The Monetary Dynamics of Hyperinflation Reconsidered

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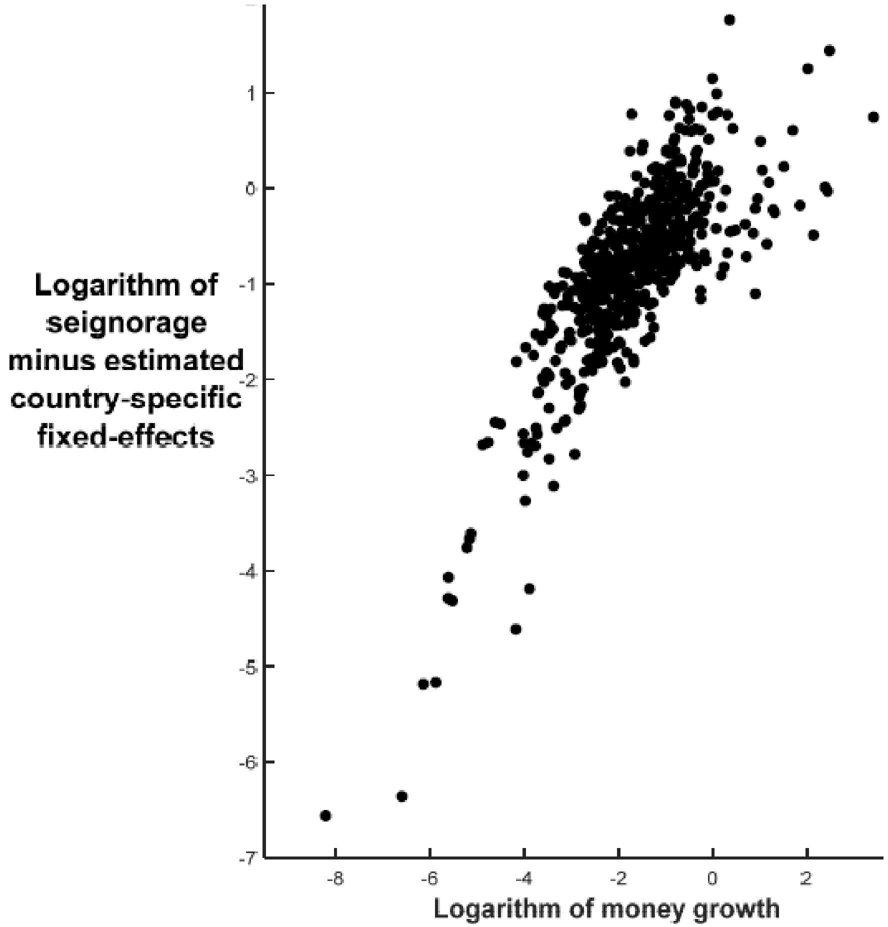
This paper: Brief overview



Contrary to what profession has believed since Cagan (1956), in entire recorded history there is <u>no evidence</u> of <u>Laffer</u> <u>curve</u> for <u>seignorage</u> …

<u>Historical relationship</u> between money growth and seignorage is on the left …

This has <u>material implications</u> along multiple dimensions …

Result was <u>implicit</u> in <u>Sargent and Wallace</u> (*IER*, 1973), but they <u>did not realize</u> it …

<u>Laffer curve</u> is <u>figment</u> of estimating money demand <u>model</u> that is, in fact, clearly <u>rejected</u> by data …

Evidence suggests Laffer curve might appear at levels of inflation materially higher than those that had been experienced historically …

Implications

My evidence is fully compatible with standard narrative accounts of hyperinflations, from Bresciani-Turroni (1937) to Sargent (1982) …

However, model-based analyses—both theoretical and empirical—should be revised, as they are both based on empirically implausible money demand specification, and therefore present incorrect view of the world …

What are hyperinflations?

Brief episodes characterized by extraordinary increases in money growth and therefore inflation …

E.g., between July 1945 and August 1946, Hungary experienced highest hyperinflation ever: in 13 months prices increased by factor of 3×10^{25} …

What are causes of hyperinflations? From Bresciani-Turroni's (1937) book on Weimar Republic's episode, to Sargent's (1982) 'The Ends of Four Big Inflations': enormous budget deficits that are financed by printing money, i.e. via the inflation tax, also known as seignorage …

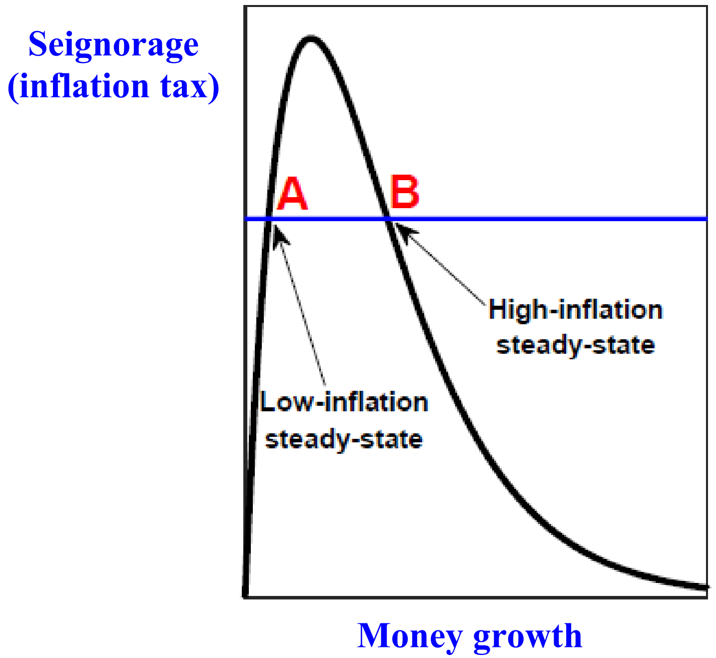
Such enormous deficits are typically product of revolutions (e.g., French Revolution), wars (e.g., WWI, WWII), or civil wars …

Standard view of relationship between money growth and seignorage

Since Cagan (1956), literature on hyperinflations has been dominated by view featuring <u>two main elements</u>:

(1) relationship between <u>money growth</u> and <u>seignorage</u> is hump-shaped—i.e., it follows <u>Laffer curve</u> (below);

(2) historically, <u>governments</u> have inflated in excess of revenue-maximizing rate, i.e., they have been on '<u>wrong side</u>' of Laffer curve (i.e., in point B) …



This view can be found in Sargent and Wallace (*IER*, 1973), Sargent (*IER*, 1977), Salemi and Sargent (*IER*, 1979), …

… and in all Ph.D. textbooks, from Blanchard and Fischer (1990) to Walsh (2017) …

How did Cagan, Sargent, etc. identify Laffer curve for seignorage?

They all estimated Cagan's 'semi-log' money demand:

$$\ln\left(\frac{M_t}{P_t Y_t}\right) = \beta - \alpha\pi_t^e$$

linking logarithm of money balances over nominal GDP to level of opportunity cost of money—here, expected inflation …

Based on estimated money demand curve, relationship between money growth and seignorage can trivially be computed …

Entire literature is based on Cagan's 'semi-log' …

However, e.g., Lucas (*Econometrica*, 2000, footnote 4): Laffer curve for seignorage is mathematical property of semi-log …

Therefore, when you estimate semi-log, you impose Laffer curve upon the data: this is what literature has been doing since 1956 …

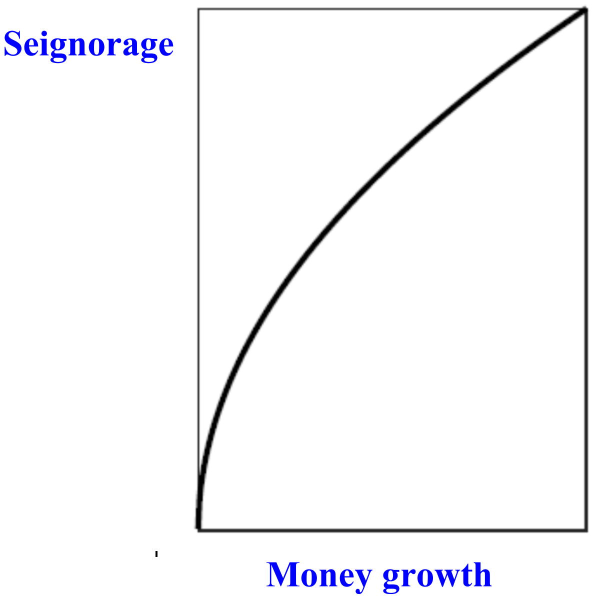
Benati, Lucas, Nicolini, and Weber (*JME*, 2021): for 44 countries since WWI most plausible money demand specification is not Cagan's, but rather Meltzer's (*JPE*, 1963) 'log-log' …

$$\ln\left(\frac{M_t}{P_t Y_t}\right) = \phi - \gamma \ln(\pi_t^e)$$

… which makes logarithm of money balances over nominal GDP depend on logarithm of opportunity cost of money …

In that paper we exclusively focused on very long samples …

In present work I show data's preference for 'log-log' is especially stark for hyperinflations …



Does this have implications for our purposes?

Yes: If elasticity smaller than 1—which empirically has been the case—log-log implies uniformly positive relationship between money growth and seignorage for all values of money growth (left) …

Therefore, question: Is Laffer curve for real, or is it figment of literature's exclusive focus on 'semi-log'?

This paper

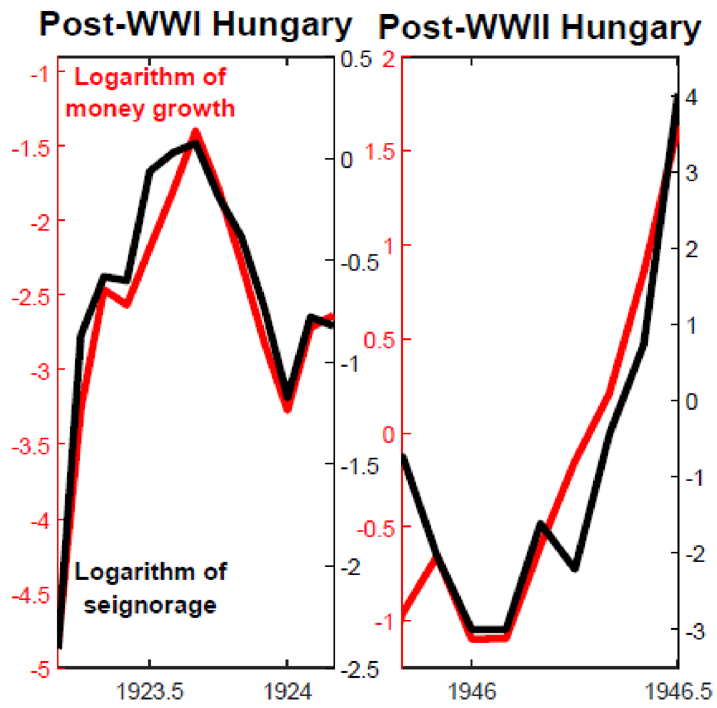
I reconsider monetary dynamics of hyperinflations based on data from 20 episodes, from French Revolution to Venezuela under Nicolas Maduro …

I find no evidence of Laffer curve for seignorage: relationship between money growth and seignorage has been positive at all levels of money growth …

On the right, evidence for Hungary: as I will show, evidence for other countries is qualitatively the same …

There is no evidence that, beyond a threshold, increases in money growth cause decreases in seignorage …

Recall: post-WWII Hungary is highest hyperinflation ever recorded …



Natural explanation: hyperinflation data show clear preference for 'log-log' specification—or functional form close to it, such as Benati et al.'s (*JME* 2021) …

I show this has material implications for

(1) theoretical analyses of hyperinflations: e.g., compared to semi-log, with log-log steady-states' stability properties are reversed …

(2) Correctly characterizing time-series properties of hyperinflations: semi-log suggests largest root in the system is around 1.7, log-log that it is 1 …

Hyperinflations have been unit root processes, not wildly explosive ones …

(3) Interpretation of specific episodes: as corollary of (1), within context of Weimar Republic's episode I provide interpretation of impact of invasion of Ruhr …

The dataset

20 episodes, from French Revolution to Venezuela under Nicolas Maduro …

Beyond standard datasets—e.g. Cagan's, and Barro's (*JPE*, 1970)—I consider several additional episodes, for which I obtained data from primary sources …

This is the case, e.g., for

- **French Revolution: data are from books published in early XIX century …**
- **Chile, Argentina, Bolivia, Brazil, etc. : data are from central banks and national statistical agencies …**

Table in next slide reports key features of dataset …

Following convention in literature, inflation is measured as monthly log-difference of price level …

Table 1 Maximum, mean, and median inflation during hyperinflationary episodes				
	Number of observations	Maximum	Mean	Median
France (January 1794-June 1796)	27	0.761	0.183	0.154
Based on Cagan's (1956) data				
Austria (January 1921-August 1922)	20	0.852	0.258	0.212
Germany (September 1920-November 1923)	39	5.885	0.616	0.157
Hungary (July 1922-February 1924)	20	0.683	0.248	0.215
Hungary (July 1945-July 1946)	13	33.670	4.909	1.677
Poland (April 1922-January 1924)	22	1.322	0.368	0.302
Russia (January 1922-February 1924)	26	1.142	0.451	0.420
Greece (January 1943-November 1944)	23	13.659	1.378	0.470
Based on Barro's (1970) data				
Austria (January 1921-December 1922)	24	0.852	0.220	0.149
Poland (January 1922-January 1924)	25	1.229	0.349	0.289
Germany (January 1921-August 1923)	32	2.931	0.339	0.136
Hungary (October 1921-February 1924)	29	0.683	0.191	0.152
Other datasets				
Germany (December 1919-December 1923)	60	5.736	0.449	0.090
China (August 1945-May 1949)	45	4.446	0.552	0.240
Chile (January 1972-November 1974)	35	0.575	0.120	0.106
Bolivia (February 1983-August 1986)	43	1.039	0.220	0.155
Argentina (January 1987-April 1991)	52	0.992	0.185	0.135
Brazil (August 1988-March 1991)	33	0.592	0.235	0.199
Peru (January 1987-September 1991)	57	1.512	0.211	0.147
Yugoslavia (January 1991-January 1994)	37	11.290	1.386	0.496
Congo (May 1991-July 1995)	51	2.144	0.296	0.241
Angola (December 1995-January 1998)	26	0.610	0.167	0.061
Bulgaria (January 1996-February 1998)	26	1.230	0.128	0.039
Zimbabwe (January 2004-June 2008)	53	3.912	0.434	0.300
Venezuela (January 2016-March 2019)	39	1.313	0.333	0.200

Inflation is computed as the month-on-month log-difference of the price level.

Range of variation in inflation is extraordinary, from post-WWII Hungary's peak of 33.67 …

… to second and third most extreme episodes, Greece and Yugoslavia, with peaks of 13.66 and 11.29 …

… down to the mildest, Chile, with peak of 0.58 …

Such wide range of variation allows for strong identification …

Empirical relationship between money growth and seignorage

Seignorage estimates for Hungary I showed before are from Sargent and Wallace (*IER*, 1973) …

Sargent and Wallace computed estimates, but did not plot them together with money growth …

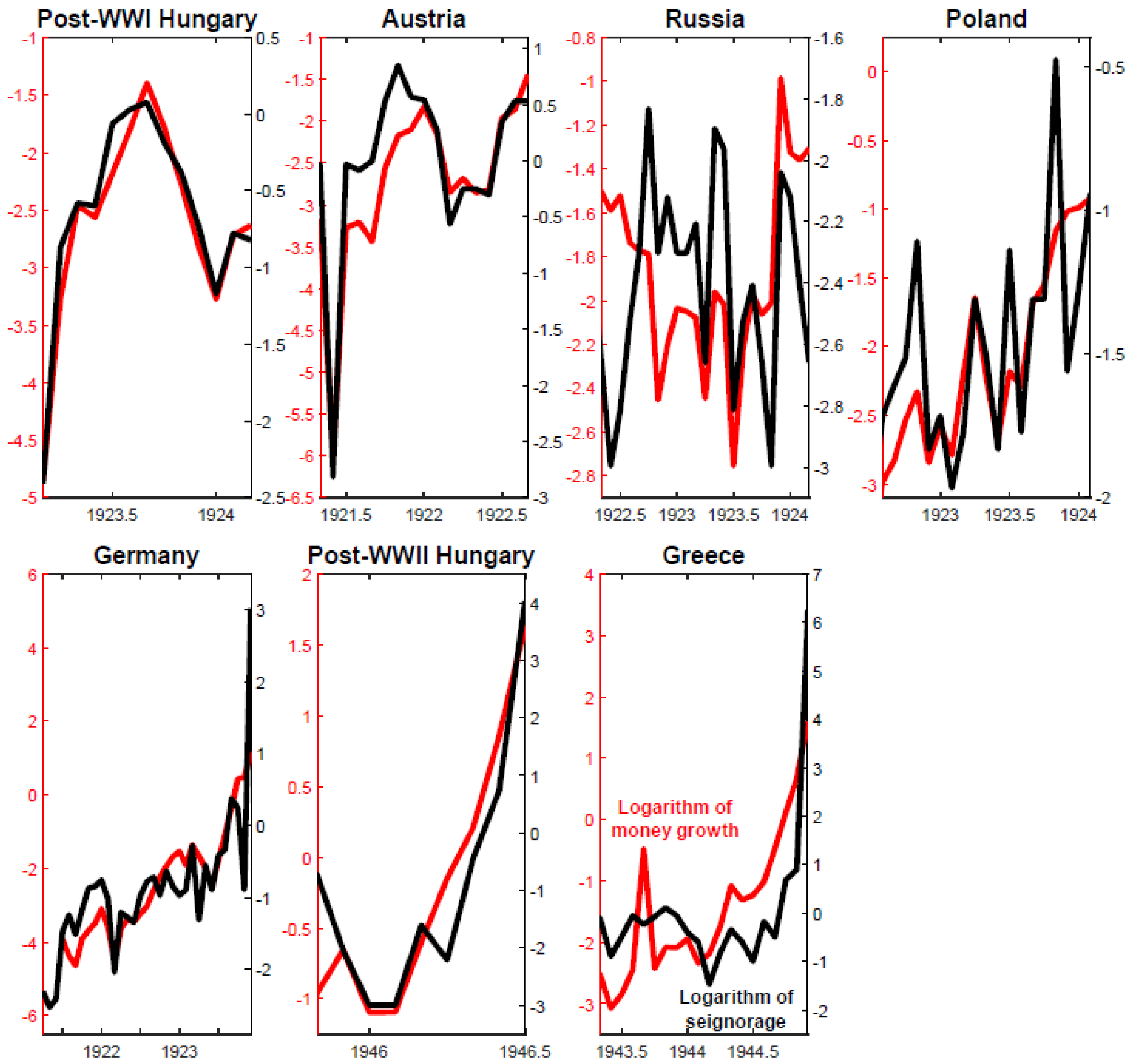
If they had done this, they would have seen Laffer curve is not in the data …

My findings have been hiding in plain sight since 1973 …

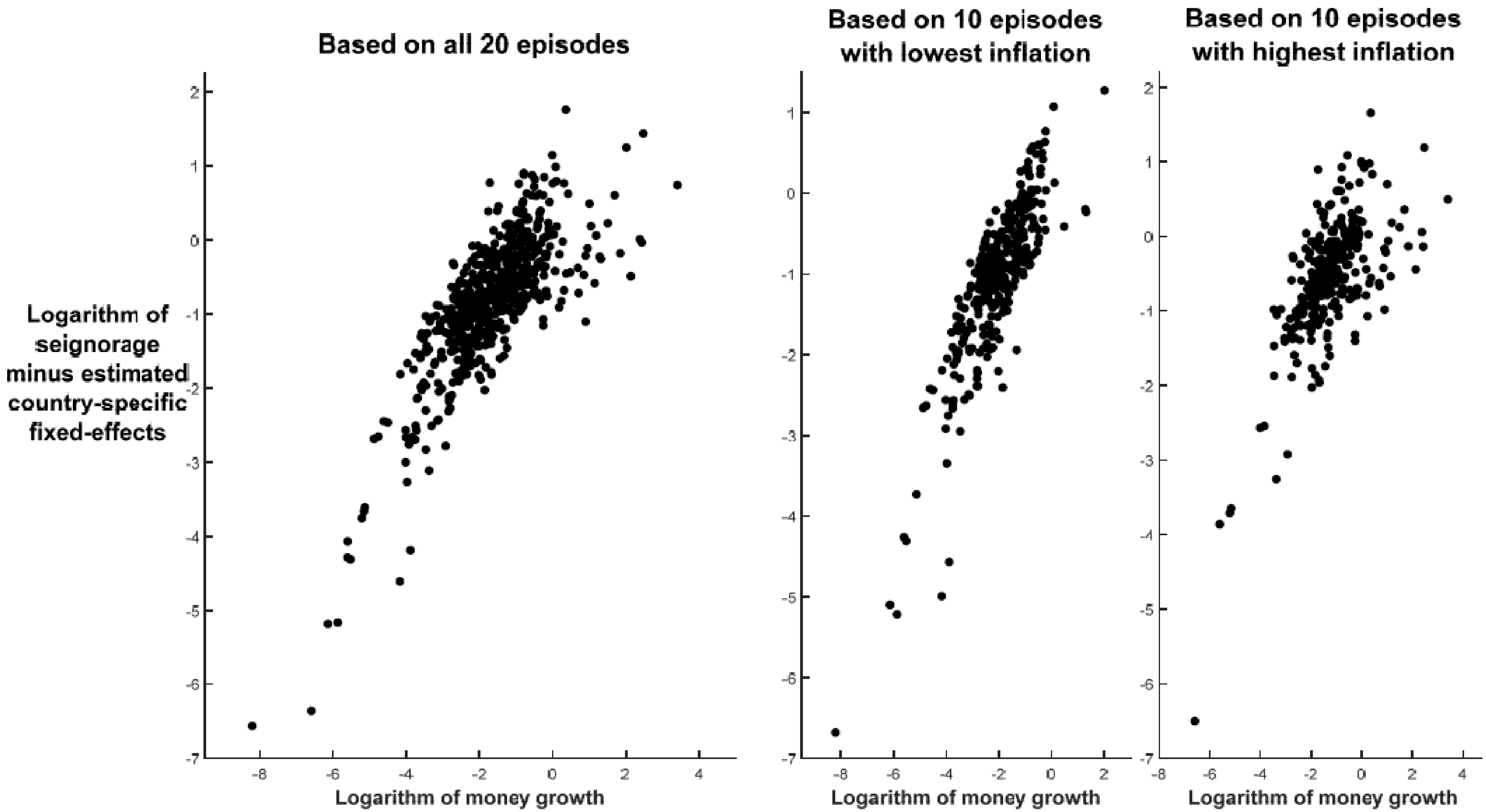
Next two slides show

- **all of Sargent and Wallace's estimates for Cagan's dataset, and**
- **joint evidence for all episodes in my dataset, controlling for country-specific fixed effects …**

No evidence of Laffer curve, i.e. that beyond a threshold, increases in money growth cause decreases in seignorage …



The logarithm of Sargent and Wallace's (1973) measure of seignorage plotted against the logarithm of money growth from Cagan (1956)



Evidence from regressing log seignorage on log money growth: logarithm of seignorage minus estimated country-specific fixed effects

What is most plausible money demand specification?

Semi-log implies Laffer curve, whereas log-log implies what you see in the data …

Therefore, alternative way of addressing this issue is exploring which specification is more empirically plausible ...

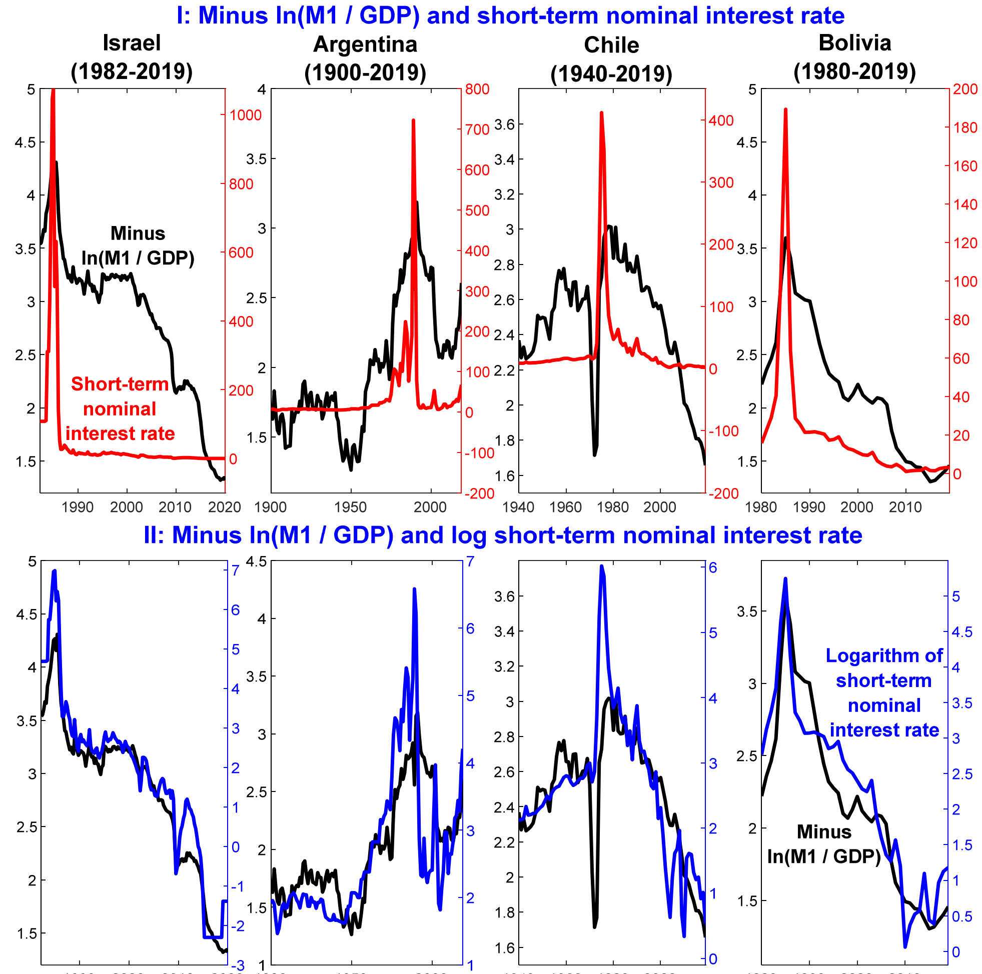
Benati, Lucas, Nicolini, and Weber (*JME*, 2021): in dataset of 44 countries since WWI data clearly prefer log-log …

We have lots of statistical evidence, but next slide shows simple visual evidence for selected countries, plotting minus log money balances over nominal GDP together with

- **top row: level of nominal interest rate**
- **bottom row: logarithm of nominal interest rate**

Top row corresponds to semi-log, bottom row to log-log …

Evidence is very clear: log-log is significantly more plausible than semi-log …



We did not specifically analyze hyperinflations, which typically last at most 2 years: What about them?

<u>Table</u> in <u>next slide</u> shows results from <u>model comparison exercise</u> based on panel VARs with country-specific fixed-effects: semi-log versus log-log …

<u>Evidence</u> is <u>overwhelming</u>: minimum of distribution of log-likelihood based on log-log is uniformly greater than maximum of distribution based on semi-log …

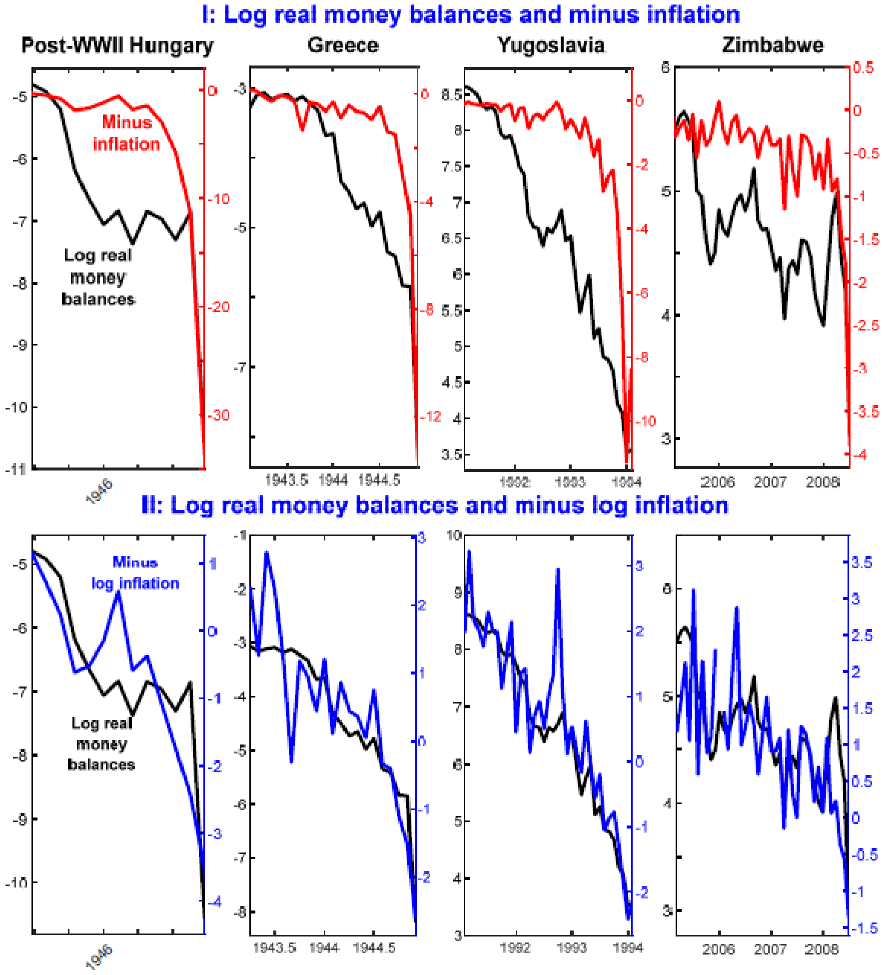
Table 2 Evidence from the model comparison exercise: minima, maxima, and medians of the distributions of the log-likelihoods

	I: Based on semi-log			II: Based on log-log		
	Min	Median	Max	Min	Median	Max
	Based on all 20 episodes					
p=2	303.1	324.8	337.1	557.9	583.1	596.2
p=4	219.5	245.4	259.7	470.0	494.6	508.4
	Based on 10 episodes with highest median inflation					
p=2	-6.4	9.6	20.3	102.0	119.0	129.0
p=4	-34.9	-13.3	-1.6	81.0	109.0	122.2
	Based on 10 episodes with lowest median inflation					
p=2	599.8	625.4	635.5	660.0	678.8	689.3
p=4	549.5	570.7	582.9	646.6	667.9	678.8

Right: **log real money balances and minus either inflation (i.e., semi-log) or log inflation (i.e., log-log): again, for hyper-inflations log-log is clearly more empirically plausible …**

Evidence for Yugoslavia is especially stark …

Clear evidence that empirically most plausible functional form for hyperinflations is log-log …



Implications, I: Time-series properties of hyperinflations

Evidence from panel VARs for the logarithm of real money balances and either inflation or log inflation: point estimates for the four largest eigenvalues, and fractions of bootstrap replications for which the eigenvalues are greater than 1

	Point estimates for four largest eigenvalues				Fractions of replications for which $\lambda_i<1$			
	λ_1	λ_2	λ_3	λ_4	λ_1	λ_2	λ_3	λ_4
	I: Based on Cagan's (1956) semi-log							
p=2	0.330	0.330	0.937	1.728	0.876	0.816	0.534	0.124
p=4	0.631	0.631	0.924	1.680	0.795	0.746	0.550	0.063
	II: Based on Meltzer's (1963) log-log							
p=2	0.314	0.314	0.672	0.980	1.000	1.000	1.000	0.770
p=4	0.582	0.582	0.834	0.981	1.000	1.000	0.992	0.741

Left: evidence from <u>panel VARs</u> with country-specific fixed-effects: semi-log versus log-log …

With <u>semi-log</u> <u>largest eigenvalue</u> is explosive, and very large, <u>around 1.7</u> …

With <u>log-log</u> it is indistinguishable from <u>exact unit root</u> …

Alternative functional forms provide different characterization of time-series properties of hyperinflations …

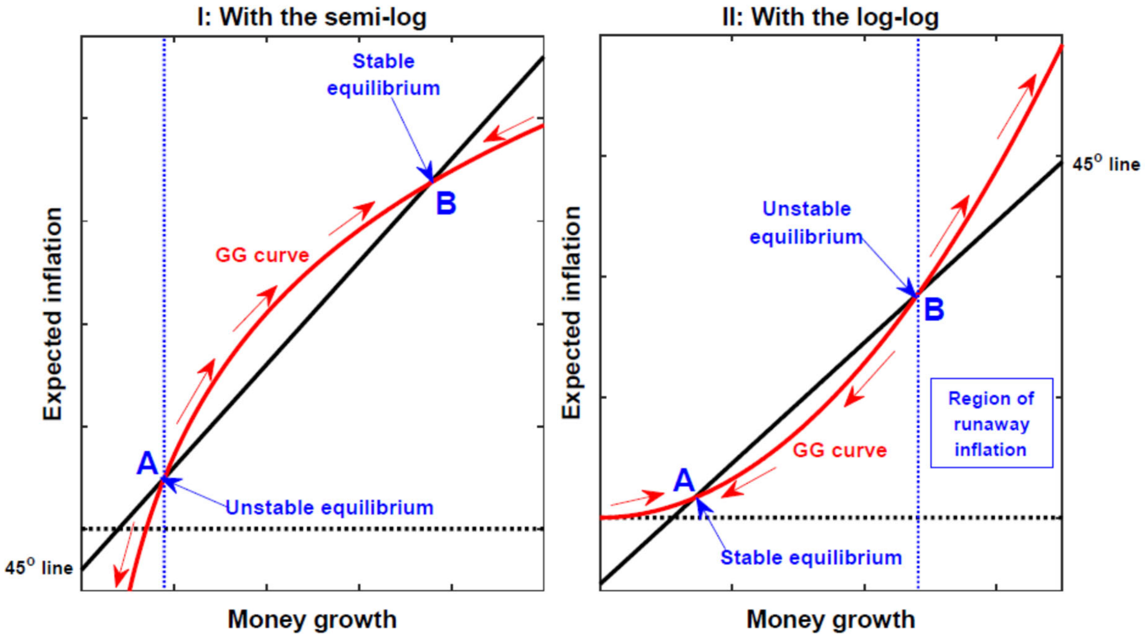
<u>Most plausible</u> characterization: hyperinflations driven by <u>random-walk</u> process …

What was it? On this, more later …

Implications, II: Theoretical analyses of hyperinflations

First implication: as mentioned, semi-log implies Laffer curve for seignorage, whereas **log-log** implies **positive relationship** at **all levels** of money growth and inflation …

Second implication: **equilibria stability** properties get **reversed**, with high-inflation equilibrium becoming unstable …



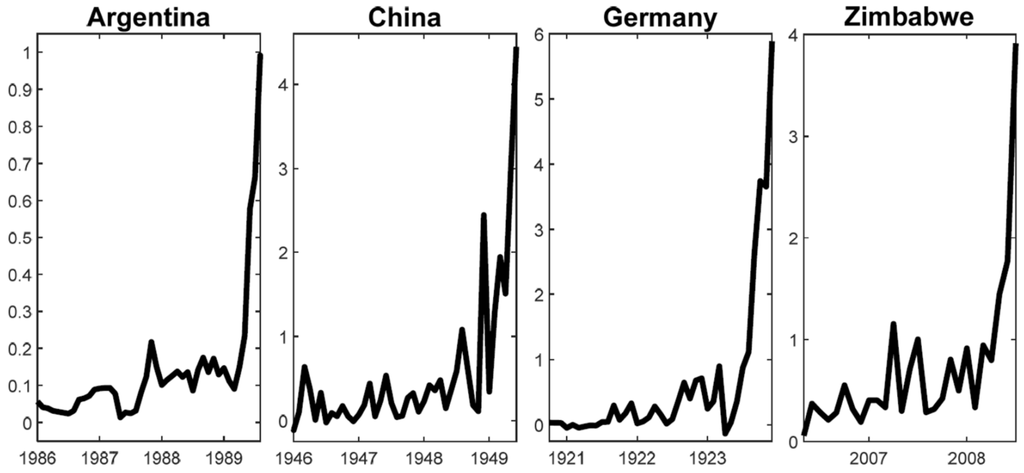
Left: **GG** curve is government **budget constraint** …

Economy is **always** on GG curve, where government finances **budget deficit** by **printing money** …

Equilibrium is at **intersection** of GG curve and **45° line**, where (up to constant) expected inflation is equal to money growth …

With semi-log, under rational expectations high-inflation steady-state is stable: Sargent and Wallace's (*IER*, 1973) classic result …

However, with log-log high-inflation steady-state is unstable: beyond it lies region of 'runaway inflation' …

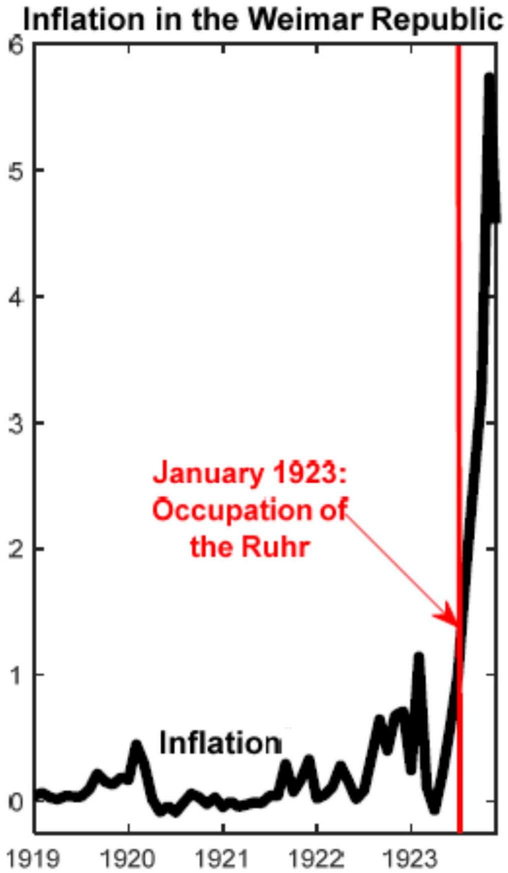


The level of inflation in selected episodes

This provides simple explanation for empirical fact: in several cases, inflation literally 'takes off' towards latest stages of hyperinflation (see left) …

Possible explanation: large shock causes economy to jump beyond high-inflation steady-state B, into region of 'runaway inflation' …

Implications, III: Interpretation of historical episodes



Evidence for Weimar Republic is compatible with this interpretation …

Until end of 1922, inflation had been high, but not explosive …

In January 1923, France occupied Ruhr: as pointed out by Bresciani-Turroni (1937), this *'gave the coup de grâce to the national finances and the German mark. Because of it some important sources of income were lost to the State.'*

Possible interpretation of occupation of Ruhr—and resulting explosion of budget deficit financed *via* seignorage—is as shock that pushed economy into region of 'runaway inflation' …

Is relationship between money growth and seignorage consistently positive for all levels of money growth?

We saw that, historically, relationship between money growth and seignorage has consistently been positive for all levels of money growth …

This, however, cannot be true for any level of money growth …

In the limit, if inflation explodes to infinity agents switch to barter, and seignorage collapses to zero …

Therefore, even with log-log there ought to be a level of inflation beyond which relationship turns negative …

Recall: with log-log relationship between money growth and seignorage is positive (negative) if elasticity of money demand is (in absolute value) smaller (greater) than 1 …

Table on next slide explores how elasticity changes with inflation based on panel cointegrated VARs …

Table 4 Evidence from panel cointegrated VARs for the logarithms of real money balances and inflation: point estimates of the elasticity, 90% bootstrapped confidence interval, and fractions of bootstrap replitions for which the elasticity is smaller than -1

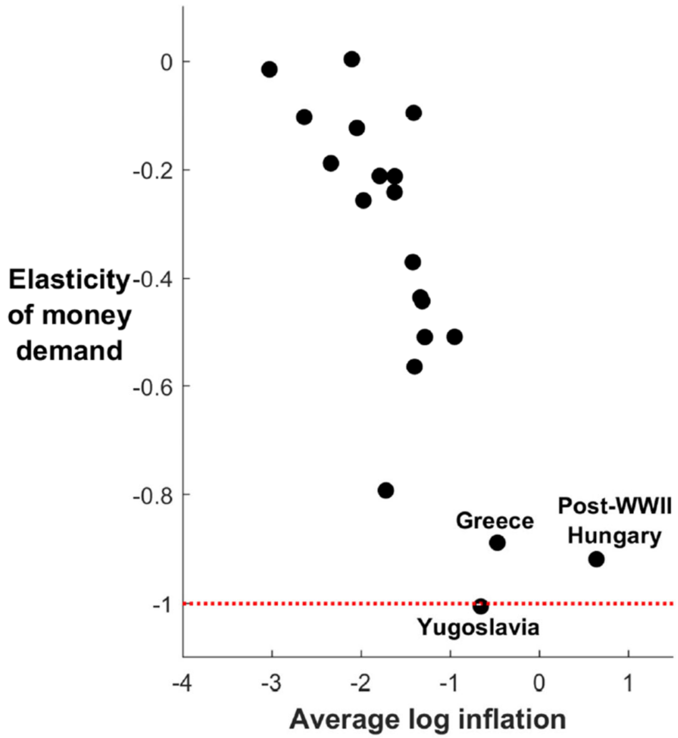
	Point estimates, and 90% confidence interval	Fractions of bootstrap replications for which elasticity is below -1
	Based on all 20 episodes	
k=1	-0.502 [-0.593; -0.412]	0.000
k=2	-0.554 [-0.656; -0.454]	0.000
	Based on 10 episodes with highest median inflation	
k=1	-0.792 [-0.896; -0.681]	0.001
k=2	-0.926 [-1.041; -0.796]	0.143
	Based on 10 episodes with lowest median inflation	
k=1	-0.295 [-0.392; -0.191]	0.000
k=2	-0.312 [-0.412; -0.202]	0.000

Splitting 20 episodes into 2 groups with highest and lowest median inflation suggests that higher inflation is associated with larger (in absolute value) elasticity …

Confidence intervals for highest and lowest median inflation do not overlap: elasticity estimates are clearly different …

Consistent with what we saw before, evidence that elasticity has been greater than 1 (in absolute value) is weak-to-non-existent …

However, evidence in the table suggests that for levels of inflation higher than those experienced historically, at some point the elasticity will become greater than 1, and a Laffer curve will appear …



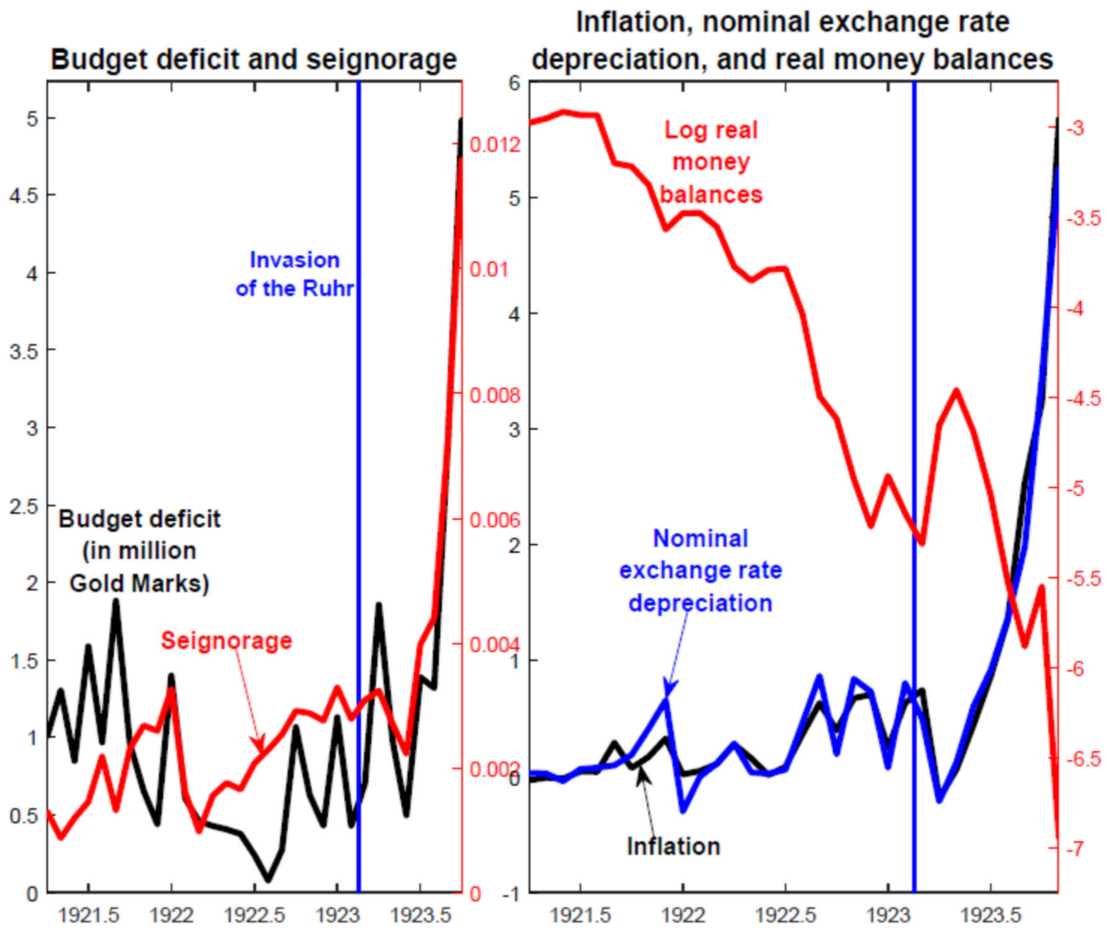
Elasticity estimates for <u>individual countries</u> (left) are <u>consistent</u> with this …

<u>Higher inflation</u> consistently associated with <u>larger elasticities</u> (in absolute value) …

Evidence suggests post-WWII Hungary quite close to inflation threshold beyond which relationship between money growth and seignorage turns negative, and Laffer curve appears …

Yugoslavia is probably an outlier …

An estimated model of Weimar Republic's hyperinflation



Left, key data: government <u>budget deficit</u> in Gold Marks (i.e., in <u>real</u> terms) and <u>money</u> balances (as fraction of income) from Bresciani-Turroni (1937); and <u>inflation</u>, <u>seignorage</u>, and <u>exchange rate</u> depreciation …

In line with Bresciani-Turroni (1937) and Sargent (1982), I assume key <u>driver</u> of hyperinflation is evolution of <u>budget deficit</u>, which is financed by printing money …

Logarithm of budget deficit, d_t, is sum of 2 orthogonal components, a random-walk with drift and a stationary AR(1):

$$d_t = d_t^P + d_t^T$$

$$d_t^P = \mu_d + d_{t-1}^P + \epsilon_t^P$$
$$d_t^T = \rho_T d_{t-1}^T + \epsilon_t^T$$

with $\epsilon_t{}^P$ and $\epsilon_t{}^T$ being white noise orthogonal shocks …

Seignorage is computed based on the geometric average

$$\xi_t \equiv \theta_t \left[\left(\frac{M_{t-1}}{P_{t-1}} \right)^{\omega} \left(\frac{M_t}{P_t} \right)^{1-\omega} \right]$$

where M_t and P_t are the nominal money stock and the price level, θ_t is money growth, and $\omega = 0.5$ …

Bresciani-Turroni (1937) conjectured that since hyperinflations follow exponential dynamics, geometric averages are more appropriate …

I show mathematically that Bresciani-Turroni was right …

Based on previous figure showing raw data, I assume that government sets log seignorage equal to permanent component of budget deficit, d_t^P:

$$\ln \xi_t = d_t^P$$

The demand for real money balances as fraction of income takes Meltzer's log-log form

$$\ln\left(\frac{M_t}{P_t Y_t}\right) = A - B \ln \pi_{t+1|t} + u_t$$

with velocity shock u_t following a stationary AR(1)

$$u_t = \rho_u u_{t-1} + v_t.$$

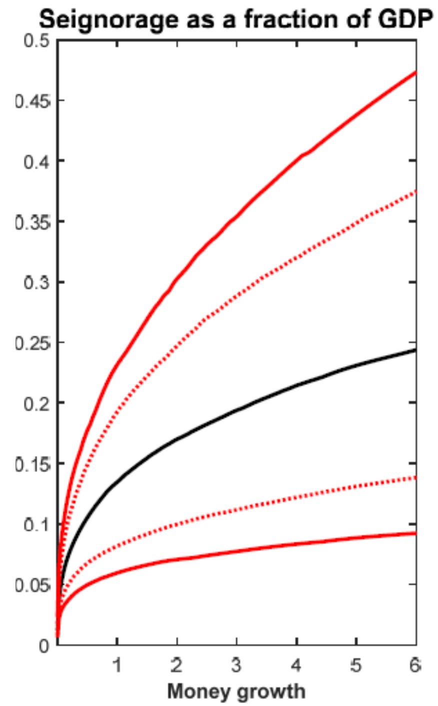
Finally, nominal exchange rate depreciation, Δe_t, is equal to inflation, π_t, plus a PPP white noise disturbance:

$$\Delta e_t = \pi_t + \varepsilon_t$$

Everything is driven by random-walk in budget deficit, d_t^P: all variables are therefore <u>stationarized</u> by d_t^P …

In estimation I impose <u>determinacy</u>: future extensions will allow for either (*i*) <u>indeterminacy</u>, or (*ii*) <u>temporarily explosive</u> paths as in Ascari, Bonomolo, and Lopes (*AER*, 2019) …

Right: estimated relationship between money growth and seignorage as fraction of GDP …



At <u>peak</u> of hyperinflation, <u>money growth</u> was equal to <u>about 6</u> on log scale: this suggests <u>government</u> was <u>collecting</u> nearly <u>25%</u> of <u>output</u> via inflation tax …

To put this into perspective: based on <u>semi-log</u>, estimates of <u>maximum</u> amount of seignorage government can raise in steady-state are <u>around 10%</u> …

Summing up

Data for 20 hyperinflations provide no evidence of Laffer curve: historically, relationship between money growth and seignorage has been positive for all values of money growth …

Natural explanation: entire literature based on Cagan's semi-log functional form, which imposes Laffer curve upon data …

However, data clearly prefer log-log, which—if elasticity smaller than 1, as historically has been the case—implies positive relationship between money growth and seignorage for all values of money growth …

Evidence suggests at inflation levels higher than those experienced historically Laffer curve might appear, since elasticity of money demand becomes greater than 1 …

Background slides

I: Additional results from Benati, Lucas, Nicolini, and Weber (*JME*, 2021)

Table 2
Results from the Wright (2000) test: 90% coverage confidence intervals for the second element of the normalized cointegration vector.

Country	*Period*	Bootstrapped process: cointegrated VECM	
		Selden-Latané	*Log-log*
United Kingdom	1922–2016	**[-0.529; -0.417]**	NCD
US - M1 + MMDAs	1915–2017	**[-0.613; -0.393]**	**[-0.352; -0.108]**
US - M1	1915–2017	NCD	**[-0.506; -0.029]**
Argentina	1914–2009	**[-0.107; -0.087]**	**[-0.513; -0.245]**
Brazil	1934–2014	**[-0.065; -0.009]**	[-1.366; 0.276]
Canada	1926–2006	**[-1.490; -1.053]**	**[-0.719; -0.607]**
	1967–2017	**[-0.578; -0.494]**	**[-0.389; -0.345]**
Colombia	1960–2017	NCD	NCD
Guatemala	1980–2017	**[-0.752; -0.448]**	**[-0.678; -0.414]**
New Zealand	1934–2017	NCD	**[-0.589; -0.312]**
Switzerland	1948–2005	NCD	NCD
Bolivia	1980–2013	**[-0.369; -0.193]**	**[-0.520; -0.388]**
Israel	1983–2016	NCD	**[-0.388; -0.320]**
Mexico	1985–2014	**[-0.260; -0.184]**	**[-0.422; -0.314]**
Belgium	1946–1990	**[-0.465; -0.289]**	**[-1.146; -0.710]**
Belize	1977–2017	**[-0.840; -0.692]**	**[-2.567; 1.433]**
Austria	1970–1998	[-0.601; 0.080]	[-1.040; 0.618]
Bahrain	1980–2017	NCD	**[-0.254; -0.194]**
Barbados	1975–2016	**[-2.006; -0.748]**	**[-2.899; 0.101]**
Ecuador	1980–2011	NCD	NCD
Netherlands	1950–1992	**[-0.394; -0.290]**	**[-0.483; -0.331]**
South Korea	1970–2017	**[-0.617; -0.521]**	**[-0.639; -0.338]**
Thailand	1979–2016	NCD	**[-0.498; -0.386]**
Venezuela	1962–1999	NCD	[-0.249; 0.287]
Australia	1941–1989	**[-0.691; -0.526]**	**[-0.808; -0.704]**
	1969–2017	**[-0.484; -0.404]**	**[-0.506; -0.314]**
Chile	1940–1995	**[-0.140; -0.028]**	**[-0.382; -0.278]**
Finland	1946–1985	**[-0.530; -0.414]**	**[-2.693; -1.780]**
Japan	1955–2017	**[-0.520; -0.312]**	**[-0.513; -0.125]**
Spain	1941–1989	**[-0.163; -0.159]**	NCD
Taiwan	1962–2017	**[-0.449; -0.341]**	**[-0.453; -0.253]**
Turkey	1968–2017	NCD	NCD
West Germany	1960–1989	[-0.963; 0.931]	[-0.489; 0.692]
Italy	1949–1996	[0.032; 0.204]	[0.159; 0.511]
Norway	1946–2014	[-0.961; 0.985]	[-0.227; 1.043]
Paraguay	1962–2015	[-0.328; 0.125]	**[-0.200; -0.024]**
Peru	1959–2017	[-0.042; 0.026]	[-0.493; 0.692]
Portugal	1914–1998	[-0.340; 0.433]	[-0.018; 0.210]
South Africa	1965–2015	[-0.170; 0.427]	[-0.052; 1.065]

NCD = No cointegration detected.

Evidence from Wright's (2000) cointegration tests

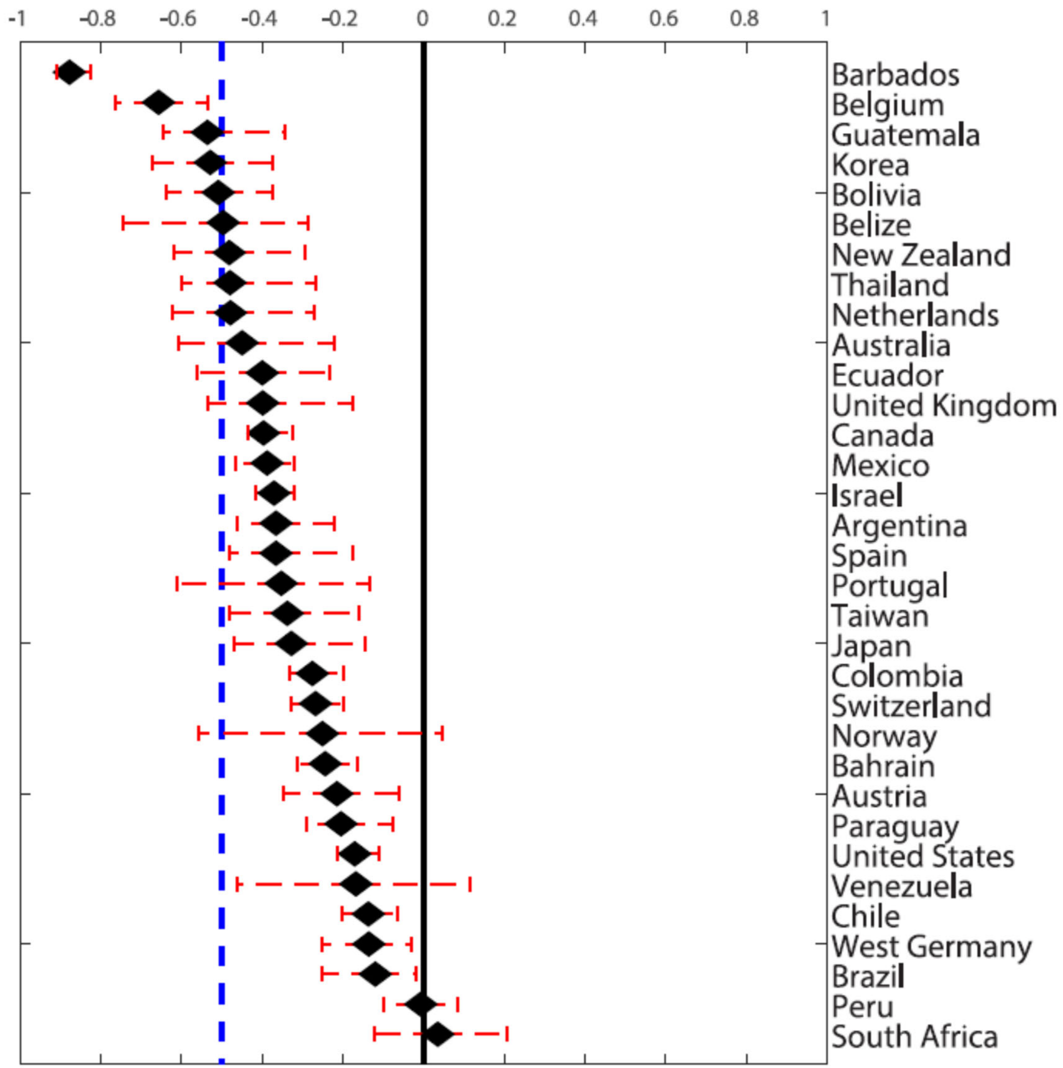
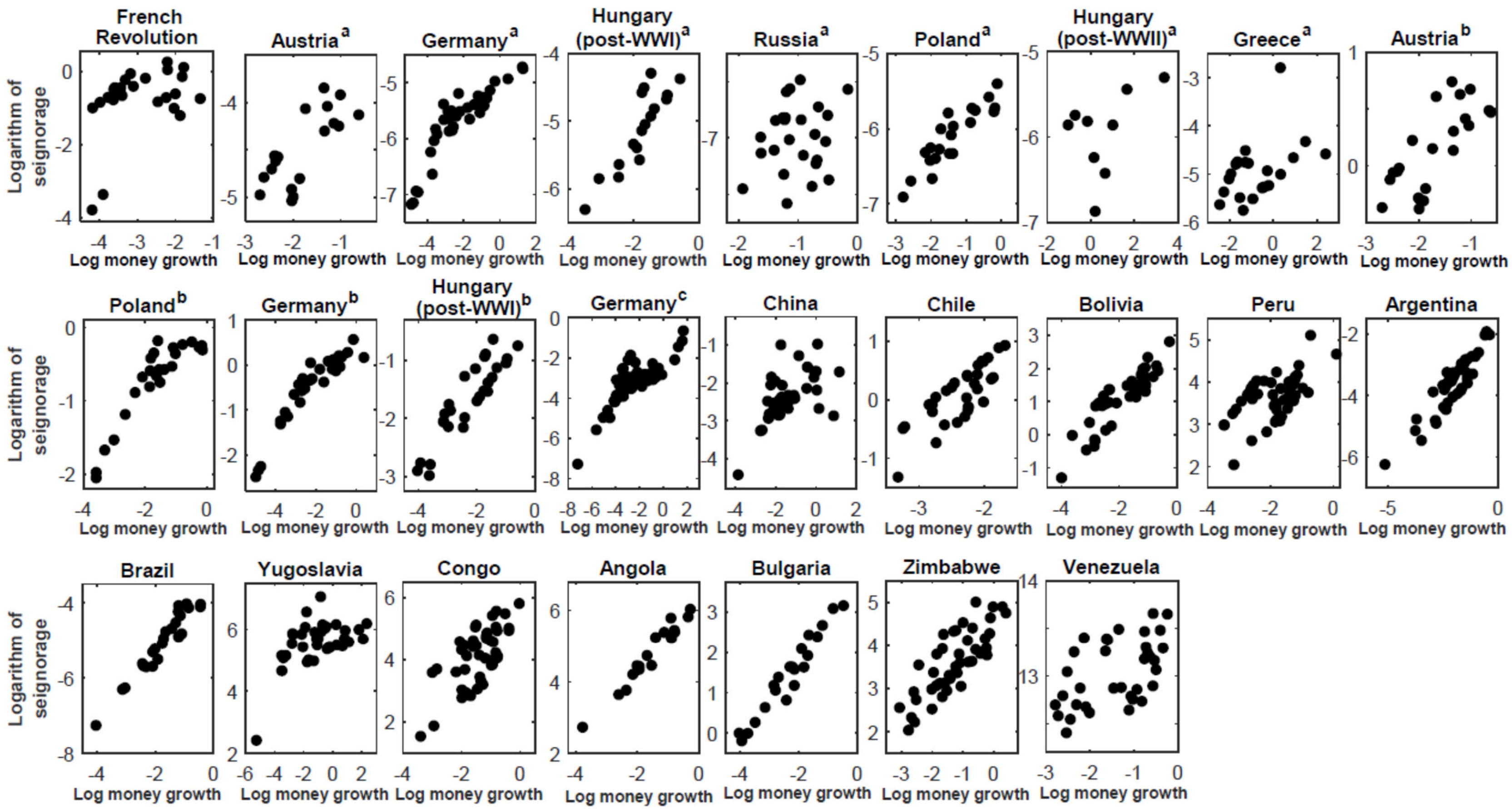


Fig. 5. Estimation results using the procedure from Stock and Watson (1993).

Estimated elasticity of money demand based on Meltzer's (1963) 'log-log' functional form

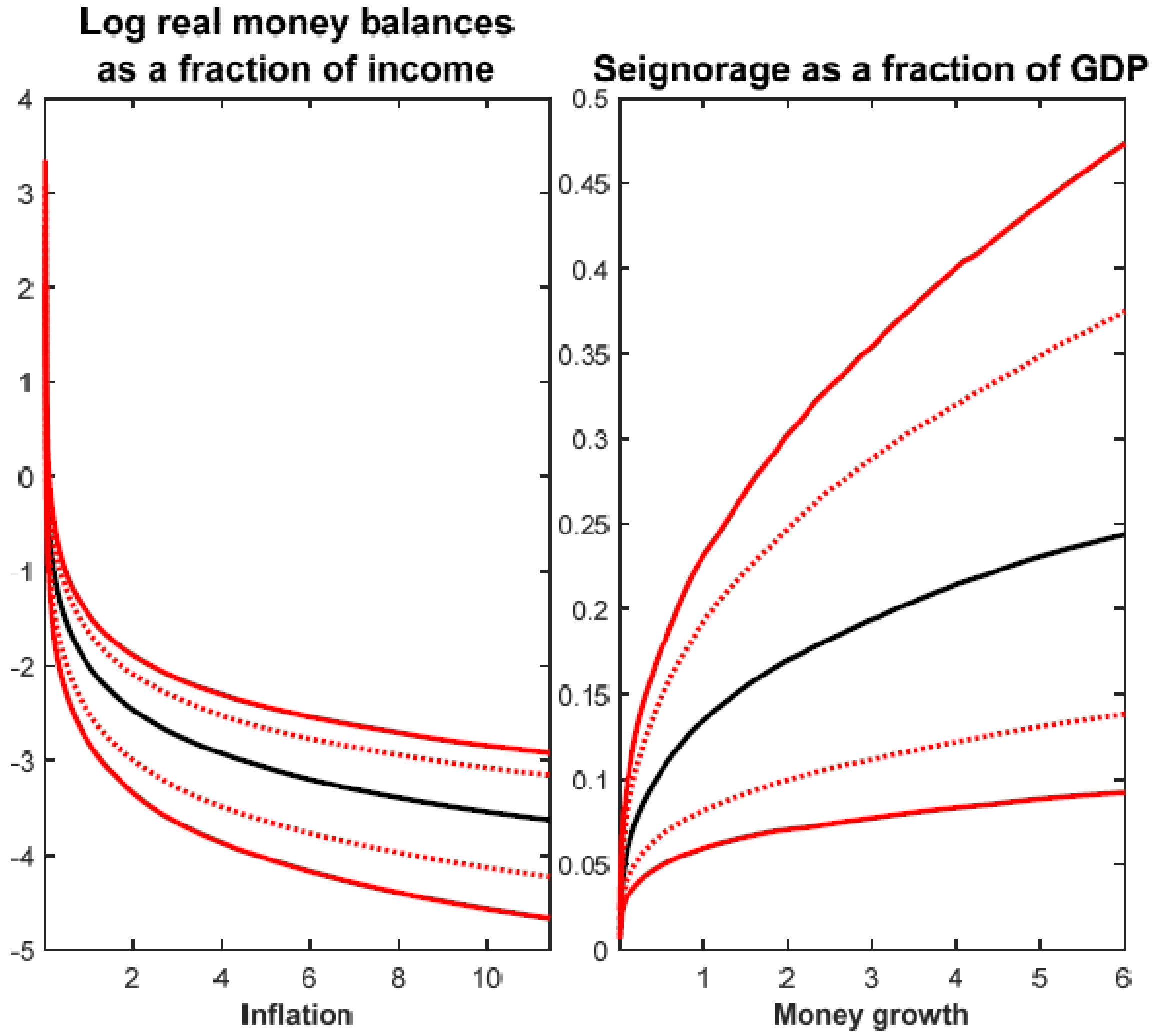
II: Additional evidence from the present work

The logarithms of money growth and seignorage for all 20 hyperinflations

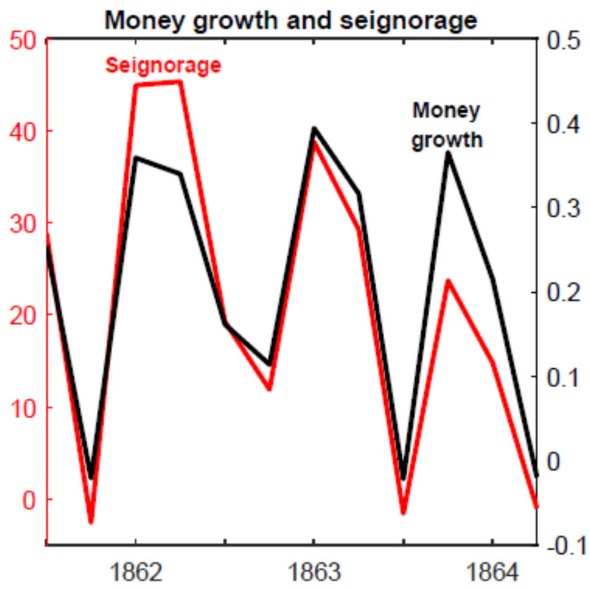


[a]Based on Cagan's data;[b]Based on Barro's data;[c]Based on Graham's data;

Evidence from an estimated DSGE model for the Weimar Republic: the demand for real money balances and the relationship between money growth and seignorage (median, and 16-84 and 5-95 credible sets)



Money growth and seignorage in the Confederacy during the U.S. Civil War



Germany, June 1921-November 1923 (based on weekly data)

Definition of seignorage

By defining the money stock, real GDP, and the price level as M_t, Y_t, and P_t, the instantaneous revenue from money creation—i.e. seignorage—expressed as a fraction of GDP is defined as

$$\xi_t \equiv \frac{dM_t}{dt}\frac{1}{P_tY_t} = \underbrace{\left(\frac{dM_t}{dt}\frac{1}{M_t}\right)}_{\theta_t}\frac{M_t}{P_tY_t} = \theta_t\frac{M_t}{P_tY_t} \tag{1}$$

where θ_t is instantaneous money growth, and $M_t/(P_tY_t)$ is the demand for real money balances expressed as a fraction of GDP.

Benati, Lucas, Nicolini and Weber's (JME, 2021) money demand

Appendix A describes in detail the model of the transaction demand for money proposed by Benati et al. (2021). The model generalizes the framework proposed by Baumol (1952) and Tobin (1956) by allowing for several alternative functional forms for the cost of making transactions as a function of the number of 'trips to the bank', n_t. Notice that within this framework n_t is the velocity of money, i.e. the ratio between nominal GDP and nominal money balances, and its inverse is therefore the demand for money balances as a fraction of GDP:

$$\frac{1}{n_t} = \frac{M_t}{P_t Y_t}. \tag{8}$$

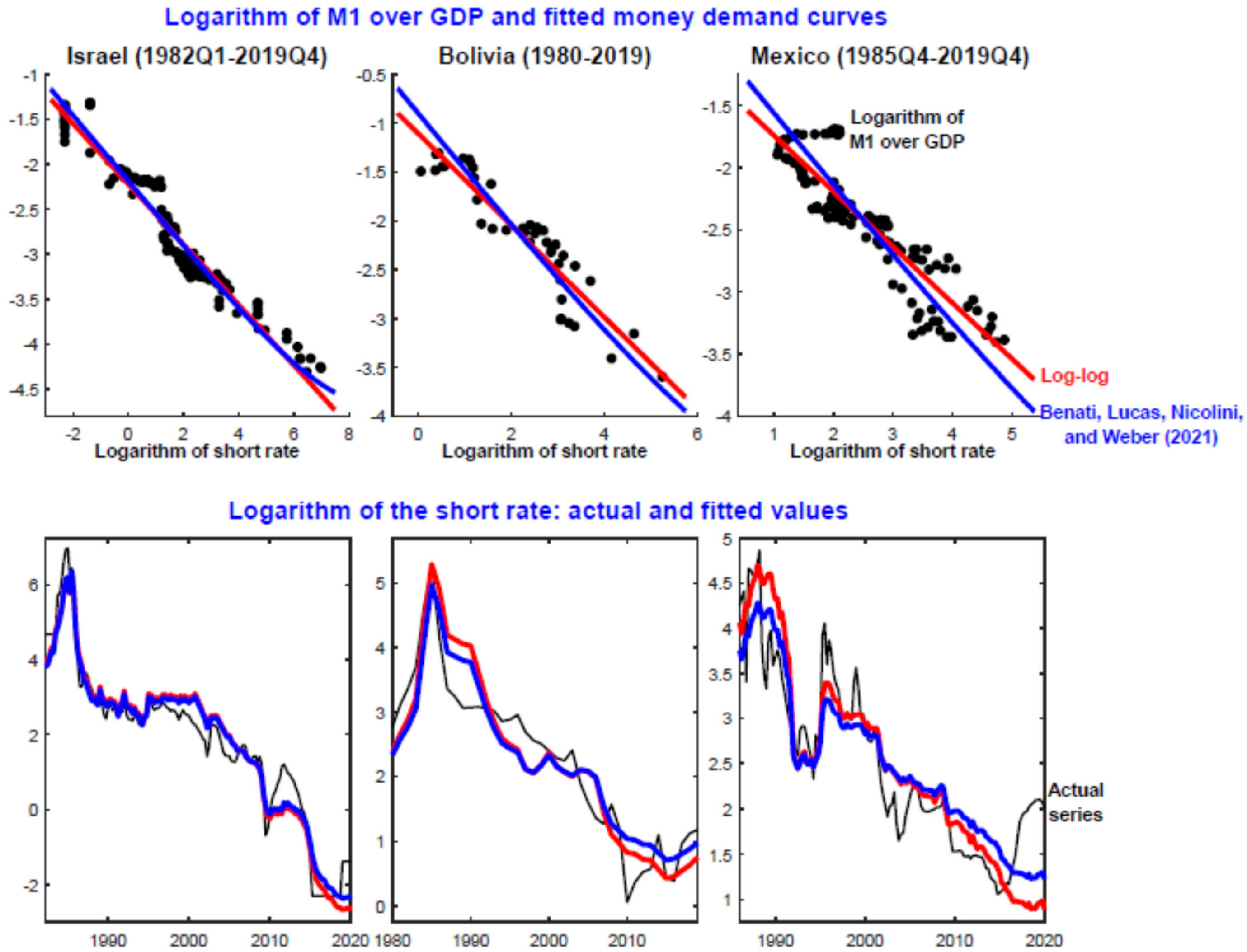
Whereas Baumol and Tobin assumed that the cost of making transactions increases linearly with n_t, Benati et al.'s (2021) benchmark functional form is given by

$$\theta(n_t) = \psi n_t^{\sigma} \tag{9}$$

where ψ and σ are positive constants (with $\sigma = 1$ we obtain the Baumol-Tobin case). Notice that ψn_t^{σ} is the welfare cost of inflation expressed as a fraction of maximum potential output.

When the cost of making transactions is described by (9), the solution for n_t is

$$\sigma\psi \frac{n_t^{\sigma+1}}{1-\psi n_t^{\sigma}} = R_t = \pi_t^e \tag{10}$$



Evidence on the similarity between Meltzer's (1963) log-log and Benati, Lucas, Nicolini and Weber's (2021) money demand

In order to replicate the fall in real money balances associated with increases in the inflation rate, and therefore in expected inflation, in expression (10) it ought to be the case that $1 - \psi n_t^{\sigma} > 0$, which implies that the welfare cost of inflation expressed as a fraction of maximum potential output ought to be smaller than one. Since all of the 20 hyperinflations analyzed herein have been characterized by dramatic collapses in real money balances, in what follows I assume that this condition is satisfied.

By combining expressions (1) and (8), log seignorage is given by

$$\ln \xi_t = \ln \theta_t - \ln n_t \tag{12}$$

Taking logarithms of (10), and then taking derivatives with respect to time, we obtain

$$\frac{d \ln \pi_t^e}{dt} = \frac{(1+\sigma) - \psi n_t^{\sigma}}{1 - \psi n_t^{\sigma}} \left[\frac{d \ln n_t}{dt} \right] \tag{13}$$

By the same token, taking logarithms of (8), and then taking derivatives with respect to time, we obtain

$$\frac{d \ln n_t}{dt} = \pi_t - \theta_t \tag{14}$$

Combining (13) and (14) we obtain

$$\frac{d \ln \pi_t^e}{dt} = \underbrace{\frac{(1+\sigma) - \psi n_t^{\sigma}}{1 - \psi n_t^{\sigma}}}_{\Psi(n_t)} [\pi_t - \theta_t] = \Psi(n_t) [\pi_t - \theta_t] \tag{15}$$

As previously discussed, the empirically relevant case is $1 - \psi n_t^\sigma = \Psi(n_t) > 0$. The solution for money growth, money velocity (and therefore its inverse, the demand for real money balances as a fraction of GDP), inflation, and seignorage is fully characterized by equations (10), (12), and (15).

2.2.1 Steady-state and dynamics

In the steady-state $d \ln \pi_t^e / dt = 0$ and $\pi_t = \pi_t^e$, so that once again expression (7), $\pi_t = \pi_t^e = \theta_t$. holds. Within the present context the GG curve becomes

$$\pi_t^e = \sigma\psi \frac{\theta_t^{\sigma+1} \xi^{-(\sigma+1)}}{1 - \psi\theta_t^\sigma \xi^{-\sigma}} \tag{16}$$

where once again ξ, which is assumed to be exogenously given, acts as a shifter for the GG curve. The steady-state equilibrium lies at the intersection between this curve and the 45 degree line (7).

It can be trivially shown that, from a qualitative point of view, both the shape of the GG curve, and the dynamical properties of the system, are exactly the same as those for the log-log that are shown in the right hand-side panel of Figure 1. In particular, since in equation (15) $\Psi(n_t) > 0$, this expression implies once again that when the economy's position on the GG curve is below (above) the 45 degrees line, so that $\pi_t - \theta_t < 0$ (> 0), $d \ln \pi_t^e / dt < 0$ (> 0). Once again the implication is that the steady-state A is stable, whereas the steady-state B is unstable, and beyond it lies a region of explosive inflation.

2.2.2 Money growth, velocity and seignorage in the steady-state

Based on the solution (10) and the fact that in the steady-state $n_t = n$ and $\pi_t^e = \theta$, by differentiang we obtain

$$\frac{dn}{d\theta} = \left[(\sigma + 1)\sigma\psi n_t^\sigma + \left(\frac{\sigma\psi n_t^\sigma}{1 - \psi n_t^\sigma} \right)^2 \right]^{-1} > 0 \tag{17}$$

which implies that velocity is uniformly increasing in money growth. Finally, by the same token

$$\frac{d\xi}{d\theta} = \frac{(\sigma + \xi)\psi^{\frac{1}{\sigma}}(1 + \frac{\sigma}{\xi})^{\frac{1}{\sigma}}}{1 + \sigma + \xi} > 0 \tag{18}$$

which implies that in the steady-state seignorage is also uniformly increasing in money growth. I now turn to a brief overview of the literature.

3.1 Cagan (1956) on the most appropriate functional form for the demand for real money balances

Cagan (1956) did not derive the semi-log specification (2) within a micro-founded framework, but rather simply postulated it.[4] In reaction to the empirical shortcoming of the postulated specification for the latests stages of hyperinflations, for which the models' fit had typically been worse than for the initial stages,[5] he speculated that an alternative functional form may be needed in order to provide a better characterization of the data. In particular, he conjectured[6]

> '[...] that the function that determines the demand for real cash balances does not conform to [the semi-log functional form]. To be consistent with the data, this hypothesis requires that all observations that lie to the right of the linear regression shall fall in order along some curved regression function [...].'

In practice, this means that the alternative specification he was speculating about should have been either a log-log, or a functional form close to it. In the end, however, Cagan's solution[7] was neither to use a log-log, nor a specification close to it, but rather to simply exclude, in some cases, the latest observations from the empirical analysis:

> 'The periods covered by the statistical analysis exclude some of the observations near the end of the hyperinflations. The excluded observations are from the German, Greek, and second Hungarian hyperinflations [...]. All the excluded observations lie considerably to the right of the regression lines, and their inclusion in the statistical analysis would improperly alter the estimates of α and β derived from the earlier observations of the hyperinflation.'

It is to be noticed that the three episodes whose latest observations Cagan excluded from the analysis are the *most extreme* in his dataset, i.e. those which, for the purpose of discriminating between the semi-log and the log-log, are the *most informative.*

6.3 Evidence from a VAR-based model comparison exercise

The evidence in the previous two sub-sections is especially persuasive because it is based on the raw data. In this section I complement it with the following model comparison exercise. Based on both all of the 20 episodes considered jointly, and the 10 episodes with either the highest or the lowest median inflation rates, I estimate *via* maximum likelihood the following two specifications for the joint dynamics of the logarithm of real money balances and inflation:

$$\begin{bmatrix} \tilde{m}_{i,t} \\ \pi_{i,t} \end{bmatrix} = \begin{bmatrix} c_i^{\tilde{m}} + A(L)\tilde{m}_{i,t\text{-}1} + B(L)\pi_{i,t\text{-}1} + \epsilon_{i,t}^{\tilde{m}} \\ c_i^{\pi} + C(L)\tilde{m}_{i,t\text{-}1} + D(L)\pi_{i,t\text{-}1} + \epsilon_{i,t}^{\pi} \end{bmatrix} \quad (22)$$

and

$$\begin{bmatrix} \tilde{m}_{i,t} \\ \pi_{i,t} \end{bmatrix} = \begin{bmatrix} c_i^{\tilde{m}} + A(L)\tilde{m}_{i,t\text{-}1} + B(L)\tilde{\pi}_{i,t\text{-}1} + \epsilon_{i,t}^{\tilde{m}} \\ c_i^{\pi} + \exp\left\{C(L)\tilde{m}_{i,t\text{-}1} + D(L)\tilde{\pi}_{i,t\text{-}1}\right\} + \epsilon_{i,t}^{\pi} \end{bmatrix} \quad (23)$$

where i indexes the country; t indexes the month; $\tilde{m}_{i,t} \equiv \ln(M_{i,t}/P_{i,t})$; $\tilde{\pi}_{i,t} \equiv \ln(\pi_{i,t})$; $A(L)$, $B(L)$, $C(L)$, and $D(L)$ are polynomials in the lag operator, L; $c_i^{\tilde{m}}$ and c_i^{π} are country-specific intercepts; and $\epsilon_{i,t}^{\tilde{m}}$ and $\epsilon_{i,t}^{\pi}$ are country-specific residuals, which I postulate to follow a bivariate normal distribution with a non-diagonal covariance matrix. Equation (22) describes a panel VAR model with country-specific fixed-effects for log real money balances and inflation, and it therefore corresponds to the semi-log specification. Expression (23), on the other hand, postulates that, up to country-specific dummies and random disturbances, the joint dynamics of real money balances and inflation is described by

$$\begin{bmatrix} \tilde{m}_{i,t} \\ \tilde{\pi}_{i,t} \end{bmatrix} = \begin{bmatrix} A(L) & B(L) \\ C(L) & D(L) \end{bmatrix} \begin{bmatrix} \tilde{m}_{i,t\text{-}1} \\ \tilde{\pi}_{i,t\text{-}1} \end{bmatrix} \quad (24)$$

corresponding to the log-log specification. By casting (24) into the form (23)—i.e, taking as the dependent variable, in the second equation, the *level* of inflation, rather than its logarithm—it is possible to meaningfully compare, in terms of log-likelihood, which of the two functional forms provides the most plausible description of the data. As we will see, evidence overwhelmingly favors the log-log.[24]

Estimating the elasticity of money demand based on the 'log-log'

Table 4 Evidence from panel cointegrated VARs for the logarithms of real money balances and inflation: point estimates of the elasticity, 90% bootstrapped confidence interval, and fractions of bootstrap replications for which the elasticity is smaller than -1

	Point estimates, and 90% confidence interval	Fractions of bootstrap replications for which elasticity is below -1
	Based on all 20 episodes	
k=1	-0.502 [-0.593; -0.412]	0.000
k=2	-0.554 [-0.656; -0.454]	0.000
	Based on 10 episodes with highest median inflation	
k=1	-0.792 [-0.896; -0.681]	0.001
k=2	-0.926 [-1.041; -0.796]	0.143
	Based on 10 episodes with lowest median inflation	
k=1	-0.295 [-0.392; -0.191]	0.000
k=2	-0.312 [-0.412; -0.202]	0.000