-form model.

additional negative effect on productivity, as it reallocates workers away from high-rent jobs where they have a high marginal revenue product and towards low-rent jobs where the marginal revenue product of labor is low. This reallocation worsens *allocative efficiency* and makes the net impact of automation on TFP and welfare ambiguous.

Our second contribution is to explore the implications of the new rent dissipation mechanism using reduced-form evidence for US workers. We rely on the measure of direct task displacement from Acemoglu and Restrepo (2022), which summarizes the extent to which detailed demographic groups of US workers were exposed to automation from 1980 to 2016. We introduce two different strategies to estimate the importance of rent dissipation. First, we provide evidence showing that the impact of automation on wages within detailed demographic groups takes the pattern predicted by theory. In particular, we document that worker groups more exposed to automation saw a reduction in within-group wage dispersion relative to others, with a more pronounced wage decline in wages at medium-to-high quantiles of the within-group distribution. Second, using several proxies for rents, we show that worker groups exposed to automation have been pushed away from high-rent jobs. Our reduced-form evidence suggests that rent dissipation explains 20–25% of the reduction in group relative wages attributed to automation.

Our third contribution is to quantitatively explore the aggregate implications of rent dissipation for wage levels, TFP, and welfare. We extend the methodology from Acemoglu and Restrepo (2022) to estimate the general equilibrium effects of automation when labor markets are distorted. We provide explicit formulas for the change in wages, output, and productivity in terms of the direct task displacement experienced by demographic groups, the amount of rent dissipation, and the cost-saving gains from automation. Our formulas show that, in general equilibrium, the effects of automation (and any other shock) on one group of workers affects other groups via *ripple effects*. These ripple effects can be summarized by two matrices: the *propagation matrix* encodes information on the strength of direct and indirect competition for tasks between different groups of workers, and the *rent impact matrix* encodes information on how task reallocation changes rents across groups.

We use our formulas to compute the general equilibrium effects of automation on wages and productivity by combining our measures of direct task displacement across demographic groups with estimates of the effects of automation on rents, estimates of the propagation and rent impact matrices, and external estimates of cost savings from automation. According to our estimates, automation can account for 60% of the rise in between-group wage inequality since 1980. Of these, 44 percentage points are due to the baseline "competitive"effects of automation.[3] The remaining 16 percentage points are due to the rent dissipation mechanism. Rent dissipation is particularly relevant for explaining the decreasing real wages of US workers without a college degree.

Moreover, we estimate that the effect of automation via cost-saving gains was to increase TFP by 3% between 1980 and 2016. However, the worsening allocative efficiency brought by rent dissipation implied that on the net, automation reduced aggregate TFP by 0.7% between 1980 and 2016. In terms of welfare, we estimate a reduction in aggregate consumption of 1% over this period, as an increased share of the final good is used for investment in automation equipment.

Related Literature: Our work contributes to the literature exploring the determinants of the rise in between-group and within-group inequality. Among others, Bound and Johnson (1992), Katz and Murphy (1992) and Card and Lemieux (2001). Our work is closest to papers exploring the effects of automation and lower equipment prices on inequality and wages, including Autor et al.'s 2003 work on the effects of computers and automation technologies on routine tasks, and the literature on capital-skill complementarity, for example, Krueger (1993), Autor et al. (1998), Krusell et al. (2000), and Burstein et al. (2019). Our previous work (Acemoglu and Restrepo, 2022) contributed to this literature by quantifying the effects of automation on the US wage structure but assumed competitive labor markets.

Relative to previous work, this paper emphasizes how the interplay between automation and labor market distortions affects the wage structure. For example, we show that the loss of high-rent jobs is important in explaining the lack of wage growth for non-college men. We also explore the implications of automation for within-group wage inequality, which, apart from recent work by Kogan et al. (2021), have received little attention. We show that rent dissipation can reduce wage dispersion within groups of workers exposed to automation and provide reduced form evidence in line with this prediction. This is in contrast to a conventional view that emphasizes the *fractal* nature of the increase in inequality (meaning that it increases at all levels of aggregation) and concludes from this that a common set of forces drive the rise in between-group and within-group inequality. Our theory suggests a more nuanced pattern, where automation can cause an increase in wage inequality between groups and a *decline* within groups impacted by automation.

[3]The 44% number is lower than the general equilibrium estimate in Acemoglu and Restrepo (2022) of 50%. This is because our framework separates the distinct role of rent dissipation from these baseline competitive effects.

Our work also contributes to the literature on worker rents—understood as wages that exceed their outside option—and the implications of rents for wage dispersion and efficiency. On the empirical front, Katz and Summers (1989) provide evidence for the existence of rents at the industry level and argue that these wage differences cannot be explained away as compensating differentials. On the theoretical front, this literature has proposed several reasons why workers might earn rents, ranging from efficiency wage considerations (see Shapiro and Stiglitz, 1984; Bulow and Summers, 1986), to the use of wages to recruit and retain workers under imperfect information about workers (see Stiglitz, 1985; Lang, 1991).[4]

Our research recognizes the presence of worker rents and investigates their effects on automation decisions and their impact on productivity and wages. Instead of emphasizing a specific micro-foundation for rents, we build on the more recent literature on misallocation (i.e., Hsieh and Klenow, 2009; Restuccia and Rogerson, 2008; Hsieh et al., 2019, and others) and use wedges as a reduced-form way of modeling the distortionary effect of rents on labor markets. Wedges capture the key economic elements common to various micro-foundations for rents: they generate wage dispersion among equal workers, and they distort firm hiring decisions, with high-rent high-wage jobs being rationed and understaffed in equilibrium.[5]

Finally, our work contributes to the growing literature that studies issues of aggregation and how technology affects productivity and welfare in inefficient economies (see, for example, Baqaee and Farhi, 2020; Basu et al., 2022; Dávila and Schaab, 2023). Our contribution here is to provide formulas for the aggregate effects of automation on wages, output, TFP, and welfare for economies with labor market distortions and use these formulas to estimate the impacts of automation on the US economy since 1980.

Organization of the paper: Section 2 provides our theoretical framework. Section 3 presents reduced form evidence in support of the rent dissipation mechanism. Section 4 outlines our approach for estimating the general equilibrium effects of automation.

[4]Some of these papers also show that rents generate misallocation, with too little employment from an efficiency point of view in high-rent jobs. Katz and Summers (1989) and Bulow and Summers (1986) argue that worker rents justify using industrial and trade policies designed to increase employment in the most distorted sectors or jobs. The flip side of these claims is that trade and globalization can reduce welfare if they reduce employment at high-rent high-wage jobs. This insight goes back to work on *immiserizing growth* in the presence of distortions by Bhagwati (1968) and relates to our finding that automation can worsen allocative efficiency when it pushes workers away from high-rent jobs.

[5]In that way, our work differs from recent papers exploring how the threat of automation affects bargaining between firms and workers (see Arnoud, 2019; Leduc and Liu, 2022).

2 Theory: Automation and Labor Market Distortions

This section presents our conceptual framework and derives our theoretical results. We first study a one-sector model and then extend our results to a multi-sector economy.[6]

2.1 Single-Sector Model

Production: A unique final good y is produced combining tasks y_x. Tasks are indexed by x, which summarizes their attributes, and belong to a measurable set $\mathcal{T} \subset \mathbb{R}^k$ of mass M.[7] Tasks are aggregated with a constant elasticity of substitution (CES) $\lambda \geq 0$, so that

$$y = \left(\frac{1}{M}\int_{x\in\mathcal{T}} (M \cdot y_x)^{\frac{\lambda-1}{\lambda}} \cdot dx\right)^{\frac{\lambda}{\lambda-1}}.$$

There is a discrete set of labor types or groups indexed by g, where $g \in \mathcal{G} = \{1, 2, \ldots, G\}$. Workers in a given group share the same productivity across tasks. In addition to labor, tasks can also be produced using task-specific capital, equipment, or software, denoted by k_x. In particular, the total quantity of task x produced equals

$$y_x = \psi_{kx} \cdot k_x + \sum_g \psi_{gx} \cdot \ell_{gx}.$$

Here, ℓ_{gx} is the amount of labor of type g allocated to task x, while k_x is the amount of task-specific capital used for this task. Here, ψ_{gx} and ψ_{kx} represent the productivity of different factors in the production of task x. We set $\psi_{gx} = 0$ if a labor type g cannot perform task x.

There is a fixed supply ℓ_g of workers of type g to be allocated across tasks, so that

$$\int_{x\in\mathcal{T}} \ell_{gx} \cdot dx \leq \ell_g.$$

On the other hand, we treat the capital used for task x, k_x, as a pure intermediate good and assume it is is produced within the same period using the final good at a constant unit

[6] The task model used here builds on the literature that studies the effects of technology and trade using task models, including Zeira (1998), Acemoglu and Zilibotti (2001), Acemoglu and Autor (2011), and Acemoglu and Restrepo (2018), as well as Grossman and Rossi-Hansberg's (2008) model of offshoring.

[7] Throughout, we let "dx" denote the Lebesgue measure over $\mathbb{R}^k$ and $\int_{x\in\mathcal{T}} m_x \cdot dx$ denote the Lebesgue integral of the function m_x over $\mathcal{T}$. The set $\mathcal{T}$ is assumed measurable.

cost $1/q_x$. This implies that total consumption is equal to net output

$$c = y - \int_{x \in \mathcal{T}} (k_x/q_x) \cdot dx.$$

For some tasks, we let $q_x = 0$, which means there is no technology available to produce them with capital. We will then consider the effects of advances in automation technologies, which enable the production of some of these tasks with capital.

The Labor Market: Firms must pay workers from group g a task-specific wage

$$w_{gx} = \mu_{gx} \cdot w_g$$

in task x, where w_g is the *base wage* of the group and μ_{gx} is an exogenous wedge that might vary across tasks and groups. We treat wedges μ_{gx} as exogenous attributes of tasks, jobs, or the labor market that force firms to pay workers wages that are artificially high (relative to what the same worker would earn at other tasks).[8] We normalize wedges for all $g \in \mathcal{G}$ so that $\mu_{gx} \geq 1$ and assume that there is a positive mass of tasks for which $\mu_{gx} = 1$.[9] This normalization implies that base wages can be interpreted as the real wage that workers from group g earn in jobs that pay no rents.

The labor market works as follows: given task-specific wages, firms decide how many workers from each group to hire for task x. Workers from group g would prefer to be employed at high-rent jobs. However, these jobs are rationed in equilibrium: firms hire workers only until their marginal revenue product of labor equals the wage they must pay (i.e., firms are on their labor demand curves). The labor market works by assigning workers from group g randomly to tasks until firms' labor demands are satisfied. The base wage w_g adjusts to ensure all workers are employed.

Appendix B provides two micro-foundations for worker rents, based on efficiency wage and bilateral bargaining models. It shows that both forms of labor market distortions can be mapped to the wedges in our model.

[8] Our notation is flexible enough to capture multiple dimensions of labor market rents. For example, a subset of tasks could represent jobs at a given firm that must pay workers rents because they are unionized. Or a subset of tasks could represent jobs at a given industry or region where wages are artificially high because of licenses. Finally, a subset of tasks could share technological attributes that make monitoring workers challenging and create informational rents.

[9] The assumption that $\mu_{gx} \geq 1$ for some tasks is without loss of generality. Because the supply of labor is inelastic, only relative rents matter for outcomes.

Equilibrium and Characterization: A *market equilibrium* is given by a vector of base wages $\{w_g\}$, output y, an allocation of tasks $\{\mathcal{T}_g\}_g, \mathcal{T}_k$, task prices p_x, hiring plans ℓ_{gx}, and capital production plans k_x such that:[10]

E1 Tasks prices equal the minimum unit cost of producing the task:

$$p_x = \min\left\{\frac{1}{q_x \cdot \psi_{kx}}, \left\{\frac{w_g \cdot \mu_{gx}}{\psi_{gx}}\right\}_g\right\}.$$

E2 Tasks are allocated in a cost-minimizing way. The set of tasks

$$\mathcal{T}_g = \left\{x : p_x = \frac{w_g \cdot \mu_{gx}}{\psi_{gx}}\right\}$$

will be produced by workers of type g, and the set of tasks

$$\mathcal{T}_k = \left\{x : p_x = \frac{1}{q_x \cdot \psi_{kx}}\right\}$$

will be produced by capital.

E3 Quantities of labor and capital are given by

$$\ell_{gx} = y \cdot \frac{1}{M} \cdot \psi_{gx}^{\lambda-1} \cdot (\mu_{gx} \cdot w_g)^{-\lambda} \text{ for } x \in \mathcal{T}_g,$$
$$k_x = y \cdot \frac{1}{M} \cdot \psi_{kx}^{\lambda-1} \cdot q_x^{\lambda} \qquad \text{for } x \in \mathcal{T}_k.$$

E4 The labor market clears for all $g \in \mathcal{G}$, $\int_{x \in \mathcal{T}} \ell_{gx} \cdot dx = \ell_g$. Workers from group g are assigned randomly to tasks in $\mathcal{T}_g$, with ℓ_{gx} of them assigned to task x.

E5 The ideal-price index condition holds:

$$1 = \left(\frac{1}{M} \int_{x \in \mathcal{T}} p_x^{1-\lambda} \cdot dx\right)^{\frac{1}{1-\lambda}}.$$

The Appendix provides sufficient conditions for the existence of an equilibrium and shows that, when these conditions are met, the equilibrium is unique. In the rest of the paper, we assume there is a unique equilibrium and study its properties.

[10] To ensure the uniqueness of the task allocation, we assume that when indifferent, tasks are allocated to capital or to the group with the highest index g. This tie-breaking rule is unimportant for aggregates.

PROPOSITION 1 *The equilibrium is inefficient. Relative to the efficient allocation, there is too little employment in high-rent jobs.*

The inefficiency arises because rents distort firms' hiring decisions. Firms hire workers until their marginal revenue product equals the task-specific wage, $\psi_{gx} \cdot p_x = \mu_{gx} \cdot w_g$. As a result, the marginal product of a worker at high-rent tasks exceeds the marginal product of the same type of worker at other tasks. The economy would produce more output by reallocating workers towards high-rent tasks, but worker rents prevent firms from expanding employment in these jobs.[11] This inefficiency will be important when interpreting the effects of automation on TFP and welfare.

2.2 Automation and Rent Dissipation

This subsection provides a formal definition of advances in automation technology and shows that these technologies tend to be adopted for high-rent tasks.

We model *advances in automation technologies* as an increase in q_x from zero to a positive level $q'_x > 0$ for tasks in $\mathcal{A}^T$. We also let $\mathcal{A}^T_g = \mathcal{A}^T \cap \mathcal{T}_g$ denote the set of tasks assigned to group g that can now be automated. The sets $\mathcal{A}^T_g$ represents new opportunities for automation affecting different groups of workers, and we take these sets and the new productivity of the capital-producing sector q'_x as exogenous. For example, advances in robotics in the 1980s and 1990s made it possible to complete industrial tasks such as welding or painting with robots, while before, these tasks had to be performed by human workers. These advances increased q_x from zero to a positive level for the subset of industrial tasks that robots can perform.

Advances in automation technologies are then adopted at some (but not all) tasks in $\mathcal{A}^T_g$. Firms will endogenously automate tasks in which the cost of producing with the new capital is below the cost of producing with labor. We define the set $\mathcal{A}_g$ as the set of tasks

[11] Not all sources of frictional wage dispersion or all forms of rents generate this type of misallocation. This is only true if rents affect firms' hiring decisions, which holds in the micro-foundations discussed above, where firms are on their labor demand curve and cut back hiring in response to rents. This is not the case in models of efficient rent sharing or in models of wage posting a-la Burdett and Mortensen (1998) where more productive firms pay higher wages (and so there is frictional wage dispersion) but these play no allocative role. An alternative notion of rents is that these arise in jobs where wages exceed workers' marginal revenue product, for example, because of government subsidies. Government subsidies play the opposite role of rents in our model. They reduce the cost of hiring workers below their competitive level and generate too much employment in subsidized jobs.

performed by workers in group g that can be now profitably automated:

$$\mathcal{A}_g = \left\{ x \in \mathcal{A}_g^T : \frac{w_g \cdot \mu_{gx}}{\psi_{gx}} \geq \frac{1}{q'_x \cdot \psi_{kx}} \right\}.$$

The definition of $\mathcal{A}_g$ highlights that, all else equal, tasks in $\mathcal{A}_g^T$ with high rents are more likely to be automated. This captures a key economic force in our model: tasks that are more expensive to produce with labor are more profitable to automate, and hence higher rents encourage the adoption of automation. As the following proposition shows, if advances in automation technologies are not biased towards low-rent tasks, this force ensures that automation will be adopted for tasks that pay high rents *on average.*

Proposition 2 (Rent Dissipation)
Let $\bar{F}_g(\mu|\mathcal{B})$ denote the share of group g employment at tasks in $\mathcal{B}$ that pay a rent above μ. If some tasks in $\mathcal{A}_g^T$ are not automated and:

i. $\bar{F}_g(\mu|\mathcal{A}_g^T) \geq \bar{F}_g(\mu|\mathcal{T}_g)$ for $\mu \geq 1$;

ii. $\bar{F}_g(\mu|\{x \in \mathcal{A}_g^T : q'_x = \tilde{q}, \psi_{kx} = \tilde{\psi}_k, \{\psi_{gx} = \tilde{\psi}_g\}_g\}) = \bar{F}_g(\mu|\mathcal{A}^T)$ for all $\tilde{q}, \tilde{\psi}_k, \{\tilde{\psi}_g\}_g$ and $\mu \geq 1$,

adoption is targeted at high-rent tasks, in the sense that $\bar{F}_g(\mu|\mathcal{A}_g) > \bar{F}_g(\mu|\mathcal{T}_g)$ for $\mu > 1$. Within a group, the distribution of rents in newly automated tasks first-order stochastically dominates the distribution of rents at other tasks.

Condition (i) says that advances in automation are not biased towards low-rent tasks. Condition (ii) says that, among the tasks that can now be automated, rents are orthogonal to workers' comparative advantage. These conditions are sufficient to ensure that automation opportunities do not occur predominantly in low-rent tasks, which would mechanically lead to the automation of low-rent tasks. One can see the role of the endogenous adoption margin by considering a case with $\bar{F}_g(\mu|\mathcal{A}_g^T) = \bar{F}_g(\mu|\mathcal{T}_g)$, so that advances in automation technology are orthogonal to rents. The proposition shows that, even in this case, adoption will be endogenously targeted at high-rent tasks.

Proposition 2 provides sufficient conditions for automation to displace workers from high-rent tasks on average. We refer to this phenomenon as *rent dissipation.* In the next sections, we assume that automation generates rent dissipation and work out the implications of this mechanism for within-group wage dispersion and aggregates. Our empirical exercise will then provide reduced-form evidence in support of rent dissipation.

2.3 Within-group Wage Differentials

Rents generate wage dispersion among equally productive workers. In particular, assume that workers perform a single task on their jobs so that the distribution of wages within group g is the same as the distribution of rents μ_{gx} across tasks in $\mathcal{T}_g$. The following proposition explains how the automation of tasks in $\mathcal{A}_g$ affects within-group wage dispersion. The proposition provides a partial equilibrium characterization that abstracts from how changes in base wages reallocate tasks in general equilibrium—a force that we study in the next subsection.

Proposition 3 (Automation and Within-Group Wage Differentials)
Suppose that not all tasks in $\mathcal{T}_g$ can be automated and the conditions from Proposition 2 hold so that $\bar{F}_g(\mu|\mathcal{A}_g) > \bar{F}_g(\mu|\mathcal{T}_g)$ for $\mu > 1$. Let $M_g \in (0,1)$ denote the mass of workers from group g at jobs that pay no rents before the advancements in automation. Let $\ln w_g^p$ denote the p-th quantile of the distribution of (log) wages in group g. Automating tasks in $\mathcal{A}_g$ shifts $\ln w_g^p$ as follows:

- $d \ln w_g^p = d \ln w_g$ *for* $p \in [0, M_g]$;
- $d \ln w_g^p < d \ln w_g$ *for* $p \in (M_g, 1)$;
- $d \ln w_g^p = d \ln w_g$ *for* $p \to 1$.

Figure 1 illustrates the U-shaped pattern of within-group wage changes. The lowest wage workers in g are employed at non-rent paying jobs and earn the base wage w_g. Automation impacts their wage through the general equilibrium change in the base wage. Quantiles above M_g are populated by workers at high-rent jobs, who are more likely to see their jobs automated. This manifests as a more pronounced decline in $d \ln w_g^p$ for $p \in (M_g, 1)$. Finally, at the top of the group, there will be some workers at high-rent jobs that cannot be automated. Automation only affects these top wage earners through the base wage.[12]

Proposition 3 contrasts the widely-held idea that technological progress increases inequality across every dimension, including within groups. Instead, our theory makes the distinctive prediction that automation can generate rising wage compression within groups of otherwise equal workers via the rent dissipation mechanism.

[12] The result that wages at the top quantile only change due to base wages is a consequence of the assumption that not all tasks in $\mathcal{T}_g$ are automated and condition (ii) in Proposition 2. As we will see, there is support in the data for this pattern.

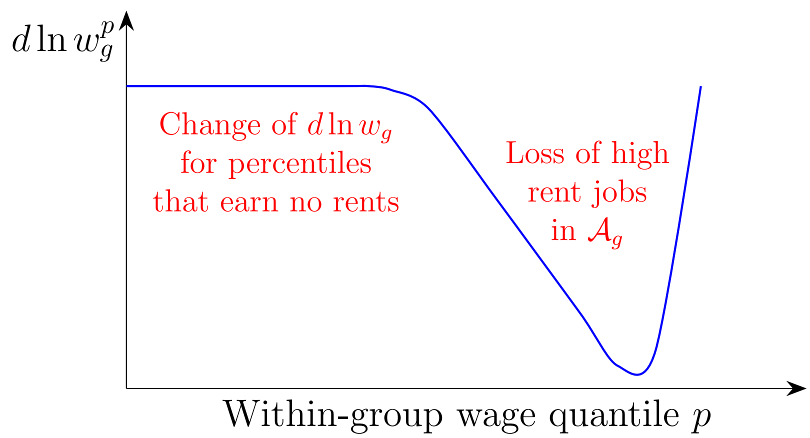

FIGURE 1: PREDICTED CHANGES IN WITHIN-GROUP WAGE QUANTILES DUE TO AUTOMATION.

2.4 Group Wages and Productivity

We now characterize the effects of automation on group-level wages, TFP, and output. We first provide formulas for group-level wages and then derive the first-order effects of an automation shock on aggregates accounting for general equilibrium forces.

Given the unique equilibrium allocation, define the *task share* of worker group g as

$$\Gamma_g = \frac{1}{M} \int_{x \in \mathcal{T}_g} \psi_{gx}^{\lambda-1} \cdot \mu_{gx}^{-\lambda} \cdot dx,$$

where the integrals are taken over the set of tasks where g (or k) is the lowest-cost producer. Task shares summarize the economic value of tasks assigned in equilibrium to each group of workers. Likewise, given the unique equilibrium allocation, define the group g rent as

$$\mu_g = \frac{1}{\ell_g} \int_{x \in \mathcal{T}_g} \ell_{gx} \cdot \mu_{gx} \cdot dx.$$

Task shares and group rents are endogenous equilibrium objects that summarize how tasks are allocated and how this assignment shapes group-level wages.

PROPOSITION 4 *Given the equilibrium allocation $\langle \{\mathcal{T}_g\}_g, \mathcal{T}_k \rangle$ and the resulting task shares and group rents, base wages by group and average wages by group are*

(1) $$w_g = \left(\frac{y}{\ell_g}\right)^{\frac{1}{\lambda}} \cdot \Gamma_g^{\frac{1}{\lambda}},$$

(2) $$\bar{w}_g = \left(\frac{y}{\ell_g}\right)^{\frac{1}{\lambda}} \cdot \Gamma_g^{\frac{1}{\lambda}} \cdot \mu_g.$$

Equation (1) shows that base wages depend on output per worker and task shares. A higher task share for group g implies that this group contributes more to output, translating into a higher base wage. Equation (2) shows that group g average wages—denoted by $\bar{w}_g$—also depend on the rents it earns across tasks. The Appendix shows that other aggregates, such as GDP, consumption, and investment, can also be computed in terms of task shares and group rents.

Consider an exogenous advance in automation technologies taking place at a small set $\mathcal{A}^T$ (formally of measure ϵ for ϵ small) with $\mathcal{A}_g^T$ in the interior of $\mathcal{T}_g$. Suppose that these advances lead to the automation of tasks in $\mathcal{A}_g$ across different groups. Following Acemoglu and Restrepo (2022), define the *direct task displacement* generated by these advances as

$$d\ln\Gamma_g^d = \frac{\int_{x\in\mathcal{A}_g}\ell_{gx}\cdot dx}{\int_{x\in\mathcal{T}_g}\ell_{gx}\cdot dx}.$$

This measures the reduction in group g's task share resulting from the automation of tasks in $\mathcal{A}_g$. Likewise, denote the average cost-saving gains at automated tasks from group g by

$$\pi_g = \frac{\int_{x\in\mathcal{A}_g}\ell_{gx}\cdot\mu_{gx}\cdot\pi_{gx}\cdot dx}{\int_{x\in\mathcal{A}_g}\ell_{gx}\cdot\mu_{gx}\cdot dx} > 0,$$

where $\pi_{gx} > 0$ is the percent cost reduction of automating task x.[13] This is positive by definition, or these tasks would not be automated. Finally, let

$$\mu_{\mathcal{A}_g} = \frac{\int_{x\in\mathcal{A}_g}\ell_{gx}\cdot\mu_{gx}\cdot dx}{\int_{x\in\mathcal{A}_g}\ell_{gx}\cdot dx}$$

denote the *average rent in automated tasks* $\mathcal{A}_g$ for group g.

The objects $\langle\{d\ln\Gamma_g\}_g, \{\pi_g\}_g, \{\{\mu_{\mathcal{A}_g}\}_g\}\rangle$ can be computed from the initial allocation of workers to tasks and contain all the relevant information about the underlying advances in automation that is needed to compute their effects on aggregates.

In response to the direct task displacement generated by advances in automation across groups, we will have an endogenous reassignment of tasks across workers. We refer to the reassignment as *ripple effects*, since they propagate the effects of automation to other

[13] Formally, this can be computed as

$$\pi_{gx} = \frac{1}{\lambda-1}\left[\left(\frac{q_x'\cdot\psi_{kx}\cdot w_g\cdot\mu_{gx}}{\psi_{gx}}\right)^{\lambda-1} - 1\right] \approx \ln\left(\frac{w_g\cdot\mu_{gx}}{\psi_{gx}}\right) - \ln\left(\frac{1}{q_x'\cdot\psi_{kx}}\right) \geq 0.$$

groups of workers. Figure 2 illustrates this reallocation. The left panel provides an example of a task space and an equilibrium allocation of tasks to g, g', and capital. The right panel shows that, following the displacement of workers of group g from $\mathcal{A}_g$, there is an endogenous reassignment of tasks. This reassignment determines whether the incidence of the displacement effects from automation falls on group g or is shared by group g'.

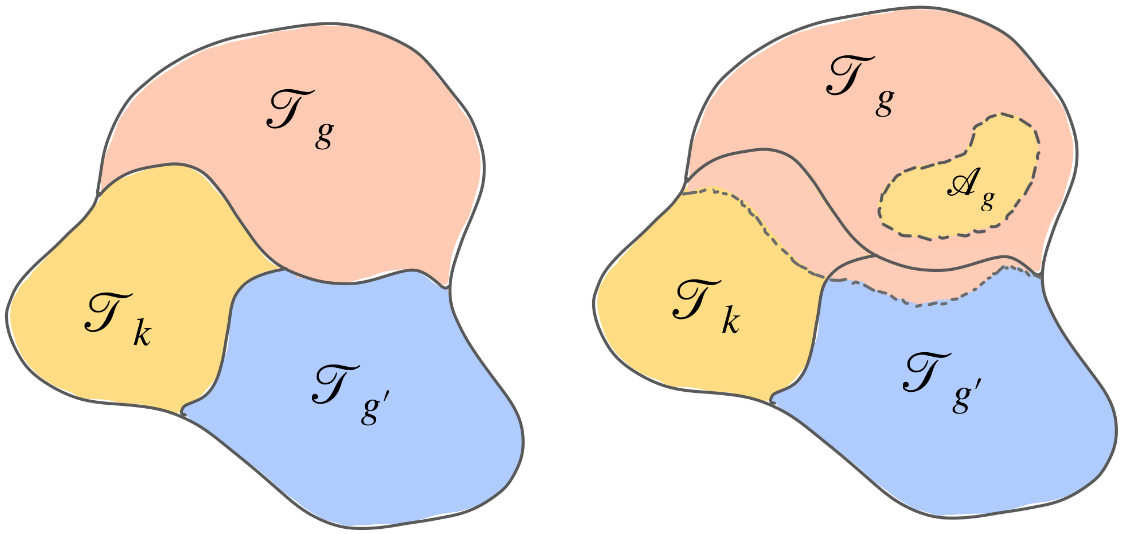

Figure 2: The Task Allocation and the Effects of Automation. The left panel provides an example of a task space and an equilibrium allocation of tasks to g, g', and capital. The right panel illustrates the direct task displacement effect and ripple effects.

We capture the impact of ripple effects on task shares and rents using two matrices. The first is the *propagation matrix* (as in Acemoglu and Restrepo, 2022), which summarizes how the direct task displacement experienced by a group propagates to others:

$$\Theta = \left(\mathbb{I} - \frac{1}{\lambda}\frac{\partial \ln \Gamma}{\partial \ln w}\right)^{-1}.$$

The entry $\theta_{gg'}$ in this matrix is non-negative and captures the extent to which shocks that reduce group g' base wages also reduce group g wages via the ripple effects.[14] The second

[14] As discussed in Acemoglu and Restrepo (2022), the propagation matrix plays a similar role to a Leontief inverse: it accumulates both direct and indirect ripples resulting from subsequent rounds of reallocation. Acemoglu and Restrepo (2022) show that this Leontief inverse is always well defined and has all its eigenvalues in $[0, 1)$. This means that ripple effects will play an equalizing role and dampen the direct effects of automation. The propagation matrix also captures the fact that, even though worker groups are perfect substitutes at the task level, they will not be perfect substitutes on the aggregate. The entries of the propagation matrix are also informative of how substitutable groups are and relate this to how much groups compete for tasks, both directly and indirectly.

is the *rent impact matrix*, which captures how task reallocation impact group rents:

$$\mathcal{M} = \left(\frac{\partial \ln \mu}{\partial \ln w}\right) \times \left(\mathbb{I} - \frac{1}{\lambda}\frac{\partial \ln \Gamma}{\partial \ln w}\right)^{-1}.$$

This matrix is similar to the propagation matrix, but its entries can now be negative and account for whether the competition between group g' and g takes place at tasks where g workers earn above-average rents (in which case the entry is positive) or below-average rents (in which case it is negative).

In these definitions $\frac{\partial \ln \Gamma}{\partial \ln w}$ is a $G \times G$ Jacobian with entry g, g' given by $\frac{d \ln \Gamma_g}{d \ln w_{g'}} > 0$. Likewise, $\frac{\partial \ln \mu}{\partial \ln w}$ is a $G \times G$ Jacobian with entry g, g' given by $\frac{d \ln \mu_g}{d \ln w_{g'}}$. These partial derivatives capture the change in task allocations and the resulting change in average rents and task shares following an increase in $w_{g'}$ holding all other wages constant.

The following proposition provides formulas for the change in all economic aggregates in terms of the direct effects of automation (summarized by $\langle\{d \ln \Gamma_g\}_g, \{\pi_g\}_g, \{\{\mu_{\mathcal{A}_g}\}_g\}\rangle$), elasticities, the propagation and rent impact matrices, and initial shares. In our formulas, we let stack(x_j) denote the column vector $(x_1, x_2, \dots, x_{\mathcal{G}})$. This proposition and its extension to a multi-sector economy in the next subsection provide the basis for our empirical analysis of the US data.

Proposition 5 *Consider an exogenous advance in automation technologies in a small set $\mathcal{A}^T$ with direct effects $\langle\{d \ln \Gamma_g\}_g, \{\pi_g\}_g, \{\{\mu_{\mathcal{A}_g}\}_g\}\rangle$. These advances change wages by*

$$(3) \qquad d \ln w_g = \Theta_g \cdot stack\left(\frac{1}{\lambda} \cdot d \ln y - \frac{1}{\lambda} \cdot d \ln \Gamma_j^d\right)$$

$$(4) \qquad d \ln \mu_g = \mathcal{M}_g \cdot stack\left(\frac{1}{\lambda} \cdot d \ln y - \frac{1}{\lambda} \cdot d \ln \Gamma_j^d\right) - \left(\frac{\mu_{\mathcal{A}_g}}{\mu_g} - 1\right) \cdot d \ln \Gamma_g^d$$

$$(5) \qquad d \ln \bar{w}_g = (\Theta_g + \mathcal{M}_g) \cdot stack\left(\frac{1}{\lambda} \cdot d \ln y - \frac{1}{\lambda} \cdot d \ln \Gamma_j^d\right) - \left(\frac{\mu_{\mathcal{A}_g}}{\mu_g} - 1\right) \cdot d \ln \Gamma_g^d,$$

where the change in output (and wage levels) is pinned down by the dual Solow formula

$$(6) \qquad \sum_g s_g \cdot d \ln \bar{w}_g = d \ln tfp.$$

Here, $s_g = \bar{w}_g \cdot \ell_g / y$ is the share of g in GDP and $d \ln tfp$ is the change in aggregate TFP

$$(7) \qquad d \ln tfp = \sum_g s_g \cdot d \ln \Gamma_g^d \cdot \frac{\mu_{\mathcal{A}_g}}{\mu_g} \cdot \pi_g + \sum_g s_g \cdot d \ln \mu_g.$$

Equations (3), (4), and (5) characterize the response of wages to advances in automation. Wage changes are given in terms of the output $d\ln y$, which is pinned down by equations (6) and (7). These equations show that average wage changes equal the change in TFP and provide a formula for computing the effects of automation on TFP. In our model, $d\ln tfp = (1 - s_k) \cdot d\ln c$, which implies that changes in TFP are also informative of changes in aggregate consumption and utilitarian social welfare.[15]

To explain the formulas, let us first consider an economy without ripple effects, where worker groups can produce disjoint sets of tasks, and capital produces all tasks for which $q_x > 0$.[16] In this economy, the propagation matrix is the identity, and the rent impact matrix is zero. Equation (5) for average wage changes for group g simplifies to

$$d\ln \bar{w}_g = \underbrace{\frac{1}{\lambda} \cdot d\ln y - \frac{1}{\lambda} \cdot d\ln \Gamma_g^d}_{\text{effects in competitive labor market}} \underbrace{- \left(\frac{\mu_{\mathcal{A}_g}}{\mu_g} - 1\right) \cdot d\ln \Gamma_g^d}_{\text{rent dissipation}} . \tag{8}$$

Equation (8) shows that group g average wages depend on three effects. The first two already exist in competitive models, such as Acemoglu and Restrepo (2022): a positive *productivity effect*—as output expands, all wages increase—and a negative *displacement effect*—the decline in group g task share following the automation of tasks in $\mathcal{A}_g$. The third effect is new and captures the negative contribution of rent dissipation to group g average wages. When $\mu_{\mathcal{A}_g} > \mu_g$, automation pushes workers away from high-rent tasks and towards tasks that pay lower rents. This shift in the composition of tasks available to workers from a given group reduces their average wage.

Turning to the general case with ripples, equation (5) in the Proposition shows similar forces at play. The difference is that now group g wages also depend on whether other groups that compete against it for high-rent tasks are being displaced by automation. In particular, the direct task displacement experienced by other groups affects group g base wage via the propagation matrix and its rents via the rent impact matrix.

One important difference between the displacement effect and the new rent dissipation mechanism is in their propagation. Once a group is displaced from their tasks, their relative base wage declines, and firms substitute for them in marginal tasks, propagating

[15]We define changes in TFP by the usual Solow residual $d\ln tfp = d\ln y - s_k \cdot d\ln k$. As equation (6) shows, changes in the Solow residual continue to be informative of changes in households' income and welfare despite the presence of distortions in the labor market (see also Basu et al., 2022).

[16]Formally, this holds when workers produce non-overlapping sets of tasks (i.e., if $\psi_{gx} > 0$, then $\psi_{g'x} = 0$ for all other g') and ψ_{kx} exceeds a threshold $\underline{\psi}_k$. This corresponds to Assumption 1 in Acemoglu and Restrepo (2022).

the incidence of task displacement to other groups. Instead, groups bear the full incidence of rent dissipation. This is because rent dissipation works by shifting the composition of tasks assigned to a group toward low-rent tasks without altering its base wage or triggering further reassignments of tasks across groups. One implication of this result is that rent dissipation can have sizable impacts on inequality that are not dampened by ripple effects.

Let's now turn to the implications of automation for productivity and welfare. In the absence of ripples, equation (7) simplifies to

$$(9) \qquad d\ln tfp = \underbrace{\sum_g s_g \cdot d\ln\Gamma_g^d \cdot \frac{\mu_{\mathcal{A}_g}}{\mu_g} \cdot \pi_g}_{\text{Direct technological gains a-la Hulten}} - \underbrace{\sum_g s_g \cdot \left(\frac{\mu_{\mathcal{A}_g}}{\mu_g} - 1\right) \cdot d\ln\Gamma_g^d}_{\text{Changes in allocative efficiency}}.$$

This decomposition for changes in TFP builds on the work of Baqaee and Farhi (2020). The first term represents the direct technological benefits of automation, and it is always positive. This term has a logic similar to Hulten's theorem: reducing the cost of producing automated tasks increases aggregate TFP by (i) the share of these tasks in output (i.e., their Domar weights) and (ii) the cost-saving gains $\pi_g > 0$. The second term reflects changes in *allocative efficiency* induced by automation. This term captures how changes in the allocation of labor affect output. In a competitive labor market, this term is zero because the allocation of labor across tasks maximizes output (an implication of the envelope theorem). However, this is not true more generally when there are labor market distortions.

The important new result in equation (9) is that the rent dissipation effects of automation worsen allocative efficiency and reduce aggregate TFP. As a result, the cost-saving gains from automation overstate the true contribution of these technological advances to TFP. The intuition for this result is as follows. Suppose that $\mu_{\mathcal{A}_g} > \mu_g$ so that automation is targeted at high-rent jobs. The automation of these tasks generates two effects. On the one hand, it reduces the cost of producing these tasks by π_g, which raises aggregate TFP. On the other hand, it reallocates workers away from high-wage tasks where they also had a high marginal revenue product to other tasks where their marginal revenue product is lower. The reallocation of workers towards low marginal revenue product tasks has a first-order negative impact on output and reduces TFP.

To illustrate this point, consider an example with a single type of worker who can perform two tasks: high-rent tasks ("welding") and no-rent tasks ("delivery driver"). Suppose welding jobs pay a rent of 20% ($\mu_{\text{welding}} = 1.2$) above delivery drivers ($\mu_{\text{delivery}} = 1$) and let w be the base wage of the group. Because firms are on their labor demand curve, they initially hire welders up to the point at which their marginal revenue product at welding exceeds the marginal revenue product of a delivery driver by 20%. From an efficiency point of view, we had too many delivery drivers and not enough welders. Imagine now that advances in robotics allow firms to automate some welding jobs at a cost of κ per robotic welding system, and to simplify the calculations, assume that all displaced workers become delivery drivers. The automation of a welding job generates positive cost savings of $\pi = \mu_{\text{welding}} \cdot w - \kappa > 0$ per job. However, the marginal revenue product of welders who are now pushed to become delivery drivers falls by 20%, reducing output per worker and surplus. The change in surplus from automating a welding job is, therefore, $\pi - 0.2 \cdot w = w - \kappa$, which is below π.

The example highlights several important aspects of the economics behind the allocative efficiency term in equation (9):

1. The result depends on the fact that firms are on their labor demand curve, so they under-hired welders to begin with. As discussed in the introduction, this requires rents to distort firms' hiring decisions.

2. The worsening allocative efficiency due to rent dissipation can dominate cost savings and reduce TFP on the net. In the example, this is true when $\mu_{\text{welding}} \cdot w > \kappa > w$. In this case, firms will still automate welding jobs since $\pi = \mu_{\text{welding}} \cdot w - \kappa > 0$.

3. There is a divergence between private and social incentives to automate. In the example, a firm automates welding jobs if $\pi = \mu_{\text{welding}} \cdot w - \kappa > 0$. Instead, from a social point of view, automation decisions should be based on a comparison between the *opportunity cost of welders* vs. that of the robotic welding system, $w - \kappa$. The allocative efficiency term in the TFP equation (9) can be seen as adjusting for the divergence between private cost savings internalized by firms and the difference in opportunity cost between welders and the robotic system.[17]

Turning to the general case with ripples, the expression for TFP changes in equation (7)

[17] This divergence is also helpful for understanding why a common counterargument to the logic in equation (7) is incorrect. The counterargument goes as follows: "rents constrain the production of welding, making the quantity of welding a bottleneck for productivity. An automated welding system that circumvents rents should therefore bring even larger productivity gains; we do not really care if the much-needed welding is done by machines or humans so long as we do more of it." This argument is incorrect because it ignores opportunity costs. We *do care* if the extra welding is done by machines or humans since they have different opportunity costs. The opportunity cost of the machine is the extra consumption we gave up to produce it (κ in the example). The opportunity cost of the welder is their marginal revenue product working as a delivery driver (w in the example). If the opportunity cost of the worker is lower than the cost of the machine, we want workers, not machines, to do the welding.

shows that allocative efficiency changes due to rent dissipation or because ripples reallocate workers away from high-rent tasks. This is summarized by the term $\sum_g s_g \cdot d\ln\mu_g$, where $d\ln\mu_g$ gives the change in group rents coming from rent dissipation or task reallocation.

2.5 Multi-Sector Economy

We conclude the theory by extending Proposition 5 to a multi-sector economy. This exercise clarifies how automation in one sector generates indirect effects not just via the ripple effects but also by changing sectoral prices. This can also affect rents if rents vary across sectors (as in Katz and Summers, 1989). This extension also guides our measurement of the direct task displacement experienced by workers, which varies significantly across industries.

The setup follows our one-sector model, except that there are multiple sectors indexed by i with production functions

$$y_i = \left(\frac{1}{M_i}\int_{x\in\mathcal{T}_i}(M_i\cdot y_x)^{\frac{\lambda-1}{\lambda}}\cdot dx\right)^{\frac{\lambda}{\lambda-1}}.$$

The sets of tasks in each sector $\mathcal{T}_i$ are non-overlapping. The production of task x is the same as before. The output from these sectors is combined into a final good that can be used for consumption or to build machinery and equipment. We denote this final good by y so that the resource constraint of the economy is again $y = c + k$, where $k = \sum_i \int_{x\in\mathcal{T}_i}(k_x/q_x)\cdot dx$. The transformation of sectoral output into the final good is described by a CES production function with elasticity of substitution $\eta \geq 0$. We denote sectoral prices by p_i and normalize the price of the final good to 1.

The labor market is similar to our one-sector model, with the difference that each worker group is now allocated to tasks across multiple sectors. In particular, an equilibrium is now given by an industry-specific allocation of tasks $\{\mathcal{T}_{gi}\}_{i,g}, \{\mathcal{T}_{ik}\}_i$ to workers and capital.

Given this allocation, define group g task share (Γ_g) and industry i task share (Γ_{gi}) as

$$\Gamma_g = \sum_i s_{y_i}\cdot p_i^{\lambda-1}\cdot\Gamma_{gi}, \qquad \Gamma_{gi} = \frac{1}{M_i}\int_{x\in\mathcal{T}_{gi}}\psi_{gx}^{\lambda-1}\cdot\mu_{gx}^{-\lambda}\cdot dx,$$

where s_{y_i} denotes the share of sector i in value added. Likewise, define group g rents (μ_g) and industry i rents (μ_{gi}) by

$$\mu_g = \frac{1}{\ell_g}\sum_i \ell_{gi}\cdot\mu_{gi}, \qquad \mu_{gi} = \frac{1}{\ell_{gi}}\int_{x\in\mathcal{T}_{gi}}\ell_{gx}\cdot\mu_{gx}\cdot dx,$$

where ℓ_{gi} is group employment in industry i.

Proposition A2 in the Appendix shows that, with these definitions in place, group base and average wages are given by equations (1) and (2).

As before, we introduce several objects that summarize the effects of an exogenous advance in automation in a small set of tasks $\mathcal{A}^T$ across groups of workers and industries. Denote by $\mathcal{A}_{gi}$ the set of automated tasks in industry i performed by g:

$$\mathcal{A}_{gi} = \left\{ x \in \mathcal{A}^T \cap \mathcal{T}_{gi} : \frac{w_g \cdot \mu_{gx}}{\psi_{gx}} \geq \frac{1}{q'_x \cdot \psi_{kx}} \right\}$$

Denote the direct task displacement in industry i experienced by group g by

$$d \ln \Gamma^d_{gi} = \frac{\int_{x \in \mathcal{A}_{gi}} \ell_{gx} \cdot dx}{\int_{x \in \mathcal{T}_{gi}} \ell_{gx} \cdot dx},$$

and the direct task displacement experienced by group g across all industries by

$$d \ln \Gamma^d_g = \sum_i \frac{\ell_{gi}}{\ell_g} \cdot d \ln \Gamma^d_{gi}.$$

Denote the cost-saving gains from automating tasks from group g in industry i by

$$\pi_{gi} = \frac{\int_{x \in \mathcal{A}_{gi}} \ell_{gx} \cdot \mu_{gx} \cdot \pi_{gx} \cdot dx}{\int_{x \in \mathcal{A}_{gi}} \ell_{gx} \cdot \mu_{gx} \cdot dx} \geq 0.$$

Finally, denote the average rent in automated tasks $\mathcal{A}_{gi}$ for group g in industry i by

$$\mu_{\mathcal{A}_{gi}} = \frac{\int_{x \in \mathcal{A}_{gi}} \ell_{gx} \cdot \mu_{gx} \cdot dx}{\int_{x \in \mathcal{A}_{gi}} \ell_{gx} \cdot dx},$$

and the average rent in automated tasks for group g averaged across all industries by

$$\mu_{\mathcal{A}_g} = \sum_i \frac{\ell_{gi}}{\ell_g} \cdot \mu_{\mathcal{A}gi}.$$

Proposition 6 *Consider an exogenous advance in automation technologies in a small set* $\mathcal{A}^T$ *with direct effects* $\left\langle \{d \ln \Gamma_{gi}\}_{g,i}, \{\pi_{gi}\}_{g,i}, \{\{\mu_{\mathcal{A}_{gi}}\}_{g,i}\} \right\rangle$ *across groups and industries. This* *advance changes wages by*

$$d\ln w_g = \Theta_g \cdot stack\left(\frac{1}{\lambda}\cdot d\ln y - \frac{1}{\lambda}\cdot d\ln\Gamma^d_j + \frac{1}{\lambda}\cdot\sum_i \frac{\ell_{ij}}{\ell_j}\cdot d\ln\zeta_i\right) \tag{10}$$

$$d\ln\mu_g = \mathcal{M}_g \cdot stack\left(\frac{1}{\lambda}\cdot d\ln y - \frac{1}{\lambda}\cdot d\ln\Gamma^d_j + \frac{1}{\lambda}\cdot\sum_i \frac{\ell_{ij}}{\ell_j}\cdot d\ln\zeta_i\right) - \left(\frac{\mu_{\mathcal{A}_g}}{\mu_g}-1\right)\cdot d\ln\Gamma^d_g + \sum_g\left(\frac{\mu_{gi}}{\mu_g}-1\right)\cdot\frac{\ell_{gi}}{\ell_g}\cdot d\ln\zeta_i \tag{11}$$

$$d\ln\bar{w}_g = (\Theta_g + \mathcal{M}_g)\cdot\ stack\left(\frac{1}{\lambda}\cdot d\ln y - \frac{1}{\lambda}\cdot d\ln\Gamma^d_j + \frac{1}{\lambda}\cdot\sum_i \frac{\ell_{ij}}{\ell_j}\cdot d\ln\zeta_i\right) - \left(\frac{\mu_{\mathcal{A}_g}}{\mu_g}-1\right)\cdot d\ln\Gamma^d_g + \sum_g\left(\frac{\mu_{gi}}{\mu_g}-1\right)\cdot\frac{\ell_{gi}}{\ell_g}\cdot d\ln\zeta_i. \tag{12}$$

The sectoral shifters satisfy $d\ln\zeta_i = (\lambda-\eta)\cdot\left(\sum_g s_{gi}\cdot d\ln w_g - \sum_g s_{gi}\cdot\frac{\mu_{\mathcal{A}gi}}{\mu_{gi}}\cdot d\ln\Gamma^d_{gi}\cdot\pi_{gi}\right)$*, where* s_{gi} *is the share of labor* g *in industry* i *value added. Average wage changes are then pinned down by the dual Solow formula*

$$\sum_g s_g\cdot d\ln\bar{w}_g = d\ln tfp, \tag{13}$$

and the change in aggregate TFP is given by

$$d\ln tfp = \sum_i s_{y_i}\cdot\sum_g s_{gi}\cdot\frac{\mu_{\mathcal{A}gi}}{\mu_{gi}}\cdot d\ln\Gamma^d_{gi}\cdot\pi_{gi} + \sum_g s_g\cdot d\ln\mu_g. \tag{14}$$

Relative to Proposition 5, the extended formulas capture the possibility that automation can also affect group wages and rents by shifting the sectoral composition of the economy.

To illustrate this point, consider again an economy with no ripple effects. Equation (5) shows that average wage changes for group g are given by

$$d\ln\bar{w}_g = \underbrace{\frac{1}{\lambda}\cdot d\ln y - \frac{1}{\lambda}\cdot d\ln\Gamma^d_g + \frac{1}{\lambda}\cdot\sum_i\frac{\ell_{ig}}{\ell_g}\cdot d\ln\zeta_i}_{\text{effects in competitive labor market}} \underbrace{-\left(\frac{\mu_{\mathcal{A}_g}}{\mu_g}-1\right)\cdot d\ln\Gamma^d_g}_{\substack{\text{rent dissipation}\\ \text{due to automation}}} + \underbrace{\sum_i\left(\frac{\mu_{gi}}{\mu_g}-1\right)\cdot\frac{\ell_{gi}}{\ell_g}\cdot d\ln\zeta_i.}_{\substack{\text{rent changes due to}\\ \text{sectoral shifts}}} \tag{15}$$

The sectoral shifts induced by automation and captured by the shifters $d\ln\zeta_i$'s affect wages via two channels: by changing the demand for labor (the third term in (15)) and by reallocating workers across sectors with different rent levels (the last term).[18]

Proposition 6 provides formulas for the effects of advances in automation on wages, sectoral prices, output, and welfare in terms of the direct effects summarizing the shock $\left\langle \{d\ln\Gamma_{gi}\}_{g,i}, \{\pi_{gi}\}_{g,i}, \{\{\mu_{\mathcal{A}_{gi}}\}_{g,i}\}\right\rangle$, elasticities, the propagation and rent impact matrices, and initial shares. These formulas form the basis for our quantitative analysis in Section 4.

3 Reduced-Form Evidence

This section presents reduced-form evidence on the impact of automation on US worker groups between 1980 and 2016. Our analysis examines various outcomes, including average group wages, wage dispersion within groups, and rents, measured using various proxies.

We build on the empirical work in Acemoglu and Restrepo (2022) and draw on their data sources.[19] We conduct our analysis for 500 detailed demographic groups in the US, defined by education (5 categories), gender (2 categories), age (5 categories), race or ethnicity (5 categories), and immigrant status (2 categories). For each of these groups, we obtain average real hourly wages in 1980 and 2016 from the Census and the American Community Survey. In addition, for each group we compute the p-th percentile of real (log) hourly wages $d\ln w_g^p$ for $p = 5, 10, \ldots, 95, 99$ both in 1980 and in 2016 among employed workers.

We first focus on the effects of automation on groups directly exposed to it, disregarding ripple and general equilibrium effects, which we explore in Section 4. Equation (15) shows that with no ripples the outcomes of group g depend on the direct task displacement it experiences. Motivated by this equation, we estimate specifications of the form

$$\text{Change in group } g \text{ outcome 1980–2016} = \beta \cdot \text{task displacement}_g^d + X_g \cdot \gamma + u_g, \tag{16}$$

where task displacement$_g^d$ is a measure of the direct task displacement experienced by group g between 1980 and 2016 (the empirical analog of $d\ln\Gamma_g^d$ in the theory) and X_g is a vector of group-level covariates. We interpret β as the reduced-form relationship between the direct

[18]In the multi-sector economy, the propagation and rent impact matrices capture the change in task allocations and the resulting change in average rents and task shares following an increase in $w_{g'}$ holding sectoral prices constant. The new terms in equation (15) capture the role of changing sectoral prices.

[19]Because our data sources are the same as in Acemoglu and Restrepo (2022), we refer the reader to that paper for more details on data sources and construction.

task displacement from automation experienced by a group and its outcomes.[20]

3.1 Measuring direct task displacement

Our first objective is measuring the direct task displacement created by automation on US demographic groups between 1980 and 2016. We follow the strategy in Acemoglu and Restrepo (2022), which relies on the following assumption:

ASSUMPTION 1 (MEASUREMENT ASSUMPTION) *During 1980–2016, only routine tasks were automated and, within an industry, different groups of workers were displaced by automation from routine tasks at a common rate.*

Appendix C shows that under this assumption, the direct task displacement experienced by a group can be estimated as

$$\text{td}_g^d = \sum_i \frac{\ell_{gi}}{\ell_g} \cdot \text{RCA}_{gi}^{\text{routine}} \cdot \frac{-d\ln s_{\ell i}^d}{1 + s_{\ell i} \cdot (\lambda - 1) \cdot \pi_i} \cdot \frac{\mu_{gi}}{\mu_{\mathcal{A}_{gi}}}. \tag{17}$$

This formula expresses the direct task displacement experienced by a group as a sum across industries of three terms:

- ℓ_{gi}/ℓ_g represents group g's exposure to industry i, which captures how important this industry is for the group's employment.
- $\text{RCA}_{gi}^{\text{routine}}$ is a measure of the revealed comparative advantage of group g at routine jobs in industry i. This term apportions the incidence of automation in the industry across groups of workers based on who performs more routine tasks. This apportioning rule follows from Assumption 1, which we see as a reasonable description of the capabilities of automation technologies during this period.
- $-d\ln s_{\ell i}^d$ is a summary measure of automation at the industry level. It is the percent reduction of the labor share in industry i due to advances in automation from 1980 to 2016, which in our model is tightly linked to the task displacement generated by automation in the industry. The labor share decline is re-scaled by two terms: $1+s_{\ell i}\cdot(\lambda-1)\cdot\pi_i$, which captures the effects of automation on the industry labor share working through substitution across tasks (and not due to the direct displacement

[20]We provide weighted OLS estimates, with groups weighted by their 1980 share in the US workforce. We also report robust standard errors that account for heteroskedasticity in u_g.

of workers at automated tasks); and the term $\mu_{\mathcal{A}_{gi}}/\mu_{gi}$, which gives the average rent dissipation in industry i and accounts for the fact that part of the decline in the labor share is due to rent dissipation.

In our reduced-form analysis we construct our measure of direct task displacement by setting $\lambda = 0.5$, $\mu_{\mathcal{A}_{gi}}/\mu_{gi} = 1.5$, and $\pi_i = 30\%$ for all industries. These choices are explained in our quantitative analysis in the next section.

We measure direct task displacement using data for 49 industries that we can trace consistently in the US Census and the BEA Integrated Industry-Level Production Accounts. We obtain employment shares by industry ℓ_{gi}/ℓ_g from the 1980 US Census. We also obtain $\text{RCA}_{gi}^{\text{routine}}$ from the 1980 Census and after defining routine jobs as the top 33% occupations with the highest routine content according to ONET. Finally, following Acemoglu and Restrepo (2022), we estimate $d \ln s_{\ell i}^{d}$ from a cross-industry regression of percent changes in the labor share by industry (from the BEA, from 1987 to 2016 and re-scaled to a 36-year change) against three proxies for automation: the adjusted penetration of industrial robots (from Acemoglu and Restrepo, 2020), the increase in the share of specialized software services in value added, and the increase in the share of dedicated machinery in value added (from BLS Total Multifactor Productivity tables).

The left panel of Figure 3 summarizes the industry-level data. The black bars show the observed reduction in industry's labor shares in percent terms. The orange bars show the component attributed to our three proxies of automation, which jointly explain 50% of the cross-industry changes in labor shares since 1987.[21]

The right panel of Figure 3 provides our measure of direct task displacement from automation for all US demographic groups, sorted against their baseline hourly wages in 1980 in the horizontal axis. Groups with post-college degrees experienced almost no direct task displacement between 1980–2016. Direct task displacement concentrates on groups of workers at the middle and lower middle of the wage distribution, where some groups are estimated to have lost 15% to 20% of the tasks they performed in 1980 to automation.[22]

[21] As shown in Acemoglu and Restrepo (2022), our proxies for automation continue to explain near 50% of the cross-industry variation after controlling for other determinants of the labor share decline, including declining unionization rates, estimates of changes in markups, and changes in industry concentration.

[22] The measure of direct task displacement in (17) is the same as the one in Acemoglu and Restrepo (2022), except for the term $\mu_{\mathcal{A}_{gi}}/\mu_{gi}$, since our previous work did not consider the role of rents. There are also minor differences in weights that are explained in Appendix C.

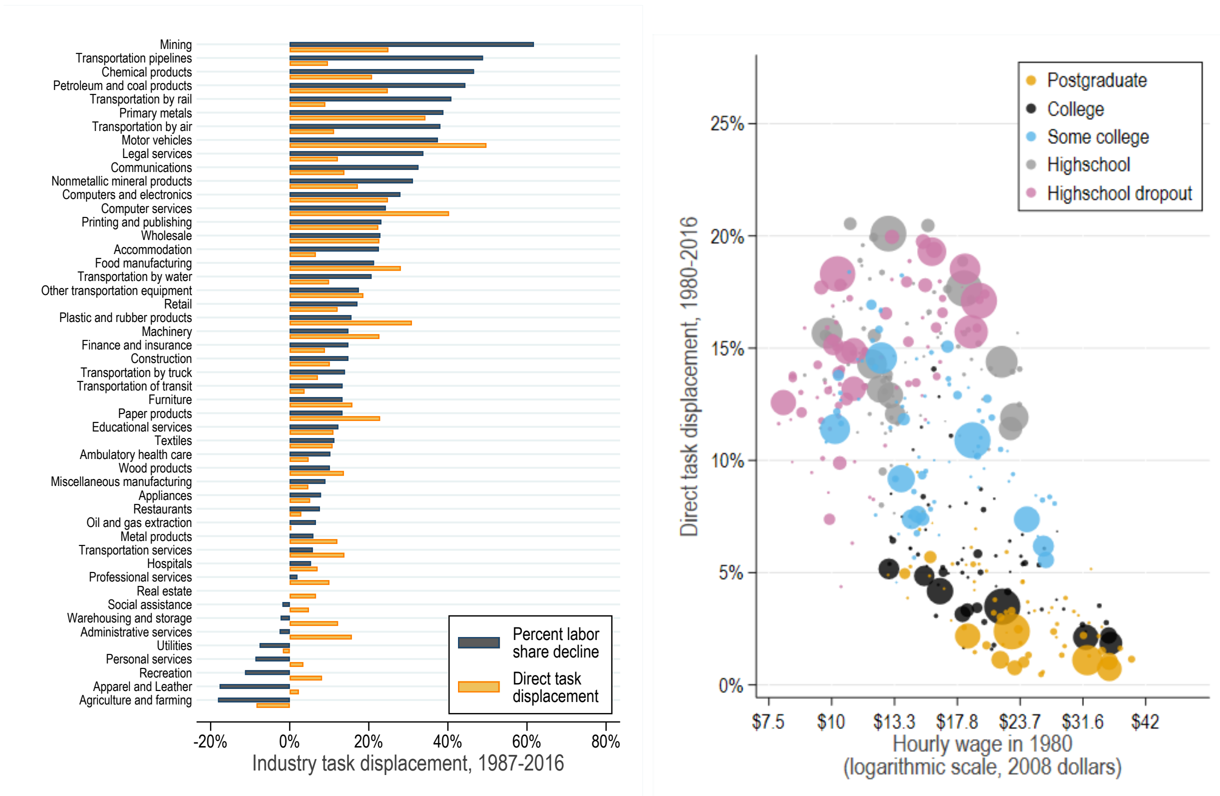

FIGURE 3: DIRECT TASK DISPLACEMENT DUE TO AUTOMATION ACROSS INDUSTRIES AND GROUPS. the left panel plots the observed labor share decline from 1987 to 2016 in percent terms across US industries. The orange bars denote the component attributed to our three proxies of automation. The right panel plots our measure of direct task displacement from equation (17) across 500 detailed demographic groups of US workers between 1980–2016.

3.2 Automation and average group wages

As a starting point, we inspect the relationship between the change in average group wages and the direct task displacement experienced by US demographic groups. We estimate equation (16) using the change in the log of average group wages between 1980 and 2016 as the outcome variable. This specification essentially reproduces the analysis in Acemoglu and Restrepo (2022) and is helpful for bench-marking our new reduced-form findings.

The left panel in Figure 4 plots the bivariate relation between change in average group wages and direct task displacement. We see a sizable and robust negative relationship. A 10 percentage point increase in direct task displacement is associated with a 24% reduction in group relative wages. In a reduced-form sense, our measure of task displacement explains 66% of the variation in wages between demographic groups in the US since 1980.

The right panel in Figure 4 plots the relation between change in average group wages

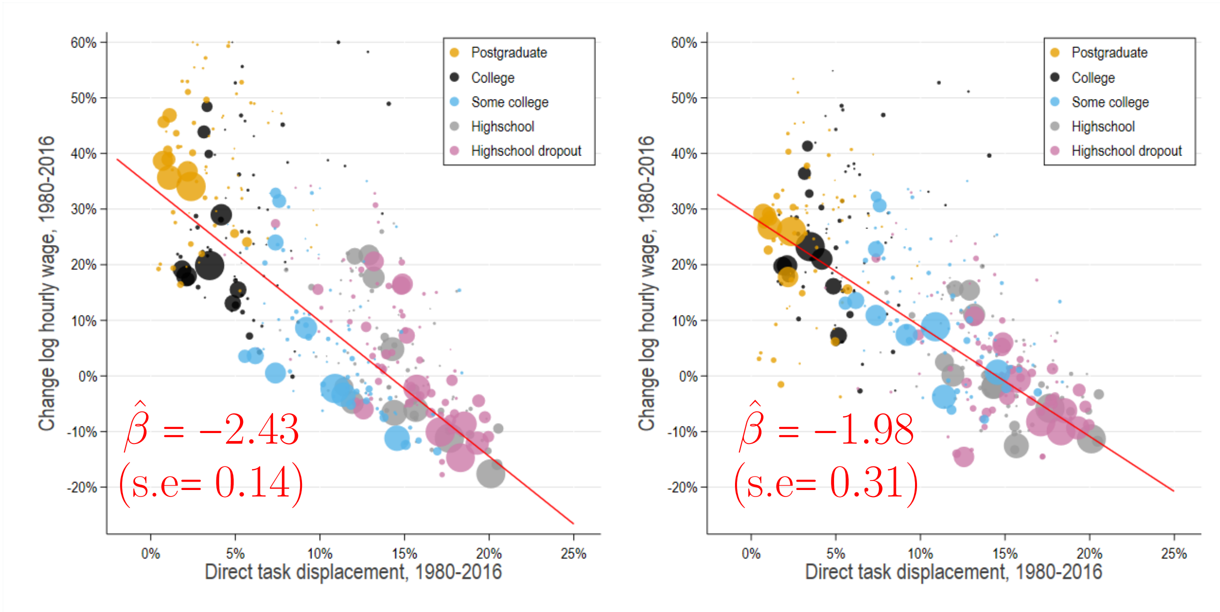

FIGURE 4: REDUCED-FORM RELATIONSHIP BETWEEN AVERAGE GROUP-LEVEL WAGE CHANGES AND TASK DISPLACEMENT. The left panel plots the bivariate relationship between change in group average wages and task displacement. The right panel partials out covariates, including gender and education dummies, sectoral demand and rent shifters, and the manufacturing employment share of groups in 1980.

and direct task displacement controlling for covariates. We control for gender and education dummies, which account for other forms of skill biased technical change favoring college educated workers. Motivated by equation (15), we control for two measures that summarize the influence of changes in the sectoral composition of the economy (presumably due to forces other than automation) on the wage structure. These include a measure of groups' exposure to expanding industries, constructed as

$$\text{sectoral demand shifters}_g = \sum_i \frac{\ell_{gi}}{\ell_g} \cdot \Delta \ln(p_i \cdot y_i),$$

which proxies for the term $\sum_i (\ell_{gi}/\ell_g) \cdot d \ln \zeta_i$ in (15), and a measure of groups' change in rents due to sectoral shifts, constructed as

$$\text{sectoral rent shifters}_g = \sum_i \frac{\ell_{gi}}{\ell_g} \cdot \left(\frac{\bar{w}_{gi}}{\bar{w}_g} - 1 \right) \cdot \Delta \ln(p_i \cdot y_i),$$

where $\bar{w}_{gi}/\bar{w}_g$ denotes the ratio between the average wage earned by the group in industry i and the average group wage.[23] This proxies for the term $\sum_i \left(\frac{\mu_{gi}}{\mu_g} - 1 \right) \cdot \frac{\ell_{gi}}{\ell_g} \cdot d \ln \zeta_i$ in (15).

[23]For the sectoral shifters, we measure ℓ_{gi}/ℓ_g as the share of hours worked in each industry by the group in 1980, and the change in value added in the industry $\Delta \ln(p_i \cdot y_i)$ using the BEA industry accounts for

We also control for the employment share of each group in manufacturing in 1980, just to show that we are not simply capturing common shocks hitting that sector.

We continue to estimate a negative relationship between group wages and direct task displacement. The point estimate of $\hat{\beta} = -1.98$ implies that a 10 percentage point increase in task displacement from automation is associated with a 20% reduction in groups' relative wages. In a reduced-form sense, our measure of direct task displacement now explains 53% of the wage variation across US demographic groups since 1980.[24]

The evidence in Figure 4 supports the idea that automation reduces the (relative) wages of directly exposed groups. This can be due to its displacement effect or the new rent dissipation mechanism. We now provide evidence in support of a significant role for rent dissipation. First, we show that automation creates a decline in wage dispersion within exposed groups of workers, as predicted by Proposition 3. Second, we construct different proxies for group rents and show that automation is associated with a decline in rents.

3.3 Automation and declining within-group wage dispersion

This section documents a novel reduced-form relationship between groups' exposure to automation and reductions in within-group wage dispersion. Motivated by Proposition 3, we estimate a version of equation (16) where the outcome variable is the change in log wages at the p-th percentile of the within-group wage distribution between 1980 and 2016, $\Delta d \ln w_g^p$, for a range of percentiles.[25]

Figure 5 reports the estimated change in log wages at the $5th, 10th, \ldots, 95th, 99th$ percentiles of the within-group wage distribution associated with the direct task displacement experienced by the group. To facilitate the interpretation, we report all estimates relative to the change at the 30th percentile. The black line reports estimates from a specification that controls for sectoral demand shifters. The remaining lines report estimates that control for sectoral rent shifters (orange line), gender and education dummies (blue line), and groups' exposure to manufacturing (pink line)

1987–2016. As with our measures of direct task displacement, this is then converted to a 36-year equivalent change. For the sectoral rent shifter, wages are measured using the 1980 Census so that they reflect the initial distribution of rents.

[24]Table A1 in Appendix C summarizes our empirical findings and provides additional specifications.

[25]This is equivalent to an unconditional group-quantile regression, as in Chetverikov et al. (2016). It is important to clarify here that the estimates obtained with this approach are informative of where wage reductions take place at the interior of an exposed group (i.e., is automation reducing within-group wage dispersion). They are not indicative of where in the *national* wage distribution automation is displacing workers (i.e., is automation reducing overall wage inequality).

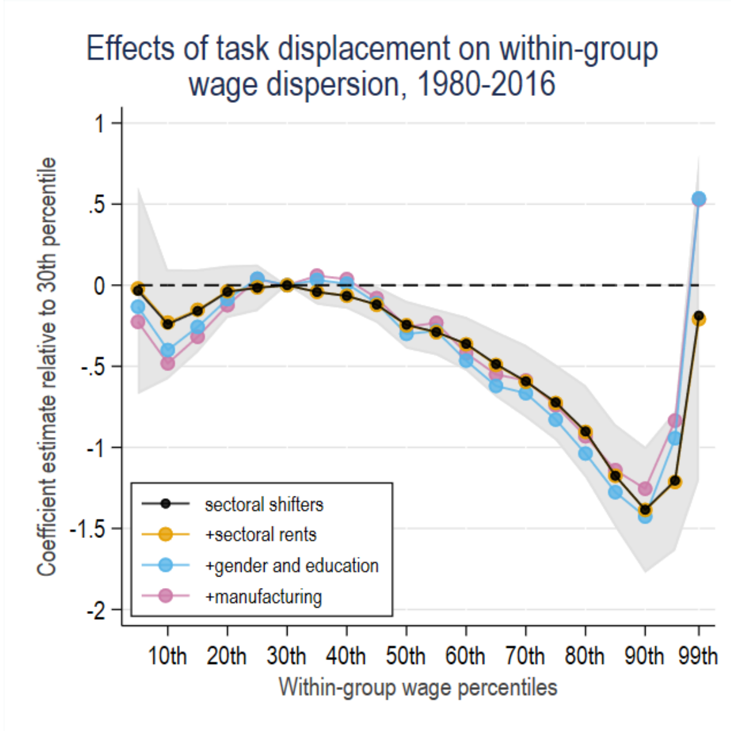

Figure 5: Reduced-form relationship between wage changes across percentiles of the within-group wage distribution and task displacement. The figure plots estimates from a group quantile regression of changes in $d \ln w_g^p$ against task displacement for percentiles p ranging from the 5th to the 99th relative to the 30th percentile. The lines provide estimates for different specifications. The confidence intervals test for a difference in effects relative to the 30th percentile.

Our estimates show that groups directly exposed to automation saw a more pronounced decline in wages among workers in the 70th to 95th percentile of the within-group wage distribution. A 10 percentage point increase in direct task displacement is associated with a 5% to 15% reduction in wages earned by workers in the 70th to 95th percentile of the within-group distribution relative to the 30th percentile. Instead, we estimate less pronounced declines at the bottom and very top of the within-group distribution.[26]

The results from Figure 5 support the idea that automation reduces within-group wage dispersion via rent dissipation. Even though the pattern in the data is precisely what our theory predicts, this is not what many would have expected. It is often assumed that automation (and technological progress in general) primarily benefits the highest-paid

[26]The main concern here is that our results could be driven by unobserved heterogeneity within groups. For example, one alternative explanation for the U-pattern is that low-wage groups are composed of minimum wage workers at the bottom (whose wages cannot change by much) and more productive workers at the top in mid-pay jobs that are more likely to be automated. To address this concern, we re-estimated the quantile regression for groups with an average real wage above \$13 in 1980 (i.e., all groups to the right of the hump in Figure 5) and obtained similar findings, reported in Figure A1 in Appendix C.

workers at the top of exposed groups. Our finding of the opposite pattern makes our evidence for rent dissipation more telling.

3.4 Automation and rents

The rent dissipation mechanism implies that groups directly exposed to automation will be pushed away from high-rent jobs and towards low-rent paying jobs. We now explore the association between groups' exposure to automation and different proxies for rents.

Our first proxy for rents is motivated by the group-quantile regressions in Figure 5. The figure shows a more pronounced decline in wages starting at the 30th percentile of the within-group wage distribution, and a more uniform decline below these percentiles. The gray confidence bars in Figure 5 confirm this and show that the wage reductions at the 30th percentile do not differ systematically from those below it or at the very top, but are above the reductions estimated at the 50th to 95th percentiles.[27] From the viewpoint of our model, wage changes at the 30th percentile of the within-group distribution can be interpreted as the effects of automation on groups' base wage, while the more pronounced decline at higher percentiles would be interpreted as the loss of high-rent jobs.

Motivated by this reasoning, we proxy the change in group rents by

$$\Delta \ln \mu_g = \Delta \ln \bar{w}_g - \Delta \ln w_g^{30th}.$$

This measure interprets rising wage compression around the 30th percentile of the within-group wage distribution as evidence of rent dissipation.

Figure 6 provides estimates of equation (16) using this proxy for rents as dependent variable. The left panel starts with the bivariate relationship and the right panel shows that the same relationship applies when we control for gender and education dummies, sectoral demand and rent shifters, and the share of employment of each group in manufacturing in 1980. Both panels show a sizable and robust negative relationship between direct task displacement and our proxy for rents, suggesting that automation is associated with rising wage compression. According to this proxy, a 10 percentage point increase in task displacement for a demographic group is associated with a 4% reduction in group rents. The point estimate of $\hat{\beta} = -0.4$ in the right panel implies that automated jobs pay an average rent of

[27] Our estimates show a minor non-monotonic pattern at the bottom. Figure A1 shows that this is driven by low-wage groups for whom minimum wages or other forms of wage floors might be binding. Excluding groups with an average real wage below $13 dollars in 1980 (i.e., all groups to the left of the hump in Figure 5) leads to a flatter pattern below the 30th percentile.

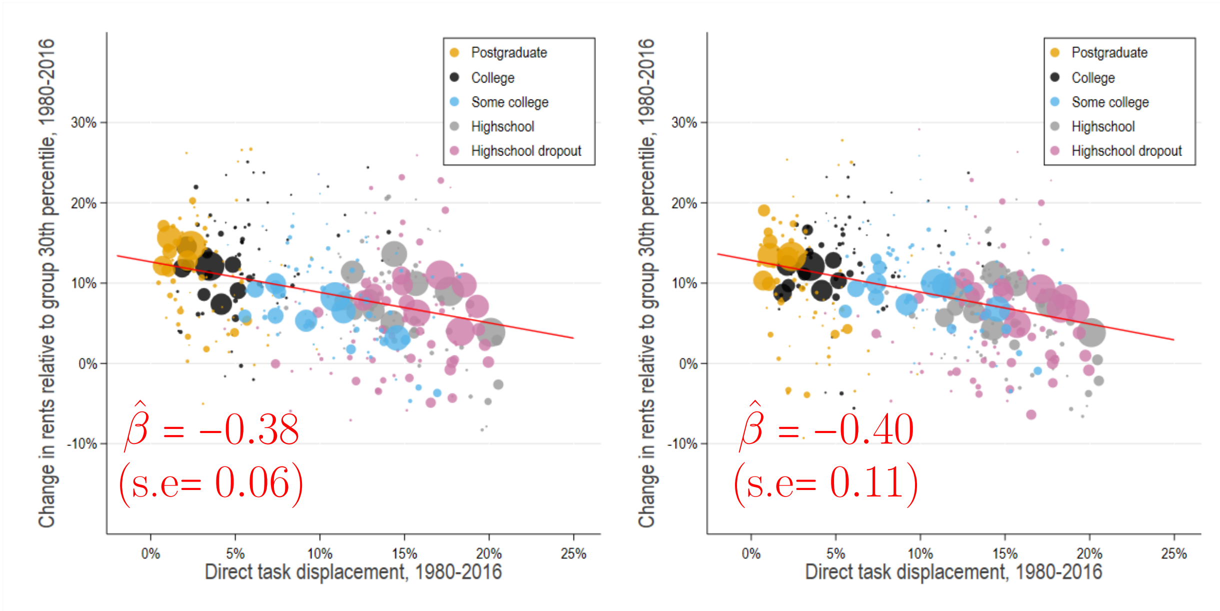

FIGURE 6: REDUCED-FORM RELATIONSHIP BETWEEN RENTS AND TASK DISPLACEMENT. The left panel plots the bivariate relationship between change in group rents and task displacement. The right panel partials out covariates, including gender and education dummies, sectoral demand and rent shifters, and the manufacturing employment share of groups in 1980. Rents are proxied by the gap between mean wages and wages at the 30th percentile of the within-group distribution, $\Delta \ln \mu_g = \Delta \ln \bar{w}_g - \Delta \ln w_g^{30th}$.

40% above others, which point to a sizable rent dissipation due to automation.

Our second strategy for measuring the importance of rent dissipation uses proxies for rents paid in 1980. Our theory predicts that group rents decline because automation shifts the composition of employment away from high-rent paying jobs. We measure part of the contribution of this reallocation away from rent-paying jobs to group wages by

$$\Delta \ln \mu_g^{\text{reallocation}} = \sum_{i,o} \left(\frac{\bar{w}_{gio}}{\bar{w}_g} - 1 \right) \cdot \Delta \ell_{g,i,o},$$

where $\frac{\bar{w}_{gio}}{\bar{w}_g}$ are estimates of the rents earned by group g at jobs in industry i and occupation o in 1980 and $\Delta \ell_{g,i,o}$ is the change in hours worked at industry i and occupation o by worker group g between 1980 and 2016.[28]

[28]In our model, $\frac{\bar{w}_{gio}}{\bar{w}_g}$ equals the average rent paid to group g in industry i and occupation o. More generally, these baseline wage differences within detailed demographic groups could also reflect selection of workers with unobserved skills into different jobs or compensating differentials. However, following a substantial literature in this area (Katz and Summers, 1989; Stansbury and Summers, 2020), it is reasonable to presume that these wage differences also reflect rents. For example, Katz and Summers (1989) show that industries that paid above average wages see lower quit rates and have longer worker queues, which is consistent with a rent interpretation. An alternative approach for future work, would use matched employer-employee data to estimate firm-specific rents and explore if automation shifts workers away from

Figure 7 provides estimates of equation (16) using $\Delta \ln \mu_g^{\text{reallocation}}$ as dependent variable. The left panel starts with the bivariate relationship and the right panel shows that the same relationship applies when we control for gender and education dummies, sectoral demand and rent shifters, and the share of employment of each group in manufacturing in 1980. Both panels show a sizable and robust negative relationship between direct task displacement and rents. A 10 percentage point increase in direct task displacement is associated with a 4.1% reduction in rents. The point estimate of $\hat{\beta} = -0.41$ in the right panel implies that automated jobs pay an average rent of 41% above others, which align with the estimates obtained for our first proxy.[29]

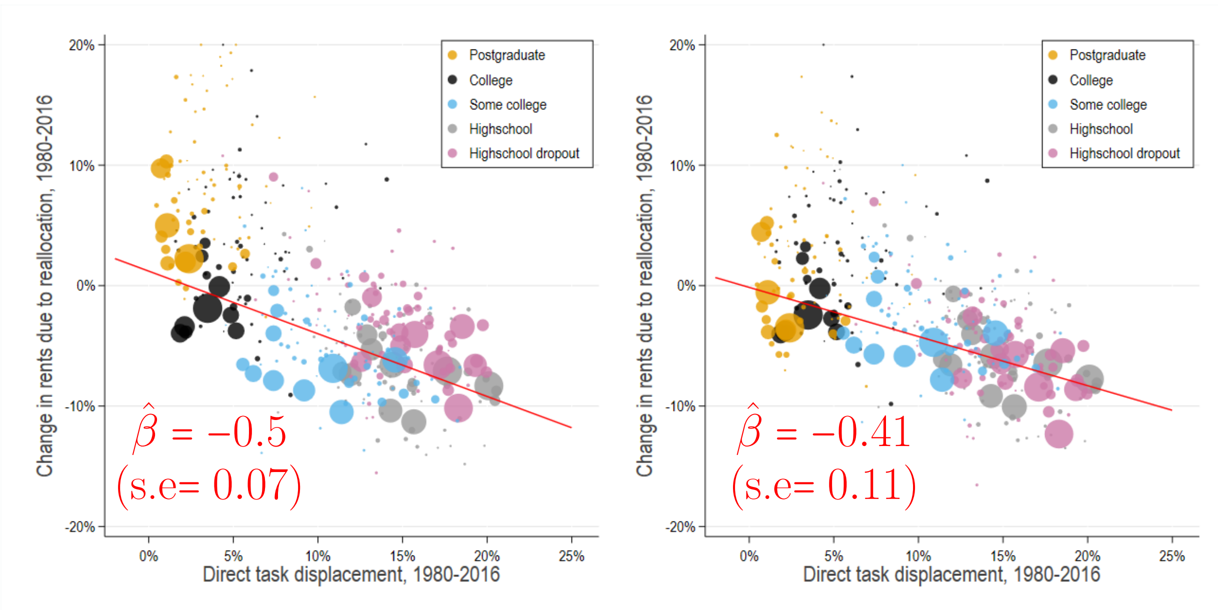

FIGURE 7: REDUCED-FORM RELATIONSHIP BETWEEN RENTS AND TASK DISPLACEMENT. The left panel plots the bivariate relationship between change in group rents and task displacement. The right panel partials out covariates, including gender and education dummies, sectoral demand and rent shifters, and the manufacturing employment share of groups in 1980. Rents are proxied by the change in employment at industry and occupations that paid groups above-average wages in 1980, $\Delta \ln \mu_g^{\text{reallocation}} = \sum_{i,o} \left(\frac{\bar{w}_{gio}}{\bar{w}_g} - 1 \right) \cdot \Delta \ell_{g,i,o}$.

An interesting pattern revealed by Figure 7 is that most worker groups have shifted away from high pay industries and occupations over time, with the exception of workers with a post-college degree. Our theory and evidence suggest that automation might be an important driver of this phenomenon. Our measure of direct task displacement explains

high-rent paying firms.

[29]We obtained very similar results if we use a measure of average rents paid by industry and occupation in 1980 across all groups instead of group-specific rents. This alleviates concern about measurement error in $\frac{\bar{w}_{gio}}{\bar{w}_g}$ for small cells. These results are reported in Table A1 in Appendix C.

38% of the variation in the shift away from high-rent jobs across worker groups in the left panel and 31% in the right panel.

Taking stock, our reduced form estimates suggest that a 10 pp increase in task displacement due to automation is associated with a reduction in relative wages of 20% for groups directly exposed. Of these, 4 pp are due to rent dissipation. Our rent proxies provide complementary information. Our first proxy shows that automation compresses group wages around its 30th percentile. The second proxy shows that this compression is driven by the shift of exposed groups away from high-rent paying jobs. Taken together, the evidence in this section supports the view that the reduction or stagnation in real wages experienced by some groups of US workers since 1980 could be due to the loss of high-rent jobs brought by automation. The next section quantifies these effects accounting for ripples and general equilibrium forces.

4 General Equilibrium Effects of Automation: A Quantitative Exploration

Proposition 6 provides formulas for the general equilibrium effects of automation on average group wages, rents, output, TFP, and welfare. The formulas show that these effects can be computed from:

(i) measures of direct task displacement experienced by all groups, both on aggregate $d\ln\Gamma_g^d$ and within industries $d\ln\Gamma_{gi}^d$;

(ii) measures of rent dissipation created by automation, summarized by $\mu_{\mathcal{A}_g}/\mu_g$;

(iii) measures of cost-saving gains from automation, π_{gi};

(iv) estimates of $\{\lambda, \eta\}$, the propagation matrix, Θ and the rent impact matrix $\mathcal{M}$.

We think of the quantities in (i)–(iii) as describing the features of the "automation shock" or "impulse" experienced by the US economy since 1980. The elasticities λ, η and the matrices $\{\Theta, \mathcal{M}\}$ then allow us to compute the effects of this shock on group wages.

This section uses our measures of direct task displacement introduced in equation (17) and estimates of (ii)-(iv) to compute the general equilibrium effects of automation using the formulas in Proposition 6. The formulas provide an approximation to the full non-linear effects. When interpreting our findings, keep in mind that the approximation error is $\mathcal{O}(\epsilon^2)$, where ϵ is the measure of the set of tasks experiencing advances in automation.

4.1 Measuring Task Displacement

We continue to use equation (17) to measure direct task displacement, but we now explicitly confront the adjustment terms. For this purpose, we make a number of simplifying assumptions. We assume that $\pi_{gi} = \pi_g = \pi_i = \pi$ across all industries and groups, and we set $\pi = 30\%$. This number corresponds to the average cost-savings from automation, and we proxy it with the average cost-savings from the adoption of industrial robots in manufacturing (see Acemoglu and Restrepo, 2020). We also assume that $\mu_{\mathcal{A}_{gi}}/\mu_{gi} - 1 = \rho$, where we treat the common rents' ratio ρ as a factor to be estimated. These simplifications are imposed by data limitations but still allow us to explore several dimensions of the general equilibrium effects of automation. Finally, we take $\lambda = 0.5$ from Humlum (2020).

Given these assumptions, task displacement can be measured as

$$d\ln\Gamma_g^d = \mathrm{td}_g^d = \sum_i \frac{\ell_{gi}}{\ell_g} \cdot \mathrm{RCA}_{gi}^{\text{routine}} \cdot \frac{-d\ln s_{\ell i}^d}{1 + s_{\ell i}\cdot(\lambda-1)\cdot\pi} \cdot \frac{1}{1+\rho},$$

$$d\ln\Gamma_{gi}^d = \mathrm{td}_{gi}^d = \mathrm{RCA}_{gi}^{\text{routine}} \cdot \frac{-d\ln s_{\ell i}^d}{1 + s_{\ell i}\cdot(\lambda-1)\cdot\pi} \cdot \frac{1}{1+\rho}.$$

4.2 Estimating the Propagation and Rent Impact Matrices

A key step in our quantitative analysis involves estimating the propagation and rent impact matrices. We can write the equation for the change in baseline wages and rents in response to shocks as:

$$(18) \qquad \Delta\ln w_g = \frac{1}{\lambda}\cdot\Theta_g\cdot\mathrm{stack}(d\ln y - \mathrm{td}_j^d + \beta\cdot Z_j + u_j)$$

$$(19) \qquad \Delta\ln\mu_g = -\rho\cdot\mathrm{td}_g^d + \frac{1}{\lambda}\cdot\mathcal{M}_g\cdot\mathrm{stack}(d\ln y - \mathrm{td}_j^d + \beta\cdot Z_j + u_j) + \beta^\mu\cdot Z_g^\mu + e_g.$$

Here, Z_j denotes observable shocks affecting the demand for workers from group j directly, and Z_j^μ denotes any direct effects that these shocks might have on rents. For example, Z_j might stand for sectoral shifters and Z_j^μ for sectoral rent shifters. In our preferred specification, we let Z_j include sectoral shifters, and gender and educational dummies. Likewise, we let Z_j^μ include sectoral rent shifters, and gender and educational dummies. The error term u_g captures unobserved labor demand shocks, and e_g captures unobserved forces affecting group g rents. In our estimation, $d\ln y$ takes the role of a constant.

We base the estimation of Θ and $\mathcal{M}$ on the moment conditions:

$$\text{td}_j^d, Z_j, Z_j^\mu \perp u_g, e_g \text{ for all } g, j.$$

In principle, one could use these moment conditions and various shocks to estimate the matrices $\{\Theta, \mathcal{M}\}$ non-parametrically. However, these matrices have a large number of entries, and any such estimation would run the risk of overfitting the data.

To avoid these concerns, we adopt an explicit parameterization of the entries of these matrices. Recall that $\Theta = \left(\mathbb{I} - \frac{1}{\lambda}\frac{\partial \ln \Gamma}{\partial \ln w}\right)^{-1}$ and $\mathcal{M} = \left(\frac{\partial \ln \mu}{\partial \ln w}\right) \times \left(\mathbb{I} - \frac{1}{\lambda}\frac{\partial \ln \Gamma}{\partial \ln w}\right)^{-1}$. We impose the following parametrization of the *Jacobian matrices* $\frac{\partial \ln \Gamma}{\partial \ln w}$ and $\frac{\partial \ln \mu}{\partial \ln w}$, which then provide a parametric family for the propagation and rent impact matrices:[30]

- Common diagonal terms $\partial \ln \Gamma_g / \partial \ln w_g = -\theta_{own}$ and $\partial \ln \mu_g / \partial \ln w_g = -\theta^\mu_{own}$.
- For task shares, off diagonal terms $g \neq j$ are parametrized as:

$$\begin{aligned}\frac{1}{s_j} \cdot \frac{\partial \ln \Gamma_g}{\partial \ln w_j} =& \theta_e \cdot \text{Education and age proximity}_{jg} \\ &+ \theta_o \cdot \frac{1}{1 + \left(1/\text{occupation distance}_{jg} - 1\right)^{-\kappa}} \\ &+ \theta_i \cdot \frac{1}{1 + \left(1/\text{industry distance}_{jg} - 1\right)^{-\kappa}}\end{aligned}$$

  Here Education and age proximity$_{jg}$ takes a value of 1 if both groups have the same age and educational level. This term captures the higher substitutability among workers of similar age and education, as in Card and Lemieux (2001). The term occupational distance$_{jg}$ is given by the dissimilarity between the vector of employment shares for group j across occupations and the vector of employment shares for group g across occupations in 1980. The term industry distance$_{jg}$ is given by the dissimilarity between the vector of employment shares for group j across industries and the vector of employment shares for group g across industries in 1980. These terms are computed from the 1980 US Census.

  Our parametrization assumes that groups employed in similar industries and occupations and groups of similar age and education will create stronger (first order) ripple

[30] We prefer to impose restrictions directly on the Jacobians, which capture first-order effects, rather than the propagation and rent impact matrices, which include both first-order and higher-order ripple effects. The advantage of this strategy is that first-order effects are more easily interpretable in terms of economic fundamentals.

effects on each other.

- For rents, off-diagonal terms $g \neq j$ are parametrized as functions of group similarity, adjusting for whether groups overlap at high-wage occupations or industries:

$$\frac{1}{s_j} \cdot \frac{\partial \ln \mu_g}{\partial \ln w_j} = \theta_o^\mu \cdot \frac{\text{occupation wage overlap}_{jg}}{1 + \left(1/\text{occupation distance}_{jg} - 1\right)^{-\kappa}} + \theta_i^\mu \cdot \frac{\text{industry wage overlap}_{jg}}{1 + \left(1/\text{industry distance}_{jg} - 1\right)^{-\kappa}}.$$

  The new terms occupation wage overlap$_{jg}$ and industry wage overlap$_{jg}$ are computed as the percent difference between a similarity measure of occupations (industries) employing groups j and g in 1980 and a weighted similarity measure that weights occupations (industries) by the wage paid to group g in that occupation relative to group g's average wage, also measured from the 1980 Census. Intuitively, these terms adjust for the possibility that j may compete against g in occupations or industries where g earns rents (as evidenced from above-average wages). When this is the case, competition from j will push g away from high-rent jobs.

- In both cases, we fix the curvature parameter to $\kappa = 2$, though this choice does not have a major impact on our results.

- We impose two restrictions motivated by the theory: the θ's should be non-negative and the rows of the Jacobian $\frac{1}{\lambda}\frac{\partial \ln \Gamma}{\partial \ln w}$ must have a negative sum.

With these restrictions in place, we estimate $\{\rho, \theta_{own}, \theta_e, \theta_o, \theta_i, \theta^\mu_{own}, \theta^\mu_e, \theta^\mu_o, \theta^\mu_i, \beta, \beta^\mu\}$ in equations (18) and (19) via GMM. Following our reduced-form empirical strategy in the previous section, we take the 30th quantile of within-group wages to be the baseline wage of that demographic group, $d \ln w_g$. This then enables us to interpret the gap between the mean and the 30th quantile as a proxy for average group-level rents, $d \ln \mu_g$.

Our estimation produces the following results:

- We estimate a common rent dissipation coefficient $\hat{\rho} = 0.5$ (s.e.=0.15). This estimate aligns with the reduced-form evidence from Section 3 but is slightly larger due to the different set of covariates in equation (19).

- The estimated propagation matrix has a common diagonal of 1.4, and its off-diagonal terms sum to 0.4 on average over a row. This implies that workers from a group directly exposed to automation bear half the incidence of its displacement effects.

This explains why the reduced-form models in Section 3 work well even though they ignore ripple effects. But this also implies that those models are misspecified, since the propagation matrix is not estimated to be diagonal.

- The estimated rent impact matrix has small terms, suggesting that ripples have a small impact on groups' rents.

4.3 General Equilibrium Effects of Automation on Wages and Rents

Figure 8 plots our estimates of the change in average wages for each of the 500 demographic groups in our analysis, computed using equation (12) from proposition 6. The figures plot wage changes from 1980–2016 due to automation against groups' hourly wages in 1980.

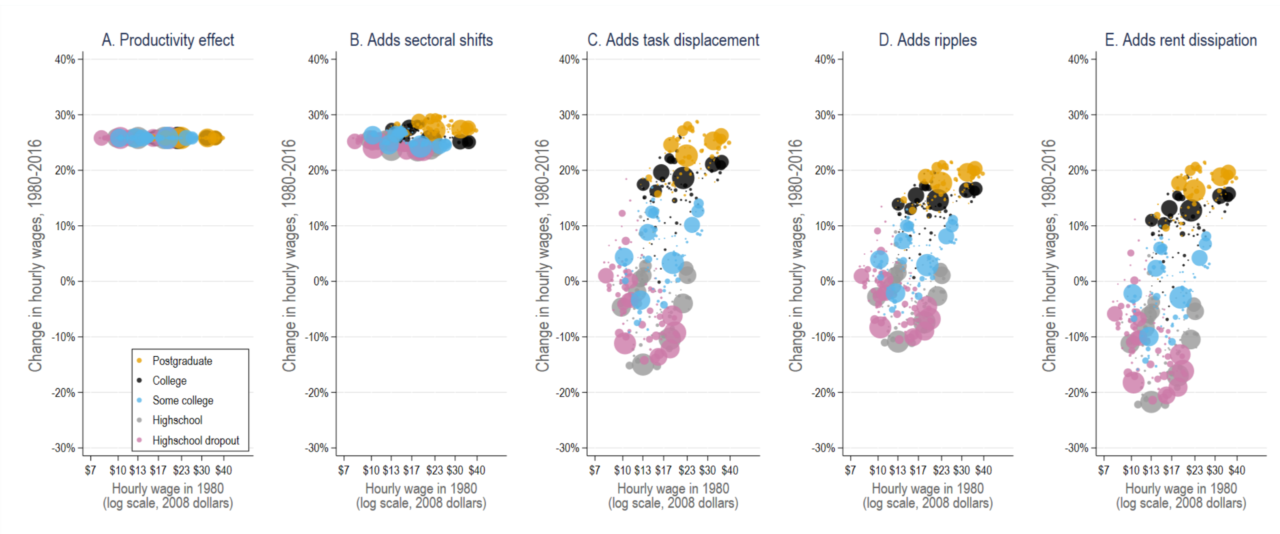

FIGURE 8: WAGE EFFECTS FROM AUTOMATION. The figure plots estimates of the estimated effects of automation on between-group wage changes. In all panels, these results are plotted against groups' baseline hourly wages in 1980.

For exposition purposes, we add different effects sequentially, starting from the productivity effect $(1/\lambda) \cdot d\ln y$ in Panel A. Using the formulas from Proposition 6 and an estimate for π of 30%, we estimate an expansion in output (GDP) of 13% over 1980–2016 in response to automation. By itself, this increase in output raises all wages by 26%.

Panel B adds sectoral shifts induced by automation and plots $(1/\lambda) \cdot d\ln y + (1/\lambda) \cdot \sum_i (\ell_{ij}/\ell_j) \cdot d\ln \zeta_i$. When computing the sectoral shifts, we take an elasticity of substitution between sectors of $\eta = 0.2$ from Buera et al. (2015). This effect is estimated to be small and does not appear as a major channel via which automation affects the wage structure.

Panel C adds in the direct displacement effects from automation and plots $(1/\lambda)\cdot d\ln y-(1/\lambda)\cdot d\ln\Gamma_g^d+(1/\lambda)\cdot\sum_i(\ell_{ij}/\ell_j)\cdot d\ln\zeta_i$. This gives the direct effects of automation that we would observe without ripple effects and rent dissipation. Automation continues to generate large wage gains for workers not exposed to it and large real wage declines for those directly exposed to it.

Panel D factors in the ripple effects and plots $\Theta_g\cdot\text{stack}((1/\lambda)\cdot d\ln y-(1/\lambda)\cdot d\ln\Gamma^d+(1/\lambda)\cdot\sum_i(\ell_{i\cdot}/\ell_j)\cdot d\ln\zeta_i)$. Ripple effects compress the effects of automation on group wages. This makes intuitive sense: directly affected workers try to reallocate to other tasks for which they have a competitive advantage, and in doing so, spread the incidence of the automation shock onto other groups.

Panel E adds the change in rents, $d\ln\mu_g$ which we compute using equation (11) in Proposition 6. As expected from the small entries estimated for the rent impact matrix, 99% of the variation in the change in rents across groups due to automation comes from the rent dissipation mechanism. Our reduced-form estimates and the estimates of ρ in this section suggest that, on average, automation displaces workers from jobs where they earned a 50% rent. This implies a sizable wage reduction for exposed workers due to rent dissipation. At the bottom and middle of the wage distribution, rent dissipation creates an additional 10% decline in group wages.

As discussed in the theory section, directly exposed groups bear the full incidence of rent dissipation. In contrast to ripple effects which "democratized" the inequality implications of automation, by spreading it to other groups, the new rent dissipation mechanism deepens the income losses of directly-impacted groups.

To further illustrate the role of rent dissipation, Figure 9 plots the estimated wage changes from our model in the horizontal axis against observed wage changes in the vertical axis for 1980–2016. The left panel ignores the effects of automation on rents through rent dissipation. This panel shows the predicted effects of automation in a competitive economy. The right panel plots the full wage effects accounting for rent changes.

The baseline competitive effects of automation in the left panel explain 44% of the observed wage changes between groups. Moreover, we see that many groups are below the 45° diagonal, which shows that the competitive effects of automation cannot fully account for the declining or stagnant real wages of groups without college.

More importantly, when accounting for rent dissipation, we estimate that automation now explains 60% of the observed wage changes between groups. Moreover, almost all groups of workers are now above the 45° diagonal, which shows that rent dissipation is

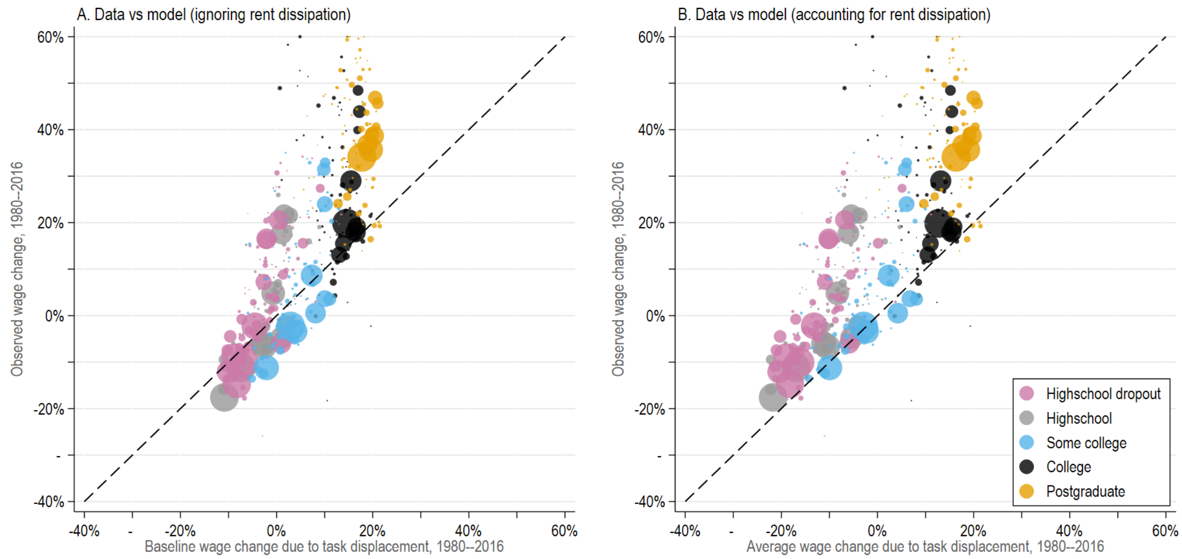

FIGURE 9: WAGE EFFECTS FROM AUTOMATION: MODEL VS DATA. The left panel plots the predicted wage changes in our model, ignoring the change in rents and the observed wage changes from 1980–2016. The right panel plots the predicted wage changes in our model accounting for the change in rents and the observed wage changes from 1980–2016.

important in accounting for the declining or stagnant real wages of groups without college.

To further illustrate this point, consider the case of men without a college degree. In the US data, their wages decreased by 6.5% from 1980–2016. The baseline competitive effects of automation in the left panel generate a 2.5% decline in their real wages. However, once we account for rent dissipation, our model generates a 10.4% decline in their real wages.

4.4 Effects of Automation on TFP and Welfare

We compute the implications of automation for TFP using the formulas in Proposition 6. Using a value of $\pi = 30\%$, we find that automation increased TFP by 3% via cost savings. This is already a small contribution to TFP. Moreover, our formula for TFP shows that this needs to be discounted by the fact that some of these cost-savings came from automating jobs where workers earned rents, and that as a result, automation worsened allocative efficiency. Using our estimates of rent dissipation, we find that automation reduced allocative efficiency by 3.7%. On net, we estimate that automation brought a 0.7% decrease in TFP from 1980 to 2016.

Because in our model $d\ln c = \frac{y}{c} \cdot d\ln tfp$, these estimates also enable us to compute the implied increase in consumption and (utilitarian) welfare due to automation. We estimate that automation reduced aggregate consumption by 1% from 1980 to 2016. Even though we find negative effects on aggregate TFP and consumption, we estimate an output expansion of 13%. This reflects increased resources used for investment k and not higher welfare.

The conclusion that automation reduced TFP and consumption is sensitive to the values of the cost-saving gains π and average rent dissipation ρ used. A higher value of $\pi = 50\%$ and our estimate of $\rho = 50\%$ imply that automation increased TFP by 1.2% from 1980 to 2016. A lower value of $\rho = 30\%$ and our estimate of $\pi = 30\%$ imply that automation increased TFP by 0.5% from 1980 to 2016. The robust conclusion from our exercise is that the reduction in allocative efficiency created by rent dissipation is of a similar magnitude to the cost-saving gains from automation, making the net effect of automation on TFP and consumption small or negative.

5 Conclusion

This paper developed a framework for studying the effects of automation technologies in economies where workers earn rents on their jobs. The distinguishing feature of our model is task-specific labor market rents, which implies that identical workers earn different wages depending on which tasks they perform.

We used this framework to study the interplay between labor market rents and automation technologies. We showed that high-rent tasks will be targeted for automation first, creating a new rent dissipation mechanism. This mechanism has important implications that we explored theoretically and empirically:

1. *Within-group wage effects of automation:* Because high-rent tasks are automated first, automation reduce within-group wage dispersion. This prediction contrasts the view that inequality increases within groups impacted by technological changes.

2. *Between-group wage effects of automation:* The rent dissipation mechanism implies a more negative effect on the wages of groups exposed to automation as they are pushed into low-rent paying jobs.

3. *Productivity effects:* the automation of high-rent jobs worsens *allocative efficiency*. This is because high-rent jobs are those where the value of the marginal product of labor is highest, and automation displaces labor from these tasks. Consequently, automation may reduce TFP and (utilitarian) welfare.

Our reduced-form econometric work provides support for the rent dissipation mechanism. We complement this evidence with a quantitative exercise that estimates the importance of rent dissipation on aggregates. This exercise suggests that the baseline ("competitive") effects of automation account for 44% of the increase in between-group inequality in the United States since 1980, while rent dissipation adds another 16% to this number. We also estimate that because of worsening allocative efficiency, automation had a small negative effect on TFP and utilitarian welfare since 1980.

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A Proofs of Results in the Main Text

A.1 Existence and Uniqueness

Preliminaries: We first derive the equilibrium conditions E3 and E4. The production of the final good is perfectly competitive, so tasks are priced at their marginal product. This implies $p_x = M^{-\frac{1}{\lambda}} \cdot (y/y_x)^{\frac{1}{\lambda}}$, which can be rearranged as

$$y_x = \frac{1}{M} \cdot y \cdot p_x^{-\lambda}. \tag{A1}$$

For tasks in $\mathcal{T}_g$, equation (A1) implies

$$\ell_{gx} \cdot \psi_{gx} = \frac{1}{M} \cdot y \cdot \left(w_g \cdot \frac{\mu_{gx}}{\psi_{gx}} \right)^{-\lambda},$$

which can be rearranged into the labor demand equation in E3.

For tasks in $\mathcal{T}_k$, equation (A1) implies

$$k_x \cdot \psi_{kx} = \frac{1}{M} \cdot y \cdot \left(\frac{1}{q_x \cdot \psi_{kx}} \right)^{-\lambda},$$

which can be rearranged into the capital demand equation in E3.

Finally, multiplying equation (A1) by p_x and integrating over x yields

$$y = \int_{x \in \mathcal{T}} p_x \cdot y_x \cdot dx = \frac{1}{M} \cdot y \cdot \int_{x \in \mathcal{T}} p_x^{1-\lambda} \cdot dx.$$

Canceling y on both sides of this equation yields the ideal-price index condition in E4.

Existence and uniqueness: Given a vector of positive baseline wages $w = \{w_1, w_2, \ldots, w_g\}$, we define the *task share* of worker group g and the *task share* of capital as

$$\Gamma_g(w) = \frac{1}{M} \int_{x \in \mathcal{T}_g} \psi_{gx}^{\lambda-1} \cdot \mu_{gx}^{-\lambda} \cdot dx \quad \text{for all } g,$$
$$\Gamma_k(w) = \frac{1}{M} \int_{x \in \mathcal{T}_k} (\psi_{kx} \cdot q_x)^{\lambda-1} \cdot dx,$$

where the integrals are taken over the set of tasks where g (or k) is the lowest cost producer when wages are given by w, so that task shares depend on baseline wages w through the task allocation.

The following assumption provides sufficient conditions for the existence and uniqueness of an equilibrium.

Assumption A1 (Restrictions on the task space) *The task elasticity of substitution λ is below 1 and task productivities are such that:*

- *The functions $\Gamma_g(w)$ and $\Gamma_k(w)$ are differentiable. This holds if for any two groups of workers $g \neq g'$ and any constants $a, b > 0$, the set of tasks for which $\psi_{gx}/\psi_{g'x} = a$ and $q_x \cdot \psi_{kx}/\psi_{gx} = b$ are of measure zero.*
- *For each g the range of $\Gamma_g(w)$ is $[\underline{\Gamma}_g, \bar{\Gamma}_g]$ with $0 < \underline{\Gamma}_g < \bar{\Gamma}_g < \infty$.*
- *The function $\Gamma_k(w)$ satisfies $\lim_{w \to 0} \Gamma_k(w) < 1$.*

The assumption that $\lambda < 1$ implies that tasks are gross complements, which is the empirically relevant case. The functions $\Gamma_g(w)$ and $\Gamma_k(w)$ are defined as integrals, and thus are continuous. The assumption that they are differentiable is natural. It follows if different demographic groups have distinct comparative advantages. The remaining assumptions are Inada-style conditions that ensure existence and uniqueness of an equilibrium with positive and finite output where all workers produce some tasks.

We first provide a lemma regarding the Jacobian of task shares as a function of wages that will be used in our proofs.

Lemma A1 *Let $\Sigma = \mathbb{I} - \frac{1}{\lambda}\frac{\partial \ln \Gamma(w)}{\partial \ln w}$. For all wage vectors w, the matrix Σ is non-singular. Moreover, Σ is a $P-$matrix of the Leontief type (i.e., with non-positive off-diagonal entries) whose inverse Θ has all entries non-negative.*

Proof. Assumption A1 ensures that task shares are a continuous and differentiable function of wages. We now establish the properties of Σ.

First, because $\partial \Gamma_g / \partial w_{g'} \geq 0$, all off-diagonal entries in Σ are negative. This implies that Σ is a $Z-$matrix.

Second, Σ has a positive dominant diagonal. This follows from the fact that

$$\Sigma_{gg} = 1 - \frac{1}{\lambda}\frac{\partial \ln \Gamma_g}{\partial \ln w_g} > 0, \qquad \text{and} \qquad \Sigma_{gg} - \sum_{g' \neq g} |\Sigma_{gg'}| = 1 - \sum_{g'} \frac{1}{\lambda}\frac{\partial \ln \Gamma_g}{\partial \ln w_{g'}} > 1.$$

This last inequality follows because $\sum_{g'} \frac{\partial \ln \Gamma_g}{\partial \ln w_{g'}} \leq 0$, which is true since when all wages rise by the same amount, workers lose tasks to capital but do not experience task reallocation among them.

Third, all eigenvalues of Σ have a real part that exceeds 1. This follows from the Gershgorin circle theorem, which states that for each eigenvalue ε of Σ, we can find a dimension g such that $\|\varepsilon - \Sigma_{gg}\| < \sum_{g' \neq g} |\Sigma_{gg'}|$. This inequality implies

$$\Re(\varepsilon) \in \left[\Sigma_{gg} - \sum_{g' \neq g} |\Sigma_{gg'}|, \Sigma_{gg} + \sum_{g' \neq g} |\Sigma_{gg'}| \right].$$

Because $\Sigma_{gg} - \sum_{g' \neq g} |\Sigma_{gg'}| > 1$ for all g, as shown above, all eigenvalues of Σ have a real part that is greater than 1.

Fourth, since Σ has negative off-diagonal elements and all of its eigenvalues have a positive real part, we can conclude that it is a non-singular $M-$ matrix, and also a $P-$matrix. Moreover, the inverse of a non-singular $M-$matrix exists and has non-negative and real entries, $\theta_{gg'} \geq 0$. ■

Proposition A1 *When Assumption A1 holds, the market equilibrium exists, is unique, and features positive and finite output.*

Proof. Aggregating the labor demand equation in E3 over tasks in $\mathcal{T}_g$, we obtain $\ell_g = y \cdot \Gamma_g(w) \cdot w_g^{-\lambda}$. This can be rewritten as the market-clearing condition

$$w_g = \left(\frac{y}{\ell_g} \right)^{\frac{1}{\lambda}} \cdot \Gamma_g(w)^{\frac{1}{\lambda}} \text{ for } g = 1, 2, \ldots, G. \tag{A2}$$

We first show that, given a level for output y, there is a unique set of wages $\{w_g(y)\}_g$ that satisfies the market clearing conditions in (A2).

Assumption A1 implies that the mapping $\mathbb{T}w$ defined by

$$\mathbb{T}w_g = \left(\frac{y}{\ell_g} \right)^{\frac{1}{\lambda}} \cdot \Gamma_g(w)^{\frac{1}{\lambda}} \text{ for } g = 1, 2, \ldots, G$$

is a continuous mapping from the compact convex set $\mathbb{X} = \prod_{g=1}^{G} [(y/\ell_g)^{\frac{1}{\lambda}} \cdot \underline{\Gamma}_g^{\frac{1}{\lambda}}, (y/\ell_g)^{\frac{1}{\lambda}} \cdot \bar{\Gamma}_g^{\frac{1}{\lambda}}]$ onto itself. The existence of a positive set of vectors $\{w_g(y)\}_g$ follows from Brouwer's fixed-point theorem.

We now turn to uniqueness. We can rewrite the system of equations $\{w_g(y)\}_g$ defining $\{w_g(y)\}_g$ in logs as

$$\ln w_g - \frac{1}{\lambda} \cdot \ln \Gamma_g(\exp(\ln w)) = \frac{1}{\lambda} \cdot (\ln y - \ln \ell_g),$$

or equivalently as

$$F(x) = \frac{1}{\lambda} \cdot \text{stack}(\ln y - \ln \ell_j),$$

where $x = (\ln w_1, \ldots, \ln w_G)$ and $F(x) = (f_1(x), \ldots, f_G(x))$ with $f_g(x) = x_g - \frac{1}{\lambda} \cdot \ln \Gamma_g(x)$.

The Jacobian of F is given by the Leontief-type matrix Σ. Theorem 5 from Gale and Nikaido (1965) shows that the solution to $F(x) = a$ is unique. The Theorem also shows that the unique solution $x(a)$ increases in a. As a result, the unique solution to the system of equations in (A2) is given by a wage vector $\{w_g(y)\}_g$ with all wages strictly increasing in y.

To conclude, we show that there is a unique y that satisfies the ideal-price index condition. This condition can be written as $c_u(y) = 1$, where

$$c_u(y) = \left(\frac{1}{M} \int_{x \in \mathcal{T}} \left[\min \left\{ \min_g \left\{ w_g(y) \cdot \frac{\mu_{gx}}{\psi_{gx}} \right\}, \frac{1}{q_x \cdot \psi_{kx}} \right\} \right]^{1-\lambda} \cdot dx \right)^{\frac{1}{1-\lambda}}.$$

Because wages are strictly increasing in y, $c_u(y)$ is an increasing function of y.

Note that

$$(y/\ell_g)^{1/\lambda} \cdot \underline{\Gamma}_g^{1/\lambda} \le w_g(y) \le (y/\ell_g)^{1/\lambda} \cdot \bar{\Gamma}_g^{1/\lambda}.$$

As a result, $w_g(y) \to \infty$ as $y \to \infty$, and $w_g(y) \to 0$ as $y \to 0$.

The function $c_u(y)$ can be written as

$$c_u(y) = \left(\Gamma_k(w(y)) + \sum_g \Gamma_g(w) \cdot \mu_g(w) \cdot w_g(y)^{1-\lambda} \right)^{\frac{1}{1-\lambda}}.$$

As $w_g(y) \to \infty$, we have that $\Gamma_g(w) \cdot \mu_g(w) w_g(y)^{1-\lambda} \to \infty$ (since $\Gamma_g(w)$ is bounded from below, $\mu_g(w) \ge 1$, and $\lambda < 1$). This then implies that $c_u(y) \to \infty$.

As $w_g(y) \to 0$, we have that $\Gamma_g(w) \cdot w_g(y)^{1-\lambda} \to 0$ (since $\Gamma_g(w)$ is bounded from above, $\mu_g(w) < \bar{\mu}$, and $\lambda < 1$). This then implies that $c_u(y) \to \lim_{w \to 0} \Gamma_k(w)^{\frac{1}{1-\lambda}} < 1$.

These observations show that there is a unique level of output $y \in (0, \infty)$ that satisfies

$c_u(y) = 1$ and, therefore, a unique equilibrium with wages $w_g = w_g(y)$. This argument shows there is a unique vector of equilibrium wages w_g. These unique equilibrium wages w_g and the tie-breaking rule in footnote 10 uniquely determine the task allocation.

Our argument for uniqueness also shows that, under Assumption A1, the unique equilibrium features finite output, positive wages, and positive task shares for all workers. Moreover, from $c_u(y) = 1$, we obtain that, in equilibrium, $1 - \Gamma_k > 0$. ■

Remarks on Assumption A1: The assumption that $\Gamma_g(w)$ is bounded from above is sufficient to ensure existence of a non-negative wage vector that solves (A2). One can show that all wages must be positive even if we drop the requirement that $\Gamma_g(w)$ is bounded from below (if wages are zero for some group, they will get some positive mass of tasks under the weaker assumption that they have positive productivity for a positive mass of tasks).

The assumption that $\Gamma_g(w)$ is bounded from below ensures that all workers are needed for production. This is sufficient to ensure finite output when $\lambda < 1$, but weaker conditions can be produced. An alternative sufficient condition for finite output that works independently of whether $\lambda \lesseqgtr 1$ is that $\lim_{w\to\infty} \Gamma_k(w)^{1/(\lambda-1)} < 1$.

Finally, the assumption that $\lim_{w\to 0} \Gamma_k(w) < 1$ ensures the economy produces at least some output. This is only needed for $\lambda < 1$. This holds in particular if labor can produce all tasks with some positive productivity.

A.2 Proofs of Results in Main Text

We now provide proofs for the results in the text.

Proof of Proposition 1. As we showed in Acemoglu and Restrepo (2022), the efficient allocation $\{\ell^*_{kx}, \{\ell^*_{gx}\}_g\}_x$ coincides with the market allocation of our economy for $\mu_{gx} = 1$. In fact, $\mu_{gx} = \mu_g$ suffices for efficiency. When μ_{gx} varies across tasks, we have two types of inefficiencies. First, too little labor is assigned to tasks in $\mathcal{T}_g$ with high wedges. This follows from the fact that

$$\frac{\ell_{gx}}{\ell_{gx'}} = \frac{\ell^*_{gx}}{\ell^*_{gx'}} \cdot \left(\frac{\mu_{gx}}{\mu'_{gx}}\right)^{-\lambda} \text{ for } x, x' \in \mathcal{T}_g.$$

Second, the equilibrium allocation of tasks to factors will differ from the efficient one because, among tasks with $a = w_g \cdot \psi_{gx}/(q_x \cdot \psi_{kx})$ with $a > 1$, those with high $\mu_{gx} > a$ will be assigned to capital; while those with $\mu_{gx} \in [1, a]$ will be assigned to labor. Instead, in the efficient allocation, all these tasks are automated or all of them are produced by labor. ■

Proof of Proposition 2. The set of tasks $\mathcal{T}$ defines a joint probability distribution over task attributes and rents. Let $\Psi = \langle q, \psi_k, \{\psi_g\}_g \rangle$ be a vector of the technological attributes of a task. For any set $\mathcal{B} \subseteq \mathcal{T}$ of the task space, define $H(\Psi|\mathcal{B})$ by

$$H(\Psi|\mathcal{B}) = \Pr(x : q'_x \le q, \psi_{kx} \le \psi_k, \{\psi_{gx} \le \psi_g\}_g | x \in \mathcal{B}) = \frac{\int_{\mathcal{B} \cap \{x: q'_x \le q, \psi_{kx} \le \psi_k, \{\psi_{gx} \le \psi_g\}_g\}} dx}{\int_{\mathcal{B}} dx},$$

and denote its density by $h(\Psi|\mathcal{B})$.

We first compute $\bar{F}(\mu|\mathcal{A}_g^T)$ and $\bar{F}(\mu|\mathcal{A}_g)$, where both distributions are defined using the initial allocation of employment. For all $\mu \ge 1$, we have

$$\bar{F}_g(\mu|\mathcal{A}_g^T) = \frac{\int_\Psi \int_\mu^\infty \bar{f}_g(u|\Psi, \mathcal{A}_g^T) \cdot h(\Psi|\mathcal{A}_g^T) \cdot du \cdot d\Psi}{\int_\Psi \int_1^\infty \bar{f}_g(u|\Psi, \mathcal{A}_g^T) \cdot h(\Psi|\mathcal{A}_g^T) \cdot du \cdot d\Psi}.$$

Using condition (ii) we simplify this as

$$\bar{F}_g(\mu|\mathcal{A}_g^T) = \frac{\int_\mu^\infty \bar{f}_g(u|\mathcal{A}_g^T) \cdot du}{\int_1^\infty \bar{f}_g(u|\mathcal{A}_g^T) \cdot du}.$$

Turning to $\bar{F}_g(\mu|\mathcal{A}_g)$, we have

$$\bar{F}_g(\mu|\mathcal{A}_g) = \frac{\int_\Psi \int_{\max\{\mu, \rho(\Psi)\}\}}^\infty \bar{f}_g(u|\Psi, \mathcal{A}_g^T) \cdot h(\Psi|\mathcal{A}_g^T) \cdot du \cdot d\Psi}{\int_\Psi \int_{\max\{1, \rho(\Psi)\}}^\infty \bar{f}_g(u|\Psi, \mathcal{A}_g^T) \cdot h(\Psi|\mathcal{A}_g^T) \cdot du \cdot d\Psi}.$$

Here, $\rho(\Psi) = \frac{1}{w_g} \cdot \frac{\psi_g}{q' \cdot \psi_k}$ gives a threshold wedge above which tasks with technological attributes Ψ will be automated.

Using condition (ii) we can write this as

$$\bar{F}_g(\mu|\mathcal{A}_g) = \frac{\int_\Psi \int_{\max\{\mu, \rho(\Psi)\}\}}^\infty \bar{f}_g(u|\mathcal{A}_g^T) \cdot h(\Psi|\mathcal{A}_g^T) \cdot du \cdot d\Psi}{\int_\Psi \int_{\max\{1, \rho(\Psi)\}}^\infty \bar{f}_g(u|\mathcal{A}_g^T) \cdot h(\Psi|\mathcal{A}_g^T) \cdot du \cdot d\Psi}.$$

We now show that, for all ρ and $\mu \geq 1$, we have

$$\text{(A3)} \qquad \frac{\int_{\max\{\mu,\rho\}\}}^{\infty} \bar{f}_g(u|\mathcal{A}_g^T) \cdot du}{\int_{\max\{1,\rho\}}^{\infty} \bar{f}_g(u|\mathcal{A}_g^T) \cdot du} \geq \frac{\int_{\mu}^{\infty} \bar{f}_g(u|\mathcal{A}_g^T) \cdot du}{\int_{1}^{\infty} \bar{f}_g(u|\mathcal{A}_g^T) \cdot du} = \bar{F}_g(\mu|\mathcal{A}_g^T),$$

with strict inequality if $\rho > 1$. For $\rho < 1$ both sides are equal. For $\rho \in (1,\mu]$, the inequality becomes

$$\frac{\int_{\mu}^{\infty} \bar{f}_g(u|\mathcal{A}_g^T) \cdot du}{\int_{\rho}^{\infty} \bar{f}_g(u|\mathcal{A}_g^T) \cdot du} > \frac{\int_{\mu}^{\infty} \bar{f}_g(u|\mathcal{A}_g^T) \cdot du}{\int_{1}^{\infty} \bar{f}_g(u|\mathcal{A}_g^T) \cdot du}.$$

Comparing denominators we can see that the strict inequality holds. Finally, for $\rho > \mu$, the inequality becomes

$$1 > \frac{\int_{\mu}^{\infty} \bar{f}_g(u|\mathcal{A}_g^T) \cdot du}{\int_{1}^{\infty} \bar{f}_g(u|\mathcal{A}_g^T) \cdot du}.$$

Comparing the numerator and denominator in the RHS we see that the stric inequality holds.

To conclude the proof, re-write (A3) as

$$\int_{\max\{\mu,\rho\}\}}^{\infty} \bar{f}_g(u|\mathcal{A}_g^T) \cdot du \geq \bar{F}_g(\mu|\mathcal{A}_g^T) \cdot \int_{\max\{1,\rho\}}^{\infty} \bar{f}_g(u|\mathcal{A}_g^T) \cdot du$$

Letting $\rho = \rho(\Psi)$ and integrating over the task space, we get

$$\int_{\Psi} \int_{\max\{\mu,\rho(\Psi)\}\}}^{\infty} \bar{f}_g(u|\mathcal{A}_g^T) \cdot h(\Psi|\mathcal{A}_g^T) \cdot du \cdot d\Psi$$
$$> \bar{F}_g(\mu|\mathcal{A}_g^T) \cdot \int_{\Psi} \int_{\max\{1,\rho(\Psi)\}}^{\infty} \bar{f}_g(u|\mathcal{A}_g^T) \cdot h(\Psi|\mathcal{A}_g^T) \cdot du \cdot d\Psi.$$

The resulting inequality is strict because (A3) holds with strict inequality for a positive mass of tasks. This inequality can then be rearranged as $\bar{F}_g(\mu|\mathcal{A}_g) > \bar{F}_g(\mu|\mathcal{A}_g^T)$. Condition (i) then implies $\bar{F}_g(\mu|\mathcal{A}_g) > \bar{F}_g(\mu|\mathcal{T}_g)$ as wanted. ■

Proof of Proposition 3. Proposition 2 showed that $\bar{F}_g(\mu|\mathcal{A}_g) > \bar{F}_g(\mu|\mathcal{T}_g)$. This implies $\bar{F}_g(\mu|\mathcal{T}_g) > \bar{F}_g(\mu|\mathcal{T}_g - \mathcal{A}_g)$. That is, the distribution of rents among workers in $\mathcal{T}_g$ dominates (in the first order stochastic sense) the distribution of rents among workers in $\mathcal{T}_g - \mathcal{A}_g$.

Following the automation of tasks in $\mathcal{A}_g$, workers are reallocated proportionally to tasks in $\mathcal{T}_g - \mathcal{A}_g$ (recall that this proposition does not account for ripples and holds the boundaries of $\mathcal{T}_g$ constant). As a result, the distribution of wages across workers after the tasks in $\mathcal{A}_g$ are automated equals the distribution of wages in $\mathcal{T}_g - \mathcal{A}_g$.

Consider the quantile function for wages in $\mathcal{T}_g$ and in $\mathcal{T}_g - \mathcal{A}_g$. Below M_g, both quantile functions are equal to w_g, since the share of workers earning no rents in $\mathcal{T}_g - \mathcal{A}_g$ exceeds the share of workers earning no rents in $\mathcal{T}_g$. This shows that $d\ln w_g^p = d\ln w_g$ for all $p \leq M_g$.

Moreover, because $\bar{F}_g(\mu|\mathcal{T}_g) > \bar{F}_g(\mu|\mathcal{T}_g - \mathcal{A}_g)$, the quantile function for wages in $\mathcal{T}_g - \mathcal{A}_g$ must be strictly below the quantile function for wages in $\mathcal{T}_g$ for all $\mu > 1$. This shows that $d\ln w_g^p < d\ln w_g$ for all $p > M_g$.

Finally, because not all tasks in $\mathcal{T}_g$ can be automated, there is a positive mass of tasks with technological attributes Φ in $\mathcal{T}_g - \mathcal{A}_g$. Condition (ii) in Proposition 2 implies that the distribution of rents in these tasks has the same maximum as the distribution of rents in $\mathcal{T}_g$. This implies that the distribution of rents in $\mathcal{T}_g - \mathcal{A}_g$ and the distribution of rents in $\mathcal{T}_g$ have the same maximum. This shows that $d\ln w_g^p = d\ln w_g$ as $p \to 1$. ∎

Proof of Proposition 4. By definition, $\bar{w}_g = w_g \cdot \mu_g$. Using equation (A2), we obtain

$$\bar{w}_g = \left(\frac{y}{\ell_g}\right)^{\frac{1}{\lambda}} \cdot \Gamma_g(w)^{\frac{1}{\lambda}} \cdot \mu_g \text{ for } g = 1, 2, \ldots, G. \tag{A4}$$

In addition, the demand for capital for tasks in $\mathcal{T}_k$ given in E3 can be written as

$$\frac{k_x}{q_x} = y \cdot \frac{1}{M} \cdot (\psi_{kx} \cdot q_x)^{\lambda - 1}.$$

Integrating over $\mathcal{T}_k$ yields

$$k = y \cdot \Gamma_k. \tag{A5}$$

There are no profits in the economy, and so all income accrues to capital or labor. As a result $y = k + \sum_g \bar{w}_g \cdot \ell_g$. Substituting the expression for wages from equation (A4) and the expression for k in (A5), we obtain

$$y = y \cdot \Gamma_k + \sum_g \left(\frac{y}{\ell_g}\right)^{\frac{1}{\lambda}} \cdot \Gamma_g(w)^{\frac{1}{\lambda}} \cdot \mu_g \cdot \ell_g.$$

Solving for y using this equation yields the expression

$$\text{(A6)} \qquad y = (1-\Gamma_k)^{\frac{\lambda}{1-\lambda}} \cdot \left(\sum_g \Gamma_g^{\frac{1}{\lambda}} \cdot \mu_g \cdot \ell_g^{\frac{\lambda-1}{\lambda}} \right)^{\frac{\lambda-1}{\lambda}}.$$

■

Before turning to the proof of Proposition 5, we formally define the notion of small automation shocks that we will use in our derivation. In what follows, we define $B_{j,g}$ as the set of boundary tasks between j and g.

DEFINITION 1 *An automation shock in $\mathcal{A}^T$ is of order ϵ if the sets $\mathcal{A}_g^T$ have density $\mathcal{O}(\epsilon)$ and the boundary set of tasks*

$$\mathcal{B}_{gi}(w) = \left\{ x \in \mathcal{A}_{gi}^T : \frac{w_g \cdot \mu_{gx}}{\psi_{gx}} = \frac{1}{q_x' \cdot \psi_{kx}} \right\}$$

has arc-length $\mathcal{O}(\epsilon)$ for all w_g. In addition, we say that an automation shock is interior if $\mathcal{A}_g^T$ is in the interior of $\mathcal{T}_g$.

Our derivations assume we have a small an interior automation shock. The shock is assumed to be interior to ensure that it does not alter substitution patterns by automating boundary tasks.

Proof of Proposition 5. Consider an interior automation shock in $\mathcal{A}_g^T$ of order ϵ. For functions over the task space, $F(w)$, we denote by $F^{\mathcal{A}}(w)$ the new function obtained after q_x increases from zero to q_x' in $\mathcal{A}^T$.

Effects on baseline wages $d \ln w_g$: we first derive equation (3) in the Proposition. Lemma A2 shows that we can do a "Taylor expansion" of equation (1) (in logs) to express the change in equilibrium wages as

$$\text{(A7)} \quad d\ln w_g = \frac{1}{\lambda} \cdot d\ln y + \frac{1}{\lambda} \cdot \left(\ln \Gamma_g^{\mathcal{A}}(w) - \ln \Gamma_g(w) \right) + \frac{1}{\lambda} \cdot \frac{\partial \ln \Gamma_g(w)}{\partial \ln w} \cdot \text{stack}(d\ln w) + \mathcal{O}(\epsilon^2)$$

This expansion decomposes the effects of wages into the productivity effect, the direct effect of automation on task shares, and the reallocation of tasks in response to wages.

We now approximate $\ln \Gamma_g^{\mathcal{A}}(w) - \ln \Gamma_g(w)$. Let $dL_g(x) = \psi_{gx}^{\lambda-1} \cdot \mu_{gx}^{-\lambda} \cdot dx$. This is a new measure over the space of tasks that accounts for employment. We have

$$\begin{aligned}\ln \Gamma_g^{\mathcal{A}}(w) - \ln \Gamma_g(w) =& \frac{\Gamma_g^{\mathcal{A}}(w) - \Gamma_g(w)}{\Gamma_g(w)} + \mathcal{O}(\epsilon^2)\\ =& -\frac{\int_{x\in\mathcal{A}_g} dL_g(x)}{\int_{x\in\mathcal{T}_g} dL_g(x)} + \mathcal{O}(\epsilon^2)\\ =& -d\ln \Gamma_g^d + \mathcal{O}(\epsilon^2).\end{aligned}$$

The first-line follows from an approximation of log changes. The second line uses the definition of task shares and of $\Gamma_g^{\mathcal{A}}(w)$. The last line is the definition of $d\ln \Gamma_g^d$.

Plugging our approximations for $\ln \Gamma_g^{\mathcal{A}}(w) - \ln \Gamma_g(w)$ into equation (A7), we obtain

$$\text{(A8)} \qquad d\ln w_g = \frac{1}{\lambda}\cdot d\ln y - \frac{1}{\lambda}\cdot d\ln \Gamma_g^d + \frac{1}{\lambda}\cdot \frac{\partial \ln \Gamma_g(w)}{\partial \ln w}\cdot \text{stack}(d\ln w) + \mathcal{O}(\epsilon^2).$$

Lemma A1 implies that this system has the unique solution (to a first-order approximation)

$$\text{(A9)} \qquad d\ln w_g = \Theta_g \cdot \text{stack}\left(\frac{1}{\lambda}\cdot d\ln y - \frac{1}{\lambda}\cdot d\ln \Gamma_j^d\right) + \mathcal{O}(\epsilon^2).$$

Effects on group rents $d\ln \mu_g$: we now derive the effects of automation on group rents, summarized by equation (4) in the Proposition. Lemma A2 implies

$$d\ln \mu_g = \ln \mu_g^{\mathcal{A}}(w) - \ln \mu_g(w) + \frac{\partial \ln \mu_g(w)}{\partial \ln w}\cdot \text{stack}(d\ln w) + \mathcal{O}(\epsilon^2).$$

We can rewrite the direct effect of automation on rents, $\ln \mu_g^{\mathcal{A}}(w) - \ln \mu_g(w)$, as

$$\begin{aligned}\ln \mu_g^{\mathcal{A}}(w) - \ln \mu_g(w) =& \frac{\mu_g^{\mathcal{A}}(w) - \mu_g(w)}{\mu_g(w)} + \mathcal{O}(\epsilon^2)\\ =& \frac{\frac{\int_{x\in\mathcal{T}_g} \mu_{gx}\cdot dL_g(x) - \int_{x\in\mathcal{A}_g} \mu_{gx}\cdot dL_g(x)}{\int_{x\in\mathcal{T}_g} dL_g(x) - \int_{x\in\mathcal{A}_g} dL_g(x)} - \mu_g}{\mu_g} + \mathcal{O}(\epsilon^2)\\ =& \frac{\frac{\mu_g\cdot\int_{x\in\mathcal{T}_g} dL_g(x) - \mu_{\mathcal{A}_g}\cdot\int_{x\in\mathcal{A}_g} dL_g(x)}{\int_{x\in\mathcal{T}_g} dL_g(x) - \int_{x\in\mathcal{A}_g} dL_g(x)} - \mu_g}{\mu_g} + \mathcal{O}(\epsilon^2)\\ =& \frac{\int_{x\in\mathcal{A}_g} dL_g(x) - \frac{\mu_{\mathcal{A}_g}}{\mu_g}\cdot\int_{x\in\mathcal{A}_g} dL_g(x)}{\int_{x\in\mathcal{T}_g} dL_g(x) - \int_{x\in\mathcal{A}_g} dL_g(x)} + \mathcal{O}(\epsilon^2).\end{aligned}$$

The first line approximates the change in logs. The second line uses the definition of $\mu_g^{\mathcal{A}}(w)$

and the fact that $\mu_g(w) = \mu_g$. The third line uses the definition of average group rents and average group rents at automated jobs, $\mu_{\mathcal{A}_g}$. The last line divides by μ_g and cancels terms. By definition $d\ln\Gamma_g^d = \frac{\int_{x\in\mathcal{A}_g} dL_g(x)}{\int_{x\in\mathcal{T}_g} dL_g(x)}$. Using this expression, we obtain

$$\ln\mu_g^{\mathcal{A}}(w) - \ln\mu_g(w) = -\left(\frac{\mu_{\mathcal{A}_g}}{\mu_g} - 1\right)\cdot\frac{d\ln\Gamma_g^d}{1-d\ln\Gamma_g^d} + \mathcal{O}(\epsilon^2) = -\left(\frac{\mu_{\mathcal{A}_g}}{\mu_g} - 1\right)\cdot d\ln\Gamma_g^d + \mathcal{O}(\epsilon^2)$$

The last equality uses the fact that $\frac{d\ln\Gamma_g^d}{1-d\ln\Gamma_g^d} = d\ln\Gamma_g^d + \mathcal{O}(\epsilon^2)$.

These derivations show that

$$d\ln\mu_g = -\left(\frac{\mu_{\mathcal{A}_g}}{\mu_g} - 1\right)\cdot d\ln\Gamma_g^d + \frac{\partial\ln\mu_g(w)}{\partial\ln w}\cdot\text{stack}(d\ln w) + \mathcal{O}(\epsilon^2).$$

Using the solution for baseline wages, we obtain

$$\text{(A10)}\quad d\ln\mu_g = -\left(\frac{\mu_{\mathcal{A}_g}}{\mu_g} - 1\right)\cdot d\ln\Gamma_g^d + \frac{\partial\ln\mu_g(w)}{\partial\ln w}\cdot\Theta\cdot\text{stack}\left(\frac{1}{\lambda}\cdot d\ln y - \frac{1}{\lambda}\cdot d\ln\Gamma_j^d\right) + \mathcal{O}(\epsilon^2).$$

Effects on group average wages $d\ln\bar{w}_g$: The expression for $d\ln\bar{w}_g$ in equation (5) follows from combining our formula for baseline wages in (A9) and our formula for rent changes $d\ln\mu_g$ in equation (A10), and using the fact that $d\ln\bar{w}_g = d\ln w_g + d\ln\mu_g$.

Effects on TFP $d\ln tfp$: To conclude, we derive the expression for the change in TFP in equation (7). First, we prove the dual version of the Solow residual. Because all income accrues to capital or labor, we have $y = \sum_g \bar{w}_g\cdot\ell_g + k$. Differentiating this expression yields

$$d\ln y = s_k\cdot d\ln k + \sum_g s_g\cdot d\ln\bar{w}_g \quad\Rightarrow \underbrace{d\ln y - s_k\cdot d\ln k}_{\equiv d\ln tfp} = \sum_g s_g\cdot d\ln\bar{w}_g.$$

Second, we turn to the ideal-price index condition, written as $C(w) = 1$, where

$$C(w) = \Gamma_k(w) + \sum_g \Gamma_g(w)\cdot\mu_g(w)\cdot w_g^{1-\lambda}.$$

Lemma A2 shows that we can expand $C(w)$ as

$$\text{(A11)}\qquad dC = C^{\mathcal{A}}(w) - C(w) + C(w)\cdot\frac{\partial\ln C(w)}{\partial\ln w}\cdot d\ln w + \mathcal{O}(\epsilon^2).$$

Note that $\frac{\partial\ln C(w)}{\partial\ln w}\cdot d\ln w$ captures the effect of a change in wages on the cost of producing the final good at the initial equilibrium allocation. Because tasks are allocated in a cost-minimizing way (given wedges), the envelope theorem implies

$$\frac{\partial \ln C(w)}{\partial \ln w} \cdot d \ln w = (1-\lambda) \cdot \sum_g s_g \cdot d \ln w_g + \mathcal{O}(\epsilon^2)$$

The term $C^{\mathcal{A}}(w) - C(w)$, on the other hand, captures the cost saving gains from automating tasks in $\mathcal{A}_g$ holding wages constant. We have

$$\begin{aligned}
C^{\mathcal{A}}(w) - C(w) =& \Gamma_k^{\mathcal{A}}(w) - \Gamma_k(w) + \sum_g \Gamma_g^{\mathcal{A}}(w) \cdot \mu_g^{\mathcal{A}}(w) \cdot {w_g}^{1-\lambda} - \sum_g \Gamma_g(w) \cdot \mu_g(w) \cdot {w_g}^{1-\lambda} \\
=& \sum_g \left[\frac{1}{M} \cdot \int_{x \in \mathcal{A}_g} (q'_x \cdot \psi_{kx})^{\lambda-1} \cdot dx - \frac{1}{M} \cdot \int_{x \in \mathcal{A}_g} (\psi_{gx}/\mu_{gx})^{\lambda-1} \cdot {w_g}^{1-\lambda} dx \cdot \right] \\
=& \sum_g \frac{1}{M} \int_{x \in \mathcal{A}_g} \left[(q'_x \cdot \psi_{kx})^{\lambda-1} - (\psi_{gx}/\mu_{gx})^{\lambda-1} \cdot {w_g}^{1-\lambda} \right] \cdot dx \\
=& \sum_g \frac{1}{M} \int_{x \in \mathcal{A}_g} (\psi_{gx}/\mu_{gx})^{\lambda-1} \cdot {w_g}^{1-\lambda} \cdot \left[\left(\frac{q'_x \cdot \psi_{kx} \cdot w_g \cdot \mu_{gx}}{\psi_{gx}} \right)^{\lambda-1} - 1 \right] \cdot dx \\
=& \sum_g s_g \cdot d \ln \Gamma_g^d \cdot \frac{\mu_{\mathcal{A}_g}}{\mu_g} \cdot \frac{\frac{1}{M} \int_{x \in \mathcal{A}_g} (\psi_{gx}/\mu_{gx})^{\lambda-1} \cdot \left[\left(\frac{q'_x \cdot \psi_{kx} \cdot w_g \cdot \mu_{gx}}{\psi_{gx}} \right)^{\lambda-1} - 1 \right] \cdot dx}{\frac{1}{M} \int_{x \in \mathcal{A}_g} (\psi_{gx}/\mu_{gx})^{\lambda-1} \cdot dx} \\
=& (\lambda - 1) \cdot \sum_g s_g \cdot d \ln \Gamma_g^d \cdot \frac{\mu_{\mathcal{A}_g}}{\mu_g} \cdot \pi_g.
\end{aligned}$$

In the last step, we used the fact that $\ell_{gx} \cdot \mu_{gx} \propto (\psi_{gx}/\mu_{gx})^{\lambda-1}$ (from equilibrium condition E3), which gives the expression for π_g in the main text.

Because real wages satisfy $C(w) = 1$, we have $dC = 0$. Equation (A11) then implies

$$\sum_g s_g \cdot d \ln w_g = \sum_g s_g \cdot d \ln \Gamma_g^d \cdot \frac{\mu_{\mathcal{A}_g}}{\mu_g} \cdot \pi_g + \mathcal{O}(\epsilon^2).$$

Using the fact that $d \ln \bar{w}_g = d \ln w_g + d \ln \mu_g$, we obtain

$$\sum_g s_g \cdot d \ln \bar{w}_g = \sum_g s_g \cdot d \ln \Gamma_g^d \cdot \frac{\mu_{\mathcal{A}_g}}{\mu_g} \cdot \pi_g - \sum_g s_g \cdot d \ln \mu_g + \mathcal{O}(\epsilon^2).$$

Finally, combining this expression for the change in wages with the dual version of the Solow residual yields the formula for TFP in the proposition. ■

A.3 Proofs and details for the multi-sector model.

A *market equilibrium* is again given by a vector of base wages $\{w_g\}$, output y, sectoral prices p_i, an allocation of tasks $\{\mathcal{T}_{gi}\}_{i,g}, \{\mathcal{T}_{ik}\}_i$, task prices p_x, hiring plans ℓ_{gx}, and capital production plans k_x such that:[31]

E1' Tasks prices equal the minimum unit cost of producing the task

$$p_x = \min\left\{\frac{1}{q_x \cdot \psi_{kx}}, \left\{w_g \cdot \frac{\mu_{gx}}{\psi_{gx}}\right\}_g\right\}.$$

E2' Tasks are allocated in a cost-minimizing way. The set of tasks

$$\mathcal{T}_{gi} = \left\{x \in \mathcal{T}_i : p_x = w_g \cdot \frac{\mu_{gx}}{\psi_{gx}}\right\}$$

will be produced by workers of type g, and the set of tasks

$$\mathcal{T}_{ik} = \left\{x \in \mathcal{T}_i : p_x = \frac{1}{q(x) \cdot \psi_k(x) \cdot A_k}\right\}$$

will be produced by capital.

E3' Quantities of labor and capital are given by

$$\begin{aligned} \ell_{gx} &= y \cdot s_{y_i} \cdot p_i^{\lambda-1} \cdot \frac{1}{M_i} \cdot \psi_{gx}^{\lambda-1} \cdot (\mu_{gx} \cdot w_g)^{-\lambda} && \text{for } x \in \mathcal{T}_{gi}, \\ k_x &= y \cdot s_{y_i} \cdot p_i^{\lambda-1} \cdot \frac{1}{M_i} \cdot \psi_{kx}^{\lambda-1} \cdot q_x^{\lambda} && \text{for } x \in \mathcal{T}_{ik}. \end{aligned}$$

E4' Sectoral prices are given by

$$p_i = \left(\frac{1}{M_i}\int_{x \in \mathcal{T}_i} p_x^{1-\lambda} \cdot dx\right)^{\frac{1}{1-\lambda}}$$

E5' The ideal-price index condition holds

$$1 = c_f(\{p\}_i).$$

[31] As in the single-sector model (see footnote 10), we assume that, when indifferent, tasks are allocated to capital or to the group with the highest index g. This tie-breaking rule ensures uniqueness of the task allocation.

Preliminaries: We first derive the equilibrium conditions E3' and E4'. The production of the final good is perfectly competitive, and so tasks are priced at their marginal product. This implies $p_x = p_i \cdot M_i^{-\frac{1}{\lambda}} \cdot (y_i/y_x)^{\frac{1}{\lambda}}$, which can be rearranged as

$$y_x = \frac{1}{M_i} \cdot y \cdot s_{y_i} \cdot p_i^{\lambda-1} \cdot p_x^{-\lambda}. \tag{A12}$$

For tasks in $\mathcal{T}_g$, equation (A1) implies

$$\ell_{gx} \cdot \psi_{gx} = \frac{1}{M} \cdot y \cdot s_{y_i} \cdot p_i^{\lambda-1} \left(w_g \cdot \frac{\mu_{gx}}{\psi_{gx}} \right)^{-\lambda},$$

which can be rearranged into the labor demand equation in E3'.

For tasks in $\mathcal{T}_k$, equation (A1) implies

$$k_x \cdot \psi_{kx} = \frac{1}{M} \cdot y \cdot s_{y_i} \cdot p_i^{\lambda-1} \left(\frac{1}{q_x \cdot \psi_{kx}} \right)^{-\lambda},$$

which can be rearranged into the capital demand equation in E3'.

Finally, multiplying equation (A12) by p_x and integrating over x yields

$$s_{y_i} \cdot y = \int_{x \in \mathcal{T}} p_x \cdot y_x \cdot dG(x) = \frac{1}{M_i} \cdot s_{y_i} \cdot y \cdot p_i^{\lambda-1} \cdot \int_{x \in \mathcal{T}} p_x^{1-\lambda} \cdot dG(x).$$

Canceling $s_{y_i} \cdot y$ on both sides of this equation yields the sectoral-price index condition E4'.

PROPOSITION A2 *Given the equilibrium allocation $\langle \{\mathcal{T}_{gi}\}_{g,i}, \{\mathcal{T}_{ki}\}_i \rangle$ and the resulting task shares and group rents, output, sectoral prices, baseline wages by group and average wages by group satisfy*

$$w_g = \left(\frac{y}{\ell_g} \right)^{\frac{1}{\lambda}} \cdot \Gamma_g^{\frac{1}{\lambda}}, \tag{A13}$$

$$\bar{w}_g = \left(\frac{y}{\ell_g} \right)^{\frac{1}{\lambda}} \cdot \Gamma_g^{\frac{1}{\lambda}} \cdot \mu_g \tag{A14}$$

$$p_i = \left(\Gamma_{ki} + \sum \Gamma_{gi} \cdot \mu_g \cdot w_g^{1-\lambda} \right)^{\frac{1}{1-\lambda}}, \tag{A15}$$

$$1 = c^f(p_i). \tag{A16}$$

Proof of Proposition A2. Aggregating the labor demand equation in E3' over tasks in

$\mathcal{T}_{gi}$ for all industries, we obtain $\ell_g = y \cdot \Gamma_g \cdot w_g^{-\lambda}$. This can be rewritten as (A13). Equation (A14) then follows from the definition of average group rents.

Equation E4' implies that sectoral prices satisfy

$$p_i = \left(\frac{1}{M_i}\int_{x\in\mathcal{T}_i} p_x^{1-\lambda}\cdot dx\right)^{\frac{1}{1-\lambda}} = \left(\Gamma_{ki} + \sum_g \Gamma_{gi}\cdot\mu_{gi}\cdot w_g^{1-\lambda}\right)^{\frac{1}{1-\lambda}}.$$

Here, we used the fact that

$$\begin{aligned}\frac{1}{M_i}\int_{x\in\mathcal{T}_{gi}} p_x^{1-\lambda}\cdot dx &= \frac{1}{M_i}\int_{x\in\mathcal{T}_{gi}}\left(w_g\cdot\frac{\mu_{gx}}{\psi_{gx}}\right)^{1-\lambda}\cdot dx\\ &= \underbrace{\left(\frac{1}{M_i}\int_{x\in\mathcal{T}_{gi}}\psi_{gx}^{\lambda-1}\cdot\mu_{gx}^{-\lambda}\cdot dx\right)}_{\Gamma_{gi}}\cdot\underbrace{\left(\frac{\int_{x\in\mathcal{T}_{gi}}\psi_{gx}^{\lambda-1}\cdot\mu_{gx}^{1-\lambda}}{\int_{x\in\mathcal{T}_{gi}}\psi_{gx}^{\lambda-1}\cdot\mu_{gx}^{-\lambda}}\right)}_{\mu_{gi}}\cdot w_g^{1-\lambda}.\end{aligned}$$

Finally, equation E5' implies $1 = c_f(\{p_i\}_i)$. ■

Proof of Proposition 6. We make the dependence of task shares on wages and sectoral prices explicit. In particular, define the industry i task share when wages are w by

$$\begin{aligned}\Gamma_{gi}(w) &= \frac{1}{M_i}\int_{x\in\mathcal{T}_{gi}}\psi_{gx}^{\lambda-1}\cdot\mu_{gx}^{-\lambda}\cdot dx \quad \text{for all } g,\\ \Gamma_{ik}(w) &= \frac{1}{M_i}\int_{x\in\mathcal{T}_{ik}}(\psi_{kx}\cdot q_x)^{\lambda-1}\cdot dx.\end{aligned}$$

and the aggregate task share when wages are w and sectoral prices are p by

$$\begin{aligned}\Gamma_g(w,p) &= \sum_i s_{y_i}\cdot p_i^{\lambda-1}\cdot\frac{1}{M_i}\int_{x\in\mathcal{T}_{gi}}\psi_{gx}^{\lambda-1}\cdot\mu_{gx}^{-\lambda}\cdot dx \quad \text{for all } g,\\ \Gamma_k(w,p) &= \sum_i s_{y_i}\cdot p_i^{\lambda-1}\cdot\frac{1}{M_i}\int_{x\in\mathcal{T}_{ik}}(\psi_{kx}\cdot q_x)^{\lambda-1}\cdot dx.\end{aligned}$$

Note that s_{y_i} is a function of sectoral prices, since the aggregator f is assumed to be homothetic.

Consider an automation shock in $\mathcal{A}_g^T$ of order ϵ. For functions over the task space, $F(w,p)$, denote by $F^{\mathcal{A}}(w,p)$ the function obtained after q_x increases from zero to q_x' in $\mathcal{A}_g^T$.

Effects on baseline wages $d\ln w_g$: we first derive equation (10) in the Proposition. Lemma A2 shows that we can do a "Taylor expansion" of equation (A13) (in logs) to express the change in equilibrium wages as

(A17)
$$d\ln w_g = \frac{1}{\lambda}\cdot d\ln y + \frac{1}{\lambda}\cdot\left(\ln\Gamma_g^{\mathcal{A}}(w) - \ln\Gamma_g(w)\right) + \frac{1}{\lambda}\cdot\frac{\partial\ln\Gamma_g(w,p)}{\partial\ln w}\cdot\text{stack}(d\ln w)$$
$$+\frac{1}{\lambda}\cdot\frac{\partial\ln\Gamma_g(w,p)}{\partial\ln p}\cdot\text{stack}(d\ln p) + \mathcal{O}(\epsilon^2)$$

This expansion decomposes the effects of wages into the productivity effect, the direct effect of automation on task shares, the reallocation of tasks in response to wages, and the effect of changes in sectoral prices on task shares.

We now approximate $\ln\Gamma_g^{\mathcal{A}}(w,p) - \ln\Gamma_g(w,p)$. Letting $dL_g(x) = \psi_{gx}^{\lambda-1}\cdot\mu_{gx}^{-\lambda}\cdot dx$, we have

$$\begin{aligned}
\ln\Gamma_g^{\mathcal{A}}(w,p) - \ln\Gamma_g(w,p) &= \frac{\Gamma_g^{\mathcal{A}}(w,p) - \Gamma_g(w,p)}{\Gamma_g(w,p)} + \mathcal{O}(\epsilon^2)\\
&= -\frac{\sum_i s_{y_i}\cdot p_i^{\lambda-1}\cdot\int_{x\in\mathcal{A}_{gi}} dL_g(x)}{\sum_i s_{y_i}\cdot p_i^{\lambda-1}\cdot\int_{x\in\mathcal{T}_{gi}} dL_g(x)} + \mathcal{O}(\epsilon^2)\\
&= -\sum_i \frac{s_{y_i}\cdot p_i^{\lambda-1}\cdot\int_{x\in\mathcal{T}_{gi}} dL_g(x)}{\sum_{i'} s_{y_{i'}}\cdot p_{i'}^{\lambda-1}\cdot\int_{x\in\mathcal{T}_{gi'}} dL_g(x)}\cdot\frac{\int_{x\in\mathcal{A}_{gi}} dL_g(x)}{\int_{x\in\mathcal{T}_{gi}} dL_g(x)} + \mathcal{O}(\epsilon^2)\\
&= -\sum_i \frac{\ell_{gi}}{\ell_g}\cdot d\ln\Gamma_{gi}^d + \mathcal{O}(\epsilon^2).
\end{aligned}$$

The first-line follows from an approximation of log changes. The second line uses the definition of task shares and of $\Gamma_g^{\mathcal{A}}(w,p)$. The last line is the definition of $d\ln\Gamma_g^d$.

We now turn to the effects of sectoral prices on task shares, $\frac{\partial\ln\Gamma_g(w,p)}{\partial\ln p}\cdot\text{stack}(d\ln p)$. This is given by

$$\frac{\partial\ln\Gamma_g(w,p)}{\partial\ln p}\cdot d\ln p = \sum_i \frac{s_{y_i}\cdot p_i^{\lambda-1}\cdot\Gamma_{gi}}{\sum_{i'} s_{y_{i'}}\cdot p_{i'}^{\lambda-1}\cdot\Gamma_{gi'}}\cdot d\ln\zeta_i = \sum_i \frac{\ell_{gi}}{\ell_g}\cdot d\ln\zeta_i,$$

which uses the definition of sectoral shifters $d\ln\zeta_i = d\ln(s_{y_i}\cdot p_i^{\lambda-1})$.

Plugging our approximation for $\ln\Gamma_g^{\mathcal{A}}(w,p) - \ln\Gamma_g(w,p)$ and our formula for $\frac{\partial\ln\Gamma_g(w,p)}{\partial\ln p}\cdot$ stack$(d\ln p)$ into equation (A17), we obtain

(A18)
$$d\ln w_g = \frac{1}{\lambda}\cdot d\ln y - \frac{1}{\lambda}\cdot d\ln\Gamma_g^d + \frac{1}{\lambda}\cdot\sum_i\frac{\ell_{gi}}{\ell_g}\cdot d\ln\zeta_i + \frac{1}{\lambda}\cdot\frac{\partial\ln\Gamma_g(w)}{\partial\ln w}\cdot\text{stack}(d\ln w) + \mathcal{O}(\epsilon^2).$$

Lemma A1 implies that this system has the unique solution (to a first-order approximation)

$$d\ln w_g = \Theta_g \cdot \text{stack}\left(\frac{1}{\lambda}\cdot d\ln y - \frac{1}{\lambda}\cdot d\ln\Gamma_j^d + \frac{1}{\lambda}\cdot\sum_i \frac{\ell_{ji}}{\ell_j}\cdot d\ln\zeta_i\right) + \mathcal{O}(\epsilon^2). \tag{A19}$$

Effects on group rents $d\ln\mu_g$: we now derive equation (11) in the Proposition. Using Lemma A2 we get

$$\begin{aligned} d\ln\mu_g = \ln\mu_g^{\mathcal{A}}(w,p) - \ln\mu_g(w,p) + \frac{\partial\ln\mu_g(w,p)}{\partial\ln w}\cdot\text{stack}(d\ln w) \\ + \frac{\partial\ln\mu_g(w,p)}{\partial\ln p}\cdot\text{stack}(d\ln p) + \mathcal{O}(\epsilon^2). \end{aligned}$$

We can rewrite the direct effect of automation on rents, $\ln\mu_g^{\mathcal{A}}(w,p) - \ln\mu_g(w,p)$, as

$$\begin{aligned} &\ln\mu_g^{\mathcal{A}}(w,p) - \ln\mu_g(w,p) \\ &\quad= \frac{\mu_g^{\mathcal{A}}(w,p) - \mu_g(w,p)}{\mu_g(w,p)} + \mathcal{O}(\epsilon^2) \\ &\quad= \frac{\dfrac{\sum_i s_{y_i}\cdot p_i^{\lambda-1}\cdot\int_{x\in\mathcal{T}_{gi}}\mu_{gx}\cdot dL_g(x) - \sum_i s_{y_i}\cdot p_i^{\lambda-1}\cdot\int_{x\in\mathcal{A}_{gi}}\mu_{gx}\cdot dL_g(x)}{\sum_i s_{y_i}\cdot p_i^{\lambda-1}\cdot\int_{x\in\mathcal{T}_{gi}} dL_g(x) - \sum_i s_{y_i}\cdot p_i^{\lambda-1}\cdot\int_{x\in\mathcal{A}_{gi}} dL_g(x)} - \mu_g}{\mu_g} + \mathcal{O}(\epsilon^2) \\ &\quad= \frac{\dfrac{\mu_g\cdot\sum_i s_{y_i}\cdot p_i^{\lambda-1}\cdot\int_{x\in\mathcal{T}_{gi}} dL_g(x) - \mu_{\mathcal{A}_g}\cdot\sum_i s_{y_i}\cdot p_i^{\lambda-1}\cdot\int_{x\in\mathcal{A}_{gi}} dL_g(x)}{\sum_i s_{y_i}\cdot p_i^{\lambda-1}\cdot\int_{x\in\mathcal{T}_{gi}} dL_g(x) - \sum_i s_{y_i}\cdot p_i^{\lambda-1}\cdot\int_{x\in\mathcal{A}_{gi}} dL_g(x)} - \mu_g}{\mu_g} + \mathcal{O}(\epsilon^2) \\ &\quad= \frac{\sum_i s_{y_i}\cdot p_i^{\lambda-1}\cdot\int_{x\in\mathcal{A}_{gi}} dL_g(x) - \frac{\mu_{\mathcal{A}_g}}{\mu_g}\cdot\sum_i s_{y_i}\cdot p_i^{\lambda-1}\cdot\int_{x\in\mathcal{A}_{gi}} dL_g(x)}{\sum_i s_{y_i}\cdot p_i^{\lambda-1}\cdot\int_{x\in\mathcal{T}_{gi}} dL_g(x) - \sum_i s_{y_i}\cdot p_i^{\lambda-1}\cdot\int_{x\in\mathcal{A}_{gi}} dL_g(x)} + \mathcal{O}(\epsilon^2). \end{aligned}$$

The first line approximates the change in logs. The second line uses the definition of $\mu_g^{\mathcal{A}}(w,p)$ and the fact that $\mu_g(w,p) = \mu_g$. The third line uses the definition of average group rents and average group rents at automated jobs, $\mu_{\mathcal{A}_g}$. The last line divides by μ_g and cancels terms. By definition $d\ln\Gamma_g^d = \frac{\sum_i s_{y_i}\cdot p_i^{\lambda-1}\cdot\int_{x\in\mathcal{A}_{gi}} dL_g(x)}{\sum_i s_{y_i}\cdot p_i^{\lambda-1}\cdot\int_{x\in\mathcal{T}_{gi}} dL_g(x)}$. Using this, we obtain

$$\begin{aligned} \ln\mu_g^{\mathcal{A}}(w,p) - \ln\mu_g(w,p) &= -\left(\frac{\mu_{\mathcal{A}_g}}{\mu_g} - 1\right)\cdot\frac{d\ln\Gamma_g^d}{1 - d\ln\Gamma_g^d} + \mathcal{O}(\epsilon^2) \\ &= -\left(\frac{\mu_{\mathcal{A}_g}}{\mu_g} - 1\right)\cdot d\ln\Gamma_g^d + \mathcal{O}(\epsilon^2) \end{aligned}$$

The last equality uses the fact that $\frac{d\ln\Gamma_g^d}{1-d\ln\Gamma_g^d} = d\ln\Gamma_g^d + \mathcal{O}(\epsilon^2)$.

We now turn to the effects of sectoral prices on group rents. First, observe that group rents can be written as

$$\mu_g(w,p) = \frac{\sum_i s_{y_i} \cdot p_i^{\lambda-1} \cdot \mu_{gi} \cdot \int_{x \in \mathcal{T}_{gi}} dL_g(x)}{\sum_i s_{y_i} \cdot p_i^{\lambda-1} \cdot \int_{x \in \mathcal{T}_{gi}} dL_g(x)}.$$

Therefore, we can compute the effect of sectoral prices on rents as

$$\frac{\partial \ln \mu_g(w,p)}{\partial \ln p} \cdot d\ln p = \sum_i \frac{s_{y_i} \cdot p_i^{\lambda-1} \cdot \mu_{gi} \cdot \int_{x \in \mathcal{T}_{gi}} dL_g(x)}{\sum_i s_{y_i} \cdot p_i^{\lambda-1} \cdot \mu_{gi} \cdot \int_{x \in \mathcal{T}_{gi}} dL_g(x)} \cdot d\ln \zeta_i - \sum_i \frac{s_{y_i} \cdot p_i^{\lambda-1} \cdot \int_{x \in \mathcal{T}_{gi}} dL_g(x)}{\sum_i s_{y_i} \cdot p_i^{\lambda-1} \cdot \int_{x \in \mathcal{T}_{gi}} dL_g(x)} \cdot d\ln \zeta_i.$$

We can rewrite this as

$$\frac{\partial \ln \mu_g(w,p)}{\partial \ln p} \cdot d\ln p = \sum_i \frac{\mu_{gi}}{\mu_g} \cdot \frac{\ell_{gi}}{\ell_g} d\ln \zeta_i - \sum_i \frac{\ell_{gi}}{\ell_g} \cdot d\ln \zeta_i = \sum_i \left(\frac{\mu_{gi}}{\mu_g} - 1\right) \cdot \frac{\ell_{gi}}{\ell_g} d\ln \zeta_i.$$

These derivations show that

$$d\ln \mu_g = -\left(\frac{\mu_{\mathcal{A}_g}}{\mu_g} - 1\right) \cdot d\ln \Gamma_g^d + \sum_i \left(\frac{\mu_{gi}}{\mu_g} - 1\right) \cdot \frac{\ell_{gi}}{\ell_g} d\ln \zeta_i + \frac{\partial \ln \mu_g(w)}{\partial \ln w} \cdot \text{stack}(d\ln w) + \mathcal{O}(\epsilon^2).$$

Using the solution for baseline wages, we obtain

$$\text{(A20)} \quad d\ln \mu_g = -\left(\frac{\mu_{\mathcal{A}_g}}{\mu_g} - 1\right) \cdot d\ln \Gamma_g^d + \sum_i \left(\frac{\mu_{gi}}{\mu_g} - 1\right) \cdot \frac{\ell_{gi}}{\ell_g} d\ln \zeta_i + \frac{\partial \ln \mu_g(w)}{\partial \ln w} \cdot \Theta \cdot \text{stack}\left(\frac{1}{\lambda} \cdot d\ln y - \frac{1}{\lambda} \cdot d\ln \Gamma_j^d + \sum_i \frac{\ell_{ji}}{\ell_j} d\ln \zeta_i\right) + \mathcal{O}(\epsilon^2).$$

Effects on average wages $d\ln \bar{w}_g$: The expression for $d\ln \bar{w}_g$ in equation (12) follows from combining our formula for baseline wages in (A19) and our formula for rent changes $d\ln \mu_g$ in equation (A20), and using the fact that $d\ln \bar{w}_g = d\ln w_g + d\ln \mu_g$.

Effects on sectoral prices $d\ln p_i$: we now derive the effects of automation on sectoral prices. Equation (A15) implies $p_i^{\lambda-1} = C_i(w)$, where

$$C_i(w) = \Gamma_{ki}(w) + \sum_g \Gamma_{gi}(w) \cdot \mu_{gi}(w) \cdot w_g^{1-\lambda}.$$

Lemma A2 shows that we can expand $C_i(w)$ as

$$dC_i = C_i^{\mathcal{A}}(w) - C_i(w) + C_i(w) \cdot \frac{\partial \ln C_i(w)}{\partial \ln w} \cdot d\ln w + \mathcal{O}(\epsilon^2). \tag{A21}$$

Note that $\frac{\partial \ln C_i(w)}{\partial \ln w} \cdot d\ln w$ captures the effect of a change in wages on the cost of producing the final good at the initial equilibrium allocation. Because tasks are allocated in a cost-minimizing way (given wedges), the envelope theorem implies

$$\frac{\partial \ln C_i(w)}{\partial \ln w} \cdot d\ln w = (1-\lambda) \cdot \sum_g s_{gi} \cdot d\ln w_g + \mathcal{O}(\epsilon^2)$$

The term $C^{\mathcal{A}}(w) - C(w)$ captures the cost saving gains from automating tasks in $\mathcal{A}_{gi}$ holding wages constant. We have

$$\begin{aligned}
C_i^{\mathcal{A}}(w) - C_i(w) =& \Gamma_{ki}^{\mathcal{A}}(w) - \Gamma_{ki}(w) + \sum_g \Gamma_{gi}^{\mathcal{A}}(w) \cdot \mu_{gi}^{\mathcal{A}}(w) \cdot {w_g}^{1-\lambda} - \sum_g \Gamma_{gi}(w) \cdot \mu_{gi}(w) \cdot {w_g}^{1-\lambda} \\
=& \sum_g \left[\frac{1}{M_i} \cdot \int_{x \in \mathcal{A}_{gi}} (q'_x \cdot \psi_{kx})^{\lambda-1} \cdot dx - \frac{1}{M_i} \cdot \int_{x\in\mathcal{A}_{gi}} (\psi_{gx}/\mu_{gx})^{\lambda-1} \cdot {w_g}^{1-\lambda} dx \cdot \right] \\
=& \sum_g \frac{1}{M_i} \int_{x\in\mathcal{A}_{gi}} \left[(q'_x \cdot \psi_{kx})^{\lambda-1} - (\psi_{gx}/\mu_{gx})^{\lambda-1} \cdot {w_g}^{1-\lambda} \right] \cdot dx \\
=& \sum_g \frac{1}{M_i} \int_{x\in\mathcal{A}_{gi}} (\psi_{gx}/\mu_{gx})^{\lambda-1} \cdot {w_g}^{1-\lambda} \cdot \left[\left(\frac{q'_x \cdot \psi_{kx} \cdot w_g \cdot \mu_{gx}}{\psi_{gx}}\right)^{\lambda-1} - 1 \right] \cdot dx \\
=& C_i(w) \cdot \sum_g s_{gi} \cdot d\ln \Gamma_{gi}^d \cdot \frac{\mu_{\mathcal{A}_{gi}}}{\mu_{gi}} \cdot \frac{\frac{1}{M_i} \int_{x\in\mathcal{A}_{gi}} (\psi_{gx}/\mu_{gx})^{\lambda-1} \cdot \left[\left(\frac{q'_x \cdot \psi_{kx} \cdot w_g \cdot \mu_{gx}}{\psi_{gx}}\right)^{\lambda-1} - 1 \right] \cdot dx}{\frac{1}{M_i} \int_{x\in\mathcal{A}_{gi}} (\psi_{gx}/\mu_{gx})^{\lambda-1} \cdot dx} \\
=& C_i(w) \cdot (\lambda - 1) \cdot \sum_g s_{gi} \cdot d\ln \Gamma_{gi}^d \cdot \frac{\mu_{\mathcal{A}_{gi}}}{\mu_{gi}} \cdot \pi_{gi}.
\end{aligned}$$

In the last step, we used the fact that $\ell_{gx} \cdot \mu_{gx} \propto (\psi_{gx}/\mu_{gx})^{\lambda-1}$ (from equilibrium condition E3'), which gives the expression for π_{gi} in the main text.

Putting together our formulas for $\frac{\partial \ln C_i(w)}{\partial \ln w} \cdot d\ln w$ and $C^{\mathcal{A}}(w) - C(w)$ in (A21), we obtain

$$dC_i = C_i(w) \cdot (\lambda - 1) \cdot \sum_g s_{gi} \cdot d\ln \Gamma_{gi}^d \cdot \frac{\mu_{\mathcal{A}_{gi}}}{\mu_{gi}} \cdot \pi_{gi} + C_i(w) \cdot (1-\lambda) \cdot \sum_g s_{gi} \cdot d\ln w_g + \mathcal{O}(\epsilon^2),$$

which is equivalent to

$$d\ln p_i = \sum_g s_{gi} \cdot d\ln w_g - \sum_g s_{gi} \cdot d\ln \Gamma_{gi}^d \cdot \frac{\mu_{\mathcal{A}_{gi}}}{\mu_{gi}} \cdot \pi_{gi}. \tag{A22}$$

Effects on sectoral shifters $d\ln\zeta_i$: The expression for $d\ln zeta_i$ in the proposition follows from the fact that, with a CES demand system, $d\ln s_{y_i}\cdot p_i^{\lambda-1} = (\lambda-\eta)\cdot d\ln p_i$.

Effects on TFP $d\ln tfp$: The expression for the change in TFP in equation (14) is derived in the same way as before.

We now turn to the ideal-price index condition, which can be written as $c_f(\{p_i\}_i) = 1$. The envelope theorem applied to this cost function implies

$$0 = \sum_i s_{y_i}\cdot d\ln p_i.$$

Substituting the expression for $d\ln p_i$ in (A22) and rearranging yields

$$\sum_g s_g\cdot d\ln w_g = \sum_i s_{y_i}\cdot\sum_g s_{gi}\cdot d\ln\Gamma_{gi}^d\cdot\frac{\mu_{\mathcal{A}_{gi}}}{\mu_{gi}}\cdot\pi_{gi}.$$

Adding $\sum_g s_g\cdot d\ln\mu_g$ to both sides yields

$$\sum_g s_g\cdot d\ln\bar{w}_g = \sum_i s_{y_i}\cdot\sum_g s_{gi}\cdot d\ln\Gamma_{gi}^d\cdot\frac{\mu_{\mathcal{A}_{gi}}}{\mu_{gi}}\cdot\pi_{gi} + \sum_g s_g d\ln\mu_g.$$

Using the dual version of the Solow residual in equation (13) to substitute for the left-hand side, we obtain the formula for TFP in equation (14). ■

A.4 Approximation Lemma

One important step in the proofs of Proposition 5 and 6 involves the approximation of the effects of automation in three parts: the effects of the automation shock holding prices constant, the effect of prices governed by the Jacobians of task shares with respect to prices, and a small approximation error. This is similar to a first-order Taylor expansion, but instead of considering a change in real arguments, we are also considering the effects of a direct change in task allocations generated by automation. The following Lemma shows that this expansion provides a valid approximation for automation shocks of order ϵ. We give a general version of the Lemma that accommodates the multisector economy. Its application to the single-sector economy follows as a corollary.

Lemma A2 (Taylor Expansions of Functions on Task Space) *Consider a func-*

tion of the form

$$f(w,z) = h\left(\left\{ \int_{x\in\mathcal{T}_{gi}(w)} N(\mu_{gx},\psi_{gx},\psi_{kx},q_x)\cdot dx \right\}_{g,i}, z\right).$$

where $\mathcal{T}_g(w)$ is defined by E1 and E2, N is a continuous vector function of task attributes to $\mathbb{R}^n$ that is bounded in $\mathcal{T}_g$, z is a vector of inputs of dimension m and h is a continuously differentiable function from $G\times I\times\mathbb{R}^n+\mathbb{R}^m$ to $\mathbb{R}$.

Let $\mathcal{T}^{\mathcal{A}}_{gi}(w)$ denote the equilibrium task allocation after a small automation shock of order ϵ when wages are w. Define

$$f^{\mathcal{A}}(w,z) = h\left(\left\{ \int_{x\in\mathcal{T}^{\mathcal{A}}_{gi}(w)} N(\mu_{gx},\psi_{gx},\psi_{kx},q_x)\cdot dx \right\}_{g,i}, z\right).$$

Suppose that the automation shock changes z and w by dz and dw, both of which are $\mathcal{O}(\epsilon)$. Then the total effect of this shock on f can be approximated as

$$df = f^{\mathcal{A}}(w,z) - f(w,z) + \frac{\partial f}{\partial w}\cdot dw + \frac{\partial f}{\partial z}\cdot dz + \mathcal{O}(\epsilon^2) \tag{A23}$$

Proof. Let $w' = w + dw$ and $z' = z + dz$ be the new values of w, z. The total change in f can be written as

$$\begin{aligned} df =& f^{\mathcal{A}}(w',z') - f(w,z) \\ =& f^{\mathcal{A}}(w,z) - f(w,z) + f^{\mathcal{A}}(w',z') - f^{\mathcal{A}}(w,z) \\ =& f^{\mathcal{A}}(w,z) - f(w,z) + \frac{\partial f^{\mathcal{A}}(w,z)}{\partial w}\cdot dw + \frac{\partial f^{\mathcal{A}}(w,z)}{\partial z}\cdot dz + \mathcal{O}(\epsilon^2), \end{aligned}$$

where the last line does a first-order Taylor expansion of $f^{\mathcal{A}}(w',z')$ around (w,z).

We now show that $\frac{\partial f^{\mathcal{A}}(w,z)}{\partial w} = \frac{\partial f(w,z)}{\partial w} + \mathcal{O}(\epsilon^2)$. Let $a'_{gi} = \int_{x\in\mathcal{T}^{\mathcal{A}}_{gi}(w)} N(\mu_{gx},\psi_{gx},\psi_{kx},q_x)\cdot dx$ and $a_{gi} = \int_{x\in\mathcal{T}_{gi}(w)} N(\mu_{gx},\psi_{gx},\psi_{kx},q_x)\cdot dx$. Because $\mathcal{A}_{gi}$ is of measure $\mathcal{O}(\epsilon)$, $a'_{gi} = a_{gi} + \mathcal{O}(\epsilon)$.

Moreover,

$$\begin{aligned}
\frac{\partial f^{\mathcal{A}}(w,z)}{\partial w} =& \sum_{g,i} \frac{\partial h(\{a'_{gi}\}_{g,i},z)}{\partial a_{gi}} \cdot \frac{\partial}{\partial w} \int_{x \in \mathcal{T}^{\mathcal{A}}_{gi}(w)} N(\mu_{gx},\psi_{gx},\psi_{kx},q_x) \cdot dx \\
=& \sum_{g,i} \frac{\partial h(\{a_{gi}\}_{g,i},z)}{\partial a_{gi}} \cdot \frac{\partial}{\partial w} \int_{x \in \mathcal{T}^{\mathcal{A}}_{gi}(w)} N(\mu_{gx},\psi_{gx},\psi_{kx},q_x) \cdot dx + \mathcal{O}(\epsilon) \\
=& \sum_{g,i} \frac{\partial h(\{a_{gi}\}_{g,i},z)}{\partial a_{gi}} \cdot \frac{\partial}{\partial w} \int_{x \in \mathcal{T}_{gi}(w)} N(\mu_{gx},\psi_{gx},\psi_{kx},q_x) \cdot dx \\
& - \sum_{g,i} \frac{\partial h(\{a_{gi}\}_{g,i},z)}{\partial a_{gi}} \cdot \frac{\partial}{\partial w} \int_{x \in \mathcal{T}_{gi}(w) \setminus \mathcal{T}^{\mathcal{A}}_{gi}(w)} N(\mu_{gx},\psi_{gx},\psi_{kx},q_x) \cdot dx + \mathcal{O}(\epsilon) \\
=& \frac{\partial f(w,z)}{\partial w} + \mathcal{O}(\epsilon).
\end{aligned}$$

The first line uses the chain rule. The second line exploits the fact that the derivatives of h are continuous and therefore $\frac{\partial h(\{a'_{gi}\}_{g,i},z)}{\partial a_{gi}} = \frac{\partial h(\{a_{gi}\}_{g,i},z)}{\partial a_{gi}} + \mathcal{O}(\epsilon)$. The third line decomposes the integral over $\mathcal{T}^{\mathcal{A}}_{gi}$ in three terms. Our focus on small and interior shocks implies that

$$\sum_{g,i} \frac{\partial h(\{a_{gi}\}_{g,i},z)}{\partial a_{gi}} \cdot \frac{\partial}{\partial w} \int_{x \in \mathcal{T}_{gi}(w) \setminus \mathcal{T}^{\mathcal{A}}_{gi}(w)} N(\mu_{gx},\psi_{gx},\psi_{kx},q_x) \cdot dx = \mathcal{O}(\epsilon).$$

To conclude, we show that $\frac{\partial f^{\mathcal{A}}(w,z)}{\partial z} = \frac{\partial f(w,z)}{\partial z} + \mathcal{O}(\epsilon^2)$. We have

$$\begin{aligned}
\frac{\partial f^{\mathcal{A}}(w,z)}{\partial z} =& \frac{\partial h(\{a'_{gi}\}_{g,i},z)}{\partial z} \\
=& \frac{\partial h(\{a_{gi}\}_{g,i},z)}{\partial z} + \mathcal{O}(\epsilon) \\
=& \frac{\partial f(w,z)}{\partial z} + \mathcal{O}(\epsilon),
\end{aligned}$$

where the second line exploits the fact that the derivatives of h are continuous and therefore $\frac{\partial h(\{a'_{gi}\}_{g,i},z)}{\partial z} = \frac{\partial h(\{a_{gi}\}_{g,i},z)}{\partial z} + \mathcal{O}(\epsilon)$.

Using these approximations in the equation for df gives (A23). ■

Remark 1: One can generalize the proof to non-interior shocks. In this case, the approximation $df = \frac{\partial f}{\partial w} \cdot dw + \frac{\partial f}{\partial z} \cdot dz + f^{\mathcal{A}}(w,z) - f(w,z)$ is still valid, but the error is now $\mathcal{O}(\epsilon \cdot \epsilon_b)$, where ϵ_b is an upper bound on the arc-length of boundary tasks that overlap with $\mathcal{A}^T$.

Remark 2: The requirement that the boundary set of tasks

$$\mathcal{B}_{gi}(w) = \left\{ x \in \mathcal{A}_{gi}^{T} : \frac{w_g \cdot \mu_{gx}}{\psi_{gx}} = \frac{1}{q'_x \cdot \psi_{kx}} \right\}$$

has arc-length $\mathcal{O}(\epsilon)$ is also needed to ensure that advances in automation do not introduce a sizable mass of new marginal tasks that change substitution patterns in response to wage changes. In Acemoglu and Restrepo (2022), this was not needed because we assumed that all tasks in which advances in automation occurred where automated. Our definition of small shocks in this paper as well as the assumed nature of the shocks in Acemoglu and Restrepo (2022) require that the wage of group g cannot fall by more than π_{gi} in equilibrium. We check this requirement directly in our quantitative section.

B Microfoundations for wedges.

B.1 Efficiency wage considerations

We consider a static version of an efficiency wage model (i.e. Shapiro and Stiglitz, 1984; Bulow and Summers, 1986).

On the one hand, there is a positive mass of tasks where workers earn a wage w_g and do not have to be monitored or receive extra incentives to work. Workers can always take these jobs freely.

On the other hand, there is a positive mass of tasks where workers need to be monitored and are paid an efficiency wage w_{gx}. In these tasks, workers have two options. They can stick to their duties, produce, and obtain a wage w_{gx}. Or they can shirk. In this case they put no effort on their main job and collect some income $e \cdot w_g$ by moonlighting in the no-rent sector. If not found, they obtain an income $w_{gx} + e \cdot w_g$. However, workers who shirk are detected with probability P_{gx}, fired, and forced to take a job that pays no rents. The no shirking condition is then

$$w_{gx} \geq (1 - P_{gx}) \cdot (w_{gx} + e \cdot w_g) + P_{gx} \cdot w_g.$$

This can be rearranged as

$$w_{gx} = \left(e \cdot \frac{1 - P_{gx}}{P_{gx}} + 1 \right) \cdot w_g.$$

This model thus provides a micro-foundation for wedges $\mu_{gx} = e \cdot \frac{1-P_{gx}}{P_{gx}} + 1$ derived from efficiency wage considerations. Our treatment assumes there are no other contracts that can solve the monitoring problem.

B.2 Bargaining models

Consider a one-shot model where firms must make an investment to create a position before matching with a worker.

A firm producing task x can create ℓ_{gx} positions for workers of type g. Creating each position takes up $\kappa \in (0,1)$ units of labor, which implies that the total amount of labor available for production is $\ell_{gx} \cdot (1-\kappa)$. The firm must pay this cost in advance, which implies that once workers are matched to their positions, there is a surplus to bargain over.

The firm obtains a surplus of $p_x \cdot \psi_{gx} - w_{gx}$ if the negotiation succeeds and 0 otherwise. The worker obtains a surplus of w_{gx} if the negotiation succeeds and w_g otherwise. As before, we assume that there is a positive mass of jobs that pay no rents at which workers can always access. The wage w_{gx} is determined by Nash bargaining, with workers' bargaining power given by $\beta_{gx} \in (0, 1-\kappa)$.

Lemma A3 (Representation result) *The equilibrium of the bargaining economy coincides with that of our baseline model by taking* $\widetilde{\psi}_{gx} = \psi_{gx} \cdot (1-\kappa)$ *and* $\mu_{gx} = \frac{(1-\kappa)\cdot(1-\beta_{gx})}{1-\kappa-\beta_{gx}} \geq 1$.

Proof. Free entry for firms implies

$$(1-\beta_{gx}) \cdot (p_x \cdot \psi_{gx} - w_g) \leq \kappa \cdot p_x \cdot \psi_{gx}.$$

This can be written as

$$p_x \leq w_g \cdot \frac{\mu_{gx}}{\psi_{gx} \cdot (1-\kappa)},$$

which coincides with E1 and E2 for $\widetilde{\psi}_{gx} = \psi_{gx} \cdot (1-\kappa)$. Thus, the bargaining model gives the same rule for allocating tasks across workers and capital than our baseline model with exogenous wedges.

Moreover market clearing for task $x \in \mathcal{T}_g$ requires

$$\psi_{gx} \cdot (1-\kappa) \cdot \ell_{gx} = y \cdot \frac{1}{M} \cdot (\psi_{gx} \cdot (1-\kappa))^{\lambda} \cdot (\mu_{gx} \cdot w_g)^{-\lambda}$$

which coincides with E3 for $\widetilde{\psi}_{gx} = \psi_{gx} \cdot (1 - \kappa)$. Thus, the bargaining model gives the same allocation of labor by tasks as our baseline model with exogenous wedges.

Turning to wages paid to workers, we have

$$w_{gx} = \beta_{gx} \cdot p_x \cdot \psi_{gx} + (1 - \beta_{gx}) \cdot w_g = \mu_{gx} \cdot w_g.$$

This implies the bargaining model gives the same wage payments by task as our baseline model with exogenous wedges. ■

C Measurement and robustness checks.

C.1 Measuring task displacement

This subsection derives the measure of direct task displacement in equation (17).

First, let $\mathcal{R}_{gi}$ denote the set of routine tasks in industry i assigned to group g. Define

$$\Gamma_{gi}^{\text{routine}} = \int_{\mathcal{R}_{gi}} \psi_{xg}^{\lambda-1} \cdot \mu_{xg}^{-\lambda} \cdot dx,$$

as the task share of group g in routine jobs at industry i. As discussed in the main text, we assume that all groups experience the same displacement from routine jobs in industry i. Formally, this implies $d \ln \Gamma_{gi}^{\text{routine},d} = \chi_i^{\text{routine}}$. In addition, the assumption that non-routine jobs are not automated implies

$$d \ln \Gamma_{gi}^{d} = (\ell_{gi}^{\text{routine}} / \ell_{gi}) \cdot \chi_i^{\text{routine}}, \tag{A24}$$

where $\ell_{gi}^{\text{routine}} / \ell_{gi}$ is the share of employment of group g in industry i earned in routine jobs (out of all employment of group g in industry i).

Let's now turn to the labor share in industry i. This is given by

$$s_{\ell i} = \frac{\sum_g \Gamma_{gi} \cdot \mu_{gi} \cdot w_g^{1-\lambda}}{p_i^{1-\lambda}}. \tag{A25}$$

The direct effect of automation on the labor share $s_{\ell i}$ holding wages constant is

$$d \ln s_{\ell i}^{d} = -\sum_g \frac{s_{gi}}{s_{\ell i}} \cdot d \ln \Gamma_{gi}^{d} \cdot \frac{\mu_{\mathcal{A}_{gi}}}{\mu_g} - (1 - \lambda) \cdot d \ln p_i.$$

Using the formula for $d\ln p_i$ in (A22), we obtain

$$d\ln s_{\ell i}^d = -\sum_g \frac{s_{gi}}{s_{\ell i}} \cdot d\ln \Gamma_{gi}^d \cdot \frac{\mu_{\mathcal{A}_{gi}}}{\mu_{gi}} + (1-\lambda) \cdot \sum_g s_{gi} \cdot d\ln \Gamma_{gi}^d \cdot \frac{\mu_{\mathcal{A}_{gi}}}{\mu_{gi}} \cdot \pi_{gi}$$
$$= -\sum_g \frac{s_{gi}}{s_{\ell i}} \cdot d\ln \Gamma_{gi}^d \cdot \frac{\mu_{\mathcal{A}_{gi}}}{\mu_{gi}} \cdot (1 - s_{\ell i} \cdot (1-\lambda) \cdot \pi_{gi}).$$

Define the average cost-saving gains and average rent dissipation in industry i as

$$\pi_i = \frac{\sum_g s_{gi} \cdot d\ln \Gamma_{gi}^d \cdot \frac{\mu_{\mathcal{A}_{gi}}}{\mu_{gi}} \cdot \pi_{gi}}{\sum_g s_{gi} \cdot d\ln \Gamma_{gi}^d \cdot \frac{\mu_{\mathcal{A}_{gi}}}{\mu_{gi}}}, \qquad 1 + \rho_i = \frac{\sum_g s_{gi} \cdot d\ln \Gamma_{gi}^d \cdot \frac{\mu_{\mathcal{A}_{gi}}}{\mu_{gi}}}{\sum_g s_{gi} \cdot d\ln \Gamma_{gi}^d}.$$

Using these definitions, we can write the change in labor shares as

$$d\ln s_{\ell i}^d = -(1+\rho_i) \cdot (1 - s_{\ell i} \cdot (1-\lambda) \cdot \pi_i) \cdot \sum_g \frac{s_{gi}}{s_{\ell i}} \cdot d\ln \Gamma_{gi}^d.$$

Using equation (A24), we can rewrite the change in the labor share as

$$\text{(A26)} \qquad d\ln s_{\ell i}^d = -(1+\rho_i) \cdot (1 - s_{\ell i} \cdot (1-\lambda) \cdot \pi_i) \cdot \sum_g \frac{s_{gi}}{s_{\ell i}} \cdot (\ell_{gi}^{\text{routine}} / \ell_{gi}) \cdot \chi_i^{\text{routine}}.$$

Using this equation, we can solve for the common rate of automation $\omega_i^{\text{routine}}$ as

$$\chi_i^{\text{routine}} = \frac{1}{\sum_g \frac{s_{gi}}{s_{\ell i}} \cdot (\ell_{gi}^{\text{routine}} / \ell_{gi})} \cdot \frac{1}{1+\rho_i} \cdot \frac{-d\ln s_{\ell i}^d}{1 - s_{\ell i} \cdot (1-\lambda) \cdot \pi_i}.$$

A second use of equation (A24) then implies

$$d\ln \Gamma_{gi}^d = \text{RCA}_{gi}^{routine} \cdot \frac{1}{1+\rho_i} \cdot \frac{-d\ln s_{\ell i}^d}{1 - s_{\ell i} \cdot (1-\lambda) \cdot \pi_i},$$

where the revealed comparative advantage measure is constructed as

$$\text{(A27)} \qquad \text{RCA}_{gi}^{routine} = \frac{\ell_{gi}^{\text{routine}} / \ell_{gi}}{\sum_{g'} \frac{s_{g'i}}{s_{\ell i}} \cdot (\ell_{g'i}^{\text{routine}} / \ell_{g'i})}.$$

C.2 Robustness checks

Acemoglu and Restrepo (2022) report a vast range of robustness checks for the reduced-form relationship between group average wages and their direct task displacement due to automation. Here, instead, we provide robustness checks for the relationship between automation and within-group wage dispersion and rents, which is the novel empirical aspect in this paper.

Figure A1 provides a robustness check for the U-shaped pattern of within group wage declines explored in Section 3.3. The left panel reports estimates by percentile in levels and not relative to the 30th percentile as in the main text. The right panel reports estimates constraining the estimation sample to groups with an average real wage in 1980 above \$13 dollars. The results in the right panel show that, once we focus on this group, wage changes become flat below the 30th percentile. They also show that the bigger drop at top percentiles is not driven by top workers in low-pay groups, but can also be seen among top workers at highly paid groups.

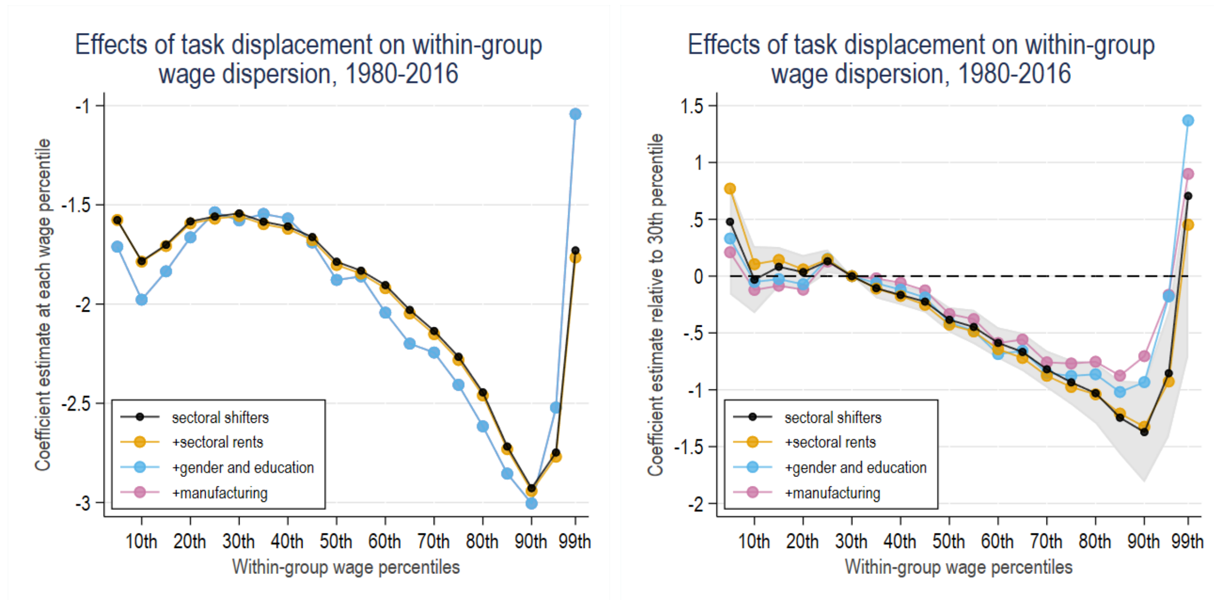

Figure A1: Reduced-form relationship between wage changes across percentiles of the within-group wage distribution and task displacement. the left panel plots estimates from a group quantile regression of changes in $d\ln w_g^p$ against task displacement for percentiles p ranging from the 5th to the 99th. The lines provide estimates for different specifications. The right panel excludes worker groups with an average hourly wage below \$13 dollars in 1980. This panel reports estimates relative to the 30th percentile.

Figure A2 provides a robustness check for the estimates in Section 3.3 exploring the association between direct task displacement and group rents. In this case, rents are proxied by the change in employment at industry and occupations that paid above-average wages nationally in 1980,

$$\Delta \ln \mu_g^{\text{reallocation}} = \sum_{i,o} \left(\frac{\bar{w}_{io}}{\bar{w}} - 1 \right) \cdot \Delta \ell_{g,i,o}.$$

The term $\frac{\bar{w}_{io}}{\bar{w}}$ is a weighted average of $\frac{\bar{w}_{gio}}{\bar{w}_g}$ across all groups, where the weights are given by group wage payments from industry i and occupation o.

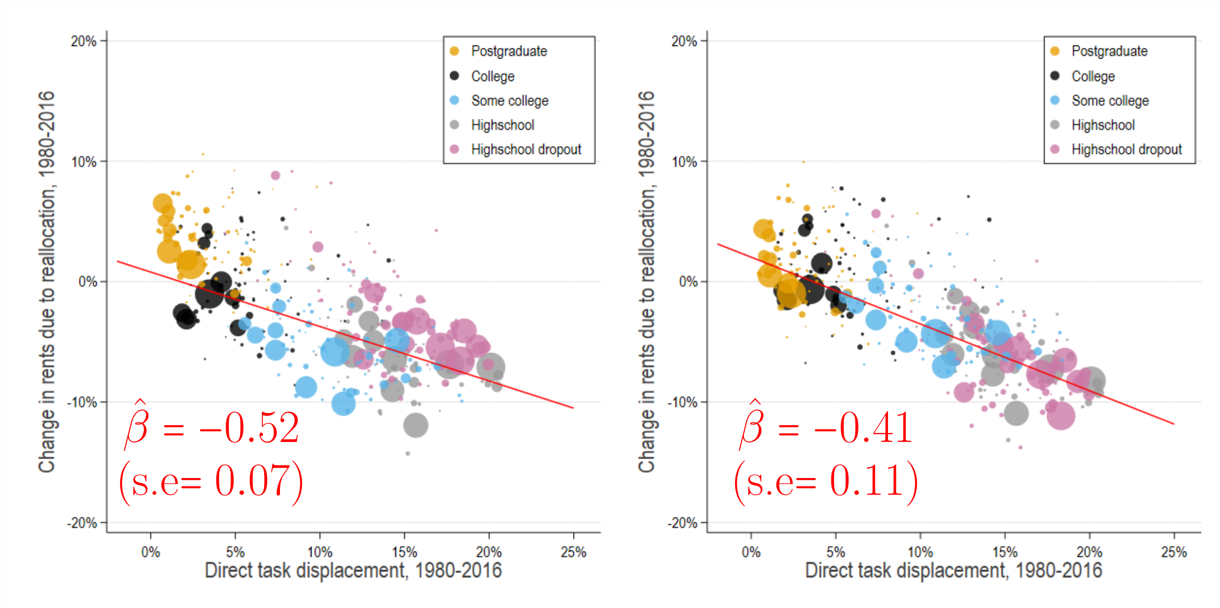

FIGURE A2: REDUCED-FORM RELATIONSHIP BETWEEN RENTS AND TASK DISPLACEMENT. The left panel plots the bivariate relationship between change in group rents and task displacement. The right panel partials out covariates, including gender and education dummies, sectoral demand and rent shifters, and the manufacturing employment share of groups in 1980. Rents are proxied by the change in employment at industry and occupations that paid above-average wages nationally in 1980, $\Delta \ln \mu_g^{\text{reallocation}} = \sum_{i,o} \left(\frac{\bar{w}_{io}}{\bar{w}_g} - 1 \right) \cdot \Delta \ell_{g,i,o}$.

Finally, Table A1 provides a summary of the reduced-form estimates linking direct task displacement to group wages and rents. Table A2 provides various robustness checks for our results for rent dissipation. Panel A measures wage compression relative to the 20th percentile. Panel B measures rent compression relative to the 40th percentile. Panel C measures rent compression relative to the 50th percentile. Panel D excludes groups with an average wage below 13 dollars. Panel E estimates the reduced-form model for rent dissipation for workers with no college degree. Panel F estimates the reduced-form model for rent dissipation for workers with a college degree. These two panels show that rent dissipation is visible among college and non-college workers, even though the estimates become imprecise when separated by group.

Table A1: Summary of reduced-form evidence, 1980-2016.

	(1)	(2)	(3)	(4)
	Panel A. Dependent variable: percent change in group average wage, $\Delta \ln \bar{w}_g$			
Direct task displacement	-2.43 (0.14)	-2.11 (0.26)	-2.11 (0.28)	-1.98 (0.31)
Share variance task displacement	0.66	0.57	0.57	0.53
R-squared	0.66	0.83	0.83	0.83
	Panel B. Dependent variable: change in group rents, $\Delta \ln \mu_g = \Delta \ln \bar{w}_g - \Delta \ln w_g^{30th}$			
Direct task displacement	-0.38 (0.06)	-0.57 (0.13)	-0.53 (0.11)	-0.40 (0.11)
Share variance task displacement	0.24	0.36	0.34	0.25
R-squared	0.24	0.38	0.39	0.44
	Panel C. Dependent variable: change in group rents due to reallocation, $\Delta \ln \mu_g^{\text{reallocation}} = \sum_{i,o} \left(\frac{\bar{w}_{gio}}{\bar{w}_g} - 1 \right) \cdot \Delta \ell_{g,i,o}$			
Direct task displacement	-0.50 (0.07)	-0.37 (0.11)	-0.36 (0.11)	-0.41 (0.11)
Share variance task displacement	0.38	0.28	0.28	0.31
R-squared	0.38	0.61	0.61	0.61
	Panel D. Dependent variable: change in group rents (measured at national level) due to reallocation, $\Delta \ln \mu_g^{\text{reallocation}} = \sum_{i,o} \left(\frac{\bar{w}_{io}}{\bar{w}} - 1 \right) \cdot \Delta \ell_{g,i,o}$			
Direct task displacement	-0.52 (0.07)	-0.38 (0.11)	-0.38 (0.11)	-0.41 (0.11)
Share variance task displacement	0.41	0.30	0.29	0.32
R-squared	0.41	0.64	0.64	0.64
Covariates:				
Education and gender		✓	✓	✓
Sectoral demand shifters		✓	✓	✓
Sectoral rent shifters			✓	✓
Manufacturing share				✓

Notes: This table presents estimates of the relationship between the direct task displacement due to automation and the change in hourly wages and rents for 500 demographic groups, defined by gender, education, age, race, and native/immigrant status. The dependent variable is indicated in the panel headers. Column 2 controls gender and education dummies and sectoral demand shifters. Column 3 controls for sectoral rent shifters. Column 4 controls for the manufacturing employment share of groups in 1980. All regressions are weighted by total hours worked by each group in 1980. Standard errors robust to heteroskedasticity are reported in parentheses.

Table A2: Robustness checks for rent dissipation, 1980-2016.

	(1)	(2)	(3)	(4)
	Panel A. Dependent variable: change in group rents, $\Delta \ln \mu_g = \Delta \ln \bar{w}_g - \Delta \ln w_g^{20th}$			
Direct task displacement	-0.45	-0.48	-0.45	-0.21
	(0.08)	(0.20)	(0.18)	(0.17)
	Panel B. Dependent variable: change in group rents, $\Delta \ln \mu_g = \Delta \ln \bar{w}_g - \Delta \ln w_g^{40th}$			
Direct task displacement	-0.33	-0.59	-0.54	-0.44
	(0.07)	(0.15)	(0.12)	(0.11)
	Panel C. Dependent variable: change in group rents, $\Delta \ln \mu_g = \Delta \ln \bar{w}_g - \Delta \ln w_g^{50th}$			
Direct task displacement	-0.12	-0.26	-0.23	-0.21
	(0.06)	(0.09)	(0.08)	(0.08)
	Panel D. Dependent variable: change in group rents, $\Delta \ln \mu_g = \Delta \ln \bar{w}_g - \Delta \ln w_g^{30th}$ excluding low income groups			
Direct task displacement	-0.27	-0.33	-0.34	-0.30
	(0.07)	(0.14)	(0.12)	(0.14)
	Panel E. Dependent variable: change in group rents, $\Delta \ln \mu_g = \Delta \ln \bar{w}_g - \Delta \ln w_g^{30th}$ for non-college groups			
Direct task displacement	-0.24	-0.58	-0.51	-0.34
	(0.10)	(0.15)	(0.13)	(0.13)
	Panel F. Dependent variable: change in group rents, $\Delta \ln \mu_g = \Delta \ln \bar{w}_g - \Delta \ln w_g^{30th}$ for college groups			
Direct task displacement	-0.71	-0.35	-0.36	-0.39
	(0.25)	(0.31)	(0.36)	(0.37)
Covariates:				
Education and gender		✓	✓	✓
Sectoral demand shifters		✓	✓	✓
Sectoral rent shifters			✓	✓
Manufacturing share				✓

Notes: This table presents estimates of the relationship between the direct task displacement due to automation and various proxies of rents for 500 demographic groups, defined by gender, education, age, race, and native/immigrant status. The details of each specification are indicated in the panel headers. Panel A measures wage compression relative to the 20th percentile. Panel B measures rent compression relative to the 40th percentile. Panel C measures rent compression relative to the 50th percentile. Panel D excludes groups with an average wage below 13 dollars ($N = 364$). Panel E estimates the reduced-form model for rent dissipation for workers with no college degree ($N = 300$). Panel F estimates the reduced-form model for rent dissipation for workers with a college degree ($N = 200$). Column 2 controls gender and education dummies and sectoral demand shifters. Column 3 controls for sectoral rent shifters. Column 4 controls for the manufacturing employment share of groups in 1980. All regressions are weighted by total hours worked by each group in 1980. Standard errors robust to heteroskedasticity are reported in parentheses.