Bonus Question: Does Flexible Incentive Pay Dampen Unemployment Fluctuations?

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Abstract

This paper introduces rich dynamic incentive contracts into a benchmark model of unemployment fluctuations and presents three results. First, wage cyclicality due to incentives does not dampen unemployment fluctuations: unemployment dynamics are first-order equivalent in an economy with flexible incentive pay and without bargaining, and in an economy with rigid real wages (Hall, 2005). Second, wage cyclicality due to bargaining does dampen unemployment fluctuations through the standard mechanism. Third, calibrating the model suggests 40% of wage cyclicality in the data arises from incentives. A standard model without incentives, calibrated to weakly pro-cyclical wages, matches unemployment dynamics in our incentive pay model, calibrated to substantially more pro-cyclical wages.

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1 Introduction

Macroeconomists have long argued that limited adjustment of wages is important for business cycles (Keynes, 1937). If wages are stable, then profits per worker and labor demand are volatile. Equipped with this insight, modern labor search models incorporate rigidities that reduce wage cyclicality and lead to large unemployment fluctuations (Hall, 2005; Gertler and Trigari, 2009).

However mapping the theory to data is difficult because compensation is complex. In particular, incentive pay – such as piece-rates, bonuses, profit sharing, commissions and stock options – is prevalent. Around half of all workers receive some incentive pay, including around 30% of bottom decile earners (Lemieux et al., 2009; Makridis and Gittleman, 2018). Longer-term incentives, such as promotions, are also common. Furthermore, incentive pay seems to be relatively flexible. Bonuses are raised and lowered frequently at the micro level (Grigsby et al., 2021) and are strongly pro-cyclical in some studies, though this fact is controversial (Bils, 1985; Devereux, 2001; Shin and Solon, 2007; Swanson, 2007).

This paper asks how flexible incentive pay affects unemployment dynamics. We consider a rich dynamic incentive contract with moral hazard and persistent idiosyncratic shocks which we embed in the Diamond-Mortensen-Pissarides (DMP) labor search model. In the model, risk neutral firms match with risk averse workers in a frictional labor market, and produce output as a function of idiosyncratic and aggregate productivity, as well as worker effort. Firms observe aggregate productivity but cannot distinguish between idiosyncratic productivity and effort. Therefore firms propose *flexible incentive pay* to overcome the resulting moral hazard problem, by conditioning wages on output to balance insurance with incentivizing effort. We also allow the contract to promise the worker higher ex ante utility during expansions, due either to bargaining or cyclicality in the outside option.[1]

Our model has two appealing features. First, it allows cyclical and flexible incentive pay, consistent with micro evidence. If the marginal product of effort falls during recessions, then firms find effort less valuable and lower expected wages. By contrast, standard labor search models without moral hazard (e.g. Shimer, 2005) attribute all wage cyclicality to bargaining. Second, our dynamic environment recognizes that employment is a long-term relationship (Barro, 1977).

Our first analytical result is that wage cyclicality due to incentives does not dampen unemployment fluctuations. We study a version of the flexible incentive pay economy without

[1]We consider a protocol that nests Nash, Hall and Milgrom (2008) bargaining and their versions in Chodorow-Reich and Karabarbounis (2016); and allows cyclical unemployment benefits (Hagedorn et al., 2013). Our model of bargaining also evokes a notion of unemployment as a "worker discipline device" as in Shapiro and Stiglitz (1984).

bargaining, in which all fluctuations in wages are due to incentives. Then we prove an equivalence result: the first order dynamics of unemployment are the same in the flexible incentive pay economy without bargaining as in an economy with exogenously fixed real wages as in Hall (2005), so long as both models are calibrated to the same steady state labor share. Therefore incentive wages, no matter how procyclical, do not mute the response of unemployment to business cycle shocks — since a model with fixed wages has the same unemployment response.

Our result may be surprising. After all, a standard argument is that flexible bonus pay dampens unemployment fluctuations, by reducing marginal costs during contractions (Weitzman, 1986). The intuition for our different result comes from incentives. In our model, the response of profits to aggregate shocks determines unemployment fluctuations. With flexible incentive pay, wages fall after a contraction, which dampens the response of profits — the standard marginal cost effect. However there is a less standard *incentive effect* of wage changes. If wages fall, then workers may have weaker incentives. If so, they lower their effort, which amplifies the fall in profits and offsets the effect of lower marginal costs. For the optimal incentive contract, the incentive and marginal cost effects of wage changes on profits cancel out exactly, due to the envelope theorem. Put differently, at the optimal contract, the firm trades off the cost of providing incentives with the marginal product of effort. Small movements in exogenous productivity generate offsetting changes in effort and wages. Therefore profits in the flexible incentive pay economy behave *as if* neither wages nor effort had responded to the aggregate shock.

The irrelevance of incentive pay for unemployment fluctuations is general: for instance it applies when utility is non-separable in effort and consumption, and for idiosyncratic shock processes with arbitrary persistence.[2] This generality is surprising, since dynamic incentive contracts are hard to characterize outside special cases (e.g. Holmstrom and Milgrom, 1987). We sidestep this difficulty by characterizing the dynamics of profits *without* characterizing the optimal contract. To do so, we implement a non-standard envelope theorem, using results from the applied mathematics literature on "sensitivity analysis" (Bonnans and Shapiro, 2013). This argument may be more broadly useful when introducing mechanism design into business cycles.

Our second analytical result shows that wage cyclicality due to bargaining or outside option fluctuations dampens unemployment fluctuations, as in standard labor search models without incentives. Reintroducing bargaining into the flexible incentive pay model, we show that only the portion of wage fluctuations associated with changes in the utility promised to

[2]We also establish a similar result in a richer environment with endogenous separations (Mortensen and Pissarides, 1994). Our incentive model also nests incentive contracts such as tournaments.

the worker, which we dub "bargained wage cyclicality," enters the equation characterizing unemployment's response to business cycle shocks. Intuitively, wage cyclicality due to bargaining dampens unemployment fluctuations for a standard reason: lower bargained wages during a contraction do not imply an offsetting fall in effort, and stabilize profits as a result.

The analytical results urge researchers to estimate the portion of wage cyclicality that is due to bargaining or outside option cyclicality, and filter out wage cyclicality due to incentives. In the final part of our paper, we pursue one path towards this goal by calibrating a version of our model to micro moments. We extend the tractable contracting environment of Edmans et al. (2012a) to incorporate aggregate risk and develop a method to simulate the model. We calibrate the model to match micro moments of wage adjustment, such as the variance of incumbent wage growth and the pass-through of idiosyncratic profitability shocks – both of which inform the strength of incentives. We also target new hire wage cyclicality, which informs the cyclicality of workers' outside options and their bargaining power. We conservatively target a low estimate of pass-through, to reduce the importance of incentives. The model thus gives an upper bound on the share of overall wage cyclicality that is due bargaining or cyclical outside options.

Our third result is therefore numerical: we find that bargained wage cyclicality accounts for approximately 60% of overall wage cyclicality. Therefore models without incentive pay should target wage cyclicality for new hires which is 60% of the overall wage cyclicality in the data: a number like -0.6. As a result, a simple version of our model with bargaining but no incentives — similar to standard labor search models — generates the same unemployment dynamics as the full model, if it is calibrated to wage cyclicality of -0.6.

Taken together, our three results suggest that researchers may work with simple and standard models without dynamic incentive contracts, so long as these models are calibrated to match only bargained wage cyclicality. Our numerical approach suggests that these simple models should target wage cyclicality that is weakly procyclical, compared to measures of overall wage cyclicality in the data. However, we stress that our numerical results are tentative: we hope future empirical work will carefully measure wage cyclicality that is due to bargaining and not incentives.

Let us make three caveats. First, our mechanism depends on procyclical effort, consistent with the available time series evidence.[3] Nevertheless, cyclical fluctuations in effort are hard to measure and an important topic for future research. Second, we do not consider on the job search. In that case, incentive pay may also affect recruitment and retention (e.g. Balke

[3]For instance, diverse measures of worker effort—from time use surveys, variable capacity utilization, and information on workplace injuries—all seem to fall during recessions (Burda et al., 2020; Fernald, 2014; Galí and Van Rens, 2021). Furthermore, the pass-through of idiosyncratic firm shocks to wages is pro-cyclical (Chan et al., 2023), which is consistent with firms seeking to incentivize more effort during booms.

and Lamadon, 2022; Elsby et al., 2023), which is beyond the scope of this paper. Finally, our result concerns the responsiveness of unemployment to exogenous productivity fluctuations. Models with incentive pay may yield output dynamics which differ from simpler models due to movements in effort, evoking a notion of capacity utilization.

Related Literature. A growing literature studies the adjustment patterns of wages over the business cycle in microdata. One common result is that measures of wages that include incentives, such as annual earnings per hour or bonus pay, often seem more flexible—whereas measures of pay excluding incentives, such as base pay, tend to be rigid. This result seems true not only for job stayers' wages (e.g. Solon et al., 1997), but also for new hire wages, which are allocative for unemployment in standard models (Pissarides, 2009). For instance, studying base wages for new hires from online vacancy postings and from administrative payroll data, both of which contain detailed job level information, Hazell and Taska (2022) and Grigsby et al. (2021) find limited responsiveness of nominal and real wages. Studying wages for new hires from survey data that do not separately report non-base pay, papers such as Bils et al. (2022a) find procyclical real wages.[4,5] One requires a model to determine the relevant notion of wage cyclicality for unemployment dynamics given bonuses, since bonuses arise in part due to incentive problems. Our contribution is to provide such a model which can be calibrated to microdata, and to clarify that wage cyclicality arising from incentives does not mute the response of unemployment to exogenous shocks.

Second, a large literature relates wage rigidity to unemployment dynamics. Many papers study wage setting with exogenous and fixed effort by workers.[6] One theme is that wage rigidity leads to large unemployment fluctuations, while flexible wages dampen these fluctuations. Our contribution is to study wage setting with endogenous and variable effort, via flexible incentive pay contracts. We show that large unemployment fluctuations can coexist with flexible and cyclical wages, so long as incentives determine wage cyclicality.

Several papers consider unemployment dynamics with incentives and relate closely to ours. First, Moen and Rosén (2011) considers an elegant model with static incentive contracts and wage posting, finding numerically that incentives amplify unemployment fluctuations.[7]

[4]See Kudlyak (2014); Basu and House (2016) and Bellou and Kaymak (2021) for related papers on the cyclicality of the wage for new hires.

[5]Grigsby et al. (2021) also find that bonus wages are cut frequently, but are not cyclical, largely studying a different time period from Bils et al. (2022a).

[6]An incomplete list of papers from this vast literature includes Shimer (2005); Hall (2005); Hall and Milgrom (2008); Gertler and Trigari (2009); Elsby (2009); Brügemann and Moscarini (2010); Rudanko (2009); Kennan (2010); Fukui (2020); Gertler et al. (2020); Elsby and Gottfries (2022) and Blanco et al. (2022). Some papers within this literature study implicit contracts, in which firms insure workers against wage risk, while effort is exogenous (e.g. Baily, 1974; Azariadis, 1975; Beaudry and Dinardo, 1991; Krusell et al., 2010; Broer et al., 2023).

[7]See Zhou (2022) for a related model.

Second, Fongoni (2020) considers a labor search model with wage posting, in which wages affect effort due to exogenous reference dependent preferences. In steady state, the paper astutely notes how effort changes can offset wage flexibility. Our contribution is to offer a model with dynamic incentive contracts and bargaining power, which allows a tight mapping to the micro evidence. Moreover our approach lets us analytically characterize unemployment dynamics, to connect to simple models with wage rigidity, and to illustrate an envelope result which underlies the amplified fluctuations in unemployment.

A third related paper studying unemployment dynamics is Bils et al. (2022b), who show that large employment fluctuations can exist despite flexible new hire wages if incumbent workers' wages are rigid and effort is flexible. In their model, contractions do not lower incumbent workers' wages, so firms demand higher effort from continuing workers and reduce hiring. Our paper is complementary to theirs and echoes their key theme—effort is crucial for employment fluctuations. However our focus is different: we link a canonical model of dynamic incentive pay to micro facts about wage adjustment and argue that one should measure only wage cyclicality arising from bargaining. On the other hand, Bils et al. (2022b) study a setting with Nash bargained observable effort.

Third, our paper relates to a literature that studies contracts between firms and workers with moral hazard. These optimal contracting problems are challenging, because the firm must maximize expected profits among a hard-to-characterize continuum of incentive compatible contracts. Holmstrom and Milgrom (1987) show that the wage contract is linear if the worker has exponential utility and there are normal idiosyncratic shocks. Sannikov (2008) shows, in continuous time, that incentives can be provided by increasing the volatility of the worker's expected utility. Edmans et al. (2012a) solve for an explicit incentive pay contract by adding further restrictions on the state-dependence of contracts. Doligalski et al. (2023) show how the pass-through of output shocks to wages varies with effort. We contribute to this literature in three ways. First, we analytically study the aggregate implications of moral hazard frictions, in a setting with aggregate risk. Second, we introduce an extensive margin of unemployment and bargaining over the promised utility of the contract. Third, we derive our main result without relying on an explicit form of the optimal contract, by applying an envelope theorem to the principal's objective—therefore, our results apply under general assumptions.[8]

Finally, our results are reminiscent of work on price setting and aggregate nominal rigidity. This literature has developed models that are consistent with micro-evidence on price setting, but tractable enough to study aggregate rigidity (Alvarez et al., 2016; Auclert et al.,

[8]Our work also relates to papers such as Li and Williams (2015) and Veracierto (2022), who study optimal unemployment insurance contracts with moral hazard and aggregate risk.

2022). Similarly, we develop a model that is consistent with micro-evidence on wage setting, but tractable for examining aggregate rigidity. In parallel, other papers have investigated which micro moments on price setting are most relevant for aggregate rigidity—for instance, concluding that sales are irrelevant (e.g. Kehoe and Midrigan, 2008; Eichenbaum et al., 2011). In this spirit, we isolate which micro moments on wage setting are relevant wage flexibility—that is, wage changes related to bargaining or cyclical outside options, but not due to incentives.[9]

Layout. Section 2 contains a static model similar to Holmstrom (1979), which provides intuition for the role of incentive effects and the irrelevance of incentive wage cyclicality for unemployment fluctuations. Section 3 explores similar ideas with a standard labor search model and a general dynamic incentive contract. In Section 4, we provide numerical results on the share of wage cyclicality due to incentives versus bargaining. Section 5 concludes.

2 Illustrative Static Model

This section explains our results in an illustrative framework that combines a static Diamond-Mortensen-Pissarides (DMP) labor search model with two alternative models of wage setting. The first model features a standard static incentive contract as in Holmstrom (1979), which results in pro-cyclical and flexible wages. The second economy has exogenously rigid wages and effort, as in Hall (2005). Then we explain our two analytical results. First, wage cyclicality due to incentives does not dampen unemployment fluctuations. Second, wage cyclicality due to bargaining does dampen unemployment fluctuations. In section 3 below, we relax many of the assumptions of this section in a rich dynamic model.

2.1 Environment of the Static Model

Frictional labor Markets. There is a unit measure of workers who begin the period unemployed. Workers randomly search for vacancies in a frictional labor market. Workers end the period employed if they match with a vacancy and otherwise end the period unemployed. There is a continuum of risk neutral firms. Each firm can post a vacancy at a cost κ per vacancy. θ is the measure of vacancies posted. Since a unit measure of workers are unemployed at the start of the period, θ is also market tightness—the ratio of vacancies to unemployed workers. Given search frictions, the probability that an individual vacancy

[9] The literature on nominal price rigidity finds that sales do not matter for aggregate rigidity because they are transient, staggered and acyclical (Nakamura and Steinsson, 2008). We find that incentive pay does not matter for aggregate rigidity even if incentive wage changes are persistent, simultaneous and cyclical.

matches with a worker is $q(\theta) \equiv \psi\theta^{-\nu}$, a decreasing and isoelastic function of the measure of vacancies posted.

Workers. Workers have risk averse preferences over consumption c and labor effort a, given by a utility function $u(c, a)$ that is strictly increasing and strictly concave in c, but weakly decreasing and concave in a. If workers end the period unemployed, they consume an unemployment benefit $b(z)$, and exert no effort. They thus attain utility $U(z) \equiv u(b(z), 0)$. If employed, the worker exerts effort and is paid a wage w, which they consume.

Technology. If a firm and worker match, they produce the numeraire good with a production function $y(a, \eta, z) = z(a + \eta)$. Here, z is an exogenous aggregate productivity term which affects all firms, a is effort of the employed worker, and η is an exogenous idiosyncratic shock to production, which we assume to be normally distributed with mean zero and standard deviation σ_η. We term η "noise."

Information. Aggregate productivity z is common knowledge. Firms are able to observe their worker's output; however, they do not observe effort a and noise η separately. Workers choose effort before noise η is realized. As such, firms' expected profits from a filled vacancy are $J(z) \equiv \mathbb{E}_\eta[z(a + \eta) - w]$, where the expectation is over values of η inferred by the firm.

Free entry. Free entry requires that expected profits from posting a vacancy must equal the cost of posting the vacancy, which implies

$$\kappa = q(\theta)J(z). \tag{1}$$

Now, we introduce two models of wage and effort setting.

Flexible incentive pay economy of Holmstrom (1979). When a firm and worker match, the firm offers a contract to the worker which specifies a suggested effort level $a(z)$ and wages as a function of output realizations $w(z, y)$. Crucially the firm cannot condition wages directly on effort, which is unobservable, leading to a moral hazard problem. Therefore the firm maximizes profits subject to an incentive compatibility and participation constraint. The incentive compatibility (IC) constraint states that the suggested effort level is an optimal choice for the worker given the wage contract offered by the firm. The participation constraint (PC) states that the worker's expected utility under the contract is at least $\mathcal{B}(z)$. $\mathcal{B}(z)$ is a function mapping the aggregate state z to the worker's "promised utility" from the contract and captures bargaining and outside option cyclicality in reduced form. For instance, if the firm makes take-it-or-leave-it (TIOLI) offers to the worker and workers' outside option is acyclical $b(z) = b$, then the worker's promised utility is the value of unemployment benefits, so $\mathcal{B} = U \equiv u(b, 0)$. If there is Nash bargaining over the output of a match, then $\mathcal{B}(z)$ will be an increasing function of z.[10]

The firm's problem after meeting a worker is

$$J^{\text{Incentive}}(z) \equiv \max_{a(z),w(z,y)} \mathbb{E}_\eta[z(a(z)+\eta) - w(z,y)] \tag{2}$$

subject to
$$a(z) \in \arg\max_{\tilde{a}(z)} \mathbb{E}_\eta \left[u(w(z,y), \tilde{a}(z))\right] \qquad [IC]$$
$$\mathbb{E}_\eta \left[u(w(z,y), a(z))\right] \geq \mathcal{B}(z). \qquad [PC]$$

Our notation makes explicit that effort and wages may depend on realizations of both z and y (and thus the idiosyncratic component of output $a+\eta$), but the firm is uncertain over the realized value of η. Let $a^*(z)$ and $w^*(z,y)$ denote the contracted effort and wage levels as a function of productivity and output realizations.[11,12]

As usual, this contract implies a tradeoff between incentives and insurance. Absent moral hazard, firms would fully insure workers against wage risk. With moral hazard, firms will pass idiosyncratic noise shocks through to workers' wages, in order to provide incentives. This simple and standard model allows for "flexible pay," since the firm can freely adjust wages subject to the IC and PC constraints without further constraints. In particular the firm can freely vary wages with z, potentially leading to pro-cyclical wages.

Rigid wage economy of Hall (2005). In this benchmark model, wages and effort are exogenously fixed at $\bar{a}$ and $\bar{w}$, irrespective of the value of aggregate productivity z. Let J^{Rigid} be the value of a filled vacancy in this economy. There are no nominal frictions, and we study real wage rigidity.

Equilibrium. In the equilibrium of the model, (i) wages and effort are set according to either the rigid wage or the flexible incentive pay economy, and (ii) vacancy creation is determined by the free entry condition.

2.2 The Role of Incentives in Employment Fluctuations

We now establish that cyclical changes in incentives can be an important determinant of employment fluctuations.

In a preliminary step, we show that fluctuations in labor market tightness are driven by profits. This result holds regardless of whether wages are set according to the rigid wage or incentive pay models, and is standard in DMP search models with free entry. To see

[10] We formally prove this claim in the dynamic version of our model in Section 3.

[11] Though the mapping is not exact, one can informally think of a "bonus" as the component of wages associated with incentives; whereas "base pay" is the component of wages associated with bargaining.

[12] In an alternative formulation that is equivalent, firms observe idiosyncratic noise and workers alter the distribution of noise by exerting effort that is unobservable to the firm.

this point, totally differentiate the free entry condition (1) with respect to log aggregate productivity $\ln z$ and rearrange to obtain

$$\frac{d\ln\theta}{d\ln z} = \frac{1}{\nu} \cdot \frac{d\ln J}{d\ln z}. \tag{3}$$

That is, the elasticity of market tightness with respect to aggregate productivity z is proportional to the elasticity of expected profits per worker to z, where the constant of proportionality depends on the elasticity of vacancy filling rates with respect to vacancies. Moreover, the employment rate n is determined by the job finding rate $f(\theta)$, which is proportionate to vacancies and given by $f(\theta) = \psi\theta^{1-\nu}$. Therefore, to understand fluctuations in employment, we can simply study the response of profits per worker to aggregate productivity.

Next, we show that cyclical changes in incentives lead to fluctuations in profits. Differentiating expected profits $J(z) \equiv \mathbb{E}_\eta[z(a+\eta) - w]$ with respect to z implies

$$\frac{dJ(z)}{dz} = \overbrace{\mathbb{E}_\eta\left[a\right]}^{\substack{\text{direct}\\ \text{productivity}}} \overbrace{- \mathbb{E}_\eta\left[\frac{dw}{dz}\right]}^{\substack{\text{marginal}\\ \text{costs}}} + \overbrace{z\mathbb{E}_\eta\left[\frac{da}{dz}\right]}^{\text{incentives}} \tag{4}$$

The first order response of profits to aggregate productivity may be decomposed into three terms. The first is the "direct productivity effect:" production rises with productivity, ceteris paribus. Second is the "marginal cost effect:" when productivity rises, wages may also increase, which lowers profits all else equal. The third effect is an "incentive effect:" effort may respond to aggregate productivity shocks. The direct productivity and marginal cost effects are common in DMP search models. If wages are pro-cyclical and dw/dz is large, then profits and employment may respond little to productivity shocks (Shimer, 2005).

The incentive effect is less standard. In particular, if the effort response is large and positive to a positive productivity shock, then profit fluctuations may be large even if expected wages are pro-cyclical. As such, pro-cyclical incentives might offset the effect of wages on profits, leading to large employment fluctuations despite pro-cyclical wages. Wage cyclicality only dampens unemployment's response to productivity shocks insofar as wages move *more* than effort.

The point of this subsection—that incentives matter for employment fluctuations—does not depend on a specific model of wage or effort setting. Equation (4) remains true regardless of the contracting environment or whether contracts are set optimally. Different models merely imply a different direct productivity, marginal cost and/or incentive effect. Next, we endogenize a and w in the flexible incentive pay economy of Holmstrom (1979) and rigid wage economy of Hall (2005) to gauge the incentive and marginal cost effects in each model.

2.3 Incentive Wage Cyclicality and Unemployment Fluctuations

Now we derive our first key result: wage cyclicality due to incentives does not dampen unemployment fluctuations. To a first order, the response of employment to labor productivity shocks is the same in a flexible incentive pay economy without bargaining power, compared to an appropriately calibrated rigid wage economy—even if incentive pay is highly pro-cyclical.

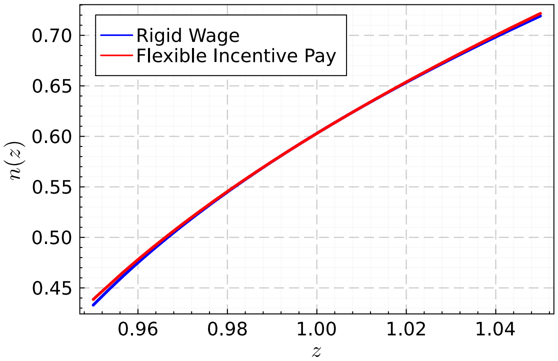
First, consider the response of profits to z in the rigid wage economy. Here, both the marginal cost and incentive effects of wage and effort changes on profits, as in equation (4), are trivially zero because neither effort nor wages respond to z. Therefore the response of profits to productivity is just the direct productivity effect. That is, we have $dJ^{\text{Rigid}}(z)/dz = \bar{a}$, where $J^{\text{Rigid}}(z)$ is the value of filled vacancies if wages are rigid.

Next, we temporarily study a special case of the flexible incentive pay economy in which $\mathcal{B}(z)$ is constant and promised utility to the worker is acyclical. This economy is a natural benchmark in which all wage cyclicality is due to incentives, because there is no bargaining and outside options are constant. Differentiating profits in the incentive pay economy (equation 2) and applying the classic envelope theorem of Milgrom and Segal (2002), we see that $dJ^{\text{Incentive}}/dz = a^*(z)$. Only the direct productivity effect affects the response of profits to productivity shocks z, exactly as in the rigid wage economy. In light of equation (4), this result might be surprising—why have the marginal cost and incentive effects vanished? The result holds because the marginal cost and incentive effects are equally sized under the optimal contract so that their effects on profits "cancel out," leaving only the direct productivity effect. Although wages and effort may fluctuate, these fluctuations do not affect the profit of a firm which is optimally choosing effort and wages. The equivalence holds even if wages are pro-cyclical under the optimal contract, so that dw/dz is large. If so, then incentives generate equally large and offsetting effects on profits, so da/dz is also large.

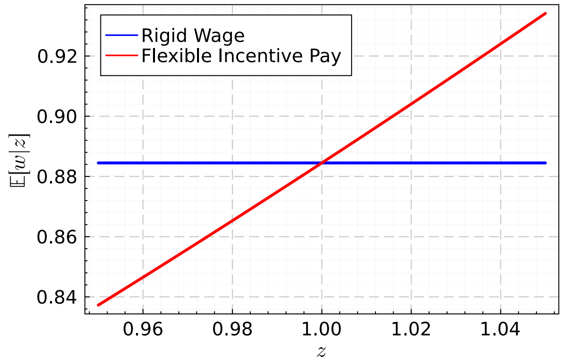
To gain intuition, observe that effort and aggregate productivity are complements, which leads to procyclical wages and effort. Increases in z lead the firm to encourage the worker to provide more effort by raising the pass-through of idiosyncratic output into wages. All else equal, higher effort raises profits. However the worker faces more risk when pass-through rises, for which they must be compensated with a higher expected wage. Therefore wages are procyclical and flexible. All else equal, higher wages lower profits. In the optimal incentive contract, the firm is indifferent at the margin between increasing expected wages and increasing worker effort. This yields the envelope logic: changes in effort and wage induced by a small change in z have exactly offsetting effects on expected profits. Expected profits—and thus vacancies and employment—respond to labor productivity shocks as if neither wages nor effort had changed, just as in the rigid wage economy.

With either rigid wages or with flexible incentive pay, only the direct productivity effect

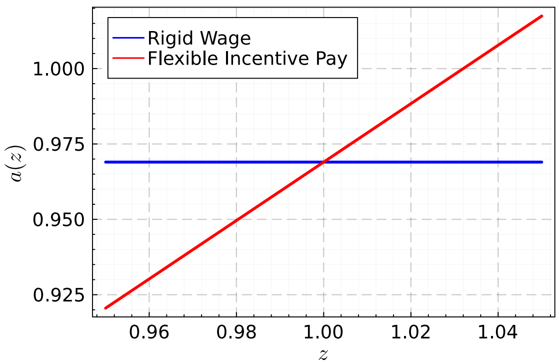
Figure 1: Employment, wage and effort fluctuations in static model



PANEL A: EMPLOYMENT $n(z)$



PANEL B: EXPECTED WAGES $\mathbb{E}[w|z]$



PANEL C: EFFORT $a(z)$

Notes: These figures plot the level of employment (Panel A), expected wages (Panel B) and effort (Panel C) as a function of aggregate productivity in the static model. The red line plots these functions for the flexible incentive pay economy. The blue line plots these functions for the rigid wage economy, calibrated to have the same wage and effort as the flexible incentive pay economy when aggregate productivity is equal to 1.

matters for the response of profits to a labor productivity shock. Therefore, first order profit dynamics — and thus unemployment dynamics — are *identical* in the two economies if they have the same direct productivity effect $\bar{a} = a^*$. This equivalence result suggests that greater wage cyclicality from incentives does not dampen unemployment fluctuations. Even though incentive pay is flexible and cyclical, unemployment responds to labor productivity shocks "as if" wages were rigid.

A numerical example illustrates the equivalence of employment dynamics. Figure 1 plots the behavior of the rigid wage economy (blue line) and the flexible incentive pay economy (red line). Both economies are calibrated to have the same expected wage and effort (and thus profits and employment) when $z = 1$.[13] The horizontal axis of each plot represents

[13]For the purposes of this illustration, we assume: workers have exponential preferences $u(c, a) = -\exp(-r(c - \frac{a^2}{2}))$ and output is $y = z(a + \eta)$. The unemployment benefit b is calibrated to be 0.4, the standard deviation of η shocks is 0.2, and the parameter governing risk aversion r is 0.8. For simplicity,

exogenous labor productivity z, while the vertical axis plots model-implied employment (Panel A), expected wages (Panel B) or effort (Panel C).

Panel A shows equivalent employment dynamics: the rigid wage and flexible incentive pay economies generate identical responses to aggregate labor productivity z, in a neighborhood of $z = 1$. The two models also generate nearly identical employment movements in response to 5% fluctuations in aggregate productivity. This result illustrates the envelope theorem in practice: profit dynamics depend only the direct productivity effect, which is locally the same in both economies under our calibration.

Panel B shows that wages are pro-cyclical in the incentive pay economy. Therefore employment dynamics are the same even though in the incentive pay economy, marginal costs fall significantly during contractions. Panel C shows the countervailing force: effort also responds strongly to z in the incentive pay economy. Therefore incentives offset the stabilizing effect of marginal costs on profits. As such, in the incentive pay economy, large employment fluctuations coexist with pro-cyclical wages.[14]

2.4 Bargained Wage Cyclicality and Unemployment Fluctuations

We now explain our second analytical result: wage cyclicality due to bargaining or a cyclical outside option dampens unemployment fluctuations, as in standard labor search models without incentives. To make this point, we augment the flexible incentive pay economy by allowing $\mathcal{B}(z)$ to vary with z. Differentiating the Lagrangian associated with problem (2) implies that the response of profits to aggregate productivity is

$$\frac{dJ^{\text{Incentive}}}{dz} = a^*(z) - \mu^*(z)\mathcal{B}'(z) \tag{5}$$

where $\mu^*(z)$ is the Lagrange multiplier on the participation constraint at the optimum.

Equation (5) shows that bargaining power stabilizes profits. With bargaining or a cyclical outside option, not only the direct effect of productivity on profits, $a^*(z)$, matters. There is an additional term $\mu^*(z)\mathcal{B}'(z)$ capturing promised utility fluctuations. By comparing equations (4) and (5) we can rewrite the promised utility term as

$$\mu^*(z)\mathcal{B}'(z) = \mathbb{E}_\eta\left[\frac{dw^*}{dz} - z\frac{da^*}{dz}\right]. \tag{6}$$

following Holmstrom and Milgrom (1987), we solve for the optimal linear (in output) contract.

[14] In our model effort and aggregate labor productivity are complements, which makes both wages and effort pro-cyclical in the optimal incentive contract. Without complementarity, wages and effort could be acyclical or countercyclical. However, employment will still have the same first order response in the rigid wage and flexible incentive pay economies.

As such, we term $\mu^*(z)\mathcal{B}'(z)$ "bargained wage cyclicality" (BWC). It is equal to wage cyclicality in excess of movements in production due to effort and incentives. BWC is different from zero if and only if promised utility is cyclical, so $\mathcal{B}'(z) \neq 0$. Only the bargaining component of wage cyclicality dampens profit fluctuations. Intuitively, an increase in wages associated with higher promised utility does not require workers to offer higher effort. Therefore the increase in wages reduces profits all else equal.

Our result relates to the standard DMP model with exogenous effort (e.g. Shimer, 2005). In our model, as in the standard model, wage cyclicality associated with bargaining power dampens profit fluctuations. However, in the standard model all wage fluctuations are due to bargaining, since $da^*/dz = 0$ by assumption. Thus, wage cyclicality always dampens profit and unemployment fluctuations. By contrast, wage cyclicality does not necessarily dampen profit fluctuations in our flexible incentive pay model since wages are no longer a sufficient statistic for workers' utility under the contract.

We stress that the static model only establishes an equivalence for the response of employment to exogenous labor productivity shocks. The response of output may still be different with rigid wages compared with incentive pay. In the incentive pay economy, output per worker varies endogenously because of effort, whereas output per worker is exogenous in the rigid wage economy. With flexible incentive pay, the endogenous component of output per worker is pro-cyclical when wages are pro-cyclical, suggesting a notion of variable capacity utilization.

Taking the two analytical results together, we have seen that wage cyclicality arising from incentives does not mute unemployment fluctuations, but wage cyclicality arising from bargaining does. The next section proves these results in a rich dynamic environment. Different from the static model, the dynamic model recognizes that labor contracts are long term relationships and that incentives are dynamic (e.g. Barro, 1977; Sannikov, 2008).

3 A Dynamic Model of Incentive Pay with Bargaining

This section studies a dynamic labor search model with rich long-term incentive contracts permitting, for instance, persistent idiosyncratic shocks and non-separable utility. We derive our main analytical results in this setting: first, wage cyclicality due to incentives does not dampen unemployment fluctuations; second, wage cyclicality due to bargaining does dampen unemployment dynamics.

3.1 Economic Environment

3.1.1 Labor Market

Time is discrete. The labor market follows the standard Diamond-Mortensen-Pissarides model. A large measure of risk-neutral firms match with workers and produce output. A unit mass of workers is either employed or unemployed and searching for a job. Let n_t denote the measure of employed workers at the start of period t, while $u_t \equiv 1 - n_t$ is the measure of unemployed workers looking for jobs. Fluctuations in labor market variables are driven by technology, which follows a Markov process $\{z_t\}_{t=0}^{\infty}$ with lower and upper bounds $\underline{z}$ and $\overline{z}$. We will denote the history of this Markov process until t by $\{z^t\} = \{z_0, ..., z_t\}$.

Firms post v_t vacancies to recruit unemployed workers. The number of matches made in period t is given by a constant returns matching function $m(u_t, v_t)$; conditions are summarized by market tightness $\theta_t = v_t/u_t$, with a job finding rate $\phi(\theta_t) = m(u_t, v_t)/u_t$ and a vacancy filling rate $q_t \equiv q(\theta_t) = m(u_t, v_t)/v_t$. Let $\nu_t \equiv d \log q_t / d \log \theta_t$ denote the period t elasticity of the job-filling rate with respect to θ_t. Keeping a vacancy open has a flow cost κ.

At the end of period $t-1$ an exogenous fraction s of workers separate from employment and enter unemployment. The unemployed search for new jobs, so u_t evolves as

$$u_t = u_{t-1} + s(1 - u_{t-1}) - \phi(\theta_{t-1})u_{t-1}(1 - s). \tag{7}$$

3.1.2 Preferences and Consumption

Workers have time-separable risk-averse preferences over consumption $c_t \in [\underline{c}, \overline{c}]$ and effort $a_t \in [\underline{a}, \overline{a}]$, and discount future payoffs by a factor $\beta \in (0, 1)$. Preferences are summarized by $u(c, a)$ where u is strictly increasing and strictly concave in c, strictly decreasing and strictly concave in a, and Lipschitz continuous.

Employed workers consume their wage in each period, with newly hired workers producing output and receiving a wage in the period in which they are hired. Workers who are not hired in the current period exert no effort and are paid an unemployment benefit $b(z_t)$, a differentiable function of the aggregate state, receiving flow payoff $\xi(z_t) \equiv u(b(z_t), 0)$.

Therefore the value of an unemployed worker at the start of period t is

$$U(z_t) = \phi(\theta_t) \mathcal{E}(z_t) + (1 - \phi(\theta_t)) (\xi(z_t) + \beta \mathbb{E}[U(z_{t+1}) | z_t]) \tag{8}$$

where $\mathcal{E}(z)$ is the worker's value if they begin employment when aggregate productivity is z.

3.1.3 Firms and Wage Setting

Firms are risk neutral and maximize expected profit with discount factor β. Consider a firm i that successfully matches with a worker at time 0 and starts producing in the same period. The firm's output in period t is $y_{it} = f(z_t, \eta_{it})$ where f is strictly increasing and differentiable in all of its arguments and η_{it} is an idiosyncratic shock to the firm's output that is independently distributed across firms. Henceforth, we omit i subscripts to ease notation.

At the beginning of the period, before the current value of η_t is realized, the worker exerts effort a_t that affects the distribution of idiosyncratic shocks. We assume a general process for η_t, which allows for arbitrary persistence and depends on the worker's effort. The process has bounds $\underline{\eta}$ and $\overline{\eta}$. Define a history of idiosyncratic shocks $\eta^t = \{\eta_0, ..., \eta_t\}$. We characterize the process for η_t by a probability measure $\pi_t(\eta_t|\eta^{t-1}, a^t)$, which gives the probability of realizing η_t given the history η^{t-1} of past idiosyncratic shocks, the history of past and present aggregate shocks z^t and the worker's history of actions $a^t = \{a_0, \ldots, a_t\}$. Let the marginal distribution of the history of aggregate productivity shocks through time t be $\hat{\pi}_t(z^t|z_0)$.

The value of a firm of posting a vacancy at time 0 is then

$$\Pi_0 = q(\theta_0)\mathbb{E}_0\left[\sum_{t=0}^{\infty}(\beta(1-s))^t(f(z_t,\eta_t)-w_t)\right]-\kappa, \tag{9}$$

where $\mathbb{E}_0$ conditions on the firm's information set at time 0 prior to meeting a worker. If a firm meets a worker, the value to the firm is the expected present value of the difference between production and wage payments, discounted by the firm's discount factor β and given separation risk s. A vacancy is filled with probability $q(\theta)$ and costs κ, yielding the above net vacancy value. Free entry guarantees that this value is zero in equilibrium. We entertain two possibilities for wage setting.

Flexible Incentive Pay Economy

In this economy, wages are set according to a dynamic incentive contract. The firm observes the initial value of z_0 and all realizations of aggregate shocks $\{z_t\}_{t=0}^{\infty}$. Firms additionally observe idiosyncratic shocks η_t in every period of the match. However, they do not observe workers' effort a_t. They thus cannot observe whether output is high because the worker exerted high effort or received a lucky idiosyncratic shock, a classic moral hazard problem.

When a firm and worker meet, the firm offers the worker a contract to incentivize effort and maximize firm value. A contract specifies a wage function mapping idiosyncratic shocks and aggregate productivity to realized wages. The contract does not condition on workers' effort, which is unobservable to the firm, but "recommends" a level of effort given the history of aggregate and idiosyncratic shocks. The worker chooses effort before the realization of the idiosyncratic shock to firm output.[15]

Thus the contract may be summarized by functions $w_t(\eta^t, z^t) \in [\underline{w}, \bar{w}]$ and $a_t(\eta^{t-1}, z^t) \in [\underline{a}, \bar{a}]$ for all t and all realizations of η^t and z^t. Let $(\mathbf{w}, \mathbf{a})$ denote a contract, with $\mathbf{w} \equiv \{w_t(\eta^t, z^t)\}_{t=0,\eta^t,z^t}^{\infty}$ and $\mathbf{a} \equiv \{a_t(\eta^{t-1}, z^t)\}_{t=0,\eta^{t-1},z^t}^{\infty}$, so that the contract is dynamic and state contingent. Let $\mathcal{X}$ denote the space of feasible contracts.

Value of a Filled Vacancy. Under the contract $(\mathbf{w}, \mathbf{a})$, and initial productivity z_0, the firm's expected present value of profits from posting a vacancy is

$$V(\mathbf{w}, \mathbf{a}; z_0) = \sum_{t=0}^{\infty} (\beta(1-s))^t \int\int \left(f(z_t, \eta_t) - w_t(\eta^t, z^t)\right) \tilde{\pi}_t \left(\eta^t, z^t | z_0, \mathbf{a}\right) d\eta^t dz^t, \tag{10}$$

where $\tilde{\pi}_t(\eta^t, z^t|\mathbf{a}) \equiv \prod_{\tau=0}^{t} \pi_\tau \left(\eta_\tau | \eta^{\tau-1}, a^\tau(\eta^{\tau-1}, z^\tau)\right) \hat{\pi}_\tau(z^\tau|z_0)$ is the probability of observing a realization of η^t and z^t given the initial z_0 and the contracted effort function $\mathbf{a}$; $a^\tau(\eta^{\tau-1}, z^\tau)$ is the sequence of effort from periods 0 to τ.

Firms' flow profits are the difference between output and wages. The firm forms an expectation over flow profit realizations by integrating over the distribution of both aggregate and idiosyncratic shocks, the latter of which depends on effort. The risk-neutral firm discounts period t profits by the economy-wide discount rate β^t and the probability $(1-s)^t$ that the match survives t periods.

The contract maximizes the value of a filled vacancy

$$J(z_0) = \max_{\{w_t(\eta^t, z^t), a_t(\eta^{t-1}, z^t)\}_{t=0,\eta^t,z^t}^{\infty} \in \mathcal{X}} V(\mathbf{w}, \mathbf{a}; z_0) \tag{11}$$

subject to the incentive (IC) and participation (PC) constraints described below.

Incentive Constraints. The worker chooses effort $\tilde{\mathbf{a}} \equiv \{\tilde{a}_t(\eta^{t-1}, z^t)\}_{t=0,\eta^{t-1},z^t}^{\infty}$ to maximize utility under the contract. The wage depends on idiosyncratic shocks and not worker effort, which is unobservable to the firm, imposing a constraint on the contracting problem. Therefore, the effort suggested under the contract by the firm must be incentive compatible; that is, the recommended effort $\mathbf{a}$ must be what is chosen by the worker, given the wage that the firm offers the worker. Specifically:

[15] An alternative notation has effort directly affect production, while the firm cannot distinguish effort from η_t. A second alternative notation has contracts mapping from idiosyncratic *output* and aggregate productivity to wages. Our results are also unchanged with the "noise before action" assumption of Edmans et al. (2012a).

$$[\mathbf{IC}]: \mathbf{a} \in \underset{\{\tilde{a}_t(\eta^{t-1},z^t)\}_{t=0,\eta^t,z^t}^{\infty}}{\operatorname{argmax}} \sum_{t=0}^{\infty} (\beta(1-s))^t \Bigg[\int\int u\Big(w_t(\eta^t, z^t), \tilde{a}_t(\eta^{t-1}, z^t)\Big) \tilde{\pi}_t\left(\eta^t, z^t | z_0, \tilde{\mathbf{a}}\right) d\eta^t dz^t$$
$$+ \beta s \int U\left(z_{t+1}\right) \hat{\pi}_{t+1}\left(z^{t+1}|z_0\right) dz^{t+1} \Bigg]. \tag{12}$$

Equation (12) is the value of an employed worker at time 0; the IC constraint requires that the recommended effort maximizes the worker's value given the wage contract offered by the firm. The worker discounts period t flow payoffs by β^t. Their value is the sum of two terms. The first is their value conditional on the contract surviving. The realized flow payoff to the worker under the contract is their utility from consuming the wage offered by the contract and providing effort. The wage and effort level depend on the realizations of aggregate productivity z^t and idiosyncratic productivity η^t. Workers' expected utility integrates over the distribution of aggregate and idiosyncratic productivity shocks. When making their effort choice, workers trade off the disutility of higher effort with the increased probability of realizing a high output draw and thus a high wage. The match survives through period t with probability $(1-s)^t$.

The second term of the worker's utility under the contract is the value conditional on separation. If the contract separates in period t, the worker receives the value of unemployment at the prevailing aggregate productivity z_t. The contract exogenously separates in period t with probability $(1-s)^{t-1}s$.

Participation Constraint. The second constraint on problem (11) is that the contract must promise the worker a value of at least $\mathcal{E}(z_0)$, the ex ante "promised utility" of the contract. This promised utility may fluctuate with z_0 either due to bargaining between a matched firm and worker, or due to fluctuations in the value of unemployment. The constraint is

$$[\mathbf{PC}]: \sum_{t=0}^{\infty} (\beta(1-s))^t \Bigg[\int\int u\Big(w_t(\eta^t, z^t), a_t(\eta^{t-1}, z^t)\Big) \tilde{\pi}_t\left(\eta^t, z^t | z_0, \mathbf{a}\right) d\eta^t dz^t$$
$$+ \beta s \int U\left(z_{t+1}\right) \hat{\pi}_{t+1}\left(z^{t+1}|z_0\right) dz^{t+1} \Bigg] \geq \mathcal{E}\left(z_0\right). \tag{13}$$

The left hand side of inequality (13) is the worker's value under the contract: it is the objective function in equation (12) evaluated at the effort choices suggested by the contract.

Bargaining and promised utility. To close the flexible incentive pay economy, we must determine promised utility $\mathcal{E}(z_0)$, which we assume is given by a reduced form "bargaining schedule" $\mathcal{B}(z_0)$.[16] Firms commit to providing workers with a utility $\mathcal{B}(z_0)$ over the life of the contract. If $\mathcal{B}(z_0)$ increases with z_0 then there is non-zero bargaining power. During booms in labor demand, when z_0 is high, workers receive greater utility. Common bargaining protocols in the labor search literature implicitly define different functions for $\mathcal{B}(z_0)$. For instance, if firms make take-it-or-leave-it offers to workers, the value of employment is equal to the value of non-employment: $\mathcal{B}(z_0) = \sum_t \beta^t \mathbb{E}[\xi(z_t)|z_0]$, where $\xi(z_t)$ is the flow value of unemployment. This nests the case in which unemployment benefits or the opportunity cost of unemployment are cyclical (Hagedorn et al., 2013; Chodorow-Reich and Karabarbounis, 2016; Mitman and Rabinovich, 2019). Nash bargaining also implicitly defines an increasing function for $\mathcal{B}(z_0)$, as we prove in Appendix A.1, as do other bargaining protocols such as that in Hall and Milgrom (2008). Our formulation also evokes a notion of unemployment as a "worker discipline device" (Shapiro and Stiglitz, 1984): if the value of employment is low because unemployment at present or in the future is costly, workers will offer higher effort at lower wages.

The reduced form approach has two advantages. First, our conclusions about the role of bargaining will be robust to a specific protocol. Second, we will be able to tractably incorporate bargaining into dynamic incentive contract models. The disadvantage of this approach is that $\mathcal{B}(z_0)$ is a reduced form object, which is not invariant to changes in the primitives of the environment.

Rigid Wage Economy. We also consider a model with rigid wages and effort following Hall (2005). Wages and effort take exogenous constant values $w_t = \bar{w}$ and $a_t = \bar{a}$ for all firms and all t, regardless of realizations of η^t or z^t. The worker's value of employment is the utility from the match, and the continuation value if the match separates, which is

$$\mathcal{E}\left(z_0\right) = \sum_{t=0}^{\infty} \left(\beta\left(1-s\right)\right)^t \left(u\left(\bar{w}, \bar{a}\right) + \int \beta s U\left(z_{t+1}\right) \hat{\pi}_t\left(z^{t+1}|z_0\right) dz^{t+1}\right). \tag{14}$$

Meanwhile, the firm's value of a filled vacancy is exogenous and given by

$$J(z_0) = \sum_{t=0}^{\infty} (\beta(1-s))^t \int \left(f(z_t, \eta_t) - \bar{w}\right) \tilde{\pi}_t(\eta^t, z^t|z_0, \bar{\mathbf{a}}) d\eta^t dz^t. \tag{15}$$

That is, the value of a filled vacancy is given by the expected present discounted value of production minus the rigid wage, where the expectation is taken over realizations of aggregate and idiosyncratic shocks at a fixed effort $\bar{a}$ in all dates and states.

[16] See Blanchard and Galí (2010) and Michaillat (2012) for this approach in search models without effort.

3.1.4 Equilibrium

Given initial unemployment u_0 and a stochastic process $\{z_t, \eta_t\}_{t=0}^{\infty}$, an equilibrium is a collection of stochastic processes $\{\theta_t, u_t\}_{t=0}^{\infty}$ and functions $J(z), U(z), \mathcal{E}(z)$, and contract $(\mathbf{w}, \mathbf{a})$ such that for all firms: (i) θ_t satisfies the free entry condition so that Π_t, given in equation (9), is equal to 0 for all t; (ii) u_t satisfies the law of motion for unemployment (7); (iii) wage and effort functions $(\mathbf{w}, \mathbf{a})$ satisfy the IC and PC constraints in the flexible incentive pay economy (12)-(13), or $w_t = \bar{w}$ and $a_t = \bar{a}$ in the rigid wage economy; (iv) the value of unemployment $U(z)$ is given by (8); (v) the value of employment is given by equation (14) in the rigid wage economy, or $\mathcal{E}(z) = \mathcal{B}(z)$ in the flexible incentive pay economy; and (vi) the value of a filled vacancy $J(z)$ is given by (11) in the flexible incentive pay economy or (15) in the rigid wage economy.

3.2 The Role of Incentives in Employment Fluctuations

We now study the response of employment to exogenous aggregate productivity shocks in the flexible incentive pay economy. As is standard in DMP models, employment fluctuations are determined by fluctuations in market tightness, which in turn is governed by fluctuations in firms' expected profits per worker (e.g. Ljungqvist and Sargent, 2017). Therefore it suffices to study how profits per worker $J(z_0)$ fluctuate with z_0.

To study profits, we combine the IC and PC constraints into a functional $G(\mathbf{w}, \mathbf{a})$, defined such that G such that $G(\mathbf{w}, \mathbf{a}) \leq 0$ holds if and only if the incentive constraints (12) and participation constraint (13) are satisfied. Let $\lambda(z_0)$ denote the co-state functional on these constraints. We write the value of a filled job using the functional Kuhn-Tucker Lagrangian

$$J(z_0) = V(\mathbf{w}^*, \mathbf{a}^*; z_0) - \langle G(\mathbf{w}^*, \mathbf{a}^*; z_0), \lambda^* \rangle \tag{16}$$

where the star superscripts indicate values chosen under the optimal contract offered at z_0.[17]

We now show how fluctuations in z might affect profits. Aggregate productivity shocks directly move production, but may also affect the optimal contract or the values of the constraints and co-states. For instance, both effort and wages might move with aggregate shocks, though this may be hard to characterize. The following proposition decomposes the response of firm profits to z_0, generalizing decomposition (2) from Section 2.

Proposition 1. *Assume that a relevant constraint qualification is satisfied on the optimal contract. Then the response of firm profits to aggregate shocks in the flexible incentive pay*

[17] As part of the proof of the below propositions, Appendix A shows that the firm's problem may be expressed using a Lagrangian.

economy is

$$\frac{dJ\left(z_0\right)}{dz_0} = \underbrace{\frac{\partial}{\partial z_0} V\left(\mathbf{w}^*, \mathbf{a}^*; z_0\right)}_{\text{(A) direct productivity effect on profits}} - \underbrace{\left\langle \frac{\partial}{\partial z_0} G\left(\mathbf{w}^*, \mathbf{a}^*; z_0\right), \lambda^*\left(z_0\right)\right\rangle}_{\text{(B) direct effect on participation and incentives}} \tag{17}$$

$$+ \underbrace{\sum_{x \in \{\mathbf{w}^*, \mathbf{a}^*\}} \left[\nabla_x V\left(\mathbf{w}^*, \mathbf{a}^*; z_0\right) - \left\langle \nabla_x G\left(\mathbf{w}^*, \mathbf{a}^*; z_0\right), \lambda^*\left(z_0\right)\right\rangle\right] \cdot \frac{dx}{dz_0} - \left\langle G\left(\mathbf{w}^*, \mathbf{a}^*; z_0\right), \frac{d\lambda^*(z_0)}{dz_0}\right\rangle}_{\text{(C) indirect effects on optimal contract and co-states}}$$

where ∇_x represents the vector of partial derivatives with respect to x.

The proof of this and all other propositions and theorems is in Appendix Section A. The direct productivity effect A measures how shocks to initial productivity affect the expected present value of output in all periods, where the expectation conditions on initial productivity z_0 and contracted effort a^*. This is given by the marginal effect of increasing z_0 on current and expected future y_t, which evaluates to

$$\frac{\partial}{\partial z_0} V(\mathbf{w}^*, \mathbf{a}^*; z_0) = \sum_{t=0}^{\infty} \left(\beta\left(1-s\right)\right)^t \frac{\partial}{\partial z_0} \mathbb{E}\left[f(z_t, \eta_t) | z_0, \mathbf{a}^*\right]. \tag{18}$$

Term B captures the direct effect of exogenous productivity movements on participation constraints, which relates to bargaining power. If higher z raises the utility the firm must promise the worker (i.e. $\mathcal{B}(z)$ is increasing in z), then the firm's profits from vacancy posting will rise by less, since the firm receives a combination of lower effort or higher wages when $\mathcal{B}(z)$ rises. Mathematically, the first order contribution of this term to profit fluctuations is given by

$$-\lambda^*_{PC}\left(z_0\right) \left[\frac{\partial}{\partial z_0} \mathcal{B}\left(z_0\right) - \sum_{t=0}^{\infty} \left(\beta\left(1-s\right)\right)^t \beta s \frac{\partial}{\partial z_0} \mathbb{E}\left[U\left(z_{t+1}\right)|z_0\right]\right], \tag{19}$$

where λ^*_{PC} is the Lagrange multiplier on the participation constraint. This term is zero if both the value of employment and unemployment are acyclical—for instance if unemployment benefits are acyclical and firms make take-it-or-leave-it offers to workers. In general, however, the term will be non-zero if workers' promised utility is cyclical, either due to a cyclical value of unemployment or due to bargaining.[18]

The C term captures the effects that the shock has on profits through changes in the firm's choice variables. The C term has three pieces. First, the shock may shift the optimal contract's wage function $\mathbf{w}^*$, holding fixed the values of the constraints, and thus the worker's promised utility. This is the marginal cost effect: the wage paid for each future realization

[18] Under the assumptions of our model, z_0 does not affect incentive constraints directly.

of η^t and z^t may differ for contracts signed at different initial aggregate productivity levels z_0. Second, the shock may increase the optimal contract's recommended effort function $\mathbf{a}^*$, which raises output. This is the incentive effect. Finally, the shock may shift the value of the co-states on the participation and incentive constraints.[19]

3.3 Unemployment Dynamics and Incentive Wage Cyclicality

We now show that wage cyclicality from incentives do not dampen unemployment fluctuations. As in the static model, the argument proceeds in two steps. First, we use an envelope logic to show that the C term in Proposition 1 — capturing the effect on profits via changes in optimal wages and effort — is zero. Second, to focus on incentives, we temporarily make assumptions that remove bargaining power, so that the B term in Proposition 1 is also zero.

The main technical challenge for the proof is therefore to transform the problem so that an envelope theorem applies. Existing general envelope theorems (e.g. Milgrom and Segal, 2002) are not well-suited to studying problems with a continuum of non-convex constraints.[20] The firm's problem has this feature, since there is a continuum of incentive compatibility constraints, which are not generally convex. Below, we provide a set of sufficient conditions under which an envelope theorem can be applied to our problem when $\mathcal{B}(z_0)$ does not vary.

Assumption 1. *Consider the following assumptions:*

(i) The set of feasible contracts that satisfy the incentive and participation constraints is non-empty.

(ii) The set of feasible contracts $(\mathbf{w}, \mathbf{a}) \in \mathcal{X}$ *that are incentive compatible and satisfy participation constraints is compact.*

Assumption (i) is a minimal assumption that lets us characterize the optimal contract. Assumption (ii) is a condition that lets us apply a theorem from the applied mathematics literature on "sensitivity analysis" (Bonnans and Shapiro, 2000). This envelope theorem directly applies when there is a continuum of constraints that may not be convex so that the first order approach may not be valid.[21]

We will need to define an "impulse response" in order to present our results. Denote $z_t = \mathbb{E}[z_t|z_0] + \varepsilon_t$, where by definition, ε_t is the cumulative innovation to the process for z

[19]The proof of Proposition 1 states the relevant constraint qualification, which is analogous to the Mangasarian-Fromowitz constraint qualification.

[20]Existing general envelope theorems are typically applied to the agent's objective, whereas we apply an envelope theorem to the principal's objective.

[21]Lemma 5 in the Appendix shows that the set of feasible contracts is compact if contracts are restricted to being continuous and twice differentiable in their arguments $\{\eta^t, z^t\}$, with uniformly bounded first and second derivatives.

between 0 and t and ε_0 is known to be 0. We will study the response of market tightness to changes z_0 while holding fixed ε_t for all t, which is the "impulse response" of market tightness to changes in initial productivity z_0.

Our next analytical result considers a benchmark in which all wage cyclicality is due to incentives. To that end, we will consider a version of the flexible incentive pay economy in which firms make take-it-or-leave-it offers to workers and unemployment benefits are acyclical. In this economy, all wage fluctuations are due to incentives rather than bargaining, and the B term from Proposition 1 that relates to bargaining is eliminated.

Theorem 2. *Suppose that (i) Assumption 1 holds, and (ii) the firm makes take it or leave it offers to workers and the flow value of unemployment is constant $\xi(z_t) = \xi$. Then the first-order impulse response of market tightness to aggregate shocks is*

$$\frac{d\log\theta_0}{d\log z_0} = \frac{1}{\nu_0}\frac{\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t \frac{\partial}{\partial\log z_0}\mathbb{E}\left[f(z_t,\eta_t)|z_0,\mathbf{a}^*\right]}{\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t \mathbb{E}\left[f(z_t,\eta_t)-w_t^*|z_0,\mathbf{a}^*\right]}, \tag{20}$$

in the flexible incentive pay economy, where $\mathbf{a}^*$ and $\mathbf{w}^*$ are effort and wages under the firm's optimal incentive wage contract, and ν_0 is the elasticity of job filling with respect to tightness. The first-order impulse response of market tightness to aggregate shocks in a rigid wage economy with $w=\bar{w}$ and $a=\bar{a}$ is

$$\frac{d\log\theta_0}{d\log z_0} = \frac{1}{\nu_0}\frac{\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t \frac{\partial}{\partial\log z_0}\mathbb{E}\left[f(z_t,\eta_t)|z_0,\bar{\mathbf{a}}\right]}{\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t \mathbb{E}\left[f(z_t,\eta_t)-\bar{w}|z_0,\bar{\mathbf{a}}\right]}. \tag{21}$$

Moreover, assume further that (i) the production function f is homogeneous of degree 1 in aggregate productivity z and (ii) z_t is a driftless random walk. Then the response of market tightness to z in both economies, in the neighborhood of a non-stochastic steady state for z, is equal to

$$\frac{d\log\theta_0}{d\log z_0} = \frac{1}{\bar{\nu}}\left(\frac{1}{1-\Lambda}\right) \tag{22}$$

In both economies, Λ is the steady state labor share defined as

$$\Lambda \equiv \frac{\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t \mathbb{E}_0 w_t}{\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t \mathbb{E}_0 f(\bar{z},\eta_t)}, \tag{23}$$

where expectations are evaluated in a steady state with constant aggregate productivity $z_t = \bar{z}$, and $\bar{\nu}$ is the steady state elasticity of job filling with respect to tightness.

The insight of the theorem is that wage cyclicality due to incentives does not dampen unemployment fluctuations. The impulse response of market tightness — and thus unemployment — to exogenous productivity shocks is the same in two economies. The first economy has exogenously fixed wages and effort. The second economy has flexible incentive pay but no bargaining power. Equation (20) characterizes the impulse response of tightness to labor productivity shocks with flexible incentive pay. This impulse response is simply the "direct productivity effect" in the numerator, scaled by the present value of profits from job creation in the denominator. Similarly equation (21) characterizes the same impulse response in the rigid wage economy—which is, again, the direct productivity effect scaled by the present value of profits. Therefore market tightness fluctuations are equivalent in both economies if they feature the same direct productivity effect and the same present value of profits. Since there is no bargaining power, by the assumptions of the theorem, all wage fluctuations in the incentive pay economy are due to incentives. Moreover, wages can be highly procyclical in this economy. Thus, wage cyclicality that arises due to incentives does not *per se* mute unemployment fluctuations.

There are two key steps in the proof of this Theorem, which is presented in Appendix A. First, as in the static model, the free entry condition ensures that changes in profits per worker determine tightness and hence unemployment fluctuations. Second, applying an envelope theorem to the firm's optimal contracting problem leads to an outcome equivalent to wage rigidity. To see why, consider Proposition 1. Since workers' promised utility is fixed, the B term measuring the effect of bargaining power on profits is zero. An appropriate envelope theorem then implies that the C term – the combined first order effect of wage and effort changes on profits – is also zero. Thus, as in the static model, the flexible incentive pay economy only features the direct productivity effect when there is no bargaining. The same is true for the rigid wage economy where by definition, there is neither bargaining power, nor changes in wages or effort. Thus profit and market tightness fluctuations are equivalent in both economies if they feature the same direct productivity effect. This equivalence holds even though the flexible incentive pay economy features potentially highly pro-cyclical present value of wage payments to new hires. The effect of higher wage payments on profits are exactly offset by higher worker effort, along the optimal contract.

The final part of the theorem refines the sense in which the flexible incentive pay economy and the rigid wage economy have the same dynamics. Both economies must be calibrated to the same steady state labor share. To see the role of the labor share, we make assumptions to simplify the expression for $d\log\theta/d\log z_0$ from equations (20) and (21). Suppose the production function is homogeneous of degree 1, z_t is a driftless random walk, and each economy is in a neighborhood of a steady state for z.[22] Then the impulse of market tightness

[22] These assumptions are made only for exposition. For a different production function, a different rigid wage $\hat{w}$ and effort $\hat{a}$ economy such that effort matches the numerator of $d\log\theta/d\log z_0$ and the wage matches

in both economies is

$$\frac{d\log\theta_0}{d\log z_0} = \frac{1}{\nu_0}\frac{\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t \mathbb{E}_0 f\left(z_t,\eta_t\right)}{\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t \left(\mathbb{E}_0 f\left(z_t,\eta_t\right) - \mathbb{E}_0 w_t\right)}$$

The numerator is the expected output while the denominator is the excess output after wage payments. Dividing numerator and denominator by the expected present value of output, one can then see that the dynamics of unemployment are governed by

$$\frac{d\log\theta_0}{d\log z_0} = \frac{1}{\nu_0}\frac{1}{1-\text{labor share}}.$$

If wages and effort lead to the same labor share in the rigid wage and incentive pay economies, then both economies feature the same dynamics of market tightness $d\log\theta_0/d\log z_0$.[23]

Our result that incentive wage flexibility does not dampen unemployment fluctuations is general. Characterizing the optimal dynamic contract is difficult in our setting, due to features such as persistent idiosyncratic shocks and potentially non-separable utility between consumption and effort. Our use of the envelope theorem means we can characterize the response of profits to labor demand shocks *without* characterizing the optimal contract, so our result holds with more general assumptions than used in standard incentive contracts. For instance, Theorem 2 remains true even if incentive constraints are not convex, so that the "first order approach" might be invalid.

3.4 Unemployment Dynamics and Bargained Wage Cylicality

This section reintroduces bargaining power. We argue that only "bargained wage cyclicality" dampens unemployment fluctuations in a setting with both incentives and bargaining.

Let $\mathcal{Y}(\mathbf{a}^*(z_0), z_0)$ denote the expected present discounted value of output from a match which originates under aggregate productivity z_0 given the optimal effort function $\mathbf{a}^*(z_0)$:

$$\mathcal{Y}(\mathbf{a}^*(z_0), z_0) \equiv \sum_{t=0}^{\infty}(\beta(1-s))^t \int f(z_t,\eta_t)\tilde{\pi}_t(\eta^t, z^t|z_0, \mathbf{a}^*(z_0))d\eta^t dz^t$$

Likewise, let $\mathcal{W}(z_0)$ denote the present discounted value of wage payments under the optimal

the denominator, is suitable. The simplifying assumption of a random walk is common because labor productivity is highly persistent and innovations have small amplitude (e.g. Michaillat, 2012). Though we assume a steady state in aggregate variables, we still allow for idiosyncratic risk.

[23]The labor share is thus the "fundamental surplus" in this economy, in the sense of Ljungqvist and Sargent (2017). However the dynamics of wages and effort in our flexible incentive pay economy may be completely different from the economies studied by Ljungqvist and Sargent (2017).

wage contract:

$$\mathcal{W}(z_0) \equiv \sum_{t=0}^{\infty} (\beta(1-s))^t \int w_t^*(\eta^t, z^t) \tilde{\pi}_t(\eta^t, z^t | z_0, \mathbf{a}^*(z_0)) d\eta^t dz^t.$$

With these definitions, one can write the value to the firm of a filled match as $J(z_0) = \mathcal{Y}(\mathbf{a}^*(z_0), z_0) - \mathcal{W}(z_0)$: the difference between the present discounted values of output and wages. Differentiating $J(z_0)$ with respect to z_0 yields the following expression

$$\frac{dJ(z_0)}{dz_0} = \frac{\partial \mathcal{Y}(\mathbf{a}^*(z_0); z_0)}{\partial z_0} - \left(\frac{d\mathcal{W}(z_0)}{dz_0} - \partial_{\mathbf{a}} \mathcal{Y}(\mathbf{a}^*(z_0); z_0) \frac{d\mathbf{a}^*}{dz_0} \right) \tag{24}$$

This expression for the response of profits to z_0 is given by two terms. The first term is the direct productivity effect on output: the partial derivative of $\mathcal{Y}$ with respect to z. The second term measures the extent to which the present value of wages responds to labor productivity shocks by more than the present value of effort does. The term $\partial_{\mathbf{a}} \mathcal{Y}(\mathbf{a}^*(z_0); z_0)$ rescales cyclical effort movements $d\mathbf{a}^*/dz_0$ so that they are in the same units as wage movements. Movements in wages in excess of effort change workers' utility, and hence represent bargaining or outside option fluctuations. As such, let us define Bargained Wage Cyclicality (BWC) as

$$\frac{\partial \mathcal{W}^{\text{bargained}}(z_0)}{\partial \log z_0} \equiv \frac{d\mathcal{W}(z_0)}{dz_0} - \partial_{\mathbf{a}} \mathcal{Y}(\mathbf{a}^*(z_0); z_0) \frac{d\mathbf{a}^*}{dz_0}. \tag{25}$$

Our next analytical result requires one more definition. Denote $\tilde{\mathcal{B}}(z)$ to be "bargained utility", the utility promised to the worker, net of their continuation value should they separate to unemployment:

$$\tilde{\mathcal{B}}(z) \equiv \mathcal{B}(z) - \sum_{t=0}^{\infty} (\beta(1-s))^t \beta s \mathbb{E}[U(z_{t+1})|z_0].$$

Characterizing the response of market tightness to productivity in this setting is made more difficult in the presence of bargaining, as the space of contracts satisfying the participation constraint now moves directly with z_0. To make progress, we therefore introduce one additional assumption which guarantees that the so-called first-order approach (FOA) offers a valid solution to the contracting problem:

Assumption 2. *The set of feasible contracts* $(\mathbf{w}, \mathbf{a}) \in \mathcal{X}$ *is compact and convex. Furthermore, the worker's optimal effort choices are determined by the first order condition to* *problem* (12)*, and the density of* η_t *can be expressed as*

$$\pi_t\left(\eta_t|\eta^{t-1}, a^t\right) = \pi_t\left(\eta_t|\eta_{t-1}, a_t\right).$$

Under this assumption, the incentive compatibility constraint may be written as the first order condition to the worker's problem, and the firm's contracting problem may be expressed recursively. This assumption permits the derivation of our second analytical result: bargained wage cyclicality mutes unemployment fluctuations.

Proposition 3. *Assume that Assumptions 1 and 2 hold. The impulse response of market tightness to aggregate shocks in the flexible incentive pay economy is*

$$\frac{d\log\theta_0}{d\log z_0} = \frac{1}{\nu_0}\frac{\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t \frac{\partial}{\partial\log z_0}\mathbb{E}\left[f(z_t,\eta_t)|z_0,\mathbf{a}^*\left(z_0\right)\right] - \frac{\partial\mathcal{W}^{bargained}(z_0)}{\partial\log z_0}}{\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t\mathbb{E}\left[f(z_t,\eta_t) - w_t^*(z_0)|z_0,\mathbf{a}^*\left(z_0\right)\right]} \tag{26}$$

where $\partial W^{bargained}(z_0)/\partial\log z_0$ *is defined in equation* (25)*. Moreover,*

$$\frac{\partial\mathcal{W}^{bargained}\left(z_0\right)}{\partial\log z_0} > 0 \quad\iff\quad \tilde{\mathcal{B}}'\left(z_0\right) > 0,$$

that is, bargained wage cyclicality is positive if and only if bargained utility is procyclical.

Proposition 3 shows that wage cyclicality due to bargaining dampens unemployment fluctuations. Equation (26) has an extra term relative to Theorem 2, which is bargained wage cyclicality. When bargained wage cyclicality is high, the impulse response of tightness is small. The proposition also shows that what we have defined as bargained wage cyclicality corresponds to the cyclicality of workers' utility — bargained wage cyclicality is only positive if the utility promised to workers is procyclical.

Intuitively, suppose that bargained utility is procyclical. Then during a boom, as z_0 increases, workers' wages increase by more than their effort. As a result, workers' utility increases during booms. At the same time, profits increase by less as z_0 rises, since workers capture part of the surplus through higher wages or lower effort. As a result, tightness is less responsive to business cycle shocks.

The proof is presented in Appendix A. A sketch of the proof is as follows. Under the conditions of the theorem, one can verify that the firm's problem may be expressed using a Lagrangian. After applying the Envelope theorem used in the proof of Theorem 2, one observes that the derivative of profits per worker with respect to z is given by the direct productivity effect minus the B term of equation (19), which reflects changes in the utility promised to the worker. However, equation (24) shows that the response of profits to z is also the direct productivity effect net of bargained wage cyclicality. Thus bargained wage cyclicality measures the cyclicality of promised utility. Appealingly, the proof does not require us to take a stand on why promised utility is cyclical. Various bargaining protocols or cyclicality in the value of unemployment benefits can lead to cyclical promised utility, all of which manifest as positive bargained wage cyclicality.[24]

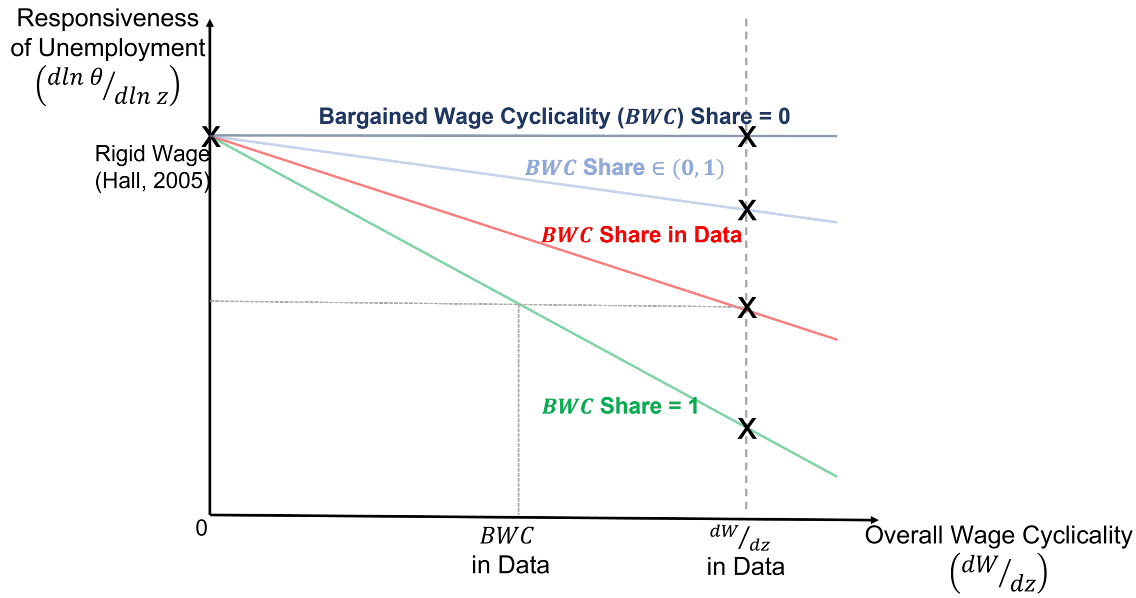
Our result relates to standard labor search models without incentives such as Shimer (2005). In these models, bargained wage cyclicality dampens unemployment fluctuations. However all wage cyclicality is due to bargaining, so that overall wage cyclicality is sufficient to measure the responsiveness of unemployment to shocks. Theorem 2 and Proposition 3 show that when wages vary due to incentives, overall and bargained wage cyclicality are no longer equal. To clarify this point, Figure 2 summarizes our analytical results and places them in context of the existing literature. The horizontal axis plots the degree of overall wage cyclicality. The vertical axis plots the responsiveness of market tightness to exogenous productivity shocks. The figure highlights four lines, each corresponding to a different model for the origins of wage cyclicality. All four lines intersect the vertical axis in the same place: when wage cyclicality is zero, we return to the rigid wage model of Hall (2005) in which market tightness is highly responsive to exogenous productivity shocks.

Figure 2 shows that models with and without incentives imply different unemployment dynamics given wage cyclicality in the data. Suppose overall wage cyclicality in the data is given by the vertical gray dashed line. Consider first the dark blue line at the top, labeled "Bargained Wage Cyclicality (BWC) Share = 0." This line corresponds to the model in which all wage cyclicality is due to incentives: bargained wage cyclicality is zero. As Proposition 3 makes clear, this model corresponds to the case in which promised utility does not vary with the cycle. Theorem 2 proves that this line is horizontal at the rigid wage line: even if wages are highly procyclical in this economy, the responsiveness of market tightness to aggregate productivity is the same as if wages and effort were exogenously held fixed.

Next consider the green line at the bottom, which corresponds to the case in which all wage cyclicality arises due to bargaining: the BWC share is equal to one. In this case, more cyclical wages dampen the response of market tightness to productivity shocks, as argued in Proposition 3. This is the classic result of Pissarides (2009) and holds in standard labor search models without incentives. Therefore models with and without incentives – with BWC shares equal to zero or one – match the same overall wage cyclicality in the data but have drastically different implications for unemployment volatility. Likewise, intermediate values for the share of overall wage cyclicality accounted for by bargaining generate lines

[24]Note that changes in promised utility also affect effort. Therefore our model can potentially generate countercyclical effort—during recessions, workers may exert more effort because their outside option is worse.

Figure 2: Illustration of The Relationship Between Wage and Market Tightness Cyclicality, by Share of Wage Cyclicality Accounted for by Bargaining



which are between the bargaining-only (green) and incentives-only (dark blue) lines. This is illustrated by the light blue and red lines on the plot.

Proposition 3 and Figure 2 offer guidance to researchers who wish to avoid working with complex models of incentive pay. Suppose that the red line corresponds to the share of bargained wage cyclicality that prevails in the data, which we seek to estimate below. This model generates a responsiveness of market tightness given by the horizontal dashed line on the graph. A model in which all wage fluctuations are accounted for by bargaining, such as the standard DMP model used in much of the literature, will generate the same unemployment dynamics as the full model with both incentives and bargaining so long as it is calibrated appropriately. In particular, one needs to calibrate a bargaining-only model in such a way that total wage cyclicality in that model is equal to bargained wage cyclicality in the data. We return to this point in Section 4.4 below.

3.5 Discussion

Next Steps. The analytical results of this section establish that wage cyclicality due to incentives does not dampen unemployment fluctuations, though wage cycliclaity due to bargaining does. The natural next question is: "what share of wage cyclicality in the data reflects bargained wage cyclicality?" Answering this question is challenging and should be the focus of future empirical work. To do so, one must measure the cyclicality of either utility promised to the worker, or the cyclicality of wages holding fixed the effort of the worker. One possibility would be to separately measure proxies for incentives and bargaining, such as the cyclicality of bonus and base pay. However, bonuses may not solely reflect incentive provision. For example, some workers may expect to receive a minimum bonus irrespective of their performance, while stock options reward aggregate stock market appreciations over which individual managers have little control. Similarly, bonuses do not reflect the full range of incentives firms may provide: longer term incentives such as promotions are ubiquitous and also seem to be cyclical (e.g. Méndez and Sepúlveda, 2012). The next section makes progress by calibrating a structural model of incentive pay to match micro moments of wage adjustment from the recent literature.

First order results. Our analytical results on the irrelevance of incentive wage cyclicality and the importance of bargaining hold to a first order rather than globally. Below, we study a globally solved numerical model with consonant results.

User cost of labor and present value of wages. Our argument is different from the emphasis on new hire wages or the user cost of labor (Kudlyak, 2014). The irrelevance of flexible incentives pay holds even if the *present value* of new hires' incentive wages is arbitrarily cyclical.

Endogenous separations. The irrelevance of incentive wage cyclicality continues to hold when separations are endogenous and efficient. Intuitively, separations are another margin over which the firm can optimize, in order to maximize the profits of a job. Therefore after an aggregate shock, changes in the firm level separation rate have no first order effect on profits. Appendix Section A.6 introduces endogenous separations into the incentive pay model and derives equivalence for the impact elasticity of tightness to productivity shocks.[25]

4 Numerical Analysis

This section considers a tractable contract that can simulated. We use this numerical model to infer what share of overall wage cyclicality in the data is due to bargaining. We also explain how researchers can use our quantitative results to properly calibrate simpler models of unemployment fluctuations without incentive pay.

[25] Pissarides (2009) proves a related result for the impact elasticity in the DMP model with efficient endogenous separations but without incentive pay.

4.1 A Tractable Contract

We now derive a tractable contract by specializing the production function, utility function and information structure of our dynamic model to follow Edmans et al. (2012a). We also parameterize workers' promised utility. All other aspects of the environment are the same as the flexible incentive pay economy of Section 3.

Production function. The firm's production function is $y = z(a + \eta)$. Let $\mathcal{Y}(\mathbf{a}, z_0) \equiv \mathbb{E}_0\left[\sum_{t=0}^{\infty}(\beta(1-s))^t y_{it} | \mathbf{a}, z_0\right]$ be the present discounted value of output of a match, given that aggregate productivity at time 0 is z_0. Idiosyncratic profit shocks η are assumed to be i.i.d. over time and across individuals and normally-distributed with zero mean and standard deviation σ_η. σ_η determines the extent to which firms can infer workers' effort, which is key for incentive pay.

Preferences. We assume that workers have log utility over consumption, with an isoelastic disutility of labor that is separable from consumption. Therefore we have $u(c, a) \equiv \ln c - \frac{a^{1+1/\epsilon}}{1+1/\epsilon}$, where ϵ governs the Frisch elasticity of effort. ϵ determines how costly the provision of effort is to workers.

Information structure. We make the "effort after noise" assumption as in Edmans et al. (2012a): workers observe the idiosyncratic profit shock η before making their effort choice. Thus there is an incentive compatibility constraint for each value of η. Following Edmans et al. (2012a), we assume that a unique level of effort $a(z^t)$ is implemented irrespective of the idiosyncratic shock η.[26] We stress that effort is allowed to vary with the history of aggregate productivity z^t.

Promised utility. We assume that firms make take-it-or-leave-it offers to workers who face cyclical unemployment benefits. Workers' flow unemployment benefits take the form $b(z) = \gamma z^\chi$. Here, γ specifies the level of unemployment benefits when $z = 1$, while χ determines the elasticity of unemployment benefits to aggregate productivity. This specification is first-order equivalent to any model in which workers and firms explicitly bargain over promised utility under the contract. However, this specification is numerically tractable by abstracting from complications of bargaining and ensuring that unemployed workers' value is given by the present discounted value of expected unemployment flow benefits. The parameter χ is a stand-in for bargaining in that it shifts the utility promised to the workers under the contract – it can reflect changes in promised utility due to fluctuations either in the worker's outside option (changes in the value of unemployment) or inside option (bargained utility). Indeed, χ determines the cyclicality of "bargained wages." In effect, we are

[26] Edmans et al. (2012a) provide sufficient conditions for this to be true. In an optimal contract, the firm will be able to infer the level of η that was realized because the worker will implement the level of effort recommended by the firm, given incentive compatibility.

assuming a particular parameterization for the reduced form bargaining rule $\mathcal{B}(z)$.

We now characterize the optimal contract following Edmans et al. (2012b).

Proposition 4. *The earnings schedule in the optimal contract satisfies the following difference equation (given initial productivity z_0):*

$$\log(w_t(a_t, \eta^t|z^t)) = \log(w_{t-1}(a_{t-1}, \eta^{t-1}|z^{t-1})) + \psi h'(a_t)\eta_t - \frac{1}{2}(\psi h'(a_t)\sigma_\eta)^2 \tag{27}$$

where $\psi = 1 - \beta(1-s)$ and $w_{-1}(z_0)$, which initializes this difference equation, is given by

$$w_{-1}(z_0) \equiv \psi \left(Y(z_0) - \frac{\kappa}{q(\theta_0)} \right). \tag{28}$$

Worker's utility under the contract $\mathcal{E}(z_0)$ is equal to their value of non-employment, so that

$$\frac{\log w_{-1}}{\psi} - \mathbb{E}_0 \left[\sum_{t=0}^{\infty} (\beta(1-s))^{t-1} \Big(\frac{\psi}{2}(h'(a_t)\sigma_\eta)^2 + h(a_t) - \beta s U(z_{t+1}) \Big) |z_0 \right] = U(z_0) \equiv \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \ln b(z_t) \right]. \tag{29}$$

In addition, the optimal effort level satisfies

$$a_t(z_t) = \left[\frac{z_t a_t(z_t)}{\psi \left(Y(z_0) - \frac{\kappa}{q(\theta_0)} \right)} - \frac{\psi}{\epsilon}(h'(a_t)\sigma_\eta)^2 \right]^{\frac{\epsilon}{1+\epsilon}}. \tag{30}$$

A proof is provided in Appendix A and closely follows that of Edmans et al. (2012a). In the contract, the passthrough of idiosyncratic shocks to wages corresponds to incentives. Intuitively, to satisfy the incentive constraint as cheaply as possible, the firm increments wages in a manner consistent with the worker's inverse Euler equation, which gives rise to the log difference equation (27). Idiosyncratic shocks η partially pass-through into wages since the firm must make workers' wages responsive to output fluctuations to incentivize effort. If the marginal disutility of effort is high, there must be a high pass-through from η to wages in order to induce workers to supply the optimal effort level. In order to maintain dynamic incentives, the passthrough of idiosyncratic productivity shocks to wages is scaled down by a quantity ψ which reflects discounting.

Initial wages correspond to promised utility at the start of the match. Exponentiating equation (27), one observes that wages are a random walk: the expectation of wages in period $t + h$ is equal to the level of wages in period t. The random walk property is a consequence of the standard "inverse Euler equation" (Rogerson, 1985). Thus the initialization of the difference equation, w_{-1}/ψ, is equal to the expected present discounted value (EPDV) of wage payments. Free entry into vacancy posting guarantees that the EPDV wage payments are the difference between the endogenous EPDV of output $Y(z_0)$ and the expected cost of filling a vacancy $\kappa/q(\theta)$. Calculating the expected utility under the contract (the left hand side of equation (29)) relies on solving forward the wage equation. Effort is determined by taking the first order condition of the worker's utility maximization problem; that is, by setting the derivative of the left hand side of equation (29) with respect to a_t equal to zero.

We make two advances relative to Edmans et al. (2012a). First, we introduce aggregate risk z_t which may cause the worker's effort level to fluctuate over the life of the contract. Second, we develop a global solution algorithm to efficiently simulate this model with labor market search. The details of this algorithm are described in Appendix B.

4.2 Calibration—Separating Bargaining from Incentives

The goal of our calibration is to infer the role of bargaining versus incentives in determining wage setting. We will disentangle these forces with two moments: the cyclicality of the wage for new hires, which inform bargaining power; and the pass through of idiosyncratic firm output shocks into wages, as well as the variance of workers' wage growth, both of which inform incentives.

We calibrate the parameters of the labor search block following the standard practice of Petrosky-Nadeau and Zhang (2017).[27] Productivity is assumed to follow an AR(1) process in logs, with autocorrelation parameter ρ, innovation $\zeta_t \sim \mathcal{N}(0, \sigma_z^2)$, and mean μ_z. We set μ_z such that $\mathbb{E}[z_t] = 1$. To account for effort fluctuations' effects on labor productivity, we calibrate our monthly process for z such that the log of the quarterly average of z_t matches the autocorrelation and standard deviation of the quarterly log TFP series described in Fernald (2014), which accounts for variable capacity utilization in labor. We view the TFP series net of variable capacity utilization as a reasonable proxy for exogenous productivity, as labor utilization is a concept highly related to effort.[28] This procedure implies a monthly autocorrelation $\rho = 0.966$ and a standard deviation of shocks $\sigma_z = 0.0056$.[29]

This leaves four parameters to calibrate: $\sigma_\eta, \gamma, \chi$, and ϵ, which we internally calibrate. We target the variance of incumbent wage growth, the pass-through of firm shocks into wages, the cyclicality of new hire wages, and the average unemployment rate. While we estimate all parameters jointly, these moments have intuitive mappings to particular parameters, which

[27]These parameters are the discount rate, the vacancy creation cost, the matching function, and the separation rate. We discuss the details in Appendix Section B.1.

[28]Basu and Kimball (1997) find that variable capacity utilization explains approximately 40-60 percent of fluctuations in unadjusted TFP and that capacity utilization is procyclical.

[29]We HP-filter the TFP data and model simulated series with a smoothing parameter of $\lambda = 10^5$, following Shimer (2005), which removes a very low frequency trend.

we explore below.

First, the variance of wage growth naturally informs the variance of idiosyncratic profit shocks σ_η. To see this, note that re-arranging equation (27) shows that the monthly wage growth of job-stayers is given by

$$\Delta \log w_t = \psi h'(a_t)\eta_t - \frac{1}{2}\left(\psi h'(a_t)\sigma_\eta\right)^2.$$

At an aggregate non-stochastic steady state, $a_t = a^{SS}$, the cross-sectional variance of wage growth is given by

$$Var(\Delta \log w) = \psi^2 h'(a^{SS})^2 \sigma_\eta^2,$$

which is closely tied to the value of σ_η.[30] The firm provides *intertemporal incentives* by exposing the worker to wage-growth risk[31] as in Sannikov (2008). We target a standard deviation of year-over-year earnings per hour growth of job-stayers of 0.064 to match that inferred by Grigsby et al. (2021), where we calculate year-over-year wage growth in the model with stochastic z_t by iterating on equation (27) for job-stayers.[32]

Second, the pass-through of firm-specific shocks to wages is informative of the parameter governing the disutility of effort ϵ. Since η_t shocks are independent across time, one can calculate how wages change with η by differentiating equation (27):

$$\frac{\partial \log w_t}{\partial \eta_t} = \psi h'(a_t). \tag{31}$$

Differentiating the production function yields $\partial \log y / \partial \eta = (a_t + \eta)^{-1}$. Dividing equation (31) by this term reveals that the expected pass-through from idiosyncratic output shocks to the wages of job-stayers is given by

$$\mathbb{E}\left[\frac{\partial \log w}{\partial \log y}\right] = \mathbb{E}\left[\psi h'(a)(a+\eta)\right] \tag{32}$$

which is directly affected by $h'(a)$. The firm provides *intratemporal incentives* with the pass-through of output to wages. Intuitively, if $h'(a)$ is high, then workers would prefer not to supply more effort. In order to induce the worker to supply more effort, the firm must

[30] With stochastic z_t, an analogous expression holds, with an additional term related to the variance of the disutility of effort stemming from fluctuations in z.

[31] This captures the idea that in a model with job ladders the firm can provide intertemporal incentives with promotions

[32] Hours are observable and thus contractible. We therefore consider earnings per hour – inclusive of base pay, bonus and overtime – to be the correct empirical counterpart of w_t.

provide high powered incentives via a high pass-through of output to wages. Pass-through is therefore linked to ϵ.

A large literature seeks to estimate the pass-through of firm-specific profitability shocks on job-stayers' wages; Card et al. (2018) provide a comprehensive survey. This literature has found estimates of pass-through elasticities from firm-level shocks ranging from 0.02 to 0.156. These estimates arise from a variety of strategies, such as specifying stochastic processes for firm productivity (Guiso et al., 2005; Card et al., 2016) or estimating the effect of identified firm shocks such as government contract awards (Cho, 2018) or patent awards (Kline et al., 2019). Many of these strategies use firm variation that is likely partially persistent: a patent award affects profitability for more than one month, for instance. In contrast, our η shocks are perfectly transitory and i.i.d. through time, which thus suggests the pass-through of η shocks to wages is likely lower than the higher ranges commonly estimated in the literature. We therefore target a pass-through of firm level output shocks to wages of 0.039, which is estimated in Martins (2009) and is on the low end of the range reported by Card et al. (2018). Targeting a low pass-through is likely to be conservative, as it suggests that incentives are not high powered, and therefore are a relatively unimportant determinant of wage variation.

Third, we identify γ, which pins down the level of unemployment benefits, from the stochastic mean of unemployment. Mean unemployment is determined by workers' job-finding rates, which in turn are determined by expected profits per worker. γ determines expected profits, because it governs workers' value of unemployment and shifts the level of the required wage payments to workers. We target an average unemployment rate of 6%, consistent with average U.S. unemployment between 1951 and 2019.

Fourth, we target the cyclicality of new hire wages to inform the cyclicality of non-employment benefits χ. Conditional on the parameters governing incentives—the disutility of labor and idiosyncratic shocks from η—the cyclicality of new hire wages is highly informative of χ. Intuitively, if the worker's outside option is highly pro-cyclical, so too is their promised utility, and thus so too will be their wage payments. Since wages are a random walk in the optimal contract, the cyclicality of new hire wages is highly informative of the cyclicality of the present discounted value of wage payments, and thus the cyclicality of promised utility. Mathematically, substituting our expression for the flow value of unemployment $b(z_t)$ into equation (29), which characterizes promised utility at the start of the match, yields

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t(\log\gamma+\chi\log z_t)\right]=\frac{\log w_{-1}}{\psi}-\mathbb{E}_0\left[\sum_{t=0}^{\infty}(\beta(1-s))^{t-1}\Big(\frac{\psi}{2}(h'(a_t)\sigma_\eta)^2+h(a_t)-\beta sU(z_{t+1})\Big)\right]$$

Given the wage schedule defined in equation (27), expected new hire wages $w_0(z_0)$ are equal to $w_{-1}(z_0)$. The above equation shows a close relationship between expected new hire wages w_0 and χ, conditional on the disutility of effort. We target a semi-elasticity of new hire wages to the unemployment rate of -1, which is at the high range of what is found by Grigsby et al. (2021), and explore robustness to this choice.

In summary, variation in wage fluctuations that occur after start of the contract inform the parameters governing the strength of incentives, while wage fluctuations at the beginning of the contract informs the strength of bargaining. Appendix B presents details of the estimation algorithm, how we produce moments within the model and the data, and how we calculate the share of wages due to bargained wage cyclicality.

4.3 Numerical Results

Table 1 summarizes our calibration, while Table 2 examines the model's fit to both targeted and untargeted moments. We estimate that the elasticity of the disutility of effort ϵ is equal to 2.4. Note that standard estimates of micro labor supply elasticities, such as those computed by Chetty (2012), consider how hours vary with wages. Since hours are observable and contractible by the firm, the lower elasticities of hours need not have any relationship with the elasticity of unobservable effort. Intuitively, one might expect the elasticity of effort to be larger than that of hours: while many jobs have a fixed number of hours over which the worker has little control (e.g. they must work 40 hours per week to remain employed), workers may be able to adjust unobserved effort more elastically.

We find the level of unemployment benefits γ to be 0.474. This value is between the value chosen by Shimer (2005) to match the replacement rate of unemployment benefits (0.4) and that chosen by Hagedorn and Manovskii (2008) (0.955) to match aggregate wage cyclicality.[33]

We estimate the standard deviation of idiosyncratic profit shocks to be $\sigma_\eta = 0.536$, similar to other labor search calibrations with idiosyncratic shocks (e.g. Schaal, 2017). This, coupled with a sizable elasticity of effort disutility, suggests that incentive provision is a relatively important consideration for the firm. We estimate the cyclicality of flow unemployment benefits χ to be 0.516.[34] This implies that promised utility to the worker is moderately elastic to the business cycle. Table 2 compares key moments in both the calibrated model (Column 1) and Data (Column 2). The top panel reports the moments we target in the

[33] Note, however, that unemployed workers do not need to supply effort in this model, which increases the effective flow unemployment value.

[34] Chodorow-Reich and Karabarbounis (2016) estimate the cyclicality of the value of unemployment to be around 0.8 using consumption data. This is not directly comparable to our results since part of the value of unemployment in our model is the fact that workers do not need to supply effort, and because workers do not have access to financial assets.

Table 1: Calibrated parameter values

Parameter	Description	Value	Source/Target
Externally Calibrated			
β	Discount Rate	$0.990^{1/3}$	Petrosky-Nadeau and Zhang (2017)
κ	Vacancy Creation Cost	0.450	Petrosky-Nadeau and Zhang (2017)
s	Separation Rate	0.031	CPS E-U Flow Rate
ρ	Autocorrelation: agg. productivity	0.966	Autocorrelation: Fernald (2014) TFP
σ_z	S.D. of agg. productivity innovations	0.006	S.D.: Fernald (2014) TFP
Internally Calibrated			
γ	Level: unemployment benefit	0.474	Average Unemployment Rate
ϵ	Elasticity: Disutility of Effort	2.385	Pass-through: profits to wages
σ_η	S.D.: Idiosyncratic Profit η	0.536	S.D.: Job-Stayer Log Wage Growth
χ	Cyclicality: Promised Utility to Worker	0.516	New Hire Wage Cyclicality

Table 2: Model Fit to Data Moments

Moment	Description	Data (1)	Model (2)
Targeted			
$d\mathbb{E}[\log w_0]/du$	Cyclicality of new hire wages	-1.000	-1.000
$\partial \log w_t / \partial \log y_{it}$	Within-job pass-through of idiosyncratic shock	0.039	0.035
$\text{std}(\Delta \log w_t)$	std(% change in earnings/hour - job stayers)	0.064	0.064
$\bar{u}_t$	Mean unemployment	0.060	0.060
Untargeted			
std $(\log u_t)$	Volatility of unemployment (quarterly)	0.157	0.103
BWC Share	Share of wage cyclicality due to bargaining	–	0.599

estimation. The model is able to fit the targeted moments very well. Most notably, we match the the cyclicality of new hire wages exactly and, if anything, under-estimate the pass-through of firm shocks to wages, suggesting that our estimate of the importance of incentives for wage cyclicality is likely a lower bound on its true importance.

The bottom panel of the table reports the behavior of unemployment in both the model and the data. The model generates around two-thirds of the fluctuation in aggregate unemployment observed in the data, which is an appropriate figure because labor productivity is not the sole shock determining unemployment fluctuations (Pissarides, 2009) and some wage cyclicality is due to bargaining. Matching the micro moments of wage adjustment therefore generates significant unemployment volatility, the reasons for which we explore below.

This model calibration reveals in the final row of Table 2 that approximately 60% of total

Table 3: Model Moments: Alternative Calibrations

		Model: source of wage flexibility			
		(1)	(2)	(3)	(4)
Moment	Data	Incentives + Bargaining	Incentives	Bargaining	Bargaining: $\partial \mathbb{E}[\log w_0]/\partial u = -0.6$
$\partial \mathbb{E}[\log w_0]/\partial u$	-1.00	-1.00	-0.69	-1.00	-0.60
std($\log u_t$)	0.157	0.103	0.150	0.078	0.103
$\partial \log \theta_0/\partial \log z_0$	-	13.3	18.0	10.4	13.0
$\mathcal{W}_0/\mathcal{Y}_0$	-	0.96	0.96	0.96	0.96
$\partial \log \mathcal{W}_0/\partial \log z_0$	-	0.44	0.43	0.32	0.26
$\partial \log \mathcal{Y}_0/\partial \log z_0$	-	0.68	0.92	0.51	0.51
BWC share	-	0.60	0.00	1.00	1.00

Notes: New hire wage cyclicality is targeted, while the second set of moments are untargeted. Column (1) is our baseline model. Column (2) sets $\chi = 0$ and does not target the cyclicality of new hire wages. Columns (3) and (4) fix effort $a = 1$, set wages to be constant within the contract, and do not target the standard deviation of log wage growth or the pass-through. Column (4) targets a cyclicality of new hire wages of -0.6. The standard deviation of log unemployment is computed at the quarterly frequency. x_0 denotes the value of variable x, evaluated at $\log z = \mu_z$. $\mathcal{W}$ and $\mathcal{Y}$ refer to the expected present value of wage payments and output, respectively. BWC share is the share of wage cyclicality that is due to bargaining for $\log z_0 = \mu_z$.

wage cyclicality is due to bargaining and outside option cyclicality. Conversely, incentives account for the remaining 40% of total wage cyclicality. The share of wage cyclicality due to incentives may seem surprisingly large. For most workers non-base compensation, which is presumably associated with incentives, is relatively small for most workers. However, what matters for wage cyclicality is whether the *marginal* dollar of wages paid is due to incentives or bargaining. If, for instance, only 2% of compensation reflects incentive pay in steady state but only incentive pay is cut in response to small output declines, then the share of wage cyclicality due to incentives is 100%.

Table 3 shows our model's ability to match data moments when we consider models which load all wage cyclicality onto either incentives or bargaining. Column (1) reproduces the baseline model as in Table 2, but includes a number of additional model-implied moments. First, note that our model generates large unemployment volatility because market tightness responds greatly to exogenous productivity shocks: the elasticity of market tightness to aggregate productivity is 13.3. Thus market tightness will be substantially more volatile than aggregate productivity, as in the data.

The high volatility of market tightness arises due to a few factors. First, the model implies a labor share (defined as $\mathcal{W}_0/\mathcal{Y}_0$) of 0.964, in line with, for instance, Hall (2005).[35] As discussed in Ljungqvist and Sargent (2017), the combination of this labor share calibration and fixed real wages delivers volatile profits and thus unemployment.

[35]Since our model does not have capital, the labor share corresponds to the labor share of payroll and rents from search frictions, excluding capital (Pissarides, 2000).

Our model, however, delivers volatile unemployment even though real wages are cyclical. The elasticity of the present value of expected wage payments with respect to productivity is 0.44. However as we have discussed in previous sections, the stabilizing effect on unemployment of cyclical wages is offset by the amplifying effect of effort movements. The response of the present value of output, $\mathcal{Y}_0$, to TFP shocks is 0.68. We shall see that this response is significantly larger than the standard DMP model, due to the endogenous response of effort. As a result, profit fluctuations—and thus market tightness and employment fluctuations—are large despite procyclical wages.

It is instructive to consider versions of our model which load all wage cyclicality in the data onto one of these two sources. We consider a calibration with only incentives and without bargaining in Column (2). For this version of the model, we assuming that the cyclicality of promised utility is zero, and re-calibrate with $\chi = 0$. We do not target wage cyclicality, and so the model remains exactly identified. In this calibration, the labor share of 0.96 is the same as the baseline. However, unemployment is far more volatile: the standard deviation of log unemployment is 0.15, compared with 0.157 in the data and 0.103 in the baseline calibration. This is a manifestation of two of our results. First, this incentives-only version of the model behaves as if wages and effort were exogenously fixed as in Hall (2005); thus it is able to generate the volatility of employment seen in the data. Second, it illustrates that bargained wage flexibility—captured by $\chi > 0$—would reduce the volatility of unemployment.

Nevertheless, the incentives-only model still generates large wage cyclicality, despite cyclical profits. As we have discussed, as z rises, so too does desired effort, due to the complementarity between effort and z in the production function. In column (2), the elasticity of $\mathcal{Y}$ to TFP shocks is a relatively large value of 0.92. To induce this effort, the firm must incentivize the worker, by making their wage more responsive to realized output. This exposes the worker to risk, for which they must be compensated. Thus, expected wages become fairly pro-cyclical, even in the absence of cyclical promised utility to the worker.

Next, we consider a version of the model without incentives and with only bargaining in Column (3). Here, we switch off incentives and variable effort, by setting the variance of the idiosyncratic profitability shocks to $\sigma_\eta = 0$, exogenously fixing effort $a = 1$, setting $\epsilon = 1$, and setting wages to be fixed within a contract. We no longer target the variance of log wage growth nor the pass through of firm shocks to wages and attribute all wage cyclicality in the data to the cyclicality of promised utility, governed by χ. This calibration of the model is closer to common practice in job search models without incentive provision and implies that the bargained wage cyclicality share is 100%.

The version of our model in which wage cyclicality only reflects bargaining generates unemployment volatility around 25% less (0.078) than the full model with both bargaining and incentives (0.103). This is because the estimated value of χ rises substantially to 0.633 (from 0.516). Therefore wage cyclicality is high, however, there is no offsetting movement in effort. As a result, the elasticity of market tightness to exogenous productivity falls to 10.4 from 13.3. This confirms what has been known since Shimer (2005): the benchmark labor search model of Mortensen and Pissarides (1994) generates far less unemployment fluctuations than are observed in the data when wages are set according to Nash bargaining and moderately pro-cyclical. Both the bargaining-only and full model have a labor share of 0.96, meaning that differences in the "fundamental surplus" cannot explain different unemployment dynamics in the two models (Ljungqvist and Sargent, 2017).

Taking stock, we find that a relatively large share of wage cyclicality in the data is due to incentives despite a conservative calibration. Therefore our model generates large unemployment fluctuations despite cyclical wages.

4.4 Discussion: A User Guide

Wage cyclicality due to incentive provision is not relevant for unemployment fluctuations. Here, we discuss how to calibrate a simple model without incentives to replicate the unemployment dynamics of our richer incentive pay model. The hope is to create a "user guide" for researchers who wish to appropriately calibrate models while avoiding the complexities of incentive pay.

To do so, consider the simple version of our model in which there is no production noise ($\sigma_\eta = 0$) and effort a is exogenously fixed at 1. This model therefore has neither incentive problems nor variable effort and is thus akin to much of the literature working with DMP search models. Wage cyclicality in this version of the model exclusively arises from fluctuations in promised utility, either due to bargaining or fluctuations in the value of unemployment. How might a researcher calibrate this simple model in order to appropriately filter out wage fluctuation due to incentives from the data?

We argue that the simple model should target a new hire wage cyclicality given by only bargained wage cyclicality. To illustrate this point, we re-estimate this bargaining-only version of the model targeting a new hire wage cyclicality of -0.6, which we previously inferred from the data as wage cyclicality due to bargaining.[36]

Column (4) of Table 3 presents the results of this exercise. This version of the model features unemployment volatility that is nearly identical to the baseline model (Column 1). Both models generate a standard deviation of log unemployment rates of 0.103, and a

[36]We normalize $\epsilon = 1$ for this exercise and solve for fixed wages within the contract. We also drop the standard deviation of log wage growth and the pass-through of firm shocks as targeted parameters.

Table 4: Varying cyclicality of new hire wages: moments

		Model: $\partial\mathbb{E}[\log w_0]/du$ target					
Moment	Data	-0.25	-0.5	-0.75	-1.0	-1.25	-1.5
$\partial\mathbb{E}[\log w_0]/\partial u$	-1.00	-0.26	-0.50	-0.75	-1.00	-1.25	-1.50
std($\log u_t$)	0.157	0.156	0.162	0.121	0.103	0.087	0.074
$\partial \log \theta_0/\partial \log z_0$	-	17.5	17.8	14.8	13.3	11.7	10.1
BWC share	-	0.42	0.32	0.56	0.60	0.66	0.73
Incentive Wage Cyclicality	-	-0.15	-0.34	-0.33	-0.40	-0.42	-0.40

Notes: The first set of moments are targeted; the second set of moments are untargeted. The standard deviation of log unemployment is computed at the quarterly frequency. x_0 denotes the value of variable x, evaluated at $\log z = \mu_z$. BWC share is the share of wage cyclicality that is due to bargaining. Incentive wage cyclicality is defined as one minus the BWC share multiplied by $\partial\mathbb{E}[\log w_0]/\partial u$.

response of market tightness to exogenous productivity of approximately 13. These are our analytical results in practice. Also consistent with the analytical results, promised utility cyclicality is similar in the simple model and in the incentive pay model, even though overall wage flexibility is different. In our simple model we internally calibrate an elasticity of unemployment benefits $\chi = 0.517$, nearly identical to that found under the full model (0.516).[37]

Researchers wishing to abstract from incentive contracts in labor search models may calibrate a simpler model without incentives to match wage cyclicality in the data that is due to bargaining. Doing so will generate an identical unemployment response to a fuller model which accounts for micro moments of wage adjustment and incentive pay. Our numerical exercise reveals that the simpler model should target weakly procyclical wages, because a substantial share of overall wage cyclicality in the data is due to incentives.

Sensitivity to Alternative Wage Cyclicality Targets. Our baseline calibration targets a semi-elasticity of new hire wages with respect to the unemployment rate of -1. We view this as being towards the high end estimates found in the literature.[38] Nevertheless, there remains some disagreement over exactly how cyclical are new hire wages. We thus test the sensitivity of our numerical exercise to different targets of new hire wages.

Table 4 reports the model-implied moments when we target different values of wage cyclicality ranging from -0.25 to -1.5. In each case, the model matches the targeted moments very well.[39] The average labor share also remains roughly constant at 96%. We find that

[37]We emphasize that the bargaining only model generates different output dynamics to the model with bargaining and incentive pay, therefore our framework also has quantitative implications for the cyclicality of variable capacity utilization, which is fertile ground for future work.

[38]Grigsby et al. (2021) argue that this high estimate may be partly due to composition. Bils et al. (2022a) report estimates of larger magnitude, though these are at lower frequency.

[39]Appendix Table B1 reports the estimated parameters given these targets.

the share of wage cyclicality attributable to incentives declines as we increase the target cyclicality of new hire wages. However, incentive wage cyclicality – that is, one minus the share of wage cyclicality from bargaining multiplied by the new hire wage cyclicality $\partial\mathbb{E}[\log w_0]/\partial u$ – is relatively stable between -0.33 and -0.42. A simple rule of thumb to sweep out wage cyclicality due to incentives is therefore to subtract 0.4 from one's preferred estimate of wage cyclicality.

5 Conclusion

The cyclicality of wages is important for macroeconomic fluctuations. Recent empirical work shows that business cycle fluctuations in wages are complicated, due to the complex nature of incentive pay.

This paper studies the role of incentive pay for unemployment dynamics. Embedding a dynamic principal-agent problem into a benchmark labor search model leads to two results. First, wage cyclicality due to incentives does not dampen unemployment fluctuations. Indeed, a model in which wage cyclicality arises solely due to incentives features identical first order unemployment dynamics as a model with exogenously fixed wage and effort as in Hall (2005). The equivalence raises because endogenous fluctuations in effort exactly offset movements in the wage under the optimal contract. Second, wage fluctuations which shift the utility promised to the worker, which we dub "bargained wage cyclicality," do mute unemployment's response to productivity shocks as in standard models.

These analytical results urge careful measurement of bargained wage cyclicality in order to calibrate models of unemployment fluctuations. We offer one attempt at measurement through a calibrated model. We find that approximately 60% of wage cyclicality observed in the data is due to bargaining, with 40% arising due to a cyclical desire to incentivize worker effort. Models which do not feature incentive pay should therefore target a semi-elasticity of new hire wages to unemployment which is around 0.4 lower than total wage cyclicality measured in the data.

There remains much work to be done. The relevance of incentive pay for inflation dynamics remains an interesting area for future research. Likewise, future work may be able to use our framework to develop a quantitative theory of capacity utilization. Finally, we hope future reduced form work will attempt to measure bargained wage cyclicality to complement our structural approach.

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A Analytic Appendix

A.1 Implicit Definition of $\mathcal{B}(z_0)$ with Nash Bargaining

This subsection shows that in the flexible incentive pay economy of the main text, Nash bargaining implicitly defines a functional form for $\mathcal{B}(z_0)$. Suppose that the firm and worker engage in generalized Nash bargaining over the surplus of the match, and φ is the firm's bargaining power. Promised utility $\mathcal{B}(z_0)$ is implicitly defined by

$$\mathcal{B}(z_0) = \arg\max_{\overline{\mathcal{B}}} J\left(z_0, \overline{\mathcal{B}}\right)^{\varphi} \left(\overline{\mathcal{B}} - U(z_0)\right)^{1-\varphi}.$$

Here, as in the main text, $U(z_0)$ is the value of unemployment at time 0. $J\left(z_0, \overline{\mathcal{B}}\right)$ is defined by equations (11)-(13) in the main text, replacing $\mathcal{E}(z_0)$ with $\overline{\mathcal{B}}$ in equation (13). Therefore $\mathcal{B}(z_0)$ is the solution of the standard Nash bargaining problem, albeit in an environment with dynamic incentive pay. The solution is

$$\varphi \frac{\frac{\partial J(z_0, \mathcal{B}(z_0))}{\partial \overline{\mathcal{B}}}}{J(z_0, \mathcal{B}(z_0))} + \frac{(1-\varphi)}{\mathcal{B}(z_0) - U(z_0)} = 0.$$

Recall equation (8) also relates the value of employment and unemployment as

$$U(z_t) = \phi(\theta_t)\mathcal{B}(z_t) + (1-\phi(\theta_t))\left(\xi(z_t) + \beta\mathbb{E}\left[U(z_{t+1})|z_t\right]\right),$$

where as in the main text, $\xi(z_t) \equiv u(b(z_t), 0)$ is the flow utility unemployment benefits. Finally, the free entry condition and equation (60) implicitly define a relationship between $J(z_t, \mathcal{B}(z_t))$ and θ_t. The preceding three equations implicitly define functions $U(z_t)$, $J(z_t)$ and $\mathcal{B}(z_t)$.

A.2 Proof of Proposition 1

The proof has two steps. First, we write the firm's problem as a Kuhn-Tucker Lagrangian

$$J(z_0) = \sum_{t=0}^{\infty} (\beta(1-s))^t \int \sum_{\eta^t \in \Xi^t} \left(f(z_t, \eta_t) - w_t^*(\eta^t, \varepsilon^t; z_0)\right) \tilde{\pi}_t\left(\eta^t, \varepsilon^t | \mathbf{a}^*(z_0)\right) d\varepsilon^t + \langle \lambda^*, G(\mathbf{w}, \mathbf{a}) \rangle \tag{33}$$

Second, we differentiate the Lagrangian to yield the Proposition.

Stating the constraint qualification of the Lagrangian is non-trivial in our setting as there is a continuum of constraints. We state the constraint qualification corresponding to the Generalized Kuhn-Tucker Lagrangian Theorem of Luenberger (1969). To state the constraint qualification, first let $\delta G(\mathbf{w}, \mathbf{a}; h)$ denote the Gateaux derivative of $G(\mathbf{w}, \mathbf{a})$ with increment h, that is

$$\delta G(\mathbf{w}, \mathbf{a}; h) = \lim_{\alpha \to 0} \frac{1}{\alpha} \left[G\left((\mathbf{w}, \mathbf{a}) + \alpha h\right) - G(\mathbf{w}, \mathbf{a}) \right]$$

for h an element in the feasible contract space $h \in \mathcal{X}$. Next, define $(\mathbf{w}_0, \mathbf{a}_0) \in \mathcal{X}$ to be a "regular point" of the inequality $G(\mathbf{w}, \mathbf{a}) \leq 0$, if:

1. $G(\mathbf{w}_0, \mathbf{a}_0) \leq 0$; and
2. There is a direction h such that $G(\mathbf{w}_0, \mathbf{a}_0) + \delta G(\mathbf{w}_0, \mathbf{a}_0; h) < 0$, where the strict inequality denotes an interior point.

That is, a regular point is a point in the feasible contract space that satisfies the constraints and which is arbitrarily close to another point in the contract space for which the constraint does not bind. The optimal contract must be a regular point to satisfy the constraint qualification.

The theorem of Luenberger (1969) states that if a contract $(\mathbf{w}^*, \mathbf{a}^*)$ solves the firm's problem and is a regular point of the constraint set, then there exists a linear functional λ^* such that the Lagrangian (33) is stationary at $(\mathbf{w}^*, \mathbf{a}^*)$ and $\langle \lambda^*, G(\mathbf{w}^*, \mathbf{a}^*) \rangle = 0$. Put simply, if the optimal contract is a regular point of the constraint set, then the problem can be expressed using a Kuhn-Tucker Lagrangian.

Differentiating the Lagrangian (33) yields Proposition 1:

$$\begin{aligned} \frac{dJ(z_0)}{dz_0} &= \frac{\partial}{\partial z_0} V(\mathbf{w}^*, \mathbf{a}^*; z_0) - \left\langle \frac{\partial}{\partial z_0} G(\mathbf{w}^*, \mathbf{a}^*; z_0), \lambda^*(z_0) \right\rangle \\ &\quad + \sum_{x \in \{\mathbf{w}^*, \mathbf{a}^*\}} \left[\nabla_x V(\mathbf{w}^*, \mathbf{a}^*; z_0) - \langle \nabla_x G(\mathbf{w}^*, \mathbf{a}^*; z_0), \lambda^*(z_0) \rangle \right] \cdot \frac{dx}{dz_0} - \left\langle G(\mathbf{w}^*, \mathbf{a}^*; z_0), \frac{d\lambda^*(z_0)}{dz_0} \right\rangle \end{aligned}$$

as required. □

A.3 Proof of Theorem 2

To begin, take logs and differentiate the free entry condition (1) with respect to $\ln z_0$ to see

$$\frac{d \log J(z_0)}{d \log z_0} = \nu_0 \frac{d \log \theta_0}{d \log z_0}. \tag{34}$$

That is, the response of market tightness to aggregate productivity shocks is proportional to the response of the value of a filled job, as in the static model.

Tightness dynamics in the rigid wage economy, with $w_t = \bar{w}$ and $a_t = \bar{a}$ are trivially given by equation (34) and

$$\frac{d\log\theta_0}{d\log z_0} = \frac{1}{\nu_0}\frac{dJ^{\text{rigid}}(z_0)}{dz_0}\frac{z_0}{\sum_{t=0}^{\infty}(\beta(1-s))^t\,\mathbb{E}\left[f(z_t,\eta_t)-\bar{w}|z_0,\bar{\mathbf{a}}\right]}$$

$$\implies \frac{d\log\theta_0}{d\log z_0} = \frac{1}{\nu_0}\frac{z_0\sum_{t=0}^{\infty}(\beta(1-s))^t\,\mathbb{E}\left[f_z(z_t,\eta_t)|\bar{\mathbf{a}}(z_0)\right]\frac{\partial\mathbb{E}[z_t|z_0]}{\partial z_0}}{\sum_{t=0}^{\infty}(\beta(1-s))^t\,\mathbb{E}\left[f(z_t,\eta_t)-\bar{w}|z_0,\bar{\mathbf{a}}\right]}$$

as required by equation (21) from the main text, and where the second implication follows from totally differentiating equation (15).

To derive $d\ln J(z_0)/d\ln z_0$ in the flexible incentive pay economy, we seek to apply an envelope theorem. However, it is not trivial to show that an envelope theorem applies in our setting because the firm faces a continuum of constraints which may be non-convex. We therefore must show that the firm's problem may be re-written in a way that satisfies the conditions of an envelope theorem. We pursue two different strategies in service of this goal. First, we rewrite the firm's problem using recursive contracts and a first order approach (i.e., assuming that the incentive compatibility constraint may be summarized by the first order condition to the worker's problem). This approach relies on part (ii) of Assumption 1, but not part (iii). Our second strategy does not rely on the first order approach, and only requires that the set of feasible contracts is compact. This second strategy relies on part (iii) of Assumption 1, but not part (ii). Both strategies maintain part (i) of Assumption 1. Finally, after applying an envelope theorem, it is straightforward to derive the expression for the elasticity of market tightness in the flexible incentive pay economy with acyclical promised utility using Proposition 1.

A.3.1 Proof Strategy 1: First Order Approach and Recursive Formulation

The application of the envelope theorem proceeds in three steps in this strategy. First we derive a first-order approach to simplify incentive constraints into local incentive constraints as in Farhi and Werning (2013) or Pavan et al. (2014). Then we develop a recursive formulation of the problem. Finally, we use these constructions to prove our main theorem.

First Order Approach and Recursive Formulation The first order condition for a_t in the worker's problem (12) given a contract is

$$\int\int\left[u_a\Big(w_t(\eta^t,z^t),a_t(\eta^{t-1},z^t)\Big)\tilde{\pi}_t\left(\eta^t,z^t|z_0,\mathbf{a}\right)+u\Big(w_t(\eta^t,z^t),a_t(\eta^{t-1},z^t)\Big)\frac{\partial}{\partial a_t}\tilde{\pi}_t\left(\eta^t,z^t|z_0,\mathbf{a}\right)\right]d\eta^t dz^t$$

$$= 0$$

Note that this holds for every t and realization of z^t. Thus one can remove the outer integral to write first-order incentive constraints as

$$\int \left[u_a \bigg(w_t(\eta^t, z^t), a_t(\eta^{t-1}, z^t) \bigg) \pi_t \left(\eta_t | z^t, \eta^{t-1}, \mathbf{a} \right) + u \bigg(w_t(\eta^t, z^t), a_t(\eta^{t-1}, z^t) \bigg) \frac{\partial}{\partial a_t} \pi_t \left(\eta_t | z^t, \eta^{t-1}, \mathbf{a} \right) \right] d\eta_t = 0$$

We will work with the relaxed problem and develop a recursive formulation of the firm's problem. Notationally, let the value of some variable X in the period t problem be given by X, the value of X in $t-1$ be given as X_- and the value of X in $t+1$ be given by X'. Suppressing explicit dependence of the problem on initial productivity z_0 for notational convenience, the recursive formulation of the firm's problem is then (we drop the history dependence with the assumption that the process for η is a Markov process):

$$\begin{aligned} J(v_-, \eta_-, z_-, t) = \max_{a(\eta_-, z), w(\eta, z), v(\eta, z)} \int \int \Big[& f(\eta, z) - w(\eta, z) \\ & + \beta (1-s) J(v(\eta, z), \eta, z, t+1) \Big] \pi(\eta | z, \eta_-, a(\eta_-, z)) \hat{\pi}(z | z_-) d\eta dz \end{aligned} \tag{35}$$

subject to

$$\omega(\eta, z) = \quad u(w(\eta, z), a(\eta_-, z)) + \beta \left[(1-s) v(\eta, z) + s \textstyle\int U(z') \hat{\pi}(z' | z) dz' \right] \tag{36}$$

for all η, z and

$$[\lambda]: \quad v_- \leq \int \int \omega(\eta, z) \pi(\eta | z, \eta_-, a(\eta_-, z)) \hat{\pi}(z | z_-) d\eta dz \tag{37}$$

And the first-order incentive constraints:

$$\int \left[u_a \bigg(w(\eta, z), a(\eta_-, z) \bigg) \pi(\eta | z, \eta_-, a) + u \bigg(w(\eta, z), a(\eta_-, z) \bigg) \frac{\partial}{\partial a} \pi(\eta | z, \eta_-, a) \right] d\eta = 0 \tag{38}$$

We now explain this problem. The firm begins period t knowing the prior realization of shocks z_- and η_- and inherits a utility it must promise to the worker over the remaining life of the contract, which we denote v_-. The firm's flow profits are the expected output $f(\eta, z)$ minus their expected wage payments $w(\eta, z)$. Firms additionally receive a continuation value with probability $1-s$, which they discount at rate β. The firm maximizes the sum of flow profits and continuation values by choosing the suggested effort and wage functions for every realization of η and z, as well as a function for next period's promised utility to the worker $v(\eta, z)$, subject to some constraints that we now describe.

The worker's value under the contract given a realization (η, z) is given by $\omega(\eta, z)$ and defined in equation (36). It is equal to the worker's flow utility $u(w(\eta, z), a(\eta_-, z)$ plus a continuation value. With probability s, the match dissolves and the worker receives the value of unemployment. With probability $1-s$, the match survives and the worker's continuation value is $v(\eta, z)$.

The recursive version of the participation constraint states that the worker's expected value under the contract must be at least the value promised to them v, and is given by equation (37). Note that v_- in the initial period of the match maps to the utility promised to the worker overall $\mathcal{B}(z_0)$ in the non-recursive formulation of the problem. For periods after the start of the contract, equation (37) may be interpreted as a promise-keeping constraint. Equation (38) is the relaxed incentive constraint described above.

Towards An Envelope Theorem Let the Lagrangian of the recursive problem be defined by $\int \int \mathcal{L}(\cdot) d\eta dz$ for

$$\begin{aligned}
\mathcal{L} \equiv & [f(\eta, z) - w(\eta, z; z_0)] \pi(\eta | z, \eta_-, a(\eta_-, z)) \hat{\pi}(z|z_-) \\
& + \beta(1-s) \Big[J(v(\eta, z), \eta, z, t+1) \Big] \pi(\eta | z, \eta_-, a(\eta_-, z)) \hat{\pi}(z|z_-) \\
& - \lambda [v_- - \omega(\eta, z) \pi(\eta | z, \eta_-, a(\eta_-, z)) \hat{\pi}(z|z_-)] \\
& - \gamma(z) \left[u_a \Big(w(\eta, z), a(\eta_-, z) \Big) \pi(\eta | z, \eta_-, a) + u \Big(w(\eta, z), a(\eta_-, z) \Big) \frac{\partial}{\partial a} \pi(\eta | z, \eta_-, a) \right]
\end{aligned} \tag{39}$$

where λ is the Lagrange multiplier on the participation constraint and $\gamma(z)$ is the multiplier on the incentive constraint given aggregate productivity z. Again, we suppress dependence on z_0, but the firm's choice variables and the distribution of z and η may all depend on z_0.

Next, we introduce the change of variable with the notation $z_t = \mathbb{E}[z_t | z_0] + \varepsilon_t$, where by definition, ε_t is the cumulative innovation to the process for z between 0 and t and ε_0 is known to be 0. Given Assumption 1, one can write the Lagrangian as:

$$\begin{aligned}
\mathcal{L} = & [f(\eta, \mathbb{E}[z|z_0] + \varepsilon) - w(\eta, \varepsilon)] \pi(\eta | \varepsilon, \eta_-, a(\eta_-, \varepsilon)) \hat{\pi}(\varepsilon | \varepsilon_-) \\
& + \beta(1-s) \Big[J(v(\eta, \varepsilon), \eta, \varepsilon, t+1) \Big] \pi(\eta | \varepsilon, \eta_-, a(\eta_-, \varepsilon)) \hat{\pi}(\varepsilon | \varepsilon_-) \\
& - \lambda [v_- - \omega(\eta, \varepsilon) \pi(\eta | \varepsilon, \eta_-, a(\eta_-, \varepsilon)) \hat{\pi}(\varepsilon | \varepsilon_-)] \\
& - \gamma(\varepsilon) \left[u_a \Big(w(\eta, \varepsilon), a(\eta_-, \varepsilon) \Big) \pi(\eta | \varepsilon, \eta_-, a) + u \Big(w(\eta, \varepsilon), a(\eta_-, \varepsilon) \Big) \frac{\partial}{\partial a} \pi(\eta | \varepsilon, \eta_-, a) \right]
\end{aligned} \tag{40}$$

We impose a few additional technical assumptions that permit application of Theorem 1 of Marcet and Marimon (2019):

Technical Assumptions:

TA1. The set $\mathcal{X}$ of feasible allocations is convex, and f, u, π, u_a, and π_a are continuous functions of $\{a, w, z_0\}$

TA2. The constraint set $\Gamma(z_0) = \{(\mathbf{w}, \mathbf{a}) \in \mathcal{X} : G(a, w, ; z) \leq 0\}$ is compact for every $z \in Z$, a neighborhood of z_0, and there exists a contract $(\mathbf{w}, \mathbf{a})$ such that the participation constraint (37) is slack.

TA3. Let Ξ denote the space of realizations for η. The correspondence $\Gamma : Z \times \Xi \to \mathcal{X}$ is continuous and the optimal controls are unique.

These technical assumptions are non-stringent conditions for assumptions A1-A6 of Theorem 1 of Marcet and Marimon (2019)[40] to hold and sufficient conditions for uniqueness of saddle-point solution of the Lagrangian, which guarantees that the left and right derivatives of the Lagrangian are equal at z_0.

One can now apply the Envelope theorem of Marcet and Marimon (2019) to argue that the derivative of the value function with respect to all variables the firm chooses and co-states – a^*, w^*, v^*, λ^*, and γ^* – sum to zero. Therefore, differentiating the Lagrangian (40) with respect to z_0 and substituting in for $\omega(\eta, \varepsilon)$ yields:

$$\begin{aligned}\frac{\partial J(v, \eta_-, z_-, t)}{\partial z_0} &= \int\int \frac{\partial}{\partial z_0}[f\left(\eta, \mathbb{E}[z|z_0] + \varepsilon\right)]\pi\left(\eta|\varepsilon, \eta_-, a^*(\eta_-, \varepsilon)\right)\hat{\pi}(\varepsilon|\varepsilon_-)d\eta d\varepsilon \\ &+ \beta\left(1-s\right)\int\int \frac{\partial}{\partial z_0}\Big[J\left(v^*(\eta, \varepsilon), \eta, \varepsilon, t+1\right)\Big]\pi\left(\eta|\varepsilon, \eta_-, a^*\right)\hat{\pi}(\varepsilon|\varepsilon_-)d\eta d\varepsilon \\ &+ \beta s\lambda^*\int\int \frac{\partial}{\partial z_0}U\left(\mathbb{E}[z'|z_0] + \varepsilon'\right)\hat{\pi}\left(\varepsilon'|\varepsilon\right)\hat{\pi}(\varepsilon|\varepsilon_-)d\varepsilon' d\varepsilon\end{aligned} \tag{41}$$

This is essentially a refinement of a recursive version of Proposition 1: the first order impact of aggregate productivity on the value of a filled job is given by the sum of the direct effect on the firm's flow and continuation values, plus the direct effect on the constraints. Two terms are missing from the fuller decomposition in Proposition 1. First, the "B-term" has no direct effect of aggregate productivity on the incentive constraints. This arises from the assumption that the distribution of η and ε do not directly depend on z_0. Second, the "C-term" – the indirect effect on firm value that arises from changes in the contracted wages or effort – does not appear because we have written the problem in such a way that the envelope theorem of Marcet and Marimon (2019) applies.

[40] This is better suited for our purposes than Corollary 5 of Milgrom and Segal (2002) since it does not require compactness assumptions on the support of the shocks.

We can write explicitly the sequence of participation constraints from time 0 as:

$$\lambda_-(z_0): \qquad \mathcal{E}(z_0) \leq v$$

$$\lambda_{t-1}(\eta^{t-1}, z^{t-1}): \qquad v_{t-1}(\eta^{t-1}, z^{t-1}) \leq \int\int \omega(\eta^t, z^t)\pi\left(\eta_t|z^t, \eta^{t-1}, a(\eta^{t-1}, z^t)\right)\hat{\pi}(z_t|z^{t-1})d\eta_t dz_t, \ \forall t \geq 1$$

where

$$\omega(\eta^t, z^t) = \quad u\left(w(\eta^t, z^t), a(\eta^{t-1}, z^t)\right) + \beta\left[(1-s)\, v(\eta^t, z^t) + s\int U\left(z^{t+1}\right)\hat{\pi}\left(z_{t+1}|z^t\right)dz_{t+1}\right]$$

The corresponding sequential participation constraints are:

$$[\lambda_-(z_0)]: \quad \mathcal{B}\left(z_0\right) \leq \sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t\Bigg[\int\int u\Bigg(w_t(\eta^t, \varepsilon^t; z_0), a_t(\eta^{t-1}, \varepsilon^t; z_0)\Bigg)\tilde{\pi}_t\left(\eta^t, \varepsilon^t|\mathbf{a}(z_0)\right)d\eta^t d\varepsilon^t$$
$$+\beta s\int U\left(\mathbb{E}[z_{t+1}|z_0] + \varepsilon_{t+1}\right)\hat{\pi}_{t+1}\left(\varepsilon^{t+1}\right)d\varepsilon^{t+1}\Bigg]$$
$$[\lambda_\tau(\eta^\tau, \varepsilon^\tau; z_0)]: \quad \sum_{t=\tau+1}^{\infty}\left(\beta\left(1-s\right)\right)^{t-\tau-1}\Bigg[\int\int u\Bigg(w_t(\eta^t, \varepsilon^t; z_0), a_t(\eta^{t-1}, \varepsilon^t; z_0)\Bigg)\tilde{\pi}_t\left(\eta^t, \varepsilon^t|\mathbf{a}(z_0)\right)d\eta^t d\varepsilon^t$$
$$+\beta s\int U\left(\mathbb{E}[z_{t+1}|z_0] + \varepsilon_{t+1}\right)\hat{\pi}_{t+1}\left(\varepsilon^{t+1}\right)d\varepsilon^{t+1}\Bigg] \geq v_\tau(\eta^\tau, \varepsilon^\tau; z_0), \quad \forall\tau = 0, \ldots, +\infty \tag{42}$$

Now we apply the envelope theorem to the problem recursively, replacing $\mathcal{E}$ with its equilibrium value $\mathcal{B}$ to obtain

$$\frac{\partial J}{\partial z_0} = \sum_{t=0}^{+\infty}\Big[\int\int\left(\beta\left(1-s\right)\right)^t\frac{\partial f\left(\mathbb{E}[z_t|z_0] + \varepsilon_t, \eta_t\right)}{\partial z_0}\tilde{\pi}_t\left(\eta^t, \varepsilon^t|\mathbf{a}\left(z_0\right)\right)d\eta^t d\varepsilon^t \tag{43}$$
$$-\lambda_-(z_0)\Big[\frac{\partial\mathcal{B}(z_0)}{\partial z_0} - \beta s\sum_{t=1}^{+\infty}\int\int\left(\beta\left(1-s\right)\right)^{t-1}\frac{\partial U\left(\mathbb{E}[z_t|z_0] + \varepsilon_t\right)}{\partial z_0}\hat{\pi}\left(\varepsilon^t\right)d\varepsilon^t\Big]$$
$$+\sum_{\tau=0}^{\infty}\int\int\lambda_\tau(\eta^\tau, \varepsilon^\tau; z_0)\beta s\Big[\sum_{t=\tau+2}^{+\infty}\int\int\left(\beta\left(1-s\right)\right)^{t-\tau-2}\frac{\partial U\left(\mathbb{E}[z_t|z_0] + \varepsilon_t\right)}{\partial z_0}\hat{\pi}\left(\varepsilon^t|\varepsilon^\tau\right)d\varepsilon^t\Big]\times$$
$$\tilde{\pi}_\tau\left(\eta^\tau, \varepsilon^\tau|\mathbf{a}\left(z_0\right)\right)d\eta^\tau d\varepsilon^\tau$$

Now the get the final expression when the outside option of the worker is acyclical and TIOLI, we have:

$$\frac{\partial J\left(z_0\right)}{\partial z_0} = \sum_{t=0}^{\infty}\Big[\int\int\left(\beta\left(1-s\right)\right)^t\frac{\partial f\left(\mathbb{E}[z_t|z_0] + \varepsilon_t, \eta_t\right)}{\partial z_0}\tilde{\pi}_t\left(\eta^t, \varepsilon^t|\mathbf{a}^*\left(z_0\right)\right)d\eta^t d\varepsilon^t \tag{44}$$

as desired.

A.3.2 Proof Strategy 2: Sequence Problem

We seek to apply Theorem 4.13 of Bonnans and Shapiro (2000), which is reproduced below:

Bonnans and Shapiro (2000) Theorem 4.13 *Consider the following optimization problem:*

$$\min_{x \in \mathcal{X}} V(x, z) \quad \textit{subject to } x \in \Phi$$

where z is a member of a Banach space Z, $\mathcal{X}$ is a Hausdorff topological space, $\Phi \subset \mathcal{X}$ is nonempty and closed, and $V : \mathcal{X} \times Z \to \mathbb{R}$ is continuous. Let the value function be defined as

$$J(z) \equiv \inf_{x \in \Phi(z)} V(x, z)$$

and the optimal control set be given by

$$x^*(z) \equiv \arg \min_{x \in \Phi(z)} V(x, z).$$

Suppose that $z_0 \in Z$ and

1. *For all $x \in \mathcal{X}$ the function $V(x, \cdot)$ is Gateaux differentiable*
2. *$V(x, z)$ and its partial Fréchet derivative with respect to z, given by $D_z V(x, z)$, are continuous on $\mathcal{X} \times Z$*
3. *There exists $\alpha \in \mathbb{R}$ and a compact set $C \subset \mathcal{X}$ such that for every z near z_0 the set $A(z) \equiv \{x \in \Phi : V(x, z) \leq \alpha\}$ is non-empty and contained in C.*

Then the optimal value function $z(\cdot)$ is Fréchet directionally differentiable at z_0 and

$$J'(z_0, d) = \inf_{x \in x^*(z_0)} D_z V(x, z_0) d,$$

where d is the direction of the Fréchet derivative.

In words, this theorem provides conditions under which the total derivative of the value function with respect to some parameter z is equal to the partial derivative of the value function with respect to that parameter.[41] To prove our Lemma, we must verify the conditions of this theorem.

First, the space of possible aggregate productivities Z is clearly a Banach space, and the set of feasible contracts $\mathcal{X}$ is trivially a Hausdorff topological space. By Assumption 1 (i), Φ

[41] Note that we have altered the original notation of Bonnans and Shapiro (2000) so that it is consistent with our problem.

is non-empty. In addition, the firm's objective function $V(x,z)$ is continuous and Gateaux differentiable since effort is assumed to continuously influence the measure of idiosyncratic profit shocks η. So too is its partial Fréchet derivative.

Thus all that remains to verify is that: (i) the constraint set does not depend directly on z and (ii) condition 3 of the theorem of Bonnans & Shapiro holds. To verify that the constraint set does not depend directly on z, note that by inspection, the incentive constraints (12) do not depend on z. With take it or leave it wage offers and acyclical unemployment benefits, as in the assumption of the theorem, the participation constraint (13) simplifies to

$$[\mathbf{PC}] : \sum_{t=0}^{\infty} \left(\beta\left(1-s\right)\right)^{t} \Bigg[\int\int u\Big(w_t(\eta^t, z^t), a_t(\eta^{t-1}, z^t)\Big) \tilde{\pi}_t \left(\eta^t, z^t | z_0, \mathbf{a}\right) d\eta^t dz^t \\ + \beta s \int U \hat{\pi}_{t+1} \left(z^{t+1}|z_0\right) dz^{t+1} \Bigg] \geq \mathcal{E}, \tag{45}$$

where now, by assumption, U and $\mathcal{E}$ are independent of z. Likewise, the bounds on w and a in each date and state do not depend on z. Therefore z does not directly enter the participation constraint.

Since Φ is compact, as argued below in Lemma 5, we have now validated the conditions of Bonnans and Shapiro (2000) Theorem 4.13 and this envelope theorem applies to our problem. Doing so yields the following expression for the derivative of the firm's value function with respect to z evaluated at z_0:

$$\frac{dJ\left(z_0\right)}{dz_0} = \frac{\partial}{\partial z_0} \left[\sum_{t=0}^{\infty} \left(\beta\left(1-s\right)\right)^t \int \sum_{\eta^t \in \Xi^t} f(z_t, \eta_t) \tilde{\pi}_t \left(\eta^t, \varepsilon^t | \mathbf{a}^*\left(z_0\right)\right) d\varepsilon^t \right]. \tag{46}$$

Equation (21), the main statement in the theorem, arises by substituting equation (46) into equation (34).

Finally we can derive equation (22) from the main text, which studies deviations from aggregate steady states. Start from equation (21) from the main text and observe that in a neighborhood of a steady state $\bar{z}$, and if z_t is a random walk, then approximately

$$\begin{aligned}\frac{d\log\theta_0}{d\log z_0} &= \frac{1}{\bar{\nu}}\frac{\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t\mathbb{E}\left[\bar{z}f_z(\bar{z},\eta_t)|\bar{z},\mathbf{a}^*\left(\bar{z}\right)\right]\frac{\partial\mathbb{E}[z_t|z_0]}{\partial z_0}}{\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t\mathbb{E}\left[f(\bar{z},\eta_t)-w_t^*(\bar{z})|\bar{z},\mathbf{a}^*\left(\bar{z}\right)\right]}\\ &= \frac{1}{\bar{\nu}}\frac{\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t\mathbb{E}\left[f(\bar{z},\eta_t)|\bar{z},\mathbf{a}^*\left(\bar{z}\right)\right]}{\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t\mathbb{E}\left[f(\bar{z},\eta_t)-w_t^*(\bar{z})|\bar{z},\mathbf{a}^*\left(\bar{z}\right)\right]}\\ &= \frac{1}{\bar{\nu}}\frac{1}{1-\Lambda}\qquad\Lambda=\frac{\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t\mathbb{E}\left[w_t^*(\bar{z})|\bar{z},\mathbf{a}^*\left(\bar{z}\right)\right]}{\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t\mathbb{E}\left[f(\bar{z},\eta_t)|\bar{z},\mathbf{a}^*\left(\bar{z}\right)\right]}.\end{aligned}$$

This derivation completes the proof of Theorem 2. □

Lemma 5. *The space of feasible contracts that satisfy the IC constraints, Φ, is compact, if contracts are restricted to being continuous and twice differentiable in their arguments $\{\eta^t, z^t\}$, with uniformly bounded first and second derivatives.*

Proof. We will show that Φ is equicontinuous.[42] Consider a set of functions that are continuously differentiable on $[0,1]$ and such that both the functions and their first and second derivatives are uniformly bounded. This means there exists some real number M such that for every function f in the set and every $x \in [0,1]$, $|f(x)| \leq M$ and $|f'(x)| \leq M$.

Given $\epsilon > 0$, choose $\delta = \epsilon/2M$. Then for any function f in Φ and any points x and y in $\Xi \times Z$ such that $||x-y|| < \delta$, by the mean value theorem, we have $||f(x) - f(y)||_\infty = |f'(c)| \cdot ||x-y||$ for some c in the line $xt + (1-t)y$, $t \in [0;1]$. Since $|f'(c)| \leq M$ and $|x-y| < \delta = \epsilon/2M$, we get $||f(x) - f(y)||_\infty < \epsilon/2$.

Similarly we can apply the mean value theorem to the first derivative of f and since the second derivatives are bounded, an analogous argument to that above yields $||f'(x) - f'(y)||_\infty < \epsilon/2$. Therefore $||f(x) - f(y)||_{C^1} \equiv ||f(x) - f(y)||_\infty + ||f'(x) - f'(y)||_\infty < \epsilon$ and we have shown that Φ is equicontinuous. By the Ascoli Theorem, any sequence in Φ thus has a subsequence that converges. Therefore Φ is compact. □

A.4 Proof of Proposition 3

The first part of Proposition 3 is a direct result of substituting equation (25) into equation (24) and plugging in the expression for $\partial\mathcal{Y}(\mathbf{a}^*(z_0);z_0)/\partial z_0$. To prove the "moreover" statement that bargained wage cyclicality is positive if and only if promised utility is pro-cyclical, one need simply compare equation (26) with equation (19) to note that

[42] Φ is said to be equicontinuous at a point $x \in \Xi \times Z$ if, for every $\epsilon > 0$, there exists a $\delta > 0$ such that for every function f in Φ and every point y in $\Xi \times Z$, if $||x-y|| < \delta$ then $||f(x) - f(y)||_{C^1} < \epsilon$, where $||\cdot||_{C^1}$ is the C^1 norm.

$$\lambda^*_{PC}(z_0)\left[\frac{\partial \tilde{\mathcal{B}}(z_0)}{\partial z_0}\right] = \frac{\partial \mathcal{W}^{bargained}(z_0)}{\partial z_0}.$$

Since $\lambda^*_{PC}(z_0) > 0$, it must be that $\partial \mathcal{W}^{bargained}(z_0)/\partial z_0$ if and only if $\dfrac{\partial \tilde{\mathcal{B}}(z_0)}{\partial z_0} > 0$. $\square$

A.5 Proof of Proposition 4

The contracting environment is nearly identical to that of Edmans et al. (2012a) (without private savings), and the derivation of the optimal contract is thus very similar; therefore, we leave some of the technical details of the proof to that paper. First, note that as is standard in dynamic agency problems without private savings and separable preferences over consumption and effort (Rogerson, 1985; Farhi and Werning, 2013), an Inverse Euler Equation (IEE) holds. With log utility and given that we assume firms and workers share β as a common discount factor, the IEE in our setting reads

$$w_t(\eta^t, \mathbf{a}|z^t) = \mathbb{E}_t[w_{t+1}(\eta^{t+1}, \mathbf{a}|z^{t+1})]. \tag{47}$$

The inverse of the agent's discounted marginal utility — which is simply the wage in this case with log utility — is the marginal cost of delivering utility to the worker. Equation (47) states that the expected marginal cost of delivering utility to the worker is equalized across periods, otherwise the principal would deliver utility to the worker in relatively low cost periods. Note that this equation dictates that wages are a martingale process and implies that the optimal contract smooths worker consumption.

We begin by solving for the optimal difference wage schedule (27). To do so, we begin by considering a finite horizon contract, with duration T, and then take the limit as $T \to \infty$.

Differentiating worker's incentive compatibility constraint with respect to a_T (wiith binding local constraints) given realizations of η^T and z^T yields

$$\frac{1}{w_T(y^T, z^T)} \frac{\partial w_T(y^T, z^T)}{\partial a_T} = h'(a_T).$$

Since the firm cannot distinguish η_T from a_T, it must be the case that $\partial w_T/\partial \eta_T = \partial w_T/\partial a_T$. Substituting this into the above first order condition yields

$$\frac{1}{w_T(y^T, z^T)} \frac{\partial w_T(\eta^T, z^T)}{\partial \eta_T} = h'(a_T).$$

Fixing η^{T-1} and integrating over all possible realizations of η_T gives

$$\ln w_T(y^T, z^T) = h'(a_T)\eta_T + K^{T-1}(\eta^{T-1}, z^T). \tag{48}$$

That is, log wages are a linear function of realizations of η_T, plus some function of past output and z_t: $K^{T-1}(\eta^{T-1}, z^T)$. This immediately implies

$$\frac{\partial \ln w_T(y^T, z^T)}{\partial \eta_{T-1}} = \frac{\partial K^{T-1}(\eta^{T-1}, z^T)}{\partial \eta_{T-1}}. \tag{49}$$

Likewise, a binding period $T-1$ incentive constraint implies

$$\frac{1}{w_{T-1}(y^{T-1}, z^{T-1})}\frac{\partial w_{T-1}(\eta^{T-1}, z^{T-1})}{\partial \eta_{T-1}} + \frac{\beta(1-s)}{w_T(y^T, z^T)}\frac{\partial w_T(\eta^T, z^T)}{\partial \eta_{T-1}} = h'(a_{T-1}).$$

Using (49), fixing η_{T-2}, and once again integrating with respect to η_{T-1} gives

$$\ln w_{T-1}(y^{T-1}, z^{T-1}) = h'(a_{T-1})\eta_{T-1} + K^{T-2}(\eta^{T-2}, z^{T-1}) - \beta(1-s)K^{T-1}(\eta^{T-1}, z^T). \tag{50}$$

Since the IEE implies that wages are a martingale, we must have

$$e^{h'(a_{T-1})\eta_{T-1}} e^{K^{T-2}(\eta^{T-2}, z^{T-1})} e^{-\beta(1-s)K^{T-1}(\eta^{T-1}, z^T)} = e^{K^{T-1}(\eta^{T-1}, z^T)}\mathbb{E}_{T-1}\left[e^{h'(a_T)\eta_T}\right] \tag{51}$$

Taking logs, using properties of the normal distribution, and simplifying yields

$$(1+\beta(1-s))K^{T-1}(\eta^{T-1}, z^T) = h'(a_{T-1})\eta_{T-1} + K^{T-2}(\eta^{T-2}, z^{T-1}) - \frac{(\sigma_\eta h'(a_T))^2}{2} \tag{52}$$

Thus, $K^{T-1}(\eta^{T-1}, z^T)$ (and thus workers' realized utility) is linear in η_{T-1}. It can be verified that utility in each period is a linear function of the performance shock in every past period. Substituting equation (52) into equation (50) gives

$$K^{T-1}(\eta^{T-1}, z^T) = \ln w_{T-1}(y^{T-1}, z^{T-1}) - \frac{(\sigma_\eta h'(a_T))^2}{2}. \tag{53}$$

Substituting this expression for $K^{T-1}(\eta^{T-1}, z^T)$ into equation (48) gives

$$\ln w_T = \ln w_{T-1} + h'(a_T)\eta_T - \frac{(\sigma_\eta h'(a_T))^2}{2}.$$

Pursuing a similar strategy, it can be verified that

$$\ln w_t = \ln w_{t-1} + \psi_t h'(a_t)\eta_t - \frac{(\psi_t \sigma_\eta h'(a_t))^2}{2} \tag{54}$$

for all $t \leq T$, where $\psi_t \equiv \left(\sum_{\tau=0}^{T-t} (\beta(1-s))^\tau \right)^{-1}$. Taking the limit of equation (54) as $T \to \infty$ yields equation (27), resulting in a constant sensitivity $\psi_t \equiv \psi = 1 - \beta(1-s)$ of log wages to idiosyncratic output shocks.

To solve for the constant $w_{-1}(z_0)$ that initializes this difference equation, note that free entry into vacancy posting requires that the firm's expected profits from posting vacancies must be zero if a positive measure of vacancies are posted in equilibrium. This implies that

$$\sum_{t=0}^{\infty} (\beta(1-s))^t \mathbb{E}[z_t a_t^*(z_t) - w_t^*(\eta^t, z^t)|z_0] = \frac{\kappa}{q(\theta_0)}.$$

However, wages are a martingale process ($\mathbb{E}[w_t^*(\cdot)|z_0] = \mathbb{E}[w_0^*(\cdot)|z_0]$), so that

$$\frac{\mathbb{E}[w_0^*(\cdot)|z_0]}{1-\beta(1-s)} = \sum_{t=0}^{\infty} (\beta(1-s))^t \mathbb{E}[z_t a_t^*(z_t)|z_0] - \frac{\kappa}{q(\theta_0)}.$$

From the definitions of $\mathcal{Y}(\mathbf{a}^*(z_0); z_0)$ and ψ, we obtain the following expression for $w_{-1}(z_0)$

$$w_{-1}(z_0) = \psi \left(\mathcal{Y}(\mathbf{a}^*(z_0); z_0) - \frac{\kappa}{q(\theta_0)} \right). \tag{55}$$

Cumulating equation (27) then yields the following expression for the log wage at time t:

$$\log w_t(a_t, \eta^t | z^t) = \log w_{-1}(z_0) + \sum_{s=0}^{t} \psi h'(a_s)\eta_s - \frac{1}{2} \sum_{s=0}^{t} (\psi h'(a_s)\sigma_\eta).^2 \tag{56}$$

The worker's utility under the contract is equal to the expected present discounted value (EPDV) of log wage payments minus the EPDV of disutility from effort, plus the continuation value should the worker separate to unemployment. First, let us focus on characterizing the worker's expected lifetime utility from consumption. From equation (56), we have

$$E_0 \left[\sum_{t=0}^{\infty} (\beta(1-s))^t \log(w_t(\eta^t, z^t | \mathbf{a}(z^t))|z_0 \right] = \frac{1}{\psi} \log w_{-1}(z_0) - E_0 \left[\sum_{t=0}^{\infty} (\beta(1-s))^t \frac{1}{2} \sum_{\tau=0}^{t} (\psi h'(a_\tau)\sigma_\eta)^2 \right],$$

where the second term on the right hand side can be simplified as

$$
\begin{aligned}
E_0\left[\sum_{t=0}^{\infty}(\beta(1-s))^t\frac{1}{2}\sum_{\tau=0}^{t}(\psi h'(a_\tau)\sigma_\eta)^2\right] &= \frac{1}{E}_0\left[\sum_{t=0}^{\infty}\sum_{\tau=t}^{\infty}(\beta(1-s))^\tau\frac{1}{2}(\psi h'(a_t)\sigma_\eta)^2\right]\\
&= E_0\left[\sum_{t=0}^{\infty}\frac{1}{2}(\psi h'(a_t)\sigma_\eta)^2\sum_{\tau=t}^{\infty}(\beta(1-s))^t(\beta(1-s))^{\tau-t}\right]\\
&= \frac{1}{2}E_0\left[\sum_{t=0}^{\infty}(\beta(1-s))^t(\psi h'(a_t)\sigma_\eta)^2\sum_{\tau=t}^{\infty}(\beta(1-s))^{\tau-t}\right]\\
&= \frac{1}{2\psi}E_0\left[\sum_{t=0}^{\infty}(\beta(1-s))^t(\psi h'(a_t)\sigma_\eta)^2\right]. \qquad (57)
\end{aligned}
$$

Note that the worker will be paid a higher expected wage if they exert a higher effort. Subtracting off the disutility of effort and adding the continuation value of separating to unemployment, the value to the worker of the contract is therefore

$$
\mathcal{E}(z_0) = \frac{1}{\psi}\log w_{-1}(z_0) - E_0\left[\sum_{t=0}^{\infty}(\beta(1-s))^t\left(\frac{1}{2\psi}(\psi h'(a_t)\sigma_\eta)^2 + h(a_t(z_t)) - \beta s U(z_{t+1})\right)\right]. \qquad (58)
$$

Given that the firm makes take it or leave it offers, $\mathcal{E}(z_0)$ is equated to the value of unemployment $U(z_0)$ in equilibrium. This observation yields equation (29).

All that remains is to derive the optimal effort choice $a_t(z_t)$. Following Edmans et al. (2012a), we assume that this effort choice does not vary with η_t. Taking the first order condition of equation (58) with respect to $a_t(z_t)$ yields

$$
\frac{1}{\psi}\frac{d\log w_{-1}(z_0)}{da_t(z_t)} - \beta(1-s)^t\left(h'(a_t(z_t)) + \psi\sigma_\eta^2 h'(a_t)h''(a_t)\right) = 0
$$

Substituting in using the assumed expression for $h(a)$ and equation (55) and suppressing dependence of a_t on z_t for notational simplicity:

$$
\frac{1}{\psi}\frac{z_t}{\mathcal{Y}(\mathbf{a}^*(z_0);z_0) - \frac{\kappa}{q(\theta_0)}} - a_t^{1/\epsilon} - \epsilon\psi\sigma_\eta^2 h'(a_t)a_t^{\frac{1-\epsilon}{\epsilon}} = 0
$$

Multiplying by a_t and rearranging terms yields

$$
a_t^{\frac{\epsilon+1}{\epsilon}} = \frac{1}{\psi}\frac{z_t a_t}{\mathcal{Y}(\mathbf{a}^*(z_0);z_0) - \frac{\kappa}{q(\theta_0)}} - \epsilon\psi(\sigma_\eta h'(a_t))^2
$$

Raising this equation to the power $\epsilon/(1+\epsilon)$ yields equation (30) as desired. $\square$

A.6 A Model with Endogenous Separations

A.6.1 Economic Environment

This section introduces efficient endogenous separations into the baseline environment. To economize, we only discuss the parts of the model that change due to efficient separations. Where not stated, the model is the same as the flexible incentive pay economy of the main text.

Labor Market As in the baseline model of the main text, a large measure of risk-neutral firms match with workers and produce output. A unit mass of workers is either employed or unemployed and searching for a job. Let n_t denote the measure of employed workers at the start of period t, while $u_t \equiv 1 - n_t$ is the measure of unemployed workers looking for jobs. Fluctuations in labor market variables are driven by technology, which follows a Markov process $\{z_t\}_{t=0}^{\infty}$ with lower and upper bounds $\underline{z}$ and $\overline{z}$. We will denote the history of this Markov process until t by $\{z^t\} = \{z_0, ..., z_t\}$.

Firms post v_t vacancies to recruit unemployed workers. The number of matches made in period t is given by a constant returns matching function $m(u_t, v_t)$; conditions are summarized by market tightness $\theta_t = v_t/u_t$, with a job finding rate $\phi(\theta_t) = m(u_t, v_t)/u_t$ and a vacancy filling rate $q_t \equiv q(\theta_t) = m(u_t, v_t)/v_t$. Let $\nu_t \equiv d\log q_t/d\log\theta_t$ denote the period t elasticity of the job-filling rate with respect to θ_t. Keeping a vacancy open has a flow cost κ.

At the end of period $t-1$ an endogenous fraction s_t of workers separate from employment and enter unemployment. The unemployed search for new jobs, so u_t evolves as

$$u_t = u_{t-1} + s_t(1 - u_{t-1}) - \phi(\theta_{t-1})u_{t-1}. \tag{59}$$

Preferences and Consumption Workers' preferences are identical to the model of the main text, we omit a description for brevity.

Firms and Wage Setting Firms are risk neutral and maximize expected profit with discount factor β. Consider a firm that successfully matches with a worker at time 0 and starts producing in the same period. The firm's output in period t is $y_{it} = f(z_t, \eta_{it})$ where f is strictly increasing and differentiable in all of its arguments and η_{it} is an idiosyncratic shock to the firm's output. Henceforth, we omit i subscripts to ease notation.

At the beginning of the period, before the current value of η_t is realized, the worker exerts effort a_t that affects the distribution of idiosyncratic shocks. We assume a general process for η_t, which allows for arbitrary persistence, and depends both on aggregate shocks and the worker's effort. The process has bounds $\underline{\eta}$ and $\overline{\eta}$. Define a history of idiosyncratic shocks $\eta^t = \{\eta_0, ..., \eta_t\}$ and assume that η_0 is fixed. We characterize the process for η_t by a probability measure $\pi_t\left(\eta_t|\eta^{t-1}, a^t\right)$, which gives the probability of realizing η_t given the history η^{t-1} of past idiosyncratic shocks, the history of past and present aggregate shocks z^t and the worker's history of actions $a^t = \{a_0, \ldots, a_t\}$. Let the marginal distribution of the history of aggregate productivity shocks through time t be $\hat{\pi}_t(z^t|z_0)$, and assume that z_t is Markovian, so that we can characterize its law of motion by $\pi\left(z_{t+1}|z_t\right)$, the one-step-ahead probability.

The value of a firm of posting a vacancy at time 0 is then

$$\Pi_0 = q(\theta_0)\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\Pi_{j=1}^t\left(1-s_j\right)\left(f\left(z_t,\eta_t\right)-w_t\right)\right]-\kappa, \tag{60}$$

where $\mathbb{E}_0$ conditions on the firm's information set at time 0 prior to meeting a worker. If a firm meets a worker, the value to the firm is the expected present value of the difference between production and wage payments, discounted by the firm's discount factor β as well as separation risk. Here, $\Pi_{j=1}^t\left(1-s_j\right)$ is the endogenous probability that a match survives until period t, which cumulates the probability $1-s_j$ that a match survives period j.

A vacancy is filled with probability $q(\theta)$ and costs κ, yielding the above net vacancy value. Free entry guarantees that this value is zero in equilibrium. We entertain two possibilities for wage setting.

Flexible Incentive Pay Economy The firm observes the initial value of z_0 and all realizations of aggregate shocks $\{z_t\}_{t=0}^{\infty}$. Firms additionally observe idiosyncratic shocks η_t in every period of the match. However, they do not observe workers' effort a_t. They thus cannot observe whether output is high because the worker exerted high effort or received a lucky idiosyncratic shock. For notational reasons, we assume "action before noise" timing, so the worker must choose their effort before the realization of the idiosyncratic shock to firm output.

When a firm and worker meet, the firm offers the worker a contract to incentivize effort and maximize firm value. However the firm now has the additional option to vary the probability that the match separates in each date and state. For instance, if the expected present value of profits has turned negative, the firm may choose to terminate the contract. Thus the contract may be summarized by functions $w_t(\eta^t, z^t) \in [\underline{w}, \bar{w}]$, $a_t(\eta^{t-1}, z^t) \in [\underline{a}, \bar{a}]$ and a separation probability $s_t\left(\eta^t, z^t\right) \in [0, 1]$ for all t and all realizations of η^t and z^t. Let $(\mathbf{w}, \mathbf{a}, \mathbf{s})$ denote a contract, with $\mathbf{w} \equiv \{w_t(\eta^t, z^t)\}_{t=0,\eta^t,z^t}^{\infty}$, $\mathbf{a} \equiv \{a_t(\eta^{t-1}, z^t)\}_{t=0,\eta^{t-1},z^t}^{\infty}$ and

$\mathbf{s} \equiv \{s_t(\eta^t, z^t)\}_{t=0,\eta^t,z^t}^{\infty}$. Let $\mathcal{X}$ denote the space of feasible contracts.

Value of a Filled Vacancy. Under the contract $(\mathbf{w}, \mathbf{a}, \mathbf{s})$, and initial productivity z_0, the firm's expected present value of profits from posting a vacancy is

$$V(\mathbf{w}, \mathbf{a}, \mathbf{s}; z_0) = \sum_{t=0}^{\infty} \int \int \beta^t \mathcal{S}_t(\eta^t, z^t) \left(f(z_t, \eta_t) - w_t(\eta^t, z^t)\right) \tilde{\pi}_t\left(\eta^t, z^t | z_0, \mathbf{a}\right) d\eta^t dz^t, \quad (61)$$

where $\mathcal{S}_t(\eta^t, z^t) \equiv \Pi_{j=1}^t (1 - s_{t-j}(\eta^{t-j}, z^{t-j}))$ is the probability that a match survives the sequence η^t, z^t; and $\tilde{\pi}_t(\eta^t, z^t|z_0, \mathbf{a}) \equiv \prod_{\tau=0}^t \pi_\tau(\eta_\tau|\eta^{\tau-1}, a_\tau(\eta^{\tau-1}, z^\tau)) \hat{\pi}_\tau(z^\tau|z_0)$ is the probability of observing a realization of η^t and z^t given the initial z_0 and the contracted effort function $\mathbf{a}$. Firms' flow profits are the difference between output and wages. The firm forms an expectation over flow profit realizations by integrating over the distribution of both aggregate and idiosyncratic shock distributions, the latter of which depends on effort. The risk-neutral firm discounts period t profits by the economy-wide discount rate β^t and the probability $\Pi_{j=1}^t (1 - s_j)$ that the match survives t periods.

The contract maximizes the value of a filled vacancy

$$J(z_0) = \max_{\{w_t(\eta^t, z^t), a_t(\eta^{t-1}, z^t), s_t(\eta^t, z^t)\}_{t=0,\eta^t,z^t}^{\infty} \in \mathcal{X}} V(\mathbf{w}, \mathbf{a}, \mathbf{s}; z_0) \quad (62)$$

subject to the incentive (IC) and participation (PC) constraints described below.

Incentive Constraints. The incentive compatibility condition is similar to the main text, but now accounts for endogenous separation risk

$$\begin{aligned}[\mathbf{IC}] : \mathbf{a} \in \underset{\{\tilde{a}_t(\eta^{t-1}, z^t)\}_{t=0,\eta^t,z^t}^{\infty}}{\operatorname{argmax}} \sum_{t=0}^{\infty} \Bigg[\int \int \beta^t \mathcal{S}_t(\eta^t, z^t) \Bigg[u\Big(w_t(\eta^t, z^t), \tilde{a}_t(\eta^{t-1}, z^t)\Big) - \psi(s(\eta^t, z^t)) \\ + \beta s(\eta^t, z^t) \Bigg] \int U(z_{t+1}) \pi(z_{t+1}|z_t) dz^{t+1} \Bigg] \tilde{\pi}_t\left(\eta^t, z^t | z_0, \tilde{\mathbf{a}}\right) d\eta^t dz^t., \end{aligned} \quad (63)$$

where $\psi(s_j)$ represents a convex utility cost of searching for a new job.

Participation Constraint. Likewise, the participation constraint must also account for separation risk, and becomes

$$\begin{aligned}[\mathbf{PC}] : \sum_{t=0}^{\infty} \Bigg[\int \int \beta^t \mathcal{S}_t(\eta^t, z^t) \Bigg[u\Big(w_t(\eta^t, z^t), \tilde{a}_t(\eta^{t-1}, z^t)\Big) - \psi(s(\eta^t, z^t)) \\ + \beta s(\eta^t, z^t) \Bigg] \int U(z_{t+1}) \pi(z_{t+1}|z_t) dz^{t+1} \Bigg] \tilde{\pi}_t\left(\eta^t, z^t | z_0, \tilde{\mathbf{a}}\right) d\eta^t dz^t \Bigg] \geq \mathcal{E}(z_0). \end{aligned} \quad (64)$$

Bargaining and promised utility. To close the flexible incentive pay economy, we must determine promised utility $\mathcal{E}(z_0)$, which we again assume is given by a reduced form "bargaining schedule" $\mathcal{B}(z_0)$.

Rigid Wage Economy The rigid wage economy is identical to the rigid wage economy of the main text, including the assumption of an exogenous separation rate s.

Equilibrium Given initial unemployment u_0 and a stochastic process $\{z_t, \eta_t\}_{t=0}^{\infty}$, an equilibrium is a collection of stochastic processes $\{\theta_t, u_t\}_{t=0}^{\infty}$ and functions $J(z), U(z), \mathcal{E}(z)$, and $(\mathbf{w}, \mathbf{a})$ such that for all firms: (i) θ_t satisfies the free entry condition so that Π_t, given in equation (60), is equal to 0 for all t; (ii) u_t satisfies the law of motion for unemployment (59); (iii) wage and effort functions $(\mathbf{w}, \mathbf{a}, \mathbf{s})$ satisfy the flexible incentive pay economy equations (62)-(64), or $w_t = \bar{w}$, $a_t = \bar{a}$ and $s_t = s$ in the rigid wage economy; (iv) the value of unemployment $U(z)$ is defined in the same way as the main text; (v) the value of employment is defined the same way as the main text for the in the rigid wage economy, or $\mathcal{E}(z) = \mathcal{B}(z)$ in the flexible incentive pay economy; and (vi) the value of a filled vacancy $J(z)$ is given by (62) in the flexible incentive pay economy or the same way as the main text for the rigid wage economy.

A.6.2 Equivalence of Rigid and Incentive Pay with Endogenous Separations

This subsection shows that, without bargaining power, the first order response of market tightness is the same in the rigid wage economy, and the flexible incentive pay economy with endogenous separations.

Proposition 6. *Assume that the set of feasible contracts that satisfies the incentive constraints (63) and the participation constraint (64) is non-empty and compact. Also assume that the production function is homogeneous of degree 1 in aggregate productivity z and z_t is a driftless random walk. Finally, assume that the firm makes take it or leave it offers to workers and the flow value of unemployment is constant. Then the impact elasticity of market tightness to shocks to z_t is*

$$\frac{d \log \theta_0}{d \log z_0} = \frac{1}{\bar{\nu}} \frac{1}{1 - \Lambda} \tag{65}$$

where Λ is the steady state labor share defined as

$$\Lambda \equiv \frac{\sum_{t=0}^{\infty} \mathbb{E}_0 \beta^t \Pi_{j=1}^{t} \left(1 - s_j^*\right) w_t^*}{\sum_{t=0}^{\infty} \mathbb{E}_0 \beta^t \Pi_{j=1}^{t} \left(1 - s_j^*\right) f\left(\bar{z}, \eta_t\right)}$$

where s_j^* and w_t^* denote choices of separations and wages along the optimal contract, where the expectation $\mathbb{E}_0$ is evaluated along the optimal contract, and $\bar{z}$ is the value of z_t at the aggregate steady state.

This theorem shows that the flexible incentive pay economy, with endogenous separations, has as an equivalent response of tightness on impact to the rigid wage economy of the main text. Note that the dynamics of the rigid wage economy are still given by equation (22). Therefore incentive wage cyclicality does not affect the impact dynamics of tightness with endogenous separations. Again, the flexible incentive pay economy and the rigid wage economy must be calibrated to the same steady state labor shares. In the incentive pay economy with endogenous separations, the labor share depends on the optimal choice of separation rates, as well as the factors from the model of the main text such as wages and effort.

We stress that this result leads to equivalence for impact elasticities, as (Pissarides, 2009) discusses. In general the response of tightness to labor productivity shocks after impact will be different in the rigid wage and flexible incentive pay economies, because endogenous separations leads to additional dynamics of unemployment after the impact of the shock.

Let us briefly explain the intuition for why endogenous separations do not alter the main message of our results. Intuitively, in the model with efficient endogenous separations, separations are an additional choice of the firm, over which the firm can optimize. However, changes in the optimal separation choice after TFP shocks have no first order effect on profits—just as neither changes in optimally chosen effort nor wages affect profits. This logic is again due to the envelope theorem.

A.6.3 Proof of Proposition 6

The free entry condition in the flexible incentive pay economy is

$$\frac{\kappa}{q\left(\theta\right)} = J\left(z_0\right),$$

where $J\left(z_0\right)$ is defined in equation (62). Taking derivatives and rearranging implies

$$\begin{aligned}\frac{d\log\theta_0}{d\log z_0} &= \frac{1}{\nu_0}\frac{d\log J\left(z_0\right)}{d\log z_0}\\ &= \frac{1}{\nu_0}\frac{z_0}{J\left(z_0\right)}\frac{dJ\left(z_0\right)}{dz_0}.\end{aligned} \tag{66}$$

With ψ convex, the conditions are ripe for the optimal separation rates s_j^* to be interior. Under the assumptions of the proposition, z_0 enters neither the incentive constraints nor the participation constraints directly. Therefore we have

$$\begin{aligned}
\frac{dJ\left(z_0\right)}{dz_0} &= \frac{\partial J\left(z_0\right)}{\partial z_0} \\
&= \frac{\partial}{\partial z_0}\sum_{t=0}^{\infty}\int\int \beta^t \mathcal{S}_t\left(\eta^t, z^t\right)\left(f(z_t,\eta_t) - w_t(\eta^t, z^t)\right)\tilde{\pi}_t\left(\eta^t, z^t|z_0, \mathbf{a}\right)d\eta^t dz^t \\
&= \frac{\partial}{\partial z_0}\sum_{t=0}^{\infty}\mathbb{E}_0\beta^t\Pi_{j=1}^t\left(1-s_j^*\right)\left(f(z_t,\eta_t) - w_t\right) \\
&= \sum_{t=0}^{\infty}\mathbb{E}_0\beta^t\Pi_{j=1}^t\left(1-s_j^*\right)\left(f_z(z_t,\eta_t)\frac{\partial z_t}{\partial z_0}\right) \\
&= \sum_{t=0}^{\infty}\mathbb{E}_0\beta^t\Pi_{j=1}^t\left(1-s_j^*\right)\left(f_z(z_t,\eta_t)\right) \qquad (67)
\end{aligned}$$

where the first equality invokes the envelope theorem, using the same argument as Appendix Section A.3.2; the second argument substitutes in the definition of profits from equation (61), and exploits that terms involving the participation or incentive constraints vanish because z_0 does not enter them directly; the third equality rewrites using the notation from the theorem; and the final equality uses that z_t is a random walk.

Substituting in equations (66) and (67) implies

$$\begin{aligned}
\frac{d\log\theta_0}{d\log z_0} &= \frac{1}{\nu_0}\frac{z_0}{J\left(z_0\right)}\frac{dJ\left(z_0\right)}{dz_0} \\
&= \frac{1}{\nu_0}\frac{z_0\sum_{t=0}^{\infty}\mathbb{E}_0\beta^t\Pi_{j=1}^t\left(1-s_j^*\right)\left(f_z(z_t,\eta_t)\right)}{\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\Pi_{j=1}^t\left(1-s_j^*\right)\left(f\left(z_t,\eta_t\right)-w_t^*\right)\right]} \\
&= \frac{1}{\nu_0}\frac{\bar{z}\sum_{t=0}^{\infty}\mathbb{E}_0\beta^t\Pi_{j=1}^t\left(1-s_j^*\right)\left(f_z(\bar{z},\eta_t)\right)}{\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\Pi_{j=1}^t\left(1-s_j^*\right)\left(f\left(\bar{z},\eta_t\right)-w_t^*\right)\right]} \\
&= \frac{1}{\nu_0}\frac{\sum_{t=0}^{\infty}\mathbb{E}_0\beta^t\Pi_{j=1}^t\left(1-s_j^*\right)\left(f(\bar{z},\eta_t)\right)}{\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\Pi_{j=1}^t\left(1-s_j^*\right)\left(f\left(\bar{z},\eta_t\right)-w_t^*\right)\right]} \\
&= \frac{1}{\nu_0}\frac{1}{1-\frac{\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\Pi_{j=1}^t\left(1-s_j^*\right)w_t^*\right]}{\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\Pi_{j=1}^t\left(1-s_j^*\right)\left(f(\bar{z},\eta_t)\right)\right]}},
\end{aligned}$$

where we use the assumption of an aggregate steady state in $\bar{z}$. We have derived equation (65) of proposition 6. □

B Numerical Appendix

B.1 Preliminaries

We calibrate the model such that t represents a month. Specifically, we set the discount rate β to $0.99^{1/3}$, the vacancy creation cost to 0.45 and employ a matching function given by $m(u,v) = uv(u^\iota + v^\iota)^{-1/\iota}$ so that $q(\theta) = (1+\theta^\iota)^{1/\iota}$, which is bounded between 0 and 1. We set $\iota = 0.9$ by a nonlinear least squares approach to match the empirical relationship between aggregate market tightness and job-finding rates. We set the exogenous separation rate $s = 0.031$ as the average monthly separation rate in the Current Population Survey (CPS) from 1951 to 2019. This implies that the pass-through parameter $\psi = 0.033$. Separation rates and job-finding rates are both adjusted for time aggregation following Shimer (2005). We compute market tightness in the data from monthly household unemployment in the CPS and vacancies from the Job Openings and Labor Turnover Survey (JOLTS) from 2000 to 2019 (JOLTS begins in 2000).

We discretize the AR(1) productivity process for $\log z_t$ onto a finite grid: $z \in \mathcal{Z} = [\underline{z}, ..., \overline{z}]$ following Rouwenhorst (1995). We set the number of gridpoints to 13.

We rewrite the key equations in our quantitative model recursively, given the Markovian structure for productivity. Let $\pi(z'|z)$ denote the probability of aggregate productivity transitioning from z to z'. Recall that optimal effort, given z_0 and current z, satisfies

$$a\left(z|z_0\right) = \left[\frac{za\left(z|z_0\right)}{\psi\left(Y\left(z_0\right) - \frac{\kappa}{q(\theta(z_0))}\right)} - \frac{\psi}{\epsilon}\left(h'\left(a\left(z\right)\right)\sigma_\eta\right)^2\right]^{\frac{\epsilon}{1+\epsilon}}.$$

Let $\tilde{Y}(z|z_0)$ denote the EPDV of future output, conditional on effort $a(\cdot|z_0)$ and current z, given by

$$\tilde{Y}\left(z|z_0\right) = za\left(z|z_0\right) + \sum_{z'\in\mathcal{Z}} \beta\left(1-s\right)\tilde{Y}\left(z'|z_0\right)\right)\pi\left(z'|z\right).$$

It follows that $Y(z_0) = \tilde{Y}(z_0|z_0)$. Note that the optimal effort depends on z_0 through $Y(z_0)$ and $\theta(z_0)$, which are both equilibrium objects. Define the worker's expected present discounted utility from starting work at z_0, taking as given the effort schedule $a(\cdot|z)$ and the wage schedule $w(\cdot|z)$, defined in Section 4:

$$\tilde{\mathcal{E}}(z_0|z) = \frac{1}{\psi}\log w_{-1}(z) + \mathbb{E}_z\Bigg[-\sum_{k=0}^{\infty}[\beta(1-s)]^k \frac{1}{\psi}\frac{1}{2}(\psi h'(a(z_k|z_0))\sigma_\eta)^2 - \\ \sum_{k=0}^{\infty}[\beta(1-s)]^k h(a(z_k|z_0)) + \sum_{k=0}^{\infty}[\beta(1-s)]^k \beta s\omega(z_{k+1})\Bigg],$$

where

$$\omega(z) = \mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t \log b(z_t) \mid z_0 = z\right].$$

It follows that $\mathcal{E}(z_0) = \tilde{\mathcal{E}}(z_0|z_0)$. It is helpful to define the term in brackets in the expression for $\tilde{\mathcal{E}}(z_0|z)$ recursively as

$$W(z_0|z) = -\frac{1}{\psi}\frac{1}{2}(\psi h'(a(z_0|z))\sigma_\eta)^2 - h(a(z_0|z)) + \sum_{z'\in\mathcal{Z}}\beta s\omega(z')\pi(z'|z_0) + \sum_{z'\in\mathcal{Z}}\beta(1-s)W(z'|z)\pi(z'|z_0).$$

Finally, we define an implicit, auxiliary function for effort $\tilde{a}$ with arguments z, $\tilde{Y}$, and $\tilde{q}$ (ignoring dependence on z_0) that will be useful when solving the model numerically:

$$\tilde{a}\left(z, \tilde{Y}, \tilde{q}\right) = \left[\frac{z\tilde{a}}{\psi\left(\tilde{Y} - \kappa/\tilde{q}\right)} - \frac{\psi}{\epsilon}\left(h'(\tilde{a})\sigma_\eta\right)^2\right]^{\frac{\epsilon}{1+\epsilon}}.$$ [43]

B.2 Algorithm to solve for the optimal contract, given z_0

Fix an initial $z_0 \in \mathcal{Z}$.

1. Set $n = 1$. Set $\underline{q}^n = 0$, and $\bar{q}^n = 1$.
2. Set $q^n(z_0) = \frac{1}{2}(\underline{q}^n + \bar{q}^n)$.
3. Set $k = 1$. Make initial guess for $Y^{k,n}(z|z_0)$ for $z \in \mathcal{Z}$.
4. Update $Y^{k+1,n}(\cdot|z_0)$ as

$$Y^{k+1,n}(z|z_0) = z\tilde{a}\left(z, Y^{k,n}(z|z_0), q^n(z_0)\right) + \sum_{z'\in\mathcal{Z}}\beta(1-s)Y^{k,n}(z'|z_0)\pi(z'|z)$$

5. Repeat (4) until $\|Y^{k,n+1}(\cdot|z_0) - Y^{k,n}(\cdot|z_0)\| < \delta_1$ for some small tolerance $\delta_1 > 0$. Define the object $Y^n(z_0) = Y^{k,n}(z_0|z_0)$. Define $\tilde{a}^n(z) = \tilde{a}(z, Y^n(z_0), q^n(z_0))$.

[43] For general ϵ, we numerically solve for a_t using a root-finder, restricting attention to positive roots.

6. Solve for $w_{-1}^{n}(z_0)$ using the free entry condition

$$w_{-1}^{n}(z_0) = \psi\left(Y^{n}(z_0) - \frac{\kappa}{q^{n}(z_0)}\right).$$

7. Set $j = 1$. Make initial guess for $W^{j,n}(z_0|z)$.

8. Update $W^{j+1,n}(\cdot|z_0)$ as

$$W^{j+1,n}(z|z_0) = -\frac{1}{\psi}\frac{1}{2}(\psi h'(\tilde{a}^{n}(z))\sigma_\eta)^2 - h(\tilde{a}^{n}(z)) + \sum_{z'\in\mathcal{Z}} \beta s \omega(z')\pi(z'|z) + \sum_{z'\in\mathcal{Z}} \beta(1-s) W^{j,n}(z'|z)\pi(z'|z)$$

9. Repeat (8) until $\|W^{j,n+1}(\cdot|z_0) - W^{j,n}(\cdot|z_0)\| < \delta_2$ for some small tolerance $\delta_2 > 0$. Define $\mathcal{E}^{n}(z_0) = \frac{1}{\psi}\log w_{-1}^{n}(z_0) + W^{j,n}(z_0|z_0)$.

10. If $\mathcal{E}^{n}(z_0) > \omega(z_0)$ then set $\bar{q}^{n+1} = q^{n}(z_0)$. If $\mathcal{E}^{n}(z_0) < \omega(z_0)$, then set $\underline{q}^{n+1} = q^{n}(z_0)$. Recall that with TIOLI offers, $\mathcal{E}(z_0) = \omega(z_0)$. Note that $\omega(z_0)$ can be computed by a simple value function iteration.

11. Repeat steps (2)-(10) until $|\mathcal{E}^{n}(z_0) - \omega(z_0)| < \delta_3$ for some small tolerance $\delta_3 > 0$.

We repeat this procedure for all possible values of $z_0 \in \mathcal{Z}$ to obtain the equilbrium objects $Y(z_0)$, $w_{-1}(z_0)$, and $a(\cdot|z_0)$. Finally, define $\theta(z_0) = q^{-1}(q^{n}(z_0))$, where $q(\theta) = \frac{1}{(1+\theta^{\iota})^{1/\iota}}$.

B.3 Additional Details on Simulation

Our set of targeted moments includes two moments that depend on within-contract, idiosyncratic realizations: the standard deviation of annual (YoY) wage growth (std($\Delta \log w_{it}$)) and the pass-through from idiosyncratic shocks to firm profits to wages ($\partial \log w_{it}/\partial \log y_{it}$), and two moments which can be computed from aggregate time series simulated in the model: the cyclicality of new hire wages ($\partial \mathbb{E}[\log w_0]/\partial u$) and average unemployment ($\bar{u}_t$). To compute these moments for a given set of parameters $\Omega := \{\epsilon, \sigma_\eta, \chi, \gamma\}$, we solve the model for each initial $z_0 \in \mathcal{Z}$ following the procedure outlined in Section B.2 to obtain $a(\cdot|z_0)$, $w_{-1}(z_0)$, and $\theta(z_0)$. We simulate the economy with aggregate shocks and compute all moments in a stochastic economy, as opposed to in a non-stochastic steady state.

B.3.1 Simulating std($\Delta \log w_{it}$) and ${}^{\partial \log w_{it}}/_{\partial \log y_{it}}$

We simulate a panel of $I = 10,000$ idiosyncratic η_{it} shocks of length $T = 2,500$ (and one sequence of aggregate z_t shocks of length T).[44] For each period t and worker i, we simulate separations and job-finding shocks consistent with the exogenous probability of separation s and endogenous job-finding probability $\phi(\theta(z_t))$.[45] All workers are employed at the beginning of $t = 0$. During job spells and given realizations of z_t and η_{it}, we can compute log wages and the pass-through for each worker according to the equations derived in Section 4. For job spells that last at least 13 months, we can compute YoY log wage growth as $\log w_{i,t+12} - \log w_{it}$ (for each year of employment). We then compute the pooled variance of YoY log wage growth and the average monthly pass-through across all job spells/periods of employment for $t \geq t_{\text{burn-in}}$ (we discard the first $t_{\text{burn-in}} = 1,000$ periods as a burn-in period). Note that cross-sectional and longitudinal data on job spells/periods of employment (job-stayers) are interchangeable in this setting.

B.4 Simulating ${}^{\partial \mathbb{E}[\log w_0]}/_{\partial u}$ and $\bar{u}_t$

We simulate 10,000 z_t sequences of length $T = 1,828$ periods (corresponding to a burn-in period of 1,000 periods, and monthly observations for the 1951-2019 period). For each z_t path, we can compute the path for unemployment as

$$u_{t+1} = u_t + s(1 - u_t) - \phi(\theta(z_t))u_t{}^{46},$$

given initial condition $u_0 = 0.06$. The expected log wage of new hire wages are

$$\mathbb{E}[\log w_0](z_t) = \log w_{-1}(z_t) - \frac{1}{2}(\psi h'(a(z_t|z_t))\sigma_\eta)^2.$$

We compute $\bar{u}_t$ as the average unemployment u_t for $t \geq t_{\text{burn-in}}$. We measure ${}^{\partial \mathbb{E}[\log w_0]}/_{\partial u}$ in the model by running an OLS regression of $\mathbb{E}[\log w_0](z_t)$ on u_t in the simulated data for $t \geq t_{\text{burn-in}}$. We report cross-simulation averages for both moments.

[44]We restrict the simulations of z_t, so that $\log z_t$ always lies within three unconditional standard deviations of μ_z. We still solve the model on a potentially larger grid for greater accuracy.

[45]This procedure includes composition effects of initial z_0 on the employment contracts, and computed moments are pooled across all job spells.

[46]For simplicitiy, we omit the $s(f(\theta(z_t))u_t$ term in all of our numerical results, i.e. we do not allow for $U \to E \to U$ transitions within t and $t + 1$.

B.5 Estimation Algorithm

We implement the Tik-Tak algorithm, a multi-start global optimization algorithm, as described by Arnoud et al. (2019), to minimize the following objective function

$$J(\Omega) = (\tilde{m}(\Omega) - m)' W(m) (\tilde{m}(\Omega) - m),$$

where Ω contains the parameters to be estimated, $\tilde{m}(\Omega)$ are the targeted moments computed using the model simulated data given the parameter vector Ω, and m is the vector of targeted moments in the actual data. The weight matrix W is set such that $W_{j,j} = |1/m_j|$ for each targeted moment j (and 0, otherwise). Thus, the objective function to minimize is the sum of squared percentage differences between simulated and empirical moments to account for differences in scale between the targeted moments. We have experimented with different derivative-free local optimization algorithms, such as BOBYQA and the Nelder-Mead Simplex Algorithm, for the local optimization step. All estimation results reported in the paper correspond to solutions obtained using the BOBYQA algorithm from the NLopt Library with 1,000 initial points. In practice, we also implement a pre-testing stage to detect "promising" regions of the parameter space by evaluating the objective function at 50,000 initial points drawn from Sobol sequences; we select the 1,000 points that yield the lowest values of the objective function as the initial points in the global search stage.

B.6 Calculating Bargained Wage Cyclicality

Bargained wage cyclicality reflects fluctuations in the "B-term" of Proposition 1: that is movements in the promised utility of workers. For a given calibration, we calculate how the value of a filled job moves with exogenous productivity $dJ(z_0)/dz_0$. The "direct effect" of z_0 on the expected present discounted value of profits per worker, given the AR(1) process for $\log z$, can be approximated as

$$\frac{dJ(z_0)}{dz_0}^{Direct} = \sum_{t=0}^{\infty} (\beta(1-s))^t \frac{\mathbb{E}_0[a^*(z_t)\rho^t z_t]}{z_0}$$

that is, the direct effect is the effect that z has on profits holding fixed the optimally contracted choice of effort and wages. Following Proposition 1, we calculate bargained wage cyclicality – the "B-term" – as

$$\text{BWC}(z_0) = \frac{dJ(z_0)}{dz_0} - \frac{dJ(z_0)}{dz_0}^{Direct}.$$

The share of wage fluctuations attributable to bargaining is then the negative of BWC(z_0) divided by the cyclicality of the expected present discounted value of wage payments ${}^{d\mathcal{W}^*(z_0)}/_{dz_0}$.

B.7 Robustness: Cyclicality of New Hire Wages

In Tables B1 and 4, we report estimation results for different target values of the cyclicality of new hire wages. Each column corresponds to a re-calibration of the model.

Table B1: Varying cyclicality of new hire wages: estimated parameters

	${}^{\partial\mathbb{E}[\log w_0]}/_{du}$ target					
Parameter	-0.25	-0.5	-0.75	-1.0	-1.25	-1.5
σ_η	0.522	0.531	0.534	0.536	0.535	0.534
χ	0.148	0.232	0.435	0.516	0.587	0.651
γ	0.752	0.501	0.523	0.474	0.467	0.491
ϵ	0.311	1.830	1.530	2.385	2.580	2.040