

Firm Export Dynamics in Interdependent Markets*

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June 22, 2023

Abstract

We introduce a model of firm export dynamics featuring cross-country export complementarities. Firms internalize the impact selling in a country has on export costs in other countries. To solve the firm's decision problem, we develop an algorithm that overcomes the computational challenges inherent to the large dimensionality of the state space and choice set. According to our estimates, firms enjoy cost reductions when exporting to countries geographically or linguistically close to each other, or that share deep trade agreements; and countries, especially small ones, sharing these traits with attractive destinations receive significantly more exports than in the absence of complementarities.

JEL Classifications: F12, F13, F14.

Keywords: export dynamics, integer programming problem, complementarities.

*We thank Rodrigo Adão, Stéphane Bonhomme, Klaus Desmet, Natalia Ramondo, Esteban Rossi-Hansberg, Felix Tintelnot, and Conor Walsh, for insightful conversations, and seminar participants at Banco de España, Bocconi, CEBRA, CEMFI, University of Chicago, University of Copenhagen, CREI, Deusto, ETOS, Federal Reserve Board, Georgetown, NUS, NYU, Purdue University, Princeton University, SAEs, SED, and World Bank, for very helpful comments. All errors are our own. The views expressed herein are solely those of the authors and are not necessarily those of BCCR. All results have been reviewed by BCCR to ensure no confidential information is disclosed. Email: alfaroua@bccr.fi.cr, castro.vincenzi@princeton.edu, fanelli@cemfi.es, ecmorale@princeton.edu.

1 Introduction

Since Baldwin (1988) and Baldwin and Krugman (1989), a large literature focuses on modeling the dynamics of firms in export markets; see Alessandria et al. (2021a) for a review. This literature nearly unanimously assumes a firm’s export decisions in a foreign country are unaffected by its decisions in other countries. There is however growing evidence questioning this assumption, supporting instead the hypothesis that there are cross-country complementarities in firm exports, such that exporting to a country makes a firm more likely to export to other countries (Chaney, 2014; Morales et al., 2019; Albornoz et al., 2021b).

The idea of cross-country export complementarities resonates often in policy discussions of trade agreements. First, it is behind claims that preferential trade agreements (PTAs) increase exports from their members to non-member countries.¹ Second, the belief the regulatory convergence deep PTAs impose on their members is a source of complementarities between them (Grossman et al., 2021) has supported claims that these agreements attract exports from third countries (Baldwin, 2011; Mattoo et al., 2022), thus counteracting the trade diversion effect of shallow PTAs predicted by models à la Eaton and Kortum (2002) or Anderson and van Wincoop (2003). Specifically, the potential trade creation effect of deep PTAs has featured prominently in analyses of Brexit.²

These policy discussions and the prior evidence supporting the existence of cross-country export complementarities raise the question of how quantitatively important these are in determining firm exports, in particular in reaction to trade policy changes. In a first step towards answering this question, we extend a canonical partial equilibrium model of firm export dynamics featuring fixed and sunk export costs (as in Das et al., 2007) to allow for complementarities in a firm’s export decision across countries. In our model, the firm chooses its per-period set of export destinations as the solution to a single-agent dynamic combinatorial discrete choice problem, and we build on Jia (2008) and Arkolakis et al. (2021) to develop a new algorithm to solve such problems. We estimate our model using firm-country-year level data on the universe of exports from Costa Rica during 2005-2015 and, using the estimated model, we show that, in the absence of complementarities, the number of firm-country-year combinations with positive exports in our sample would have been close to 12% smaller, and total exports would have decreased in approximately 5%. When evaluating the impact on Costa Rica of a Brexit-driven hypothetical regulatory divergence between the UK and the EU, our model predicts total exports and the number of exporters to the UK to decrease in around 4% on average in the ten year window post Brexit. Analogous predictions for the EU as a whole are below 0.5%, reflecting that, everything else equal, cross-country complementarities

¹In an example involving Costa Rica, whose data we use in our analysis, its government has argued that the PTA with Singapore lets Costa Rica increase its exports throughout Asia (Ruiz, 2013). Similarly, the Australian government has defended the PTA with Peru asserting it “provides Australian businesses a gateway to Latin America” (Australian Government, 2020). Similar claims have been made in relation to, e.g., the PTAs between India and the UAE (Jayaswal, 2021), China and Uruguay (Werner, 2021), or Canada and Morocco (Canadian Government, 2022), the last two currently under negotiation.

²For e.g., UNCTAD (2020) claims that “the positive third-country effect could be diminished by increasing regulatory divergence. If the UK’s regulations divert over time from the EU’s, trade costs would rise for third countries due to production process adjustment costs and potential duplication of proofs of compliance.”

have a larger impact on exports to smaller markets. Finally, we predict the impact of Costa Rica joining the Comprehensive and Progressive Agreement for Trans-Pacific Partnership (CPTPP), and show that researchers using a model analogous to ours but that excludes the possibility of complementarities would have predicted an increase in Costa Rican exports to CPTPP members only slightly smaller than that implied by our model. The difference would however be larger if the CPTPP included a large potential export destination, such as the US, among its members, as in this case the increase in exports to this large destination that would result from Costa Rica joining the CPTPP would have significant spillovers on the other members.³

Consistently with findings in the prior literature, the firm in our sample tends to export to countries geographically or linguistically close to, or that share a deep PTA with, its other concurrent export destinations. This correlation in export choices decreases only marginally when controlling for sector-destination-year and firm-year fixed effects and, thus, is mostly due to factors varying at the firm-country level. Although cross-country complementarities in firm exports could explain this correlation pattern, it may be caused instead by firm- and country-specific unobserved export profit (e.g., demand) shifters that are positively correlated across countries. To guide the separate identification of cross-country complementarities and correlation in unobserved export profit shifters, and to quantify the role the former play in determining firm exports, we build a model of export dynamics that allows for cross-country complementarities in firm choices.

In our model, monopolistically competitive firms featuring constant marginal production costs face destination- and period-specific variable, fixed, and sunk export costs. We model variable costs as “iceberg” costs and, building on Roberts and Tybout (1997), assume firms face a sunk entry cost if they export to a destination to which they did not export in the previous period. All export costs in a destination are allowed to depend on its geographic and linguistic distance to, and the deepness of its PTAs with, the firm’s home country. The fixed cost a firm faces in a country and period may additionally depend on the firm’s other export destinations in the same period. Specifically, a firm may face a smaller fixed cost in a country if it concurrently exports to other countries, and the extent of this cost reduction may depend on the geographic or linguistic proximity between both countries, as well as on the deepness of the PTAs of which both are members. To discipline the estimation of the parameters determining the extent to which a firm’s fixed cost in a country depend on the firm’s export choices in other countries, our model also allows this cost to depend on a term unobserved to the researcher and potentially correlated across destinations according to a correlation coefficient that may also depend on their geographic or linguistic proximity, or on the deepness of the PTAs of which they are members.

The inclusion of sunk costs and our modeling of fixed costs imply a firm’s static export profits in a country and period are weakly larger if the firm exported to the same country in the previous period, or if it exports to other countries in the same period. The firm internalizes the impact its export choice in a country and period has on profits in other countries and periods. Specifically,

³Costa Rica formally requested in 2022 to join the CPTPP. This is an agreement among Australia, Brunei, Canada, Chile, Japan, Malaysia, Mexico, New Zealand, Peru, Singapore, and Vietnam. It evolved from the Trans-Pacific Partnership (TPP), which had the US among its members and never entered into force due to the US withdrawal.

firms select each period’s optimal set of export destinations after solving an infinite-horizon dynamic combinatorial discrete choice problem. We assume for tractability firms have perfect foresight on most payoff-relevant variables, but allow for firm uncertainty about future realizations of a country- and period-specific “blocking” or “exit” shock that, if realized, prevents the firm from exporting to a country in a period. As in Eaton et al. (2016) and Caliendo et al. (2019), we assume all payoff-relevant variables on which firms have perfect foresight are constant after a terminal period.

Given commonly available computational capabilities, the optimization problem determining the firm’s export path cannot be solved using standard dynamic programming algorithms. The reason is that the cardinality of the per-period choice set and state space grow exponentially in the number of possible export destinations: given J feasible destinations, the choice set includes 2^J elements (each element being a J -dimensional vector of binary variables indicating the set of countries to which the firm exports) and the state space includes 2^{2J} elements (each element indicating the firm’s export bundle in the previous period and the current realization of the blocking shocks in every country). To compute the firm’s optimal export path, we develop a novel algorithm that solves a series of increasingly complex problems that put gradually tighter bounds on the firms’ optimal decision. Our algorithm exploits the supermodularity of the firm’s objective function; i.e., exporting to a country in a period and state weakly increases the returns to exporting in every other country, future period, and possible state. It thus builds on previous work that has leveraged the supermodularity of the objective function to solve otherwise intractable static optimization problems (see, e.g., Jia, 2008, Antràs et al., 2017, Arkolakis et al., 2021), and it extends the set of supermodular problems that are computationally feasible to solve to a family of optimization problems featuring dynamics and firms’ uncertainty about future payoffs.

The problem of separately identifying the parameters governing the sensitivity of a firm’s country-specific fixed costs to its concurrent export destinations from the parameters determining the cross-country correlation in fixed costs’ unobserved determinants is an instance of the general problem of separately identifying “path” dependence from correlated unobservables; in our case, across countries within a period. For any given measure of proximity between countries, be it geographic or linguistic proximity, or whether they share a deep PTA, we show these may be separately identified combining two types of moment conditions. First, moments capturing how the correlation in firms’ export choices in any two countries depends on their proximity. Second, moments capturing the impact exogenous determinants of the firm’s export participation in countries close to a potential destination have on the probability the firm exports to such destination.⁴ While the first type of moments is particularly sensitive to the parameters determining the correlation in unobserved fixed cost shocks, the second type is specially sensitive to the parameters determining the impact exporting to a country has on fixed costs in other countries. In our model, both types of moments identify the parameters of interest.

⁴We use a country’s export potential as exogenous determinant of firms’ export participation in it; thus, the second type of moments relates the firm’s export participation in a country to the aggregate export potential of the countries close to it. To measure a country’s export potential, we use the importer fixed effect in a standard gravity equation estimated using sectoral trade data for all country pairs that do not include Costa Rica as importer or exporter.

Our estimates reveal there is a large heterogeneity across country pairs in the impact exporting to one of them has on fixed costs in the other one. This heterogeneity reflects geographic and linguistic distances between countries, as well as the deepness of the PTAs tying together their regulations. For e.g., exporting to Korea reduces fixed costs in China in 0.3%, exporting to Canada brings down fixed costs in the US in 3.5%, and exporting to France reduces fixed costs in Germany in 9%. These cost savings accumulate as the firm incorporates more destinations to its export bundle; e.g., for a firm exporting to France, adding Switzerland to its export bundle increases the reduction in fixed costs in Germany from 9% to 16%. More generally, members of the European Common Market, being geographically close to each other and sharing a deep PTA, have fixed export costs that are particularly sensitive to firms' other export destinations among their members.

We use our estimated model to perform three types of analysis. First, to quantify the role cross-country complementarities play in determining firm exports, we compare the choices of all sample firms during 2005-2015 predicted by our estimated model to those predicted by an alternative model that differs from ours only in that a firm's fixed costs in a country no longer depend on the firm's other export destinations. Complementarities increase the total number of firm-country-periods with positive exports in 11.8%, and total export revenues in 5.1%. Of the three possible sources of cross-country complementarities we account for, geographical proximity plays a larger role, causing by itself a 2.7% increase in export sales, while allowing deep PTAs to generate cross-country complementarities increases exports in 1.6%, and linguistic proximity does so in only 0.9%. These numbers mask a large heterogeneity across destinations: most EU members see exports from Costa Rica increase in at least 10% (with some countries in Central and Eastern Europe experiencing increases above 25%), and exports to large countries such as the US, China, or Russia, are largely unaffected by the complementarities implied by our estimated model.

Second, to measure the third-country effect of cross-country complementarities arising from deep PTAs, we quantify the impact of Brexit on exports from Costa Rica to the UK and the EU. Specifically, we use our estimated model to compare firms' exports in a setting in which the UK and the EU share no deep PTA post Brexit to those in a counterfactual setting in which the UK is still a member of the European Common Market and, thus, still shares a deep PTA with the EU. Trade barriers between Costa Rica and every other country are kept the same in both scenarios; thus, our analysis captures only the third-country effect of Brexit, and a partial-equilibrium model such as ours that rules out cross-country complementarities would predict identical export flows in both scenarios. In our model, in the four years between the Brexit referendum and the effective UK withdrawal from the EU, firms anticipate the future reduction in UK-EU complementarities, causing the number of firm-periods with positive exports to the UK to decrease in 1.4%, and total exports to decrease in 0.5%. In the ten years subsequent to the effective withdrawal, both the number of firm-periods with positive exports and total exports to the UK drop in close to 4%. Conversely, the impact on export flows to the EU is minimal.

Third, and finally, we study the impact of Costa Rica joining the CPTPP on its exports, and compare the predictions of our estimated model to those of a re-estimated model analogous to ours

in every aspect except in that it assumes away the possibility of cross-country complementarities. According to our estimated model, exports to CPTPP members are predicted to increase in 28%. Researchers using a model analogous to ours but that excludes the possibility of complementarities would have predicted a slightly smaller increase of 25.7%. The reason why both models yield similar predictions for this counterfactual analysis is that, given our estimates, CPTPP members either exhibit low levels of complementarities with every other country or the impact of Costa Rica joining CPTPP on its exports to them is too small to generate significant spillovers in other countries. To illustrate this point, we also compare the predictions of both models for the impact of Costa Rica joining a counterfactual CPTPP that additionally includes the US among its members. In this case, the two models yield different predictions. While the prediction of the model without complementarities for the increase in exports to actual CPTPP members is unaffected by the addition of the US to this trade bloc, the model with complementarities now predicts a much larger increase in exports (close to 40%) to actual CPTPP members. Intuitively, in this counterfactual scenario, there is an increase in exports to the US and, in the model with complementarities, this itself causes a large increase in exports to actual CPTPP members. Thus, whether models that allow for complementarities yield counterfactual predictions similar to models that do not depends on the particular change in trade policy being studied.

Our paper is related to several strands of the literature. First, it relates to the literature on firm export dynamics. This one has traditionally studied the firm’s decision in an aggregate export market (Roberts and Tybout, 1997; Das et al., 2007; Alessandria and Choi, 2007; Arkolakis, 2016; Ruhl and Willis, 2017) or in independent foreign markets (Fitzgerald et al., 2022).⁵ Exceptions are Schmeiser (2012), Chaney (2014), Alborno et al. (2016), and Morales et al. (2019), which allow for cross-country complementarities in firm exports. Our approach differs from that in Chaney (2014) in that we model firms’ export decisions as the outcome of combinatorial binary-choice optimization problems. Alborno et al. (2016) study analytically the implications of cross-country complementarities for export survival. The models in Schmeiser (2012) and Morales et al. (2019) are closer to ours, but while the latter does not attempt to solve the model, the former does so only for a small number of destinations.⁶ Our contribution is twofold: first, we provide an algorithm to solve a partial-equilibrium model of firm export dynamics that allows for cross-country complementarities in firm export decisions; second, we use the estimated model to quantify the role complementarities play in determining firms’ responses to trade policy changes.

Second, our paper also relates to the reduced-form literature that identifies interdependencies in firm sales across markets. While there is a large literature documenting non-zero correlation patterns in firm sales across markets (e.g., Evenett and Venables, 2002; Lawless, 2009, 2013; Alborno

⁵Other work studying firm export dynamics in aggregate or independent markets includes Eaton et al. (2008, 2021a,b); Impulliti et al. (2013); Alessandria and Choi (2014a,b); Timoshenko (2015); Alborno et al. (2016); Fitzgerald and Haller (2018); Dickstein and Morales (2018); Berman et al. (2019); Gumpert et al. (2020); Piveteau (2021); Alessandria et al. (2021b). There is also work studying dynamics in firm imports and in multinational production in independent markets; e.g., Conconi et al. (2016); Ramanarayanan (2017); Garetto et al. (2021); Lu et al. (2022).

⁶Besides Morales et al. (2019), other papers using moment inequalities to measure interdependencies across firms’ decisions, without solving the underlying dynamic model, are Holmes (2011), Houde et al. (2022), and Hoang (2022).

et al., 2012, 2016; Morales et al., 2019; Albornoz et al., 2021a), there is a more recent literature using instrumental variables to separately identify cross-market interdependencies in firm sales from cross-market correlation in unobserved determinants of these sales (e.g., Defever et al., 2015; Berman et al., 2015; Almunia et al., 2021; Albornoz et al., 2021b; Mattoo et al., 2022). Our contribution is to allow for complementarities in a dynamic firm entry model, to estimate the model parameters determining the strength of these complementarities using an approach that builds on the literature using instruments for the identification of these complementarities, and to quantify the role these complementarities play in determining the firm’s export decisions.

Third, our paper relates to the literature studying combinatorial discrete choice problems. This literature has focused nearly exclusively on static problems and, to solve them, has implemented several approaches: evaluating all choices (Tintelnot, 2017); modeling combinatorial choices as an aggregation of multinomial ones (Hendel, 1999); approximating the discrete problem as a choice over a continuous variable (Oberfield et al., 2022); or, devising algorithms that exploit the super- or sub-modularity of the objective function (Jia, 2008; Antràs et al., 2017; Arkolakis et al., 2021).⁷ We build on this last approach, and introduce a novel algorithm to solve rational-expectations single-agent combinatorial dynamic discrete choice problems in which all choices are complements.⁸

The rest of the paper proceeds as follows. Section 2 describes our data. Section 3 documents correlation patterns in firm exports. Section 4 introduces a dynamic model that allows for cross-country export complementarities, and sections 5 and 6 describe how we solve and estimate the model, respectively. In Section 7, we present the model estimates, and we discuss counterfactual results in Section 8. Section 9 concludes.

2 Data

Our analysis relies mainly on two types of data: firm-level data on the domestic sales and exports of firms located in Costa Rica, and data on the characteristics of foreign countries as potential export destinations of Costa Rican firms.

Our firm-level data covers the years 2005 to 2015, and comes from three sources. The first one is the Costa Rican customs database, which we use to measure export revenue by foreign country for the universe of Costa Rican firms. The second one is an administrative dataset that, for all firms located in Costa Rica, contains information on the firm’s sector, total revenue, and expenditure in materials. We combine the information on these two datasets and construct our measure of firm domestic revenue by subtracting total export revenue from total revenue. The third one was built by Alfaro-Ureña et al. (2022), who use information from different sources to identify the Costa Rican firms that belong to a foreign multinational corporation. We merge these three datasets using firm identifiers provided by Alfaro-Ureña et al. (2022), and restrict the set of

⁷For a paper that incorporates dynamics, see Zheng (2016), who groups choices in clusters such that, within each cluster, the choice depends only on an aggregate of the choices made in other clusters.

⁸Problems exhibiting both complementarities and substitutabilities across choices are relatively unexplored even in static settings; see Antràs et al., 2022 and Castro-Vincenzi et al., 2022.

firms in our sample to include only manufacturing firms (firms whose main activity is in sectors 10 to 33 according to ISIC Rev. 4) that are not part of a multinational corporation.

The resulting dataset includes 7,203 firms. Approximately 8% of these firms export in a typical year. While exporting firms often export to a single destination (this being the case for approximately 40% of exporters), approximately 25% of them export to at least four destinations, 10% of them export to at least seven, and 5% of them export to at least ten. By sector, most export participation events are concentrated in the manufacturing of other food products (sector 1079 in the ISIC Rev. 4 classification) and of plastic products (sector 2220). The most popular destinations are either countries that are geographically close to Costa Rica (e.g., Nicaragua) or relatively large (e.g., the United States). We provide additional descriptive statistics in Appendix B.1.

We complement our firm-level data with data on country characteristics. We obtain information on the languages spoken in each country from Ethnologue (Eberhard et al., 2021), on the geographical distance between countries from CEPII’s GeoDist (Mayer and Zignago, 2011), on the tariffs applied to exports from Costa Rica from Barari and Kim (2021), on the content of PTAs from Hofmann et al. (2019), and on countries’ GDP from the World Bank. Among other purposes, we use these data to build geographical, linguistic, and regulatory distances between countries.

We denote the geographical distance between two countries j and j' as $n_{jj'}^g$. As in Head and Mayer (2002), we construct $n_{jj'}^g$ as a weighted average of the distances between largely populated cities located in countries j and j' ; i.e.,

$$n_{jj'}^g \equiv \sum_{k \in j} \sum_{k' \in j'} \frac{pop_k pop_{k'}}{pop_j pop_{j'}} dist_{kk'}, \quad (1)$$

where k and k' respectively index cities in countries j and j' , pop_k and $pop_{k'}$ denote the population of cities k and k' , pop_j and $pop_{j'}$ denote the total population of the cities in countries j and j' used to calculate $n_{jj'}^g$, and $dist_{kk'}$ is the distance between k and k' in thousands of kilometers. Two features of the measure $n_{jj'}^g$ are worth noting. First, it accounts for the distribution of population within a country; e.g., according to this measure, Russia is closer to Germany (2,290 km) than to China (4,984 km). Second, large countries tend to appear isolated; e.g., while the distance between Switzerland and the UK is 872 km, that between the US and Canada is 1,154 km.

We denote the linguistic distance between countries j and j' as $n_{jj'}^l$, and measure it as the probability two randomly selected individuals from j and j' do not speak a common language; i.e.,

$$n_{jj'}^l \equiv \max \left\{ 0, 1 - \sum_{k=1}^K s_{jk} s_{j'k} \right\}, \quad (2)$$

where s_{jk} is the share of country j ’s population that speak language k as either their first or second language, and K is the total number of languages considered in Ethnologue.⁹ Relative to

⁹Ethnologue provides information by country on the population shares speaking a language as their first or second language, but it does not provide information on the distribution of second language speakers conditional on their first language. The measure $n_{jj'}^l$ assumes a joint distribution of first and second languages in each country such that the distance between countries is minimized. See Desmet et al. (2012) for another application of Ethnologue data.

distance measures relying only on the commonality of official languages between countries, $n_{jj'}^l$ reflects the actual prevalence of each language in each country, and thus accounts for the fact that certain languages are popular in countries in which they are not official; e.g., although the UK and Denmark share no official language, they are linguistically close according to our measure, as a large share of the Danish population reports speaking English as their second language.¹⁰

Our third distance measure between countries j and j' in a year t is an inverse measure of the breadth of the regulatory harmonization imposed by the PTAs of which j and j' are members in t , if any. We denote this measure as $n_{jj't}^a$, refer to it as the regulatory distance between j and j' in t , and build it using the data in Hofmann et al. (2019), which indicates whether a PTA contains provisions in each of 52 policy areas. We focus on the seven (out of the 52) areas that concern regulatory harmonization, and count in how many of them a PTA includes some provision.¹¹ When two countries are cosignatories of more than one PTA in a year t , we consider only the agreement containing provisions in the largest number of harmonization-focused policy areas, and compute:

$$n_{jj't}^a = 1 - \frac{1}{7} \left\{ \begin{array}{l} \text{number harmonization-focused policy areas in which} \\ \text{the PTA between } j \text{ and } j' \text{ in } t \text{ includes some provision} \end{array} \right\}. \quad (3)$$

This measure is between zero and one. For e.g., the European Common Market contains provisions in all seven harmonization areas of interest and, thus, $n_{jj't}^a = 0$ between their members. Conversely, NAFTA contains provisions in five of the seven areas and, thus, $n_{jj't}^a = 0.29$ between their members.

In Appendix sections B.2 to B.4, we provide additional information on the distance measures introduced in equations (1) to (3).

3 Cross-country Correlation in Export Participation Decisions

If geographical, linguistic, or regulatory proximity are sources of cross-country complementarities in firm exports, a firm's export probability in a country j and year t will, all else equal, be larger if it concurrently exports to countries close to j according to any of these three distance measures. To explore whether firm exports in our sample exhibit these correlation patterns, for each firm i , country j , and year t , and for each of the three distance measures above, we compute a dummy variable that equals one if firm i exports in year t to at least one country close to j . For e.g., for the case of geographical distance, we compute

$$Y_{ijt}^g = \mathbb{1} \left\{ \sum_{j' \neq j} \mathbb{1} \{ n_{jj'}^g \leq \bar{n}_g \} y_{ij't} > 0 \right\}, \quad (4)$$

where $\mathbb{1}\{\cdot\}$ is an indicator function, $n_{jj'}^g$ is defined in equation (1), \bar{n}_g is a threshold determining whether we classify two countries as geographically close to each other, and $y_{ij't}$ is a dummy variable

¹⁰The linguistic distance between the UK and Denmark is 0.11; i.e., we measure the probability a randomly selected individual from Denmark does not understand a randomly selected individual from the UK to be 11%.

¹¹These areas cover the harmonization of: sanitary or phytosanitary measures; technical barriers to trade; intellectual property rights; environmental standards; consumer protection laws; statistical methods; competition laws.

Table 1: Conditional Export Probabilities

	<i>Panel A:</i> <i>No Controls</i>				<i>Panel B:</i> <i>Controlling for Firm-Year Fixed Effects</i>			
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
Y_{ijt}^g	0.2622 ^a (0.0092)			0.2082 ^a (0.0079)	0.2226 ^a (0.0089)			0.1957 ^a (0.0081)
Y_{ijt}^l		0.1617 ^a (0.0076)		0.0752 ^a (0.0054)		0.1220 ^a (0.0067)		0.0718 ^a (0.0055)
Y_{ijt}^a			0.0857 ^a (0.0037)	0.0386 ^a (0.0021)			0.0517 ^a (0.0026)	0.0259 ^a (0.0018)
Obs.			3,859,618				3,859,618	
	<i>Panel C:</i> <i>Controlling for Sector-Country-Year Fixed Effects</i>				<i>Panel D:</i> <i>Controlling for Firm-Year & Sector-Country-Year Fixed Effects</i>			
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
Y_{ijt}^g	0.2462 ^a (0.0089)			0.1955 ^a (0.0076)	0.2043 ^a (0.0086)			0.1809 ^a (0.0078)
Y_{ijt}^l		0.1572 ^a (0.0074)		0.0764 ^a (0.0052)		0.1160 ^a (0.0066)		0.0720 ^a (0.0054)
Y_{ijt}^a			0.0809 ^a (0.0035)	0.0363 ^a (0.0019)			0.0473 ^a (0.0026)	0.0207 ^a (0.0018)
Obs.			3,859,618				3,859,618	

Note: ^a denotes 1% significance. Standard errors are clustered by firm. The dependent variable is a dummy that equals 1 if firm i exports to country j in year t . The covariates of interest are $Y_{ijt}^x = \mathbb{1}\{\sum_{j' \neq j} \mathbb{1}\{n_{jj'}^x \leq \bar{n}_x\} y_{ij't} > 0\}$ for $x \in \{g, l\}$, and $Y_{ijt}^a = \mathbb{1}\{\sum_{j' \neq j} \mathbb{1}\{n_{jj't}^a \leq \bar{n}_a\} y_{ij't} > 0\}$, with $\bar{n}_g = 790$ km, $\bar{n}_l = 0.11$ and $\bar{n}_a = 0.43$.

that equals one if firm i exports to country j' in year t . Thus, Y_{ijt}^g is a dummy that equals one if i exports in t to at least one country whose geographical distance to j is smaller than \bar{n}_g . In our baseline analysis, we set \bar{n}_g such that we classify two countries as close if their distance is less than 790 km, which is the 2.5 percentile of the distribution of distances across all country pairs.

We use expressions analogous to that in equation (4) to define two dummy variables, Y_{ijt}^l and Y_{ijt}^a , that equal one if firm i exports in year t to at least one country sufficiently close to j according to the distance measures $n_{jj'}^l$ or $n_{jj't}^a$, respectively. In our baseline analysis, we classify two countries as linguistically close if the probability two randomly selected individuals from both countries speak a common language is at least 0.89 (i.e., if $n_{jj'}^l < 0.11$, where 0.11 is the 2.5 percentile of the distribution of linguistic distances across all country pairs), and we classify two countries as regulatory close if they are cosignatories of a PTA including provisions in at least four of the seven areas listed in footnote 11 (i.e., if $n_{jj't}^a < 0.43$).¹² In Appendix B.5, we present results analogous to those in this section, but that rely on looser thresholds for classifying two countries as close to each other on the basis of their geographical, linguistic, or regulatory distance.

Table 1 presents OLS estimates of regressions of a dummy variable that equals one if firm i exports to a country j in a year t on the covariates Y_{ijt}^g , Y_{ijt}^l , and Y_{ijt}^a , controlling for different sets of

¹²According to these thresholds, e.g., Argentina and Spain (but not France and Switzerland) are linguistically close; and all members of the EU, NAFTA, CAFTA, or Mercosur are close in their regulations.

fixed effects. In Panel A, we include estimates of regression specifications that do not include fixed effects. The results in column (1) indicate exporting in year t to a destination geographically close to a country j increases in 26.2% the probability the firm exports to j in t . The results in columns (2) and (3) indicate this probability increase is 16.2% when the destination is linguistically close to j , and 8.6% when it shares a sufficiently deep PTA with j , respectively. As the results in column (4) show, these point estimates decrease only slightly when including all three covariates of interest in a regression, and they all remain significant at the 1% significance level. Quantitatively, these estimates reflect a very strong correlation in firm export participation decisions across countries close to each other, as the average probability a firm exports to a country in a year is below 1%.

In panels B, C, and D in Table 1, we present estimates analogous to those in Panel A but for regression specifications that control for firm-year fixed effects, sector-country-year fixed effects, or both, respectively. The estimates in these panels are only moderately smaller than those in Panel A. The results in Table 1 thus show that firms' export participation decisions in countries geographically or linguistically close to each other, or cosignatories of a deep PTA, are positively correlated, and that factors varying at the firm-year level (e.g., firm productivity) or at the sector-destination-year level (e.g., market size, or total number of exporters, potentially by origin, in a destination) are not the main drivers of this correlation.

Although consistent with them, the correlation patterns described in Table 1 are not evidence of the presence of cross-country complementarities in firm exports, as these patterns may be due instead to firm-country specific export profit shifters (e.g., demand shifters) being positively correlated across countries geographically or linguistically close to each other, or that are cosignatories of a deep PTA. To guide the identification of cross-country complementarities, and to quantify the role these play in determining firms' export choices, we present below a model of firm exports that accounts for potential cross-country export complementarities as well as for cross-country correlation in unobserved export determinants.

4 Dynamic Export Model With Complementarities

We present here a partial-equilibrium model in which firms choose every period the bundle of countries they export to among a large set of potential destinations. When exporting to a country, firms face variable, fixed, and sunk costs. We allow the fixed costs a firm faces in a destination and period to be smaller if the firm also exports to other countries in the same period. The model thus allows for *static cross-country complementarities* in firm exports: a firm's profits when exporting to multiple countries in a period may be higher than the sum of the profits of exporting to each of them individually. Consistently with the correlation patterns in Section 3, we allow the strength of the complementarities between any two countries to depend on the geographical and linguistic proximity between them, as well as on the deepness of any PTA of which they are both members. Sunk costs make a firm's export choice in a country and period impact export profits in that country in the subsequent period. This creates *dynamic within-country complementarities* in firm

exports: a firm’s profits when exporting two consecutive periods to a destination are higher than the sum of the profits of exporting in each of the two periods individually.

In the presence of static and dynamic complementarities, a firm’s export choice in a country and period impacts its export profits in other countries and periods. When choosing its export bundle in a period, we assume the firm takes into account how its choice impacts export profits across countries and periods. Specifically, firms determine their optimal export bundle in any given period after solving an infinite-horizon dynamic combinatorial discrete-choice problem.

We incorporate into our model several shocks that allow export profits to vary flexibly across firms, countries and periods. To make the optimization problem of potential exporters computationally tractable, we assume firms have perfect foresight on most (but not all) of these shocks, and follow Eaton et al. (2016) and Caliendo et al. (2019) in assuming all payoff-relevant variables on which firms have perfect foresight stay constant after a terminal period T .¹³

4.1 Setup

Firms produce in a country h . Time and locations are discrete. We index periods by $t \geq 0$, firms by i , and foreign countries by j . Firm i is born exogenously at period \underline{t}_i and, once born, is active forever. We denote the first and last sample periods as \underline{t} and \bar{t} , respectively, and assume $T > \bar{t}$.

4.2 Marginal Costs, Demand Function, and Market Structure

Firm i has at period t constant marginal production costs w_{it} . Selling in a market requires incurring in extra variable “iceberg” costs. Specifically, firm i must ship τ_{ijt} units of output for a unit to reach country j at period t , and its marginal export cost in j at t is thus $\tau_{ijt}w_{it}$. The marginal cost of selling in the home market is $\tau_{ht}w_{it}$.

Firms face an isoelastic demand in every country. Conditional on exporting to country j at period t , the quantity sold by firm i , q_{ijt} , depends on the price p_{ijt} it sets, the price index, P_{jt} , and the total market expenditure, Y_{jt} , according to the function $q_{ijt} = p_{ijt}^{-\eta} P_{jt}^{\eta-1} Y_{jt}$. Firms face an analogous demand at home. Firms set optimal prices in every market taking as given the market’s total expenditure and price index and, thus, prices equal a constant markup over the marginal cost of selling in a market; e.g., firm i ’s price in country j at period t is $p_{ijt} = (\eta/(\eta - 1))\tau_{ijt}w_{it}$.

4.3 Potential Export Revenues

The assumptions in Section 4.2 imply that the potential export revenue of firm i in country j at period t is

$$r_{ijt} = \left[\frac{\eta}{\eta - 1} \frac{\tau_{ijt}w_{it}}{P_{jt}} \right]^{1-\eta} Y_{jt}. \quad (5)$$

¹³Our approach is similar to that in Kehoe et al. (2018), who assume that, after a terminal period, all variables on which agents have perfect foresight converge deterministically to a balance growth path. Our approach is also similar to that in Igami (2017, 2018) and Igami and Uetake (2019), who solve a finite-horizon model with a terminal value analogous to the continuation value firms in our model have in terminal period T .

We model the impact of variable trade costs on potential export revenues as

$$(\tau_{ijt})^{1-\eta} = \exp(\xi_y y_{ijt-1} + \xi_s + \xi_{jt} + \xi_a \ln(a_{sjt}) + \xi_w \ln(w_{it})), \quad \text{with} \quad \xi_y \geq 0, \quad (6)$$

where y_{ijt-1} is a dummy variable that equals one if firm i exports to country j at period $t-1$, ξ_s is a term specific to the sector s to which firm i belongs, ξ_{jt} is a country-period term that accounts for trade barriers common to all firms located in country h , a_{sjt} equals one plus the average tariffs country j imposes at period t on exports from h in sector s , and, as indicated above, w_{it} denotes marginal production costs. By allowing $(\tau_{ijt})^{1-\eta}$ to depend on w_{it} , we account for determinants of variable trade costs that may vary across firms of different productivities in a systematic way.¹⁴

Equations (5) and (6) imply we can write potential export revenues as

$$r_{ijt} = \exp(\alpha_y y_{ijt-1} + \alpha_s + \alpha_{jt} + \alpha_a \ln(a_{sjt}) + \alpha_r \ln(r_{iht})), \quad (7)$$

where α_s and α_{jt} are sector and country-period specific terms, respectively, and r_{iht} is firm i 's domestic sales at t . The dependency of r_{ijt} on the export participation dummy y_{ijt-1} accounts for the limited sales firms often obtain upon entering a new market.¹⁵ The term α_s accounts for the impact of the sector-specific trade cost component ξ_s , and the country-period term α_{jt} accounts for the impact of the foreign price index P_{jt} , market size Y_{jt} , and variable trade cost component ξ_{jt} . The term $\alpha_a \ln(a_{sjt})$ accounts for the impact of tariff barriers, and domestic sales r_{iht} proxy for the impact of the firm's marginal production cost, w_{it} . See Appendix C for details.

According to equation (7), potential export revenues in a country and period depend on the lagged export participation dummy y_{ijt-1} and four exogenous variables: the time-invariant term α_s and three time-varying terms comprising of the country-period component α_{jt} , log domestic sales $\ln(r_{iht})$, and tariff barriers a_{sjt} . The time-invariant term and the in-sample realized values of the three time-varying terms are either observed or consistently estimated (see sections 2 and 6.1). Out-of-sample, we impose the following restrictions on the distribution of the three time-varying exogenous determinants of export revenues.¹⁶

Concerning α_{jt} and $\ln(r_{iht})$, we assume they are constant after terminal period T and, for all $t \leq T$, follow stationary AR(1) processes with *iid* normal shocks and intercepts that may vary by country and firm, respectively. Formally, for any country j and period $t \leq T$, we assume $\alpha_{jt} = (X_{jt}^\alpha)' \beta_\alpha + \rho_\alpha \alpha_{jt-1} + e_{jt}^\alpha$, where X_{jt}^α is a vector including a constant, market j 's log GDP at t , and the geographic, linguistic, and regulatory distances between h and j ; β_α and ρ_α are unknown parameters with $|\rho_\alpha| < 1$; and, e_{jt}^α is *iid* normally distributed with mean zero and variance σ_α^2 . Similarly, for any firm i and period $t \leq T$, $\ln(r_{iht}) = (X_i^r)' \beta_r + \rho_r \ln(r_{iht-1}) + e_{iht}^r$, where X_i^r is

¹⁴This could be due to more productive firms having larger buyer networks in a destination (Bernard et al., 2018).

¹⁵These limited sales may be due to a lack of information or limited customer capital in the destination (see, e.g., Eaton et al., 2008; Alborno et al., 2012; Ruhl and Willis, 2017; Berman et al., 2019; Piveteau, 2021; Fitzgerald et al., 2022) or to partial-year effects (see, e.g., Bernard et al., 2017; Gumpert et al., 2020).

¹⁶The need to restrict the out-of-sample distribution of the exogenous determinants of export revenues is due to our model featuring sunk export costs and forward-looking firms with rational expectations, which implies firms' optimal export choices in-sample depend on their expected potential export revenues out-of-sample (see Section 4.6).

a vector including dummies for firm i 's sector and region of location in country h ; β_r and ρ_r are unknown parameters with $|\rho_r| < 1$; and e_{it}^r is *iid* normally distributed with mean zero and variance σ_r^2 . Concerning a_{sjt} , we assume it is constant out-of-sample; i.e., for all j and s , $a_{sjt} = a_{sj\bar{t}}$ if $t \leq \bar{t}$, and $a_{sjt} = a_{sj\bar{t}}$ if $t \geq \bar{t}$. Finally, we assume the time series of these three time-varying determinants of revenues are independent of each other and of any other determinant of firm export profits.

4.4 Fixed and Sunk Export Costs

Firms may face fixed and sunk export costs, which differ from variable costs in that, conditional on selling in a market, are independent of the quantity sold. Fixed and sunk costs differ in that the former are paid every period a firm exports to a country, and the latter are only paid if the firm did not export to it in the previous period. We model fixed costs as the sum of four terms:

$$f_{ijt} = g_{jt} - eg_{ijt} + \nu_{ijt} + \omega_{ijt}. \quad (8)$$

The first term captures the impact of all distance measures between countries h and j ,

$$g_{jt} = \gamma_0^F + \sum_{x=\{g,l\}} \gamma_x^F n_{hj}^x + \gamma_a^F n_{hjt}^a. \quad (9)$$

The second term captures static complementarities in export destinations:

$$eg_{ijt} = \sum_{j' \neq j} y_{ij't} c_{jj't}, \quad (10)$$

where the complementarities between countries j and j' are modeled as

$$c_{jj't} = \sum_{x=\{g,l\}} \gamma_x^E (1 + \varphi_x^E n_{hj}^x) \exp(-\kappa_x^E n_{jj'}^x) + \gamma_a^E (1 + \varphi_a^E n_{hjt}^a) \exp(-\kappa_a^E n_{jj't}^a), \quad (11)$$

with $\gamma_x^E (1 + \varphi_x^E n_{hj}^x) \geq 0$ for $x = \{g, l, a\}$ and every foreign country j . For all three distance measures we consider, equation (11) allows the static complementarities a firm enjoys in a market j if it also exports to a market j' to depend on the distance between j and j' and between j and the firm's home market h . For e.g., for $x = g$, a firm exporting to country j' experiences a reduction in fixed costs in country j equal to the product of a constant γ_g^E , a function $1 + \varphi_g^E n_{hj}^g$ of the distance between countries h and j , and a function $\exp(-\kappa_g^E n_{jj'}^g)$ of the distance between j and j' .

Imposing $\gamma_x^E (1 + \varphi_x^E n_{hj}^x) \geq 0$ for $x = \{g, l, a\}$ implies $c_{jj't} \geq 0$ for any markets j and j' and period t , ruling out the possibility that adding an export destination may increase fixed export costs in other countries. As discussed in Section 5.1, in conjunction with the rest of the model, this sign restriction on $c_{jj't}$ for all (j, j', t) implies the firm's country-specific export participation decisions are not substitutable, and is a necessary assumption for our algorithm to correctly solve the optimization problem determining firms' export bundles.¹⁷

¹⁷Blum et al. (2013) and Almunia et al. (2021) show evidence of within-firm substitutabilities between the domestic

The third determinant of fixed export costs, ν_{ijt} , is an unobserved firm-country-period variable whose distribution in all periods prior to terminal period T is independent of all other determinants of firms' export profits and satisfies the following restrictions:

$$\nu_{ijt} \sim \mathbb{N}(0, \sigma_\nu^2), \quad \text{for all } i, j, \text{ and } t, \quad (12a)$$

$$\nu_{ijt} \perp \nu_{i'j't'}, \quad \text{if } i \neq i' \text{ or } t \neq t', \quad (12b)$$

$$\rho_{jj't} = \sum_{x=\{g,l\}} \gamma_x^N \exp(\kappa_x^N n_{jj'}^x) + \gamma_a^N \exp(\kappa_a^N n_{jj't}^a), \quad \text{if } j \neq j', \quad (12c)$$

where $\rho_{jj't}$ is the correlation coefficient between ν_{ijt} and $\nu_{i'j't}$ for period t and countries j and j' . From T onwards, ν_{ijt} is constant; i.e., $\nu_{ijt} = \nu_{ijT}$ for $t \geq T$. By allowing for a firm-country specific unobserved fixed cost term potentially correlated across countries, we allow for the correlation patterns in firm exports discussed in Section 3 to be due not to cross-country export complementarities but to correlated unobserved determinants of export profits.

The fourth term in equation (8), ω_{ijt} , is an *iid* unobserved variable whose distribution is independent of all other determinants of profits and has two points of support, $\underline{\omega}$ and $\bar{\omega}$. Formally,

$$\omega_{ijt} \perp \omega_{i'j't'} \quad \text{if } i \neq i', j \neq j' \text{ or } t \neq t', \quad (13a)$$

$$P(\omega_{ijt} = \omega) = \begin{cases} p & \text{if } \omega = \underline{\omega}, \\ 1 - p & \text{if } \omega = \bar{\omega}. \end{cases} \quad (13b)$$

To simplify the model estimation, we set $(\underline{\omega}, \bar{\omega}) = (0, \infty)$ and, thus, we model ω_{ijt} as a ‘‘blocking’’ shock preventing firm i from exporting to country j in period t . Equation (13) characterizes the distribution of $\omega_{it} \equiv (\omega_{i1t}, \dots, \omega_{iJt})$ in all periods; thus, ω_{it} may vary over time even after T .

We model sunk export costs in a more parsimonious way than fixed costs. Specifically, we assume sunk costs in a market j and period t only depend on distance between countries h and j :

$$s_{jt} = \gamma_0^S + \sum_{x=\{g,l\}} \gamma_x^S n_{hj}^x + \gamma_a^S n_{hjt}^a. \quad (14)$$

Sunk costs allow for possible dynamic complementarities in firms' export decisions within a country.

4.5 Static Export Profits

The assumptions in Section 4.2 imply potential export revenues net of variable trade costs equal $\eta^{-1}r_{ijt}$. Netting out also fixed and sunk export costs, and using the expressions in equations (7), (8) and (10), potential export profits of firm i in country j at period t may be written as

$$\pi_{ijt}(y_{it}, y_{ijt-1}, \omega_{ijt}) = u_{ijt}(y_{ijt-1}, \omega_{ijt}) + \sum_{j' \neq j} y_{ij't} c_{jj't}, \quad (15)$$

and aggregate export markets. We know of no work showing evidence of within-firm export substitutabilities across foreign countries. Conversely, Eaton et al. (2008), Alborno et al. (2012, 2021a), Chaney (2014), Morales et al. (2019) and, specially, Alborno et al. (2021b) provide evidence of complementarities in these decisions.

with

$$u_{ijt}(y_{ijt-1}, \omega_{ijt}) = \eta^{-1} r_{ijt} - (g_{jt} + \nu_{ijt} + \omega_{ijt}) - (1 - y_{ijt-1}) s_{jt}, \quad (16)$$

where $y_{it} \equiv (y_{i1t}, \dots, y_{iJt})$ identifies the bundle of export destinations of firm i at period t . Denoting by J the number of feasible export destinations, total export profits of firm i at period t are

$$\pi_{it}(y_{it}, y_{it-1}, \omega_{it}) = \sum_{j=1}^J y_{ijt} \pi_{ijt}(y_{it}, y_{ijt-1}, \omega_{ijt}). \quad (17)$$

Equations (15) and (17) highlight two model properties. First, for any y_{it} , total profits depend on all export destinations at $t-1$, y_{it-1} , and all blocking shocks at t , ω_{it} , but profits in a country j only depend on the lagged export status, y_{ijt-1} , and blocking shock, ω_{ijt} , in that country j . Second, as $c_{jj't} \geq 0$ for any j, j' and t , potential export profits in a country j at t are weakly increasing in y_{it} .

4.6 Optimal Export Choice

Firms choose every period the bundle of export destinations maximizing their expected discounted sum of current and future profits. The information set of firm i at period t is

$$\mathcal{J}_{it} = (\{x_{it'}\}_{t' \geq t}, y_{it-1}, \omega_{it}) \quad \text{with} \quad x_{it'} = (\nu_{it'}, \alpha_{t'}, a_{st'}, r_{iht'}) \text{ for all } t'. \quad (18)$$

The vector x_{it} includes all period- t export profit shocks whose realizations are known to firm i at any period prior to t . Every firm i thus knows at any t the value of all exogenous determinants of current and future potential export profits except for the future fixed costs shocks $\{\omega_{it'}\}_{t' > t}$.¹⁸ The problem firm i solves when choosing its period t export bundle may thus be written as

$$V_{it}(y_{it-1}, \omega_{it}) = \max_{y_{it} \in \{0,1\}^J} \mathbb{E}_{it} \left[\pi_{it}(y_{it}, y_{it-1}, \omega_{it}) + \delta V_{it+1}(y_{it}, \omega_{it+1}) \right], \quad (19)$$

where $\delta < 1$ is the discount factor, $\mathbb{E}_{it}[\cdot]$ is the expectation operator with respect to the data generating process conditional on \mathcal{J}_{it} (i.e., firms' expectations are rational), and $V_{it}(y_{it-1}, \omega_{it}) = V(\{x_{it'}\}_{t' \geq t}, y_{it-1}, \omega_{it})$ is firm i 's value function at t . Given the profit function in equation (15) and the information set \mathcal{J}_{it} in equation (18), we rewrite the firm's problem as

$$V_{it}(y_{it-1}, \omega_{it}) = \max_{y_{it} \in \{0,1\}^J} \left\{ \sum_{j=1}^J y_{ijt} (u_{ijt}(y_{ijt-1}, \omega_{ijt}) + \sum_{j' \neq j} y_{ij't} c_{jj't}) + \delta \mathbb{E}_{it} V_{it+1}(y_{it}, \omega_{it+1}) \right\}. \quad (20)$$

For all possible values of $\{x_{it'}\}_{t' \geq t}$, $V_{it}(\cdot)$ is bounded and, thus, a solution to the problem in equation (20) exists (see Appendix E.2.2). We denote firm i 's optimal policy function at t as

$$o_{it}(y_{it-1}, \omega_{it}) = (o_{i1t}(y_{it-1}, \omega_{it}), \dots, o_{iJt}(y_{it-1}, \omega_{it})) \quad (21)$$

¹⁸Additionally, the firm knows the relevant distance measures between all countries and the value of all parameters.

where $o_{ijt}(\cdot)$ is a function that equals one if firm i exports to country j at t (and zero otherwise) depending on the vectors y_{it-1} and ω_{it} . The subindex it reflects the dependency of the optimal export bundle on $\{x_{it'}\}_{t' \geq t}$. As x_{it} is constant from period T onwards (see sections 4.3 and 4.4), it holds that $o_{it}(\cdot) = o_{iT}(\cdot)$ and $V_{it}(\cdot) = V_{iT}(\cdot)$ for all $t \geq T$. The problem is thus non-stationary until terminal period T , and stationary henceforth.

5 Solution Algorithm

We describe here the computational challenges entailed in solving the problem in equation (20), and present a solution algorithm that overcomes them. We discuss formally the algorithm's properties in Appendix A. In Appendix D.2, we illustrate in a simple setting how the algorithm works.

For any arbitrary sequence of export profit shocks $\{x_{it'}\}_{t' \geq t}$, the firm's optimization problem in equation (20) has three properties that make solving for the value of the policy function $o_{it}(y_{it-1}, \omega_{it})$ in equation (21) at every possible state vector (y_{it-1}, ω_{it}) computationally challenging:

- P.1** *Large discrete choice set.* The choice set $\{0, 1\}^J$ is discrete and has cardinality 2^J .
- P.2** *Integration over a discrete random variable with many points of support.* For any choice y_{it} , evaluating the firm's objective function requires integrating numerically next period's value function, $V_{it+1}(y_{it}, \omega_{it+1})$, over ω_{it+1} , whose support includes 2^J points.
- P.3** *Large state space.* As y_{it-1} and ω_{it} may take 2^J values, the state space includes 2^{2J} points.

These properties imply the choice set, the support of the random variable one must integrate over, and the state space grow exponentially in J . Incorporating into the analysis a reasonable set of countries thus makes solving the firm's optimization problem computationally challenging. Specifically, given the non-stationarity of the firm's problem prior to T , property P.3 implies one must solve $2^{2J} \sum_{i=1}^M (T - \underline{t}_i + 1)$ distinct problems to obtain the optimal export bundles of a set of firms $i = 1, \dots, M$ in all periods in which they are active and in all points in the state space. Properties P.1 and P.2 make finding the solution to each of these problems computationally challenging.¹⁹

Given the large dimensionality of the state space, for each firm i and period t , we compute the value of the policy function $o_{it}(y_{it-1}, \omega_{it})$ only at a particular state $(\check{y}_{it-1}, \check{\omega}_{it})$. Specifically, we consider each firm i in a set (e.g., those in our estimation sample) independently and, for any two periods t_I and t_F (e.g., the first and last sample periods), determine the value of the functions $\{o_{it}(\cdot)\}_{t=t_I}^{t_F}$ only at the states $\{(\check{y}_{it-1}, \check{\omega}_{it})\}_{t=t_I}^{t_F}$ the firm reaches if it chooses the optimal export bundle at every period and all exogenous determinants of export profits follow specific paths of interest $\{\check{x}_{it'}\}_{t' \geq \underline{t}_i}$ and $\{\check{\omega}_{it'}\}_{t'=\underline{t}_i}^{t_F}$ (e.g., the observed or simulated paths). Formally, given $\{\check{x}_{it'}\}_{t' \geq \underline{t}_i}$ and $\{\check{\omega}_{it'}\}_{t'=\underline{t}_i}^{t_F}$, our algorithm yields for every $t \in [t_I, t_F]$ the value of $o_{it}(\check{y}_{it-1}, \check{\omega}_{it})$, where \check{y}_{it-1} is

$$\check{y}_{it'} = o_{it'}(\check{y}_{it'-1}, \check{\omega}_{it'}), \quad \text{for } t' = \underline{t}_i, \dots, t-1, \text{ with initial condition equal to } 0_J. \quad (22)$$

¹⁹If firms' discount factor was zero, one could compute firms' optimal export bundles using the tools in Jia (2008) and Arkolakis et al. (2021). These tools have been applied to static discrete choice problems with large choice sets featuring complementarities (Jia, 2008; Arkolakis et al., 2021) and substitutabilities (Arkolakis et al., 2021).

Thus, given paths of exogenous determinants of a firm's potential export profits, our algorithm evaluates the firm's policy function only at the states reached along the firm's optimal export path. As discussed in sections 6 and 8, we use this algorithm in the estimation of the model parameters and to compute model predicted export choices in counterfactual scenarios.

As our model features dynamic complementarities and forward-looking firms uncertain about future values of the blocking shocks ω_{it} , solving the optimization problem of firm i at period t at a state $(\check{y}_{it-1}, \check{\omega}_{it})$ requires *some knowledge* of how the firm will subsequently behave at any state reached with positive probability from $(\check{y}_{it-1}, \check{\omega}_{it})$. However, solving the optimization problem of firm i at t at $(\check{y}_{it-1}, \check{\omega}_{it})$ may not require knowing *exactly* the firm's optimal export bundle in all states that may be subsequently reached; e.g., if a firm's potential export profits in a market j and period t are sufficiently high, its optimal decision may be to export to j at t regardless of its optimal export bundle at any state that may be reached in the future. Our algorithm uses this idea and yields the optimal choice of a firm i , period t , and state $(\check{y}_{it}, \check{\omega}_{it})$ using information on *bounds* on the firm's optimal choice at every future state that is reached with positive probability.

Our algorithm consists of several steps. In each step, we compute upper and lower bounds on the solution to the firm's optimization problem at the periods and states of interest. If both bounds coincide, they must coincide with the solution as well. If they do not, we move on to the next step. As we advance through steps, our bounds become tighter but harder to compute. We describe here the first two steps, and the remaining ones in Appendix D.1.

Step 1. Consider a hypothetical scenario in which we knew firm i 's optimal policy function $o_{ij't}(\cdot)$ for every country $j' \neq j$ and period $t \geq \underline{t}_i$. Absent computational constraints, we could then compute country j 's optimal policy $o_{ijt}(\cdot)$ for every $t \geq \underline{t}_i$ by solving the following country j -specific problem

$$V_{ijt}(y_{it-1}, \omega_{it}) = \max_{y_{ijt} \in \{0,1\}} \left\{ y_{ijt}(u_{ijt}(y_{ijt-1}, \omega_{ijt}) + \sum_{j' \neq j} o_{ij't}(y_{it-1}, \omega_{it})(c_{jj't} + c_{j'jt})) \right. \\ \left. + \delta \mathbb{E}_{it} V_{ijt+1}((o_{i1t}(y_{it-1}, \omega_{it}), \dots, y_{ijt}, \dots, o_{iJt}(y_{it-1}, \omega_{it})), \omega_{it+1}) \right\}. \quad (23)$$

In every period and state, the value of y_{ijt} solving this problem coincides with that implied by the solution to the general optimization problem in equation (20).²⁰ However, we cannot solve the problem in equation (23) for two reasons. First, we do not know the firm's optimal policy function in any country or period. Second, even if we knew, solving the problem in equation (23) requires overcoming some of the computational challenges affecting the original problem in equation (20): it requires integrating over ω_{it+1} and dealing with a state space of dimension 2^{2J} .

Assume now we know for every country $j' \neq j$ and period $t \geq \underline{t}_i$ a constant upper bound $\bar{b}_{ij't}$ such that $\bar{b}_{ij't} \geq o_{ij't}(y_{it-1}, \omega_{it})$ for all (y_{it-1}, ω_{it}) . We may then solve the following problem

$$\bar{V}_{ijt}(y_{ijt-1}, \omega_{ijt}) = \max_{y_{ijt} \in \{0,1\}} \left\{ y_{ijt}(u_{ijt}(y_{ijt-1}, \omega_{ijt}) + \sum_{j' \neq j} \bar{b}_{ij't}(c_{jj't} + c_{j'jt})) + \delta \mathbb{E}_{it} \bar{V}_{ijt+1}(y_{ijt}, \omega_{ijt+1}) \right\}. \quad (24)$$

²⁰In equation (23), we include in period- t 's expected static profits only the terms relevant to the choice of y_{ijt} .

The static and dynamic complementarities in our model imply the solution to this problem is, for all periods and states, an upper bound on the firm's optimal export choice in country j . Formally, at any t , the solution to the problem in equation (24) is a function $\bar{o}_{ijt}(\cdot)$ such that $\bar{o}_{ijt}(y_{ijt-1}, \omega_{ijt}) \geq o_{ijt}(y_{ijt-1}, \omega_{ijt})$ for all feasible $(y_{ijt-1}, \omega_{ijt})$. Importantly, the problem in equation (24) does not have any of the three properties that makes solving the original problem in equation (20) computationally challenging: the control variable y_{ijt} is binary, one only needs to integrate over the binary variable ω_{ijt+1} , and the vector $(y_{ijt-1}, \omega_{ijt})$ only takes four values.²¹ Given constant upper bounds $\bar{b}_{it} = (\bar{b}_{i1t}, \dots, \bar{b}_{iJt})$ for all $t \geq \underline{t}_i$, we may solve the problem in equation (24) for every country $j = 1, \dots, J$, and obtain in this way for each $\underline{t}_i \leq t \leq T$ an upper-bound policy function

$$\bar{o}_{it}(y_{it-1}, \omega_{it}) = (\bar{o}_{i1t}(y_{i1t-1}, \omega_{i1t}), \dots, \bar{o}_{iJt}(y_{iJt-1}, \omega_{iJt})). \quad (25)$$

The upper-bound policy function we obtain depends on the values of the constant upper bounds we use: the tighter these are, the tighter the resulting upper-bound policy function will be.

To initialize our algorithm, we use constant upper bounds for all countries and periods implying the firm always exports. We denote these bounds with a zero superscript (i.e., $\bar{b}_{it}^{[0]} = 1_J$ for all t) and use them to solve the problem in equation (24) for every country, obtaining in this way upper bound policies $\bar{o}_{it}^{[0]}(y_{it-1}, \omega_{it})$ for all $\underline{t}_i \leq t \leq T$ and all values of (y_{it-1}, ω_{it}) in the state space. Using these policies, we compute new constant upper bounds, which we use to solve again the problem in equation (24) and obtain new upper-bound policy functions. Generally, for all $n \geq 1$, we use the policies computed at iteration $n - 1$ to compute new constant upper bounds, and we use these bounds to solve the problem in equation (24) and obtain the iteration- n policies. Specifically, to compute the iteration- n constant upper bounds for a period t , we evaluate the iteration- $(n - 1)$ upper-bound policy at the feasible state at which the firm is most likely to export at t . This is the state reached if the blocking shocks equal the smallest point in their support (i.e., $\omega_{it'} = \omega_J$) for all $t' \leq t$ and the firm chooses the bundle prescribed by the policy $\bar{o}_{it'}^{[n-1]}(\cdot)$ in all $t' < t$. Formally,

$$\bar{b}_{it'}^{[n]} = \bar{o}_{it'}^{[n-1]}(\bar{b}_{it'-1}^{[n]}, \omega_J), \quad \text{for } t' = \underline{t}_i, \dots, t, \text{ with initial condition equal to } 0_J. \quad (26)$$

These bounds get tighter with every iteration and converge after a finite number of iterations; see Appendix A. We denote as $\bar{o}_{it}^*(\cdot)$ the converged upper-bound policies, and similarly obtain lower-bound policies $\underline{o}_{it}^*(\cdot)$. We use these policies to obtain upper and lower bounds on the optimal choices along a specific path $\{\tilde{\omega}_{it'}\}_{t'=\underline{t}_i}^{t_F}$. Formally, for any t , we compute bounds \check{y}_{it} and \check{y}_{it} as

$$\check{y}_{it'} = \bar{o}_{it'}^*(\check{y}_{it'-1}, \tilde{\omega}_{it'}), \quad \text{for } t' = \underline{t}_i, \dots, t, \text{ with initial condition equal to } 0_J, \quad (27a)$$

$$\check{y}_{it'} = \underline{o}_{it'}^*(\check{y}_{it'-1}, \tilde{\omega}_{it'}), \quad \text{for } t' = \underline{t}_i, \dots, t, \text{ with initial condition equal to } 0_J. \quad (27b)$$

If both bounds coincide for all $t \in [t_I, t_F]$, they identify the firm's optimal export path along the

²¹These are $(0, \underline{\omega})$, $(0, \bar{\omega})$, $(1, \underline{\omega})$, and $(1, \bar{\omega})$. We use value function iteration to solve for period T value and policy functions $V_{ijT}(\cdot)$ and $\bar{o}_{ijT}(\cdot)$, and backward induction to solve for $\{\bar{V}_{ijt}(\cdot)\}_{t=\underline{t}_i}^{T-1}$ and $\{\bar{o}_{ijt}(\cdot)\}_{t=\underline{t}_i}^{T-1}$. As $\bar{\omega} = \infty$ in our application, $o_{ijt}(0, \bar{\omega}) = o_{ijt}(1, \bar{\omega}) = 0$ for all i, j and t , and we only need to compute $o_{ijt}(0, \underline{\omega})$ and $o_{ijt}(1, \underline{\omega})$.

path of interest $\{\tilde{\omega}_{it}\}_{t=t_i}^{t_F}$. If they differ for at least one period, we proceed to step 2.

Step 2. Denote by τ the smallest t with $\check{y}_{it} > \underline{y}_{it}$. In step 2, we tighten our bounds at τ . Our procedure here differs from that in step 1 in that we now condition on the state the firm reaches at τ at the path of interest. Specifically, when solving the problem in equation (24) for every country, we now do so only for $t \geq \tau$ and condition on the state $(\check{y}_{i\tau-1}, \check{\omega}_{i\tau})$. This means that, for all $t \geq \tau$, the step 2 initial constant upper bounds, which we denote as $\bar{b}_{it|\tau}^{[0]}$, equal the firm's choice if: (a) its state at τ is $(\check{y}_{i\tau-1}, \check{\omega}_{i\tau})$; (b) for $t' \in [\tau + 1, t]$, the blocking shocks equal the smallest value in their support; and, (c) for $t' \in [\tau, t]$, the firm makes the choices prescribed by the step 1 upper-bound policy functions after convergence; i.e., $\bar{o}_{it'}^*(\cdot)$. Formally, $\bar{b}_{i\tau|\tau}^{[0]} = \bar{o}_{i\tau}^*(\check{y}_{i\tau-1}, \check{\omega}_{i\tau})$ and, for all $t > \tau$,

$$\bar{b}_{it'|\tau}^{[0]} = \bar{o}_{it'}^*(\bar{b}_{it'-1|\tau}^{[0]}, \underline{\omega}_J), \quad \text{for } t' = \tau + 1, \dots, t, \text{ with initial condition equal to } \bar{b}_{i\tau|\tau}^{[0]}. \quad (28)$$

Solving for all countries the problem in equation (24) with these constant upper bounds, we obtain new upper-bound policies $\bar{o}_{it|\tau}^{[0]}(\cdot)$ for all $t \geq \tau$. As in step 1, we use these policies to compute new constant upper bounds, which we use to solve again the problem in equation (24) and obtain in this way new upper-bound policies. Specifically, for all $n \geq 1$, the iteration- n constant upper bounds for a period $t \geq \tau$ equal the firm's choice when: (a) the state at τ is $(\check{y}_{i\tau-1}, \check{\omega}_{i\tau})$; (b) for $\tau < t' \leq t$, the blocking shocks equal the smallest value in their support; and, (c) for $\tau \leq t' \leq t$, the firm makes the choices prescribed by iteration- $(n-1)$ upper-bound policies. Formally, $\bar{b}_{it|\tau}^{[n]} = \bar{o}_{it}^{[n-1]}(\check{y}_{i\tau-1}, \check{\omega}_{i\tau})$ and, for all $t > \tau$,

$$\bar{b}_{it'|\tau}^{[n]} = \bar{o}_{it'}^{[n-1]}(\bar{b}_{it'-1|\tau}^{[n]}, \underline{\omega}_J), \quad \text{for } t' = \tau + 1, \dots, t, \text{ with initial condition equal to } \bar{b}_{i\tau|\tau}^{[n]}. \quad (29)$$

We implement this procedure until the guaranteed convergence (see Appendix A), denoting as $\bar{o}_{it|\tau}^*(\cdot)$ the resulting upper-bound policy for any $t \geq \tau$. We then use these upper-bound policies in combination with similarly computed lower-bound policies $\underline{o}_{it|\tau}^*(\cdot)$ to obtain bounds on the firm's optimal choice at period τ at the path of interest. Specifically, we compute

$$\check{y}_{i\tau|\tau} = \bar{o}_{i\tau}^*(\check{y}_{i\tau-1}, \check{\omega}_{i\tau}), \quad \text{and} \quad \underline{y}_{i\tau|\tau} = \underline{o}_{i\tau}^*(\check{y}_{i\tau-1}, \check{\omega}_{i\tau}). \quad (30)$$

If these bounds coincide, they also equal the optimal choice at τ at $(\check{y}_{i\tau-1}, \check{\omega}_{i\tau})$. If so, we proceed to the next period τ' at which the bounds in equation (27) differ, implementing again the step 2 procedure to tighten the bounds at τ' . If the bounds in equation (30) do not coincide, we implement additional steps that we describe in Appendix D.1.

5.1 Discussion

Two features of the model described in Section 4 are necessary for the algorithm introduced in Section 5 to provide valid and computationally feasible bounds on the firm's optimal choices at a path of interest.

First, the function the firm maximizes when making choices at any period and state is super-

modular; i.e., the objective function in the optimization problem in equation (20) is supermodular. Supermodularity of the objective function implies we can compute upper and lower bounds on the firm’s optimal policy function by iteratively solving for the firm’s optimal policy in a subset of countries while conditioning on upper and lower bounds, respectively, on the firm’s optimal choices in all other countries. In our model, the objective function is supermodular because of possible complementarities in export choices across countries within a period (due to fixed costs being weakly smaller when firms concurrently export to several destinations) and across periods within a country (due to weakly positive sunk costs). The specific source of complementarities is however irrelevant for the validity of the solution algorithm.

Second, given bounds on the firm’s optimal choices in all other countries, the firm’s dynamic optimization problem for one country (or a small set of them) is computationally tractable. For this, the dimensionality of the state vector in the country-specific problem in equation (24) must be small. In our model, this vector takes only four values, as $y_{ijt-1} \in \{0, 1\}$ and $\omega_{ijt} \in \{\underline{\omega}, \bar{\omega}\}$ for all i, j , and t . Conditional on the state space of the country-specific problem being small, our solution algorithm is however still feasible if, e.g., ω_{ijt} has a distribution with more than two points of support; export profits in a country j and period t depend on multiple lags of the firm’s export participation in j ; and, the firm’s information is more limited than indicated in equation (18).

As discussed in Appendix D.3, for our sample of firms, periods, and potential destinations, the share of export choices solved in each step of the algorithm, and the associated computing time, depend on the model parameter values. When these equal the baseline estimates (see Section 7), our algorithm finds in less than 13 minutes the solution to 99.89% of the more than 22 million choices we solve for when computing the model’s predictions in our sample.²² The unsolved choices are concentrated in countries sharing complementarities with a large number of other potential destinations; i.e., according to our estimates, those sharing deep PTA with many other countries (e.g., members of the EU). At each step of the algorithm, the share of choices solved increases, and the computing time decreases, as the gravity component in fixed or sunk costs gets larger (i.e., as the value of the parameters entering equations (9) or (14) increase) and as cross-country complementarities get smaller (i.e., as γ_x^E or φ_x^E decrease, or as κ_x^E increases, for $x = \{g, l, a\}$).

6 Estimation Procedure

We estimate the model in two steps. In the first one, we estimate the demand elasticity and time series process of potential export revenues. In the second step, we estimate fixed and sunk costs.

6.1 First Step

We assume $r_{ijt}^{obs} = (r_{ijt} + \epsilon_{ijt})y_{ijt}$, where r_{ijt}^{obs} denotes observed export revenues, ϵ_{ijt} accounts for measurement error and, as a reminder, r_{ijt} is the potential export revenue of firm i in country j at t ,

²²The algorithm takes close to two minutes to find the solution to 99.72% of all choices. These times are measured at Princeton University’s Della cluster using 44 processors with 20 GB of memory each.

and y_{ijt} is a dummy variable that equals one if i exports to j at t . Using d_s and d_{jt} to denote vectors of sector and country-year dummies, respectively, we assume $\mathbb{E}[\varepsilon_{ijt}|y_{ijt-1}, d_s, d_{jt}, a_{sjt}, r_{iht}, y_{ijt} = 1] = 0$ and use a Poisson pseudo-maximum likelihood estimator and data on the sample of firms, countries, and years for which $y_{ijt} = 1$ to obtain consistent estimates of the parameters entering the expression for potential export revenues in equation (7); i.e., $(\alpha_y, \alpha_a, \alpha_r, \{\alpha_{jt}\}_{jt}, \{\alpha_s\}_s)$.²³

We also assume $r_{it}^{obs} = r_{it} + \varepsilon_{it}$, where r_{it}^{obs} denotes the observed total sales of firm i in year t , r_{it} is this variable's true value, and ε_{it} accounts for measurement error. As firms are monopolistically competitive and face in all markets a demand function with constant elasticity equal to η , it holds that $r_{it} = (\eta/(\eta - 1))vc_{it}$, where vc_{it} is the total variable costs of firm i in year t , which we measure as the sum of the wage bill and total expenditure in materials. Assuming $\mathbb{E}[\varepsilon_{it}|vc_{it}] = 0$, we use a non-linear least squares estimator to obtain a consistent estimate of η .

Finally, given estimates of α_{jt} for all sample countries and years, and data on domestic sales, r_{iht} , for all sample firms and years, we use OLS to compute consistent estimates of the parameters of the first-order autoregressive models for α_{jt} and $\ln(r_{iht})$ imposed in Section 4.3.

6.2 Second Step

Given first-step estimates, we use a Simulated Method of Moments (SMM) estimator to obtain consistent estimates of the fixed and sunk cost parameters; see equations (9) to (14). In Section 6.2.1, we use a simple example to illustrate the approach we follow to separately identify the parameters that, according to equation (11), determine the strength of cross-country complementarities from those that, according to equation (12c), determine the strength of the cross-country correlation in unobserved export determinants. In Section 6.2.2, we describe in detail our SMM estimator.

6.2.1 Identification of Cross-Country Export Complementarities

Without loss of generality, consider a setting with one sector and three foreign countries. Firms are heterogeneous only in their fixed export costs; countries 1 and 2 are identical in their export revenue shifters and distance to the firm's home country, but different in their distance to country 3. Specifically, country 3's export complementarities and correlation in the fixed cost term ν_{ijt} equal zero with country 1, but equal possibly nonzero values \bar{c} and $\bar{\rho}$, respectively, with country 2. See Appendix F.1 for extra details on this simple setting.

To focus the identification of \bar{c} and $\bar{\rho}$, consider a researcher that knows the value of all other parameters and, in addition to the variables described in Section 2, observes potential export revenues for all firms, countries, and periods. Then, the parameters \bar{c} and $\bar{\rho}$ are identified by:

$$m_1 = \mathbb{E}[y_{i2t} - y_{i1t}] \quad \text{and} \quad m_2 = \mathbb{C}[y_{i2t}, y_{i3t}], \quad (31)$$

²³We assume firms' export participation decisions do not depend on unobserved determinants of potential export revenues. Instead, we may assume firms select into exporting on the basis of such unobserved determinants, but computational reasons would force us in this case to limit the number of parameters entering potential export revenues; e.g., we would need to substitute the fixed effects $\{\alpha_{jt}\}_{jt}$ and $\{\alpha_s\}_s$ by random effects and functions of observed covariates and a small number of parameters.

where, generally, m_1 is a moment that captures the difference in export probabilities in two export destinations that differ only in the size of the countries “connected” to each of them (i.e., country 2 is connected to other countries while country 1 is not), and m_2 is a moment that captures the correlation in the firm’s export choices in “connected” countries (i.e., countries 2 and 3). As Table F.1 in Appendix F.1 shows, both moments equal zero when there are no export complementarities and the term ν_{ijt} is independent across countries (i.e., when $\bar{c} = \bar{\rho} = 0$). Correlation in unobservables in the absence of complementarities (i.e., $\bar{\rho} > 0$ and $\bar{c} = 0$) yields correlated export choices without affecting the difference in export probabilities between connected and isolated countries (i.e., $m_2 > 0$ and $m_1 = 0$). Conversely, export complementarities alone (i.e., $\bar{c} > 0$ with $\bar{\rho} = 0$) make both moments positive. This seems to suggest an identification strategy in which m_1 identifies the strength of cross-country complementarities and, given these, m_2 identifies the strength of the correlation in unobserved determinants of export profits. This logic is however incorrect, as m_1 is also affected by the strength of the correlation in unobserved determinants of export profits whenever export complementarities are non-zero; i.e., m_1 is also affected by $\bar{\rho}$ whenever $\bar{c} > 0$. What is true is that m_1 and m_2 are differentially affected by \bar{c} and $\bar{\rho}$, and jointly identify them; see Figure F.1 in Appendix F.1. When estimating the model described in Section 4, we use moments analogous to m_1 and m_2 , but adjusted to account for many foreign countries, for firms that are heterogeneous not only in fixed costs but also in productivity, and for the fact that no two countries in the data are identical in every dimension except the size of their “connected” countries.

6.2.2 Details on SMM Estimator

Consider a vector z_i that includes all first-step estimates (see Section 6.1) and all observed (to the researcher) firm i ’s payoff-relevant variables. That is, besides first-step estimates, z_i includes, for all sample years, firm i ’s domestic sales and exports by destination, tariffs by sector and destination, and, for all country pairs, the distance measures in equations (1) to (3). Consider also a vector χ_i including all unobserved firm i ’s payoff-relevant variables: all fixed cost shocks $\nu_{it} \equiv (\nu_{i1t}, \dots, \nu_{iJt})$ and ω_{it} and, for non-sample years, foreign countries’ export revenue shifters $\alpha_t \equiv (\alpha_{1t}, \dots, \alpha_{Jt})$ and firm i ’s domestic sales. Finally, consider vectors y_i^{obs} and $y_i^s(\theta)$ of observed and model-implied, respectively, export choices in all countries and sample years. Specifically, $y_i^s(\theta)$ includes the choices implied by the model described in Section 4 given the vector of observed covariates and first-step estimates z_i , a vector θ of values for all parameters estimated in the second step, and a draw χ_i^s from the distribution of χ_i conditional on z_i .²⁴ We can then write each of the $k = 1, \dots, K$ moments we use in our SMM estimator as

$$\frac{1}{M} \sum_{i=1}^M \{m_k(y_i^{obs}, z_i, x) - \frac{1}{S} \sum_{i=1}^S m_k(y_i^s(\theta), z_i, x)\} = 0, \quad (32)$$

where M is the number of sample firms, $m_k(\cdot)$ is k ’s moment function, and x is a vector that includes an exogenous measure of market size for every sector, foreign country and sample year.

²⁴The vector of second-step parameters is defined as $\theta \equiv (\gamma_0^F, \gamma_0^S, \sigma_\nu, p, \{(\gamma_x^F, \gamma_x^E, \psi_x^E, \kappa_x^E, \gamma_x^N, \kappa_x^N, \gamma_x^S)\}_{x=\{g,l,a\}})$.

By allowing the generic formulation of the moments in equation (32) to depend on x , we account for moments that capture how the probability of exporting to a country j depends on the total market size of *other* foreign countries that are geographically or linguistically close to j , or that share a deep PTA with j . As discussed in Section 6.2.1, this type of moments allow us to identify the parameters determining the strength of cross-country export complementarities separately from those determining the strength of the cross-country correlation in unobserved export determinants.

As market size measure included in x , we use the export potential of a country in a sector and year, measured as the importer fixed effect in a gravity equation estimated using sectoral trade data for all country pairs that do not include Costa Rica as importer or exporter. See Appendix F.2.2 for more information on these export potentials and for reduced-form evidence showing that, controlling for the export potential of a foreign country, firms are indeed more likely to export to those countries whose neighbors' (geographical, linguistic or regulatory) export potential is larger.

In our estimation, we use 89 moments that, for expositional purposes, can be organized in three blocks. In a first block, with the goal of identifying the parameters that determine the level of fixed and sunk costs and the impact on them of the distance between the firm's home country and each destination (i.e., γ_0^F , γ_0^S , and $\{(\gamma_x^F, \gamma_x^S)\}_{x=\{g,l,a\}}$), we use moments capturing firms' export participation and export survival probabilities by groups of destinations that differ in their distances to the firm's home country. In a second block, with the goal of identifying the parameters that determine the strength of cross-country complementarities (i.e., $\{(\gamma_x^E, \psi_x^E, \kappa_x^E)\}_{x=\{g,l,a\}}$), we use moments, similar to m_1 in the previous subsection, capturing firms' export probabilities by groups of destinations that are similar in their distances to the firm's home country but different in the total export potential of the *other* countries that are close to them geographically or linguistically, or that share with them a deep PTA. Finally, with the goal of identifying the parameters of the distribution of the unobserved fixed cost terms ν_{it} and ω_{it} (i.e., σ_ν , p , and $\{(\gamma_x^N, \kappa_x^N)\}_{x=\{g,l,a\}}$), we use moments, in the spirit of m_2 , that capture the correlation across firms and countries in firms' export participation decisions, and moments that capture the frequency with which we observe short-lived changes in a firm's export status in a destination.

We include in Appendix F.3 the full list of moments we use in our estimation. We provide in Appendix F.4 additional details on our SMM estimator. In Appendix F.5, we explore the robustness of our estimates to alternative realizations of the simulation draws χ_i^s we use to build our moments.

7 Estimation Results

We summarize here our parameter estimates. Additional details are presented in Appendix F.6.

7.1 First-step Estimates: Potential Export Revenue Parameters

We estimate the parameters entering equation (7) using information on the 13,293 firm-country-year sample observations with positive export revenues. The estimate of α_γ is 1.856 (robust s.e. equal to 0.066), implying firms' potential export revenues grow approximately 6 times between the

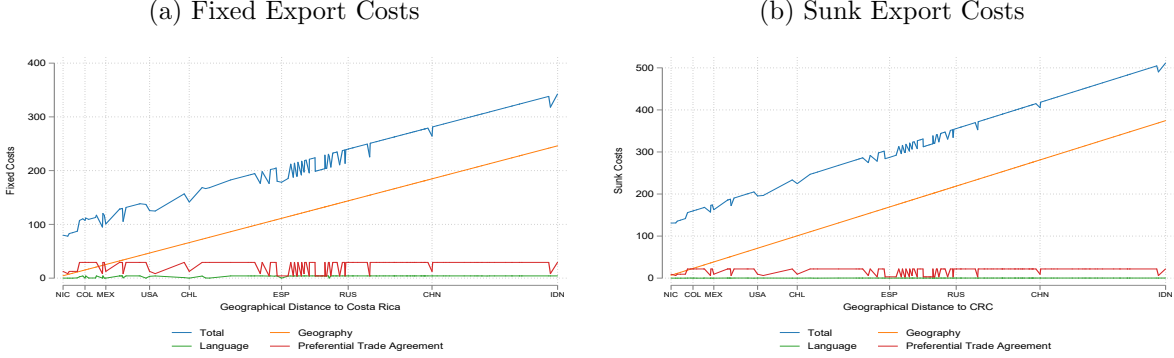
first and second year of exports to a destination. The estimate of α_a , which equals the elasticity of potential export revenues to tariffs, is -3.832 (s.e. equal to 0.066). If trade costs moved one-to-one with tariffs, this estimate would imply that the demand elasticity η equals 4.832 . When estimating η as described in Section 6.1 (i.e., using information on total revenues and variable costs for all 44,785 firm-year sample observations), we obtain an estimate of 5.713 (robust s.e. equal to 0.489). As this estimate does not rely on assuming the passthrough of tariffs to trade costs is perfect, we adopt it as our baseline. The estimate of α_r , the elasticity of potential export revenues to domestic sales, is 0.285 (s.e. equal to 0.041), reflecting that firms that are larger in the domestic market also tend to have larger potential export revenues.

The estimation of the parameters in equation (7) also yields estimates of the sector and country-year specific effects $\{\alpha_s\}_s$ and $\{\alpha_{jt}\}_{jt}$. As shown in Figure F.8, countries with large estimated values of α_{jt} tend to be geographically close to Costa Rica (e.g., Guatemala) or large economies (e.g., the United States), and countries with small estimated values tend to be geographically far from Costa Rica (e.g., Russia) or small economies (e.g., Oman). When using the 467 estimated values $\{\hat{\alpha}_{jt}\}_{jt}$ to estimate the parameters of the stochastic process of α_{jt} assumed in Section 4.3, we obtain an estimate of its autocorrelation parameter ρ_α equal to 0.686 (s.e. clustered by destination equal to 0.059), an estimate of the standard deviation σ_α of its innovations equal to 0.630 , and estimates implying the mean of α_{jt} for each country j increases in its GDP and geographical proximity to Costa Rica (with the effect of linguistic and regulatory distances not statistically significant at the 5% level). Similarly, when using information on the 44,785 sample observations of firms' domestic sales $\{r_{iht}\}_{it}$ to estimate the parameters of its autoregressive process according to Section 4.3, we obtain an estimate of its autocorrelation parameter ρ_r equal to 0.857 (s.e. clustered by firm equal to 0.012), and an estimate of the standard deviation σ_r of its innovations equal to 0.865 .

7.2 Second-step Estimates: Fixed and Sunk Costs Parameters

As shown in Figure 1, the estimates of the fixed and sunk cost parameters (reported in Table F.4) imply mean fixed costs for single-destination exporters (as modeled in equation (9)) and sunk costs are well approximated by a constant (which equals $\$63,000$ in the case of fixed costs and $\$115,000$ in the case of sunk costs) plus a term that increases in the geographical distance between the firm's home country and each destination. The estimated impact of linguistic distance is small and not statistically significant, while the differences in fixed and sunk costs between a destination with whom Costa Rica has a deep PTA and one with whom it has no agreement are only $\$29,000$ and $\$22,000$, respectively. Adding all terms, for single-destination exporters, the estimated mean fixed and sunk costs to, e.g., the US are close to $\$125,000$ and $\$200,000$, respectively. The estimates for, e.g., Mexico are $\$100,000$ and $\$175,000$, and for China these are $\$180,000$ and $\$400,000$. Comparing these estimates to the distribution of observed export revenues in those countries, mean fixed costs in the US and Mexico are between the median and the 75 percentile (and below average), while they are between the 75 and the 95 percentile (and close to the mean) in China. One should bear in mind that the actual fixed cost a firm faces in a destination (as modeled in equation (8)) will

Figure 1: Estimates of Fixed and Sunk Export Costs



Note: In both figures, countries are identified by their ISO 3166-1 alpha-3 code, and placed in the horizontal axis by their distance to Costa Rica. The vertical axis indicates the estimated cost in thousands of 2010 USD.

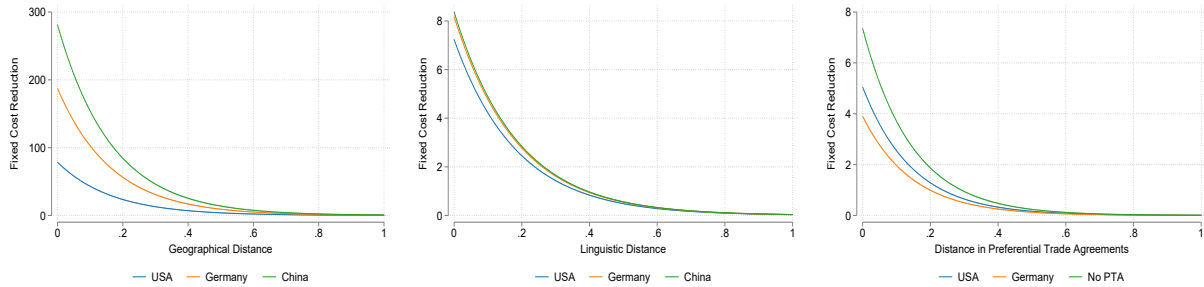
differ from the mean fixed cost for single-destination exporters due to the unobserved term ν_{ijt} and the effect of cross-country export complementarities. Everything else equal, firms exporting to a country j at a period t will on average have relatively low values of ν_{ijt} . As ν_{ijt} is normal and its estimated standard derivation is close to \$81,000 (see Table F.4), there is a large firm heterogeneity in fixed costs, and actual exporters to a destination (even if they do not export anywhere else in the same period) likely face realized fixed costs that are much below their mean level.

In Figure 2, we represent the estimated cross-country export complementarities. In each panel, we plot, for the corresponding index x in $\{g, l, a\}$, the function $\hat{\gamma}_x^E (1 + \hat{\varphi}_x^E n_{hj}^x) \exp(-\hat{\kappa}_x^E n_{jj'}^x)$ for three different destinations j (i.e., the US, Germany, and China) against their distance to any other country j' , $n_{jj'}^x$. Panel (a) shows that complementarities arising from geographic proximity are large between countries that are close to each other (they imply a reduction in fixed costs in up to \$90,000 for firms simultaneously exporting to countries that are 200 km apart) but decrease quickly, being close to zero between countries whose distance is 800 km or more. The strength of these geographical complementarities is heterogeneous across destinations depending on their distance to the firm's home country: for any given value of $n_{jj'}^g$, complementarities are larger for China than for Germany, and for Germany than for the US, reflecting their ranking in terms of distance to Costa Rica. Panel (b) shows that linguistic complementarities are always small, reaching a maximum of close to \$8,000 for country pairs whose linguistic distance is zero; i.e., country pairs whose residents understand each other with probability one. Finally, panel (c) shows that complementarities due to common participation in PTAs are close to zero unless these agreements are sufficiently deep; i.e., the complementarities between members of a PTA that does not impose any regulatory harmonization are the same as if they did not belong to any common PTA. Among members whose regulatory distance is zero, the reduction in fixed costs in one of them for a firm that also exports to the other is nearly \$8,000 if the destination does not share any PTA with Costa Rica, and below \$4,000 if the destination also has a deep PTA with Costa Rica.

The estimates in Figure 2 are compatible with complementarities due to geographical, linguistic,

Figure 2: Estimates of Sources of Static Complementarities

(a) Due to Geographical Proximity (b) Due to Linguistic Proximity (c) Due to Regulatory Proximity

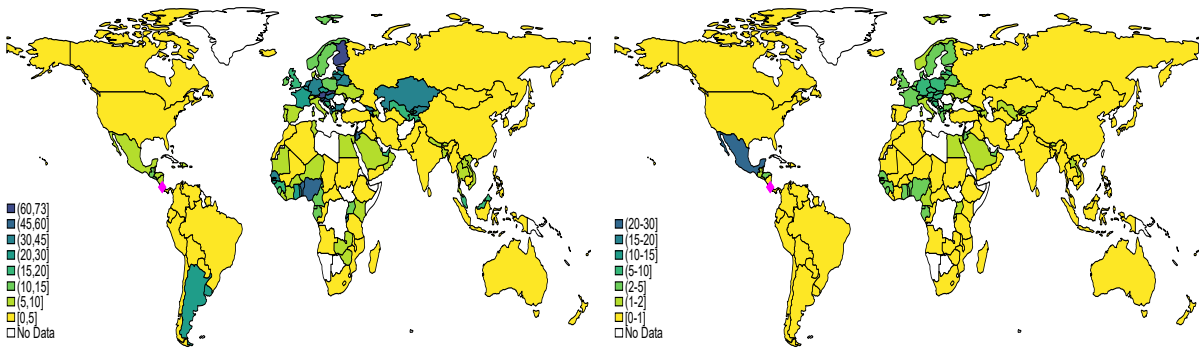


Note: In panels (a) to (c), the horizontal axis corresponds to the distance measures defined in equations (1) to (3), respectively. The vertical axis indicates the estimated reduction in fixed export costs in thousands of 2010 USD.

and regulatory proximity potentially playing an important role in firm exports. This is true even if, as shown in panels (b) and (c), linguistic and regulatory complementarities between any two countries are never large. As each country may share language or deep PTAs with many other countries, a firm may export to several destinations that are linguistically or regulatory close to each other and, in this case, benefit from large cumulated reductions in fixed costs in each of these destinations. This is captured in Figure 3. In panel (a), we show a firm’s fixed costs in a destination are up to 73% smaller if the firm simultaneously exports to the country with whom complementarities are the largest; for complementarities between two countries to be this strong, both must be geographically close and share a deep PTA (e.g., Austria and Slovakia). In panel (b), we show there are certain countries (e.g., Mexico) that, although do not share strong complementarities with any one country in particular (as shown in panel (a), Mexico’s closest neighbor reduces fixed costs in it in less than 10%), benefit from sharing a moderate level of

Figure 3: Implications of Estimated Static Complementarities

(a) Fixed Costs Reduction from Closest Neighbor (b) Num. Neighbors Reducing Fixed Costs in $\geq 5\%$



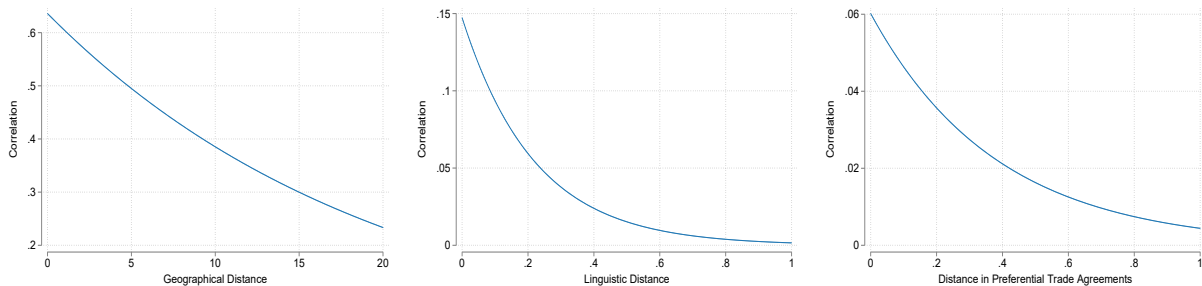
Note: In panel (a), we illustrate, for each destination j , the percentage reduction in fixed export costs a firm would enjoy if it also exports to its closest neighbor j' ; i.e., the country for which the value of $c_{jj't}$ in equation (11) is the largest. In panel (b), we illustrate, for each destination country j , the number of other foreign countries $j' \neq j$ for whom $c_{jj't}/g_{jt} \geq 5\%$.

complementarities with many other countries (Mexico shares common language and membership in deep PTAs with many other countries). Thus, a firm exporting simultaneously to several countries that share common language or deep PTAs with, e.g., Mexico may ultimately be able to export to this country while facing very small fixed costs in it. In Figure F.9, for the case of the US, China, Germany and Spain, we illustrate their bilateral complementarities with any other country.

In Figure 4, we represent the estimated cross-country correlation in the fixed cost term ν_{ijt} within a firm-period. In each panel, we plot, for the corresponding index x in $\{g, l, a\}$, the function $\gamma_x^N \exp(\kappa_x^N n_{jj'}^x)$ against the distance $n_{jj'}^x$. The figure shows there is a large correlation in ν_{ijt} , and the key determinant of the correlation coefficient between any two countries is their geographic proximity, although their linguistic proximity also plays a role. It is thus potentially important to allow for possibly correlated unobserved export profit shifters when estimating cross-country export complementarities. For the US, China, Germany and Spain, we illustrate in Figure F.9 the correlation coefficient in ν_{ijt} vis-a-vis any other country.

Figure 4: Estimates of Correlation Coefficient in Fixed Export Cost Shock

(a) Due to Geographical Proximity (b) Due to Linguistic Proximity (c) Due to Regulatory Proximity



Note: In panels (a) to (c), the horizontal axis indicates the distance measures defined in equations (1), to (3), respectively. The vertical axis indicates the estimated correlation in ν_{ijt} .

8 Counterfactual Analysis

We perform three counterfactual experiments. In Section 8.1, we quantify the importance of cross-country complementarities in firm exports by comparing the predictions of the estimated model to versions of the model in which some or all of the complementarities between countries are set to zero. In sections 8.2 and 8.3, we use the estimated model to compute the impact on Costa Rican exports of a Brexit-induced increase in the regulatory distance between the UK and current EU members, and of Costa Rica joining the CPTPP, respectively.

8.1 Quantitative Importance of Cross-country Complementarities

To quantify the impact of complementarities on exports, we compute for each firm in the sample and all sample years model-implied export decisions for 200 simulations of the vector χ_i of un-

observed payoff-relevant variables (see Section 6.2.2). We do so for the estimated model and for models that differ from it only in that some or all of the parameters determining the strength of the complementarities according to equation (11) are set to zero, and report in Table 2 the differences across models in the predicted number of firm-country-years with positive exports (i.e., export events) and total export revenue. The results in the column “All” show that including all complementarities causes the number of export events and total exports to increase in 12% and 5%, respectively, vis-a-vis a model that sets all $\{(\gamma_x^E, \psi_x^E, \kappa_x^E)\}_{x=\{g,l,a\}} = 0$. The results in the remaining columns show that the most important source of cross-country complementarities is spatial proximity: setting γ_g^E, ψ_g^E and κ_g^E to their estimated values, while keeping all the other parameters at 0, causes export events and total exports to increase in 7% and 3%, respectively. Complementarities due to linguistic and regulatory proximity explain each around 2.5% of the export events predicted by the estimated model, and around 0.9% and 1.6%, respectively, of total exports.

Table 2: Impact of Cross-country Complementarities

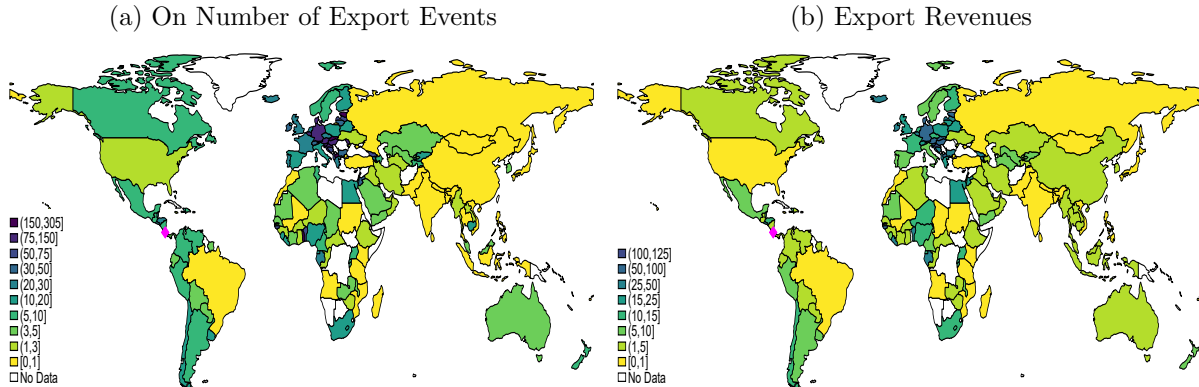
Percentage Increase in:	Sources of Complementarities Included:			
	All	Geographic Proximity	Linguistic Proximity	Common Deep PTA
Number of Export Events:	11.78%	6.57%	2.35%	2.57%
Export Revenues:	5.14%	2.74%	0.86%	1.58%

Note: In the column labeled *All*, we report the percentage difference in the number of export events and export revenues between our estimated model and a model in which the parameters $\{(\gamma_x^E, \psi_x^E, \kappa_x^E)\}_{x=\{g,l,a\}}$ are all set to zero. In the other three columns, we compare models in which only the subset of these parameters indicated by the corresponding label is set to their estimated values, whereas the other ones are set to 0, with a model in which the parameters $\{(\gamma_x^E, \psi_x^E, \kappa_x^E)\}_{x=\{g,l,a\}} = 0$.

The smaller impact of complementarities on total exports relative to its impact on the number of export events in Table 2 is partly due to complementarities having, all else equal, a larger impact on smaller countries. To gain intuition on this model property, consider a setting with two foreign countries *A* and *B* identical except for one of them having larger revenue shifters than the other; e.g., assume $\alpha_{At} > \alpha_{Bt}$ for all *t*. As shown in Appendix G, introducing complementarities in this context increases exports to *B* more than to *A*. The reason is that, without complementarities, exports to *A* are larger than to *B* and, with complementarities, firms benefit from a reduction in fixed costs in *B* only if they also export to *A*. Thus, complementarities push firms to export to both countries, but this implies the growth in exports to the smaller country must be larger, as it started from a lower baseline level of exports in the setting without complementarities.

Besides size, the geographical, linguistic, and regulatory proximity of each foreign country to every other country also matters for the impact complementarities have on exports to it. As a result, as shown in Figure 5, there is a large heterogeneity across countries in the impact of complementarities. In a large number of them, these play a minimal role; conversely, for some, several of which are located in Central Europe, complementarities cause the number of export events and total exports from Costa Rica to increase in more than 50%. The reason is that these

Figure 5: Impact of Eliminating Cross-country Complementarities



Note: In Panel (a), we illustrate, for each destination and all firms and years in the sample, the percentage reduction in the total number of firm-year pairs with positive exports predicted by our model when we set the parameters $\{(\gamma_x^E, \psi_x^E, \kappa_x^E)\}_{x=\{g,l,a\}}$ to zero. In Panel (b), we provide analogous information for the total predicted export revenues.

Central European countries are typically small, geographically close to many other destinations, and members of deep PTAs that also include many other countries.

8.2 Third-Market Effects of Regulatory Differences Due to Brexit

A possible implication of Brexit is that regulations in the UK and in the EU will drift apart. To quantify the third-country effect of this Brexit implication, we use our estimated model to evaluate the impact on Costa Rican exports of a permanent increase in 2021 (expected since the 2017 referendum, but unexpected before) in the regulatory distance, $n_{jj't}^a$, between the UK and all EU members from zero (its pre-Brexit actual value) to one (its maximum value). Specifically, for all sample firms and these two sets of values of the regulatory distances, we compute model-implied export choices for 200 simulations of the vector χ_i , and report in Table 3 the relative differences in the expected number of export events and total exports for the period 2021-2030; i.e., the 10 years subsequent to the UK withdrawal from the EU.

As shown in Table 3, our model predicts exports from Costa Rica to the UK will fall as a consequence of the increase in the regulatory distance between the UK and the EU. Specifically, the predicted fall both in export events and total exports in the 10 years subsequent to Brexit is around 4%. In the four years between the Brexit referendum and the UK's effective withdrawal from the EU, firms anticipate the subsequent change in policy, and the number of export events and total exports to the UK fall in approximately 1.4% and 0.5%, respectively. Although the reduction in complementarities between the UK and the EU is symmetric, the effect on exports to the UK is much larger than that on exports to the EU, where the drop is always below 0.5%.

Zooming in on the impact on individual EU members, our model predicts that the countries geographically close to the UK will be more affected than those further away; e.g., in comparison to the 0.45% reduction in total exports to the EU as a whole, total exports fall between 2021 and 2030 in 1.96% and 0.87% in Belgium and Ireland, respectively. To understand the large effects on these

Table 3: Impact of Regulatory Differences Due to Brexit

Countries:	Percentage Reduction in:			
	Export Events		Export Revenues	
	<i>2017-20</i>	<i>2021-30</i>	<i>2017-20</i>	<i>2021-30</i>
United Kingdom	-1.38%	-4.13%	-0.52%	-3.81%
European Union	-0.19%	-0.46%	-0.07%	-0.45%
<i>In particular:</i>				
Belgium	-0.58%	-1.66%	-0.23%	-1.96%
Ireland	-0.19%	-0.80%	-0.09%	-0.87%

Note: For the geographic area indicated in the column “Countries,” we report the relative change for the periods 2017-20 and 2021-30 in the number of export events and total exports of all sample firms caused by a permanent change in 2021 (expected since 2017) in the regulatory distances between the UK and every EU member from zero to one.

two countries, one should bear in mind that the cross-country complementarities we have estimated imply that the reduction in exports to the UK as a result of its regulatory isolation from the EU will have subsequent effects on all countries geographically close to the UK, such as Belgium and Ireland. For the same reason, exports to countries other than the UK with large English-speaking populations will also be affected by the increase in the UK-EU regulatory distance, but these effects will be small as linguistic complementarities are estimated to be small (see Section 7.2).

A partial-equilibrium model (such as ours) without cross-country export complementarities would predict Costa Rican exports to be unaffected by changes in trade barriers (regulatory or otherwise) between the UK and the EU. Standard general equilibrium trade models à la Eaton and Kortum (2002) or Anderson and van Wincoop (2003) imply exports of different origins are substitutes and, thus, predict Costa Rican exports to the UK and the EU to *increase* in reaction to the increase in the UK-EU trade barriers.²⁵ The third-market effects implied by the complementarities in our model are thus different from those in standard trade models.

8.3 Impact of Costa Rica Becoming a CPTPP Member

In 2022, Costa Rica applied for membership in the CPTPP. To quantify the impact of Costa Rica joining this trade bloc, we use the estimated model to evaluate the effect on Costa Rican exports of a permanent reduction in the tariffs and regulatory barriers Costa Rican firms face when exporting to CPTPP members. Specifically, for all sample firms, the ten-year period 2025-35, and 200 simulations of the vector χ_i , we compute model-implied firm exports in a setting in which trade barriers do no change and in one in which, from 2025 onwards (expected since 2022), we set to zero the tariffs Costa Rica faces in CPTPP members, and to 0.143 the regulatory distance between Costa Rica and these members: for any member j and $t \geq 2025$, we set $a_{sjt} = 0$ and $n_{hjt}^a = 0.143$.

²⁵Adão et al. (2017) and Lind and Ramondo (2022) allow for more flexible elasticities of substitution across export countries, but maintain the assumption that different export countries are substitutes in any given destination. For a framework that allows for positive third-market effects, see Fajgelbaum et al. (2021).

As shown in columns (1) and (2) of Panel (a) in Table 4, the estimated model predicts the number of firm-year pairs with positive exports and total exports to CPTPP members to increase in 18.3% and 28%, respectively. As illustrated by the results in columns (3) and (4), these effects are mostly due to the drop in tariffs Costa Rican exporters experience if their home country becomes a CPTPP member; the reduction in the regulatory distance between Costa Rica and CPTPP members is predicted to cause only a 5.7% and 2.3% increase in export events and total exports, respectively. Columns (5) to (8) reveal that a researcher using a model analogous to ours but in which cross-country complementarities are assumed away (see Appendix F.7) would have predicted a growth in Costa Rican exports to CPTPP members only slightly smaller than that predicted by our model. The reason why cross-country complementarities play a small role in determining the impact of Costa Rica becoming a CPTPP member is that current members exhibit small complementarities both with each other and with non-members. Thus, the growth in exports in any member country has small spillovers on other countries.

In other contexts, the predictions of a model that assumes away cross-country complementarities may differ from those of our estimated model. To illustrate this point, we compute in Panel (b) the impact of Costa Rica joining a hypothetical CPTPP that also includes the US among its members. In this case, while the estimated model predicts the number of export events and total exports in current member countries to grow in 22.8% and 40.1%, respectively, the re-estimated model without complementarities predicts these growth rates to be approximately a third smaller. The reason for

Table 4: Impact of Costa Rica Becoming a CPTPP Member

Countries:	Model With Cross-Country Complementarities				Model Without Cross-Country Complementarities			
	With Tariff Changes		Without Tariff Changes		With Tariff Changes		Without Tariff Changes	
	Export Events	Export Revenues	Export Events	Export Revenues	Export Events	Export Revenues	Export Events	Export Revenues
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
<i>Panel (a): If Costa Rica Joins the CPTPP</i>								
Members	18.30%	28.01%	5.67%	2.33%	15.56%	25.73%	4.80%	1.92%
Others	0.24%	0.30%	0.20%	0.28%	0%	0%	0%	0%
United States	0.17%	0.08%	0.13%	0.06%	0%	0%	0%	0%
<i>Panel (b): If Costa Rica Joins the CPTPP (with the US as member)</i>								
Members	22.88%	40.10%	6.49%	3.13%	15.56%	25.73%	4.80%	1.92%
Others	0.24%	0.32%	0.21%	0.30%	0%	0%	0%	0%
United States	10.03%	15.67%	6.02%	2.82%	5.63%	8.43%	4.42%	1.46%

Note: “Members” denotes current CPTPP members, which are Australia, Brunei, Chile, Japan, Malaysia, Mexico, New Zealand, Peru, Singapore and Vietnam; “Others” denotes all other countries. The results in columns (1) to (4) are computed using our estimated model; those in columns (5) to (8) are computed using the model described in Appendix F.7. The results in columns (1), (2), (5), and (6) determine the impact of counterfactual changes both in tariffs (set to zero in every CPTPP member) and regulatory distances (set to 0.143 for every CPTPP member). The results in columns (3), (4), (7), and (8) only evaluate the impact of the counterfactual changes in regulatory distances.

the significant disparity in model predictions in this case is that the US is a large destination, and the spillovers to current CPTPP members of the 15.7% growth in exports to the US (see column (2)) imply a large growth in exports to those members. The model without complementarities assumes away these spillovers and, thus, predicts a much smaller export growth to CPTPP members.

9 Conclusion

We estimate and solve a partial-equilibrium dynamic model of firm exports featuring cross-country complementarities. In our model, the firm has rational expectations and chooses every period the bundle of export destinations that maximizes its expected discounted sum of current and future profits. We introduce a novel algorithm to solve the firm’s combinatorial dynamic discrete choice optimization problem. Our estimates reveal substantial heterogeneity in complementarities across country pairs. Fixed export costs in several Central European countries are reduced in more than 50% if the firm also exports to these countries’ closest neighbors. Conversely, for the US or China, exporting to their closest neighbor reduces fixed costs in these countries in less than 5%.

We quantify the impact of the estimated cross-country complementarities to be non-negligible. We predict Costa Rica’s total exports are approximately 5% larger due to these complementarities, reflecting a 12% increase in the number of firm-country-period triplets with positive exports. We also use our estimated model to quantify the impact Brexit has on Costa Rican exports to the UK and the EU as a result of both countries no longer sharing a deep PTA: although bilateral trade barriers between Costa Rica and every foreign country are held constant in this counterfactual experiment, exports to the UK and the EU drop in 4% and 0.5%, respectively, illustrating that deep PTA generate significant positive trade *creation* effects, specially towards smaller destinations. Finally, using Costa Rica’s request to join CPTPP as motivating example, we show that researchers that assume away the presence of complementarities when predicting the impact of counterfactual changes in trade policy will obtain predictions similar to those of our estimated model when the policy changes affect isolated countries, and quite different predictions when the policy changes affect a large destination that exhibits important complementarities with other destinations.

We provide a first quantification of the role cross-country complementarities play in firms’ optimal export decisions in a dynamic framework, and develop tools that have the potential to help quantify the relevance of complementarities across alternatives in other dynamic discrete-choice settings. Our paper is an early step towards merging two literatures, the literature on firm export dynamics, which has a long tradition within international trade, and the more recent literature exploring interdependencies across countries in firm exports. Natural next steps in this literature are to further depart from the perfect foresight assumption when modeling firms’ choices, to allow for sources of cross-country interdependencies beyond those in our framework (e.g., non-constant marginal production costs), or to study the impact complementarities have in a general-equilibrium framework. In the context of dynamic models, these extra steps involve potentially substantial methodological contributions beyond those in our paper.

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A General Optimization Problem and Solution Algorithm

In Appendix A.1, we characterize an optimization problem encompassing that in equation (20). In Appendix A.2, we propose a solution algorithm covering those used in the steps described in Section 5 and Appendix D.1, and state several of its properties when applied to problems of the kind characterized in Appendix A.1. To simplify notation and without loss of generality, we focus on an agent born at period $t = 0$.

A.1 General Optimization Problem

Consider an agent that, in every period $t \geq 0$, makes J simultaneous binary choices with the goal of maximizing the expected discounted sum at birth of infinite per-period (static) payoffs.

Per-period payoffs in any t depend on a shock ω_t taking values in a set Ω_t according to a distribution $Q_t(\omega_t|\omega_{t-1}, \dots, \omega_0)$. We denote as $z^t = \{\omega_{t'}\}_{t'=0}^t$ the history of shocks in all periods $t' \leq t$, and as $Z^t = \times_{t'=0}^t \Omega_{t'}$ the set of all possible period- t histories. We denote as $y_j(z^t) \in \{0, 1\}$ a generic choice at z^t for alternative j , as $y(z^t) \in \{0, 1\}^J$ a generic vector of choices at z^t for all J alternatives, and as $\mathbf{y} \in Y$ a generic vector of choices for all $t \geq 0$, all $z^t \in Z^t$, and all alternatives; i.e.,

$$Y = \times_{t=0, z^t \in Z^t}^{\infty} \{0, 1\}^J. \quad (\text{A.1})$$

Considering only optimization problems where the solution exists and is unique, we can write

$$\mathbf{o} = \underset{\mathbf{y} \in Y}{\operatorname{argmax}} \Pi_0(\mathbf{y}), \quad (\text{A.2})$$

where $\Pi_0(\mathbf{y})$ is the agent's objective function and \mathbf{o} is the optimal choice for all $t \geq 0$, all $z^t \in Z^t$, and all alternatives.²⁶ Thus, using $o(z^t)$ to denote the agent's optimal choice at z^t for all alternatives, it holds

$$\mathbf{o} = \{o(z^t)\}_{t=0, z^t \in Z^t}^{\infty}. \quad (\text{A.3})$$

The following assumption establishes a list of sufficient conditions on the objective function $\Pi_0(\cdot)$.

Assumption 1 *Assume:*

1. (*Additive separability*) The function $\Pi_0(\cdot)$ satisfies

$$\Pi_0(\mathbf{y}) = \pi_0(y(z^0), 0_J, \omega(z^0)) + \sum_{t=1}^{\infty} \delta^t \mathbb{E}[\pi_t(y(z^t), y(z^{t-1}), \omega(z^t))], \quad (\text{A.4})$$

where the expectation is over $\{z^t\}_{t=1}^{\infty}$, 0_J is a $J \times 1$ vector of zeros, $\delta \in (0, 1)$ and, for all $t \geq 0$,

$$\pi_t(y(z^t), y(z^{t-1}), \omega(z^t)) = \sum_{j=1}^J (\hat{\pi}_{jt}(y_j(z^t), y_j(z^{t-1}), \omega(z^t)) + \tilde{\pi}_{jt}(y(z^t), y(z^{t-1}))) \quad (\text{A.5})$$

where $\hat{\pi}_{jt}: \{0, 1\} \times \{0, 1\} \times \Omega_t \rightarrow \mathbb{R} \cup \{-\infty\}$ and $\tilde{\pi}_{jt}: \{0, 1\}^J \times \{0, 1\}^J \rightarrow \mathbb{R}$.

2. (*Supermodularity*) For all $t \geq 0$ and $\omega_t \in \Omega_t$, π_t is supermodular in $(y(z^t), y(z^{t-1}))$ on $\{0, 1\}^J \times \{0, 1\}^J$.
3. (*Inaction*) For all $j = 1, \dots, J$, $t \geq 0$, and $z^t \in Z^t$, there exists $y_j(z^t) \in \{0, 1\}$ such that, defining the set $X_t \equiv \{0, 1\} \times \Omega_t$, it holds that $\hat{\pi}_{jt}(y_j(z^t), x) \geq -K$ for all $x \in X_t$ and a real number $K \geq 0$.
4. (*Markov with finite state space*) For all $t \geq 0$, Ω_t is finite and $Q_t(\omega_t|\omega_{t-1}, \dots, \omega_0) = Q_t(\omega_t|\omega_{t-1})$.
5. (*Stationarity*) There exists T such that, for all $t \geq T$ and all $j = 1, \dots, J$, it holds that $\Omega_t = \Omega_T$, $Q_t = Q_T$, $\hat{\pi}_{jt} = \pi_{jT}$ and $\tilde{\pi}_{jt} = \tilde{\pi}_{jT}$.

As shown in Appendix E.1, equating agents to firms and alternatives to potential export destinations, the model described in Section 4 satisfies all restrictions in Assumption 1.

²⁶The restrictions in Section 4 imply the solution to the problem in equation (20) exists and is unique almost surely.

A.2 General Solution Algorithm

We describe here an iterative algorithm that yields upper bounds on the solution to the problem in equation (A.2) when the function $\Pi_0(\cdot)$ satisfies the restrictions listed in Assumption 1. An algorithm that yields lower bounds may be devised in an analogous fashion.

As a preliminary step, partition the J alternatives into U groups indexed by u . Denote as $M_u \subseteq \{1, \dots, J\}$ the set of alternatives included in group u , and denote as M_u^c the complement of M_u ; i.e., the set including all alternatives not in M_u . E.g., if $J = 4$ and $U = 3$, we can form the subsets $M_1 = \{1, 2\}$, $M_2 = \{3\}$, and $M_3 = \{4\}$, and the corresponding complements are $M_1^c = \{3, 4\}$, $M_2^c = \{1, 2, 4\}$, and $M_3^c = \{1, 2, 3\}$.

For each set M_u and each iteration $n = 1, 2, 3, \dots$ of the algorithm, we solve

$$\bar{o}_{M_u}^{(n)} = \operatorname{argmax}_{\mathbf{y}_{M_u} \in Y_{M_u}} \Pi_0(\mathbf{y}_{M_u}, \bar{\mathbf{y}}_{M_u^c}^{(n)}), \quad (\text{A.6})$$

where \mathbf{y}_{M_u} is a generic vector of export choices for every alternative in the set M_u , all periods $t \geq 0$, and every history z^t that may be reached at t , and the set Y_{M_u} includes all feasible values of \mathbf{y}_{M_u} ; i.e.,

$$Y_{M_u} = \times_{t=0, z^t \in Z^t}^{\infty} \{0, 1\}^{J_u},$$

where J_u is M_u 's cardinality. The second argument of the function $\Pi_0(\cdot)$ in equation (A.6) is an upper bound on the firm's optimal choice in every alternative not in M_u , all periods $t \geq 0$, and all histories $z^t \in Z^t$; i.e.,

$$\bar{\mathbf{y}}_{M_u^c}^{(n)} = \{\bar{y}_{M_u^c}^{(n)}(z^t)\}_{t=0, z^t \in Z^t}^{\infty}, \quad \text{with} \quad \bar{y}_{M_u^c}^{(n)}(z^t) \geq o_{M_u^c}(z^t) \text{ for all } t \geq 0 \text{ and } z^t \in Z^t,$$

where $o_{M_u^c}(z^t)$ is the vector of optimal choices at period t and history z^t in all alternatives not in M_u .

Solving the problem in equation (A.6) for any group u at any iteration n requires specifying first the upper-bounds included in the vector

$$\bar{\mathbf{y}}_{M_u^c}^{(n)}.$$

For computational reasons (see discussion in Section 5), we set the upper bound corresponding to any country j , period t , and history z^t , to a value that does not vary across histories; i.e., we set

$$\bar{y}_j^{(n)}(z^t) = \bar{b}_{jt}^{(n)} \text{ for all } z^t \in Z^t, \quad (\text{A.7})$$

In the first iteration (i.e., for $n = 1$), we set each of these upper bounds to its largest value within the feasible choice set; i.e., for every j , every $t \geq 0$, and every $z^t \in Z^t$, we set

$$\bar{b}_{jt}^{(1)} = 1. \quad (\text{A.8})$$

In all subsequent iterations (for all $n > 1$), we set

$$\bar{b}_{jt}^{(n)} = \max_{z^t \in Z^t} \bar{o}_j^{(n-1)}(z^t), \quad (\text{A.9})$$

where $\bar{o}_j^{(n-1)}(z^t)$ is the element corresponding to alternative j , period t , and history z^t of the vector $\bar{o}_{M_u}^{(n-1)}$ for the set of alternatives M_u including j . Equation (A.9) shows that, to compute the iteration- n upper bound on the firm's optimal choice in alternative j at history z^t , we use the outcome of the optimization problem in equation (A.6) at iteration $n - 1$ for the set M_u including j . Specifically, as shown in equation (A.9), we assign to every j , t , and z^t , the max of the outcomes obtained for j and t across every $z^t \in Z^t$.

Theorem 1 establishes certain properties of the iterative algorithm defined in equations (A.6) to (A.9)

Theorem 1 *Let $\bar{b}_{jt}^{(n)}$ be defined by equations (A.6) to (A.9), and let $o_j(z^t)$ be the element of the vector \mathbf{o} defined in equation (A.2) that corresponds to alternative j and history z^t . Then, for all $j = 1, \dots, J$, $t = 1, 2, \dots$, $z^t \in Z^t$, and $n = 1, 2, 3, \dots$, it holds that*

1. $\bar{b}_{jt}^{(n)} \geq o_j(z^t)$.

2. $\bar{b}_{jt}^{(n)} \leq \bar{b}_{jt}^{(n-1)}$.

3. *There exists $N < \infty$ such that $\bar{b}_{jt}^{(n)} = \bar{b}_{jt}^{(n-1)}$ for all $n \geq N$.*

Theorem 1 establishes that the values $\{\bar{b}_{jt}^{(n)}\}_{j=1, t=0}^{J, \infty}$ computed according to equations (A.6) to (A.9) are an upper bound on the firm's optimal choice at any feasible history, get tighter with every iteration; and, converge after a finite number of iterations. See Appendix E for a proof of Theorem 1.

Property 3 of Theorem 1 does not imply that the upper bound defined by equations (A.6) to (A.9) (nor the analogous lower bound) converges to the solution of the firm's optimization problem in equation (A.2). However, as the partition of the J alternatives into U subgroups gets coarser, the upper bound defined by equations (A.6) to (A.9) (and the analogous lower bound) gets tighter. In the limiting case in which $U = 1$ and, therefore, $M_u = \{1, 2, \dots, J\}$, the optimization problem in equation (A.6) coincides with that in equation (A.1) and, thus, solving this optimization problem is equivalent to solving the firm's problem.

The algorithms implemented in each of the steps described in Section 5 and Appendix D.1 are special cases of the algorithm defined in equations (A.6) to (A.9). For e.g., the algorithm implemented in step 1 is a case in which: (a) $U = J$ and, for $u = 1, \dots, J$, the set M_u is a singleton; and (b) period $t = 0$ corresponds to the birth year of the firm (i.e., $t = t_i$). The algorithm implemented in step 2 is a case in which: (a) $U = J$ and, for $u = 1, \dots, J$, M_u is a singleton; and, (b) period $t = 0$ corresponds to the first period at which the step 1 upper and lower bounds differ. The algorithm implemented in step 5 is a case in which: (a) $U < J$, and for some $u = 1, \dots, U$, the set M_u includes more than one country; and, (b) period $t = 0$ corresponds to the first period at which the upper and lower bounds computed in previous steps differ.

Online Appendix for “Firm Export Dynamics in Interdependent Markets”

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January 2023

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B Additional Reduced-Form Results

B.1 Firm-level Data: Sample Descriptive Statistics

We provide here descriptive statistics for the firm-level data introduced in Section 2. In Table B.1, we report information for every sample year on total manufacturing exports, total number of exporting firms, and total number of foreign countries to which Costa Rican manufacturing firms exported in the corresponding year. Total manufacturing exports are measured in thousands of 2013 dollars. While the total number of exporters remained stable at a number between approximately 400 and 450 (with a minimum of 395 exporters in 2015 and a maximum of 459 exporters in 2012), and the total number of export destinations remained stable at around 90 destinations, the total export volume grew significantly in real terms between 2005 and 2015.

Table B.1: Aggregate Statistics

Years	Total Exports	Number of Exporters	Number of Destinations
2005	262,549.6	400	95
2006	303,344.6	415	96
2007	332,929.1	422	91
2008	371,202.9	419	91
2009	328,435.2	438	87
2010	347,235.1	432	96
2011	431,820.7	456	91
2012	479,806.0	459	90
2013	450,472.3	437	84
2014	494,083.5	436	84
2015	479,485.1	395	90

Notes: *Total Exports* are reported in thousands of 2013 dollars. All numbers in this table are obtained by aggregating the firm-level data introduced in Section 2.

In Table B.2, we report the mean and median domestic sales across all firms and across exporters. We measure domestic sales by subtracting total export revenue (from the Customs dataset) from total revenue. As it is common in datasets similar to ours, the distribution of domestic sales is skewed to the right (mean domestic sales are much larger than median domestic sales), and exporters are larger on average than non-exporters (mean domestic sales in the subpopulation of exporters is larger than in the overall population).

Table B.2: Firm-level Statistics

Years	Domestic Sales (All Firms)		Domestic Sales (Exporters)		Exports		Number of Destinations (Exporters)		
	Average	Median	Average	Median	Average	Median	Average	Median	95th/99th perc.
2005	684.4	119.4	3,312.0	822.9	656.4	63.4	3.38	2	10/17
2006	695.4	118.4	3,553.2	772.6	731.0	63.1	3.28	2	10/18
2007	782.4	131.7	3,864.6	904.3	788.9	63.7	3.35	2	10/16
2008	889.6	147.0	4,693.6	1,160.0	885.9	66.4	3.30	2	9/18
2009	839.1	126.4	4,682.5	1,033.4	749.9	43.4	3.19	2	10/18
2010	937.2	139.2	5,256.7	1,161.1	803.8	56.7	3.28	2	9/18
2011	1,031.9	147.4	5,601.4	1,201.7	947.0	56.3	3.25	2	9/19
2012	1,067.5	154.1	5,663.2	1,091.7	1,045.3	65.9	3.22	2	9/19
2013	1,098.9	158.1	5,922.9	1,178.6	1,030.8	78.2	3.35	2	10/17
2014	1,043.8	147.4	5,793.3	1,208.3	1,133.2	59.7	3.28	2	10/18
2015	1,166.0	155.8	6,809.5	1,566.5	1,213.9	80.5	3.62	2	11/20

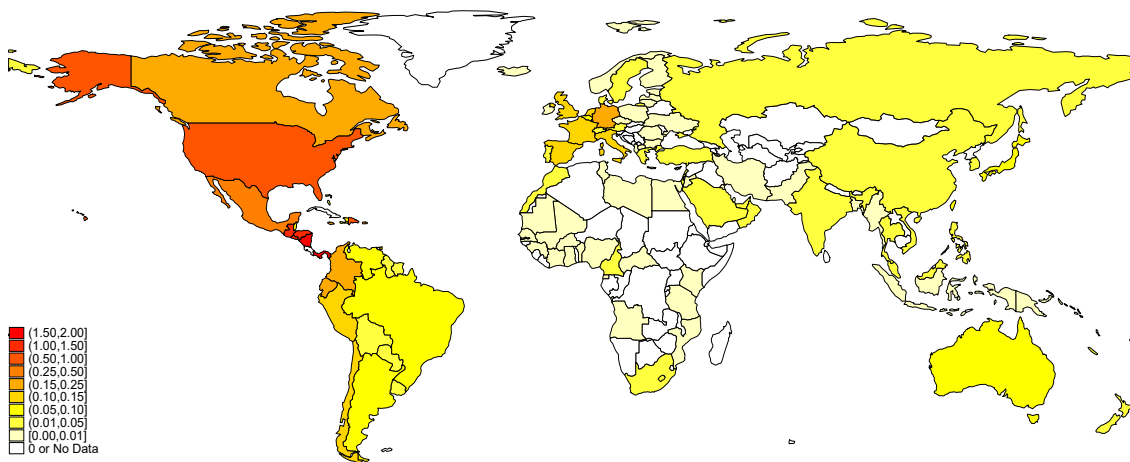
Notes: *Domestic sales* and *Exports* are reported in thousands of 2013 dollars.

In Table B.2, we also report export revenues for the mean and median exporters in each sample year. Consistently with the fact that, between 2005 and 2015, total exports grew significantly while the total number of exporters remained roughly constant, we observe that the aggregate export revenue of the mean exporting firm also grew during the same period. Specifically, while total exports grew by 82% between 2005 and 2015, total export revenues for the average exporter grew at nearly the same rate, 85%.

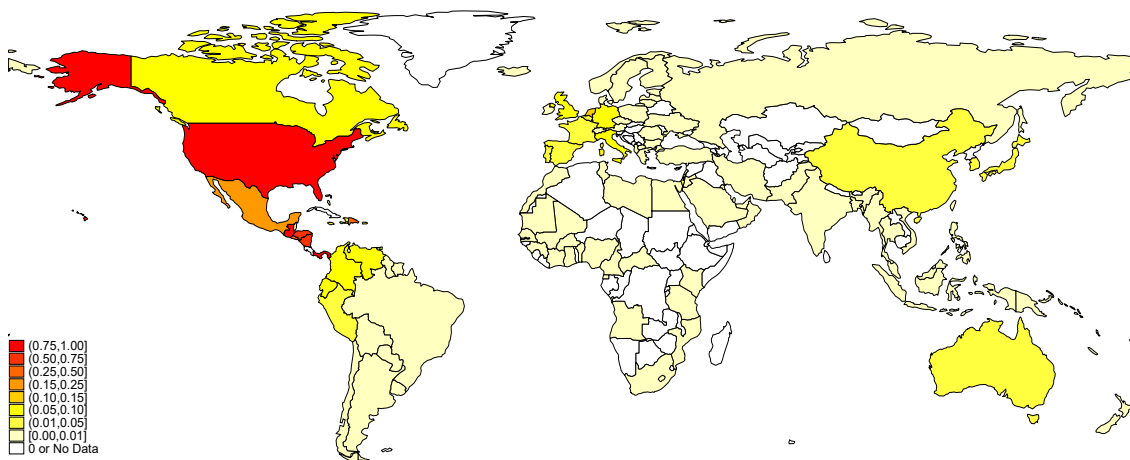
The last three columns in Table B.2 report several statistics of the distribution of the number of export destinations across firms. Three features of this distribution are apparent. First, it is very skewed. Note that the difference in the number of destinations between the median exporter and that at the 95th percentile (approximately 8 destinations) is the same as the difference between the exporter at the 95th percentile and that at the 99th percentile. Second, some firms export to a large number of destinations; the 95% percentile of the distribution of the number of export destinations is approximately 10, and the 99th percentile oscillates between 17 and 20. Third, the distribution of the number of export destinations is very stable over time. Consequently, the growth in average and median exports documented in Table B.2 is not explained by a hypothetical growth in the number of export destinations.

Figure B.1: Export Activity by Destination Country During Period 2005-2015

(a) Total Number of Export Events



(b) Total Volume of Exports



Notes: Panel (a) shows the total number of firm-year pairs with positive exports relative to that in the United States. Panel (b) shows the total volume of manufacturing exports relative to that in the United States.

In terms of the distribution of export activity across destinations, the maps in Figure B.1 reflect the total number of export events (i.e., firm-year pairs with positive exports) and the total volume of exports by destination during the sample period 2005-2015, in both cases relative to the corresponding magnitude in the United States. Both maps show that the most popular destination for Costa Rican manufacturing exports are countries in North and Central America, followed by China, Australia, and countries in Western Europe. Specifically, the top 5 destinations by total volume of exports during the sample period are the United States, Guatemala, Panama, Nicaragua and Honduras.

In Table B.3, we present the mean and several percentiles of the distribution of annual firm-level exports to several countries over the period 2005-2015. The distribution of annual firm-level exports by market is very skewed to the right; e.g., while median exports to the US are approximately \$28,000, mean exports are close to \$600,000. The second feature to remark is that there is a large dispersion in annual firm-level exports by destination over the sample period; while the 25th percentile of the distribution of annual firm-level exports is below \$10,000 for all destinations considered in Table B.3, the 95th percentile is either above \$1,000,000 or close to it.

Table B.3: Distribution of Export Sales in Several Markets

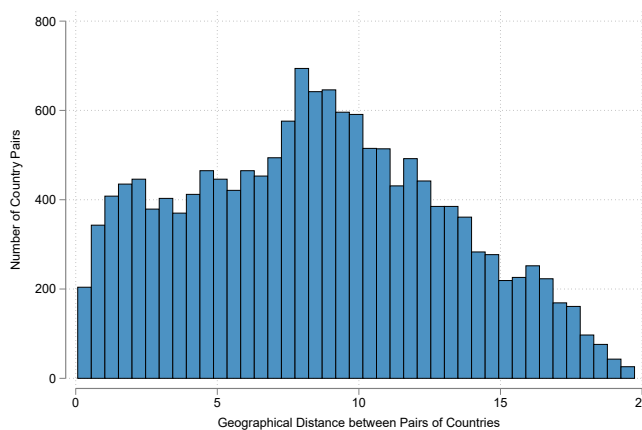
Country	Average	Percentiles					
		5	25	50	75	95	99
United States	597.6	0.4	5.0	28.1	227.4	3,477.9	9,615.9
Panama	271.4	1.2	7.4	32.5	138.6	1,013.6	5,022.9
Germany	350.8	0.3	6.3	54.0	419.5	1,844.9	3,015.5
Nicaragua	209.8	1.2	8.7	37.6	134.5	879.5	3,013.9
Mexico	295.4	0.4	9.0	51.0	284.2	1,224.8	2,637.1
China	128.8	0.2	3.9	21.8	68.9	713.7	1,584.0

Notes: All numbers in this table are reported in thousands of 2013 dollars.

B.2 Geographical Distance

In Figure B.2, we present a histogram of the geographical distance, computed according to the formula in equation (1), between any pair of countries. As Figure B.2 reveals, the most typical distance is approximately 7,000 kilometers, but there is a wide disparity in geographical distance across country pairs.

Figure B.2: Histogram of Bilateral Geographic Distances

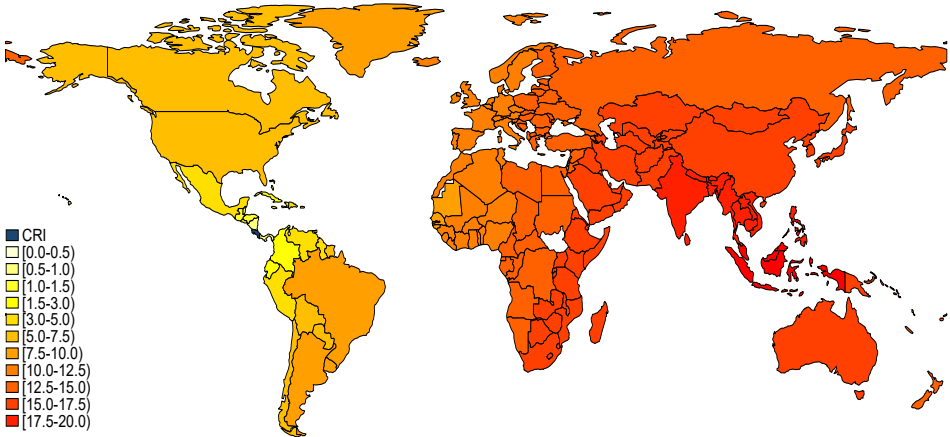


Notes: The vertical axis indicates the number of country pairs whose geographical distance according to equation (1) falls in the corresponding bin. The horizontal axis denotes geographical distance in thousands of kilometers.

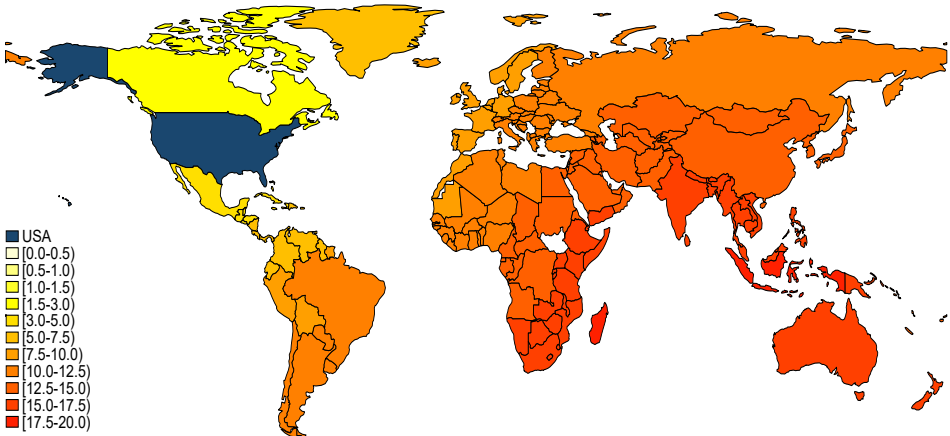
In Figure B.3, we represent in maps the geographical distance from Costa Rica (in Figure B.3a), the United States (in Figure B.3b), France (in Figure B.3c) and China (in Figure B.3d), respectively, to any other country of the world.

Figure B.3: Geographical Distances From Certain Countries

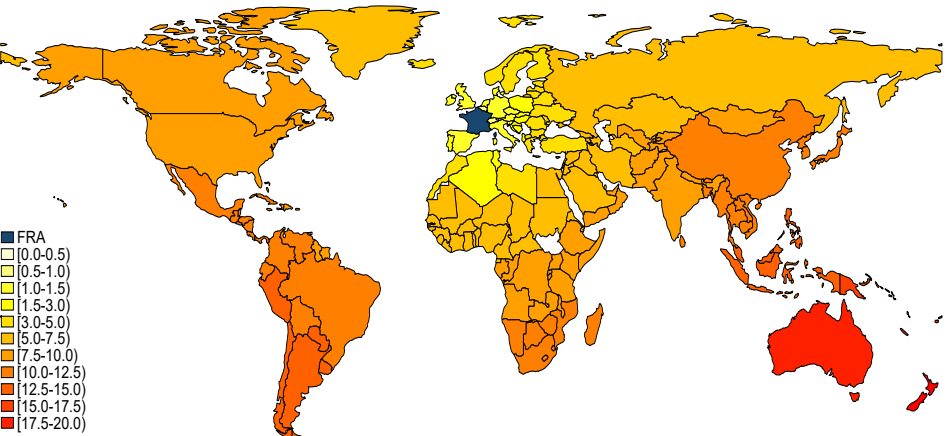
(a) From Costa Rica



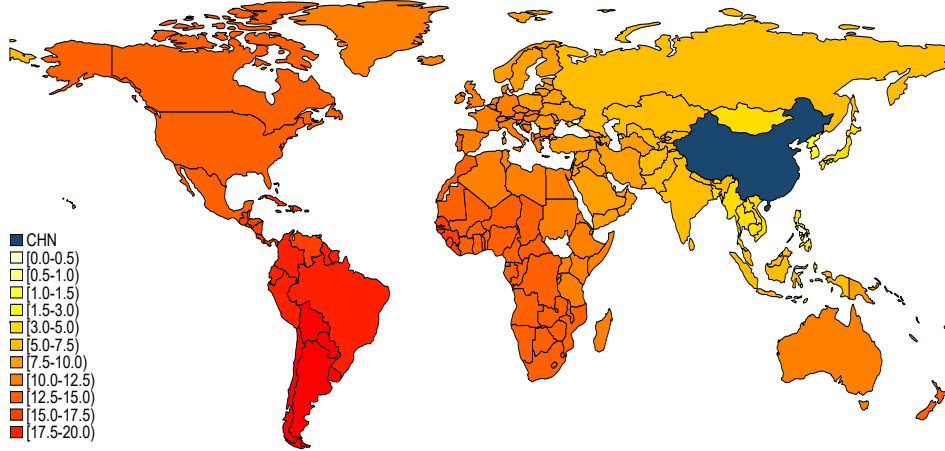
(b) From the United States



(c) From France



(d) From China

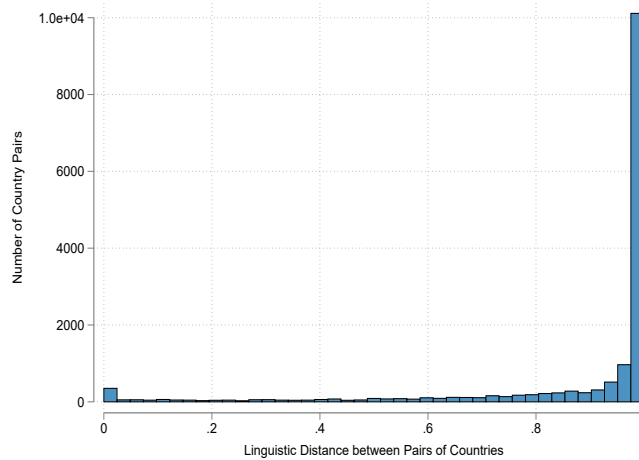


Notes: Each of the panels in this figure indicate the geographical distance (computed according to the expression in equation (1)) between a particular country (Costa Rica in Panel (a), the US in Panel (b), France in Panel (c), and China in Panel (d)) and any other country in the world. All distances are reported in thousands of kilometers.

B.3 Linguistic Distance

The value of the linguistic distance measure n_{jj}^l , introduced in equation (2) depends on the language definition. Ethnologue defines languages according to 15 aggregation levels; e.g., at the 1st level, all Indo-European languages are considered the same language; at the 15th level, Spanish and Extremaduran are distinct. We use the 9th aggregation level, the first one classifying Portuguese and Spanish as distinct. In Figure B.4, we present a histogram of the implied bilateral linguistic distance measures. As Figure B.4 reveals, for most country pairs, a randomly selected resident of one of the two countries will not share either first or second language with a randomly selected resident of the other country. Therefore, for most country pairs, their linguistic distance equals 1, which is the maximum possible value of the linguistic distance measure introduced in equation (2).

Figure B.4: Histogram of Bilateral Linguistic Distances

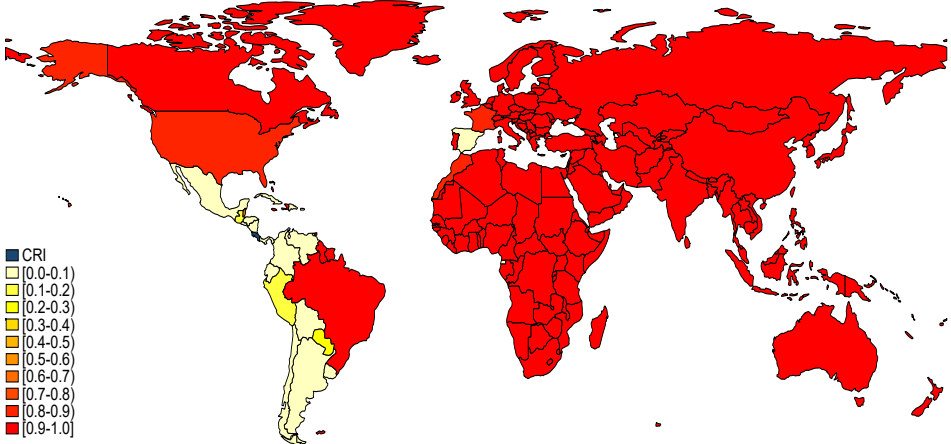


Notes: The vertical axis indicates the number of country pairs whose linguistic distance according to the formula in equation (2) falls in the corresponding bin. The horizontal axis denotes the corresponding linguistic distance.

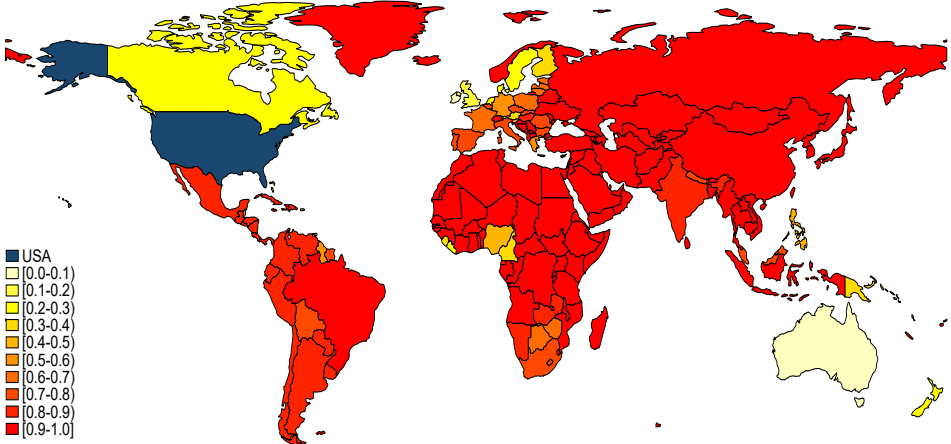
In Figure B.5, we represent bilateral linguistic distance measures from Costa Rica (in Figure B.5a), the US (in Figure B.5b), France (in Figure B.5c) and China (in Figure B.5d) to any other country of the world.

Figure B.5: Bilateral Linguistic Distances From Certain Origin Countries

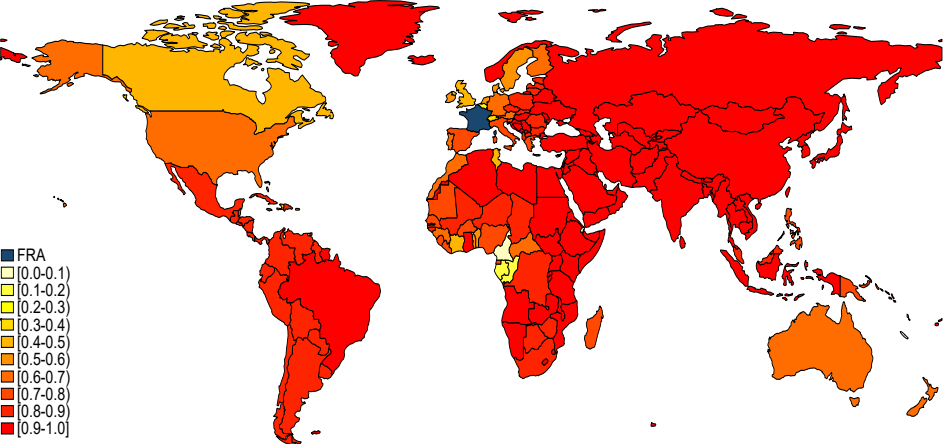
(a) From Costa Rica



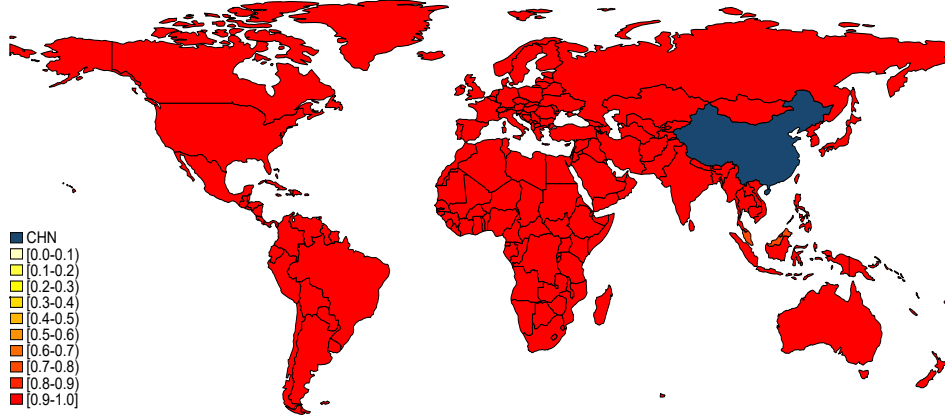
(b) From the United States



(c) From France



(d) From China



Notes: Each of the four panels in this figure indicate the linguistic distance (computed according to the expression in equation (2)) between a particular country (Costa Rica in Panel (a), the US in Panel (b), France in Panel (c), and China in Panel (d)) and any other country in the world.

The distance measures in Figure B.5a reflect the extent of the network of countries where Spanish is the most commonly spoken language; as the map illustrates, with the only exception of Spain, these countries are geographically close to Costa Rica. The measures in Figure B.5b reveal that the popularity of the English language as second language in many European countries implies that, according to the distance measure in equation (2), countries such as the Netherlands, Denmark, or Sweden, are linguistically very close to the US. Interestingly, as Figure B.5c reveals, the popularity of the English language as second language makes that certain countries that do not have English as official language (e.g., France and Sweden, France and Denmark) are linguistically close, the main reason being their residents can often communicate in English. Finally, Figure B.5d reveals that countries such as China, in which a large share of their residents speak neither English nor Spanish, will be generally isolated from a linguistic perspective. Specifically, China only exhibits some linguistic proximity with Malaysia.

As discussed in footnote 9, Ethnologue provides information by country on the population shares that speak any given language as first and second language, but it does not provide information on the distribution of second language speakers conditional on their first language. The measure in equation (2) assumes a joint distribution of first and second languages in each country such that the linguistic distance between any two countries is minimized. To illustrate this point, consider a simplified setting in which there are only two languages in the world, k_1 and k_2 . In this setting, the probability that two individuals i and i' randomly selected from any two given countries j and j' , respectively, speak a common language is:

$$\begin{aligned}
& P(\{i \text{ speaks } k_1\} \cap \{i' \text{ speaks } k_1\}) \cup (\{i \text{ speaks } k_2\} \cap \{i' \text{ speaks } k_2\})) \\
& = \\
& P(\{i \text{ speaks } k_1\} \cap \{i' \text{ speaks } k_1\}) + P(\{i \text{ speaks } k_2\} \cap \{i' \text{ speaks } k_2\}) - \\
& P(\{i \text{ speaks } k_1\} \cap \{i' \text{ speaks } k_1\}) \cap (\{i \text{ speaks } k_2\} \cap \{i' \text{ speaks } k_2\})).
\end{aligned}$$

Using the notation in the main text, we can rewrite this expression as

$$\begin{aligned}
& P(\{i \text{ speaks } k_1\} \cap \{i' \text{ speaks } k_1\}) \cup (\{i \text{ speaks } k_2\} \cap \{i' \text{ speaks } k_2\})) = \\
& s_{jk_1}s_{j'k_1} + s_{jk_2}s_{j'k_2} - P(\{i \text{ speaks } k_1\} \cap \{i' \text{ speaks } k_1\}) \cap (\{i \text{ speaks } k_2\} \cap \{i' \text{ speaks } k_2\})),
\end{aligned}$$

and we can thus write the probability that two randomly selected individuals from countries j and j' do not speak a common language as

$$1 - s_{jk_1}s_{j'k_1} - s_{jk_2}s_{j'k_2} + P(\{i \text{ speaks } k_1\} \cap \{i' \text{ speaks } k_1\}) \cap (\{i \text{ speaks } k_2\} \cap \{i' \text{ speaks } k_2\})).$$

As the Ethnologue data does not contain information on the joint distribution of first and second languages

within a country, we cannot compute

$$P(\left(\{i \text{ speaks } k_1\} \cap \{i' \text{ speaks } k_1\}\right) \cap \left(\{i \text{ speaks } k_2\} \cap \{i' \text{ speaks } k_2\}\right)).$$

Given information on s_{jk_1} , s_{jk_2} , $s_{j'k_1}$, and $s_{j'k_2}$, we can however obtain a lower bound on this probability; denoting this lower bound as $LB_{jj'}$, it holds that

$$LB_{jj'} = \begin{cases} 0 & \text{if } s_{jk_1}s_{j'k_1} - s_{jk_2}s_{j'k_2} \leq 1, \\ s_{jk_1}s_{j'k_1} + s_{jk_2}s_{j'k_2} - 1 & \text{if } s_{jk_1}s_{j'k_1} + s_{jk_2}s_{j'k_2} > 1, \end{cases}$$

or, equivalently,

$$LB_{jj'} = \max\{0, s_{jk_1}s_{j'k_1} + s_{jk_2}s_{j'k_2} - 1\}.$$

Consequently, we can obtain a lower bound on the probability that two randomly selected individuals from countries j and j' do not speak a common language as

$$1 - s_{jk_1}s_{j'k_1} - s_{jk_2}s_{j'k_2} + LB_{jj'} = 1 - s_{jk_1}s_{j'k_1} - s_{jk_2}s_{j'k_2} + \max\{0, s_{jk_1}s_{j'k_1} + s_{jk_2}s_{j'k_2} - 1\}$$

or, equivalently,

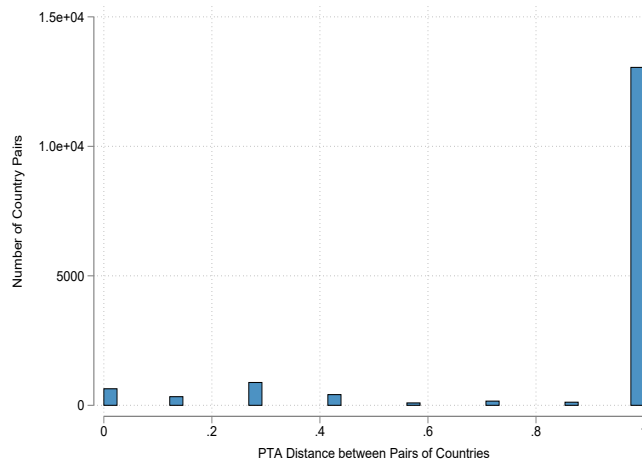
$$\max\{0, 1 - s_{jk_1}s_{j'k_1} - s_{jk_2}s_{j'k_2}\}.$$

This expression corresponds to that in equation (2) for the simple case in which there are only two languages in the world, k_1 and k_2 .

B.4 Measures of Regulatory Distance

In Figure B.6, we present a histogram of an inverse measure of the breadth of the regulatory harmonization imposed by preferential trade agreements (PTAs), computed according to the formula in equation (3).

Figure B.6: Histogram of Bilateral Distances in PTAs



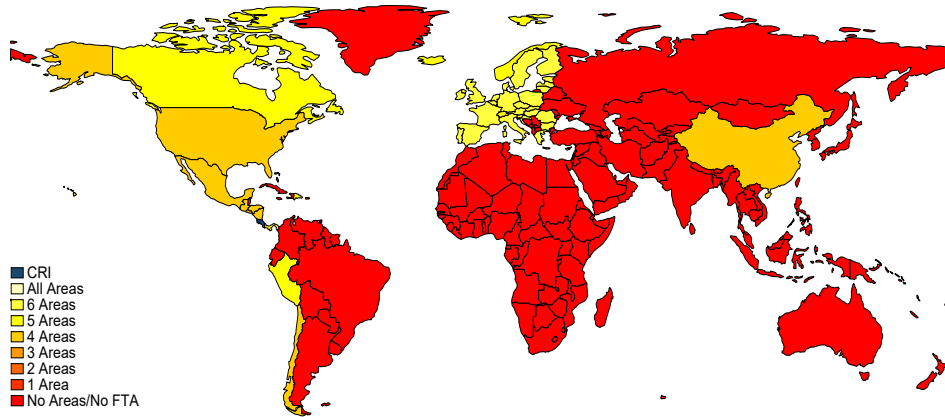
Notes: The vertical axis indicates the number of country pairs whose distance according to the formula in equation (3) falls in the corresponding bin. The horizontal axis denotes the value of the distance measure in equation (3).

As Figure B.6 reveals, most country pairs do not share any PTA that contains a provision in at least one of the seven policy areas listed in footnote 11. Therefore, for most country pairs, the distance measure introduced in equation (3) equals one, which is the maximum possible value this distance measure may take.

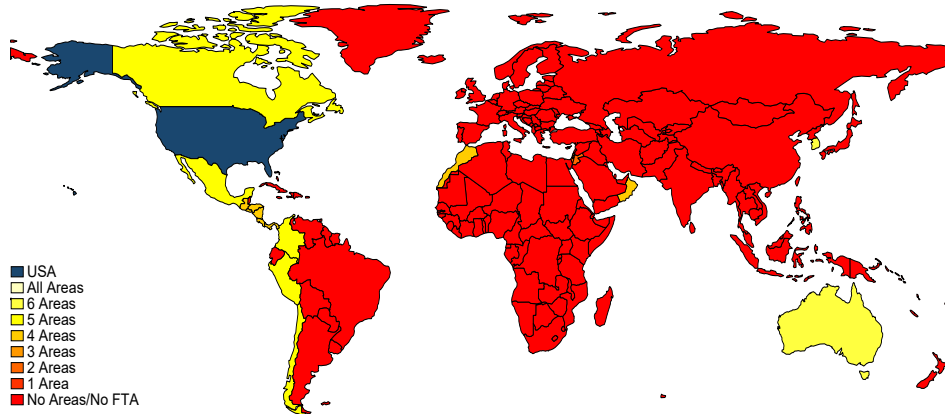
In Figure B.7, we illustrate the countries with which Costa Rica, the United States, France, and China, respectively, share in 2015 a PTA containing provisions in at least one of the seven policy areas listed in footnote 11. Whenever two countries had signed a PTA with a provision in one of these seven areas, Figure B.7 also indicates in how many of these areas the corresponding PTA includes a provision.

Figure B.7: Bilateral Regulatory Distances From Certain Origin Countries

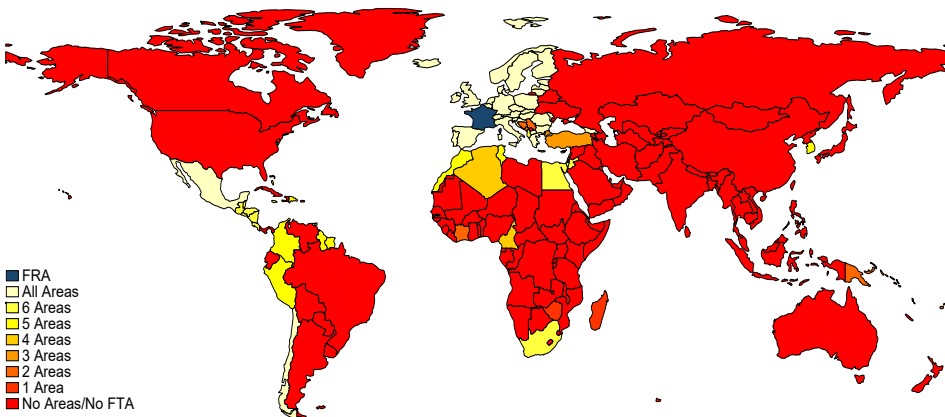
(a) From Costa Rica



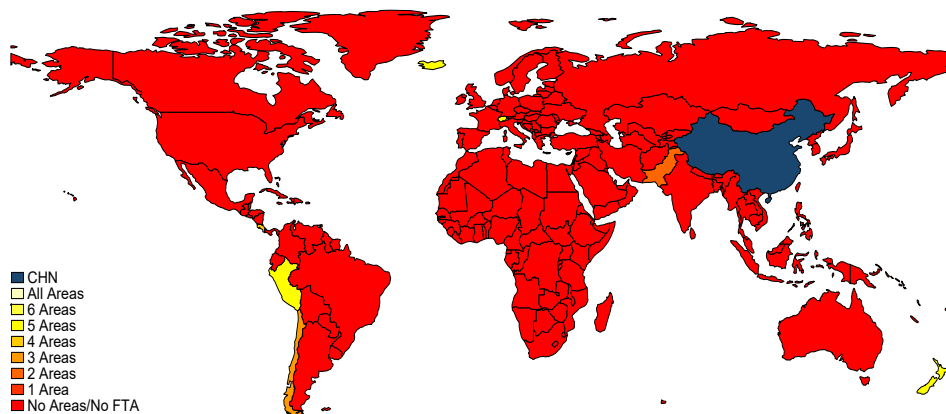
(b) From the United States



(c) From France



(d) From China



Notes: Each of the four panels in this figure illustrate the countries with which Costa Rica (in panel (a)), the United States (in panel (b)), France (in panel (c)), and China (in panel (d)) share in 2015 a PTA containing provisions in at least one of the seven policy areas listed in footnote 11. If it does, it indicates in how many of the seven policy areas listed in footnote 11 the corresponding preferential trade agreement contains some provision.

Figure B.7a reveals that Costa Rica has very deep integration agreements with Canada, members of the European Common Market, Panama, the Dominican Republic, and Peru, and slightly less deep agreements with China, Chile, and other Central and North American countries. Figure B.7b shows that the US has a relatively deep PTA with Canada and Mexico (the NAFTA agreement), as well as with Colombia, Peru, Chile and Australia (these four are bilateral trade agreements), and a relatively less deep agreement with Central American countries (the CAFTA agreement). In the case of France, Figure B.7c illustrates that it has deep trade integration agreements not only with the other members of the European Common Market, but also with countries in North America (Mexico), Central America (e.g., Guatemala, Honduras, or Costa Rica), South America (e.g., Colombia, Peru, or Chile), Africa (e.g., Morocco, Tunisia, Egypt, or South Africa), and Asia (South Korea). Conversely, Figure B.7d illustrates that China has deep trade integration agreements with comparatively few and smaller countries (e.g., Iceland, Switzerland, Peru, or New Zealand).

In sum, the four panels in Figure B.7 show that countries differ significantly in the number and identity of the potential trade partners with whom they have signed deep PTAs. Furthermore, it is common for countries to sign deep PTAs with other countries that are neither geographically nor linguistically close (e.g., Costa Rica and China, the US and South Korea, France and South Africa, or China and Iceland).

B.5 Correlation in Export Participation Decisions: Additional Results

We present here estimates analogous to those in Section 3, but for alternative threshold values \bar{n}_g , \bar{n}_l , and \bar{n}_a . While we set $\bar{n}_g = 0.79$ (or 790 km), $\bar{n}_l = 0.11$, and $\bar{n}_a = 0.43$ in the main text, we set here instead $\bar{n}_g = 1.153$ (or 1,153 km), $\bar{n}_l = 0.5$ and $\bar{n}_a = 0.78$. The values of \bar{n}_g and \bar{n}_l we use here equal the 5th percentile of the distribution of the corresponding distance measure between any pair of countries in our sample; the value $\bar{n}_a = 0.72$ is equivalent to characterizing as deep any PTA that contains a provision in at least two of the seven policy areas listed in footnote 11.

In Table B.4, we present OLS estimates analogous to those in Table 1. A comparison of the estimates in these two tables reveals that, as we increase the set of countries classified as being geographically or linguistically close to a destination j , or as being cosignatories of a deep PTA with j , the impact that exporting to at least one of these countries has on the probability of exporting to destination j decreases.

Comparing the estimate of the parameter on Y_{ijt}^g in column (4) of Panel D in Table 1 to that in Table B.4, we observe that the difference in the predicted export probability to any given destination is 18.1% when comparing firms that export to at least one country that is less than 790 km away from it to those that do not, but only 11.2% when comparing firms that export to at least one country that is less than 1153 km away from it to those that do not. This is consistent with the correlation between a firm's export

Table B.4: Conditional Export Probabilities

	<i>Panel A:</i> <i>No Controls</i>				<i>Panel B:</i> <i>Controlling for Firm-Year Fixed Effects</i>			
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
Y_{ijt}^g	0.1904 ^a (0.0072)			0.1345 ^a (0.0059)	0.1529 ^a (0.0068)			0.1217 ^a (0.0060)
Y_{ijt}^l		0.1334 ^a (0.0057)		0.0733 ^a (0.0038)		0.1091 ^a (0.0050)		0.0760 ^a (0.0041)
Y_{ijt}^a			0.0825 ^a (0.0037)	0.0297 ^a (0.0016)			0.0517 ^a (0.0026)	0.0222 ^a (0.0018)
Obs.			3,859,618				3,859,618	
	<i>Panel C:</i> <i>Controlling for Sector-Country-Year Fixed Effects</i>				<i>Panel D:</i> <i>Controlling for Firm-Year & Sector-Country-Year Fixed Effects</i>			
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
Y_{ijt}^g	0.1785 ^a (0.0069)			0.1269 ^a (0.0057)	0.1384 ^a (0.0065)			0.1116 ^a (0.0057)
Y_{ijt}^l		0.1277 ^a (0.0054)		0.0706 ^a (0.0035)		0.1013 ^a (0.0048)		0.0721 ^a (0.0039)
Y_{ijt}^a			0.0779 ^a (0.0035)	0.0283 ^a (0.0015)			0.0431 ^a (0.0025)	0.0169 ^a (0.0017)
Obs.			3,859,618				3,859,618	

Note: ^a denotes 1% significance. Standard errors clustered by firm. The dependent variable in all specifications is a dummy that equals one if firm i exports to country j in year t . The covariates are $Y_{ijt}^x = \mathbb{1}\{\sum_{j' \neq j} \mathbb{1}\{n_{jj'}^x \leq \bar{n}_x\} y_{ij't} > 0\}$ for $x \in \{g, l\}$, and $Y_{ijt}^a = \mathbb{1}\{\sum_{j' \neq j} \mathbb{1}\{n_{jj't}^a \leq \bar{n}_a\} y_{ij't} > 0\}$, with $\bar{n}_g = 1.153$, $\bar{n}_l = 0.5$ and $\bar{n}_a = 0.78$.

participation decisions in any two countries decreasing in the geographic distance between both countries.

Similarly, comparing the estimate of the parameter on Y_{ijt}^a in column (4) of Panel D in Table 1 to that in Table B.4, we observe that the difference in the predicted export probability to any given destination between firms that export to at least one country that shares a deep PTA with it and those that do not decreases from 2.1% to 1.7% as we loosen the requirements that a PTA must satisfy for us to classify it as “deep.” This is consistent with the correlation between a firm’s export participation decisions in any two countries increasing in the deepness of the PTAs linking both countries.

Finally, the estimate of the parameter on Y_{ijt}^l in column (4) of Panel D in Table 1 is very similar to that in Table B.4. In this case, the correlation between a firm’s export participation decisions in any two countries seems not to vary much depending on whether the probability that two randomly chosen individuals, one from each country, understand each other is at least 0.89 (i.e., $\bar{n}_l = 0.11$, the threshold imposed in Table 1) or 0.5 (i.e., $\bar{n}_l = 0.5$, the threshold imposed in Table B.4). A possible explanation for this fact is that exporters select into their workforce workers knowledgeable of the languages spoken in the countries where they export and, consequently, the general prevalence of a language in a country is an imperfect predictor of the language barriers that exporting firms experience.

C Equation for Potential Export Revenues: Details

We derive in three steps the expression in equation (7).

First Step. As firm i 's marginal cost of selling in the home market h at period t is $\tau_{ht}w_{it}$ (see Section 4.2), the revenue firm i obtains in h at a period t is

$$r_{iht} = \left[\frac{\eta}{\eta - 1} \frac{\tau_{ht}w_{it}}{P_{ht}} \right]^{1-\eta} Y_{ht}. \quad (\text{C.1})$$

Combining equations (5) and (C.1), we rewrite the potential export revenues of firm i in country j at period t as a function of its revenue in the domestic market:

$$r_{ijt} = \left[\frac{\tau_{ijt} P_{ht}}{\tau_{ht} P_{jt}} \right]^{1-\eta} \frac{Y_{jt}}{Y_{ht}} r_{iht}. \quad (\text{C.2})$$

Second Step. Substituting $(\tau_{ijt})^{1-\eta}$ in equation (C.2) by its expression in equation (6), we obtain

$$r_{ijt} = \exp(\xi_y y_{ijt-1} + \check{\xi}_{jt} + \alpha_s + \alpha_a \ln(a_{s jt}) + \xi_w \ln(w_{it}) + \ln(r_{iht})), \quad (\text{C.3})$$

with $\alpha_s = \xi_s$, $\alpha_a = \xi_a$, and

$$\check{\xi}_{jt} = \xi_{jt} + (1 - \eta) \ln(P_{ht}/P_{jt}) + \ln(Y_{jt}/Y_{ht}) - (1 - \eta) \ln(\tau_{ht}). \quad (\text{C.4})$$

Third Step. Taking the logarithm of both sides of equation (C.1) and rearranging terms, we obtain

$$\ln(w_{it}) = \frac{1}{1 - \eta} (\ln(r_{iht}) - \ln(Y_{ht})) + \ln(\eta - 1) - \ln(\eta) + \ln(P_{ht}) - \ln(\tau_{ht}).$$

Plugging this expression into equation (C.3), we obtain the expression for r_{ijt} in equation (7) with

$$\alpha_{jt} = \check{\xi}_{jt} + \xi_w (-1/(1 - \eta)) \ln(Y_{ht}) + \ln(\eta - 1) - \ln(\eta) + \ln(P_{ht}) - \ln(\tau_{ht}), \quad (\text{C.5a})$$

$$\alpha_r = 1 + \xi_w / (1 - \eta). \quad (\text{C.5b})$$

D Solution Algorithm: Additional Details

D.1 Additional Steps

We discuss here how we tighten the upper bounds on firm choices at a period τ ; the procedure for the lower bound being analogous. Once the optimal choice at the path of interest at τ is determined, we proceed to apply the step 2 algorithm to the next period τ' at which the bounds in equation (27) differ.

Step 3. In this step, we tighten further the bounds at period τ . To do so, for every country j for which the bounds in equation (30) do not coincide, we solve a problem that differs from that in equation (24) in that, for period $\tau + 1$ and a subset of countries M that does not include j , we condition on *functional* (instead of constant) upper bounds. Specifically, for any j such that $\check{y}_{ij\tau|\tau} > \check{y}_{ij\tau|\tau}$, we find the solution to

$$\max_{y_{ij\tau}} \left\{ y_{ij\tau} (u_{ij\tau}(\check{y}_{ij\tau-1}, \check{\omega}_{ij\tau}) + \sum_{j' \neq j} \check{y}_{ij'\tau|\tau} (c_{jj'\tau} + c_{j'j\tau})) + \delta \mathbb{E}_{i\tau} \tilde{V}_{ij\tau+1}(y_{ij\tau}, \omega_{ij\tau+1}, \{\omega_{ij'\tau+1}\}_{j' \in M}) \right\}, \quad (\text{D.1})$$

with

$$\begin{aligned} \tilde{V}_{ij\tau+1}(y_{ij\tau}, \omega_{ij\tau+1}, \{\omega_{ij'\tau+1}\}_{j' \in M}) &= \max_{y_{ij\tau+1}} \left\{ y_{ij\tau+1} (u_{ij\tau+1}(y_{ij\tau}, \omega_{ij\tau+1}) + \delta \mathbb{E}_{i\tau+1} \bar{V}_{ij\tau+2}(y_{ij\tau+1}, \omega_{ij\tau+2})) \right. \\ &\quad \left. + \sum_{j' \in M} \bar{b}_{ij'\tau+1|\tau}(\omega_{ij'\tau+1})(c_{jj'\tau+1} + c_{j'j\tau+1}) + \sum_{j' \notin M} \mathbb{1}\{j \neq j'\} \bar{b}_{ij'\tau+1|\tau}^*(c_{jj'\tau+1} + c_{j'j\tau+1}) \right\}. \end{aligned} \quad (\text{D.2})$$

The function $\tilde{V}_{ij\tau+2}(y_{ij\tau+1}, \omega_{ij\tau+2})$ is country j 's value function when the firm's choice in every period $t \geq \tau + 2$ and every country other than j is set to the constant upper bounds obtained in the last iteration of the step 2 procedure. For every country j' other than j , equation (D.2) imposes the upper bounds

$$\bar{b}_{ij'\tau+1|\tau}(\omega_{ij'\tau+1}) = \bar{o}_{ij'\tau+1|\tau}^*(\check{y}_{ij\tau|\tau}, \omega_{ij'\tau+1}), \quad \text{if } j' \in M, \quad (\text{D.3a})$$

$$\bar{b}_{ij'\tau+1|\tau}^* = \bar{o}_{ij'\tau+1|\tau}^*(\check{y}_{ij\tau|\tau}, \omega), \quad \text{if } j' \notin M, \quad (\text{D.3b})$$

where $\bar{o}_{ij'\tau+1|\tau}^*(\cdot)$ and $\check{y}_{ij\tau|\tau}$ are computed in step 2. By definition, $\bar{b}_{ij'\tau+1|\tau}(\omega_{ij'\tau+1}) \leq \bar{b}_{ij'\tau+1|\tau}^*$ for any j' , and, thus, the bounds computed in step 3 are tighter than those computed in step 2, and they will be tighter the larger the set M . However, solving the problem in equation (D.1) requires computing an expectation over the vector $(\omega_{ij\tau+1}, \{\omega_{ij'\tau+1}\}_{j' \in M})$, a step that is computationally more complicated the larger the cardinality of M . In our application, for each country j , we choose M as the 16 countries that are geographically closer to j . If the step 3 upper and lower bounds do not coincide at $(\check{y}_{i\tau-1}, \check{\omega}_{i\tau})$ at τ , we proceed to step 4.

Step 4. In this step, we tighten further the bounds at period τ . To do so, we solve an optimization problem that differs from those solved in steps 1 to 3 in that, instead of computing policy functions iteratively country by country, we do so for several countries simultaneously.

Consider a set M of countries for which step 3 upper and lower bounds on the firm's optimal choices at the path of interest do not coincide at τ . For any $t \geq \tau$, define vectors y_{iMt} and ω_{iMt} that, for t and all countries j in M , include firm i 's export choice y_{ijt} and blocking shock ω_{ijt} , respectively. Define also

$$\bar{V}_{iM\tau+h}(y_{iM\tau+h-1}, \omega_{iM\tau+h}) = \sum_{j \in M} \bar{V}_{ij\tau+h}(y_{ij\tau+h-1}, \omega_{ij\tau+h}), \quad (\text{D.4})$$

where, as above, $\bar{V}_{ij\tau+h}(\cdot)$ is the country j 's value function that results from equating the firm's choice in all periods $t \geq \tau + h$ and all countries other than j to the constant upper bounds obtained in the last iteration of the step 2 procedure. In step 4, we solve by backward induction for all $t \in [\tau, \tau + h - 1]$ the problem

$$\begin{aligned} \bar{V}_{iMt}(y_{iMt-1}, \omega_{iMt}) &= \max_{y_{iMt} \in \{0,1\}^M} \left\{ \sum_{j \in M} \left\{ y_{ijt} (u_{ijt}(y_{ij\tau-1}, \omega_{ijt}) + \sum_{j' \in M} y_{ij't} c_{j'jt}) \right. \right. \\ &\quad \left. \left. + \sum_{j' \notin M} \mathbb{1}\{j \neq j'\} \bar{b}_{ij'\tau|\tau}^*(c_{jj't} + c_{j'jt}) \right\} + \delta \mathbb{E}_{it} \bar{V}_{iM\tau+1}(y_{iMt}, \omega_{iM\tau+1}) \right\}, \end{aligned} \quad (\text{D.5})$$

with $\bar{b}_{ij'\tau|_\tau}^*$ and $\bar{V}_{iM\tau+h}(\cdot)$ defined as in equations (D.3b) and (D.4), respectively. Solving this problem is computationally more complicated the larger the set M and the horizon h are. In our application, if there are less than ten countries for which step 3 upper and lower bounds on the optimal choice at the path of interest at period τ differ, we include them all in M . If there are more than ten countries for which the step 3 bounds differ, we solve the problem in equation (D.5) repeatedly for different sets of ten countries, grouping together in these sets countries that are geographically close to each other. Concerning h , we solve first the problem for $h = 1$, and increase progressively the value of h until $h = 10$.

Step 5. In this step, we tighten further the bounds at τ . To do so, we compute the firm's optimal export paths in a set M of countries fixing the firm's choices in all countries not in M to constant upper bound. Specifically, in step 5, we first solve the following period- T problem for every value of $(y_{iMT-1}, \omega_{iMT})$:

$$\bar{V}_{iMT}(y_{iMT-1}, \omega_{iMT}) = \max_{y_{iMT} \in \{0,1\}^M} \left\{ \sum_{j \in M} \{y_{ijT}(u_{ijT}(y_{ijT-1}, \omega_{ijT}) + \sum_{j' \in M} y_{ij'T}c_{jj'T} + \sum_{j' \notin M} \mathbb{1}\{j \neq j'\} \bar{b}_{ij'T|_\tau}^*(c_{jj'T} + c_{j'jT}))\} + \delta \mathbb{E}_{iT} \bar{V}_{iMT+1}(y_{iMT}, \omega_{iMT+1}) \right\}. \quad (\text{D.6})$$

As this problem is stationary, we use value-function iteration to solve for the value function $\bar{V}_{iMT}(\cdot)$. Given $\bar{V}_{iMT}(\cdot)$, we use backward induction to solve for the optimal policy function in M for all $t \in [\tau, T]$.

If M includes all J foreign countries, the problem in equation (D.6) coincides with that in equation (20) and, thus, its solution yields the firm's optimal policy function. Solving the problem in equation (20) for a large set M is however computationally infeasible. In our application, we choose M according to the following rules. If there are less than six countries for which step 4 upper and lower bounds on the optimal choice at the path of interest at period τ differ, we include them all in M . If there are more than six countries for which the step 4 bounds differ, we implement the step 5 algorithm repeatedly for different sets of six countries grouping together countries that are geographically close to each other.

Closing the algorithm. If there are countries for which the upper and lower bound on the optimal choice at the path of interest at period τ differ after step 5, we assume the optimal choice is to not export to those countries at τ at the state of interest.

D.2 Illustration of Algorithm in a Two-Country and Three-Period Setting

We illustrate here our algorithm in an example with two countries (A and B) and three periods. We use trees to represent graphically all possible paths of ω_{ijt} . With the letters L (with stands for low) and H (which stands for high), we denote the events in which the blocking shock ω_{ijt} respectively equals the smallest, $\underline{\omega}$, and largest, $\bar{\omega}$, values in their support. For e.g., in Figure D.1, the orange path is one in which blocking shocks in A are low in all three periods while, in B , these are low in periods 1 and 3, and high in period 2.

Figure D.1: Possible Paths of Fixed Cost Shocks

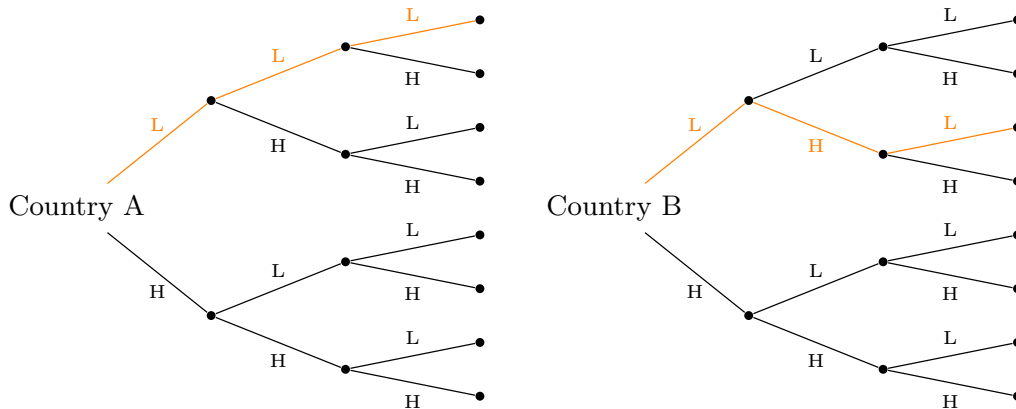
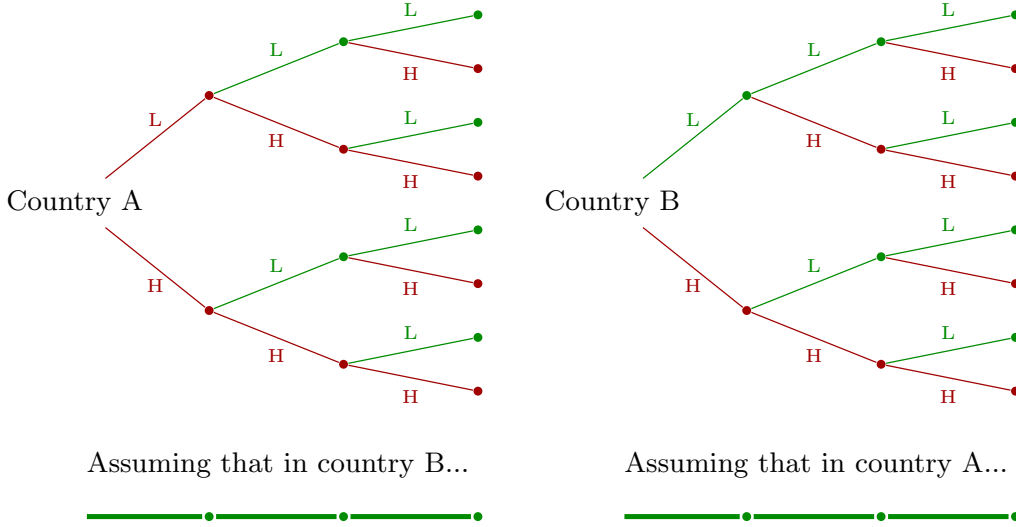


Figure D.2: Initial Upper-Bound Policy Functions



Step 1. In Figure D.2, we illustrate the first iteration of step 1 of the algorithm (see Section 5). The left panel illustrates the solution to the optimization problem in equation (24) for country A when setting $\bar{b}_{iBt} = 1$ for all three time periods; the right panel is analogous but for country B . Using the notation in Section 5, Figure D.2 thus illustrates the upper-bound policy function

$$\bar{o}_{it}^{[0]}(y_{it-1}, \omega_{it}) = (\bar{o}_{iAt}^{[0]}(y_{iAt-1}, \omega_{iAt}), \bar{o}_{iBt}^{[0]}(y_{iBt-1}, \omega_{iBt})), \quad \text{for all } t = \{1, 2, 3\}. \quad (\text{D.7})$$

Specifically, in all figures in this section, we use green to identify branches at which the firm exports, and red to identify branches at which it does not. The left panel in Figure D.2 thus shows that, conditional on the firm exporting to B in all periods and states (as reflected by the three green segments under “Assuming that in country $B \dots$ ”), the firm chooses not to export to A at $t = 1$ regardless of whether ω_{iA1} is high or low (as reflected by the two red segments branching out from the “Country A ” label), and chooses to export to A at $t = 2$ and $t = 3$ if and only if ω_{iAt} in the corresponding period t is low (as reflected by the L -segments being green and the H -segments being red). Similarly, the right panel in Figure D.2 shows that, if the firm exports to A in all periods and states (as reflected by the three green segments under “Assuming that in country $A \dots$ ”), the firm chooses to export to B in any given period if and only if ω_{iBt} in the corresponding period t is low (as reflected by the L -segments being green and the H -segments being red).

In Figure D.3, we evaluate the upper-bound policy in equation (D.7), as represented in Figure D.2, at the path of shocks in which these equal their lowest possible value in every country and period (i.e., the path marked by thick lines in each tree’s top branch). Doing so, we obtain new constant upper bounds on the firm’s choice in all countries and periods. E.g., as the upper-bound policy represented in Figure D.2 prescribes the firm not to export to A at $t = 1$ even $\omega_{iA1} = \omega$, we update from one to zero the constant upper bound in A at $t = 1$ (as reflected in the change in color of the segment labeled “Update”). Using the notation in Section 5, it is thus the case that

$$(\bar{b}_{iA1}^{[1]}, \bar{b}_{iA2}^{[1]}, \bar{b}_{iA3}^{[1]}) = (0, 1, 1) \quad \text{and} \quad (\bar{b}_{iB1}^{[1]}, \bar{b}_{iB2}^{[1]}, \bar{b}_{iB3}^{[1]}) = (1, 1, 1). \quad (\text{D.8})$$

We represent in Figure D.4 the new upper-bound policy function we obtain by solving again the optimization problem in equation (24) but now conditioning on the constant upper bounds illustrated at the bottom of both Figure D.3 and Figure D.4, and listed in equation (D.8). Comparing figures D.2 and D.4, we observe that the change in the constant upper bound in country A at period $t = 1$ drives a change in the upper-bound policy function in country B at $t = 1$ at the low fixed cost shock segment, whose color switches to red. As country B ’s constant upper bounds in figures D.2 and D.4 coincide, the upper-bound policy function in country A remains the same.

Figure D.3: Updated Constant Upper Bounds

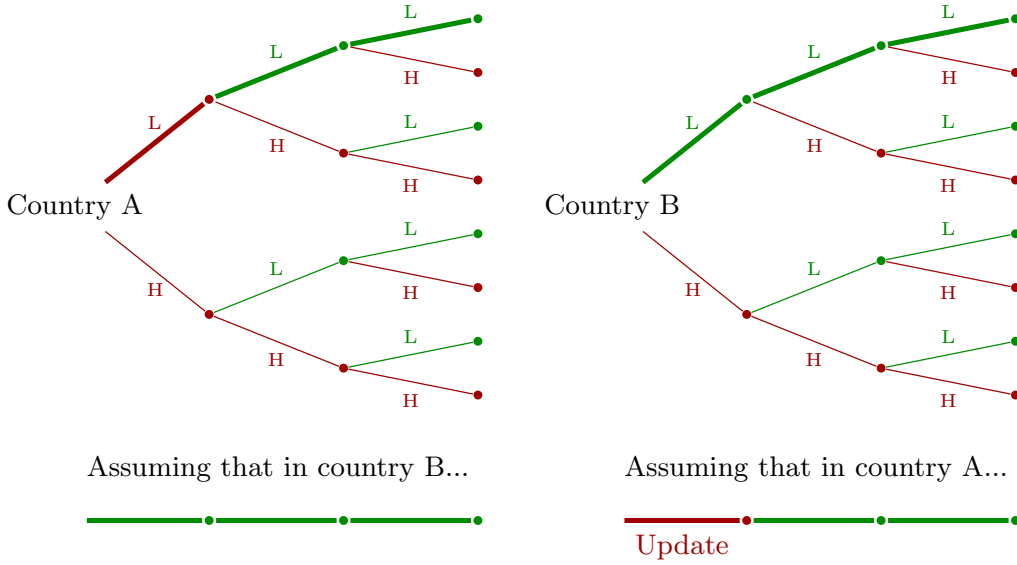
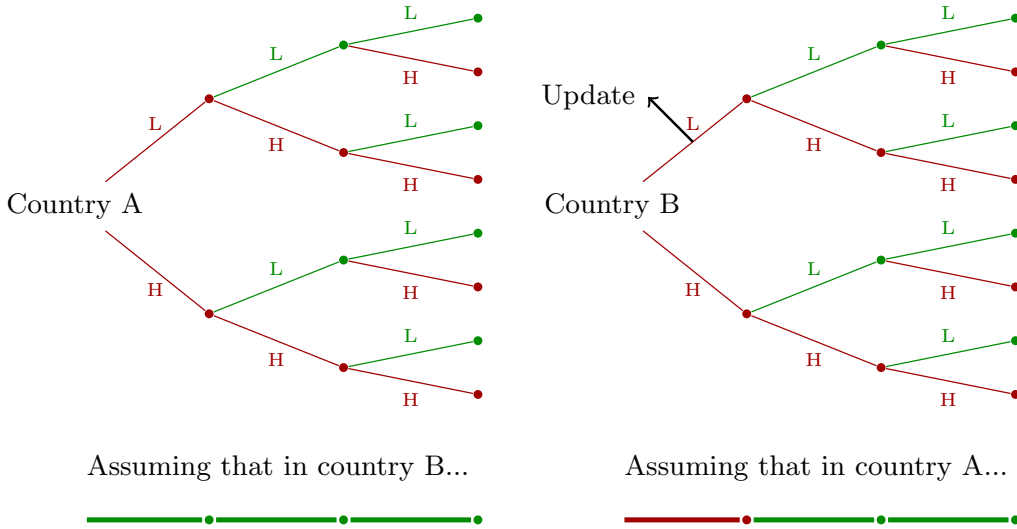


Figure D.4: Updated Upper-Bound Policy Functions



In Figure D.5, we evaluate the updated upper-bound policy illustrated in Figure D.4 at the path of shocks in which these equal their lowest possible value in every country and period, represented in Figure D.3 by the thick lines in each tree's top branch. Comparing figures D.3 and D.5, we observe that the update in the upper-bound policy in Figure D.4 with respect to that in Figure D.2 allows to update from one to zero the constant upper bound in B at $t = 1$ (as reflected in the change in color of the segment labeled "Update"). Using the notation in Section 5, it is then the case that

$$(\bar{b}_{iA1}^{[2]}, \bar{b}_{iA2}^{[2]}, \bar{b}_{iA3}^{[2]}) = (0, 1, 1) \quad \text{and} \quad (\bar{b}_{iB1}^{[2]}, \bar{b}_{iB2}^{[2]}, \bar{b}_{iB3}^{[2]}) = (0, 1, 1). \quad (\text{D.9})$$

Continuing with the iterative procedure prescribed by our algorithm, we solve again the optimization problem in equation (24) but now conditioning on the updated constant upper bounds illustrated at the bottom of Figure D.5 and listed in equation (D.9). The solution is an upper-bound policy function identical

Figure D.5: Updated Constant Upper Bounds

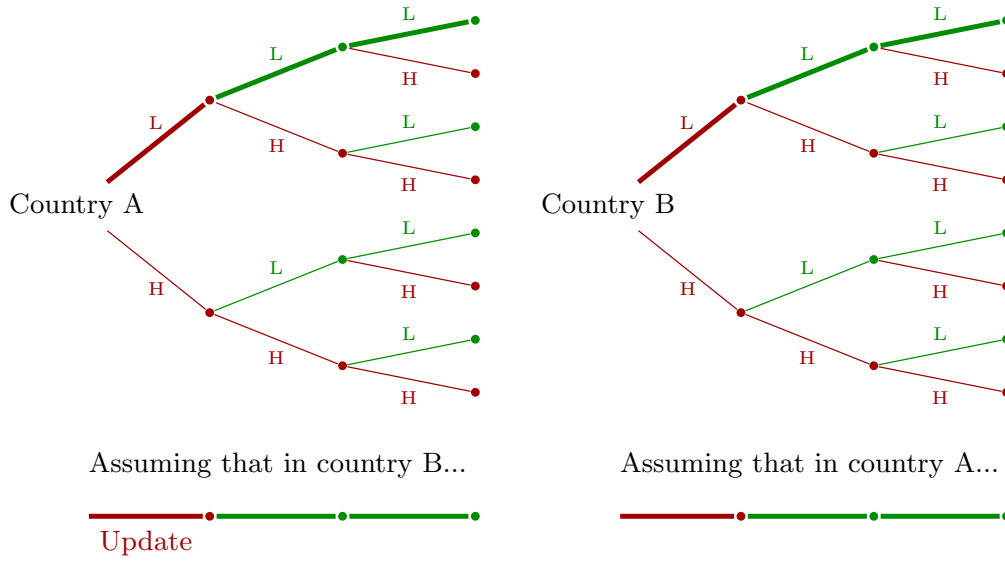


Figure D.6: Upper-Bound Policy Functions After Convergence

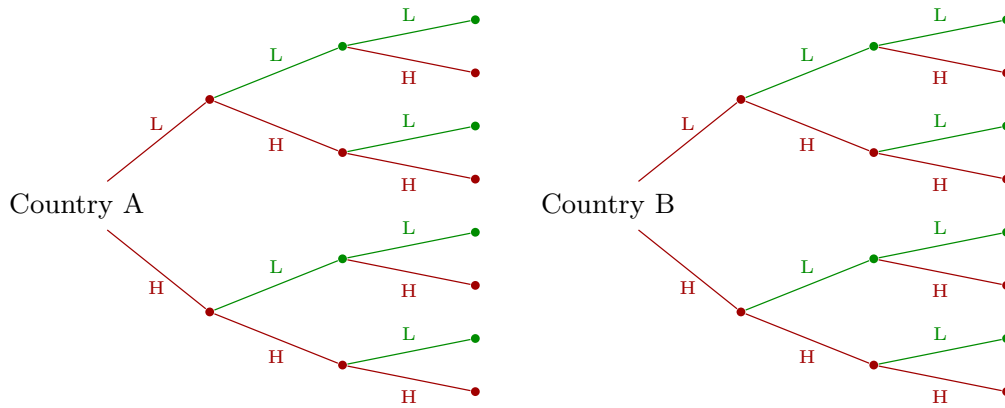


Figure D.7: Lower-Bound Policy Functions After Convergence

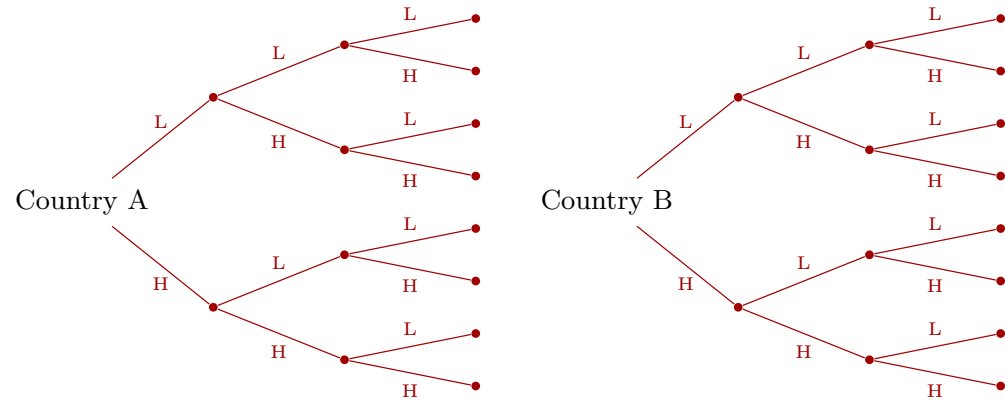
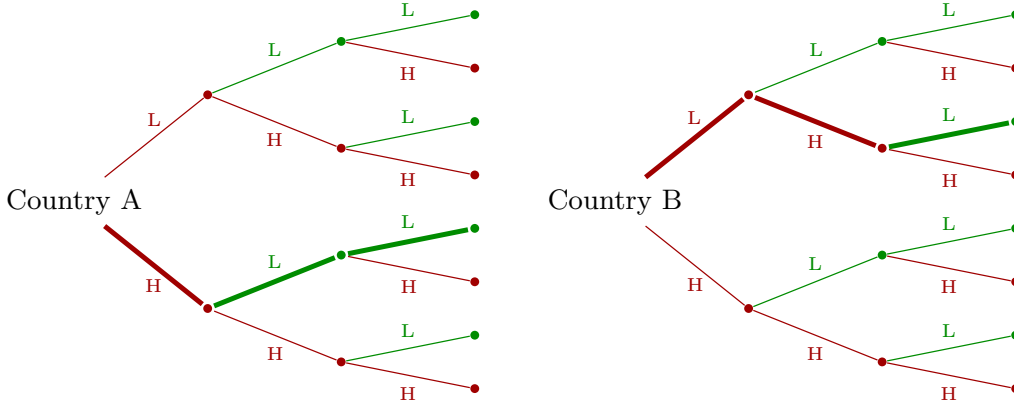


Figure D.8: Evaluating Upper-Bound Policy Functions at Path of Interest



to that obtained in the previous iteration; i.e., that in Figure D.4. Intuitively, as the upper-bound policy in Figure D.4 already prescribes the firm not to export to A at $t = 1$ regardless of the value of ω_{iA1} , the update in the constant upper bound in B at $t = 1$ does not change the upper-bound policy function in A . Thus, after two iterations, the step 1 upper-bound policy function has converged to that represented in Figure D.6.

We follow analogous steps to compute lower-bound policy functions. Assume for simplicity the converged lower-bound policies prescribe the firm not to export to any country in any period regardless of the value of ω_{ijt} for any j and t . The converged lower-bound policy thus corresponds to that in Figure D.7.

The final stage in step 1 of our algorithm is to evaluate the converged lower- and upper-bound policy functions at a specific path of interest. Assume, e.g., this path is:

$$(\hat{\omega}_{iA1}, \hat{\omega}_{iA2}, \hat{\omega}_{iA3}) = (\bar{\omega}, \omega, \omega) \quad \text{and} \quad (\hat{\omega}_{iB1}, \hat{\omega}_{iB2}, \hat{\omega}_{iB3}) = (\omega, \bar{\omega}, \omega), \quad (\text{D.10})$$

where, as a reminder, ω and $\bar{\omega}$ are represented by L and H , respectively, in all figures in this section.

Figure D.8 is identical to Figure D.6 except that the path of interest is highlighted. The colors of the highlighted branches indicate the upper bounds on the firm's optimal choices at the path of interest; i.e.,

$$(\check{y}_{iA1}, \check{y}_{iA2}, \check{y}_{iA3}) = (0, 1, 1) \quad \text{and} \quad (\check{y}_{iB1}, \check{y}_{iB2}, \check{y}_{iB3}) = (0, 0, 1). \quad (\text{D.11})$$

Similarly, given the converged lower-bound policy function in Figure D.7, the lower bounds on the firm's optimal choices at the path of interest are

$$(\check{y}_{iA1}, \check{y}_{iA2}, \check{y}_{iA3}) = (0, 0, 0) \quad \text{and} \quad (\check{y}_{iB1}, \check{y}_{iB2}, \check{y}_{iB3}) = (0, 0, 0). \quad (\text{D.12})$$

Upper and lower bounds coincide at $t = 1$ for both countries; thus, the optimal choices at $t = 1$ at the path of interest are $(\check{y}_{iA1}, \check{y}_{iB1}) = (0, 0)$. At $t = 2$, both bounds differ in their prescribed choice in country A .

Step 2. In this step, we tighten the bounds at $t = 2$. To do so, we first compute new constant upper bounds that condition on the state reached at $t = 2$ at the path of interest; i.e., we evaluate the policy function in Figure D.6 along a path that, for $j = \{A, B\}$, sets $\omega_{ijt} = \hat{\omega}_{ijt}$ for $t \leq 2$, and $\omega_{ijt} = \omega$ for $t > 2$. In Figure D.9, we recover the upper-bound policy in Figure D.6, fade all branches that cannot be reached from the path of interest at $t = 2$ and mark with a wide line the relevant path. Conditioning on the path of interest up to $t = 2$ permits updating the constant upper bound in B at $t = 2$ (as reflected in the change in color of the segment labeled “Update” in Figure D.9). Using the notation in Section 5, it then holds that

$$(\bar{b}_{iA2|2}^{[0]}, \bar{b}_{iA3|2}^{[0]}) = (1, 1) \quad \text{and} \quad (\bar{b}_{iB2|2}^{[0]}, \bar{b}_{iB3|2}^{[0]}) = (0, 1). \quad (\text{D.13})$$

We represent in Figure D.10 the upper-bound policy function obtained by solving the optimization problem in equation (24) for $t \geq 2$ with the new constant upper bounds represented at the bottom of Figure D.9 and listed in equation (D.13). Figure D.10 shows that the upper-bound policy in A at $t = 2$ is updated.

Figure D.9: Initial Constant Upper Bounds That Condition on Path of Interest for $t \leq 2$

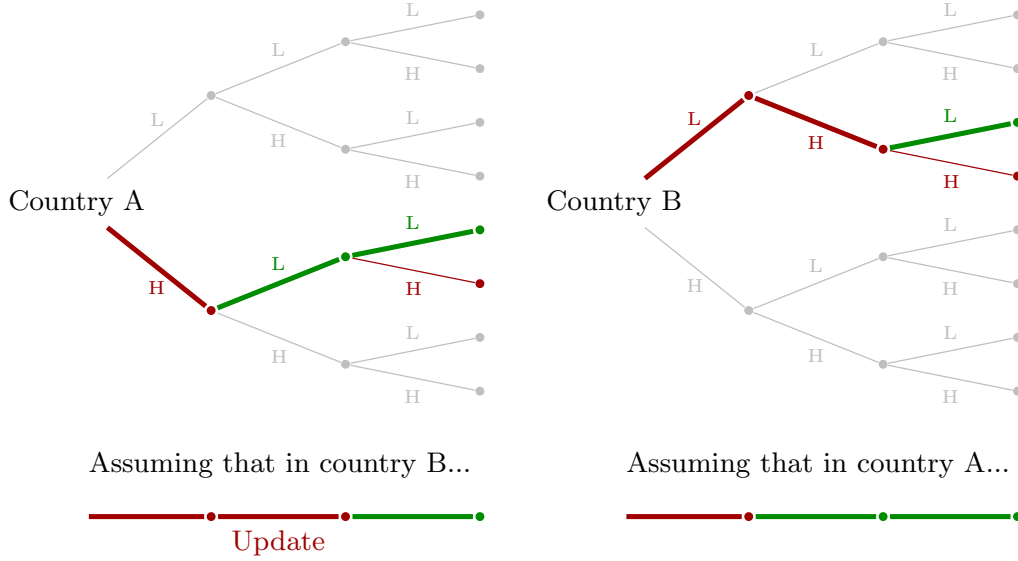
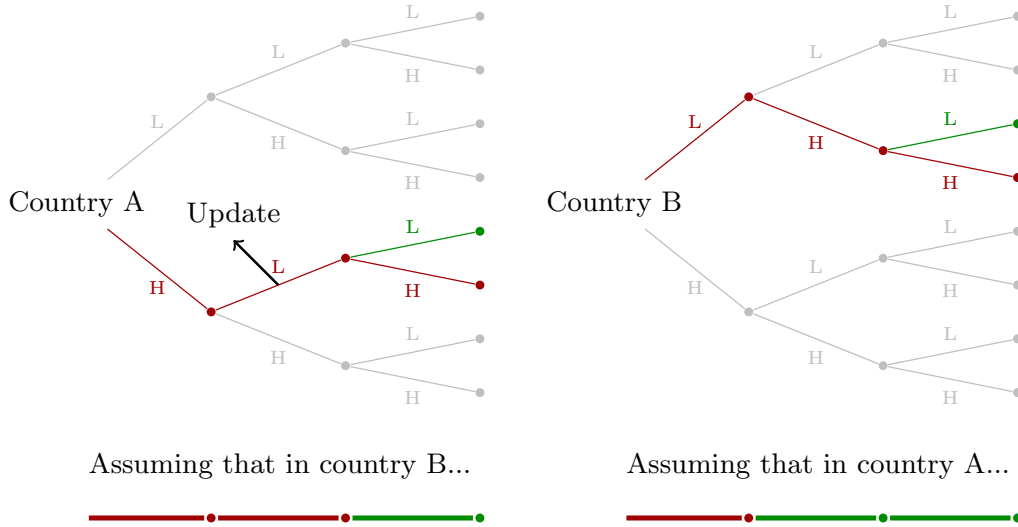


Figure D.10: Upper-Bound Policy Functions That Condition on Path of Interest for $t \leq 2$



Next, we evaluate the updated upper-bound policies in Figure D.10 along the path that, for $j = \{A, B\}$, sets $\omega_{ijt} = \tilde{\omega}_{ijt}$ for $t \leq 2$ and $\omega_{ijt} = \omega$ for $t > 2$, represented in Figure D.11 by thick lines. Comparing figures D.9 and D.11, we observe that the update in the upper-bound policy in Figure D.10 relative to that in Figure D.8 allows us to update the constant upper bound in A at $t = 2$ (see the red segment over the label “Update” in Figure D.11). In the notation introduced in Section 5, it is then the case that

$$(\bar{b}_{iA2|2}^{[1]}, \bar{b}_{iA3|2}^{[1]}) = (0, 1) \quad \text{and} \quad (\bar{b}_{iB2|2}^{[1]}, \bar{b}_{iB3|2}^{[1]}) = (0, 1). \quad (\text{D.14})$$

Continuing with this iterative procedure, we solve again the optimization problem in equation (24) for periods $t \geq 2$, but now conditioning on the new constant upper bounds in equation (D.14) (see also bottom of Figure D.11). The solution to this problem yields upper-bound policy functions identical to those obtained in the previous iteration. Intuitively, as the upper-bound policy in Figure D.10 already prescribes the firm

Figure D.11: Updated Constant Upper Bounds That Condition on Path of Interest for $t \leq 2$

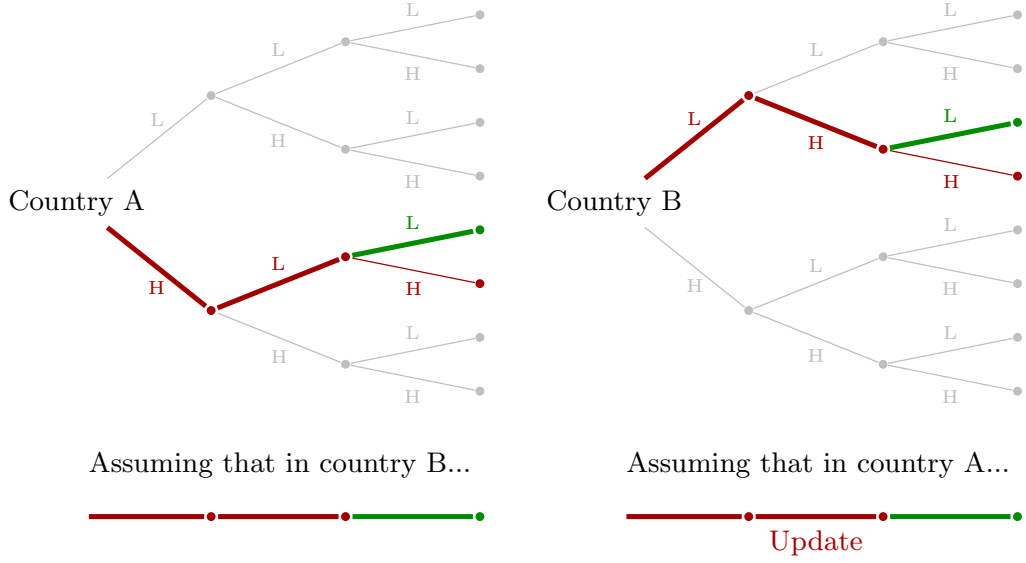
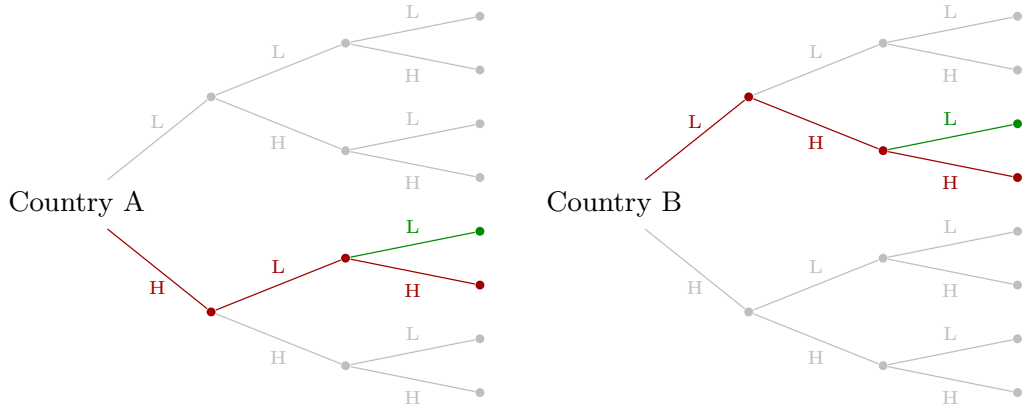


Figure D.12: Upper-Bound Policy Functions That Condition on Path for $t \leq 2$ After Convergence



not to export to B at $t = 2$ at the path of interest, the change in the constant upper bound in A at $t = 2$ does not change the upper-bound policy function. Thus, at this point, the step 2 upper-bound policy functions has converged; we represent it in Figure D.12.

We follow similar steps to compute a lower-bound policy function that conditions on the path of interest up to $t = 2$. As the lower-bound policy that converged in step 1 (see Figure D.7) prescribe the firm not to export to any country at any period or state, the resulting constant lower bounds are

$$(b_{iA2|2}^{[0]}, b_{iA3|2}^{[0]}) = (0, 0) \quad \text{and} \quad (b_{iB2|2}^{[0]}, b_{iB3|2}^{[0]}) = (0, 0). \quad (\text{D.15})$$

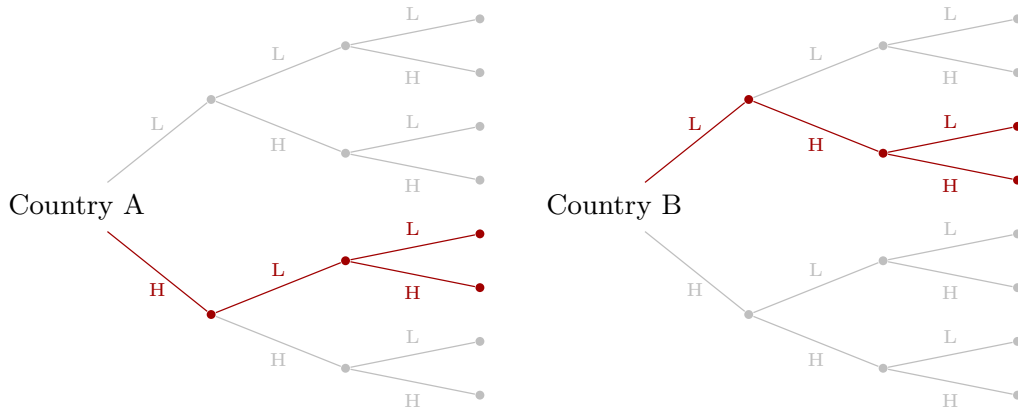
Given these, the lower-bound policy function cannot be updated further; we represent it in Figure D.13.

Evaluating the lower- and upper-bound policy functions in figures D.12 and D.13 at the path of interest at period $t = 2$, we obtain the following bounds on the firm's optimal export choices

$$\check{y}_{iA2|2} = \check{\underline{y}}_{iA2|2} = 0 \quad \text{and} \quad \check{y}_{iB2|2} = \check{\underline{y}}_{iB2|2} = 0. \quad (\text{D.16})$$

As the bounds coincide, the firm's optimal choice at $t = 2$ at the path of interest is $(\check{y}_{iA2}, \check{y}_{iB2}) = (0, 0)$.

Figure D.13: Lower-Bound Policy Functions That Condition on Path at $t = 2$ After Convergence



Additional steps. At this point in the algorithm, we have computed the firm’s optimal choice at the path of interest for $t \leq 2$. However, the step 1 bounds, described in equations (D.11) and (D.12), also differ at the path of interest at $t = 3$. Our algorithm thus proceeds by trying to tighten these bounds. To do so, we first implement a step 2 procedure analogous to the one just described, but now conditioning on the state reached at $t = 3$ along the path of interest. To save on space, we do not describe here how the step 2 algorithm is applied at $t = 3$. It suffices to say that it is not successful at tightening further the bounds on the firm’s optimal choice along the path of interest at $t = 3$. Thus, we proceed to implement the extra steps described in Appendix D.1. Specifically, computing the firm’s optimal choice at the state of interest at $t = 3$ requires solving jointly for the firm’s optimal choices in A and B at this period.

D.3 Performance of the Algorithm

We present here summary statistics of the performance of the algorithm described in Section 5 and Appendix D.1. For all 4,709 firms in the sample, all 74 foreign countries we use in our estimation, 13 periods, and 5 simulation draws of ω_{ijt} for each i, j and t , we measure at the end of each step of the algorithm the percentage of all 22,650,290 ($4,709 \times 74 \times 13 \times 5$) choices solved and the cumulated running time (measured at Princeton University’s Della cluster using 44 processors with 20 GB of memory per processor).

The statistics in Table D.1 are computed setting all parameter values to the baseline estimates reported in tables F.3 and F.4 in Appendix F.6. As reported in the first row in Table D.1, the step 1 of the algorithm (see Section 5 for a description) runs in slightly over two minutes, and provides the solution to 99.72% of the 22,650,290 choices considered. The 0.28% of choices that remain unsolved after step 1 of the algorithm are concentrated in a few countries but dispersed across firms and simulation draws; thus, the number of firms and draws whose choices in every country and period are solved in step 1 is only 78.51%.

Steps 2 and 3 increase the overall share of choices solved to 99.85%, and the share of firms and draws whose complete set of choices is solved to 93.07%. Furthermore, this is attained with a relatively small cost in terms of computing time, as step 3 is completed after less than 4 minutes of running time. In steps 4 and 5, we solve optimization problems that consider multiple countries simultaneously. As the last row in Table D.1 reveals, these steps are the slowest ones: approximately 70% of the 741 seconds it takes to run completely our algorithm are spent in steps 4 and 5. These steps are however useful at raising the share of choices solved to nearly 99.9%, and the share of firms and simulations entirely solved to nearly 96%.

The choices that remain unsolved after step 5 of the algorithm is finished are concentrated in countries that share cross-country complementarities with a large set of other potential export destinations. E.g., of all unsolved choices, nearly 7% are for Mexico, close to 6.5% are for Belgium, between 5% and 6% correspond to The Netherlands and Germany, and between 4% and 5% correspond to Sweden and France. These are all countries that share deep PTA (or regulatory proximity) with a number of other countries larger than the cardinality of the sets of destinations that we solve jointly in steps 4 and 5 of our algorithm: while we consider sets of 10 and 6 destinations in steps 4 and 5, respectively, both Mexico and all members of the

Table D.1: Performance of Algorithm at Baseline Estimates

	Percentage of Firms Solved	Percentage of Choices Solved	Time (in seconds)
Step 1	78.51%	99.72%	131
Step 2	82.74%	99.75%	163
Step 3	93.07%	99.85%	218
Steps 4 & 5	95.80%	99.89%	741

European Common Market share deep PTA with more than 10 destinations.

In Table D.2, we present statistics analogous to those presented in the last row of Table D.1, but for alternative parameterizations in which we change the value of the model parameters one at a time. Specifically, we present results for parameterizations in which we increase in 20% the value of the parameter indicated in the column labeled “Parameter,” leaving all other parameters at their baseline estimates.

The results in Table D.2 show the performance of the algorithm improves (i.e., the percentage of firms and simulations for which all choices are solved increases, and the running time decreases) as we increase the value of those parameters that have a positive impact on the gravity component of fixed and sunk costs; i.e., the parameters entering the expressions in equations (9) and (14). Conversely, the performance of the algorithm worsens as we increase the value of the parameters that have a positive impact on the magnitude of the complementarities between countries (i.e., $(\gamma_x^E, \varphi_x^E)$ for $x = \{g, l, a\}$), and improves as we increase the value of the parameters that determine the speed at which the complementarities between any two countries decay in the distance between them (i.e., κ_x^E for $x = \{g, l, a\}$). The performance of the algorithm varies very little with the value of the parameters that determine the cross-country correlation in the fixed cost shock ν_{ijt} ; i.e., the parameters entering the expression in equation (12c). Finally, when we increase the standard deviation of ν_{ijt} or the probability that ω_{ijt} equals $\underline{\omega} = 0$ (i.e., when we increase σ_ν or p), the performance of the algorithm worsens.

Table D.2: Performance of Algorithm at Estimates 20% Higher than Baseline Ones

Parameter	Percentage of Firms Solved	Time (in seconds)	Parameter	Percentage of Firms Solved	Time (in seconds)
γ_0^F	97.18%	606	κ_l^E	96.03%	703
γ_g^F	97.25%	479	γ_a^E	91.28%	1256
γ_l^F	95.89%	710	φ_a^E	94.70%	935
γ_a^F	96.21%	628	κ_a^E	96.35%	647
γ_0^S	96.77%	582	γ_g^N	95.67%	795
γ_g^S	96.59%	569	κ_g^N	95.86%	742
γ_l^S	95.80%	719	γ_l^N	95.67%	687
γ_a^S	95.96%	692	κ_l^N	95.83%	689
γ_g^E	93.27%	1119	γ_a^N	95.77%	702
φ_g^E	93.59%	1070	κ_a^N	95.81%	686
κ_g^E	97.33%	479	σ_ν	93.88%	841
γ_l^E	95.52%	790	p	82.29%	2841
φ_l^E	95.65%	749			

Note: The *Percentage of Firms Solved* and *Time* are measured after Step 5 of the algorithm has concluded.

E General Optimization Problem: Mapping to Model and Proofs

E.1 Mapping Between Framework in Appendix A.1 and Model in Section 4

We show in this section that, equating agents to firms and alternatives to potential export destinations, the model described in Section 4 satisfies all restrictions in Assumption 1.

As part of the *first restriction*, equation (A.4) assumes agents maximize the expected infinite-horizon discounted sum of a sequence of static payoffs that exhibit one-period dependence. Equation (A.5) restricts these payoffs to be additively separable across alternatives and, in every alternative j , additively separable in the vector of shocks $\omega(z^t)$ and in the vector of choices in every alternative other than j . Finally, the restriction that the domain of the functions $\hat{\pi}_{jt}$ and $\tilde{\pi}_{jt}$ is finite and that these never equal infinity in their domain implies both $\hat{\pi}_{jt}$ and $\tilde{\pi}_{jt}$ are bounded from above. Additionally, $\tilde{\pi}_{jt}$ is also bounded from below.

Our model satisfies the first restriction in Assumption 1. Specifically, equation (A.4) is satisfied as equation (19) implies firms maximize the infinite-horizon expected discounted sum of static profits. Equation (A.5) is also satisfied as equations (15) to (17) imply that model-implied static profits are

$$\pi_t(y(z^t), y(z^{t-1}), \omega(z^t)) = \sum_{j=1}^J (y_j(z^t) u_{jt}(y_j(z^{t-1}), \omega_j(z^t))) + \sum_{j'=1}^J y_j(z^t) y_{j'}(z^t) c_{jj't}, \quad (\text{E.1})$$

where $\omega(z^t)$ equals a vector $(\omega_1(z^t), \dots, \omega_J(z^t))$, $c_{jj't}$ is defined in equation (11) for $j' \neq j$ (with $c_{jj} = 0$), and u_{jt} is defined in equation (16). Static profits may thus be written as in equation (A.5) with

$$\begin{aligned} \hat{\pi}_{jt}(y_j(z^t), y_j(z^{t-1}), \omega(z^t)) &= y_j(z^t)(\eta^{-1} \exp(\alpha_y y_j(z^{t-1}) + \alpha_{jt} + \alpha_s + \alpha_a \ln(a_{s_{jt}}) + \alpha_r \ln(r_{iht})) \\ &\quad - (g_{jt} + \nu_{ijt} + \omega_j(z^t)) - (1 - y_j(z^{t-1}))s_{jt}), \end{aligned} \quad (\text{E.2a})$$

$$\tilde{\pi}_{jt}(y(z^t), y(z^{t-1})) = \sum_{j'=1}^J y_j(z^t) y_{j'}(z^t) c_{jj't}. \quad (\text{E.2b})$$

Finally, these model-implied functions $\hat{\pi}_{jt}$ and $\tilde{\pi}_{jt}$ satisfy the restrictions on their domain and range imposed in Assumption 1. Specifically, as $y_j(z^t) \in \{0, 1\}$, $y_j(z^{t-1}) \in \{0, 1\}$ and $\omega_j(z^t) \in \{0, \infty\}$ for all j and t , $\hat{\pi}_{jt}$ and $\tilde{\pi}_{jt}$ are bounded from above for any realization of ν_{jt} as long as the parameter space is finite.²⁷

The *second restriction* in Assumption 1 imposes the function π_t is supermodular on the sets of choices at $t-1$ and t . As these sets are finite, Corollary 2.6.1 in Topkis (1998) implies one can prove π_t is supermodular by proving it has increasing differences in $y(z^t)$ and $y(z^{t-1})$. For any alternative j and period t , we denote as D_{jt} the change in π_t when changing the value of the choice in j at t , y_{jt} , from zero to one. Given equations (E.1) and (E.2), the expression for D_{jt} in the model described in Section 4 is

$$\begin{aligned} D_{jt} &= \eta^{-1} \exp(\alpha_y y_j(z^{t-1}) + \alpha_{jt} + \alpha_s + \alpha_a \ln(a_{s_{jt}}) + \alpha_r \ln(r_{ht})) \\ &\quad - (g_{jt} + \nu_{jt} + \omega_j(z^t)) - (1 - y_j(z^{t-1}))s_{jt} + 2 \sum_{j' \neq j} y_{j'}(z^t) c_{jj't}. \end{aligned}$$

Since $\alpha_y \geq 0$ and $s_j \geq 0$ for every j , D_{jt} is increasing in $y_j(z^{t-1})$. Since $c_{jj't} \geq 0$ for any j, j' , and t , D_{jt} is also increasing in $\{y_{j'}(z^t)\}_{j' \neq j}$. Finally, D_{jt} is invariant to $y_{j'}(z^{t-1})$ if $j' \neq j$. Thus, π_t has increasing differences on the sets of export choices at $t-1$ and t and, consequently, π_t is supermodular on these sets. The second restriction in Assumption 1 is thus satisfied by the model described in Section 4.

The *third restriction* in Assumption 1 imposes that there exists a feasible strategy such that, if chosen by the agent, the functions $\{\hat{\pi}_{jt}\}_j$ entering static profits are bounded from below no matter the value of the shock ω_t . In the model in Section 4, not exporting to country j ensures $\hat{\pi}_{jt}$ equals zero; i.e., $\hat{\pi}_{jt}(0, x, \omega) = 0$ for any $x \in \{0, 1\}$ and $\omega \in \Omega_t$. Thus, the third restriction in Assumption 1 is satisfied.

The *fourth restriction* imposes Ω_t is finite and the sequence of shocks $\{\omega_{jt}\}_{t \geq 0}$ is Markovian. In the

²⁷As equation (E.2a) shows, the model-implied function $\hat{\pi}_{jt}$ depends on $\omega(z^t)$ only through a scalar $\omega_j(z^t)$. While this is not relevant for the algorithm's theoretical properties (and, thus, is not imposed in Assumption 1), it is critical for its computational tractability.

model in Section 4, Ω_t includes only two elements and ω_t is independent over time (see equation (13)); thus, this fourth restriction is satisfied.

Finally, the *fifth restriction* imposes that the firm's problem becomes stationary after a terminal period T ; i.e., the functions $\{\hat{\pi}_{jt}\}_j$ and $\{\tilde{\pi}_{jt}\}_j$, the distribution of ω_t , and the set Ω_t become constant at T . In the model described in Section 4, Ω_t and the distribution of ω_t are time-invariant, and the functions $\hat{\pi}_{jt}$ and $\tilde{\pi}_{jt}$ become constant at T for every country j . Thus, the fifth restriction in Assumption 1 is satisfied.

E.2 Proof of Theorem 1: Preliminary Results

We prove here two preliminary results that we use in Appendix E.3 as part of the proof of Theorem 1. First, we show that restrictions 1 and 2 in Assumption 1 imply that the solution to the optimization problem in equation (A.6) for any given set of alternatives M_u is increasing in the second argument of the objective function Π_0 ; i.e., increasing in the upper bounds on the firm's optimal choice in every alternative not in M_u . Second, we show restrictions 1 and 3 to 5 in Assumption 1 imply there exists a solution to the optimization problem in equation (A.6), and that it attains the maximum. Additionally, we provide an algorithm to compute this solution. Finally, as a corollary, we show the solution of the optimization problem in equation (A.2) exists and the maximum is attained.

In our proofs, we use Lemma 2.6.1 and Theorem 2.8.1 in Topkis (1998), which we re-state here.

Lemma E.1 (*Topkis, 1998, Lemma 2.6.1*) *Suppose X is a lattice. Then,*

1. *If $f(x)$ is supermodular on X and $\alpha > 0$, then $\alpha f(x)$ is supermodular on X .*
2. *If $f(x)$ and $g(x)$ are supermodular on X , then $f(x) + g(x)$ is supermodular on X .*
3. *If $f_k(x)$ is supermodular on X for $k = 1, 2, \dots$ and $\lim_{k \rightarrow \infty} f_k(x) = f(x)$ for each $x \in X$, then $f(x)$ is supermodular on X .*

Theorem E.1 (*Topkis, 1998, Theorem 2.8.1*) *If X is a lattice, T is a partially ordered set, S_t is a subset of X for each t in T , S_t is increasing in t on T , $f(x, t)$ is supermodular in x on X for each t in T , and $f(x, t)$ has increasing differences in (x, t) on $X \times T$, then $\operatorname{argmax}_{x \in S_t} f(x, t)$ is increasing in t on $\{t : t \in T, \operatorname{argmax}_{x \in S_t} f(x, t) \text{ is non-empty}\}$.*

E.2.1 First Preliminary Result

We prove here that, for any set of alternatives M_u and iteration n , if it exists, the solution $\bar{o}_{M_u}^{(n)}$ to the optimization problem in equation (A.6) is increasing in the set of upper bounds on alternatives not in M_u ; i.e., the solution to the optimization problem in equation (A.6) is increasing in

$$\bar{\mathbf{y}}_{M_u^c}^{(n)} = \{\bar{y}_{M_u^c}^{(n)}(z^t)\}_{t=0, z^t \in Z^t}, \quad \text{with} \quad \bar{y}_{M_u^c}^{(n)}(z^t) \geq o_{M_u^c}(z^t) \text{ for all } t \geq 0 \text{ and } z^t \in Z^t.$$

Our proof has two steps. First, we show the agent's objective function according to equation (A.2), $\Pi_0(\mathbf{y})$, is supermodular in \mathbf{y} on Y ; see equation (A.1) for the definition of Y . Second, we show this implies that the solution to the optimization problem in equation (A.6) is increasing in the set of upper bounds on alternatives not in M_u .

Lemma E.2 *Assumption 1 implies $\Pi_0(\mathbf{y})$ is supermodular in \mathbf{y} on Y .*

Proof. The second restriction in Assumption 1 in Appendix A.1 states that, for every period t and every feasible history z^t , $\pi_t(y(z^t), y(z^{t-1}), \omega(z^t))$ is supermodular in $(y(z^t), y(z^{t-1}))$ on $\{0, 1\}^J \times \{0, 1\}^J$. Define $\tilde{\pi}_t(\mathbf{y}, z^t) = \pi_t(y(z^t), y(z^{t-1}), \omega(z^t))$, where, as indicated in Appendix A.1, \mathbf{y} is a generic vector of agent choices at every history $z^t \in Z^t$ and every period $t \geq 0$. Therefore, $\tilde{\pi}_t(\cdot)$ is identical to $\pi_t(\cdot)$, but written as a function of the whole vector of choices in every period and feasible history.

First, we show that $\tilde{\pi}_t(\mathbf{y}, z^t)$ is supermodular in \mathbf{y} . More specifically, we show that, for any two vectors $\mathbf{y}' \in Y$ and $\mathbf{y}'' \in Y$, it holds that $\tilde{\pi}_t(\mathbf{y}', z^t) + \tilde{\pi}_t(\mathbf{y}'', z^t) \leq \tilde{\pi}_t(\mathbf{y}' \vee \mathbf{y}'', z^t) + \tilde{\pi}_t(\mathbf{y}' \wedge \mathbf{y}'', z^t)$, where the "join"

\vee takes the maximum element by element, and the “meet” \wedge takes the minimum element by element. To prove this result, note that

$$\begin{aligned}\tilde{\pi}_t(\mathbf{y}', z^t) + \tilde{\pi}_t(\mathbf{y}'', z^t) &= \pi_t(y'(z^t), y'(z^{t-1}), \omega(z^t)) + \pi_t(y''(z^t), y''(z^{t-1}), \omega(z^t)) \\ &\leq \pi_t(y'(z^t) \vee y''(z^t), y'(z^{t-1}) \vee y''(z^{t-1}), \omega(z^t)) \\ &\quad + \pi_t(y'(z^t) \wedge y''(z^t), y'(z^{t-1}) \wedge y''(z^{t-1}), \omega(z^t)) \\ &= \tilde{\pi}_t(\mathbf{y}' \vee \mathbf{y}'', z^t) + \tilde{\pi}_t(\mathbf{y}' \wedge \mathbf{y}'', z^t),\end{aligned}$$

where the two equalities follow from the relationship between the functions π_t and $\tilde{\pi}_t$, and the inequality follows from the supermodularity of $\pi_t(y'(z^t), y'(z^{t-1}), \omega(z^t))$ in $\{y(z^s), y(z^{s-1})\}$ on $\{0, 1\}^J \times \{0, 1\}^J$.

Second, we define a function $\Pi_0^\tau(\mathbf{y})$ as the expected discounted sum of static profits between periods $t = 0$ and $t = \tau$, and show that the supermodularity of $\tilde{\pi}_t(\mathbf{y}, z^t)$ in \mathbf{y} on Y implies $\Pi_0^\tau(\mathbf{y})$ is supermodular in \mathbf{y} on Y . As the set Ω_t is finite for every period t (see restriction 4 in Assumption 1), we can write

$$\begin{aligned}\Pi_0^\tau(\mathbf{y}) &= \pi_0(y(z^0), 0_J, \omega(z^0)) + \sum_{t=1}^{\tau} \sum_{z^t \in Z^t} \delta^t \pi_t(y(z^t), y(z^{t-1}), \omega(z^t)) \Pr(z^t), \\ &= \tilde{\pi}_0(\mathbf{y}, z^0) + \sum_{t=1}^{\tau} \sum_{z^t \in Z^t} \delta^t \tilde{\pi}_t(\mathbf{y}, z^t) \Pr(z^t).\end{aligned}$$

Since $\tilde{\pi}_t(\mathbf{y}, z^t)$ is supermodular in \mathbf{y} on Y for every period t and history z^t , and the finite sum of supermodular functions is supermodular (see part 2 of Lemma E.1), then $\Pi_0^\tau(\mathbf{y})$ is supermodular in \mathbf{y} on Y .

Finally, noting restriction 1 in Assumption 1 implies $\Pi_0(\mathbf{y}) = \lim_{\tau \rightarrow \infty} \Pi_0^\tau(\mathbf{y})$, we apply part 3 in Lemma E.1 to conclude that the supermodularity of $\Pi_0^\tau(\mathbf{y})$ in \mathbf{y} on Y implies $\Pi_0(\mathbf{y})$ is supermodular in \mathbf{y} on Y . ■

Lemma E.3 *Assumption 1 implies that, for every set of alternatives M_u and every iteration n of the algorithm described in Appendix A.2, if the solution to the optimization problem in equation (A.6) exists, it is increasing in the export strategy in every alternative not in M_u .*

Proof. This lemma states that, if it exists, $\bar{\mathbf{o}}_{M_u}^{(n)}$ is increasing in $\bar{\mathbf{y}}_{M_u}^{(n)}$. This lemma is thus implied by Theorem E.1 and the supermodularity of $\Pi(\mathbf{y})$ in \mathbf{y} on Y . ■

E.2.2 Second Preliminary Result

We prove here that, for every subset of alternatives M_u and iteration n , the solution $\bar{\mathbf{o}}_{M_u}^{(n)}$ to the optimization problem in equation (A.6) exists and the maximum is attained. More specifically, Lemma E.4 below establishes the existence of the solution to the problem in equation (A.6), and that the maximum is attained, for every $t \geq T$; that is, for all periods after the terminal period T , when the problem of the firm becomes stationary according to the restriction 5 in Assumption 1. Given Lemma E.4, establishing the existence of the solution to the problem in equation (A.6), and that the maximum is attained, for every $0 \leq t < T$ is straightforward by backward induction, as there are a finite number of feasible choices.

For any set of alternatives M_u and any vector $\bar{b}_{M_u^c} \in \{0, 1\}^{J-J_u}$, we define the firm’s expected discounted sum of static payoffs at T conditional on setting $\bar{y}_{M_u^c}(z^t) = \bar{b}_{M_u^c}$ for all $t \geq T$ and all $z^t \in Z^t$ as

$$\begin{aligned}\Pi_T(\mathbf{y}_{M_u}, \bar{b}_{M_u^c}, y(z^{T-1}), \omega(z^T)) &= \pi_T((y_{M_u}(z^T), \bar{b}_{M_u^c}), (y_{M_u}(z^{T-1}), y_{M_u^c}(z^{T-1})), \omega(z^T)) \\ &\quad + \sum_{t=T+1}^{\infty} \delta^{t-T} \mathbb{E}_T \left[\pi_T((y_{M_u}(z^t), \bar{b}_{M_u^c}), (y_{M_u}(z^{t-1}), \bar{b}_{M_u^c}), \omega(z^t)) \right],\end{aligned}$$

where $\pi_T(\cdot)$ equals the payoff function in equation (A.5) for $t = T$, $y(z^{T-1}) = (y_{M_u}(z^{T-1}), y_{M_u^c}(z^{T-1}))$, and \mathbf{y}_{M_u} includes a generic set of choices for all alternatives in M_u , all $t \geq T$, and all $z^t \in Z^t$. We can then define the period- T value function

$$V_{TM_u}(\bar{b}_{M_u^c}, y(z^{T-1}), \omega(z^T)) = \sup_{\mathbf{y}_{M_u}} \Pi_T(\mathbf{y}_{M_u}, \bar{b}_{M_u^c}, y(z^{T-1}), \omega(z^T)). \quad (\text{E.3})$$

Lemma E.4 For any set of alternatives M_u and any vector $\bar{b}_{M_u^c} \in \{0, 1\}^{J-J_u}$, Assumption 1 implies the solution to the problem in equation (E.3) exists and the maximum is attained.

Proof. For any set of alternatives M_u and any vector $\bar{b}_{M_u^c} \in \{0, 1\}^{J-J_u}$, we define the payoff function

$$\begin{aligned} \check{\Pi}_T(\mathbf{y}_{M_u}, \bar{b}_{M_u^c}, y(z^{T-1}), \omega(z^T)) = \\ \sum_{j \in M_u} \hat{\pi}_{jT}(y_j(z^T), y_j(z^{T-1}), \omega(z^T)) + \sum_{j=1}^J \tilde{\pi}_{jT}((y_{M_u}(z^T), \bar{b}_{M_u^c}), (y_{M_u}(z^{T-1}), y_{M_u^c}(z^{T-1}))) + \\ \sum_{t=T+1}^{\infty} \delta^{t-T} \mathbb{E}_T \left[\sum_{j \in M_u} \hat{\pi}_{jT}(y_j(z^t), y_j(z^{t-1}), \omega(z^t)) + \sum_{j=1}^J \tilde{\pi}_{jT}((y_{M_u}(z^t), \bar{b}_{M_u^c}), (y_{M_u}(z^{t-1}), \bar{b}_{M_u^c})) \right], \end{aligned}$$

and the associated value function

$$\check{V}_{TM_u}(\bar{b}_{M_u^c}, y(z^{T-1}), \omega(z^T)) = \sup_{\mathbf{y}_{M_u}} \check{\Pi}_T(\mathbf{y}_{M_u}, \bar{b}_{M_u^c}, y(z^{T-1}), \omega(z^T)). \quad (\text{E.4})$$

The functions $\check{\Pi}_T(\cdot)$ and $\Pi_T(\cdot)$ differ from each other in that the former only includes those terms entering the latter that depend on \mathbf{y}_{M_u} . Thus, $\check{\Pi}_T(\cdot)$ and $\Pi_T(\cdot)$ differ in a term that is invariant to the choice of \mathbf{y}_{M_u} and, consequently, a vector \mathbf{y}_{M_u} will solve the optimization problem in equation (E.4) if and only if it also solves the optimization problem in equation (E.3).

Restriction 1 in Assumption 1 implies the functions $\hat{\pi}_{jT}(\cdot)$ and $\tilde{\pi}_{jT}(\cdot)$ are bounded from above. As $\delta < 1$, we can then conclude that the value function $\check{V}_{TM_u}(\cdot)$ in equation (E.4) is bounded from above. Restriction 3 in Assumption 1 implies there is a feasible value of the choice vector \mathbf{y}_{M_u} such that $\hat{\pi}_{jT}(\cdot)$ is bounded from below for all $j \in M_u$. As restriction 1 in Assumption 1 also implies that the function $\tilde{\pi}_{jT}(\cdot)$ is bounded from below, we can then conclude that the value function $\check{V}_{TM_u}(\cdot)$ in equation (E.4) is bounded from below. In sum, restrictions 1 and 3 in Assumption 1 imply that $\check{V}_{TM_u}(\cdot)$ is bounded from above and from below.

Theorem 4.2 in Stokey and Lucas Jr. (1989) implies we can write $\check{V}_{TM_u}(\cdot)$ as the solution to the following functional equation,

$$\begin{aligned} \check{V}_{TM_u}(\bar{b}_{M_u^c}, (y_{M_u}, y_{M_u^c}), \omega) = \\ \sup_{y'_{M_u}} \left\{ \sum_{j \in M_u} \hat{\pi}_{jT}(y'_j, y_j, \omega) + \sum_{j=1}^J \tilde{\pi}_{jT}((y'_{M_u}, \bar{b}_{M_u^c}), (y_{M_u}, y_{M_u^c})) + \delta \mathbb{E}[\check{V}_{TM_u}(\bar{b}_{M_u^c}, (y'_{M_u}, \bar{b}_{M_u^c}), \omega)] \right\} \quad (\text{E.5}) \end{aligned}$$

Since $\check{V}_{TM_u}(\cdot)$ is bounded from above and from below, equation (E.5) maps bounded functions into bounded functions. Additionally, it also satisfies the monotonicity and discounting properties of Blackwell's sufficient conditions for a contraction of modulus δ . Therefore, there is a unique bounded function $\hat{V}_{TM_u}(\cdot)$ that solves the problem in equation (E.5); see Theorem 3.3 in Stokey and Lucas Jr. (1989). Since the solution to the problem in equation (E.5) is unique, then it must also be a solution to the sequence problem in equation (E.4). Furthermore, as the solution to the sequence problems in equations (E.3) and (E.4) coincide, we can conclude that the solution to the optimization problem in equation (E.3) *exists*. Finally, as the choice variable y'_{M_u} in equation (E.5) may only take finitely many values, the maximum is attained. ■

Lemma E.5 Assumption 1 implies the solution to the problem in equation (A.2) exists and the maximum is attained.

Proof. It is an implication of Lemma E.4 when applied to the specific set M_u that includes all possible alternatives; i.e., $M_u = \{1, \dots, J\}$. ■

E.3 Proof of Theorem 1

E.3.1 Proof of Part 1 of Theorem 1

We prove part 1 of Theorem 1 by induction.

As the *base case*, note that, according to equation (A.8), $\bar{b}_{jt}^{(1)} = 1$ for all $j = 1, \dots, J$ and, therefore,

$$\bar{b}_{jt}^{(n)} \geq o_j(z^t) \quad \text{for } n = 1, j = 1, \dots, J, t \geq 0, \text{ and } z^t \in Z^t.$$

As the *step case*, suppose that, for some arbitrary n , $\bar{b}_{jt}^{(n)} \geq o_j(z^t)$ for all $j = 1, \dots, J$, $t \geq 0$, and $z^t \in Z^t$. For any group of alternatives M_u , denote as

$$\bar{\mathbf{b}}_{M_u}^{(n)}$$

the vector that assigns the value of $\bar{b}_{jt}^{(n)}$ to every alternative j in M_u , every $t \geq 0$, and every $z^t \in Z^t$; i.e.,

$$\bar{\mathbf{b}}_{M_u}^{(n)} = \{\bar{y}_j^{(n)}(z^t)\}_{t=0, z^t \in Z^t, j \in M_u}, \quad \text{with } \bar{y}_j^{(n)}(z^t) = \bar{b}_{jt}^{(n)} \text{ for all } t \geq 0, \text{ all } j \in M_u, \text{ and all } z^t \in Z^t.$$

Thus, $\bar{\mathbf{b}}_{M_u}^{(n)} \geq \mathbf{o}_{M_u}$, where \mathbf{o}_{M_u} is the vector containing the agent's optimal choice for every $j \in M_u$, every $t \geq 0$, and every $z^t \in Z^t$. For any alternative j and period t , equations (A.6) and (A.9) further imply that

$$\bar{b}_{jt}^{(n+1)} = \max_{z^t \in Z^t} \bar{o}_j^{(n)}(z^t),$$

where, for a set M_u including alternative j , $\bar{o}_j^{(n)}(z^t)$ is the corresponding element of $\bar{\mathbf{o}}_{M_u}^{(n)}$, defined as

$$\bar{\mathbf{o}}_{M_u}^{(n)} = \operatorname{argmax}_{\mathbf{y}_{M_u} \in Y_{M_u}} \Pi_0(\mathbf{y}_{M_u}, \bar{\mathbf{b}}_{M_u}^{(n)}).$$

To prove that $\bar{b}_{jt}^{(n+1)} \geq o_j(z^t)$ for all $j = 1, \dots, J$, $t \geq 0$, and $z^t \in Z^t$, it is thus enough to prove that

$$\bar{\mathbf{o}}_{M_u}^{(n)} \geq \mathbf{o}_{M_u}. \quad (\text{E.6})$$

For any group of destinations M_u , we can write \mathbf{o}_{M_u} as

$$\mathbf{o}_{M_u} = \operatorname{argmax}_{\mathbf{y}_{M_u} \in Y_{M_u}} \Pi_0(\mathbf{y}_{M_u}, \mathbf{o}_{M_u^c}). \quad (\text{E.7})$$

Lemma E.4 implies $\bar{\mathbf{o}}_{M_u}^{(n)}$ and \mathbf{o}_{M_u} exist, and Lemma E.3 implies $\bar{\mathbf{o}}_{M_u}^{(n)} \geq \mathbf{o}_{M_u}$. Thus, it holds that

$$\bar{b}_{jt}^{(n+1)} \geq o_j(z^t),$$

for all $j = 1, \dots, J$, $t \geq 0$, and $z^t \in Z^t$. ■

E.3.2 Proof of Part 2 of Theorem 1

We prove part 2 of Theorem 1 by induction.

As *base case*, note that equation (A.8) implies $\bar{b}_{jt}^{(1)} = 1$ for every alternative j and period t . As, naturally,

$$\bar{o}_j^{(1)}(z^t) \in \{0, 1\}$$

for every alternative j , period $t \geq 0$, and history $z^t \in Z^t$, it must be the case that $\bar{b}_{jt}^{(2)}$, defined according to equation (A.9), is also either 0 or 1 for every alternative j and period t . Consequently,

$$\bar{b}_{jt}^{(2)} \leq \bar{b}_{jt}^{(1)}, \quad \text{for all } j = 1, \dots, J \text{ and } t \geq 0.$$

As the *step case*, suppose that, for some arbitrary n , $\bar{b}_{jt}^{(n)} \leq \bar{b}_{jt}^{(n-1)}$ for all $j = 1, \dots, J$ and $t \geq 0$. Given

the definition of $\bar{\mathbf{y}}_{M_u}^{(n)}$ in equation (A.7), it is then the case that, for any set of alternatives M_u , it holds that

$$\bar{\mathbf{y}}_{M_u}^{(n)} \leq \bar{\mathbf{y}}_{M_u}^{(n-1)}. \quad (\text{E.8})$$

Given the definition of $\bar{\mathbf{o}}_{M_u}^{(n)}$ in equation (A.6), Lemma E.4 guarantees $\bar{\mathbf{o}}_{M_u}^{(n)}$ and $\bar{\mathbf{o}}_{M_u}^{(n-1)}$ exist. Given equations (A.6) and (E.8), Lemma E.3 implies that

$$\bar{\mathbf{o}}_{M_u}^{(n)} \leq \bar{\mathbf{o}}_{M_u}^{(n-1)}.$$

Since, according to equation (A.9), $\bar{b}_{jt}^{(n+1)} = \max_{z^t \in Z^t} \bar{o}_j^{(n)}(z^t)$ for every t, j , and z^t , it then holds that

$$\bar{b}_{jt}^{(n+1)} \leq \bar{b}_{jt}^{(n)},$$

for all $j = 1, \dots, J$, $t \geq 0$, and $z^t \in Z^t$. ■

E.3.3 Proof of Part 3 of Theorem 1

As shown in the proof of Lemma E.4, Assumption 1 implies that, for any arbitrary iteration n , $\bar{b}_{jt}^{(n)} = \bar{b}_{jT}^{(n)}$ for every alternative j and period $t \geq T$; this is a consequence of the agent's optimization problem becoming stationary after period T . Therefore, we can summarize the infinite set of upper bounds

$$\{\bar{b}_{jt}^{(n)}\}_{j=1, t \geq T}^J$$

in a vector that belongs to the set $\{0, 1\}^J$; i.e., in a vector with a finite number of coordinates. For every period $t < T$ and an arbitrary iteration n , it is the case that

$$\bar{b}_{jt}^{(n)} \in \{0, 1\}^J.$$

Therefore, for any arbitrary iteration, computing the full set of upper-bounds $\{\bar{b}_{jt}^{(n)}\}_{j=1, t \geq 0}^J$ implies computing the value of $(T + 1)J$ unknowns, each of whom may equal either 0 or 1.

Part 2 of Theorem 1 indicates that, at every iteration n , the value of each of these upper bounds either decreases or remains constant. As there is a finite number $(T + 1)J$ of upper bounds to solve for at each iteration n , and each of these upper bounds may equal either 0 or 1 (i.e., they are bounded from below by 0), it must then be the case that these bounds converge in a finite number of steps.

F Estimation: Additional Details

F.1 Identification of Cross-Country Complementarities: Details

Consider a simplified version of the model described in Section 4 in which we impose the following restrictions. First, there are only three foreign countries. Second, in terms of the parameters entering the expression for potential export revenues in equation (7), assume that $\alpha_y = \alpha_a = \alpha_r = 0$, $\alpha_s = 0$ for every sector s , and, for every period t , $\alpha_{1t} = \alpha_{2t} = \bar{\alpha} = \ln(20)$ and $\alpha_{3t} = \alpha_3$. Third, in terms of the fixed export costs determined in equations (8) to (13), assume that

$$\begin{aligned} f_{i1t} &= \gamma_0^F + \nu_{i1t} + \omega_{i1t}, \\ f_{i2t} &= \gamma_0^F - y_{i3t}\bar{c} + \nu_{i2t} + \omega_{i2t}, \\ f_{i3t} &= \gamma_0^F - y_{i2t}\bar{c} + \nu_{i3t} + \omega_{i3t}, \end{aligned}$$

with $\gamma_0^F = 80$, $\bar{c} = 15$, ν_{ijt} drawn according to the distribution in equation (12) with $\sigma_\nu = 80$ and, for every period t ,

$$\rho_{12t} = \rho_{13t} = 0, \quad \text{and} \quad \rho_{23t} = \bar{\rho},$$

and ω_{ijt} drawn according to the distribution in equation (13) with $p = 0.7$. Fourth, in terms of the sunk export cost determined in equation (14), assume that, for every country $j \in \{1, 2, 3\}$ and period t , $s_{jt} = \gamma_s^0 = 120$.

In this simplified framework, we first show how the values of the moments m_1 and m_2 in equation (31) change as we change the market size of country 3 (i.e., α_3) and the values of the parameters determining the strength of export complementarities between countries 2 and 3 (i.e., the value of \bar{c}), and the strength of the correlation in ν_{ijt} between countries 2 and 3 (i.e., the value of $\bar{\rho}$). With that goal in mind, for any value of interest of $(\alpha_3, \bar{c}, \bar{\rho})$, we simulate the model for over 2,351,500 firms, set terminal period $T = 120$, and compute moments m_1 and m_2 using the information on y_{ijt} for all firms and periods $50 \leq t \leq 64$.

In Table F.1, we set $\alpha_3 = \bar{\alpha}$ and compute the values of $m_1 = \mathbb{E}[y_{i2t} - y_{i1t}]$ and $m_2 = \mathbb{C}[y_{i2t}, y_{i3t}]$ for four different combinations for the parameters $(\bar{c}, \bar{\rho})$.

In the first row in Table F.1, we set $(\bar{c}, \bar{\rho}) = (0, 0)$, obtaining in this case that $m_1 = m_2 = 0$. Intuitively, as countries 1 and 2 are identical in every respect except in their potential export complementarities with country 3, export probabilities in both countries must be equal when the parameter that determines the strength of those export complementarities is set to 0; i.e., when $\bar{c} = 0$. Similarly, as all firms are identical in every respect except in the fixed cost unobserved terms ν_{ijt} and ω_{ijt} , the within-firm covariance in export choices in countries 2 and 3 will equal zero when the parameter determining the potential correlation in these unobserved terms for these two countries equals zero; i.e., when $\bar{\rho} = 0$.

In the second row, we introduce positive export complementarities between countries 2 and 3 by setting $\bar{c} = 30$. These export complementarities increase the export probability in country 2 (and in country 3, although this is not relevant for Table F.1), while they do not affect the export probability in country 1 (as country 1 is isolated from any other potential export destination); therefore, moment m_1 increases as \bar{c} increases. Even if $\bar{c} > 0$, firms will enjoy a reduction in fixed export costs in country 2 if and only if they export in the same time period to country 3 (and vice versa); therefore, an increase in the strength of the export complementarities, as determined by the value of the parameter \bar{c} , makes firms more likely to simultaneously export to countries 2 and 3 and, consequently, moment m_2 also increases as \bar{c} increases.

In the third row in Table F.1, we set the value of the parameter \bar{c} back to zero (as in the first row) but introduce positive correlation in ν_{ijt} between countries 2 and 3. When there are no cross-country export complementarities, the within-firm positive correlation between countries 2 and 3 in fixed export costs does not affect the (marginal) export probability in any country; therefore, moment m_1 does not depend on the value of $\bar{\rho}$ when $\bar{c} = 0$. However, the within-firm positive correlation between countries 2 and 3 in fixed export costs affects the joint probability that firms export simultaneously to those two countries, which increases; therefore, moment m_2 increases in the value of $\bar{\rho}$ when $\bar{c} = 0$.

In the fourth row in Table F.1, we set both \bar{c} and $\bar{\rho}$ to positive values. When comparing the results in the second and fourth rows, we observe that introducing positive correlation in ν_{ijt} between countries 2 and

3 in a baseline setting with cross-country export complementarities (i.e., in a baseline setting with $\bar{c} > 0$) affects not only the joint probability that firms export simultaneously to countries 2 and 3 (i.e., the value of m_2) but also the difference in the export probabilities between countries 2 and 3 (i.e., the value of m_1).

In unreported simulation results, we observe that the features described in Table F.1 hold generally as we change the values of the parameters \bar{c} and $\bar{\rho}$ between different numbers and as we set the size of country 3, α_3 , to different values.

Table F.1: Impact of Complementarities and Correlation in Unobservables on Moment Conditions

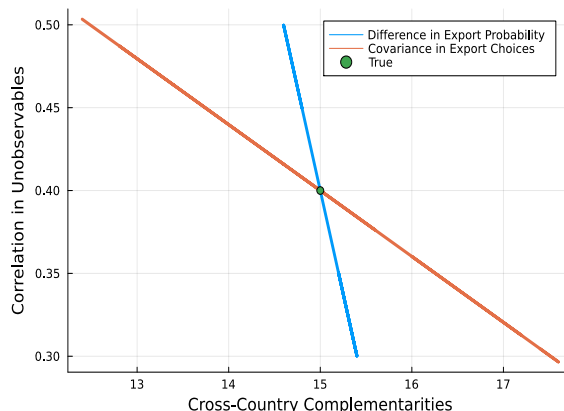
Parameters		Moments	
\bar{c}	$\bar{\rho}$	$\mathbb{E}[y_{i2t} - y_{i1t}]$	$\mathbb{C}[y_{i2t}, y_{i3t}]$
0	0	0	0
Positive	0	0.15	0.05
0	Positive	0	0.02
Positive	Positive	0.17	0.07

Note: by the label “Positive” in the first column, we denote cases in which $\bar{c} = 30$. By the label “Positive” in the second column, we denote cases in which $\bar{\rho} = 0.8$.

In Figure F.1, we perform a different exercise that more directly illustrates the capacity of moments m_1 and m_2 to identify the parameters \bar{c} and $\bar{\rho}$. We simulate data from a “true” model in which we set $\alpha_3 = \bar{\alpha}$, $\bar{c} = 15$, and $\bar{\rho} = 0.4$, and we then compare how the values of moments m_1 and m_2 corresponding to the “true” model compare to those generated under alternative values of the parameters \bar{c} and $\bar{\rho}$. More specifically, the green dot represents the true values of the parameters $\bar{\rho}$ and \bar{c} , and the blue and the orange lines represent all values of $(\bar{\rho}, \bar{c})$ for which moments m_1 and m_2 , respectively, equal their respective values in the “true” model. The slope of the orange line, e.g., shows we can keep moment m_2 at its true value as we increase the value of the parameter $\bar{\rho}$ if we simultaneously decrease the value of the parameter \bar{c} . The blue line indicates the same is true for moment m_1 . Thus, neither moment alone allows to identify the parameter vector $(\bar{\rho}, \bar{c})$, but the fact that the orange and blue lines have different slopes implies that both moments jointly identify $(\bar{\rho}, \bar{c})$.

In unreported simulation results, we observe that the features described in Figure F.1 hold generally as we change the true values of the parameters \bar{c} and $\bar{\rho}$ and as we set the size of country 3, α_3 , to different values.

Figure F.1: Impact of Complementarities and Correlation in Unobservables on Moment Conditions



Notes: The axis labeled “Correlation in Unobservables” includes values of the parameter $\bar{\rho}$. The axis labeled “Cross-country Complementarities” includes values of the parameter \bar{c} . The green dot represents the true values of the parameters $\bar{\rho}$ and \bar{c} ; i.e., $(\bar{\rho}, \bar{c}) = (15, 0.4)$. The blue and the orange lines represent all values of $(\bar{\rho}, \bar{c})$ for which the moments m_1 and m_2 , respectively, equal their respective values in the “true” model.

F.2 Export Potential Measures

We define export potential in Appendix F.2.1. In Appendix F.2.2, we present summary statistics on the gravity equation estimates used to compute these export potentials, on the resulting export potential measures, and on the aggregate export potential of the countries geographically or linguistically close to each foreign country j , or that share a deep PTA with it. In Appendix F.2.3, we present reduced-form evidence showing firm export participation choices in a foreign country correlate with the aggregate export potential of the other countries that are geographically or linguistically close to it, or that share a deep PTA with it.

F.2.1 Definiton and Estimation of Export Potential Measures

We use country-to-country sector-specific trade flows, and the distance measures in equations (1) to (3), to compute measures of the *export potential* of Costa Rica in each sector, destination and year.²⁸ Specifically, we first compute Poisson pseudo-maximum-likelihood estimates of the parameters of the gravity equation

$$X_{odt}^s = \exp(\Psi_{ot}^s + \Xi_{dt}^s + \lambda_g^s n_{od}^g + \lambda_l^s n_{od}^l + \lambda_a^s n_{odt}^a) + u_{odt}^s, \quad (\text{F.1})$$

where X_{odt}^s denotes the export volume from origin o to destination d in sector s and year t ; Ψ_{ot}^s and Ξ_{dt}^s are sector-origin-year and sector-destination-year unobserved effects, respectively; n_{od}^g , n_{od}^l , and n_{odt}^a are the distance measures described in equations (1) to (3); λ_g^s , λ_l^s , and λ_a^s are sector-specific parameters; and u_{odt}^s is an unobserved term. Denoting parameter estimates with a hat, we measure Costa Rica's export potential in a sector s , destination j , and year t as

$$E_{jt}^s = \exp(\hat{\Xi}_{jt}^s + \hat{\lambda}_g^s n_{hj}^g + \hat{\lambda}_l^s n_{hj}^l + \hat{\lambda}_a^s n_{hjt}^a), \quad (\text{F.2})$$

where n_{hj}^g , n_{hj}^l , and n_{hjt}^a denote distances between Costa Rica and country j .²⁹

F.2.2 Gravity-Equation Estimates and Export Potential Measures: Statistics

In Figure F.2, we include boxplots summarizing the distribution across sectors of the parameter estimates $\hat{\lambda}_g^s$ (in green), $\hat{\lambda}_l^s$ (in orange), and $\hat{\lambda}_a^s$ (in blue). The estimates of λ_g^s are negative for all sectors and centered around -1 . The estimates of λ_l^s and λ_a^s are also nearly always negative, although they tend to be smaller in absolute value than the estimates of λ_g^s .

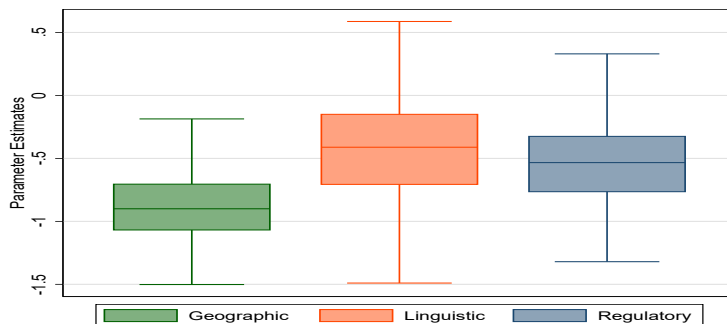
In Figure F.3, we present boxplots summarizing the distribution across sectors and years of the export potential measures E_{jt}^s for the ten destination countries with the largest (in Figure F.3a) and smallest (in Figure F.3b) mean export potentials. The US is the country with the largest mean value of E_{jt}^s . The distribution of E_{jt}^s for the US is actually distinctively different from that corresponding to all other destinations, with the first quartile of the distribution for the US being similar to the third quartile of the distribution of export potentials in Mexico, which is the country with the second largest mean export potential. Other destinations with large mean export potentials are countries that are geographically or linguistically close to Costa Rica (e.g., Panama, Colombia, Venezuela, Spain), or countries that are large importers (e.g., Canada, Germany, Brazil, China). As Figure F.3b shows, the ten destination countries with the smallest mean export potentials (e.g., Bhutan, the Central African Republic, Seychelles, or Burundi) are all small, distant from Costa Rica geographically and linguistically, and do not share any PTA with Costa Rica.

In Figure F.4, we show a color map displaying, for each foreign country j , the mean value of E_{jt}^s across the sectors s and years t in the sample. Most countries in North America, Central America, South America and Europe are in the top three deciles. Also in the top three deciles are Australia, Russia, China and India. On the contrary, most countries in Africa, several in South Asia, and the former Soviet republics are in the bottom deciles.

²⁸The BACI data by CEPII reports country-to-country trade flows at the HS-6 level; see Gaulier and Zignago (2010) for details. Using a concordance provided by WITS (https://wits.worldbank.org/product_concordance.html), we aggregate this product-level data to generate sector-level flows, with sectors defined at the four-digit level according to ISIC Rev. 3. We use a concordance provided by UNSD (<https://unstats.un.org/unsd/classifications/Econ/ISIC.cshtml>) to further convert the data to four-digit sectors defined according to the ISIC Rev. 4.

²⁹In estimating equation (F.1), we exclude all observations in which Costa Rica is the origin or destination country.

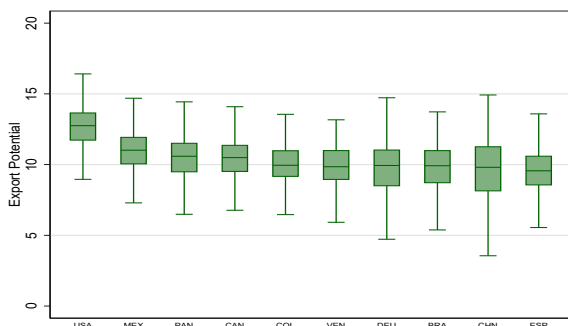
Figure F.2: Estimates of Gravity Equation Parameters



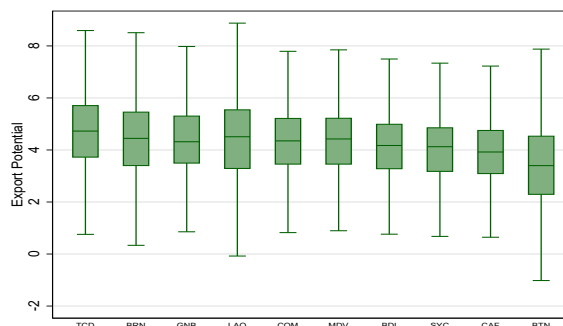
Notes: These boxplots represent the distribution of $\hat{\lambda}_g^s$ (geographic), $\hat{\lambda}_l^s$ (linguistic) and $\hat{\lambda}_a^s$ (regulatory) across sectors.

Figure F.3: Export Potential - Distributions by Country for Top 10 Destinations

(a) Top 10 Destinations

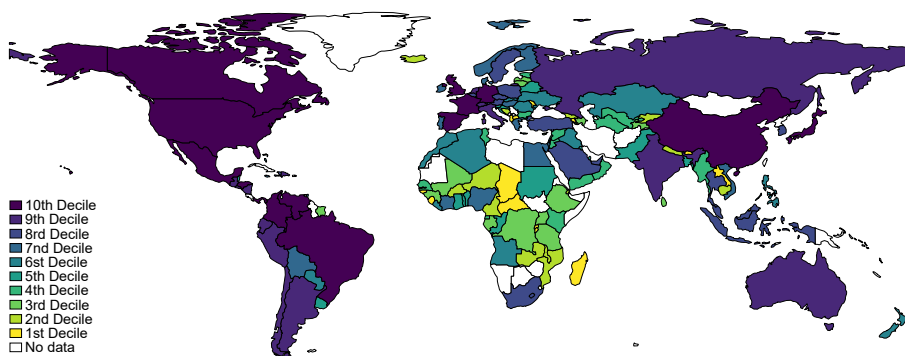


(b) Bottom 10 Destinations



Notes: These boxplots summarize the distribution of E_{jt}^s (see equation (F.2)) for the 10 destination countries with the largest (Figure F.3a) and smallest (Figure F.3b) mean export potentials, where the mean is computed across sectors and years in the period 2005-2015. Countries are listed according to their alpha-3 ISO code.

Figure F.4: Mean Export Potential by Destination Country



Notes: Map of the mean (across sectors and years in the period 2005-2015) E_{jt}^s by country.

In terms of the distribution of export potentials E_{jt}^s across sectors, we find that the five sectors with the largest mean export potentials are the manufacturing of pharmaceuticals, medicinal chemical and botanical products (sector 2100 according to the 4-digit ISIC Rev. 4), the building of ships and floating structures (sector 3011), the manufacturing of computers and peripheral equipment (sector 2620), the manufacturing of motor vehicles (sector 2910), and the manufacturing of basic iron and steel (sector 2410). The manufacturing of plastic products (sector 2220), which is one of the top sectors by aggregate volume of exports from Costa Rica during the period 2005-2015, is also in the top 10 of sectors by their mean export potential.

For each foreign country j , sector s , and year t , we use the export potential measures E_{jt}^s of countries other than j to construct the aggregate export potential of the countries geographically or linguistically close to j , or that share a deep PTA with it. Denoting the aggregate export potential of the countries that, e.g., are geographically close to a destination j as $AE_{jt,g}^s$, we compute it as the sum of the sector- and year-specific export potentials of all countries whose geographical distance to j is smaller than some threshold \bar{n}_g :

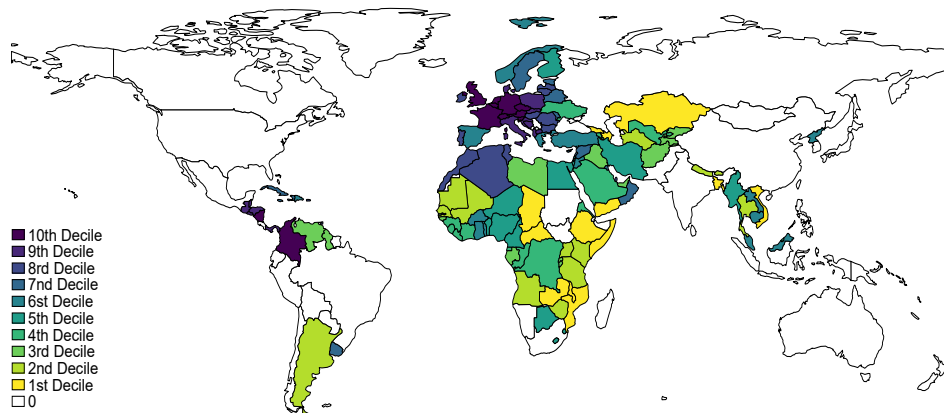
$$AE_{jt,g}^s = \sum_{j' \neq j} \mathbb{1}\{n_{jj'}^g \leq \bar{n}_g\} E_{j't}^s. \quad (\text{F.3})$$

We build similar measures for countries linguistically close to j , or cosignatories of a deep PTA with j , denoted respectively as $AE_{jt,l}^s$ and $AE_{jt,a}^s$. We use as thresholds $\bar{n}_g = 0.79$ (790km), $\bar{n}_l = 0.11$, or $\bar{n}_a = 0.43$.

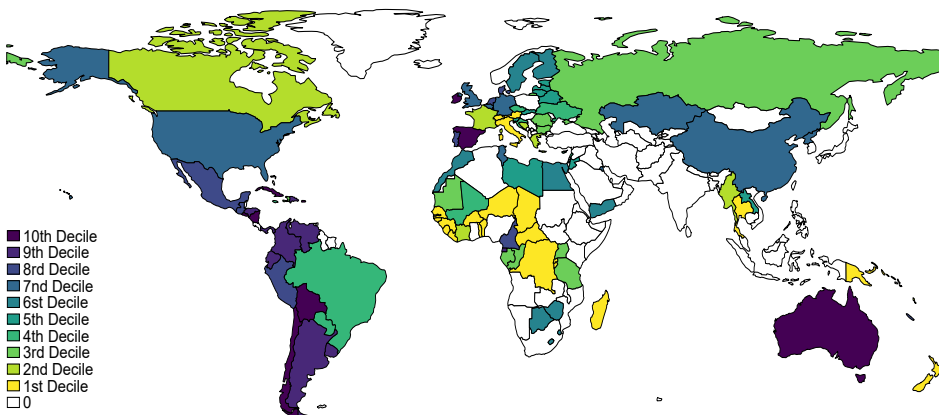
We describe in Figure F.5 the mean (across sectors and years in the period 2005-2015) value of $AE_{jt,g}^s$ (in Panel (a)), $AE_{jt,l}^s$ (in Panel (b)), and $AE_{jt,a}^s$ (in Panel (c)), for every destination country j in the sample.

Figure F.5: Aggregate Export Potential Measures

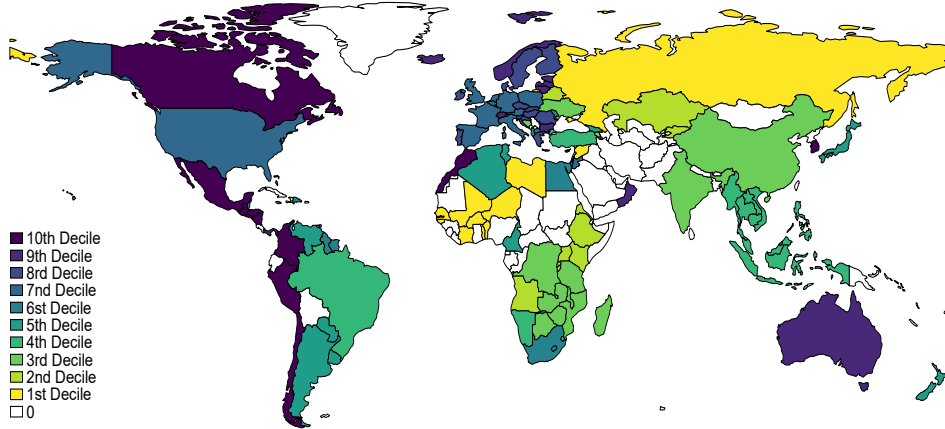
(a) Based on Geographic Distance



(b) Based on Linguistic Distance



(c) Based on Common Membership in a Deep PTA



Notes: Each of the three panels in this figure displays the mean (across sectors and sample years) for each destination country of the aggregate export potential measures $AE_{jt,g}^s$ (in Panel (a)), $AE_{jt,l}^s$ (in Panel (b)), and $AE_{jt,a}^s$ (in Panel (c)).

As discussed in Section 2, our measure of the geographic distance between any two countries j and j' is a weighted average of the distances between largely populated cities located in j and j' and, thus, large countries tend to be geographically isolated. This explains why the United States, Canada, Russia, China or India have a zero value of the aggregate export potential measure $AE_{jt,g}^s$; these countries have no other country such that their bilateral geographic distance $n_{jj'}^g$ is below the threshold $\bar{n}_g = 790$ km we use in our baseline analysis to classify two countries as geographic neighbors. Conversely, as illustrated in Figure F.5a, countries located in Central America and in Central Europe have many geographic neighbors with relatively large export potentials and, thus, their value of $AE_{jt,g}^s$ is large. The aggregate export potential of their geographic neighbors is smaller for countries in Africa (which tend to have many neighbors, but small in terms of their own export potential) and for countries in the European periphery (who have fewer geographic neighbors than those in Central Europe).

The map in Figure F.5b shows that countries with a large share of Spanish speakers (e.g., Spain and several countries in South and Central America) and countries with a large share of English speakers (e.g., countries such as Australia and the UK in which English is an official language, but also countries in which English is not an official language such as Germany or Denmark) exhibit large values of $AE_{jt,l}^s$.

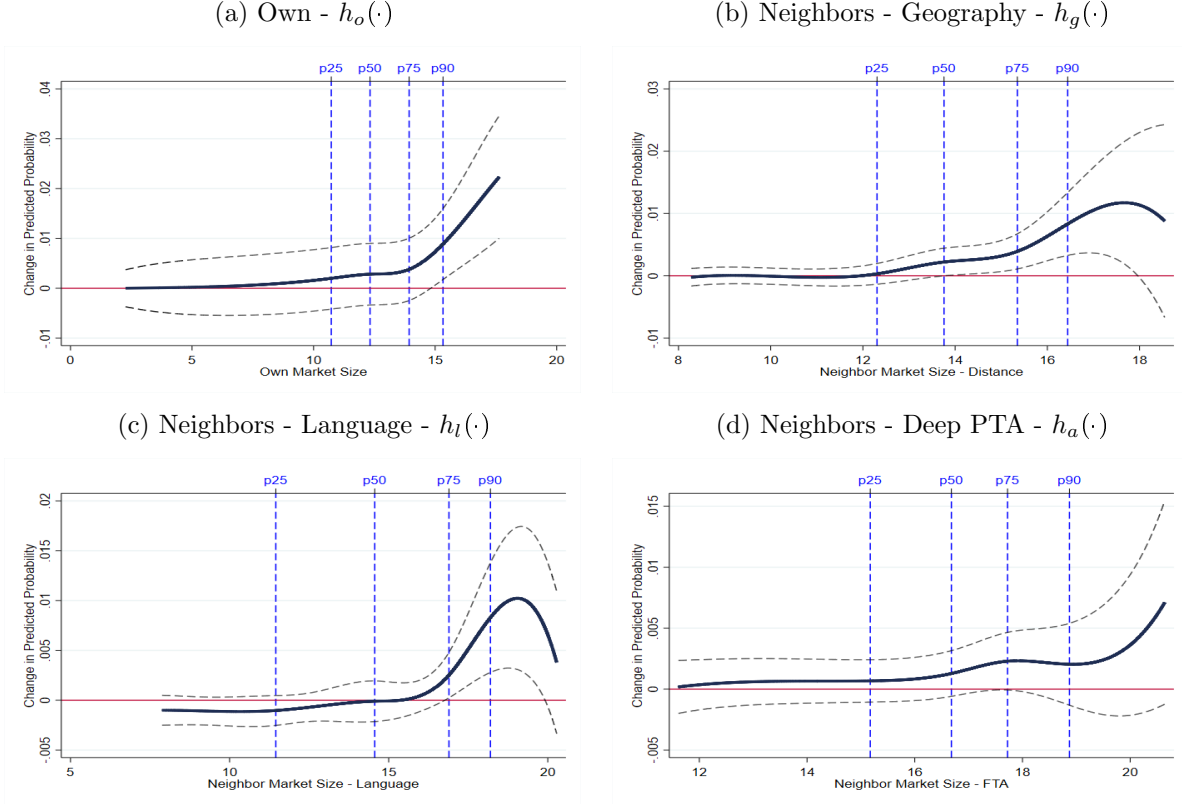
Finally, the map in Figure F.5c shows that countries belonging to the EEC, NAFTA or CAFTA, and countries that have deep PTA with one or more of these trade blocs (e.g., Morocco and Australia) have large values of $AE_{jt,a}^s$. Conversely, countries that are not members of a deep PTA with other large countries (e.g., Russia, China, and most countries in Africa and Central Asia) have small values of $AE_{jt,a}^s$.

F.2.3 Correlation Between Export Potential Measures and Firms' Export Choices

As illustrated in Section 6.2.1, if geographical or linguistic proximity, or common participation in a deep PTA, are a source of cross-country complementarities in export participation decisions, a firm's export probability in a country j and year t will, all else equal, increase in the aggregate market size of the countries geographically or linguistically close to j , or that share a deep PTA with it. To test this implication, we use the aggregate export potential measures introduced in Appendix F.2.2 as proxy for the aggregate market size of the countries close to j , and compute OLS estimates of a regression of a dummy variable that equals one if firm i exports to country j in year t on flexible functions of country j 's log export potential (introduced only as a control variable) and the log of the aggregate export potential of the countries geographically or linguistically close to j , or that share a deep PTA with it. Specifically, given the estimating equation

$$y_{ijt} = h_o(\ln(E_{sjt})) + \sum_{x=\{g,l,a\}} \mathbb{1}\{AE_{jt,x}^s > 0\} h_x(\ln(AE_{jt,x}^s)) + \beta_{it} + u_{ijt}, \quad (\text{F.4})$$

Figure F.6: Impact of Own and Neighbors' Export Potential



Notes: Panels (a), (b), (c), and (d) show the point estimate and 95% confidence intervals for the cubic splines $h_o(\cdot)$, $h_g(\cdot)$, $h_l(\cdot)$, and $h_a(\cdot)$, respectively, in equation (F.4). The marks $p25$, $p50$, $p75$, and $p90$ correspond to the 25th, 50th, 75th, and 90th percentiles of the corresponding covariate; i.e., $E_{s_{jt}}$ for panel (a), $AE_{jt,g}^s$ for panel (b), $AE_{jt,l}^s$ for panel (c), and $AE_{jt,a}^s$ for panel (d). Standard errors are clustered by country.

where $h_x(\cdot)$ for $x = \{o, g, l, a\}$ are cubic splines, and β_{it} is a firm-year fixed effect, panels (a) to (d) in Figure F.6 respectively show OLS estimates of the functions $h_o(\cdot)$, $h_g(\cdot)$, $h_l(\cdot)$, and $h_a(\cdot)$.³⁰

The estimates in Figure F.6 imply that the effect of a country's own export potential as well as the effect of the aggregate export potential of a country's neighbors is highly non-linear, with effects being generally not statistically different from zero until we reach the destination that is at the 75th percentile of the distribution of the corresponding variable. From the 75th percentile onwards, the firm's export probability in a destination increases in the destination's own export potential and in the aggregate export potential of the countries geographically or linguistically close to it.

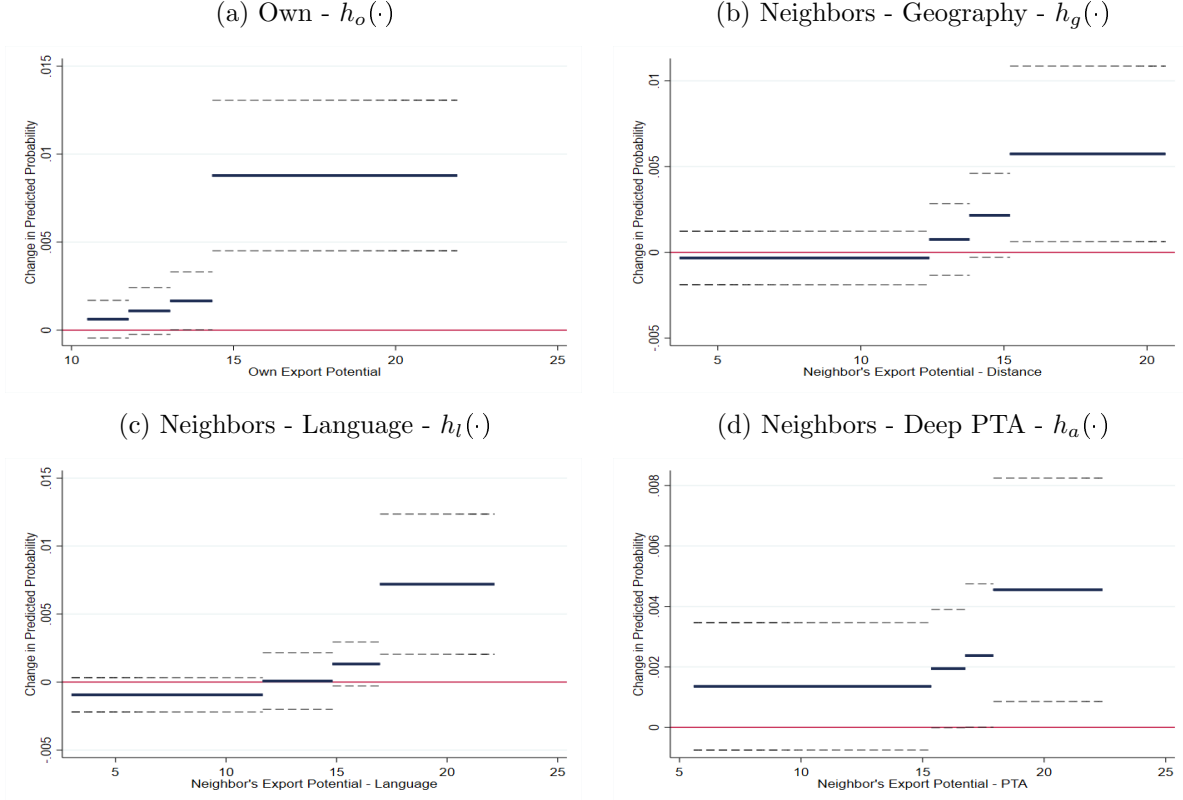
To test the robustness of the findings in Figure F.6, we also compute estimates of a regression similar to that in equation (F.4), but in which we capture the effect of $E_{s_{jt}}$, $AE_{jt,g}^s$, $AE_{jt,l}^s$, and $AE_{jt,a}^s$ on y_{ijt} through step functions (instead of through cubic splines). Given the estimating equation

$$y_{ijt} = \tilde{h}_o(E_{s_{jt}}) + \sum_{x=\{g,l,a\}} \tilde{h}_x(AE_{jt,x}^s) + \beta_{it} + u_{ijt}, \quad (\text{F.5})$$

where $\tilde{h}_x(\cdot)$ for $x = \{o, g, l, a\}$ are step functions, and β_{it} is a firm-year fixed effect, panels (a) to (d) in Figure F.7 respectively show OLS estimates of the functions $\tilde{h}_o(\cdot)$, $\tilde{h}_g(\cdot)$, $\tilde{h}_l(\cdot)$, and $\tilde{h}_a(\cdot)$. More specifically,

³⁰More specifically, for any $x = \{o, g, l, a\}$ and any variable z , $h_x(z) = (1, z, z^2, z^3, (z-q_{25})_+^3, (z-q_{50})_+^3, (z-q_{75})_+^3)' \beta_x$ where β_x is a parameter vector, q_Q is the Q th percentile of the distribution of z , and $(z - q_Q)_+^3 = (\max\{z - q_Q, 0\})^3$.

Figure F.7: Impact of Own and Neighbors' Export Potential



Notes: Panels (a), (b), (c), and (d) show the point estimate and 95% confidence intervals for the step functions $\tilde{h}_o(\cdot)$, $\tilde{h}_g(\cdot)$, $\tilde{h}_l(\cdot)$, and $\tilde{h}_a(\cdot)$, respectively, in equation (F.5). Standard errors are clustered by country.

the function $\tilde{h}_o(E_{sjt})$ is defined as

$$\begin{aligned} \tilde{h}_o(E_{sjt}) = & \beta_{o,1} \mathbb{1}\{0 \leq E_{sjt} \leq q_{o,25}\} + \beta_{o,2} \mathbb{1}\{q_{o,25} \leq E_{sjt} \leq q_{o,50}\} \\ & + \beta_{o,3} \mathbb{1}\{q_{o,50} \leq E_{sjt} \leq q_{o,75}\} + \beta_{o,4} \mathbb{1}\{q_{o,75} \leq E_{sjt}\}, \end{aligned} \quad (\text{F.6})$$

where $(\beta_{o,1}, \beta_{o,2}, \beta_{o,3}, \beta_{o,4})$ is a vector of unknown parameters, and $q_{o,Q}$ is the Qth percentile of the distribution of E_{sjt} . Similarly, for any $x = \{g, l, a\}$, the function $\tilde{h}_x(AE_{jt,x}^s)$ is defined as

$$\begin{aligned} \tilde{h}_x(AE_{jt,x}^s) = & \beta_{x,1} \mathbb{1}\{0 \leq AE_{jt,x}^s \leq q_{x,25}\} + \beta_{x,2} \mathbb{1}\{q_{x,25} \leq AE_{jt,x}^s \leq q_{x,50}\} \\ & + \beta_{x,3} \mathbb{1}\{q_{x,50} \leq AE_{jt,x}^s \leq q_{x,75}\} + \beta_{x,4} \mathbb{1}\{q_{x,75} \leq AE_{jt,x}^s\}, \end{aligned} \quad (\text{F.7})$$

where $(\beta_{x,1}, \beta_{x,2}, \beta_{x,3}, \beta_{x,4})$ is a vector of unknown parameters, and $q_{x,Q}$ is the Qth percentile of the distribution of $AE_{jt,x}^s$ conditional on $AE_{jt,x}^s > 0$. The regression estimates described in Figure F.7 are very similar to those described in Figure F.6.

F.3 List of Moment Conditions

As discussed in Section 6.2, all moment conditions we use in our SMM estimator take the form

$$\frac{1}{M} \sum_{i=1}^M \{m_k(y_i^{obs}, z_i, x) - \frac{1}{S} \sum_{i=1}^S m_k(y_i^s(\theta), z_i, x)\} = 0, \quad (\text{F.8})$$

where y_i^{obs} includes the observed firm i 's export participation decisions in every country $j = 1, \dots, J$ and every sample period t in which the firm is active (i.e., every t in $[t_i, \bar{t}]$); z_i includes all observed payoff-relevant variables and all estimates computed in the first step of our estimation procedure (see Section 6.1); x includes the export potential measures in equation (F.2) for all foreign countries and sample period; and $y_i^s(\theta)$ includes all model-implied export participation decisions for given values of z_i and the parameter vector θ , and a draw χ_i^s from the distribution of χ_i conditional on z_i . Specifically, we can write z_i , x , and χ_i as

$$z_i \equiv (\hat{\alpha}_y, \hat{\alpha}_a, \hat{\alpha}_r, \hat{\beta}_\alpha, \hat{\rho}_\alpha, \hat{\sigma}_\alpha, \hat{\beta}_r, \hat{\rho}_r, \hat{\sigma}_r, \{\hat{\alpha}_{jt}\}_{j=1, t=\underline{t}}^{J, \bar{t}}, \hat{\alpha}_s, \{r_{iht}\}_{t=t_i}^{\bar{t}}, \{a_{st}\}_{t=\underline{t}}^{\bar{t}}, \quad (\text{F.9})$$

$$\{(n_{jj'}^g, n_{jj'}^l)\}_{j=1, j'=1}^{J, J}, \{n_{jj't}^a\}_{j=1, j'=1, t=\underline{t}}^{J, J, \bar{t}}, \{(n_{hj}^g, n_{hj}^l)\}_{j=1}^J, \{n_{hjt}^a\}_{j=1, t=\underline{t}}^{J, \bar{t}}, \quad (\text{F.10})$$

$$x = \{E_{jt}^s\}_{j=1, t=\underline{t}}^{J, \bar{t}}, \quad (\text{F.11})$$

$$\chi_i \equiv (\{\alpha_{jt}\}_{j=1, t=\underline{t}}^{J, t=\underline{t}-1}, \{\alpha_{jt}\}_{j=1, t=\bar{t}+1}^{J, t=T}, \{r_{iht}\}_{t=\underline{t}}^{t=\underline{t}-1}, \{r_{iht}\}_{t=\bar{t}+1}^{t=T}, \{\nu_{ijt}\}_{j=1, t=\underline{t}}^{J, T}). \quad (\text{F.12})$$

Each moment function $m_k(\cdot)$ is an average over foreign countries and periods of a function $\tilde{m}_{k,jt}(\cdot)$. Specifically, both for $y_i = y_i^{obs}$ and for $y_i = y_i^s(\theta)$, it holds that

$$m_k(y_i, z_i, x) \equiv \frac{1}{J(\bar{t} - t_i)} \sum_{j=1}^J \sum_{t=t_i}^{\bar{t}} \tilde{m}_{k,jt}(y_i, z_i, x) \quad (\text{F.13})$$

where $t_i = \max\{\underline{t}, \underline{t}_i\}$ is the first year firm i is observed. As a reminder, J is the number of potential export destinations, \underline{t} and \bar{t} respectively denote the first and last sample periods, and \underline{t}_i denotes firm i 's birth year.

We use in our SMM estimator 89 moments of the type defined by equations (F.8) and (F.13) for different functions $m_{k,jt}(\cdot)$. For expositional purposes, we can classify these in three different blocks.

The *first* block includes moments targeted to identify the parameters determining the level of fixed and sunk export costs as well as the impact on them of the distance between the firm's home country and each potential export destinations. Specifically, the first block of moments targets the identification of the parameters

$$(\gamma_0^F, \gamma_0^S, \{(\gamma_x^F, \gamma_x^S)\}_{x=\{g,l,a\}}),$$

which enter the model through the expressions in equations (9) and (14). A first set of moments in this block captures firms' export participation choices by groups of destinations that differ in their distances to the firm's home country. More specifically, these moments are defined by

$$\tilde{m}_{k,jt}(y, z, x) = y_{ijt} \mathbb{1}\{n_{hj}^{x_1} < \bar{n}_{x_1}\} \mathbb{1}\{n_{hjt}^{x_2} < \bar{n}_{x_2}\} n_{hjt}^{x_1} n_{hjt}^{x_2}, \quad (\text{F.14a})$$

$$\tilde{m}_{k,jt}(y, z, x) = y_{ijt} \mathbb{1}\{n_{hj}^{x_1} \geq \bar{n}_{x_1}\} \mathbb{1}\{n_{hjt}^{x_2} < \bar{n}_{x_2}\} n_{hjt}^{x_1} n_{hjt}^{x_2}, \quad (\text{F.14b})$$

$$\tilde{m}_{k,jt}(y, z, x) = y_{ijt} \mathbb{1}\{n_{hj}^{x_1} < \bar{n}_{x_1}\} \mathbb{1}\{n_{hjt}^{x_2} \geq \bar{n}_{x_2}\} n_{hjt}^{x_1} n_{hjt}^{x_2}, \quad (\text{F.14c})$$

$$\tilde{m}_{k,jt}(y, z, x) = y_{ijt} \mathbb{1}\{n_{hj}^{x_1} \geq \bar{n}_{x_1}\} \mathbb{1}\{n_{hjt}^{x_2} \geq \bar{n}_{x_2}\} n_{hjt}^{x_1} n_{hjt}^{x_2}, \quad (\text{F.14d})$$

for all (x_1, x_2) in $\{(g, l), (g, a), (l, a)\}$. As a reminder, $n_{hj}^g = n_{hj}^g$ and $n_{hjt}^l = n_{hj}^l$ for all t , and n_{hj}^g , n_{hj}^l , and n_{hjt}^a respectively denote the geographic, linguistic and regulatory distances between the firm's home country h and the foreign country j . The constants \bar{n}_{x_1} and \bar{n}_{x_2} are thresholds that split destination countries into two groups depending on whether their distance to the firm's home market h is larger or smaller than the corresponding threshold; specifically, we set $\bar{n}_g = 6$ (i.e., 6,000 km), $\bar{n}_l = 0.5$, and $\bar{n}_a = 1$. According to these thresholds, we split countries roughly depending on whether they are in the Americas (in which case $n_{hj}^g < 6$), on whether at least 50% of their population speak Spanish (in which case $n_{hj}^l < 0.5$), and on whether they have any sort of deep PTA with Costa Rica (in which case $n_{hjt}^a < 1$). E.g., the moment given the function in equation (F.14a) for $(x_1, x_2) = (g, l)$ is

$$\frac{1}{MJ(\bar{t} - \underline{t})} \sum_{i=1}^M \sum_{j=1}^J \sum_{t=\underline{t}}^{\bar{t}} (y_{ijt}^{obs} - \frac{1}{S} \sum_{i=1}^S y_{ijt}^s(\theta)) \mathbb{1}\{n_{hj}^g < 6\} \mathbb{1}\{n_{hj}^l < 0.5\} n_{hj}^g n_{hj}^l = 0. \quad (\text{F.15})$$

For the foreign countries less than 6,000 km away from Costa Rica and with linguistic distance to Costa Rica below 0.5, this moment captures the difference between the observed sample and the average across the S simulated samples in the average (across firms, countries and periods) product of the geographic and linguistic distances of the actual export destinations.

A second set of moments still within this first block are defined by the following functions:

$$\tilde{m}_{k,jt}(y_i, z_i, x) = y_{ijt}y_{ijt-1} \mathbb{1}\{n_{hjt}^{x_1} < \bar{n}_{x_1}\} \mathbb{1}\{n_{hjt}^{x_2} < \bar{n}_{x_2}\} n_{hjt}^{x_1} n_{hjt}^{x_2}, \quad (\text{F.16a})$$

$$\tilde{m}_{k,jt}(y_i, z_i, x) = y_{ijt}y_{ijt-1} \mathbb{1}\{n_{hjt}^{x_1} \geq \bar{n}_{x_1}\} \mathbb{1}\{n_{hjt}^{x_2} < \bar{n}_{x_2}\} n_{hjt}^{x_1} n_{hjt}^{x_2}, \quad (\text{F.16b})$$

$$\tilde{m}_{k,jt}(y_i, z_i, x) = y_{ijt}y_{ijt-1} \mathbb{1}\{n_{hjt}^{x_1} < \bar{n}_{x_1}\} \mathbb{1}\{n_{hjt}^{x_2} \geq \bar{n}_{x_2}\} n_{hjt}^{x_1} n_{hjt}^{x_2}, \quad (\text{F.16c})$$

$$\tilde{m}_{k,jt}(y_i, z_i, x) = y_{ijt}y_{ijt-1} \mathbb{1}\{n_{hjt}^{x_1} \geq \bar{n}_{x_1}\} \mathbb{1}\{n_{hjt}^{x_2} \geq \bar{n}_{x_2}\} n_{hjt}^{x_1} n_{hjt}^{x_2}, \quad (\text{F.16d})$$

for all (x_1, x_2) in $\{(g, l), (g, a), (l, a)\}$. These functions differ from those in equation (F.14) in that they depend not on whether a firm i exports to a country j at a period t (as captured by the dummy y_{ijt}) but on whether a firm i continues exporting at t to a country j to which it was exporting at $t - 1$ (as captured by the dummy $y_{ijt}y_{ijt-1}$). E.g., the moment given the function in equation (F.16a) for $(x_1, x_2) = (g, l)$ is

$$\frac{1}{MJ(\bar{t} - \underline{t})} \sum_{i=1}^M \sum_{j=1}^J \sum_{t=\underline{t}}^{\bar{t}} (y_{ijt}^{obs} y_{ijt-1}^{obs} - \frac{1}{S} \sum_{i=1}^S y_{ijt}^s(\theta) y_{ijt-1}^s(\theta)) \mathbb{1}\{n_{hj}^g < 6\} \mathbb{1}\{n_{hj}^l < 0.5\} n_{hj}^g n_{hj}^l = 0. \quad (\text{F.17})$$

The interpretation of this moment is analogous to that in equation (F.15), with the only difference that it focuses in export survival events instead of export participation events.

Equations (F.14) and (F.16) list four moments each for each (x_1, x_2) in $\{(g, l), (g, a), (l, a)\}$. Thus, the first block of moments includes 24 moments in total.

The *second* block includes moments targeted to identify the parameters determining the strength of export complementarities. Specifically, this block of moments targets the identification of the parameters

$$\{(\gamma_x^E, \psi_x^E, \kappa_x^E)\}_{x=\{g,l,a\}},$$

which enter the model through the expression in equation (11). The functions defining the moments included in this second block capture firms' export probabilities by groups of destinations that differ in the aggregate export potential of the *other* countries that are at a given geographical, linguistic, or regulatory distance to them. A key variable in these moments is thus the aggregate export potential of the countries that are within certain distance thresholds of each potential export destination; we define these as

$$AE_{jt,x_1}^{s,x_2} = \sum_{j' \neq j} \mathbb{1}\{n_{x_1}^{x_2} \leq n_{jj't}^{x_1} < \bar{n}_{x_1}^{x_2}\} E_{j't}^s, \quad (\text{F.18})$$

where the index x_1 identifies the distance measure, and the index x_2 identifies the distance interval over which we are summing the export potential measures $E_{j't}^s$. The index x_1 takes values in the set $\{g, l, a\}$, with $x_1 = g$ denoting the geographical distance in equation (1), $x_1 = l$ denoting the linguistic distance in equation (2), and $x_1 = a$ denoting the regulatory distance in equation (3). The index x_2 takes values in the set $\{1, 2, 3\}$, and it determines the distance thresholds according to the following rules. For the case in which $x_1 = g$, the distance thresholds are

$$(n_g^{x_2}, \bar{n}_g^{x_2}) = \begin{cases} (0, 426) & \text{if } x_2 = 1, \\ (426, 790) & \text{if } x_2 = 2, \\ (790, 1153) & \text{if } x_2 = 3. \end{cases} \quad (\text{F.19})$$

For the case in which $x_1 = l$, the distance thresholds are

$$(n_l^{x_2}, \bar{n}_l^{x_2}) = \begin{cases} (0, 0.01) & \text{if } x_2 = 1, \\ (0.01, 0.11) & \text{if } x_2 = 2, \\ (0.11, 0.50) & \text{if } x_2 = 3. \end{cases} \quad (\text{F.20})$$

Finally, for the case in which $x_1 = a$, the distance thresholds are

$$(\bar{n}_a^{x_2}, \bar{n}_a^{x_2}) = \begin{cases} (0, \frac{1}{7}) & \text{if } x_2 = 1, \\ (\frac{1}{7}, \frac{3}{7}) & \text{if } x_2 = 2, \\ (\frac{3}{7}, \frac{6}{7}) & \text{if } x_2 = 3. \end{cases} \quad (\text{F.21})$$

Then, for example, the variables $AE_{jt,g}^{s,1}$, $AE_{jt,g}^{s,2}$, and $AE_{jt,g}^{s,3}$ denote the aggregate export potential in sector s of all countries j' other than country j which are less than 426 km away from j , between 426 km and 790 km away from j , and between 790 km and 1153 km away from j , respectively. There is a close connection between the variables defined in equations (F.18) to (F.21) and those used as regressors in equation (F.4). Specifically, for any x_1 in $\{g, l, a\}$, the thresholds $\bar{n}_{x_1}^2$ defined in equations (F.19) to (F.21) coincide with the thresholds \bar{n}_{x_1} used to compute the aggregate export potentials displayed in Figure F.5. Thus,

$$AE_{jt,x_1}^s = AE_{jt,x_1}^{s,1} + AE_{jt,x_1}^{s,2} \quad \text{for } x_1 = \{g, l, a\}.$$

Given AE_{jt,x_1}^{s,x_2} for $x_1 = \{g, l, a\}$ and $x_2 = \{1, 2, 3\}$, the moments in this second block are defined by

$$\tilde{m}_{k,jt}(y_i, z_i, x) = y_{ijt} \mathbb{1}\{n_{hjt}^{x_1} < \bar{n}_{x_1}\} \mathbb{1}\{AE_{jt,x_1}^{s,x_2} = 0\}, \quad (\text{F.22a})$$

$$\tilde{m}_{k,jt}(y_i, z_i, x) = y_{ijt} \mathbb{1}\{n_{hjt}^{x_1} < \bar{n}_{x_1}\} \mathbb{1}\{0 < AE_{jt,x_1}^{s,x_2} \leq p_{66}(AE_{jt,x_1}^{s,x_2})\}, \quad (\text{F.22b})$$

$$\tilde{m}_{k,jt}(y_i, z_i, x) = y_{ijt} \mathbb{1}\{n_{hjt}^{x_1} < \bar{n}_{x_1}\} \mathbb{1}\{p_{66}(AE_{jt,x_1}^{s,x_2}) < AE_{jt,x_1}^{s,x_2}\}, \quad (\text{F.22c})$$

$$\tilde{m}_{k,jt}(y_i, z_i, x) = y_{ijt} \mathbb{1}\{n_{hjt}^{x_1} \geq \bar{n}_{x_1}\} \mathbb{1}\{AE_{jt,x_1}^{s,x_2} = 0\}, \quad (\text{F.22d})$$

$$\tilde{m}_{k,jt}(y_i, z_i, x) = y_{ijt} \mathbb{1}\{n_{hjt}^{x_1} \geq \bar{n}_{x_1}\} \mathbb{1}\{0 < AE_{jt,x_1}^{s,x_2} \leq p_{66}(AE_{jt,x_1}^{s,x_2})\}, \quad (\text{F.22e})$$

$$\tilde{m}_{k,jt}(y_i, z_i, x) = y_{ijt} \mathbb{1}\{n_{hjt}^{x_1} \geq \bar{n}_{x_1}\} \mathbb{1}\{p_{66}(AE_{jt,x_1}^{s,x_2}) < AE_{jt,x_1}^{s,x_2}\}, \quad (\text{F.22f})$$

where $p_{66}(\cdot)$ denotes the 66th percentile of the random variable in parenthesis. As a reminder, $n_{hjt}^{x_1}$ denotes for any x_1 in $\{g, l, a\}$ the corresponding distance between the firm's home country h and the foreign country j , and \bar{n}_{x_1} is a threshold value we use to split foreign countries into two groups depending on whether their distance to the home market is larger or smaller than the corresponding threshold; specifically, we set $\bar{n}_g = 6$, $\bar{n}_l = 0.5$, and $\bar{n}_a = 1$, which are the same threshold values we use to define the moments in equations (F.14) and (F.16). E.g., the moment given by the function in equation (F.22a) for $(x_1, x_2) = (g, 1)$ is

$$\frac{1}{MJ(\bar{t} - \underline{t})} \sum_{i=1}^M \sum_{j=1}^J \sum_{t=\underline{t}}^{\bar{t}} (y_{ijt}^{obs} - \frac{1}{S} \sum_{i=1}^S y_{ijt}^s(\theta)) \mathbb{1}\{n_{hj}^g < 6\} \mathbb{1}\{AE_{jt,g}^{s,1} = 0\} = 0. \quad (\text{F.23})$$

This moment captures, for those foreign countries that are less than 6,000 km away from Costa Rica and have no country closer than 426 km to them, the difference between the export probability in the observed sample and the average export probability across the S simulated samples. Similarly, the moment given by the function in equation (F.22b) for $(x_1, x_2) = (g, 1)$ is

$$\frac{1}{MJ(\bar{t} - \underline{t})} \sum_{i=1}^M \sum_{j=1}^J \sum_{t=\underline{t}}^{\bar{t}} (y_{ijt}^{obs} - \frac{1}{S} \sum_{i=1}^S y_{ijt}^s(\theta)) \mathbb{1}\{n_{hj}^g < 6\} \mathbb{1}\{0 < AE_{jt,g}^{s,1} \leq p_{66}(AE_{jt,g}^{s,1})\} = 0. \quad (\text{F.24})$$

This moment captures, for foreign countries that are less than 6,000 km away from Costa Rica and have countries located less than 426 km away from them whose aggregate export potential is positive by below the 66th percentile of the corresponding distribution, the difference between the export probability in the observed sample and the average export probability across the S simulated samples.

Equation (F.22) lists six moments for each x_1 in $\{g, l, a\}$ and each x_2 in $\{1, 2, 3\}$. Thus, this block of moments could include 54 moments in total, each of them defined as the difference between the observed and simulated export probabilities in a subset of countries selected on the basis of their geographic, linguistic, or regulatory, distance to Costa Rica and of the aggregate export potential of the other potential destinations located within some pre-specified distance interval from those countries. However, 2 of these 54 moments

select subsets of countries that are empty. As a result, the second block includes 52 moments in total.

The *third* block includes moments targeted to identify the parameters determining the distribution of the unobserved (to the researcher) terms ν_{it} and ω_{it} . Specifically, this block targets the identification of

$$(\sigma_\nu, p, \{(\gamma_x^N, \kappa_x^N)\}_{x=\{g,l,a\}}),$$

which enter the model through the expressions in equations (12) and (13). With the aim of identifying the variance of the fixed cost shock ν_{ijt} , σ_ν^2 , we use moments defined by the following two functions

$$\tilde{m}_{k,jt}(y_i, z_i, x) = y_{ijt} \sum_{i' \neq i} y_{i'jt} \mathbb{1}\{Q(r_{iht}) = Q(r_{i'ht})\}, \quad (\text{F.25a})$$

$$\tilde{m}_{k,jt}(y_i, z_i, x) = \mathbb{1}\left\{\sum_{j=1} y_{ijt} > 0\right\}, \quad (\text{F.25b})$$

where $Q(\cdot) : \mathbb{R}^+ \rightarrow \{1, 2, 3, 4\}$ is a function that maps the firm's domestic revenue level into its corresponding quartile. The moment defined by the function in equation (F.25a) captures, on average across periods and pairs of firms i and i' whose domestic sales belong to the same quartile of the distribution, the similarity in the sets of export destinations of these two firms in the corresponding period. The function in equation (F.25b) captures whether firm i is an exporter at period t . These two moments help identify σ_ν .

With the aim of identifying p , we use moments defined by the following two functions

$$\tilde{m}_{k,jt}(y_i, z_i, x) = y_{ijt}(1 - y_{ijt-1})y_{ijt-2} + y_{ijt}(1 - y_{ijt-1})(1 - y_{ijt-2})y_{ijt-3}, \quad (\text{F.26a})$$

$$\tilde{m}_{k,jt}(y_i, z_i, x) = (1 - y_{ijt})y_{ijt-1}(1 - y_{ijt-2}) + (1 - y_{ijt})y_{ijt-1}y_{ijt-2}(1 - y_{ijt-3}). \quad (\text{F.26b})$$

The function in equation (F.26a) captures short (one or two periods) spells outside of an export market. The function in equation (F.26a) captures short export spells. As our model features firms that have perfect foresight on all payoff-relevant variables other than the fixed cost shock ω_{ijt} , short-lived transitions in and out of an export market will be largely driven by unexpected realizations of this fixed cost shock. The functions in equation (F.26) measure the frequency with which these short-lived transition take place.

Finally, with the aim of identifying $\{(\gamma_x^N, \kappa_x^N)\}_{x=\{g,l,a\}}$, we use moments defined by the following functions

$$m_k(y, z, x) = y_{ijt} \sum_{j'=1}^J y_{i'jt} \mathbb{1}\{y_{ijt-1} = y_{i'jt-1}\} \mathbb{1}\{Q(E_{ijt}) = Q(E_{i'jt})\} \mathbb{1}\{n_{x_1}^{x_2} \leq n_{j'jt}^{x_1} < \bar{n}_{x_1}^{x_2}\} \quad (\text{F.27})$$

for any value of x_1 in $\{g, l, a\}$ and any value of x_2 in $\{1, 2, 3\}$, where $Q(\cdot) : \mathbb{R}^+ \rightarrow \{1, 2, 3, 4\}$ is a function that maps a country's export potential into its corresponding quartile. For any value of x_1 in $\{g, l, a\}$ and any value of x_2 in $\{1, 2, 3\}$, the thresholds $\bar{n}_{x_1}^{x_2}$ and $\underline{n}_{x_1}^{x_2}$ are determined as in equations (F.19) to (F.21). E.g., the moment built using the function in equation (F.27) for $(x_1, x_2) = (g, 1)$ captures, on average across firms and time periods, the frequency with which firms simultaneously export to any two countries j and j' in which they had the same export status in the previous period (as imposed by the condition that y_{ijt-1} and $y_{i'jt-1}$ should coincide), that have similar export potentials (as imposed by the condition that E_{ijt} and $E_{i'jt}$ should fall in the same quartile), and that are less than 426 km apart from each other. Intuitively, the function in equation (F.27) for $(x_1, x_2) = (g, 1)$ captures the correlation in firms' export participation decisions across countries of similar market size that are geographically very close to each other.

The function in equation (F.27) for $(x_1, x_2) = (g, 2)$ is analogous to that for $(x_1, x_2) = (g, 1)$, differing exclusively in that, instead of focusing on pairs of countries that are less than 426 km apart, it focuses on pairs of countries whose bilateral distance is larger than 426 km and smaller than 790 km. Similarly, the function in equation (F.27) for $(x_1, x_2) = (g, 3)$ focuses instead on pairs of countries whose bilateral distance is larger than 790 km and smaller than 1,153 km. Thus, the functions in equation (F.27) for $x_1 = g$ and all three possible values of x_2 allow us to identify the parameters determining the correlation between ν_{ijt} and $\nu_{i'jt}$ as a function of the geographical distance between countries j and j' .

Equations (F.25) and (F.26) list two moments each. Equation (F.27) lists one moment for each x_1 in $\{g, l, a\}$ and each x_2 in $\{1, 2, 3\}$. Thus, the third block of moments includes 13 moments in total.

F.4 Additional Details on SMM Estimator

We provide here additional details on two aspects of our SMM estimator. In Appendix F.4.1, we describe how we compute the vector of simulated choices $y_i^s(\theta)$ that enter the moment conditions; see equation (32). In Appendix F.4.2, we describe how we compute our SMM estimates given the vector of moment conditions.

F.4.1 Computing Vector of Simulated Choices

Given any value of the parameter vector θ defined in footnote 24, we describe here the steps we follow to compute each of the moment conditions (see equation (32)) we use in our estimation.

First step. For each firm i in the sample, we take $S = 5$ draws of the vector of unobserved payoff-relevant variables χ_i defined in equation (F.12). Specifically, for each draw, we implement the following procedure.

First, if we observe firm i in the first sample year, \underline{t} , then we treat its birth year, \underline{t}_i , as unknown, and we draw it randomly from the empirical distribution of firm ages in Costa Rica in 2010, as reported in World Bank (2012). If we do not observe firm i in \underline{t} , then we assume its birth year coincides with the first year it appears in the sample. The firm's birth year is thus observed, and not randomly drawn, in this case.³¹

Second, we draw firm i 's domestic revenue shocks e_{iht}^r for every period t in the interval $[\underline{t}_i, T]$. To do so, we draw $T - \underline{t}_i + 1$ independent standard normal variables, which we then multiply by σ_r . If $\underline{t}_i < \underline{t}$, we use the draws of e_{iht}^r for every t in $[\underline{t}_i, \underline{t}]$, together with the firm's observed domestic sales in the first sample year, r_{iht} , to generate values of the firm's log domestic sales for every t in $[\underline{t}_i, \underline{t}]$. In this case, $\ln(r_{iht})$ operates as a terminal condition of the corresponding AR(1) process, and we use the unconditional mean of this process as initial condition. Similarly, we use the draws of e_{iht}^r for every t in $[\bar{t} + 1, T]$, together with the firm's observed domestic sales in the last sample year, r_{iht} , to generate values of the firm's log domestic sales for every t in $[\bar{t} + 1, T]$. In this case, $\ln(r_{iht})$ operates as an initial condition of the corresponding process.

Third, we draw firm i 's fixed cost shocks ν_{ijt} and ω_{ijt} for every country $j = 1, \dots, J$ and every t in $[\underline{t}_i, T]$. To obtain these draws of ν_{ijt} , we first draw $J(T - \underline{t}_i + 1)$ independent standard normal random variables, which we then multiply by the Cholesky decomposition of the variance matrix in equation (12). To obtain these draws of ω_{ijt} , we first draw $J(T - \underline{t}_i + 1)$ independent random variables distributed uniformly between 0 and 1; we then set $\omega_{ijt} = \omega$ if the draw corresponding to country j and period t is smaller than the parameter p introduced in equation (13), and $\omega_{ijt} = \bar{\omega}$ otherwise.

Fourth, for each country j , we draw α_{jt} for every period t between the earliest birth year in the corresponding simulated sample and the initial sample year, and for every t between the last sample year and the terminal period; i.e., for all t in $[\min_i\{\underline{t}_i\}, \underline{t}] \cup [\bar{t}, T]$. To do so, we first obtain $T - \min_i\{\underline{t}_i\} + 1$ draws of the shocks e_{jt}^α , which we do by drawing an equal number of independent standard normal variables that we then multiply by σ_α . We then use the draws of e_{jt}^α for every t in $[\min_i\{\underline{t}_i\}, \underline{t}]$, together with the value of α_{jt} observed in the first sample year, $\alpha_{j\underline{t}}$, to generate values of α_{jt} for every t in $[\min_i\{\underline{t}_i\}, \underline{t}]$. In this case, $\alpha_{j\underline{t}}$ operates as a terminal condition of the corresponding AR(1) process, and we use the unconditional mean of this process as initial condition. Similarly, we use the draws of e_{jt}^α for every t in $[\bar{t} + 1, T]$, together with the value of α_{jt} observed in the last sample year, $\alpha_{j\bar{t}}$, to generate values of α_{jt} for every t in $[\bar{t} + 1, T]$. In this case, $\alpha_{j\bar{t}}$ operates as an initial condition of the corresponding AR(1) process.

Second step. For each firm i in the sample, we use the S draws of χ_i generated according to the procedure described above, the vector z_i of observed payoff-relevant variables, and a value of the parameter vector θ , to compute the vector of model-implied firm i 's optimal export choices $y_i^s(\theta)$ for all $s = 1, \dots, S$ simulated samples. We do so implementing the algorithm described in Section 5.

F.4.2 Computing SMM Estimates

Denote the vector of K moment conditions as $\mathcal{M}(Z, x; \theta) = (\mathfrak{m}_1(Z, x; \theta), \dots, \mathfrak{m}_K(Z, x; \theta))'$ where

$$\mathfrak{m}_k(y^{obs}, Z, x; \theta) = \frac{1}{M} \sum_{i=1}^M \left\{ \frac{1}{J(\bar{t} - \underline{t}_i)} \sum_{j=1}^J \sum_{t=\underline{t}_i}^{\bar{t}} \left\{ m_k(y_i^{obs}, z_i, x) - \frac{1}{S} \sum_{i=1}^S m_k(y_i^s(\theta), z_i, x) \right\} \right\},$$

³¹A firm will appear in our dataset as long as it has positive domestic sales, regardless of whether it exports.

with $y^{obs} = \{y_i^{obs}\}_{i=1}^M$ and $Z = \{x_i\}_{i=1}^M$. Given $\mathcal{M}(Z, x; \theta)$ and a $K \times K$ positive semi-definite matrix W , we compute our SMM estimate of θ as the solution to the following minimization problem

$$\min_{\theta} \mathcal{M}(Z, x; \theta) W \mathcal{M}(Z, x; \theta)'. \quad (\text{F.28})$$

To solve this minimization problem numerically, we use a two-step algorithm: first, we use the TikTak global optimizer proposed in Arnaud et al. (2019) with 5,000 starting points, using BOBYQA as the local optimizer; second, we polish the outcome of the global optimizer using a Subplex local optimizer.

In practice, we compute a two-stage SMM estimate. In the first stage, we obtain estimates of θ , which we denote as $\hat{\theta}_1$, minimizing the objective function in equation (F.28) for a diagonal weight matrix $W = W_1$ in which every diagonal element $k = 1, \dots, K$ equals

$$W_{1,k} = \frac{1}{(\mathfrak{m}_k^{obs}(y^{obs}, Z, x))^2}, \quad \text{with} \quad \mathfrak{m}_k^{obs}(y^{obs}, Z, x) \equiv \frac{1}{M} \sum_{i=1}^M \left\{ \frac{1}{J(\bar{t} - t_i)} \frac{1}{S} \sum_{j=1}^J \sum_{t=t_i}^{\bar{t}} \sum_{i=1}^S m_k(y_i^{obs}, z_i, x) \right\}. \quad (\text{F.29})$$

In the second stage, we obtain estimates of θ , which we denote as $\hat{\theta}_2$, minimizing the function in equation (F.28) for a diagonal weight matrix $W = W_2$ in which every diagonal element $k = 1, \dots, K$ equals $W_{2,k} = (\hat{V}_k(y^{obs}, Z, x; \hat{\theta}_1))^{-1}$, with $\hat{V}_k(y^{obs}, Z, x; \hat{\theta}_1)$ the clustered-robust variance of the moment $\mathcal{M}_k(y^{obs}, Z, x; \hat{\theta}_1)$, with each cluster defined as a firm-year combination (see Section 11 in Hansen and Lee, 2019, for details). We present heteroskedasticity-robust, clustered at the firm-year level, and clustered at the firm level, standard error estimates. We compute each of these applying the formulas in Section 11 of Hansen and Lee (2019), with the adjustment for simulation noise in Gourieroux et al. (1993).

F.5 Alternative Simulation Draws

We evaluate here the sensitivity of our estimates of the vector θ defined in footnote 24 to the set of $S = 5$ draws of χ_i (see equation (F.12)) we use to compute those estimates. We take 50 independent sets of 5 draws of χ_i and, for each of them, we compute a new SMM estimate of θ . For each parameter in θ , we compute a non-parametric density of the estimates obtained in the 50 sets of simulations, and report in Table F.2 the mode of this density as well as our baseline estimate; see Table F.4. Our baseline estimates are generally close to the mode of the distribution of the estimates obtained for different draws of χ_i , the only exception

Table F.2: Sensitivity of Baseline SMM Estimates to Alternative Simulation Draws

Parameters	Baseline Estimates	Alternative Estimates	Parameters	Baseline Estimates	Alternative Estimates
γ_0^F	62.92	63.53	κ_l^E	5.40	5.53
γ_g^F	13.11	17.68	γ_a^E	3.32	3.29
γ_l^F	4.14	2.79	φ_a^E	1.21	1.26
γ_a^F	29.28	28.99	κ_a^E	6.85	6.68
γ_0^S	114.76	115.09	γ_g^N	0.64	0.66
γ_g^S	19.95	19.88	κ_g^N	0.05	0.10
γ_l^S	0.23	0.26	γ_l^N	0.15	0.15
γ_a^S	21.83	21.07	κ_l^N	4.54	4.60
γ_g^E	9.83	10.79	γ_a^N	0.06	0.06
φ_g^E	1.96	1.98	κ_a^N	2.61	2.57
κ_g^E	6.02	6.03	σ_ν	80.04	79.98
γ_l^E	0.98	1.06	p	0.72	0.72
φ_l^E	2.74	2.76			

Note: the number in the ‘‘Baseline Estimates’’ column is the estimate reported in Table F.4; that in the ‘‘Alternative Estimates’’ column is the mode of the non-parametric density of the estimates obtained when reestimating our model using 50 alternative sets of draws of χ_i^s .

being the estimate of γ_g^F , which is 25% smaller than the mode of the density of the corresponding estimates.

F.6 Estimation Results: Additional Details

F.6.1 First-step Estimates: Potential Export Revenue Parameters

In Table F.3, we present point estimates and standard errors for all parameters affecting the evolution over time of potential export revenues (see Section 4.3).

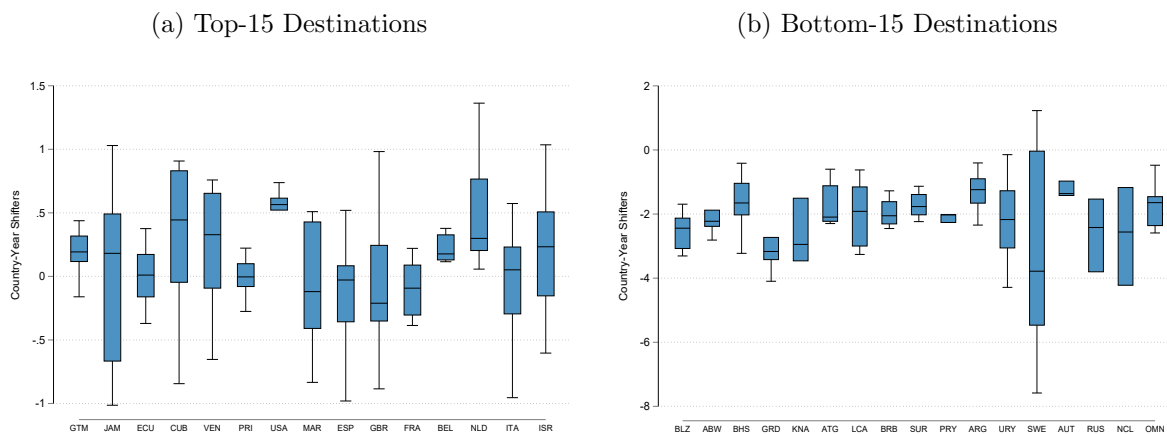
Table F.3: Estimates of Potential Export Revenue Parameters and Their Process

Potential Export Revenue Parameters		Process for Country- and Year-Specific Rev. Shifter		Process for Log Domestic Sales	
Parameter	Estimate (Standard Error)	Parameter	Estimate (Standard Error)	Parameter	Estimate (Standard Error)
α_y	1.856 ^a (0.066)	$\beta_{\alpha,g}$	-0.117 ^b (0.037)	ρ_r	0.857 ^a (0.012)
α_a	-3.832 ^a (0.066)	$\beta_{\alpha,l}$	-0.047 (0.071)	σ_r	0.865
α_r	0.285 ^a (0.041)	$\beta_{\alpha,a}$	-0.109 (0.079)		
		$\beta_{\alpha,gdp}$	0.079 ^a (0.019)		
		ρ_α	0.686 ^a (0.059)		
		σ_α	0.630		
Observations	13,293	Observations	467	Observations	43,300

Note: ^a denotes significance at 1%, ^b denotes significance at 5%. In parenthesis, standard error estimates. The results for *Potential Export Revenue Parameters* include country-year and sector fixed effects, and the displayed standard errors are heteroskedasticity robust standard errors. The results for *Process for Country- and Year-Specific Rev. Shifter* include no fixed effects, and the displayed standard errors are clustered by country. The results for *Process for Log Domestic Sales* include fixed effects for the firm's sector and province of location, and the displayed standard errors are clustered by firm.

In Figure F.8, we present box plots summarizing the distribution of the estimated values of α_{jt} across all sample periods for several specific countries. Specifically, panels (a) and (b) contain information for the 15 countries with the largest and smallest median estimates of α_{jt} , respectively.

Figure F.8: Estimates of Country- and Year-Specific Export Revenue Shifters



Note: In both figures, countries are identified by their ISO 3166-1 alpha-3 code, and ordered in the horizontal axis by their distance to Costa Rica. For each country, the corresponding box plot represents (from top to bottom) the max, third quartile, median, first quartile and min of the estimated values of α_{jt} across all sample periods. Panel (a) displays box-plots of the estimates of $\{\alpha_{jt}\}_t$ for the 15 countries with the largest median estimates. Panel (b) displays analogous information for the 15 countries with the lowest median estimates.

F.6.2 Second-Step Estimates: Fixed and Sunk Costs Parameters

In Table F.4, we present point estimates and standard errors for all parameters entering fixed and sunk export costs (see Section 4.4).

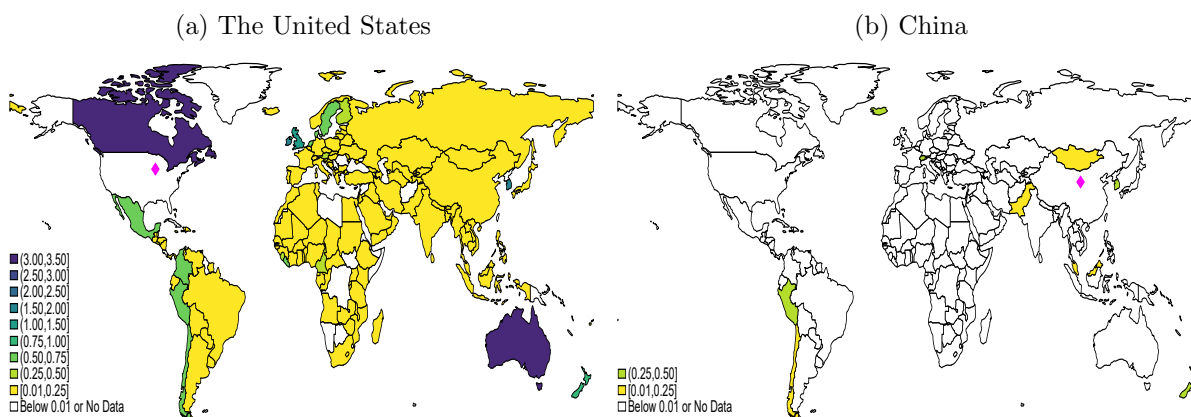
Table F.4: SMM Estimates of Fixed and Sunk Cost Parameters

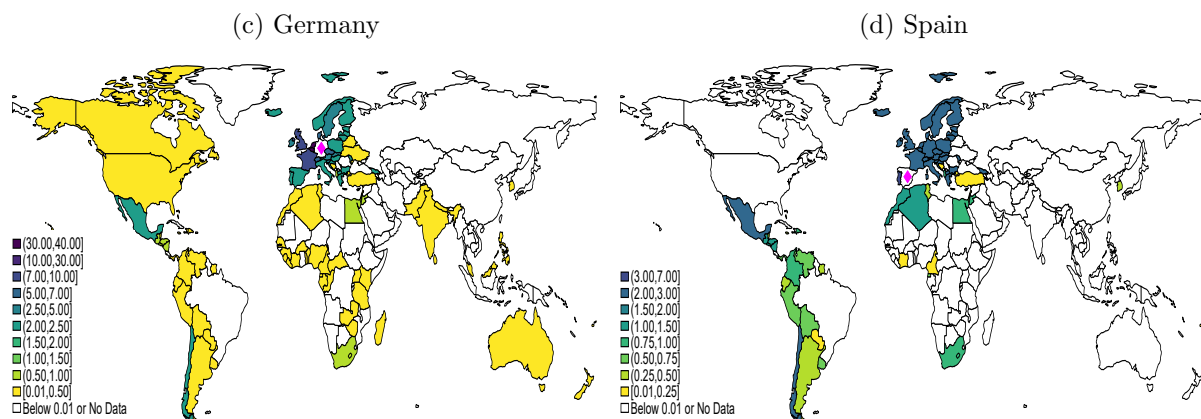
Parameter	Estimate (Standard Errors)	Parameter	Estimate (Standard Errors)
γ_0^F	62.92 ^a (1.11)(1.34)(2.77)	κ_l^E	5.40 (6.05)(7.84)(19.56)
γ_g^F	13.11 ^a (0.38)(1.17)(3.43)	γ_a^E	3.32 ^a (0.04)(0.04)(0.06)
γ_l^F	4.14 ^a (0.99)(1.71)(4.71)	φ_a^E	1.21 (0.52)(0.73)(1.51)
γ_a^F	29.28 ^a (0.78)(0.62)(1.09)	κ_a^E	6.85 ^a (1.02)(1.48)(3.18)
γ_0^S	114.76 ^a (3.18)(3.09)(6.03)	γ_g^N	0.64 ^a (0.00)(0.00)(0.01)
γ_g^S	19.95 ^a (0.92)(1.10)(2.80)	κ_g^N	0.05 ^a (0.00)(0.00)(0.01)
γ_l^S	0.23 (3.56)(4.43)(8.36)	γ_l^N	0.15 ^a (0.00)(0.00)(0.01)
γ_a^S	21.83 ^a (1.04)(0.83)(1.46)	κ_l^N	4.54 ^a (0.29)(0.31)(0.50)
γ_g^E	9.83 ^a (2.33)(2.85)(6.42)	γ_a^N	0.06 ^a (0.01)(0.01)(0.01)
φ_g^E	1.96 ^a (0.50)(0.66)(1.55)	κ_a^N	2.61 ^a (0.00)(0.00)(0.00)
κ_g^E	6.02 ^a (0.28)(0.49)(0.66)	σ_ν	80.72 ^a (0.51)(0.79)(2.05)
γ_l^E	0.98 ^a (0.08)(0.07)(0.11)	p	0.72 ^a (0.00)(0.00)(0.00)
φ_l^E	2.74 (2.88)(3.79)(7.16)		

Note: ^a denotes significance at 1%. In parenthesis, robust standard errors, standard errors clustered by firm-year, and standard errors clustered by firm, respectively. Displayed markers of statistical significance are determined on the basis of the standard errors clustered by firm-year.

In Figure F.9, for the case of the US, China, Germany and Spain, we plot the value of $c_{jj't}/g_{jt}$ multiplied by 100 for all other destinations j' .

Figure F.9: Estimated Static Complementarities

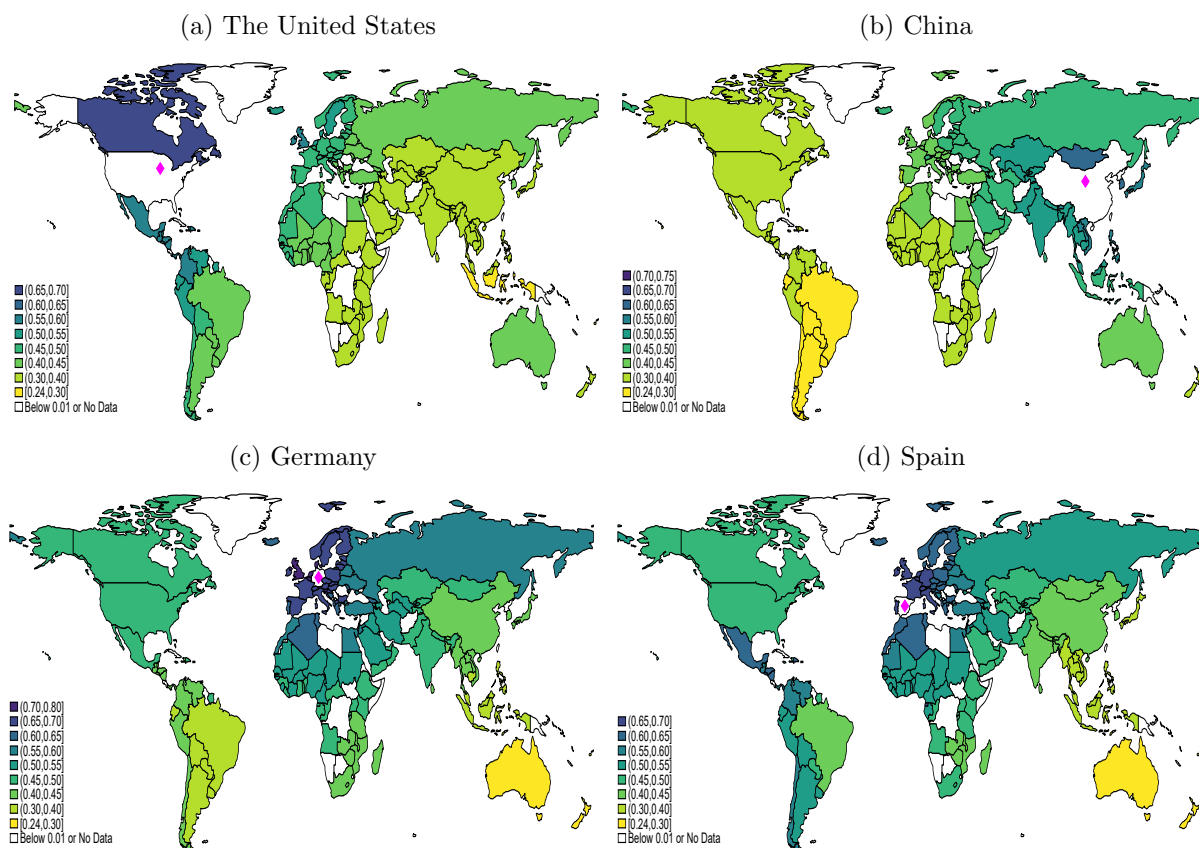




Note: In Panels (a), (b), (c) and (d) we illustrate, for the cases of the United States, China, Germany, and Spain, respectively, the percentage reduction in fixed costs of exporting to these countries if the firm simultaneously also exports to each of the other possible export destinations.

In Figure F.10, for the case of the US, China, Germany and Spain, we plot the value of $\rho_{jj't}$ for all other destinations j' .

Figure F.10: Estimated Pairwise Correlation Coefficients in Fixed Cost Shocks



Note: In Panels (a), (b), (c) and (d) we illustrate, for the cases of the United States, China, Germany, and Spain, respectively, the correlation coefficient in the fixed cost shock ν_{ijt} between the corresponding country and every other country in the world.

F.7 Model Without Cross-Country Complementarities

We present here the estimates of a model analogous to that in Section 4 except for the additional restriction that the complementarity term in equation (11) equals zero for all countries and periods. Fixed and sunk costs in this restricted model thus only depend on the parameters $\theta_R \equiv (\gamma_0^F, \gamma_0^S, \sigma_\nu, p, \{(\gamma_x^F, \gamma_x^N, \kappa_x^N, \gamma_x^S)\}_{x=\{g,l,a\}})$.

In this restricted model, the estimation approach in Section 7.1 is still valid; thus, the estimates of the demand elasticity and the parameters entering potential export revenues coincide with those described in Section 7.1. Concerning the estimation of θ_R , we follow an approach analogous to that in Section 6.2, using the same moments described in Section F.3. We present in Table F.5 the resulting estimates.

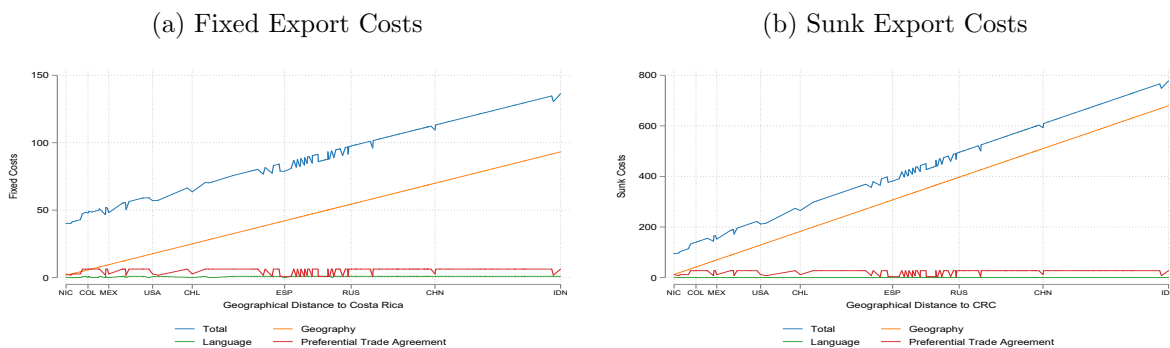
Table F.5: Estimates of Fixed and Sunk Cost Parameters in Model Without Complementarities

Parameter	Estimate (Standard Errors)	Parameter	Estimate (Standard Errors)
γ_0^F	35.81 ^a (4.78)(7.93)(19.89)	γ_g^N	0.64 ^a (0.01)(0.01)(0.01)
γ_g^F	4.97 ^a (0.41)(0.75)(1.77)	κ_g^N	0.04 ^a (0.00)(0.00)(0.01)
γ_l^F	0.96 (2.64)(3.87)(9.59)	γ_l^N	0.18 ^a (0.03)(0.03)(0.07)
γ_a^F	6.32 (3.62)(6.05)(16.11)	κ_l^N	0.38 (0.52)(0.70)(1.59)
γ_0^S	70.70 ^a (6.17)(9.24)(21.09)	γ_a^N	0.10 ^a (0.01)(0.01)(0.03)
γ_g^S	36.21 ^a (0.22)(0.31)(0.31)	κ_a^N	0.42 ^a (0.05)(0.04)(0.10)
γ_l^S	0.16 (5.25)(9.93)(25.32)	σ_ν	41.59 ^a (0.76)(1.33)(3.36)
γ_a^S	27.39 ^a (4.48)(8.92)(24.36)	p	0.65 ^a (0.00)(0.00)(0.00)

Note: ^a denotes significance at 1%. In parenthesis, robust standard errors, standard errors clustered by firm-year, and standard errors clustered by firm, respectively. Markers of statistical significance are determined on the basis of the standard errors clustered by firm-year.

Figure F.11 is analogous to Figure 1. The mean fixed cost function implied by the estimates in Table F.5 is smaller than the estimated mean fixed cost function for single-destination exporters displayed in panel (a) of Figure 1 for our general model with complementarities. This is to be expected, as the estimated mean fixed export costs in the restricted model without cross-country complementarities likely approximate a weighted average of the mean fixed export costs faced by different firms depending on their export bundles, with weights given by the frequency with which different export bundles are observed in the data.

Figure F.11: Fixed and Sunk Costs Estimates in Model Without Cross-Country Complementarities



Note: In both figures, countries are identified by their ISO 3166-1 alpha-3 code, and placed in the horizontal axis by their distance to Costa Rica. The vertical axis indicates the estimated cost in thousands of 2010 USD.

G Properties of Estimated Model: Additional Details

We consider here a simplified version of the model in Section 4 with the goal of understanding the role cross-country complementarities play on firm export choices. Specifically, we impose on the model in Section 4 the following additional restrictions: (a) there are two markets, A and B ; (b) for both markets, the fixed cost gravity term g_{jt} and sunk costs s_{jt} are constant over time; (c) the complementarity term in fixed costs c_{ABt} is constant over time; (d) $\omega_{ijt} = 0$ for every i, j and t ; (e) $\alpha_y = 0$ and all other determinants of export revenues are constant over time, implying that r_{ijt} is constant over time for every firm i and market j .

Dropping the t subscript from all constant variables, and denoting the complementarities between markets A and B as c , firm i will thus solve the following optimization problem at $t = 0$:

$$\max_{\{y_{jt}\}_{j \geq 0}} \sum_{t \geq 0} \left\{ \delta^t (y_{iAt} \pi_{iA} - (1 - y_{iAt-1}) s_A + y_{iBt} \pi_{iB} - (1 - y_{iBt-1}) s_B + y_{iAt} y_{iBt} c) \right\} \quad (\text{G.1})$$

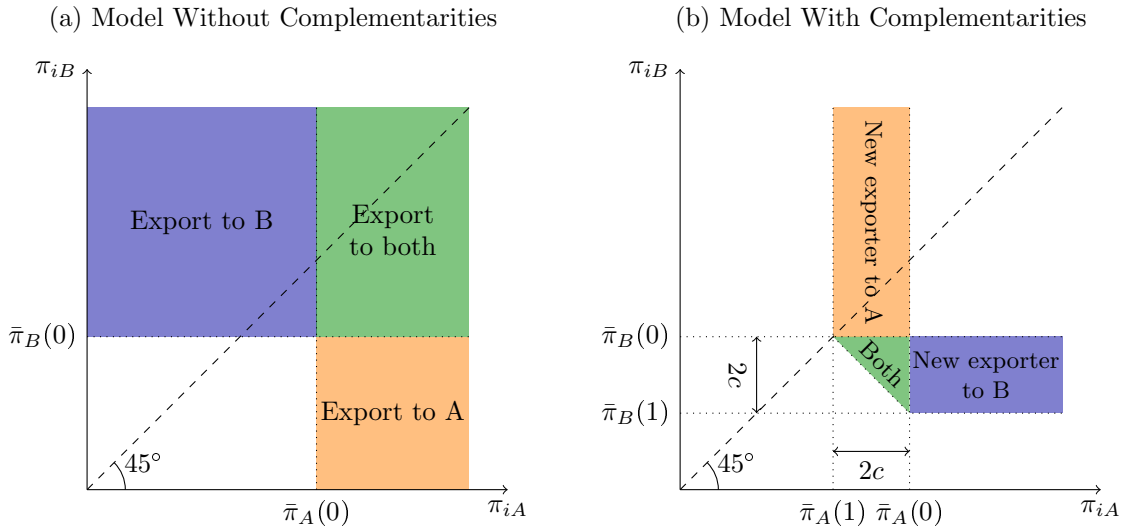
where, for any country j , $\pi_{ij} = \eta^{-1} r_{ij} - g_j - \nu_{ij}$ is the potential export profits of firm i in j net of all components of fixed export costs other than the complementarity term; i.e., net of g_j and ν_{ij} . As no firm can export before the first period of activity, it holds that $y_{iAt-1} = y_{iBt-1} = 0$ when $t = 0$.

To understand the role complementarities play on firm choices, we consider two cases: one in which $c = 0$, and one in which $c > 0$. Without loss of generality, we keep all throughout the assumption that sunk export costs are lower in country B than in country A ; i.e., $s_B < s_A$.

Case 1: no complementarities. In this case, $c = 0$ and the firm's export decision is independent across countries. As the problem in equation (G.1) is stationary, a firm exports to any country $j = \{A, B\}$ at any period $t \geq 0$ if and only if $\pi_{ij} \geq \bar{\pi}_j(0)$, for $\bar{\pi}_j(0) \equiv (1 - \delta) s_j$. Thus, as shown in panel (a) in Figure G.1, firms with $\pi_{iA} < \bar{\pi}_A$ and $\pi_{iB} \geq \bar{\pi}_B$ export only to B ; firms with $\pi_{iA} \geq \bar{\pi}_A$ and $\pi_{iB} < \bar{\pi}_B$ export only to A ; and, firms with $\pi_{iA} \geq \bar{\pi}_A$ and $\pi_{iB} \geq \bar{\pi}_B$ export to both countries. Consistently with the parametrization that $s_B < s_A$, the plot in panel (a) of Figure G.1 assumes that $\bar{\pi}_B(0) < \bar{\pi}_A(0)$.

Case 2: positive complementarities. In this case, $c > 0$ and the firm's export decision is not independent across countries. Conditional on exporting to country $j' \neq j$, exporting to j is optimal if and only if $\pi_{ij} \geq \bar{\pi}_j(1)$ with $\bar{\pi}_j(1) = (1 - \beta) s_j - 2c$. Note that $\bar{\pi}_j(1) < \bar{\pi}_j(0)$ for any $c > 0$. Panel (b) in Figure G.1 illustrates the new exporters that emerge when c becomes positive. These new exporters are of two kinds. First, "natural exporters" to one of the markets (i.e., firms that export to one of the markets even when $c = 0$) and that, as complementarities become more important (i.e, as the value of c increases), start exporting to the other one. These are firms whose value of (π_{iA}, π_{iB}) falls in the orange and blue areas in

Figure G.1: Export Choices Models With and Without Complementarities

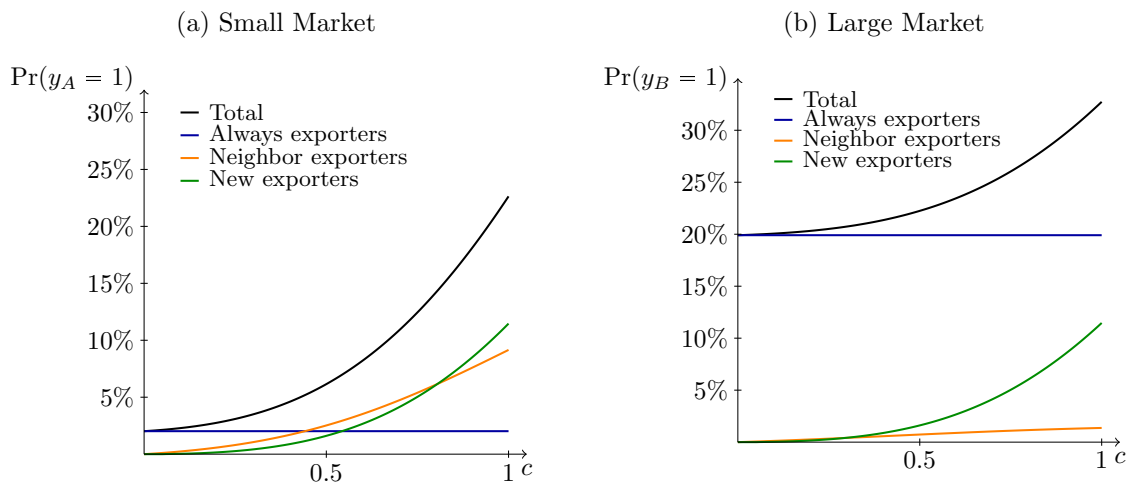


panel (b). Second, firms that do not export when $c = 0$, but export to both markets at the new level of c . These are firms whose value of (π_{iA}, π_{iB}) falls in the green area in panel (b).

Panel (b) in Figure G.1 shows how a firm i , depending on the values of (π_{iA}, π_{iB}) , changes its set of destinations when c switches from being equal to zero to being positive. To determine how the share of firms exporting to either country changes as we change the value of c , we need to impose assumptions on the distribution of (π_{iA}, π_{iB}) . In Figure G.2, we show how country-specific export shares change as we change the value of c when, for $j = \{A, B\}$, π_{ij} is normally distributed with mean μ (common in both markets) and variance equal to 1. We further assume that π_{iA} and π_{iB} are independent of each other. We impose values of μ , δ , s_A and s_B such that, when $c = 0$, the export share to A equals 2%, and the export share to B equals 20%. Thus, we can characterize markets A and B as being “small” and “large”, respectively.

We extract several conclusions from Figure G.2. First, as reflected in the black lines in both panels, the effect on export shares of changes in c is non-linear: export shares are convex in c . Second, when comparing the export shares for positive values of c to those for $c = 0$, both the absolute and the relative increase in the export share is larger in the “small” export market (i.e., country A) than in the large one (i.e., country B). More specifically, when measuring the change in export shares as the value of c switches from zero to one, we observe that the percentage point increase in export shares in markets A and B is 21 pp. and 13 pp., respectively, and the relative increase in export shares in markets A and B is 11.5 (which equals $23\%/2\%$) and 1.65 (which equals $33\%/20\%$), respectively. Third, the reason for the larger impact of changes in c on export shares in A than in B is that there are many more firms that exported only to B in the case with $c = 0$ and add market A as export destination when c increases, than there are firms that exported only to A in the case with $c = 0$ and add market B as export destination when c increases; i.e., the probability that the vector (π_{iA}, π_{iB}) is in the area painted in orange in panel (b) of Figure G.1 is larger than the probability that it is in the area painted in blue in the same graph.

Figure G.2: Export Share and Cross-Country Complementarities



Note: In panel (a), for each value of c , “Total” denotes the share of firms that export to A at that value of c ; “Always exporters” denotes the share of firms that export to A at that value of c and also export to A when $c = 0$; “Neighbor exporters” denotes the share of firms that export to A at that value of c , do not export to A when $c = 0$, and export to B when $c = 0$; and “New exporters” denotes the share of firms that export to A at that value of c and export neither to A nor to B when $c = 0$. The interpretation of the labels for panel (b) is analogous.

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