A Model of Credit, Money, Interest, and Prices*

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Abstract

This paper integrates an implementation of monetary policy through the banking system into an incomplete-markets economy with nominal rigidity. Monetary policy sets policy rates and alters the supply of reserves. These tools grant independent control over credit spreads and an interest-rate target. Through these tools, monetary policy affects the evolution of real interest rates, credit, output, and the wealth distribution. We decompose their effects into a combination of the interest and credit channels that depend on the size of the central bank's balance sheet. The model provides insights regarding when are counter-cyclical central bank balance sheets ideal. This model highlights a trade-off between worse micro economic insurance (insurance across agents) and better macroeconomic insurance (insurance across states).

Keywords: Monetary Economics, Monetary Policy, Credit Channel.

JEL: E31-2, E41-4, E52-2

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1. Introduction

In modern economies, monetary policy (MP) is implemented by setting a policy-rate corridor and supplying reserves.[1] The textbook view is that these tools implement a desired nominal interest rate, and, ultimately, this solely matters to stabilize aggregate demand. In practice, there is more to MP. During the latest crises, all major central banks expanded their balance sheets and reduced their corridor rates—see Figure 1—with the clear intention to ease credit conditions. However, this unprecedented expansion was carried out instinctively, without the backing of a consensual theoretical framework. To this day, there is still an ongoing debate among central bankers regarding the principles that should guide their balance sheet policies, (Schnabel, 2023, e.g.).

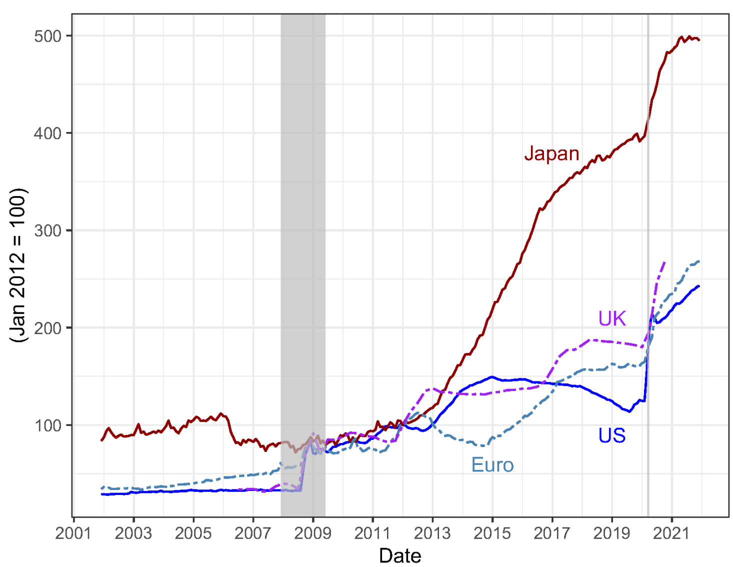


Figure 1: Total Asset Holdings of Major Central Banks

Although there is empirical evidence of the effects of balance sheet policies on credit markets (Kashyap and Stein, 2000, e.g.,), the theoretical foundations of these effects are still a work in progress. This paper is part of the ongoing research program to build theoretical foundations that can be useful in ongoing policy debates. The paper examines MP in an economy where banks face settlement frictions, and the central bank (CB) can control both the quantity of reserves and the interest on reserves (IOR). The study analyzes these effects in an incomplete-markets economy with aggregate-demand externalities.

[1] A policy-rate corridor is defined by an interest rate on reserves, the rate paid to banks holding central bank reserves, and a discount rate, the rate charged to banks that borrow reserves from a central bank.

The paper has two goals. First, to articulate how the supply of reserves and IOR affect credit, interest rates, inflation, and output in this economy. Second, to prescribe guidelines for ideal CB balance sheet management. A new policy insight is that the CB should operate a counter-cyclical balance sheet. This recommendation results from a trade-off between micro and macroeconomic insurance. On the one hand, a small balance sheet induces a higher credit spread, harming idiosyncratic risk insurance (henceforth, micro insurance). On the other hand, a smaller balance sheet increases the sensitivity of aggregate demand to MP and reduces its sensitivity to aggregate shocks (henceforth, macro insurance).

The model includes households that face the risk of being laid off and demand credit to smooth consumption during unemployment. Price and wage rigidities create an aggregate demand externality, where the unemployment rate adjusts to produce an output gap that is consistent with aggregate demand conditions, as in Blanchard and Gali (2010). This unemployment risk generates a precautionary motive, which leads to an aggregate demand amplification.

The transmission of MP includes banks that issue deposits and loans and use CB reserves to settle deposit transfers. The potential shortage of reserves induces a frictional interbank market and results in liquidity premia for nominal deposit and loan rates relative to the IOR. Critically, liquidity premia depend on the size of the CB balance sheet. The CB can control the level of interest rates by settings its IOR and control the credit spread by conducting open-market operations (OMO).

There are three different regimes in which MP operates. In the first regime, reserves are scarce, and the CB runs a corridor system. In this regime, the CB can target both the real savings rate and the credit spread. Reductions in the real savings rate and the credit spread stimulate demand via the interest-rate channel and a credit channel, respectively. In the second regime, reserves are abundant, and the CB runs a floor system. In this system, the CB can only target the real savings rate because OMO are neutral. In the third regime, the IOR is so negative that the nominal deposit rate is fixed at a deposit-zero lower bound (DZLB). In this liquidity trap regime, OMO are also neutral, but reducing the IOR becomes pervasive as they only increase the credit spread. Because in a liquidity trap regime, MP responses are ineffective, the CB balance sheet policy must be designed considering macroeconomic insurance against adverse aggregate shocks.

Macroeconomic insurance stems from the ability of the CB to affect the real interest rates in the long run. In the long run, the CB balance sheet influences the credit spread and, thus, the real interest rate and quantity of credit. By influencing the amount of credit, a counter-cyclical balance sheet policy can enhance macroeconomic insurance by reducing the number of households exposed to credit shocks and increasing their sensitivity to reductions in the IOR. Moreover, starting from higher spreads allows the CB to reduce spreads more during crises. The paper finds that the effectiveness of macroeconomic insurance increases with higher spreads through a sensitivity and a scale effect. On the one hand, higher spreads increase the sensitivity of aggregate demand to interest rates and decrease its sensitivity to credit shocks. On the other hand, increasing spreads leaves more room to lowering rate in the future.

The ability to provide macroeconomic insurance suggests that running counter-cyclical balance sheets may be desirable. In the normative section of the paper, we compare the individual household welfare gains from a reduction in the CB balance sheet. Even though households can anticipate a credit crunch event and, therefore, can self-insure, a smaller balance sheet brings welfare gains to households that value more macro-insurance than micro-insurance. These households belong to the middle class, the majority of the population. An ideal CB balance sheet policy should trade off better macro insurance against the worse micro insurance, taking into consideration which households benefit and which are hurt from this trade off.

Connection with the Literature This paper connects with two common frameworks used to analyze MP. One approach emphasizes the relation between *money and prices* and the other between *interest and prices*. In the money and prices approach, (Lucas and Stokey, 1987; Lagos and Wright, 2005), there is a tight connection between prices and outside money. In the interest and prices approach, the new-Keynesian model, MP controls real rates directly because prices are rigid. Neither framework emphasizes the direct effect of MP on credit. The model here establishes a connection between *money and credit* and studies how these affect the connection between *interest and prices*.[2]

Since 2008, there's been an interest in how MP influences credit markets. A variety of emerging incomplete-market models with room for MP emerged as a result. In fact, a first generation of heterogeneous-agent models, Lucas (1980) and Bewley (1983) were about MP, not about heterogeneity per se, but credit was absent in those models. Credit, of course, has a tradition in heterogeneous-agent models (Huggett, 1993; Aiyagari, 1994, see the early work of), but the literature evolved away from its initial interest in MP. A more recent generation introduced nominal rigidities into these models and shows how MP responses

[2]Recent work in the new-Monetarist approach introduces credit to models where money is a medium of exchange (Berentsen, Camera and Waller, 2007; Williamson, 2012; Gu, Mattesini, Monnet and Wright, 2013, see for example). Rocheteau, Weill and Wong (2021) studies an incomplete-market economy like the one here.

depend on the wealth distribution. For example, Auclert (2019) decomposes the responses to policy into different forces, Kaplan, Moll and Violante (2018) introduce illiquid assets, Werning (2015) and Bilbiie (2020) study amplification, and Guerrieri and Lorenzoni (2017) investigate credit crunches.[3] In all of these studies, MP uses a single instrument, the interest rate. Here, we connect the CB balance sheet to the volume of credit and how this relates to macroeconomic insurance.[4]

The MP implementation here is similar to the one in Bianchi and Bigio (2022) and other models with a realistic implementation of MP.[5] Relative to those papers, the banking side is simplified while the emphasis is put on macroeconomic insurance. Korinek and Simsek (2016) and Farhi and Werning (2016) study how debt limits are a macro-prudential tool to mitigate demand recessions. Here, we argue that the CB balance sheet also has a macro-prudential function. Moreover, we show that by limiting the amount of credit, the CB balance sheet can increase the power of its tools after the arrival of shocks.

On the normative front, Bhandari, Evans, Golosov and Sargent (2021) study optimal interest-rate policies to balance aggregate demand stabilization against insurance considerations. The normative message that MP should actively target spreads is controversial: Curdia and Woodford (2016) and Arce, Nuno, Thaler and Thomas (2020) suggests that a floor system is ideal, but they do so in models where spreads do not bring benefits. Like us, Lee, Luetticke and Ravn (2020) argue that greater spreads bring about macro-economic stability, but in their case, spreads are induced by financial regulation not liquidity regulation. Historically, financial stability has been conceived as a crucial element of MP, as noted in Stein (2012), for example. Here, we showcase that controlling spreads allows MP to enhance the power of its tools and bring macro-prudential benefits.

Organization. Section 2 lays out the core model. Section 3 describes the determination of spreads and real rates and catalogs the MP regimes. Section 4 studies the benefits of countercyclical balance sheet policies. Section 5 concludes.

2. Environment

[3]Other papers study mortgage-refinancing, i.e. Greenwald (2018) and Wong (2019).

[4]Brunnermeier and Sannikov (2012) study outside money in a Bewley-like economy with aggregate shocks. Similarly, Lippi, Ragni and Trachter (2015) study the optimal helicopter drops in a two-agent economy. Other models with cross-agent risk-sharing include Silva (2020) and Buera and Nicolini (2020).

[5]In the representative-agent new-Keynesian model, Piazzesi, Rogers and Schneider (2019) compare floor with corridor regimes, Benigno and Benigno (2021) study how the reserve supply impacts aggregate-demand, and Niepelt (2022) compares reserves with digital currency.

2.1 From Central Bank Balance Sheets to Credit Spreads

In the following model, we embed bank intermediation into a general equilibrium setting with households, firms, and a CB. In this section, we present the banking block. We derive a formula that maps the CB balance sheet size into a credit spread. Appendix A presents the bank balance sheet flows and a timeline corresponding to this block.

Notation. Individual-level variables are denoted with lowercase letters. Aggregate nominal state variables are denoted with capital letters. Aggregate real variables are written in capital calligraphic font. For example, a_t denotes a nominal deposit amount, A_t is the aggregate level of deposits, and $\mathcal{A}_t$ is the aggregate real deposits. We use an ss subscript to denote steady-state objects. The superscripts b, h, and cb denote the aggregate variables held by banks, households, and the CB. We avoid superscripts when the reference is unnecessary.

Banks. We consider a portfolio decision within an interval of time Δ. We take the limit as $\Delta \to 0$ in the general equilibrium block. Because there are no aggregate shocks within the Δ interval, the bank's objective is to maximize expected profits. There is free entry and perfect competition among banks.

The CB supplies a quantity of reserves, M^b, sets an interest on reserves i^m, and a penalty spread over the interest on reserves, ι, at which banks can borrow reserves from an overnight facility. These policy variables are exogenous in this section.

At the start of the Δ interval, banks chose nominal deposits, a, nominal loans, l, and reserve holdings, m. Deposits, loans, and reserves earn corresponding rates i^a, i^l, and i^m. Loan and deposit rates are equilibrium objects; i^m is a policy choice. After the portfolio decision is made, banks face random payment shocks (as in Bianchi and Bigio, 2022). Payment shocks ω are i.i.d. and take two possible values with equal probability, $\omega \in \{-\delta, +\delta\}$ where $\delta \leq 1$. If $\omega = \delta$, a bank receives δa deposits from other banks. If $\omega = -\delta$, the bank transfers δa deposits to other banks. In the aggregate, deposits remain within the banking system. As a result of this movement of liabilities, an asset transfer is needed to settle the transfers. A fundamental assumption is that loans are illiquid in the sense that they must remain with the bank that issues the loan during the Δ interval. As a result, deposit flows are settled with reserves.

After the payment shock, the reserve balance of a bank is:

$$b = m + \min\{\omega, 0\}\, a.$$

That is, if the bank suffers a withdrawal, its reserve balance is reduced and could be negative if m is sufficiently small. If the bank experiences an inflow of deposits, its balance is unchanged until its position is settled. The underlying assumption is that a bank that receives a deposit inflow cannot lend the reserves it is owed even though deposits never leave the banking system. Since ω is random the reserve balance is not controlled perfectly. A bank with a negative balance must close its negative reserve position, either by borrowing reserves from other banks or the CB discount facility.

The aggregate deficit and the surplus balances are the sum of all the positive and negative balances:

this is just Mb if half the banks want to borrow

$$B^{-} \equiv \frac{1}{2}\max\left\{\delta A^{b} - M^{b}, 0\right\} \quad \text{and} \quad B^{+} \equiv \frac{1}{2}\left(M^{b} + \max\left\{M^{b} - \delta A^{b}, 0\right\}\right).$$

The max operator in the surplus balance captures the possibility of a positive balance after an outflow.

Interbank Market. After reserve balances are determined, an interbank market opens. If a bank has a surplus b, it lends the fraction ψ^{+} to other banks; the remainder balances $(1-\psi^{+})\,b$ remain idle. If a bank has a deficit, $-b$, it only borrows the fraction ψ^{-} from other banks; the remainder deficit, $-(1-\psi^{-})\cdot b$ is borrowed from the CB discount facility. The cost of borrowing from the CB is $i^{m}+\iota$.[6]

Clearing in the interbank market requires that the total amount of reserve balances lent is equal to the amount borrowed, $\psi^{-}B^{-} = \psi^{+}B^{+}$. The trading probabilities $\{\psi^{+}, \psi^{-}\}$ result from trading frictions.[7] We follow Afonso and Lagos (2015) and assume that these frictions result from the interbank market's over-the-counter (OTC) nature. Here, we adopt the formulation in Bianchi and Bigio (2017) that renders analytic expressions. In particular, let $\theta = B^{-}/B^{+} \leq 1$ be the interbank "market tightness which measures the ratio of the demand for funds relative to the available funds. Then, the trading probabilities are given by $\{\psi^{+}, \psi^{-}\} = \{1-\exp(-\lambda), \theta^{-1}(1-\exp(-\lambda))\}$ where λ captures the extent of the friction. If $\lambda \to \infty$, the setting collapses to a Walrasian market.[8] In turn, the average rate at which

[6] The CB faces two solvency restrictions: $\iota \geq 0$ and $\iota + i^{m} \geq 0$. Without these constraints, banks could earn arbitrage profits at the expense of the CB.

[7] Trading frictions in the interbank market are well-documented (Ashcraft and Duffie, 2007; Afonso and Lagos, 2014).

[8] When $\lambda \to \infty$, the interbank market becomes Walrasian, and the interbank-market rate converges to i^{m}.

banks lend and borrow reserve is:

$$\bar{i}^f(\theta, i^m) = i^m + \iota \cdot \frac{(\theta + (1-\theta)\exp(\lambda))^{1/2} - 1}{(1-\theta)(\exp(\lambda) - 1)}. \tag{1}$$

Given the trading probabilities, the average interbank rate, and the cost of borrowing from the discount window, the marginal cost of closing a reserve deficit and marginal benefit of a reserve surplus are given by:

$$\chi^- = \psi^-\left(\bar{i}^f - i^m\right) + (1-\psi^-)\cdot\iota, \text{ and } \chi^+ = \psi^+\left(\bar{i}^f - i^m\right).$$

We use these marginal costs to obtain a function that accounts for the average benefit (cost) of a reserve balance, b:

$$\chi(b;\theta,\iota) = \begin{cases} \chi^-(\theta)\, b & \text{if } b \leq 0 \\ \chi^+(\theta)\, b & \text{if } b > 0 \end{cases}. \tag{2}$$

We present further details of the interbank market in Appendix B. For now, all we need to know is that a given market tightness maps into an average cost of closing negative reserve positions and benefits from lending a surplus, as given by χ . This is critical to obtain an equilibrium spread as a function of the CB balance sheet.

The Bank Problem and Equilibrium Spreads. We turn to the bank's problem.

Problem 1 *[Bank's Problem] A bank maximizes its instantaneous expected profits:*

$$\max_{\{l,m,a\}\in\mathbb{R}^3_+} i^l \cdot l + i^m \cdot m - i^a \cdot a + \mathbb{E}\left[\chi(b;\theta,\iota)\right]$$

subject to the budget constraint $l + m = a$.

This bank problem is piece-wise linear. At the individual level, banks will be indifferent among a subset of portfolios—within a cone in the $\{m, a\}$-space. However, at the aggregate level, the ratio of reserves to deposits pins down $\{i^l, i^a\}$ consistent with zero expected profits.[9]

Next, we explain how the ratio of CB liabilities to private liabilities, determines the equilibrium loan and deposit rates. Define the aggregate *liquidity ratio* as $\Lambda \equiv M^b/A^b$, a ratio

[9] This feature is analog to what occurs when firms compete and operate constant returns to scale technologies: firms earn zero profits, their scale is indeterminate, but the ratio of inputs pins down relative prices.

that measures how well bank deposits are covered with reserves. Employing definitions, the interbank market tightness can be expressed in terms of Λ:

$$\theta\left(\Lambda\right) \equiv \max\left\{\frac{\delta}{\Lambda} - 1, 0\right\}. \tag{3}$$

Observe that the interbank tightness decreases with Λ: the higher the liquidity ratio, the lower the fraction of banks that will be in deficit on average.[10] If we substitute (3) into (2) and exploit the zero-profit condition of banks, we obtain the equilibrium borrowing and lending rates.

Proposition 1 *[Nominal Rates and Real Spread] Given $\{\Lambda, i^m\}$, the nominal loan and deposit rates are:*

$$i^l \equiv i^m + \underbrace{\frac{1}{2}\left(\chi^+ + \chi^-\right)}_{\text{liquidity premium}} \quad \text{and} \quad i^a \equiv i^m + \frac{1}{2}\left(\chi^+ + \chi^-\right) \underbrace{-\frac{\delta}{2}\chi^-}_{\text{liquidity risk}}. \tag{4}$$

In all cases, banks earn zero-expected profits (reserve surplus profits net out with reserve deficit losses).

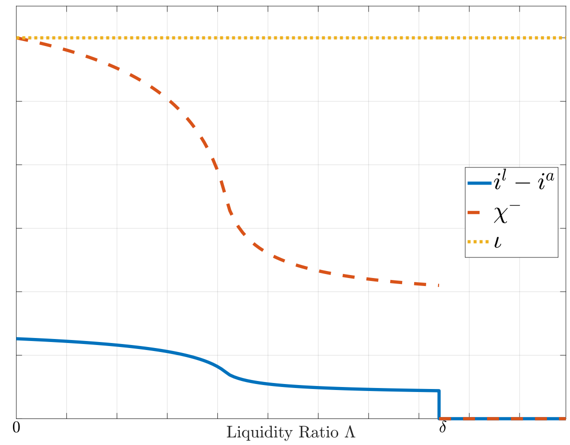
Proposition 1 establishes that the nominal borrowing and lending rates carry different liquidity premia above the IOR. By holding an additional reserve, the bank earns the IOR plus an expected liquidity service: If the bank ends in surplus, the liquidity service is the marginal expected return from lending reserves, χ^+. If the bank ends in deficit, the liquidity service is the value of avoiding the average borrowing cost, χ^-. Since each scenario occurs with equal likelihood, the liquidity service is $\frac{1}{2}\left(\chi^+ + \chi^-\right)$. Loans earn a premium over reserves to compensate for the lack of a liquidity service. In turn, the liquidity premium of deposits also captures the liquidity risk of deposits. If the bank ends in deficit, on the margin, deposits increase the deficit by δ and cost χ^-. Since the probability of a deficit scenario is one-half, the liquidity risk of deposits is $\frac{\delta}{2}\chi^-$.

The loan deposit (credit) spread follows directly:

$$\Delta r = i^l - i^a = \frac{\delta}{2}\chi^-\left(\max\left\{\frac{\delta}{\Lambda} - 1, 0\right\}\right). \tag{5}$$

Figure 2 depicts Δr as functions of Λ. The credit spread is positive when the liquidity ratio is below the amount needed to satisfy the clearing of deficit of all banks $\Lambda < \delta$. Within

[10] Note that, $\theta = 0$ for any $\Lambda \geq \delta$ which reflects that if the liquidity ratio is above δ, no bank is in deficit.



(b) Equilibrium spread given ι and Λ

Figure 2: Borrowing-Lending Spread as Function of Λ(CB Balance Sheet)

Note: The figure plots the components of the equilibrium spread. The figure is constructed using the calibration presented in Section 3.

$[0, \delta]$, the spread decreases with Λ, reflecting that providing more reserves reduces the need to access the discount facility. When reserves are ample, $\Lambda > \delta$, the interbank market shuts down and $\{\chi^+, \chi^-\} = 0$ because no bank is ever in deficit.[11]

The following section embeds the intermediation block into the GE model. Before we proceed, we discuss some features.

Digression: model assumptions. Several modeling features merit discussion. First, a critical assumption is that loans are illiquid. If banks could easily sell loans to cover deposit outflows, there would be no spread. However, there are many reasons why loans are less easy to sell than reserves, such as private information, technical specialization, and legal or transaction costs. Second, banks operate without equity or liquidity constraints. These features are left out for parsimony. While these constraints could change the sensitivity of spreads to the CB balance sheet, they would not alter the paper's conclusion.[12] Third, the CB keeps a constant penalty spread ι. In principle, ι could be an independent instrument, but it is not used in practice. Appendix F.2 presents a discussion of alternative implementations of spreads through MP and relates these alternative implementations to actual CB

[11] If $\Lambda > \delta$, then $i^l = i^a = i^m$. In the knife-edge case where $\Lambda = \delta$, then any rates that satisfy $i^l - i^a \in \left[0, \frac{\delta}{2}\chi^-(0)\right]$ and $i^a = i^m + \left(i^l - i^a\right) \cdot (1-\delta)/\delta\Delta r$ are possible. We abstract away from this case.

[12] If the model features capital requirements or limited capital mobility, bank equity becomes a state variable. While effective reserve requirements are small, current bank regulation imposes minimum liquidity requirements. See den Heuvel (2002) for an early model of bank equity capital. Wang (2019) studies the pass-through of MP as a function of the level of bank equity and liquidity requirements.

practices. Finally, other missing elements are government bonds and non-bank intermediaries. In practice, CB conducts OMO by purchasing government bonds. Here, negative holdings of L^f can be interpreted as government bonds.[13]

2.2 General Equilibrium

We now present the general equilibrium setting. Time is now indexed by $t \in [0, \infty)$. We take a continuous time limit of the bank's problem, letting $\Delta \to 0$, and work directly with the formulas for the deposits and loan rates.[14] The non-financial sector features a measure-one continuum of households, monopolistically competing retailers, and competitive intermediate input firms. Implicitly, banks issue loans to borrower households and issue deposits to lender households. Credit emerges to smooth unemployment spells. Since banks make zero profits, they are simple pass-through entities. It is understood that households own banks, so we do not include bank profits in their household budget constraints since these are zero. The price of the good in terms of money is P_t, and the rate of inflation is $\pi_t \equiv \dot{P}_t / P_t$. As mentioned above, the CB sets the IOR and the money supply.

The Demand-Side Block: Households. Households face a consumption-saving problem with instantaneous utility $U(c_t) \equiv \left(c_t^{1-\gamma} - 1\right) / (1-\gamma)$ over a bundle of differentiated final goods differentiated, $\left\{y_t^j\right\}_{j \in [0,1]}$:

$$c_t = \left(\int_0^1 \left(y_t^j\right)^{1-1/\varepsilon} dj \right)^{\frac{1}{1-1/\varepsilon}}.$$

The individual demand for retail goods follows from the standard cost minimization assumptions: $y_t^i = \left(P_t / p_t^i\right)^{\varepsilon} c_t$ where p_t^j is the price of good j and $P_t = \left(\int_0^1 \left(p_t^j\right)^{1/\varepsilon} dj \right)^{\varepsilon}$. Households only differ in their employment status $z \in \{e, u\}$, where e and u refer to employment and unemployment states. The transition from employment to unemployment, Γ_t^{ue}, is the exogenous constant ξ. By contrast, the transition rate from unemployment to employment, Γ_t^{eu}, is endogenous.

[13] Typically, non-bank intermediaries hold bonds as a settlement instrument. Thus, the non-bank and bank institutions interact through the bond market. The present model can be extended to incorporate government bonds along those lines. Bianchi and Bigio (2022) introduce bonds that are more liquid than loans but less liquid than reserves.

[14] Within Δ time interval, average profits are $\Delta \cdot \pi^b$—all rates are scaled by Δ and the objective is linear. Since bank policy functions are independent of Δ, the equilibrium rates of Proposition 1 also scale with Δ, even as $\Delta \to 0$. The reserve balance, b_t, is a random variable. If we were to track b_t as a function of time, this stochastic process would not be well defined. However, treating $b_{t+\Delta}$ as the single realization of the random variable is well defined, and so is the limit of the deposit and loan rates.

The income of the employed and unemployed are:

$$w_t(e) = \left(1-\tau^l\right) w_t + T_t, \qquad w_t(u) = b + T_t.$$

The real wage of the employed is w_t and is taxed at rate τ^l. b is an exogenous unemployment benefit, and T_t are transfers (taxes) that balance the government's budget. The unemployment benefit b provides a subsistence income to the unemployed when transfers are zero.

Households store wealth in deposits, a_t^h, or currency, m_t^h, and can borrow loans, l_t^h. We introduce currency to articulate a DZLB that limits the stabilization power of interest rates and provides a normative role to the CB balance sheet. By convention, assets are nominal and $\left\{a_t^h, m_t^h, l_t^h\right\} \geq 0$. The real rates on deposits and loans are $r_t^a \equiv i^a - \pi_t$ and $r_t^l \equiv i^l - \pi_t$, respectively. Currency does not earn interest; its real return is $-\pi_t$. Households chose $\left\{a_t^h, m_t^h, l_t^h\right\}$ that satisfy a budget constraint:

$$\left(a_t^h + m_t^h\right)/P_t = s_t + l_t^h/P_t,$$

where s_t is the household's real financial wealth, their only state variable. The law of motion of s_t is:

$$ds_t = \left(r_t^a \frac{a_t^h}{P_t} - \pi_t \frac{m_t^h}{P_t} - r_t^l \frac{l_t^h}{P_t} - c_t + w_t(z)\right) dt. \tag{6}$$

Two immediate observations simplify this law of motion. First, currency and deposits are perfect substitutes. Hence, unlike models with a transaction demand, currency is only held when the nominal deposit rate is less than or equal to zero. The second observation is that households will never hold deposits and loans simultaneously. Combining these observations, we rewrite (6) as:

$$ds_t = \left(r_t(s)\, s - c_t\right) dt + dw_t \text{ where } r_t(s) \equiv \begin{cases} r_t^a \text{ if } s_t > 0 \\ \\ r_t^l \text{ if } s_t \leq 0. \end{cases} \tag{7}$$

An important feature is that employment risk cannot be perfectly insured. In particular, credit is limited by an exogenous *debt limit* $\bar{s} \leq 0$. That is, if $s = \bar{s}$, then $ds_t \geq 0$. In addition to the *debt limit* $\bar{s}$, we introduce a time-varying *borrowing limit,* $\tilde{s}_t \in [\bar{s}, 0]$. If households exceed the borrowing limit, $s_t \leq \tilde{s}_t$, they cannot increase their debt principal, although they can roll over the principal. Thus, $c_t \leq w_t(z)$ in $s \in [\bar{s}, \tilde{s}_t]$. We distinguish between borrowing and debt limits for mathematical convenience and economic appeal.[15]

The solution to the household's problem produces a joint distribution $f(z, s, t)$ with cumulative distribution denoted by F. Denote the fraction of unemployed households as $\mathcal{U}_t$. The distribution satisfies the consistency condition $\mathcal{U}_t = \int_{\bar{s}}^{\infty} f(u, s, t)\, ds = 1 - \int_{\bar{s}}^{\infty} f(e, s, t)\, ds$. The household's problem and the evolution of wealth are summarized by the following HJB-KFE system:

Problem 2 *[Demand Block]*

I) The household's value and policy functions are the solutions to the HJB equation:

$$\rho V(z, s, t) = \max_{\{c\}} U(c) + V_s' \cdot \mu(z, s, t) + \Gamma_t^{z'z}\left[V(z', s, t) - V(z, s, t)\right] + \dot{V}_t, \qquad z' \neq z, \quad (8)$$

where $\mu(z, s, t) \equiv r_t(a)\, s - c + w_t(z)$ *s.t. (i)* $\dot{s} \geq 0$ *at* $s = \bar{s}$*, and (ii)* $c_t \leq w_t(z)$ *in* $s \in [\bar{s}, \tilde{s}_t)$.

II) The wealth distribution f *satisfies:*

$$\frac{\partial}{\partial t} f(z, s, t) = -\frac{\partial}{\partial s}\left[\mu(u, z, t)\, f(u, z, t)\right] - \Gamma_t^{zz'} \cdot f(u, z, t) + \Gamma_t^{z'z} \cdot f(e, z, t), \qquad z' \neq z, \quad (9)$$

with the boundary condition $\lim_{s \to \infty} \sum_{z \in \{u, e\}} F(z, s, t) = 1$.

The problem described in the demand block is common to models where the distribution of wealth is a state variable. The solution to the household HJB equation yields the dynamics of savings, whereas the associated KFE governs the dynamics of f equations. As noted by Achdou, Han, Lasry, Lions and Moll (2021), this model admits a positive mass of agents at the debt limit, $F(z, \bar{s}, t) \geq 0$.

The Supply-Side Block: Employment, Production, and Prices. The supply-side block features nominal price rigidity combined with labor-market frictions. Intermediate goods firms use labor as their sole input and produce identical intermediate goods. Retailers buy the input and sell the differentiated final goods to households in a monopolistically

[15]The borrowing limit allows studying a credit crunch. An unexpected jump in the debt limit is not well-defined mathematically because a positive mass of households may violate debt limits if these increase unexpectedly. This does not apply to the borrowing limit $\tilde{s}_t$. The economic motivation is that banks may want to roll over debt during crises while not increasing debt principals. If loan repayment is suddenly forced, it can trigger a default which may lead to costly underwritings. Although banks may be unwilling to extend loans, rollovers do not consume regulatory capital.

competitive setting. The price adjustment cost arises at the retail level while the labor-matching friction arises at the intermediate-good level.[16] For simplicity, we assume that risk-neutral managers control firms. Managers maximize profits and use a discount rate ρ. We introduce these managers so that the steady-state unemployment rate is independent of the asset market.[17] Profits are taxed and distributed back to households, so the presence of managers is irrelevant.

Retailer j purchases intermediate inputs x_t^j to produce $y_t^j = x_t^j$. The retailer's problem is:

Problem 3 *[Retailer's Problem] Retailer j chooses the change in its individual price $\dot{p}_\tau^j$ to maximize*

$$q\left(p_t^j, t\right) = \max_{\left\{\dot{p}_\tau^j\right\}} \int_t^\infty \exp\left(-\rho\tau\right)\left[\frac{p_\tau^j y_\tau^j - p_\tau x_\tau^j}{P_\tau} - \frac{\Theta}{2}\left(\frac{\dot{p}_\tau^j}{p_\tau^j}\right)^2 Y_\tau\right] d\tau,$$

subject to its individual demand, $y_t^i = \left(P_t/p_t^i\right)^\varepsilon c_t$, and production function $y_t^j = x_t^j$.

The retailer takes the price of the intermediate good, p_t, and the price of the aggregate final good bundle, P_t, as given. The retailer chooses $\dot{p}_\tau^j$ to maximize the present discounted value of real profits minus a Rotemberg price-adjustment cost with coefficient Θ. The state variable of the retailer is p_t^j. The real retailer marginal cost is $mc_t \equiv p_t/P_t$, measured in terms of final goods. The price-adjustment cost are in terms of effort and do not consume resources. The real marginal cost appears in the following standard Phillips curve:

Lemma 1 *The path of prices satisfies the following Phillips curve:*

$$\left(\rho - \frac{\dot{Y}_t}{Y_t}\right)\pi_t = \frac{\epsilon}{\Theta}\left(mc_t - \frac{\epsilon-1}{\epsilon}\right) + \dot{\pi}_t. \tag{10}$$

An intermediate good firm produces one good per worker. Its labor force evolves according to $\dot{n}_t = j_t v_t - \xi n_t$, where ξ is the exogenous job-separation rate, j_t is the per-vacancy job-filling rate, and v_t is the firm's vacancy postings. Vacancies cost μ in real terms, and the firm takes the hiring rate as given. The intermediate good firm's problem is:

[16] We could subsume the intermediate-goods sector into the final goods sector, but we keep sectors separate for a better exposition.

[17] If households owned firms, we would need to assign a firm discount factor. In incomplete-market economies, there is no standard way of assigning a discount factor to the firm. To avoid this complication, we assume a constant firm discount factor.

Problem 4 *[Intermediate Producer's Problem] The intermediate goods firm chooses*

$$G(n,t) = \max_{\{v_\tau\}} \int_t^\infty \exp(-\rho\tau) \left[\frac{p_\tau}{P_\tau} x_\tau - \frac{w_\tau}{P_\tau} n_\tau - v_\tau \mu \right] d\tau,$$

subject to its production function, $x_t = n_t$, and its worker flows $\dot{n}_t = j_t v_t - \xi n_t$.

The intermediate good firm chooses vacancies to maximize its real operational profits net of hiring costs. For that, the firm must post vacancies to offset its worker losses. The state of the intermediate-good firm is its workers.

Wages are determined ex-post. As in Caballero and Hammour (1998), workers can abscond the fraction $(1-\eta)$ of the intermediate good output. If they do so, the firm loses all its production. To avoid losing its production, wages are renegotiated after production, which results in an intermediate-goods labor share of $1-\eta$. Thus, $w_t = (1-\eta) p_t$.[18] Neither workers nor managers can credibly commit to a compensation package or to separation contingencies. The following Lemma characterizes the intermediate good firm's problem:

Lemma 2 *The value of the intermediate-good firm is $G(n,t) = g_t \cdot n_t$ where g_t is the value of a worker:*

$$g_t = \eta \times \int_t^\infty \exp(-(\tau - t)(\rho + \xi)) \, mc_\tau d\tau. \tag{11}$$

Equilibrium Job-Transition Probabilities. The flow of job matches depends on a homogeneous-of-degree-1 matching function, $\Xi(\mathcal{U}_t, v_t)$. It is convenient to define two auxiliary functions:

$$\phi(x) \equiv \Xi(1,x) \quad \text{and} \quad \mathcal{J}(x) \equiv \Xi\left(x^{-1}, 1\right).$$

The equilibrium job-filling rates and job-hiring rates are:

$$j_t = \mathcal{J}\left(\frac{v_t}{\mathcal{U}_t}\right), \qquad \Gamma_t^{eu} = \phi\left(\frac{v_t}{\mathcal{U}_t}\right). \tag{12}$$

The job-finding and job-filling rates are, correspondingly, increasing and decreasing in the vacancy-to-unemployment ratio.

[18] A limit can approximate this construction. Fix a sequence of time intervals $\Delta t, 2\Delta t$, ... For every interval, assume that once workers are hired, workers may threaten the firm to divert $1-\eta$ of output, in which case the firms get zero. With ex-post negotiations, the labor share is $1-\eta$.

Given these rates, aggregate unemployment evolves according to

$$\dot{\mathcal{U}}_t = \xi \left(1 - \mathcal{U}_t\right) - \Gamma_t^{ue} \cdot \mathcal{U}_t. \tag{13}$$

Since labor is indivisible, output is $Y_t \equiv 1 - \mathcal{U}_t$. From the wholesale firm, we obtain a relationship between the vacancy-to-unemployment ratio and the value of a worker.

Proposition 2 *[Beveridge Curve] Fix g_t. Then, the equilibrium vacancy-to-unemployment rate is:*

$$\frac{v_t}{\mathcal{U}_t} = \mathcal{J}^{-1}\left(\frac{\mu}{g_t}\right). \tag{14}$$

Government Block. The CB has two instruments: the money supply, M_t, and the IOR, i_t^m. The CB holds, L_t^{cb} as assets which it matches with its liabilities, $M_t = L_t^{cb}$. The CB uses OMO to alter its balance sheet, $dM_t = dL_t^{cb}$. The money supply is held as reserves, M_t^b, and currency, $M0_t$, so $M_t = M0_t + M_t^b$. Because of interest rate differentials and discount lending, the CB generates revenues:

$$\Pi_t^{cb} = \overbrace{i_t^l L_t^{cb} - i_t^m \left(M_t - M0_t\right)}^{\text{portfolio profits}} + \overbrace{\iota \left(1 - \psi_t^-\right) B_t^-}^{\text{operational revenue}}. \tag{15}$$

The CB earns i_t^l on L_t^{cb}, and pays i_t^m on its liabilities (net of currency), and earns $\iota \cdot \left(1 - \psi_t^-\right) B_t^-$ from its discount loans. The revenues and the surplus of income taxes minus the unemployment benefit are distributed through lump-sum taxes/transfers:

$$P_t T_t = \Pi_t^{cb} + P_t \left(\tau^l \cdot \left(1 - \mathcal{U}_t\right) - b \cdot \mathcal{U}_t\right). \tag{16}$$

T_t adjusts passively. It is convenient to express the government budget constraint in real terms:

$$T_t = \underbrace{\Delta r_t \cdot \sum_{z \in \{u,e\}} \int_0^\infty s f\left(z, s, t\right) ds}_{\text{CB operational revenue } \Pi_t^{cb}/P_t} + \underbrace{\tau^l \left(1 - \mathcal{U}_t\right) - b \cdot \mathcal{U}_t}_{\text{fiscal surplus}}. \tag{17}$$

Markets. There are various markets: the market for reserves and currency, the interbank market, the deposit and loan markets, the intermediate goods market, the final goods market, and the labor market—Appendix C presents the corresponding market clearing conditions. However, a single clearing condition in real wealth summarizes equilibrium in all markets:

Lemma 3 *[Real Wealth Clearing and Walras's Law] Let nominal rates be given by (4) and let transfers be given by (16). Then, market clearing in real wealth,*

$$0 = \sum_{z \in \{u,e\}} \int_{\bar{s}}^{\infty} s f(z,s,t)\, ds \quad \text{for } t \in [0,\infty), \tag{18}$$

implies market clearing in all asset markets. Furthermore, Walras's law holds.

Walras's law implies that we can guarantee to clear the goods market,

$$Y_t = C_t \equiv \sum_{z \in \{u,e\}} \int_{\bar{s}}^{\infty} c_t(z,s) f(s,t)\, ds$$

if (18) is satisfied, and vice-versa.

Equilibrium. A price path is the function $\left\{P(t), i^l(t), i^a(t)\right\} : [0,\infty) \to \mathbb{R}_+^3$. A policy path is the function $\{i_t^m, M_t\} : [0,\infty) \to \mathbb{R}_+^2$. Next, we define an equilibrium.

Definition 1 *[Perfect Foresight Equilibrium.] Given an initial condition for the distribution of wealth, f_0, and an initial price level, P_0, a policy path, a perfect-foresight equilibrium is given by (a) a price path, (b) a path for the real wealth distribution f, (c) a path of aggregate bank holdings $\left\{L_t^b, M_t^b, A_t^b\right\}_{t \geq 0}$, (d) unemployment flows, and (e) household's policy $\left\{c, m^h\right\}$ and value functions $\{V\}_{t \geq 0}$, such that:*

1. *The path of aggregate bank holdings solves the static bank's problem (1),*
2. *The household's policy rule and value functions solve the household's problem (2),*
3. *The unemployment transitions satisfy (13),*
4. *The government's policy path satisfies the budget constraint (15),*
5. *All markets clear (18) and the law of motion for f is consistent with individual decisions.*

A **steady state** occurs when $\frac{\partial}{\partial t} f(z,s,t) = 0$ and $\left\{r_t^a, r_t^l\right\}$ are constant. For the rest of the paper, we consider steady states with zero inflation.

Digression: Model Assumptions. The model marries new-Keynesian price rigidities with labor-market frictions to create unemployment flows resulting from insufficient demand. This differs from the standard new-Keynesian model, which focuses on the intensive margin of hours. The model's emphasis on unemployment allows for business cycle dynamics driven by firing and hiring rates (Davis, Faberman and Haltiwanger, 2006).[19] We use ex-post bargaining, following Caballero and Hammour (1998), so that wages do not depend on the worker's outside option. The labor-market feedback to aggregate demand is similar to that found in other studies.[20]

3. Positive Analysis

A spread between two nominal rates equals the spread between their corresponding real counterparts. This observation implies that if a CB can control credit spreads, it can control real credit. If that is the case, the CB can have long-run real effects. This section investigates the effects of the CB's tools. We start by describing the implementation of credit spreads and then discuss these instruments' long-run and short-run effects.

3.1 From Instruments to Rates and Spreads

From (5), we know that the credit spread is a function Λ_t. A natural question is when does OMO induce effects on Λ_t? The answer depends on the level of the IOR and the CB balance sheet. To articulate this point, we focus on a given instant and momentarily suppress the t sub-index. Toward the characterization, holding an IOR fixed, we compute, $\underline{\theta}(i^m)$, the lowest possible interbank market tightness consistent with a positive deposit rate:

Problem 5 *[Minimal Market Tightness]*

$$\underline{\theta}(i^m) \equiv \min_{\theta \in [0,\infty)} \theta \text{ subject to } \underbrace{i^m + \frac{1}{2}\left(\chi^+(\theta) + (1-\delta)\,\chi^-(\theta)\right)}_{i^a} \geq 0.$$

There is a lower bound on the interbank market tightness because households can exchange deposits for currency. Thus, deposit rates may never drop below the return on currency.

[19] This is the motivation in other new-Keynesian models with unemployment such as Blanchard and Gali (2010); Gertler and Trigari (2009), and Christiano, Trabandt and Walentin (2011).

[20] Michaillat and Saez (2015) obtain a similarly tractable model assuming all matches are destroyed immediately after production. Importantly, unemployment risk feeds back into aggregate demand conditions, a feedback also found in Ravn and Sterk (2020) and Challe (2020).

From the bank's first-order conditions, this induces a lower bound on the interbank-market tightness. The solution to this problem is trivial: when $i^m \geq 0$, $\underline{\theta}(i^m) = 0$. When $i^m < 0$, then $\underline{\theta}(i^m) > 0$ is exactly the tightness consistent with a zero deposit rate. With $\underline{\theta}$ we can obtain the real currency balances as a function of i^m and the ratio of the size of CB balance sheet, $\mathcal{L}^{cb}$, relative to private savings, $\sum_{z\in\{u,e\}} \int_0^\infty sfds$:

$$\mathcal{M}0\left(i^m, \mathcal{L}^{cb}\right) = \frac{\mathbb{I}_{[i^m<0]}}{1+\underline{\theta}(i^m)-\delta} \cdot \max\left\{(1+\underline{\theta}(i^m)) \cdot \mathcal{L}^{cb} - \delta \cdot \int_0^\infty sf(s,t)\,ds, 0\right\}. \quad (19)$$

Thus, the IOR must be sufficiently negative and the balance sheet sufficiently large for currency to be held. The liquidity ratio is then:

$$\Lambda\left(\mathcal{L}^f, i^m\right) = \frac{\mathcal{L}^{cb} - \mathcal{M}0\left(i^m, \mathcal{L}^{cb}\right)}{\sum_{z\in\{u,e\}} \int_0^\infty sfds - \mathcal{M}0\left(i^m, \mathcal{L}^{cb}\right)}. \quad (20)$$

Since the liquidity ratio determines the credit spread, we have a map from $\left\{i^m, \mathcal{L}^{cb}\right\}$ to $\{\Delta r, i^a\}$.

The analytic expression for the liquidity ratio allows us to construct a taxonomy MP regimes, as depicted in Figure 3. The policy instruments, $\left\{i^m, \mathcal{L}^{cb}\right\}$, have different qualitative effects in each regime.

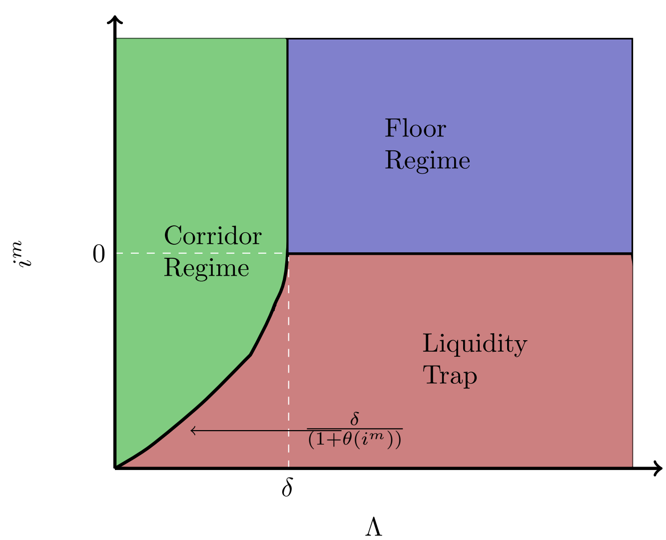


Figure 3: Three Monetary Policy Regimes

Note: The figure presents the MP regimes as a function of Λ and i^m.

The following Proposition characterizes these regimes.

Proposition 3 *[Properties of Equilibrium Rates and Spreads] Consider a distribution of real wealth f and a price level P. The effects of policy instruments are given by:*

Corridor Regime. *If $\Lambda\left(\mathcal{L}^f, i^m\right) < \delta/\left(1+\underline{\theta}\left(i^m\right)\right)$, then $i^l > i^a > i^m$ and $M0 = 0$. Furthermore,*

$$\frac{di^a}{di^m} = 1, \quad \frac{d\Delta r}{di^m} = 0, \quad \frac{di^a}{d\mathcal{L}^{cb}} = -\frac{1}{2\mathcal{L}^{cb}} \frac{\left(\chi_\theta^+ + (1-\delta)\chi_\theta^-\right)}{\Lambda} < 0, \frac{d\Delta r}{d\mathcal{L}^{cb}} = -\frac{\delta}{2\mathcal{L}^{cb}} \frac{\chi_\theta^-}{\Lambda} < 0.$$

Floor system (satiation). *If $i^m > 0$ and $\Lambda\left(\mathcal{L}^{cb}, i^m\right) > \delta$, then $i^l = i^a = i^m$, $\Delta r = 0$ and $M0 = 0$. Furthermore,*

$$\frac{di^a}{di^m} = 1, \quad \frac{d\Delta r}{di^m} = \frac{di^a}{d\mathcal{L}^{cb}}\mathcal{L}^{cb} = \frac{d\Delta r}{d\mathcal{L}^{cb}}\mathcal{L}^{cb} = 0.$$

Liquidity Trap. *If $i^m < 0$ and $\Lambda > \delta/\left(1+\underline{\theta}\left(i^m\right)\right)$, then $i^l > i^a = 0$ and $M0 > 0$. Furthermore,*

$$\frac{d\Delta r}{di^m} = -\delta \frac{\chi_\theta^-\left(\delta\mu - 1\right)}{\left(\chi_\theta^+\left(\delta\mu-1\right) + (1-\delta)\chi_\theta^-\left(\delta\mu-1\right)\right)} < 0, \quad \frac{di^a}{di_t^m} = \frac{di^a}{d\mathcal{L}^{cb}}\mathcal{L}^{cb} = \frac{d\Delta r}{d\mathcal{L}^f}\mathcal{L}^{cb} = 0.$$

and $\frac{d}{d\mathcal{L}^{cb}}\left[M0/P\right] > 0$.

When $\Lambda_t < \delta/\left(1+\underline{\theta}\left(i^m\right)\right)$, MP is in a *corridor system*. In this regime, the passthrough of changes in i^m to the nominal deposit and loan rates is one and, therefore, do not affect the credit spread. In turn, OMO reduces the spread and move both rates differentially. MP is in a corridor system when the CB balance sheet is small and the IOR is sufficiently high.

MP policy is in a *floor system or reserve satiation* when $i_t^m > 0$ and $\Lambda_t > \delta$. When $\Lambda_t > \delta$, no bank faces a reserve deficit, so banks are satiated with reserves, and the interbank market is inoperative. The regime is implemented with a large enough CB balance sheet. In this regime, OMOs are irrelevant in the sense of Wallace (1981) because all assets have equal rates of return. Hence, the CB loses the ability to affect spreads in a *floor system.*

A *liquidity trap* occurs when $i_t^m < 0$ and $\Lambda_t > \delta/\left(1+\theta^{lb}\left(i^m\right)\right)$. In a liquidity trap, the deposit rate is zero. Moreover, OMOs are irrelevant in a different sense from Wallace (1981): When the CB increases reserves by buying loans, the private sector responds by reducing deposits and increasing currency, exactly as required to keep the liquidity ratio constant. We can observe this effect by inspecting the equations (19) and (20). While OMOs are irrelevant in a liquidity trap, a reduction in i_t^m provokes an increase in the loan rate, contrary to the effects in the other regimes: Since the deposit rate cannot fall below zero, banks charge a higher loans rate to break even.[21] Appendix F.2 presents a three dimensional plot that shows how different combinations of i_t^m and Λ_t induce different combinations of deposit and loan rates.

3.2 From Monetary Instruments to Transmission Channels

The previous section characterizes how MP instruments influence the nominal deposit rate and the credit spread. Next, we discuss how these changes impact aggregate output and the wealth distribution. To do so, we derive an aggregate-supply schedule and a generalized Euler equation that renders an analysis akin to the two-equation system of the new-Keynesian model.

Supply Side Block-Aggregate Supply. The aggregate-supply schedule maps an output path into an inflation path consistent with the labor-market flows and the Phillips curve. These are given in the following proposition.

Proposition 4 *[Equilibrium inflation] Fix an initial value of $Y_0 = 1 - \mathcal{U}_0$. Then, for a given path of aggregate output, $\gamma_t^s \equiv \frac{\dot{Y}(t)}{Y(t)}$, the path of inflation is:*

$$\pi_t = \frac{\epsilon}{\Theta} \int_t^{\infty} e^{-(\rho - \gamma_\tau^s)(\tau - t)} \left(mc_t - \frac{\epsilon - 1}{\epsilon} \right) d\tau \tag{21}$$

where the marginal cost, the value of a worker, and the job finding rate is given by:

$$mc_t = g_t \cdot \frac{\overbrace{(\rho + \xi)}^{\text{flow}} - \overbrace{\frac{\alpha}{1-\alpha} \left(\frac{\xi + \dot{\gamma}_\tau^s}{\xi + \gamma_\tau^s} + \gamma_\tau^s \left[\frac{1}{1 - Y_0 \exp\left(\int_0^\tau \gamma_z^s dz\right)} \right] \right)}^{\text{change in worker value}}}{\eta},$$

$$g_t = \frac{\mu}{\underbrace{\eta \mathcal{J} \left(\phi \left(\frac{\xi + \gamma_\tau^s}{1/Y_0 \exp\left(\int_0^\tau \gamma_z^s dz\right) - 1} \right)^{-1} \right)}_{\text{job-filling rate}}}, \quad \text{and} \quad \Gamma_t^{eu} = \frac{\xi + \gamma_t^s}{\exp\left(-\int_0^t \gamma_\tau^s d\tau\right) / Y_0 - 1}. \tag{22}$$

[21] The interest-rate reversal near a DZLB is documented by Heider, Saidi and Schepens (2019); Eggertsson, Juelsrud, Summers and Wold (2019). Koby and Brunnermeier (2022) and Ulate (2020) present models of bank capital and monopoly power where this effect is present.

Inflation is the present value of the difference between marginal costs and an ideal markup. In turn, the value of a worker is the present value of the revenue flows obtained by filling a job in the intermediate good sector. Because revenue flows are proportional to the marginal cost, the marginal cost can be expressed in terms of the current value of a worker and its rate of change. The value of the worker relates to the job-finding rate through the intermediate firm's optimal-hiring decision (14). In equilibrium, the job-finding rate must be consistent with the path of aggregate output. Hence, we have a mapping from aggregate output to inflation.

Intuitively, when output increases above its steady-state level, the value of a worker also increases to induce companies to hire more and keep up with the demand. This results in higher marginal costs, which, in turn, adds inflationary pressure.

Demand Side Block - A Generalized Euler Equation. An aggregate-demand equation maps an inflation path to an aggregate consumption path according to a generalized Euler equation.

Proposition 5 *[Generalized Euler Equation] Given a path of $\pi(t)$, $\Delta r(t)$, and $i^a(t)$, the growth rate of aggregate consumption is:*

$$\gamma(t) \equiv \frac{\dot{C}(t)}{C(t)} = \sum_{z \in \{e,u\}} \int_{\bar{s}}^{\infty} \frac{\dot{c}(z,s,t)}{c(z,s,t)} \cdot g(z,s,t)\, ds + \sum_{z \in \{e,u\}} \frac{\dot{c}(z,s,t)}{c(z,s,t)} \cdot G(z,\bar{s},t),$$

where $g(z,s,t) \equiv \frac{c(z,s,t)}{C(z,s,t)} . f(z,s,t)$ is the distribution of expenditures with CDF, $G(z,s,t)$. Consumption growth among unconstrained agents is:

$$\frac{\dot{c}(z,s,t)}{c(z,s,t)} = \frac{\overbrace{r_t(s) - \rho}^{substitution} + \overbrace{\Gamma_t^{z'z}}^{amplification} \times \overbrace{J(z,s,t)}^{risk}}{\sigma} \quad \text{for } s > \bar{s} \tag{23}$$

where $z' \neq z$ and J is the jump in marginal utility after an employment-status change:

$$J(z,s,t) = U'\left(\frac{c(z',s,t)}{c(z,s,t)}\right) - 1. \tag{24}$$

Among debt and borrowing constraint agents, consumption growth is:

$$\frac{\dot{c}(z,s,t)}{c(z,s,t)} = \frac{\dot{w}_t(z)}{w_t(z)}, \; [\bar{s},\tilde{s}_t) \text{ and } \frac{\dot{c}(z,\bar{s},t)}{c(z,\bar{s},t)} = \frac{\dot{w}_t(z) + (\dot{r}^a + \Delta\dot{r})\,\bar{s}}{w_t(z) + (r^a + \Delta r)\,\bar{s}}. \tag{25}$$

Proposition 5 establishes that aggregate consumption growth is a weighted average of individual consumption growth with weights given by the expenditure distribution. The individual growth rates depend on the household's employment and financially-constrained status. Importantly, the evolution of the wealth distribution, given by the KFE (9), is part of this aggregate demand block as it determines the expenditure weights.

Among *unconstrained* households the Euler equation (23) has three terms. The first term, which appears in linearized models, is the difference between the real rate and the discount rate, scaled by the elasticity of intertemporal substitution, $(r_t(s) - \rho)/\sigma$. Here, the relevant real-interest rate depends on the debtor status. The second term, J, is a risk adjustment that accounts for imperfect insurance. The adjustment is given by the jump in marginal utilities after a change in the employment status. The third term scales the risk adjustment, $\Gamma^{z'z}$. This term captures an aggregate-demand externality because the hiring rate, given by (22), depends on aggregate demand.

Constrained households behave differently. They consume their labor income if they are either borrowing constrained or debt constrained.

Steady State effects of CB Balance Sheet and Micro Insurance. In this model, MP has long-run real effects. Because a spread between two nominal rates equals the spread between the corresponding real interest rates, the control over spreads grants MP the power to affect real long-run rates. We next describe these long-run effects.

When steady-state inflation is zero, output is constant and independent from spreads:

Proposition 6 *[Steady-State Supply] Let $\pi_{ss} = 0$. Steady-state output, unemployment, and the job-finding rate are:*

$$Y_{ss} = \frac{\Gamma^{eu}_{ss}}{\xi + \Gamma^{eu}_{ss}}, \quad \mathcal{U}_{ss} = \frac{\xi}{\xi + \Gamma^{eu}_{ss}}, \quad \Gamma^{eu}_{ss} = \phi\left(\left(\mathcal{J}^{-1}\left(\frac{\mu}{\eta}\frac{\epsilon}{\epsilon - 1}(\rho + \xi)\right)\right)^{-1}\right).$$

To achieve a zero inflation target, the CB sets a consistent IOR. Hence the long-term IOR is restricted by an inflation target. By contrast, the CB can choose its balance sheet freely. Since labor flows are independent of household decisions, the CB balance sheet only affects the extent of implicit insurance in the long run. To see this, we express the savings drift as

$$\mu(z, s, t) = \frac{V_s}{-V_{ss}}\left(r_t(s) - \rho + \Gamma^{z'z} J(z, s, t)\right). \tag{26}$$

Noticing that $-V_s/V_{ss}$ is positive and exploiting the sign of J for the employed and unemployed, we obtain the following relationship:

Corollary 1 *[Savings Drift] In steady state, $\mu_{ss}(e,s) \geq 0 \geq \mu_{ss}(u,s)$ and $s \geq \bar{s}$ binds only for the unemployed.*

This result reveals that households save to self-insure against unemployment spells. Proposition 3 establishes that the CB's balance sheet, $\mathcal{L}^{cb}$, affects the steady-state spread. Thus, even though steady-state unemployment is fixed, credit *is* affected by the CB steady-state balance sheet, as the spread enters in the wealth drifts through $r_t(s)$. Holding r^a fixed, the wealth drift for borrowers increases with the spread, meaning that they take on less debt with greater spreads.

We illustrate the long-run effects of different $\mathcal{L}^{cb}_{ss}$ through Figure 4. The figure reports, for different values of Δr_{ss} induced by a different level of $\mathcal{L}^{cb}_{ss}$, the real wealth distribution for employed and unemployed households (Panels a and b, respectively) and the corresponding real rates (Panel c). Larger spreads compress the wealth distribution, reduce the wealth mass at $\bar{s}$, and depress the real savings rate. Intuitively, the spread is isomorphic to a credit tax.[22]

Equating the of the employed and unemployed to zero at zero wealth, we obtain a lower bound for the spread $\Delta\bar{r}$ such that borrowing is suppressed entirely:

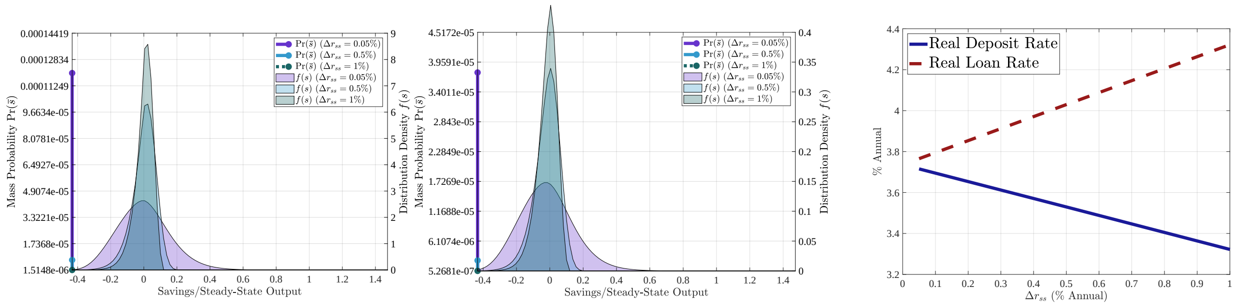
$$\Delta\bar{r} = \xi\left[U'\left(\frac{b}{1-\tau}\right) - 1\right] + \Gamma^{eu}_{ss}\left[1 - U'\left(\frac{1-\tau}{b}\right)\right].$$

If $\iota > \Delta\bar{r}$, the CB can compress the wealth distribution up to the point where credit is entirely repressed. While repressing credit leads to the most equal society, this is the society with the highest consumption risk. In general, by running a corridor system, the CB can implement any spread $\Delta r \in [0, \Delta\bar{r}]$. While not as extreme as repressing credit entirely, positive spreads compress the wealth distribution and limit micro insurance.

Long-run effects are prevalent in classic incomplete-market models, Lucas (1980), Bewley (1983) , and Aiyagari and McGrattan (1998). In those models, the net supply of government debt increases real interest rates. Here, the net supply is zero, but the gross supply has effects because the liquidity properties of assets have effects on spreads.[23]

[22]Like all taxes, the spread has an incidence ton borrowers and lenders hat depends on the deposit supply and loan demand elasticities. From Panel (c), the incidence is evidently higher for borrowers: the increase in the borrowing rate is higher than the decrease in the savings rate. This reflects borrowers are less responsive to changes in real interest rates.

[23]In Lucas (1980) and Bewley (1983) real-long run effects occur because the inflation rate determines the equilibrium level of real balances. In Aiyagari and McGrattan (1998) credit co-exists with government debt and the mix affects real rates.



(a) Employed (b) Unemployed (c) Deposit and Loan Rates

Figure 4: Steady State Effects of Real Spreads.

Note: In panels (a) and (b), the measure of households with assets $\bar{s}$ is a probability mass (left scale), and the measure of households with $s > \bar{s}$ is a probability density (right scale). In panel (c), deposit and loan rates are expressed in annual percentage terms.

Short-Run Equilibrium Dynamics. Next, we discuss the short-term effects of the CB policy instruments. We assume the CB sets i^m according to a Taylor rule:

$$i_t^m = \bar{i}_t^m + \eta_t \cdot (\pi_t - \pi_{ss}), \tag{27}$$

where η_t is a time-varying response to inflation. We allow for time-varying coefficients to isolate the direct effects of policy rates from the endogenous policy response. We let $\{\bar{i}_t^m, \eta_t\}$ to eventually converge and abide by the Taylor principle.

We express the real borrowing and deposit rates as a function i^m and $\mathcal{L}^{cb}$. From Proposition 1, both nominal rates are functions of i^m and the market tightness, $\theta\left(\Lambda\left(\mathcal{L}^{cb}, i^m\right)\right)$. Thus $r_t^l(s) = i^l\left(i^m, \theta\left(\Lambda\left(\mathcal{L}^{cb}, i^m\right)\right)\right) - \pi_t$ if and $r_t^a(s) = i^a\left(i^m, \theta\left(\Lambda\left(\mathcal{L}^{cb}, i^m\right)\right)\right) - \pi_t$.

An equilibrium transition is a fixed point: Proposition 4 delivers a path of inflation $\pi_t[\{\gamma_t^s\}, Y_0]$ and the job-finding rate $\Gamma_t^{ue}[\{\gamma_t^s\}, Y_0]$ consistent with aggregate output. In turn, Proposition 5 delivers a path for aggregate consumption $\gamma_t^d(\{\pi_t\}, \{\Gamma_t^{ue}\}, \{X_t\})$, given the path of inflation, job-finding rate, and any path of policy or borrowing limit. We solve

$$\gamma_t^d(\{\pi_t[\{\gamma_t^s\}, Y_0]\}, \{\Gamma_t^{ue}[\{\gamma_t^s\}, Y_0]\}, \{X_t\}) = \gamma_t^s$$

numerically to describe the effects of policy instruments and explain results through this mathematical structure.

In each experiment, we initiate at a steady state and study an unexpected event at $t = 0$. For each variable $x_t \in X_t$, we initiate its value at some x_{0^+} and assume that the variable

Scenario	Shock x	x_{ss}	x_0	T_x	$\bar{\zeta}_x$
I. IOR	η	1.5	0	1	0.2
	$\bar{i}^m$	0.24%	-2%	1	50
II. Spread	η	1.5	0	1	0.2
	Δr	1%	-1%	1	50
III. Credit Crunch	η	1.5	0	1	0.2
	$\tilde{s}$	$\bar{s}$	$0.03\bar{s}$	1	5

Table 1: Logistic Path - Changes in the IOR.

Note: This table lists the baseline calibration of parameters in the logistic paths in each experiments.

stays put for T_x periods, and then follows a logistic path:

$$x_t = \begin{cases} x_0, & \text{if } t \in [0, T_x] \\ x_{ss} + (x_0 - x_{ss}) \cdot \exp\left(-\bar{\zeta}_x (t - T_x)\right), & \text{if } t > T_x. \end{cases} \tag{28}$$

We next study shocks to $x \in \{\bar{i}^m, \Delta r, \tilde{s}\}$ where $\bar{\zeta}_x > 0$ controls the speed of reversion to steady state. Under each experiment, we shock a variable and the Taylor rule coefficient η to conveniently isolate the effect of the shock from the endogenous response of the Taylor rule. The shocks in the experiment are summarized in Table 1. We use the calibration of the model that we describe in the following section, assuming a constant common discount factor.

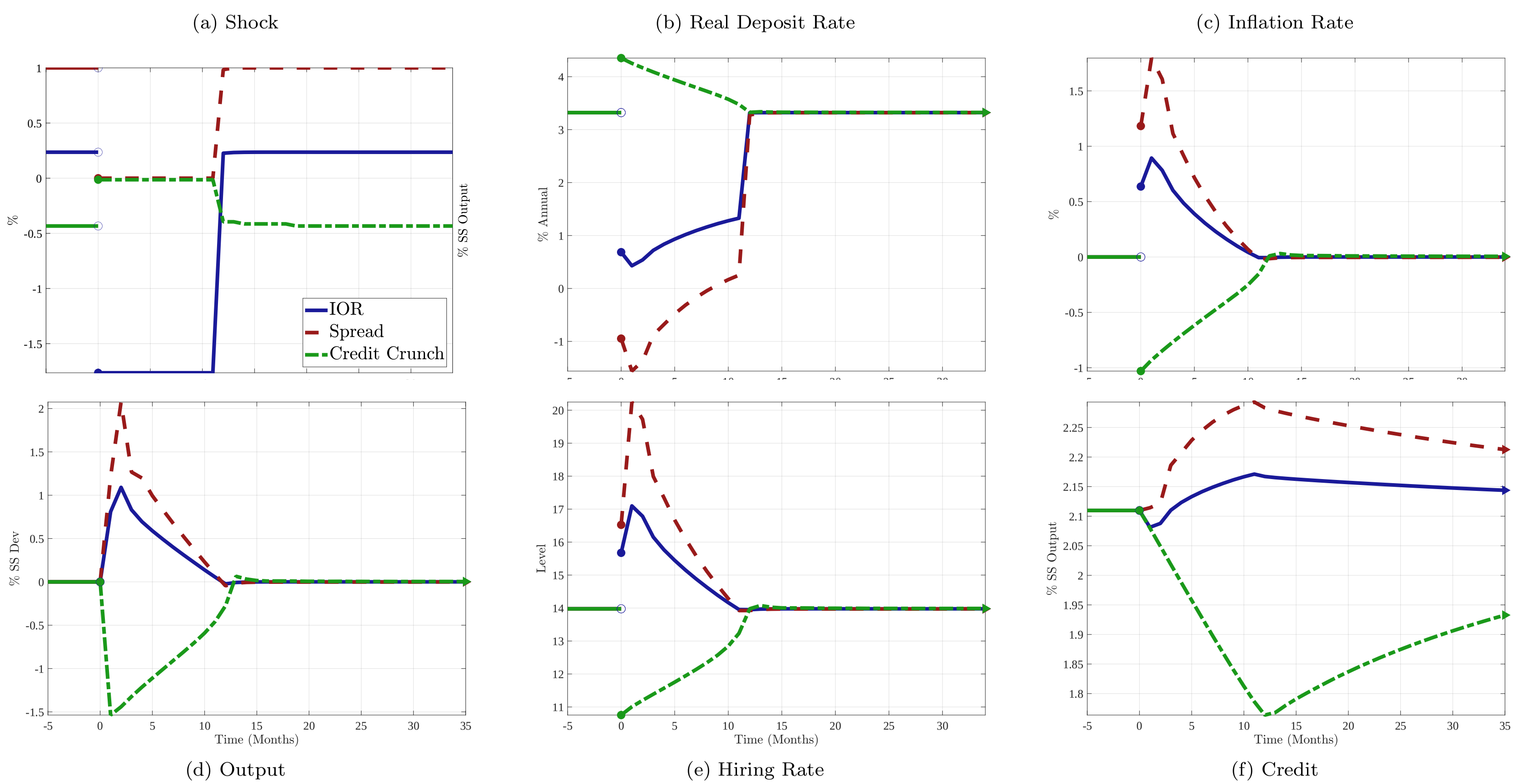


Figure 5: Transition Dynamics after a Credit Crunch.

Note: The figure reports the real wealth distribution, and the responses to credit, rates, inflation and output after an unanticipated credit crunch. In panels (a) and (b), the measure of households with assets $\bar{s}$ is in mass probability (left scale), and the measure of households with $s > \bar{s}$ is in probability density (right scale). The credit crunch is unanticipated at time zero. The net income from credit spread is returned back to households as lump sum transfer. All the rates are expressed in annual percentages, the aggregate output is expressed in percentage deviations from the steady state, and the aggregate credit is expressed in the credit-to-steady state output ratio.

Effects of changes in the IOR and OMO. In a corridor system, the CB can move i^m and Δr. We associate changes in i^m to the interest-rate channel and changes in Δr to the credit channel. We first consider the effects of changes in i^m in which the CB performs OMO to maintain Δr fixed and $\tilde{s}_t = \bar{s}$. Figure 5 presents a transition after a temporary reduction in i^m depicted in Panel (a). The solid lines show the responses corresponding to this exercise. Due to nominal rigidities, the policy induces an identical decline in the real savings and loan rates. Because production is demand determined, the reduction in real rates provokes an output expansion and an immediate inflation response. In tandem, an increase in marginal costs creates inflationary pressure. Increases in marginal costs are necessary to induce the required hires and accommodate the greater demand. During the transition, unemployment falls below its steady-state level. A virtue of the environment is that the output response is hump-shaped even though the policy impulse is a step function.

Changes in the IOR carry real effects that resemble those in incomplete-market new-Keynesian models, e.g. Kaplan et al. (2018); Auclert (2019). In those models, the effects of changes in interest are produced by the direct effects that stimulate aggregate demand and an indirect feedback effect via real wages that further stimulate demand. The real wage also responds here as it is proportional to the marginal cost, mc_t. An indirect effect not present in those models is the aggregate demand feedback to Γ^{eu} captured in the generalized Euler equation. The increase in the job-finding rate amplifies the responses by reducing the precautionary household behavior. Yet another feature is the countercyclical response of credit induced by the decline in unemployment.[24] Another virtue of the model is that the countercyclical response of credit breaks the short-term relationship between the money aggregate M1 and the price level.

A way to understand the quantitative effects of changes in the IOR is to consider the direct impulse on aggregate demand. Because the distribution of consumption is a slow-moving object, the direct impact on aggregate demand of changes in the IOR, $\mathcal{D}_t^i$, approximately is:[25]

$$\mathcal{D}_t^i \approx \underbrace{\sigma^{-1} \times (i_t^m - i_{ss}^m) \times (1 - G(u, \bar{s}, ss))}_{\text{interest-rate channel}}. \tag{29}$$

The expression captures how the change in the policy rate by $(i_t^m - i_{ss}^m)$ affects consumption

[24]The reduction in credit corresponds to a decline in deposits. If the CB does not conduct reverse OMO, the stock of reserves would lead to a decline in Λ. The reduction in credit has ambiguous effects on the CB's profits, but the implied non-Ricardian effects are negligible.

[25]The expression is obtained by computing the change in aggregate demand from a change $(i_t^m - i_{ss}^m)$ in interest rates.

demand scaled by the mass of unconstrained agents, $(1 - G(u, \bar{s}, ss))$ and the intertemporal-substitution coefficient. Indirect effects scale with the direct effects.

To understand the credit channel, we consider a reduction in Δr_t while keeping the IOR constant.[26] Panel (a) shows the path of spreads; the dashed lines show the responses corresponding to this exercise. The reduction in the spread is produced by an OMO that increases the liquidity ratio. Since both rates carry premia over the IOR that decrease with the liquidity ratio, the deposit and savings rates fall. However, the reduction in the lending rate is greater. The effects of these reductions in real rates enter the same way as reductions in the IOR, but in different magnitudes between the borrowers and lenders, as dictated by the Euler equation (23). The credit channel works only under a corridor system.

The direct effect of an OMO tailored to increase the spread by $\Delta r_t - \Delta r_{ss}$ is:

$$\begin{aligned} \mathcal{D}_t^{\Delta r} \approx\ & \underbrace{\sigma^{-1}\left(\int_{\Delta r_{ss}}^{\Delta r_t} \frac{\partial i_t^a/\partial \mathcal{L}_t^{cb}}{\partial \Delta r/\partial \mathcal{L}_t^{cb}} d\Delta r\right) \times (1 - G(u, \bar{s}, ss))}_{\text{credit channel (all unconstrained)}} \\ & + \underbrace{\sigma^{-1}(\Delta r_t - \Delta r_{ss}) \times \left(\sum_{z \in \{e,u\}} G(z, 0, ss) - G(u, \bar{s}, ss)\right)}_{\text{credit channel (unconstrained borrowers)}}. \end{aligned} \quad (30)$$

In this case, the expression captures how the consumption demand of all unconstrained agents, the fraction $(1 - G(u, \bar{s}, ss))$, responds to the change in the deposit rate, i_t^a. The change in i_t^a results from how spreads impact the savings rate, which, from from Proposition 3, we know falls. This reduction in the nominal savings rate also stimulates the consumption of unconstrained borrowers because the loan rate is the savings rate plus the spread. Unconstrained borrowers, $\left(\sum_{z \in \{e,u\}} G(z, 0, ss) - G(u, \bar{s}, ss)\right)$ contribute an extra boost to aggregate demand because the reduction in the spread impacts the loans rate in addition to the effect on the deposit rate. The power to stabilize output by using the IOR and OMO is proportional to $\mathcal{D}_t^i$ and $\mathcal{D}_t^{\Delta r}$, respectively.

Effects of Credit Crunches. We now turn to a credit crunch; the dot-dashed lines show the responses corresponding to this exercise. Starting from $\tilde{s}_{ss} = \bar{s}$, we study a temporary decrease in $\tilde{s}_t$ also depicted in Panel (a). The immediate effect is a decline in the consumption of households that become borrowing-constrained. Aggregating across

[26] In the exercise, an OMO is engineered so Δr_t is brought down to the DZLB during a year, $T_{\Delta r}$, following the logistic path. We isolate the effects from the endogenous response of the Taylor rule by setting $\eta = 0$ during the year of the response.

them, the shock provokes a drop in aggregate demand, leading to an output decline and an unemployment increase. Output falls because deflation does not offset the decline in marginal costs.

For the credit crunch, the direct effect is:

$$\mathcal{D}^{\tilde{s}} \approx \left[\sum_{z \in \{e,u\}} \int_{\bar{s}}^{\tilde{s}_t} \left[\frac{\dot{w}_t(u)}{w_t(u)} - \frac{r_t^a + \Delta r - \rho + \Gamma_{ss}^{z'z} \times J(u,s,ss)}{\sigma} \right] \cdot g(u,s,ss)\, ds \right].$$

This direct effect measures the consumption drop of households that become borrowing constrained, the unemployed with $s \in [\bar{s}, \tilde{s}_t]$.

4. Normative Analysis

We now build the case for a countercyclical CB balance sheet by conducting a welfare analysis. We adapt the model in two ways. First, we incorporate discount-factor shocks to produce a skewed wealth distribution. Second, after we analyze macro-insurance, we let the credit crunch be anticipated. With this extension, the household's self-insurance behavior is taken into account when evaluating their welfare gains.

Discount-factor Shocks and Ex-Ante Heterogeneity. Realistic employment flows are insufficient to produce a meaningful wealth dispersion. While the spirit of this section is to remain parsimonious by using a small number of parameters, we also want to the model have a shot at reproducing the sufficient statistics that govern the macro insurance. From Hubmer, Krusell and Jr. (2020), we know that discount factor shocks allow us to produce much more dispersion in the wealth distribution. Thus, for this section, households have a state-dependent discount factor $\rho(x)$ where $x \in \{\ell, h\}$ and $\rho(\ell) < \rho(h)$ and transition between states with Poisson intensity $\Gamma^{x'x}$.

Calibration. Next, we present a calibration of the model's steady state to the US economy summarized in Table 2. Rates are annualized. The supply side's calibration is autonomous while the demand side's calibration depends on the supply side's. *Job flows:* We calibrate the matching job-destruction rate and the vacancy posting costs to produce realistic job flows. We set the exogenous job separations to $\Gamma^{ue} = 0.8$, following Weingarden (2020).[27]

[27] This figure only includes employer-induced separations and is, therefore, lower than measures that include separations from job-to-job transitions that are not present in the model.

We set the unemployment benefit, b, to 40% of real wages to reproduce the last decade's average replacement rate.[28] The labor tax τ^l is set to 0.3, the average labor income tax.[29] We use a symmetric Cobb-Douglas matching function with unit efficiency. The job finding rate depends on the ratio of the labor-share coefficient to the job posting cost, η/μ. We set this ratio to 13.2 to obtain a steady-state unemployment rate of 5.4%, a figure close to the average unemployment rate from 1948 to 2020 for expansion periods.[30] While this ratio is fixed, we take the limit as $\{\eta, \mu\} \to 0$ to eliminate intermediate firm profits. The elasticity of substitution across goods, ε, is set to induce a 10% retail-sector markup. We set the price-adjustment cost Θ to induce a slope of the Phillips curve equal to 35 which is the estimate in Schaab (2020).

While the labor flows determine the household's income process, their preferences and their debt constraints determine the wealth distribution. *Preferences:* We set the power-utility coefficient γ to 2, a standard value. We assume symmetry in the transition rates between discount-factor states, $\Gamma^{\ell h} = \Gamma^{h\ell} = 1/20$. The rate is chosen so that switches on average occur every 20 years, to mimic life-cycle patterns (Hubmer et al., 2020, as in). We set $\left\{\rho^{\ell}, \rho^{h}\right\}$ to target a real savings rate of 2.4% and to produce sufficient statistics for the right skew tail of the wealth distribution. We set the debt limit to $\bar{s} = -w\left(u\right)$ so that the debt-to-annual-income is one among the unemployed. In the baseline calibration $\tilde{s} = \bar{s}$.

MP parameters: The IOR implements a zero inflation target. The coefficient of the Taylor rule is set to 1.5. For the benchmark calibration, we target a zero credit spread. In the exercises, we move spreads by conducting OMO, and for that, we must also calibrate the banking block. We set the interbank-market market parameters following Bianchi and Bigio (2022).[31]

In choosing the debt limit $\bar{s}$ and the ratio $\rho\left(\ell\right)/\rho\left(h\right)$ we strike a balance between different moments. For the debt limit $\bar{s}$, we balance the debt-constrained population which in the model is 0.25% with the the population with debt, which is close to 58%. For $\rho\left(\ell\right)/\rho\left(h\right)$, we balance the held wealth and the Pareto Tail of the top wealth decile, 50% and 3.1 respec-

[28]The UI replacement rate is the ratio of the claimants' weekly benefit amount (WBA) to the claimants' average weekly wage. The average weekly wage is based on the hourly wage of a usual job claim, normalized to a 40-hour work week. The data is from https://oui.doleta.gov/unemploy/ui_replacement_rates.asp.

[29]The average labor income tax is equal to the U.S. average tax wedge for a single worker from 2000 to 2019. The data is from the OECD's database https://data.oecd.org/tax/tax-wedge.htm.

[30]The interpretation of this ratio η/μ is a 6% recruiting relative to firm profits. With these targets, the hiring rate adjusts accordingly.

[31]We set the payment shock δ to produce the same steady-state interbank market tightness as in that paper, λ, to 2.1. The penalty ι is 10% to account for missing elements such as collateral and stigma (De Fiore, Hoerova and Uhlig, 2018). The implied CB operational revenue over output is 0.15% of GDP, close to the operating profits of the Federal Reserve.

tively. As we explain below, macro-insurance critically depends on the mass of constrained agents and the skewness of the wealth distribution. Regarding the debt-constrained households, there is less mass than in Kaplan et al. (2018). Also, there is less debt in the lower wealth quantiles, but we view this as reasonable because the model lacks housing, collateralized debt or other sources of social insurance. The model further abstracts away from other important features such as illiquid assets (Kaplan et al., 2018, as in), consumption commitments (Chetty and Szeidl, 2007, as in), entrepreneurial activities, capital, life-cycle, health risks, etc. While these features are important, the goal is only to provide a sense of the importance of macro-insurance by reproducing the sufficient statistics that govern it with a minimal set of assumptions.

Parameters			
	Value	**Description**	**Reference**
γ	2	risk aversion	standard calibration
ρ^{ℓ}	0.0175	low discount	real-interest rate / Pareto Tail
ρ^{h}	0.0375	high discount	real-interest rate / Pareto Tail / 4th quantile
$\Gamma^{\ell h}, \Gamma^{h\ell}$	0.05	discount-factor shock	life-cycle approximation
Γ^{ue}	1.2	job separation rate	Weingarden (2020)
ε	10	CES coefficient	average markup
Θ	350	Phillips curve coefficient of 35	Schaab (2020)
b	0.41	replacement rate	standard calibration
η/μ	13.5	profit share/vacancy cost	5.4 unemployment rate
τ^{l}	0.3	labor tax rate	budget balance
Δr_{ss}	0%	steady-state credit spread	benchmark
$\bar{s}$	$-1.0w\,(u)$	credit limit	Indebted and debt-constraint pops.

Table 2: Parameter Values

Note: The table lists the calibrated values of parameters and the corresponding reference/target of calibration.

Macro Insurance and its Sources. We now analyze the macro insurance properties of different CB balance sheets. As emphasized above, expansions in the CB balance sheet are irrelevant under satiation or liquidity-trap regimes. Moreover, reductions in the IOR are contractionary once MP hits the DZLB. Because MP has these limits, it is important to understand how the CB's balance sheet enhances the power of these tools (the stabilization power) and mitigates the effects of a credit crunch (the macro-prudential effect).

To showcase the macro-insurance role of balance-sheet policies, we discuss how the effects of MP tools and the credit crunch shock, quantitatively change with the initial steady-state spreads induced by different CB balance sheets. Figure 6 displays the output response induced by (a) a reduction in the IOR all the way to the DZLB, (b) the same reduction in the IOR together with an OMO that takes the spread to zero, and, finally, (c) during a credit crunch absent a policy response—respectively depicted in Panels (a), (b), and (c). The different output responses correspond to different steady-state values of Δr. The takeaway is that the higher the initial spread, the greater the macro insurance across all margins: Indeed, Panels (a-b) show that a wider spread increases the stabilization power of MP tools. In this exercise, higher spreads increase the power of MP primarily by strengthening the credit channel, Panel (b). In turn, Panel (c) demonstrates the macro-prudential benefit: the output decline after a credit crunch is mitigated with a higher spread.

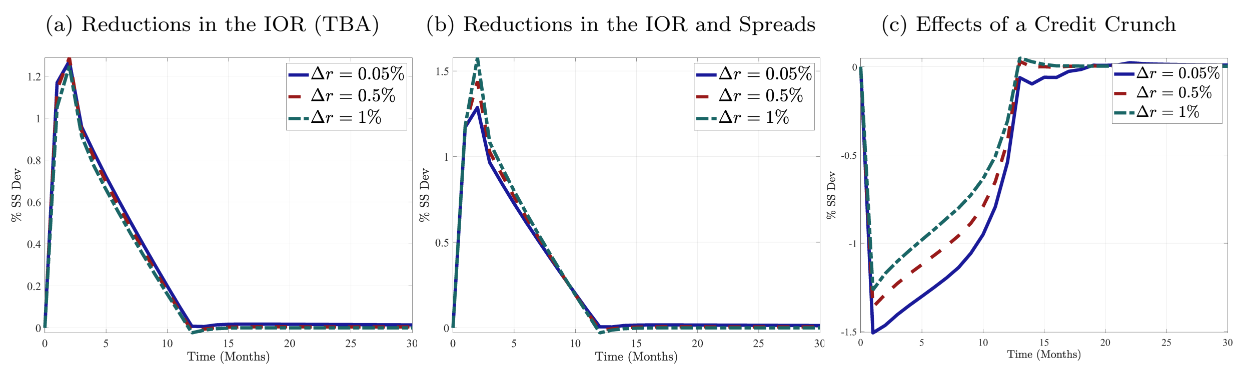


Figure 6: Policy and Credit Crunch for Different Spreads.

Note: The figure reports the responses of aggregate output after an unanticipated reduction in IOR, an unanticipated reduction in IOR and spread, and an unanticipated credit crunch. Aggregate output is expressed in percentage deviations from the steady state. In each panel we simulate the paths under four different steady-state spreads: $\Delta r_{ss} = \{0.05\%, 0.5\%, 1\%\}$.

Why does the CB balance sheet enhance macro insurance? We summarize the mechanisms through the flow chart in Figure 7 and discuss the sources of greater macro insurance analytically. The most expansionary MP response possible occurs when $\Delta r = i_t^m = 0$, the policy studied in Panel (b). We can compute how the change in the sum of the direct effects of changes in the IOR and spreads, change with Δr. Taking derivatives to (29) and (30) and adding both direct effects, we re-arrange terms to obtain:

$$
\begin{aligned}
\frac{\partial \mathcal{D}_t^{\Delta r}}{\partial \Delta r_{ss}} + \frac{\partial \mathcal{D}_t^{i}}{\partial \Delta r_{ss}} \approx \; -\sigma^{-1} \Bigg[& \underbrace{r_{ss}^{l} \left(\sum_{z \in \{e,u\}} \frac{\partial G\left(z,0,ss\right)}{\partial \Delta r_{ss}} - \frac{\partial G\left(u,\bar{s},ss\right)}{\partial \Delta r_{ss}} \right)}_{\text{enhanced loan-rate sensitivity}} \\
& \underbrace{- \, r_{ss}^{a} \left(\sum_{z \in \{e,u\}} \frac{\partial G\left(z,0,ss\right)}{\partial \Delta r_{ss}} \right)}_{\text{enhanced saving-rate sensitivity}} \\
+ \quad & \underbrace{\frac{\partial r_{ss}^{l}}{\partial \Delta r_{ss}} \left(\sum_{z \in \{e,u\}} G\left(z,0,ss\right) - G\left(u,\bar{s},ss\right) \right)}_{\text{loan-rate scale}} \\
+ \quad & \underbrace{\frac{\partial r_{ss}^{a}}{\partial \Delta r_{ss}} \left(1 - \sum_{z \in \{e,u\}} G\left(z,0,ss\right) \right)}_{\text{saving-rate scale}} \Bigg]. \qquad (31)
\end{aligned}
$$

The decomposition reveals the sources of macro insurance: Recall that more negative direct effects reflect a greater expansionary MP shock. Thus, the scale of the expansionary effect is modulated by $-\sigma^{-1}$. Equation (31) presents four terms: the first two terms capture the change in the spread changes the steady-state expenditure distribution, thereby changing the aggregate sensitivity to interest rates. The first term corresponds to the change in the mass of interest rate sensitive borrowers, $\sum_{z\in\{e,u\}} \frac{\partial G(z,0,ss)}{\partial \Delta r_{ss}} - \frac{G(u,\bar{s},ss)}{\partial \Delta r_{ss}}$, times the interest on loans. The second term subtracts the change in the mass switching from a borrower to a lender status times the deposit rate.

The third and fourth terms capture a scale effect. The scale effect captures that if the loan and deposit rates start off at higher levels, the demand stimulus will be greater once both rates are taken all the way to zero in the future. That is, the higher the starting point, the greater the demand stimulus. The scale effects are weighted by the population masses sensitive to the corresponding rates. While the scale effect of the loan rate is positive, the scale effect of the savings rate is negative.

This decomposition showcases the features that contribute to greater macro insurance. First, the mass of debt-constrained households must fall substantially with the spread, $\frac{\partial G(u,\bar{s},ss)}{\partial \Delta r_{ss}} < 0$. In fact, the terms in $\frac{\partial \mathcal{D}_t^{i}}{\partial \Delta r_{ss}}$ only appear in the first two terms. Hence, impacting the mass of debt-constrained agents is key to enhancing the direct effect of the interest-rate channel. Second, the mass of debt-constrained households must fall substantially with the spread, $\frac{\partial G(u,\bar{s},ss)}{\partial \Delta r_{ss}} < 0$. In the calibration, the mass of hand-to-mouth agents is small to start with, leaving little room to move the mass of debt-constrained households, and the population with debt does not change substantially, which is why the responses in Panel (a) are alike for different initial spreads. Of course, the increase in the stabilization power induced by greater spreads increases in alternative calibrations where the wealth distribution is much more sensitive to the spread.

From the decomposition, we can also conclude that to enhance the credit and interest-rate channels with small balance sheets, an economy must feature substantial right skewness of the wealth distribution and must produce a difference in the incidence of the spread on the loans and deposit rates. To see this, assume that the mass of debt-constrained households is zero. If the wealth distribution is insensitive to the spread, the interest-rate sensitivity is not enhanced; the first two terms cancel. In particular, enhancing the interest-rate channel requires that the fraction of the population in debt and debt constrained to be large to begin with. If furthermore, the incidence of the spread on the lending rate and the borrowing rate is the same, $\frac{\partial r^l_{ss}}{\partial \Delta r_{ss}} = -\frac{\partial r^a_{ss}}{\partial \Delta r_{ss}}$, and $\sum_{z\in\{e,u\}} G(z,0,ss) \approx 1/2$, the scale effects cancel out. In other words, leaving more room to reduce the loan rate in the future cancels with the effect of leaving less room to lower the deposit rate if the distribution is symmetric. The decomposition shows that an asymmetry in the incidence of the spread on the loan and deposit rate and a right-tailed wealth distribution are key to strengthening the credit channel. Large differences in interest-rate elasticities result from strong precautionary behavior.

In the current calibration, CB balance sheets strengthen the credit channel but not the interest rate channel as is evident from Panels (a) and (b). While the masses of households in debt and debt constraints do not vary substantially with the initial spread, indicating that the enhanced sensitivity effects are small, the scale effects do vary significantly higher spreads.

Finally, the macro-prudential effect of CB balance sheets is captured by:

$$\begin{aligned}\frac{\partial \mathcal{D}_t^{\tilde{s}}}{\partial \Delta r_{ss}} \approx & \sum_{z\in\{e,u\}} \int_{\bar{s}}^{\tilde{s}_t} \left[\frac{\dot{w}_t(u)}{w_t(u)} - \frac{r^l_t - \rho + \Gamma^{z'z}_{ss} \times J(u,s,ss)}{\sigma}\right] \cdot \frac{\partial g(u,s,ss)}{\partial \Delta r_{ss}} ds \\ & -\sigma^{-1} \frac{\partial r^l_t}{\partial \Delta r_{ss}} G(u,\tilde{s},ss).\end{aligned}$$

The source of greater macro-insurance stems from the change in the mass of agents impacted

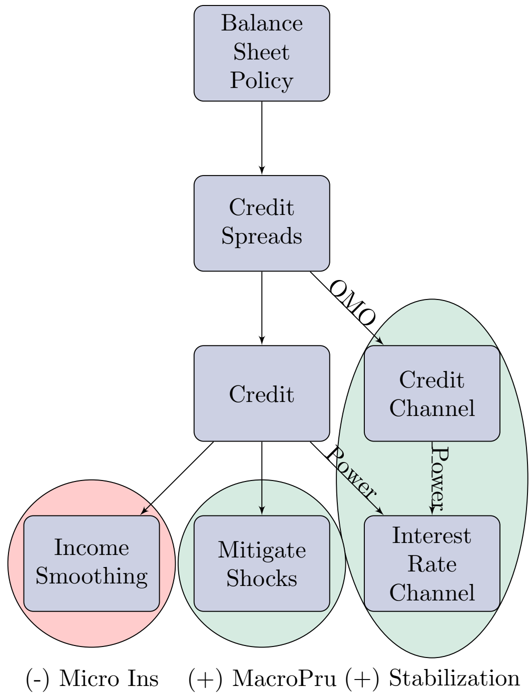


Figure 7: Flow Diagram of Forces in the Model.

by the shock.

So far, we have established that higher spreads enhance the stabilization power of MP and have macro-prudential benefits. In the calibrated version, a stronger credit channel and a weaker effect of credit crunch shocks bring about greater macro economic insurance. However, earlier, we also noted that spreads induce worse micro insurance. Hence, there is a trade-off between better macro insurance and worse micro insurance, which we investigate next through a welfare analysis.

Risky Steady State Welfare. Letting households anticipate an aggregate shock is key to analyzing their welfare. If households do not anticipate the aggregate shock, we cannot establish whether macro insurance is necessary because households will self-insure against the shock. However, letting households anticipate an aggregate shock leads to a common technical challenge: in models where the wealth distribution is a state variable, computing exact solutions is unfeasible. For that reason, the literature often deviates from rational expectations equilibria and studies models where households use approximate models to make forecasts. Here, we want to study different ex-ante CB balance sheets that affect ex-post stabilization. Because we want to compare across ex-ante policies, and model approximations can also differ as we vary the ex-ante policies, we favor an approach where we can compute exact rational expectations equilibria, albeit sacrificing statistical realism.

In this section, we compute the exact model solution when the credit crunch is anticipated, but the credit crunch is a single-shock event. That is, we employ the risky steady-state (RSS) approach of Coeurdacier, Rey and Winant (2011) used to understand the anticipation of risk in representative agent environments. In our context, the RSS is the asymptotic limit of the economy where the credit crunch is expected but has not yet occurred. After the realization of the shock, the transition is deterministic that takes the RSS wealth distribution as an initial condition. Although a single-shock event is a restrictive assumption, it is an approximation to the case of recurrent rare "disaster" events.[32] Moreover, the RSS does not add further complexity to the analysis nor to the computation, as we discuss in Appendix G.[33]

The RSS with idiosyncratic discount shocks is characterized as follows. Given a Poisson arrival rate for a credit crunch, Φ, and a real spread, Δr_{rss}, the RSS is characterized by a modified household HJB equation and a wealth distribution:

Definition 2 *[RSS] A RSS is an equilibrium with the following modified conditions:*

a) the solution to the household's value and policy functions at the RSS are the solutions to:

$$\begin{aligned}\rho(x)\tilde{V}(x,z,s) &= \max_{\{c\}} U(c)+\tilde{V}_s'\cdot\tilde{\mu}(z,s)+\Gamma_{rss}^{z'z}\left[\tilde{V}(x,z',s)-\tilde{V}(x,z,s)\right] \\ &+ \Gamma^{x'x}\left[\tilde{V}(x',z,s)-\tilde{V}(x,z,s)\right]+\Phi\left[V_0(x,z,s)-\tilde{V}(x,z,s)\right],\end{aligned} \tag{32}$$

and $\dot{s}\geq 0$ at $s=\bar{s}$ where $\tilde{\mu}(z,s)\equiv r_{rss}s-c+w_{rss}(z)+T_{rss}$, $x\in\{\ell,h\}$, $z\in\{e,u\}$.

b) after the credit-crunch shock households solve the problem without shocks:

$$\begin{aligned}\rho(x)V_t(x,z,s) &= \max_{\{c\}} U(c)+V_s'\cdot\mu_t(x,z,s)+\Gamma_t^{z'z}\left[V_t(x,z',s)-V(x,z,s)\right] \\ &+ \Gamma^{x'x}\left[V_t(x',z,s)-V_t(x,z,s)\right]+\dot{V},\end{aligned} \tag{33}$$

[32] Disasters are large infrequent shocks, see Barro and Ursúa (2012). Hence, it is reasonable to disregard the effect of a second shock on agent behavior. We conjecture that for sufficiently high discounting and far apart events, the deterministic behavior after the single shock can approximate well the behavior under recurrent shocks—akin to the Turnpike Theorem.

[33] The main challenge is that the distribution of wealth at the start of the crunch, is unknown. However, we no longer need to solve for a fixed point in the space of distributions, but only in the space of functions—the problem is of equal computational complexity as a perfect-foresight transition. To see this, observe that to solve (32), all we need to obtain is a RSS value for the real rate, r_{rss}. With r_{rss}, we obtain consumption rules that solve (32), and from these, we obtain $\tilde{f}_{rss}$ according to an analog of (9). Then, $\tilde{f}_{rss}$ is the initial condition for a transition that converges to a steady state.

subject to the same constraints as the original problem, $x \in \{\ell, h\}$, $z \in \{e, u\}$.

c) the RSS distribution of wealth and employment status, $\tilde{f}_{rss}$, is given by the analog KFE in (9).

d) After the shock, the equilibrium is computed as a deterministic economy with $\tilde{f}_{rss}$ as an initial condition.

e) Given a spread target, Δr, a real deposit rate r^a that solves (18) and T_{rss} is given by (17)—using the distribution $\tilde{f}$.

Notice that the HBJ equation (32) is identical to a steady-state HJB, but one where with intensity Φ, values jump to the time-zero value of the HJB equation (33). In turn, the KFE of the RSS is analog to (9), taking RSS distribution, $\tilde{f}_{rss}$ as an initial condition after the shock. We use this equilibrium concept to compute the utility gains associated with different balance sheet policies.

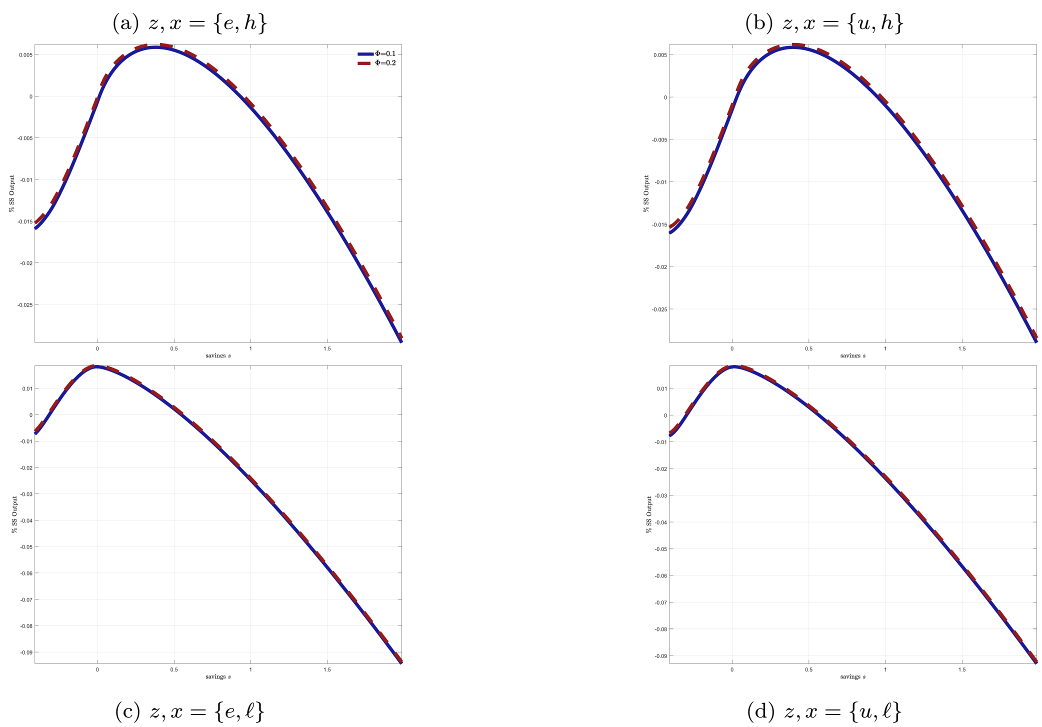


Figure 8: RSS Change in Values from increases in spreads

Note: This figure reports the differences $\beta_{rss}(x, z, s, 0) - \beta_{rss}(x, z, s, 0.1)$. The welfare loss is measured in % deviation of aggregate CE from steady-state output.

Ex-ante CB balance sheets impact the welfare of households depending on their characteristics. Because solving for an optimal RSS spread escapes the scope of this paper, here we only compare the individual welfare of increasing spreads from zero to a positive number for households, as a function of their state. These calculations give us a sense of the trade off that a policymaker should confront when designing optimal policies. To compare the individual welfare gains associated with different balance sheet policies, we compute different RSS associated with different initials spreads Δr. Each RSS yields a value function $V_{rss}(x, z, s; \Delta r)$ for each household. We translate the value of each household into a risk-free annuity (in terms of deviations from the steady-state output) by solving for $\beta_{rss}(x, z, s, \Delta r_{ss})$:

$$U\left(Y_{ss}\left(1 - \beta_{rss}\left(x, z, s, \Delta r_{ss}\right)\right)\right) = \rho\left(x\right) V_{rss}\left(x, z, s; \Delta r\right).$$

Using this welfare metric, we can compare the steady-state welfare gains/losses from raising spreads for different households. Figure 8 plots four panels, each associated with one of the households $\{x, z\} \in \{h, \ell\} \times \{e, u\}$. For each level of savings (shown in terms of annual GDP), the figure reports the change in β_{rss} starting from a spread of zero to a spread of 0.1. That is, we plot $\Delta\beta_{rss}(x, z, s) \equiv \beta_{rss}(x, z, s, 0) - \beta_{rss}(x, z, s, 0.1)$. This small change in the welfare metric gives us a sense of the rate of change of the welfare of different households for a small increase in the spread. We report $\Delta\beta_{rss}(x, z, s)$ for two intensity levels of the aggregate shocks, Φ. A positive value of $\Delta\beta_{rss}$ indicates welfare gains from the spread increase.

Several takeaways emerge from Figure 8. These takeaways reveal the trade-offs involved in the design of optimal balance sheet policies. First, an increase in the spread is detrimental for households at the extremes of the wealth distribution, regardless of $\{x, z\}$. Indeed, spreads lower the savings rate and increase the loans rate, hurting agents at the extremes. This feature reflects worse micro-insurance. By contrast, middle-class benefits from a higher spread. Their welfare gains stem from the greater macro-insurance and the fiscal rebates, which that more than compensate for the spreads they may confront in the future. Comparing across employment status, we find that the unemployed need more savings to benefit from a higher spread. This is because the unemployed expect to continue depleting their savings so the higher loan rate impacts the unemployed more. However, for moderate wealth levels, the unemployed gain because macro-insurance stabilizes output through the hiring rate. Comparing across different shock intensities, higher spreads increase welfare with the intensity of the shock. If we compare high with low-discount households, we find that patient agents experiences greater gains on average. This is because patient agents value more the future benefits from greater macro-insurance—recall that in the RSS, the credit crunch happens in the future so the macro-insurance benefit is discounted.

While this takeaways do not tell us what is the ideal optimal spread, they point toward the trade-offs that a policy maker should confront.

5. Conclusion

The paper shows how the supply of reserves and the IOR affect credit, interest rates, inflation, and output in an economy with credit market frictions, nominal rigidities, and aggregate demand externalities. The model highlights the importance of CB balance sheet policies in shaping the economy's responses to policy and aggregate shocks. The paper identifies three different regimes in which MP can operate, depending on the level of reserves in the banking system, and discusses the implications of each regime for macroeconomic outcomes. Overall, the paper provides insights into the ideal management of the CB balance sheet and how it can be used to promote macroeconomic stability.

The paper's message is presented in the context of a credit crunch, but aggregate discount factor shocks can also be a source of aggregate demand shocks—see Eggertsson and Krugman (2012). Discount factor shocks can capture deeper phenomena like sectoral shocks such as those of the Covid-19 crisis—–see (Guerrieri, Lorenzoni, Straub and Werning, 2022; Bigio, Zhang and Zilberman, 2020). Earlier versions of this paper presented a similar analysis in the context of aggregate discount factor shocks, and the message remains the same: limiting credit is desirable to save monetary policy for the future. We expect similar benefits to be present in models with more interesting financial intermediaries. In either version, we would like to use this framework to solve for optimal CB balance sheets in the future.

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Online Appendix

Not for Publication

Appendix I: Formulas Used in the Paper

A. List of Acronyms and Accounting Identities

List of Acronyms in the Paper. Along the paper we used the following acronyms:

- MP: Monetary Policy
- CB: Central Bank
- DZLB: Deposit Zero Lower Bound
- IOR: Interest on Reserves
- OMO: Open-Market Operation

Household Balance Sheet. The household's balance sheet in in nominal terms is structured as:

Assets	Liabilities
m_t^h	l_t^h
a_t^h	$P_t s_t$

.

Bank Balance Sheet. The balance sheet of an individual bank is structured as:

Assets	Liabilities
m_t^b	a_t^b
l_t^b	

.

CB Balance Sheet. The balance sheet of the CB is structured as:

Assets	Liabilities
L_t^f	M_t

Monetary Aggregates. The monetary aggregates are given by, M_t, the monetary base, $M0_t$, the currency and $M1_t \equiv A_t^b + M0_t$, the highest monetary aggregate.

Money Multiplier. The money multiplier MM_t is the inverse of the liquidity ratio, $MM_t = \Lambda_t^{-1} = A_t^b / (M_t - M0_t)$.

Timeline of interbank transactions. Figure 9 presents the accounting for banks, within a Δ time interval. Unlucky banks get hit by negative withdrawal shocks, which can lead them to a negative balance of reserves in the period. That bank must cover the position by the end of the interval by borrowing funds from other banks, or from the discount window.

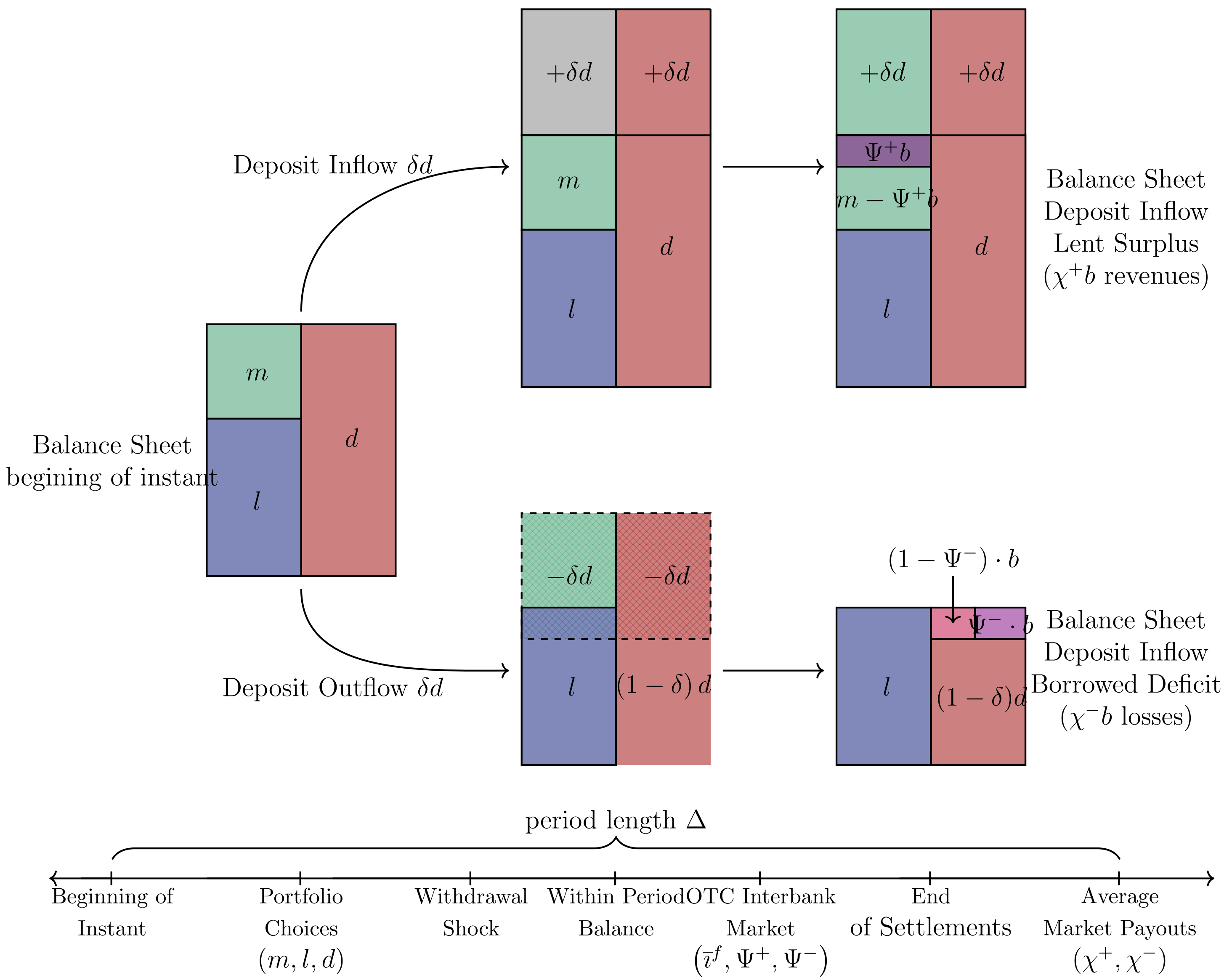


Figure 9: Timeline diagram and banks' balance sheet.

B. Formulas for the Interbank-Market Payments

We adopt the formulation in Bianchi and Bigio (2017) that renders analytic expressions. The interbank market works as follows: The market operates in a sequence of n trading rounds. Given the initial positions $\left\{B_0^-, B_0^+\right\} \equiv \left\{B^-, B^+\right\}$, surplus and deficit positions are matched randomly. When a match is formed between two banks, they agree on an interbank market rate for the transaction. The remaining surplus and deficit positions define a new balance, $\left\{B_1^-, B_1^+\right\}$. New matches are formed, and a new interbank market rate emerges. The process is repeated n times, defining a sequence $\left\{B_j^-, B_j^+\right\}_{j\in 1:n}$ until a final round is reached. Whatever deficit remains is then borrowed from the CB at a cost given by ι.

The interbank market rate of a given trading round is determined by a bargaining problem in which banks take into consideration the matching probabilities and trading terms of future rounds. This produces an endogenous average interbank rate, $\bar{i}^f$. Given trading probabilities, the policy rates and the average rate $\bar{i}^f$, the average rates earned on negative and positive positions are respectively:

$$\chi^- = \psi^- \left(\bar{i}^f - i^m\right) + (1-\psi^-)\cdot\iota, \text{ and } \chi^+ = \psi^+ \left(\bar{i}^f - i^m\right).$$

Banks take into account these costs and benefits when forming their portfolios. To express $\{\chi^-, \chi^+\}$, Bianchi and Bigio (2017) assume that matches are formed on a per-position basis and according to a Leontief matching technology, $\frac{\lambda}{n}\min\left\{B_j^-, B_j^+\right\}$, where λ captures the trading efficiency. Let $\theta = B^-/B^+$ define an initial interbank "market tightness." If $\theta \leq 1$, in the limit $n \to \infty$, trading probabilities across all trading rounds, $\{\psi^+, \psi^-\}$, converge to $\psi^+(\theta) = \theta\left(1-\exp(-\lambda)\right)$ and $\psi^-(\theta) = 1-\exp(-\lambda)$, two expressions consistent with market clearing. With equal bargaining power, the average interbank market rate $\bar{i}^f$ solves

$$\bar{i}^f(\theta, i^m) - i^m = \iota \cdot \frac{\left((\theta + (1-\theta)\exp(\lambda))^{1/2} - 1\right)}{(1-\theta)(\exp(\lambda)-1)}. \tag{34}$$

Bianchi and Bigio (2017) show that iwth a Leontief matching function, the trading probabilities for surpluses and deficit positions along a trading session are:

$$\psi^+(\theta) \equiv \theta\left(1-e^{-\lambda}\right), \qquad \psi^-(\theta) \equiv 1 - e^{-\lambda}.$$

In turn, the expected interbank payments that follow from the sequence of bargaining problems at different trading rounds are given by:

$$\chi^-(\theta,\eta) = \iota\frac{(\theta + (1-\theta)\exp(\lambda))^{1-\eta} - \theta}{(1-\theta)\exp(\lambda)},$$

and

$$\chi^+(\theta,\eta) = \iota\frac{\theta\,(\theta + (1-\theta)\exp(\lambda))^{1-\eta} - \theta}{(1-\theta)\exp(\lambda)}.$$

The resulting average interbank market rate is:

$$\bar{i}^{f}\left(\theta,\eta\right)\equiv i^{m}+\iota\frac{\left(\theta+\left(1-\theta\right)\exp\left(\lambda\right)\right)^{1-\eta}-1}{1-\exp\left(\lambda\right)}. \tag{35}$$

In the paper we set $\eta=1/2$.

C. Markets

Clearing in intermediate goods requires:

$$\int_0^1 x_t^j dj = x_t.$$

The aggregate currency stock is:

$$M0_t \equiv \sum_{z\in\{u,e\}} \int_{\bar{s}}^{\infty} m_t^h (z,s) f (z,s,t) ds.$$

The supply of money is the sum of reserves and currency. Thus the money market clears when:

$$M0_t + M_t^b = M_t. \tag{36}$$

The credit market has two sides: deposit and loan markets. The deposit and loans market clear when:

$$A_t^b = \sum_{z\in\{u,e\}} \int_0^{\infty} a_t^h (z,s) f (z,s,t) ds, a \text{ and } L_t^b + L_t^{cb} = \sum_{z\in\{u,e\}} \int_{\bar{s}}^{0} l_t^h (z,s) f (z,s,t) ds. \tag{37}$$

Finally, the goods market clears when:

$$Y_t = C_t \equiv \sum_{z\in\{u,e\}} \int_{\bar{s}}^{\infty} c_t (z,s) f (s,t) ds. \tag{38}$$

The corresponding labor-clearing conditions are given by the labor flow equations.

Appendix II: Main Proofs

D. Proofs

D.1 Proof of Proposition 1

Preliminary Steps. We are interested in solutions that satisfy $\{l, m, a\} > 0$. An individual bank takes $\{\chi^+, \chi^-, \theta\}$ and the interest rates $\{i^a, i^l\}$ as given. Consider the bank's problem:

$$\pi^b = \max_{\{l,m,a\} \in \mathbb{R}^3_+} \left(i^l \cdot l + i^m \cdot m - i^a \cdot a + \mathbb{E}\left[\chi\left(b; \theta, \iota\right)\right]\right)$$

subject to the budget constraint $l + m = a$ and the law of motion for reserve balances at the CB:

$$b\left(a, m\right) = \begin{cases} m \text{ with probability } 1/2 \\ \\ m - \delta \cdot a \text{ with probability } 1/2 \end{cases}.$$

The objective is homogeneous of degree 1. Hence, profits should be zero, otherwise the solution is unbounded or zero. Although the solution is unbounded, we can determine the equilibrium portfolio shares consistent with given rates. We also know that the objective is piece-wise linear. Thus, it can be transformed into a linear program. However, here we characterize the solution through the principle of optimality.

To obtain a solution, we substitute out l from the budget constraint to obtain a modified problem:

$$\pi\left(m, a\right) \equiv \max_{\{m,a\}} \left(\left(i^m - i^l\right) \cdot m + \left(i^l - i^a\right) \cdot a + \frac{1}{2}\chi^+ m + \frac{1}{2}\left(\chi^+ \cdot \mathbb{I}_{\left[\frac{m}{a} > \delta\right]} + \chi^- \cdot \mathbb{I}_{\left[\frac{m}{a} \leq \delta\right]}\right)\left(m - \frac{\delta}{2}a\right)\right),$$

subject to $a \geq 0$ and $m \in [0, a]$. In a solution with $a > 0$, we can factor deposits and write the objective as:

$$\pi\left(m, a\right) \equiv \max_{a \in \mathbb{R}_{++}} a \cdot \left[\left(i^l - i^a\right) + \vartheta\right]$$

where

$$\vartheta \equiv \max_{\mu \in [0,1]} \left[-\frac{\delta}{2}\left(\chi^- \cdot \mathbb{I}_{[\mu \leq \delta]} + \chi^+ \cdot \mathbb{I}_{\left[\frac{m}{a} > \delta\right]}\right) + \left(\left(i^m - i^l\right) + \frac{1}{2}\chi^+ + \frac{1}{2}\left(\chi^+ \cdot \mathbb{I}_{[\mu > \delta]} + \chi^- \cdot \mathbb{I}_{[\mu \leq \delta]}\right)\right)\mu\right].$$

and we recover $m = \mu \cdot a$. We further write ϑ as:

$$\vartheta \equiv \max\left\{\vartheta^{scarcity}, \vartheta^{satiation}\right\},$$

where

$$\vartheta^{scarcity} \equiv \sup_{\mu \in [0,\delta)} -\frac{\delta}{2}\chi^- + \left(\left(i^m - i^l\right) + \frac{1}{2}\left(\chi^+ + \chi^-\right)\right)\mu$$

and

$$\vartheta^{satiation} = \max_{\mu \in [\delta,1]} -\frac{\delta}{2}\chi^+ + \left(\left(i^m - i^l\right) + \chi^+\right)\mu.$$

Thus, we break ϑ into two sub-problems, one corresponding to the case where the bank has enough reserves to meet the withdrawal shock and always end with a positive balance (satiation) and another where the bank ends with a reserve deficit if it faces a shock (scarcity).

We have three possible cases depending on the three possible signs of $\left(i^m - i^l\right) + \frac{1}{2}\chi^+$. We describe each of these cases next.

Case 1 (not an equilibrium). If $\left(i^m - i^l\right) + \frac{1}{2}\chi^+ > 0$, we argue that this condition cannot occur in equilibrium. In this case, the solution to $\vartheta^{satiation}$ is to set μ as large as possible. Thus, $\vartheta^{satiation} = \left(i^m - i^l\right) + \left(1 - \frac{\delta}{2}\right)\chi^+$ with $\mu = 1$. Since $\chi^- \geq \chi^+$, the solution to $\vartheta^{scarcity}$ is also to set μ as large as possible, which then yields: $\vartheta^{scarcity} = -\frac{\delta}{2}\chi^- + \left(\left(i^m - i^l\right) + \frac{1}{2}\left(\chi^+ + \chi^-\right)\right)\delta = \left(i^m - i^l\right) + \frac{\delta}{2}\chi^+$. Note that

$$\vartheta^{satiation} - \vartheta^{scarcity} = \chi^+\left(1 - \frac{\delta}{2}\right) - \frac{\delta}{2}\chi^+ = \left(1 - \delta\right)\chi^+ > 0,$$

where the inequality follows form $\delta < 1$.

Thus, under the stated case, it is optimal for the bank to be satiated. Therefore, the solution to the bank's problem is to set, $\vartheta = \vartheta^{satiation}$ with $\mu = 1$. However, since $\mu = 1$, this implies that $a = m$. This cannot occur in an equilibrium with positive loans. Hence, in equilibrium, $\left(i^m - i^l\right) + \chi^+ \leq 0$, two cases we evaluate next.

Case 2 (equilibrium with satiation). Assume that $\left(i^m - i^l\right) + \chi^+ = 0$. Then, $\vartheta^{satiation} = -\frac{\delta}{2}\chi^+ \leq 0$ for any $\mu \in [\delta, 1]$. Also, because $\left(i^m - i^l\right) + \chi^+ = 0$, the value of holding a portfolio with reserves scarcity is:

$$\vartheta^{scarcity} \equiv \sup_{\mu \in [0,\delta)} -\frac{\delta}{2}\chi^- + \frac{1}{2}\chi^-\mu.$$

The objective is increasing in μ, and thus, $\vartheta^{scarcity} = 0$ with a solution $\mu \to \delta$. Hence, $\vartheta = \vartheta^{scarcity} \geq \vartheta^{satiation}$.

We now consider the aggregate conditions, setting $\Lambda = \mu$. Since for any $\mu = \Lambda \geq \delta$, we have that $\theta = 0$, we verify that in any case $\chi^+ = 0$. Thus, $\vartheta^{scarcity} = \vartheta^{satiation} = 0$, and any $[\delta, 1]$ is a solution.Thus, from the stated condition we obtain that:

$$\left(i^m - i^l\right) + \chi^+ = 0 \rightarrow \chi^+ = 0, i^m = i^l.$$

We now turn to the deposit rate. Given that $\vartheta = 0$, and that $\pi = 0$ is an equilibrium condition for any a, it must be that

$$i^m = i^a.$$

Thus, if $\mu \geq \delta$, all banks are satiated and all nominal rates are equal:

$$i^m = i^a = i^l. \tag{39}$$

This case corresponds to the solution under satiation where $\Lambda \geq \delta$.

Case 3 (knife-edge and scarcity equilibria). Finally, assume that $\left(i^m - i^l\right) + \chi^+ < 0$. In this case, the solution to $\vartheta^{satiation}$ is attained when $\mu = \delta$. Thus,

$$\vartheta^{satiation} = \left(i^m - i^l + \frac{1}{2}\chi^+\right)\delta < 0.$$

Now, let's consider the value of $\vartheta^{scarcity}$.

Again, we have to separate the analysis case into three possible cases depending now on the sign of $\left(i^m - i^l\right) + \frac{1}{2}\left(\chi^+ + \chi^-\right)$. We do so in the following steps:

Case 3.a (not an equilibrium). Assume that $\left(i^m - i^l\right) + \chi^+ < 0$ and, in addition, that $i^m - i^l + \frac{1}{2}\left(\chi^+ + \chi^-\right) < 0$. Then, the solution to $\vartheta^{scarcity} = -\frac{\delta}{2}\chi^-$, which is obtained when $\mu = 0$. Therefore,

$$\vartheta^{scarcity} - \vartheta^{satiation} = -\left(i^m - i^l + \frac{1}{2}\left(\chi^+ + \chi^-\right)\right)\delta > 0.$$

The inequality follows by hypothesis. Thus, the bank would chose to remain with a reserve scarcity and set $\mu = 0$. However, this solution implies that $m = 0$. Hence, the case cannot occur with positive reserve holdings.

We furthermore know that:

$$\left(i^l - i^m\right) \in \left[\chi^+, \frac{1}{2}\left(\chi^+ + \chi^-\right)\right],$$

because neither case 3.a nor case 1 can occur in equilibrium.

Case 3.b (knife edge case). Assume that $\left(i^m - i^l\right) + \chi^+ < 0$ and$i^m - i^l + \frac{1}{2}\left(\chi^+ + \chi^-\right) > 0$. Then, the solution to $\vartheta^{scarcity} = \left(i^m - i^l + \frac{1}{2}\chi^+\right)\delta$ obtained $\lim \mu \to \delta$. Thus, $\vartheta^{scarcity} = \vartheta^{satiation}$ and hence, the solution to ϑ requires $\mu = \delta$. Since for $\mu = \delta$, we have that $\chi^+ = 0$ then $\vartheta = -\left(i^m - i^l\right)\delta > 0$. Thus,

$$i^m < i^l.$$

From the condition that requires banks ear zero profits, we obtain:

$$i^l - i^a = \left(i^l - i^m\right)\delta.$$

Thus, clearing this condition we obtain:

$$\Delta r \equiv i^l - i^a = \left(i^a - i^m\right)\frac{\delta}{1-\delta}.$$

We re-write the solution to i^a as:

$$i^a = i^m + \Delta r\left(1 - \delta\right)/\delta. \tag{40}$$

Then, from $\left(i^{m}-i^{l}\right)+\frac{1}{2}\chi^{-}(0)>0$ we obtain that

$$i^{m}+\frac{1}{2}\chi^{-}(0)>i^{l}=i^{a}+\dot{\Delta r}\rightarrow\frac{1}{2}\chi^{-}(0)>\frac{i^{a}-i^{m}}{1-\delta}=\frac{\Delta r}{\delta}.$$

Thus, in the point where $\Lambda=\delta$, we have that the spread is given by:

$$\Delta r\in\left[0,\frac{\delta}{2}\chi^{-}(0)\right]. \tag{41}$$

We arrive at the final case next.

Case 3.c (scarcity solutions). Assume that $\left(i^{m}-i^{l}\right)+\chi^{+}<0$ and in addition $i^{m}-i^{l}+\frac{1}{2}\left(\chi^{+}+\chi^{-}\right)=0$. Then, the solution to $\vartheta^{scarcity}=-\frac{1}{2}\chi^{+}\delta$ obtained by any $\mu\in[0,\delta]$. Thus,

$$\vartheta^{satiation}-\vartheta^{scarcity}=\left(i^{m}-i^{l}+\frac{1}{2}\chi^{+}+\frac{1}{2}\chi^{-}\right)\delta=0,$$

where the equality follows by hypothesis. Thus, the bank is indifferent between level of reserves from $\mu\in[0,\delta]$. Therefore, in this case we have that:

$$i^{l}=i^{m}+\frac{1}{2}\left(\chi^{+}+\chi^{-}\right). \tag{42}$$

From the condition that requires $\pi=0$, we obtain:

$$\left(i^{l}-i^{a}\right)+\vartheta=0\rightarrow\left(i^{l}-i^{a}\right)=\frac{\delta}{2}\chi^{-}$$

and, thus, we obtain:

$$i^{a}=i^{m}+\frac{1}{2}\left(\chi^{+}+\chi^{-}\right)-\frac{\delta}{2}\chi^{-}. \tag{43}$$

Summary. Thus, taken together, we know that

$$\left\{i^{l},i^{a}\right\}\in\left[i^{m},i^{m}+\frac{1}{2}\left(\chi^{+}+\chi^{-}\right)\right]$$

If an equilibrium features scarcity of reserves, it must fall in case 3.c and satisfy (42) and (43), as stated in the proposition. If the satiation is strict in the sense that $\Lambda>\delta$, then we are in case 2, and the solution is given by (39). Finally, a knife edge case occurs when $\Lambda=\delta$ the satiation is weak in the sense that $\Lambda=\delta$. In this case, there's a range of values as given by Case 3.b. and equations (41) and (40)**. QED.**

D.2 Proof of Proposition 3

We now derive the effects of policy instruments, $\left\{\mathcal{L}_{t}^{f},i_{t}^{m}\right\}\in\mathbb{R}_{+}\times\mathbb{R}$, on the interest rates and the quantity of currency. Recall that $\{P_{t}\}$ and $\{f_{t}\}$, are pre-determined. Thus, we focus on the instantaneous impact of policies, holding fixed prices and wealth. Since effects are static, for the rest of the proof we avoid time subscripts.

Equilibrium conditions. Recall that from Proposition 1, that we have the following subsystem of equilibrium conditions:

$$i^{l} = i^{m} + \frac{1}{2}\left(\chi^{+}(\theta) + \chi^{-}(\theta)\right) \tag{44}$$

and

$$i^{a} = i^{m} + \frac{1}{2}\left(\chi^{+}(\theta) + (1-\delta)\chi^{-}(\theta)\right) \geq 0. \tag{45}$$

In turn, the spread is given by:

$$\Delta r = \frac{\delta}{2}\chi^{-}(\theta). \tag{46}$$

Also, note that

$$\theta(\Lambda) \equiv \max\left\{\frac{\delta}{\Lambda} - 1, 0\right\}, \tag{47}$$

with derivative $\theta_{\Lambda}(\Lambda) = -\frac{\delta}{\Lambda^2} < 0$.

The liquidity ratio of banks, considering the monetary base and currency holdings, is given by:

$$\Lambda = \frac{M/P}{D/P} = \frac{M/P - M0/P}{\sum_{z\in\{u,e\}}\int_0^{\infty} sf(z,s,t)\,ds - M0/P} = \frac{\mathcal{L}^{f} - M0/P}{\sum_{z\in\{u,e\}}\int_0^{\infty} sf(z,s,t)\,ds - M0/P}. \tag{48}$$

The second equality is obtained by replacing the money-market clearing condition and the deposit-market clearing condition, and then, by replacing the CB balance sheet. Given Λ, the market tightness is given by:

$$\theta(\Lambda) = \delta\left(\frac{\sum_{z\in\{u,e\}}\int_0^{\infty} sf(z,s,t)\,ds - M0/P}{\mathcal{L}^{f} - M0/P}\right) - 1. \tag{49}$$

From the household's problem, we also have that

$$M0 \geq 0 \text{ with strict equality if } i^{a} > i^{m}. \tag{50}$$

Equations (44-50), represent a subsystem of equilibrium conditions.

Organizing results into Policy Regimes. We study the effects of policy changes in three possible regimes, defined as follows:

- We say that MP is in a *Corridor Regime* if $\left\{\mathcal{L}^{f}, i^{m}\right\}$ is such that $i^{a} > i^{m}$ and $M0 = 0$.
- We say that MP is in a *Satiation Regime* if $\left\{\mathcal{L}^{f}, i^{m}\right\}$ is such that $i^{a} = i^{m} > 0$.
- We say that MP is in a *Liquidity Trap* if $\left\{\mathcal{L}^{f}, i^{m}\right\}$ is such that $i^{a} = 0 > i^{m}$ and $M0 > 0$.

These regions do not overlap and cover the space of policies $\{\mathcal{L}^f, i^m\} \in \mathbb{R}_+ \times \mathbb{R}$: By definition, the parameters corridor and satiation regimes do not overlap because $i^a > i^m$ and $i^a = i^m$ cannot occur together and $\chi^+(\theta) + (1 - \delta)\chi^-(\theta)$ is a monotone function in θ, thus separating the space in Λ. By definition, a liquidity trap occurs if $\{\mathcal{L}^f, i^m\}$ induce either (a) $i^a = 0 > i^m$ and $M0 > 0$ or (b) $i^a = i^m = 0$ and $M0 \geq 0$.

We consider the policy effects at the strict interior of these regions. At the boundaries, the system is not differentiable. It is convenient to define the money multiplier as the inverse of the liquidity ratio:

$$\mu \equiv \frac{\sum_{z\in\{u,e\}} \int_0^\infty sf(z,s,t)\,ds}{\mathcal{L}^f}.$$

Case 1 ($i^m > 0$). Assume that $i^m > 0$. Then, because $i^a \geq i^m > 0$, it must be that $M0 = 0$ for any $\mathcal{L}^f$. Note that since $M0 = 0$, combining the CB budget constraint, (15), the money-market clearing condition, (36), and the bank's budget constraint, we obtain that:

$$d\mathcal{L}^f = d\mathcal{M} = d\mathcal{M}^b = -d\mathcal{L}^b, d\mathcal{M}^h = 0.$$

Because $\mathcal{M}^h = 0$, we obtain that (49) is:

$$\Lambda = \frac{\mathcal{L}^f}{\sum_{z\in\{u,e\}} \int_0^\infty sf(z,s,t)\,ds}.$$

By Proposition 1, we know that if $\Lambda < \delta$ then, banks must face a reserve scarcity and thus, an equilibrium must feature:

$$i^l > i^a > i^m.$$

We have reserve scarcity only when:

$$\mathcal{L}^f < \delta \int_0^\infty sf(s,t)\,ds.$$

We now consider the effects of policy variables when there is scarcity and when there isn't.

Case 1.a ($i^m > 0$ and $\Lambda < \delta$). We now consider the policy effects of changes in $\mathcal{L}_t^f$ and i_t^m when $i^m \geq 0$ and $\mathcal{L}^f < \delta \int_0^\infty sf(s,t)\,ds$.

Let's consider first the effects of changes in i_t^m. Taking the differential with respect to i^m in (42) and (43) we obtain:

$$\frac{\partial i^l}{\partial i^m} = 1, \quad \frac{\partial i^a}{\partial i^m} = 1, \quad \frac{\partial \Delta r}{\partial i^m} = 0.$$

Now, let's consider the differential with respect to $\mathcal{L}^f$. Using (43) we obtain:

$$di^a = \frac{1}{2}\left(\chi_\theta^+(\theta) + (1 - \delta)\chi_\theta^-(\theta)\right)\frac{\partial \theta}{\partial \Lambda}\frac{\partial \Lambda}{\partial \mathcal{L}^f}d\mathcal{L}^f.$$

Substituting derivatives yields:

$$di^a = -\frac{1}{2}\left(\chi_\theta^+(\theta) + (1-\delta)\chi_\theta^-(\theta)\right)\frac{\delta}{\Lambda}\frac{d\mathcal{L}^f}{\mathcal{L}^f} < 0.$$

Similarly, the change in the equilibrium spread is:

$$d\Delta r = -\frac{\delta}{2}\chi_\theta^-(\delta\mu - 1)\cdot\mu\cdot\frac{d\mathcal{L}^f}{\mathcal{L}^f} < 0.$$

Finally, we obtain:

$$di^l = di^l + d\Delta r = -\frac{\delta}{2}\left(\chi_\theta^+(\delta\mu-1) + \chi_\theta^-(\delta\mu-1)\right)(\delta\mu-1)\frac{d\mathcal{L}^f}{\mathcal{L}^f} < 0.$$

From this expression, we obtain the semi elasticities displayed in the proposition, for the corridor system regime.

Case 1.b ($i^m > 0$ and $\Lambda \geq \delta$). We now consider the policy effects of changes in $\mathcal{L}_t^f$ and i_t^m when $i^m \geq 0$ and $\mathcal{L}^f > \delta\int_0^\infty sf(s,t)\,ds$. Thus, we have that in this case:

$$\frac{\partial i^l}{\partial i^m} = 1,\; \frac{\partial i^a}{\partial i^m} = 1,\; \frac{\partial \Delta r}{\partial i^m} = 0.$$

Therefore, the effects of changes in the IOR are again given by:

$$\frac{\partial i^l}{\partial \mathcal{L}^f}\mathcal{L}^f = 0,\; \frac{\partial i^a}{\partial \mathcal{L}^f}\mathcal{L}^f = 0,\; \frac{\partial \Delta r}{\partial \mathcal{L}^f}\mathcal{L}^f = 0.$$

Case 2 ($i^m < 0$). Assume that $i^m < 0$. In this case, banks cannot be satiated because $0 > i^m = i^a$ implies that banks would not hold deposits, a situation that we do not considered in body of the paper. However, it is possible to construct equilibria where deposits are zero. In that case, households only hold currency and the stock of loans is held by the CB. This is ruled out in the paper.

We now define implicitly $\theta^{lb}(i^m)$. Thus function maps the IOR to an interbank market tightness that takes the deposit rate to exactly zero using (43):

$$0 \equiv i^m + \frac{1}{2}\left(\chi^+\left(\theta^{lb}(i^m)\right) + (1-\delta)\chi^-\left(\theta^{lb}(i^m)\right)\right) \geq 0. \tag{51}$$

Also, we define,

$$\Xi = \left(\chi^+(0) + (1-\delta)\chi^-(0)\right).$$

If $i^m \in [-\Xi, 0]$, then since $(\chi^+(\theta) + (1-\delta)\chi^-(\theta))$ is strictly increasing and bounded between $[0,\Xi]$, we obtain that $\theta^{lb}(i^m)$ is well defined in $i^m \in [-\Xi, 0]$. We now substitute (49) with $M0$ into (51) to obtain:

$$\frac{1}{2}\left(\chi^+\left(\delta\left(\frac{\sum_{z\in\{u,e\}}\int_0^\infty sf(z,s,t)\,ds}{\mathcal{L}^f}\right) - 1\right) + (1-\delta)\chi^-\left(\delta\left(\frac{\sum_{z\in\{u,e\}}\int_0^\infty sf(z,s,t)\,ds}{\mathcal{L}^f}\right) - 1\right)\right) = -i^m \leq \Xi.$$

From here, we define $\mathcal{L}^{lb}(i^m, f)$ as the CB balance sheet size such that without currency, the interbank market tightness is exactly $\theta^{lb}(i^m)$. We obtain:

$$\delta\left(\frac{\int_0^\infty sf(s,t)}{\mathcal{L}^{lb}(i^m,f)}\right)-1=\theta^{lb}(i^m).$$

Rearranging yields:

$$\mathcal{L}^{lb}(i^m,f)\geq\frac{\delta}{1+\theta^{lb}(i^m)}\int_0^\infty sf(s,t).$$

We now analyze the effects of policy in two cases, depending on whether $\Lambda\leq\mathcal{L}^{lb}(i^m,f)$ or $\Lambda>\mathcal{L}^{lb}(i^m,f)$.

Case 2.a ($i^m<0$ and $\mathcal{L}^f<\mathcal{L}^{lb}(i^m,f)$). Next, we show that if $\mathcal{L}^f<\mathcal{L}^{lb}(i^m,f)$ and $i^m<0$, the effects of policy are identical to those of Case 1.a. First, observe that,

$$\theta^{lb}(i^m)=\delta\left(\frac{\int_0^\infty sf(s,t)}{\mathcal{L}^{lb}(i^m,f)}\right)-1<\delta\left(\frac{\sum_{z\in\{u,e\}}\int_0^\infty sf(z,s,t)\,ds-M0/P}{\mathcal{L}^f-M0/P}\right)-1=\delta\mu-1=\theta,$$

for any $M0$ where the inequality follows because:

$$\begin{aligned}\frac{\partial}{\partial M0}\left[\delta\left(\frac{\sum_{z\in\{u,e\}}\int_0^\infty sf(z,s,t)\,ds-M0/P}{\mathcal{L}^f-M0/P}\right)\right] &= \\ \frac{1}{P}\mu\left(\frac{1}{\mathcal{L}^f-M0/P}-\frac{1}{\sum_{z\in\{u,e\}}\int_0^\infty sf(z,s,t)\,ds-M0/P}\right) &> 0.\end{aligned}$$

and $\mathcal{L}^f<\mathcal{L}^{lb}$. Thus, we have that $\mathcal{L}^f<\mathcal{L}^{lb}$ implies $\theta>\theta^{lb}(i^m)$. Since this is the case, from Proposition 1, and $\{\chi^-,\chi^+\}$ are increasing in θ, we have that $i^a>i^m$, from the bank's problem. Since $i^a>i^m$ implies that $M0/P=0$, the equilibrium is characterized by the conditions of case 1.a.

Case 2.b ($i^m<0$ and $\mathcal{L}^f\geq\mathcal{L}^{lb}(i^m,f)$). Next, we show that if $\mathcal{L}^f\geq\mathcal{L}^{lb}(i^m,f)$ and $i^m<0$, the effects of MP are modified. OMO lead to an increase in currency and reductions in rates to an interest rate reversal. Observe that if $\mathcal{L}^f>\mathcal{L}^{lb}(i^m,f)$ and $M0/P=0$,then the corresponding market tightness would be:

$$\tilde{\theta}=\delta\left(\frac{\sum_{z\in\{u,e\}}\int_0^\infty sf(z,s,t)\,ds}{\mathcal{L}^f}\right)-1\leq\delta\left(\frac{\sum_{z\in\{u,e\}}\int_0^\infty sf(z,s,t)\,ds}{\mathcal{L}^{lb}(i^m,f)}\right)-1=\theta^{lb}(i^m).$$

Now, since $\{\chi^+,\chi^-\}$ are increasing in θ, the equilibrium deposit rate obtained from Proposition 1 would be negative if the market tightness is indeed $\tilde{\theta}<\theta^{lb}(i^m)$. Thus, it must be the case that if $\mathcal{L}^f>\mathcal{L}^{lb}(i^m,f)$, $M0>0$ to obtain a tightness such that $\theta=\theta^{lb}(i^m)$.

In particular, it must be the case that:

$$\theta^{lb}(i^m)=\delta\left(\frac{\sum_{z\in\{u,e\}}\int_0^\infty sf(z,s,t)\,ds-M0/P}{\mathcal{L}^f-M0/P}\right)-1,\tag{52}$$

and solving $M0/P$, we obtain that:

$$M0/P = \frac{1+\theta^{lb}(i^m)}{1-\delta+\theta^{lb}(i^m)}\mathcal{L}^f - \frac{\delta}{1-\delta+\theta^{lb}(i^m)}\int_0^\infty sf(s,t) > 0. \tag{53}$$

We now consider the change in currency balances and markets rates in relation to changes in $\mathcal{L}^f$. We have that taking differentials in (53) we obtain:

$$dM0/P = \frac{\mu}{\mu-1}d\mathcal{L}^f.$$

where we used that:

$$\frac{\mu}{\mu-1} = \frac{\delta\mu}{\delta\mu-\delta} = \frac{1+\theta}{\theta+1-\delta} = \frac{1+\theta^{lb}(i^m)}{1-\delta+\theta^{lb}(i^m)}.$$

Next, we produce the effects of policy instruments on the equilibrium rates. We obtain

$$\frac{\partial}{\partial\mathcal{L}^f}\left[\theta^{lb}(i^m)\right] = 0,$$

and thus, it must be the case that:

$$\frac{\partial i^l}{\partial\mathcal{L}^f}\mathcal{L}^f = 0,\ \frac{\partial i^a}{\partial\mathcal{L}^f}\mathcal{L}^f = 0,\ \frac{\partial\Delta r}{\partial\mathcal{L}^f}\mathcal{L}^f = 0.$$

Next, consider the effects of changes in the IOR on the market rates and currency holdings. Note that we have from (51) that if the deposit rate is zero:

$$di^m = -\frac{1}{2}\left(\chi_\theta^+ + (1-\delta)\chi_\theta^-\right)d\theta < 0.$$

Then, from the expression for the spread, we have that:

$$d\Delta r = \frac{\delta}{2}\chi_\theta^- d\theta = -\delta\frac{\chi_\theta^-}{\left(\chi_\theta^+ + (1-\delta)\chi_\theta^-\right)}di^m < 0.$$

Thus:

$$\frac{\partial\Delta r}{\partial i^m} = -\delta\frac{\chi_\theta^-(\delta\mu-1)}{\left(\chi_\theta^+(\delta\mu-1) + (1-\delta)\chi_\theta^-(\delta\mu-1)\right)} < 0.$$

Finally, the effect on currency holdings is given by is:

$$d\theta = \delta\mu\left(\frac{1}{\mathcal{L}^f - M0/P} - \frac{1}{\int_0^\infty sf(s,t) - M0/P}\right)dM0/P > 0.$$

Thus, we obtain that increases in the IOR produce increases in the reserve balances:

$$\frac{\partial}{\partial i^m}[M0/P] = -\frac{\delta\mu\left(\frac{1}{\mathcal{L}^f-M0/P} - \frac{1}{\int_0^\infty sf(s,t)-M0/P}\right)}{\frac{1}{2}\left(\chi_\theta^+ + (1-\delta)\chi_\theta^-(\theta)\right)} < 0.$$

General solution to currency balances. We obtain a general solution to the currency holdings. The solution is given by:

$$M0/P=\mathbb{I}_{[i^m<0]}\cdot\max\left\{\frac{1+\theta^{lb}\left(i^m\right)}{1-\delta+\theta^{lb}\left(i^m\right)}\mathcal{L}^f-\frac{\delta}{1-\delta+\theta^{lb}\left(i^m\right)}\int_0^\infty sf\left(s,t\right),0\right\}.$$

Note that when $i^m<0$, then $\theta^{lb}\left(i^m\right)>0$. Thus, the term on first entry is positive when:

$$\mathcal{L}^f>\frac{\delta}{1+\theta^{lb}\left(i^m\right)},$$

which coincides with (53) when i^m.

Regimes that are not considered in the paper. There could be equilibria where $\mathcal{L}^f=\mathcal{L}^b=\frac{M0}{P}=\sum_{z\in\{u,e\}}\int_0^\infty sf\left(z,s,t\right)ds$. That is, equilibria in which the CB does all the intermediation. In this case, deposits are zero and bank balance sheets are empty. This will occur if the interest on reserves is so low that banks cannot hold deposits, $-i^m>\frac{\iota}{2}$. At that point, the deposit rate is zero and the loans rate is also zero. We do not consider this case.

QED.

D.3 Proof of Lemma 1 - Phillips Curve Derivation (10)

Derivation of the Retailers of Phillips Curve. We factor out the aggregate price level P_τ from the instantaneous return in the expression $q(p,t)$. We obtain:

$$q(p,t) = \max_{\{\dot{p}^j_\tau\}} \int_t^\infty \frac{\exp(-\rho\tau)}{P_\tau} \left(\left(p^j_\tau - p_t\right) y^j_\tau - P_\tau \frac{\Theta}{2} \left(\frac{\dot{p}^j_\tau}{p^j_\tau}\right)^2 Y_\tau \right) d\tau,$$

$$s.t. \quad y^j_\tau = \left(p^j_\tau\right)^{-\epsilon} P^\epsilon_\tau Y_\tau.$$

Substituting:

$$P_\tau = P_t \exp\left(\int_t^\tau \pi_s ds\right),$$

we express the objective as:

$$q(p,t) P_t = \max_{\{\dot{p}^j_\tau\}} \int_t^\infty \exp\left(\int_t^\tau (-\pi_s - \rho) ds\right) \left(\left(p^j_\tau - p_t\right) y^j_\tau - P_\tau \frac{\Theta}{2} \left(\frac{\dot{p}^j_\tau}{p^j_\tau}\right)^2 Y_\tau \right) d\tau.$$

$$s.t. \quad y^j_\tau = \left(p^j_\tau\right)^{-\epsilon} P^\epsilon_\tau Y_\tau$$

We recast this equation into the following HJB equation for the nominal value of the retailer's problem, $Q(p,t) \equiv q(p,t) P_t$. The corresponding equation is:

$$(\rho + \pi_t) Q(p,t) = \max_{\{\dot{p}^j_t\}} \left(p^j_t - p_t\right) \left(p^j_t\right)^{-\epsilon} P^\epsilon_t Y_t - P_t \frac{\Theta}{2} \left(\frac{\dot{p}^j_t}{p^j_t}\right)^2 Y_t + Q_p \dot{p}^j_t + \dot{Q}.$$

Next, we obtain the first-order condition:

$$Q_p - \Theta \frac{\dot{p}^j_t}{\left(p^j_t\right)^2} P_t Y_t = 0. \tag{54}$$

Deriving the first-order condition with respect to time yields:

$$Q_{pp}\dot{p} + \dot{Q}_p = \Theta \left(\frac{\ddot{p}^j_t}{\left(p^j_t\right)^2} P_t Y_t - 2 \left(\frac{\dot{p}^j_t}{p^j_t}\right)^2 \frac{P_t}{p^j_t} Y_t \right) + \Theta \frac{\dot{p}^j_t}{\left(p^j_t\right)^2} \dot{P}_t Y_t + \Theta \frac{\dot{p}^j_t}{\left(p^j_t\right)^2} P_t \dot{Y}_t. \tag{55}$$

Next, we produce the envelope condition. For that, we take the derivative of $Q(p,t)$ with respect to the individual price p^j to obtain:

$$(\rho + \pi_t) Q_p = -\epsilon \left(p^j_t - p_t\right) \left(p^j_t\right)^{-(\epsilon+1)} P^\epsilon_t Y_t + p^{-\epsilon}_{j,t} P^\epsilon_t Y_t + \Theta \left(\frac{\dot{p}^j_t}{p^j_t}\right)^2 \frac{P_t}{p^j_t} Y_t + Q_{pp} \dot{p}^j_t + \dot{Q}_p. \tag{56}$$

Substituting (55) and (54) into (56), we obtain:

$$(\rho+\pi_t)\,\Theta\frac{\dot{p}_t^j}{\left(p_t^j\right)^2}P_tY_t = -\epsilon\left(p_t^j-p_t\right)\left(p_t^j\right)^{-1}y_t^j+y_t^j+\Theta\left(\frac{\dot{p}_t^j}{p_t^j}\right)^2\frac{P_t}{p_t^j}Y_t+$$

$$\Theta\left(\frac{\ddot{p}_t^j}{\left(p_t^j\right)^2}P_t-2\left(\frac{\dot{p}_t^j}{p_t^j}\right)^2\frac{P_t}{p_t^j}\right)Y_t+\Theta\frac{\dot{p}_t^j}{\left(p_t^j\right)^2}\dot{P}_tY_t+\Theta\frac{\dot{p}_t^j}{\left(p_t^j\right)^2}P_t\dot{Y}_t.$$

Using that all managers act identically, we substitute, $p_t^j = P_t$ and $y_{j,t} = Y_t$, and obtain:

$$(\rho+\pi_t)\,\Theta\frac{\dot{P}_t}{P_t}Y_t = \left(1-\epsilon\left(P_t-p_t\right)P_t^{-1}\right)Y_t+\Theta\left(\frac{\ddot{P}_t}{P_t}-2\left(\frac{\dot{P}_t}{P_t}\right)^2\right)Y_t+2\Theta\frac{\dot{P}_t^2}{P_t^2}Y_t+\Theta\frac{\dot{P}_t}{P_t}\dot{Y}_t.$$

Recall that inflation and the price acceleration are:

$$\pi_t=\frac{\dot{P}_t}{P_t}\to\dot{\pi}_t=\left(\frac{\ddot{P}_t}{P_t}-\left(\frac{\dot{P}_t}{P_t}\right)^2\right).$$

Replacing these conditions, in the condition above, we arrive that the following condition:

$$\rho\Theta\pi_tY_t+\Theta\pi_t^2Y_t = \left\{1-\epsilon\left(1-\frac{p_t}{P_t}\right)\right\}Y_t+\Theta\dot{\pi}_tY_t+\Theta\pi_t^2Y_t+\Theta\pi_t\dot{Y}_t$$

Dividing both sides by ΘY_t, we simplify things to:

$$\rho\pi_t = \frac{1-\epsilon\left(1-\frac{p_t}{P_t}\right)}{\Theta}+\dot{\pi}_t+\pi_t\frac{\dot{Y}_t}{Y_t}$$

Recall that $mc_t \equiv \frac{p_t}{P_t}$ represents the real marginal cost for retailers. Then, we arrive at the Phillips curve:

$$\left(\rho-\frac{\dot{Y}_t}{Y_t}\right)\pi_t = \frac{\epsilon}{\Theta}\left(mc_t-\frac{\epsilon-1}{\epsilon}\right)+\dot{\pi}_t. \tag{57}$$

In present value form, this equation is given by:

$$\pi_t=\frac{\epsilon}{\Theta}\int_t^{\infty}e^{-(\tau-t)\rho}\frac{Y_\tau}{Y_t}\left(mc_s-\frac{\epsilon-1}{\epsilon}\right)ds.$$

In the current framework, retailers make positive profits. Flow real profit in the symmetric equilibrium is given by

$$\left(\frac{p_t^j}{P_t}-\frac{p_t}{P_t}\right)y_t^j=(1-mc_t)\,Y_t.$$

We assume that profits earned by retailers are paid out uniformly to all households, via the government's lump sum rebate τ_t^{lump}.

D.4 Proof of Lemma 2 - Value per Worker Derivation (11)

Derivation of the value per worker. Here we derive the marginal value per worker. Consider the wholesale firm's problem 4. We obtain:

$$G(n,t) = \max_{\{v_\tau\}} \int_t^\infty \exp(-\rho\tau) \left[\frac{p_\tau}{P_\tau} n_\tau - \frac{w_\tau}{P_\tau} n_\tau - v_\tau \mu \right] d\tau,$$

$$s.t. \quad \dot{n}_\tau = j_\tau v_\tau - \xi n_\tau.$$

This expression leads to the following HJB equation for the intermediate good firm's problem. The corresponding equation is:

$$\rho G(n,t) = \max_{\{v_t\}} \left(\frac{p_t}{P_t} - \frac{w_t}{P_t} \right) n_t - v_t \mu + G_n \dot{n}_t + \dot{G}.$$

$$s.t. \quad \dot{n}_t = j_t v_t - \xi n_t$$

Substituting the constraint on $\dot{n}_t$, we obtain:

$$\rho G(n,t) = \max_{\{v_t\}} \left(\frac{p_t}{P_t} - \frac{w_t}{P_t} \right) n_t - v_t \mu + G_n (j_t v_t - \xi n_t) + \dot{G}.$$

The first-order condition with respect to vacancies, v_t, is:

$$j_t G_n = \mu. \tag{58}$$

We conjecture the value function as follows:

$$G(n,t) = g_t n_t \tag{59}$$

We verify this guess below. Under this assumption, we obtain that the first order condition is:

$$j_t g(t) = \mu \tag{60}$$

This relationship must hold in a solution with finite vacancies, if indeed the conjectur is to be verified. We replace the condition into the HJB equation and using our guess obtain:

$$(\rho + \xi) g_t n_t = \left(\frac{p_t}{P_t} - \frac{w_t}{P_t} \right) n_t + + \overbrace{(j_t g_t - \mu)}^{=0} v_t + \dot{g}_t n_t,$$

which verifies the conjecture. Hence, we obtain:

$$(\rho + \xi) g_t = \left(\frac{p_t}{P_t} - \frac{w_t}{P_t} \right) + \dot{g}_t. \tag{61}$$

The integral solution to this equation is:

$$g(t) = \int_t^\infty \exp\left(-(\tau - t)(\rho + \xi)\right)\left(\frac{p_\tau}{P_\tau} - \frac{w_\tau}{P_\tau}\right) d\tau.$$

Assuming an ex-post hold up problem by workers in the intermediate-good firm, we obtain that $\frac{w_\tau}{P_\tau} = (1-\eta)\frac{p_\tau}{P_\tau}$, because of the holdup problem. Under this assumption, the real value per worker is given by

$$g(t) = (1-\eta)\int_t^\infty \exp\left(-(\tau - t)(\rho + \xi)\right)\frac{p_\tau}{P_\tau} d\tau$$

Using the definition of marginal cost relative to final goods, $mc_t \equiv \frac{p_t}{P_t}$, we get

$$g(t) = \eta\int_t^\infty \exp\left(-(\tau - t)(\rho + \xi)\right) mc_\tau d\tau.$$

D.5 Flow of Funds Identity

In the proofs that follow, we make use of the following Lemma.

Lemma 4 *If the deposit, loan and money markets clear, then:*

$$P_t \sum_{z\in\{u,e\}} \int_{\bar{s}}^\infty sf(z,s,t)\, ds = 0. \tag{62}$$

Proof. The deposits and loan markets clearing conditions require:

$$A_t^b = \sum_{z\in\{u,e\}} \int_0^\infty a_t^h(s,z) f(s,z,t) ds \tag{63}$$

$$L_t^b + L_t^f = \sum_{z\in\{u,e\}} \int_{\bar{s}}^0 l_t^h(s,z) f(s,z,t) ds, \tag{64}$$

and clearing in the money market requires:

$$M_t^b + M0_t = M_t. \tag{65}$$

If we aggregate the budget constraint—the balance sheet identity—of banks, we obtain:

$$A_t^b = L_t^b + M_t^b. \tag{66}$$

Once we combine (63), (64), and (65) into (66), we obtain:

$$\sum_{z\in\{u,e\}} \int_0^\infty a_t^h(s) f(s,z,t) ds = \sum_{z\in\{u,e\}} \int_{\bar{s}}^0 l_t^h(s) f(s,z,t) ds + M_t - M0_t - L_t^f. \tag{67}$$

Nominal deposits and currency are related to real wealth via:

$$P_t \sum_{z\in\{u,e\}} \int_0^{\infty} sf(s,z,t)ds = \sum_{z\in\{u,e\}} \int_0^{\infty} a_t^h(s,z)f(s,z,t)ds + M0_t. \tag{68}$$

and, similarly for loans:

$$-P_t \sum_{z\in\{u,e\}} \int_{\bar{s}}^{0} sf(s,z,t)ds = \sum_{z\in\{u,e\}} \int_{\bar{s}}^{0} l_t^h(s)f(s,z,t)ds. \tag{69}$$

This condition can be expressed in terms of real household wealth, with the use of (68) and (69):

$$P_t \sum_{z\in\{u,e\}} \int_0^{\infty} sf(s,z,t)ds - M0_t = P_t \sum_{z\in\{u,e\}} \int_{\bar{s}}^{0} sf(s,z,t)ds + M_t - M0_t - L_t^f.$$

Thus, using that $M_t = L_t^f$, we obtain:

$$\sum_{z\in\{u,e\}} \int_0^{\infty} sf(s,z,t)ds = \sum_{z\in\{u,e\}} \int_{\bar{s}}^{0} sf(s,z,t)ds.$$

Thus, clearing in all nominal asset markets implies clearing in a single real asset market, (62). **QED.**

D.6 Proof of Government Real Bugdet (equation 15)

The nominal profits of the CB are given by:

$$\Pi_t^{CB} = i_t^l L_t^f - i_t^m (M_t - M0_t) + \iota_t \left(1 - \psi_t^-\right) B_t^-.$$

Note that the earnings from discount window loans equal the average payment in the interbank market, and thus:

$$\iota_t \left(1 - \psi_t^-\right) B_t^- = -\mathbb{E}\left[\chi_t \left(b\left(A_t, A_t - L_t\right)\right)\right]. \tag{70}$$

By Proposition 1, banks earn zero profits in expectation. Thus,

$$-\mathbb{E}\left[\chi_t \left(b\left(A_t, A_t - L_t\right)\right)\right] = i_t^l L_t^b + i_t^m M_t^b - i_t^a A_t^b. \tag{71}$$

Thus, substituting (70) and (71) into the expression for π_t^f above yields:

$$\begin{aligned}\Pi_t^{CB} &= i_t^l L_t^f - i_t^m (M_t - M0_t) + i_t^l L_t^b + i_t^m M_t^b - i_t^a A_t^b. \\ &= i_t^l L_t^h - i_t^a A_t^h,\end{aligned}$$

where we used the clearing condition in the money market, $M_t^b + M_t^0 = M_t$, the deposit market, $A_t^b = A_t^h$, and the loans market, $L_t^h = L_t^b + L_t^f$. Now, observe that:

$$\Pi_t^{CB} = -i_t^l P_t \sum_{z\in\{u,e\}} \int_{\bar{s}}^{0} sf(z,s,t)ds - i_t^a \left(P_t \sum_{z\in\{u,e\}} \int_{0}^{\infty} sf(z,s,t)ds - M0_t. \right),$$

but we know from the household's problem that $i_t^a M0_t = 0$. Hence, profits are given by:

$$\Pi_t^{CB} = -i_t^l P_t \sum_{z\in\{u,e\}} \int_{\bar{s}}^{0} sf(z,s,t)ds - i_t^a P_t \sum_{z\in\{u,e\}} \int_{0}^{\infty} sf(z,s,t)ds.$$

Then, since $i^a = i^l - \Delta r$, we have that:

$$\Pi_t^{CB} = -i_t^l P_t \left(\sum_{z\in\{u,e\}} \int_{\bar{s}}^{\infty} sf(z,s,t)ds \right) + \Delta r_t P_t \sum_{z\in\{u,e\}} \int_{0}^{\infty} sf(z,s,t)ds.$$

Thus, from Lemma 4, we obtain:

$$\Pi_t^{CB} = P_t \cdot \Delta r_t \sum_{z\in\{u,e\}} \int_{0}^{\infty} sf(z,s,t)ds.$$

Now, we turn to the government's budget constraint, (16), we have that:

$$P_t T_t = P_t \cdot \left(\Delta r_t \sum_{z\in\{u,e\}} \int_{0}^{\infty} sf(z,s,t)ds \right) + P_t \left(\tau^l \cdot (1 - \mathcal{U}_t) - b \cdot \mathcal{U}_t \right),$$

and dividing by the price level we obtain

$$T_t = \Delta r_t \sum_{z\in\{u,e\}} \int_{0}^{\infty} sf(z,s,t)ds + \tau^l \cdot (1 - \mathcal{U}_t) - b \cdot \mathcal{U}_t$$

as stated by the proposition. **QED.**

D.7 Proof of Single-Clearing condition (equation 18)

Proof of Clearing in all markets. Lemma 4 shows that if all asset markets clear, then there is clearing in real wealth (18). We now prove the converse. That is, if (18) holds, then, the deposit, loans, and money markets must clear.

The proof is by contradiction. We start by taking (18) as given. Next, we multiply by P_t on both sides and, by definition, obtain:

$$\sum_{z\in\{u,e\}} \int_{\bar{s}}^{0} l_t^h(s) f(z,s,t)ds = \sum_{z\in\{u,e\}} \int_{0}^{\infty} a_t^h(s) f(z,s,t)ds + M0_t.$$

From the central bank's balance sheet, we obtain that:

$$\sum_{z\in\{u,e\}}\int_{\bar{s}}^{0} l_t^h(s)f(z,s,t)ds = \sum_{z\in\{u,e\}}\int_{0}^{\infty} a_t^h(s)f(z,s,t)ds + M_t - M_t^b, \text{ for } t\in[0,\infty). \tag{72}$$

We now substitute the balance sheet of the CB, $M_t = L_t^f$, and the consolidated balance sheet of banks, $M_t^b = A_t^d - L_t^b$, to obtain:

$$\sum_{z\in\{u,e\}}\int_{\bar{s}}^{0} l_t^h(s)f(z,s,t)ds - L_t^f - L_t^b = \sum_{z\in\{u,e\}}\int_{0}^{\infty} a_t^h(s)f(z,s,t)ds - A_t^d.$$

This equation guarantees that if there is no clearing in the loans market, there is no clearing in the deposit market by that same amount. Assume there is a deviation from market clearing in the amount ε. Then, an income $\Delta r\cdot\varepsilon$ would not be accounted for by the equation. However, since all the spread is earned by the CB, following Proposition (17), it must be that $\varepsilon = 0$. **QED.**

Proof of Walras's Law. Next, we prove that if (18) holds, then the goods market clears, which is a derivation of Walras's law in the continuous-time setting.

Recall that f satisfies the following KFE equations:

$$\begin{aligned}
\frac{\partial}{\partial t}f(e,s,t) &= -\frac{\partial}{\partial s}\left[\mu(e,s,t)f(e,s,t)\right] - \Gamma_t^{eu}\cdot f(e,s,t) + \Gamma_t^{ue}\cdot f(u,s,t), \text{ and}\\
\frac{\partial}{\partial t}f(u,s,t) &= -\frac{\partial}{\partial s}\left[\mu(u,s,t)f(u,s,t)\right] - \Gamma_t^{ue}\cdot f(u,s,t) + \Gamma_t^{eu}\cdot f(e,s,t).
\end{aligned}$$

A similar KFE holds for the cumulative distributions:

$$\begin{aligned}
\frac{\partial}{\partial t}F(e,s,t) &= -\mu(e,s,t)f(e,s,t) - \Gamma_t^{eu}\cdot F(e,s,t) + \Gamma_t^{ue}\cdot F(u,s,t), \text{ and}\\
\frac{\partial}{\partial t}F(u,s,t) &= -\mu(u,s,t)F(u,s,t) - \Gamma_t^{ue}\cdot F(u,s,t) + \Gamma_t^{eu}\cdot F(e,s,t).
\end{aligned}$$

Recall that the integrals in the clearing conditions, are Lebesgue integrals. It is convenient to be explicit about the mass points at the debt limit in (18):

$$0 = \sum_{z\in\{u,e\}}\left[\bar{s}F(z,\bar{s},t) + \lim_{\sigma\to\bar{s}^+}\int_{\sigma}^{\infty} sf(z,s,t)ds\right],$$

so that the first integral is in the Riemann sense. Then, taking time derivatives:

$$0 = \sum_{z\in\{u,e\}}\left[\sum_{z\in\{u,e\}}\bar{s}\frac{\partial}{\partial t}F(z,\bar{s},t) + \frac{\partial}{\partial t}\left[\lim_{\sigma^+\to\bar{s}}\int_{\sigma}^{\infty} sf(z,s,t)ds\right]\right]. \tag{73}$$

Substituting the KFE equations into the first term, we obtain:

$$\sum_{z\in\{u,e\}} \bar{s}\frac{\partial}{\partial t}F(z,\bar{s},t) = -\bar{s}\cdot\sum_{z\in\{u,e\}}\Big(\mu(z,s,t)\,f(z,\bar{s},t)+\Gamma_t^{zz'}\cdot F(z,s,t)-\Gamma_t^{z'z}\cdot F(z',s,t)\Big). \\ = -\sum_{z\in\{u,e\}}\bar{s}\mu(z,s,t)\,f(z,\bar{s},t). \tag{74}$$

The second line follow from: $\sum_{z\in\{u,e\}}\Gamma_t^{zz'}\cdot F(z,s,t)-\Gamma_t^{z'z}\cdot F(z',s,t)=0.$[34]

Substituting the KFE equations into the second term of (73), we obtain:

$$\sum_{z\in\{u,e\}}\frac{\partial}{\partial t}\left[\lim_{\sigma\to\bar{s}}\int_{\sigma}^{\infty}sf(z,s,t)ds\right]=\sum_{z\in\{u,e\}}\int_{\bar{s}}^{\infty}\left[\underbrace{-s\frac{\partial}{\partial s}\left[\mu(s,t)\,f(z,s,t)\right]}_{\mathcal{A}\equiv}-s\underbrace{\left(\Gamma_t^{zz'}\cdot f(z,s,t)+\Gamma_t^{z'z}\cdot f(z',s,t)\right)}_{\mathcal{B}\equiv}\right]ds.$$

We analyze each term in the integral. First, notice that $\mathcal{B}=0$, because again:

$$\sum_{z\in\{u,e\}}\left[\Gamma_t^{zz'}\cdot f(z,s,t)-\Gamma_t^{z'z}\cdot f(z',s,t)\right]=0,$$

Second, we use integration by parts, to obtain that $\mathcal{A}$ is given by:

$$-\sum_{z\in\{u,e\}}\lim_{\sigma\to\bar{s}}\int_{\sigma}^{\infty}s\frac{\partial}{\partial s}\left[\mu(s,t)\,f(z,s,t)\right]ds=\sum_{z\in\{u,e\}}\underbrace{-s\cdot\mu(s,t)\,f(z,s,t)|_{\bar{s}}^{\infty}}_{\mathcal{A}.1\equiv}+\sum_{z\in\{u,e\}}\underbrace{\int_{\bar{s}}^{\infty}\mu(s,t)\,f(z,s,t)\,ds}_{\mathcal{A}.2\equiv}.$$

Importantly, to use integration by parts, in evaluating the definite integral, we use the Lebesgue integral. Thus, $\mathcal{A}.2$ is in the Lebesgue sense again.

Evaluating the terms$\mathcal{A}.1$ yields,

$$\lim_{s\to\infty}f(z,s,t)=0 \text{ and } \lim_{\sigma\to\bar{s}}\sigma\cdot\mu(\sigma,t)\,f(z,\sigma,t)=\bar{s}\frac{\partial}{\partial t}F(z,s,t).$$

Thus, summing the terms (74) and $\mathcal{A}.1$, we obtain that (73), is equivalent to:

$$0=\sum_{z\in\{u,e\}}\int_{\bar{s}}^{\infty}\mu(s,t)\,f(z,s,t)\,ds.$$

Next, we compute the integral $\mathcal{A}.2$. Recall that:

$$\mu(s,t)=\Big[r_t(s)\left(s-m^h(z,s,t)/P_t\right)-\dot{P}_t/P_t\cdot m^h(z,s,t)/P_t-c(z,s,t)+w_t(z)\Big].$$

From the household's problem, $i_t^a\cdot m^h(z,s,t)=0$ for $s>0$ and $m^h(z,s,t)=0$ for any $s\leq 0$. Hence, we

[34] The employment status is independent of z and population is preserved. Thus, the condition says that within a wealth level, the mixing from employment to unemployment does not change wealth.

have that:

$$\left(r_t(s) + \dot{P}_t/P_t\right) m^h(z,s,t)/P_t = 0.$$

Thus, we can freely add the term above into the drift, since this term is always zero, hence:

$$\mu(s,t) = \left[r_t(s)\cdot s - c(z,s,t) + w_t(z)\right].$$

Thus, $\mathcal{A}.2$ reduces to:

$$\sum_{z\in\{u,e\}}\int_{\bar{s}}^{\infty}\mu(s,t)\,f(z,s,t)\,ds = \sum_{z\in\{u,e\}}\int_{\bar{s}}^{\infty}\left[r_t(s)\,s - c(z,s,t) + w_t(z)\right] f(z,s,t)\,ds. \tag{75}$$

We have that:

$$\sum_{z\in\{u,e\}}\int_{\bar{s}}^{\infty}\left(w_t(z) - c(s,t)\right) f(z,s,t)\,ds = \left(1-\tau^l\right)(1-\mathcal{U}_t) + b\cdot\mathcal{U}_t + T_t - C_t,$$

and using $Y_t = (1-\mathcal{U}_t)$ we have:

$$\sum_{z\in\{u,e\}}\int_{\bar{s}}^{\infty}\left(w_t(z) - c(s,t)\right) f(z,s,t)\,ds = Y_t - C_t - \tau^l\cdot(1-\mathcal{U}_t) + b\cdot\mathcal{U}_t + T_t$$

In turn, we have that:

$$\begin{aligned}\sum_{z\in\{u,e\}}\int_{\bar{s}}^{\infty} r_t(s)\,s\cdot f(z,s,t)\,ds &= r_t^l\sum_{z\in\{u,e\}}\int_{\bar{s}}^{\infty} s\cdot f(z,s,t)\,ds - \Delta r_t\sum_{z\in\{u,e\}}\int_{0}^{\infty} sf(z,s,t)\,ds\\ &= -\Delta r_t\sum_{z\in\{u,e\}}\int_{0}^{\infty} sf(z,s,t)\,ds.\end{aligned}$$

where the second line follows from the market clearing condition, Lemma 4.

Thus, summing the last two equations above, we obtain that (75) is:

$$\begin{aligned}0 &= \sum_{z\in\{u,e\}}\int_{\bar{s}}^{\infty}\left[r_t(s)\,s - c(z,s,t) + w_t(z)\right] f(z,s,t)\,ds\\ &= Y_t - C_t + T_t - \tau^l\cdot(1-\mathcal{U}_t) + b\cdot\mathcal{U}_t - \Delta r_t\sum_{z\in\{u,e\}}\int_{0}^{\infty} sf(z,s,t)\,ds,\end{aligned}$$

but recall that (17) implies that $T_t = \tau^l\cdot(1-\mathcal{U}_t) - b\cdot\mathcal{U}_t + \Delta r_t\sum_{z\in\{u,e\}}\int_0^{\infty} sf(z,s,t)\,ds$. Thus, we obtain that (75) implies:

$$0 = Y_t - C_t.$$

This expression verifies Walras's Law.

D.8 Proof of Corollary 2

The discount window profits are equal to $\Delta r_t \int_0^\infty s f(s,t)\, ds$ since banks are competitive and earn zero profits. Given the same real credit spread Δr_t, the equilibrium real wealth distribution $f(s,t)$ is also same. Thus Corollary 2 is established. **QED.**

E. Steady State

Steady State: Supply-Side Block. Here, we present aggregate output in steady state when MP guarantees that steady-state inflation is zero.

Proposition 7 *[Steady-State Supply] When $\pi_{ss} = 0$ steady-state output is given by:*

$$Y_{ss} = \frac{\Gamma^{eu}_{ss}}{\xi + \Gamma^{eu}_{ss}}, \quad \mathcal{U}_{ss} = \frac{\xi}{\xi + \Gamma^{eu}_{ss}}, \quad \Gamma^{eu}_{ss} = \Xi\left(1, \left(\mathcal{J}^{-1}\left(\frac{\mu}{\eta}\frac{\epsilon}{\epsilon - 1}(\rho + \xi)\right)\right)^{-1}\right),$$

where the function $\mathcal{J}$ is defined as $\mathcal{J}(x) \equiv \Xi(x, 1)$.

In this proposition, we assume that monetary policy targets zero inflation rate, i.e., $\pi_{ss} = 0$. Nominal policy rate adjusts accordingly to achieve $\pi_{ss} = 0$. At steady state, we have that π_t and $\dot{\pi}_t$ are zero. Thus, as a result, the Phillips curve (10), requires:

$$\left(\rho - \frac{\dot{Y}_t}{Y_t}\right)\pi_t|_{\pi_t = 0} = \frac{\epsilon}{\Theta}\left(mc_t - \frac{\epsilon - 1}{\epsilon}\right) + \dot{\pi}_t|_{\dot{\pi}_t = 0} \Longrightarrow mc_{ss} = \frac{\epsilon - 1}{\epsilon}. \tag{76}$$

We can pin down mc_{ss}.

When the marginal cost is constant, then, using (11) we obtain that:

$$g_{ss} = \eta \times \int_0^\infty \exp\left(-(\rho + \xi)\tau\right) mc_{ss} d\tau \Longrightarrow g_{ss} = \eta \frac{mc_{ss}}{(\rho + \xi)}.$$

Using (76), we find that:

$$g_{ss} = \frac{\eta(\epsilon - 1)}{\epsilon(\rho + \xi)}. \tag{77}$$

Next, using (14-12) and the above expression for g_{ss}, we can pin down the equilibrium job filling rate j_{ss}:

$$j_{ss} = \frac{\mu}{\eta}\frac{\epsilon}{\epsilon - 1}(\rho + \xi). \tag{78}$$

Define the function:

$$\mathcal{J}(x) \equiv \Xi(x, 1),$$

We obtain the unemployment-to-vacancy ratio using the matching function:

$$\Xi\left(\frac{\mathcal{U}_{ss}}{v_{ss}},1\right)=\mathcal{J}\left(\frac{\mathcal{U}_{ss}}{v_{ss}}\right)=j_{ss}$$

Inverting the expression above and using (78) we obtain:

$$\frac{\mathcal{U}_{ss}}{v_{ss}}=\mathcal{J}^{-1}\left(\frac{\mu}{\eta}\frac{\epsilon}{\epsilon-1}\left(\rho+\xi\right)\right). \tag{79}$$

Recall that the flow of unemployment is given by (13), that at steady state, we have that:

$$0=\xi\left(1-\mathcal{U}_{ss}\right)-\mathcal{U}_{ss}\Gamma_{ss}^{eu}.$$

In turn, recall that we can use the homogeneity of Ξ to obtain:

$$\Gamma_{ss}^{eu}=\Xi\left(1,\frac{v_{ss}}{\mathcal{U}_{ss}}\right)=\Xi\left(1,\left(\mathcal{J}^{-1}\left(\frac{\mu}{\eta}\frac{\epsilon}{\epsilon-1}\left(\rho+\xi\right)\right)\right)^{-1}\right),$$

where the second equality follows from (79). Thus, the steady-state unemployment rate is:

$$\mathcal{U}_{ss}=\frac{\xi}{\xi+\Xi\left(1,\left(\mathcal{J}^{-1}\left(\frac{\mu}{\eta}\frac{\epsilon}{\epsilon-1}\left(\rho+\xi\right)\right)\right)^{-1}\right)}. \tag{80}$$

Aggregate output is given by

$$Y_{ss}=1-\mathcal{U}_{ss}=\frac{\Xi\left(1,\left(\mathcal{J}^{-1}\left(\frac{\mu}{\eta}\frac{\epsilon}{\epsilon-1}\left(\rho+\xi\right)\right)\right)^{-1}\right)}{\xi+\Xi\left(1,\left(\mathcal{J}^{-1}\left(\frac{\mu}{\eta}\frac{\epsilon}{\epsilon-1}\left(\rho+\xi\right)\right)\right)^{-1}\right)}.$$

When consider the limiting case where $\eta\to 0$, $\mu\to 0$, and $\frac{\eta}{\mu}\to\kappa>0$ hold so that we don't have to consider profits by the intermediate good producers. Under this limiting case, real wage is equal to real marginal cost, i.e., $\frac{w_{ss}}{P_{ss}}=mc_{ss}=\frac{\epsilon-1}{\epsilon}$. In this case, the expressions simplify to:

$$Y_{ss}=\frac{\Gamma_{ss}^{eu}}{\xi+\Xi\left(1,\left(\mathcal{J}^{-1}\left(\frac{\epsilon\kappa}{\epsilon-1}\left(\rho+\xi\right)\right)\right)^{-1}\right)},\quad \mathcal{U}_{ss}=\frac{\xi}{\xi+\Xi\left(1,\left(\mathcal{J}^{-1}\left(\frac{\epsilon\kappa}{\epsilon-1}\left(\rho+\xi\right)\right)\right)^{-1}\right)},$$

and

$$\Gamma_{ss}^{eu}=\Xi\left(1,\left(\mathcal{J}^{-1}\left(\frac{\epsilon\kappa}{\epsilon-1}\left(\rho+\xi\right)\right)\right)^{-1}\right).$$

Steady State and Long-Run MP Effect. To solve the rest of the equilibrium, we search for the real interest rate r^a such that consumption of goods is equal to the production of goods. Since we know all the relevant prices, we can solve household's problem and derive aggregate demand. Notice that there are flow profits by retailers represented by $(1 - mc_{ss}) Y_{ss}$. We assume that these profits are paid out uniformly to all households, via the government's lump sum rebate. Consider a steady state. Let the CB target a long-run credit spread Δr_{ss}. At steady state, the disturbance in job-separation ϕ_t must be zero because this is the only possibility consistent with a Phillips curve with constant inflation. We also know that inflation has no effect in a steady state—here MP is super-neutral, unlike in a standard new-Keynesian model. Thus, at steady-state, the real interest rate r^a_{ss} solves:

$$0 = \sum_{z \in \{u,e\}} \int_{\bar{s}}^{\infty} s f_{ss}(z, s)\, ds.$$

Once we obtain an equilibrium r^a_{ss} in steady state, which corresponds to the real interest rate, inflation is given by the corresponding Fisher's equation:

$$\pi_{ss} = \pi_{\infty} = i^m_{\infty} - r^a_{\infty} + \frac{1}{2}\left[\chi^+_{\infty} + (1-\delta)\,\chi^-_{\infty}\right].$$

Once inflation is obtained, all nominal variables grow at the rate of inflation. To implement Δr_{ss}, the path of M_t must be consistent with the Λ_{ss} that produces Δr_{ss} according to (5).

Transitions. Along a transition, things work differently. In particular, π_t is given by (21). Then, given i^m_t and Λ_t, i^l_t and i^a_t are determined by (4). The real rates r^l_t and r^a_t follow from the Fisher's equation,

$$r^x_t = i^x_t - \pi_t \text{ for } x \in \{l, a\}. \tag{81}$$

Then, to satisfy clearing in the asset market, ϕ_t adjusts to satisfy (18).

Appendix F.1 discusses the equilibrium restriction imposed on MP along a transition. That appendix also connects the monetary properties of this model with the monetary properties of classic Bewley models, in connection to fiscal and monetary interactions. Appendix G explains how transitions are calculated numerically.

F. Implementation of Desired Spreads

In this section we discuss alternative MP implementations. We then discuss the implementability condition of a spread in the model and finally provide a discussion on fiscal policy considerations.

F.1 Implementation Conditions

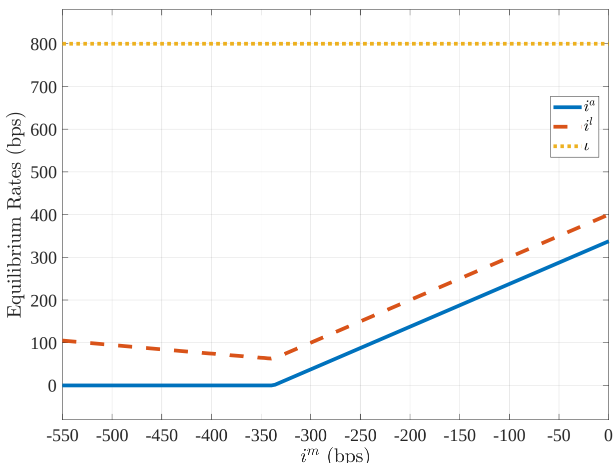
In the body of the text we lay out the general equilibrium block and work under the premise that the CB directly controls spreads. The following proposition describes the set of allocations that can be achieved by a policy with a stationary inflation path where the spread is treated as an endogenous variable and we work directly with the policy instruments $\left\{i_t^m, \mathcal{L}_t^f\right\}$.

Proposition 8 *[Implementation Conditions] Consider a desired equilibrium path for* $\{r_t^a, \Delta r_t, \pi_t, \Gamma_t^{eu}\}_{t\geq 0}$. *To implement the equilibrium path, the CB chooses* $\left\{i_t^m, \mathcal{L}_t^f\right\}$ *subject to the following restrictions:*

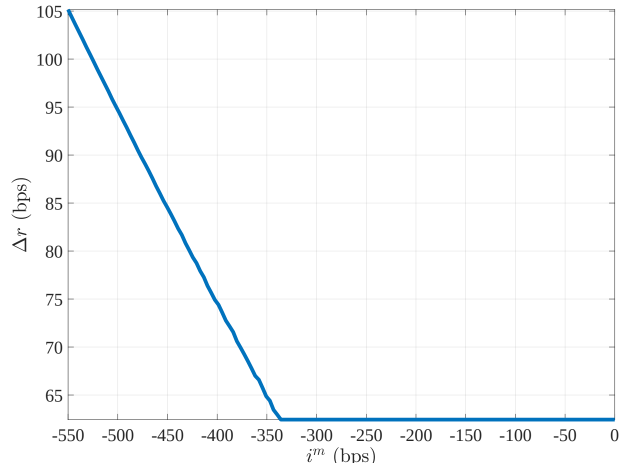
1. $\mathcal{L}_t^f \leq -\int_{\bar{s}}^0 sf(s,t)ds$,
2. *The equilibrium liquidity ratio is* $\Lambda_t = \min\left\{\Lambda^{zlb}\left(i_t^m, \iota_t\right), \Lambda^{MB}\left(f_t, \mathcal{L}_t^f\right)\right\}$,
3. *The real transfer,* T_t*, adjusts to satisfy (17),*
4. *The real spread,* Δr_t*, satisfies (5) given* Λ_t*,*
5. *Given* i_t^m *and* Λ_t*, the nominal rates* $\left\{i_t^l, i_t^a\right\}$ *are given by (4),*
6. *Given* $\Gamma_t^{z'z}$*, the unemployment rate* $\mathcal{U}_t$ *satisfies (13),*
7. *Inflation is consistent with the Phillips curve, (1),*
8. *The real rates* $\left\{r_t^l, r_t^a\right\}$ *are the corresponding nominal rates minus inflation,*
9. *The distribution of wealth,* f*, evolves according to (9);* f_0 *given,*
10. *Given* f*, the job separation* $\Gamma_t^{z'z}$ *guarantees the real asset market-clearing condition (18).*

Proposition 8 describes the allocations that can be induced by the CB. These allocations are affected by the CB because it controls the spread and the IOR. The implementation constraint $\mathcal{L}_t^f \leq -\int_{\bar{s}}^0 sf(s,t)ds$ simply tells that there must be enough private liabilities to set $\mathcal{L}_t^f$. The proof is immediate and only requires the use the of the equilibrium conditions where the liquidity ratio is treated exogenously.

MP implementation in a liquidity trap: Spread and Negative Interest on Reserves. The implementability conditions above generalize to liquidity traps. Figure 2 that depicts a map from the liquidity ratio to borrowing and lending rates. That figure is valid when police variables are set within the corridor system regime. In Figure 10, we keep Λ^{MB} constant and show borrowing and lending rates, as we vary i^m. As we can observe, there's an interval of values for i^m such that the spread is constant and both rates move in parallel. Once i^m reaches a sufficiently low value, further reductions in i^m begin to increase spreads while the deposit rate stays fixed. Beyond that point, currency is held by households.



(a) Equilibrium Rates

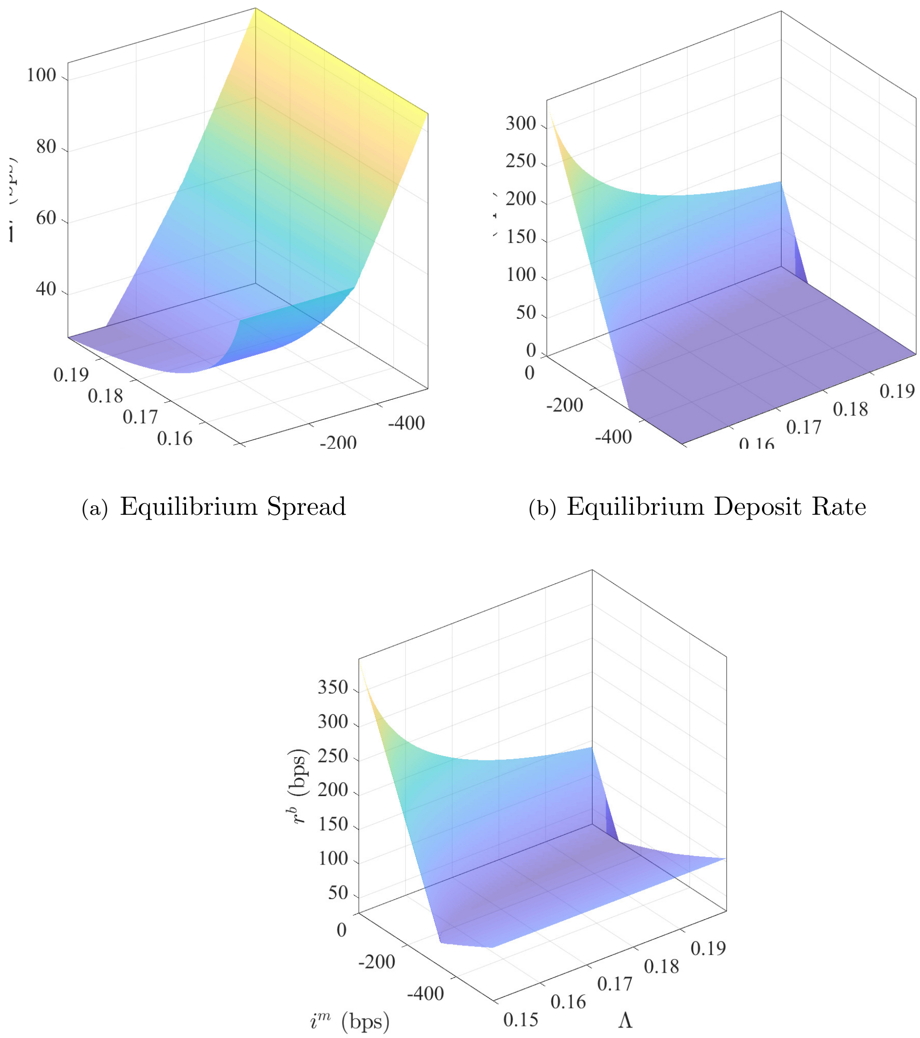


(b) Equilibrium Spread

Figure 10: Negative Interest on Reserves and the DZLB.

Note: This figure depicts the equilibrium rates and spread as a function of interest on reserves under the DZLB. All the rates and spread are expressed in basis points.

In 11, we vary i^m and Λ^{MB} together. Panel (a) shows the spread as a function of both policy variables. There are many combinations that allow us to implement the same spread at the DZLB. Panels (b-c) show the corresponding deposit and loans rates.



(a) Equilibrium Spread (b) Equilibrium Deposit Rate

(c) Equilibrium Loan Rate

Figure 11: Negative Interest on Reserves, Liquidity Ratio and the DZLB.

Note: This figure presents the equilibrium spread, deposit rate and loan rate as functions of the liquidity ratio and interest on reserves under the DZLB. All the rates and spread are expressed in basis points.

F.2 Alternative Implementations

Fiscal Consequences of different tool configurations. In the current formulation, the CB has two tools, $\{i_t^m, M_t^{cb}\}$. We observed that i^m controls inflation directly and that the size of the balance sheet can achieve a desires spread. We took as given the spread ι. In principle, a desired spread can also be implemented by moving ι, while keeping M^{cb} fixed. We could be tempted to argue that these instruments have different fiscal consequences, but they don't:

Corollary 2 *[No Fiscal Consequence of an implementation choice] Consider two policies $\left\{\iota_t, M_t^{cb}\right\}$ and $\left\{i_t^m, M_t^{cb}\right\}$ that implement the same spread, Δr_t. Both are consistent with the same government budget constraint.*

Thus, in the context of this model, moving $\left\{\iota_t, M_t^{cb}\right\}$ or $\left\{i_t^m, M_t^{cb}\right\}$ most implement the same set of allocations.

Interbank Rate Targets. It is also worth discussing alternative MP implementations (Bindseil, 2014, reviews cross-country practices.). One way to the control the spread directly is through OMO that targets the interbank rate, $\bar{i}^f$. Because there is a map from $\bar{i}^f$ to Δr_t, a target for the interbank rate also implements a spread independently of i^m. In practice, most CBs have an explicit interbank rate target, but restrict the way in which they achieve that target: targeting an interbank market at the middle of the corridor, $\bar{i}^f = i^m + \frac{1}{2}\iota$.

Other countries keep the rate on reserves at zero, but move ι and maintaining a constant distance between the discount rate and the target. With these additional constraints, CBs simultaneously spreads and inflation when they change policy rates—perhaps inadvertently.

Numerical Appendix

G. Solution Algorithm

The computational method follows (Achdou et al., 2021) closely. The main differences in the approach is the spread and the conceptual approach of the RSS. Equations 4, 17, and 4 are provide the equilibrium set of equations we need to solve the model. They allow us to solve the model entirely by solving for the equilibrium path of a single object Γ_t. The spread Δr_t follows immediately from Proposition 1 if we know the path for ι_t and Λ_t set by the CB. The real spread gives us r_t^l. To solve the household's problem, we need the path for $\left\{r_t^a, r_t^l, T_t\right\}$. The path for T_t must be consistent with (17). Then, the evolution of $f\left(s, t\right)$ obtained from the household's problem yields the right-hand side of equation (18). The equilibrium rate r_t^a must be the one that solves (4) implicitly.

Note that in the steady state of the model, given the real credit spread Δr, the HJB equation (8), KFE equation (9) and the real market clearing condition (18) imply that the equilibrium solution to the real markets is independent of implementation and nominal variables.

Thus, we split the exposition of the solution algorithm into two parts: the part of real market and the implementation block. For the part of real market, the path of credit spreads is taken as given. For the part of implementation and nominal variables, we take the IOR i^m as given and use the equations in (5) and the Fisher equation to pin down the steady-state interbank market tightness θ, nominal deposit rate i^a and inflation rate π. However, in solving the transition dynamics, the real market variables are connected to implementation and nominal variables via the Phillips curve and the Taylor Rule. Therefore, given the initial IOR i_0^m and the path of the real credit spreads Δr_t, we solve the real deposit rate r_t^a and the endogenous labor- market adjustment rate Γ_t jointly using the real market clearing condition and the Fisher equation.

G.1 Solution Algorithm: Stationary (Steady-State) Equilibrium

The supply side block is entirely determined by the equilibrium labor flows described in E. That block can be solved analytically. For the demand side, we need to compute the value of the real deposit rate that satisfies the real market clearing condition (18) in steady state. We focus on the stationary equilibrium where the stead-state job finding rate and job separation rate are the natural rates calibrated in Table 2.

The demand side requires to solve the steady-state real deposit rate r_{ss}^a. For that we use an iteration algorithm that proceeds as follows. Let us denote $z \in \{e, u\}$ as the household's employment status with switch, $z' = \{e, u\}$, and $s \in [\bar{s}, \infty)$ as the household's asset holdings. First, we take the real credit spread Δr as given, consider an initial guess of deposit rate $r^{a,0}$ and fiscal transfer $T^{0,0}$, and set the iteration index $j, k := 0$. Then:

1. **Individual household's problem.** Given $r^{a,k}$ and $T^{j,k}$, for each $z \in \{e, u\}$, solve the household's value function $V^{j,k}\left(z, s\right)$ from HJB equation (8) using a finite difference method. Calculate the consumption function $c^{j,k}\left(z, s\right)$ and the asset accumulation rate $\mu^{j,k}\left(z, s\right) = r^k\left(s\right) \cdot s - c^{j,k}\left(z, s\right) + w^{j,k}\left(z\right)$, where

$$r^k\left(s\right) = \begin{cases} r^{a,k}, & \text{if } s > 0, \\ r^{a,k} + \Delta r_{ss}, & \text{if } s \leq 0, \end{cases}$$

$w^{j,k}\left(e\right) = 1 - \tau^l + T^{j,k}$, and $w^{j,k}\left(u\right) = b + T^{j,k}$.

2. **Aggregate distribution.** Given $\mu^{j,k}(z,s)$ and $c^{j,k}(z,s)$, solve the KF equation (9) for $f^{j,k}(z,s)$ using a finite difference method.

3. **Fiscal transfer and total output.** Given $c^{j,k}(z,s)$, $f^{j,k}(z,s)$, calculate fiscal transfer

$$T^{j+1,k} = \Delta r_{ss} \cdot \int_0^\infty s\left[f^{j,k}(e,s) + f^{j,k}(u,s)\right] ds + \tau^l \cdot e_{ss} - b \cdot u_{ss}.$$

If $T^{j+1,k}$ is close enough to $T^{j,k}$, proceed to 4. Otherwise, set $j := j+1$ and proceed to 1.

4. **Equilibrium real deposit rate.** Given $f^{j,k}(z,s)$, compute the net supply of real financial claims

$$S\left(r^{a,k}\right) = \int_{\bar{s}}^\infty s\left[f^{j,k}(e,s) + f^{j,k}(u,s)\right] ds$$

and update the interest rate: if $S\left(r^{a,k}\right) > 0$, decrease it to $r^{a,k+1} < r^{a,k}$ and vice versa. If $S\left(r^{a,k}\right)$ is close enough to 0, stop. Otherwise, set $k := k+1$ and $j = 0$, and proceed to 1.

5. **Equilibrium implementation and nominal variables.** Given the exogenous credit spread Δr_{ss} and IOR rate i^m_{ss}, the steady-state interbank market tightness θ_{ss}, the nominal deposit rate i^a_{ss} and inflation rate π_{ss} are given by

$$\begin{cases} \Delta r_{ss} = \frac{\delta}{2}\chi^-(\theta_{ss}), \\ i^a_{ss} = i^m_{ss} + \frac{1}{2}\left[\chi^+(\theta_{ss}) + (1-\delta)\chi^-(\theta_{ss})\right], \\ \pi_{ss} = i^a_{ss} - r^a_{ss}. \end{cases}$$

G.1.1 Solution to the HJB equation

The household's HJB equation is solved using an upwind finite difference scheme similar to Achdou et al. (2021). It approximates the value function $V(z,s)$ on a finite grid with step $\Delta s : s \in \{s_1, ..., s_I\}$, where $s_i = s_{i-1} + \Delta s = s_1 + (i-1)\Delta s$ for $2 \le i \le I$. The bounds are $s_1 = \bar{s}$ and $s_I = s^{max}$, such that $\Delta s = (s^{max} - \bar{s})/(I-1)$. The upper bound s^{max} is an arbitrarily large number such that $f(z,s,t) = 0$ for all $s > s^{max}$. We use the short-hand notation $V_{z,i} \equiv V(z,s_i)$, and similarly for the policy function $c_{z,i}$ and $\mu_{z,i}$.

Note that the HJB involves the first and second derivatives of the value function, $V'_{z,i} = V'_s(z,s_i)$ and $V''_{z,i} = V''_s(z,s_i)$. The first derivative is approximated with either a forward (F) or a backward (B) approximation,

$$V'_{z,i} \approx \partial_F V_{z,i} \equiv \frac{V_{z,i+1} - V_{z,i}}{\Delta s}, \tag{82}$$

$$V'_{z,i} \approx \partial_B V_{z,i} \equiv \frac{V_{z,i} - V_{z,i-1}}{\Delta s}. \tag{83}$$

The second-order derivative is approximated by a central difference:

$$V''_{z,i} \approx \partial_{ss} V_{z,i} \equiv \frac{V_{z,i+1} - 2V_{z,i} + V_{z,i-1}}{(\Delta s)^2}. \tag{84}$$

Let the superscript n be the iteration counter. The HJB equation is approximated by the following upwind scheme,

$$\frac{V_{z,i}^{n+1}-V_{z,i}^{n}}{\Delta}+\rho V_{z,i}^{n+1}=U\left(c_{z,i}^{n}\right)+\partial_F V_{z,i}^{n+1}\cdot\left(\mu_{z,i,F}^{n}\right)^{+}+\partial_B V_{z,i}^{n+1}\cdot\left(\mu_{z,i,B}^{n}\right)^{-}+\Gamma^{zz'}\left[V_{z',i}^{n+1}-V_{z,i}^{n+1}\right], \quad (85)$$

where

$$\mu_{z,i,F}^{n}=r\left(s_i\right)\cdot s_i-\left(\partial_F V_{z,i}^{n}\right)^{-1/\gamma}+w\left(z\right), \quad (86)$$

$$\mu_{z,i,B}^{n}=r\left(s_i\right)\cdot s_i-\left(\partial_B V_{z,i}^{n}\right)^{-1/\gamma}+w\left(z\right). \quad (87)$$

The optimal consumption is set to

$$c_{z,i}^{n}=\left(\partial V_{z,i}^{n}\right)^{-1/\gamma}, \quad (88)$$

where

$$\partial V_{z,i}^{n}=\partial_F V_{z,i}^{n}\mathbf{1}_{\mu_{z,i,F}^{n}>0}+\partial_B V_{z,i}^{n}\mathbf{1}_{\mu_{z,i,B}^{n}<0}+\partial\bar{V}_{z,i}^{n}\mathbf{1}_{\mu_{z,i,F}^{n}\leq 0}\mathbf{1}_{\mu_{z,i,B}^{n}\geq 0}.$$

In the above expression, $\partial\bar{V}_{z,i}^{n}=\left(\bar{c}_{z,i}^{n}\right)^{-\gamma}$ where $\bar{c}_{z,i}^{n}$ is the consumption level such that $\mu_{z,i}^{n}=0$, i.e.,

$$\bar{c}_{z,i}^{n}=r\left(s_i\right)\cdot s_i+w\left(z\right).$$

Substituting the definition of the derivatives (82), (83) and (84), equation (85) is

$$\frac{V_{z,i}^{n+1}-V_{z,i}^{n}}{\Delta}+\rho V_{z,i}^{n+1}=U\left(c_{z,i}^{n}\right)+\frac{V_{z,i+1}^{n+1}-V_{z,i}^{n+1}}{\Delta s}\cdot\left(\mu_{z,i,F}^{n}\right)^{+}+\frac{V_{z,i}^{n+1}-V_{z,i-1}^{n+1}}{\Delta s}\cdot\left(\mu_{z,i,B}^{n}\right)^{-}+\Gamma^{zz'}\left[V_{z',i}^{n+1}-V_{z,i}^{n+1}\right].$$

Collecting terms with the same subscripts on the right-hand side

$$\begin{cases}\frac{V_{z,i}^{n+1}-V_{z,i}^{n}}{\Delta}+\rho V_{z,i}^{n+1}=U\left(c_{z,i}^{n}\right)+\alpha_{z,i}^{n}V_{z,i-1}^{n+1}+\beta_{z,i}^{n}V_{z,i}^{n+1}+\zeta_{z,i}^{n}V_{z,i+1}^{n+1}+\Gamma^{zz'}V_{z',i}^{n+1}\\ \alpha_{z,i}^{n}=-\frac{\left(\mu_{z,i,B}^{n}\right)^{-}}{\Delta s}\\ \beta_{z,i}^{n}=-\frac{\left(\mu_{z,i,F}^{n}\right)^{+}}{\Delta s}+\frac{\left(\mu_{z,i,B}^{n}\right)^{-}}{\Delta s}-\Gamma^{zz'}\\ \zeta_{z,i}^{n}=\frac{\left(\mu_{z,i,F}^{n}\right)^{+}}{\Delta s}\end{cases} \quad (89)$$

Note that $\alpha_1=0$, and we set $\zeta_I=0$ for the stability of the algorithm. Equation (89) is a system of $2I$ linear equations which can be written in the following matrix form:

$$\frac{1}{\Delta}\left(\mathbf{V}^{n+1}-\mathbf{V}^{n}\right)+\rho\mathbf{V}^{n+1}=\mathbf{U}^{n}+\mathbf{A}^{n}\mathbf{V}^{n+1}$$

where

$$\mathbf{A}^n = \begin{bmatrix} \beta^n_{e,1} & \zeta^n_{e,1} & 0 & \cdots & 0 & \Gamma^{eu} & 0 & 0 & \cdots & 0 \\ \alpha^n_{e,2} & \beta^n_{e,2} & \zeta^n_{e,2} & 0 & \cdots & 0 & \Gamma^{eu} & 0 & 0 & \cdots \\ 0 & \alpha^n_{e,3} & \beta^n_{e,3} & \zeta^n_{e,3} & 0 & \cdots & 0 & \Gamma^{eu} & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \ddots & \ddots & \alpha^n_{e,I} & \beta^n_{e,I} & 0 & 0 & 0 & 0 & \Gamma^{eu} \\ \Gamma^{ue} & 0 & 0 & 0 & 0 & \beta^n_{u,1} & \zeta^n_{u,1} & 0 & 0 & 0 \\ 0 & \Gamma^{ue} & 0 & 0 & 0 & \alpha^n_{u,2} & \beta^n_{u,2} & \zeta^n_{u,2} & 0 & 0 \\ 0 & 0 & \Gamma^{ue} & 0 & 0 & 0 & \alpha^n_{u,3} & \beta^n_{u,3} & \zeta^n_{u,3} & 0 \\ 0 & 0 & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\ 0 & \cdots & \cdots & 0 & \Gamma^{ue} & 0 & \cdots & 0 & \alpha^n_{u,I} & \beta^n_{u,I} \end{bmatrix}, \quad (90)$$

and

$$\mathbf{V}^{n+1} = \begin{bmatrix} V^{n+1}_{e,1} \\ \vdots \\ V^{n+1}_{e,I} \\ V^{n+1}_{u,1} \\ \vdots \\ V^{n+1}_{u,I} \end{bmatrix}, \ \mathbf{U}^n = \begin{bmatrix} U\left(c^n_{e,1}\right) \\ \vdots \\ U\left(c^n_{e,I}\right) \\ U\left(c^n_{u,1}\right) \\ \vdots \\ U\left(c^n_{u,I}\right) \end{bmatrix}.$$

The system in turn can be written as

$$\mathbf{B}^n\mathbf{V}^{n+1} = \mathbf{d}^n \quad (91)$$

where $\mathbf{B}^n = \left(\frac{1}{\Delta} + \rho\right)\mathbf{I} - \mathbf{A}^n$ and $\mathbf{d}^n = \mathbf{U}^n + \frac{1}{\Delta}\mathbf{V}^n$.

Version with Idiosyncratic Discount Shocks. In this note, we introduce discount rate heterogeneity into the standard household problem. The HJB equation by household is given by

$$\rho V(z,s,\rho,t) = \max_{\{c\}} U(c) + V_s^{'}\mu(z,s,\rho,t) + \Gamma_t^{z^{,}z}\left(V(z^{,},s,\rho,t) - V(z,s,\rho,t)\right) + \Gamma_t^{\rho^{,}\rho}\left(V(z,s,\rho^{,},t) - V(z,s,\rho,t)\right) + \dot{V}_t$$

where z captures the employment status, s captures the asset holding status, and ρ captures the discount rate status. To simplify our analysis, we assume that ρ takes either ρ_1 or ρ_2. $\mu(z,s,\rho,t)$ is given by

$\mu(z,s,\rho,t) = r_t s - c + w_t(z)$ subject to the following constraint $\dot{s} \geq 0$ at $s = \overline{s}$ and $c_t \leq w_t(z)$ in $s \in [\overline{s}, s_t)$. We approximate the value function $V(z,s)$ on a finite grid with step Δs: $s \in \{s_1, \cdots, s_I\}$,where $s_i = s_1 + (i-1)\Delta s$.

HJB involves the first and second derivatives of the value function. The first derivative is approximated with either a forward (F) or a backward (B) approximation,

$$V'_{z,i,\rho} \approx \partial_F V_{z,i,\rho} = \frac{V_{z,i+1,\rho} - V_{z,i,\rho}}{\Delta s}$$

$$V'_{z,i,\rho} \approx \partial_B V_{z,i,\rho} = \frac{V_{z,i,\rho} - V_{z,i-1,\rho}}{\Delta s}$$

where z captures the employment status, i captures the state of asset holding, and ρ captures the state of discount rate. The second order difference is approximated by a central difference:

$$V''_{z,i,\rho} \approx \partial_{ss} V_{z,i,\rho} = \frac{V_{z,i+1,\rho} - 2V_{z,i,\rho} + V_{z,i-1,\rho}}{(\Delta s)^2}.$$

Let the superscript n be the iteration counter. The HJB equation is approximated by the following upward scheme,

$$\begin{aligned} \frac{V^{n+1}_{z,i,\rho_1} - V^n_{z,i,\rho_1}}{\Delta} + \rho_1 V^{n+1}_{z,i,\rho_1} &= U\left(c^n_{z,i,\rho_1}\right) + \partial_F V^{n+1}_{z,i,\rho_1}\left(\mu^n_{z,i,\rho_1,F}\right)^+ \\ &\quad + \partial_B V^{n+1}_{z,i,\rho_1}\left(\mu^n_{z,i,\rho_1,B}\right)^- + \Gamma^{z'z}_t\left(V^{n+1}_{z',i,\rho_1} - V^{n+1}_{z,i,\rho_1}\right) + \Gamma^{\rho_2\rho_1}_t\left(V^{n+1}_{z,i,\rho_2} - V^{n+1}_{z,i,\rho_1}\right) \end{aligned}$$

where

$$\mu^n_{z,i,\rho_1,F} = r(s_i)s_i - \left(\partial_F V^n_{z,i,\rho_1}\right)^{-1/\gamma} + w(z)$$

$$\mu^n_{z,i,\rho_1,B} = r(s_i)s_i - \left(\partial_B V^n_{z,i,\rho_1}\right)^{-1/\gamma} + w(z)$$

and

$$\begin{aligned} \frac{V^{n+1}_{z,i,\rho_2} - V^n_{z,i,\rho_2}}{\Delta} + \rho_2 V^{n+1}_{z,i,\rho_2} &= U\left(c^n_{z,i,\rho_2}\right) + \partial_F V^{n+1}_{z,i,\rho_2}\left(\mu^n_{z,i,\rho_2,F}\right)^+ \\ &\quad + \partial_B V^{n+1}_{z,i,\rho_2}\left(\mu^n_{z,i,\rho_2,B}\right)^- + \Gamma^{z'z}_t\left(V^{n+1}_{z',i,\rho_2} - V^{n+1}_{z,i,\rho_2}\right) + \Gamma^{\rho_1\rho_2}_t\left(V^{n+1}_{z,i,\rho_1} - V^{n+1}_{z,i,\rho_2}\right) \end{aligned}$$

where

$$\mu^n_{z,i,\rho_2,F} = r(s_i)s_i - \left(\partial_F V^n_{z,i,\rho_2}\right)^{-1/\gamma} + w(z)$$

$$\mu^n_{z,i,\rho_2,B} = r(s_i)s_i - \left(\partial_B V^n_{z,i,\rho_2}\right)^{-1/\gamma} + w(z).$$

The optimal consumption is set to

$$c^n_{z,i,\rho} = \left(\partial V^n_{z,i,\rho}\right)^{-1/\gamma}$$

where

$$\partial V^n_{z,i,\rho} = \partial_F V^n_{z,i,\rho} \mathbf{1}_{\mu^n_{z,i,\rho,F}>0} + \partial_B V^n_{z,i,\rho} \mathbf{1}_{\mu^n_{z,i,\rho,B}<0} + \partial_F \overline{V}^n_{z,i,\rho} \mathbf{1}_{\mu^n_{z,i,\rho,F}\leq 0} \mathbf{1}_{\mu^n_{z,i,\rho,B}\geq 0}$$

In the above expression, $\partial_F \overline{V}^n_{z,i,\rho} = \left(\overline{c}^n_{z,i,\rho}\right)^{-\gamma}$ where $\overline{c}^n_{z,i,\rho}$ is the consumption level such that $\mu^n_{z,i,\rho} = 0$, i.e.,

$$\overline{c}^n_{z,i,\rho} = r\left(s_i\right) s_i + w\left(z\right)$$

Substituting everything into the HJB equation, we get

$$\begin{aligned}
\frac{V^{n+1}_{z,i,\rho_1} - V^n_{z,i,\rho_1}}{\Delta} + \rho_1 V^{n+1}_{z,i,\rho_1} &= U\left(c^n_{z,i,\rho_1}\right) + \frac{V^{n+1}_{z,i+1,\rho_1} - V^{n+1}_{z,i,\rho_1}}{\Delta s}\left(\mu^n_{z,i,\rho_1,F}\right)^+ \\
&+ \frac{V^{n+1}_{z,i,\rho_1} - V^{n+1}_{z,i-1,\rho_1}}{\Delta s}\left(\mu^n_{z,i,\rho_1,B}\right)^- + \Gamma^{z'z}\left(V^{n+1}_{z',i,\rho_1} - V^{n+1}_{z,i,\rho_1}\right) + \Gamma^{\rho_2\rho_1}\left(V^{n+1}_{z,i,\rho_2} - V^{n+1}_{z,i,\rho_1}\right)
\end{aligned}$$

and

$$\begin{aligned}
\frac{V^{n+1}_{z,i,\rho_2} - V^n_{z,i,\rho_2}}{\Delta} + \rho_2 V^{n+1}_{z,i,\rho_2} &= U\left(c^n_{z,i,\rho_2}\right) + \frac{V^{n+1}_{z,i+1,\rho_2} - V^{n+1}_{z,i,\rho_2}}{\Delta s}\left(\mu^n_{z,i,\rho_2,F}\right)^+ \\
&+ \frac{V^{n+1}_{z,i,\rho_2} - V^{n+1}_{z,i-1,\rho_2}}{\Delta s}\left(\mu^n_{z,i,\rho_2,B}\right)^- + \Gamma^{z'z}\left(V^{n+1}_{z',i,\rho_2} - V^{n+1}_{z,i,\rho_2}\right) + \Gamma^{\rho_1\rho_2}\left(V^{n+1}_{z,i,\rho_1} - V^{n+1}_{z,i,\rho_2}\right).
\end{aligned}$$

Collecting terms with the same subscripts on the right-hand side

$$\frac{V^{n+1}_{z,i,\rho_1} - V^n_{z,i,\rho_1}}{\Delta} + \rho_1 V^{n+1}_{z,i,\rho_1} = U\left(c^n_{z,i,\rho_1}\right) + \alpha^n_{z,i,\rho_1} V^{n+1}_{z,i-1,\rho_1} + \beta^n_{z,i,\rho_1} V^{n+1}_{z,i,\rho_1} + \zeta^n_{z,i,\rho_1} V^{n+1}_{z,i+1,\rho_1} + \Gamma^{z'z} V^{n+1}_{z',i,\rho_1} + \Gamma^{\rho_2\rho_1} V^{n+1}_{z,i,\rho_2}$$

where

$$\begin{aligned}
\alpha^n_{z,i,\rho_1} &= -\frac{\left(\mu^n_{z,i,\rho_1,B}\right)^-}{\Delta s} \\
\beta^n_{z,i,\rho_1} &= -\frac{\left(\mu^n_{z,i,\rho_1,F}\right)^+}{\Delta s} + \frac{\left(\mu^n_{z,i,\rho_1,B}\right)^-}{\Delta s} - \Gamma^{zz'} - \Gamma^{\rho_2\rho_1} \\
\zeta^n_{z,i,\rho_1} &= \frac{\left(\mu^n_{z,i,\rho_1,F}\right)^+}{\Delta s}
\end{aligned}$$

and

$$\frac{V^{n+1}_{z,i,\rho_2} - V^n_{z,i,\rho_2}}{\Delta} + \rho_2 V^{n+1}_{z,i,\rho_2} = U\left(c^n_{z,i,\rho_2}\right) + \alpha^n_{z,i,\rho_2} V^{n+1}_{z,i-1,\rho_2} + \beta^n_{z,i,\rho_2} V^{n+1}_{z,i,\rho_2} + \zeta^n_{z,i,\rho_2} V^{n+1}_{z,i+1,\rho_2} + \Gamma^{z'z} V^{n+1}_{z',i,\rho_2} + \Gamma^{\rho_1\rho_2} V^{n+1}_{z,i,\rho_1}$$

where

$$\begin{aligned}
\alpha^n_{z,i,\rho_2} &= -\frac{\left(\mu^n_{z,i,\rho_2,B}\right)^-}{\Delta s} \\
\beta^n_{z,i,\rho_2} &= -\frac{\left(\mu^n_{z,i,\rho_2,F}\right)^+}{\Delta s} + \frac{\left(\mu^n_{z,i,\rho_2,B}\right)^-}{\Delta s} - \Gamma^{zz'} - \Gamma^{\rho_1\rho_2}
\end{aligned}$$

$$\zeta^n_{z,i,\rho_2} = \frac{\left(\mu^n_{z,i,\rho_2,F}\right)^+}{\Delta s}$$

This is a system of $2 \times 2 \times I$ linear equations which can be written in the following matrix form:

$$\frac{1}{\Delta}\left(\boldsymbol{V}^{n+1} - \boldsymbol{V}^n\right) + \boldsymbol{\rho}\boldsymbol{V}^{n+1} = \boldsymbol{U}^n + \boldsymbol{A}^n\boldsymbol{V}^{n+1}$$

where

$$
\boldsymbol{A}^n = \begin{bmatrix}
\beta^n_{e,1,\rho_1} & \zeta^n_{e,1,\rho_1} & 0 & \cdots & 0 & \Gamma^{ue} & 0 & 0 & \cdots & 0 & \Gamma^{\rho_2\rho_1} & 0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 \\
\alpha^n_{e,2,\rho_1} & \beta^n_{e,2,\rho_1} & \zeta^n_{e,2,\rho_1} & 0 & \cdots & 0 & \Gamma^{ue} & 0 & \cdots & 0 & 0 & \Gamma^{\rho_2\rho_1} & 0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \vdots \\
0 & \alpha^n_{e,3,\rho_1} & \beta^n_{e,3,\rho_1} & \zeta^n_{e,3,\rho_1} & 0 & \cdots & 0 & \Gamma^{ue} & \cdots & 0 & \vdots & 0 & \ddots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \vdots \\
\cdots & \ddots & \ddots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \cdots & \cdots & \cdots & \cdots & \cdots & \vdots \\
0 & \cdots & \cdots & \alpha^n_{e,I,\rho_1} & \beta^n_{e,I,\rho_1} & 0 & 0 & 0 & 0 & \Gamma^{ue} & \vdots & \vdots & \vdots & \vdots & \ddots & \cdots & \cdots & \cdots & \cdots & \vdots \\
\Gamma^{eu} & 0 & 0 & \cdots & 0 & \beta^n_{u,1,\rho_1} & \zeta^n_{u,1,\rho_1} & 0 & \cdots & 0 & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \cdots & \cdots & \cdots & \vdots \\
0 & \Gamma^{eu} & 0 & \cdots & \vdots & \alpha^n_{u,2,\rho_1} & \beta^n_{u,2,\rho_1} & \zeta^n_{u,2,\rho_1} & 0 & \cdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \cdots & \cdots & \vdots \\
0 & 0 & \Gamma^{eu} & \cdots & \vdots & 0 & \alpha^n_{u,3,\rho_1} & \beta^n_{u,3,\rho_1} & \zeta^n_{u,3,\rho_1} & 0 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \cdots & \vdots \\
\vdots & \vdots & \vdots & \ddots & \vdots & \cdots & \ddots & \ddots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & \cdots & \cdots & 0 & \Gamma^{eu} & 0 & \cdots & \cdots & \alpha^n_{u,I,\rho_1} & \beta^n_{u,I,\rho_1} & 0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \Gamma^{\rho_2\rho_1} \\
\Gamma^{\rho_1\rho_2} & 0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & \beta^n_{e,1,\rho_2} & \zeta^n_{e,1,\rho_2} & 0 & \cdots & 0 & \Gamma^{ue} & 0 & 0 & \cdots & 0 \\
0 & \Gamma^{\rho_1\rho_2} & 0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \vdots & \alpha^n_{e,2,\rho_2} & \beta^n_{e,2,\rho_2} & \zeta^n_{e,2,\rho_2} & 0 & \cdots & 0 & \Gamma^{ue} & 0 & \cdots & 0 \\
\vdots & 0 & \ddots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \vdots & 0 & \alpha^n_{e,3,\rho_2} & \beta^n_{e,3,\rho_2} & \zeta^n_{e,3,\rho_2} & 0 & \cdots & 0 & \Gamma^{ue} & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \cdots & \cdots & \cdots & \cdots & \cdots & \vdots & \cdots & \ddots & \ddots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \ddots & \cdots & \cdots & \cdots & \cdots & \vdots & 0 & \cdots & \cdots & \alpha^n_{e,I,\rho_2} & \beta^n_{e,I,\rho_2} & 0 & 0 & 0 & 0 & \Gamma^{ue} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \cdots & \cdots & \cdots & \vdots & \Gamma^{eu} & 0 & 0 & \cdots & 0 & \beta^n_{u,1,\rho_2} & \zeta^n_{u,1,\rho_2} & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \cdots & \cdots & \vdots & 0 & \Gamma^{eu} & 0 & \cdots & \vdots & \alpha^n_{u,2,\rho_2} & \beta^n_{u,2,\rho_2} & \zeta^n_{u,2,\rho_2} & 0 & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \cdots & \vdots & 0 & 0 & \Gamma^{eu} & \cdots & \vdots & 0 & \alpha^n_{u,3,\rho_2} & \beta^n_{u,3,\rho_2} & \zeta^n_{u,3,\rho_2} & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \cdots & \ddots & \ddots & \ddots & \vdots \\
0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \Gamma^{\rho_1\rho_2} & 0 & \cdots & \cdots & 0 & \Gamma^{eu} & 0 & \cdots & \cdots & \alpha^n_{u,I,\rho_2} & \beta^n_{u,I,\rho_2}
\end{bmatrix},
$$

$$
\boldsymbol{V}^{n+1} = \begin{bmatrix} V_{e,1,\rho_1}^{n+1} \\ \vdots \\ V_{e,I,\rho_1}^{n+1} \\ V_{u,1,\rho_1}^{n+1} \\ \vdots \\ V_{u,I,\rho_1}^{n+1} \\ V_{e,1,\rho_2}^{n+1} \\ \vdots \\ V_{e,I,\rho_2}^{n+1} \\ V_{u,1,\rho_2}^{n+1} \\ \vdots \\ V_{u,I,\rho_2}^{n+1} \end{bmatrix}
$$

,

$$
\boldsymbol{U}^{n} = \begin{bmatrix} U\left(c_{e,1,\rho_1}^{n}\right) \\ \vdots \\ U\left(c_{e,I,\rho_1}^{n}\right) \\ U\left(c_{u,1,\rho_1}^{n}\right) \\ \vdots \\ U\left(c_{u,I,\rho_1}^{n}\right) \\ U\left(c_{e,1,\rho_2}^{n}\right) \\ \vdots \\ U\left(c_{e,I,\rho_2}^{n}\right) \\ U\left(c_{u,1,\rho_2}^{n}\right) \\ \vdots \\ U\left(c_{u,I,\rho_2}^{n}\right) \end{bmatrix}
$$

, and

$$\boldsymbol{\rho} = \begin{bmatrix} \rho_1 & 0 & \cdots & \cdots & \cdots & 0 \\ 0 & \ddots & 0 & \vdots & \vdots & \vdots \\ \vdots & 0 & \rho_1 & 0 & \vdots & \vdots \\ \vdots & \vdots & 0 & \rho_2 & 0 & \vdots \\ \vdots & \vdots & \vdots & 0 & \ddots & 0 \\ 0 & \cdots & \cdots & \cdots & 0 & \rho_2 \end{bmatrix}$$

The system in turn can be written as

$$\frac{1}{\Delta}\left(\boldsymbol{V}^{n+1} - \boldsymbol{V}^{n}\right) + \boldsymbol{\rho}\boldsymbol{V}^{n+1} = \boldsymbol{U}^{n} + \boldsymbol{A}^{n}\boldsymbol{V}^{n+1}$$

$$\boldsymbol{B}^{n}\boldsymbol{V}^{n+1} = \boldsymbol{d}^{n}$$

where $\boldsymbol{B}^n = \frac{1}{\Delta}\boldsymbol{I} + \boldsymbol{\rho} - \boldsymbol{A}^n$ and $\boldsymbol{d}^n = \boldsymbol{U}^n + \frac{1}{\Delta}\boldsymbol{V}^n$.

Summary Algorithm - HJB. The algorithm to solve the HJB is as follows. We take the interest rate $\{r(s_i)\}_{i=1}^{I}$ and fiscal transfer T as given and begin with an initial guess $\left\{V_{e,i}^0, V_{u,i}^0\right\}_{i=1}^{I}$. Set $n = 0$. Then:

1. Compute $\left\{\partial_F V_{z,i}^n, \partial_B V_{z,i}^n\right\}_{i=1}^{I}$ using (82) and (83).
2. Compute $\left\{c_{z,i}^n\right\}_{i=1}^{I}$ using (88) and $\left\{\mu_{z,i,F}^n, \mu_{z,i,B}^n\right\}_{i=1}^{I}$ using (86) and (87).
3. Find $\left\{V_{z,i}^n\right\}_{i=1}^{I}$ solving the linear system of equations (91).
4. If $\left\{V_{z,i}^{n+1}\right\}$ is close enough to $\left\{V_{z,i}^n\right\}$, stop. Otherwise set $n := n+1$ and proceed to step 1.

G.1.2 Solve KFE in Stationary Equilibrium

The stationary distribution of real wealth satisfies the Kolmogorov Forward equation:

$$0 = -\frac{\partial}{\partial s}\left[\mu(z,s) f(z,s)\right] - \Gamma^{zz'} \cdot f(z,s) + \Gamma^{z'z} \cdot f(z',s), \tag{92}$$

$$1 = \int_{\bar{s}}^{\infty} \left[f(e,s) + f(u,s)\right] ds. \tag{93}$$

We also solve the equation using a finite difference scheme. We use the notation $f_{z,i} \equiv f(z, s_i)$ .The system can be expressed as

$$0 = -\frac{f_{z,i}\left(\mu_{z,i,F}^n\right)^+ - f_{z,i-1}\left(\mu_{z,i-1,F}^n\right)^+}{\Delta s} - \frac{f_{z,i+1}\left(\mu_{z,i+1,B}^n\right)^- - f_{z,i}\left(\mu_{z,i,B}^n\right)^-}{\Delta s}$$
$$- \Gamma^{zz'} f_{z,i} + \Gamma^{z'z} f_{z',i},$$

or equivalently

$$f_{z,i-1}\zeta_{z,i-1} + f_{z,i}\beta_{z,i} + f_{z,i+1}\alpha_{z,i+1} + f_{z',i}\Gamma^{z'z} = 0.$$

The linear equations system can be written as

$$\mathbf{A^T f} = \mathbf{0}, \tag{94}$$

where $\mathbf{A^T}$ is the transpose of $\mathbf{A} = \lim_{n\to\infty} \mathbf{A}^n$. Notice that $\mathbf{A}^n$ is the approximation of the operator $\mathcal{A}$ and $\mathbf{A^T}$ is the approximation of the adjoint operator $\mathcal{A}^*$. In order to impose the normalization constraint (93), we replace one of the entries of the zero vector in equation (94) by a positive constant. We solve the system (94) and obtain a solution $\hat{\mathbf{f}}$. Then we renormalize as

$$f_{z,i} = \frac{\hat{f}_{z,i}}{\sum_{i=1}^{I} \sum_{z\in\{e,u\}} \hat{f}_{z,i}\Delta s}.$$

The same is true of the case with idiosyncratic shocks.

Summary Algorithm - KFE. The algorithm to solve the stationary distribution is as follows.

1. Given the interest rate $\{r(s_i)\}_{i=1}^{I}$ and fiscal transfer T, solve the HJB equation to obtain an estimate of the matrix $\mathbf{A}$.

2. Given $\mathbf{A}$, find the aggregate distribution $\mathbf{f}$.

G.2 Solution Algorithm: Transition Dynamics

For transitions, again we split the solution into the supply and demand side blocks.

G.2.1 Solving the Supply Side Block

1. We conjecture the sequence for Γ_t^{eu} hiring rate.
2. The evolution of the unemployed workers is

   $$\dot{u} = \xi(1-u_t) - u_t\Gamma_t^{eu}.$$

   This is an autonomous ODE, with initial condition u_0. Given the initial guess, Γ_t^{eu} it produces an entire sequence of u_t.
3. Using $Y_t = n_t = 1 - u_t$, the solution to step 2 produces a sequence of output.
   (a) this step also produce a series for $\frac{\dot{Y}_t}{Y_t} = \frac{\dot{n}_t}{n_t}$
4. We backout the number of vacancies using u_t and $\Xi\left(1, \frac{v_t}{u_t}\right) = \Gamma_t^{eu}$.
5. Using $\Xi\left(\frac{u_t}{v_t}, 1\right) = j_t$ we obtain a sequence of job filling rates.

6. By using (60), we get the sequence of $g(t)$ valuations of filled jobs by the firm:

$$g_t = \frac{\mu}{j_t}.$$

7. We use the ODE representation of g_t (61) to generate a sequence for the real intermediate good price denoted by $mc_t \left(= \frac{p_t}{P_t}\right)$:

$$mc_t = \frac{g_t(\rho+\xi) - \dot{g}(t)}{1-\eta}.$$

8. We use the backward solution to the (10),

$$r^n \pi_t = \frac{\epsilon}{\Theta}\left(mc_t - \frac{\epsilon-1}{\epsilon}\right) + \dot{\pi}_t$$

to solve for inflation π_t.

$$\pi_t = \frac{\epsilon}{\Theta}\int_t^{\infty} \exp\left(-r^n(\tau-t)\right)\left(mc_\tau - \frac{\epsilon-1}{\epsilon}\right)d\tau$$

where

$$r^n \equiv \left(\rho - \frac{\dot{Y}_t}{Y_t}\right).$$

- To numerically solve the forwards equation. We treat the transition as lasting for T discrete time intervals. And treat the T part as if we have reached steady state

9. Using the Taylor rule, we generate a sequence of nominal rates (because we know π_t and $\dot{Y}_t/Y_t$). The combination of i_t^m and inflation, gives us a sequence for r_t^m.

10. We pin down a liquidity ratio using a desired spread Δr_t.

11. We obtain: r_t^a and r_t^ℓ as given by the liquidity ratio and i_t^m.

12. Given r_t^a and r_t^ℓ we obtain a sequence of aggregate demand C_t.

    (a) wages are pinned down by mc_t

    (b) we also know the sequence of price adjustment costs

    (c) we also know sequence of transfer due to firm profits.

13. We iterate over the Γ_t^{eu} to solve for the goods or asset market clearing condition.

G.2.2 Solving the Demand-Side Block

The equilibrium transition path is solved in finite horizon $\left[0, \bar{T}\right]$, assuming that the terminal state of the economy is steady state. The finite horizon is discretized evenly into $N_{\bar{T}}$ points in time dimension. We use an iterative algorithm as follows. Given the initial distribution of real wealth $f_0(z,s)$ and the path of exogenous shocks (e.g., equation (28), guess a path of real deposit rate $r_t^{a,0}$, endogenous adjustment rate Γ_t^0 total output Y_t, and fiscal transfer T_t, and set the iteration index $j,k := 0$. Then

0. **The asymptotic steady state.** The asymptotic steady-state value function and real wealth distribution are calculated from Section G.1.

1. **The aggregate output, employment and unemployment.** Given the path of $\Gamma_t^{eu(k)}$ and the terminal condition $\mathcal{U}_{\bar{T}}^k = u_{ss}$, solve the law of motion of unemployed mass (13) backwards in time to compute the path of unemployed mass $\mathcal{U}_t^k$. Calculate the path of aggregate output $Y_t^k = 1 - \mathcal{U}_t^k$.

2. **Individual household's problem.** Given $r_t^{a,k}$, Γ_t^k, $\mathcal{U}_t^k$ and $T_t^{j,k}$, and the terminal condition $V^{j,k}\left(z,s,\bar{T}\right) = V_{ss}\left(z,s\right)$, solve the HJB equation (8) backwards in time to compute the path of $V^{j,k}\left(z,s,t\right)$. Calculate the consumption policy function $c^{j,k}\left(z,s,t\right)$ and the rate of asset accumulation $\mu^{j,k}\left(z,s,t\right)$.

3. **Aggregate distribution.** Given $c^{j,k}\left(z,s,t\right)$ and $\mu^{j,k}\left(z,s,t\right)$, solve the Kolmogorov Forward equation (9) with initial condition $f^{j,k}\left(z,s,0\right) = f_0\left(z,s\right)$ forward in time to compute the path for $f^{j,k}\left(z,s,t\right)$.

4. **Fiscal transfer and total output.** Given $c^{j,k}\left(z,s,t\right)$, $f^{j,k}\left(z,s,t\right)$ and $\mathcal{U}_t^k$ calculate the path of fiscal transfer

$$T_t^{j+1,k} = \Delta r_t \cdot \int_0^\infty s\left[f^{j,k}\left(e,s,t\right) + f^{j,k}\left(u,s,t\right)\right] ds + \tau^l \cdot \left(1 - \mathcal{U}_t^k\right) - b \cdot \mathcal{U}_t^k.$$

If $\left\{T_t^{j+1,k}\right\}_{t=0}^{\bar{T}}$ is close enough to $\left\{T_t^{j,l}\right\}_{t=0}^{\bar{T}}$, proceed to 5. Otherwise, set $j := j+1$ and proceed to 2.

5. **Equilibrium inflation rate and nominal deposit rate.** Given the path of aggregate unemployed mass $\mathcal{U}_t^k$ and the terminal condition of inflation $\pi_{\bar{T}}^k = \pi_{ss}$, solve the Phillips curve (21) backwards in time to compute the path of the inflation rate π_t^k. Next, given the paths of discretionary rate $\bar{i}_t^m$, Taylor parameter η_t and the inflation rate π_t^k, use the Taylor rule (27) to calculate the path of IOR $i_t^{m,k}$. Then given the path of credit spread Δr_t, back out the path of interbank market tightness θ_t using

$$\Delta r_t = \frac{\delta}{2}\chi^-\left(\theta_t\right).$$

Finally, compute the nominal deposit rate using the implementation equation (4), i.e.,

$$i_t^{a,k} = i_t^{m,k} + \frac{1}{2}\left[\chi^+\left(\theta_t\right) + \left(1-\delta\right)\chi^-\left(\theta_t\right)\right].$$

6. **Equilibrium real deposit rate and endogenous adjustment rate.** Given $f^{j,k}\left(z,s,t\right)$, $i_t^{a,k}$ and π_t^k, calculate

$$S^r\left(r_t^{a,k}, \Gamma_t^{eu(k)}, t\right) = \int_{\bar{s}}^\infty s\left[f^{j,k}\left(e,s,t\right) + f^{j,k}\left(u,s,t\right)\right] ds$$

and

$$S^\phi\left(r_t^{a,k}, \Gamma_t^{eu(k)}, t\right) = i_t^{a,k} - r_t^{a,k} - \pi_t^k.$$

We update $\left\{r_t^{a,k}, \Gamma_t^{eu(k)}\right\}_{t=0}^{\bar{T}}$ to $\left\{r_t^{a,k+1}, \Gamma_t^{eu(k1)}\right\}_{t=0}^{\bar{T}}$ using the Broyden's method. However, one can use alternative numerical methods for finding roots in $2N_{\bar{T}}$ variables. If

$$\max_t \left\{ \max \left\{ \left| S^r \left(r_t^{a,k}, \Gamma_t^{eu(k)}, t \right) \right|, \left| S^\Gamma \left(r_t^{a,k}, \Gamma_t^{eu(k)}, t \right) \right| \right\} \right\}$$

is close enough to 0, stop. Otherwise, set $k := k+1$ and $j = 0$, and proceed to 1.

G.2.3 Solution to the HJB Equation

The dynamic HJB equation (8) can be approximated using an upwind scheme as

$$\rho \mathbf{V}^n = \mathbf{U}^{n+1} + \mathbf{A}^{n+1}\mathbf{V}^n + \frac{1}{\Delta t}\left(\mathbf{V}^{n+1} - \mathbf{V}^n\right),$$

where $\mathbf{A}^{n+1}$ is defined in an analogous fashion to (90), and $\Delta t = T/N$ denotes the time length of each discrete period. We start with the terminal condition $\mathbf{V}^N = \mathbf{V}_{ss}$ and solve the path of value function backward, where $\mathbf{V}_{ss}$ denotes the solution to stationary equilibrium obtained from Section G.1. For each $n = 0, 1, ..., N-1$, define $\mathbf{B}^n = \left(\frac{1}{\Delta t} + \rho\right)\mathbf{I} - \mathbf{A}^{n+1}$ and $\mathbf{d}^{n+1} = \mathbf{U}^{n+1} + \frac{1}{\Delta}\mathbf{V}^{n+1}$, and we can solve

$$\mathbf{V}^n = \left(\mathbf{B}^n\right)^{-1}\mathbf{d}^{n+1}.$$

G.2.4 Solution to the KF Equation

Let $\{\mathbf{A}^n\}_{n=1}^{N-1}$ be the solution obtained from Section G.2.3. It is the approximation to the operator $\mathcal{A}$. Using a finite difference scheme similar to the one we employed in Section G.1.2, we obtain:

$$\frac{\mathbf{f}^{n+1} - \mathbf{f}^n}{\Delta t} = \left(\mathbf{A}^n\right)^{\mathbf{T}}\mathbf{f}^{n+1},$$

which implies

$$\mathbf{f}^{n+1} = \left(\mathbf{I} - \Delta t \left(\mathbf{A}^n\right)^{\mathbf{T}}\right)^{-1}\mathbf{f}^n, \ n = 0, 1, ..., N-1. \tag{95}$$

We start from the initial period condition $\mathbf{f}^0 = \mathbf{f}_0$ and solve the KFE forward using (95).

G.3 Solution Algorithm: Risky Steady State Equilibrium

The risky steady state equilibrium consists of the post-shock transition path and the pre-shock steady state. We solve the two parts simultaneously based on the algorithm in Section G.1 and G.2 as follows. Set the iteration index $k := 0$. Then

1. Use the algorithm in Section G.1 to solve the post-shock steady state. Use the post-shock steady-state distribution $f_{ss}(z,s)$ as the guess of initial wealth distribution $f^k(z,s,0)$ at time 0, and use the algorithm in Section G.2 to solve the post-shock transition path and time-0 value function $V^k(z,s,0)$.

2. Use $V^k(z,s,0)$ in step 1 as the input into the following risky steady state HJB:

$$\rho V_{rss}(z,s) = \max_{\{c\}} U(c) + \frac{\partial V_{rss}(z,s)}{\partial s} \cdot \mu(z,s) + \Gamma^{zz'}\left[V_{rss}(z',s) - V_{rss}(z,s)\right] + \chi_{rss}\left[V(z,s,0) - V_{rss}(z,s)\right].$$

Solve the risky steady state solution $V^k_{rss}(z,s)$ and $f^k_{rss}(z,s)$ using the above HJB together with the KF equation (9) and the real market clearing condition (18) according to the algorithm in Section G.1.

3. Set $f^{k+1}(z,s,0) = f^k_{rss}(z,s)$ as the initial wealth distribution at time 0, and then use the algorithm in Section G.2 to solve the post-shock transition path and time-0 value function $V^{k+1}(z,s,0)$.

4. Iterate step 2 and 3 until $\left\{f^k_{rss}(z,s), V^k_{rss}(z,s)\right\}$ converges.