

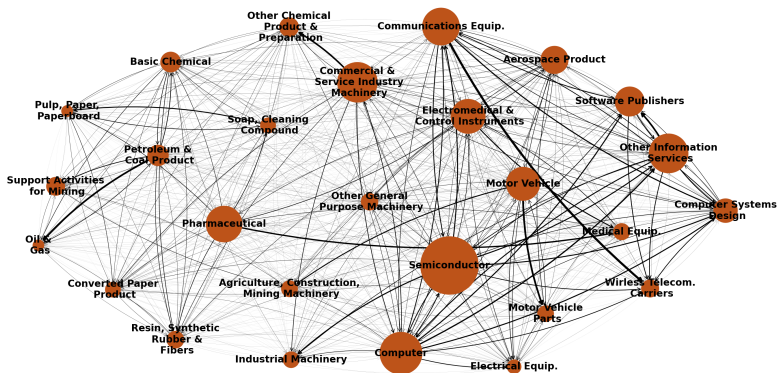
# Innovation Networks and R&D Allocation

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*Princeton and NBER*

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- Innovation is the source of long-run growth
- How to optimally allocate R&D resources to stimulate technological innovation?
  - many economies have dedicated government agencies for innovation policy
  - existing literature focuses on *over-time* or *within-sector* allocation of R&D resources
- **This paper: *cross-sector* allocation of R&D resources in the presence of innovation network**



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- $q_{it}$ : a sector's knowledge stock (state variable); can be improved through R&D

Flow innovation output:  $n_{it} = s_{it} \chi_{it}$ ,  $\chi_{it} \equiv \eta_i \prod_{j=1}^K q_{jt}^{\omega_{ij}}$

- $s_{it}$ : amount of R&D resources used in sector  $i$
- $\chi_{it}$ : R&D productivity; an aggregator of prior knowledge that is useful for R&D in sector  $i$
- $\Omega \equiv [\omega_{ij}]$  defines the **innovation network**; row-sum normalized to one

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- $\Omega \equiv [\omega_{ij}]$  defines the **innovation network**; row-sum normalized to one
- Flow innovation  $n_{it}$  improves knowledge stock  $q_{it}$  according to the law of motion
$$\dot{q}_{it}/q_{it} = \lambda \ln(n_{it}/q_{it})$$
  - without cross-sector spillover ( $\Omega = I$ ), law of motion collapses to  $\dot{q}_{it}/q_{it} = \lambda \ln(\eta_i s_{it})$
- Given total production and R&D resources  $(\bar{\ell}, \bar{s})$ , how to allocate across sectors  $(\ell_{it}, s_{it})$ ?

## Optimal R&D allocation: planner's optimal control problem

$$V(\{q_{i0}\}) \equiv \max_{\{s_{it}, \ell_{it}\}} \int_0^{\infty} e^{-\rho t} \sum_i \beta_i (\psi \ln q_{it} + \ln \ell_{it}) dt$$

$$\text{s.t. } \dot{q}_{it}/q_{it} = \lambda \left( \ln \eta_i + \ln s_{it} + \sum_j \omega_{ij} (\ln q_{jt} - \ln q_{it}) \right), \quad \sum_i s_{it} = \bar{s}, \quad \sum_i \ell_{it} = \bar{\ell}.$$

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**Proposition.** For any  $q_0$ , the optimal allocation of resources is time-invariant:  $\ell_{it} = \beta_i \bar{\ell}$  for all  $t$ , and

$$s_{it} = \gamma_i \bar{s} \text{ for all } t, \quad \text{where } \gamma' \propto \beta' \left( \mathbf{I} - \frac{\mathbf{\Omega}}{1 + \rho/\lambda} \right)^{-1} \equiv \beta' \left( \mathbf{I} + \frac{\mathbf{\Omega}}{1 + \rho/\lambda} + \left( \frac{\mathbf{\Omega}}{1 + \rho/\lambda} \right)^2 + \dots \right).$$

- Planner incorporates (and discounts by  $\rho/\lambda$ ) future network spillover effects
  - myopic planner:  $\lim_{\rho/\lambda \rightarrow \infty} \gamma = \beta$
  - patient planner:  $\lim_{\rho/\lambda \rightarrow 0} \gamma = \mathbf{a}$  (eigenvector centrality of  $\mathbf{\Omega}$ ; growth-maximizing allocation)

## Welfare gains from R&D reallocation

**Proposition.** Consumption-equivalent welfare gain of reallocating R&D from  $b$  to  $\gamma$  is

$$\mathcal{L}(b) = \exp\left(\frac{\psi\lambda}{\rho} \times \gamma'(\ln \gamma - \ln b)\right)$$

- Consumer indifferent between  $\gamma$  vs.  $b$  with consumption multiplied by  $\mathcal{L}(b)$  at all times



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Extensions: (1) production network; (2) factor mobility (btwn.  $\bar{\ell}$  and  $\bar{s}$ );  
(3) time-varying, exogenous  $\Omega_t$ ; (4) semi-endogenous growth;  
(5) general function forms (endogenous  $\Omega$ ); (6) foreign spillovers

## Model extension: general functional forms (endogenous $\Omega$ )

	Baseline model	General functional form
preferences	$\int_0^\infty e^{-\rho t} \sum_i \beta_i \ln y_{it} dt$	$\int_0^\infty e^{-\rho t} \ln \mathcal{Y}(y_{it}) dt, \mathcal{Y}$ CRTS
law of motion	$\dot{q}_{it}/q_{it} = \lambda \ln \left( s_{it} \eta_i \prod_j q_{jt}^{\omega_{ij}} / q_{it} \right)$	$\dot{q}_{it}/q_{it} = \lambda \ln (s_{it} \mathcal{X}_i(\{q_{jt}\}))$

- In a BGP with R&D allocation  $\mathbf{b}$ , define local elasticities (in BGP,  $\mathcal{X}_i$  is locally homog. of deg 0)

$$\beta_i \equiv \frac{\partial \ln \mathcal{Y}(\{q_{it} \ell_{it}\})}{\partial \ln q_{it}}, \quad \omega_{ij} \equiv \frac{\partial \ln \mathcal{X}_i(\{q_{jt}\})}{\partial \ln q_{jt}}, \quad \omega_{ii} = 1 - \frac{\partial \ln \mathcal{X}_i(\{q_{jt}\})}{\partial \ln q_{jt}}$$

- Define  $\gamma' = \frac{\rho}{\rho+\lambda} \beta' \left( \mathbf{I} - \frac{\Omega}{1+\rho/\lambda} \right)^{-1}$

**Proposition. (General Functional Forms)** To first-order, around the observed BGP, the consumption-equivalent welfare gain of moving R&D allocation from  $\mathbf{b}$  to  $\tilde{\mathbf{b}}$  is

$$\exp \left( \frac{\psi \lambda}{\rho} \times \gamma' \left( \ln \tilde{\mathbf{b}} - \ln \mathbf{b} \right) \right).$$

## Model extension: unilaterally optimal R&D with foreign spillovers

- Suppose the economy benefits from foreign spillovers:  $\chi_{it} \equiv \eta_i \prod_j \left[ (q_{jt})^{x_{ij}} (q_{jt}^f)^{1-x_{ij}} \right]^{\omega_{ij}}$ 
  - $x_{ij}$ : share of domestic contribution of spillovers from  $j$  to  $i$
- **Unilaterally optimal**: maximize domestic welfare, taking the path of foreign knowledge as given

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$$\text{Optimal R\&D allocation: } \gamma' \propto \beta' \left( \mathbf{I} - \frac{\boldsymbol{\Omega} \circ \mathbf{X}}{1 + \rho/\lambda} \right)^{-1}$$

- An economy reliant on foreign knowledge (lower  $x$ ) should choose R&D as if impatient (high  $\rho/\lambda$ )
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- An economy reliant on foreign knowledge (lower  $x$ ) should choose R&D as if impatient (high  $\rho/\lambda$ )
  - countries with self-contained network  $\Rightarrow$  invest more in innovation-central sectors
- Open economy log-welfare gains from optimal R&D reallocation is

$$\ln \mathcal{L}(\mathbf{b}, \xi) = \underbrace{\xi}_{\text{R\&D self-sufficiency}} \times \frac{\psi\lambda}{\rho} \underbrace{\gamma' (\ln \gamma - \ln \mathbf{b})}_{\text{misallocation}}$$

- more foreign dependent economies (lower  $\xi$ ) have less to gain from optimal R&D allocations
- $\xi \equiv \frac{\rho}{\rho+\lambda} \beta' \left( \mathbf{I} - \frac{\boldsymbol{\Omega} \circ \mathbf{X}}{1+\rho/\lambda} \right)^{-1} \mathbf{1}$ .  $\xi$  is decreasing in foreign-reliance;  $\xi = 1$  only if  $x_{ij} = 1 \forall i, j$

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# Map to Empirical Applications

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$\gamma$	Optimal Allocation	$\gamma' \propto \beta' \left( \mathbf{I} - \frac{\mathbf{\Omega} \circ \mathbf{X}}{1 + \rho/\lambda} \right)^{-1}$
$\ln \mathcal{L}(\mathbf{b}, \xi)$	Potential Welfare Gains	$\ln \mathcal{L}(\mathbf{b}, \xi) = \xi \times \frac{\psi \lambda}{\rho} \gamma' (\ln \gamma - \ln \mathbf{b})$

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- Key Data:

- $\beta$ : sectoral value-added
- $\mathbf{\Omega}$ : innovation network
- $\mathbf{X}$ : self-dependence on innovation production
- $\mathbf{b}$ : real-world R&D allocation

- Parametrization:  $\rho = 0.05$ ,  $\lambda = 0.17$ ,  $\psi = 0.06$

- optimal allocation  $\gamma$  is robust to alternative parameter values; so is the relative entropy  $\gamma' (\ln \gamma - \ln \bar{\mathbf{b}})$
- welfare effect sensitive to  $\psi$
- calibrated so that  $\frac{dg^y}{d \ln \bar{s}} = 0.01$  (semi-elasticity of BGP consumption growth to R&D stock  $\bar{s}$ )

# Innovation Data: Domestic and International

To construct the innovation network  $\Omega$ , we rely on patent citations

$$\text{(baseline definition)} \quad \omega_{ij} \equiv \frac{Citations_{ij}}{\sum_k Citations_{ik}}$$

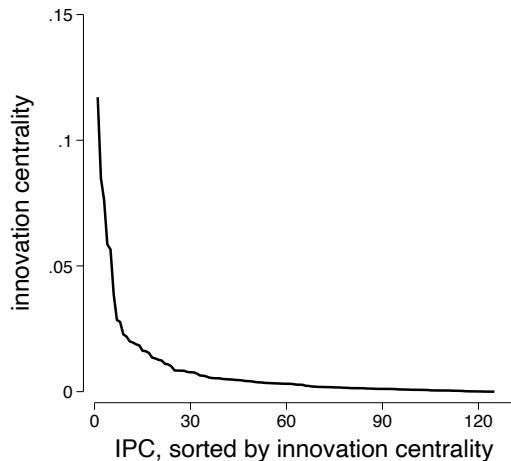
- Domestic U.S. Patent Data from USPTO: 6.9 Million Patents, 1975–2020
  - key information: filing year, assignee, technology class (IPC), citation relations
- International Patent Data from Google Patents: 36 million patents from 42 countries, 1976–2020
  - combines patent data from more than twenty major patent offices (US, Japan, China, EPO, ...)
  - identify unique innovation (origin country and sectors) from multiple patent filings (“patent family”)

Production-side information: WIOD

R&D data from firm-level data sets (Compustat, Worldscope, Datastream) and OECD-ANBERD



## Innovation centrality $\alpha$ is highly heterogeneous



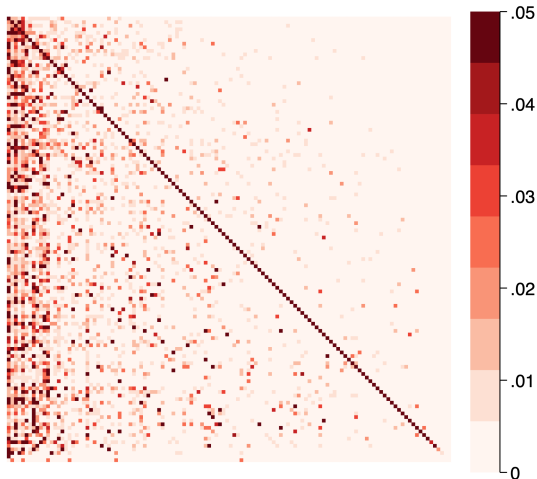
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1	A61	medical or veterinary science; hygiene
2	G06	computing; calculating or counting
3	H01	basic electric elements
4	H04	electric communication technique
5	G01	measuring; testing
6	B60	vehicles in general
7	G02	optics
8	B01	physical or chemical processes or apparatus in general
9	C08	organic macromolecular compounds; their preparation or chemical working-up; compositions based thereon
10	F16	engineering elements or units; general measures for producing and maintaining effective functioning of machines or installations; thermal insulation in general

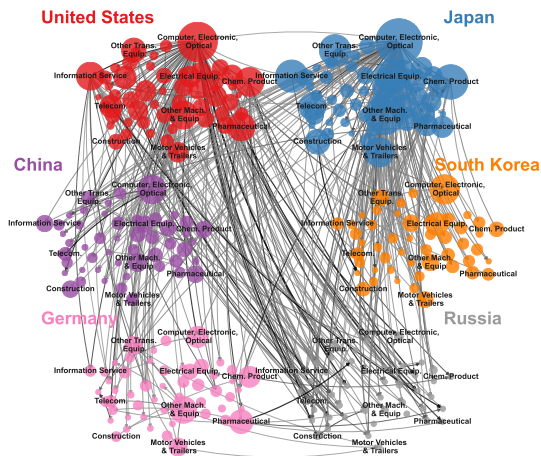
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# Innovation network $\Omega$ visualization, IPC-to-IPC

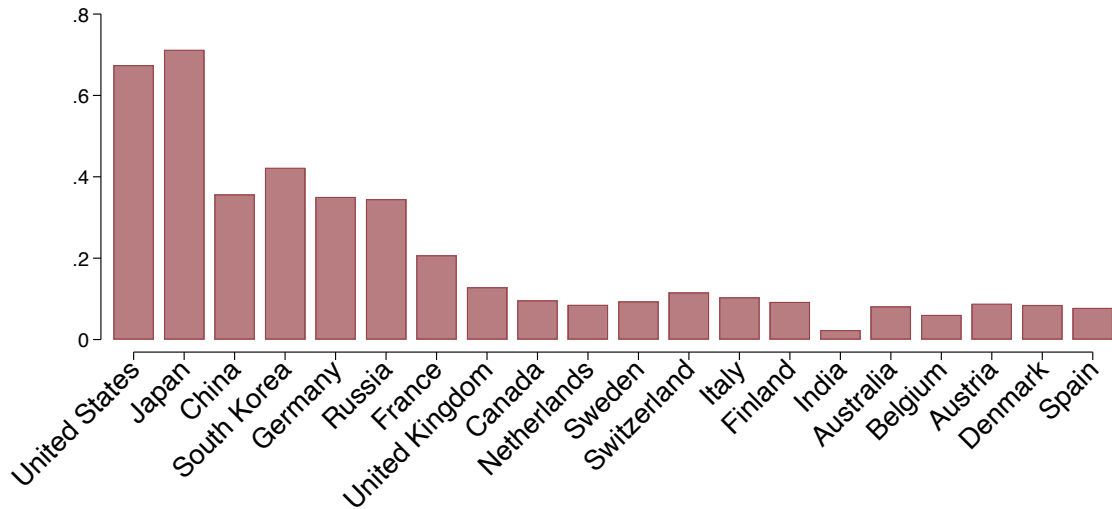
(a) IPC-to-IPC (131×131) network  $\Omega$



(b) The global innovation network



$x_{ij}$ : Domestic citation shares across 20 countries



- The innovation network  $\Omega$  is highly stable across countries and time
  - only weakly correlated with production network

Countries	All	US	Japan	China	South Korea	Germany	Russia	France	UK	Canada	Netherlands
All		0.98	0.87	0.87	0.84	0.89	0.63	0.86	0.92	0.88	0.81
US	0.95		0.84	0.86	0.82	0.88	0.64	0.85	0.92	0.88	0.80
Japan	0.86	0.83		0.88	0.89	0.85	0.63	0.87	0.86	0.84	0.83
China	0.85	0.86	0.87		0.88	0.85	0.66	0.85	0.87	0.86	0.82
South Korea	0.78	0.77	0.83	0.84		0.84	0.64	0.84	0.85	0.82	0.84
Germany	0.85	0.87	0.81	0.80	0.72		0.64	0.83	0.87	0.83	0.81
Russia	0.62	0.63	0.62	0.62	0.55	0.61		0.65	0.64	0.64	0.66
France	0.91	0.86	0.79	0.77	0.72	0.82	0.57		0.86	0.85	0.83
UK	0.87	0.89	0.85	0.85	0.80	0.86	0.64	0.80		0.88	0.82
Canada	0.86	0.88	0.79	0.81	0.71	0.81	0.59	0.80	0.81		0.81
Netherlands	0.84	0.85	0.79	0.82	0.75	0.79	0.58	0.78	0.79	0.81	

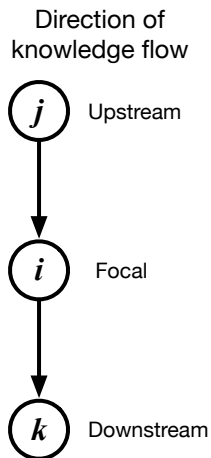
Time Period	All years	2020	2010	2000	1990	1980
All years		0.98	0.98	0.97	0.90	0.89
2020	0.95		0.97	0.93	0.86	0.85
2010	0.96	0.97		0.96	0.88	0.87
2000	0.93	0.92	0.96		0.92	0.90
1990	0.90	0.80	0.84	0.90		0.91
1980	0.81	0.77	0.81	0.87	0.89	

- Significant cross-country variation in optimal R&D allocation  $\gamma$

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## Evidence for knowledge spillover, building on Acemoglu, Akcigit, and Kerr (PNAS 2016)



- Upstream patents foster sector  $i$ 's future innovation; effect weakens over time

$$\ln n_{it} = \ln \eta_i + \ln s_{it} + \lambda \underbrace{\sum_{j=1}^K \omega_{ij} \left( \int_0^{\infty} e^{-\lambda s} \ln n_{jt-s} ds \right)}_{\text{knowledge from upstream sectors}}$$

- We construct the empirical counterpart to “knowledge from upstream”:

$$\text{Knowledge}_{it}^{Up} \equiv \sum_{j=1, j \neq i}^K \omega_{ij} \sum_{s=1}^{10} \log \text{Patent}_{j,t-s}$$

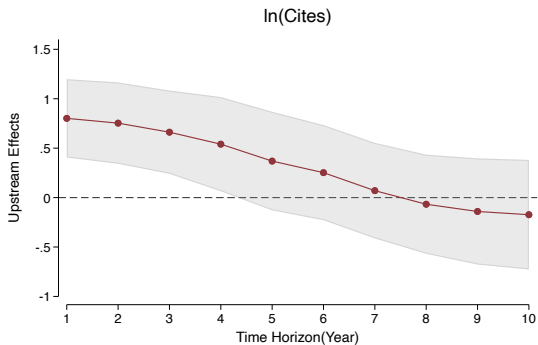
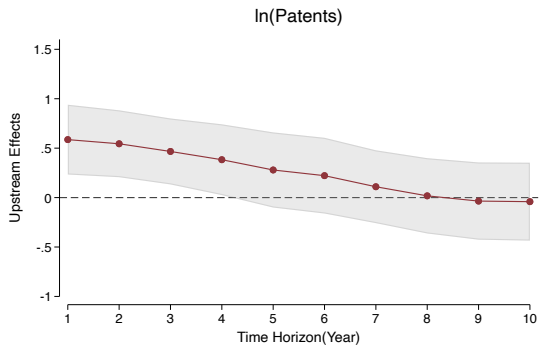
- We show  $\text{Knowledge}_{it}^{Up}$  predicts sector  $i$ 's innovation, effects decay over time
  - holds in both the U.S. domestic & the global innovation networks
- To rule out “common shock”, we show:
  1. “knowledge” from downstream doesn't predict sector  $i$ 's innovation
  2. “knowledge” aggregated through input-output linkages doesn't either
  3. results robust to using tax-induced R&D cost variations as IV (Bloom et al. 2013)

# Upstream knowledge fosters new innovation in focal sector

$$Innovation_{it} = \beta \times Knowledge_{it}^{Up} + \xi_i + \xi_t + control_{it} + \varepsilon_{it}$$

Y=	ln(Patents)				ln(Cites)			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$Knowledge_{it}^{Up}$	0.586*** (0.180)	0.600*** (0.205)	0.508*** (0.174)	0.679** (0.266)	0.802*** (0.202)	0.830*** (0.218)	0.743*** (0.196)	0.974*** (0.279)
$ln(R\&D)_{i,t-1}$	0.275*** (0.063)	0.274*** (0.062)	0.279*** (0.060)	0.269*** (0.070)	0.258*** (0.086)	0.256*** (0.086)	0.261*** (0.086)	0.174** (0.082)
$Knowledge_{it}^{Down}$		-0.029 (0.157)				-0.058 (0.098)		
$Knowledge_{it}^{Up,IO}$			0.363** (0.173)				0.268 (0.205)	
Specification	OLS	OLS	OLS	IV 2nd Stage	OLS	OLS	OLS	IV 2nd Stage
IV 1st Stage $F$ -statistics				465.9				465.9
$R^2$	0.915	0.915	0.917	0.152	0.901	0.901	0.902	0.099
No. of Sectors	94	94	94	94	94	94	94	94
No. of Obs	1847	1847	1847	1113	1847	1847	1847	1113
Fixed Effects		Sector, Year				Sector, Year		

# Impulse response shows upstream spillover effect weakens over longer lags

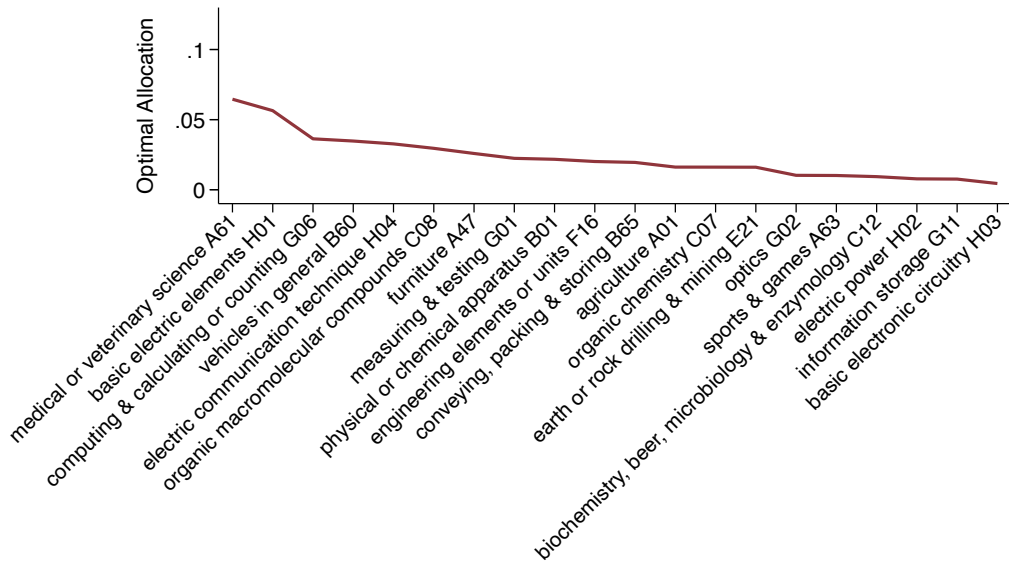




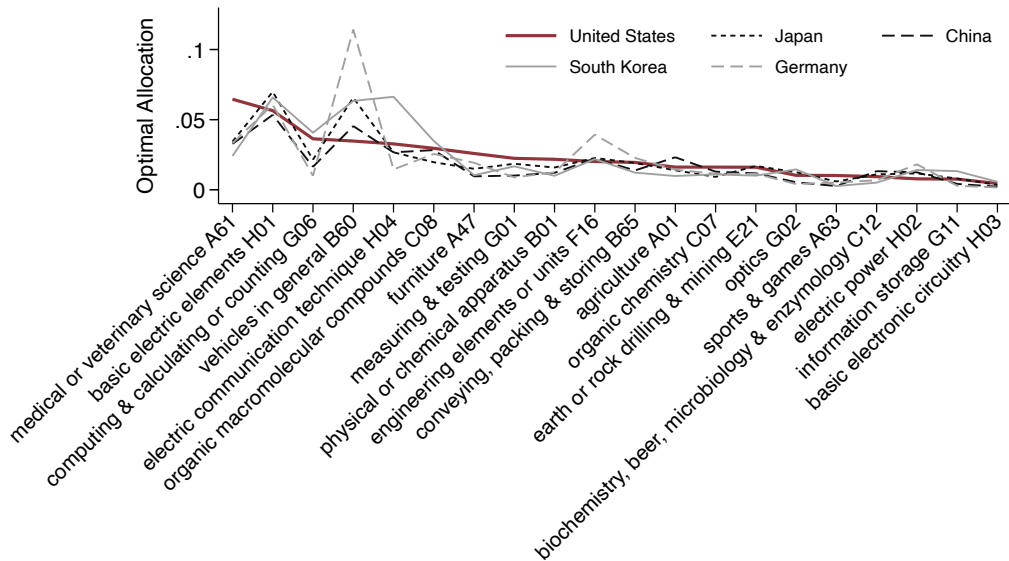
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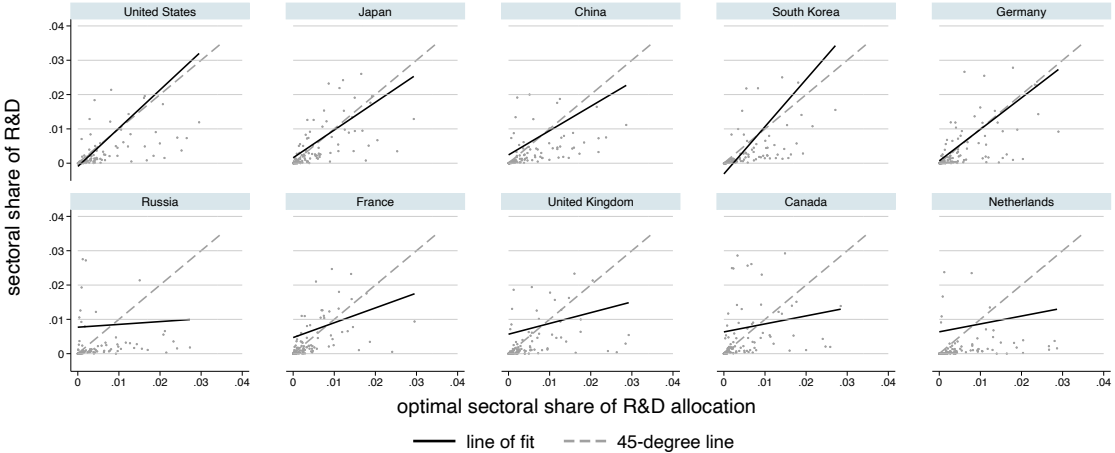
## Optimal R&D Allocation in US



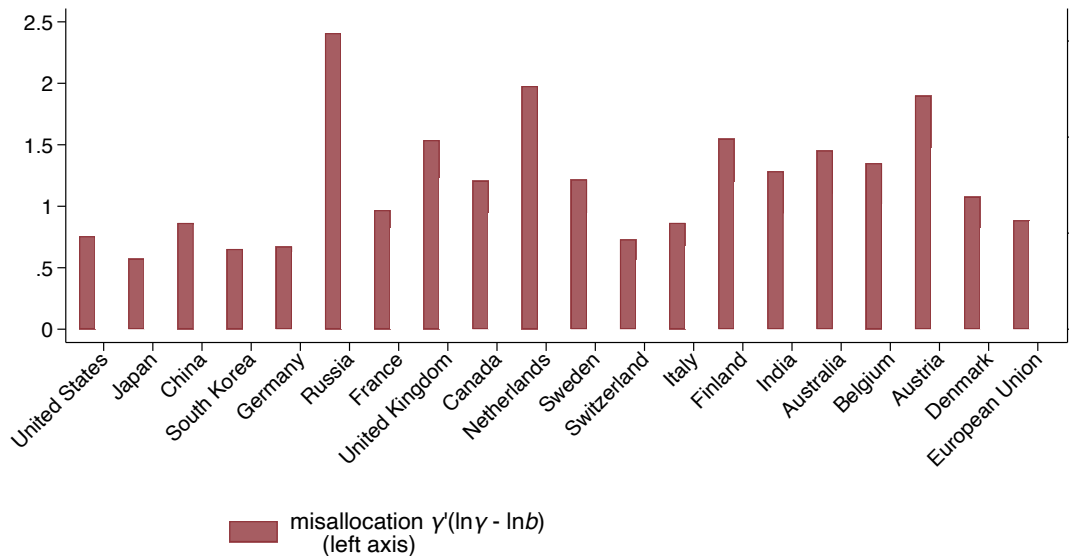
# Optimal R&D Allocation in Top Innovative Economies



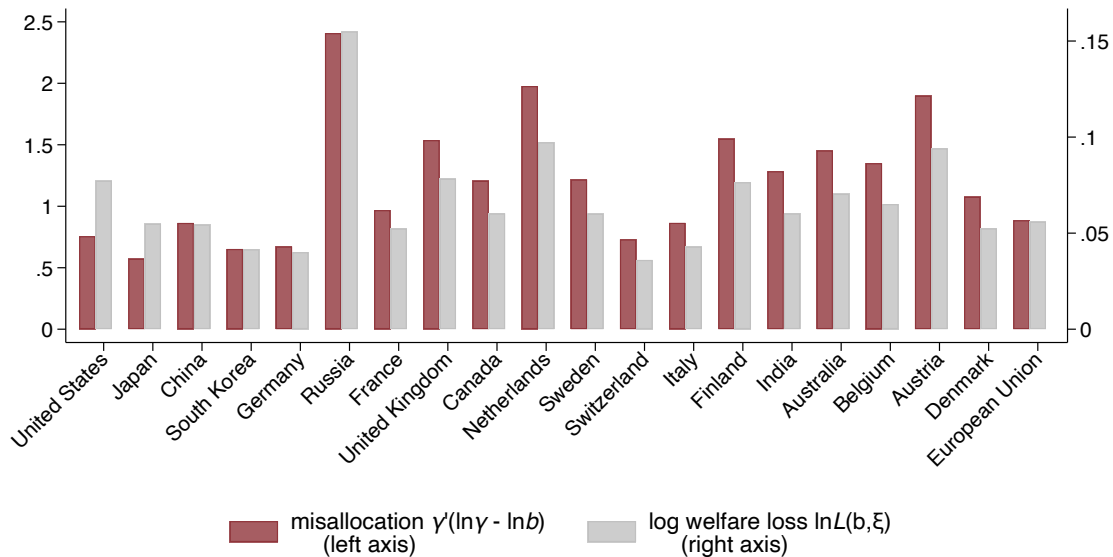
# Actual vs optimal R&D allocations across countries (2010–2014)



## R&D allocative inefficiency in the data



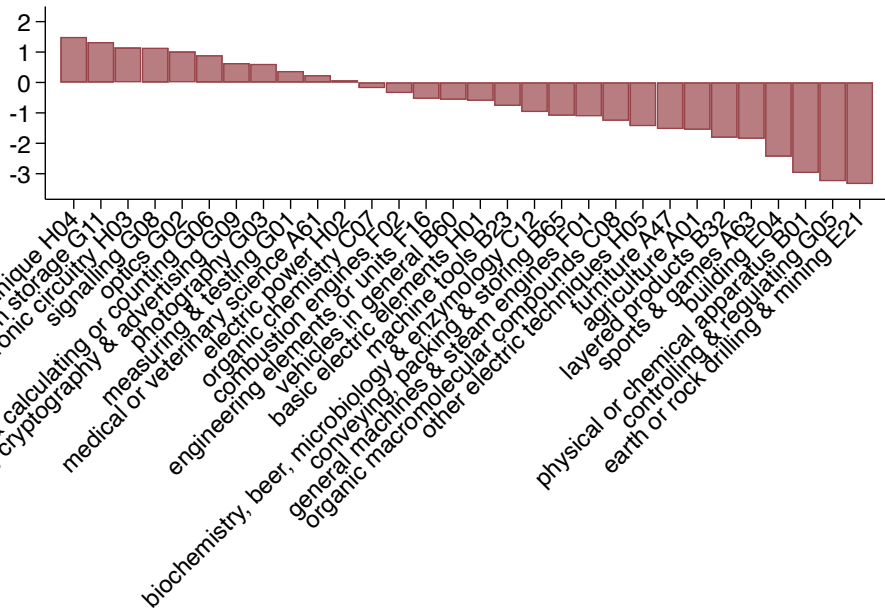
## R&D allocative inefficiency in the data



## Consumption-equivalent welfare gains

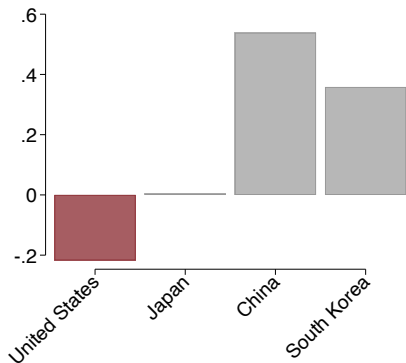
	US	Japan	China	South Korea	Germany	Russia	France	UK	Canada	Netherlands
2000	9.98	4.24	5.78	5.25	4.79	13.70	5.17	7.55	7.22	6.70
2005	8.85	5.04	5.26	3.92	4.11	11.18	5.38	8.17	7.29	5.45
2010	8.04	5.64	5.60	4.24	4.09	16.76	5.38	8.15	6.21	10.22
	Sweden	Switzerland	Italy	Finland	India	Australia	Belgium	Austria	Denmark	European Union
2000	6.65	5.18	5.04	5.39	10.91	5.72	5.72	6.52	5.93	5.91
2005	5.53	4.10	4.57	5.63	8.33	4.19	5.62	8.50	5.30	5.04
2010	6.20	3.67	4.40	7.95	6.21	7.30	6.73	9.87	5.39	5.76

## Log-difference between actual and optimal R&D allocation, US, top 30 IPCs



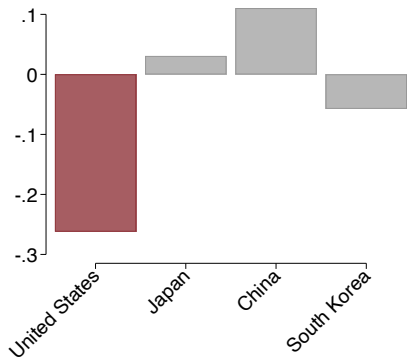


## Examples of Misallocation (1): Semiconductors



- US underfunds semi-conductor R&D by about 21%
- South Korea and China invest more aggressively
- Policy Relevance
  - CHIPS for America Act
  - Facilitating American-Built Semiconductors Act

## Examples of Misallocation (2): Green Innovation



- Policy Relevance
  - Green innovation grants, tax credit, ...
  - Impact investment
- In Our Calculation: US green-innovation R&D
  - Under-funded by about 25%
  - While other countries have milder misallocation

# Conclusion

- Theory: optimal innovation allocation in innovation networks
  - sufficient statistics for optimal R&D & misallocation accounting in closed & open economies
  - planner should direct R&D towards more fundamental sectors, but incentive muted in open economics
- Construct the global patent citation network; empirical validation of knowledge spillover dynamics
- Japan, US, South Korea, Germany are the most allocatively efficient among advanced economies, but welfare cost of R&D misallocation in other economies mitigated by foreign spillovers
- Moving to efficient allocation  $\implies$  consumption-equivalent gains of 8% in the US in 2010