ESG: A Panacea for Market Power?*

Philip Bond† **Doron Levit**‡

June 26, 2023

Abstract

We study the equilibrium effects of the “S” dimension of ESG in a model of imperfect competition in labor (and product) markets. All else equal, a profit maximizing firm can benefit from adopting ESG policies that give a competitive edge in attracting workers; “Doing Well by Doing Good” applies in our setting. ESG policies are strategic complements, and in equilibrium, they are adopted by all firms resulting in higher worker welfare but lower shareholder value. Thus, profit maximizing firms benefit from coordinating on low impact ESG policies, raising anti-trust concerns from the adoption of industry-wide ESG standards. A purposeful firm (led by a socially conscious board) benefits from such ESG policies, and imperfect competition between purposeful firms obtains the first best in equilibrium. Thus, the social purpose of the corporation is a panacea to excessive marker power. More broadly, our analysis relates the adoption of ESG policies to the nature of competition between firms and their model of corporate governance.

Keywords: ESG, Shareholder Primacy, Stakeholder Capitalism, Corporate Social Responsibility, Corporate Governance, Market Power

JEL classifications: D74, D82, D83, G34, K22

---

*We are grateful to Daniel Green, Deeksha Gupta, Robert Marquez, Martin Oehmke, Johann Reindl, conference participants at the UNC-Duke Corporate Finance Conference 2023, FTG Webinar 2023, Adam Smith Workshop in Corporate Finance 2023, the Financial Intermediation Research Society 2023, The Fourth BI Conference on Corporate Governance, and seminar participants at the University of British Columbia, the University of Bonn, the Federal Reserve Board, Frankfurt School of Finance and Management, the University of Geneva, and HKUST for helpful comments and discussions.

†University of Washington. Email: apbond@uw.edu.

‡University of Washington and ECGI. Email: dlevit@uw.edu.

1 Introduction

There is a long-running debate in academic and policy circles over whether the purpose of the corporation is or, should be, to maximize value for shareholders or, instead, to operate in the interest of all of its various stakeholders. These questions have far-reaching implications, including whether and how companies and boards take into account Environmental, Social and Governance (ESG) considerations when developing and delivering products and services, making business decisions, managing risk, developing long-term strategies, recruiting and retaining talent and investing in the workforce, implementing compliance programs, and crafting public disclosures. A growing number of empirical studies have examined whether firms indeed pursue ESG policies, whether these policies achieve their putative aims, and whether equity markets reward such policies. Theoretical studies have also examined whether and how shareholder actions incentivize firms to behave in socially responsible ways. However, largely absent from the literature is an examination of how firms' ESG policies affect equilibrium outcomes in the real input and output markets that they operate in. Our paper aims to fill this gap.

We develop a benchmark model of the equilibrium effects of corporate social responsibility, thereby focusing on the "S" component of ESG in labor and product markets. In our framework, two oligopolistic firms interact in either the labor or product markets. Imperfect competition and constrained regulation leave room for meaningful corporate social responsibility.[1] We model an ESG policy as a constraint that the firm's board of directors places on the firm's manager to treat workers/customers/suppliers well. The firm's manager maximizes profits (i.e., the shareholder value) subject to satisfying the constraints imposed by the firm's ESG policies. For example, in the context of labor markets, an ESG policy is a commitment to pay employees above market wages, provide generous benefits, invest in worker training, and create a friendly work environment; the manager then chooses how many workers to hire, taking these commitments as given. In the context of product markets, an ESG policy is a commitment to offer products with low environmental impact, high safety standards, protection of customer privacy, cybersecurity, etc.; the then manager chooses production levels, taking these commitments as given. For concreteness, we focus on the labor market application, while

[1] We focus on externalities imposed by firms on industry participants such as competitors, suppliers, and customers. Corporate responsibility with respect to the environment or society at large can be meaningful even under perfect competition.

emphasizing that the product- and input-market applications are isomorphic.

Our analysis highlights two natural consequences of ESG policies. First, an ESG policy potentially strengthens a firm's competitive position with respect to its competitors. Specifically, a promise to treat workers well reduces a manager's ability to exercise monopsony power, thereby increasing hiring. But second, an ESG policy potentially weakens a firm's competitive position by increasing the cost of hiring employees. We characterize how these two different effects play out.

We start by characterizing the equilibrium in labor markets, taking firms' ESG policies as given. If a firm adopts a moderate ESG policy, the pro-competitive effect dominates. Consequently, a moderate ESG firm raises both its market share and its profits, at the expense of competitors. Workers at the ESG firm benefit; the firm hires more of them, and at more generous terms. Workers at non-ESG competitors also benefit, via competition in the labor market.

In contrast, if a firm adopts an aggressive ESG policy then the anti-competitive effect dominates. Such a firm hires fewer workers, though it treats this smaller workforce better. Because the firm's workforce is both smaller and more expensive, profits and market share shrink, benefiting competitors. Workers at these competitors are worse off, because of reduced competition in the labor market.

This first set of equilibrium results illustrates several key points. First, firms can benefit from adopting moderate ESG policies even absent any "warm glow" social preferences of its shareholders or corporate decision makers. Put differently: no matter the reason behind an adoption of ESG policies, we should not be surprised to see that such policies sometimes increase profits. Second, ESG policies that target a firm's stakeholders affect other firms' stakeholders also, and hence have broader welfare implications. Third, the non-monotonic relationship between the strength of a firm's ESG policies and their impact on social welfare underscores that more isn't necessarily better when it comes to ESG, and an externally imposed one-size-fits-all ESG standard could be counter-productive. Fourth, our analysis highlights a novel benefit to firms from publicizing their ESG policies (or pretending to adopt such policies, i.e., social-washing); it gives them a competitive advantage in input and output markets. Without a proper disclosure, the strategic effect of the firm's ESG policies is muted. Finally, the benefit from adopting and advertising an ESG policy depends on the firm's market power and the competitiveness of the markets in which it operates.

Next, we build on our characterization of the labor market equilibrium to study what ESG policies firms adopt, including the strategic interaction with competitors' ESG policies. We first consider the shareholder primacy model in which a firm's board of directors sets ESG policy with the objective of maximizing shareholder value. We show that at moderate levels of ESG, firms' choices are strategic complements. Intuitively, each firm benefits from at least marginally outdoing its competitors' ESG policies, as a means of attracting workers and gaining market share. However, as ESG policies become more extreme, the cost to a firm of being more generous to workers than its competitors is too high, and firms' ESG choices are instead strategic substitutes. Specifically: Although a firm increases its profits by marginally outdoing its competitors' ESG policies, it does even better by instead abandoning ESG policies so that it can compete in an unconstrained way. In equilibrium, profit maximizing firms adopt ESG policies that result in higher wages, higher employment, and in some cases higher social welfare, but lower total shareholder value. Interestingly, larger and more productive firms tend to adopt more aggressive ESG policies, and when they do, the effect on social welfare is typically positive. In contrast, when smaller and unproductive firms use ESG policies as a means to create a competitive advantage in real markets, they create distortions that are beneficial to their shareholders but can be costly from a social perspective.

While the unintended consequences of profit-motivated ESG policies can be socially beneficial, equilibrium ESG policies are always too moderate to fully remove market power distortions, and equilibrium social surplus falls short of the first best. Importantly, profit maximizing firms would benefit from coordinating on low impact ESG policies, raising anti-trust concerns related to the adoption of industry-wide ESG standards.

Nothing that we have said so far requires either shareholders or board members to have preferences that extend beyond the traditional assumption of profit maximization. But in practice, such concerns are likely to lie behind at least some ESG-adoption decisions, and be driven in part by socially conscious investors and/or directors. We conclude our analysis by asking: If a firm sets ESG policies to maximize its total surplus—that is, the sum of profits and employee surplus—then what policy does it set? We label such firms as "purposeful" firms, as their objectives internalize the effect of their policies on other stakeholders of the firm, in our case, workers. Importantly, we maintain the assumption that the firm's manager makes hiring decisions to maximize profits; as such, we distinguish between corporate decision makers who set the firm's ESG policies (i.e., the board of directors) and those who execute them (i.e., managers).

Loosely speaking, purposeful firms want to be large, and as one might expect, they adopt more aggressive ESG policies than profit-maximizing firms. When a purposeful firm competes against profit-maximizing firms, its optimal ESG policy can also benefit its own shareholders. Thus, "Doing Well by Doing Good" applies in our setting. Nevertheless, a purposeful firm adopts excessively aggressive ESG policies, and grows too large relative to other firms, both from the perspective of total industry surplus. Intuitively, purposeful firms do not internalize how their ESG policies affect the hiring decisions and the surplus of other firms. In this case, a purposeful firm would do more social good (i.e., generate a labor-market equilibrium with higher industry surplus) if it were less purposeful, that is, if it weighted shareholder value more heavily than worker welfare, for example, by changing the composition of the company's board of directors. In some cases, the industry surplus created by a profit-maximizing firm can even be higher than the one created by a purposeful firm.

Alternatively, the distortions introduced by a purposeful firm are also mitigated by competition with other purposeful firms. We show that ESG policies are always strategic complements for purposeful firms. Intuitively, and similar to profit-maximizing firms, a purposeful firm always benefits from at least marginally outdoing its competitors' ESG policies. Unlike profit-maximizing firms, however, a purposeful firm is never tempted to undercut its competitors by abandoning ESG policies. In this case, we obtain a striking welfare theorem: Competing purposeful firms pick equilibrium ESG policies that lead to the first-best outcome in labor markets. In other words, competition in ESG policies between purposeful firms entirely eliminates the oligopolistic distortion and maximizes industry surplus. This is true even though each individual firm aims only to maximize its own surplus, which as discussed above, can have adverse welfare effects when only a subset of firms are purposeful.

We have discussed our model's predictions in terms of labor markets. But we re-emphasize a point that we noted early, namely that our analysis applies equally to ESG policies in imperfectly competitive product markets, and generates parallel implications for that setting.

Overall, the social purpose of the corporation is a panacea to excessive marker power. More broadly, our analysis relates the adoption of ESG policies to the nature of competition between firms and their model of corporate governance.

Related literature

At an abstract level, the idea of firms' ESG choices affecting subsequent equilibrium outcomes under imperfect competition is related to literature studying the effects of other types of firm decisions, including, for example, Brander and Lewis (1986)'s analysis of debt choices and Sklivas (1987)'s analysis of managerial contracts. A central theme in much of this literature is that firms can effectively commit to compete more aggressively via decisions made prior to product market interactions, and that doing so is a potential source of advantage. Perhaps surprisingly, this same effect operates in our setting also—after all, it isn't obvious whether committing to pay workers more leads a firm to compete more or less aggressively. More generally, the application of the idea that commitment helps in imperfect competition settings to the specific context of ESG yields numerous insights, including the extent to which competition in ESG firms pushes the equilibrium outcome towards the socially optimal one.

The literature on the consequences of ESG policies for the equilibria of the real markets in which firms operate, and in turn for the ESG choices of competing firms, is relatively small. Closest to our paper is the recent working paper of Stoughton et al (2020), which similarly characterizes the consequences of firms committing to ESG policies before interacting in imperfectly competitive product or labor markets, and shows that profit-maximizing firms typically individually benefit from this commitment. Relative to Stoughton et al we model ESG as a clear commitment to deliver a minimum level of utility to worker or customers, as opposed to committing the manager to the more diffuse objective of putting weight on worker or customer surplus. This difference in how we conceptualize ESG policies has important implications for our analysis, including, for example, the observations that aggressive ESG policies hurt stakeholders in other firms; that there is a strong force pushing each firm to marginally out-do the ESG policies of its competitors; and that a firm's best response to its competitors adopting aggressive ESG policies is to abandon ESG altogether. Moreover, this distinction allows to investigate differences in optimal ESG policies adopted by purposeful and profit maximizing firms.

Xiong and Yang (2022) explore a different motive for ESG policies by profit-maximizing firms that operates for network goods, namely that since each customer benefits from an increase in the total number of customers, an ESG policy can increase a firms' profits by incentivizing a firm to charge lower prices, thereby attracting more customers.

Albuquerque et al (2018) conceptualize ESG very differently, and in particular, as a characteristic that directly impacts consumer demand by decreasing consumers' elasticity of substitution. As such, ESG policies raise profit margins, and reduces exposure to shocks.

In a non-ESG setting, Rey and Tirole (2019) study the use of price caps by firms selling complementary goods, and show that such price caps can alleviate double-marginalization problems for firms. In their analysis, firms collectively agree to price-cap arrangements

A sizeable literature has addressed the topic of a firm's objectives. See, for example, Tirole (2001); or for a recent survey, Gorton et al (2022). Allcott et al (2022) quantitatively estimate the relative importance of firm's profits, consumer surplus, worker surplus, and a subset of externalities including carbon emissions.

While the theoretical literature on the effects of ESG policies on product and labor market is small, a larger theoretical literature considers responsible investing. Heinkel, Kraus, and Zechner (2001) show that, when some investors automatically exclude a brown stock, this lowers its number of shareholders, meaning that each individual shareholder has to bear more risk, in turn reducing its stock price. Davies and Van Wesep (2018) demonstrate that the resulting lower price raises the number of shares granted to the manager if his equity-based pay is fixed in dollar terms, paradoxically rewarding him. Oehmke and Opp (2020) show that responsible investing is only effective if responsible investors are affected by externalities regardless of whether they own the emitting companies, and if they can co-ordinate. Pedersen, Fitzgibbons, and Pomorski (2021) focus on the asset pricing implications of responsible investing and solve for the ESG-efficient frontier. Goldstein et al. (2022) show that responsible investors can increase the cost of capital, because their trades reflect ESG rather than financial performance, thus making the stock price less informative about financials. Pastor, Stambaugh, and Taylor (2021) model how greater taste for green companies increases their valuation and reduces equilibrium expected returns. Edmans, Levit, and Schneemeier (2022) study the optimal socially responsible divestment strategy and show that a tilting strategy whereby a responsible investor holds only the best-in-class brown firm, can be superior to a blanket exclusion strategy whereby all brown firms are sold, as the former gives brown firms incentives to reform. Landier and Lovo (2020) find that the more money investors put into ESG funds, the more important it is for an industry to reduce its externalities to obtain financing. Green and Roth (2021) show that investors targeting social welfare should consider how other commercially-focused investors will react to their portfolio decisions. Chowdhry et al (2019) study co-investment by "impact" and profit-motivated investors.

Our analysis takes as a building block a standard Cournot model of imperfect competition, which makes transparent the role of the novel aspect of our analysis, viz., firms' ESG commitments to treat their stakeholders well. The Cournot model has the specific advantages of allowing for a clear separation between ESG commitments (expressed in terms of price) and subsequent actions in the imperfect-competition game (in Cournot, quantities). It also naturally generates the pro- and anti-competitive effects of ESG policies that are central to our analysis.[2]

2 Set-up

There are two firms. Each firm $i \in \{1,2\}$ deploys labor $l_i \in [0,1]$ to produce $f_i(l_i)$, where $f_i(\cdot)$ is strictly increasing and concave. Throughout, we assume firms choose interior levels of l; that is, we either impose the Inada conditions $f_i'(0) = \infty$ and $f_i'(1) = 0$, or in the case of a linear specification adopt appropriate parameter values. The productivity of the two firms is unambiguously ordered, i.e., the comparison between $f_1'(l)$ and $f_2'(l)$ is independent of l. Without loss, firm 1 is weakly more productive,

$$f_1'(\cdot) \geq f_2'(\cdot). \tag{1}$$

[2] We follow much of the literature of work with a Cournot model in which firms' quantity decisions are strategic substitutes; see (2) below. While this feature matters for our results, a satisfactory analysis of the case of strategic substitutes is beyond the scope of the current paper. We also note that all although the distinction between actions as strategic substitutes and complements is sometimes related to quantity versus price competition, the two notions are separate; quantity competition can generate strategic complementarity, while price competition can generate strategic substitutability. Indeed, in models of price competition based on firm "location," this last point is often overlooked because many analyses focus for simplicity on the case in which all consumers buy from at least one firm; see, for example, the discussion in Mas-Colell et al (1995), and especially exercise 12.c.14.

We write $L \equiv l_1 + l_2$ for total labor employed at all firms. There is a continuum of workers, with a measure normalized to 1, and ordered on $[0,1]$ by outside option $W(l)$ for worker $l \in [0,1]$, where $W'(\cdot) > 0$. Hence the inverse labor supply curve is $W(L)$. We assume

$$W''(L)\,L + W'(L) > 0, \tag{2}$$

which ensures both that firms' reaction functions to other firms' hiring decisions slope down (see formal result below) and that the employment cost $W(L)\,L$ faced by a monopsonistic firm is convex (i.e., $W''(L)\,L + 2W'(L) > 0$). For example, this assumption holds if $W(L)$ is linear or $W(L) = KL^{\frac{1}{\epsilon}}$, where K and ϵ are positive constants.[3]

Firms compete in Cournot fashion. That is, the manager of each firm simultaneously announces employment l_i and the market wage is determined by $W(L)$. There is significant evidence that employers enjoy market power in labor markets; see, for example, Lamadon et al. (2022). The objective of the manager of each firm is to maximize its profits.[4]

The key innovation of our analysis is the assumption that firms can adopt ESG policies. Specifically, before hiring decisions are made, the board of directors of each firm publicly commits its firm to pay its workers a minimum level of $\omega_i \geq 0$, that is, an ESG policy is a ω_i. A firm that has adopted such a policy pays its workers $\max\{\omega_i, W(L)\}$. The manager of the firm maximizes the firm's profits subject to this constraint. Intuitively, the board of directors of the firm sets a minimum wage policy that can be monitored and enforced (wages and benefits are observable and verifiable), but the hiring decision is made by executives who have incentives to maximize profit. Notice that ω_i may also include non-pecuniary benefits to employees.

3 Preliminaries

In this section we state several basic results and definitions that are used in the core analysis.

[3] In this example, the supply curve $\left(\frac{W}{K}\right)^{\epsilon}$ has constant elasticity, where ϵ is the elasticity of labor supply.

[4] Kreps and Scheinkman (1983) show that, under some circumstances, the Cournot outcome arises if firms first choose maximum capacities, and then subsequently engage in price competition. Similarly, we conjecture that equilibria in our setting coincide with the outcomes of a game in which (i) boards of directors set ESG policies; (ii) profit-maximizing managers make capacity decisions; (iii) profit-maximizing managers engage in price competition.

3.1 First-best benchmark

The first best allocation maximizes industry surplus which is defined by the firms' total output net of the outside options of the workers they employ. It is given by

$$S(l_1, l_2) \equiv f_1(l_1) + f_2(l_2) - \int_0^{l_1+l_2} W(l)\, dl. \tag{3}$$

Thus, the first best allocation is l_i^{**} such that for $i \in \{1, 2\}$ we have

$$f_i'(l_i^{**}) = W^{**} \equiv W(l_1^{**} + l_2^{**}). \tag{4}$$

Notice that l_i^{**} would be the equilibrium outcome if all firms were controlled by a single owner whose objective is to maximize surplus rather than profit. It is also immediate that the first best allocation would be achieved if the labor market was fully competitive, so that each firm acts as a price-taker. Indeed, let

$$\lambda_i(W) \equiv \arg\max_l \; f_i(l) - Wl \tag{5}$$

be firm i's profit-maximizing employment decision if facing a constant wage W. Then, $l_i^{**} = \lambda_i(W^{**})$. Notice that $\lambda_i(\cdot)$ is a decreasing function. We use this notation throughout. Since firm 1 is weakly more productive it hires more workers under the first-best allocation, $l_1^{**} \geq l_2^{**}$. Nevertheless, the marginal productivity of both firms is identical, $f_1'(l_1^{**}) = f_2'(l_2^{**})$.

3.2 No-ESG benchmark

Suppose firms cannot commit to ESG policies. Firm i takes firm $-i$'s hiring l_{-i} as given and maximizes profits, generating firm i's reaction function $r_i(l_{-i}; 0)$. Here, 0 denotes No-ESG policy ($\omega_i = 0$). Formally,

$$r_i(l_{-i}; 0) \equiv \max_l \; f_i(l) - lW(l + l_{-i}). \tag{6}$$

Lemma 1 *The reaction function $r_i(l_{-i}; 0)$ is strictly decreasing in l_{-i} and $r_i(l_{-i}; 0) + l_{-i}$ is strictly increasing in l_{-i}.*

All omitted proofs are in the Appendix. Lemma 1 establishes if firm $-i$ hires more then firm i hires less, because firm $-i$'s increased hiring raises wages. However, firm i reduces its employment by less than the increase firm $-i$'s employment, so that overall employment increases. To see the latter point, notice that if firm i instead reduces its employment by the same amount that firm $-i$ increases its, then wages would remain unchanged, while firm i's marginal productivity is higher (since f is concave), implying that firm i isn't optimizing.

Next, we characterize the equilibrium of the No-ESG benchmark.

Lemma 2 *A unique equilibrium of the No-ESG benchmark exists. In equilibrium, each firm $i \in \{1, 2\}$ hires $l_i^B = r_i\left(l_{-i}^B, 0\right)$ which is given by the solution of*

$$f_i'\left(l_i^B\right) = W'\left(l_1^B + l_2^B\right) l_i^B + W\left(l_1^B + l_2^B\right). \tag{7}$$

Moreover, $l_1^B \geq l_2^B$,

$$l_1^B + l_2^B < l_1^{**} + l_2^{**}, \tag{8}$$

and both firms pay their workers

$$W^B \equiv W\left(l_1^B + l_2^B\right) < W^{**}. \tag{9}$$

As in the first-best benchmark, the more productive firm hires more workers, $l_1^B \geq l_2^B$. However, unlike the first-best benchmark, the larger firm has a higher marginal productivity, $f_1'\left(l_1^B\right) \geq f_2'\left(l_2^B\right)$. Intuitively, monopsony power leads firms to not fully internalize the social benefit from increasing employment, and the larger firm fails to internalize it to a larger extent.[5]

Lemma 2 confirms that the usual monopsony distortion arises, so that total employment and wages are below first-best levels. Forcing both firms to hire more and pay higher wages would move the economy closer to efficiency. Regulators who aim to maximize social welfare would be tempted to impose a minimum wage on the industry. However, such an intervention would need to be tailored to industry-specific conditions that are likely to be hard for a regulator

[5] In the proof of Lemma 2 we show that $l_1^B < l_1^{**}$, i.e., the larger firm is always distorted down. However, in general, $l_2^B < l_2^{**}$ is not guaranteed. Intuitively, if the smaller firm is sufficiently unproductive, it hires very few employees in the first place, and hence, the first order determinant of its hiring decision is the market wage, which is lower in equilibrium (relative to the first best) due to the incentives of the larger firm to distort down its own employment.

to observe. In contrast, firms have a better knowledge of the industry in which they operate, motivating our interest in studying their incentives to self-impose ESG policies.

3.3 An ESG firm's reaction function $r_i(\cdot;\omega_i)$

Suppose that before hiring, the board of firm i commits the firm to pay a minimum level of ω_i. Recall that a firm that has adopted such a policy pays its workers $\max\{\omega_i, W(L)\}$. Given the announced ESG policies, the manager of firm i chooses l_i to maximize its profits. This subsection characterizes firm i's hiring response l_i to firm $-i$'s hiring l_{-i}, given firm i's ESG policy ω_i—that is, firm i's reaction function.

Firm i's profits given employment decisions l_i and l_{-i} and firm i's ESG policy ω_i is

$$\pi_i(l_i, l_{-i}; \omega_i) \equiv f_i(l_i) - \max\{W(l_i + l_{-i}), \omega_i\} l_i. \tag{10}$$

Note that firm i's profits are affected by firm $-i$'s ESG policy only via firm $-i$'s hiring decision l_{-i}. As such, firm i's reaction function is independent of firm $-i$'s ESG policy:

$$r_i(l_{-i}; \omega_i) \equiv \arg\max_l \pi_i(l, l_{-i}; \omega_i). \tag{11}$$

To characterize $r_i(l_{-i}; \omega_i)$, we first define $\Lambda_i(\omega)$ as the solution to

$$\Lambda + r_{-i}(\Lambda; 0) = W^{-1}(\omega). \tag{12}$$

In words, $\Lambda_i(\omega)$ is the level of hiring by firm i such if firm $-i$ is a non-ESG firm and responds optimally then the resulting wage is ω. Define $\Lambda_i(\omega) = 0$ if $W(r_i(0;0)) > \omega$ and $\Lambda_i(\omega) = \infty$ if $W(\Lambda + r_i(\Lambda; 0)) < \omega$ for all Λ. Note that $\Lambda_i(\omega) = 0$ is well-defined because, by Lemma 1, the left hand side of (12) is strictly increasing in Λ, so at most one solution exists. For use below, note that Lemma 1 also implies that $\Lambda_i(\cdot)$ is strictly increasing.

Lemma 3 *Firm i's reaction function is given by*

$$r_i\left(l_{-i};\omega_i\right) = \begin{cases} \lambda_i\left(\omega_i\right) & \text{if } l_{-i} \leq W^{-1}\left(\omega_i\right) - \lambda_i(\omega_i) \\ W^{-1}\left(\omega_i\right) - l_{-i} & \text{if } l_{-i} \in \left(W^{-1}\left(\omega_i\right) - \lambda_i\left(\omega_i\right), \Lambda_{-i}\left(\omega_i\right)\right) \\ r_i\left(l_{-i};0\right) & \text{if } l_{-i} \geq \Lambda_{-i}\left(\omega_i\right) \end{cases} \tag{13}$$

$$= \min\left\{\lambda_i\left(\omega_i\right), \max\left\{W^{-1}\left(\omega_i\right) - l_{-i}, r_i\left(l_{-i};0\right)\right\}\right\}. \tag{14}$$

The solid line in Figure 1 graphically illustrates Lemma 3, and in particular shows the three regions of firm i's ESG reaction function. As one would expect, the reaction function is weakly decreasing in l_{-i}. In the first region, where $l_{-i} \leq W^{-1}\left(\omega_i\right) - \lambda_i\left(\omega_i\right)$, we have $r_i\left(l_{-i};\omega_i\right) = \lambda_i\left(\omega_i\right)$ and $W\left(r_i\left(l_{-i};\omega_i\right) + l_{-i}\right) \leq \omega_i$. Since the demand by firm $-i$ is relatively low, the market wage is below firm $i's$ self-imposed minimum wage ω_i. Hence, firm i pays its employees above the market wage as if it faces a perfectly elastic supply at ω_i.[6] In other words, the ESG policy mutes the monopsony distortion of the manager, who acts as a price taker. We label it as the *competitive* region.

In the second region, where $l_{-i} \in \left(W^{-1}\left(\omega_i\right) - \lambda_i\left(\omega_i\right), \Lambda_{-i}\left(\omega_i\right)\right)$, we have $r_i\left(l_{-i};\omega_i\right) = W^{-1}\left(\omega_i\right) - l_{-i}$, which implies $W\left(r_i\left(l_{-i};\omega_i\right) + l_{-i}\right) = \omega_i$. That is, the market wage is equal to firm i's self-imposed minimum wage. In this region, demand by firm $-i$ is higher, and if firm i were to hire as if it faces a perfectly elastic supply at ω_i, the resulting market wage would be higher than its self-imposed minimum wage, which in turn would incentivize firm i to hire less, as if it faces no minimum wage constraint. However, since firm $-i$'s demand isn't so high, if firm i were to hire as if it has no constraints, that is $l_i = r_i\left(l_{-i};0\right)$, then the resulting market wage would be lower than its self-imposed minimum wage, which in turn, would incentivize it to hire more aggressively, as if it faces perfectly elastic supply at ω_i. Therefore, the best response of the firm is to choose the residual level of demand such that the resulting market wage exactly equals its self-imposed minimum wage. Put differently, the manager of firm i ignores the monopsony distortion as long as there are enough workers who are willing to accept a wage of ω_i. Notice that while firm i is not paying above the market wage, its ESG policy increases the market wage above the level that would have emerged if it were to set

[6] If $\omega_i > W\left(L\right)$ then firm i may face excess supply. In this case, the employment in firm i is rationed and workers are randomly allocated to firm i until l_i of them are hired.

$\omega_i = 0$. We label this region as the *residual* region.

In the third region, where $l_{-i} > \Lambda_{-i}(\omega_i)$, firm i's ESG policy isn't binding, i.e., $r_i(l_{-i};\omega_i) = r_i(l_{-i};0)$. To see this, note that $l_{-i} > \Lambda_{-i}(\omega_i)$ is equivalent to $W(l_{-i} + r_i(l_{-i};0)) > \omega_i$, which says that if firm i's profit maximizing response to l_{-i} pushes the market wage above ω_i even absent ESG. We label this as the *non-binding* region.

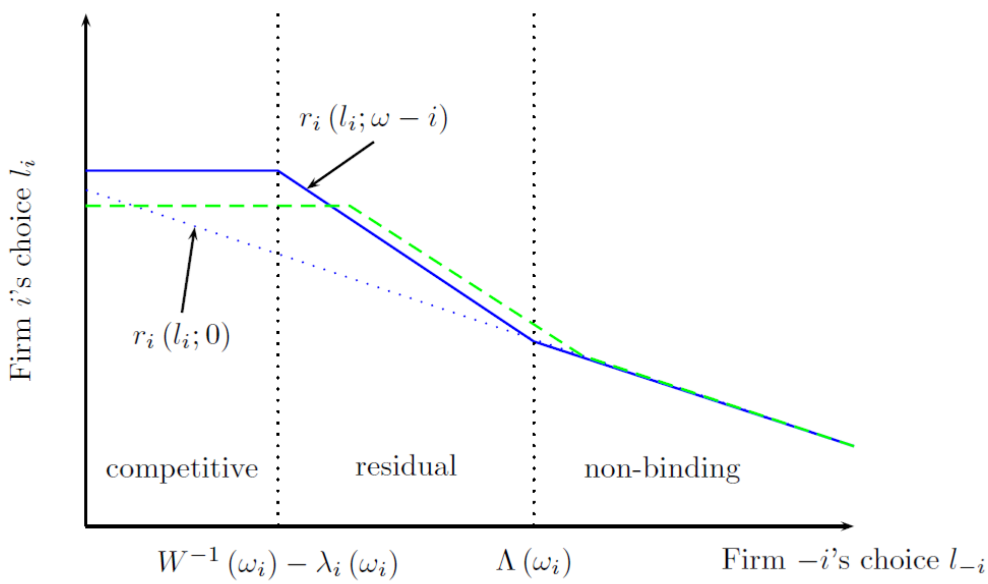


Figure 1 $-$ *ESG firm′s labor reaction function*

Figure 1 also shows how firm i's reaction function shifts as its ESG policy grows more aggressive; this is the shift from the solid blue line to the dashed green line. The competitive, residual, and non-binding regions all shift to the right. For intermediate hiring by firm $-i$, roughly the residual region, a more aggressive ESG policy ω_i leads firm i to hire more, and the reaction function shifts up. This is the pro-competitive effect of ESG; a more aggressive ESG policy extends the perfectly elastic portion of the supply curve that firm i's manager faces. But for low hiring by firm $-i$, roughly the competitive region, a more aggressive ESG policy ω_i leads firm i to hire less, and the reaction function shifts down. This is the anti-competitive effect of ESG; a more ESG policy makes workers more expensive, and the manager hires less.

4 Labor market equilibrium

We next characterize the labor market equilibrium arising from an arbitrary pair of ESG policies. In equilibrium, $l_i^* = r_i\left(l_{-i}^*; \omega_i\right)$ for $i \in \{1,2\}$, and firm i pays its workers $W_i^* = \max\left\{W\left(l_1^* + l_2^*\right), \omega_i\right\}$.

Proposition 1 *For a given (ω_1, ω_2), a labor market equilibrium always exists.*

(i) If $\max_i \omega_i \leq W^B$ then the unique equilibrium is the No-ESG Benchmark as characterized by Lemma 2.

(ii) If $\min_i \omega_i \geq W^{**}$ then the unique equilibrium is $l_i^* = \lambda_i(\omega_i)$ and $W_i^* = \omega_i$ for firms $i = 1, 2$.

(iii) If $\omega_i = \omega_{-i} = \omega \in (W^B, W^{**})$ then for any $i = 1, 2$ and

$$l^* \in \left[W^{-1}(\omega) - \min\left\{\Lambda_{-i}(\omega), \lambda_{-i}(\omega)\right\}, \min\left\{\Lambda_i(\omega), \lambda_i(\omega)\right\}\right] \tag{15}$$

there is an equilibrium in which $\left(l_i^*, l_{-i}^*\right) = \left(l^*, W^{-1}(\omega) - l^*\right)$ and $W_i^* = W_{-i}^* = \omega$. No other equilibrium exists.

(iv) If $\omega_i > \omega_{-i}$, $\omega_i > W^B$ and $\omega_{-i} < W^{**}$ then the unique equilibrium is $l_i^* = \min\left\{\Lambda_i(\omega_i), \lambda_i(\omega_i)\right\}$, $l_{-i}^* = r\left(l_i^*; \omega_{-i}\right)$, $W_i^* = \omega_i$ and $W_{-i}^* = \max\left\{\omega_{-i}, W\left(l_i^* + r_{-i}\left(l_i^*; \omega_{-i}\right)\right)\right\}$. If $\omega_i < W^{**}$ and either firms are symmetric or $i = 1$, then $l_i^* > l_{-i}^*$.

Proposition 1 has several important takeaways. First, according to part (i), if both firms adopt ESG-policies milder than W^B, then the labor market equilibrium coincides with the No-ESG benchmark outcome. Intuitively, these mild ESG policies are non-binding and don't effect the labor market equilibrium. Second, according to part (ii), if both firms adopt ESG-policies that are more aggressive than the first-best wage W^{**}, then in equilibrium each firm pays its self-imposed minimum wage and hires as if it faces a perfectly elastic supply at that level. Both firms pay higher wages than what would have been set by the market absent their self-imposed ESG policies.[7] An immediate implication of this result is that if both firms commit to an ESG

[7] If $\omega_i > W^{**}$ then $\lambda_i(\omega_i) < \lambda_i(W^{**})$, and hence, $W(\lambda_1(\omega_1) + \lambda_2(\omega_2)) < W(\lambda_1(W^{**}) + \lambda_2(W^{**})) = W^{**} < \omega_i$.

policy of W^{**} then the first-best is obtained. The left and right panels of Figure 2 depict the reaction functions and the resulting labor market equilibrium of two symmetric firms when $\max_i \omega_i \leq W^B$ and $\omega_1 = \omega_2 = W^{**}$, respectively.

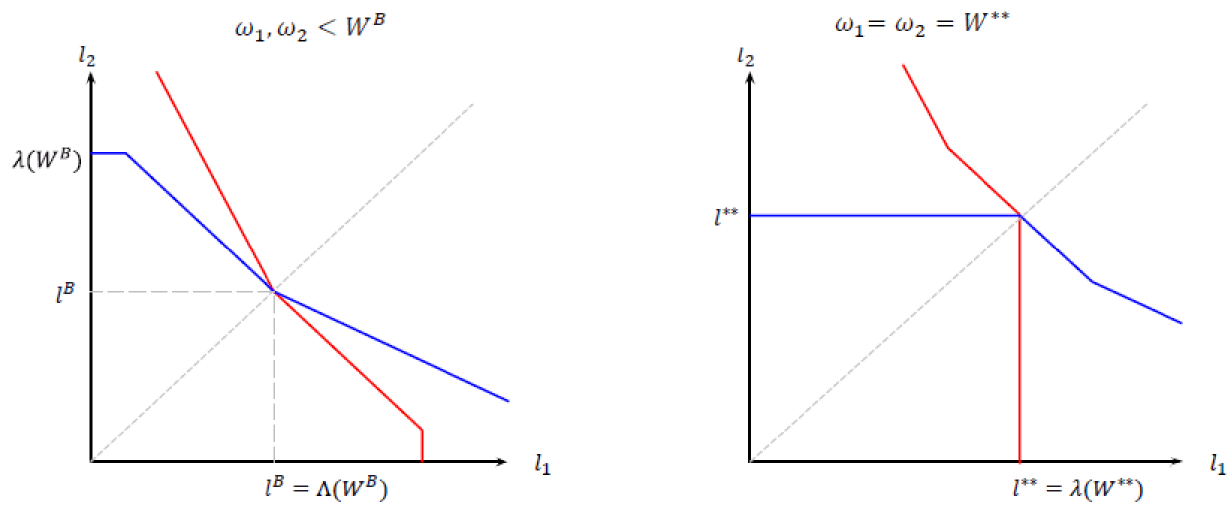


Figure 2 – labor reaction functions under ESG policies that induce the No – ESG benchmark (left panel) and the first best benchmark (right panel)

Third, according to part (iii), if both firms adopt the same ESG-policy then multiple equilibria exist. In all of these equilibria, both firms pay the market wage, which is equal to their identical self-imposed minimum wage ω, and total employment equals $W^{-1}(\omega)$. Although firms pay the market wage, the paid wage and total employment are both higher than their counterparts in the No-ESG benchmark. The different equilibria differ only in the number of workers that each firm hires. The multiplicity stems from the fact that the reaction functions always intersect in the residual-demand region, which has a slope of -1. There, both firms have incentives to hire just enough workers such that the market wage equals the self-imposed minimum wage. Indeed, neither firm has incentives to hire more, since doing which would increase the wage it has to pay (the monopsony effect). At the same time, no firm has incentives to hire less, since doing so would push the market wage below its self-imposed minimum wage.[8]

[8] Notice, this region is non-trivial even though firms are asymmetric; indeed, in the residual-demand region a firm's hiring decision is independent of its production function.

Last, according to part (iv), if the competing firms are similar, the firm that adopts a more aggressive ESG-policy hires more workers in equilibrium. Intuitively, an aggressive ESG-policy commits a firm to hire more and consequently push its competitor to hire less. If the more productive firm also adopts a more aggressive ESG policy, then it will be more aggressive in the labor market both due to its ESG policy and its inherent higher productivity. If the less productive firm adopts a more aggressive ESG policy, then the two forces operate in opposite directions, and the ranking with respect to the ESG policies is ambiguous.[9]

The left panel of Figure 3 depicts the reaction functions of the symmetric firms when they adopt the same moderate ESG policy ($\hat{W}_i$ is in the interval (W^B, W^{**}) and defined in (17) below). The overlapping 45-degree lines is the graphical representation of equilibrium multiplicity. The right panel shows how the equilibrium set shifts to the green dot when firm 2 increases its ESG policy above its opponent ($\omega_2' > \omega_2 = \omega_1$). In this case, the equilibrium is unique, and firm 2 increases its employment while firm 1 reduces its.

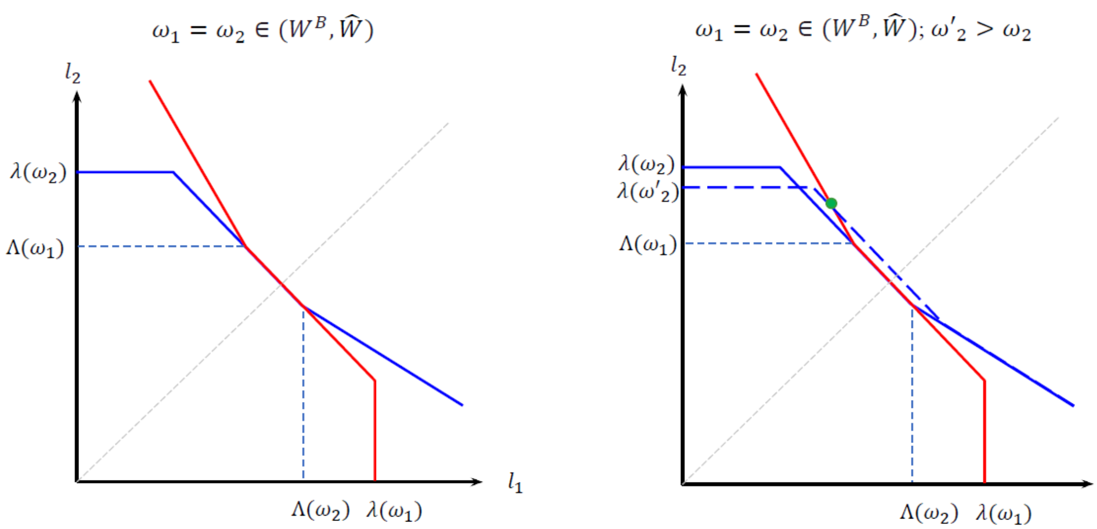


Figure 3 – labor reaction functions under moderate ESG policies that induce multiple equilibria

[9] Interestingly, it is possible that $\omega_i < W^{**}$ and yet $l_i^* > l_i^{**}$. That is, firms can adopt ESG policies that commit them to pay less than the first best wage, and nevertheless, end up hiring more than the first best level in equilibrium. If $\omega_i = \hat{W}_i$, where $\hat{W}_i \in (W_B^*, W^{**})$ is defined by (17) below then $l_i^* = \min\{\Lambda_i(\hat{W}_i), \lambda_i(\hat{W}_i)\} = \lambda_i(\hat{W}_i) > \lambda_i(W^{**}) = l_i^{**}$.

Figure 4 is similar to Figure 3 with the exception that the two firms adopt a relatively extreme ESG policy ($\omega_1, \omega_2 \in (\hat{W}, W^{**})$). The right panel shows how the equilibrium set shifts to the green dot when firm 2 decreases its ESG policy below its opponent's. In this case, the equilibrium is unique, and firm 2 decreases its employment, while firm 1 (weakly) increases its.

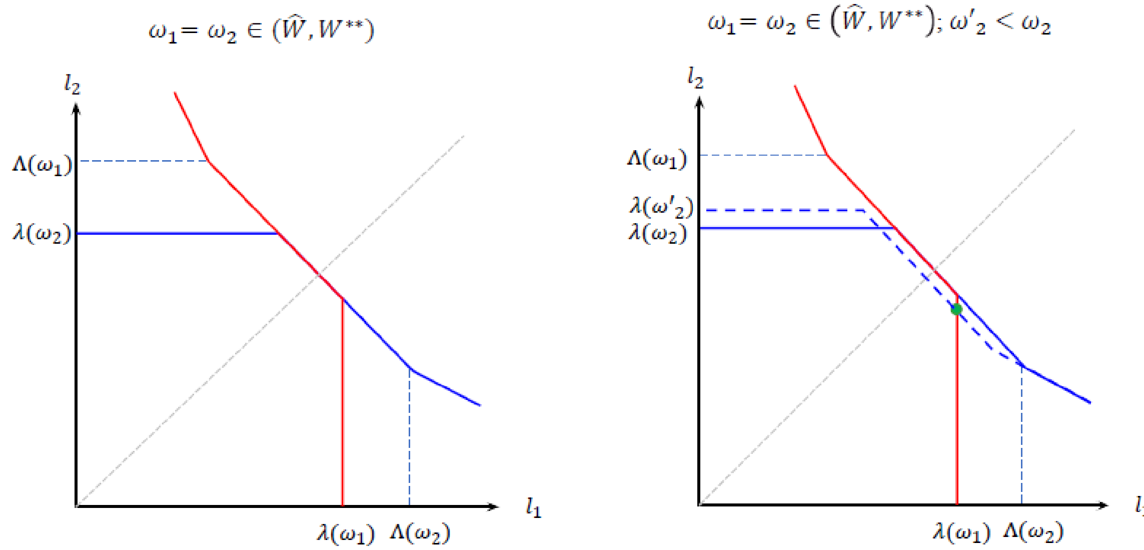


Figure 4 – labor reaction functions under extreme ESG policies that induce multiple equilibria

5 ESG equilibrium

In this section and the next we consider the optimal choice of ESG policies by firms' boards of directors. We first assume that a board's objective is to maximize firm profits, i.e., shareholder value. Section 6 in turn considers the case of boards maximizing a broader measure of a firm's impact on the economy, namely total surplus created by a firm.

5.1 Shareholder value maximizing ESG policies

We consider first the case in which only firm i adopts the an ESG policy. For example, only firm i is able to credibly commit to treat its workers well; or alternatively, firm i is a "thought leader" and considers a policy that hasn't occurred to firm $-i$. In the following subsection, we in turn allow firm $-i$ to respond to firm i's ESG policy by optimally choosing ω_{-i}, and given the expected reaction of firm $-i$, we analyze firm i's optimal ESG policy. The following corollary of Proposition 1 gives the labor market equilibrium that arises in this case.

Corollary 1 *If $\omega_{-i} = 0$ then the equilibrium is unique and characterized as follows:*

(i) If $\omega_i \leq W^B$ then the No-ESG benchmark is obtained.

(ii) If $\omega_i > W^B$ then $l_i^* = \min\{\Lambda_i(\omega_i), \lambda_i(\omega_i)\}$, $l_{-i}^* = r_{-i}(l_i^*; 0)$, $W_i^* = \omega_i$, and $W_{-i}^* = W(l_i^* + r_{-i}(l_i^*; 0))$.

From Corollary 1, the ESG firm's hiring is $l_i^* = \min\{\Lambda_i(\omega_i), \lambda_i(\omega_i)\}$. The two terms in the minimand correspond, respectively, to the equilibrium falling in the residual and competitive regions of firm i's reaction function. As firm i's ESG policy ω_i becomes more aggressive, the first term $\Lambda_i(\omega_i)$ increases, while the second term $\lambda_i(\omega_i)$ decreases, corresponding to the pro- and anti-competitive effects of ESG discussed above. At the non-ESG benchmark W^B we know $\Lambda_i(W^B) = l_i^B$; while the monopsony distortion in the non-ESG benchmark implies $l_i^B < \lambda_i(W^B)$. Consequently, if firm i adopts an ESG policy moderately above W^B then it hires $l_i^* = \Lambda_i(\omega_i) > l_i^B$, which in increasing in the ESG policy ω_i.

As firm i continues to increase its ESG policy the anti-competitive effect eventually dominates, and $l_i^* = \lambda_i(\omega_i)$. In particular, we know the anti-competitive effect dominates as ω_i approaches the first-best wage level W^{**}, because the monopsony distortion and the definition of W^{**} imply

$$\lambda_i(W^{**}) + r_{-i}(\lambda_i(W^{**}); 0) < \lambda_i(W^{**}) + \lambda_{-i}(W^{**}) = W^{-1}(W^{**}), \tag{16}$$

in turn implying (Lemma 1) $\lambda_i(\omega_i) < \Lambda_i(\omega_i)$.

It follows that the ESG policy that maximizes firm i's employment is $\hat{W}_i \in (W^B, W^{**})$, defined as the (unique) intersection of the functions Λ_i and λ_i:

$$\Lambda_i\left(\hat{W}_i\right) = \lambda_i\left(\hat{W}_i\right). \tag{17}$$

In words, $\hat{W}_i$ is the ESG level at which pro-competitive effects end and anti-competitive effects begin. Figure 5 graphically depicts this. Moreover: since total employment $l_i + r_{-i}(l_i; 0)$ increases in l_i, the same figure qualitatively applies to industry employment.

Modest ESG policies increase profits for the adopting firm. The reason is that a modest ESG policy effectively commits firm i to compete more aggressively in the labor market. Given this commitment, the competing firm $-i$ retreats. By definition, if firm i hires l_i^B then the marginal benefits and costs of hiring more workers are exactly balanced, conditional on firm $-i$ hiring l_{-i}^B. But if firm i can commit to its hiring choice, the marginal cost of additional hiring is reduced because firm $-i$ retreats and hires less, reducing the wage impact of firm i's additional hiring.

Modest ESG policies are profitable for the same reason that a firm benefits from commitment in Cournot settings. However, the commitment attainable with ESG policies is limited; as discussed above, the maximal employment that firm i can achieve is $\lambda_i\left(\hat{W}_i\right)$. But if firm i is adopting ESG policies purely in order to maximize profits, then the limited commitment power they generate is more than enough. Specifically, a profit-maximizing firm i would adopt an ESG policy strictly below $\hat{W}_i$. This is readily seen from the following expression for firm i's marginal profits from committing to increase hiring l_i:

$$f_i'\left(l_i\right) - W\left(l_i + r_{-i}\left(l_i;0\right)\right) - \left(1 + r_{-i}'\left(l_i;0\right)\right) W'\left(l_i + r_{-i}\left(l_i;0\right)\right). \tag{18}$$

This expression is negative at $l_i = \lambda_i\left(\hat{W}_i\right)$. The third term is the monopsony distortion, and is negative. Evaluated at $l_i = \lambda_i\left(\hat{W}_i\right)$, the combination of the first two terms is 0, because by definition $f_i'\left(\lambda_i\left(\hat{W}_i\right)\right) = \hat{W}_i$.

The next result characterizes the ESG policy that maximizes shareholder value, and compares the properties of the equilibrium that unfolds to the No-ESG benchmark. Notationally, let φ_i^* denote the shareholder-value maximizing choice of ESG.

Proposition 2 *Suppose firm i's opponent adopts the No-ESG policy (i.e., $\omega_{-i} = 0$). Then, the shareholder value maximizing ESG policy of firm i satisfies $\varphi_i^* \in (W^B, \hat{W}_i)$. Under the optimal ESG policy φ_i^*, $l_i^* = \Lambda_i\left(\varphi_i^*\right)$, $l_{-i}^* = r_{-i}\left(\Lambda_i\left(\varphi_i^*\right);0\right)$, and $W_i^* = W_{-i}^* = \varphi_i^*$. Relative to the No-ESG benchmark, worker welfare, industry employment, and firm i's employment and profit are higher. In contrast, firm $-i$'s employment and profit are lower. Both firms pay the same wage, which is higher than the No-ESG benchmark.*

The reasoning above has two interesting implications. First, if the labor market was competitive (i.e., labor supply is perfectly elastic) then the shareholder-value maximizing ESG policy would be the No-ESG policy. Second, the firm's ESG policies have an effect only if the firm's competitors are aware of these policies. Thus, it's in the best interest of firms to credibly advertise their ESG policies; regulations that facilitate transparency and disclosure of ESG policies would contribute to their effectiveness and widespread adoption.

As noted above, the shareholders of firm i benefit from their firm's ESG policy at the expense of the shareholders of firm $-i$. But the employees of both firms gain from firm i's ESG policy. Indeed, in equilibrium, both firms pay their employees a higher wage of $\varphi_i^* > W^B$.[10] Moreover, while the employment of firm i increases at the expense of firm $-i$'s employment (i.e., $l_i^* > l_i^B$ and $l_{-i}^* < l_{-i}^B$), total employment increases (i.e., $l_i^* + l_{-i}^* > l_i^B + l_{-i}^B$). That is, firm i increases its employment by more than firm $-i$ reduces it. Therefore, worker welfare always increases relative to the No-ESG benchmark. In this respect, the unintended consequences of a profit-motivated ESG policy are beneficial to workers. Interestingly, since there is no pay difference in equilibrium between ESG and non-ESG firms, without additional information about the productivity of each firm, identifying the ESG firms based on how they treat their workers is empirically challenging.[11]

Nonetheless, firm i's adoption of ESG never raises industry employment to its first-best level. By Lemma 1, industry employment is maximized by firm i maximizing its own employment, which it does by adopting $\hat{W}_i$ and hiring $l_i = \Lambda_i\left(\hat{W}_i\right)$. By the definition of Λ_i, firm $-i$'s best response leads to a market wage $\hat{W}_i$, and industry employment of $W^{-1}\left(\hat{W}_i\right)$. Since $\hat{W}_i < W^{**}$, this establishes that industry employment is below its first-best level, as claimed.

The effect of firm i's ESG policy on industry profits and surplus is more nuanced. In the Appendix, we show that if firm i is the (weakly) less-productive firm (i.e., $i = 2$), then total industry profits decrease relative to the No-ESG benchmark. That is, the increase in the profitability of firm i is lower than the decline in the profitability of firm $-i$. Intuitively, as firm i increases employment at the expense of its more productive opponent, production is shifted the "wrong" way, toward the firm with the lower marginal productivity and a smaller

[10] Since $W_{-i}^* = W\left(\Lambda_i\left(\omega_i\right) + r_{-i}\left(\Lambda_i\left(\omega_i\right);0\right)\right)$, by the definition of $\Lambda_i\left(\cdot\right)$, $W_{-i}^* = \omega_i$.

[11] Notice that if firms were symmetric then the ESG firm would be larger than the non-ESG firm since it employs more workers. However, in general, when firms are asymmetric, it is hard to identify which one is the ESG firm since less productive firms can adopt ESG policy and still hire less.

monopsony distortion in the first place. This force also explains why industry surplus could decline due to firm i's ESG policy. In this respect, when unproductive firms use ESG policies to gain a competitive advantage in real markets, they create distortions that are beneficial to the firm's shareholders but can be costly from a social perspective. In contrast, if firm i is the more productive firm (i.e., $i = 1$), then it is possible that total industry profits increase relative to the No-ESG benchmark. In this case industry surplus also increases (since worker welfare is always higher). In fact, industry surplus can increase in those cases even if industry profitability declines. Intuitively, when the more productive firm uses ESG to enhance its competitive advantage, production is shifted the "right" way and toward the firm whose monopsony distortion creates a larger social cost (and hence, increasing production is marginally more valuable).

Under relatively mild conditions (specifically, $W(\cdot)$ log-concave and production functions having the standard log-linear form), we show that the more productive firm has stronger incentives to adopt ESG, that is, the derivative of profits with respect to ESG is greater for the more productive firm in the neighborhood of the no-ESG benchmark W^B. Intuitively, the more productive firm has more to gain from higher production (since $f_1'(\cdot) > f_2'(\cdot)$). As noted above, industry surplus rises when it is the more productive firm that adopts ESG.

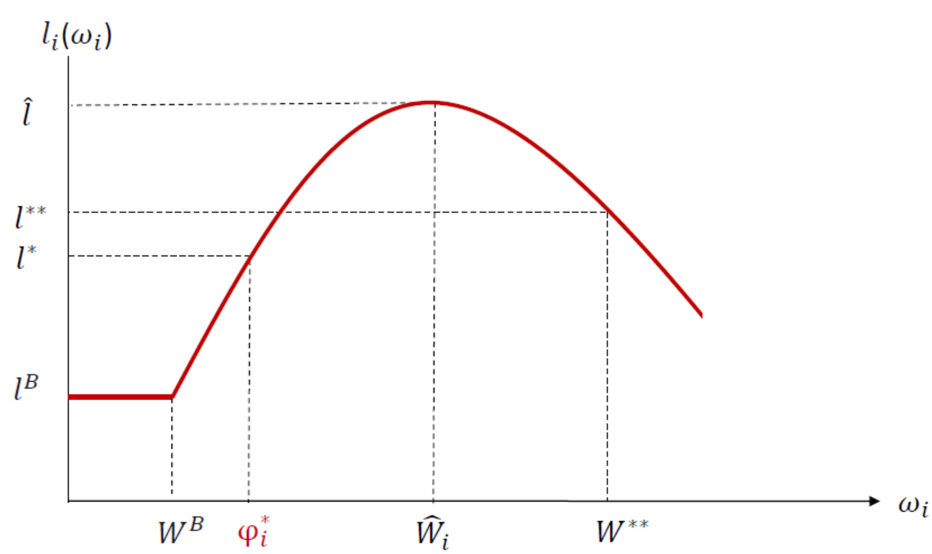


Figure 5— firm's employment as a function of its ESG policy

5.2 Competition in ESG policies

Only firm i has the capacity to adopt ESG policies in the analysis above. We next consider what ESG policies firm $-i$ would adopt in response to firm i's ESG choice, where we continue to focus on the case in which boards seek to maximize shareholder value.

Lemma 4 *Suppose firm i adopts ESG policy ω_i. There exists $\check{W}_{-i} \in \left(\hat{W}_i, W^{**}\right)$ such that the shareholder-value maximizing ESG policy of firm $-i$ is*

$$\varphi_{-i}^{SH}(\omega_i) \equiv \begin{cases} \varphi_{-i}^* & \text{if } \omega_i < \min\{\varphi_{-i}^*, \check{W}_{-i}\} \\ \omega_i + \varepsilon & \text{if } \omega_i \in [\min\{\varphi_{-i}^*, \check{W}_{-i}\}, \check{W}_{-i}) \\ 0 & \text{if } \omega_i \geq \check{W}_{-i}. \end{cases} \tag{19}$$

Lemma 4 shows that ESG policies are strategic complements when the policies are moderate and strategic substitutes when they are extreme. If firm i's ESG policy is sufficiently mild ($\omega_i < \min\{\varphi_{-i}^*, \check{W}_{-i}\}$), then firm $-i$ simply responds by picking $\omega_{-i} = \varphi_{-i}^*$, which by definition is the ESG policy that it would adopt if firm i hadn't adopted any ESG policy at all. In this case, the "leader" firm i's ESG policy has no effect on the "follower" firm's choice.

If firm i's ESG policy is intermediate ($\varphi_{-i}^* < \omega_i < \check{W}_{-i}$) then if firm $-i$ simply adopts φ_{-i}^* it has a less aggressive ESG policy than firm $-i$. From Proposition 1's characterization of the labor-market equilibrium, a firm gains nothing from adopting an ESG policy that is less aggressive than its competitor. So instead, firm $-i$ responds by marginally outdoing firm i's ESG policy. So in this case, as firm i's ESG choice becomes more aggressive, it pushes firm $-i$ to respond by in turn adopting progressively more and more aggressive ESG policies.

Finally, if firm i's ESG policy is sufficiently aggressive ($\omega_i > \check{W}_{-i}$) the benefit to firm $-i$ of outdoing ω_i is too small to justify the cost of higher wages. This is immediate once ω_i crosses the first-best level W^{**}, since in this case firm $-i$'s hiring shrinks if it outdoes firm i's ESG policy, while its labor costs increase (Proposition 1). By continuity, this conclusion extends to an interval of firm i's ESG policies below W^{**}. Conditional on not outdoing firm i ESG choice, firm $-i$ is best-off abandoning ESG (or, strictly speaking, picking an ESG policy so moderate that it has no effect on its behavior).

The next result characterizes the equilibrium when shareholder value maximizing firms compete in ESG policies. Specifically, firm i chooses ω_i and then firm $-i$ responds by choosing

ω_{-i}. Given ESG policies (ω_i, ω_{-i}), the firms compete in the labor market. We present our results for ESG competition for the case of symmetric firms, i.e., $\varphi_1^* = \varphi_2^* = \varphi^*$ and $\check{W}_1 = \check{W}_2 = \check{W}$. However, analysis of a parameterized example suggests that our conclusions also extend to the case of asymmetric firms.

Proposition 3 *Suppose firms are symmetric and choose ESG policies to maximize their shareholder value. Then,*

- *(i) If $\Lambda(\varphi^*) \leq \lambda(\check{W})$ then firm i chooses ESG policy φ^* and of firm $-i$ chooses policy $\varphi^* + \varepsilon$. The equilibrium is payoff equivalent to the equilibrium that emerges when firm i adopts the No-ESG policy (i.e., $\omega_i = 0$) and firm $-i$ adopts policy φ^*, as characterized by Proposition 2.*

- *(ii) If $\Lambda(\varphi^*) > \lambda(\check{W})$ then firm i chooses ESG policy $\check{W}$ and firm $-i$ chooses the No-ESG policy (i.e., $\omega_{-i} = 0$).*

In part (i) of Proposition 3, firm i adopts a moderate ESG policy that is too moderate to deter firm $-i$, who in turn outdoes firm i's ESG policy and obtains an advantage in the labor market. In contrast, in part (ii), firm i adopts an ESG policy that is aggressive enough to deter firm $-i$ from matching it, and firm i consequently retains its advantage in the labor market.

The condition $\Lambda(\varphi^*) \leq \lambda(\check{W})$ is a comparison between two hiring levels. To understand this condition, note first that if firm i "concedes" and adopts an ESG policy that it knows its competitor will outdo, then firm i's payoff is the same as from simply playing the non-ESG best-response to firm $-i$ hiring $\Lambda(\varphi^*)$. Second, $\lambda(\check{W})$ is firm i's hiring if it adopts $\check{W}$ and deters firm $-i$. By the definition of $\check{W}$, if firm i adopts this ESG policy then firm $-i$ is indifferent between marginally outdoing it and hiring $\lambda(\check{W})$ itself; and abandoning ESG and simply playing the non-ESG best-response to $\lambda(\check{W})$. By symmetry, it follows that firm i's profits from adopting $\check{W}$ equal its profits from playing the non-ESG best response to $\lambda(\check{W})$.

Consequently, firm i's choice between concession and deterrence can be re-expressed as: would it prefer to play a non-ESG best-response to a competitor hiring $\Lambda(\varphi^*)$ or hiring $\lambda(\check{W})$? Naturally, firm i would prefer to play against the competitor with the lower hiring level, yielding the condition in Proposition 3.

The analysis above suggests that the implications of Proposition 2 with respect to worker welfare, industry profitability and surplus extend to cases in which profit-maximizing firms compete in ESG policies. Therefore, competition in ESG policies always benefits workers but it may in fact reduce industry surplus once firm's profitability and shareholder valuation is accounted for. As before, the misallocation of labor across firms due to competition in ESG policies can be determinantal from a social perspective. As we shall see below, this conclusion will be overturned when purposeful firms compete in ESG policies.

If competition in ESG policies reduces industry profitability (see discussion above) and there is ex-ante uncertainty about which firm is the first-mover in the ESG-game, then both firms have incentives to coordinate on low impact ESG policies if possible. Ideally firms would like to commit to abstain from ESG altogether. But in practice this may not be possible, since the gain to deviation would be highest in this case, and firms may instead have to settle on coordinating on mild ESG in order to reduce deviation-incentives. This conclusion raises anti-trust concerns for the seemingly benevolent adoption of industry-wide ESG standards, and for moves by large asset managers ("common owners") to promote ESG.

Finally: Proposition 3 uses the best-ESG-response characterization of Lemma 4 to characterize a leader-follower game. One can also ask: What happens if the two firms choose ESG policies independently, without observing the other's choice? An implication of Lemma 4 is that no pure-strategy equilibrium of this simultaneous-move game exists. At least for the symmetric case, we have explicitly characterized the mixed-strategy equilibrium of this game for many cases;[12] it consists of both firms mixing over an interval of ESG policies immediately above the one-ESG-firm profit-maximizing policy φ^*, and bounded away from $\hat{W}$. The implications of this mixed-strategy equilibrium for employment and worker welfare, and industry profits and surplus, are qualitatively similar to those for the equilibrium of the leader-follower game.

[12] Specifically, we characterize the mixed-strategy equilibrium under the following condition: Define $\bar{\omega}$ as the ESG policy such that firm i is indifferent between playing $\omega_i = \bar{\omega}$ against $\omega_{-i} = 0$ and playing $\omega_i = 0$ against $\omega_{-i} = \varphi^*$. The condition is: $\bar{\omega} \leq \hat{W}$. This condition is satisfied in all parameterized examples that we have examined.

6 Purposeful firms

Thus far, we have assumed that boards seek to maximize shareholder value, i.e., profits. In this section, we analyze what happens if boards care about the total surplus created by the firm, which in this setting equals the sum of profits and workers' compensation net of their outside options. We label such firms as *purposeful.*

Importantly, we continue to assume that managers are profit-maximizing, subject to the constraints imposed by ESG policies.[13] Leading cases in which purposeful firms potentially emerge are if shareholders are socially conscious, or if workers gain board representation.

Section 5 established that profit-maximizing firms adopt less-than-"maximal" ESG policies. Specifically, a single-ESG firm adopts milder ESG than size-maximizing level $\hat{W}_i$; while a competing ESG firm doesn't match aggressive ESG policies of competitors, and instead abandons ESG. In contrast, here we show that purposeful firms fully exploit ESG policies, so that both of the above statements are reversed. Indeed, the board of a purposeful firm would like to have additional tools beyond ESG at its disposal.

A purposeful firm cares directly about worker surplus, which in turn depends on the outside options of the workers it employs. Calculating these outside options requires assumptions on how workers are allocated across firms. The minimum and maximum values of the combined outside options of firm i's workers are, respectively, $\int_0^{l_i} W(l)\,dl$ and $\int_{l_{-i}}^{l_i+l_{-i}} W(l)\,dl$. We define firm i's surplus using a weighted average of these possibilities, with weight $\mu \in (0,1)$.[14]

$$S_i(l_i, l_{-i}) \equiv f_i(l_i) - \mu \int_0^{l_i} W(l)\,dl - (1-\mu) \int_{l_{-i}}^{l_i+l_{-i}} W(l)\,dl. \tag{20}$$

Purposeful firms are "narrow" consequentialists. They care about the immediate outcomes of their actions, but not the equilibrium implications for the surplus created by other firms.

Notationally, let $\varphi_i^P(\omega_{-i})$ be the ESG policy that maximizes firm i's surplus, given that firm $-i$ adopts ESG policy ω_{-i}.

[13] Effectively, we assume the board of the firm (or its investors) cannot directly alter the incentives of the manager to internalize the welfare of the firm's employees.

[14] Our results hold for any $\mu \in [0,1]$. If $\mu = \frac{1}{2}$ then $S_i(l_i, l_j) + S_j(l_j, l_i) = S(l_i, l_j)$, that is, the sum of individual firms' surplus equals the industry surplus.

6.1 Optimal purposeful ESG policy

As in Section 5, we first consider the case in which only firm i adopts an ESG policy, and analyze the consequences both for surplus created by firm i, and for other welfare measures. In the following subsection, we in turn allow firm $-i$ to respond, under the assumption that both firms are purposeful.

We start by characterizing by characterizing firm i's surplus-maximizing ESG policy, $\varphi_i^P(0)$:

Proposition 4 *Suppose firm i's opponent adopts the No-ESG policy (i.e., $\omega_{-i} = 0$). Then, the optimal purposeful ESG policy of firm i is $\varphi_i^P(0) = \hat{W}_i$. Under optimal ESG policy $\hat{W}_i$, $l_i^* = \Lambda_i\left(\hat{W}_i\right) = \lambda_i\left(\hat{W}_i\right)$, $l_{-i}^* = r_{-i}\left(\Lambda_i\left(\hat{W}_i\right); 0\right)$, and $W_i^* = W_{-i}^* = \hat{W}_i$. Relative to the No-ESG benchmark, worker welfare, industry employment, and firm i's employment are higher. Firm $-i$'s employment and profit are lower. Both firms pay the same wage, which is higher than the No-ESG benchmark.*

Proposition 4 is similar to Proposition 2, with the exception of $\varphi_i^P(0) > \varphi_i^{SH}(0) = \varphi_i^*$, that is, purposeful firms adopt more aggressive ESG policies than their shareholder value maximizing counterparts. Intuitively, in order to maximize its own surplus, a purposeful firm wants to be large, even at the expense of its profitability.

It is worth highlighting that the purposeful firm i would like to be even larger than the size $\lambda_i\left(\hat{W}_i\right)$ that it attains with ESG policy $\hat{W}_i$. The reason is that the marginal worker hired produces zero profits, since $f_i'\left(\lambda_i\left(\hat{W}_i\right)\right) = \hat{W}_i$; but strictly positive worker surplus, since firm i evaluates the marginal worker's outside option as $\mu W(l_i) + (1-\mu)\hat{W}_i < \hat{W}_i$.

This observation has two significant implications. First, and in contrast to the case of a board seeking to maximize profits, a purposeful firm's board wishes it had additional tools at its disposal beyond an ESG promise to treat its workers well. But under the assumption that this is the only tool available, increases in ESG ω_i beyond $\hat{W}_i$ backfire, because they reduce firm i's hiring. Second, the implication that a purposeful firm adopts $\hat{W}_i$ is robust to perturbing the weights it attaches to shareholder profits and to worker welfare.

Returning to Proposition 4, it follows that firm i's hiring and total industry employment are both maximized under the optimal purposeful ESG policy, whereas firm $-i$'s hiring is minimized. Since total employment is higher than under the optimal ESG policy of a shareholder-value maximizing firm and the wages that both firms pay their workers are also higher, employees of both companies benefit more from the optimal purposeful ESG policy.

As in the case of a shareholder-value maximizing firm, the profitability of firm $-i$ is lower under the optimal purposeful ESG policy relative to the No-ESG benchmark. However, it is not guaranteed that the profitability of firm i is higher relative to this benchmark. After all, a purposeful firm's ESG policy is not chosen to maximize profitability; and indeed, since it produces at a level where the marginal productivity equals the wage, the firm could increase its marginal profitability by choosing a slightly less generous ESG policy. That said, since the ESG policy of firm i deters hiring by firm $-i$ and enables it to hire more workers at a lower cost, in many cases the profitability of the purposeful firm under its optimal ESG is still higher relative to the No-ESG benchmark. We give an example of this case in the Appendix. In this respect, the ESG policy of a purposeful firm can benefit its own shareholders even though it was not necessarily intended to. Thus, "Doing Well by Doing Good" applies in our setting as well.

At the same time, relative to the optimal ESG policy of a shareholder-value maximizing firm, combined industry profits are lower under the optimal purposeful ESG policy. Moreover, and interestingly, the optimal purposeful ESG policy fails to maximize the industry surplus.

Corollary 2 *The optimal purposeful ESG policy of firm i does not maximize industry surplus. The industry-surplus maximizing ESG policy of firm i leads to less employment at firm i and more employment at firm $-i$, relative to the optimal purposeful ESG policy $\varphi_i^P(0)$.*

Intuitively, purposeful firms do not fully internalize how their ESG policies affect the hiring decisions of other firms. In particular, since under the optimal purposeful ESG policy of firm i we have $f_i'(l_i^*) = \hat{W}_i < f_{-i}'(l_{-i}^*)$,[15] that is, the marginal productivity of firm i's employees is lower than of firm $-i$'s employees. Therefore, industry surplus can increase if firm i hires fewer employees while firm $-i$ hires more employees. However, since $\varphi_i^P(0)$ only maximizes the surplus of firm i, it does not account for this welfare gain. In this respect, the ESG policy of a purposeful firm is too aggressive from a social perspective. Recall that shareholder value maximizing firms adopt a less aggressive ESG policy (i.e., $\varphi_i^{SH}(0) < \varphi_i^P(0)$). Thus, to maximize

[15] Firm $-i's$ production always reflects the monopsony distortion and hence marginal productivity is above the paid wage.

industry surplus, a purposeful firm must overweight shareholders relative to other stakeholders of the firm, for example, by giving shareholders larger representation on the company's board of directors. By doing so, the firm will reduce hiring due to a partial monopsony distortion, which would be socially beneficial given that the marginal productivity of the purposeful firm is endogenously lower.

6.2 Competition in ESG policies between purposeful firms

We next allow purposeful firm $-i$ to respond to firm i's ESG policy by adopting its own ESG policy. We start by characterizing the best response ESG policy of one firm to another.

Lemma 5 *Suppose firm i adopts ESG policy ω_i. Then, the optimal purposeful ESG policy of firm $-i$ is*

$$\varphi_{-i}^{P}(\omega_i) = \begin{cases} \hat{W}_{-i} & \textit{if } \omega_i < \hat{W}_{-i} \\ \omega_i + \varepsilon & \textit{if } \omega_i \in [\hat{W}_{-i}, W^{**}) \\ W^{**} & \textit{if } \omega_i \geq W^{**}. \end{cases} \tag{21}$$

Parallel to Lemma 4, the ESG-best-response function has three regions; but in contrast to this prior result, ESG policies are always strategic complements for purposeful firms.

If firm i's ESG policy is sufficiently mild ($\omega_i < \hat{W}_{-i}$), then firm $-i$ simply chooses $\omega_{-i} = \hat{W}_{-i}$, which is the optimal ESG policy of a purposeful firm when the other firm does not adopt an ESG policy. In this range, and just as in the analogous case in Lemma 4, the leader's ESG choice doesn't affect the follower's response.

If firm i's ESG policy is intermediate ($\hat{W}_{-i} < \omega_i < W^{**}$) then firm $-i$ has incentives to just out-do the policy. This is again similar to the analogous case in Lemma 4, though with an important difference: now, the purposeful firm $-i$ outdoes firm i ESG policy ω_i even as ω_i approaches the first-best level W^{**}. Specifically: Marginally outdoing firm i's ESG is attractive from firm $-i$ because it discretely boosts firm $-i$'s hiring (at the expense of firm i). But in contrast to the profit-maximizing case, firm $-i$ is no longer tempted to undercut firm i's ESG policy; doing so reduces firm $-i$'s hiring, which is unattractive to a purposeful firm; while the "benefit" of reduced wages is simply a transfer of worker surplus to shareholder surplus, and so isn't valued by a purposeful firm.

Finally, if firm i's ESG policy is very aggressive ($\omega_i > W^{**}$) then firm $-i$ simply adopts $\omega_i = W^{**}$ and hires $\lambda_{-i}(W^{**})$. Parallel to the discussion above, this is another case in which firm $-i$'s board wishes it had more tools at its disposal, since the marginal worker hired produces strictly positive surplus for firm $-i$. However, if firm $-i$'s board adopts more aggressive ESG policies that W^{**} then its manager responds by hiring less, and so aggressive ESG policies backfire from the purposeful board's perspective. Hence the purposeful firm never chooses an ESG policy above the first-best level W^{**}.

The next result characterizes the result of competition in ESG policies by purposeful firms.

Proposition 5 *In the unique equilibrium, both purposeful firms adopt ESG policy W^{**}, leading to the first-best outcome.*

Proposition 5 is striking: competition in ESG policies between purposeful firms entirely eliminates the monopsony distortion and delivers the first-best industry surplus. This is true even though each individual firm's objective is to maximize only its own surplus, which as Corollary 2 shows can have adverse welfare effects because firms don't internalize the externalities that they inflict on competitors' surplus.

To understand the intuition behind this result, note that firm i anticipates firm $-i$'s best response $\varphi^P_{-i}(\omega_i)$. While firm i would like to adopt an ESG policy that induces its manager to be more aggressive in the labor market than firm $-i$, it cannot achieve this because firm $-i$ will always respond with a more aggressive policy, $\omega_{-i} = \omega_i + \varepsilon$. Thus, the best firm i can do is to adopt an ESG policy that maximizes its employment; it has incentives to grow larger. In principle, since purposeful firms do not internalize the externalities they inflict on their competitors, they have incentives to grow larger even above the first-best employment level. However, since the hiring decision is made by a profit-maximizing manager and the firm cannot commit to an employment level, the second-best is to choose the highest employment such that marginal productivity is equal to the minimum wage imposed by its ESG policy. This force pushes both firms to adopt the first-best wages as their equilibrium ESG policies. Put differently, the strategic complementarity in ESG policies between competing purposeful firms achieves the first-best outcome. In this respect, ESG is a panacea to market power.

Proposition 5's conclusion that purposeful competition in ESG policies delivers the first best outcome equilibrium is robust to perturbing the weights that a purposeful firm puts on shareholder and worker surplus. Specifically, as long as a purposeful firm puts sufficient weight on worker welfare, then it has incentives to marginally outdo any ESG choice by its competitor that is less than W^{**}. Moreover, as long as a purposeful firm's hiring decision is made by a profit-maximizing manager, the board of a purposeful firm will never set an ESG policy that is more aggressive that W^{**}.

We have established Proposition 5 in the same leader-follower framework that we used to analyze competition between profit-maximizing ESG firms. But exactly the same outcome arises if two purposeful firms select ESG firms independently, as in a simultaneous-move game.

7 Concluding remarks

In this paper we study the equilibrium effects of the "S" dimension of ESG in a model of imperfect competition in labor (and product) markets. All else equal, a profit-maximizing firm can benefit from adopting ESG policies that give a competitive edge in attracting workers; "Doing Well by Doing Good" applies in our setting. ESG policies are strategic complements, and in equilibrium, they are adopted by all firms resulting with higher worker welfare but lower shareholder value. Thus, profit maximizing firms benefit from coordinating on low impact ESG policies, raising anti-trust concerns from the adoption of industry-wide ESG standards. A purposeful firm (lead by a socially conscious board) benefits from such ESG policies, and imperfect competition between purposeful firms obtains the first best in equilibrium. Thus, the social purpose of the corporation is a panacea to excessive marker power. More broadly, our analysis relates the adoption of ESG policies to the nature of competition between firms and their model of corporate governance.

References

[1] Allcott, Hunt, Giovanni Montanari, Bora Ozaltun, and Brandon Tan (2022), "An Economic View of Corporate Social Impact." *Working Paper*.

[2] Albuquerque, Rui, Yrjö Koskinen, and Chendi Zhang (2019): "Corporate Social Responsibility and Firm Risk: Theory and Empirical Evidence." *Management Science* 65, 4451-4469.

[3] Baker, Steven D., Burton Hollifield, and Emilio Osambela (2022): "Asset Prices and Portfolios with Externalities." *Review of Finance*, forthcoming.

[4] Berk, Jonathan and Jules H. van Binsbergen (2021): "The Impact of Impact Investing." Working Paper, Stanford University.

[5] Berg, Florian, Julian F. Kölbel, and Roberto Rigobon (2022): "Aggregate Confusion: The Divergence of ESG Ratings." *Review of Finance*, forthcoming.

[6] Bolton, Patrick and Marcin Kacperczyk (2021): "Do Investors Care About Carbon Risk?" *Journal of Financial Economics* 142, 517–549.

[7] Brander, James A. and Tracy R. Lewis (1986): "Oligopoly and Financial Structure: The Limited Liability Effect" *American Economic Review* 76, 956–970.

[8] Cohen, Lauren, Umit G. Gurun, and Quoc Nguyen (2021): "The ESG-Innovation Disconnect: Evidence from Green Patenting." Working Paper, Harvard University.

[9] Chowdhry, Bhagwan, Shaun William Davies, and Brian Waters (2019): "Investing for Impact." *Review of Financial Studies* 32 (3):864-904.

[10] Davies, Shaun William and Edward Dickersin Van Wesep (2018): "The Unintended Consequences of Divestment." *Journal of Financial Economics* 128, 558–575.

[11] Edmans, Alex, Doron Levit, and Jan Schneemeier (2022): "Socially Responsible Divestment." Working Paper, University of Washington

[12] Gantchev, Nickolay, Mariassunta Giannetti, and Rachel Li (2022): "Does Money Talk? Divestitures and Corporate Environmental and Social Policies." *Review of Finance*, forthcoming.

[13] Gibson, Rajna, Simon Glossner, Philipp Krueger, Pedro Matos, and Tom Steffen (2022): "Do Responsible Investors Invest Responsibly?" *Review of Finance*, forthcoming.

[14] Goldstein, Itay, Alexandr Kopytov, Lin Shen, and Haotian Xiang (2022): "On ESG Investing: Heterogeneous Preferences, Information, and Asset Prices." Working Paper, University of Pennsylvania

[15] Gorton, Gary, Jillian Grennan, and Alexander Zentefis (2022). "Corporate Culture." *Annual Review of Financial Economics*, 14:5, pp 1–27.

[16] Green, Daniel and Benjamin Roth. (2021): "The Allocation of Socially Responsible Capital." SSRN Working Paper No. 3737772.

[17] Hartzmark, Samuel M. and Abigail B. Sussman (2019): "Do Investors Value Sustainability? A Natural Experiment Examining Ranking and Fund Flows." *Journal of Finance* 74, 2789–2837.

[18] Heath, Davidson, Daniele Macciocchi, Roni Michaely, and Matthew C. Ringgenberg (2020): "Does Socially Responsible Investing Change Firm Behavior?" Working Paper, University of Utah.

[19] Heinkel, Robert, Alan Kraus and Josef Zechner (2001): "The Effect of Green Investment on Corporate Behavior." *Journal of Financial and Quantitative Analysis* 36, 431–449.

[20] Hong, Harrison and Marcin Kacperczyk (2009): "The Price of Sin: The Effects of Social Norms on Markets." *Journal of Financial Economics* 93, 15–36.

[21] Kim, Soohun and Aaron Yoon (2022): "Assessing Active Managers' Commitment to ESG Evidence from United Nations Principles for Responsible Investment." *Management Science*, forthcoming.

[22] Kreps, David M. and Jose A. Scheinkman (1983): "Quantity Precommitment and Bertrand Competition Yield Cournot Outcomes." *Bell Journal of Economics*, Vol. 14, No. 2, pp. 326-337.

[23] Lamadon, Thibaut, Magne Mogstad, and Bradley Setzler (2022): "Imperfect Competition, Compensating Differentials, and Rent Sharing in the US Labor Market." *American Economic Review* 112, 169–212.

[24] Landier, Augustin and Stefano Lovo (2020): "ESG Investing: How to Optimize Impact?" Working Paper, HEC Paris.

[25] Liang, Hao, Lin Sun, and Melvyn Teo (2022): "Responsible Hedge Funds." *Review of Finance*, forthcoming.

[26] Mackintosh, James (2022): "ESG Investing Can Do Good or Do Well, but Don't Expect Both." *Wall Street Journal*, January 24, 2022.

[27] Mas-Colell, Andreu, Michael D. Whinston, and Jerry R. Green (1995): "Microeconomic Theory ." Oxford University Press.

[28] Oehmke, Martin and Marcus Opp (2020): "A Theory of Socially Responsible Investment." Working Paper, London School of Economics.

[29] Pedersen, Lasse Heje, Shaun Fitzgibbons, and Lukas Pomorski (2021): "Responsible Investing: The ESG-Efficient Frontier." *Journal of Financial Economics* 142, 572–597.

[30] Pastor, Lubos, Robert F. Stambaugh, and Lucian A. Taylor (2021): "Sustainable Investing in Equilibrium." *Journal of Financial Economics* 142, 550–571.

[31] Rey, Patrick and Jean Tirole (2019): "Price Caps as Welfare-Enhancing Coopetition." *Journal of Political Economy* 127, 3018–3069.

[32] Sklivas, Steven D. (1987): "The Strategic Choice of Managerial Incentives." *RAND Journal of Economics* 18, 452–458.

[33] Stoughton, Neal M., Kit Pong Wong, and Long Yi (2020): "Competitive Corporate Social Responsibility," *Working Paper.*

[34] Teoh, Siew Hong, Ivo Welch, and C. Paul Wazzan (1999): "The Effect of Socially Activist Investment Policies on the Financial Markets: Evidence from the South African Boycott." *Journal of Business* 72, 35–89.

[35] Tirole, Jean (2001): "Corporate Governance." *Econometrica* 69, 1–35.

[36] Xiong, Yan and Liyan Yang (2022): "A Product-Based Theory of Corporate Social Responsibility." *Working Paper.*

A Appendix

A.1 Proofs for Section 3

Proof of Lemma 1. It is convenient to rewrite firm i's maximization problem as

$$\max_L f_i\left(L-l_{-i}\right)-W\left(L\right)\left(L-l_{-i}\right).$$

We first note that $W\left(L\right)\left(L-l_{-i}\right)$ is strictly convex. If W is weakly convex then this is immediate. Otherwise, consider any L such that $W''\left(L\right)<0$, and note that

$$\frac{\partial^2 W\left(L\right)\left(L-l_{-i}\right)}{\partial L^2}=W''\left(L\right)\left(L-l_{-i}\right)+2W'\left(L\right)>W''\left(L\right)L+2W'\left(L\right)>0,$$

where the final inequality follows from (2). It follows that the firm's objective is strictly concave, and hence has a unique maximizer.

Next, we establish that $r_i\left(l_{-i},0\right)$ is decreasing. This follows from the FOC

$$f_i'\left(l_i\right)=W'\left(l_i+l_{-i}\right)l_i+W\left(l_i+l_{-i}\right).$$

The derivative of the RHS with respect to l_{-i} is

$$W''\left(l_i+l_{-i}\right)l_i+W'\left(l_i+l_{-i}\right)=W''\left(L\right)\left(L-l_{-i}\right)+W'\left(L\right),$$

which is strictly positive: this is immediate if $W''\left(L\right)\geq 0$, and follows from (2) if $W''\left(L\right)<0$. The result follows.

Finally, we establish that $r_i\left(l_{-i},0\right)+l_{-i}$ is strictly increasing in l_{-i}. This follows from the single-crossing property applied to firm i profits $f_i\left(L-l_{-i}\right)-W\left(L\right)\left(L-l_{-i}\right)$. Specifically, consider L and $\tilde{L}>L$ such that

$$f_i(\tilde{L}-l_{-i})-W(\tilde{L})(\tilde{L}-l_{-i})\geq f_i\left(L-l_{-i}\right)-W\left(L\right)\left(L-l_{-i}\right).$$

Then for any $\widetilde{l}_{-i}>L_{-i}$, we claim

$$f_i(\tilde{L}-\widetilde{l}_{-i})-W(\tilde{L})(\tilde{L}-\widetilde{l}_{-i})>f_i(L-\widetilde{l}_{-i})-W\left(L\right)\left(L-\widetilde{l}_{-i}\right).$$

This holds because

$$\begin{aligned}
f_i(\tilde{L}-\widetilde{l}_{-i})-f_i(L-\widetilde{l}_{-i}) &> f_i(\tilde{L}-l_{-i})-f_i\left(L-l_{-i}\right)\\
&\geq W(\tilde{L})(\tilde{L}-l_{-i})-W\left(L\right)\left(L-l_{-i}\right)\\
&> W(\tilde{L})(\tilde{L}-\widetilde{l}_{-i})-W\left(L\right)\left(L-\widetilde{l}_{-i}\right),
\end{aligned}$$

where the first inequality follows from the concavity of f_i, and the third inequality follows from W being strictly increasing.[16] ■

[16] Local argument: Recall $r_i\left(l_{-i};0\right)$ satisfies $f_i'\left(r\right)=W'\left(r+l_{-i}\right)r+W\left(r+l_{-i}\right)$. By the implicit function

Proof of Lemma 2. In equilibrium, l_i^B solves $l = r_i\left(r_{-i}\left(l, 0\right), 0\right)$. Since the slopes of $r_i\left(\cdot, 0\right)$ and $r_{-i}\left(\cdot, 0\right)$ are smaller than one, the slope of $r_i\left(r_{-i}\left(\cdot, 0\right), 0\right)$ is strictly less than one, and hence, l_i^B is unique. Inada condition ensures existence. If on the contrary $l_1^B + l_2^B \geq l_1^{**} + l_2^{**}$, then

$$f_i'\left(l_i^B\right) = W'\left(l_1^B + l_2^B\right) l_i^B + W\left(l_1^B + l_2^B\right) > W\left(l_1^{**} + l_2^{**}\right) = f_i'\left(l_i^{**}\right),$$

which implies $f_i'\left(l_i^B\right) > f_i'\left(l_i^{**}\right) \Leftrightarrow l_i^B < l_i^{**}$, contradicting $l_1^B + l_2^B \geq l_1^{**} + l_2^{**}$.

Next, notice $f_1' \geq f_2'$ implies $r_1\left(l; 0\right) \geq r_2\left(l; 0\right)$. Since $r_i\left(l; 0\right)$ is a decreasing function,

$$l_1^B = r_1\left(r_2\left(l_1^B; 0\right); 0\right) \geq r_1\left(r_1\left(l_1^B; 0\right); 0\right) \geq r_2\left(r_1\left(l_1^B; 0\right); 0\right) = l_2^B,$$

as required. Therefore,

$$\begin{aligned} f_1'\left(l_1^B\right) &= W'\left(l_1^B + l_2^B\right) l_1^B + W\left(l_1^B + l_2^B\right) \\ &\geq W'\left(l_1^B + l_2^B\right) l_2^B + W\left(l_1^B + l_2^B\right) \\ &= f_2'\left(l_2^B\right), \end{aligned}$$

as required.

Notice $l_1^B \geq l_2^B$. If in contrast $l_1^B \geq l_1^{**}$, then $l_1^B + l_2^B < l_1^{**} + l_2^{**}$ implies $l_2^B < l_2^{**}$. Therefore,

$$W'\left(l_1^B + l_2^B\right) l_2^B + W\left(l_1^B + l_2^B\right) > W\left(l_1^{**} + l_2^{**}\right)$$

and

$$W'\left(l_1^B + l_2^B\right) l_1^B + W\left(l_1^B + l_2^B\right) \leq W\left(l_1^{**} + l_2^{**}\right)$$

which implies

$$W'\left(l_1^B + l_2^B\right) l_2^B + W\left(l_1^B + l_2^B\right) > W'\left(l_1^B + l_2^B\right) l_1^B + W\left(l_1^B + l_2^B\right),$$

and a contradiction to $l_2^B \leq l_1^B$. Therefore, it must be $l_1^B < l_1^{**}$.

Notice, we cannot rule out $l_2^B \geq l_2^{**}$. For example, suppose the productivity of firm 2 is arbitrarily low (e.g., $f_2\left(l\right) = A_2 l^\alpha$, where $\alpha \in (0, 1)$ and $A_2 \approx 0$). Then, $l_2^{**} \approx l_2^B \approx 0$. First order conditions imply

$$\begin{aligned} f_2'\left(l_2^B\right) &\approx W\left(l_1^B\right) \\ f_2'\left(l_2^{**}\right) &\approx W\left(l_1^{**}\right) \end{aligned}$$

(where $f_2'\left(l_1^B\right) \approx W'\left(l_1^B\right) l_1^B + W\left(l_1^B\right)$ and $f_2'\left(l_1^{**}\right) \approx W\left(l_1^{**}\right)$). Since $l_1^{**} > l_1^B$ and $f_2' < 0$, it must be $l_2^{**} < l_2^B$. $\blacksquare$

---

theorem, $\frac{\partial r_i}{\partial l_{-i}} = \frac{W''(r_i + l_{-i})r + W'(r_i + l_{-i})}{f''(r_i) - W''(r_i + l_{-i})r - W'(r_i + l_{-i}) - W'(r_i + l_{-i})}$. The assumption $W''\left(L\right) L + W'\left(L\right) > 0$ implies the denominator is negative and the numerator is positive. Notice $\frac{\partial r_i}{\partial l_{-i}} > -1 \Leftrightarrow f_i''\left(r\right) < W'\left(r_i + l_{-i}\right)$, which holds given $f_i'' < 0 < W'$.

Proof of Lemma 3. Let

$$\pi_i^c(l_i;\omega_i) \equiv f_i(l_i) - \omega_i l_i.$$

We can write

$$\begin{aligned}\pi_i(l_i,l_{-i};\omega_i) &= \min\left\{\pi_i(l_i,l_{-i};0),\pi_i^c(l_i;\omega_i)\right\}\\ &= \min\left\{f_i(l_i) - W(l_i+l_{-i})\,l_i, f_i(l_i)-\omega_i l_i\right\}.\end{aligned}$$

Notice that profits $\pi_i(l_i,l_{-i};\omega_i)$ are concave in l_i since it is the lower envelope of two concave functions. We make two useful observations:

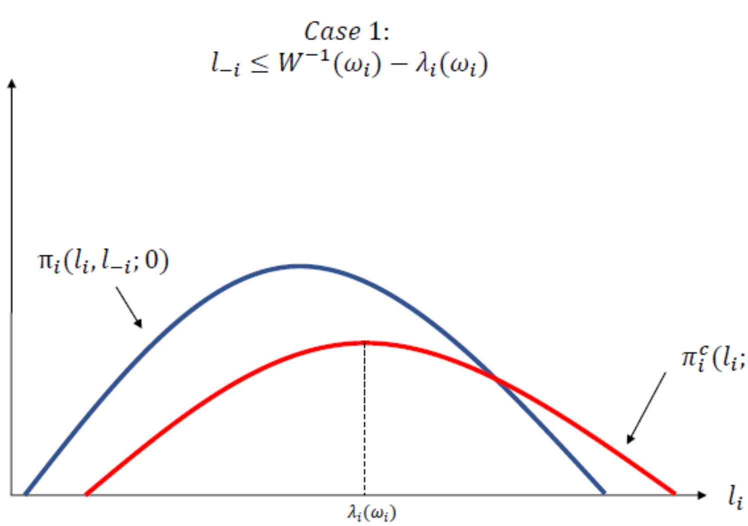
1. Recall $\lambda_i(\omega_i) = \arg\max_{l_i}\pi_i^c(l_i;\omega_i)$ and $r_i(l_{-i};0) = \arg\max_{l_i}\pi_i(l_i,l_{-i};0)$.
2. Note that $\pi_i^c(l_i;\omega_i) > \pi_i(l_i,l_{-i};0) \Leftrightarrow W(l_i+l_{-i}) > \omega_i$. If $W(l_i+l_{-i}) = \omega_i$ then $\pi_i(l_i,l_{-i};0) = \pi_i^c(l_i;\omega_i)$ and at this point,

$$\begin{aligned}\frac{\partial\pi_i(l_i,l_{-i};0)}{\partial l_i} &= f_i'(l_i) - W(l_i+l_{-i}) - W'(l_i+l_{-i})\,l_i\\ &< f_i'(l_i) - W(l_i+l_{-i}) = \frac{\partial\pi_i^c(l_i;\omega_i)}{\partial l_i}.\end{aligned}$$

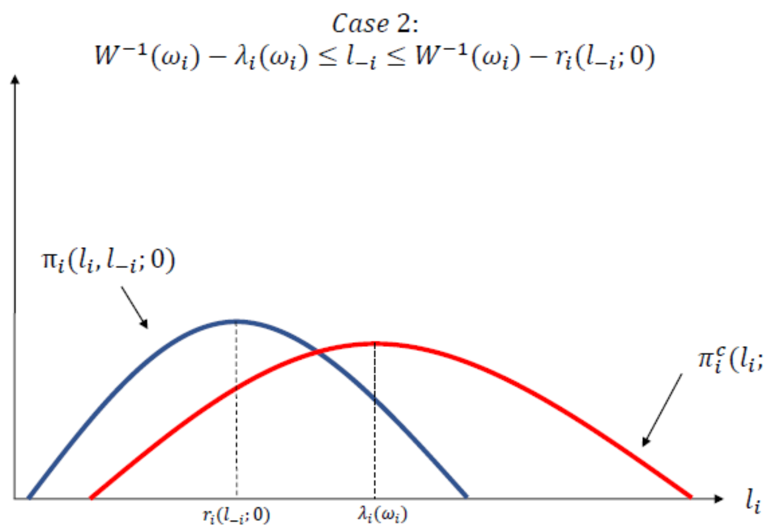
   Hence $\pi_i(l_i,l_{-i};0)$ crosses $\pi_i^c(l_i;\omega_i)$ from above.

There are three cases to consider:

1. Suppose $W(\lambda_i(\omega_i)+l_{-i}) \le \omega_i$, which holds if and only if $l_{-i} \le W^{-1}(\omega_i) - \lambda_i(\omega_i)$. At $l_i = \lambda_i(\omega_i)$, $W(l_i+l_{-i}) \le \omega_i$ and so $\pi_i^c(l_i;\omega_i) \le \pi_i(l_i,l_{-i};0)$. So $\pi_i(l_i,l_{-i};0)$ crosses $\pi_i^c(l_i;\omega_i)$ from above to the right of $\lambda_i(\omega_i)$, which is the maximizer of $\pi_i^c(l_i;\omega_i)$. Hence the maximum of $\pi_i(l_i,l_{-i};\omega_i)$ is $l_i = \lambda_i(\omega_i)$.

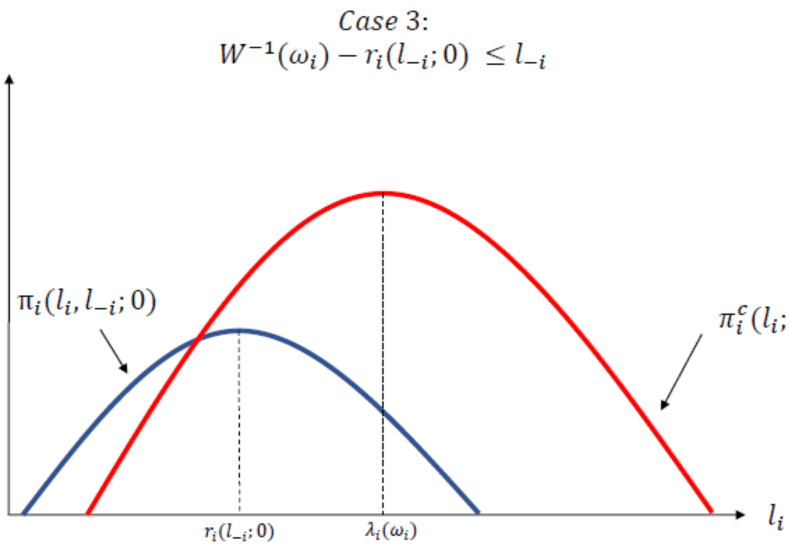


2. Suppose $W\left(r_i\left(l_{-i};0\right)+l_{-i}\right)\leq\omega_i\leq W\left(\lambda_i\left(\omega_i\right)+l_{-i}\right)$, which holds if and only if $W^{-1}\left(\omega_i\right)-\lambda_i\left(\omega_i\right)\leq l_{-i}\leq W^{-1}\left(\omega_i\right)-r_i\left(l_{-i};0\right)$. Note that, in this case, $r\left(l_{-i};0\right)\leq\lambda_i\left(\omega_i\right)$. At $l_i=r_i\left(l_{-i};0\right)$, $W\left(l_i+l_{-i}\right)\leq\omega_i$ and so $\pi_i^c\left(l_i;\omega_i\right)\leq\pi_i\left(l_i,l_{-i};0\right)$. At $l_i=\lambda_i\left(\omega_i\right)$, $\omega_i\leq W\left(\lambda_i\left(\omega_i\right)+l_{-i}\right)$, and so $\pi_i\left(l_i,l_{-i};0\right)\leq\pi_i^c\left(l_i;\omega_i\right)$. Hence the crossing point of the functions $\pi_i^c\left(l_i;\omega_i\right)$ and $\pi_i\left(l_i,l_{-i};0\right)$ occurs in the interval $\left[r_i\left(l_{-i};0\right),\lambda\left(\omega_i\right)\right]$, with $\pi_i^c\left(l_i;\omega_i\right)\leq(\geq)\,\pi_i\left(l_i,l_{-i};0\right)$ to the left (right) of the crossing point. Hence $\min\left\{\pi_i^c\left(l_i;\omega_i\right),\pi_i\left(l_i,l_{-i};0\right)\right\}$ is strictly increasing up to the crossing point, and strictly decreasing after the crossing point, and so is maximized at the crossing point. The crossing point l_i satisfies $W\left(l_i+l_{-i}\right)=\omega_i$, i.e., $l_i=W^{-1}\left(\omega_i\right)-l_{-i}$.



3. Suppose $\omega_i\leq W\left(r_i\left(l_{-i};0\right)+l_{-i}\right)$, which holds if and only if $l_{-i}\geq W^{-1}\left(\omega_i\right)-r_i\left(l_{-i};0\right)$. At $l_i=r_i\left(l_{-i};0\right)$, $\omega_i\leq W\left(l_i+l_{-i}\right)$, and so $\pi_i\left(l_i,l_{-i};0\right)\leq\pi_i^c\left(l_i;\omega_i\right)$. If $\pi_i\left(l_i,l_{-i};0\right)\leq\pi_i^c\left(l_i;\omega_i\right)$ for all l_i, it is immediate that the maximizer of $\min\left\{\pi_i^c\left(l_i;\omega_i\right),\pi_i\left(l_i,l_{-i};0\right)\right\}$ is $r_i\left(l_{-i};0\right)$. Otherwise, $\pi_i\left(l_i,l_{-i};0\right)$ crosses $\pi_i^c\left(l_i;\omega_i\right)$ from above at a point to the left of $r_i\left(l_{-i};0\right)$. Hence $\pi_i^c\left(l_i;\omega_i\right)$ is increasing up to this crossing point, and the maximizer of

$\min\{\pi_i^c(l_i;\omega_i), \pi_i(l_i, l_{-i}; 0)\}$ is again $r_i(l_{-i}; 0)$.



Observe that it cannot be $W(\lambda_i(\omega_i) + l_{-i}) \leq \omega_i \leq W(r_i(l_{-i}; 0) + l_{-i})$. If it did, then $W(\lambda_i(\omega_i) + l_{-i}) \leq W(r_i(l_{-i}; 0) + l_{-i})$ implies $\lambda_i(\omega_i) < r_i(l_{-i}; 0)$, $W(\lambda_i(\omega_i) + l_{-i}) \leq \omega_i$ implies $\pi_i^c(\lambda_i(\omega_i); \omega_i) \leq \pi_i(\lambda_i(\omega_i), l_{-i}; 0)$, and $\omega_i \leq W(r_i(l_{-i}; 0) + l_{-i})$ implies $\pi_i^c(r_i(l_{-i}; 0); \omega_i) > \pi_i(r_i(l_{-i}; 0), l_{-i}; 0)$. Since $\pi_i^c(r_i(l_{-i}; 0); \omega_i) \leq \pi_i^c(\lambda_i(\omega_i); \omega_i)$, the above implies $\pi_i(r_i(l_{-i}; 0), l_{-i}; 0) < \pi_i(\lambda_i(\omega_i), l_{-i}; 0)$, which contradicts the observation that $r_i(l_{-i}; 0)$ is the maximizer of $\pi_i(l_i, l_{-i}; 0)$.

Finally, we rewrite the condition on l_{-i} from the second case. Note that

$$\pi_i\left(\lambda_i(\omega_i), W^{-1}(\omega_i) - \lambda_i(\omega_i); 0\right) = \pi_i^c(\lambda_i(\omega_i); \omega_i) = \max_{l_i} \pi_i^c(l_i; \omega_i),$$

implying $r_i(W^{-1}(\omega_i) - \lambda_i(\omega_i); 0) < \lambda_i(\omega_i)$. Hence

$$W^{-1}(\omega_i) - \lambda_i(\omega_i) + r_i\left(W^{-1}(\omega_i) - \lambda_i(\omega_i); 0\right) < W^{-1}(\omega_i),$$

i.e., at $l_{-i} = W^{-1}(\omega_i) - \lambda_i(\omega_i)$,

$$l_{-i} + r_i(l_{-i}; 0) < W^{-1}(\omega_i).$$

Hence

$$W^{-1}(\omega_i) - \lambda_i(\omega_i) < \Lambda_{-i}(\omega_i).$$

Hence the condition on l_{-i} is equivalent to

$$l_{-i} \in \left[W^{-1}(\omega_i) - \lambda_i(\omega_i), \Lambda_{-i}(\omega_i)\right].$$

This completes the proof of the first equality in the statement of the result. The second equality follows from the property (Lemma 1) that $r_i(l_{-i}, 0) + l_{-i}$ is strictly increasing. ■

A.2 Proofs for Section 4

The next sequence of auxiliary results will be used for the proof of Proposition 1.

Lemma 6 *If $\omega_1 \neq \omega_2$ then there is at most one labor market equilibrium.*

Proof. Note that (l_1, l_2) is a labor market equilibrium if and only if l_2 is a solution to

$$r_2\left(r_1\left(l_2; \omega_1\right); \omega_2\right) = l_2.$$

and $l_1 = r_1\left(l_2; \omega_1\right)$. From Lemma 3, it is immediate that the function $r_2\left(r_1\left(\cdot; \omega_1\right); \omega_2\right)$ has the following properties: It is continuous and weakly increasing. It is differentiable at all but at most four points. The set of points at which the function has slope 1 is an interval. Everywhere outside this interval the slope is strictly less than 1. And finally, if the slope is 1 then

$$\begin{aligned} r_1\left(l_2; \omega_1\right) &= W^{-1}\left(\omega_1\right) - l_2 \\ r_2\left(r_1\left(l_2; \omega_1\right); \omega_2\right) &= W^{-1}\left(\omega_2\right) - r_1\left(l_2; \omega_1\right). \end{aligned}$$

From these properties, equilibrium multiplicity occurs only if

$$W^{-1}\left(\omega_2\right) - \left(W^{-1}\left(\omega_1\right) - l_2\right) = l_2,$$

has more than one solution, i.e., only if $\omega_1 = \omega_2$. ■

Lemma 7 *If $\max_i \omega_i \leq W^B$ then in any equilibrium, $l_i^* = l_i^B$ and $W_1^* = W_2^* = W^B$.*

Proof. To show that $l_i^* = l_i^B$ is an equilibrium, notice $\lambda_i\left(W^B\right) > l_i^B = W^{-1}\left(W^B\right) - l_{-i}^B = r_i\left(l_{-i}^B; 0\right)$. Notice $\omega_i \leq W^B \Rightarrow \lambda_i\left(\omega_i\right) \geq r_i\left(l_{-i}^B; 0\right)$. Also notice $\omega_i \leq W^B$ and $W^{-1}\left(W^B\right) - l_{-i}^B = r_i\left(l_{-i}^B; 0\right)$ imply $W^{-1}\left(\omega_i\right) - l_{-i}^B < r_i\left(l_{-i}^B; 0\right)$. Based on Lemma 3, $r_i\left(l_{-i}^B; \omega_i\right) = r_i\left(l_{-i}^B; 0\right)$. Thus, if firm $-i$ picks $l_{-i} = l_{-i}^B$ then firm i's best response is $r_i\left(l_{-i}^B; \omega_i\right) = l_i^B$.

It remains to show that this is the unique equilibrium. Suppose to the contrary there is a second equilibrium $(\tilde{l}_1, \tilde{l}_2)$. By Lemma 6 it must be $\omega_2 = \omega_1 = \omega$ for some $\omega \leq W^B$, and by its proof, it must be $\tilde{l}_1 + \tilde{l}_2 = W^{-1}(\omega)$.

Since $r_i(\cdot; \omega)$ is weakly decreasing, if $\tilde{l}_i \leq l_i^B$ then $\tilde{l}_{-i} = r_{-i}(\tilde{l}_i, \omega) \geq r_{-i}(l_i^B, \omega) = l_{-i}^B$ (the last equality follows from the observation above that $l_i^* = l_i^B$ is an equilibrium). Hence for some i, $\tilde{l}_i \geq l_i^B$. Moreover, $\tilde{l}_i > l_i^B$, since if instead $\tilde{l}_i = l_i^B$ then $\tilde{l}_{-i} = l_{-i}^B$, a contradiction for the existence of a second equilibrium.

Observe

$$W^{-1}(\omega) = \tilde{l}_1 + \tilde{l}_2 = \tilde{l}_i + r_{-i}(\tilde{l}_i; \omega) \geq l_i^B + r_{-i}\left(l_i^B; \omega\right) = W^{-1}\left(W^B\right).$$

Indeed, the second equality follows from the definition of equilibrium, the first inequality follows from the observation that $l + r(l; \omega)$ is a weakly increasing function of l, and the third equality follows from the observation that $\left(l_i^B, l_{-i}^B\right)$ is an equilibrium when $\omega \leq W^B$. Therefore, it must be $\omega = W^B$. But notice that $l_i^B = \Lambda_i\left(W^B\right)$. And thus, $\tilde{l}_i > l_i^B$ implies $\tilde{l}_i > \Lambda_i\left(W^B\right) = \Lambda_i(\omega)$, and hence, $r_{-i}(\tilde{l}_i;\omega) = r_{-i}(\tilde{l}_i;0)$ by Lemma 3. Therefore, and since $\omega = W^B$,

$$W^{-1}(\omega) = \tilde{l}_i + r_{-i}(\tilde{l}_i;\omega) = \tilde{l}_i + r_{-i}(\tilde{l}_i;0) > l_i^B + r\left(l_i^B;0\right) = W^{-1}\left(W^B\right),$$

where the strict inequality follows from Lemma 1, a contradiction. ■

Lemma 8 *If $\omega_i \geq W^{**}$ then $l_i = \lambda_i(\omega_i)$.*

Proof. For specificity, set $i = 2$. For use at various points in the proof, note that

$$\lambda_1(\omega_2) + \lambda_2(\omega_2) \leq \lambda_1(W^{**}) + \lambda_2(W^{**}) = W^{-1}(W^{**}) \leq W^{-1}(\omega_2) \tag{22}$$

and that, if $l_i \leq \lambda_i(\omega_i)$ and $\omega_i \geq W^{**}$ then by Lemma 1,

$$\begin{aligned} l_i + r_{-i}(l_i;0) &\leq \lambda_i(\omega_i) + r_{-i}(\lambda_i(\omega_i);0) \\ &\leq \lambda_i(W^{**}) + r_{-i}(\lambda_i(W^{**});0) \\ &\leq \lambda_i(W^{**}) + \lambda_{-i}(W^{**}) = W^{-1}(W^{**}) \leq W^{-1}(\omega_i), \end{aligned}$$

i.e., if $l_i \leq \lambda_i(\omega_i)$ and $\omega_i \geq W^{**}$ then

$$r_{-i}(l_i;0) \leq W^{-1}(\omega_i) - l_i. \tag{23}$$

Notice we used $r_{-i}(\lambda_i(W^{**});0) \leq \lambda_{-i}(W^{**})$. Indeed since $r_{-i}(\lambda_i(W^{**});0)$ satisfies $f'_{-i}(r) = W'(r + \lambda_i(W^{**}))\,r + W(r + \lambda_i(W^{**}))$. Using $f'_{-i}(\lambda_{-i}(W^{**})) = W^{**}$, we have

$$\begin{aligned} & W'(\lambda_{-i}(W^{**}) + \lambda_i(W^{**}))\,\lambda_{-i}(W^{**}) + W(\lambda_{-i}(W^{**}) + \lambda_i(W^{**})) \\ = \;& W'(\lambda_{-i}(W^{**}) + \lambda_i(W^{**}))\,\lambda_{-i}(W^{**}) + W^{**} \\ = \;& W'(\lambda_{-i}(W^{**}) + \lambda_i(W^{**}))\,\lambda_{-i}(W^{**}) + f'_{-i}(\lambda_{-i}(W^{**})) \\ > \;& f'_{-i}(\lambda_{-i}(W^{**})), \end{aligned}$$

and hence, $r_{-i}(\lambda_i(W^{**});0) \leq \lambda_{-i}(W^{**})$.

First, we show that in any equilibrium $l_2 = \lambda(\omega_2)$. It suffices to show that

$$r_1(\lambda_2(\omega_2);\omega_1) \leq W^{-1}(\omega_2) - \lambda_2(\omega_2), \tag{24}$$

because in this case,

$$\begin{aligned} \lambda_2(\omega_2) &\leq W^{-1}(\omega_2) - r_1(\lambda_2(\omega_2);\omega_1) \\ &\leq \max\left\{W^{-1}(\omega_2) - r_1(\lambda_2(\omega_2);\omega_1),\, r_2(r_1(\lambda_2(\omega_2);\omega_1);0)\right\} \end{aligned}$$

thereby implying that $r_2(r_1(\lambda_2(\omega_2);\omega_1);\omega_2) = \lambda_2(\omega_2)$.

To establish (24): If $\omega_1 \geq \omega_2$ then the inequality is immediate from the combination of $r_1(\cdot;\omega_1) \leq \lambda_1(\omega_1) \leq \lambda_1(\omega_2)$ and (22). If instead $\omega_1 < \omega_2$ then note that it is sufficient to establish

$$\max\left\{W^{-1}(\omega_1)-\lambda_2(\omega_2), r_1(\lambda_2(\omega_2);0)\right\} \leq W^{-1}(\omega_2)-\lambda_2(\omega_2). \tag{25}$$

This inequality indeed holds by the combination of $\omega_1 < \omega_2$ and (23).

Next, if $\omega_1 \neq \omega_2$ then the equilibrium is unique by Lemma 6, and the proof is complete.

For $\omega_1 = \omega_2 = \omega \geq W^{**}$, simply note that $l_i \leq \lambda_i(\omega)$ for both firms and so:

$$\begin{aligned} r_i(l_{-i};\omega) &= \min\left\{\lambda_i(\omega), \max\left\{W^{-1}(\omega)-l_{-i}, r(l_{-i};0)\right\}\right\} \\ &= \min\left\{\lambda_i(\omega), W^{-1}(\omega)-l_{-i}\right\} \\ &= \lambda_i(\omega), \end{aligned}$$

where the first and second equalities follow from (23) and (22), respectively. Hence the unique equilibrium in this case is $l_i = \lambda_i(\omega)$. ■

Lemma 9 *If $\omega_i \in (W^B, \hat{W}_i]$ and $\omega_{-i} \leq \omega_i$ then $l_i^* = \Lambda_i(\omega_i)$, $l_{-i}^* = W^{-1}(\omega_i) - \Lambda_i(\omega_i)$, and $W_1^* = W_2^* = \omega_i$ is an equilibrium; and is the unique equilibrium if $\omega_{-i} < \omega_i$.*

Proof. For concreteness, we prove the lemma for $i = 1$; the same proof follows for $i = 2$. We start by arguing that the best response of firm 2 to $l_1 = \Lambda_1(\omega_1)$ is $l_2 = W^{-1}(\omega_1) - \Lambda_1(\omega_1)$. Firm 2's best response is

$$r_2(\Lambda_1(\omega_1);\omega_2) = \min\left\{\lambda_2(\omega_2), \max\left\{W^{-1}(\omega_2)-\Lambda_1(\omega_1), r_2(\Lambda_1(\omega_1);0)\right\}\right\}.$$

Observe that

$$r_2(\Lambda_1(\omega_1);0) < \lambda_2(\omega_1). \tag{26}$$

This follows because, by definition of $\Lambda_1(\omega_1)$, at $(l_1, l_2) = (\Lambda_1(\omega_1), r_2(\Lambda_1(\omega_1);0))$ the market wage is ω_1, and so the marginal effect of changing l_2 on firm 2's profits is

$$f_2'(l_2) - \omega_1 - W'(l_1 + l_2).$$

Since $f_2'(\lambda_2(\omega_1)) = \omega_1$, this expression is strictly negative for any $l_2 \geq \lambda_2(\omega_1)$, implying the optimal response of firm 2 to $l_1 = \Lambda_1(\omega_1)$ is strictly smaller than $\lambda_2(\omega_1)$, i.e., inequality (26).

Again using the definition of $\Lambda_1(\omega_1)$, $\omega_2 \leq \omega_1$, and inequality (26) implies

$$W^{-1}(\omega_2) - \Lambda_1(\omega_1) \leq W^{-1}(\omega_1) - \Lambda_1(\omega_1) = r_2(\Lambda_1(\omega_1);0) < \lambda_2(\omega_1) < \lambda_2(\omega_2).$$

Recalling

$$r_2(l_1;\omega_2) = \min\left\{\lambda_2(\omega_2), \max\left\{W^{-1}(\omega_2)-l_1, r_2(l_1;0)\right\}\right\},$$

we established $r_2(\Lambda_1(\omega_1);\omega_2) = W^{-1}(\omega_1) - \Lambda_1(\omega_1)$ as claimed.

Next, we argue that the best response of firm 1 to $l_2 = W^{-1}(\omega_1) - \Lambda_1(\omega_1)$ is $l_1 = \Lambda_1(\omega_1)$. Firm 1's best response is

$$r_1\left(W^{-1}(\omega_1)-\Lambda_1(\omega_1);\omega_1\right) = \min\left\{\lambda_1(\omega_1), \max\left\{\Lambda_1(\omega_1), r_1\left(W^{-1}(\omega_1)-\Lambda_1(\omega_1);0\right)\right\}\right\},$$

As an intermediate step, we establish that for any $\omega > W^B$,

$$W^{-1}(\omega) < \Lambda_1(\omega) + \Lambda_2(\omega). \tag{27}$$

To see why, observe that for $i = 1, 2$, $\Lambda_i(\omega) > \Lambda_i\left(W^B\right) = l_i^B$, and hence,

$$W^{-1}(\omega) = \Lambda_i(\omega) + r_{-i}(\Lambda_i(\omega); 0) < \Lambda_i(\omega) + r_{-i}\left(l_i^B; 0\right) = \Lambda_i(\omega) + l_{-i}^B.$$

Summing over $i = 1, 2$ implies

$$2W^{-1}(\omega) < \Lambda_1(\omega) + \Lambda_2(\omega) + l_1^B + l_2^B = \Lambda_1(\omega) + \Lambda_2(\omega) + W^{-1}\left(W^B\right).$$

Inequality (27) then follows from the fact that $W^{-1}(\omega) > W^{-1}\left(W^B\right)$. Combining Lemma 1 and (27) implies

$$W^{-1}(\omega_1) - \Lambda_1(\omega_1) + r_1\left(W^{-1}(\omega_1) - \Lambda_1(\omega_1); 0\right) < \Lambda_2(\omega_1) + r_1(\Lambda_2(\omega_1); 0) = W^{-1}(\omega_1),$$

and so,

$$r_1\left(W^{-1}(\omega_1) - \Lambda_1(\omega_1); 0\right) < \Lambda_1(\omega_1) \leq \lambda_1(\omega_1),$$

where the final weak inequality follows from $\omega_1 \leq \hat{W}_1$ and that fact that $\omega_1 \leq \hat{W}_1 \Leftrightarrow \Lambda_1(\omega_1) \leq \lambda_1(\omega_1)$. Therefore,

$$r_1\left(W^{-1}(\omega_1) - \Lambda_1(\omega_1); \omega_1\right) = \Lambda_1(\omega_1)$$

as claimed.

Hence, $(l_1^*, l_2^*) = (\Lambda_1(\omega_1), W^{-1}(\omega_1) - \Lambda_1(\omega_1))$ is an equilibrium. Uniqueness when $\omega_1 > \omega_2$ follows from Lemma 6. Finally, notice that

$$W(l_1^* + l_2^*) = W\left(\Lambda_1(\omega_1) + W^{-1}(\omega_1) - \Lambda_1(\omega_1)\right) = \omega_1 \geq \omega_2,$$

and hence $W_1^* = W_2^* = \omega_1$, completing the proof. ■

Lemma 10 *Suppose $\omega_i \in (\hat{W}_i, W^{**}]$ and $\omega_{-i} \leq \omega_i$. Then,*

- *(i) There is an equilibrium in which, $l_i^* = \lambda_i(\omega_i)$, $l_{-i}^* = r_{-i}(\lambda_i(\omega_i); \omega_{-i}) \leq W^{-1}(\omega_i) - \lambda_i(\omega_i)$, and $W_i^* = \omega_i$.*
- *(ii) If $\omega_{-i} < \omega_i$ then the equilibrium in part (i) is the unique equilibrium and $l_{-i}^* < W^{-1}(\omega_i) - \lambda_i(\omega_i)$. Moreover:*
  - *(a) If $W^{-1}(\omega_{-i}) - \lambda_i(\omega_i) \geq r_{-i}(\lambda_i(\omega_i); 0)$ then $l_{-i}^* = W^{-1}(\omega_{-i}) - \lambda_i(\omega_i)$ and $W_{-i}^* = \omega_{-i}$.*
  - *(b) If $W^{-1}(\omega_{-i}) - \lambda_i(\omega_i) < r_{-i}(\lambda_i(\omega_i); 0)$ then $l_{-i}^* = r_{-i}(\lambda_i(\omega_i); 0)$ and $W_{-i}^* = W(\lambda_i(\omega_i) + r_{-i}(\lambda_i(\omega_i); 0))$.*
- *(iii) If $\omega_{-i} = \omega_i$ then $l_{-i}^* = r_{-i}(\lambda_i(\omega_i); \omega_{-i}) = W^{-1}(\omega_i) - \lambda_i(\omega_i)$ and $W_{-i}^* = \omega_i$.*

Proof. For concreteness, we prove the lemma for $i = 1$; the same proof follows for $i = 2$. First, we show that if $l_2 \leq W^{-1}(\omega_1) - \lambda_1(\omega_1)$ then firm 1's best response is $r_1(l_2; \omega_1) = \lambda_1(\omega_1)$. This follows directly from $\lambda_1(\omega_1) \leq W^{-1}(\omega_1) - l_2 \leq \max\{W^{-1}(\omega_1) - l_2, r_1(l_2; 0)\}$.

Second, we show firm 2's best response to firm 1 picking $\lambda_1(\omega_1)$ is $r_2(\lambda_1(\omega_1); \omega_2) \leq W^{-1}(\omega_1) - \lambda_1(\omega_1)$. It is sufficient to establish that $\max\{W^{-1}(\omega_2) - \lambda_1(\omega_1), r_2(\lambda_1(\omega_1); 0)\} \leq W^{-1}(\omega_1) - \lambda_1(\omega_1)$. This is indeed the case since $\omega_2 \leq \omega_1$ implies $W^{-1}(\omega_2) - \lambda_1(\omega_1) \leq W^{-1}(\omega_1) - \lambda_1(\omega_1)$ and by $\lambda_1(\omega_1) < \Lambda_1(\omega_1)$ (from $\omega_1 > \hat{W}_1$) and Lemma 1,

$$\lambda_1(\omega_1) + r_2(\lambda_1(\omega_1); 0) < \Lambda_1(\omega_1) + r_2(\Lambda_1(\omega_1); 0) = W^{-1}(\omega_1),$$

and so

$$r_2(\lambda_1(\omega_1); 0) < W^{-1}(\omega_1) - \lambda_1(\omega_1).$$

Therefore, $r_2(\lambda_1(\omega_1); \omega_2) \leq W^{-1}(\omega_1) - \lambda_1(\omega_1)$.

Third, we show that

$$r_2(\lambda_1(\omega_1); \omega_2) = \max\left\{W^{-1}(\omega_2) - \lambda_1(\omega_1), r_2(\lambda_1(\omega_1); 0)\right\}.$$

Since $\omega_2 \leq \omega_1 \leq W^{**}$, we have

$$W^{-1}(\omega_1) \leq W^{-1}(W^{**}) = \lambda_1(W^{**}) + \lambda_2(W^{**}) \leq \lambda_1(\omega_1) + \lambda_2(\omega_2),$$

and so

$$W^{-1}(\omega_1) - \lambda_1(\omega_1) \leq \lambda_2(\omega_1). \tag{28}$$

The result follows from the combination of step 2, (28), and $\omega_2 \leq \omega_1$.

Fourth, from Steps 1 and 2, there is an equilibrium in which $l_1^* = \lambda_1(\omega_1)$ and $l_2 = r_2(\lambda_1(\omega_1); \omega_2) \leq W^{-1}(\omega_1) - \lambda_1(\omega_1)$ and hence $W_1^* = \omega_1$. This completes part (i). If $\omega_2 < \omega_1$ then based on Lemma 6 this is the unique equilibrium, and the characterization follows from Steps 2 and 3. This completes part (ii). Similarly, if $\omega_2 = \omega_1$ then the characterization again follows from Steps 2 and 3, completing part (iii) and the proof. ■

Proof of Proposition 1. Part (i) follows from Lemma 7. Part (ii) follows from Lemma 8.

Consider part (iii). Suppose $\omega_2 = \omega_1 = \omega \in (W^B, W^{**})$. As we show in the proof of Lemma 9, inequality (26) holds, that is

$$r_{-i}(\Lambda_i(\omega); 0) < \lambda_{-i}(\omega). \tag{29}$$

Since $\Lambda_i(\omega) + r_{-i}(\Lambda_i(\omega); 0) = W^{-1}(\omega)$, then (26) implies

$$W^{-1}(\omega) < \Lambda_i(\omega) + \lambda_{-i}(\omega).$$

Since $\omega > W^B$, repeating the arguments in the proof of Lemma 9 that shows (27), for $i = 1, 2$ we have

$$W^{-1}(\omega) < \Lambda_i(\omega) + \Lambda_{-i}(\omega). \tag{30}$$

Since $\omega < W^{**}$, we have

$$W^{-1}(\omega) < W^{-1}(W^{**}) = \lambda_i(\omega) + \lambda_{-i}(\omega).$$

Combined, these three inequalities establish the interval in (15) is not empty.

Let l^* be an element in interval (15). Then,

$$l^* \in \left[W^{-1}(\omega) - \lambda_{-i}(\omega), \Lambda_i(\omega)\right].$$

Notice $l^* \leq \Lambda_i(\omega)$ implies $W^{-1}(\omega) - l^* \geq r_{-i}(l^*; 0)$ and $W^{-1}(\omega) - \lambda_{-i}(\omega) \leq l^*$ implies $\lambda_{-i}(\omega) \leq W^{-1}(\omega) - l^*$. Thus, from Lemma 3, $r_{-i}(l^*; \omega) = W^{-1}(\omega) - l^*$. Moreover

$$l^* \in \left[W^{-1}(\omega) - \Lambda_{-i}(\omega), \lambda_i(\omega)\right]$$

and so

$$r_{-i}(l^*; \omega) = W^{-1}(\omega) - l^* \in \left[W^{-1}(\omega) - \lambda_i(\omega), \Lambda_{-i}(\omega)\right].$$

Thus, from Lemma 3

$$r_i(r_{-i}(l^*; \omega); \omega) = W^{-1}(\omega) - r_{-i}(l^*; \omega) = l^*,$$

establishing that $(l^*, W^{-1}(\omega) - l^*)$ is an equilibrium. The fact that both firms pay ω is immediate.

Finally, we show that there are no other equilibria. We have just shown that the function $r_i(r_{-i}(\cdot; \omega); \omega)$ has an interval of fixed points, and that over this interval the function has slope 1. From the proof of Lemma 6, it follows that the set of fixed points of $r_i(r_{-i}(\cdot; \omega); \omega)$ coincides with with the interval over which the function has slope 1. From the proof of Lemma 6, and from Lemma 3, this interval is defined by the pair of conditions

$$\begin{aligned} l_i &\in \left[W^{-1}(\omega) - \lambda_{-i}(\omega), \Lambda_i(\omega)\right] \\ W^{-1}(\omega) - l_i &\in \left[W^{-1}(\omega) - \lambda_i(\omega), \Lambda_{-i}(\omega)\right] \end{aligned}$$

which together is exactly the interval in (15). This completes part (iii).

Consider part (iv). If $\omega_{-i} < \omega_i$ then the equilibrium is unique based on Lemma 6. Based on Lemma 9, if $\omega_i \in (W^B, \hat{W}_i]$ then $l_i = \Lambda_i(\omega_i)$ and $W_i^* = \omega_i$. Based on Lemma 10 part (i), if $\omega_i \in (\hat{W}_i, W^{**}]$ then $l_i^* = \lambda_i(\omega_i)$ and $W_i^* = \omega_i$. Since $\omega_i \leq \hat{W}_i \Leftrightarrow \Lambda_i(\omega_i) \leq \lambda_i(\omega_i)$, this can be written as $l_i^* = \min\{\Lambda_i(\omega_i), \lambda_i(\omega_i)\}$ and $W_i^* = \omega_i$ as required. Notice l_{-i}^* and W_{-i}^* follow from the definition of equilibrium, and their explicit characterization is given in Lemmas 9 and 10.

$$W^{-1}(\omega) < \Lambda_1(\omega) + \Lambda_2(\omega). \tag{31}$$

Finally, we prove that if firms i are symmetric (i.e., have the same production functions) or $i = 1$ (the larger firm adopts a more aggressive ESG policy), then $l_i^* > l_{-i}^*$. If $\omega_i \in (W^B, \hat{W}_i]$ then based on Lemma 9, $l_i^* > l_{-i}^* \Leftrightarrow \Lambda_i(\omega_i) > W^{-1}(\omega_i) - \Lambda_i(\omega_i)$. Inequality (27) from the proof of Lemma 9 implies $\Lambda_i(\omega) + \Lambda_{-i}(\omega) > W^{-1}(\omega)$. Thus, $\Lambda_i(\omega_i) > W^{-1}(\omega_i) - \Lambda_i(\omega_i)$ must hold.

If $\omega_i \in (\hat{W}_i, W^{**}]$ then based on Lemma 10 $l_i^* = \lambda_i(\omega_i)$ and $l_{-i}^* < W^{-1}(\omega_i) - \lambda_i(\omega_i)$. Recall $\lambda_i(W^{**}) + \lambda_{-i}(W^{**}) = W^{-1}(W^{**})$. If $\omega_i < W^{**}$ and firms are symmetric or $\lambda_i(\cdot) > \lambda_{-i}(\cdot)$ then $\lambda_i(\omega_i) > W^{-1}(\omega_i) - \lambda_i(\omega_i)$. ■

A.3 Proofs for Section 5.1

Proof of Proposition 2. Based on Corollary 1, if $\omega_i \in [W^B, \hat{W}_i]$ then $l_i = \Lambda_i(\omega_i)$, which is strictly increasing in ω_i. Firm i's profits are

$$f_i(l_i) - l_i W(l_i + r_{-i}(l_i; 0)). \tag{32}$$

The derivative of (32) with respect to Λ_i is

$$f_i'(l_i) - W(l_i + r_{-i}(l_i; 0)) - l_i W'(l_i + r_{-i}(l_i; 0))\left(1 + r_{-i}'(l_i; 0)\right). \tag{33}$$

Notice $\Lambda_i\left(W^B\right) = l_i^B$, and so (33) reduces to

$$\begin{aligned}
& f_i'\left(l_i^B\right) - W^B - l_i^B W'\left(l_i^B + l_{-i}^B\right)\left(1 + r_{-i}'\left(l_i^B; 0\right)\right) \\
= \;& f_i'\left(l_i^B\right) - W^B - l_i^B W'\left(l_i^B + l_{-i}^B\right) - l_i^B W'\left(l_i^B + l_{-i}^B\right) r_{-i}'\left(l_i^B; 0\right) \\
= \;& -l_i^B W'\left(l_i^B + l_{-i}^B\right) r_{-i}'\left(l_i^B; 0\right) \\
> \;& 0
\end{aligned}$$

where the last equality follows from firm i's optimality condition in the non-ESG benchmark. Since $r_{-i}' < 0$, it follows that firm i's profits are strictly increasing in the ESG policy ω_i in the neighborhood to above W^B.

Also notice $\Lambda_i\left(\hat{W}_i\right) = \lambda_i\left(\hat{W}_i\right)$, or equivalently, $f_i'(l_i) = W(l_i + r_{-i}(l_i; 0))$. Hence (33) reduces to

$$-l_i W'(l_i + r_{-i}(l_i; 0))\left(1 + r_{-i}'(l_i; 0)\right),$$

which is strictly negative by Lemma 3. So firm i's profits are strictly decreasing in the ESG policy ω_i in the neighborhood below $\hat{W}_i$.

Based on Corollary 1, for $\omega_i \geq \hat{W}_i$, firm i hires $l_i = \lambda_i(\omega_i)$, or equivalently, firm i's profits are $\max_l f_i(l) - \omega_i l$, and so are strictly decreasing in ω_i. Therefore, $\varphi_i^* \in (W^B, \hat{W}_i)$ as required.

Next, notice $l_i^B = \Lambda_i\left(W^B\right) < \Lambda_i(\varphi_i^*)$, thus firm i's employment is higher than the No-ESG benchmark. Since ω_i is chosen to maximize firm i profit, $\omega_i \leq W^B$ is a feasible policy that is strictly inferior to $\omega_i = \varphi_i^* > W^B$, then firm i's profit is also higher than the No-ESG benchmark. Since $l_{-i}^* = r_{-i}(\Lambda_i(\varphi_i^*); 0)$ and $r_{-i}(l_i; 0) + l_i$ increases in l_i, then total industry employment is also higher than the No-ESG benchmark. Moreover, $r_{-i}'(\cdot; 0) < 0$ implies firm $-i$'s employment is lower than the No-ESG benchmark. Notice $\max_{l_{-i}} f_{-i}(l_{-i}) - l_{-i} W(l_i + l_{-i})$ is decreasing in l_i. Since firm i's employment is higher than the No-ESG, firm $-i$'s profit is lower than the No-ESG.

Consider the effect on workers' surplus,

$$\begin{aligned}
K(l_i) &\equiv (l_i + r_{-i}(l_i;0)) W(l_i + r_{-i}(l_i;0)) - \int_0^{l_i + r_{-i}(l_i;0)} W(l)\,dl \\
K'(l_i) &= \left(1 + r'_{-i}(l_i;0)\right) \left[W(l_i + r_{-i}(l_i;0)) + (l_i + r_{-i}(l_i;0)) W'(l_i + r_{-i}(l_i;0))\right] \\
&\quad - \left(1 + r'_{-i}(l_i;0)\right) W(l_i + r_{-i}(l_i;0)) \\
&= \left(1 + r'_{-i}(l_i;0)\right) (l_i + r_{-i}(l_i;0)) W'(l_i + r_{-i}(l_i;0)) > 0
\end{aligned}$$

Consider the effect on total profit.

$$\begin{aligned}
\Pi(l_i) &= f_i(l_i) - l_i W(l_i + r_{-i}(l_i;0)) \\
&\quad + f_{-i}(r_{-i}(l_i;0)) - r_{-i}(l_i;0) W(l_i + r_{-i}(l_i;0))
\end{aligned}$$

and

$$\begin{aligned}
\Pi'(l_i) &= f'_i(l_i) + r'_{-i}(l_i;0) f'_{-i}(r_{-i}(l_i;0)) \\
&\quad - \left(1 + r'_{-i}(l_i;0)\right) W(l_i + r_{-i}(l_i;0)) \\
&\quad - \left(1 + r'_{-i}(l_i;0)\right) W'(l_i + r_{-i}(l_i;0)) (l_i + r_{-i}(l_i;0)) \\
&= f'_i(l_i) - f'_{-i}(r_{-i}(l_i;0)) + \left(1 + r'_{-i}(l_i;0)\right) \left[\begin{array}{c} f'_{-i}(r_{-i}(l_i;0)) - W(l_i + r_{-i}(l_i;0)) \\ -W'(l_i + r_{-i}(l_i;0)) (l_i + r_{-i}(l_i;0)) \end{array} \right]
\end{aligned}$$

The FOC of firm $-i$ implies

$$f'_{-i}(r_{-i}(l_i;0)) - W(l_i + r_{-i}(l_i;0)) - r_{-i}(l_i;0) W'(l_i + r_{-i}(l_i;0)) = 0 \tag{34}$$

Therefore,

$$\Pi'(l_i) = f'_i(l_i) - f'_{-i}(r_{-i}(l_i;0)) - l_i \left(1 + r'_{-i}(l_i;0)\right) W'(l_i + r_{-i}(l_i;0))$$

If $f'_i\left(l_i^B\right) \leq f'_{-i}\left(l_{-i}^B\right)$ (which holds if and only if firm i is the smaller firm) then $f'_i(l_i) \leq f'_{-i}(r_{-i}(l_i;0))$ for all $l_i > l_i^B$, and hence, $\Pi'(l_i) < 0$ for the entire region. The FOC of firm i implies

$$f'_i(l_i) - W(l_i + r_{-i}(l_i;0)) - l_i W'(l_i + r_{-i}(l_i;0)) \left(1 + r'_{-i}(l_i;0)\right) = 0. \tag{35}$$

Thus, at the optimum,

$$\begin{aligned}
\Pi'(l_i) &= r'_{-i}(l_i;0) f'_{-i}(r_{-i}(l_i;0)) - r'_{-i}(l_i;0) W(l_i + r_{-i}(l_i;0)) \\
&\quad - r_{-i}(l_i;0) W'(l_i + r_{-i}(l_i;0)) \left(1 + r'_{-i}(l_i;0)\right)
\end{aligned}$$

Using (34)

$$\Pi'(l_i) = -r_{-i}(l_i;0) W'(l_i + r_{-i}(l_i;0)) < 0.$$

Consider the effect on total surplus,

$$\begin{aligned} S(l_i) &= f_i(l_i) + f_{-i}(r_{-i}(l_i;0)) - \int_0^{l_i+r_{-i}(l_i;0)} W(l)\,dl. \\ S'(l_i) &= f_i'(l_i) + r_{-i}'(l_i;0)\,f_{-i}'(r_{-i}(l_i;0)) - \left(1+r_{-i}'(l_i;0)\right) W(l_i + r_{-i}(l_i;0)) \\ &= f_i'(l_i) - f_{-i}'(r_{-i}(l_i;0)) + \left(1+r_{-i}'(l_i;0)\right)\left[f_{-i}'(r_{-i}(l_i;0)) - W(l_i+r_{-i}(l_i;0))\right] \end{aligned}$$

Since $f_1'\left(l_1^B\right) \geq f_2'\left(l_2^B\right)$, for small ESG policies $S'(l_i) > 0$.

Notice $f_1'(\cdot) > f_2'(\cdot) \Rightarrow r_1(l;0) > r_2(l;0)$. Suppose $W'' \geq 0$ and $r_1'(l;0) > r_2'(l;0)$. That is, the more productive firm is less sensitive to its opponents changes in production. Then

$$\begin{aligned} & f_1'(l) - lW(l+r_2(l;0)) - lW'(l+r_2(l;0))(1+r_2'(l;0)) \\ > \; & f_1'(l) - lW(l+r_1(l;0)) - lW'(l+r_1(l;0))(1+r_1'(l;0)) \\ > \; & f_2'(l) - lW(l+r_1(l;0)) - lW'(l+r_1(l;0))(1+r_1'(l;0)) \end{aligned}$$

Therefore, firm 1 has more, chooses more aggressive ESG policy.

Recall

$$r_i'(l;0) = \frac{W''(r_i+l)\,r_i + W'(r_i+l)}{f_i''(r_i) - W''(r_i+l)\,r_i - W'(r_i+l) - W'(r_i+l)}$$

If $W''=0$ then $f_1''(\cdot) \leq f_2''(\cdot)$ and $f_1'''(\cdot) = f_2'''(\cdot) = 0 \Rightarrow r_1'(l;0) \geq r_2'(l;0)$. More generally, we need $f_1''(\cdot)$ to be sufficiently smaller than $f_2''(\cdot)$

$$\begin{aligned} r_1'(l;0) &> r_2'(l;0) \Leftrightarrow \\ & f_i''(r_{-i}) < f_i''(r_i)\,\frac{W''(r_{-i}+l)\,r_{-i} + W'(r_{-i}+l)}{W''(r_i+l)\,r_i + W'(r_i+l)} \\ & + \frac{W''(r_i+l)\,W'(r_{-i}+l)\,r_i - W''(r_{-i}+l)\,W'(r_i+l)\,r_{-i}}{W''(r_i+l)\,r_i + W'(r_i+l)} \end{aligned}$$

■

A.4 Proofs for Section 5.2

Proof of Lemma 4. Recall from Proposition 2, $\varphi_{-i}^* \in \left(W^B, \hat{W}_{-i}\right)$. Suppose firm i chooses ω_i.

1. Consider a downward reaction: $\omega_{-i} < \omega_i$. There are two subcases to consider:

   (a) If $\omega_i \leq \hat{W}_i$ then based on Lemma 9, $l_i = \Lambda_i(\omega_i)$, $l_{-i} = W^{-1}(\omega_i) - \Lambda_i(\omega_i)$, and the wage is ω_i. Firm $-i$'s profit is invariant to ω_{-i} and given by

$$\begin{aligned} \pi_{-i} &= f_{-i}\left(W^{-1}(\omega_i) - \Lambda_i(\omega_i)\right) - \left(W^{-1}(\omega_i) - \Lambda_i(\omega_i)\right)\omega_i \\ &= f_{-i}(r_{-i}(\Lambda_i(\omega_i);0)) - r_{-i}(\Lambda_i(\omega_i);0)\,W(r_{-i}(\Lambda_i(\omega_i);0) + \Lambda_i(\omega_i)). \end{aligned}$$

   The inequality follows from the observation that $\Lambda_i(\omega_i) + r_{-i}(\Lambda_i(\omega_i);0) = W^{-1}(\omega_i)$.

(b) If $\omega_i > \hat{W}_i$ then based on Lemma 10 part (ii) $l_i = \lambda_i(\omega_i)$ regardless of ω_{-i}. So conditional on firm $-i$ choosing $\omega_{-i} < \omega_i$, firm $-i$ maximizes its profits using any ESG policy that leads to $r_{-i}(\lambda_i(\omega_i);0)$. Recall

$$\omega_i > \hat{W}_i \Rightarrow \lambda_i(\omega_i) + r_{-i}(\lambda_i(\omega_i);0) < W^{-1}(\omega_i).$$

Therefore, there exists a unique $\hat{\omega} \in \left(W^B, \omega_i\right)$ such that

$$\lambda_i(\omega_i) + r_{-i}(\lambda_i(\omega_i);0) = W^{-1}(\hat{\omega}).$$

If $\omega_{-i} \in [0, \hat{\omega}]$ then

$$\lambda_i(\omega_i) + r_{-i}(\lambda_i(\omega_i);0) \geq W^{-1}(\omega_{-i})$$

and according to Lemma 10, $l_{-i} = r_{-i}(\lambda_i(\omega_i);0)$ and the wage is $W(\lambda_i(\omega_i) + r_{-i}(\lambda_i(\omega_i);0))$. Firm $-i$'s profit is invariant to ω_{-i} and given by

$$\pi_{-i} = f_{-i}(r_{-i}(\lambda_i(\omega_i);0)) - r_{-i}(\lambda_i(\omega_i);0)\, W(r_{-i}(\lambda_i(\omega_i);0) + \lambda_i(\omega_i)).$$

If $\omega_{-i} \in (\hat{\omega}, \omega_i)$ then $\lambda_i(\omega_i) + r_{-i}(\lambda_i(\omega_i);0) < W^{-1}(\omega_{-i})$ and Lemma 10 would imply $l_{-i} = W^{-1}(\omega_{-i}) - \lambda_i(\omega_i) > r_{-i}(\lambda_i(\omega_i);0)$ which is suboptimal.

Overall, the weakly optimal downward reaction is $\omega_{-i} = 0$. Recall

$$\Lambda_i(\omega_i) < \lambda_i(\omega_i) \Leftrightarrow \omega_i < \hat{W}_i$$

and that $\Lambda_i(\omega_i)$ increases in ω_i while $\lambda_i(\omega_i)$ decreases in ω_i. Therefore, if $\omega_{-i} = 0$ then $l_i(\omega_i) = L_i(\omega_i) \equiv \min\{\Lambda_i(\omega_i), \lambda_i(\omega_i)\}$, and firm $-i$'s profit under the optimal downward reaction is

$$\pi_{-i}^{down}(\omega_i) = f_{-i}(r_{-i}(L_i(\omega_i);0)) - r_{-i}(L_i(\omega_i);0)\, W(r_{-i}(L_i(\omega_i);0) + L_i(\omega_i)).$$

Let

$$f_{-i}(r_{-i}(l_i;0)) - r_{-i}(l_i;0)\, W(r_{-i}(l_i;0) + l_i).$$

The derivative with respect to l_i is

$$\begin{aligned}
& r'_{-i}(l_i;0)\, f'_{-i}(r_{-i}(l_i;0)) - r'_{-i}(l_i;0)\, W(r_{-i}(l_i;0) + l_i) - r_{-i}(l_i;0)\, W'(r_{-i}(l_i;0) + l_i)\left(r'_{-i}(l_i;0) + \right. \\
= \; & r'_{-i}(l_i;0)\left[f'_{-i}(r_{-i}(l_i;0)) - W(r_{-i}(l_i;0) + l_i) - r_{-i}(l_i;0)\, W'(r_{-i}(l_i;0) + l_i)\right] - r_{-i}(l_i;0)\, W'(r_{-i}(\\
= \; & -r_{-i}(l_i;0)\, W'(r_{-i}(l_i;0) + l_i) < 0
\end{aligned}$$

where the second inequality follows from the observation that $r_{-i}(l_i;0)$ satisfies the FOC of firm $-i$. Therefore, $\pi_{-i}^{down}(\omega_i)$ increases in ω_i if and only if $\omega_i > \hat{W}_i$. The minimum is obtained when $\omega_i = \hat{W}_i$.

2. Consider an upward reaction: $\omega_{-i} > \omega_i$. There are two subcases to consider:

(a) If $\omega_{-i} > \max\left\{\omega_i, \hat{W}_{-i}\right\}$ then based on Lemma 10 part (ii) $l_{-i} = \lambda_{-i}(\omega_{-i})$ and $W^*_{-i} = \omega_{-i}$ regardless of ω_i. Then

$$\pi_{-i} = f_{-i}(\lambda_{-i}(\omega_{-i})) - \lambda_{-i}(\omega_{-i})\,\omega_{-i}$$

and

$$\begin{aligned}\frac{\partial \pi_{-i}}{\partial \omega_{-i}} &= \lambda'_{-i}(\omega_{-i})\,f'_{-i}(\lambda_{-i}(\omega_{-i})) - \lambda_{-i}(\omega_{-i}) - \lambda'_{-i}(\omega_{-i})\,\omega_{-i} \\ &= -\lambda_{-i}(\omega_{-i}) < 0.\end{aligned}$$

Therefore, $\omega_{-i} > \max\left\{\omega_i, \hat{W}_{-i}\right\}$ is suboptimal.

(b) If $\omega_{-i} \in (\omega_i, \hat{W}_{-i}]$ then based on Lemma 9

$$\begin{aligned}l_{-i} &= \Lambda_{-i}(\omega_{-i}) \\ l_i &= W^{-1}(\omega_{-i}) - \Lambda_{-i}(\omega_{-i}) = r_i(\Lambda_{-i}(\omega_{-i}), 0)\end{aligned}$$

and the wage is ω_{-i}. Since $\varphi^*_i < \hat{W}_{-i}$, from the optimality of φ^*_i when $l_i = r_i(l_{-i}, 0)$, the best upward deviation in this case is φ^*_i if $\omega_i < \varphi^*_i$, and $\omega_i + \varepsilon$ otherwise.

Overall, the optimal upward reaction

$$\omega^{up}_{-i} = \begin{cases} \varphi^*_{-i} & \text{if } \omega_i < \varphi^*_{-i} \\ \omega_i + \varepsilon & \text{if } \omega_i \geq \varphi^*_{-i} \end{cases}$$

Let $L_{-i}(\omega_{-i}) \equiv \min\{\Lambda_{-i}(\omega_{-i}), \lambda_{-i}(\omega_{-i})\}$, then firm $-i$'s profit under the optimal upward reaction is

$$\pi^{up}_{-i}(\omega_i) = f_{-i}\left(L_{-i}\left(\max\left\{\omega_i, \varphi^*_{-i}\right\}\right)\right) - L_{-i}\left(\max\left\{\omega_i, \varphi^*_{-i}\right\}\right)\max\left\{\omega_i, \varphi^*_{-i}\right\}.$$

Thus,if $\omega_i < \varphi^*_{-i}$ then $\pi^{up}_{-i}(\omega_i)$ is invariant to ω_i, and if $\varphi^*_{-i} < \omega_i$ then $\pi^{up}_{-i}(\omega_i)$ decreases in ω_i. .

Notice $\pi^{down}_{-i}(0) < \pi^{up}_{-i}(0)$ since in the latter firm $-i$ chooses it's optimal ESG policy φ^*_{-i} when $\omega_i = 0$. Since $\pi^{down}_{-i}(0)$ obtains its minimum at $\hat{W}_i$, if $\omega_i \leq \min\left\{\varphi^*_{-i}, \hat{W}_i\right\}$ then $\pi^{down}_{-i}(\omega_i) < \pi^{up}_{-i}(\omega_i)$ and the optimal reaction is upward: $\varphi^{SH}_{-i}(\omega_i) = \varphi^*_{-i}$.

If $\omega_i \geq \max\left\{\varphi^*_{-i}, \hat{W}_i\right\}$ then $\pi^{down}_{-i}(\omega_i)$ is increasing and $\pi^{up}_{-i}(\omega_i)$ is decreasing.

Next we argue that if $\omega_i \geq W^{**}$ then $\pi^{down}_{-i}(\omega_i) > \pi^{up}_{-i}(\omega_i)$. Based on Lemma 8 $l_i = \lambda_i(\omega_i)$ and

$$\begin{aligned}\pi^{up}_{-i}(\omega_i) &= f_{-i}(\lambda_{-i}(\omega_i)) - \lambda_{-i}(\omega_i)\,\omega_i \\ \pi^{down}_{-i}(\omega_i) &= f_{-i}(r_{-i}(\lambda_i(\omega_i); 0)) - r_{-i}(\lambda_i(\omega_i); 0)\,W(r_{-i}(\lambda_i(\omega_i); 0) + \lambda_i(\omega_i))\end{aligned}$$

Notice $r_{-i}(\lambda_i(\omega_i);0) = \arg\max_l f_{-i}(l) - lW(l+\lambda_i(\omega_i))$ and

$$\omega_i \geq W^{**} = W(\lambda_{-i}(W^{**}) + \lambda_i(W^{**})) \geq W(\lambda_{-i}(\omega_i) + \lambda_i(\omega_i)).$$

Therefore,

$$\begin{aligned}
\pi_{-i}^{up}(W^{**}) &= f_{-i}(\lambda_{-i}(\omega_i)) - \lambda_{-i}(\omega_i)\omega_i \\
&< f_{-i}(\lambda_{-i}(\omega_i)) - \lambda_{-i}(\omega_i) W(\lambda_{-i}(\omega_i) + \lambda_i(\omega_i)) \\
&< \max_l f_{-i}(l) - lW(l+\lambda_i(\omega_i)) \\
&= \pi_{-i}^{down}(\omega_i)
\end{aligned}$$

as required.

Next we argue that if $W^B < \omega_i \leq \min\left\{\hat{W}_i, \hat{W}_{-i}\right\}$ then $\pi_{-i}^{up}(\omega_i) \geq \pi_{-i}^{down}(\omega_i)$. Indeed if $\omega_{-i} = W^B$ then the allocation is $(l_i, l_{-i}) = (\Lambda_i(\omega_i), r_{-i}(\Lambda_i(\omega_i);0))$ and the wage is ω_i, and if $\omega_{-i} = \omega_i + \varepsilon$ then the allocation is (arbitrarily close to) $(l_i, l_{-i}) = (r_i(\Lambda_{-i}(\omega_i);0), \Lambda_{-i}(\omega_i))$ and the wage is ω_i. Since $W^B < \omega_i$ we have

$$\Lambda_{-i}(\omega_i) > \Lambda_{-i}\left(W^B\right) = l_{-i}^B = r_{-i}\left(l_i^B;0\right) = r_{-i}\left(\Lambda_i\left(W^B\right);0\right) > r_{-i}(\Lambda_i(\omega_i);0)$$

Therefore, $\omega_{-i} = \omega_i + \varepsilon$ is a superior reaction, that is, $\pi_{-i}^{up}(\omega_i) \geq \pi_{-i}^{down}(\omega_i)$.

Finally, there are two cases:

1. Suppose $\hat{W}_{-i} < \hat{W}_i$. If $\omega_i \leq \hat{W}_{-i}$ then $\pi_{-i}^{up}(\omega_i) \geq \pi_{-i}^{down}(\omega_i)$. If $\omega_i \in \left(\hat{W}_{-i}, \hat{W}_i\right)$ then

$$\begin{aligned}
\pi_{-i}^{up}(\omega_i) &= f_{-i}(\lambda_{-i}(\omega_i)) - \lambda_{-i}(\omega_i)\omega_i \\
\pi_{-i}^{down}(\omega_i) &= f_{-i}(r_{-i}(\Lambda_i(\omega_i);0)) - r_{-i}(\Lambda_i(\omega_i);0) W(r_{-i}(\Lambda_i(\omega_i);0) + \Lambda_i(\omega_i)) \\
&= f_{-i}(r_{-i}(\Lambda_i(\omega_i);0)) - r_{-i}(\Lambda_i(\omega_i);0)\omega_i
\end{aligned}$$

   Notice $\lambda_{-i}(\omega_i) = \arg\max_l f_{-i}(l) - l\omega_i$. Therefore, $\pi_{-i}^{up}(\omega_i) \geq \pi_{-i}^{down}(\omega_i)$ in this region as well. If $\omega_i \geq \hat{W}_i$ then $\pi_{-i}^{down}(\omega_i)$ increases in ω_i and $\pi_{-i}^{up}(\omega_i)$ decrease ω_i. Therefore, there is a unique $\check{W}_{-i} \in \left(\hat{W}_i, W^{**}\right)$ such that $\pi_{-i}^{up}(\omega_i) \geq \pi_{-i}^{down}(\omega_i) \Leftrightarrow \omega_i < \check{W}_{-i}$. Since $\varphi_{-i}^* < \hat{W}_{-i} < \hat{W}_i$, we have

$$\varphi_{-i}^{SH}(\omega_i) = \begin{cases} \varphi_{-i}^* & \text{if } \omega_i < \varphi_{-i}^* \\ \omega_i + \varepsilon & \text{if } \omega_i \in [\varphi_{-i}^*, \check{W}_{-i}) \\ 0 & \text{if } \omega_i \geq \check{W}_{-i}. \end{cases}$$

2. Suppose $\hat{W}_i \leq \hat{W}_{-i}$. If $\omega_i \leq \hat{W}_i$ then $\pi_{-i}^{up}(\omega_i) \geq \pi_{-i}^{down}(\omega_i)$. If $\omega_i > \hat{W}_i$ then $\pi_{-i}^{down}(\omega_i)$ increases in ω_i and $\pi_{-i}^{up}(\omega_i)$ (weakly) decreases in ω_i. Therefore, there is a unique $\check{W}_{-i} \in$

$\left(\hat{W}_i, W^{**}\right)$ such that $\pi_{-i}^{up}(\omega_i) \geq \pi_{-i}^{down}(\omega_i) \Leftrightarrow \omega_i < \check{W}_{-i}$. In this case

$$\varphi_{-i}^{SH}(\omega_i) = \begin{cases} \varphi_{-i}^* & \text{if } \omega_i < \min\{\varphi_{-i}^*, \check{W}_{-i}\} \\ \omega_i + \varepsilon & \text{if } \omega_i \in [\min\{\varphi_{-i}^*, \check{W}_{-i}\}, \check{W}_{-i}) \\ 0 & \text{if } \omega_i \geq \check{W}_{-i}. \end{cases}$$

■

Proof of Proposition 3. Based on the proof of Lemma 4 there is a unique $\check{W}_{-i} \in \left(\hat{W}_i, W^{**}\right)$ such that for $\varepsilon > 0$ sufficiently small, firm $-i's$ profits under ESG profile $(\omega_i, \omega_{-i}) = (\omega, \omega + \varepsilon)$ are strictly higher than under $(\omega_i, \omega_{-i}) = (\omega, 0)$ if and only if $\omega < \check{W}_{-i}$. In words, by choosing $\omega_i = \check{W}_{-i}$, the leader firm i induces the follower firm $-i$ to respond with $\omega_{-i} = 0$.

Suppose firms are symmetric, then $\check{W}_{-i} = \check{W}_i$ and $\hat{W}_i = \hat{W}_{-i}$. From this point on, we omit subscripts i and $-i$ from these cutoffs and whenever appropriate.[17]

Notice that $\check{W}$ solves

$$f(r(\lambda(\omega), 0)) - r(\lambda(\omega), 0) W(r(\lambda(\omega), 0) + \lambda(\omega)) = f(\lambda(\omega)) - \lambda(\omega)\omega. \tag{36}$$

There are a few cases to consider:

1. If $\omega_i \leq \varphi^*$ then based on Lemma 4, $\omega_{-i} = \varphi^*$, $l_{-i} = \Lambda(\varphi^*)$ and $\pi_i(\omega_i) = \max_l f(l) - lW(l + \Lambda(\varphi^*))$.

2. If $\omega_i \in \left[\varphi^*, \hat{W}\right]$ then based on Lemma 4 $\omega_{-i} = \omega_i + \varepsilon$, and based on Lemma 9, $l_{-i} = \Lambda(\omega_i)$ and

$$\begin{aligned} \pi_i(\omega_i) &= \max_l f(l) - lW(l + \Lambda(\omega_i)) \\ &= f(r(\Lambda(\omega_i), 0)) - r(\Lambda(\omega_i), 0) W(r(\Lambda(\omega_i), 0) + \Lambda(\omega_i)) \\ &= f(r(\Lambda(\omega_i), 0)) - r(\Lambda(\omega_i), 0)\omega_i. \end{aligned}$$

   Since $\omega_i \in \left[\varphi^*, \hat{W}\right] \Rightarrow \Lambda'(\omega_i) > 0$, we have $\pi_i'(\omega_i) < 0$ in this range.

3. If $\omega_i \in (\hat{W}, \check{W}]$ then based on Lemma 4, $\omega_{-i} = \omega_i + \varepsilon$, and based on Lemma 10 $l_{-i} = \lambda(\omega_i)$ and $l_i = W^{-1}(\omega_i) - \lambda(\omega_i)$. Therefore,

$$\begin{aligned} \pi_i(\omega_i) &= f\left(W^{-1}(\omega_i) - \lambda(\omega_i)\right) - \left(W^{-1}(\omega_i) - \lambda(\omega_i)\right) W\left(W^{-1}(\omega_i) - \lambda(\omega_i) + \lambda(\omega_i)\right) \\ &\leq \max_l f(l) - lW(l + \lambda(\omega_i)) \\ &= f(r(\lambda(\omega_i), 0)) - r(\lambda(\omega_i), 0) W(r(\lambda(\omega_i), 0) + \lambda(\omega_i)). \end{aligned}$$

   Notice $\max_l f(l) - lW(l + \lambda(\omega_i))$ increases in ω_i (since $\lambda'(\omega_i) < 0$). Moreover, if $\omega_i = \check{W}$

[17] To avoid open-set issues, we assume that firms choose policies from a large finite set, which includes $W^B, \varphi^*, \hat{W}, \check{W}, W^{**}$. The set of feasible choices is fine grid.

then

$$\pi_i(\check{W}) = f(\lambda(\check{W})) - \lambda(\omega_i)\check{W}$$

and by definition of $\check{W}$ we have

$$\pi_i(\check{W}) = f\left(r\left(\lambda\left(\check{W}\right),0\right)\right) - r\left(\lambda\left(\check{W}\right),0\right)W\left(r\left(\lambda\left(\check{W}\right),0\right) + \lambda\left(\check{W}\right)\right).$$

Therefore, $\check{W}$ obtains the maximum of $\pi_i(\omega_i)$ over the interval $[\hat{W},\check{W}]$.

4. If $\omega_i \geq \check{W}$ then based on Lemma 4 $\omega_{-i} = 0$, and based on Lemma 10, $l_{-i} = \lambda(\omega_i)$ and $\pi_i(\omega_i) = f(\lambda(\omega_i)) - \lambda(\omega_i)\omega_i$. Notice

$$\begin{aligned}
\pi_i'(\omega_i) &= \lambda'(\omega_i) f'(\lambda(\omega_i)) - \lambda'(\omega_i)\omega_i - \lambda(\omega_i) \\
&= \lambda'(\omega_i)\omega_i - \lambda'(\omega_i)\omega_i - \lambda(\omega_i) \\
&= -\lambda(\omega_i) < 0.
\end{aligned}$$

Therefore, it's suboptimal to choose $\omega_i > \check{W}$.

Overall, we showed that the optimal ω_i is either φ^* or $\check{W}$. In the former case, the profit of firm i is

$$\max_l f(l) - lW(l + \Lambda(\varphi^*))$$

and in the later case it is

$$\max_l f(l) - lW\left(l + \lambda\left(\check{W}\right)\right).$$

Therefore, $\varphi^* \in \left(W^B,\hat{W}\right)$ is optimal if $\Lambda(\varphi^*) \leq \lambda\left(\check{W}\right)$, and $\check{W} \in \left(\hat{W},W^{**}\right)$ is optimal otherwise.

Finally, notice that if $\omega_i = \check{W}$ then $l_i = \lambda(\check{W})$. Since $\check{W} < W^{**}$, $\lambda(\check{W}) > \lambda(W^{**}) = l^{**} > l^B$. Therefore $\lambda(\check{W}) + r(\lambda(\check{W}),0) > l^B + r(l^B,0)$, and employment in equilibrium is higher than in the No-ESG benchmark. ■

A.5 Proofs for Section 6.1

Proof of Proposition 4. Firm i's surplus is

$$f_i(l_i) - \mu\int_0^{l_i} W(l)\,dl - (1-\mu)\int_{r_{-i}(l_i;0)}^{l_i + r_{-i}(l_i;0)} W(l)\,dl, \tag{37}$$

The derivative of (37) with respect to l_i is

$$\begin{aligned}
& f_i'(l_i) - \mu W(l_i) - (1-\mu)\begin{bmatrix} \left(1 + r_{-i}'(l_i;0)\right)W(l_i + r_{-i}(l_i;0)) \\ -r_{-i}'(l_i;0)\,W(r_{-i}(l_i;0)) \end{bmatrix} \\
= \; & f_i'(l_i) - \mu W(l_i) - (1-\mu)W(l_i + r_{-i}(l_i;0)) \\
& -(1-\mu)\,r_{-i}'(l_i;0)\left[W(l_i + r_{-i}(l_i;0)) - W(r_{-i}(l_i;0))\right] \\
> \; & f_i'(l_i) - W(l_i + r_{-i}(l_i;0)),
\end{aligned} \tag{38}$$

where the inequality follows because $r'_{-i}(l_i;0) < 0$.

First, consider $\omega_i \in [W^B, \hat{W}_i)$. Increasing ω_i corresponds to increasing l_i. In this case, $l_i < \lambda_i(\omega_i)$, or equivalently, $f'_i(l_i) > \omega_i$; and $\omega_i = W(l_i + r_{-i}(l_i;0))$. Hence (38) is strictly positive. It follows that $\omega_i = \hat{W}_i$ delivers higher firm surplus than any choice in $[W^B, \hat{W}_i)$.

Second, consider $\omega_i > \hat{W}_i$. Decreasing ω_i corresponds to increasing l_i. In this case, $l_i = \lambda_i(\omega_i)$, or equivalently, $f'_i(l_i) = \omega_i$; and $\omega_i > W(l_i + r_{-i}(l_i;0))$. Hence (38) is strictly positive. It follows that $\omega_i = \hat{W}_i$ delivers higher firm surplus than any choice in $\omega_i > \hat{W}_i$.

As in the proof of Proposition 2, firm $i's$ employment, total employment, wages, and workers' surplus, are all higher in equilibrium relative to the No-ESG benchmark. Moreover, firm's $-i's$ employment and profitability are lower, and if $i = 1$ then total profitability is also lower. ■

Proof of Corollary 2. Industry surplus is

$$f_i(l_i) + f_{-i}(r_{-i}(l_i;0)) - \int_0^{l_i + r_{-i}(l_i;0)} W(l)\, dl, \tag{39}$$

The derivative of (39) with respect to l_i is

$$\begin{aligned} & f'_i(l_i) - W(l_i + r_{-i}(l_i;0)) + r'_{-i}(l_i;0)\left[f'_{-i}(r_{-i}(l_i;0)) - W(l_i + r_{-i}(l_i;0))\right] \\ < \quad & f'_i(l_i) - W(l_i + r_{-i}(l_i;0)). \end{aligned}$$

where the inequality follows from the monopsony distortion in non-ESG firm's hiring decisions, $f'_{-i}(r_{-i}(l_i;0)) > W(l_i + r_{-i}(l_i;0))$, along with the fact that $r'_{-i}(l_i;0) < 0$.

From Proposition 4, the ESG policy that maximizes firm i's surplus is $\hat{W}_i$, and the associated employment level is such that $f'_i(l_i) = \hat{W}_i = W(l_i + r_{-i}(l_i;0))$. Hence the derivative of (39) with respect to l_i is strictly negative at this point, implying that the ESG policy that maximizes industry surplus must induce strictly lower employment at firm i. (No ESG policy can induce strictly more employment.) ■

A.6 Proofs for Section 6.2

Lemma 11 *If $\omega_j = \omega_k \in \left(W^B, W^{**}\right)$ then at least one firm can profitably deviate to some $\omega > \omega_k = \omega_j$.*

Proof. Suppose $\omega_j = \omega_k = \omega \in \left(W^B, W^{**}\right)$. Based on Proposition 1 part (iii), for any $i = j, k$ and

$$l^* \in \left[W^{-1}(\omega) - \min\left\{\Lambda_{-i}(\omega), \lambda_{-i}(\omega)\right\}, \min\left\{\Lambda_i(\omega), \lambda_i(\omega)\right\}\right] \tag{40}$$

there is an equilibrium in which $\left(l_j^*, l_k^*\right) = \left(l^*, W^{-1}(\omega) - l^*\right)$ and $W_j^* = W_k^* = \omega$. For all members of the equilibrium set, the equilibrium wage is ω. Because both firms $i = k, j$ hire strictly less than $\lambda_i(\omega)$, at any equilibrium in the interior of the equilibrium set, firm i's profits and own surplus are strictly increasing in l_i. Take any equilibrium (l_k, l_j). At least one firm i has $l_i < \min\{\Lambda_i(\omega), \lambda_i(\omega)\}$. By choosing $\omega_i \in (\omega, W^{**})$ this firm i ensures the labor market equilibrium has $l_i = \min\{\Lambda_i(\omega), \lambda_i(\omega)\}$, and that it pays ω. By choosing ω_i sufficiently close to ω, firm i can achieve profits arbitrarily close to that which it would receive from hiring $l_i = \min\{\Lambda_i(\omega), \lambda_i(\omega)\}$ and paying ω, which in turn strictly exceed its equilibrium profits. ■

Proof of Lemma 5. The surplus of firm $-i$ is given by

$$S_{-i} = f_{-i}(l_{-i}) - \mu \int_0^{l_{-i}} W(l)\,dl - (1-\mu)\int_{l_i}^{l_i+l_{-i}} W(l)\,dl. \tag{41}$$

We divide the proof to three cases:

1. Suppose $\omega_i \in (W^B, \hat{W}_{-i}]$.

   (a) If $\omega_{-i} < \omega_i$ there are two cases:

      i. If $\omega_i \leq \hat{W}_i$ then based on Lemma 9, $l_i = \Lambda_i(\omega_i)$, $l_{-i} = W^{-1}(\omega_i) - \Lambda_i(\omega_i)$, and the wage is ω_i. Then, $\frac{\partial S_{-i}}{\partial \omega_{-i}} = 0$.

      ii. If $\omega_i > \hat{W}_i$ then based on Lemma 10 part (ii) $l_i^* = \lambda_i(\omega_i)$. There are two subcases. First, if $W^{-1}(\omega_{-i}) - \lambda_i(\omega_i) < r_{-i}(\lambda_i(\omega_i); 0)$ then $l_{-i}^* = r_{-i}(\lambda_i(\omega_i); 0)$ and $W_{-i}^* = W(\lambda_i(\omega_i) + r_{-i}(\lambda_i(\omega_i); 0))$. Then, firm $-i$ surplus is invariant to ω_{-i}. Second, if $W^{-1}(\omega_{-i}) - \lambda_i(\omega_i) \geq r_{-i}(\lambda_i(\omega_i); 0)$ then $l_{-i}^* = W^{-1}(\omega_{-i}) - \lambda_i(\omega_i)$ and $W_{-i}^* = \omega_{-i}$. In this case,

      $$S_{-i} = f_{-i}\left(W^{-1}(\omega_{-i}) - \lambda_i(\omega_i)\right) - \mu \int_0^{W^{-1}(\omega_{-i})-\lambda_i(\omega_i)} W(l)\,dl - (1-\mu)\int_{\lambda_i(\omega_i)}^{W^{-1}(\omega_{-i})} W(l)\,dl$$

      and

      $$\begin{aligned}\frac{\partial S_{-i}}{\partial \omega_{-i}} &= \left(W^{-1}(\omega_{-i})\right)' f'_{-i}\left(W^{-1}(\omega_{-i}) - \lambda_i(\omega_i)\right) \\ &\quad -\mu\left(W^{-1}(\omega_{-i})\right)' W\left(W^{-1}(\omega_{-i}) - \lambda_i(\omega_i)\right) \\ &= \left(W^{-1}(\omega_{-i})\right)' \begin{bmatrix} f'_{-i}(W^{-1}(\omega_{-i}) - \lambda_i(\omega_i)) \\ -\mu W(W^{-1}(\omega_{-i}) - \lambda_i(\omega_i)) \end{bmatrix} \\ &> \left(W^{-1}(\omega_{-i})\right)' \begin{bmatrix} f'_{-i}(W^{-1}(\omega_{-i}) - \lambda_i(\omega_i)) \\ -W(W^{-1}(\omega_{-i}) - \lambda_i(\omega_i)) \end{bmatrix} \\ &> 0.\end{aligned}$$

      The inequality follows from the fact that $(W^{-1}(\omega_{-i}))' > 0$ and $f'_{-i}\left(l_{-i}^*\right) \geq W_{-i}^*$ implies

      $$f'_{-i}\left(W^{-1}(\omega_{-i}) - \lambda_i(\omega_i)\right) \geq \omega_{-i} > W\left(W^{-1}(\omega_{-i}) - \lambda_i(\omega_i)\right).$$

      Therefore, $\frac{\partial S_{-i}}{\partial \omega_{-i}} > 0$.

(b) If $\omega_{-i} = \omega_i$ then as we show in Lemma 11 below, firm $-i$ is weakly better off by choosing $\omega_{-i} = \omega_i + \varepsilon$, where $\varepsilon > 0$ is arbitrarily small.

(c) If $\omega_{-i} \in (\omega_i, \hat{W}_{-i}]$ then based on Lemma 9, $l_{-i} = \Lambda_{-i}(\omega_{-i})$, $l_i = W^{-1}(\omega_{-i}) - \Lambda(\omega_{-i})$, and the wage is ω_{-i}. The surplus of firm $-i$ is

$$S_{-i} = f_{-i}(\Lambda_{-i}(\omega_{-i})) - \mu \int_0^{\Lambda_{-i}(\omega_{-i})} W(l)\, dl - (1-\mu) \int_{W^{-1}(\omega_{-i}) - \Lambda_{-i}(\omega_{-i})}^{W^{-1}(\omega_{-i})} W(l)\, dl. \tag{42}$$

Observe

$$\begin{aligned}
\frac{\partial S_{-i}}{\partial \omega_{-i}} &= \Lambda'_{-i}(\omega_{-i}) f'_{-i}(\Lambda_{-i}(\omega_{-i})) - \mu \Lambda'_{-i}(\omega_{-i}) W(\Lambda_{-i}(\omega_{-i})) \\
&\quad - (1-\mu) \left[\begin{array}{c} (W^{-1})'(\omega_{-i})\,\omega_{-i} \\ - \left((W^{-1})'(\omega_{-i}) - \Lambda'_{-i}(\omega_{-i})\right) W\left(W^{-1}(\omega_{-i}) - \Lambda_{-i}(\omega_{-i})\right) \end{array} \right] \\
&= \Lambda'_{-i}(\omega_{-i}) \left[f'_{-i}(\Lambda_{-i}(\omega_{-i})) - \mu W(\Lambda_{-i}(\omega_{-i})) - (1-\mu) W\left(W^{-1}(\omega_{-i}) - \Lambda_{-i}(\omega_{-i})\right) \right] \\
&\quad - (1-\mu) \left(W^{-1}\right)'(\omega_{-i}) \left[\omega_{-i} - W\left(W^{-1}(\omega_{-i}) - \Lambda_{-i}(\omega_{-i})\right) \right]
\end{aligned}$$

By definition

$$\Lambda_{-i}(\omega_{-i}) + r_i(\Lambda_{-i}(\omega_{-i}); 0) = W^{-1}(\omega_{-i}),$$

so that

$$\Lambda'_{-i}(\omega_{-i}) \left(\frac{\partial}{\partial \tilde{l}} \left(\tilde{l} + r_i\left(\tilde{l}; 0\right)\right) \Big|_{\tilde{l} = \Lambda_{-i}(\omega_{-i})} \right) = \left(W^{-1}\right)'(\omega_{-i}),$$

implying (since $\frac{\partial\left(\tilde{l} + r_i\left(\tilde{l}; 0\right)\right)}{\partial \tilde{l}} \in (0,1)$)

$$\Lambda'_{-i}(\omega_{-i}) > \left(W^{-1}\right)'(\omega_{-i}). \tag{43}$$

Since $\omega_{-i} - W\left(W^{-1}(\omega_{-i}) - \Lambda_{-i}(\omega_{-i})\right) > 0$, combined, we have:

$$\begin{aligned}
\frac{\partial S_{-i}}{\partial \omega_{-i}} &> \Lambda'_{-i}(\omega_{-i}) \left[f'_{-i}(\Lambda_{-i}(\omega_{-i})) - \mu W(\Lambda_{-i}(\omega_{-i})) - (1-\mu) W\left(W^{-1}(\omega_{-i}) - \Lambda_{-i}(\omega_{-i})\right) \right] \\
&\quad - (1-\mu) \Lambda'_{-i}(\omega_{-i}) \left[\omega_{-i} - W\left(W^{-1}(\omega_{-i}) - \Lambda_{-i}(\omega_{-i})\right) \right] \\
&= \Lambda'_{-i}(\omega_{-i}) \left[f'_{-i}(\Lambda_{-i}(\omega_{-i})) - \mu W(\Lambda_{-i}(\omega_{-i})) - (1-\mu)\,\omega_{-i} \right]
\end{aligned}$$

Since $W(\Lambda_{-i}(\omega_{-i})) < W(\Lambda_{-i}(\omega_{-i}) + l_i) = \omega_{-i}$ and $f'_{-i}(\Lambda_{-i}(\omega_{-i})) > \omega_{-i}$, we have $\frac{\partial S_{-i}}{\partial \omega_{-i}} > 0$.

(d) If $\omega_{-i} \in (\hat{W}_{-i}, W^{**}]$ then, based on Lemma 10, $l_{-i} = \lambda_{-i}(\omega_{-i})$ and firm $-i$ pays ω_{-i}. There are two subcases:

   i. If $W^{-1}(\omega_i) - \lambda_{-i}(\omega_{-i}) \geq r_i(\lambda_{-i}(\omega_{-i}); 0)$ then $l_i = W^{-1}(\omega_i) - \lambda_{-i}(\omega_{-i})$ and firm i pays ω_i. The surplus of firm $-i$ is

$$S_{-i} = f_{-i}\left(\lambda_{-i}\left(\omega_{-i}\right)\right) - \mu \int_0^{\lambda_{-i}(\omega_{-i})} W\left(l\right) dl - \left(1-\mu\right) \int_{W^{-1}(\omega_i)-\lambda_{-i}(\omega_{-i})}^{W^{-1}(\omega_i)} W\left(l\right) dl,$$

and

$$\begin{aligned} \frac{\partial S_{-i}}{\partial \omega_{-i}} &= \lambda_{-i}'\left(\omega_{-i}\right) f_{-i}'\left(\lambda_{-i}\left(\omega_{-i}\right)\right) - \mu \lambda_{-i}'\left(\omega_{-i}\right) W\left(\lambda_{-i}\left(\omega_{-i}\right)\right) \\ &\quad - \left(1-\mu\right) \lambda_{-i}'\left(\omega_{-i}\right) W\left(W^{-1}\left(\omega_i\right) - \lambda_{-i}\left(\omega_{-i}\right)\right) \\ &= \lambda_{-i}'\left(\omega_{-i}\right)\left[f_{-i}'\left(\lambda_{-i}\left(\omega_{-i}\right)\right) - \mu W\left(\lambda_{-i}\left(\omega_{-i}\right)\right) - \left(1-\mu\right) W\left(W^{-1}\left(\omega_i\right) - \lambda_{-i}\left(\omega_{-i}\right)\right)\right] \end{aligned}$$

which is negative given that $\lambda_{-i}'\left(\omega_{-i}\right) < 0$ and

$$\begin{aligned} f_{-i}'\left(\lambda_{-i}\left(\omega_{-i}\right)\right) &> \omega_{-i} > \omega_i \\ &> \mu W\left(\lambda_{-i}\left(\omega_{-i}\right)\right) + \left(1-\mu\right) W\left(W^{-1}\left(\omega_i\right) - \lambda_{-i}\left(\omega_{-i}\right)\right). \end{aligned}$$

Thus, in this range, $\frac{\partial S_{-i}}{\partial \omega_{-i}} < 0$.

ii. If $W^{-1}\left(\omega_i\right) - \lambda_{-i}\left(\omega_{-i}\right) < r_i\left(\lambda_{-i}\left(\omega_{-i}\right);0\right)$ then $l_i = r_i\left(\lambda_{-i}\left(\omega_{-i}\right);0\right)$ and firm i pays $W\left(\lambda_{-i}\left(\omega_{-i}\right) + r_i\left(\lambda_{-i}\left(\omega_{-i}\right);0\right)\right) > \omega_i$. The surplus of firm $-i$ is

$$S_{-i} = f_{-i}\left(\lambda_{-i}\left(\omega_{-i}\right)\right) - \mu \int_0^{\lambda_{-i}(\omega_{-i})} W\left(l\right) dl - \left(1-\mu\right) \int_{r_i(\lambda_{-i}(\omega_{-i});0)}^{\lambda_{-i}(\omega_{-i})+r_i(\lambda_{-i}(\omega_{-i});0)} W\left(l\right) dl,$$

and

$$\begin{aligned} \frac{\partial S_{-i}}{\partial \omega_{-i}} &= \lambda_{-i}'\left(\omega_{-i}\right) f_{-i}'\left(\lambda_{-i}\left(\omega_{-i}\right)\right) - \mu \lambda_{-i}'\left(\omega_{-i}\right) W\left(\lambda_{-i}\left(\omega_{-i}\right)\right) \\ &\quad - \left(1-\mu\right)\left[\begin{array}{c} \left(\lambda_{-i}\left(\omega_{-i}\right) + r_i\left(\lambda_{-i}\left(\omega_{-i}\right);0\right)\right)' W\left(\lambda_{-i}\left(\omega_{-i}\right) + r_i\left(\lambda_{-i}\left(\omega_{-i}\right);0\right)\right) \\ - \left(r_i\left(\lambda_{-i}\left(\omega_{-i}\right);0\right)\right)' W\left(r_i\left(\lambda_{-i}\left(\omega_{-i}\right);0\right)\right) \end{array}\right] \\ &= \lambda_{-i}'\left(\omega_{-i}\right)\left[\begin{array}{c} f_{-i}'\left(\lambda_{-i}\left(\omega_{-i}\right)\right) - \mu W\left(\lambda_{-i}\left(\omega_{-i}\right)\right) \\ -\left(1-\mu\right)\left[\begin{array}{c} \left(1 + r_i'\left(\lambda_{-i}\left(\omega_{-i}\right);0\right)\right) W\left(\lambda_{-i}\left(\omega_{-i}\right) + r_i\left(\lambda_{-i}\left(\omega_{-i}\right);0\right)\right) \\ -r_i'\left(\lambda_{-i}\left(\omega_{-i}\right);0\right) W\left(r_i\left(\lambda_{-i}\left(\omega_{-i}\right);0\right)\right) \end{array}\right] \end{array}\right. \end{aligned}$$

Recall $\omega_{-i} > \hat{W}_{-i}$ implies $\omega_{-i} > W\left(\lambda_{-i}\left(\omega_{-i}\right) + r_i\left(\lambda_{-i}\left(\omega_{-i}\right);0\right)\right)$ and notice

$$\begin{aligned} \left[\begin{array}{c} \left(1 + r_i'\left(\lambda_{-i}\left(\omega_{-i}\right);0\right)\right) W\left(\lambda_{-i}\left(\omega_{-i}\right) + r_i\left(\lambda_{-i}\left(\omega_{-i}\right);0\right)\right) \\ -r_i'\left(\lambda_{-i}\left(\omega_{-i}\right);0\right) W\left(r_i\left(\lambda_{-i}\left(\omega_{-i}\right);0\right)\right) \end{array}\right] &< W\left(\lambda_{-i}\left(\omega_{-i}\right) + r_i\left(\lambda_{-i}\left(\omega_{-i}\right);\right.\right. \\ W\left(\lambda_{-i}\left(\omega_{-i}\right) + r_i\left(\lambda_{-i}\left(\omega_{-i}\right);0\right)\right) &> W\left(r_i\left(\lambda_{-i}\left(\omega_{-i}\right);0\right)\right) \end{aligned}$$

Thus,

$$\mu W\left(r_i\left(\lambda_{-i}\left(\omega_{-i}\right);0\right)\right) + \left(1-\mu\right)\left[\begin{array}{c} \left(1 + r_i'\left(\lambda_{-i}\left(\omega_{-i}\right);0\right)\right) W\left(\lambda_{-i}\left(\omega_{-i}\right) + r_i\left(\lambda_{-i}\left(\omega_{-i}\right);0\right)\right) \\ -r_i'\left(\lambda_{-i}\left(\omega_{-i}\right);0\right) W\left(r_i\left(\lambda_{-i}\left(\omega_{-i}\right);0\right)\right) \end{array}\right] <$$

and since $f'_{-i}(\lambda_{-i}(\omega_{-i})) > \omega_{-i}$, we have $\frac{\partial S_{-i}}{\partial \omega_{-i}} < 0$.

Overall, $\frac{\partial S_{-i}}{\partial \omega_{-i}} > 0$ if and only if $\omega_{-i} < \hat{W}_{-i}$, and hence, the best response of firm $-i$ is $\omega_{-i} = \hat{W}_{-i}$.

2. Suppose $\omega_i \in (\hat{W}_{-i}, W^{**}]$. If $\omega_{-i} > \omega_i$ then, the argument in (1.d) shows that firm $-i$ has incentives to get as close as possible to $\hat{W}_{-i}$ from above (indeed, the conditions in Lemma 10 do not require the firm with the lower ESG policy to be above or below $\hat{W}_{-i}$). If $\omega_{-i} < \omega_i$ then there are two cases:

   (a) If $\omega_i \leq \hat{W}_i$ then as in case 1.a.i we have $\frac{\partial S_{-i}}{\partial \omega_{-i}} = 0$.

   (b) If $\omega_i > \hat{W}_i$ then as in case 1.a.ii we have $\frac{\partial S_{-i}}{\partial \omega_{-i}} > 0$.

   If $\omega_{-i} = \omega_i$ then as we show in Lemma 11 below, firm $-i$ is weakly better off by choosing $\omega_{-i} = \omega_i + \varepsilon$, where $\varepsilon > 0$ is arbitrarily small (in fact, it's a strict benefit expect for one point in the convex equilibrium set). Overall, $\frac{\partial S_{-i}}{\partial \omega_{-i}} \geq 0$ if $\omega_{-i} < \omega_i + \varepsilon$ and $\frac{\partial S_{-i}}{\partial \omega_{-i}} < 0$ otherwise. and hence, the best response of firm $-i$ is $\omega_{-i} = \omega_i + \varepsilon$.

3. Suppose $\omega_i > W^{**}$ then based on Lemma 8 $l_i = \lambda_i(\omega_i)$. Since $\omega_i > W^{**} > \hat{W}_i$, if $\omega_{-i} \leq W^{**}$ then $\omega_{-i} < \omega_i$ and as in case 1.a.ii we have $\frac{\partial S_{-i}}{\partial \omega_{-i}} > 0$. If $\omega_{-i} \geq W^{**}$ then based on Lemma 8 $l_{-i} = \lambda_{-i}(\omega_{-i})$. For a given l_{-i},

$$S_{-i} = f_{-i}(l_{-i}) - \mu \int_0^{l_{-i}} W(l)\, dl - (1-\mu) \int_{\lambda_i(\omega_i)}^{\lambda_i(\omega_i)+l_{-i}} W(l)\, dl$$

   and

$$S'_{-i} = f'_{-i}(l_{-i}) - \mu W(l_{-i}) - (1-\mu) W(\lambda_i(\omega_i) + l_{-i})$$

   and hence

$$\begin{aligned} S'_{-i}(\lambda_{-i}(\omega_{-i})) &= f'_{-i}(\lambda_{-i}(\omega_{-i})) - \mu W(\lambda_{-i}(\omega_{-i})) - (1-\mu) W(\lambda_i(\omega_i) + \lambda_{-i}(\omega_{-i})) \\ &= \omega_{-i} - \mu W(\lambda_{-i}(\omega_{-i})) - (1-\mu) W(\lambda_i(\omega_i) + \lambda_{-i}(\omega_{-i})) \\ &> \omega_{-i} - W(\lambda_i(\omega_i) + \lambda_{-i}(\omega_{-i})). \end{aligned}$$

   Notice $\omega_{-i}, \omega_i \geq W^{**}$ implies

$$W(\lambda_i(\omega_i) + \lambda_{-i}(\omega_{-i})) \leq W(\lambda_i(W^{**}) + \lambda_{-i}(W^{**})) = W^{**} \leq \omega_{-i}$$

   and hence $S'_{-i}(\lambda_{-i}(\omega_{-i})) \geq 0$. Since $\lambda'_{-i} < 0$, the optimal ω_{-i} is W^{**} as required.

■

Proof of Propositoin 5. Notice that from Lemma 5 it is immediate that in a game in which each firms simultaneously chooses ESG policies to maximize their surplus the unique equilibrium is that both firms set $\omega_i = W^{**}$, leading to the first-best outcome.

Consider the sequential game. If $\omega_i \in [W^B, \hat{W}_{-i})$, then based on Lemma 5, firm $-i$ will choose $\omega_{-i} = \hat{W}_{-i}$. Then, based on Lemma 9, $l_{-i} = \Lambda_{-i}\left(\hat{W}_{-i}\right)$, $l_i = W^{-1}(\hat{W}_{-i}) - \Lambda_{-i}\left(\hat{W}_{-i}\right)$, and the wage is $\hat{W}_{-i}$. The surplus of firm i is

$$S_i(\omega_i) = f_i(W^{-1}(\hat{W}_{-i}) - \Lambda_{-i}\left(\hat{W}_{-i}\right)) - \mu \int_0^{W^{-1}(\hat{W}_{-i}) - \Lambda_{-i}\left(\hat{W}_{-i}\right)} W(l)\, dl - (1-\mu) \int_{\Lambda_{-i}\left(\hat{W}_{-i}\right)}^{W^{-1}(\hat{W}_{-i})} W(l)\, dl. \tag{44}$$

which is independent of ω_i.

If $\omega_i \in [\hat{W}_{-i}, W^{**})$, then based on Lemma 5, firm $-i$ will choose $\omega_{-i} = \omega_i + \varepsilon$. We show that $S_i(\omega_i, \omega_i + \varepsilon)$ is increasing in ω_i. Based on Lemma 10, $l_{-i} = \lambda_{-i}(\omega_{-i})$ and firm $-i$ pays ω_{-i}. There are two cases:

1. If $W^{-1}(\omega_i) - \lambda_{-i}(\omega_{-i}) \geq r_i(\lambda_{-i}(\omega_{-i}); 0)$ then $l_i^* = W^{-1}(\omega_i) - \lambda_{-i}(\omega_{-i})$. Firm i's surplus is

$$S_i(\omega_i, \omega_i + \varepsilon) = f_i(W^{-1}(\omega_i) - \lambda_{-i}(\omega_i + \varepsilon)) - \mu \int_0^{W^{-1}(\omega_i) - \lambda_{-i}(\omega_i + \varepsilon)} W(l)\, dl - (1-\mu) \int_{\lambda_{-i}(\omega_i + \varepsilon)}^{W^{-1}(\omega_i)} W(l)\, dl$$

and the derivative with respect to ω_i is

$$\begin{aligned} S_i' &= \left[\left[W^{-1}(\omega_i)\right]' - \lambda_{-i}'(\omega_i + \varepsilon)\right] \begin{bmatrix} f_i'(W^{-1}(\omega_i) - \lambda_{-i}(\omega_i + \varepsilon)) \\ -\mu W(W^{-1}(\omega_i) - \lambda_{-i}(\omega_i + \varepsilon)) \end{bmatrix} \\ &\quad + (1-\mu)\left[\lambda_{-i}'(\omega_i + \varepsilon)\right] W(\lambda_{-i}(\omega_i + \varepsilon)) - (1-\mu)\left[W^{-1}(\omega_i)\right]' \omega_i \\ &= \left[W^{-1}(\omega_i)\right]' \begin{bmatrix} f_i'(W^{-1}(\omega_i) - \lambda_{-i}(\omega_i + \varepsilon)) \\ -\mu W(W^{-1}(\omega_i) - \lambda_{-i}(\omega_i + \varepsilon)) - (1-\mu)\omega_i \end{bmatrix} \\ &\quad - \lambda_{-i}'(\omega_i + \varepsilon) \begin{bmatrix} f_i'(W^{-1}(\omega_i) - \lambda_{-i}(\omega_i + \varepsilon)) \\ -\mu W(W^{-1}(\omega_i) - \lambda_{-i}(\omega_i + \varepsilon)) - (1-\mu) W(\lambda_{-i}(\omega_i + \varepsilon)) \end{bmatrix} \end{aligned}$$

Notice $\lambda_{-i}'(\omega_i + \varepsilon) < 0$. Therefore,

$$\begin{aligned} S_i' &> \left[W^{-1}(\omega_i)\right]' \left[f_i'(W^{-1}(\omega_i) - \lambda_{-i}(\omega_i + \varepsilon)) - \omega_i\right] \\ &\quad - \lambda_{-i}'(\omega_i + \varepsilon)\left[f_i'(W^{-1}(\omega_i) - \lambda_{-i}(\omega_i + \varepsilon)) - \omega_i\right]. \end{aligned}$$

Since $l_i^* = r_i(\lambda_{-i}(\omega_{-i}); \omega_i) \leq \lambda_i(\omega_i)$, we have $f_i'(l_i^*) \geq \omega_i$ and the RHS is positive. Therefore, $S_i' > 0$.

2. If $W^{-1}(\omega_i) - \lambda_{-i}(\omega_{-i}) < r_i(\lambda_{-i}(\omega_{-i}); 0)$ then $l_i^* = r_i(\lambda_{-i}(\omega_{-i}); 0)$. Firm i's surplus is

$$S_i(\omega_i, \omega_i + \varepsilon) = f_i(r_i(\lambda_{-i}(\omega_i + \varepsilon); 0)) - \mu \int_0^{r_i(\lambda_{-i}(\omega_i + \varepsilon); 0)} W(l)\, dl - (1-\mu) \int_{\lambda_{-i}(\omega_i + \varepsilon)}^{r_i(\lambda_{-i}(\omega_i + \varepsilon); 0) + \lambda_{-i}(\omega_i + \varepsilon)} W(l$$

and the derivative with respect to ω_i is

$$\begin{aligned} S_i' &= \lambda_{-i}'(\omega_i+\varepsilon)\left[\begin{array}{c} r_i'(\lambda_{-i}(\omega_i+\varepsilon);0)\, f_i'(r_i(\lambda_{-i}(\omega_i+\varepsilon);0)) \\ -\mu r_i'(\lambda_{-i}(\omega_i+\varepsilon);0)\, W(r_i(\lambda_{-i}(\omega_i+\varepsilon);0)) \\ -(1-\mu)[r_i'(\lambda_{-i}(\omega_i+\varepsilon);0)+1]\, W(r_i(\lambda_{-i}(\omega_i+\varepsilon);0)+\lambda_{-i}(\omega_i+\varepsilon)) \\ +(1-\mu)W(\lambda_{-i}(\omega_i+\varepsilon)) \end{array}\right] \\ &= \lambda_{-i}'(\omega_i+\varepsilon)\left[\begin{array}{c} r_i'(\lambda_{-i}(\omega_i+\varepsilon);0)\left[\begin{array}{c} f_i'(r_i(\lambda_{-i}(\omega_i+\varepsilon);0)) \\ -\mu W(r_i(\lambda_{-i}(\omega_i+\varepsilon);0)) \\ -(1-\mu)W(r_i(\lambda_{-i}(\omega_i+\varepsilon);0)+\lambda_{-i}(\omega_i+\varepsilon)) \end{array}\right] \\ +(1-\mu)(W(\lambda_{-i}(\omega_i+\varepsilon))-W(r_i(\lambda_{-i}(\omega_i+\varepsilon);0)+\lambda_{-i}(\omega_i+\varepsilon))) \end{array}\right] \end{aligned}$$

Notice $\lambda_{-i}'(\omega_i+\varepsilon), r_i'(\lambda_{-i}(\omega_i+\varepsilon);0) < 0$. Moreover, $l_i^* = r_i(\lambda_{-i}(\omega_{-i});\omega_i) \le \lambda_i(\omega_i)$ implies

$$\begin{aligned} & f_i'(r_i(\lambda_{-i}(\omega_i+\varepsilon);0)) - \mu W(r_i(\lambda_{-i}(\omega_i+\varepsilon);0)) - (1-\mu)W(r_i(\lambda_{-i}(\omega_i+\varepsilon);0)+\lambda_{-i}(\omega_i+\varepsilon)) \\ > \; & f_i'(r_i(\lambda_{-i}(\omega_i+\varepsilon);0)) - W(r_i(\lambda_{-i}(\omega_i+\varepsilon);0)+\lambda_{-i}(\omega_i+\varepsilon)), \end{aligned}$$

which is strictly positive given that $r_i(l_{-i};0)$ solves $f_i'(r_i) - W(r_i+l_{-i}) = r_i W'(r_i+l_{-i})$. Therefore, $S_i' > 0$.

Since firm i surplus is increasing in ω_i either way, then it chooses $\omega_i \ge W^{**}$. If $\omega_i \ge W^{**}$ then $\omega_{-i} = W^{**}$, and $l_i = \lambda_i(\omega_i)$ and $l_{-i} = \lambda_{-i}(W^{**})$. Firm i's surplus is

$$S_i = f_i(\lambda_i(\omega_i)) - \mu\int_0^{\lambda_i(\omega_i)} W(l)\,dl - (1-\mu)\int_{\lambda_{-i}(W^{**})}^{\lambda_i(\omega_i)+\lambda_{-i}(W^{**})} W(l)\,dl$$

and the derivative with respect to ω_i is

$$\begin{aligned} S_i' &= \lambda_i'(\omega_i)[f_i'(\lambda_i(\omega_i)) - \mu W(\lambda_i(\omega_i)) - (1-\mu)W(\lambda_i(\omega_i)+\lambda_{-i}(W^{**}))] \\ &= \lambda_i'(\omega_i)[\omega_i - \mu W(\lambda_i(\omega_i)) - (1-\mu)W(\lambda_i(\omega_i)+\lambda_{-i}(W^{**}))] \end{aligned}$$

Since $\omega_i \ge W^{**}$, we have

$$\begin{aligned} W(\lambda_i(\omega_i)) &< W(\lambda_i(\omega_i)+\lambda_{-i}(W^{**})) \\ &< W(\lambda_i(W^{**})+\lambda_{-i}(W^{**})) = W^{**} \le \omega_i. \end{aligned}$$

Since $\lambda_i'(\omega_i) < 0$, we have $S_i' < 0$. Therefore, the unique equilibrium is $\omega_i^* = \omega_{-i}^* = W^{**}$. ■