How Oil Shocks Propagate: Evidence on the Monetary Policy Channel[*]

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Abstract

We examine the effects of systematic monetary policy on oil shock transmission. Using high-frequency responses of oil futures prices to prominent oil market news, we estimate responses of macro variables to oil supply news shocks when systematic monetary policy is switched off by the zero lower bound (ZLB) and when it is not (normal periods) in Japan, the United Kingdom, and the United States. We emphasize two results. First, in these countries, negative oil supply news shocks are less contractionary at the ZLB than in normal periods. Moreover, these shocks are expansionary at the ZLB. Second, inflation expectations increase during the ZLB and normal periods, while the short nominal interest rates do not respond to oil shocks at the ZLB. Our evidence points to the importance of systematic monetary policy for the transmission of oil shocks.

Keywords: oil price shocks, high-frequency identification, zero lower bound, systematic monetary policy

JEL Classification: E5, E7, G4

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1 Introduction

Oil price shocks are among the most prominent macroeconomic disturbances. They are large and sudden, especially when caused by wars involving oil producers. They are challenging for policymakers because they move inflation and output in opposite directions (according to the conventional supply-side view), making it difficult to stabilize both with monetary policy tools. They are global, affecting all countries simultaneously because crude oil is an internationally traded commodity. The recent events, such as the oil price increase in 2021-22, the 2022 Russian invasion of Ukraine, and the burst of high inflation worldwide in 2021-22, renewed interest in understanding the magnitudes and channels of oil shock effects.

The fact that most post-WWII recessions in the United States followed oil price spikes inspired a vast empirical literature on oil shock effects (Hamilton, 1983; Kilian, 2009; Kilian and Murphy, 2012; Baumeister and Hamilton, 2019; Kanzig, 2021). This literature has documented substantial effects of oil shocks on macroeconomic variables relying on a variety of econometric approaches. At the same time, how exactly oil shocks propagate is still unclear. Theoretical literature proposes aggregate supply channels, such as variations in the cost of production, which can be amplified by markup endogeneity (Rotemberg and Woodford, 1996) and production networks nonlinearities (Baqaee and Farhi, 2019) as well as aggregate demand channels, such as the interaction between nominal price stickiness and real wage rigidity (Blanchard and Gali, 2009) and the presence of financially constrained households (Chan *et al.*, 2022; Auclert *et al.*, 2023). One aggregate demand channel stands out, and it is related to the following observation.

Most post-WWII recessions in the United States also followed monetary policy hikes (Bernanke, Gertler and Watson, 1997). This raises the possibility that the measured effects of oil shocks on real activity can work indirectly through systematic monetary policy reaction to these shocks. Such logic is central in standard New Keynesian models, where changes in systematic monetary policy can influence both the magnitudes and even signs of responses to shocks (Eggertsson, 2008; Bodenstein, Guerrieri and Gust, 2013).

In this paper, we empirically evaluate the role of monetary policy in oil shock propagation. We leverage the fact that several countries have experienced ZLB episodes recently. This allows us to estimate the effects of oil shocks, relying on the data from the ZLB periods when monetary policy did not respond actively. We start our analysis by presenting a simple theoretical framework consisting of a small-open economy and the rest of the world to illustrate how oil supply shocks propagate through aggregate supply, aggregate demand (via monetary policy), and international spillovers that affect both aggregate supply and demand. We then show that the key difference that ZLB introduces in this environment is the change in the sign of the real interest rate response, which can lead to a flip in the sing of output response.

We then examine how Japan, the United Kingdom, and the United States responds to oil shocks between 1975:1 and 2019:12, differentiating between ZLB and normal periods. Our primary focus is on Japan because it has the longest ZLB experience, which started at the end of 1995. Moreover, Japan experienced several business cycles during the ZLB period, making it possible to average out the effects of (potentially) differential impact of shocks during booms and busts. In addition, Japan was a net oil importer throughout our sample. Looking at the data in other countries, such as the United Kingdom and the United States, is helpful because the ZLB periods in different countries do not coincide, which alleviates the concern that some aggregate factor is responsible for results in all countries.

To extract exogenous variation in oil prices, we build on the recent literature that uses high-frequency data to identify unexpected and exogenous macroeconomic shocks (Kuttner, 2001; Gürkaynak, Sack and Swanson, 2004; Gertler and Karadi, 2015; Nakamura and Steinsson, 2018). In particular, we follow Kanzig (2021), who uses the changes in oil futures prices in a tight window around the Organization of the Petroleum Exporting Countries (OPEC) production announcements. The series of oil futures price changes then becomes an external instrument in an oil market vector-autoregression, allowing us to estimate structural oil supply news shocks. These shocks change oil prices on impact and the production of oil gradually.

Using a state-dependent Jorda's local projection method, we show that oil supply news shocks are less contractionary in the ZLB than in normal periods. In particular, in Japan, industrial production increases by 1.3 percent after one year during the ZLB following a shock that increases the oil price by 10 percent. Outside of the ZLB period, industrial production falls, reaching almost minus one percent 12 months after the shock. The differences in the two responses are statistically significant at conventional levels. It is particularly remarkable that the increase in industrial production during the ZLB period is significantly different from zero (at some horizons). The unemployment rate in Japan exhibits similar patterns with the opposite sign: it decreases significantly in the ZLB period, while it stays near zero or increases outside of the ZLB. Since Japan has a long period with a zero nominal interest rate, we also estimate the effects of oil supply shocks using quarterly macro variables. Consistent with the results from industrial production and the unemployment rate, oil supply news shocks cause real per capita gross domestic product (GDP), consumption, and investment to increase in the ZLB period and decrease outside of the ZLB.

We next explore the relevance of the monetary policy channel by estimating the responses of the 3-month and 5-year nominal interest rates, realized inflation, and inflation expectations. In the ZLB period, the short-term interest rate does not react to oil shocks, and the 5-year rate increases slightly. In contrast, during the normal period, both the short and 5-year rates increase considerably after an oil price spike, and these changes are statistically different from those during the ZLB period. Moreover, inflation expectations react more during the ZLB. These results are consistent with the channel that works through ex-ante real interest rate movements. At the same time, we do not observe a stronger reaction of the realized inflation during the ZLB period. In fact, the realized inflation increases more in the normal period.

Our estimated effects of oil supply news shocks in the United Kingdom and the United States are consistent with those in Japan. For example, in the US, industrial production increases by 0.9 percent after 15 months following a shock that increases the oil price by 10 percent. Outside of the ZLB period, an industrial production fall reaches 0.8 percent. The difference in the two responses is statistically significant at conventional levels, and the ZLB response is significantly above zero at the 5 percent level for multiple horizons. The unemployment rate repeats this pattern with the opposite sign. The nominal interest rate responds more at the ZLB than in the normal period, while both expected and realized inflation rates respond identically in the two periods. Still, this is consistent with the fall in the real interest rate in the ZLB period. The results in the UK are qualitatively similar. These findings imply that the patterns obtained in Japan are not a particular feature of the Japanese economy.

In the last part of the paper, we perform a sensitivity analysis of our baseline results. We start by considering alternative oil supply shocks obtained in two prominent recent studies, such as Kilian (2009) and Baumeister and Hamilton (2019). These papers rely on structural VAR identification of supply shocks. Kilian (2009) assumes that oil supply does not react to changes in oil prices in the short run to disentangle oil supply from demand shocks.[1] This identification leads to an empirical conclusion that oil supply shocks played only a minor role in historical oil price fluctuations. Baumeister and Hamilton (2019) argue that it is more realistic to assume the oil supply reacts to oil price changes even in the short run. Based on prior studies, they assign an informative prior to the oil supply elasticity and use Bayesian methods to estimate an oil VAR. Their posterior results support the view that the oil supply elasticity is economically significant and that oil supply shocks explain a significant share of oil price fluctuations in the past. We find that the difference between the ZLB and normal periods responses of the Japanese economy following oil supply shocks identified as in Kilian (2009) is not statistically significant.

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[1] Kilian (2009) estimates a 3-variable VAR where oil production is ordered first, the index of real economic activity second, and the real price of oil third. Then, it is assumed that the structural oil shocks are mapped into a reduced-form VAR shocks via a lower-triangular matrix.

This is consistent with Wieland (2019), who also used these oil supply shocks to estimate the response of the Japanese economy during and outside of the ZLB. At the same time, the difference in the responses to a Baumeister and Hamilton (2019) oil supply shock is closer to our baseline results. The discrepancy in results based on the two types of oil shocks can potentially be explained by the fact that oil supply shocks identified in Kilian (2009) virtually do not change oil prices. In other words, it may be difficult to estimate the real effects of oil supply shocks when the identified oil supply shocks have low power, generating a weak response to oil prices.

Related literature. Our paper contributes to the literature that studies the role of monetary policy in shock propagation by estimating a differential impact of shocks during ZLB (or constant nominal interest rate periods) and normal periods. On the aggregate demand side, Miyamoto, Nguyen and Sergeyev (2018) and Ramey and Zubairy (2018) estimated higher government spending multipliers in the ZLB period than outside of it in Japan and the United States, respectively. On the aggregate supply side, Bernanke, Gertler and Watson (1997) showed that most of the US macroeconomic response to oil shocks is due to a systematic monetary policy. Their analysis relies on the triangular vector-autoregression identification of oil shocks and a counterfactual simulation with a fixed policy interest rate in the spirit of Sims and Zha (2006), which aims at removing systematic monetary policy effects but can be subject to the Lucas critique.[2] Garin, Lester and Sims (2019) find that *positive* TFP shocks, defined as in Fernald (2014), are more expansionary in the ZLB than in the normal periods in the United States. As mentioned, Wieland (2019) finds no difference in Japan's output reactions to oil supply shocks identified as in Kilian (2009). Unlike these contributions, we use high-frequency identification. The identified shocks have an economically significant impact on the economy. Moreover, we use data from several countries, which is essential when studying relatively short ZLB periods. Unlike Garin, Lester and Sims (2019) and Wieland (2019), we find that shocks that depress aggregate supply increase output during the ZLB periods relative to the normal periods. Differently from Bernanke, Gertler and Watson (1997), we do not rely on counterfactual exercises and estimate our results directly from the episodes with fixed interest rates. In addition, our findings are consistent with the indirect evidence in Datta, Johannsen, Kwon and Vigfusson (2021). The authors estimate that oil prices and equity returns are more positively correlated in the recent past than before and that both variables become more responsive to macroeconomic news, consistent with the prediction of a model where the ZLB alters the economic environment.

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[2] Wolf and McKay (2022) extend the Sims and Zha (2006) methodology by incorporating the expectations effects via additional information about the responses of economic variables to *news* shocks. Importantly, this method assumes that people are aware of counterfactual policy changes.

The rest of the paper is organized as follows. Section 2 starts with a simple model highlighting the effects of oil price changes and the interaction of shocks with monetary policy. Section 3 explains how we measure the effects of oil supply news shocks on the aggregate economy and summarizes the data. We then present the main results for Japan in Section 4 and the United Kingdom and the United States in Section 5. We describe our sensitivity analysis in Section 6. Section 7 concludes.

2 Oil Shocks in an Open Economy New Keynesian Model

To set the stage for our empirical analysis, we present standard Open Economy New Keynesian model predictions about the effects of oil shocks. We use a model in Galí and Monacelli (2002) extended with oil intermediate inputs used by firms in the spirit of Bodenstein, Guerrieri and Gust (2013).[3] Our goal is to illustrate the effects of oil shocks through aggregate supply, systematic monetary policy, and the world's reaction to oil shocks.

2.1 The Model

Our environment consists of a small open home country and a foreign country representing the world economy. Each country has a representative household that consumes home and foreign goods but with home-biased preferences, saves via complete domestic and international asset markets, and supplies labor to domestic firms.[4] Monopolistically competitive firms produce differentiated goods from domestic labor and oil using a constant-returns-to-scale production function. These firms set their nominal prices in domestic currencies and change them infrequently a la Calvo. Each period, both countries receive a stochastic endowment of oil. All goods, including oil, cannot be stored. All goods are also traded globally, implying the low of one price.

We first describe the foreign country—the world economy—by showing log-linearized equilibrium conditions, which we derive in Appendix B. Aggregate consumption follows the Euler equation

$$\mathbb{E}_t\left[\widehat{c}^*_{t+1}\right] - \widehat{c}^*_t = \frac{1}{\sigma}\left(i^*_t - \mathbb{E}_t\left[\pi^*_{F,t+1}\right] - \iota\right), \tag{1}$$

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[3] Galí and Monacelli (2002) is a working paper of the classical Gali and Monacelli (2005) paper. The key difference between the two is that the working paper features two countries, a small open economy and the world economy, while the published version has a continuum of small open-economies.

[4] Chan *et al.* (2022) and Auclert *et al.* (2023) study how oil shocks affect a small-open economy in the presence of a household heterogeneity using a two agent (TANK) and heterogeneous agent New Keynesian (HANK) models, respectively. Household heterogeneity adds an extra aggregate demand effect from which we abstract here.

where asterisk denotes foreign country variables, $\widehat{c}_t^*$ is a log deviation of world consumption from its steady state, i_t^* is the nominal interest rate, $\pi_{F,t+1}^*$ is simultaneously a consumer price index (CPI) and producer price index (PPI) inflation rate because the world economy is closed, ι is the steady state nominal interest rate, and σ is the inverse of the intertemporal elasticity of substitution. The Phillips curve is

$$\pi_{F,t}^* = \kappa\zeta^*\widehat{c}_t^* + \beta\mathbb{E}_t\left[\pi_{F,t+1}^*\right] + \kappa\psi_o^*\widehat{r}_t, \tag{2}$$

where ζ^* and ψ_o^* are composite parameters that represent the sensitivities of firm's marginal costs to consumption $\widehat{c}_t^*$ and the real oil price $\widehat{r}_t$ expressed in units of foreign goods, κ is the sensitivity of inflation to changes in the marginal cost.[5] The last term in the Phillips curve represents the effect of changes in oil price on inflation. The foreign central bank sets the interest rate according to the following Taylor rule

$$i_t^* = \iota + \varphi_\pi^*\pi_{F,t}^*, \tag{3}$$

where $\varphi_\pi^* \geq 0$, the target inflation is zero, and the effective lower bound is not explicitly introduced. The equilibrium on world's oil market determines the real oil price

$$\widehat{r}_t = \phi_c\widehat{c}_t^* - \phi_o\widehat{o}_t^*, \tag{4}$$

where $\phi_c > 0$ indicates that oil price increases with world economic activity, and $\phi_o > 0$ implies that the oil price increases when oil supply drops.[6]

Home country consumption dynamics follows a similar Euler equation

$$\mathbb{E}_t\left[\widehat{c}_{t+1}\right] - \widehat{c}_t = \frac{1}{\sigma}\left(i_t - \mathbb{E}_t\left[\pi_{t+1}\right] - \iota\right), \tag{5}$$

while the home-produced goods inflation rate obeys the Phillips curve

$$\pi_{H,t} = \kappa\zeta_H\widehat{c}_t + \beta\mathbb{E}_t\left[\pi_{H,t+1}\right] + \kappa\psi_o\widehat{r}_t + \kappa\zeta_F\widehat{c}_t^* + \kappa\psi_q\widehat{q}_t, \tag{6}$$

where, in addition, to the channels present in the Phillips curve of the foreign country in equation (2), the term $\kappa\zeta_F\widehat{c}_t^*$ incorporates the effect of global aggregate demand on domestic inflation, while the last term shows the influence of the real exchange rate $\widehat{q}_t$ on the marginal costs of home firms and, hence, the inflation rate. Intuitively, when the global aggregate demand drops, foreign residents purchase fewer home goods reducing

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[5] The composite coefficients $\kappa, \zeta^*, \psi_o^*$, are expressed in equations (B.24), (B.27), and (B.28) in Appendix B through preferences and production function parameters.

[6] ϕ_c and ϕ_o are in equations (B.25) and (B.26).

marginal costs of home firms and, hence, the inflation rate. In addition, when home currency depreciates in real terms ($\widehat{q}_t$ increases), oil becomes more expensive to buy at home for any level of the real oil price $\widehat{r}_t$ denominated in foreign goods. As a result, home firms' marginal costs and the inflation rate go up. The central bank at home follows the Taylor rule

$$i_t = \iota + \varphi_\pi \pi_{H,t}, \tag{7}$$

where $\phi_\pi \geq 0$. The link between the CPI and PPI inflation rates is

$$\pi_t = \pi_{H,t} + \frac{1-\Omega}{\Omega} \Delta \widehat{q}_t, \tag{8}$$

where $\Omega \in [0,1]$ is the share of home-goods in home household spending, which represents the degree of home bias. The second term expresses the effect of real depreciation of domestic currency on the consumption basket inflation. The coefficient multiplying the change in real exchange rate is the ratio of foreign-produced goods to home-produced goods spending in the consumption basket of home household. The production of home-goods is a sum of home and foreign demand for home goods. Formally,

$$\widehat{y}_{H,t} = \frac{\Omega}{\Omega + 1 - \Omega^*} \widehat{c}_t + \frac{1-\Omega^*}{\Omega + 1 - \Omega^*} \widehat{c}_t^* + \frac{\Omega\left(1-\Omega\right) + \left(1-\Omega^*\right)}{\Omega + 1 - \Omega^*} \cdot \frac{\gamma_n}{\Omega} \widehat{q}_t. \tag{9}$$

where $1 - \Omega^*$ is the proportion of consumption that foreign households allocate to home goods.[7] The last two terms express two forces that take into account that oil shocks affect home country through the world economy. The presence of $\widehat{c}_t^*$ captures the spillover of world consumption demand to domestic goods. The term featuring the real exchange rate $\widehat{q}_t$ measures the strength of the expenditure switching effect between foreign to home goods, and it crucially depends on the elasticity of substitution between home and foreign goods γ_n.

We assume perfect risk sharing across countries. This requires the real exchange rate to be proportional to the difference in consumption in the two countries

$$\widehat{q}_t = \sigma \left(\widehat{c}_t - \widehat{c}_t^* \right). \tag{10}$$

We will look into the consequences of a one-time unanticipated negative global oil supply shock $\widehat{o}_0^* = \widehat{o}^* < 0$ that reverts back to its steady state value of zero with probability $\alpha \in [0,1]$. There are no other source of uncertainty.

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[7] As Appendix B formally shows, we assume that foreign demand for domestic goods is proportional to the size of home country. Because we consider a limit when the size of home country tends to zero (the small-open economy assumption), both home production and foreign demand for home goods shrink to zero. However, their ratio stays positive in the limit.

Finally, instead of explicitly representing the effective the lower bound (ELB) on the nominal interest rate, we assume that the nominal interest rate stays constant following an oil price shock. This assumption retains the key property of the ELB that the interest rate does not respond to developments in the economy. At the same time, it avoids the need to introduce additional notation. See Nakamura and Steinsson (2014) for a similar treatment in the context of fiscal multipliers.

2.2 The Effects of Oil Shocks

World economy. We start by presenting the responses of foreign goods production, which equals consumption, and the inflation rate. We look for a unique bounded Markov equilibrium where endogenous variables $\{\widehat{c}_t^*, \pi_{F,t+1}^*, i_t^*, \widehat{r}_t\}$ are linear functions of oil supply $\widehat{o}_t^*$. The Phillips curve and the Euler equation (together with the Taylor rule) are

$$\pi_F^* = \frac{\kappa\left(\zeta^* + \psi_o^* \phi_c\right)}{1-\beta\alpha}\widehat{c}^* + \frac{\kappa \psi_o^* \phi_o}{1-\beta\alpha}\left(-\widehat{o}^*\right), \tag{11}$$

$$\widehat{c}^* = -\frac{\left(\varphi_\pi^* - \alpha\right)\pi_F^*}{\left(1-\alpha\right)\sigma}. \tag{12}$$

Figure 1 plots the two equations under the assumption that the monetary policy is active, that is, $\varphi_\pi^* > 1$ (the left panel), and when the monetary policy does not change following the oil shock, that is, $\varphi_\pi^* = 0$ (the right panel). The upward-sloping Euler equation on the right panel is due to the fact that higher inflation in the absence of the response of the nominal interest rate reduces the real interest rate and increases aggregate consumption. For the unique bounded solution to exist, the Euler equation should be steeper than the Phillips curve when they have the same sign of the slope. Hence, we focus on the parameters that satisfy

$$\left(1-\beta\alpha\right)\left(1-\alpha\right)\sigma + \kappa\left(\varphi_\pi^* - \alpha\right)\left(\zeta^* + \psi_o^* \phi_c\right) > 0. \tag{13}$$

This inequality is trivially satisfied when the monetary policy is active. When $\varphi_\pi^* = 0$, however, inequality (13) restricts the parameter space.

A decline in the oil supply results in an upward shift in the Phillips curve in both panels of Figure 1. Under active monetary policy, the central bank increases the nominal interest rate such that the real interest rate rises reducing aggregate output. At the same time, under the constant nominal interest rate policy, the real rate falls stimulating the world economy. Formally, a decline in the oil supply that increases the oil price by one percent changes output by

$$\widehat{c}^{*}=-\frac{\kappa\left(\varphi_{\pi}^{*}-\alpha\right)\zeta^{*}}{\left(1-\beta\alpha\right)\left(1-\alpha\right)\sigma+\kappa\left(\varphi_{\pi}^{*}-\alpha\right)\zeta^{*}}\cdot\frac{\psi_{o}^{*}}{\zeta^{*}},$$

where the ratio $-\psi_o^*/\zeta^*$ is the change in world output in the absence of nominal rigidities. The last formula implies that with active monetary policy, the output response is dampened compared to the neoclassical benchmark.[8] Under the constant interest rate policy, the consumption response changes sign and the size is larger in absolute terms compared to the neoclassical case.

Small-open economy. Similarly to the world economy, we can represent the Phillips curve and Euler equation of the home economy as

$$\pi_{H}=\frac{\kappa\left(\zeta_{H}+\psi_{q}\sigma\right)}{1-\alpha\beta}\widehat{c}+\frac{\kappa}{1-\alpha\beta}\psi_{o}\widehat{r}+\frac{\kappa\left(\zeta_{F}-\psi_{q}\sigma\right)}{1-\alpha\beta}\widehat{c}^{*},\tag{14}$$

$$\widehat{c}=-\Omega\frac{\left(\varphi_{\pi}-\alpha\right)}{\sigma\left(1-\alpha\right)}\pi_{H}+\left(1-\Omega\right)\widehat{c}^{*}.\tag{15}$$

The two equations closely resemble the world equations (11) and (12), with the exception of additional terms. The final term in the Phillips curve encapsulates two global forces. Firstly, domestic marginal costs and inflation fall when global economic activity slows down. Secondly, to keep the marginal utilities of consumption equalized in both countries as prescribed by equation (10), the home currency depreciates in real terms when global economic activity falls. This makes the purchase of oil from the global market costlier, and consequently increases the marginal costs of production at home. The net effect of these two forces is ambiguous and it depends on the degree of openness of home country. The final term in the Euler equation stems from the fact that a decline in global economic activity, which is expected to revert in the future, appreciates home currency in real terms over time. This, in turn, reduces home inflation and, hence, increases the real interest rate for any level of the nominal interest rate. As a result, domestic aggregate demand falls.

Figure 2 plots equations (14) and (15) when when monetary policy in the rest of the world actively responds to the oil shock. The main take away from these diagrams is that a decline in oil supply that increases the oil price can also be expansionary when the leftward shift in the Euler equation is small enough, which is the case with a sufficiently high degree of home bias, i.e., when the coefficient Ω is close to one. Finally, we note that

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[8] Intuitively, if output fell as much as in the neo-classical limit and the inflation rate did not change as well (implying unchanged nominal interest rate), then aggregate demand would exceed aggregate supply pushing firms to increase both output and prices.

for the bounded solution to be unique, the following condition must hold

$$\sigma\left(1-\alpha\beta\right)\left(1-\alpha\right)+\kappa\left(\varphi_{\pi}-\alpha\right)\left(\zeta_{H}+\psi_{q}\sigma\right)\Omega>0. \tag{16}$$

This condition always holds when the central bank actively responds to the oil shock, i.e., $\varphi_{\pi} > 1$. However, when $\varphi_{\pi} = 0$, the above condition restricts the set of model parameters.

Illustrative calibration. To illustrate quantitative importance of the highlighted channels, we calibrate the model by choosing the values used in recent literature. The quarterly calibration parameters are listed in Table 1. Most of the parameters are taken from Gali and Monacelli (2005). The exceptions are the share of oil in production, which is an average of home and foreign oil shares in Bodenstein *et al.* (2013). The persistence of oil shocks is set to 0.58 (half life of five months) to match an estimated decline in Japanese GDP of 0.1 percent in the normal period following a shock that increases oil price by 10 percent, which we present in the empirical part. The elasticity of substitution between home and foreign goods consumption is set to 0.5 and between labor and oil in production to 0.1. We chose these somewhat lower than usual values to highlight the short run effects of oil supply shocks. The weight of home goods in home consumption basket is $\Omega = 0.8$, which is set to approximate this value in Japan.[9]

Table 1: Calibration Parameters

$\beta = 0.99$	Discount factor
$\sigma^{-1} = 1$	Intertemporal elasticity of substitution
$\varphi^{-1} = 0.5$	Frisch elasticity of labor supply
$\gamma_n = 0.5$	Elasticity of substitution between home and foreign goods
$\Omega = 0.8$	Share of home goods in home consumption basket
$\alpha = 0.58$	Persistence of oil shocks
$\varphi_{\pi} = \varphi_{\pi}^{*} = 1.5$	Taylor rule coefficient on inflation
$\gamma_y = 0.1$	Elasticity between labor and oil in production
$\omega_{oy} = 0.04$	share of oil in production
$1-\theta = 0.25$	Probability of price adjustment
Composite parameters	
$\kappa = \left(1-\theta\right)\left(1-\beta\theta\right)/\theta = 0.086$	$\zeta^{*} = \left(1-\omega_{oy}\right)\left(\sigma+\varphi\right)/\left(1+\omega_{oy}\gamma_y\varphi\right) = 2.769$
$\psi_o^{*} = \psi_o = \frac{\left(1+\gamma_y\varphi\right)\omega_{oy}}{1+\gamma_y\varphi\omega_{oy}} = 0.077$	$\phi_c = \left\{1+\gamma_y\left[\varphi+\left(1-\omega_{oy}\right)\sigma\right]\right\}/\left[\left(1-\omega_{oy}\right)\gamma_y\right] = 5.167$
$\phi_o = \frac{1+\omega_{oy}\gamma_y\varphi}{\left(1-\omega_{oy}\right)\gamma_y} = 2.167$	$\psi_q = \frac{1-\Omega}{\Omega}\cdot\frac{\left(1+\varphi\gamma_n\right)\left(1-\omega_{oy}\right)}{1+\gamma_y\varphi\omega_{oy}} = 1.27$
$\zeta_H = \frac{1-\omega_{oy}}{1+\gamma_y\varphi\omega_{oy}}\left(\sigma+\varphi\frac{\Omega}{\Omega+1-\Omega^{*}}\right) = 2.05$	$\zeta_F = \frac{1-\omega_{oy}}{1+\gamma_y\varphi\omega_{oy}}\varphi\frac{1-\Omega^{*}}{\Omega+1-\Omega^{*}} = 0.808$
$\Omega^{*} = \Omega-\omega_{oy}/\left(1-\omega_{oy}\right) = 0.558$	$\iota = -\log\beta = 0.01$

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[9] This value is lower than the one typically used for the US, but higher that that for the Euro Area countries.

Under these parameters and the shock to oil supply that increase oil price by ten percent, the world output falls by 0.09 percent under active monetary policy and increases by 0.65 percent when the nominal interest rate is constant. The home economy's output falls by 0.10 percent when monetary policy is active both at home and abroad, and home output increases by 0.69 percent when monetary policy does not respond at home but responds in the rest of the world. Using equation (9), we decompose the 0.69 percent change in output into 0.45 percent due to a higher aggregate demand at home, -0.02 percent due to a lower foreign demand, and 0.26 percent due to expenditure switching.

3 Measurement and Data

This section describes the identification of oil shocks, our empirical specification, and the data we use for estimation.

3.1 Measurement of Oil Shock Effects

High-frequency identification. To isolate exogenous and unanticipated changes in oil prices, we use a high-frequency approach, popular in monetary economics (Kuttner, 2001; Gürkaynak, Sack and Swanson, 2004; Gertler and Karadi, 2015; Nakamura and Steinsson, 2018) and recently applied to the oil market in Kanzig (2021), which we follow closely.

Next, we brief describe the high-frequency approach applied to the oil market. The main idea relies on the observation that the oil market is dominated by a few large players, one of which—the OPEC—is responsible for about half of the world's oil production. The OPEC is an intergovernmental organization that coordinates its oil production during conference meetings that end with public announcements about oil supply changes. Asset prices, particularly oil futures prices, react immediately to these announcements. Hence, looking at oil futures price changes in a narrow window of one day around OPEC announcements reveals unanticipated and arguably exogenous news about future oil supply.[10]

Following Kanzig (2021), we use the prices of the West Texas Intermediate (WTI) crude oil futures because they are the most liquid contracts with the largest sample size starting in 1983. To flexibly capture the horizon of OPEC announcements, we extract the first principle component of future prices with maturities from one to twelve months. We sum up these price changes to aggregate them to a monthly frequency. While, in principle,

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[10] The size of the window is motivated by the fact that the exact time of the announcement are unavailable.

these price changes can serve as oil supply news shocks, in practice, they are only a small share of news that affects the oil market. As a result, they will serve as an instrument.

External instruments approach. We use the oil futures price changes as an external instrument —a variable that is correlated with the shock of interest but not the other shocks—in a 4-variable oil market vector autoregression (VAR) following Stock and Watson (2012); Mertens and Ravn (2013).[11] Formally, consider a VAR(p) process of the form

$$Y_t = C + \sum_{j=1}^{p} B_l Y_{t-l} + \mathcal{U}_t,$$

where p is the number of lags, Y_t is an $n \times 1$ vector of endogenous variables, $\mathcal{U}_t$ is an $n \times 1$ vector of reduced-form errors with the variance-covariance matrix Σ, C is an $n \times 1$ vector of constants, $\{B_l\}$ are $n \times n$ matrices of coefficients. The reduced-form errors are related to an $n \times 1$ vector of uncorrelated structural shocks $\mathcal{E}_t$ linearly as $\mathcal{U}_t = S\mathcal{E}_t$, where S is an unknown $n \times n$ matrix. If we order the oil supply news shocks to be the first element of $\mathcal{E}_t$, the goal is to identify the first column of matrix S, which we denote as $S_{1:n,1}$.

Let z_t be an external instrument, that is, the oil supply surprise series. If it is a valid instrument, it must be (i) correlated with the shock of interest, i.e., $\mathbb{E}\left[z_t \mathcal{E}_{1,t}\right] \neq 0$ and uncorrelated with the other shocks, i.e., $\mathbb{E}\left[z_t \mathcal{E}_{k,t}\right] = 0$ for $k = 2, 3, \ldots, n$. Under this assumption, we get

$$S_{2:n,1} = \mathbb{E}\left[z_t \mathcal{U}_{2:n,t}\right] / \mathbb{E}\left[z_t \mathcal{U}_{1,t}\right],$$

and the whole vector $S_{1:n,1}$ equals $(x, xS_{2:n,1})'$, where x is a normalization constant which can take any value. We will normalize x so that the shock $\mathcal{E}_{1,t}$ increases oil price on impact by ten percent. Having computed $S_{1:n,1}$, we estimate the oil supply news shock $\widehat{\mathcal{E}}_{1,t}$. We will call these shocks as oil supply news shocks or oil shocks for brevity.

Main specification. To evaluate the impact of oil supply news shocks on a variable of interest y_t during and outside of the ZLB periods, we estimate a series of regressions at each horizon h from $h = 0$ to H of the form

$$\begin{aligned} y_{t+h} - y_{t-1} = {} & \mathbb{I}_{t-1} \cdot \left[\alpha^y_{A,h} + \beta^y_{A,h} P^{oil}_t + \psi^y_{A,h}(L) x_{t-1}\right] \\ & + (1 - \mathbb{I}_{t-1}) \cdot \left[\alpha^y_{B,h} + \beta^y_{B,h} P^{oil}_t + \psi^y_{B,h}(L) x_{t-1}\right] + \epsilon^y_{t+h}. \end{aligned} \tag{17}$$

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[11] The 4-variable oil vector auto-regression consists of the real price of oil, world oil production, world oil inventories, and world industrial production. Unlike Kanzig (2021), we do not include US industrial production and the US consumer price index (CPI) in the oil VAR in our main analysis because the responses of these variables can be state dependent. In Section 6, we present our results based on a 6-variable VAR.

where $\mathbb{I}_{t-1}$ is the indicator variable that takes the value of one if the economy is in the ZLB in period $t-1$, and zero otherwise, and the subscripts A and B indicate the ZLB and the normal periods, respectively, P_t^{oil} is the log real oil price, and the controls $\psi_{A,h}^{y}(L)x_{t-1}$ are twelve lags of the variable of interest, the unemployment rate, and the measure of oil supply news shock $\widehat{\mathcal{E}}_{1,t}$. In some cases, there are additional controls which we specify separately. The variables of interest, such as industrial production and the consumer price index (CPI), are in logs, while nominal interest rates, the unemployment rate, and inflation expectations are in levels. The state-dependent specification allows us to test the null hypothesis that the responses are identical in the two periods, that is, $\beta_{A,h}^{y} = \beta_{B,h}^{y}$. We instrument the log of oil price P_t^{oil} with the oil supply news shock $\widehat{\mathcal{E}}_{1,t}$, so the estimated coefficients $\beta_{A,h}^{y}, \beta_{B,h}^{y}$ are the impulse response of y at horizon h to a change in real oil price driven by oil supply news shocks.[12]

3.2 Data

We estimate the 4-variable oil VAR using the following variables. The real oil price is the WTI spot oil price deflated by the US CPI, the world industrial production is the Baumeister and Hamilton (2019) index for the OECD countries plus six major economies (Brazil, China, India, Indonesia, the Russian Federation, and South Africa), which jointly represent 75 percent of world's GDP. World oil inventories are the OECD petroleum stocks from Kilian and Murphy (2014). The world oil production is from the U.S. Energy Information Administration. Because of the oil futures data constraint, we use a shorter period between 1983:4 and 2019:12 for identification and a longer period between 1974:1 and 2019:12 to estimate of the oil supply news shock.

To estimate the effects of oil shocks, we use monthly and quarterly macro data for three advanced countries that have experienced a sufficiently long period where the nominal interest rate is at the ZLB. These countries are Japan, the United Kingdom, and the United States. The sample period is between 1975:1 and 2019:12. We exclude the COVID pandemic by dropping data after 2019:12.

The ZLB periods for Japan are October 1995 to June 2006 and January 2009 to December 2019. This definition is consistent with Wieland (2019), who follows the previous literature on the timing of the ZLB spell in Japan. As plotted in the first panel of Figure 3, this period of the ZLB in Japan coincides with a 0.5 percent cutoff before 1998 and 0.25 percent after that.[13] We define the ZLB periods in the United Kingdom and the United

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[12] Unlike in the case of generated regressions, generated instruments do not require standard error correction under very general conditions. See Section 6.1 in Wooldridge (2010) for details.

[13] The results presented with this ZLB definition are similar to those when Japan's cutoff nominal interest rate for the ZLB period is 0.5 percent.

States as the months during which each country's short-term nominal interest rate is at or below 0.5 percent. The second panel of Figure 3 shows that the ZLB period in the United States was between 2008:11 and 2016:11.[14]

The United Kingdom experienced two ZLB episodes: 2009:04–2010:09, 2012:02–2018:07. Table 1 summarizes each series' data availability and sources and defines ZLB periods in each country.

4 Oil Supply News Shocks and their Effects in Japan

This section presents oil supply news shocks and their effects on the Japanese economy when the nominal interest rate is at the zero lower bound and outside of it. Specifically, we present the results for real variables, such as monthly industrial production and the unemployment rate, and then the variables available only quarterly, such as gross domestic product, private consumption, and investment per capita. Finally, we examine the New Keynesian mechanism by estimating the responses of the nominal interest rate and inflation expectations.

4.1 Oil Supply News Shocks

The external variable approach to estimating the oil supply news shocks requires the instrument to be relevant. The first stage regression of the oil price residuals on the instrument (the oil news surprise series) yields an F-statistic of 19.9, well over of a recommended threshold of 10 (Montiel Olea *et al.*, 2021). The instrument explains 3.6 percent of the variation in the oil price residual.

The right panel of Figure 4 plots the estimated oil supply news shocks $\widehat{\mathcal{E}}_{1,t}$ from 1975:1 to 2019:12. There is no noticeable difference between the different parts of the sample in terms of the size and frequency of the shocks. There was a relatively large negative shock in February 1986, followed by a positive shock of similar size in August 1986.

Since the estimated oil supply news shocks are used as an instrument for the real oil price in equation (17), we also report the F-statistics from the first stage regression of log real oil price on this instrument. The smallest value of the F-statistics using the data from 1975:01 to 2019:12 for horizons up to 36 months is 16.3, well above 10. Nevertheless, we take account of the possibility that the instrument is weak at some horizons by reporting the differences in the responses using both standard statistics and Anderson and Rubin

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[14] This 0.5 percent cutoff results in a ZLB period similar to Ramey and Zubairy (2018). Lowering the cutoff to 0.25 percent shrinks the ZLB period to 2008:12-2015:12, however, the estimation results remain qualitatively similar.

(1949) statistics. Finally, we estimate the economy's responses to an oil supply news shock by regressing directly variables of interest on the shock using the local projection method. These additional results, reported in Appendix A, are consistent with the baseline results presented next.

Before turning to the analysis of the Japanese economy, we plot in Figure 5 the responses of the real oil prices to the oil supply news shocks using the ZLB definition for Japan. There is some evidence that the shock in the ZLB period leads to a more persistent increase in the real oil price than that in the normal period. The difference between the real oil price responses in the ZLB period and those in the normal period is statistically different at the 5 percent level at horizons between 15 and 20 months after the shock, as seen from the right panel of Figure 5. However, the responses in the two sub-periods are qualitatively similar.

4.2 Real Economy

The left panel of Figure 6 shows the responses of industrial production to a 10-percent increase in real oil price driven by the oil supply news shocks at horizons up to 36 months after the shock. The error bands are one standard deviation confidence interval. In normal times, industrial production slowly falls, declining almost one percent one year after the shock. This decline reverts after two years after the shock. In the ZLB period, the response of industrial production increases to just above one percent one year after the shock, and it is statistically different from zero at the five percent level at horizons between 11 and 14 months. The difference between the two responses of industrial production in the ZLB and normal periods are in the bottom left panel of Figure 6. This difference is statistically significant at the 10 percent level at horizons between 9 and 18 months and at the 5 percent level at horizons between 10 and 14 months after the shock. The Anderson and Rubin (1949) p-values are higher, as it allows for weak instruments, but the differencedeclining in the industrial production response is significant at the 10-percent level at horizons of 12 and 13 months.

Oil supply news shocks also lead to different labor market dynamics in the ZLB and normal periods. The right panel of Figure 6 plots the responses of the unemployment rate to a 10-percent increase in real oil price driven by the oil supply news shocks in both normal and ZLB periods. Initially, the unemployment rate responses to an increase in real oil prices are negative and insignificantly different from zero in the normal period. At the same time, it decreases slightly in the ZLB period. Then, the unemployment rate increases by up to 0.03 percentage points during the normal times at the one-year horizon, while it decreases by 0.08 percentage points in the ZLB period a year after the shock. This decrease is statistically significant at the 5 percent level at horizons between 5 and 24 months after the shock. We test the difference in responses in the normal and ZLB period, and the p-values are below 5 percent for horizons between 16 and 22 months (except month 21) after the shock. The Anderson and Rubin (1949) p-values are below 10 percent for horizons 16 to 22 months (again excluding month 21).[15]

Long ZLB spells in Japan allow us to perform estimation using quarterly data. Figure 7 plots the impulse responses of real gross domestic product (GDP), private consumption, and investment per capita in Japan to a 10-percent increase in the real oil price. We sum all the shocks within a quarter to obtain quarterly oil supply news shocks. Consistent with the monthly real variables, we observe that real GDP, consumption, and investment per capita responses are larger in the ZLB period than in the normal one. In particular, GDP virtually does not change on impact but increases by 0.4 percent after one year at the ZLB and declines gradually by 0.2 percent after two years in the normal period. The one standard deviation confidence bands of the estimated responses in the normal and the ZLB period responses do not overlap at horizons longer than three quarters after the shock.

4.3 Interest Rates, Inflation, and Inflation Expectations

One possible explanation for the observed differences in the responses of real macro variables in Japan to oil supply news shocks is that the monetary policy stance during these periods differs. To inspect this mechanism, we estimate the responses of nominal interest rates, inflation, and inflation expectations to an oil supply news shock. In the specification for the nominal interest rates, we add 12 lags of CPI inflation rate and the real oil prices in addition to the controls described in Section 3.1.

The top left panel of Figure 8 plots the impulse response of the short-term nominal interest rate to a 10-percent increase in real oil prices driven by the oil supply news shocks. In the ZLB period, the response of the nominal interest rate stays near zero and is precisely estimated, as one would expect from the ZLB period. In normal times, the short-term nominal interest rate does not change on impact, and then it gradually increases to 0.3 percentage points one year after the shock. The top right panel of the same figure presents the response of the 5-year nominal government bonds yield. This longer rate behaves qualitatively similar to the short rate, with the only difference being that it increases somewhat in the ZLB period and goes up less in the normal period than the short rate.

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[15] The above results are robust to the following modification of our baseline specification. First, we add a quadratic trend into the specification, allowing for state-dependent coefficients. Second, we control for the index of global economic activity.

The CPI inflation rate reaction to the oil shock is in the bottom left panel of Figure 8. The inflation rate increases in normal times by about 0.3 percentage points after 8 months from the shock and reverts after 17 months. In the ZLB period, the inflation rate does not react at the beginning and shows a moderate increase of 0.1 percentage points 20 months after the shock. The bottom right panel of Figure 8 demonstrates the impulse responses of the one-year ahead inflation expectations. We use the Japanese Center for Economic Research (JCER) CPI inflation forecast to proxy for inflation expectations.[16] Inflation expectations increase by 0.3 percentage points in the ZLB one quarter after the shock, which is statistically significant at the 5-percent level. The normal-period inflation expectations response is close to zero on impact and becomes negative after one year. The difference between the ZLB and normal period responses is significant at the 4, 5, and 7-quarter horizons.

The above results regarding the behavior of the realized inflation go contrary to the model presented in Section 2. At the same time, the behavior of the nominal interest rates and expected inflation suggests that the ex-ante real interest rate falls during the ZLB period and increases during normal times, consistent with the main channel in the model of Section 2.

5 Oil Shocks in United Kingdom and Unites States

This section investigates the effects of oil supply news shocks in the United Kingdom and the United States. We present the responses of real and nominal variables during and outside of the ZLB periods in these countries. The results provide additional evidence that oil supply news shocks are less contractionary in the ZLB periods than in the periods outside of the ZLB, suggesting that the results using Japanese data are not specific only to Japan or to the period when Japan was in a liquidity trap.

5.1 Real Economy

Panel (a) of Figure 9 plots the impulse responses of industrial production to a 10-percent increase in real oil price driven by the oil supply news shocks. Both in the United States and United Kingdom, the difference between the industrial production responses in the ZLB and normal periods becomes positive and statistically significant after after a few months from the shock. Moreover, in these countries, the industrial production responses

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[16] Alternative inflation expectations datasets are too short to estimate the effects of oil shocks on inflation expectations in the normal period in Japan. For example, the Consensus Economics Forecast data for Japan started in 1989.

are positive and statistically significant in the ZLB period, and they are negative and statistically significant in the normal period. These results are consistent with the behavior of Japan.

The labor market responses, which we plot in Panel (b) of Figure 9, mirror the responses of industrial production. Specifically, the unemployment rate in the United States and the United Kingdom falls after an oil shock that increases the oil price in the ZLB period. These increases statistically differ from zero and the responses in the normal periods.

5.2 Interest rate, Inflation, and Inflation Expectations

To investigate whether the monetary policy channel can be responsible for the above differences in the United States and the United Kingdom, we demonstrate the influence of oil supply news shocks on the nominal variables. Panel (a) of Figure 10 shows that ZLB responses of the short-term nominal interest rate remain zero. In contrast, the normal period responses are positive and statistically different from the ZLB responses for a few horizons. The 5-year nominal yields respond to oil shocks in the two countries during and outside the ZLB. Nevertheless, the ZLB responses are statistically different from the normal period ones at several horizons. Finally, the inflation rate behavior in the ZLB and normal period is indistinguishable in the US. In the United Kingdom, the initial inflation response in both periods is close to each. However, the ZLB inflation starts falling significantly below zero after one year.

We close this section by describing the behavior of inflation expectations in the United States.[17] In Figure 11, we plot the responses of two measures of one-year ahead inflation expectations from the Michigan Survey of Consumers and the Survey of Professional Forecasters (SPF). The former survey collects data monthly from households, while the latter is a quarterly survey of professional forecasters. As in the case of regressions with quarterly Japanese data, we construct quarterly oil supply news shocks by adding up the shocks within the same quarter. For both measures of inflation expectations, we observe an increase in inflation expectations on impact in both the ZLB and the normal periods. The response of consumers' expectations is slightly larger (above 0.1 percentage points) than that of professional forecasters (below 0.1 percentage points) when the real oil price increases by 10 percent, driven by oil supply news shocks. The differences in the responses of inflation expectations are indistinguishable between the ZLB and the normal periods in the Michigan Survey of Consumers case. There is evidence that inflation expectations react more in the normal period in the survey of professional forecasters.

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[17] For the United Kingdom, the quarterly forecast in Consensus Forecast is not sufficiently long for our analysis.

Overall these results parallel those in Japan in Section 4.3. Specifically, the reaction of the nominal interest rates and expected inflation indicate the differential reaction of the ex-ante real interest in the ZLB and normal period.

6 Alternative Oil Supply Shocks and Further Results

This section relates our findings to previous studies and further checks our results.

6.1 Alternative Oil Supply Shocks

We find that oil supply news shocks affect the real economy in Japan, among other countries, in the ZLB period compared to the normal periods differently. At the same time, Wieland (2019) does not find any differences in the responses of Japanese industrial production and unemployment rate to oil supply shocks. This section examines the effects of oil supply shocks identified in alternative ways to uncover the reasons for the differences in our and prior estimates.

We focus on two influential identification strategies for oil supply shocks. The first one is the oil supply shocks identified in Kilian (2009), also used in Wieland (2019). We will refer to these oil supply shocks as the Kilian shocks. The second set of oil supply shocks is from Baumeister and Hamilton (2019), which we will call the BH shocks. To identify oil supply shocks in a structural VAR, Kilian (2009) assumes that there is no short-run reaction of oil supply to changes in the real oil price.[18] In contrast, Baumeister and Hamilton (2019) do not restrict the oil supply elasticity to be small and estimate it to be a much larger number of 0.15. This leads to a different series of oil supply shocks that are more important in accounting for historical oil price movements. We use these two oil supply shocks in the same way as in our baseline specification described in Section 3.1.

Figure 13 plots the impulse responses of Japanese industrial production and unemployment rate to the Kilian (first row) and BH (second row) shocks. The results based on the Kilian shocks are consistent with Wieland (2019).[19] The responses of industrial production and the unemployment rate to the BH shocks are qualitatively similar to our

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[18] Kilian and Murphy (2012) later modified this assumption with an alternative that the short-run price elasticity of oil supply is small or, more specifically, less than 0.0258.

[19] Wieland (2019) does not use the Kilian shocks to instrument for the real oil price; instead, he uses the shocks directly in equation (17) instead of the oil price. We present the estimation based on this direct approach in Appendix Figure A.1 Panel (b). These responses between the ZLB and normal periods are not statistically significant. Note that our responses are not identical to the responses in Panel B and C of Figure 6 in Wieland (2019). This is because of two differences. First, we compute the Kilian shocks from the 1973:2-2019:12 sample instead of the 1973:2-2015:2015:9 sample in Wieland (2019). Second, we start the normal period in Japan in 1980:1 instead of 1986:1 in Wieland (2019).

baseline estimation using oil supply news shocks. In particular, we find that industrial production increases in the ZLB period while it decreases in the normal period, and the unemployment rate decreases in the ZLB period while it increases in the normal period. The differences between the responses of the unemployment rate to BH shocks in the ZLB period and those in the normal period are not statistically significant, and industrial production is significant at 5 percent at the horizon of 20 and 21 months after the shock and at 10 percent two more horizons are added of 18 and 22 months.

To understand why the estimates using Kilian or BH supply shocks are more noisy, we estimate the impulse responses of the real oil price to both Kilian and BH supply shocks and plot the results in Figure 12. As displayed in the second panel, Kilian shocks generate a small and insignificant response to the real oil price, consistent with the result in Kilian (2009) that oil supply shocks are not an important driver of historical oil price variations. The response of the real oil price to the BH shock is more significant but still not as precise as the response of the real oil prices to the oil supply news shocks identified in Kanzig (2021) (the first panel of Figure 12). These results suggest that the effects of Kilian and BH shocks on the real oil price are relatively weak, so the estimates of the effects of oil supply shocks on the economy can be imprecise.

We close this discussion by presenting the responses of the Japanese industrial production and the unemployment rate to a shock referred to as the oil demand shock in Kilian (2009). This shock is identified by assuming it does not affect oil production and the index of global economic activity on impact. Figure A.3 presents the results. As one can see, the results are consistent with our baseline results in Japan. A potential explanation of this finding is that the oil supply news shocks identified in Kanzig (2021) correlate with the oil demand shock in Kilian (2009). In fact, the correlation is 0.17 on the 1975:2-2007:12 sample.

6.2 Further Results

Our estimates for Japan are consistent with those in the United States and the United Kingdom, suggesting that the less contractionary effects of an oil supply news shock are not particular to Japan. We now extend our analysis in several dimensions to explore the sensitivity of our results.

Japan before the US ZLB. The advantage of showing results from several countries that experienced ZLB episodes is that these episodes may not coincide, providing evidence from different subperiods. In practice, however, the United States and United Kingdom ZLB episodes occur almost simultaneously and partly overlap with the Japanese ZLB experience. In Figure 14, we show the reaction of Japanese industrial production and the unemployment rate for the normal period and the ZLB period that stops before the start of the ZLB period in the United States and the United Kingdom. Specifically, we choose the ending date to be June of 2006, corresponding to the end of the first ZLB subperiod in Japan, as depicted in Figure 3. The results qualitatively repeat the patterns for the full sample estimation in Figure 6. The standard errors become larger for the ZLB period, which is not surprising with a smaller sample size. The difference between the industrial production responses remains significant for several horizons but not for the unemployment rate.

Information effect. A high-frequency approach to identifying monetary policy shocks is subject to criticism that it cannot distinguish between a true shock and information revelation about the state of the economy by a central bank (Nakamura and Steinsson, 2018). The same criticism can apply to the announcements by the OPEC. To check whether our results are driven by news about oil shocks instead of information revelation about the state of the economy, we use recent advances in monetary shocks identification literature (Cieslak and Schrimpf, 2019; Cieslak and Pang, 2021; Jarociński and Karadi, 2020) that was applied to the OPEC announcements in (Degasperi, 2021). The idea is to separate oil futures price movements around OPEC announcements into those that positively comove with the stock market (they will represent the information revelation effect) and those that comove negatively (they proxy a true oil supply news shock). We use the S&P 500 stock price index to approximate the stock market price. Figure 15 shows our results for Japan based on the oil supply news shocks that were estimated using the announcements that comove negatively with the stock market. The results are qualitatively similar to the baseline results: the difference in industrial production and unemployment responses in the normal and ZLB period remain significant.[20]

Canada and the Euro Area. The United States and the United Kingdom experienced clear episodes when the interest rate was constant, as depicted in Figure 3. Two more countries experienced almost constant interest rate episodes: Canada and the Euro Area. Figure A.7 plots their short-term nominal interest rates. We define the ZLB period to be between 2009:04 and 2017:04 in Canada and during 2012:07–2019:12 in the Euro Area.

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[20] One can object to such an exercise by arguing that the comovement between oil futures prices and the stock market can depend not only on the shock vs. information revelation but also on the systematic monetary policy response. After all, we observe that the US economy booms after a shock that increases oil prices during the ZLB period and contracts after a similar shock in the normal period. To address this issue, we re-estimate the results in Figure 15 by removing a period when the US interest rate was constant. Specifically, we choose a sample between 1975:1 and 2006:6. The main finding that the shock that increases the real oil price is less contractionary remains true even in this case.

Note that the interest rates continued changing even when they were close to zero in both countries. This is why it is reasonable to expect noisier results.

Figure A.8 demonstrates a lack of significant difference between responses of industrial production and unemployment in the ZLB and normal periods in Canada and the Euro Area. There is some evidence that initial output goes up more, and the unemployment rate goes down more in the normal period than in the ZLB. In Canada, while not significant, the point estimates are consistent with our results for Japan, the United States, and the United Kingdom.

Placebo countries. We have seen that Canada and the Euro Area generally do not respond differently in the normal and ZLB periods. In addition to these results, we present the responses of three more countries (Mexico, Sweden, and the Republic of Korea) that did not experience ZLB during the global financial crisis. We chose these countries from the OECD countries and ensured they are located on different continents. Figure A.11 shows that neither industrial production nor unemployment rate responds differently in the normal and ZLB period based on the US definition. Only Mexico shows some signs of the expansionary effect of oil shocks during the (US) ZLB. This, however, can be driven by an international spillover from an expansion in the United States following an increase in oil prices.

Sign of oil shocks. The strength of the effects of macroeconomic shocks can depend on the sign of a shock (Barnichon *et al.*, 2022). This can, for example, be driven by downward nominal wage rigidity that binds when a shock is contractionary but is slack when the shock is expansionary. In Figure 16, we present the results from estimating specification

$$\begin{aligned} y_{t+h} - y_{t-1} = {} & \mathbb{I}\left(\widehat{\mathcal{E}}_{1,t} \geq 0\right) \cdot \left[\alpha^{y}_{A,h} + \beta^{y}_{A,h}\widehat{\mathcal{E}}_{1,t} + \psi^{y}_{A,h}(L)x_{t-1}\right] \\ & + \mathbb{I}\left(\widehat{\mathcal{E}}_{1,t} < 0\right) \cdot \left[\alpha^{y}_{B,h} + \beta^{y}_{B,h}\widehat{\mathcal{E}}_{1,t} + \psi^{y}_{B,h}(L)x_{t-1}\right] + \epsilon^{y}_{t+h}, \end{aligned}$$

where $\mathbb{I}(\widehat{\mathcal{E}}_{1,t} < 0)$ is an indicator variable that equals one when the shock $\widehat{\mathcal{E}}_{1,t}$ is negative and zero otherwise, and $\mathbb{I}(\widehat{\mathcal{E}}_{1,t} \geq 0) = 1 - \mathbb{I}(\widehat{\mathcal{E}}_{1,t} < 0)$. We do not find evidence for a sign-dependent effects. Figure 16 shows that the difference $\widehat{\beta}^{y}_{A,h} - \widehat{\beta}^{y}_{B,h}$ is not statistically significant from zero at conventional levels.

Industrial production without oil. Our analysis extensively used the measure of industrial production that includes mining, such as oil extraction. This raises a concern that oil price spikes can mechanically increase the mining sector's industrial production. This can be particularly important in the United States, where the importance of oil production has grown in the last two decades. To address this possibility, we present the response of industrial production in the US and Japan only for the manufacturing sector in Panel (a) of Figure A.9 and for industrial production when we remove oil production for the US in Panel (b). The main findings remain unchanged.

Unconventional monetary policy. During the ZLB periods, central banks often use unconventional monetary policies, such as forward guidance (announcements about future conventional monetary policy) or quantitative easing (purchases of long-term private and public financial assets). As a result, central banks can still respond to oil price shocks even during ZLB. One way to assess this possibility is to look at a measure of monetary policy stance summarized by the show rate (Wu and Xia, 2016). Figure A.6 resents the impulse response of the shadow rate to an oil supply shock in the ZLB period in the United States. The shadow rate falls significantly following an oil price hike. This pattern is consistent with the expansion of real economic activity during the ZLB we obtained earlier. The shadow rate might fall after the oil price spike instead of increasing, as it does during the normal period, because the inflation rate during ZLB periods is typically below its target, perhaps making central bankers less concerned about inflation.

7 Conclusion

This paper presents new evidence on the effects of oil supply news shocks when the nominal interest rate is not responsive due to the ZLB constraint. We focus on Japan, which has the longest spell of the ZLB, and supplement with evidence from the United Kingdom and the United States, which experienced considerable periods with fixed interest rate. We find that oil supply shocks are less contractionary in the ZLB period than in the periods outside of the ZLB in Japan, the United Kingdom, and the United States. Moreover, we document that these economies expand following an oil price spike during the ZLB periods. In addition, inflation expectations increase following the oil supply news shock, and the nominal interest rate stays at zero in the ZLB period. In contrast, the interest rate increases in the normal period.

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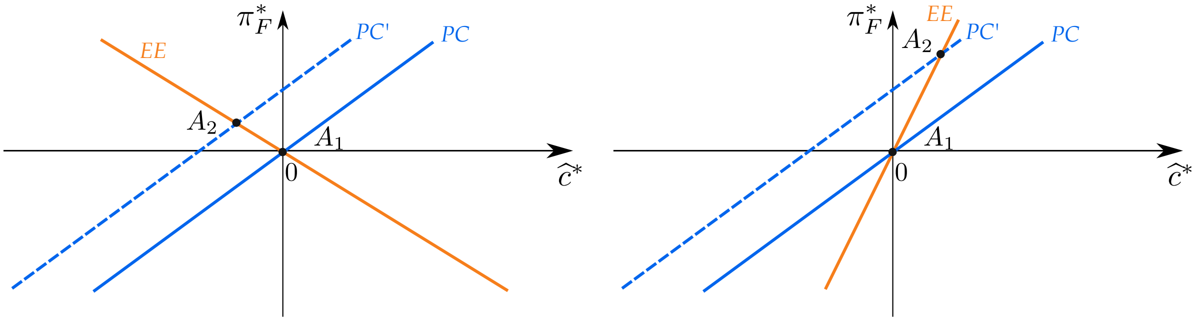
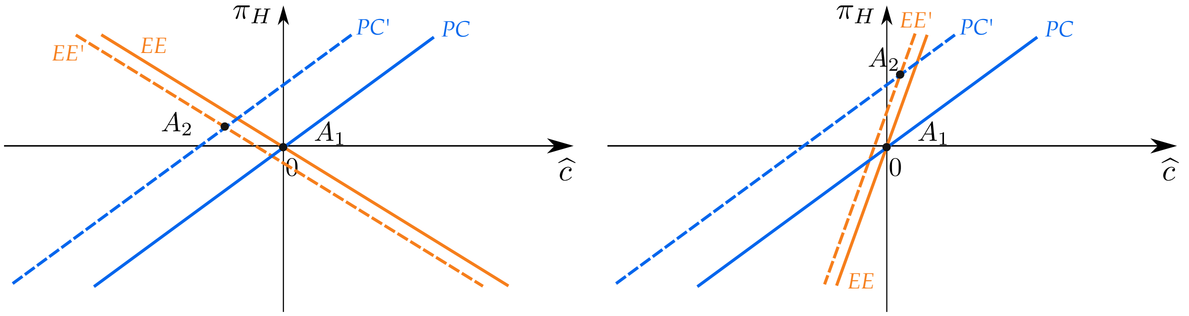
8 Main Text Figures and Tables

Figure 1: The World Economy.



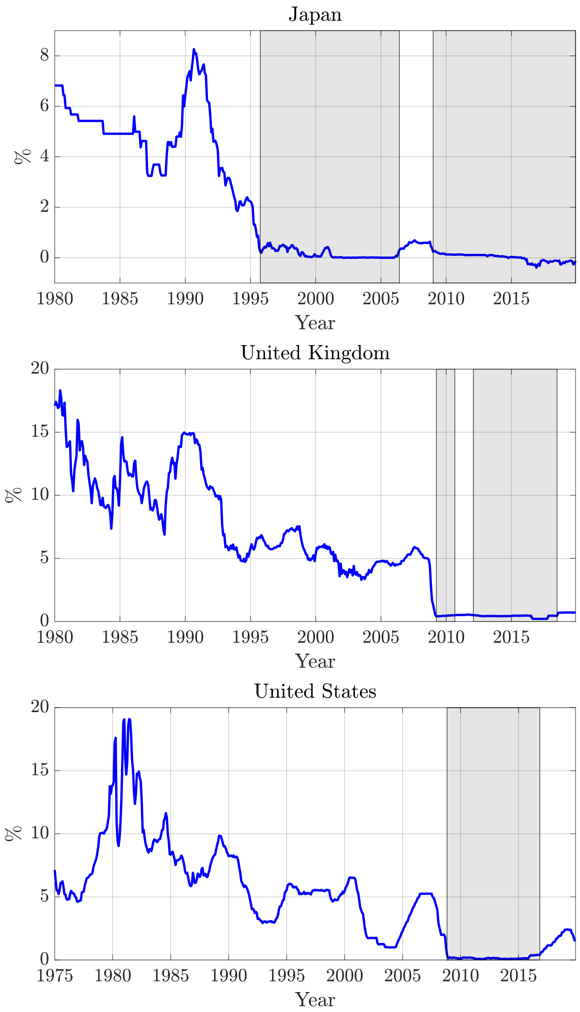
Notes: Both panel show the response of the world economy to a decline in oil supply when the monetary policy is active (the left panel) and when it does not respond following the oil shock (the right panel).

Figure 2: The Small-Open Economy.



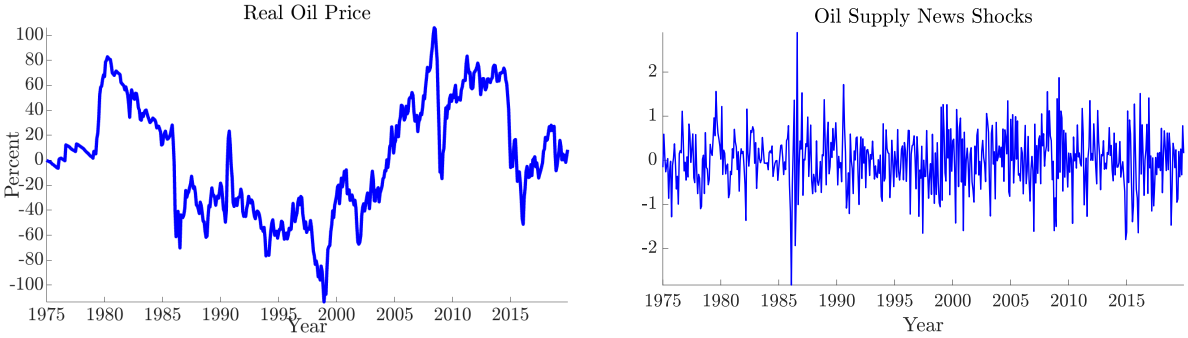
Notes: Both panel show the response of the small-open economy to a decline in oil supply when the monetary policy is active (the left panel) and when it does not respond following the oil shock (the right panel). Both panels assume that the world economy's output declines following an increase in the oil price.

Figure 3: The short-term nominal interest rates.



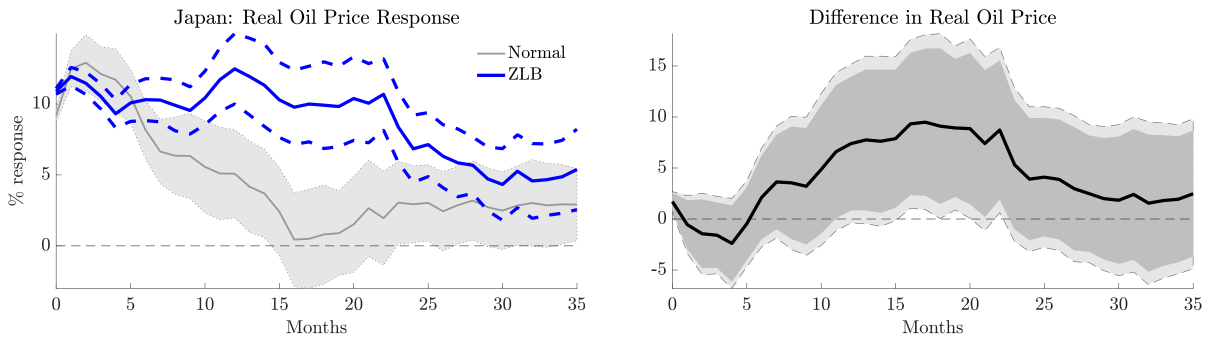
Notes: Each panel presents central bank's policy rate for each country. The shaded areas are the zero lower bound periods defined in Section 3.2.

Figure 4: The real oil price and oil supply news shock.



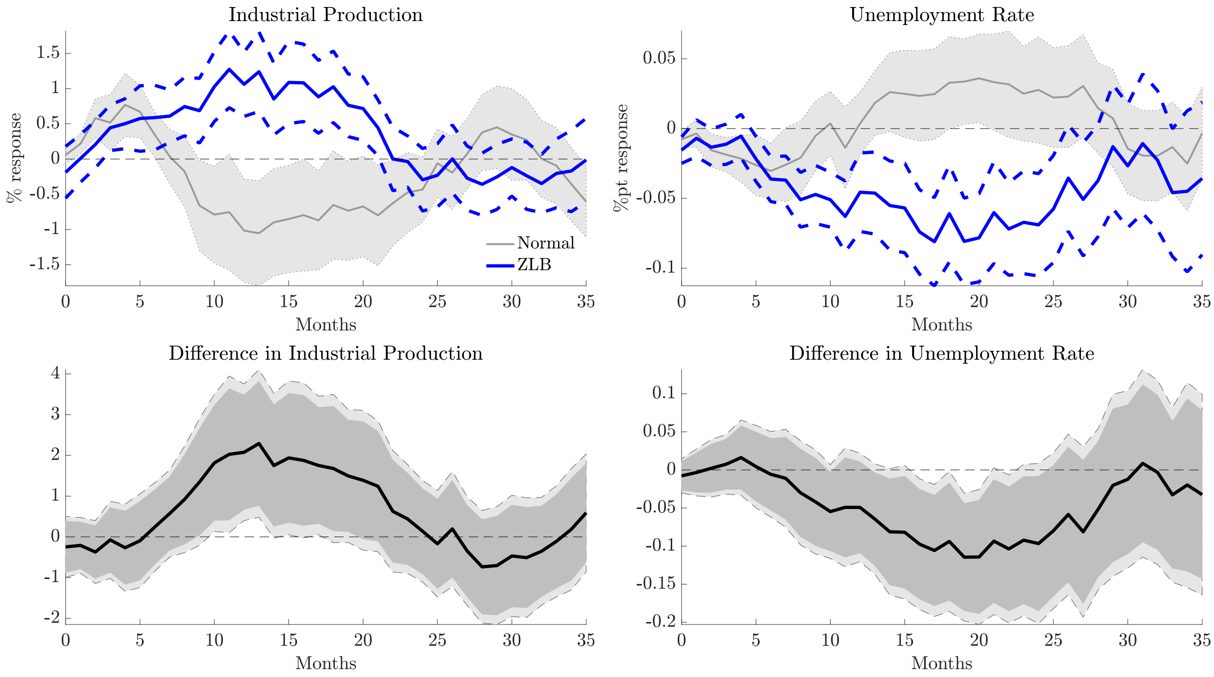
Notes: the left panel plots the real oil price defined as the WTI crude oil spot price deflated by the US CPI. The right panel presents the estimated oil supply news shock $\widehat{\mathcal{E}}_{1,t}$ from a 4-variable oil VAR instrumented by the oil supply surprise series as discussed in Section 3.1.

Figure 5: Impulse responses of real oil prices



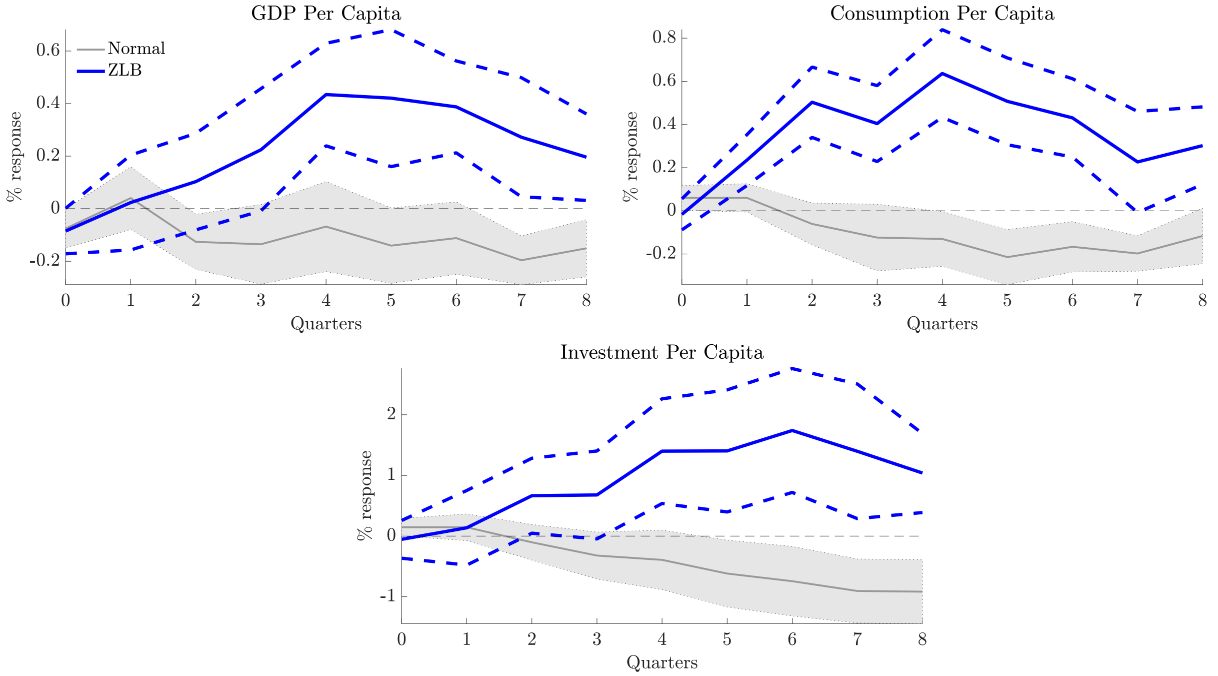
Notes: The left figure plots the mean and one standard-deviation confidence bands of the responses of the real oil price in the ZLB (blue) and in the normal (grey) periods, where the ZLB period is based on the nominal interest rate in Japan. The right figure plots the differences in the responses along with the 90% and 95% confidence bands.

Figure 6: Japanese industrial production and unemployment rate impulse responses.



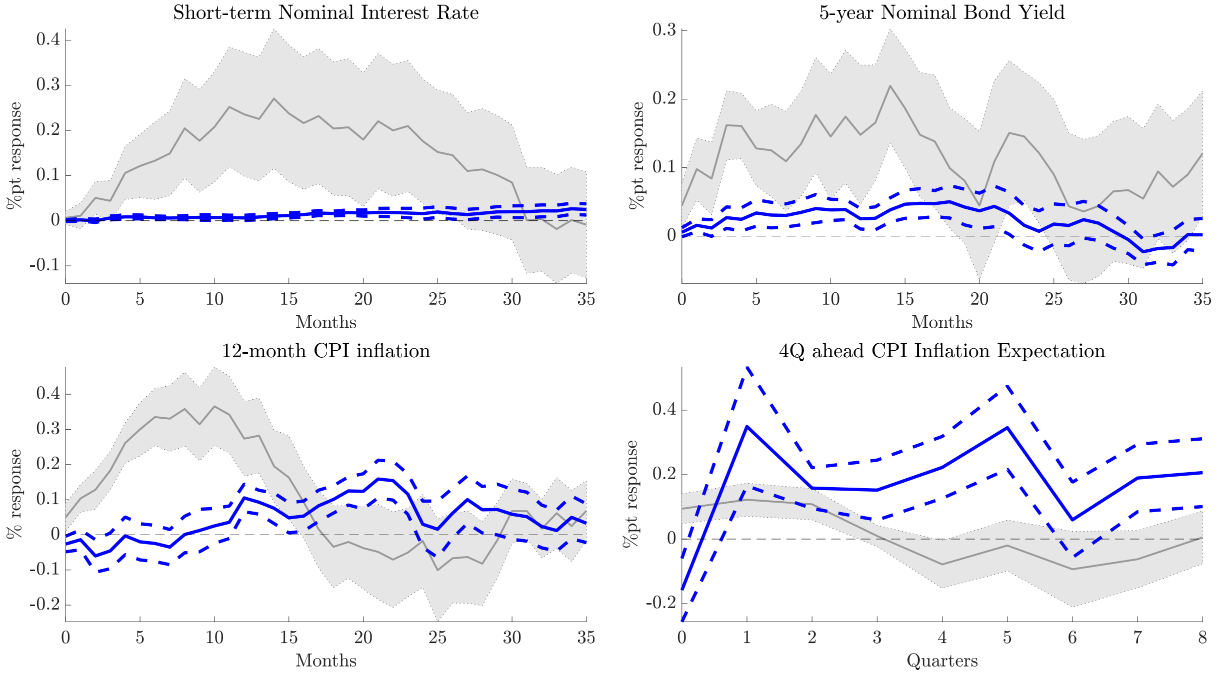
Notes: Each figure in the top panel plots the mean and one standard-deviation confidence bands of the responses of industrial production and the unemployment rate in Japan in the ZLB (blue) and in the normal (grey) periods. The lower panel figures plot the differences in the IRFs of industrial production and the unemployment rate in the normal and the ZLB period, with a 95% and 90% confidence bands.

Figure 7: Japanese quarterly variable impulse responses.



Notes: Each figure plots the mean and one standard-deviation confidence bands of the responses of real GDP per capita, real consumption per capita, and real investment per capita in Japan in the ZLB (blue) and in the normal (grey) periods.

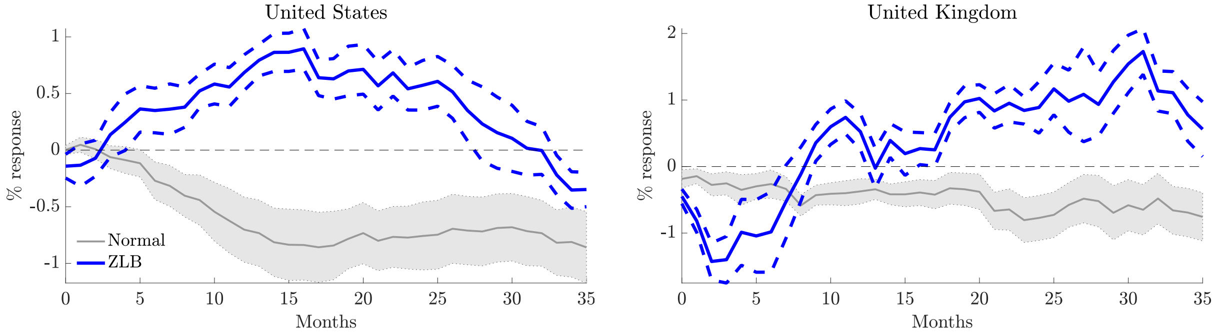
Figure 8: Interest rates, inflation, and inflation expectations responses in Japan.



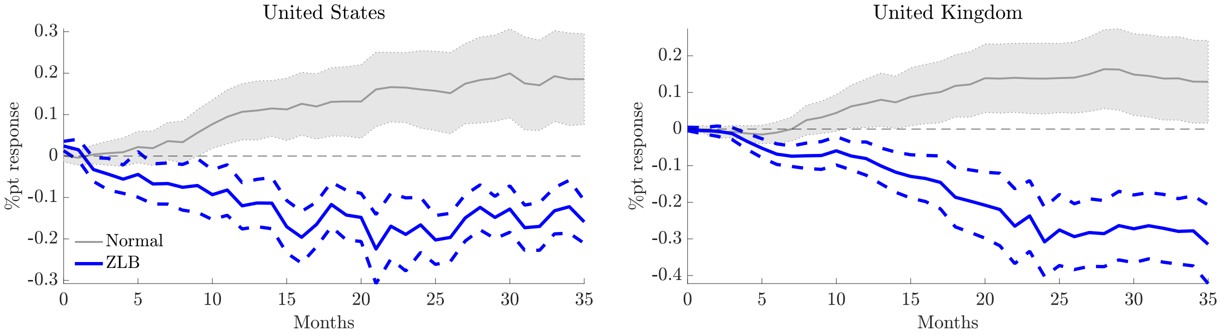
Notes: The figures plot the mean and one standard-deviation confidence bands of the responses of the short-term nominal interest rate, 5-year nominal interest rate, CPI inflation rate, and CPI inflation expectations in Japan in the ZLB (blue) and in the normal (grey) periods. The responses are to an oil supply shock that increases the real oil price by ten percent.

Figure 9: Industrial production and unemployment rate responses in the United States and United Kingdom.

(a) Industrial production



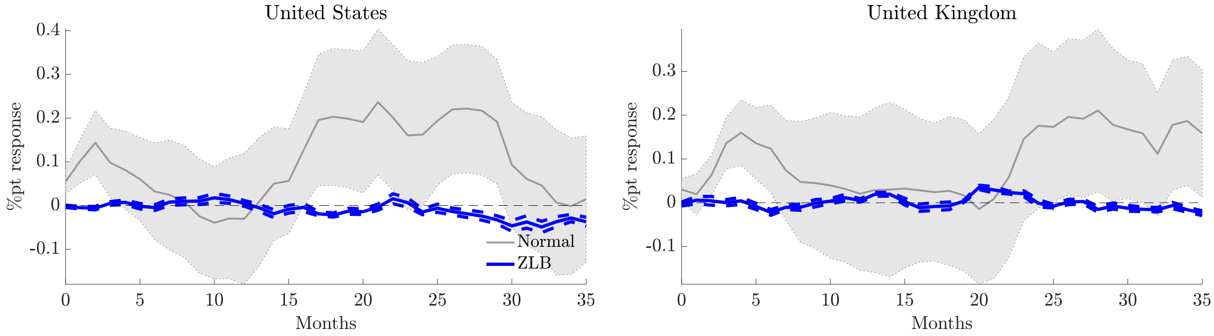
(b) Unemployment rate



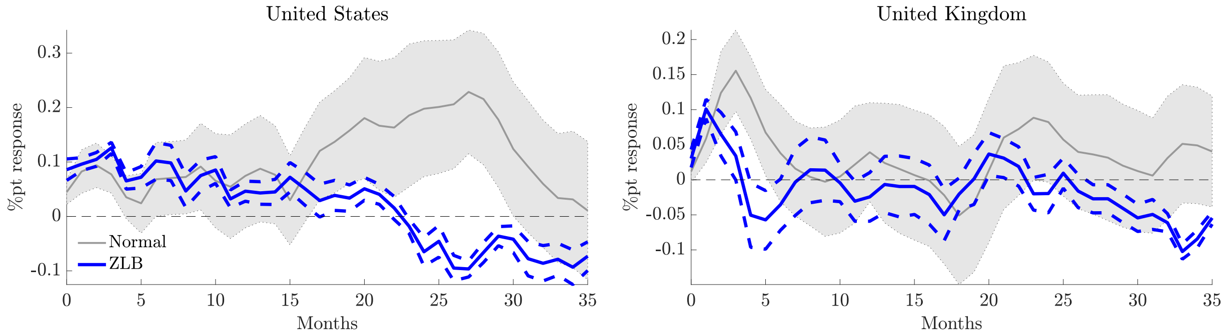
Notes: Each figure plots the mean and one standard-deviation confidence bands of the responses. Panel (a) shows industrial production responses in the United Kingdom and the Unites States in the ZLB (blue) and in the normal (grey) periods. Panel (b) plots unemployment rate responses in the United Kingdom and the Unites States in the ZLB (blue) and in the normal (grey) periods.

Figure 10: Interest rates and inflation in the the United States and United Kingdom

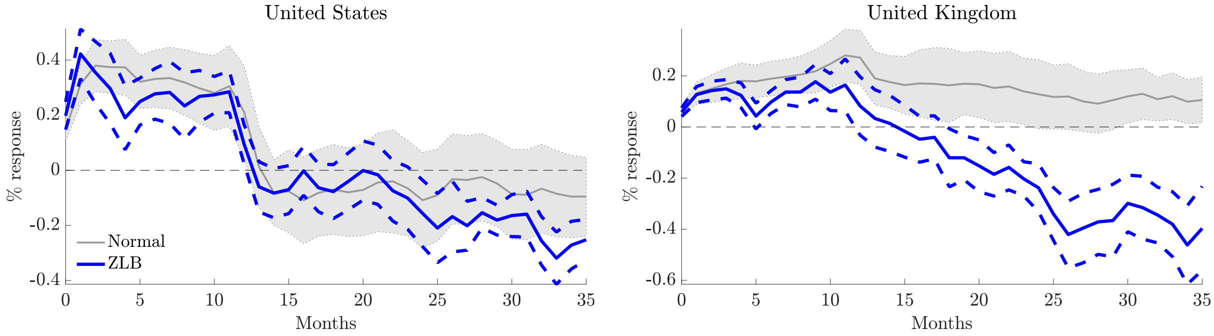
(a) Short-term nominal interest rate



(b) 5-year nominal government bond yield

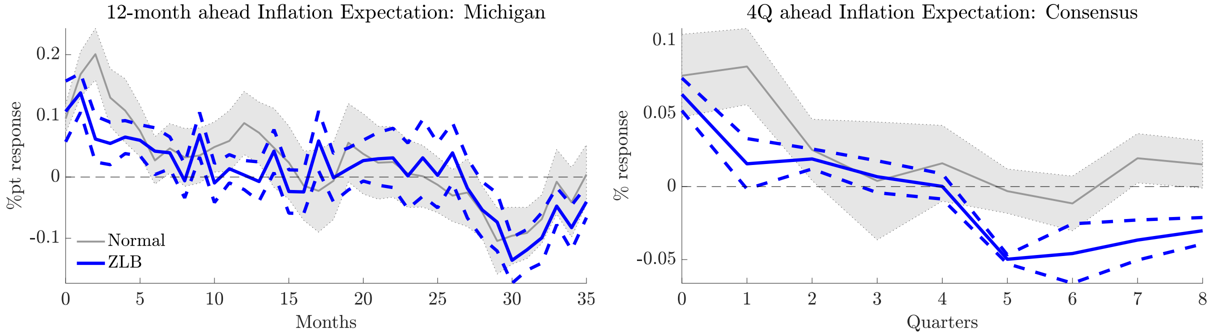


(c) CPI inflation rate



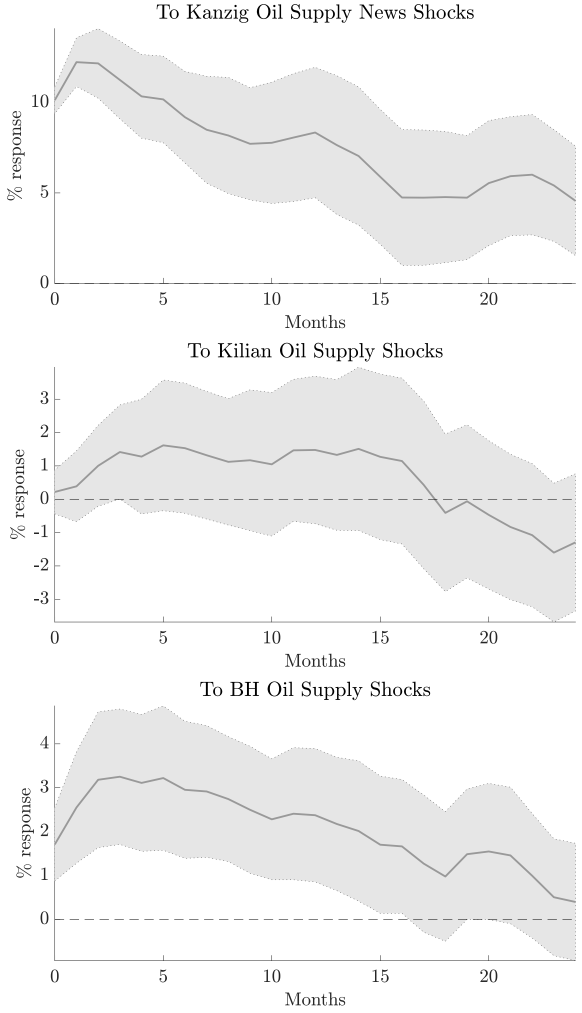
Notes: Each figure plots the mean and one standard-deviation confidence bands of the responses of CPI in the Euro Area, Canada, the United Kingdom and the Unites States in the ZLB (blue) and in the normal (grey) periods.

Figure 11: Responses of inflation expectations in the United States in the ZLB and normal periods.



Notes: Each figure plots the mean and one standard-deviation confidence bands of the responses of inflation expectations in the Unites States in the ZLB (blue) and in the normal (grey) periods.

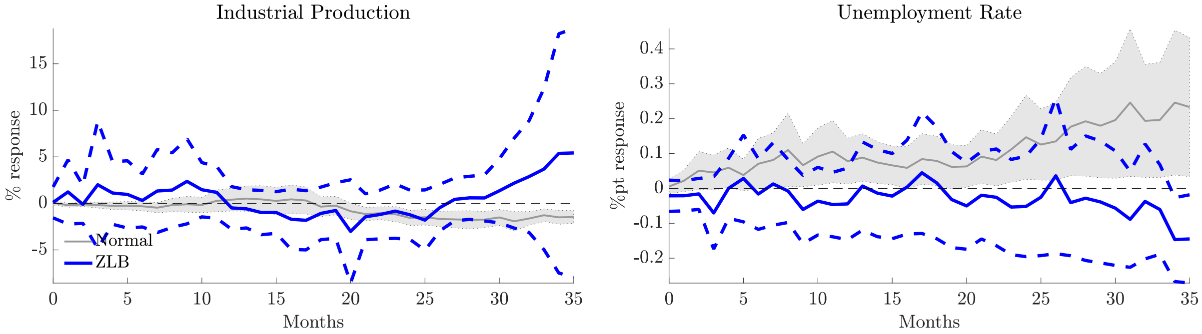
Figure 12: Real oil price response to different identified oil price shocks.



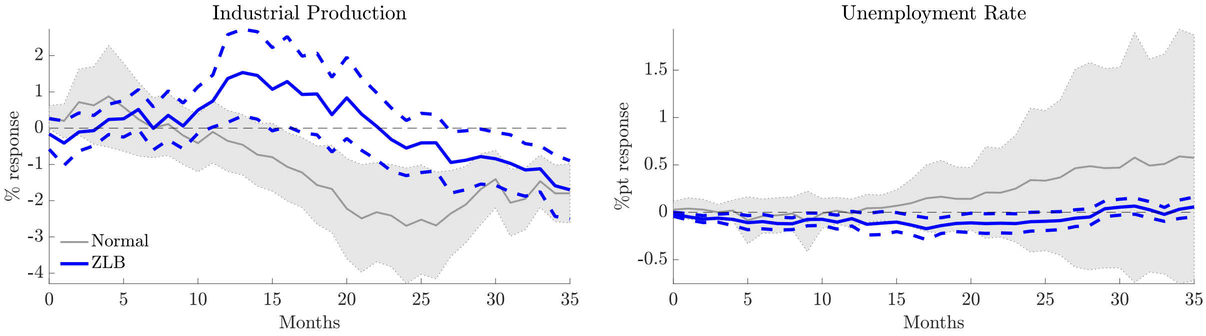
Notes: Each figure plots the mean and 90% confidence bands of the responses of oil price to oil price shocks identified in Kanzig (2021), Kilian (2009), and Baumeister and Hamilton (2019).

Figure 13: Japan industrial production and unemployment rate to different oil supply shocks.

(a) Kilian (2009) oil supply shocks

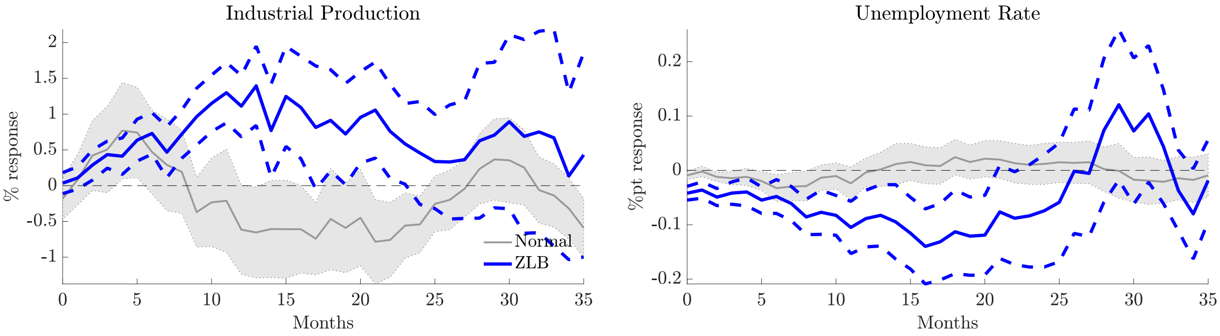


(b) Baumeister and Hamilton (2019) oil supply shocks



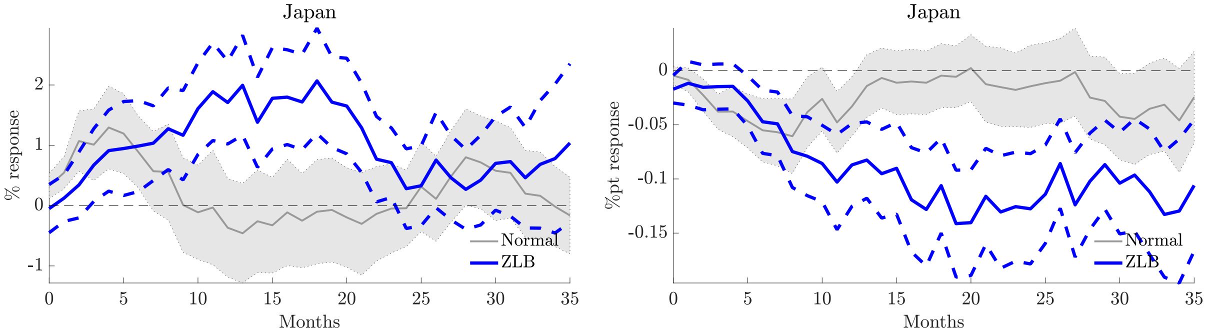
Notes: Each figure plots the mean and 68% confidence bands of the responses of industrial production and the unemployment rate in Japan to oil price shocks identified in Kanzig (2021), Kilian (2009), and Baumeister and Hamilton (2019).

Figure 14: Industrial production and unemployment rate responses in Japan when ZLB is 1995:10-2006:6.



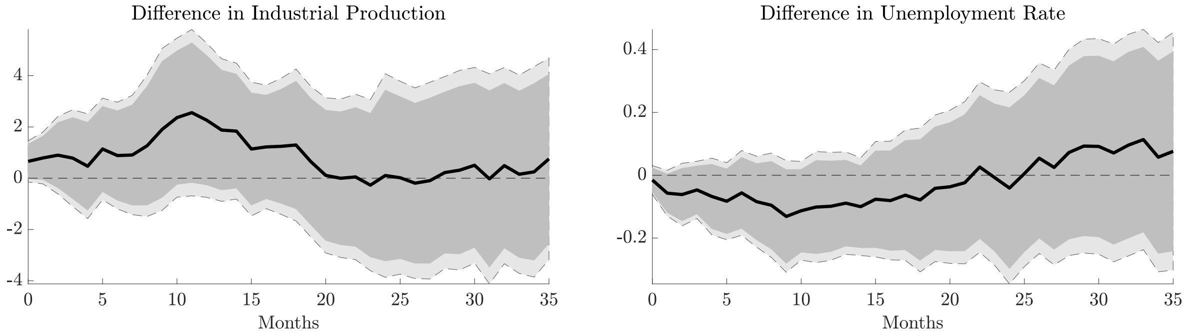
Notes: Impulse responses of industrial production (left) and unployment rate (right) to an oil supply news shock that increases oil price by 10 percent. The figures plot the mean and 90% confidence bands of the responses.

Figure 15: Industrial production and unemployment rate responses in Japan (without information revelation).



Notes: Impulse responses of industrial production (left) and unployment rate (right) to an oil supply news shock that increases oil price by 10 percent. The figures plot the mean and 90% confidence bands of the responses.

Figure 16: Positive and negative oil supply shocks.



Notes: Differences in responses to positive and negative oil supply news shocks. Industrial production is on the left panel and the unployment rate is on the right panel. The figures plot the mean and 90% confidence bands of the responses.

Table 1: Data availability.

Country	Industrial production	Unemployment rate	Short-term interest rate	Inflation expectations	CPI	ZLB period
Japan	1975:01-2019:12 (Haver)	1975:01-2019:12 (OECD)	1985:07-2019:12 (GFD)	1980:I-2019:IV (JCER) 1989Q4–2019Q4 (Consensus)	1975:01-2019:12 (OECD)	1995:10–2006:06, 2009:01–2019:12 (authors' definition)
Canada	1975:01-2019:12 (Haver)	1975:01-2019:12 (OECD)	1975:01-2019:12 (GFD)		1975:01-2019:12 (OECD)	2009:04–2017:04 (authors' definition)
Euro Area	1975:07-2019:12 (Haver)	1975:07-2019:12 (OECD)	1994:01-2019:12 (GFD)		1996:01-2019:12 (OECD)	2012:07–2019:12 (authors' definition)
United Kingdom	1975:01-2019:12 (Haver)	1975:01-2019:12 (OECD)	1975:01-2019:12 (GFD)		1975:01-2019:12 (OECD)	2009:04–2010:09, 2012:02–2018:07 (authors' definition)
United States	1975:01-2019:12 (Haver)	1975:01-2019:12 (OECD)	1975:01-2019:12 (GFD)	1989:01–2019:12 (MSC) 1989Q4–2019Q4 (Consensus)	1975:01-2019:12 (OECD)	2008:11–2016:11 (authors' definition)

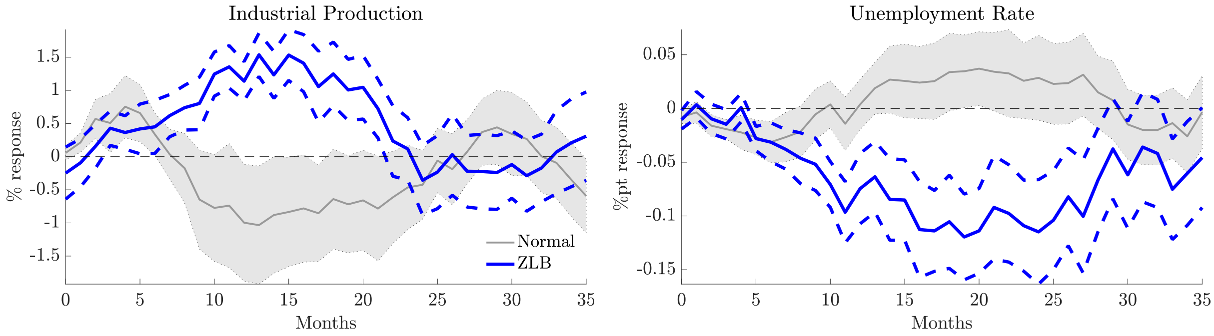
Notes: The table lists time periods and data sources (in brackets) for the variables used in the analysis.

Online Appendix

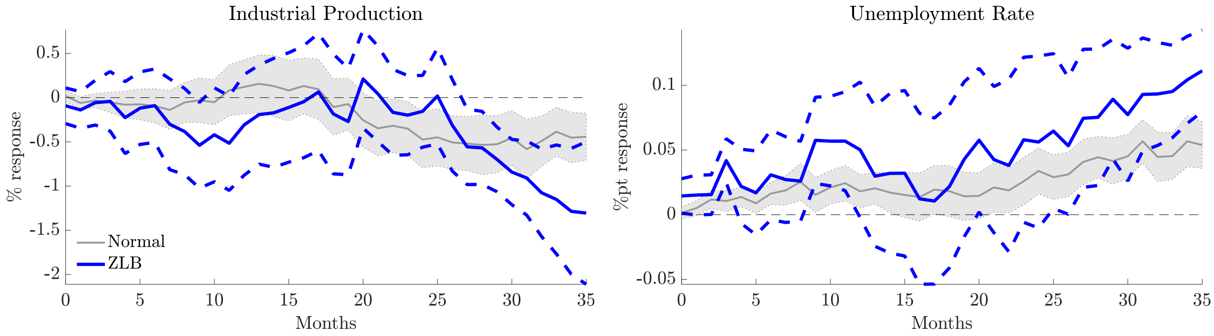
A Additional Figures and Tables

Figure A.1: Japanese real variables responses to different oil supply shocks.

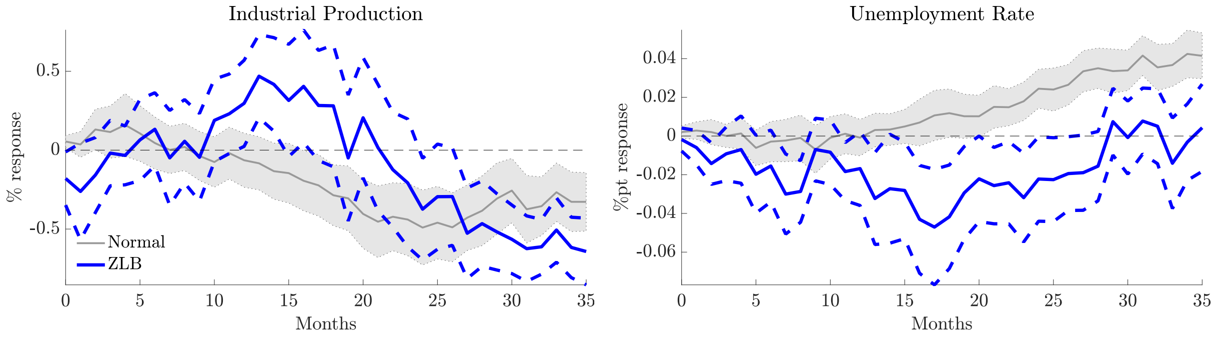
(a) The baseline oil supply shocks



(b) The Kilian oil supply shocks

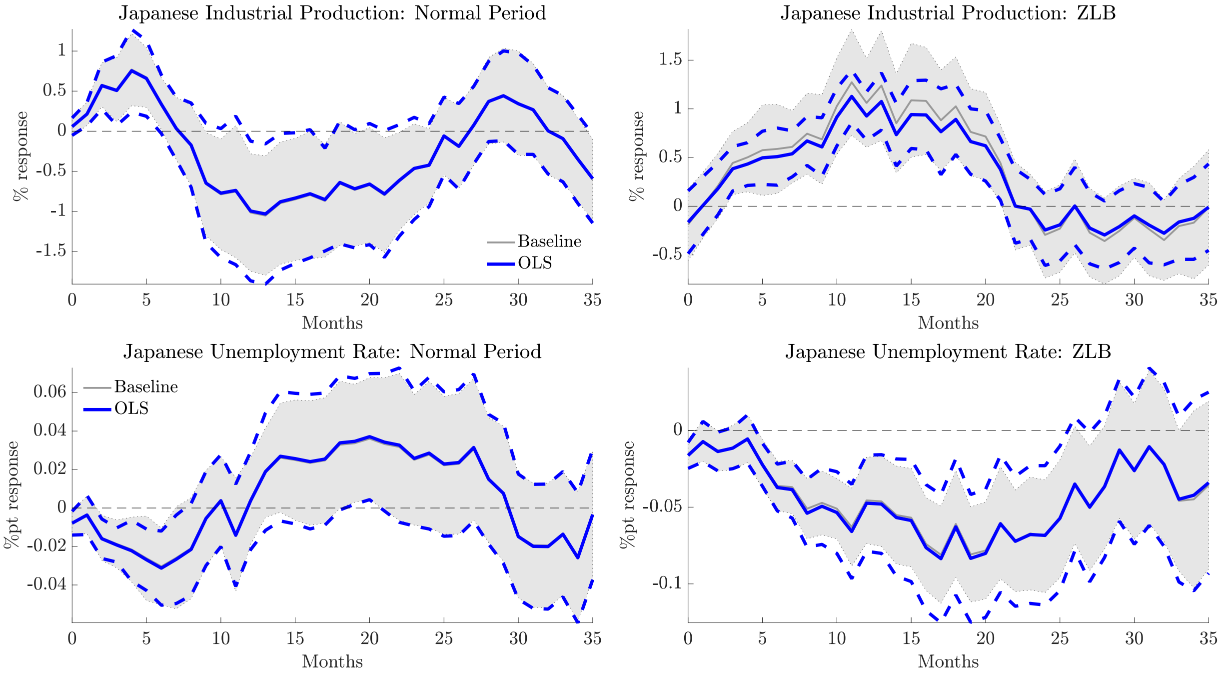


(c) The Baumeister-Hamilton oil supply shocks



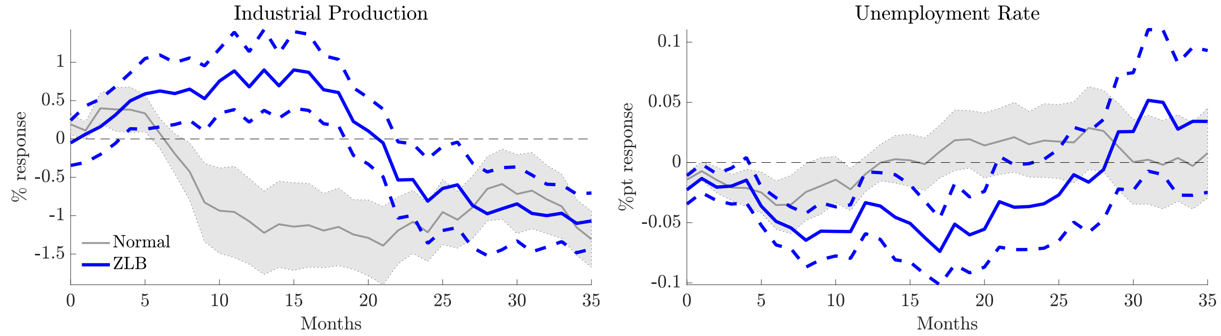
Notes: Each row plots the IRFs and 68% confidence bands of the responses of Japanese industrial proudction and unemployment rate to oil supply shocks identified as in Kanzig (2021), Kilian (2009), and Baumeister and Hamilton (2019) using the OLS estimation instead of IV. The period of estimation is between 1975 and 2017. Note that the oil supply shocks would increase real oil prices.

Figure A.2: Japan Impulse Responses using OLS vs baseline specification with instruments.



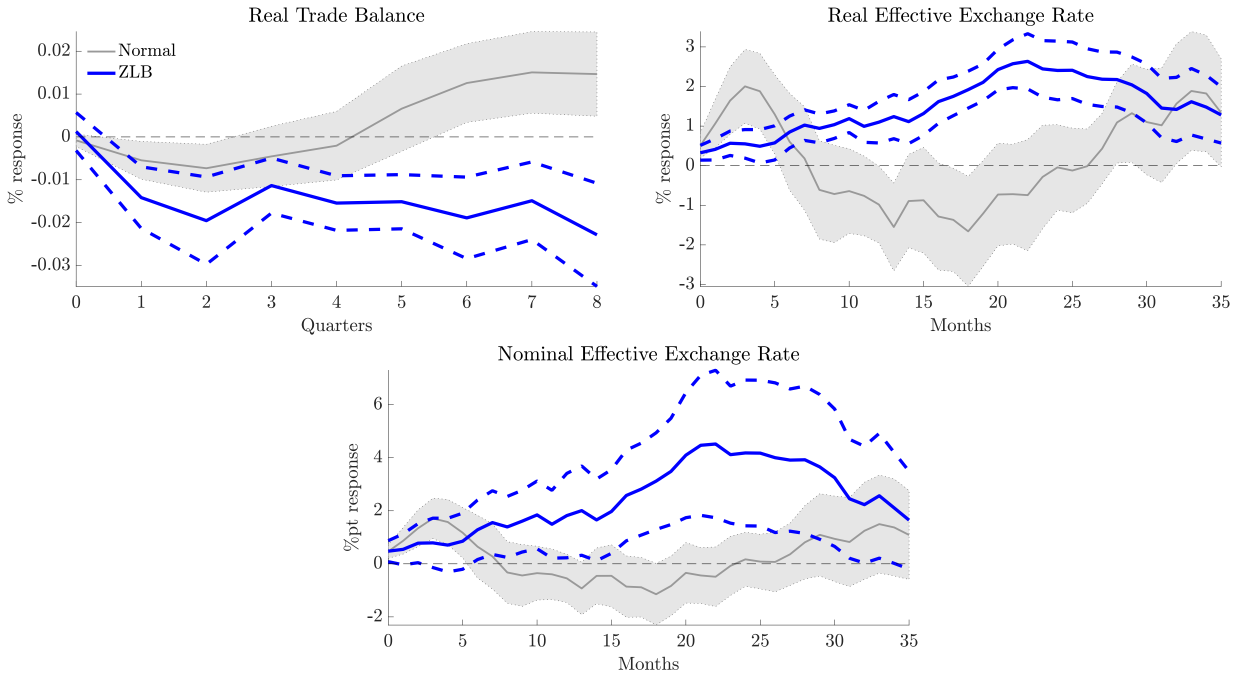
Notes: Each row plots the IRFs and 68% confidence bands of the responses of Japanese industrial proudction and unemployment rate to oil price shocks identified in Kanzig (2021): OLS estimates directly on the shocks while Baseline estimates the responses of those variables on real oil price instrumented by the shocks.

Figure A.3: Japan Impulse Responses to Kilian (2009) oil demand shocks.



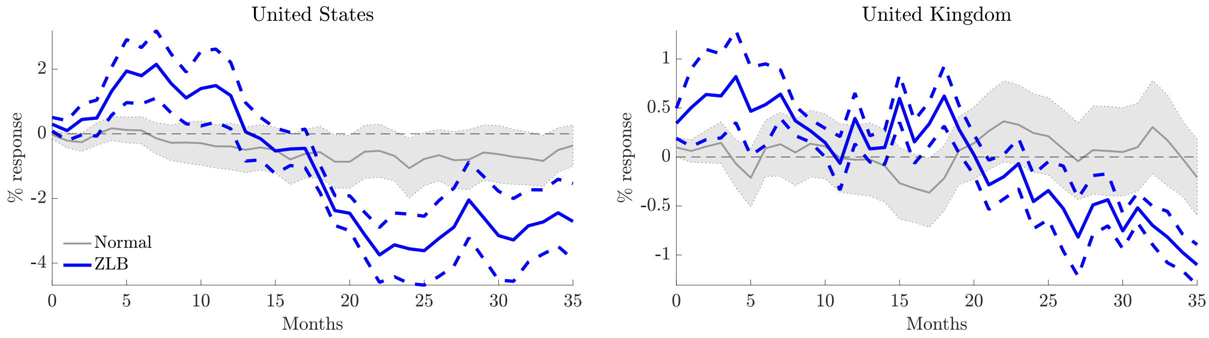
Notes: Each row plots the IRFs and 68% confidence bands of the responses of Japanese industrial proudction and unemployment rate to oil price shocks identified in Kilian (2009) oil demand shocks.

Figure A.4: Open economy variables responses in Japan.



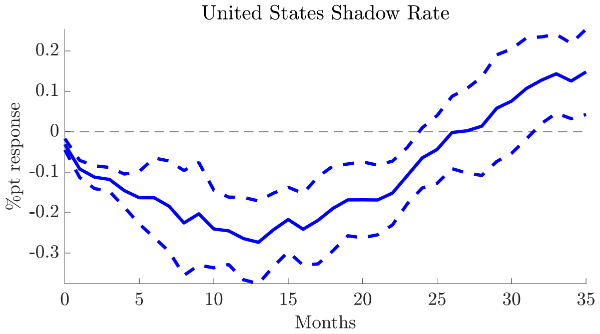
Notes: Each row plots the IRFs and 68% confidence bands of the responses of Japanese real trade balance. real effective exchange rate, nominal effective exchange rate. An increase in the real (nominal) exchange rate is a real (nominal) depreciation.

Figure A.5: Other countries' exchange rates.



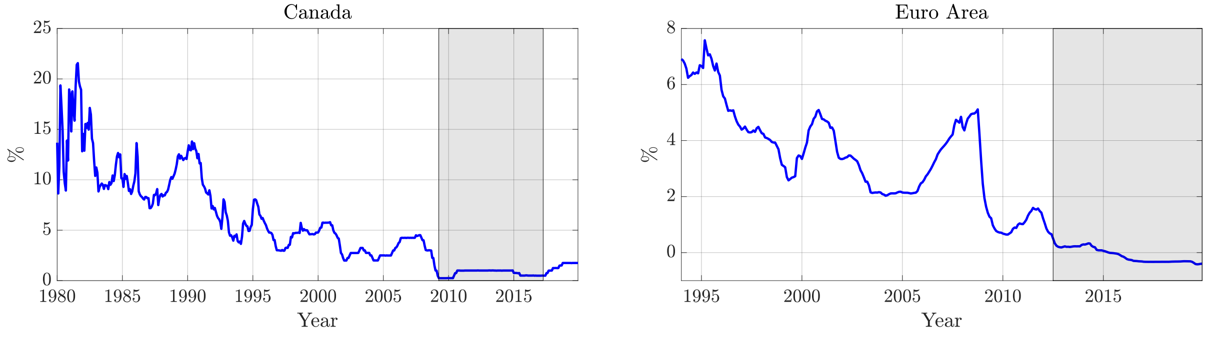
Notes: Each row plots the IRFs and 68% confidence bands of the responses of the United States and the United Kingdom real effective exchange rate. An increase in the real exchange rate is a depreciation.

Figure A.6: Shadow rate impulse responses in the United States in the ZLB period



Notes: This figure plots the mean and one standard-deviation confidence bands of the responses of the shadow interest rate in the Unites States in the ZLB (blue).

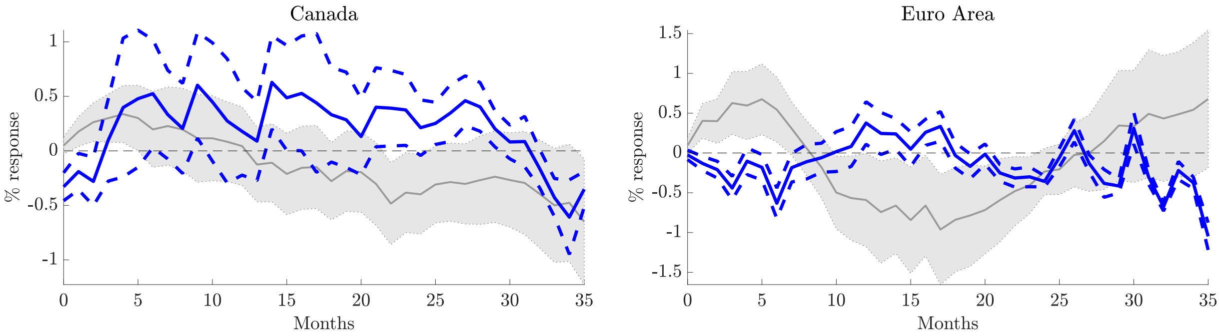
Figure A.7: The short-term nominal interest rates.



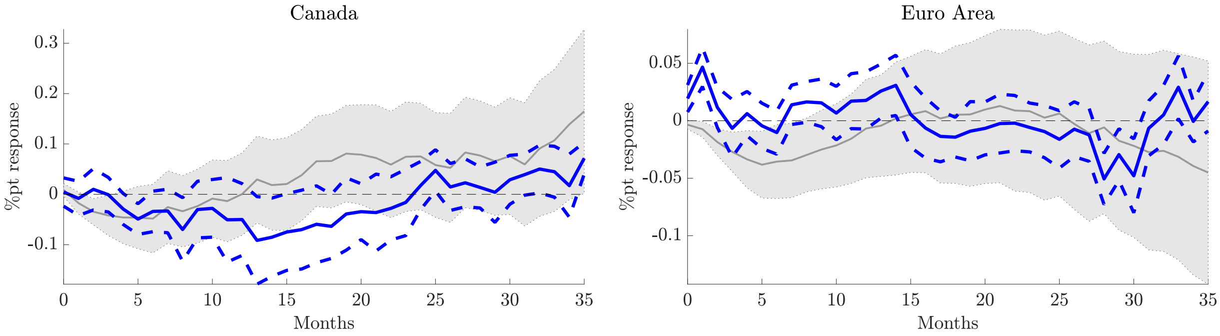
Notes: Each panel presents central bank's policy rate for each country. The shaded areas are the zero lower bound periods defined in Section refsec:Discussion.

Figure A.8: Industrial production and unemployment rate responses in Canada and Euro Area.

(a) Industrial production



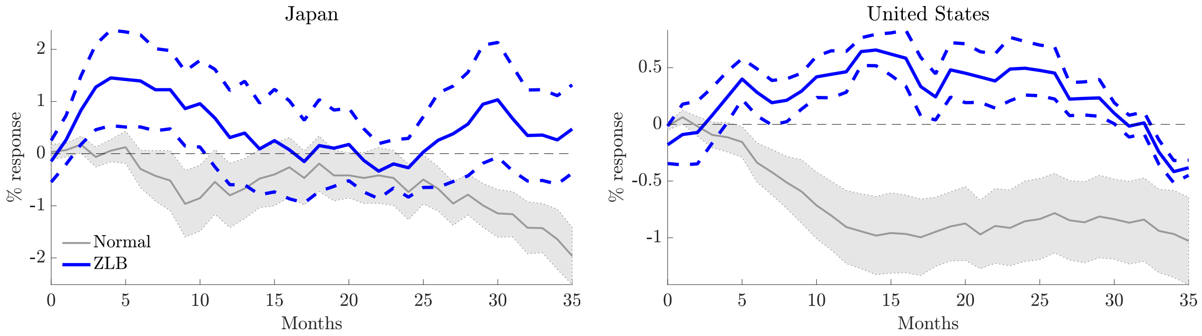
(b) Unemployment rate



Notes: Each figure plots the mean and one standard-deviation confidence bands of the responses. Panel (a) shows industrial production responses in Canada and the Euro Area in the ZLB (blue) and in the normal (grey) periods. Panel (b) plots unemployment rate responses in Canada and the Euro Area in the ZLB (blue) and in the normal (grey) periods.

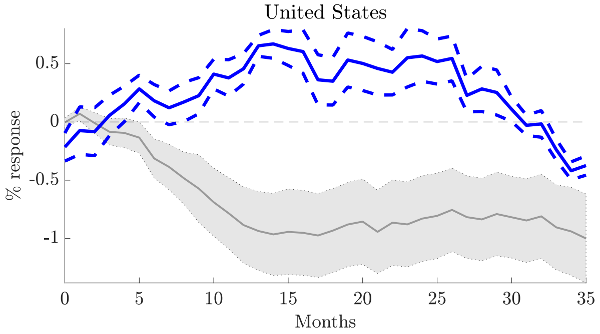
Figure A.9: Non-oil industrial production responses in Japan and the United States

(a) Manufacturing Industrial Production


Japan

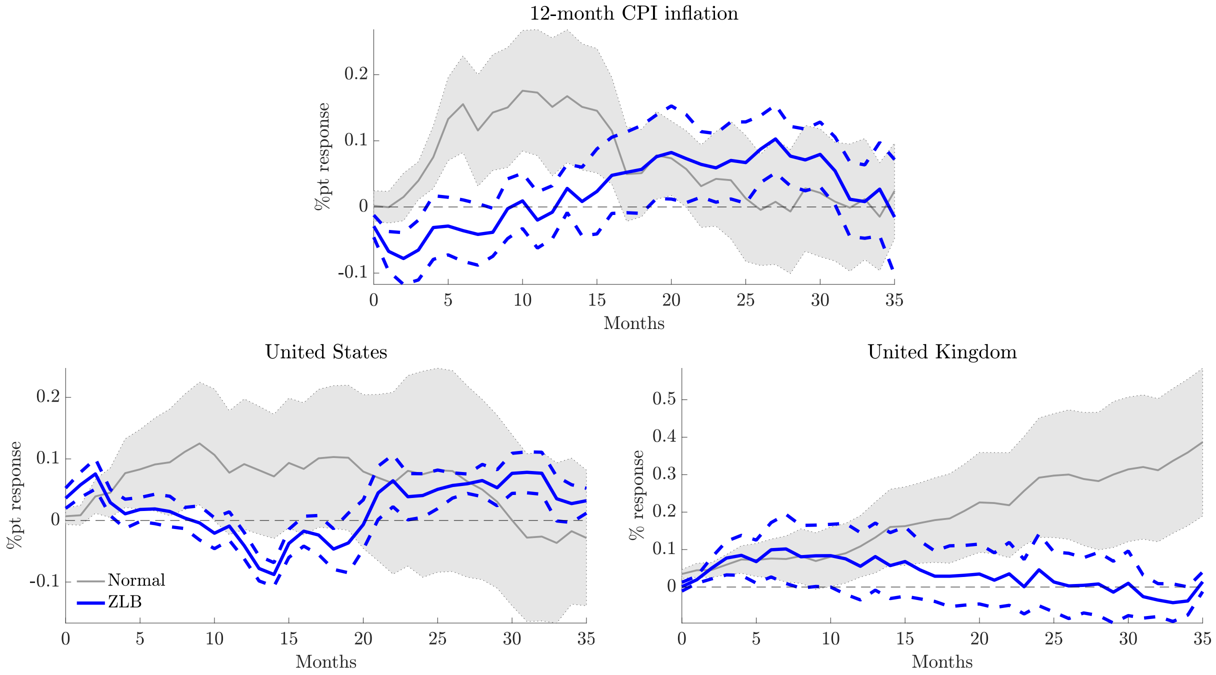
United States

(a) Non-Oil Industrial Production


United States

Notes: This figure plots the mean and one standard-deviation confidence bands of the responses of manufacturing industrial production in Japan and the Unites States in the ZLB (upper panel), and non-oil industrial production in the United States (lower panel).

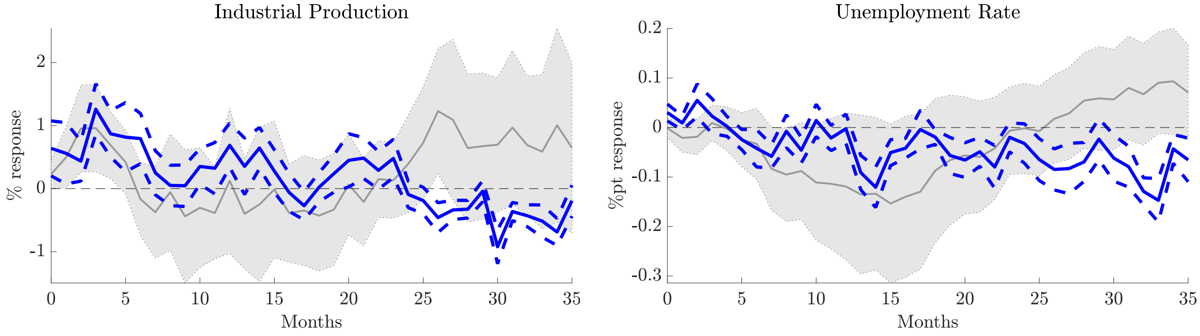
Figure A.10: Core CPI inflation rate responses in Japan, the United States, and United Kingdom.



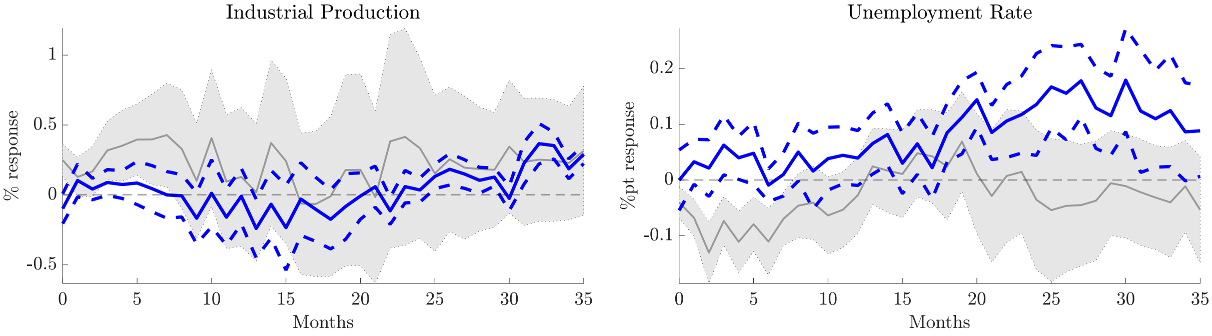
Notes: Each figure plots the mean and one standard-deviation confidence bands of the responses of the core CPI infltion rate in Japan, the United Kingdom, and the Unites States in the ZLB (blue) and in the normal (grey) periods.

Figure A.11: Responses in other countries: Korea, Mexico and Sweden.

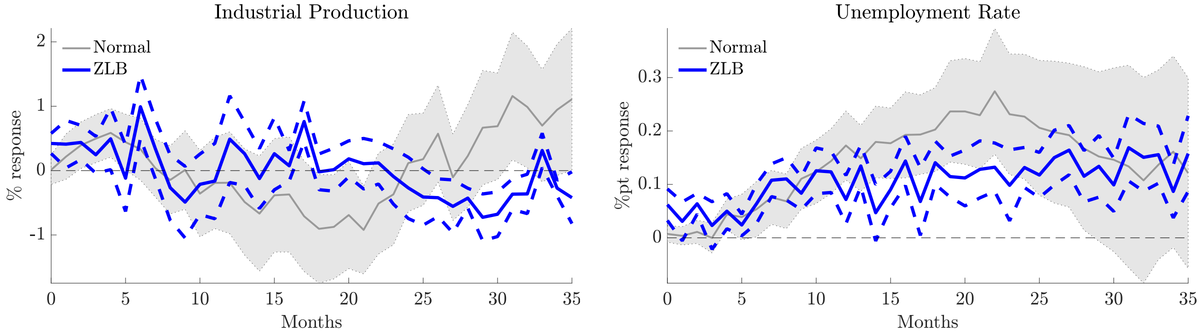
(a) Republic of Korea



(b) Mexico



(c) Sweden



Notes: Each figure plots the mean and one standard-deviation confidence bands of the responses of industrial production (left panels) and the unemployment rate (right panels) in Republic of Korea, Mexico and Sweden in the ZLB (blue) and in the normal (grey) periods, where the ZLB period is defined based on the behavior of the short term nominal interest rate in the US.

B Model Details

This section presents a standard open-economy New Keynesian model extended with demand for oil by firms. The goal of this section is to illustrate the direct effects of oil shocks on aggregate supply, systematic monetary policy, and the world economy.

There are two countries home (H) and foreign (F). We denote variables of foreign country with an asterisk. Each country is populated by measure n and $n^* \equiv 1-n$ residents. All quantity variables are expressed in per capita terms. Monopolistically competitive producers in home country produce varieties with indexes i on the interval $[0,n)$, while foreign firms on the interval $[n,1]$.

We describe the agent in home country. Foreign country agents description is symmetric.

B.1 Households

At home, there is a representative household with preferences represented by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t), \tag{B.1}$$

where

$$U(C,N) \equiv \frac{C^{1-\sigma}}{1-\sigma} - \Theta \frac{N^{1+\varphi}}{1+\varphi},$$

and N_t is labor supply, and C_t is consumption index[21]

$$C_t \equiv \left[\omega^{\frac{1}{\gamma_n}} C_{H,t}^{\frac{\gamma_n-1}{\gamma_n}} + (1-\omega)^{\frac{1}{\gamma_n}} C_{F,t}^{\frac{\gamma_n-1}{\gamma_n}} \right]^{\frac{\gamma_n}{\gamma_n-1}}.$$

The indices of home and foreign goods consumption $C_{H,t}$ and $C_{F,t}$ by home residents are

$$C_{H,t} \equiv \left[\left(\frac{1}{n}\right)^{\frac{1}{\epsilon}} \int_0^n C_{H,t}(i)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}},$$

and

$$C_{F,t} \equiv \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\epsilon}} \int_n^1 C_{F,t}(i)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}},$$

where $i \in [0,1]$ is the index of individual good variety, ϵ is the elasticity of substitution across varieties, γ_n is the elasticity of substitution between home and foreign goods, $1-\omega \equiv (1-n)(1-\Omega)$ is a share of foreign-good expenditures in total expenditures, and $\Omega \in [0,1)$ is an index of home bias. The value of $\Omega = 0$ corresponds to the case of no home bias, while $\Omega > 0$ is required for non-zero degree of home bias. For the foreign economy, $1-\omega^* \equiv n(1-\Omega^*)$ is the share of home-good expenditures in total expenditures.

The household maximizes its preferences by choosing a plan $\{\{C_{F,t}(i)\}, \{C_{H,t}(i)\}, C_{F,t}, C_{H,t}, C_t, N_t,$

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[21] The consumption index in foreign country is symmetric to home consumption index and equals: $C_t^* \equiv [(\omega^*)^{1/\gamma_n} (C_{F,t}^*)^{(\gamma_n-1)/\gamma_n} + (1-\omega^*)^{1/\gamma_n} (C_{H,t}^*)^{(\gamma_n-1)/\gamma_n}]^{\gamma_n/(\gamma_n-1)}$, where ω^* is a share of foreign goods in foreign consumption basket.

$B_{H,t+1}, B_{F,t+1}\}$ subject to the flow budget constraint

$$\int_0^n P_{H,t}(i)C_{H,t}(i)di + \mathcal{E}_t \int_n^1 P^*_{F,t}(j)C_{F,t}(j)dj + \mathbb{E}_t\left[M_{t,t+1}B_{H,t+1}\right] \\ +\mathbb{E}_t\left[\mathcal{E}_t M^*_{t,t+1}B_{F,t+1}\right] \leq B_{H,t} + \mathcal{E}_t B_{F,t} + W_t N_t + P_{O,t}O_t + \Pi_t - T_t, \tag{B.2}$$

where $P_{H,t}(i)$ and $P^*_{F,t}(j)$ are the prices of home and foreign varieties denoted in home and foreign currencies, respectively, $\mathcal{E}_t$ is the nominal exchange rate in units of domestic currency per foreign currency (an increase in $\mathcal{E}_t$ is a depreciation of home currency), $P_{O,t}$ is the oil price in units of home currency, $M_{t,t+1}$ and $M^*_{t,t+1}$ are prices of domestic and foreign state-contingent securities in home and foreign currencies, W_t is nominal wage at home, Π_t represent nominal profits, and T_t is nominal lump-sum taxes. The holdings of home and foreign state-contingent securities are $B_{H,t+1}$ and $B_{F,t+1}$. O_t represents the oil endowment at home.

Price indexes, terms of trade, and real exchange rate. We define the price index of home and foreign produced goods as $P_{H,t} \equiv [n^{-1}\int_0^n P_{H,t}(i)^{1-\epsilon}di]^{1/(1-\epsilon)}$ and $P^*_{F,t} \equiv [(1-n)^{-1}\int_n^1 P^*_{F,t}(i)^{1-\epsilon}di]^{1/(1-\epsilon)}$, and the home CPI is

$$P_t \equiv \left[\omega P_{H,t}^{1-\gamma_n} + (1-\omega)\left(\mathcal{E}_t P^*_{F,t}\right)^{1-\gamma_n}\right]^{1/(1-\gamma_n)}, \tag{B.3}$$

In addition, we assume that the law of one price holds for all goods. We define the terms of trade as the relative price of imported goods $S_t \equiv P_{F,t}/P_{H,t}$ and the real exchange rate as

$$Q_t \equiv \mathcal{E}_t P^*_t / P_t \tag{B.4}$$

The real oil price as the nominal oil price expressed in home currency divided by the foreign producers price index $P_{F,t}$ expressed in home currency:

$$R_t \equiv \frac{P_{O,t}}{P_{F,t}}.$$

Optimality conditions. Consumption

$$C_{H,t}(i) = \frac{1}{n}\left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon} C_{H,t}, \tag{B.5}$$

$$C_{F,t}(i) = \frac{1}{1-n}\left(\frac{P^*_{F,t}(i)}{P^*_{F,t}}\right)^{-\epsilon} C_{F,t}, \tag{B.6}$$

where

$$C_{H,t} = \omega\left(\frac{P_{H,t}}{P_t}\right)^{-\gamma_n} C_t, \tag{B.7}$$

$$C_{F,t} = (1-\omega)\left(\frac{\mathcal{E}_t P^*_{F,t}}{P_t}\right)^{-\gamma_n} C_t, \tag{B.8}$$

Labor supply

$$-\frac{U_2(C_t, N_t)}{U_1(C_t, N_t)} = \frac{W_t}{P_t}. \tag{B.9}$$

where $U_1(C_t, N_t)$ and $U_2(C_t, N_t)$ are partial derivatives with respect to the first and the second arguments. Assets

$$M_{t,t+1} = \beta \frac{U_1(C_{t+1}, N_{t+1})}{U_1(C_t, N_t)} \cdot \frac{P_t}{P_{t+1}}, \tag{B.10}$$

$$M^*_{t,t+1} = \beta \frac{U_1(C_{t+1}, N_{t+1})}{U_1(C_t, N_t)} \cdot \frac{P_t}{P_{t+1}} \cdot \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}. \tag{B.11}$$

Foreign households. Conditions symmetric to (B.5)-(B.11) hold for foreign country. In particular, the first-order condition with respect to $B^*_{H,t}$ imply

$$M_{t,t+1} = \beta \frac{U_1(C^*_{t+1}, N^*_{t+1})}{U_1(C^*_t, N^*_t)} \cdot \frac{P^*_t}{P^*_{t+1}} \cdot \frac{1/\mathcal{E}_{t+1}}{1/\mathcal{E}_t}. \tag{B.12}$$

Combining equations (B.10) and (B.12), we get

$$\frac{U_1(C_{t+1}, N_{t+1})}{U_1(C^*_{t+1}, N^*_{t+1})} Q_{t+1} = \frac{U_1(C_t, N_t)}{U_1(C^*_t, N^*_t)} Q_t \equiv \phi_q, \tag{B.13}$$

which implies that the ratio of marginal utility of consumption in the two countries multiplied by the real exchange rate is constant over time, which we denote as ϕ_q.

Log-linarization. We next log-linearize around the steady state where all relative goods prices are 1 and all prices and quantities are constant. Section B.8 will provide more details about steady state. For now, we obtain

$$\widehat{c}_{H,t} = -\gamma_n \left(\widehat{p}_{H,t} - \widehat{p}_t\right) + \widehat{c}_t,$$
$$\widehat{c}_{F,t} = -\gamma_n \left(\widehat{e}_t + \widehat{p}^*_{F,t} - \widehat{p}_t\right) + \widehat{c}_t,$$

where hats denote log-deviations from steady state values.

The labor supply

$$\sigma \widehat{c}_t + \varphi \widehat{n}_t = \widehat{w}_t - \widehat{p}_t.$$

The Euler equation

$$\mathbb{E}_t \left[\widehat{c}_{t+1}\right] - \widehat{c}_t = \frac{1}{\sigma} \left(i_t - \mathbb{E}_t \left[\pi_{t+1}\right] - \iota\right),$$

where $i_t \equiv -\log \mathbb{E}_t[M_{t,t+1}]$ is the safe short-term nominal interest rate and $\iota \equiv -\log \beta$.

The international risk sharing condition—a log-linear version of the second equality in equation (B.13)—is

$$\widehat{c}^*_t + \frac{1}{\sigma} q_t = \widehat{c}_t.$$

Total consumption is

$$\widehat{c}_t = \omega \widehat{c}_{H,t} + (1 - \omega) \widehat{c}_{F,t}.$$

B.2 Price indices

The log-linearization of price indexes in home and foreign countries are

$$\begin{aligned}\widehat{p}_t &= \omega \widehat{p}_{H,t} + (1-\omega)\widehat{p}_{F,t} = \widehat{p}_{H,t} + (1-\omega)\left(\widehat{p}_{F,t} - \widehat{p}_{H,t}\right), \\ \widehat{p}^*_t &= \omega^* \widehat{p}^*_{F,t} + (1-\omega^*)\widehat{p}^*_{H,t} = \widehat{p}^*_{F,t} + (1-\omega^*)\left(\widehat{p}^*_{H,t} - \widehat{p}^*_{F,t}\right).\end{aligned}$$

The real exchange rate is

$$\widehat{q}_t = \widehat{e}_t + \widehat{p}^*_t - \widehat{p}_t = (\omega + \omega^* - 1)\left(\widehat{p}_{F,t} - \widehat{p}_{H,t}\right).$$

Home relative prices are

$$\begin{aligned}\widehat{p}_{H,t} - \widehat{p}_t &= -\frac{1-\omega}{\omega + \omega^* - 1}\widehat{q}_t, \\ \widehat{p}_{F,t} - \widehat{p}_t &= \frac{\omega}{\omega + \omega^* - 1}\widehat{q}_t, \\ \widehat{p}_{O,t} - \widehat{p}_t &= \widehat{p}_{O,t} - \widehat{p}_{F,t} + \frac{\omega}{\omega + \omega^* - 1}\widehat{q}_t.\end{aligned}$$

Foreign relative prices are

$$\begin{aligned}\widehat{p}^*_{F,t} - \widehat{p}^*_t &= \frac{1-\omega^*}{\omega + \omega^* - 1}\widehat{q}_t, \\ \widehat{p}^*_{H,t} - \widehat{p}^*_t &= -\frac{\omega^*}{\omega + \omega^* - 1}\widehat{q}_t, \\ \widehat{p}^*_{O,t} - \widehat{p}^*_t &= \widehat{p}^*_{O,t} - \widehat{p}^*_{F,t} + \frac{1-\omega^*}{\omega + \omega^* - 1}\widehat{q}_t.\end{aligned}$$

B.3 Firms

A home producer of variety i combines labor $N_t(i)$ and oil inputs $O_{Y,t}(i)$ to produce good i according to the CES production function of the form

$$Y_t(i) = \left[(1-\omega_{oy})^{\frac{1}{\gamma_y}}\left(AN_t(i)\right)^{\frac{\gamma_y - 1}{\gamma_y}} + \omega_{oy}^{\frac{1}{\gamma_y}} O_{Y,t}(i)^{\frac{\gamma_y - 1}{\gamma_y}}\right]^{\frac{\gamma_y}{\gamma_y - 1}},$$

where A_t is labor productivity, the intra-temporal elasticity of substitution γ_y is positive, and $\omega_{oy} \in [0,1]$ is the share of oil in production. The firm is free to optimize its inputs every period. Because of the constant-elasticity-of-substitution assumption about the form of the production function, the *nominal* marginal cost of production does not depend on output and equals

$$MC_t = \left[(1-\omega_{oy})\left(\frac{W_t}{A}\right)^{1-\gamma_y} + \omega_{oy} P_{O,t}^{1-\gamma_y}\right]^{\frac{1}{1-\gamma_y}}, \tag{B.14}$$

and the optimal labor and oil choices are

$$O_{Y,t}(i) = \omega_{oy} \left(\frac{P_{O,t}}{MC_t} \right)^{-\gamma_y} Y_t(i), \tag{B.15}$$

$$N_t(i) = (1 - \omega_{oy}) \left(\frac{W_t / A_t}{MC_t} \right)^{-\gamma_y} \frac{Y_t(i)}{A}. \tag{B.16}$$

The firm resets its price infrequently in the spirit of Calvo. When allowed, a firm chooses the reset price $P^r_{H,t}(i)$ to maximizes an expected discounted sum of future profits

$$\mathbb{E}_t \left[\sum_{k=0}^{\infty} \theta^k M_{t,t+k} \left(P^r_{H,t}(i) - (1+\tau) MC_{t+k} \right) Y_{H,t+k|t}(i) \right]$$

where $1 - \theta \in [0, 1]$ is the probability of price reset, $Y_{H,t+k|t}(i)$ is the output in period $t + k$ conditional on the price set in period t, and τ is the government's proportional tax. The price is set in producer's currency and the firm does not differentiate between domestic and foreign consumers. In other words, the law of one price holds for individual varieties.

The demand for variety i is

$$Y_{H,t+k|t}(i) = nC_{H,t+k|t}(i) + (1-n) C^*_{H,t+k|t}(i) = \left(\frac{P^r_{H,t}(i)}{P_{H,t+k}} \right)^{-\epsilon} \left(C_{H,t+k} + \frac{1-n}{n} C^*_{H,t+k} \right),$$

where $C_{H,t+k}$ and $C^*_{H,t+k}$ are the home and foreign demand for home goods.

Firm's optimal choice of its reset price requires

$$\mathbb{E}_t \left[\sum_{k=0}^{\infty} \theta^k M_{t,t+k} Y_{N,t+k|t}(i) \left(\frac{P^r_{H,t}(i)}{P_{H,t-1}} - \frac{\epsilon}{\epsilon - 1} (1+\tau) \frac{MC_{t+k}}{P_{H,t+k}} \cdot \frac{P_{H,t+k}}{P_{H,t-1}} \right) \right] = 0, \tag{B.17}$$

where we deflated the expression by $P_{H,t-1}$.

Log-linearization. We log-linearize around a steady state with constant prices and quantities and where all relative prices equal one. The marginal costs are

$$\widehat{mc}_t = (1 - \omega_{oy}) \widehat{w}_t + \omega_{oy} \widehat{p}_{O,t}.$$

The demand for labor and oil are

$$\widehat{n}_t = -\gamma_y \left(\widehat{w}_t - \widehat{mc}_t \right) + \frac{nC_H}{nC_H + (1-n) C^*_H} \widehat{c}_{H,t} + \frac{(1-n) C^*_H}{nC_H + (1-n) C^*_H} \widehat{c}^*_{H,t}$$

$$\widehat{o}_{Y,t} = -\gamma_y \left(\widehat{p}_{O,t} - \widehat{mc}_t \right) + \frac{nC_H}{nC_H + (1-n) C^*_H} \widehat{c}_{H,t} + \frac{(1-n) C^*_H}{nC_H + (1-n) C^*_H} \widehat{c}^*_{H,t},$$

The optimal reset price satisfies

$$p^r_{H,t} - p_{H,t-1} = (1 - \beta\theta) \left(\widehat{mc}_t - \widehat{p}_{H,t} \right) + \pi_{H,t} + \beta\theta \mathbb{E}_t \left[p^r_{H,t+1} - p_{H,t} \right]. \tag{B.18}$$

B.4 Government

The home government consists of fiscal and monetary authorities. The fiscal authority undoes monopolistic competition distortion by subsidizing production. Specifically, it sets the production tax $\tau = -1/\epsilon$ and runs a balanced budget each period.

The monetary authority sets the short-term safe nominal interest rate $i_t \equiv -\log \mathbb{E}_t[M_{t,t+1}]$ according to the Taylor rule that reacts to the producers price inflation rate $\pi_{H,t} \equiv \log(P_{H,t}/P_{H,t-1})$. Formally,

$$i_t = \iota + \varphi_\pi \pi_t, \tag{B.19}$$

where $\iota \equiv -\log \beta$ and $\varphi_\pi \geq 0$. We do not explicitly introduce the zero lower bound on the interest rate here.

The foreign government acts analogous to home government. In particular, the central bank set the interest rate according to

$$i_t^* = \iota + \varphi_\pi^* \pi_{H,t}^*, \tag{B.20}$$

where $\varphi_\pi^* \geq 0$.

B.5 Oil

Oil is supplied as endowment that equals O_t and O_t^* in per capita terms, and these endowments change randomly every period. Households own oil and it cannot be stored. The oil prices $P_{O,t}$ and $P_{O,t}^*$ are perfectly flexible and the law of one price applies to oil.

B.6 Market Clearing

Local markets. The home labor supply equals home labor demand

$$N_t = \int_0^n \left(1 - \omega_{oy}\right) \left(\frac{W_t/A_t}{MC_t}\right)^{-\gamma_y} Y_t(i) di.$$

The market for every home variety $i \in [0,1]$ clears

$$Y_{H,t}(i) = nC_{H,t}(i) + (1-n)C_{H,t}^*(i). \tag{B.21}$$

The analogoes conditions hold for foreign country.

Global markets. In equilibrium, all markets clear. Specifically, all state-contingent asset markets clear

$$n\left(B_{H,t} + \mathcal{E}_t B_{F,t}\right) + (1-n)\left(B_{H,t}^* + \mathcal{E}_t B_{F,t}^*\right) = 0.$$

The oil market clears

$$nO_{Y,t} + (1-n)\, O_{Y,t}^* = nO_t + (1-n)O_t^*.$$

and its log-linear approximation is

$$nO_Y \widehat{o}_{Y,t} + (1-n)O_Y^* \widehat{o}_{Y,t}^* = nO\widehat{o}_t + (1-n)O^* \widehat{o}_t^*.$$

B.7 Aggregation

The law of motion of the price index

$$\Pi_{H,t}^{1-\epsilon} = \theta + (1-\theta)\left(\frac{P_{H,t}^{r}}{P_{H,t-1}}\right)^{1-\epsilon},$$
$$\left(\Pi_{F,t}^{*}\right)^{1-\epsilon} = \theta + (1-\theta)\left(\frac{P_{F,t}^{*,r}}{P_{F,t-1}^{*}}\right)^{1-\epsilon}.$$

The price dispersion in home country is

$$\Delta_{H,t} \equiv \frac{1}{n}\int_0^n \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon} di = \theta\left(\frac{P_{H,t-1}}{P_{H,t}}\right)^{-\epsilon}\Delta_{H,t-1} + (1-\theta)\left(\frac{P_{H,t}^{r}}{P_{H,t}}\right)^{-\epsilon}.$$

Aggregate goods market clearing in home and foreign countries

$$\left[(1-\omega_{oy})^{\frac{1}{\gamma y}}\left(A_t N_t\right)^{\frac{\gamma y-1}{\gamma y}} + \omega_{oy}^{\frac{1}{\gamma y}} O_{Y,t}^{\frac{\gamma y-1}{\gamma y}}\right]^{\frac{\gamma y}{\gamma y-1}} = \left(C_{H,t+k} + \frac{1-n}{n}C_{H,t+k}^{*}\right)\Delta_{H,t},$$
$$\left[(1-\omega_{oy})^{\frac{1}{\gamma y}}\left(A_t^{*} N_t^{*}\right)^{\frac{\gamma y-1}{\gamma y}} + \omega_{oy}^{\frac{1}{\gamma y}} \left(O_{Y,t}^{*}\right)^{\frac{\gamma y-1}{\gamma y}}\right]^{\frac{\gamma y}{\gamma y-1}} = \left(C_{F,t+k}^{*} + \frac{n}{1-n}C_{H,t+k}\right)\Delta_{F,t}^{*}.$$

Value added in terms of produced goods

$$Z_{H,t} = Y_{H,t} - \frac{P_{O,t}}{P_{H,t}}O_{Y,t} = Y_{H,t} - R_t\frac{P_{F,t}}{P_{H,t}}O_{Y,t},$$
$$Z_{F,t}^{*} = Y_{F,t}^{*} - \frac{P_{O,t}^{*}}{P_{F,t}^{*}}O_{Y,t}^{*} = Y_{F,t}^{*} - R_t O_{Y,t}^{*}.$$

where $Y_{H,t}$ and $Y_{F,t}^{*}$ is the demand for home and foreign-produced goods.

Trade balance in units of domestically produced goods over steady state output

$$\frac{NX_t}{Y_H} = \frac{1}{Y_H}\left(Y_{H,t} + \frac{P_{O,t}}{P_{H,t}}O_t - \frac{P_t}{P_{H,t}}C_t - \frac{P_{O,t}}{P_{H,t}}O_{Y,t}\right),$$
$$\frac{NX_t^{*}}{Y_F^{*}} = \frac{1}{Y_F^{*}}\left(Y_{F,t}^{*} + \frac{P_{O,t}^{*}}{P_{F,t}^{*}}O_t^{*} - \frac{P_t^{*}}{P_{F,t}^{*}}C_t^{*} - \frac{P_{O,t}^{*}}{P_{F,t}^{*}}O_{Y,t}^{*}\right).$$

This formulas takes into account that the countries trade in goods and oil.

Log-linearization. Home and foreign inflation rates are

$$\pi_{H,t} = (1-\theta)\left(\widehat{p}_{H,t}^{r} - \widehat{p}_{H,t-1}\right),$$
$$\pi_{F,t}^{*} = (1-\theta)\left(\widehat{p}_{F,t}^{*,r} - \widehat{p}_{F,t-1}^{*}\right).$$

Combining the last two equations with the optimal reset price equation at home B.18 and a similar equation abroad, we get two standard Phillips curves

$$\begin{aligned}
\pi_{H,t} &= \frac{(1-\theta)(1-\beta\theta)}{\theta}\left(\widehat{mc}_t - \widehat{p}_{H,t}\right) + \beta\mathbb{E}_t\left[\pi_{H,t+1}\right], \\
\pi^*_{F,t} &= \frac{(1-\theta)(1-\beta\theta)}{\theta}\left(\widehat{mc}^*_t - \widehat{p}^*_{F,t}\right) + \beta\mathbb{E}_t\left[\pi^*_{F,t+1}\right].
\end{aligned}$$

Goods market clearing

$$\begin{aligned}
&(1-\omega_{oy})^{\frac{1}{\gamma_y}}\left(\frac{AN}{C_H + \frac{1-n}{n}C^*_H}\right)^{\frac{\gamma_y-1}{\gamma_y}}\left(\widehat{a}_t + \widehat{n}_t\right) + \omega_{oy}^{\frac{1}{\gamma_y}}\left(\frac{O_Y}{C_H + \frac{1-n}{n}C^*_H}\right)^{\frac{\gamma_y-1}{\gamma_y}}\widehat{o}_{Y,t} \\
=&\frac{nC_H}{nC_H + (1-n)\,C^*_H}\widehat{c}_{H,t} + \frac{(1-n)\,C^*_H}{nC_H + (1-n)\,C^*_H}\widehat{c}^*_{H,t}, \\
&(1-\omega_{oy})^{\frac{1}{\gamma_y}}\left(\frac{A^*N^*}{C^*_F + \frac{n}{1-n}C_F}\right)^{\frac{\gamma_y-1}{\gamma_y}}\left(\widehat{a}^*_t + \widehat{n}^*_t\right) + \omega_{oy}^{\frac{1}{\gamma_y}}\left(\frac{O^*_Y}{C^*_F + \frac{n}{1-n}C_F}\right)^{\frac{\gamma_y-1}{\gamma_y}}\widehat{o}^*_{Y,t} \\
=&\frac{(1-n)\,C^*_F}{(1-n)\,C^*_F + nC_F}\widehat{c}^*_{F,t} + \frac{nC_F}{(1-n)\,C^*_F + nC_F}\widehat{c}_{F,t},
\end{aligned}$$

Value added

$$\begin{aligned}
\widehat{z}_{H,t} &= \frac{1}{1-\omega_{oy}}\widehat{y}_{H,t} - \frac{\omega_{oy}}{1-\omega_{oy}}\left(\widehat{r}_t + \widehat{p}_{F,t} - \widehat{p}_{H,t} + \widehat{o}_{y,t}\right), \\
\widehat{z}^*_{F,t} &= \frac{1}{1-\omega_{oy}}\widehat{y}^*_{F,t} - \frac{\omega_{oy}}{1-\omega_{oy}}\left(\widehat{r}_t + \widehat{o}^*_{y,t}\right).
\end{aligned}$$

The trade balance is

$$\widehat{nx}_t = \widehat{y}_{H,t} + \frac{O - O_Y}{Y_H}\left(\widehat{r}_t + \frac{1}{\omega+\omega^*-1}\widehat{q}_t\right) + \frac{O\widehat{o}_t - O_Y\widehat{o}_{Y,t}}{Y_H} - \frac{C}{Y_H}\left(\frac{1-\omega}{\omega+\omega^*-1}\widehat{q}_t + \widehat{c}_t\right), \tag{B.22}$$

$$\widehat{nx}^*_t = \widehat{y}^*_{F,t} + \frac{O^* - O^*_Y}{Y^*_F}\widehat{r}_t + \frac{O^*\widehat{o}^*_t - O^*_Y\widehat{o}^*_{Y,t}}{Y^*_F} - \frac{C^*}{Y^*_F}\left(-\frac{1-\omega^*}{\omega+\omega^*-1}\widehat{q}_t + \widehat{c}^*_t\right). \tag{B.23}$$

B.8 Steady State

We assume that in steady state all prices are constant and the relative prices equal one:

$$\frac{P_O}{P} = \frac{P_F}{P} = \frac{P_H}{P} = 1.$$

This implies that the real exchange rate is

$$Q = \frac{\mathcal{E}P^*}{P} = \mathcal{E}\frac{\left[(1-\omega^*)\left(P_H/\mathcal{E}\right)^{1-\gamma_n} + \omega^*\left(P^*_F\right)^{1-\gamma_n}\right]^{1/(1-\gamma_n)}}{\left[\omega P_H^{1-\gamma_n} + (1-\omega)\left(\mathcal{E}P^*_F\right)^{1-\gamma_n}\right]^{1/(1-\gamma_n)}} = 1$$

We get

$$-\frac{U_2\left(C,N\right)}{U_1\left(C,N\right)} = A$$

$$
\begin{aligned}
\frac{1}{R} &= \beta, \\
\frac{W}{P} &= A, \\
\frac{MC}{P} &= \frac{W}{AP} = 1, \\
C_H &= \omega C, \\
C_F &= (1-\omega)\, C, \\
Y_H &= C_H + \frac{1-n}{n} C_H^*, \\
Y_H &= \left[(1-\omega_{oy})^{\frac{1}{\gamma_y}} (AN)^{\frac{\gamma_y - 1}{\gamma_y}} + \omega_{oy}^{\frac{1}{\gamma_y}} O_Y^{\frac{\gamma_y - 1}{\gamma_y}} \right]^{\frac{\gamma_y}{\gamma_y - 1}}, \\
AN &= \left(1-\omega_{oy}\right) Y_H, \\
O_Y &= \omega_{oy} Y_H, \\
C &= Y_H + O - O_Y.
\end{aligned}
$$

Observe that that the last equation is the flow budget constraint of the country. Also note that if there were no oil, then we would have $C = Y_H$ and $C^* = C_F^* + \frac{n}{1-n} C_F$ and $C_H = \omega C$ and $C_F = (1-\omega)\, C$ and $C_F^* = \omega^* C^*$ and $C_H^* = (1-\omega^*)\, C^*$. Combining these together, we get $(1-\omega)\, nC = (1-n)(1-\omega^*)\, C^*$ and $(1-n)(1-\omega^*)\, C^* = n(1-\omega)\, C$. The last two equations are identical. They determine the ratio: $C/C^* = (1-n)/n \cdot (1-\omega^*)/(1-\omega) = (1-\Omega^*)/(1-\Omega)$. How can we determine steady state consumption level? The Backus-Smith condition $C = \phi_q^{-\frac{1}{\sigma}} C^* Q^{\frac{1}{\sigma}}$ implies $C/C^* = \phi_q^{-\frac{1}{\sigma}} = (1-\Omega^*)/(1-\Omega)$.

Simplifying we get 3 equations and 2 unknowns C, N as functions of parameters and foreign consumption C^*. The first two equations below unambiguously determine C and AN, while the third equation is a constraint on the parameter Θ.

$$
\begin{aligned}
nC &= \frac{(1-n)(1-\omega^*)\left(1-\omega_{oy}\right) C^* + nO}{1-\left(1-\omega_{oy}\right)\omega}, \\
AN &= \left(1-\omega_{oy}\right)\left[\omega C + \frac{1-n}{n}(1-\omega^*)\, C^*\right], \\
-\frac{U_2(C,N)}{U_1(C,N)} &= \frac{\Theta N^{\varphi}}{C^{-\sigma}} = A,
\end{aligned}
$$

We can write symmetric equations for the foreign economy. Specifically,

$$
(1-n)\, C^* = \frac{n(1-\omega)\left(1-\omega_{oy}\right) C + (1-n)\, O^*}{1-\left(1-\omega_{oy}\right)\omega^*},
$$

$$
N^* = \frac{\left(1-\omega_{oy}\right)\left(\omega^* C^* + \frac{n}{1-n}(1-\omega)\, C\right)}{A}.
$$

For the Backus-Smith condition to be satisfied, the constant ϕ_q has to take a certain value.

$$
\frac{C}{C^*} = \phi_q^{-\frac{1}{\sigma}}.
$$

Solving jointly for C and C^*, we get:

$$nC = \frac{\left[1-\left(1-\omega_{oy}\right)\omega^*\right] nO + \left(1-\omega_{oy}\right)\left(1-\omega^*\right)\left(1-n\right)O^*}{\omega_{oy}\left\{1+\left(1-\omega_{oy}\right)\left[1-\left(\omega^*+\omega\right)\right]\right\}}$$

and

$$\left(1-n\right)C^* = \frac{\left(1-\omega\right)\left(1-\omega_{oy}\right) nO + \left[1-\omega\left(1-\omega_{oy}\right)\right]\left(1-n\right)O^*}{\omega_{oy}\left\{1+\left(1-\omega_{oy}\right)\left[1-\left(\omega^*+\omega\right)\right]\right\}}.$$

The steady state share of home goods domestic demand in home good demand

$$\frac{nC_H}{nC_H+\left(1-n\right)C_H^*} = \frac{n\omega C}{n\omega C+\left(1-n\right)\left(1-\omega^*\right)C^*}$$

And

$$\frac{nC_F}{nC_F+\left(1-n\right)C_F^*} = \frac{n\left(1-\omega\right)C}{n\left(1-\omega\right)C+\left(1-n\right)\omega^* C^*}$$

Consumption normalization. We further normalize steady state to have unit consumption in home and foreign countries: $C = C^* = 1$.This normalization requires the following restriction on the parameters

$$\begin{aligned} 1 &= \left(1-\omega_{oy}\right)\left[1+\left(1-n\right)\left(\Omega-\Omega^*\right)\right]+O. \\ 1 &= \left(1-\omega_{oy}\right)\left[1+n\left(\Omega^*-\Omega\right)\right]+O^*, \end{aligned}$$

Small open economy limit. In the case when n goes to zero, we get

$$\begin{aligned} \left(1-\Omega^*\right) &= \left(1-\Omega\right)+\frac{\omega_{oy}-O}{1-\omega_{oy}}. \\ O^* &= \omega_{oy}. \end{aligned}$$

In this case, we have

$$\frac{nC_H}{nC_H+\left(1-n\right)C_H^*} = \frac{\Omega}{1-O}\left(1-\omega_{oy}\right) = \Omega\frac{1-O^*}{1-O} = \frac{\Omega}{1+\Omega-\Omega^*}.$$

No home oil production (benchmark case). As a benchmark case, we assume that home country does not produce oil, that is, $O = 0$. In this case,

$$\begin{aligned} \left(1-\Omega^*\right) &= \left(1-\Omega\right)+\frac{\omega_{oy}}{1-\omega_{oy}}. \\ O^* &= \omega_{oy}, \end{aligned}$$

and the share of domestic and foreign consumption are

$$\begin{aligned} \frac{nC_H}{nC_H+\left(1-n\right)C_H^*} &= \Omega\left(1-\omega_{oy}\right), \\ \frac{nC_F}{nC_F+\left(1-n\right)C_F^*} &= 0. \end{aligned}$$

B.9 Equilibrium

Unknowns (27).

$$\begin{aligned}
\textbf{Home quantities (6): } & \widehat{c}_t, \widehat{c}_{H,t}, \widehat{c}_{F,t}, \widehat{o}_{C,t}, \widehat{o}_{Y,t}, \widehat{n}_t, \\
\textbf{Home prices (6):} & \widehat{p}_t, \widehat{p}_{H,t}, \widehat{w}_t, i_t, \widehat{mc}_t, \pi_{H,t} \\
\textbf{Foreign quantities (6): } & \widehat{c}^*_t, \widehat{c}^*_{H,t}, \widehat{c}^*_{F,t}, \widehat{o}^*_{C,t}, \widehat{o}^*_{Y,t}, \widehat{n}^*_t, \\
\textbf{Foreign prices (6):} & \widehat{p}^*_t, \widehat{p}^*_{F,t}, \widehat{w}^*_t, i^*_t, \widehat{mc}^*_t, \pi^*_{F,t} \\
\textbf{International prices (3): } & \widehat{e}_t, \widehat{q}_t, \widehat{p}_{O,t}
\end{aligned}$$

Home conditions (11 equations). Households

$$\begin{aligned}
\widehat{c}_{H,t} &= -\gamma_n \left(\widehat{p}_{H,t} - \widehat{p}_t\right) + \widehat{c}_t, \\
\widehat{c}_{F,t} &= -\gamma_n \left(\widehat{e}_t + \widehat{p}^*_{F,t} - \widehat{p}_t\right) + \widehat{c}_t, \\
\sigma \widehat{c}_t + \varphi \widehat{n}_t &= \widehat{w}_t - \widehat{p}_t, \\
\mathbb{E}_t \left[\widehat{c}_{t+1}\right] - \widehat{c}_t &= \frac{1}{\sigma} \left(i_t - \mathbb{E}_t \left[\pi_{t+1}\right] - \iota\right).
\end{aligned}$$

The Price index

$$\widehat{p}_t = \omega \widehat{p}_{H,t} + (1-\omega)\left(\widehat{e}_t + \widehat{p}^*_{F,t}\right).$$

Firms

$$\begin{aligned}
\widehat{mc}_t &= \left(1-\omega_{oy}\right)\left(\widehat{w}_t - \widehat{a}_t\right) + \omega_o \widehat{p}_{O,t}, \\
\widehat{n}_t &= -\gamma_y \left(\widehat{w}_t - \widehat{a}_t - \widehat{mc}_t\right) - \widehat{a}_t + \frac{nC_H}{nC_H + (1-n)\,C^*_H}\widehat{c}_{H,t} + \frac{(1-n)\,C^*_H}{nC_H + (1-n)\,C^*_H}\widehat{c}^*_{H,t}, \\
\widehat{o}_{Y,t} &= -\gamma_y \left(\widehat{p}_{O,t} - \widehat{mc}_t\right) + \frac{nC_H}{nC_H + (1-n)\,C^*_H}\widehat{c}_{H,t} + \frac{(1-n)\,C^*_H}{nC_H + (1-n)\,C^*_H}\widehat{c}^*_{H,t}, \\
\pi_{H,t} &= \frac{(1-\theta)(1-\beta\theta)}{\theta}\left(\widehat{mc}_t - \widehat{p}_{H,t}\right) + \beta \mathbb{E}_t \left[\pi_{H,t+1}\right].
\end{aligned}$$

Goods market clearing

$$\left(1-\omega_{oy}\right)\left(\widehat{a}_t + \widehat{n}_t\right) + \omega_{oy}\widehat{o}_{Y,t} = \frac{nC_H}{nC_H + (1-n)\,C^*_H}\widehat{c}_{H,t} + \frac{(1-n)\,C^*_H}{nC_H + (1-n)\,C^*_H}\widehat{c}^*_{H,t}.$$

Government

$$i_t = \iota + \varphi_\pi \pi_{H,t}.$$

Foreign condition (11 equations). Households

$$\begin{aligned}\widehat{c}^{*}_{H,t} &= -\gamma_n\left(\widehat{p}_{H,t} - \widehat{e}_t - \widehat{p}^{*}_t\right) + \widehat{c}^{*}_t,\\ \widehat{c}^{*}_{F,t} &= -\gamma_n\left(\widehat{p}^{*}_{F,t} - \widehat{p}^{*}_t\right) + \widehat{c}^{*}_{N,t},\\ \sigma\widehat{c}^{*}_t + \varphi\widehat{n}^{*}_t &= \widehat{w}^{*}_t - \widehat{p}^{*}_t,\\ \mathbb{E}_t\left[\widehat{c}^{*}_{t+1}\right] - \widehat{c}^{*}_t &= \frac{1}{\sigma}\left(i^{*}_t - \mathbb{E}_t\left[\pi^{*}_{t+1}\right] - \iota\right).\end{aligned}$$

The price index

$$\widehat{p}^{*}_t = \omega^{*}\widehat{p}^{*}_{F,t} + (1-\omega^{*})\widehat{p}^{*}_{H,t}.$$

Firms

$$\begin{aligned}\widehat{mc}^{*}_t &= (1-\omega_{oy})\left(\widehat{w}^{*}_t - \widehat{a}^{*}_t\right) + \omega_o\left(\widehat{p}_{O,t} - \widehat{e}_t\right),\\ \widehat{n}^{*}_t &= -\gamma_y\left(\widehat{w}^{*}_t - \widehat{a}_t - \widehat{mc}^{*}_t\right) - \widehat{a}_t + \frac{nC_F}{nC_F + (1-n)\,C^{*}_F}\widehat{c}_{F,t} + \frac{(1-n)\,C^{*}_F}{nC_F + (1-n)\,C^{*}_F}\widehat{c}^{*}_{F,t}\\ \widehat{o}^{*}_{Y,t} &= -\gamma_y\left(\widehat{p}^{*}_{O,t} - \widehat{mc}^{*}_t\right) + \frac{nC_F}{nC_F + (1-n)\,C^{*}_F}\widehat{c}_{F,t} + \frac{(1-n)\,C^{*}_F}{nC_F + (1-n)\,C^{*}_F}\widehat{c}^{*}_{F,t}\\ \pi^{*}_{F,t} &= \frac{(1-\theta)(1-\beta\theta)}{\theta}\left(\widehat{mc}^{*}_t - \widehat{p}^{*}_{F,t}\right) + \beta\mathbb{E}_t\left[\pi^{*}_{F,t+1}\right]\end{aligned}$$

Goods market clearing

$$(1-\omega_{oy})\left(\widehat{a}^{*}_t + \widehat{n}^{*}_t\right) + \omega_{oy}\widehat{o}^{*}_{Y,t} = \frac{(1-n)\,C^{*}_F}{(1-n)\,C^{*}_F + nC_F}\widehat{c}^{*}_{F,t} + \frac{(1-n)\,C^{*}_H}{(1-n)\,C^{*}_F + nC_F}\widehat{c}_{F,t}.$$

Government

$$i^{*}_t = \iota + \varphi^{*}_{\pi}\pi^{*}_{F,t}.$$

International conditions (3 equations).

$$\begin{aligned}\widehat{c}_t - \widehat{c}^{*}_t &= \frac{\widehat{q}_t}{\sigma},\\ \widehat{q}_t &= \widehat{e}_t + \widehat{p}^{*}_t - \widehat{p}_t,\\ nO_Y\widehat{o}_{Y,t} + (1-n)\,O^{*}_Y\widehat{o}^{*}_{Y,t} &= nO\widehat{o}_t + (1-n)O^{*}\widehat{o}^{*}_t.\end{aligned}$$

B.10 Euler Equations, Phillips Curves, Oil Price, and Risk Sharing

This section reduces all the equilibrium equations to only six: two Euler equations, two Phillips curves, the Backus-Smith condition and the equilibrium on a global oil market. The unknowns are consumption $(\widehat{c}_t, \widehat{c}^{*}_t)$, inflation $(\pi_{H,t}, \pi^{*}_{F,t})$, the real exchange rate q_t and the real price of oil expressed in units of *foreign* goods $\widehat{r} \equiv \widehat{p}_{O,t} - \widehat{p}_{F,t} = \widehat{p}^{*}_{O,t} - \widehat{p}^{*}_{F,t}$.

Euler equations. Rewrite the Euler equation as

$$\mathbb{E}_t\left[\widehat{c}_{t+1}\right] - \widehat{c}_t = \frac{1}{\sigma}\left(i_t - \mathbb{E}_t\left[\pi_{H,t+1}\right] - \iota\right) - \frac{1}{\sigma}\cdot\frac{1-\omega}{\omega+\omega^{*}-1}\mathbb{E}_t\left[\Delta\widehat{q}_{t+1}\right],$$

where $\pi_{r,t} \equiv \pi_{O,t} - \pi_{F,t} = \left(\widehat{p}_{O,t} - \widehat{p}_{O,t-1}\right) - \left(\widehat{p}_{F,t} - \widehat{p}_{F,t-1}\right).$

Foreign Euler equation

$$\mathbb{E}_t\left[\widehat{c}^*_{t+1}\right] - \widehat{c}^*_t = \frac{1}{\sigma}\left(i^*_t - \mathbb{E}_t\left[\pi^*_{F,t+1}\right] - \iota\right) + \frac{1}{\sigma} \cdot \frac{1-\omega^*}{\omega+\omega^*-1}\mathbb{E}_t\left[\Delta\widehat{q}_{t+1}\right].$$

Consumption. Rewrite consumption of domestic goods through overall consumption and prices at home

$$\widehat{c}_{H,t} = \frac{1-\omega}{\omega+\omega^*-1}\gamma_n\widehat{q}_t + \widehat{c}_t,$$

and abroad

$$\widehat{c}^*_{H,t} = \frac{\omega^*}{\omega+\omega^*-1}\gamma_n\widehat{q}_t + \widehat{c}^*_t.$$

And consumption of foreign goods

$$\widehat{c}_{F,t} = -\frac{\omega}{\omega+\omega^*-1}\gamma_n\widehat{q}_t + \widehat{c}_t,$$

and

$$\widehat{c}^*_{F,t} = -\frac{1-\omega^*}{\omega+\omega^*-1}\gamma_n\widehat{q}_t + \widehat{c}^*_t.$$

Production. Domestic production of home goods

$$\begin{aligned}\widehat{y}_{H,t} =& \frac{n\omega C}{n\omega C + (1-n)\left(1-\omega^*\right)C^*}\widehat{c}_{H,t} + \frac{(1-n)\left(1-\omega^*\right)C^*}{n\omega C + (1-n)\left(1-\omega^*\right)C^*}\widehat{c}^*_{H,t} \\ =& \widehat{c}^M_t + \frac{n\omega C\left(1-\omega\right) + (1-n)\left(1-\omega^*\right)C^*\omega^*}{n\omega C + (1-n)\left(1-\omega^*\right)C^*} \cdot \frac{\gamma_n}{\omega+\omega^*-1}\widehat{q}_t.\end{aligned}$$

where

$$\widehat{c}^M_t \equiv \frac{n\omega C}{n\omega C + (1-n)\left(1-\omega^*\right)C^*}\widehat{c}_t + \frac{(1-n)\left(1-\omega^*\right)C^*}{n\omega C + (1-n)\left(1-\omega^*\right)C^*}\widehat{c}^*_t.$$

Foreign production of foreign goods

$$\begin{aligned}\widehat{y}^*_{F,t} =& \frac{nC_F}{nC_F + (1-n)\,C^*_F}\widehat{c}_{F,t} + \frac{(1-n)\,C^*_F}{nC_F + (1-n)\,C^*_F}\widehat{c}^*_{F,t} \\ =& \widehat{c}^{M*}_t - \frac{n\omega\left(1-\omega\right)C + (1-n)\left(1-\omega^*\right)\omega^*C^*}{n\left(1-\omega\right)C + (1-n)\,\omega^*C^*} \cdot \frac{\gamma_n}{\omega+\omega^*-1}\widehat{q}_t\end{aligned}$$

where

$$\widehat{c}^{M,*}_t \equiv \frac{n\left(1-\omega\right)C}{n\left(1-\omega\right)C + (1-n)\,\omega^*C^*}\widehat{c}_t + \frac{(1-n)\,\omega^*C^*}{n\left(1-\omega\right)C + (1-n)\,\omega^*C^*}\widehat{c}^*_t.$$

Home marginal costs. Replace real wage (using the labor supply equation) from the labor demand and the equation for the marginal costs

$$\widehat{mc}_t - \widehat{p}_t = \left(1-\omega_{oy}\right)\left(\sigma\widehat{c}_t + \varphi\widehat{n}_t\right) + \omega_{oy}\left(\widehat{p}_{O,t} - \widehat{p}_t\right),$$

$$\begin{aligned}\widehat{n}_t =& -\frac{\gamma_y}{1+\gamma_y\varphi}\sigma\widehat{c}_t + \frac{\gamma_y}{1+\gamma_y\varphi}\left(\widehat{mc}_t - \widehat{p}_t\right)\\ &+ \frac{1}{1+\gamma_y\varphi}\left(\frac{nC_H}{nC_H+(1-n)\,C_H^*}\widehat{c}_{H,t} + \frac{(1-n)\,C_H^*}{nC_H+(1-n)\,C_H^*}\widehat{c}_{H,t}^*\right).\end{aligned}$$

Solve for equilibrium labor and the marginal costs

$$\begin{aligned}\widehat{mc}_t - \widehat{p}_t =& \frac{1-\omega_{oy}}{1+\gamma_y\varphi\omega_{oy}}\left[\sigma\widehat{c}_t + \varphi\left(\frac{nC_H}{nC_H+(1-n)\,C_H^*}\widehat{c}_{H,t} + \frac{(1-n)\,C_H^*}{nC_H+(1-n)\,C_H^*}\widehat{c}_{H,t}^*\right)\right]\\ &+ \frac{1+\gamma_y\varphi}{1+\gamma_y\varphi\omega_{oy}}\omega_{oy}\left(\widehat{p}_{O,t} - \widehat{p}_t\right).\end{aligned}$$

and

$$\begin{aligned}\widehat{n}_t =& -\frac{\sigma\gamma_y\omega_{oy}}{1+\gamma_y\varphi\omega_{oy}}\widehat{c}_t + \frac{\gamma_y\omega_{oy}}{1+\gamma_y\varphi\omega_{oy}}\left(\widehat{p}_{O,t} - \widehat{p}_t\right)\\ &+ \frac{1}{1+\gamma_y\varphi\omega_{oy}}\left(\frac{nC_H}{nC_H+(1-n)\,C_H^*}\widehat{c}_{H,t} + \frac{(1-n)\,C_H^*}{nC_H+(1-n)\,C_H^*}\widehat{c}_{H,t}^*\right).\end{aligned}$$

Replace the CPI $\widehat{p}_t$ with $\widehat{p}_{H,t}$ in the marginal cost equation above at home

$$\begin{aligned}\widehat{mc}_t - \widehat{p}_{H,t} =& \frac{1-\omega_{oy}}{1+\gamma_y\varphi\omega_{oy}}\left[\sigma\widehat{c}_t + \varphi\left(\frac{nC_H}{nC_H+(1-n)\,C_H^*}\widehat{c}_{H,t} + \frac{(1-n)\,C_H^*}{nC_H+(1-n)\,C_H^*}\widehat{c}_{H,t}^*\right)\right]\\ &+ \frac{1+\gamma_y\varphi}{1+\gamma_y\varphi\omega_{oy}}\omega_{oy}\left(\widehat{p}_{O,t} - \widehat{p}_{F,t}\right) + \frac{\left(1+\gamma_y\varphi\right)\omega_{oy} + (1-\omega)\left(1-\omega_{oy}\right)}{\left(1+\gamma_y\varphi\omega_{oy}\right)(\omega+\omega^*-1)}\widehat{q}_t.\end{aligned}$$

Rewrite the marginal costs

$$\widehat{mc}_t - \widehat{p}_{H,t} = \frac{1-\omega_{oy}}{1+\gamma_y\varphi\omega_{oy}}\left[\sigma\widehat{c}_t + \varphi\widehat{c}_t^M\right] + \psi_o\left(\widehat{p}_{O,t} - \widehat{p}_{F,t}\right) + \psi_q\widehat{q}_t,$$

where

$$\begin{aligned}\psi_o \equiv& \frac{\left(1+\gamma_y\varphi\right)\omega_{oy}}{1+\gamma_y\varphi\omega_{oy}},\\ \psi_q \equiv& \frac{\left(1+\gamma_y\varphi\right)\omega_{oy} + (1-\omega)\left(1-\omega_{oy}\right)}{\left(1+\gamma_y\varphi\omega_{oy}\right)(\omega+\omega^*-1)}\\ &+ \frac{1-\omega_{oy}}{1+\gamma_y\varphi\omega_{oy}}\cdot\frac{1}{\omega+\omega^*-1}\cdot\frac{n\omega C\,(1-\omega) + (1-n)\,(1-\omega^*)C^*\omega^*}{n\omega C + (1-n)\,(1-\omega)C^*}\varphi\gamma_n\end{aligned}$$

Foreign marginal costs. A similar expression for foreign country marginal costs is

$$\begin{aligned}\widehat{mc}_t^* - \widehat{p}_t^* =& \frac{1-\omega_{oy}}{1+\omega_{oy}\gamma_y\varphi}\left[\sigma\widehat{c}_t^* + \varphi\left(\frac{nC_F}{nC_F+(1-n)\,C_F^*}\widehat{c}_{F,t} + \frac{(1-n)\,C_F^*}{nC_F+(1-n)\,C_F^*}\widehat{c}_{F,t}^*\right)\right]\\ &+ \omega_{oy}\frac{1+\varphi\gamma_y}{1+\omega_{oy}\gamma_y\varphi}\left(\widehat{p}_{O,t}^* - \widehat{p}_t^*\right)\end{aligned}$$

and replace $\widehat{p}_t^*$

$$\begin{aligned}\widehat{mc}_t^* - \widehat{p}_{F,t}^* =& \frac{1-\omega_{oy}}{1+\omega_{oy}\gamma_y\varphi}\left[\sigma\widehat{c}_t^* + \varphi\left(\frac{nC_F}{nC_F+(1-n)\,C_F^*}\widehat{c}_{F,t} + \frac{(1-n)\,C_F^*}{nC_F+(1-n)\,C_F^*}\widehat{c}_{F,t}^*\right)\right] \\ &+ \left[\omega_{oc} + \omega_{oy}\frac{1+\varphi\gamma_y}{1+\omega_{oy}\gamma_y\varphi}\left(1-\omega_{o,c}\right)\right]\left(\widehat{p}_{O,t}^* - \widehat{p}_{F,t}^*\right) - \frac{1-\omega_{oy}}{1+\omega_{oy}\gamma_y\varphi}\cdot\frac{1-\omega^*}{(\omega+\omega^*-1)}\widehat{q}_t\end{aligned}$$

Foreign country marginal costs are

$$\widehat{mc}_t^* - \widehat{p}_{F,t}^* = \frac{1-\omega_{oy}}{1+\omega_{oy}\gamma_y\varphi}\left[\sigma\widehat{c}_t^* + \varphi\widehat{c}_t^{M,*}\right] + \psi_o^*\left(\widehat{p}_{O,t}^* - \widehat{p}_{F,t}^*\right) + \psi_q^*\widehat{q}_t$$

where

$$\begin{aligned}\psi_o^* \equiv& \frac{\omega_{oy}\left(1+\varphi\gamma_y\right)}{1+\omega_{oy}\gamma_y\varphi} = \psi_o, \\ \psi_q^* \equiv& -\frac{1-\omega_{oy}}{1+\omega_{oy}\gamma_y\varphi}\cdot\frac{1-\omega^*}{\omega+\omega^*-1} \\ &-\frac{1-\omega_{oy}}{1+\omega_{oy}\gamma_y\varphi}\cdot\frac{1}{\omega+\omega^*-1}\cdot\frac{n\omega C_F+(1-n)\left(1-\omega^*\right)C_F^*}{nC_F+(1-n)\,C_F^*}\varphi\gamma_n.\end{aligned}$$

Note that ψ_q^* and ψ_q are not symmetric. This is because in both equations for the marginal costs the real oil price is expressed in terms of foreign goods.

Phillips curves. The home Phillips curve is

$$\pi_{H,t} = \kappa\left\{\frac{1-\omega_{oy}}{1+\gamma_y\varphi\omega_{oy}}\left[\sigma\widehat{c}_t + \varphi\widehat{c}_t^M\right] + \psi_o\widehat{r}_t + \psi_q\widehat{q}_t\right\} + \beta\mathbb{E}_t\left[\pi_{H,t+1}\right],$$

and the foreign Phillips curve is

$$\pi_{F,t}^* = \kappa\left\{\frac{1-\omega_{oy}}{1+\omega_{oy}\gamma_y\varphi}\left[\sigma\widehat{c}_t^* + \varphi\widehat{c}_t^{M,*}\right] + \psi_o^*\widehat{r}_t + \psi_q^*\widehat{q}_t\right\} + \beta\mathbb{E}_t\left[\pi_{F,t+1}^*\right],$$

where

$$\kappa \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}. \tag{B.24}$$

Oil demand. The home firms demand for oil

$$\begin{aligned}\widehat{o}_{Y,t} =& \gamma_y\frac{1-\omega_{oy}}{1+\gamma_y\varphi\omega_{oy}}\sigma\widehat{c}_t - \frac{1-\omega_{oy}}{1+\omega_{oy}\gamma_y\varphi}\gamma_y\widehat{r}_t + \frac{1+\varphi\gamma_y}{1+\gamma_y\varphi\omega_{oy}}\widehat{c}_t^M \\ &+ \left\{\left[(\omega+\omega^*-1)\,\psi_q - 1\right]\gamma_y + \frac{n\omega C\,(1-\omega)+(1-n)\,(1-\omega^*)C^*\omega^*}{n\omega C+(1-n)\,(1-\omega)C^*}\gamma_n\right\}\frac{\widehat{q}_t}{\omega+\omega^*-1}.\end{aligned}$$

The foreign firm demand for oil

$$\begin{aligned}\widehat{o}^{*}_{Y,t} =& \gamma_y \frac{1-\omega_{oy}}{1+\omega_{oy}\gamma_y\varphi}\sigma\widehat{c}^{*}_t - \frac{1-\omega_{oy}}{1+\omega_{oy}\gamma_y\varphi}\gamma_y\widehat{r}_t + \frac{1+\gamma_y\varphi}{1+\omega_{oy}\gamma_y\varphi}\widehat{c}^{M*}_t \\ &- \frac{1}{1+\omega_{oy}\gamma_y\varphi}\left(\gamma_n \frac{n\omega C_F + (1-n)\left(1-\omega^*\right)C^*_F}{nC_F + (1-n)\,C^*_F} + \gamma_y\left(1-\omega_{oy}\right)\left(1-\omega^*\right)\right)\frac{\widehat{q}_t}{\omega+\omega^*-1}\end{aligned}$$

Oil market. Oil market equilibrium condition

$$nO_Y\widehat{o}_{Y,t} + (1-n)O^*_Y\widehat{o}^*_{Y,t} = nO\widehat{o}_t + (1-n)O^*\widehat{o}^*_t,$$

which determines $\widehat{r}_t = \widehat{p}^*_{O,t} - \widehat{p}^*_{F,t}$.

B.11 A Small-Open Economy Limit

We now take the limit as the size of home economy approaches zero and the size of the foreign economy approaches one, that is, $n \to 0$ and $n^* = 1-n \to 1$. Taking into account the following definitions

$$\begin{aligned}1-\omega &= (1-n)\,(1-\Omega)\,, \\ 1-\omega^* &= n\,(1-\Omega^*)\,,\end{aligned}$$

we have that in the limit the fraction of domestic goods expenditure

$$\begin{aligned}\omega &= \Omega, \\ \omega^* &= 1.\end{aligned}$$

This implies that

$$\begin{aligned}\frac{1-\omega^*}{\omega+\omega^*-1} &= \frac{n\,(1-\Omega^*)}{1-(1-n)\,(1-\Omega)-n\,(1-\Omega^*)} = 0, \\ \frac{nC_H}{nC_H+(1-n)\,C^*_H} &= \frac{\Omega C}{\Omega C+(1-\Omega^*)\,C^*}, \\ \frac{nC_F}{nC_F+(1-n)\,C^*_F} &= 0, \\ \frac{n\omega C_F+(1-n)\,(1-\omega^*)\,C^*_F}{nC_F+(1-n)\,C^*_F} &= 0.\end{aligned}$$

The relative prices of home goods in units of domestic and foreign consumption baskets are

$$\begin{aligned}\widehat{p}_{H,t} - \widehat{p}_t &= -\frac{1-\omega}{\omega+\omega^*-1}\widehat{q}_t = -\frac{1-\Omega}{\Omega}\widehat{q}_t, \\ \widehat{p}^*_{H,t} - \widehat{p}^*_t &= -\frac{1}{\Omega}\widehat{q}_t.\end{aligned}$$

The world economy. The world equilibrium consists of six unknowns $\left(\widehat{y}_F^*, \widehat{c}_t^*, \pi_t^*, \pi_{F,t}^*, \widehat{r}_t, i_t^*\right)$ and six equations are

$$
\begin{aligned}
\widehat{r}_t &= \phi_c \widehat{c}_t^* - \phi_o \widehat{o}_t^*, \\
\pi_{F,t}^* &= \kappa \zeta^* \widehat{c}_t^* + \beta \mathbb{E}_t \left[\pi_{F,t+1}^*\right] + \kappa \psi_o^* \widehat{r}_t, \\
\mathbb{E}_t \left[\widehat{c}_{t+1}^*\right] - \widehat{c}_t^* &= \frac{1}{\sigma} \left(i_t^* - \mathbb{E}_t \left[\pi_{t+1}^*\right] - \iota\right), \\
i_t^* &= \iota + \varphi_\pi^* \pi_{F,t}^*, \\
\pi_t^* &= \pi_{F,t}^*, \\
\widehat{y}_{F,t}^* &= \widehat{c}_t^*.
\end{aligned}
$$

where $\widehat{r}_t \equiv \widehat{p}_{O,t}^* - \widehat{p}_{F,t}^*$ and

$$
\phi_o = \frac{1 + \omega_{oy} \gamma_y \varphi}{\left(1 - \omega_{oy}\right) \gamma_y}, \tag{B.25}
$$

$$
\phi_c = \frac{1 + \gamma_y \left[\varphi + \left(1 - \omega_{oy}\right) \sigma\right]}{\left(1 - \omega_{oy}\right) \gamma_y}, \tag{B.26}
$$

$$
\kappa \equiv \frac{(1 - \theta)(1 - \beta\theta)}{\theta},
$$

$$
\zeta^* \equiv \frac{1 - \omega_{oy}}{1 + \omega_{oy} \gamma_y \varphi} (\sigma + \varphi), \tag{B.27}
$$

$$
\psi_o^* = \frac{\left(1 + \gamma_y \varphi\right) \omega_{oy}}{1 + \gamma_y \varphi \omega_{oy}}. \tag{B.28}
$$

We took into account that $\psi_q^* = 0, \widehat{c}_t^{M*} = \widehat{c}_t^*$.

Substituting oil price into the Phillips curve, we get

$$
\pi_{F,t}^* = \kappa \left(\zeta^* + \psi_o^* \phi_c\right) \widehat{c}_t^* + \beta \mathbb{E}_t \left[\pi_{F,t+1}^*\right] - \kappa \psi_o^* \phi_o \widehat{o}_t^*,
$$

The value added is

$$
\widehat{z}_{F,t}^* = \frac{\left(1 - \omega_{oy} \phi_c\right) \widehat{c}_t^* - \omega_{oy} \left(1 - \phi_o\right) \widehat{o}_t^*}{1 - \omega_{oy}}.
$$

Trade balance is

$$
\widehat{nx}_t^* = 0.
$$

Small-open economy. The SOE block consists of six unknowns $(\widehat{c}_t, \widehat{y}_{H,t}, \pi_t, \pi_{H,t}, i_t, q_t)$ and six equations

$$
\begin{aligned}
\mathbb{E}_t\left[\widehat{c}_{t+1}\right]-\widehat{c}_t &= \frac{1}{\sigma}\left(i_t-\mathbb{E}_t\left[\pi_{t+1}\right]-\iota\right), \\
\pi_{H,t} &= \kappa\zeta_H\widehat{c}_t+\beta\mathbb{E}_t\left[\pi_{H,t+1}\right]+\kappa\psi_o\widehat{r}_t+\kappa\zeta_F\widehat{c}_t^*+\kappa\psi_q\widehat{q}_t, \\
\pi_t &= \pi_{H,t}+\frac{1-\Omega}{\Omega}\Delta\widehat{q}_t, \\
i_t &= \iota+\phi_\pi\pi_{H,t}, \\
\widehat{q}_t &= \sigma\left(\widehat{c}_t-\widehat{c}_t^*\right), \\
\widehat{y}_{H,t} &= \frac{\Omega}{\Omega+1-\Omega^*}\widehat{c}_t+\frac{1-\Omega^*}{\Omega+1-\Omega^*}\widehat{c}_t^*+\frac{\Omega\left(1-\Omega\right)+\left(1-\Omega^*\right)}{\Omega+1-\Omega^*}\cdot\frac{\gamma_n}{\Omega}\widehat{q}_t.
\end{aligned}
$$

where

$$
\begin{aligned}
\zeta_H &= \frac{1-\omega_{oy}}{1+\gamma_y\varphi\omega_{oy}}\left(\sigma+\varphi\frac{\Omega}{\Omega+1-\Omega^*}\right), \\
\zeta_F &= \frac{1-\omega_{oy}}{1+\gamma_y\varphi\omega_{oy}}\varphi\frac{1-\Omega^*}{\Omega+1-\Omega^*}, \\
\psi_q &= \frac{1-\Omega}{\Omega}\cdot\frac{\left(1+\varphi\gamma_n\right)\left(1-\omega_{oy}\right)}{1+\gamma_y\varphi\omega_{oy}}, \\
\psi_o &= \frac{\left(1+\gamma_y\varphi\right)\omega_{oy}}{1+\gamma_y\varphi\omega_{oy}},
\end{aligned}
$$

and we took into account the fact that $\widehat{c}_t^M=\widehat{c}_t\Omega/\left(\Omega+1-\Omega^*\right)+\widehat{c}_t^*\left(1-\Omega^*\right)/\left(\Omega+1-\Omega^*\right)$. Note that the Phillips curve has three additional terms compared to the standard closed-economy formulation. The term $\kappa\psi_o\widehat{r}_t$ shows that higher real oil price increase the marginal cost and, hence, inflation. The term $\kappa\zeta_F\widehat{c}_t^*$ is due to the fact that the world aggregate demand affects the demand for home products and increases the cost of production. The term $\kappa\psi_q\widehat{q}_t$ reflects the fact that oil prices are quoted in units of foreign goods. This implies that absence any chance in the real oil price (in units of foreign goods), a real depreciation of domestic currency (an increase in $\widehat{q}_t$) acts to increase the oil price in units of home goods, which, in turn, increases the marginal cost and inflation. Finally, note that $\zeta_H+\zeta_F=\zeta^*$.

Substituting away the real exchange rate from the Euler equation and the Phillips curve, we obtain

$$
\begin{aligned}
\mathbb{E}_t\left[\widehat{c}_{t+1}\right]-\widehat{c}_t &= \frac{\Omega}{\sigma}\left(i_t-\mathbb{E}_t\left[\pi_{H,t+1}\right]-\iota\right)+\left(1-\Omega\right)\left\{\mathbb{E}_t\left[\widehat{c}_{t+1}^*\right]-\widehat{c}_t^*\right\}, \\
\pi_{H,t} &= \kappa\left(\zeta_H+\psi_q\sigma\right)\widehat{c}_t+\beta\mathbb{E}_t\left[\pi_{H,t+1}\right]+\kappa\psi_o\widehat{r}_t+\kappa\left(\zeta_F-\psi_q\sigma\right)\widehat{c}_t^*,
\end{aligned}
$$

This form of the Euler equation implies that the home aggregate demand is affected by the rest of the world because it changes the relative price of imported goods. A booming world economy has two opposing effects on the Phillips curve. On the one hand, this boom increases demand for home goods and pushes up inflation. On the other hand, a booming world economy appreciates the home currency in real terms and makes oil cheaper at home. The net effect of these two forces is captured by the coefficient

$$
\zeta_F-\psi_q\sigma=\frac{1-\Omega^*}{\Omega+1-\Omega^*}\varphi-\frac{1-\Omega}{\Omega}\left(1+\varphi\gamma_n\right)\sigma.
$$

In general, the sign of this coefficient is ambiguos. However, under the empirical relevant parameters $\Omega=\Omega^*=2/3,\sigma=1,\varphi=2$, this coefficient is positive.

Oil demand is

Trade balance is

B.12 Neo-classical Effects

When prices are completely flexible, the Phillips curve in foreign country implies

$$\widehat{c}_t^* = \Gamma_O^{c,*} \phi_o \widehat{o}_t^*.$$

where

$$\Gamma_O^{c,*} \equiv \frac{\psi_o^*}{\zeta^* + \psi_o^* \phi_c} = \frac{\frac{\omega_{oy}}{1-\omega_{oy}} \left(1 + \varphi\gamma_y\right)}{\sigma + \varphi + \frac{\omega_{oy}}{1-\omega_{oy}} \left(1 + \varphi\gamma_y\right) \frac{1-\gamma_y\omega_{oy}\sigma+\gamma_y(\varphi+\sigma)}{\gamma_y\left(1-\omega_{oy}\right)}}.$$

The consumption effect of oil supply change

$$\begin{aligned} \Gamma_O^{c,*}\phi_o &= \frac{\frac{\omega_{oy}}{1-\omega_{oy}} \left(1 + \varphi\gamma_y\right)}{\frac{\left(1-\omega_{oy}\right)\gamma_y}{\left(1+\omega_{oy}\gamma_y\varphi\right)} \left[(\sigma + \varphi) + \frac{\omega_{oy}}{1-\omega_{oy}} \left(1 + \varphi\gamma_y\right) \frac{\left(1-\gamma_y\omega_{oy}\sigma\right)+\gamma_y(\varphi+\sigma)}{\left(1-\omega_{oy}\right)\gamma_y}\right]} \\ &= \frac{\omega_{oy}\left(1 + \varphi\gamma_y\right)}{\omega_{oy} + \gamma_y\left(\sigma + \varphi - \omega_{oy}\sigma\right)}. \end{aligned}$$

The oil price

$$\begin{aligned} \widehat{r}_t &= -\frac{(\sigma + \varphi)}{\sigma + \varphi + \frac{\omega_{oy}}{1-\omega_{oy}} \left(1 + \varphi\gamma_y\right) \frac{\left(1-\gamma_y\omega_{oy}\sigma\right)+\gamma_y(\varphi+\sigma)}{\left(1-\omega_{oy}\right)\gamma_y}} \phi_o \widehat{o}_t^* \\ &= -\frac{(\sigma + \varphi)\left(1 - \omega_{oy}\right)}{\omega_{oy} + \gamma_y\left(\sigma + \varphi - \omega_{oy}\sigma\right)} \widehat{o}_t^*. \end{aligned}$$

A change in output normalized by a change in oil price

$$\frac{\widehat{c}_t^*}{\widehat{r}_t} = \frac{\frac{\omega_{oy}\left(1+\varphi\gamma_y\right)}{\omega_{oy}+\gamma_y\left(\sigma+\varphi-\omega_{oy}\sigma\right)}}{-\frac{(\sigma+\varphi)\left(1-\omega_{oy}\right)}{\omega_{oy}+\gamma_y\left(\sigma+\varphi-\omega_{oy}\sigma\right)}} = -\frac{\omega_{oy}}{1 - \omega_{oy}} \cdot \frac{1 + \varphi\gamma_y}{\sigma + \varphi}.$$

B.13 A Two-state Markov Shock

With probability α, the shock persists, with the remaining probability it goes away. Oil shock $\widehat{o}_t^* \in \{\widehat{o}^*, 0\}$,where $\widehat{o}^* < 0$.

B.13.1 The World Economy

We start from the world economy. Using the equation for the oil price in equilibrium, we obtain the following Phillips curve

$$\pi_F^* = \frac{\kappa\left(\zeta^* + \psi_o^*\phi_c\right)}{1 - \beta\alpha} \widehat{c}^* + \frac{\kappa\psi_o^*\phi_o}{1 - \beta\alpha} \left(-\widehat{o}^*\right), \tag{B.29}$$

The Euler equation is

$$\widehat{c}^* = -\frac{\left(\varphi_\pi^* - \alpha\right)\pi_F^*}{\left(1-\alpha\right)\sigma}. \tag{B.30}$$

The response of the inflation rate that solves the above two equations is

$$\frac{d\pi_F^*}{d\left(-\widehat{o}^*\right)} = \phi_o \frac{\frac{\kappa\psi_o^*}{(1-\beta\alpha)}}{1 + \frac{\kappa(\varphi_\pi^*-\alpha)}{(1-\beta\alpha)(1-\alpha)\sigma}\left(\zeta^* + \psi_o^*\phi_c\right)} > 0. \tag{B.31}$$

There are two notable features here. First, the effect on inflation is non-negative. Second, when the shock is permanent, inflation response is zero. The response of inflation rate when oil shock is such that the oil price increases by one percent is

$$\pi_F^* = \frac{\frac{\kappa\psi_o^*}{1-\beta\alpha}}{1 + \frac{\kappa(\varphi_\pi^*-\alpha)}{(1-\beta\alpha)(1-\alpha)\sigma}\zeta^*}.$$

The response of consumption (and output) that solve the Euler equation (combined with the monetary policy rule) and the Phillips curve is

$$\frac{d\widehat{c}^*}{d\left(-\widehat{o}^*\right)} = -\phi_o \frac{\frac{\kappa(\varphi_\pi^*-\alpha)}{(1-\beta\alpha)(1-\alpha)}\psi_o^*}{\sigma + \frac{\kappa(\varphi_\pi^*-\alpha)}{(1-\beta\alpha)(1-\alpha)}\left(\zeta^* + \psi_o^*\phi_c\right)} = \frac{\left(\zeta^* + \psi_o^*\phi_c\right)\frac{\kappa(\varphi_\pi^*-\alpha)}{(1-\beta\alpha)(1-\alpha)}}{\sigma + \left(\zeta^* + \psi_o^*\phi_c\right)\frac{\kappa(\varphi_\pi^*-\alpha)}{(1-\beta\alpha)(1-\alpha)}}\Gamma_y^{nc,*} \lesseqgtr 0. \tag{B.32}$$

where $\Gamma_y^{nc,*} = -\phi_o\psi_o^* / \left(\zeta^* + \psi_o^*\phi_c\right)$ is the neo-classical response. There are several notable observations here. When the Taylor rule response to the inflation rate is strong enough, that is, $\varphi_\pi^* - \alpha > 0$, the aggregate consumption unambiguously falls following a hike in oil prices. This is because of the increase in the real interest rate and the fall in demand for oil consumption. Second, when the shock is permanent: $d\widehat{c}^*/d\left(-\underline{o}^*\right) = -\phi_o\psi_o^*/(\zeta^* + \psi_o^*\phi_c) < 0$. Third, when goods prices are completely sticky, i.e., $\kappa = 0$, or the central bank targets a fixed real rate, i.e., $\varphi_\pi^* = \alpha$, the response of consumption is zero: $d\widehat{c}^*/d\left(-\underline{o}^*\right) = 0$. Fourth, the response is smaller compared to the case of completely flexible prices.

The reaction of the oil price is

$$\widehat{r} = \frac{1 + \frac{\kappa(\varphi_\pi^*-\alpha)}{(1-\beta\alpha)(1-\alpha)\sigma}\zeta^*}{1 + \frac{\kappa(\varphi_\pi^*-\alpha)}{(1-\beta\alpha)(1-\alpha)\sigma}\left(\zeta^* + \psi_o^*\phi_c\right)}\phi_o\left(-\underline{o}^*\right). \tag{B.33}$$

Note that an oil supply decline unambiguously raises oil price when $\varphi_\pi^* - \alpha > 0$. The size of the oil shock that increase the oil price by one percent equals

$$\phi_o\left(-\widehat{o}^*\right) = \frac{1 + \frac{\kappa(\varphi_\pi^*-\alpha)}{(1-\beta\alpha)(1-\alpha)\sigma}\left(\zeta^* + \psi_o^*\phi_c\right)}{1 + \frac{\kappa(\varphi_\pi^*-\alpha)}{(1-\beta\alpha)(1-\alpha)\sigma}\zeta^*}.$$

The log-deviation of the consumption level from its steady state c^* following the oil supply shock that increases oil price by one percent, $\widehat{r} = 1$, is

$$\widehat{c}^* = -\frac{\frac{\kappa(\varphi_\pi^*-\alpha)}{\sigma(1-\beta\alpha)(1-\alpha)}\zeta^*}{1 + \frac{\kappa(\varphi_\pi^*-\alpha)}{(1-\beta\alpha)(1-\alpha)\sigma}\zeta^*} \cdot \frac{\psi_o^*}{\zeta^*}.$$

When prices are flexible, i.e., $\kappa \to \infty$,we have $\widehat{c}^* = -\psi_o^*/\zeta^*$, while when they are completely rigid, consumption does not respond, i.e., $\widehat{c}^* = 0$.

ZLB. All the above formulas can be applied to the case of the liquidity trap or inelastic interest rates by setting $\varphi_\pi^* = 0$. We have

$$\frac{d\pi_F^*}{d\left(-\widehat{o}^*\right)} = \phi_o \frac{\frac{(1-\alpha)\kappa\psi_o^*}{1-\beta\alpha}}{1 - \frac{\kappa\alpha}{(1-\beta\alpha)(1-\alpha)\sigma}\left(\zeta^* + \psi_o^*\phi_c\right)} > 0,$$

$$\frac{d\widehat{c}^*}{d\left(-\widehat{o}^*\right)} = -\phi_o \frac{-\frac{\alpha\kappa\psi_o^*}{(1-\beta\alpha)(1-\alpha)\sigma}}{1 - \frac{\alpha\kappa}{(1-\beta\alpha)(1-\alpha)\sigma}\left(\zeta^* + \psi_o^*\phi_c\right)} > 0,$$

$$\frac{d\widehat{r}}{d\left(-\widehat{o}^*\right)} = \phi_o \frac{1 - \frac{\alpha\kappa\zeta^*}{(1-\beta\alpha)(1-\alpha)\sigma}}{1 - \frac{\alpha\kappa}{(1-\beta\alpha)(1-\alpha)\sigma}\left(\zeta^* + \psi_o^*\phi_c\right)} > 0.$$

For the equilibrium to be unique, we assume that

$$1 - \frac{\alpha\kappa}{\sigma\left(1-\beta\alpha\right)\left(1-\alpha\right)}\left(\zeta^* + \psi_o^*\phi_c\right) > 0.$$

The inflation rate response is

$$\left.\frac{d\pi_F^*}{d\left(-\widehat{o}^*\right)}\right|_{\text{ZLB}} - \left.\frac{d\pi_F^*}{d\left(-\widehat{o}^*\right)}\right|_{\text{normal}}$$
$$= \frac{\kappa\left(\zeta^* + \psi_o^*\phi_c\right)\varphi_\pi^*\phi_o\left(1-\alpha\right)\kappa\psi_o^*\sigma}{\left[\left(1-\beta\alpha\right)\left(1-\alpha\right)\sigma - \alpha\kappa\left(\zeta^* + \psi_o^*\phi_c\right)\right]\left[\left(1-\beta\alpha\right)\left(1-\alpha\right)\sigma + \kappa\left(\zeta^* + \psi_o^*\phi_c\right)\left(\varphi_\pi^* - \alpha\right)\right]} > 0.$$

The consumption response is

$$\left.\frac{d\widehat{c}^*}{d\left(-\widehat{o}^*\right)}\right|_{\text{ZLB}} - \left.\frac{d\widehat{c}^*}{d\left(-\widehat{o}^*\right)}\right|_{\text{normal}} = \phi_o \frac{\frac{\kappa\varphi_\pi^*}{(1-\beta\alpha)(1-\alpha)}\psi_o^*\sigma}{\left[\sigma + \frac{-\alpha\kappa}{(1-\beta\alpha)(1-\alpha)}\left(\zeta^* + \psi_o^*\phi_c\right)\right]\left[\sigma + \frac{\kappa(\varphi_\pi^*-\alpha)}{(1-\beta\alpha)(1-\alpha)}\left(\zeta^* + \psi_o^*\phi_c\right)\right]}.$$

The absolute response

$$\left.\frac{d\widehat{c}^*}{d\left(-\widehat{o}^*\right)}\right|_{\text{ZLB}} = \phi_o \frac{\frac{\alpha\kappa}{(1-\beta\alpha)(1-\alpha)} \cdot \frac{\left(1+\gamma_y\varphi\right)\omega_{oy}}{1+\gamma_y\varphi\omega_{oy}}}{\sigma + \frac{\kappa(0-\alpha)}{(1-\beta\alpha)(1-\alpha)}\left(\zeta^* + \psi_o^*\phi_c\right)}$$

B.13.2 The SOE Economy

The Euler equation

$$\widehat{c} = -\Omega\frac{\left(\varphi_\pi - \alpha\right)}{\sigma\left(1-\alpha\right)}\pi_H + \left(1-\Omega\right)\widehat{c}^*.$$

The Phillips curve

$$\pi_H = \frac{\kappa\left(\zeta_H + \psi_q\sigma\right)}{1-\alpha\beta}\widehat{c} + \frac{\kappa}{1-\alpha\beta}\psi_o\widehat{r} + \frac{\kappa}{1-\alpha\beta}\left(\zeta_F - \psi_q\sigma\right)\widehat{c}^*.$$

The inflation rate response is

$$\pi_H = \frac{\psi_o \frac{\kappa}{1-\alpha\beta}}{1+\frac{\kappa(\varphi_\pi-\alpha)}{(1-\alpha)(1-\alpha\beta)\sigma}\left(\zeta_H+\sigma\psi_q\right)\Omega}\widehat{r} + \frac{\frac{\kappa}{1-\alpha\beta}\left[\zeta_H\left(1-\Omega\right)+\zeta_F-\psi_q\sigma\Omega\right]}{1+\frac{\kappa(\varphi_\pi-\alpha)}{\sigma(1-\alpha\beta)(1-\alpha)}\left(\zeta_H+\psi_q\sigma\right)\Omega}\widehat{c}^*,$$

The first term represents the reaction of the economy to the world oil price, while the second term represents the reaction to a change in the global economic activity. Specifically, a higher demand in the foreign economy increases the demand for home products and, at the same time, appreciates domestic real exchange rate making it less costly to produce conditional on an unchanged real oil price in the units of foreign goods.

The response of aggregate consumption is

$$\widehat{c} = \frac{1-\Omega+\frac{\kappa(\varphi_\pi-\alpha)}{\sigma(1-\alpha\beta)(1-\alpha)}\left(\zeta_H+\psi_q\sigma\right)\Omega\frac{\psi_q\sigma-\zeta_F}{\zeta_H+\psi_q\sigma}}{1+\frac{\kappa(\varphi_\pi-\alpha)}{\sigma(1-\alpha\beta)(1-\alpha)}\left(\zeta_H+\psi_q\sigma\right)\Omega}\widehat{c}^* - \frac{\frac{(\varphi_\pi-\alpha)\kappa}{\sigma(1-\alpha)(1-\alpha\beta)}\left(\zeta_H+\sigma\psi_q\right)\Omega}{1+\frac{\kappa(\varphi_\pi-\alpha)}{(1-\alpha)(1-\alpha\beta)\sigma}\left(\zeta_H+\sigma\psi_q\right)\Omega}\cdot\frac{\psi_o}{\zeta_H+\sigma\psi_q}\widehat{r}.$$

The response of aggregate consumption conditional on the oil supply shock that increases oil price by one percent

$$\widehat{c} = \frac{1-\Omega+\Omega\frac{\kappa(\varphi_\pi-\alpha)}{\sigma(1-\alpha\beta)(1-\alpha)}\left(\psi_q\sigma-\zeta_F\right)}{1+\frac{\kappa(\varphi_\pi-\alpha)}{\sigma(1-\alpha\beta)(1-\alpha)}\left(\zeta_H+\psi_q\sigma\right)\Omega}\widehat{c}^* - \frac{\frac{(\varphi_\pi-\alpha)\kappa}{\sigma(1-\alpha)(1-\alpha\beta)}\left(\zeta_H+\sigma\psi_q\right)\Omega}{1+\frac{\kappa(\varphi_\pi-\alpha)}{(1-\alpha)(1-\alpha\beta)\sigma}\left(\zeta_H+\sigma\psi_q\right)\Omega}\cdot\frac{\psi_o}{\zeta_H+\sigma\psi_q}.$$

We note that when $\varphi_\pi = \varphi_\pi^*$, we get $\widehat{c} = \widehat{c}^*$.

Home production is

$$\widehat{y}_{H,t} = \frac{\Omega}{\Omega+1-\Omega^*}\widehat{c}_t + \frac{1-\Omega^*}{\Omega+1-\Omega^*}\widehat{c}_t^* + \frac{\Omega\left(1-\Omega\right)+\left(1-\Omega^*\right)}{\Omega+1-\Omega^*}\cdot\frac{\gamma_n}{\Omega}\widehat{q}_t.$$

This formula clearly illustrates that the oil shock affects the non-oil production through three distinct channels corresponding to three terms in the formula: (i) a change in domestic aggregate demand; (ii) a change in foreign aggregate demand; (iii) an expenditure switching effect from foreign to domestic goods. We replace the real exchange rate and domestic consumption

$$\begin{aligned}\widehat{y}_{H,t} &= \frac{\Omega}{\Omega+1-\Omega^*}\widehat{c}_t + \frac{1-\Omega^*}{\Omega+1-\Omega^*}\widehat{c}_t^* + \frac{\Omega\left(1-\Omega\right)+\left(1-\Omega^*\right)}{\Omega+1-\Omega^*}\cdot\frac{\gamma_n}{\Omega}\sigma\left(\widehat{c}_t-\widehat{c}_t^*\right)\\ &= \frac{\Omega}{\Omega+1-\Omega^*}\left(1+\frac{\Omega\left(1-\Omega\right)+\left(1-\Omega^*\right)}{\Omega}\cdot\frac{\gamma_n\sigma}{\Omega}\right)\widehat{c}_t + \frac{1-\Omega^*}{\Omega+1-\Omega^*}\left(1-\frac{\Omega\left(1-\Omega\right)+\left(1-\Omega^*\right)}{1-\Omega^*}\cdot\frac{\gamma_n\sigma}{\Omega}\right)\widehat{c}_t^*.\end{aligned}$$

The response of the real exchange rate is

$$\widehat{q}_t = \sigma\left(\widehat{c}_t-\widehat{c}_t^*\right) = -\sigma\frac{\frac{\kappa(\varphi_\pi-\varphi_\pi^*)}{\sigma(1-\beta\alpha)(1-\alpha)}\psi_o\Omega}{1+\frac{\kappa(\varphi_\pi-\alpha)}{\sigma(1-\alpha\beta)(1-\alpha)}\left(\zeta_H+\psi_q\sigma\right)\Omega}\cdot\frac{1}{1+\frac{\kappa(\varphi_\pi^*-\alpha)}{(1-\beta\alpha)(1-\alpha)\sigma}\zeta^*}.$$

The last expression implies that the real exchange rate does not respond when the home and foreign central banks respond to domestic inflation in the same way, that is, $\varphi_\pi = \varphi_\pi^*$. When, however, home country is at the ZLB while foreign country actively responds to oil shock, i.e., $\varphi_\pi - \varphi_\pi^* < 0$, the real exchange rate depreciates $\widehat{q}_t > 0$.

For uniqueness of bounded ZLB solution, the following condition must hold

$$1-\frac{\alpha\kappa}{\sigma\left(1-\alpha\beta\right)\left(1-\alpha\right)}\left(\zeta_{H}+\psi_{q}\sigma\right)\Omega>0.$$