The Transmission of Keynesian Supply Shocks

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*This paper does not necessarily represent the views of the Bank of England or of any of its Committees.
What’s a Keynesian Supply Shock?

Two sectors: cinemas and popcorn

Negative supply shock affects cinemas only

- Excess demand for cinemas, price increases

→ What happens to popcorn?

- Nobody wants popcorn since people eat popcorn only at the movies (complementarity)

- Fall in demand for popcorn, price falls

→ Sectoral supply shocks can lead to fall in aggregate output and prices

Keynesian Supply Questions:

- Do the data support the notion of Keynesian supply shocks?
- Can we offer evidence on their transmission mechanism?
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Questions:
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Empirical challenge: Keynesian supply shocks ≈ Aggregate demand shocks

* Aggregate output and prices move in the same direction
This Paper

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[1] Aggregate demand: Output and prices move in same direction in all sectors
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  - [1] *Aggregate demand*: Output and prices move *same* direction in all sectors
  - [2] *Keynesian supply*: Output and prices move *opposite* direction in one or more sectors
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What we do

* Identify a shock that moves aggregate output and prices in the same direction in a VAR
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* Estimate response of sectoral output and prices to check whether [1] holds

* Evaluate empirical approach and interpretation with New Keynesian multi-sector DSGE
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What we find

* Data consistent with Keynesian supply view
* General feature of business cycles, not driven by Covid shock
* Important role for heterogeneity in price stickiness and production network
Literature and Contribution

▶ Supply shocks and complementarities
  * Corsetti, Dedola and Leduc (2008); Atalay (2017); Guerrieri, Lorenzoni, Straub and Werning (2022)

▶ Granular fluctuations and production networks
  * Gabaix (2011); Foerster, Sarte and Watson (2011); Bouakez, Cardia, Ruge-Murcia (2014); Smets, Tielens and Van Hove (2018); Baqae and Farhi (2020a, 2020b); Gabaix and Kojien (2020)

▶ Supply vs. demand shocks during Covid-19 and its recovery
  * Bekaert, Engstrom and Ermolov (2020); Brinca, Duarte and Faria-e-Castro (2021); del Rio-Chanona, Mealy, Pichler, Lafond and Farmer (2020), Bilbiie and Melitz (2020); Fornaro and Romei (2022)
Empirical Results
Data

- Aggregate and sectoral quarterly data on real gross output and its deflator (Source: BEA)
  - 64 sectors (NAICS 3-digits, ex ‘Oil and Petroleum’)
  - Sample period: 2005Q1 to 2019Q4
Data

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![Deflator Inflation: All sectors](chart1.png)

![Deflator Inflation: Aggregate](chart2.png)
A Simple VAR

VAR for aggregate output growth ($y_t$) and inflation rate of its deflator ($\pi_t$)

\[ x_t = A_0 + A_1 x_{t-1} + B e_t \]
A Simple VAR

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- Identification of \( B \) with sign restrictions:

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<th>Supply shock ( e_{t}^{Sup} )</th>
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Inference

* Sign restrictions as in Uhlig (2005) and Rubio-Ramirez, Waggoner and Zha (2010)
* Gaussian-inverse Wishart / Haar prior with 5,000 draws
Impulse Responses

(A) Demand shock

(B) Supply shock

Note. The solid line in each panel depicts the median impulse response of the specified variable to a 1 standard deviation shock. Shaded bands denote the 90 percent pointwise credible sets.
Local projection of sectoral output growth ($y_{it}$) and inflation ($\pi_{it}$) to aggregate demand shock

$$x_{i,t+h} = \alpha_h + \beta_{i,h}e_{t}^{Dem} + \Gamma_{i,h}Z_{i,t-1} + u_{it+h}$$

where

* $x_{it} \equiv [y_{it} \pi_{it}]'$
* $e_{t}^{Dem}$: aggregate demand shock from VAR
* $Z_{i,t-1}$: lags of output and prices (both sectoral and aggregate)
Sectoral Responses to Aggregate Demand Shocks

- Local projection of **sectoral** output growth \(y_{it}\) and inflation \(\pi_{it}\) to aggregate demand shock

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* \(e_{t}^{Dem}\): aggregate demand shock from VAR
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- Estimate \(\beta_{i,h}\) for each sector \((i = 1, 2, \ldots, 64)\) and each of the 5,000 sign restrictions draws
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- Estimate $\beta_{i,h}$ for each sector ($i = 1, 2, ..., 64$) and each of the 5,000 sign restrictions draws

- Plot distribution of impact responses $\beta_{i,o}$
  * Normalize output impact response to be negative (negative demand shock)
Sectoral Responses to Aggregate Demand Shocks

Two Examples: Accommodation sector vs. Apparel sector

![Graph showing the response of Accommodation sector to aggregate demand shocks]

- Accommodation sector vs. Apparel sector

Output vs. Inflation

In 16 sectors (of 64), not even one of 5,000 impact responses behaves 'demand-like'.

Introduction Empirical Results Model Mechanism Conclusions
Sectoral Responses to Aggregate Demand Shocks

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Introduction

Empirical Results

Model

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Sectoral Responses to Aggregate Demand Shocks
Two Examples: Accommodation sector vs. Apparel sector

In 16 sectors (of 64) not even one of 5,000 impact responses behaves ‘demand-like’
Across all sectors, almost 40% of inflation impact responses do not behave 'demand-like'.
Robustness

- Richer dynamics (4 lags)
- Specification in levels (4 lags)
- Including Covid data
- Value added instead of gross output
- Identify oil shocks alongside demand and supply
- Adding EBP to aggregate VAR and LPs
Empirical evidence not consistent with a *strict view* of aggregate demand shocks
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Conjecture:
- VAR mis-classifies sectoral supply shocks as aggregate demand shock

Interpretation:
- Evidence supportive of Keynesian supply transmission mechanism
Empirical evidence not consistent with a *strict view* of aggregate demand shocks

**Conjecture:**
* VAR mis-classifies sectoral supply shocks as aggregate demand shock

**Interpretation:**
* Evidence supportive of Keynesian supply transmission mechanism

**Next:** Evaluate conjecture and interpretation with a structural model
A Multi-Sector DSGE Model
A Multi-Sector DSGE Model

- Multi-sector DSGE model with (roundabout) production network
  - Similar to Pasten, Schoenle and Weber (2020) and Ghassibe (2021)
  - Heterogeneity in price stickiness
  - Asymmetric input-output linkages
Representative household maximizes present discounted value of utility

\[ \forall h_t = \mathbb{E}_t \left[ \sum_{s=0}^{\infty} \beta^s \Delta_{t+s-1} \left( \ln C_{t+s} - \frac{\sum_{k=1}^{K} N_{kt+s}^{1+\phi}}{1 + \phi} \right) \right] \]

subject to

\[ P_t C_t + \mathbb{E}_t(Q_{t,t+1}D_{t+1}) = D_t + \sum_{k=1}^{K} (W_{kt}N_{kt} + P_{kt}) \]
The overall consumption index is a CES aggregate of sectoral consumption bundles

\[ C_t \equiv \left( \sum_{k=1}^{K} (e^{m_{kt}} \omega_{ck})^{\frac{1}{\eta_c}} \frac{\eta_c}{\eta_c-1} C_{kt}^{\eta_c-1} \right)^{\frac{1}{\eta_c-1}} \]

In turn, each sectoral bundle is a CES aggregator of diversified varieties

\[ C_{kt} \equiv \left[ f_k^{-\frac{1}{\theta}} \int_0^{f_k} C_{kt}(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \]
The technology for firm $j$ in sector $k$ is

$$Y_{kt}(j) = e^{a_{kt}} Z_{kt}(j)^{\alpha_k} N_{kt}(j)^{1-\alpha_k}$$

Composite intermediate input that combines goods from all sectors of the economy

$$Z_{kt}(j) \equiv \left[ \sum_{r=1}^{K} \omega_{kr}^{1/\eta_z} Z_{krt}(j)^{\eta_z} \right]^{\eta_z \eta_z^{-1}}$$

In turn, the sectoral intermediate inputs are aggregators of varieties produced by firms

$$Z_{krt}(j) \equiv \left[ f_r^{1/\theta} \int_0^{f_r} Z_{krt}(j, \ell)^{\theta-1} d\ell \right]^{\theta \theta^{-1}}$$
Price Stickiness

▶ Firms set prices on a staggered basis as in Calvo (1983)

▶ Probability of not being able to reset prices in \( t \) for a firm in sector \( k \) is \( \xi_k \in (0, 1) \)

▶ A firm that can reset its price at time \( t \) solves

\[
V_t^f = \max_{P^*_t(j)} \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \xi_k^s Q_{t,t+s} \left[ P^*_t(j) Y_{kt+s}(j) - W_{t+s} N_{kt+s} - P^k_{t+s} Z_{kt+s}(j) \right] \right\}
\]

subject to the demand for its own good (\( P^k_t \) is the price of the intermediate input bundle)
Monetary policy & Equilibrium

- Central bank sets monetary policy following an interest rate rule

\[
\frac{R_t}{R} = \left( \frac{R_t}{R} \right)^{\rho_i} \left[ \left( \frac{P_t}{P_{t-1}} \right)^{\phi} \left( \frac{Y_t}{Y_{t-1}} \right)^{\phi_y} \right]^{1-\rho_i}
\]

- Labor markets are competitive and clear at the sectoral level

\[
N_{kt} = \int_0^{f_k} N_{kt}(j) dj
\]

- Goods market clearing implies

\[
Y_{kt}(j) = C_{kt}(j) + \sum_{r=1}^{K} \int_0^{f_r} Z_{rkt}(\ell, j) d\ell
\]
Model-Based Validation Exercise

[1] Calibrate model to same 64 sectors as in empirical analysis

* Input/output linkages and intermediates intensity (BEA)
* Frequency of price adjustment stickiness (BLS)
* Elasticity of substitution across intermediates $\eta^Z = 0.5$
* Elasticity of substitution across goods $\eta^C = 1$
* Standard values for remaining parameters

Pasten, Schoenle and Weber (2020)

Baqae and Farhi (2020a,b)
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[3] Apply our empirical methodology to simulated data

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Experiment #1: Sectoral TFP Shocks

- Simulated data driven exclusively by sectoral (uncorrelated) TFP shocks
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- **Step #1**: Aggregate VAR with sign restrictions
  - Aggregate demand-like shocks explain 50% of output forecast error variance
Experiment #1: Sectoral TFP Shocks

- Simulated data driven exclusively by sectoral (uncorrelated) TFP shocks

- Step #2: Estimation of sectoral impact responses
  - Share of wrong responses 41%, number of sectors with wrong responses 21
Experiment #2: Aggregate Demand Shocks

- Simulated data driven exclusively by aggregate demand shocks
Experiment #2: Aggregate Demand Shocks

- Simulated data driven exclusively by aggregate demand shocks

- **Step #1**: Aggregate VAR with sign restrictions
  - Aggregate demand-like shocks explain 99.7% of output forecast error variance
Experiment #2: Aggregate Demand Shocks

- Simulated data driven exclusively by aggregate demand shocks

- **Step #2**: Estimation of sectoral impact responses
  - Share of wrong responses 0%, number of sectors with wrong responses 0
Experiment #3: Sectoral Demand Shocks

- Simulated data driven exclusively by sectoral (uncorrelated) demand shocks
Experiment #3: Sectoral Demand Shocks

- Simulated data driven exclusively by sectoral (uncorrelated) demand shocks

- **Step #1**: Aggregate VAR with sign restrictions
  - Aggregate demand-like shocks explain 93% of output forecast error variance
Experiment #3: Sectoral Demand Shocks

- Simulated data driven exclusively by sectoral (uncorrelated) demand shocks

- **Step #2**: Estimation of sectoral impact responses
  - Share of wrong responses 6%, number of sectors with wrong responses 3

![Output Growth](chart)

![Inflation](chart)
Inspecting the Mechanism
What drives the Keynesian supply transmission mechanism?

Four dimensions of sectoral heterogeneity:

- **Price stickiness**: probability of being able to reset the price in each period
- **Downstreamness**: other sectors’ reliance on a sector’s intermediate goods
- **Upstreamness**: a sector’s reliance on other sectors’ intermediate goods
- **Intermediates intensity**: exponent of intermediates in the production function

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What drives the Keynesian supply transmission mechanism?

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Model exercise

1. Consider a negative sectoral TFP shock for each of the 64 sectors separately
2. Compute the impact response of aggregate inflation to each sectoral TFP shock
3. Scatter plot aggregate inflation response against dimensions of heterogeneity
Inspecting the Mechanism

**Price stickiness**: probability of being able to reset the price in each period

(A) Price Stickiness

\[ t-stat: 6.23 \]
Inspecting the Mechanism

**Downstreamness**: Other sectors’ reliance on a sector’s intermediate goods

![Graph showing the relationship between reliance on sector i’s intermediates and inflation response. The graph has a linear trend line with a t-stat of 6.3.](image-url)
Inspecting the Mechanism

- **Upstreamness**: a sector’s reliance on other sectors’ intermediate goods

![Graph showing the relationship between upstreamness and inflation response](image)

- **(C) Upstreamness**: The graph illustrates the relationship between inflation response and the reliance on other sectors’ intermediates. The t-stat is -1.78.
Inspecting the Mechanism

- **Intermediates intensity**: exponent of intermediates in the production function

![Graph showing the relationship between Intermediates intensity and the share of intermediates in production, with a t-stat of 1.54.](image-url)
Conclusions

- Data supportive of Keynesian supply transmission of sectoral shocks
  * Demand-like shocks derived from aggregate data contaminated by Keynesian supply shocks
Conclusions

- Data supportive of Keynesian supply transmission of sectoral shocks
  - Demand-like shocks derived from aggregate data contaminated by Keynesian supply shocks

- Model highlights key ingredients for Keynesian supply transmission
  - Price stickiness and a sector’s position in the production network
Conclusions

Why do we care?

* Optimal policy mix in response to sectoral shocks (like Covid-19)
  + Tilt balance in favor of fiscal policy? (Guerrieri, Lorenzoni, Straub and Werning, 2020; Woodford, 2020)

* Beyond pandemic, debate about sources of business cycle fluctuations
  + Granular shocks and production networks (Gabaix, 2011; Baqae and Farhi, 2020a, b)
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- **Future research:**
  - Identification of sectoral shocks
  - Cross-country analysis
A1: A Multi-Sector Factor-Augmented VAR
A Multi-Sector Factor-Augmented VAR

- Economy consists of $N$ sectors indexed by $i = 1, 2, \ldots, N$

- Model sectoral output growth ($y_{it}$) and inflation ($\pi_{it}$) through a VAR(1)

$$x_{it} = \Phi_{i0} + \Phi_{i1}x_{it-1} + \eta_{it}$$

where

* $x_{it} \equiv [y_{it} \pi_{it}]'$

All results extend to VAR($p$)
A Multi-Sector Factor-Augmented VAR

Economy consists of \( N \) sectors indexed by \( i = 1, 2, \ldots, N \).

Model sectoral output growth \((y_{it})\) and inflation \((\pi_{it})\) through a VAR(1) with a factor structure

\[
x_{it} = \phi_{i0} + \phi_{i1} x_{it-1} + \Gamma_i f_t + u_{it} \quad \quad i = 1, 2, \ldots, N
\]

where

* \( x_{it} \equiv [y_{it} \pi_{it}]' \)
* \( f_t \) is a vector of unobserved factors common across sectors
* \( u_{it} \) is a vector of unobserved cross-sectionally weakly correlated sectoral innovations
A Multi-Sector Factor-Augmented VAR

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* $f_t$ is a vector of unobserved factors common across sectors
* $u_{it}$ is a vector of unobserved cross-sectionally weakly correlated sectoral innovations

Note: Common factors (elements of $f_t$) capture all cross-sectional comovement in $x_{it}$ due to

[1] Truly aggregate shocks (e.g. TFP, aggregate demand, etc)

[2] Sector-specific shocks with aggregate effects (Foerster, Sarte and Watson, 2011)
A Multi-Sector Factor-Augmented VAR

- Economy consists of $N$ sectors indexed by $i = 1, 2, \ldots, N$

- Model sectoral output growth ($y_{it}$) and inflation ($\pi_{it}$) through a VAR(1) with a factor structure

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where

* $x_{it} \equiv [y_{it} \pi_{it}]'$
* $f_t$ is a vector of unobserved factors common across sectors
* $u_{it}$ is a vector of unobserved cross-sectionally weakly correlated sectoral innovations

- Recover $f_t$ ‘by aggregation’ as in Cesa-Bianchi, Pesaran and Rebucci (2020)
  - Factors can be approximated by cross-sectional averages of observables ($\bar{x}_t$)
Recovering the Common Factors

Notation:

- Consider set of sectoral weights $w = \{w_1, w_2, \ldots, w_N\}$
- Denote weighted average of generic variable $z_{it}$ across all sectors $i$ with $\bar{z}_t = \sum_{i=1}^{N} w_i z_{it}$
- Denote vector of cross-sectional weighted averages with $\bar{x}_t \equiv [\bar{y}_t \; \bar{\pi}_t]'$
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* Denote vector of cross-sectional weighted averages with \( \bar{x}_t \equiv [\bar{y}_t \, \bar{\pi}_t]' \)

**Key assumption:** Sectoral innovations \( u_{it} \) are cross-sectionally weakly correlated

\[
\bar{u}_t = \sum_{i=1}^{N} w_i u_{it} = O_p \left( N^{-\frac{1}{2}} \right)
\]
Recovering the Common Factors

- Solve for $x_{it}$ in terms of current and past common and sectoral shocks

\[ x_{it} = \mu_i + \sum_{l=0}^{\infty} \Phi_{i1}^l \Gamma_{if_t} + \sum_{l=0}^{\infty} \Phi_{i1}^l u_{it} \]
Recovering the Common Factors

- Solve for $x_{it}$ in terms of current and past common and sectoral shocks

$$x_{it} = \mu_i + \sum_{\ell=0}^{\infty} \Phi_{i1}^\ell \Gamma_i f_{t-\ell} + \sum_{\ell=0}^{\infty} \Phi_{i1}^\ell u_{it}$$

- Pre-multiply both sides by $w_i$ and sum equation by equation over $i$

$$\bar{x}_t = \bar{\mu} + \sum_{\ell=0}^{\infty} \sum_{i=0}^{N} w_i \Phi_{i1}^\ell \Gamma_i f_{t-\ell} + \sum_{\ell=0}^{\infty} \sum_{i=0}^{N} w_i \Phi_{i1}^\ell u_{it}$$
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- Weak correlation (+ regularity conditions on $\Phi$, $\Gamma_{i}$, and $w$) imply

$$\bar{x}_t = \bar{\mu} + \Omega(L) f_t + O(N^{-\frac{1}{2}})$$

See all assumptions
Recovering the Common Factors

- Solve for $x_{it}$ in terms of current and past common and sectoral shocks

$$x_{it} = \mu_i + \sum_{\ell=0}^{\infty} \phi_i^\ell \Gamma_i f_t + \sum_{\ell=0}^{\infty} \phi_i^\ell u_{it}$$

- Pre-multiply both sides by $w_i$ and sum equation by equation over $i$

$$\bar{x}_t = \bar{\mu} + \sum_{\ell=0}^{\infty} \sum_{i=0}^{N} w_i \phi_i^\ell \Gamma_i f_{t-\ell} + \sum_{\ell=0}^{\infty} \sum_{i=0}^{N} w_i \phi_i^\ell u_{it}$$

- Weak correlation (+ regularity conditions on $\Phi_i$, $\Gamma_i$, and $w_i$) imply

$$\bar{x}_t = \bar{\mu} + \Omega(L) \Gamma f_t + O(N^{-\frac{1}{2}})$$

- Approximate common factors by inverting and truncating previous expression

$$f_t = \theta + \Theta_0 \bar{x}_t + \sum_{\ell=1}^{k} \Theta_\ell \bar{x}_{t-\ell} + O(N^{-\frac{1}{2}})$$
A Multi-Sector Factor-Augmented VAR

- Economy consists of $N$ sectors indexed by $i = 1, 2, \ldots, N$

- Model sectoral output growth ($y_{it}$) and inflation ($\pi_{it}$) through a VAR(1) with a factor structure

  $$x_{it} = \Phi_{i0} + \Phi_{i1}x_{i,t-1} + \Gamma f_t + u_{it} \quad i = 1, 2, \ldots, N$$

  where

  - $x_{it} \equiv [y_{it} \; \pi_{it}]'$
  - $f_t$ is a vector of unobserved factors common across sectors
  - $u_{it}$ is a vector of unobserved cross-sectionally weakly correlated sectoral innovations

- Unobserved factor model can be approximated by plugging expression for $f_t$

  $$x_{it} = \varphi_{i0} + \Phi_{i1}x_{i,t-1} + \Xi_{i0}x_t + \sum_{\ell=1}^{k} \Xi_{i\ell}x_{t-\ell} + u_{it} \approx \Gamma f_t$$
Identification of the Common Factors

- Factors are always identified up to a rotation matrix

- Rotate $\bar{x}_t$ with a SVAR to obtain aggregate structural shocks $e_t$

$$
\bar{x}_t = A_0 + \sum_{\ell=1}^{k} A_{\ell} \bar{x}_{t-\ell} + B e_t
$$
Identification of the Common Factors

- Factors are always identified up to a rotation matrix

- Rotate $\bar{x}_t$ with a SVAR to obtain aggregate structural shocks $e_t$

$$\bar{x}_t = A_0 + \sum_{\ell=1}^{k} A_\ell \bar{x}_{t-\ell} + B e_t$$

- Plug rotated $\bar{x}_t$ back in sectoral VAR

$$x_{it} = \psi_{i0} + \Phi_{i1} x_{i,t-1} + \Lambda_i \hat{e}_t + \sum_{\ell=1}^{k} \psi_{i\ell} \bar{x}_{t-\ell} + u_{it}$$
Identification of the Common Factors

- Factors are always identified up to a rotation matrix

- Rotate $\tilde{x}_t$ with a SVAR to obtain aggregate structural shocks $e_t$

$$\tilde{x}_t = A_0 + \sum_{\ell=1}^{k} A_\ell \tilde{x}_{t-\ell} + B e_t$$

- Plug rotated $\tilde{x}_t$ back in sectoral VAR

$$x_{it} = \psi_{i0} + \Phi_{i1} x_{i,t-1} + \Lambda_{i} \hat{e}_t + \sum_{\ell=1}^{k} \psi_{i\ell} \tilde{x}_{t-\ell} + u_{it}$$

Main object of interest
Weights and Sectoral Innovations: Theory

**Weights** satisfy smallness conditions

\[ \|w\| = O(N^{-1}) \quad \text{and} \quad \frac{w_i}{\|w\|} = O(N^{-1/2}) \]

**Sectoral innovations** \( u_{it} \) are cross-sectionally weakly correlated

\[ \rho_{\text{max}}(\Sigma_u) = O(1) \]

where \( \rho_{\text{max}}(\Sigma_u) \) denotes largest eigenvalue of covariance matrix \( \Sigma_u = \text{Var}([u_{1t} \; u_{2t} \; \ldots \; u_{Nt}]') \)

Appendix

A1: Multi-sector FAVAR

A2: Additional Results

A3: Model

References

References
Weights and Sectoral Innovations: Data

(A) Weights distribution

(B) Weights over time
Common Factors, Factor Loadings and Coefficients

- The unobservable **common factors** $f_t$ have zero means and finite variances, are serially uncorrelated, and are distributed independently of sector-specific shocks $u_{it}$ for all $i$ and $t$.

- The **factor loadings** (i.e. the elements of $\Gamma_i$ for $i = 1, 2, \ldots, N$) are distributed independently across $i$ and from $f_t$ for all $i$ and $t$. Denoting a generic element of $\Gamma_i$ by $\gamma_i$, the loadings satisfy

$$\gamma = \sum_{i=1}^{N} w_i \gamma_i \neq 0 \quad \text{and} \quad \sum_{i=1}^{N} \gamma_i^2 = O(N).$$

In addition, $\Gamma \equiv \mathbb{E}[\Gamma_i]$ is invertible.

- **Coefficients:** The constants $\Phi_{i0}$ are bounded, the autoregressive coefficients $\Phi_{i1}$ are independently distributed for all $i$, the support of $q(\Phi_{ij})$ lies strictly inside the unit circle, for $i = 1, 2, \ldots, N$, and the inverse of the polynomial $\Omega(L) = \sum_{\ell=0}^{\infty} \Omega_{\ell} L^\ell$, where $\Omega_{\ell} = \mathbb{E}(\Phi_{i\ell})$, exists and has exponentially decaying coefficients, namely $\|\Omega_{\ell}\| \leq C_0 \rho^\ell$, with $0 < \rho < 1$.
A2: Additional Results
<table>
<thead>
<tr>
<th>Sector</th>
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<tbody>
<tr>
<td>Farms</td>
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<tr>
<td>Other transportation equipment</td>
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<tr>
<td>Apparel and leather and allied products</td>
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<tr>
<td>Food and beverage stores</td>
<td></td>
</tr>
<tr>
<td>Transit and ground passenger transport.</td>
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<td>Publishing industries, except internet [..]</td>
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<td>Motion picture and sound recording [..]</td>
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<tr>
<td>Broadcasting and telecommunications</td>
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<td>Data processing, internet publishing [..]</td>
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<td>Fed banks, credit intermed. [..]</td>
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<td>Insurance carriers [..]</td>
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<tr>
<td>Housing</td>
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<tr>
<td>Administrative and support services</td>
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<td>Performing arts, spectator sports [..]</td>
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<tr>
<td>Food services and drinking places</td>
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</tbody>
</table>
Factor Loadings to ‘Demand-Like’ Shock (2020Q1)
Factor Loadings to ‘Demand-Like’ Shock (Value Added)
Factor Loadings to ‘Demand-Like’ Shock (4 lags)

Output Growth

Inflation
Factor Loadings to ‘Demand-Like’ Shock (4 lags, Levels)

Output Growth

-0.02 -0.015 -0.01 -0.005 0

0 K
10 K
20 K
30 K
40 K
50 K
60 K
70 K

Inflation

-0.03 -0.02 -0.01 0 0.01 0.02

0 K
20 K
40 K
60 K
80 K
100 K
120 K
Factor Loadings to ‘Demand-Like’ Shock (Oil Shock)
Factor Loadings to ‘Demand-Like’ Shock (EBP)

Output Growth

Inflation

Appendix
A1: Multi-sector FAVAR
A2: Additional Results
A3: Model
References

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A3: Model
Multi-Sector DSGE Model: Ingredients

- Continuum of monopolistically competitive firms $j$ in sector $k$ produce one variety
- Varieties bundled into sectoral intermediate input and sectoral consumption good
- Each firm $j$ employs CES aggregate of sectoral intermediate bundles
  - Weights calibrated using input-output matrix
- Representative household consumes CES aggregate of sectoral consumption bundles
  - Weights calibrated using sectoral data
- Intermediate good producers set prices on a staggered basis (Calvo, 1983)
- Competitive labor markets clear at sectoral level
- Complete financial markets
- Central bank sets interest rate according to feedback rule (Taylor, 1993)
## Multi-Sector DSGE Model: Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
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<th>Description</th>
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<td>$\beta$</td>
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<td>$\varphi$</td>
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<td>$\omega_{ck}$</td>
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<td>Consumption shares</td>
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<td>$\omega_{kr}$</td>
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<td>Input-Output coefficients</td>
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<td>Sectoral input shares</td>
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<td>Price rigidity parameters</td>
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<td>Elasticity of substitution across sectors (intermediates)</td>
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<td>Elasticity of substitution across sectors (final good)</td>
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<td>$\phi_y$</td>
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<td>$\rho$</td>
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<td>Persistence of shocks</td>
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References


References II


