Clean Growth*

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Abstract

We provide a spatial theory of clean growth to assess the global impact of the rise of renewable energy. We model the details of the combined production and transmission network of electricity ("the grid") that determine the supply and losses of energy in space. The local rate of clean energy adoption depends on learning-by-doing, the global electricity and trade network, and regional comparative advantage in renewable resources. To quantify the contribution of renewable adoption to global growth, we collect and harmonize global data on transmission lines, power stations, trade, and regional output. We use the model to measure the aggregate and spatial implications of clean growth, as well as the role of the Inflation Reduction Act in affecting the transition.

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1 Introduction

The arrival of cheap renewable technologies is reshaping global energy use, power production, and trade. Beginning from virtually zero in the mid 2000’s, the share of global electricity produced by new solar and wind technologies has begun to rise rapidly (see top panel of Figure 1). Underpinning this trend, costs for renewables have fallen precipitously (see bottom panel of Figure 1). Wind and solar are now the among the cheapest forms of energy in many parts of the world (Roser (2020)). Furthermore, costs are expected to continue to fall with overall capacity installation, a trend the literature has attributed to technical progress and “learning by doing”.

These technological trends provide several opportunities. To begin with, they make decarbonization goals much more achievable than thought even a decade ago. Beyond this, they promise substantially cheaper energy. Governments have recently directed substantial resources to decarbonizing electricity grids and encouraging electrification. But what is the role for policy in shaping these technological trends? What are the gains from clean energy subsidies, and grid investment?

To answer these questions, we build a theory of clean growth. We take into account both the continued cost declines for renewables, and the core spatial nature of the renewable transition. To do so, we embed the details of the combined production and transmission of electricity (“the grid”) in a spatial growth model. We then use data on the global transmission grid, renewable energy potential, and existing fossil assets to quantify the impact of renewables across regions and sectors.

We start in Section 2 by modeling the details of the electricity production network. Electrical power can be generated from fossil or renewable assets in each region. We use the physical properties of the electrical grid to develop a model of transmission and losses of electricity in space, and analyze the net supply of electricity of each region. The demand for electricity is determined by a multi-sector Armington trade model, as in Armington (1969) and Anderson (1979). Prices of electricity are set for a set of regions by a local utility planer depending on the supply and demand for electricity in each node. We show that in equilibrium, the prices chosen by the utility planner are unique, and the corresponding allocations are efficient.

In Section 3, we incorporate this setup in a model of investment in renewable assets. Clean growth is driven by learning by doing in renewable capital. Local investment in renewable technology depends on local electricity prices now and in the future. The speed of adoption in a given location is determined by the spatial productivity of the renewable capital, which depends on local renewable potential, local economic conditions, and the structure of the electricity grid. As more

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1In Appendix A2 we show that the decline of the investment costs for renewables has rivaled declines for computers and software. In particular, since the introduction of the first commercial silicon-based photovoltaic panel by Bell laboratories in 1956 for use in the satellite industry (see e.g. Perlin (1999) Chapter 6), photovoltaic costs have been declining at an average rate of about 15% per year.

2See, for example, McDonald and Schrattenholzer (2001), Van Benthem et al. (2008), Baker et al. (2013), Rubin et al. (2015), Newbery (2018), Roser (2020).
Figure 1: Progress in Clean Energy

(a) Renewable Energy Adoption

(b) Solar Energy and Onshore Wind Energy Costs

Notes: The rop panels plot the share of a country’s total electricity usage that comes from solar or wind. The right panel plots the levelized cost of energy from a new solar or wind energy project across countries. Data comes from the International Renewable Energy Agency.
renewable capital is installed globally, investment prices fall continually due to global spillovers from learning by doing. This pushes down wholesale power prices on the electricity network, and drives increases in output.

To create a quantitative portrait of the transition to renewable energy, we also incorporate fossil capital investment. In addition, we model the supply of fossil fuels as arising from a dynamic fossil fuel extraction problem. This allows us to measure the income losses faced by fossil-rich countries. Finally, we assume that renewable installation may need to be paired with batteries to deal with intermittency, or face curtailment, as their electricity output is variable over both daily and seasonal horizons.

In Section 4, we complement the theory with several newly constructed datasets, harmonized at the regional level. We use a comprehensive database of the world’s power plants to measure current fossil and renewable capital. We incorporate information on renewable potential at a very fine spatial disaggregation for solar and wind technologies. Finally, we construct a new database on global electricity transmission networks and their connections.

In Section 5, we use our quantitative model to trace out the implications of clean growth for sectoral specialization and productivity growth across the globe. We find that, in our baseline case with no subsidies, renewables quickly dominate the electric grid, rising to over 80% of world power output by 2040. Their adoption leads to dramatic falls in power prices, which fall on average by over 50% in the same time. The subsequent increase in industrial output raises welfare by an average of 4.6%, though with substantial heterogeneity across the globe. The need for storage slows the transition, and in a medium storage scenario this renewables account for 50% of world power output by 2040. We additionally compare the path of the model to a world in which technical progress in renewables is exogenous under a range of scenarios.

Lastly, we consider the role of policy in shaping the transition. In particular, we model the effects of the Inflation Reduction Act (IRA) passed by the Biden administration in 2022. Among other things, this included a substantial production tax credit for new renewable energy of $26 per mWh. We incorporate this subsidy into the model and solve for the induced change in the energy transition relative to our baseline.

We find that the IRA significantly hastens the adoption of renewable energy in the US, increasing renewable penetration by around 13% by 2030. It also has global spillovers, as the increased investment drives capital prices lower, increasing adoption in other countries around the world. The budgetary cost is substantial, coming in close to the budgeted cost of $160 billion. Much of the subsidy goes to inframarginal investment, which would have been installed in the absence of the IRA. Nonetheless, even without considering any benefits of carbon pollution reduction, we find that the IRA more than pays for itself.
Overall, the picture that emerges from our analysis is of a world undergoing rapid, beneficial change. Under all scenarios we consider, world power production is likely to shift fairly quickly to a grid dominated by renewables, bringing with it cheaper power and greater industrial output. Importantly, this conclusion holds when we conduct our analysis under a fixed transmission grid. As we discuss below, the modular nature of renewables and the speed of ongoing cost falls means that significant welfare gains are likely even without additional investment in grid infrastructure. In ongoing work, we consider the role of improving transmission linkages in further hastening clean growth.

Related Literature. A large literature in energy economics has built multi-sector integrated assessment models to evaluate the medium and long-run macroeconomic effects of renewable adaption based on Computable General Equilibrium models (see Baker et al. (2013) for a review). Although we incorporate a comprehensive, multi-sector depiction of the economy, our examination takes place at the regional level, which is crucial due to the regional differences in electricity generation and efficiency of renewable energy. We furthermore contribute to this literature by considering the details of the electricity grid, modeling electricity produced using various alternative sources, the centralized pricing of electricity, renewable and fossil assets regional accumulation, and modeling the extraction problem. Lastly, but importantly, our analysis inherits the welfare-theoretical rigor and the relative tractability of modern quantitative trade and spatial models.

We contribute to the literature analyzing economic growth and the growth of the renewables sector and energy in particular. As pointed out by Stern (2019), “the core mainstream economic growth models disregard energy or other resources”. There are, however, some growth models that incorporate energy usage. For example, Tahvonen and Salo (2001) and Fröling (2011) provide a theoretical analysis of the use of renewable energy in a unified growth framework. Acemoglu et al. (2012) model the endogenous adoption and development of clean vs dirty inputs into production in a directed technical change framework (see also Acemoglu et al. (2014)). Relative to these papers, we model the physical properties of electrical transmission and generation, and stress the spatial dimension of the analysis, given the varying productivity of renewables across space and the

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3At the same time, energy usage has been recently the topic of work in industrial organization (see, for example, Asker et al., 2019, Hodgson, 2019). Other academic work, such as Golosov et al. (2014); Kortum and Weisbach (2021); Krusell and Smith Jr (2022) consider optimal taxation of fossil fuels and policies taxing CO2 emissions. Alvarez and Rossi-Hansberg (2021) model the implications of climate change for economic allocations across space with the presence of an energy sector (see also Conte et al. (2021)).

4Our paper is related to the integrated assessment model of Bohringer and Loschel (2006). They adapt a “bottom-up” approach to model the electricity sector wherein energy source selections are modeled as a mixed complementarity problem within a Computable General Equilibrium framework. We instead model the choice of energy sources as a “merit order dispatch” common to power networks in that power is scheduled from different sources in order of increasing marginal cost and used depending on overall energy needs.

5See Redding and Rossi-Hansberg, 2017 and Costinot and Rodríguez-Clare, 2014 for in-depth reviews of quantitative spatial and trade models. For example, an interesting feature of our setup is that given electricity prices, determined by the grid, it retains several of the properties of spatial models and allows for the use of the standard techniques of solving for the equilibrium and model counterfactuals (see e.g. Allen and Arkolakis (2014)). Likewise, we show how to tractably model investment dynamics in a multi-region, multi-sector setup by exploiting the modularity of renewable investment and assuming free entry into renewable investment (see Walsh (2021) for a similar approach).
need to transfer energy from regions with abundant renewable resources to heavily populated or industrialized ones.

Finally, a significant literature in development has empirically evaluated the effects of electrification on productivity either in developing countries or in the context of historical development, see for example Rud, 2012; Moretti, 2013; Allcott et al., 2016; Moneke, 2020; Fiszbein et al., 2020. In a regional model, electrification has been modeled as an exogenous increase in productivity as in Kline and Moretti, 2014; Moneke, 2020. Instead, our approach emphasizes the endogenous effects that access to electricity has on output through energy prices, and integrates recent developments in the quantitative spatial and trade literature with models that incorporate supply and demand of energy (Nordhaus et al., 1973; Kypreos and Bahn, 2003; Edelhofer et al., 2013). To these latter contributions we add a model of the electricity grid and renewable or traditional power assets. In addition, we deal with intermittency of renewables by allowing renewable investment be paired with batteries, or face curtailment. Lastly, due to the challenges involved in modeling the energy transition at a global level, we abstract from many interesting features of the microeconomics of energy markets and renewable integration; see Wolak (2021) and Tangeras and Wolak (2021).

2 Setup

We consider an electrical grid with $J$ nodes. Each node could be the location of an electricity generator, a location where other economic activity takes place and consumes power, or both. We use bold notation to denote vectors and bold with a bar to denote matrices. We start with the description of the electrical grid, the production of electricity, the associated electricity losses, and the power planning problem that determines electricity prices in each region. Subsequently, we embed this setup in a standard spatial economy setup.

2.1 The Electrical Grid

Each node has a cost of generation $M_j(Y_j^c)$, where $Y_j^c$ is power produced in node $j \in J$. The maximum output of node $j$ is denoted by $Y_j^{max}$. Each location has demand for power $D_j(p_j^E)$, where $p_j^E$ is the price of power, dictated by trade, aggregate demand and supply of labor, which will be determined by our spatial equilibrium setup. We denote by $P$ the vector of dimension $J - 1$ and its element $P_j$ to be the net power output of node $j$, such that

$$P_j = Y_j^c - D_j. \tag{1}$$

$^6$We assume that the demand for energy is continuously differentiable and decreasing in $p_j$, and diverges as $p_j \to 0$. We show that these assumptions hold from the energy demand function resulting from the quantitative trade model of Section 2.5.
We clarify below how the power in $j^{th}$ node (also called the swing bus) is determined.

The electrical grid consists of power lines that connect a subset of the power nodes. This grid is represented by a matrix $\bar{A}$ of dimension $K \times (J-1)$, with an entry of 1 in the $(k,j)$ cell and $-1$ in the $(k,i)$ cell if node $j$ is connected to node $i$ via line $k$. $K$ is the total number of lines. We consider only bilateral links. Power flows on all lines are represented by the vector $Z$ with element $Z_k$. Flows can take on positive or negative values; if $Z_k$ is positive it indicates power flowing from node $j$ through line $k$ to $i$. Each line also has a maximum amount of power that can flow on it, the transmission capacity, which we denote as $Z_{k}^{\text{max}}$, such that $|Z_k| \leq Z_{k}^{\text{max}}$. Lastly, each line has a resistance $R_k$ and an inductance $X_k$. Both are determined by physical properties of the line, including the line length and materials: Resistance measures how strongly a line resists electrical current flowing through it, while inductance is relevant for AC power networks and measures how strongly a line resists changes in electrical current.

2.2 Electricity Production

Electricity production in a node depends on the amount of power capital installed in that node. Power capital in a node $j$ is owned by the agents who live in the region $j$. It comes in two forms: renewable ($\mathcal{R}$) and fossil fuel ($\mathcal{F}$), with amounts $K_j^\mathcal{R}$ and $K_j^\mathcal{F}$, respectively. Renewable capital produces

$$Y_j^\mathcal{R} = \theta_j^\mathcal{R}K_j^\mathcal{R}$$

units of electricity per period, where $\theta_j$ is a local renewable potential. That is, renewable capital produces power with no inputs, and at zero marginal cost. There are two types of renewable capital: solar and wind, $\mathcal{R} = \{S,W\}$, that differ only in their local potential $\theta_j^\mathcal{R}$. Figure 2 plots a representation of this potential across the globe.\(^7\)

Fossil fuel capital instead requires the use of fossil fuel inputs, the total use of which in $j$ is denoted $F_j$. Producing $Y_j^\mathcal{F}$ units of power requires

$$F_j^\mathcal{F} = G(K_j^\mathcal{F}, Y_j^\mathcal{F})$$

units of fossil fuels, where $G$ is continuous and strictly increasing in $Y_j^\mathcal{F}$, and for fixed capital $K_j^\mathcal{F}$ is convex in $Y_j^\mathcal{F}$. We denote the total electricity production as $Y_j^\mathcal{E}$ so that

$$Y_j^\mathcal{E} = \sum_{\mathcal{R} = \{S,W\}} Y_j^\mathcal{R} + Y_j^\mathcal{F}. \quad (3)$$

\(^7\)We consider hydro and nuclear, with their large fixed costs and lack of recent technological progress, to be part of fossil fuel capital. We are primarily interested in modeling power prices, and not emissions. In reality, solar and wind differ also in their intermittency; solar is intermittent at diurnal frequencies, wind is variable over weekly and seasonal horizons. We address intermittency of these technologies below in Section 2.4.

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Figure 2: Global Renewable Potential

(a) Solar Potential

Notes: Panel (a) shows solar panel potential (defined as daily kWh per peak kilowatt rating of the panel) using data from the Global Solar Atlas. Panel (b) shows wind electricity potential (average daily wind speeds) using data from the Global Wind Atlas.
Fossil fuels are traded globally for a price $p^F$ in units of a final numeraire good (specified below), and can be bought anywhere for the same price. The total cost of producing power $Y^F_j$ is

$$M_j(Y^F_j) = \begin{cases} p^F G_j(K^F_j, Y^F_j - \sum_{R=(S,W)} \theta^R_j K^R_j) & \text{if } Y^F_j > \sum_{R} \theta^R_j K^R_j \\ 0 & \text{if } Y^F_j \leq \sum_{R} \theta^R_j K^R_j \end{cases}$$

This formulation reflects the process of “merit order dispatch” common to power networks: the lowest marginal cost power stations are dispatched to supply the grid (the operation of which is described below), and then power is scheduled in order of increasing marginal cost. The availability of renewables has significant effects on merit order dispatch prices, as they typically have much lower marginal costs than traditional forms of energy—see e.g. Woo et al. (2016); Clo et al. (2015) for California and Italy, respectively—.

Lastly, there is a maximum fossil fuel input $F^\text{max}_j$, such that maximum power production in a node is

$$Y^F_j \leq Y^\text{max}_j$$

This captures the notion that power stations have a rated maximum power capacity. Generation capacities are readily available across the world, and we will use them for our calibration in Section 4.

### 2.3 Electricity Losses

The distribution of electric power across the lines in the grid incurs losses. However, flows cannot be allocated to a line. They are determined from Kirchhoff’s laws and the amount of generation and consumption in each node (see Decker (2021)) and must at all times satisfy the energy balance constraint

$$\sum_j D_j + \lambda = \sum_j Y^F_j$$

so that total energy used, plus total systems losses $\lambda$, is equal to that produced across all power assets.

Based on these properties of the grid and the power flow, we show in Appendix A5 that we can approximate the loss in each line in the grid as $\lambda_k = R_k Z_k^2$. The total transmission loss function is a function of all of these flows, denoted $\lambda(Z)$ and is given by

$$\lambda(Z) = \sum_k R_k Z_k^2.$$
power in each line and the power nodes can be written as

\[ Z = \bar{\Omega} \bar{A}(\bar{A}' \bar{\Omega} \bar{A})^{-1}P \]  

(6)

where \( \bar{\Omega} \) is a \( K \times K \) diagonal matrix of “line admittances” such that \( \Omega_k = (R_k^2 + X_k^2)^{-\frac{1}{2}} \) (i.e. its elements are inversely related to resistance \( R_k \) and inductance \( X_k \)). We assume for the remainder of the paper that inductance and resistance are proportional.

Now since line losses are given by (5), we can express losses in terms of the characteristics of the network and the power generated by each asset according to the following lemma:

**Lemma 1.** Power losses in the system can be expressed in a quadratic form:

\[ \lambda = P' \bar{B} P \]  

(7)

where

\[ \bar{B} = (\bar{A}' \bar{R}^{-1} \bar{A})^{-1} \]  

(8)

is a positive semi-definite \((J - 1) \times (J - 1)\) matrix of fundamentals and \( \bar{R} \) is a \( K \times K \) diagonal matrix of line resistances.

This representation of losses — its elegance notwithstanding — is useful for two purposes. First, it allows us to express the relation between losses in the network and power generation across nodes in terms of the measurable features of the network summarized in the matrix \( \bar{B} \) in equation (8) uses information for the power network links and their associated resistances. Given the net output power of each node, this information is a sufficient statistic for the losses in each line. Second, the fact that \( \bar{B} \) is positive semi-definite is important in allowing us to characterize the problem of the power network planning problem.

### 2.4 The Power Network Planning Problem

We assume the presence of a centralized power network administrator over a collection of connected nodes \( T \). This administrator aims to maximize economy wide output net of generation costs by setting local electricity prices, while balancing supply and demand of electricity.\(^8\) For example, in the continental United States the power grid is made up of three main interconnected transmission networks: the Eastern Interconnection encompasses the area east of the Rocky Mountains and a portion of the Texas panhandle, the Western Interconnection encompasses the area to the west of

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\(^8\)We refer to the “power network administrator” as being the same entity as the “system operator”. However, these two entities are typically separate, as is the case in the European Union and United States (Decker (2021)). Oftentimes, multiple system operators set electricity prices within a single grid network managed by a power network administrator. In our theory, we can consider multiple system operators within a grid as a simple extension, so long as we assume that electricity trade among operators takes place in predetermined contracted prices.
Figure 3: The United States Transmission Grid

Notes: This Figure shows the high voltage transmission network of the United States as well as the interconnections between the networks using data from the U.S. Energy Information Administration. While there are a few small interconnections between the three networks, they are effectively independent.

the Rocky Mountains and the Electric Reliability Council of Texas (ERCOT) that covers most of Texas. While there are minimal connections between these three transmission networks, the power capacity of these connections is very small and we treat these networks as independent. Thus, the power grid mechanics described above will hold for each of these networks separately. In Figure 3 we depict the entire continental United States network, and the areas covered by the three power transmission networks are shaded with a different grey.

We assume that the network administrators have an objective for each location \( Q_j(p_j^F) \) which is strictly decreasing in the price of that location, \( p_j^F \). We put further structure on this objective below, but keep it general here for exposition. As above, \( M_j \) is the cost of generating energy output \( Y_j^E \) in node \( j \), in units of the final good. The problem of the administrator is to choose prices \( \{p_j^F\} \) in the nodes that it administers in order to solve

\[
\max_{\{p_j^F\}} \sum_j Q_j(p_j^F) - \sum_j M_j(Y_j^E(p_j^F)) \tag{9}
\]

subject to the net power production in each power node equation (1), the energy balance constraint in equation (4), equation (7) that determines the losses in the network, the power in each link equation (6), and the network constraint for each link, \( Z_k \leq Z_k^{max} \).

We assume that any profits or losses of the electricity network are rebated uniformly to the con-
sumers that inhabit in the administrative boundaries of each transmission network. While it is simple to show that the price of electricity is always weakly higher in nodes that are net users of power, it does not follow that the electricity network always makes a positive profit, since they must absorb transmission losses. The associated profits or losses from the operation of the electricity network in a region $j$ belonging to the transmission network $T$ are denoted by

$$\Pi_j^T = \sum_{i \in T} p_i^e \left( D_i - Y_i^e \right) \frac{L_j}{\sum_{i \in T} L_i}$$

where $L_j$ is population of region $j$, and $D_i$ is total power demand in region $i$.

**Intermittency.** So far we have ignored the intermittency of electricity production of renewable capital, which is a central issue in the uptake of this technology. The output of fossil fuel capital is largely predictable and can be scheduled and controlled in advance (severe weather events such as the Texas blackout of 2021 notwithstanding). This is not so true for solar and wind energy sources.

There are two dimensions to this intermittency. First, there is a strong seasonal component to electricity produced from renewable capital, which is a central issue in the uptake of this technology. The output of fossil fuel capital is largely predictable and can be scheduled and controlled in advance (severe weather events such as the Texas blackout of 2021 notwithstanding). This is not so true for solar and wind energy sources.

Second, renewable electricity production has marked variation on a daily time scale, fluctuating substantially from day to day for both wind and solar capital (depending on wind speeds and cloud cover/night respectively). Due to the complexity of doing so at a world scale, we avoid modeling the stochastic scheduling problem of each utility directly, which involves setting variable wholesale prices across different periods of the day in response to both predicted and observed weather patterns. This variation in daily prices provides substantial opportunities for battery arbitrage as battery prices continue to fall (Nykvist and Nilsson, 2015). While stand-alone merchant batteries are beginning to be observed, direct pairing with renewable generation assets is also likely.

In our calibration, we address intermittency by modeling simple scenarios for the adoption of batteries, avoiding modeling the dynamic stochastic problem of storage directly. We assume each unit of renewable capital must be paired with battery storage in a range of capacities. We also assume battery storage costs also follow a learning by doing process, in which their production costs fall as output expands. The baseline case assumes zero battery storage requirements. In addition, we calibrate curtailment as a function of renewable penetration for different countries in the world, using engineering estimates of the amount of curtailment of renewable production because of supply and demand temporal mismatch from Shaner et al. (2018); Tong et al. (2021).
2.5 A Spatial Economy with Power

We embed the energy sector in a dynamic multi-region, multi-sector spatial economy. The consumer has Cobb-Douglas preferences over goods from different sectors $s$,

$$C_j = \prod_s C_{js}^\beta_{js}$$

where $\beta_{js}$ is the consumption share of region $j$ in sector $s$.

For each sector $s$ each region $j$ produces a differentiated good as in Armington (1969) and these goods are aggregated through a standard constant elasticity aggregator with an elasticity across varieties $\sigma_s$. Labor is fully mobile across sectors within the regions, and we denote the wage rate in region $j$ by $w_j$.

The production of the differentiated good for the amount the regional good $j$ is given by

$$q_{js} = z_{js} e_{js}^1 \left( k_{js} \left( e_{js}^2 \frac{\psi}{\theta - \psi} + f_{js} \frac{\psi^2 - 1}{\theta - \psi} k_{js}^{-\psi} l_{js}^{1-\psi} \right) \right)$$

where for each region $j$, sector $s$, and time $t$, $z_{js}$ is a regional productivity shifter, $L_{js}$ is the amount of labor used, $e_{js}^1$ is the direct amount of electricity used in production, and $e_{js}^2$ is secondary use of electricity that is directly substitutable with the additional use of fossil fuels $f_{js}$. The production capital is denoted by $k_{js}$ and the total production capital in a region is denoted by $K_j^P \equiv \sum_s k_{js}$.

We assume that $v_{is}^E + v_{is}^F + v_{is}^K + v_{is}^L = 1$ for all $i$ and $s$. Lastly, there is an iceberg trade cost $\tau_{ijs}$ for shipping a unit of the good from $i$ to $j$ in sector $s$.

Electricity plays two roles in the production function. $e^1$ is direct use for purposes that cannot be typically substituted for fossil fuels, such as lighting, computation or machine power. $e^2$ is a potential input where employment of fossil fuels is feasible, such as heating or transportation, but electricity usage is also possible. The parameter $\psi$ denotes the elasticity of substitution for this input. Denote total usage of fossil fuels used in production in region $j$ as $F_j^F = \sum_s f_{js}$. Each region also has a certain supply of fossil fuels, which it sells for the world market price $p^F$ and yield profits $\Pi_j^F$. We determine the optimal extraction problem and the associated profits below.

Total sales for each sector in each region, denoted by $X_{is,t}$ are given by

$$X_{is} = \sum_j p_{ijs} Y_{ijs} = \sum_j \frac{p_{ijs}^{1-\sigma_s}}{p_{js}^{1-\sigma_s}} \beta_{ijs} E_j$$
where $P^c_{js} = \left( \sum_i p^{1-\sigma_i} \right)^{1/(1-\sigma_i)}$ is the CES price index and $E_j$ is given by

$$E_j = w_j L_j + r^c_j K^P_j + p^c_j \gamma_j - p^F j F^c_j + \Pi^T_j + \Pi^F_j$$

We now define a period equilibrium, leaving the dynamic determination of capital stocks and fossil fuel supplies—as well as time notation— for the following section.

**Definition 1.** Given capital endowments $\{K^P_j, K^F_j, K^R_j\}$, labor endowments, $\{L_j\}$ and a supply of fossil fuels $\{F^S_j\}$, a period equilibrium is prices for the sectoral intermediates $\{p_{ij,s}\}$, a vector of wages $\{w_j\}$ and returns on capital $\{r^c_j\}$, global fossil prices, $p^F$, prices of electricity $\{p^c_j\}$, and an allocation $\{k_{js}, l_{js}, f_{js}, y^c_j, e^1_j, e^2_j\}$ such that:

1. The markets for sectoral intermediates clears (equation (11))
2. The labor and production capital markets clear in each location
3. The world fossil fuel market clears
4. The electricity utility in each region $\mathcal{T}$ solves (9) subject to (1), (4), (7), (6), and for each link, $Z_k \leq Z^{max}_k$.

We now assume that the utility has an objective function that maximizes total regional Gross Domestic Product (GDP), taking output prices as given. That is, they set prices across the network in a way that maximizes the GDP of the regions they serve, given the constraints on the network.

**Proposition 1.** If the utility planner’s objective function is

$$Q_j = \sum_s p_{js} q_{js} (D_{js}(p^c_j)),$$  

where $D_{js} = e^1_{js} + e^2_{js}$, then the problem has a unique solution, and can be implemented by setting spatial prices in each node $i$ according to

$$p^c_j = \mu + \mu \frac{\partial \lambda}{\partial D_j} + \sum_k \eta_k \frac{\partial Z_k}{\partial D_j}$$

where $\eta_k$ is the shadow value of extra capacity on a constrained line $k$, and $\mu$ is the shadow value of total generation capacity.

The price of electricity in any node consists of three components. First, $\mu$, is the marginal system generating cost, determined by the last unit of electricity generation used by the grid, and is the same everywhere. Second, the utility planner adjusts prices by the marginal impact on the losses
of demanding more (net) power in a location $j$. This term may be positive or negative; it may be that increasing power demand in a node $j$ decreases flows on congested lines elsewhere, and brings losses down at the network level. The last term is determined by the overall line capacity constraints. When lines are at capacity in the system, increasing power withdrawal at node $j$ may not be possible without burning out a fully loaded transmission line. As such, demand must be discouraged by raising the price in $j$ so that the line remains fully loaded.

Pricing formulas such as (14) are known in the electricity industry as Locational Marginal Pricing (LMP), since they differ across space in ways that reflect the marginal impact of additional demand or generation, as well as transmission usage costs, on total system performance. It is worth noting that while many utilities use some version of (14), others do not.\(^9\)

Assuming that all locations in the world participate in markets that are structured in this way is no doubt a serious simplification of reality. However, it has two advantages that make it worthwhile. First, many geographies that do not currently implement full LMP are considering and studying its adoption, given the obvious efficiency advantages in a world of decentralized, renewable generation. Second, fully specifying how different county electricity markets operate renders a global analysis infeasible.

In addition, if the planner’s objective function is to maximize regional GDP, we have the following proposition.

**Proposition 2.** If the grid planner’s objective function is given by (13), then the period equilibrium is unique and efficient.

The proof shows an equivalence between the period equilibrium and the solution to a global planner’s problem under a certain set of Pareto weights. It also includes a derivation and elaboration of the equilibrium conditions in Definition 1. Crucial for this result is that the utility only considers the impact of pricing decisions on regional production, but not regional prices. That is, they take regional output prices as given.

### 3 Renewables Investment, Fossil Fuel Extraction, and Clean Growth

To analyze the growth of renewables, we now introduce investment in electricity generation capital. We explicitly introduce time, which is discrete and indexed by $t$, and characterize consumer and firm intertemporal choices.

\(^9\)In the US, around 80% of US generation assets operate under LMP. In the Europe, nodes are organized into collections called zones, with the same price charged within the zone (see ENTSO-E). This can be thought of as another form of LMP with additional constraints on prices. See Decker (2021) and Tangeras and Wolak (2021) for an in depth discussion of LMP.
**Capital.** The consumer can invest in renewable capital $K_{R,j,t}$ within their location. The law of motion of renewable capital is

$$K_{R,j,t+1} = (1 - \delta)K_{R,j,t} + Q_{R,j,t}^R,$$

(15)

where $Q_{R,j,t}$ is the total installation of new renewable capital in $j$ at time $t$. We suppose that renewable capital can be produced by transforming $I_{R,j,t}$ units of the final good according to

$$Q_{R,j,t}^R = \Omega_j \left( \sum_j \sum_{i=1}^{\infty} \left( \mu \right)^i Q_{R,j',t-i} \right) \gamma R I_{R,j,t},$$

(16)

where $I_{R,j,t}$ is total investment of the final good in technology $R = \{S, W\}$, and $\Omega_j$ is a region specific cost shifter. There is a spillover into local renewable capital production from global production in all previous periods, by which it becomes cheaper to produce a unit of renewable capital if more has been installed worldwide. $\mu < 1$ is a parameter that discounts learning from the past due to production experience depreciation (Benkard, 2000). The parameter $\gamma_R$ is termed the learning rate. This is a “learning-by-doing” formulation with frictionless spillovers across regional borders. While no doubt an approximation, it is helpful to consider the knowledge spillovers as being mostly global and largely exogenous to local decisions. While installation costs across countries show some differences in levels, they have tended to trend downwards at similar rates (see Figure 1).

We assume that the final good can also be converted into two additional types of capital; fossil fuel capital $K_{F,j,t}$ and production capital $K_{P,j,t}$. Both evolve as in 15, but without external learning economies. For fossil fuel capital, this stylized assumption appears well supported in the data: installation costs of fossil fuel capital such as coal and gas appear to have stayed stable or even increased in real terms in recent decades as pointed out in Lazard (2019).

**Money and bonds.** There are two additional assets in the world. First, there is a fixed supply of money in the world, which does not depreciate and gives a utility flow to the consumer. Money holdings are denoted $M_{j,t}$. Money is able to be costlessly transferred across the world. It serves as the numeraire good.

Second, the consumer is able to save or borrow via one-period, riskless bonds, denoted by $a_{j,t}$, which promise payment of $R_t \geq 1$ units of money in the following period. Bonds are in zero-net supply worldwide. Since money is freely transferable, the return on these bonds is common across regions. We assume that bonds are in zero net supply worldwide, and bond markets clear in each period, such that

$$\sum_j a_{j,t} = 0.$$ 

(17)

The introduction of frictionlessly tradable bonds and money is a useful addition to the model, since it allows the formation of a single world interest rate that functions to price capital investment.
decisions in all locations. It recovers the idea that returns on capital should be roughly equal across space, which can break down in spatial growth models that feature immobile, local capitalists (see, for example, Kleinman et al. (2023)).

**Fossil Fuel Extraction.** An important aspect of the transition into renewables is the corresponding (negative) income effects induced by the transition out of fossil fuels. Countries rich in fossil deposits, such as oil, natural gas, and coal may lose a stream of income associated with extracting those natural resources or licensing that extraction.

Our analysis of the income effects of fossil fuel ownership follows a long tradition in the exhaustible resources literature following the influential work of Hotelling (1931). We assume that each country $c$, a collection of regions, is endowed with a total supply of fossil fuels ordered by extraction costs. The marginal cost of extraction is $c_{c,t} > 0$, where $S_{c,t}$ is the total value of resources extracted up to period $t$ by country $c$. We also assume that the resources are extracted off-shore and that the cost is denominated in terms of units of the final good. Finally, reflecting on evidence such as those of Clark and Jacks (2007), we assume that constant technological progress in extraction takes place every year at a rate $g^F$. The extraction problem for fossil fuels at the country level is

$$
\Pi^F_{c,t} = \max_{S_{c,t}} \sum_t R_0^{-t} \left( p_t^F (S_{c,t} - S_{c,t-1}) - \left( 1 + g^F \right)^{-t} p_t^F \int_{S_{c,t-1}}^{S_{c,t}} c(q') dq' \right),
$$

where $R_0^{-t} \equiv \prod_{s=0}^t R_s$.

Each region $j$ owns a fraction of the country $c$’s fossil fuel endowment that is proportional to their population, such that $\Pi^F_{j,t} = \Pi^F_{c,t} \times L_{j,t} / L_{c,t}$. Given the presence of a common world interest rate, all regions within a country agree on the optimal rate of extraction.

**Capitalist Intertemporal Choice.** The intertemporal preferences of the household are

$$U_j = \max \left\{ \{C_{j,t}, M_{j,t}, I^C_{j,t+1}, a_{j,t+1}\} \right\} \sum_{t=0}^\infty \beta^t (\log(C_{j,t}) + \epsilon \log(M_{j,t})).$$

subject to

$$P^C_{j,t} C_{j,t} + \sum_{k \in \{F, K, R, W\}} P^k_{j,t} I^k_{j,t+1} + a_{j,t+1} = E_{j,t} + R_t a_{j,t} + (M_{j,t} - M_{j,t+1}),$$

where $a_{j,t}$ is the saving in location $r$ in a global, risk-free bond, $P^A_t$ is the price of the savings bond to yield 1 unit next period (normalized in terms of the price of the world numeraire), and $I^k_{j,t+1}$ is the amount invested in different types of capital. $\tilde{R}$ denotes composite renewable capital, determined below. Capital is sunk once installed and cannot be sold, such that $I^k_{j,t+1} \geq 0$. The first order conditions of the household with respect to consumption and savings imply a standard Euler
equation for consumption,
\[ \frac{C_{j,t+1}}{C_{j,t}} = \beta R_t \frac{P^C_{j,t}}{P^C_{j,t+1}}. \]  

(20)

**Renewable Capital Investment.** We assume that each household decides how much to invest total in solar $S$ and wind $W$ together. For a given spend in units of money they invest in renewables, they draw a pair of idiosyncratic shocks $\{\eta^S_j, \eta^W_j\}$, one for solar and one for wind, reflecting variability in local project opportunities. Once they have drawn the shocks, they choose whether to invest in a solar or wind project in that location. Owing to the fact that the marginal cost of renewables is very close to zero (see e.g. Baker et al. (2013)) and the investment is modular, the value of a unit of renewable capital of type $R$, once installed is given by

\[ V^R_{j,t}(\eta^R_j) = p^R_{j,t+1} \theta^R_j \eta^R_j + \frac{1 - \delta}{R_t} V^R_{j,t+1}(\eta^R_j). \]

Then define the return to investing one dollar in renewable technologies today as

\[ V^R_{j,t} = E[\max\{\frac{V^S_{j,t}(\eta^S_j)}{P^S_{j,t}}, \frac{V^W_{j,t}(\eta^W_j)}{P^W_{j,t}}\}]. \]

We assume that $\eta^S_j$ are distributed Frechet with shape parameter $\varphi$ and a unit scale parameter. Due to the linearity of the value function, the fraction of investment in renewables that flows to solar, denoted $\Theta^S_{j,t}$ each period will be

\[ \Theta^S_{j,t} = Prob\left( \frac{\theta^S_j \eta^S_j}{P^S_{j,t}} \geq \frac{\theta^W_j \eta^W_j}{P^W_{j,t}} \right) = \frac{\left( \theta^S_j / P^S_{j,t} \right)^\varphi + \left( \theta^W_j / P^W_{j,t} \right)^\varphi}{\left( \theta^S_j / P^S_{j,t} \right)^\varphi + \left( \theta^W_j / P^W_{j,t} \right)^\varphi}. \]

Given equation (16), the price of renewable capital of type $R$ in region $j$ time $t$ is

\[ P^R_{j,t} = \Omega_j^{-1} \left( \sum_{j' = 1}^{\infty} (\mu)^i Q^S_{j',t-i} \right)^{-\gamma} p^C_{j,t}, \]  

(21)

and the expected price of renewable capital is

\[ \bar{P}_{j,t} = \Theta^S_{j,t} P^S_{j,t} + (1 - \Theta^S_{j,t}) P^W_{j,t}. \]

Free entry into renewable capital implies

\[ \bar{P}_{j,t} \geq R_t^{-1} \left( p^C_{j,t+1} \bar{\theta}_{j,t} + (1 - \delta) \bar{P}_{j,t+1} \right), \]  

(22)

where $\bar{\theta}_{j,t} \equiv \Theta^S_{j,t} \theta^S_j + \Theta^W_{j,t} \theta^W_j$, and where this holds with equality if investment is positive in $t$. 

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Given capital prices, the price of electricity, \( p^E_{jt} \), is pinned down by this law of motion. This in turn determines electricity production \( Y^E_{jt} \), and consumption, \( D_j \).

**Definition 2.** An equilibrium is a vector of prices for the sectoral intermediates \( \{p_{ij,t}\} \), vector of wages \( \{w_{jt}\} \), a bond price \( \{P^A_t\} \), prices of fossil fuels \( \{p^F_t\} \), prices of electricity \( \{p^E_{jt}\} \), prices of the renewable capital \( \{p^R_{jt}\} \), allocations of consumption \( \{C_{jt}\} \), renewable capital stock \( \{K^R_{jt}\} \), fossil fuel capital stock \( \{K^F_{jt}\} \), production capital stock \( \{K^P_{jt}\} \), and country fossil fuel extraction \( \{S_{ct}\} \), such that for each location \( j \) and time period \( t \):

i) within each \( t \), a period equilibrium holds as defined in Definition 1, ii) the bond market clears (equation (17)), iii) the price of each type renewable capital \( \{p^S_{jt}\} \) evolves according to (21), iv) intertemporal consumption choices \( \{C_{jt}\} \) satisfy (20), v) investment in renewables and the associated capital stock \( \{K^R_{jt}\} \) satisfy the free entry condition (equation (22)) if \( I^R_{jt+1} > 0 \), with analogous conditions for production capital \( \{K^P_{jt}\} \) and fossil capital \( \{K^F_{jt}\} \), vi) The extraction of fossil fuels \( \{S_{ct}\} \) solves (18) for each country.

A steady state equilibrium is an equilibrium in which all variables are constant. In the steady state, all fossil fuels have been extracted, and electricity is entirely renewable. The existence of a long-run steady state is ensured by the forgetting parameter \( \mu \). Without it, perpetual price falls occur.

Importantly, because of the CES specification in our production function, equation 10, fossil fuels are needed in every period to produce the final good, so long as they are extracted. With such an assumption, there is always non-zero demand for fossil fuels at a finite price. The reason this occurs is the finite elasticity of substitution at the industry level \( \psi \). The extent of the timeline of the depletion of fossil reserves ultimately depends on the parameter \( \kappa_{js} \) and in our calibrations the depletion of fossil reserves does not happen for hundreds of years.\(^\text{10}\) However, even with decarbonization occurring rapidly in the power sector, all fossil fuels are eventually burned in industry. We assume that once all fossil fuels are extracted, the price of fossil fuels is infinite.

### 4 Quantitative Analysis

For our quantitative analysis we consider a set of sub-national geographies that roughly correspond to local labor market areas. In the US our regions are commuting areas in the US, and in most European countries to NUTS-3 regions. For other countries we attempt the most granular level of aggregation possible, depending on the available data.

We harmonize datasets from different sources so that they correspond to those geographical boundaries. The harmonization procedure constitutes of bringing together datasets from three

\(^{10}\text{As Covert et al. (2016) point out the reverse to consumption ratio of oil, gas, and coal has remained roughly constant over the decades. This provides some evidence that fossil fuels are indeed not going to deplete for many decades to come.}\)
disparate sets of sources: data from the electricity grid and power assets, data for output and employment, and regional trade data. We describe the data sources below, and provide data construction details in Appendix A3. Further details are provided in the Online Supplementary Material.

4.1 Global Power Grid, Electricity Generation, and Fossil Extraction

We first construct a unified dataset on the global electricity grid. We source data on the length and voltage of high-voltage transmission lines from OpenStreetMap. A full description of our methods for processing the raw data is contained in Appendix A3. Data on global power generation assets comes from the World Resources Institute Database. It contains information on the size of the power plant, the location, the date constructed and the type of fuel and detailed location information. A summary of this data is provided in Appendix A3.

We then construct a simplified representation of the electricity dataset. First, we aggregate transmission capacity between each region. We sum the total capacity of all lines which begin in one region and end in another, and compute the distance of transmission as stretching from region centroid to region centroid. Lines which begin and end in the same region are dropped. We then add the total fossil-fuel capacity and renewable capacity (by type) separately, and consider all generation to be done at the region centroid. We code hydro-electricity plants, coal, gas and nuclear as fossil-fuels \( F \), and consider renewables to be solar and wind. The result for North America is presented in Figure 4, and other major regions are presented in the Appendix.

As specified above, we assume renewable generation capacity has zero marginal cost, and the productivity of the asset is determined by the renewable potential in the area and the data. We average both daily solar insolation and daily wind speeds within a location, using data from the Global Solar Atlas and the Global Wind Atlas as displayed in Figures 2a and 2b by our national sub-divisions. Power generation using fossil assets is extracted from data from the World Resources institute, depicted in the bottom panel of Figure 4 for the US.

For the fossil fuel extraction problem, we use data for fossil fuel marginal cost curves from Welsby et al. (2021) for 16 world regions and 3 fuel sources (oil, natural gas, and coal). For each multi-country region we compute the fraction of reserves each country contributes, using the British Petroleum Statistical Review of World Energy (British Petroleum, 2022) and combine the curves of all three sources by expressing unit costs and production levels in the same energy units, following Clark and Jacks (2007, p. 468).

\[ \text{Note that as the regional partition becomes finer, this method of representation will approach the true dataset.} \]

\[ \text{We do this given their similar economics of large fixed costs and variable fuel costs, and not for their role in emitting CO2.} \]
4.2 Production, Employment, and Trade Data

Our main data source for regional and sectoral value added and employment data is the OECD iLibrary, which includes regional economic and demographic data per year at two different levels of territorial aggregation called territorial levels (TL): TL3 and TL2. The large TL2 regions correspond to political boundaries similar to states. In contrast, smaller TL3 regions, which are contained within a TL2 region, are similar to counties or commuting zones as discussed above for the US and EU. Whenever possible we aim for TL3 regions, and opt for additional data sources if these are not directly available through OECD. For our calibration we use data for 2015, because in this the latest year where most of the countries had the least amount of missing data by industry. After our selection process and the additions from other datasets we end up with a total of 40 countries and 2506 regions. These regions represent almost 80% of world GDP. We depict the regional and national borders in Figure A5.

Notes: This figure shows our representation of the electrical grid between regions of the United States using data from OpenStreetMap (transmission lines) and the World Resources Institute Database (power capital). Circle size is proportional to region generating capacity in MW. Line thickness is proportional to transmission capacity between regions. Generating capacities are total nameplate capacity of fossil and clean resources, without adjusting for capacity factors.

13We discuss the details of all the construction of the production, employment, and trade data, in our online data appendix.
14Sectoral data can be found in https://www.oecd-ilibrary.org/urban-rural-and-regional-development/data/oecd-regional-statistics_region-data-en. The ten ISIC revision 4 sectors we aggregate to in the data are i. Agriculture, forestry and fishing, Manufacturing, ii. mining and quarrying and other industrial activities, iii. Construction iv. Wholesale and retail trade, transportation and storage, Accommodation and food service activities, v. Information and communication vi. Financial and insurance activities vii. Real state activities viii. Professional, scientific,
To calibrate the trade costs of the model, we measure trade frictions using regional trade data aggregated at the same ten sectors as the gross value added and employment data. The data are based on the EUREGIO Regional Input-Output database (see Thissen et al. (2018)). The survey determines trade flows of different types of goods between regions in Europe and some non-European countries, by surveying all possible modes of transport. We aggregate the data to our geographic regions to create a precise mapping to our employment data.

To estimate trade costs, we postulate that they are given by

$$\tau_{ijs} = e^{\omega_s} (B_{\text{inter}})^{1_{C_i \neq C_j}} (B_{\text{intra}})^{1_{i \neq j, C_i = C_j}} (d_{ij})^{\beta_s}. \quad (23)$$

with distance $d_{ij}$, an intra-country border cost, $(B_{\text{intra}})^{1_{i \neq j, C_i = C_j}}$, an inter-country border cost, $(B_{\text{inter}})^{1_{C_i \neq C_j}}$, and a sector-specific cost $e^{\omega_s}$. Taking logs, we can recover the trade cost components by estimating the following gravity regression

$$\ln (X_{ijs}) = (1 - \sigma_s) \ln (B_{\text{intra}})^{1_{i \neq j, C_i = C_j}} + (1 - \sigma_s) \ln (B_{\text{inter}})^{1_{C_i \neq C_j}} + (1 - \sigma_s) \beta_s \ln (d_{ij})^{1_s} + \omega_i + \omega_j + \omega_s + \epsilon_{isj},$$

where $X_{ijs}$ is the value of the trade flow from region $i$, sector $s$, to region $j$, $\mathbb{1}_s$ is an indicator that takes the value of 1 when the sector is $s$, $\mathbb{1}_{i \neq j, C_i = C_j}$ is an indicator variable that takes the value of 1 if regions $i$ and $j$ belong to the same country (and are not the same region), $\mathbb{1}_{C_i \neq C_j}$ is an indicator variable that takes the value of 1 if regions $i$ and $j$ belong to different countries, $d_{ij}$ is the distance regions $i$ and $j$, and $\omega_i$, $\omega_j$, and $\omega_s$ are fixed effects for region of origin, destination region, and origin sector, respectively.

The estimates are reported in the Supplementary Material to the paper. We report three salient findings. First, all three coefficients differ across sectors. In particular, the coefficient on log distance shows intuitive deviations from the coefficient of $-1$ that commonly appears in aggregated gravity regressions (see e.g. Head and Mayer (2014)). The coefficient on construction is -1.34, while the coefficient on information and communication is -0.80. The coefficient on various tradable services (Financial and insurance activities as well as Real state activities/ Professional, scientific, technical, administrative and support service activities) is low, below -0.5. Rerunning the regression aggregated across sectors indeed yields a coefficient very close to $-1 \ (-1.003)$, and running the regression aggregated across countries for the ten sectors also results in a coefficient much closer to one. Second, the effects of the borders are large. Third, not surprisingly, the inter-country borders affect trade much more than intra-country borders.

15These modes include land traffic (by trucks), rail transport, air transport, freight transport, and non-motorized passenger transport.

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technical, administrative and support service activities ix. Public administration and defense, education, human health and social work activities x. Other service activities
4.3 Calibration

We use our estimated sectoral trade costs to calibrate the trade costs in the model, $\tau_{ijr}$, using centroid to centroid distances of our regions. We calibrate sectoral trade elasticities using data from Fontagné et al. (2022). We target the employment shares by region-sector and the regional output to calibrate the productivities of the model for each sector, $z_{is}$, using the full equilibrium structure of the model (Allen and Arkolakis (2014); Redding and Rossi-Hansberg (2017)).

To calibrate factor shares in the final product firms’ production function, we use data from the Global Trade Analysis Project (henceforth, GTAP) global input-output (IO) table, which features 65 sectors in 141 countries. To accommodate our sectoral definitions, we proceed by mapping each of the GTAP sectors to our 10 SNA/ISIC sectors. Using this data, we compute factor shares for fossil fuels, $v_{Fi}$, for electricity, $v_{Ei}$, for capital, $v_{Ki}$, and labor $v_{Li}$, by dividing the expenditures on each factor for a given sector-country by the total sectoral expenditures in that country. To obtain total expenditures in each of these factors, we proceed as follows. For capital and labor, we use factor endowments directly. For fossil fuels and electricity, we label each of the original GTAP sectors as fossil fuels sectors, electricity sectors, or neither. Then, we compute spending from the intermediate input trade data as total expenditure by each sector $s$ in country $c$ on fossil fuel sectors and electricity sectors, respectively.

Recall that $\psi$ is the ease of substituting between direct fossil fuel use and electricity at end use. In a meta-analysis of inter-fuel substitution elasticities in the industrial sector, Stern (2012) finds, in general, elasticities of substitution that are larger than one, though not always statistically significant. The cross-sectional meta-elasticities for oil-electricity, gas-electricity and coal-electricity are 2.5, 1.4, and 0.9, respectively.\footnote{In particular, these elasticities differ at short and long run horizons, as it indexes the possibility of “electrification”. Stern (2012) reports long-run dynamic elasticities, but points out that “none of these elasticities can be estimated with any real precision given the available data.” The history of innovation leads us to suspect that long-run substitution elasticities could be substantially higher.} In our baseline calibration we assume $\psi = 1.4$, reflecting the widespread usage of gas in industry.

We specify the production function for energy as following. For fossil fuels, we set

$$Y_j^F = (K_j^F)^{a} F_j^{1-a}, \quad Y_j^F \leq Y_j^{F,max}.$$  

We choose $a = 0.7$ to match a marginal cost curve of fossil fuel electricity that rises slowly with increased power output, roughly matching the slope of marginal cost curves in PJM and other major power markets in the US (see Sahraei-Ardakani et al. (2015)).\footnote{One limitation of this specification is that it is not able to capture sharp increases in marginal cost that are seen when regions are operating at peak output due to the use of gas and oil peaker plants.} Maximum output in a region $Y_j^{F,max}$ is given by summing the rated capacity of all fossil assets with a region in the data. We use
the information on the line length and voltage to construct a value of the resistance for each line in the model by dividing the length (which is measured in centroid to centroid distance if two regions are connected by transmission lines in the data) by the rated voltage in the data.

To calibrate the learning rate of renewable capital, we begin by using data from the International Renewable Energy Agency (IRENA) on the average total installed cost of wind and solar power. We estimate a regression of the form

$$\log(\text{Total Cost}_t) = a + \gamma \log(\text{Total Installation}_t) + \epsilon_t$$

separately by technology. The results are plotted in Figure (A6), and yield a value of 0.6 for solar and 0.27 for onshore wind. These values are very high compared to the values estimated from micro studies in the literature. The reason is clear: part of the cost declines in renewables are due to ongoing research and development and technical progress, and not the economies of scale and learning economies that come from directly increasing output. See Nordhaus (2014) for a discussion. Parameters found in the literature for $\gamma$ from micro-studies of learning in renewable technologies are typically on the order of 0.1 – 0.2. However, these studies are mostly unable to account for international spillovers. As a compromise, we choose $\gamma_S = 0.35$ and $\gamma_W = 0.2$ and conduct various robustness exercises. In addition, we chose the depreciation rate of knowledge $\mu$ to be 1%, but find our quantitative results to be mostly unchanged in the presence of higher values.

To calibrate renewable potential $\theta^R_{ij}$, we use an estimate of annual kWh/m² for solar and wind from the Global Solar Atlas and the Global Wind Atlas. This measure is taken from solar insolation satellite data and wind speed data and combined with assumptions on panel tilt for solar and turbine height for wind, to give potential at a very fine scale of aggregation. We average this measure within each of our regions.

There are four remaining parameters that we externally calibrate. We set the depreciation of renewable capital to 3%, based on the low degradation rates of new renewable photovoltaic and wind turbines that are expected to last thirty years or more (Jordan and Kurtz (2013); Wiser and Bolinger (2019)). We set the long run interest rate to be 5%, and the rate of forgetting to be $\mu = 0.01$.

Lastly, for the fossil fuel module, we allow technology improvements to reduce extraction costs over time. Following the EIA Annual Energy Outlook (U.S. Energy Information Administration, 2022), we impute a 1% year-over-year reduction in marginal costs.

4.4 Model Fit

The model does a reasonable job at reproducing spatial patterns of power prices seen in the data. We collect Locational Marginal Prices for the US from 9 Regional Transmission Organizations, and
Table 1: Overview of Parameter Choices

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<th>Value</th>
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</tbody>
</table>

Notes: This table shows the baseline parameterization of the model, as well as the sources and moments used to calibrate the model.
then average the hourly price at each node in the data for all of 2019. The result is shown in Figure A8, along with the predictions of the model for the equilibrium prices in each region of the US. The states without data do not employ Locational Marginal Pricing and thus cannot be compared to our model predictions.

The higher prices in California and New England are clearly visible. The lower band of prices across the Midwest is also apparent, as are somewhat higher prices in the Rustbelt ex. upstate New York. Some notable misses of the model include the West Texas oil fields (the dark crimson areas of Texas), which are huge consumers of electricity, and have no analogue in the model. Overall, the model can reproduce fairly well the variation in spatial prices in at least one region of the world for which we have data.

4.5 Extensions for Quantitative Analysis

We introduce two extensions to the baseline model that can partially address the intermittency of renewables indirectly, as we have sidestepped modeling their stochastic nature. In particular, in line with the discussion in Baker et al. (2013), we model backup batteries requirements and overbuilding for renewables.\footnote{As the authors point out, for solar renewables “intermittency of a solar resource can add to system costs, as additional system reserves and backup generation may be required to maintain system reliability”. The remaining unmodeled complication that intermittency brings is the covariance of demand and yield of renewables, for example high output of photovoltaic when energy is needed for cooling in the summer and low energy output in the winter when energy is needed for heating, as is considered in Lamont (2008); Baker et al. (2013). Our model can be simulated at high frequency to incorporate those effects of seasonal intermittency, something we leave for future research.}

**Batteries.** We assume every unit of renewable capital must be paired with a battery of size $B$, where size is measured in MWh. These sizes we consider are 0, 2, 4 and 8 MWh per each unit of renewable capital (recall that size is measured in MW). Batteries are produced under constant returns, and available at price $p_{j,t}^B$, where $p_{j,t}^B$ follows a learning process similar to that of renewable capital, given by

$$p_{j,t}^B = \left( \sum_{j'}^{\infty} \sum_{i=1}^{\infty} (\mu)^i Q_{j,t}^B \right)^{-\gamma_B} p_{j,t}^C. \tag{24}$$

where $Q_{j,t}^B$ is the quantity of batteries produced. We take the learning rate for batteries $\gamma_B$ to be $\gamma_B = 0.35$, roughly in line with estimates from the literature, and reflecting somewhat slower expected price declines with output than in the recent past, due to projected supply constraints for battery materials.

**Overbuilding and Curtailment.** We additionally consider the idea that one way of dealing with the intermittency of renewables is to simply build more capacity than is generally needed, so that in times of low wind speeds or sunlight, the extra capacity can compensate. In normal times,
surplus electricity is curtailed, or wasted, and generators are not paid for this lost output. With capital prices falling far enough, even with significant curtailment, positive investment can still be profitable for investors.

5 Counterfactual Exercises

In this section, we assess the impact of renewable energy on income growth, prices and welfare. We solve for a perfect foresight equilibrium, beginning with the calibrated model in 2020, and shooting forward until the model reaches a new steady state. We the consider the extensions outlined above, before finally turning to policy.

5.1 Clean Growth Across the Globe

We begin by examining the long run consequences of the energy revolution. In Figure 5 we plot the long run changes in per-capita welfare across countries induced by the energy transition, comparing the long run steady state of the model to the outcomes at the beginning of the transitional dynamics (in 2021). We decompose welfare changes into several informative components and define the change in welfare as the change in real expenditure per capita.

\[ d\ln W_j = d\ln \left( \frac{E_j}{P_j} \right) \]

where

\[ d\ln \left( \frac{E_j}{P_j} \right) \approx d\ln \left( \frac{w_j L_j}{E_j} \right) + d\ln \left( \frac{r^{K_j} k_j}{E_j} \right) + d\ln \left( \frac{\Xi_j}{E_j} \right) + d\ln \left( \frac{\Pi^F_j}{E_j} \right) \]

\[ (25) \]

where \( \Xi_j \equiv p^c_j Y^c_j - p^F_j F^c_j + \Pi^F_j \) is net electricity sales revenue. In the long-run, adoption of renewable capital can have four primary effects. First, it induces changes in specialization across industries. This can cause some regions to become relatively more well off compared to others through terms of trade effects. Falls in goods price from cheaper electricity additionally lower the price index in each region, leading to real wage gains. Second, cheaper electricity induces capital deepening, raising output. Third, electricity sales grow as more renewable capital is installed, and industry is electrified, with some fossil capital rich places losing. Lastly, countries which are currently exporting fossil fuels lose this revenue in the long run.

To plot this at the country level, we take the weighted average of regions within a country, and show this in Figure 5. Welfare gains are large. The average welfare gain is 4.1%, though is this very
heterogeneous across regions. For a point of reference, Cruz and Rossi-Hansberg (2021) estimate the costs of long run climate change in the high emissions scenario of 5.1 degrees of warming by 2100 as 6%, and for 2.9 degrees as 1%. As such, the pure economic benefits of technologies designed to alter the warming path of the world are of a similar order to the detrimental effects of climate change in a business as usual, high emission scenario.

Indeed, the gains from the renewable energy transition are even larger than is apparent here. An important factor dampening the long-run welfare gains is that in the long-run steady state of the model, fossil fuels have been completely exhausted. This loss of a factor of production is only partially ameliorated by electrification of industry, since we assume the elasticity of substitution $\psi$ is relatively low at 1.4. The loss of economically useful fossil fuels depresses the welfare gains of the clean energy transition in a way that is not immediately apparent from Figure 5. While fossil fuel exporters suffer disproportionately from the loss in fossil revenue, all countries are hurt by an increase in the price index from phasing out fossil fuels.

Figure 6 presents an aggregate summary of the energy transition transitional dynamics. The model predicts that the aggregate price of solar capital falls by almost 50% by 2040. While substantial, this is somewhat lower than the previous pace of price declines seen for solar and consistent with projections from SETO (2021). As seen in Figure A6, in the previous decade (from 2010 to 2010), the total installed price of solar capital decreased by 81%. The reason is twofold. Once capital becomes widely adopted in many regions, passing the initial threshold of no investment, proportional
increases in the total stock installed worldwide begin to slow (though the learning rate $\gamma$ remains unchanged). Second, we have taken a somewhat lower learning rate for renewable capital than is observed in the data for solar costs. The pace of the decline for wind is substantially slower, owing both to the lower learning rate and to the fact that the renewable mix shifts continually toward solar as the solar price drops faster than the wind price.

Most countries around the world achieve substantial renewable energy shares by 2040. The US and Europe rise to 80% renewable power by 2040 on the basis of capital cost declines alone. China takes much longer, owing to an abundance of fossil fuel capital and relatively cheap power prices, which discourages initial take up of renewables.

It is worth emphasizing that this is not a forecast, but a quantitative illustration of the cost effect of learning-based capital price declines on the world energy system. Still, several features of reality have been abstracted from that may quantitatively shape the transition. In particular, this is a frictionless investment model. In reality, there may be congestion effects in the construction sector that impede the ability of countries to install the amount of capital projected here. In addition, while line constraints are modeled in detail, there are technical limitations due to temporal mismatches of renewable supply and energy demand, which may become more severe with greater renewable penetration. This intermittent nature of renewables means that substantial backup is likely to be necessary, and we now consider the role of battery storage in providing this backup. In addition we model the possibility of curtailment of renewable production with and without storage. We turn to these extensions now.
Figure 7: Renewable Transition Paths Under Battery Scenarios

Notes: This figure plots the share of electricity by country coming from renewable sources in four different scenarios: the baseline, without any storage requirements, and then 3 scenarios of storage pairing with renewable capital.

Figure 8: Capital Prices under Battery Scenarios

Notes: The left panel shows the composite renewable capital price under different battery scenarios. The right panel shows the bath for battery prices under different battery scenarios.
5.2 Batteries

To set the initial level of battery costs, we use the current price of a Tesla Megapack. Currently available in modular units of 3.9 mWh and 1.9 MW power output, these packs are widely used in nascent battery projects across the world.\textsuperscript{19} As of the time writing, this cost works out to be $482 per kWh. We then let the price of batteries evolve along the transition according to the law of motion in 24. Recall that we require that each unit of renewable capital must be paired with a battery of size $h$, so that the free entry condition now becomes

\[
\left( \tilde{P}_{j,t} + h p_{j,t}^B \right) \geq R_t^{-1} \left( p_{j,t+1}^F \tilde{P}_{j,t} + (1 - \delta) \tilde{P}_{j,t+1} + (1 - \delta^B) h p_{j,t+1}^B \right)
\]

We set the depreciation rate of batteries, $\delta^B$, to 0.05, reflecting an expected lifetime of 20 years.\textsuperscript{20} The results are plotted in Figure 7. In each region, requiring the adoption of batteries substantially slows renewable penetration out to 2040. However, even in the most restrictive scenario, the world still reaches 40% renewable penetration by 2040. Ongoing capital price falls for batteries with continued uptake gradually allow renewables to dominate most areas of the grid.

We also note that mandating a larger battery market from the beginning actually does not lead to cheaper battery prices through learning by doing, as we show in Figure 8. The increased costs from mandating larger amounts of storage act as a deterrent to renewable adoption, which more than compensates for the increased requirements. In the 4 hours and 8 hours of storage scenarios, battery prices decline significantly slower than the 2 hours scenario.

5.3 Curtailment and Overbuilding

The problem of renewable intermittency may lead to excess energy generation which, without substantial storage, results in curtailment. We incorporate estimates of curtailment modeled as efficiency loses in electricity generation of renewables using the measurements of Shaner et al. (2018); Tong et al. (2021). The authors measure curtailment using precise geophysical measures of solar and wind potential and demand of countries, as well as various other geophysical constraints such as spatial variability, transmission network, demand fluctuations etc. In particular, Tong et al. (2021), do so under various battery storage scenarios for 18 major countries.

We take advantage of their estimates to construct a continuous measure of curtailment of wind and solar production as a function of penetration of each of the resources and different battery storage scenarios. We use the estimates of Tong et al. (2021) for 17 countries and apply the US estimates for the US and any country that the authors do not provide estimates for to calibrate curtailment.

\textsuperscript{19}See, for example, the Hornsdale Power reserve, which was the world’s first large scale battery installation in 2017, and more recently the Moss Landing substation for PG&E in California.

\textsuperscript{20}For example, the current warranty on a Tesla Megapack is for 20 years.
Figure 9: Considering Curtailment and Overbuilding in the Renewables Transition

Notes: The left panel shows curtailment as a function of wind and solar penetration in the country with 12 hours of storage on the grid. The right panel shows the share of electricity coming from renewables at the world level in the baseline scenario and under curtailment with 12 hours of storage.

Factors for each country in our dataset. Details are provided in Appendix A4. We illustrate the US curtailment estimates in Figure 9. For example, with 12 hours of storage, if solar energy capacity is 30% of the nominal demand and wind 30% of the nominal demand then the curtailment is about 15%. But if instead 85% of electricity is produced by solar with 15% wind contribution then the curtailment is more severe, in the order of 40%. Curtailment ameliorates substantially the more battery storage is introduced. In Figure A10 in the Appendix we plot the same figure for 4 hours and 0 hours of storage. The figure makes clear the curtailment constraints of renewables and that a mix between solar and wind can be used to diversity renewable resources, to the extent that geophysical constraints permit.

We model curtailment as reducing the amount of output from both solar and wind capital by the endogenous fraction $C_{j,t}$, using the estimates described above, such that the free entry price in any period is

$$ (P_{j,t} + hp_{j,t}^G) \geq R_{t-1} \left( P_{j,t+1}^{E} \delta_{j,t} C_{j,t} + (1 - \delta) P_{j,t+1} + (1 - \delta^B) hp_{j,t+1}^G \right) $$

This scenario is somewhat more pessimistic. In order to build a grid that is resilient to severe curtailment, around 12 hours of storage is required. This is because, in any scenario, solar quickly dominates wind, and this necessitates a large amount of battery storage to provide electricity when the sun is not shining. Such a large storage requirement significantly slows renewable penetration at the global level.

5.4 Robustness with Exogenous Technical Progress

The results in this paper are sensitive to the specification of the learning rates $\gamma_S$ and $\gamma_W$, and speculation whether the aggregate rate is stable and will remain at its current value for the foresee-
Figure 10: Renewable Transition Paths under Exogenous Technical Progress

Notes: The top left panel shows projections for the cost of solar capital, and the right for wind, using historical data from IRENA in constant $2019. The bottom left panel shows the same for batteries using data from Ziegler and Trancik (2021b), available from Ziegler and Trancik (2021a). The bottom right shows projections for the world renewable share under these different scenarios.
able future is reasonable. Though price declines in solar have been more or less constant for 50 years, this is is no guarantee that they will continue in this manner. As a robustness exercise, in this section we consider what the path for the renewable transition would look like under different assumptions about exogenous technical progress, with no learning by doing.

To do so, we estimate a linear trend for solar and wind capital costs off the last ten years of data, and then extrapolate this trend out to 2040. Beyond this, we assume technological progress stops. We consider a case where the trendline remains the same, and also scenarios where the speed of improvement slows by 50% and two thirds. The results are shown in Figure 10. We find that at current rates of technical progress in these technologies, the world energy system transitions to renewables even faster than our baseline scenario with endogenous capital price falls. Even significant slowdowns of these trends imply a fairly rapid decarbonisation.

5.5 The Inflation Reduction Act and Renewable Subsidies

We now turn to policy, and study the effects of a subsidy in the form of a production credit to renewable energy. In August 2022, the Biden Administration signed into law the Inflation Reduction Act, which contained significant spending measures designed to encourage the uptake of renewable energy. Chief among these was an production tax credit of $26 per MWh for renewable energy from any source, extended out to 2033 (H.R.5376 - Inflation Reduction Act of 2022). Certain domestic labor and materials are required to receive the full credit, and if they are not met, the credit is reduced to $5.

We model this by assuming that electricity produced for renewable sources receives a subsidy $s_t$, so that

\[ p_{jt}^R \geq R_t^{-1} \left( (p_{jt}^E + s_t)\bar{\theta}_{jt} + (1 - \delta)\bar{p}_{jt+1}^R \right) \]

The subsidy is funded by labor income taxes on all regions in the US, with a time varying (but spatially invariant) rate $\tau_t$, which solves

\[ \sum_{j \in US} w_{jt}L_j(1 - \tau_t) = s_t \sum_{j \in US} Y_j^e \]

There are no taxes in the model, and thus directly comparing this to the electricity revenue received by a unit of renewable capital is not immediate. We proceed as follows. We first note that $5 is around 15% of the average wholesale price prevailing in markets with locational market prices in 2020, and set the value of $s_t$ in the model to be 15% of the average equilibrium 2020 $p_j^e$ in the US.\textsuperscript{21} We assume that the subsidy is removed after 2032, and goes to zero in 2033. We solve for

\textsuperscript{21}It is difficult to ascertain from a reading of the bill just how restrictive the conditions are to receive the full $26. We thus model the effects of the lower amount.
Figure 11: The Impact of the Inflation Reduction Act

Notes: The top left panel shows the model’s projection for renewable power share under the IRA production tax credit, and without. The top right panel shows the total cost of the bill (in blue), and subsidies going to capital that would be installed in the absence of the subsidy. The bottom left shows GDP growth in both scenarios, and the bottom right shows the renewable capital price.

the counterfactual global equilibrium, holding all other regional fundamentals constant. In this scenario, we also do not assume any storage is required.

The results are presented in Figure 11. The production tax credit induces significant uptake of renewable energy in the US over and above the baseline case. Instead of reaching 60% of electricity production by 2030, the renewable share is 76%. The gap between the baseline and counterfactual renewable share then slowly closes as the credit is withdrawn. Again, it is worth emphasizing that this is not a forecast, given the abstractions we have made of necessity, but instead is an illustration of the power of existing renewable subsidies to shape the energy transition.

It is also worth noting that in the model, the IRA production credit can have only transitional effects. This is due to the forgetting rate $\mu$, which discounts production in the far past in affecting the price of capital today. As such, the IRA cannot effect the long run steady of the model, as infinitely far into the future its effects have been lost. It only has transitional effects, speeding up adoption in the movement towards the long run steady state.

The credit goes some way to accelerating world-wide adoption, modestly increasing adoption in other countries. This is because the US is a substantial buyer of renewable capital on the world market, and accelerating adoption plans domestically increases world production of capital, putting downward pressure on the price via learning by doing. By 2030, the world renewable capital price is 7% lower thanks to the IRA than it would be in the baseline with no subsidies.

The budgetary costs of the IRA are substantial, but within the projections of the bill itself. The IRA bill budgets for $160 billion in spending on the production tax credit. We find instead that around $90 billion is spent before the subsidy expires in 2034. Over 70% of this total goes to subsidizing inframarginal investment, or investment that would have happened absent the subsidy. This is
due to the relatively rapid adoption in the baseline case; much of the renewable transition appears likely on the basis of cost declines alone.

However, the intervention more than pays for itself. We find that the intervention generates around $970bn of additional GDP on the transition, due to cheaper power prices both in the US and around the world. This more than justifies the budgetary outlay. The reason is simple: dynamically, the world economy is not efficient, since firms do not internalize the cost benefits for other future capital producers when they expand output. Note also that this figure does not take into account any additional benefits of carbon reduction, nor does it consider the spillover effects onto the GDP of other countries.

6 Conclusions

We have developed a spatial theory of clean growth to assess the global rise of renewable energy. We find that continued growth in renewable installation is likely to lead to substantial falls in the price of power, with widespread benefits to production and real wages. Most countries benefit from this technological shift, though the long run effect depends on initial power prices and the strength of the transmission networks. Production based subsidies of the kind seen in the US with the Inflation Reduction Act of 2022 can substantially accelerate the renewable energy transition.

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Figure A1: Aggregate Capacity Additions

Notes: This figure plots the share of total new generating capacity across the globe by type. Data is from the International Renewable Energy Agency.

A1 Additional Figures and Tables

In this section we provide additional figures to the paper. Figure A2 plots the raw power station data and transmission line data over the globe. Figure A3 plots the representation of this data for Europe and China which we use in the model. Figure A6 plots the aggregate learning rates for solar and wind capital in the data. Figure A8 plots Locational Marginal Prices for the US in 2020, as well as the model generated prices in each one of the US regions in 2020.
Figure A2: Global Electricity Infrastructure

(a) Power Capital
(b) Transmission Lines

Notes: Panel (A) shows installed power capital in MW using data from the World Resources Institute. Panel (B) shows high-voltage transmission lines scraped from OpenStreetMap.
Figure A3: Network Representations for Other Major Regions

(a) China

(b) Europe

Notes: This figure shows a representation of the electrical grid between regions of China (Panel (a)) and Europe (Panel (b)) using data from Openstreetmap (transmission lines) and the World Resources Institute Database (power capital). Circle size is proportional to region generating capacity in MW. Line thickness is proportional to the number of transmission lines between regions.
Figure A4: Network Representations for Other Major Regions (2)

(a) India

Notes: This figure shows a representation of the electrical grid between regions of India (Panel (a)) and Brazil (Panel (b)) using data from Open Street Map (transmission lines) and the World Resources Institute Database (power capital). Circle size is proportional to region generating capacity in MW. Line thickness is proportional to the number of transmission lines between regions.
A2 Data on The Renewable Technological Revolution

Renewable technologies have seen consistent improvements over the past many decades. In this Appendix, we provide evidence that renewable technological improvement is a technological revolution still in the making. For our analysis we focus on photovoltaics (PV) and onshore wind. The history of photovoltaics dates back to 1839 when French scientist Edmond Becquerel discovered the photovoltaic effect. Commercial production of photovoltaics only started in 1954 when the first silicon based photovoltaics were invented by Bell labs with a 4% efficiency in conversion of light to electricity. The first commercial application of this technology in 1956 was priced at $2269 per Watt, in 2013 prices. In Figure A11 we plot the Solar PV efficiency over time with data until 2019 and with recent projections from the Department of Energy. The price per Watt has fallen to $0.38 in 2019, and the Department of Energy expects that new projects, will cost $0.15 per Watt in 2030. This extraordinary technological progress rivals any other investment price cost decrease, and matches even that of computers. In Figure A12 we plot the sectors with fastest price declines in investment costs from the Bureau of Economic Analysis Database and Solar PV and Onshore Wind Investment Costs from 1985 to 2021. In this graph we use US dollar per kilowatt for Solar PV and Onshore Wind. We normalize all prices to 1 in 1985 and illustrate relative investments from then to 2021. Our choice of initial year is dictated by the availability of onshore wind data. Computer and Peripheral Equipment prices show the fastest declines for the first two decades but the rapid speedup in Solar PV innovation in the last two decades meant that the PV have now declined more than any other technology and in a trajectory to even larger declines. Onshore wind costs have been consistently falling throughout and decrease a little bit slower than communication equipment during that period, but still faster than software.

To formally compute the cost of investment of these different sectors we measure the user cost of Solar Photovoltaics (PV) and Onshore Wind following Caunedo et al. (2021). In particular, we use the familiar arbitrage condition of Jorgenson (1963) so that User cost = adjusted price of renewable * (Interest rate – (1–depreciation rate) * relative price change). Table A1 depicts the results. Solar PV and Computers user costs drop fastest, at a rate faster than 12%. These declines swamp the other sectors with very rapidly declining costs. However, Onshore wind declines are just a little bit slower than software and communication equipment, the sectors with the slowest declining costs.
Figure A6: Aggregate Learning Rates for Power Technologies

Learning Rate for Solar PV $\gamma=0.6$

Learning Rate for Onshore Wind $\gamma=0.27$

Notes: This figure plots the total installed cost of capacity from a new solar and wind energy project against world installed capacity. Data is from the International Renewable Energy Agency. Values are in constant 2019 dollars. Data is from 2009-2019.
Figure A7: Aggregate Learning Rates for Lithium-Ion Batteries

Notes: This figure plots the total installed cost against world installed capacity for lithium-ion batteries. Values are in constant 2018 dollars. Data is from 2000-2016 using Ziegler and Trancik (2021b), available from Ziegler and Trancik (2021a).

Table A1: Asset Investment Prices and User Cost (1985-2020)

<table>
<thead>
<tr>
<th>Description</th>
<th>NIPA Code</th>
<th>Price of Investment % change</th>
<th>Usercost % change</th>
</tr>
</thead>
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<tr>
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<td>−12.14</td>
</tr>
<tr>
<td>Onshore Wind</td>
<td>N/A</td>
<td>−5.61</td>
<td>−5.46</td>
</tr>
<tr>
<td>Computers &amp; peripheral equipment</td>
<td>4</td>
<td>−11.97</td>
<td>−12.37</td>
</tr>
<tr>
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<td>−7.04</td>
<td>−6.94</td>
</tr>
<tr>
<td>Software</td>
<td>99</td>
<td>−4.06</td>
<td>−4.14</td>
</tr>
</tbody>
</table>

Notes: Column 1 describes the equipment category while column 2 presents the corresponding NIPA code in the BEA’s fixed asset tables. Column 3 reports the annual percent change in the quality-adjusted relative price of investment to consumption, while column 4 reports the annual percent change in the user cost of capital.
Figure A8: Spatial Electricity Prices

Notes: The top panel of this figures show locational marginal prices from 9 RTO/ISOs in the continental US. Data are averaged at each pricing node for 2020, and then smoothed over space. The bottom panel shows the model predictions for prices in 2020 for all US commuting zones.
Notes: This figure shows the fossil fuel resources of the globe ordered by current (2021) extraction cost for three types of fuel: coal, oil and natural gas. The source data is from Welsby et al. (2021).

Notes: The left panel shows curtailment as a function of wind and solar penetration in the country under no hours of battery storage, and the right the same figure under 4 hours of battery storage.
Figure A11: Cost per Watt of Solar Photovoltaics


Figure A12: Investment Price Index for Sectors with Fastest Declines and Renewables

Notes: This graph presents the price of investment for the three sectors with the fastest declines from BEA and renewables, with 1985 normalized to 1. Sources: The price of investment data for computers & peripheral equipment and communication equipment are from Bureau of Economic Analysis (BEA), software from FRED, solar PV from Our World in Data and Onshore Wind from IRENA.
The price indices used to compute the change in price of investment are from the BEA for computers and communication equipment U.S. Bureau of Economic Analysis, "Table 5.5.4. Price Indexes for Private Fixed Investment in Equipment by Type", from FRED for software, from Our World in Data for Solar PV and from IRENA for Onshore wind. The depreciation rates used to construct the user cost of capital are from the BEA for computers, communication equipment and software, from Wiser and Bolinger (2019) for onshore wind, and from Jordan and Kurtz (2013) for Solar PV. We use FRED data for the personal consumption expenditure index (PCEP). Solar PV depreciation rates were estimated at 5% per year between 1985 and 2005, and at 4% between 2006 and 2020 as solar PV useful life increased over more recent years (NREL) while onshore wind depreciation rates declined from 5% pre 2010 to 3.3% post 2010.

A3 Data Construction for Electricity Grid

In this section we provide details about the construction of data used in the paper. The data consists of two primary components: asset data and grid data. Greater detail is given in the Supplementary Material. Details about the other regional datasets used throughout the paper are also provided in the Supplementary Material.

A3.1 Power Assets

To generate the asset data we use the Global Power Plant (GPP) database, which consists of over 35,000 power plants in 167 countries, each with a name, primary fuel type, geolocation, and power capacity information. We cross-reference the data with official sources from numerous countries to ensure accuracy.

A3.2 Electrical Grid

We use Open Street Map (OSM) data and a Java tool called Osmosis to filter out all non-power-grid data. After filtering we perform further cleaning on the extracted data. We use the power stations and plants in the OSM data extensively in the cleaning process to determine the start and end points of transmission line segments but use the asset data from GPP as this dataset is far richer for assets. The following is a high-level summary of the operations performed by our algorithm.

At any given location, there may be multiple polygons representing different buildings or components within the same "station". We use the term "station" as a catch-all term to describe plants, stations, substations and transformers. As such, we cluster stations within 500 meters of each other. After clustering the stations, we replace line endpoints with the relevant station centroids, either stations intersecting or within 500 meters of the of line, at each step. Spot checks on Google Earth confirm that many lines that actually connect to nearby stations in reality are not captured by

See BEA https://fred.stlouisfed.org/series/B985RG3A086NBEA. Our version of the data was downloaded on November 30, 2020. This involves grouping stations within 500 meters of one another and taking the convex hull of each group as our final station.
an intersection in the OSM data; instead the lines end outside of the polygon. Incorporating nearby stations addresses this problem. After we complete the line endpoint replacement, lines which are intersected by one or more stations are split. As a result a unique line is defined as one that connects at most two stations. This is also done for stations that are within 500 meters of lines by taking the closest point on the line to the nearby station, buffering it to account for tolerance issues, and subtracting that buffered object from the line. Each line is given a new unique ID number after splitting.

We merge lines together when they have separate IDs and are connected, or within 500 meters of each other, with no station in between. For T-intersections, we take two approaches. A T-intersection is defined as an intersection between two lines A and B where A intersects (or lies within a 500 meter radius of) Line B but not at line B’s endpoint. In cases where the relationship is symmetric between A and B, we combine the lines into a single line. When the T-intersection is not symmetric (the more common case), the top of the T is split by the bottom, and an artificial station called an "auxiliary" station is constructed at that point to keep track of the intersection. Next, lines which do not intersect any other lines at endpoints are removed and all lines are straightened between their endpoints. Finally, lines which do not intersect any stations or other lines are dropped.

The post-algorithm grid data and the GPP asset data are both granular because each observation is at the line or station level. As such, we aggregate assets and grid lines using the same geographical borders as for our employment. Using the simplified network, we construct a symmetric adjacency matrix with entries indicating the number of transmission lines from one region to another.

A4  Data Construction for Curtailment

In this section we discuss the details of the construction of curtailment factors for different countries. The construction entails two steps.

In the first step, we use Tong et al. (2021) to extract data from Figure 2 of their paper, which contain the following variables: generation capacity as a function of the overall demand (e.g. 1x generation is capacity potential as much as the demand), storage level under no storage, 3 hours of storage, 12 hour storage and for the available wind and solar mix. We use a color picker, Digital Color Meter in Macintosh, to extract the color of a particular value in the figure along with a program that extracts the correct scales of the color from the figure. The output for this is the reliability of a renewable resource measured by the authors, i.e. the percentage of energy demand met by the resource.

Using these data as input we then generate the curtailment matrix for each generation capacity. Finally, we interpolate the missing points using linear interpolation, and generate heatmaps for each country for no storage, 2, 4, and 8 hours of storage.

A5  Electrical Line Losses

In this section we derive network losses as a function of power injections and withdrawals in the network. This discussion follows Bohn et al. (1984), where we have provided additional detail
for clarity. We consider lines with resistance \( R_k \) and inductance \( X_k \). The impedance magnitude is
\[ z_k = \left( X_k^2 + R_k^2 \right)^{\frac{1}{2}}, \]
which is sufficient to compute the losses, and we call
\[ \Omega_k = z_k^{-1} \tag{A1} \]
the admittance of a line. Let \( S \) denote the sending end of the line and \( R \) the receiving. Ohm’s law for AC circuits gives that the current flowing out of \( S \) is equal to
\[ I_S = \frac{V_S - V_R}{Z_k}, \]
where \( I_S \) is the current at the sending end of the line, \( V_S - V_R \) is the voltage drop across the line \( k \), and \( Z_k \) is the impedance of the line. These are complex-valued variables, and have an associated magnitude and angle, such that
\[ I_S = |V_S|e^{j\delta_S} - |V_R|e^{j\delta_R} |z_k|e^{jb} \]
where \( \delta_S \) and \( \delta_R \) are the voltage angles in \( S \) and \( R \) and \( b \) is the angle shift induced by the line itself, and following the notation of the electrical engineering literature, \( j = \sqrt{-1} \). Define \( \Delta = \delta_S - \delta_R \). The same expression holds for \( I_R \). Apparent power is defined as
\[ T_S = Z_S + jQ_S = V_S I_S^* \]
\[ T_R = Z_R + jQ_R = V_R I_R^* \]
where \( Z \) is real power (the component of power that can do useful work) and \( Q \) is reactive power. This can be written in polar form as
\[ T_S = \Omega_k |V_S| \left( |V_S|e^{j(b)} - |V_R|e^{j(b-(\Delta_S-\Delta_R))} \right) \]
\[ T_R = \Omega_k |V_R| \left( |V_S|e^{j(b-(\delta_S-\delta_R))} - |V_R|e^{jb} \right) \]
Now assuming that voltages are constant and unitized everywhere (\(|V_S| = |V_k| = 1\)), we can write real power flows out of \( S \) and into \( R \) as
\[ Z_S = \Omega_k (\cos(b_k) - \cos(b_k + \Delta_k)) \]
\[ Z_R = \Omega_k (\cos(b_k) - \cos(b_k - \Delta_k)) \]
Note that with power flowing from \( S \) to \( R \), we have \( P_S > 0 > P_R \). Total real power losses on the line are
\[ \lambda_k = Z_S + Z_R = \Omega_k 2\cos(b_k) - \Omega_k (\cos(b_k - \Delta_k) + \cos(b_k + \Delta_k)) \]
which can be written
\[ \lambda_k = \Omega_k 2\cos(b_k) - \Omega_k 2\cos(b_k)\cos(\Delta_k) = \]
\[ = \Omega_k 2\cos b_k (1 - \cos(\Delta_k)) \approx \Omega_k 2 (\cos(b_k)) \left( \frac{\Delta_k^2}{2} \right) = R_k \Omega_k^2 \Delta_k^2 \tag{A2} \]
where in the second line we have used a second-order approximation to \( 1 - \cos(\Delta) \approx \Delta^2 / 2 \), assuming that \( \Delta_k \) is small, and that \( b_k = \cos^{-1}(R_k / z_k) \) and the definition in (A1). Note also that

\[
Z_k = \Omega_k (\cos(b) - \cos(b + \Delta)) = \Omega_k 2\sin \left( \frac{2b + \Delta_k}{2} \right) \sin \left( \frac{\Delta_k}{2} \right) \approx \Omega_k^2 X_k \Delta_k
\]

where we have taken a second order expansion as \( \Delta_k \) becomes small. Now assuming that \( R_k << X_k \), we have \( \Omega_k^{-1} = z_k \approx X_k \). So we can write

\[
Z_S = \Omega_k \Delta_k \tag{A3}
\]

and using (A2) we finally arrive at

\[
\lambda_k = R_k (Z_S)^2 \tag{A4}
\]

As above, let \( \bar{A} \) be the bus-branch incidence matrix of dimension \( K \times (N - 1) \). Each row consists of a 1, a -1 and zeros. The “direction” of a power-line is arbitrary (i.e. suppose line \( k \) has a 1 for node \( m \) and -1 for node \( n \) in the matrix \( \bar{A} \). Then a positive value for \( Z_k \) indicates net real power flow from \( m \) to \( n \), while a negative value will show power flowing in the other direction). Power injections \( P_j = Y_j^2 - D_j \) must satisfy

\[
P = \bar{A}'Z \tag{A5}
\]

such that power injected equals the sum of flows out of the bus. The \( J^{th} \) node is called the “swing bus”, and power flows there are determined by the energy balance constraint, which can be written

\[
e'P + P_j - \lambda = 0
\]

where \( e \) is a \((J - 1)\) vector of ones, and \( \lambda = \sum_k \lambda_k \) is total system losses. Now, the incidence matrix \( \bar{A} \) allows us to write all the voltage differences \( \Delta \) on each line as

\[
\Delta = \bar{A} \delta
\]

where \( \delta \) is a \( N - 1 \) vector of voltage phase differences on each node from the swing bus. Now using equation (A3), we can write the \( K \times 1 \) vector of power flows \( Z \) as

\[
Z = \bar{\Omega} \Delta
\]

where \( \bar{\Omega} \) is a \( K \times K \) diagonal matrix of line admittances with \( \Omega_k = (R_k^2 + X_k^2)^{-\frac{1}{2}} \) on the diagonal and 0 on the off-diagonal elements. Combine this with (A5) to get

\[
P = \bar{A}'\bar{\Omega}\bar{A}\delta
\]

and use this to get the voltage angles

\[
\delta = (\bar{A}'\bar{\Omega}\bar{A})^{-1} P
\]

in terms of properties of the electricity network.

Finally, the flows on each line are

\[
Z = \bar{\Omega} \bar{A}(\bar{A}'\bar{\Omega}\bar{A})^{-1} P \tag{A6}
\]
Lastly, let $\bar{R}$ be a diagonal matrix with $R_k$ on the diagonal. Using (A4), we can write

$$
\lambda = Z' R Z \quad (A7)
$$

### A6  Proofs and Additional Results

#### A6.1  Lemma 1

**Proof.** With constant resistance-inductance ratio across all lines, we have $\bar{R} = a \Omega^{-1}$ for some positive $a$. Replace $Z$ in (A7) using (A6) and obtain

$$
\lambda = \left( \Omega \bar{A} (\bar{A}' \Omega \bar{A})^{-1} P \right)' a \Omega^{-1} \Omega \bar{A} (\bar{A}' \Omega \bar{A})^{-1} P
$$

$$
= a P' (\bar{A}' \Omega \bar{A})^{-1} \bar{A}' \Omega \bar{A} (\bar{A}' \Omega \bar{A})^{-1} P
$$

$$
= a P' (\bar{A}' \Omega \bar{A})^{-1} P
$$

Define

$$
\bar{B} = a (\bar{A}' \Omega \bar{A})^{-1} = (\bar{A}' \bar{R}^{-1} \bar{A})^{-1}
$$

Now note we can write

$$
\bar{A}' \bar{R}^{-1} \bar{A} = \left( \bar{R}^{-1} \bar{A} \right)' \bar{R}^{-1} \bar{A}
$$

since $\bar{R}$ is diagonal. Then since $\bar{A}$ has linearly independent columns by inspection, $\bar{R}^{-1} \bar{A}$ has linearly independent columns. Recall that a matrix is positive definite if and only if it can be written as $\bar{X} = \bar{Y}' \bar{Y}$ for some possibly rectangular matrix $\bar{Y}$ with independent columns. Since $\bar{A}' \bar{R}^{-1} \bar{A}$ is positive definite, it is invertible, and its inverse $\bar{B}$ is also positive definite. $\square$

#### A6.2  Proposition 1

**Proof.** Begin by with the problem of the planner who picks quantities $Y^C_j$ and $D_j$ to maximise the sum of consumer and producer surplus. Choose a node $b$ as the reference “swing bus”. We can rewrite the planner’s problem by substituting in the power balance constraint into the objective function to get

$$
\max_{\{P_j\},\{D_{js}\}} \sum_j \sum_s p_{js} q_{js} (D_{js}) - \sum_{j \neq b} M_j (P_j + D_j) - M_b \left( P' \bar{B} P + D_b - \sum_{j \neq b} P_j \right) \quad (A8)
$$

subject to

$$
D_j = \sum_s D_{js}
$$

$$
0 \leq P_j + D_j \leq \bar{Y}_j^{max} \quad (A9)
$$

$$
|\Omega \bar{A} (\bar{A}' \Omega \bar{A})^{-1} P| \leq Z^{max} \quad (A10)
$$
Note that $G_j(P,D) = P_j + D_j$ is convex, and $H(P,D) = P^t \tilde{B} P + D_b - \sum_{j \neq b} P_j$ is convex by the positive definiteness of $\tilde{B}$ (see Lemma 1). Then since $M_j$ and $M_b$ are non-decreasing convex functions, $\sum_{j \neq b} M_j(P_j + D_j) + M_b(P^t \tilde{B} P - \sum_{j \neq b} P_j + D_b)$ is convex. Hence the planner’s objective function maximizes a strictly concave function over a convex set defined by the linear inequality constraints in (A9) and (A10).

Then all that remains to show is that this solution can be implemented by choosing prices appropriately. The FOC’s of the original planning problem are

$$p_{js} q'_{js}(D_{js}) - \sum_k \eta_k \frac{\partial Z_k}{\partial D_{js}} - \mu \left(1 + \frac{\partial \lambda}{\partial D_{js}}\right) = 0$$

$$-M'_j(Y^c_j) - \mu_j - \sum_k \eta_k \frac{\partial Z_k}{\partial Y_j} - \mu \left(-1 + \frac{\partial \lambda}{\partial Y_j}\right) = 0$$

where $\eta_k$ are the Lagrange multipliers on additional line capacity on line $k$, $\mu$ is the multiplier on total system generation and $\mu_j$ is the multiplier on extra generating capacity at $j$. Note that by the definition of $P$, we have

$$\frac{\partial Z_k}{\partial D_{js}} = \frac{\partial Z_k}{\partial D_j} = -\frac{\partial Z_k}{\partial Y_j}$$

and

$$\frac{\partial \lambda}{\partial D_j} = -\frac{\partial \lambda}{\partial Y_j}$$

Note that the FOC of final goods producers will satisfy

$$p_{js} q'_{js}(D_{js}) = p^{\varepsilon,D}_j$$

and for unconstrained electricity generators we get

$$M'_j(Y^c_j) = p^{\varepsilon,S}_j$$

So note that setting prices equal to

$$p^{\varepsilon,D}_j = p^{\varepsilon,S}_j = \sum_k \eta_k \frac{\partial Z_k}{\partial D_j} + \mu \left(1 + \frac{\partial \lambda}{\partial D_j}\right)$$

at unconstrained generating nodes will induce optimal behavior.

\[\square\]

**A6.3 Proof of Proposition 2**

We proceed by showing that the market equilibrium with a number of utilities over connected sets of regions coincides with the solution to a global planner’s problem under a certain set of Pareto weights. The problem of the global planner is
for a fixed amount of total fossil fuel inputs $\bar{F}$, where $\Theta_j$ is the Pareto weight on region $j$, subject to

\[
\sum_j \Theta_j \prod_s \left( \sum_{j'} \frac{(q_{jj's})^{\sigma-1}}{\tau_{jj's}} \frac{\bar{f}_{jj's}}{\bar{v}_{jj's}} \right)^{\frac{\bar{f}_{jj's}}{\bar{v}_{jj's}}} \]

The first order conditions for the global planner for the amount of sectoral location goods $q_{jj's}$ shipped from $j'$ to $j$ are

\[
q_{jj's} : \quad \Theta_j \beta_{jj's} \prod_s \left( \sum_{j'} \left( \frac{q_{jj's}}{\tau_{jj's}} \right)^{\sigma-1} \frac{\bar{f}_{jj's}}{\bar{v}_{jj's}} \right)^{\frac{\bar{f}_{jj's}}{\bar{v}_{jj's}}} = \tau_{jj's} \mu_{jj's}^q
\]

where $\mu_{jj's}^q$ denotes the Lagrange multiplier on the resource constraint on sector $s$ good from $j'$. Similarly, we can derive the first order condition for total sectoral electricity demand $D_{js} = e_{js}^1 + e_{js}^2$ as

\[
D_{js} : \quad \mu_{js}^q \frac{v_{js}}{D_{js}} \left( \kappa + \left( \frac{\kappa e_j^F}{\mu_T^F} \right)^{1-\psi} \right)^{\frac{\psi - 1}{\psi}} \left( \frac{\bar{v}_{js}^E}{\bar{v}_{js}^F} \right)^{-\frac{\psi - 1}{\psi}} = \mu_T^E \left( 1 + \frac{d\lambda_T}{dD_{js}} \right) + \sum_{l \in T} \frac{\partial Z_l}{\partial D_{js}} \mu_T^F
\]

where $\mu_T^E$ is the Lagrange multiplier resource constraint on total electricity in network $T$, and $\mu_T^F$ the Lagrange multiplier on the constraint for fossil fuels, and $\mu_T^q$ the multiplier on the constraint on
capacity for line \( l \). Meanwhile, the usage of fossil fuels within sector satisfy

\[
\mu^F_T \left( 1 - \frac{d\lambda}{dY} \right) = \mu^F \frac{\partial G_j}{\partial Y_T} (K_j, Y_j^e - \theta_j K_j^R) + \mu^\text{max}_j \tag{A14}
\]

where \( \mu^F \) is the multiplier on the constraint on fossil fuel usage, and \( \mu^\text{max}_j \) is the multiplier on the maximum power output in \( j \).

The FOC’s for labor and capital allocations across sectors within \( j \) are

\[
\mu^q_{js} v_{js}^K = \mu^K_j \quad \mu^q_{js} v_{js}^L = \mu^L_j \tag{A15}
\]

and the condition for fossil fuel usage is

\[
\mu^q_{js} v_{js}^F \left( \frac{y_{js}}{k_{js}} \left( \frac{e_{js}^2}{\gamma} + f_{js}^2 \right) \right)^{\gamma-1} = \mu^F \tag{A16}
\]

Lastly, it straightforward to show that the social planner maximizes a strictly concave objective function over a convex set, and as such the solution exists and is unique. Moreover, any vector \( \{e_1^f, e_2^f, k_{js}, l_{js}, Y_j^e, f_{js}, q_{js}^e\}, \{\mu^K_j, \mu^F_j, \mu^L_j, \mu^\text{max}_j\} \) which satisfies both the FOCs above, the resource constraints and complementary slackness conditions is this solution. We now show an equivalence of these conditions with the equations governing the market equilibrium. First, the expenditure allocation problem of the consumer in \( j \) is

\[
\max_{\{q_{js}^e\}} \prod_s \left( \left( \sum_j (q_{js}^e)^\sigma-1 \right)^{\frac{1}{\sigma-1}} \right)^{\beta_{js}}
\]

subject to

\[
\sum_j \sum_s p_{js} q_{js}^e = E_j \tag{A17}
\]

which has the FOC for \( q_{js}^e \) as

\[
\beta_{js} \left( \sum_j q_{js}^{\sigma-1} \right)^{\frac{\beta_{js}}{\sigma-1} (q_{js}^e)^{\sigma-2} = p_{js} \Lambda_j, \tag{A18}
\]

where \( p_{js} \) is the price of good \( j \)’s in location \( j \) and \( \Lambda_j \) is the Lagrange multiplier on the expenditure constraint of \( j \) in A17.

Second, the problem of the utility planner is

\[
\max_{\{D_{js}, Y_j^e\}} \sum_j p_{js} q_{js} (D_{js}) - \sum_j p^F G_j (Y_j^e - \theta_j K_j^R)
\]

dsubject to

\[
\sum_j Y_j^e - \sum_j s D_{js} = P' \tilde{B}_T P
\]
\[-Z_T^{\text{max}} \leq \bar{\Omega}_T \bar{A}_T (\bar{A}_T' \bar{\Omega}_T \bar{A}_T)^{-1} P_T \leq Z_T^{\text{max}}\]

\[Y_j^E \leq Y_j^{\text{max}}\]

The FOCs for this problem are

\[p_j q_j'(D_{js}) = \bar{\mu}_T^E (1 + \frac{d\lambda_T}{dD_{js}}) + \sum_{l \in T} \frac{\partial Z_l}{\partial D_{js}} \bar{\mu}_l^Z\]

\[p_j^F \frac{\partial G_j}{\partial Y_j^E} (K_j, Y_j^E - \theta_j K_j^R) = \bar{\mu}_j^E (1 + \frac{d\lambda_j}{dY_j^E}) + \sum_{l \in T} \frac{\partial Z_l}{\partial Y_j^E} \bar{\mu}_l^Z\]

Third, the first order conditions for the representative firm satisfy, in equilibrium,

\[p_j^L v_{ls} \gamma_j = \mu_j^L\]

and the condition for fossil fuel usage is

\[p_j^F y_{js} (\kappa + \left( \frac{\kappa p_j^F}{p_j^E} \right)^{1-\psi})^{\frac{\psi}{\psi-1}} f_{js}^{\psi-1} = p_j^E\]

\[D_{js} = e_j (1 + \frac{v_j^E}{v_j^s})\]

Finally, there are three sets of market clearing conditions,

\[\sum_f q_{fjs} = z_{js} \left( e_{1js}^s \right)^{\psi-1} \left( \kappa_{js} (e_{2js})^\frac{\psi}{\psi-1} + f_{js}^{\psi-1} \right)^{\frac{\psi}{\psi-1}} k_{js}^s l_{js}^s\]

\[\sum_s l_{js} = L_{js}\]

\[\sum_s k_{js} = K_{js}\]

Now we show an equivalence between these two sets of conditions. Suppose that the Pareto weight on \(j\) is equal to the inverse marginal utility of wealth in \(j\) in the competitive allocation \(\Lambda_j^{-1}\). Since
it is straightforward to show that the planner maximize a strictly concave function over a strictly convex set, there is a single solution that solves the planning problem.

Then via relabelling, the same equations can be seen to describe the planners allocation (A13) and the competitive allocation for goods must yield the same values for the Lagrange multiplier when evaluated at the competitive equilibrium quantities.