Collateral Advantage: Exchange Rates, Capital Flows, and Global Cycles

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Abstract

We construct a two-country New Keynesian model in which US government debt has an advantage as a superior collateral asset on the balance sheets of banks. The model can account for the observed response of the US dollar and US bond returns to a global downturn, in particular when the downturn is associated with global financial stress. In our model, the U.S. enjoys an “exorbitant privilege” as its government bonds are desired by banks both in the U.S. and abroad as superior collateral. In times of global stress, the dollar appreciates and the “convenience yield” earned by U.S. government bonds increases. There is “retrenchment” - each country reduces its holdings of foreign assets - a critical determinant of which is the endogenous response of prices and returns. In addition, the model displays a U.S. real exchange rate appreciation despite that domestic absorption in the US falls relative to the rest of the world during a global downturn, thus addressing the “reserve currency paradox” highlighted by Maggiori (2017).

JEL Classification Codes: F3, F4, G1

Key words: Exchange rates, financial frictions, liquidity, convenience yield, dollar specialness

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1 Introduction

During the global financial crisis of 2007-2009, the US Federal Reserve cut interest rates more quickly and sharply than central banks of most other major economies, yet the US dollar appreciated. Analysts refer to a “flight to safety”, highlighting the high demand for liquid and safe assets in influencing exchange rates. The appreciation of the dollar has been associated with large reversal (or retrenchment) of capital flows, and with the “exorbitant duty” of the U.S. currency – meaning that foreign investors are rewarded during times of global stress by an increase in the value of their U.S. assets. This amounts to a wealth transfer from the U.S. to the rest of the world. This exorbitant duty is the complement of the “exorbitant privilege” the U.S. enjoys during quiescent times, as the average return on U.S. investments abroad exceeds that of foreign investments in the U.S.\(^1\)

Such behavior of the dollar implies that the dollar is a hedge during times of global downturn, so it seems plausible that the low rate of return on U.S. assets is attributable at least in part to their value as insurance. But this leaves open the question of why the dollar appreciates during downturns. Gourinchas and Rey (2022) provide a model in which global demand for dollar assets as insurance increases as the potential for a global crisis increases. Bianchi et al. (2021), on the other hand, attributes the appreciation to liquidity demand arising from financial intermediaries. A disproportionate amount of volatile, short-term funding globally is in dollars, and as funding becomes more uncertain, the demand by intermediaries for liquid dollar assets increases, thus appreciating the dollar. Kekre and Lenel (2020) and Jiang et al. (2021a) construct models in which the dollar appreciates because it earns a higher “convenience yield” during times of global stress. The convenience yield arises in those frameworks because agents directly derive utility from dollar assets, though this is meant to be an expedient way to capture the various factors such as liquidity services that are provided by dollar assets.\(^2\)

In this paper, we propose a framework that can explain these disparate features of the global financial crisis. We construct a two-country New Keynesian model in which US government debt is superior collateral on the balance sheets of financial intermediaries. In other words, for a balanced sheet-constrained financial intermediary, it is less costly to hold U.S. Treasury debt than safe assets of other countries. The higher pledgeability of U.S. Treasury debt creates a liquidity or “convenience” demand for the debt. When there is financial stress, the balance sheet constraint is a bigger concern, which endogenously generates a high “convenience yield” for US Treasury debt. This leads all financial intermediaries to shift demand towards US Treasury debt, causing a dollar appreciation. The appreciation of foreign bank’s U.S. assets results in an effective wealth transfer such that the U.S. financial intermediaries demand more of the liquid assets than the foreign banks. This results in an endogenous capital flow retrenchment: the US intermediaries end up holding more US assets and the foreign intermediatures hold more foreign assets. The model also features a persistent deviation from uncovered interest parity (UIP) due to a time varying convenience yield,

\(^1\)Gourinchas and Rey (2007a,b, 2022) and Eichengreen (2011) originate this idea, provide empirical evidence to support it, and build a model to account for it.
\(^2\)See also Engel (2016) and Engel and Wu (2018) for New Keynesian models in which the convenience yield comes about because bonds are in the utility function.
and simultaneously a long run “exorbitant privilege”, as US assets pay a lower interest rate than those of the rest of the world.

Our model is a two-country, New Keynesian DSGE model with a financial sector as in Gertler and Kiyotaki (2010) and Gertler and Karadi (2011). In each country, households consume, provide labor services, and save by putting deposits in banks located in their own country. Firms in each country produce output using labor and capital. Banks in each country rent capital to firms in their own country, supply deposits to households, and hold government bonds from both countries. Governments issue debt and make transfers to households. Monetary policy in each country is modeled as a simple inflation-targeting rule.

Markets or regulators constrain the banks from acquiring too large a balance sheet, because financial intermediaries (which we shall call “banks”) have private information about the value of their assets that is not observed by private agents or financial regulators. In order to prevent excessive risk taking, the leverage of banks is constrained. However, there is less private information about the value of government debt, and especially U.S. government debt that is widely traded globally in very deep markets. In our set-up, U.S. and foreign banks are less constrained in their holdings of government debt, and U.S. Treasury debt has a “comparative advantage” in the sense that the relative constraint on this debt is lower globally than on foreign government debt. The fact that the value of U.S. Treasury debt is easily assessed in global markets is one sense in which these assets are very liquid. We draw a connection between liquidity and “safety”. As Gorton (2017) states, “A safe asset is an asset that is (almost always) valued at face value without expensive and prolonged analysis. By design, there is no benefit to producing (private) information about its value, and this is common knowledge” This is precisely the motivation for our modeling of the special role of U.S. Treasury assets in bank balance sheets.

The banks face constraints requiring that a weighted average of the value of their assets must be less than some multiple of the equity value of the bank. However, the weights on the government bonds are lower than the weight on capital, implying that banks are less constrained in their holdings of government bonds. Importantly, though, banks are less constrained in their holdings of U.S. government bonds, which are considered to be better collateral. Globally, therefore, U.S. debt is especially safe/liquid.

Even abstracting from the insurance premium (as in Gourinchas and Rey (2022)) that U.S. Treasury debt might earn, the model exhibits an exorbitant privilege enjoyed by the U.S. During normal times, U.S. debt pays a low rate of monetary return because it constrains banks’ balance sheets less than other assets. In fact, the model is capable of accounting for the fact that while the U.S. is a net international debtor, its net investment income is positive, a noted feature of the U.S. balance of payments.

Moreover, the model explains the appreciation of the dollar during times of global downturn as an increase in demand by banks, both in the U.S. and abroad, for dollar Treasury debt. This appreciation is coincident with an increase in the convenience yield on U.S. government debt, measured as a deviation from uncovered interest rate parity. The global downturn may be caused
either by a financial crisis (modeled here as a uniform tightening of borrowing constraints) hitting all countries equally, or a global negative total factor productivity shock.

However, our model demonstrates an important feature of equilibrium adjustment in the global economy. While the foreign bank, ceteris paribus, demands more of the U.S. bond, we find in our baseline calibration that both the value of U.S. bonds held by the foreign bank and their share in its portfolio of assets decline. This finding highlights the importance of understanding the equilibrating process when demand for assets change. Here, there are two important features that lead the foreign bank to shed some of its holdings of U.S. bonds. First, because of the dollar appreciation, the foreign currency value of the bank’s holdings of U.S. bonds increases even without a change in the quantity that it holds. Second, the increase in U.S. demand for U.S. bonds is somewhat stronger, as it has a greater incentive to shift out of non-governmental assets. As we will explain, the advantage of U.S. government bonds relative to equity capital is greater than the equivalent for foreign bonds. The global increase in demand drives down the relative rate of return on these U.S. bonds (that is, there is an increase in the convenience yield), which ultimately leads foreign banks to decrease their holdings of U.S. bonds and increase their holdings of foreign bonds. In other words, there is retrenchment: banks both in the U.S. and abroad shed assets of the other country (bonds and equities) during times of global stress, a prediction which is consistent with the empirical findings of Milesi-Ferretti and Tille (2011), Forbes and Warnock (2012) and, with a particular focus on banks, Wang (2018).

The “reserve currency paradox” of Maggiori (2017) notes that during these times in which the dollar appreciates, the net international investment position of the U.S. deteriorates as the value of U.S. holdings of foreign assets falls relative to the value of foreign holdings of U.S. assets. The paradox is that if the real value of the dollar is determined through home bias in preferences (or through the presence of non-traded goods) by a rise in the price of goods favored by U.S. households, then a real appreciation of the dollar should be associated with an increase in relative demand by U.S. households. It is difficult to reconcile an increase in demand by U.S. households with a fall in their wealth. This paradox is resolved in our model because nominal goods prices are sticky in the currency of consumers (local-currency pricing, or LCP.) A real appreciation is not immediately associated with an increase in the relative price of U.S. goods.

We also consider the effects of monetary shocks in this framework. As the work of Rey (2015), Miranda-Agrippino and Rey (2020), and others have emphasized, there are repercussions from U.S. monetary shocks on the rest of the world that work through financial channels, and not just through the traditional channels of import demand. We consider the effects of an equal monetary contraction in the U.S. and the rest of the world. The insight from this exercise is that even when monetary policy changes are identical across countries, there is an asymmetric effect on the dollar exchange rate. If the U.S. and other countries were to follow uniform monetary contractions (for example, in response to an inflationary shock), the dollar would nonetheless appreciate. The contraction lowers aggregate demand, and tightens balance sheet constraints on banks, precipitating a global increase in...
in demand for dollar bonds and an appreciation.

Because the liquid dollar assets earn a convenience yield as well as a monetary return, the central bank effectively has two instruments that can affect macroeconomic outcomes. Its monetary policy - the rule for setting interest rates in response to inflation - works as in standard models, but the central bank can also affect the supply of liquidity. As Rogoff (2017) has argued, increasing reserves in the banking system is tantamount to greater issuance of liquid assets by the government. When reserves earn interest, there is little difference between increasing the supply of liquid short-term bonds and increasing reserves. We demonstrate how different menus of monetary policy and unconventional policies (in the form of swapping liquid bonds for less liquid assets), and, somewhat similarly, sterilized intervention, can affect the mix of inflation, currency value and other aggregate variables. For example, if the monetary contraction in the U.S. in 2021 were not accompanied by quantitative tightening, the dollar appreciation over that period may have been significantly reduced.

There are two key differences between our model and models that posit an exogenous increase in U.S. bond demand, for example coming from a demand shock from noise traders as in Itskhoki and Mukhin (2021a,b), or an exogenous switch in demand of bond investors, as in Kekre and Lenel (2021) or Jiang et al. (2020, 2021b). In our model, we posit a neutral global tightening of financial constraints. This has the effect of endogenously increasing the relative demand for U.S. bonds and increasing the convenience yield during times of global financial strain. Second, the financial constraints matter for the response of the global economy. For example, the tightening of financial constraints leads to a reduction in investment directly, but there is also a magnifying effect as financial intermediaries rebalance their portfolios away from productive capital investments toward liquid bonds backed by governments during times of financial stress. Thus, while a global financial tightening in our model does lead to a deviation from uncovered interest parity, which Miyamoto et al. (2022) find is the dominant driver of dollar exchange rates, our model does not have the counterfactual prediction of Itskhoki and Mukhin (2021a), noted by these authors, of a strong, nearly perfect, correlation of the real exchange rate and the trade balance. Additionally, as noted above, our model features endogenous retrenchment as the financial intermediaries’ reallocation of assets in their portfolios induces a decline in foreign asset positions during times of financial pressure even as demand for liquid dollar assets increases, due to the influence of equilibrium changes in expected returns, asset prices, and the exchange rate.

Related Literature

Several recent papers have found a relationship between “convenience yields” and either exchange rates or deviations from uncovered interest rate parity. These include Jiang et al. (2021a), Jiang et al. (2018), Jiang et al. (2021b), Krishnamurthy and Lustig (2019) Engel and Wu (2018), Kekre and Lenel (2020), Valchev (2020), and Bianchi et al. (2021). These papers provide evidence that deviations from UIP may be attributable in part to liquidity or convenience yields, and that this return to liquidity also influences the level of the exchange rate. Much of the literature has
noted especially the nexus between convenience yields on U.S. government bonds and the level of the dollar exchange rate as well as deviations from UIP for the dollar. These general equilibrium models follow earlier literature that takes the deviation from UIP due to the convenience yield either as exogenous, or because some assets are in the utility function, or from exogenously given bond demand functions. An exception is Bianchi et al. (2021), which models the endogenous demand for assets from financial intermediaries during times of global stress, with emphasis on the liquidity return of dollar short-term assets. That model, however, is too stylized to take to a realistic quantitative open-economy macro setting.

The “exorbitant privilege” of the United States - that it earns a greater rate of return on its foreign investments than foreigners earn on investments in the U.S. - in conjunction with the persistent U.S. trade balance and current account deficits has been intensively investigated. Gourinchas and Rey (2007b) and Gourinchas and Rey (2007a) perhaps first noted the importance of these excess returns in the global financial adjustment process. Mendoza et al. (2009), Caballero et al. (2008), and Caballero et al. (2016) build models to account for this global pattern of portfolio returns. A noted feature of this “exorbitant privilege” has been that the US net international investment position, up to the late 2000’s, was significantly less than the cumulated level of current account deficits since the early 1990’s. However, recent work by Atkeson et al. (2022) has called attention to a reversal in this situation since the global financial crisis due to the superior performance of U.S. equities relative to those in the rest of the world. It is not obvious however if this reversal has affected the convenience yield on US government bonds relative to those of other major sovereign borrowers.

The role of global financial intermediation is the focus of much contemporary research into exchange rates and capital flows. Notable contributions include Maggiori (2017), Gabaix and Maggiori (2015), Itskhoki and Mukhin (2021a), and Gopinath and Stein (2021).

Some other recent studies have provided potential explanations for how dollar bonds can earn on average a lower return than foreign bonds, and how the dollar can appreciate during global downturns, and yet provide resolutions to the reserve currency paradox. In Maggiori (2017), foreign banks face balance sheet constraints, while U.S. banks are unconstrained. This effectively makes foreigners more risk averse than investors in the U.S. In equilibrium, the U.S. borrows from abroad and invests in equities, while the foreign country buys U.S. debt which acts as insurance during global downturns. However, since the real exchange rate is determined by the relative price of goods produced in the U.S. compared to those in the foreign country due to home bias in preferences, there has to be a channel that makes this relative price increase. Maggiori assumes that there are costs

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4Earlier papers that model the convenience yield as arising from assets in the utility function include Krishnamurthy and Vissing-Jorgensen (2012), Nagel (2016), and, in an international model, Engel (2016). A prominent recent example of a model with an exogenous UIP shock is Itskhoiki and Mukhin (2021a). Greenwood et al. (2020) and Gourinchas et al. (2022) specify international portfolio choice models in which a “preferred habitat” as well as risk and return play a role in asset demand. Du et al. (2018a) provide measures of the convenience yield on U.S. bonds relative to government bonds of other countries. Garleanu and Pedersen (2011) study the asset pricing implication of a finance model of different margin requirements. Georgiadis et al. (2023) provides a related framework to understand the global financial cycle.


6See He and Krishnamurthy (2013) for a general survey of how financial constraints affect and magnify the effective risk aversion of investors.
of exporting the foreign good that rise during bad times. This leads to a switch in demand toward U.S. goods, which generates the rise in their relative price.

Gourinchas and Rey (2022) build a model in which the global economy can be in one of three states at any time: normal, fragile, and disaster. That study associates times of global stress with the fragile state. Investors in both countries can trade in equities and real bonds that pay off in terms of each country’s consumption basket. U.S. bonds are less risky because of their payoffs in the fragile and disaster states, so they earn on average a lower rate of return. During disaster periods, which are rare, there is a large drop in global output, and in addition, there is partial default on foreign bonds. When the economy enters into a fragile state from a normal state, the probability of disaster increases. To hedge the risk that occurs from holding foreign bonds in the disaster state, investors purchase home bonds during fragile times. The trade deficit of the U.S. increases during fragile times, allowing the U.S. to increase demand for its own non-traded goods, leading to a real appreciation.

Jiang et al. (2020) posit that there is an exogenously given demand from abroad for U.S. bonds because they are valued for their liquidity or some other special property. Bonds are issued by firms in the U.S., but these firms face a borrowing limit. Bonds are used to finance productive activity. The model can, for example, account for dynamics of real exchange rates in response to a monetary tightening in the U.S. As the U.S. raises interest rates, the debt issuing capacity of firms falls, reducing their output but also reducing their supply to the rest of the world of the liquid asset. The dollar appreciates both because of the increase in the interest rate and the higher liquidity return on the liquid bonds forced by the contraction in their supply. The study also considers the effects of a flight to dollars, which would also lead to an increase in the convenience yield and appreciation of the dollar. The paper emphasizes that if such a shock is accompanied by a monetary easing in the U.S., that the U.S. trade balance could run into deficit even while the dollar was appreciating. This is driven by the increase in U.S. wealth from the greater seignorage arising from the increase in the liquidity yield on bonds. In another experiment, the study considers a shock to productivity for U.S. firms. This reduces the firms’ borrowing capacity, reduces the supply of debt, and thus raises the convenience yield and appreciates the dollar. Foreign firms which hold some dollar debt have an adverse balance sheet effect, which is contractionary for their output.

Kekre and Lenel (2021) present a model of convenience yields and risk premia in an general equilibrium model of the global economy. The convenience yield arises from an ad hoc model in which agents have some preference for bonds issued by the U.S. The main drivers of global downturns are a disaster shock that is also correlated with a “safety shock” that changes the relative demand for U.S. assets. In addition, there is nominal wage stickiness and monetary policy set by a Taylor rule. The model is quite rich, but some intuition can be gleaned by considering the effect of the safety shock alone, as global demand for U.S. bonds tends to increase during global crises. The shift in demand toward U.S. bonds tends to lower their rate of return, which is expansionary in the U.S. (relative to the foreign country). This leads to higher inflation in the U.S. (again, relative to the rest of the world), which induces a monetary policy response that raises the real interest rate.
and in turn leads to a real appreciation of the dollar. On the goods market side, the drop in U.S. consumption induced by the real interest rate increase raises the real wage (given the nominal wage stickiness), which in turn induces producers to supply less. The drop in supply outstrips the drop in consumption, so the relative price of U.S. goods rises, consistent with the real appreciation given home bias in preferences.

In Akinci et al. (2022), during times of global stress, there is an increase in volatility of the productivity process. In the version of their model that aims to explain the real appreciation of the dollar during times of uncertainty, there is a financial intermediary in the U.S. that takes in saving from U.S. households and invests in U.S. equities and foreign debt. This intermediary faces balance sheet constraints, and, as in our model, the bank faces lower constraints on its holdings of debt than on capital, though there is no special role for U.S. government debt. The foreign financial sector is unconstrained, so we note that the set-up is the precise opposite of that in Maggiori (2017) in which foreign intermediaries are constrained and the U.S. financial sector operates freely. Akinci et al. (2022) consider an increase in the volatility of U.S. productivity, holding foreign volatility unchanged, which leads to an increase in risk aversion in the U.S. relative to the rest of the world. The shock causes U.S. intermediaries to lower demand for foreign bonds, and they shed some of their holdings of foreign debt, contributing to a depreciation of the foreign currency. Since the foreign country must borrow less, its consumption level declines. With home bias in preferences, it appears that the drop in foreign demand for their goods helps deliver the real depreciation of their currency.

Dahlquist et al. (2022) build a model in which agents have “deep habits” - an external habit for each good in the consumption basket. The key properties of these preferences is that they are not homothetic, and expenditure shares may vary as wealth and consumption levels change. In particular, the study assumes that U.S. households are wealthier, and thereby less risk averse than those in other countries. During periods in which income and global consumption takes a turn downward, U.S. consumption falls less than that in other countries, and because of home bias in consumption, the relative demand for goods produced in the U.S. increases. This leads to a U.S. real appreciation during bad times, which also makes U.S. bonds a good hedge against global shocks.

Our model differs from much of the literature in that we examine a global shock that hits the U.S. and the rest of the world equally. Our main focus is on a global financial tightening. The asymmetry in the model arises not from the shock, but from the property that U.S. government bonds are considered to be better collateral on banks’ balance sheets. We emphasize the increase in the liquidity yield on these bonds, and the appreciation of the dollar that occurs because these bonds become more valuable during global downturns. In most respects, our model is a standard open-economy New Keynesian model, so it does not require the introduction of any new features, other than the special role of dollar debt, to account for the puzzles. Moreover, we find that in equilibrium there is retrenchment, which accounts for another possibly puzzling feature of the data.

We find that during times of global stress, the U.S. banks are more incentivized to switch their portfolios away from capital toward their country’s safe bonds than are foreign banks. In turn, since
output is demand-determined in the short run in the model, the downturn in U.S. output is greater that for the rest of the world. In terms of the Maggiori (2017) reserve currency paradox, it might seem trivial that if the U.S. has a greater downturn, then the dollar must appreciate in real terms. In a flexible-price model in which output is exogenous, if U.S. output falls more, the relative price of U.S. goods will rise more, which will lead to a real appreciation if there is consumption home bias. But in our model, the mechanism is entirely different. Output is not exogenous, but determined endogenously, and it is demand-determined. The real appreciation does not require an increase in the relative price of U.S.-produced goods under LCP. And, importantly, we are considering global shocks that hit the U.S. and the foreign country equally. Moreover, we can find under some unrealistic calibrations that the global shock can lead to a dollar appreciation even with output falling less in the U.S., so the drop in relative U.S. output is not intrinsically necessary to deliver the real appreciation.

Section 2 presents an empirical exchange rate instrumental variable regression analysis. Section 3 presents the model. Section 3 presents the model. The calibrated parameters are described in section 4 and section 5 describes the steady state. Section 6 examines the responses to a global financial tightening, to a global productivity shock, to a monetary shock originating in the U.S., and to some unconventional monetary policy experiments. Section 8 concludes.

2 Motivating facts and empirical analysis

Figure 1 shows some stylized facts around time of the global financial crisis that helps to motivate the model. The upper panel shows the strong relationship of liquidity returns and the U.S. dollar exchange rate. The blue line represents the U.S. dollar price of the euro in the left panel and the U.S. dollar price of the average of the rest of the G9 currencies in the right panel (converted into real exchange rates by adjusting for relative consumer prices.) The red line in both graphs presents the liquidity yield measure in Engel and Wu (2018), defined as the difference between a market rate of return and the rate of return on short-term government bonds for the U.S. relative to the other country.\(^7\) The measure captures the liquidity or convenience services of government bonds. The panel clearly illustrates that the sharp appreciation of the U.S. dollar in the 2008 period is associated with a large increase in the US liquidity yield, which implies an increase in demand for U.S. Treasuries during the crisis.

The middle panel shows retrenchment during the crisis. The left (right) panel plots the ratio of capital inflows (outflows) to GDP respectively for the U.S., Germany and the rest of the G10 average.\(^8\) Before 2008, there are regular inflow and outflow of roughly 10% of GDP. (Note that a positive amount of outflows from a country is plotted as a negative number in the right-hand-side panel.) During the global financial crisis period, highlighted in gray, there is a dramatic difference.

\(^7\)This measure is used by Engel and Wu (2018) but is nearly identical to the ones that are used in the studies of Du et al. (2018a) and Jiang et al. (2021a). The exchange rate and price data is from DataStream. The “rest of the G10 means the G10 countries excluding the U.S. and euro area.

\(^8\)The capital flow data is from Bluedorn et al. (2013).
Inflows into these countries turn from positive to negative, implying that investors from other countries are shedding their holdings of foreign (i.e., U.S., German, or G10, respectively) assets. The right panel tells a similar story, that U.S., German, or G10 investors are reducing their holdings of foreign assets, as signified by the fact that the outflow numbers become positive.

The last panel reports the dynamics of real variables of the US relative to the rest of the world. We look at three different ratios: the U.S. GDP to world GDP ratio, the U.S. consumption to world consumption ratio and the U.S. investment to world investment ratio. The figure plots the first difference of these ratios. During the financial crisis, U.S. GDP fell relative to the rest of the world (blue line). U.S. consumption also fell relative to the rest of the world but less than the drop of the GDP ratio. Finally, there is a big drop in the relative U.S. investment.

In addition to the well-known data on net capital flows, our model will offer some insights into these empirical regularities.
Figure 1: Stylized facts during the Global Financial Crisis

**Upper panel**

Note: Liquidity yield is defined as in Engel and Wu (2018), which is the home to foreign market interest rate indifferential minus home to foreign government interest rate differential ($i^m_t - i^m*_t - (i^g_t - i^g*_t)$).

**Middle panel**

Note: Following the convention, a capital outflow is defined as a negative value when outflow occurs.

**Lower panel**

Note: The three ratios are US GDP/World GDP, US consumption/World consumption and US investment/World investment. Data is annual and the first difference ($\Delta$) is year-on-year difference.
Our model is motivated by empirical evidence that has linked exchange rate changes to movements in measures of the convenience yield on liquid government bonds, as in Engel and Wu (2018) and Jiang et al. (2021a). We estimate a panel regression for the U.S. dollar exchange rate relative to the other G10 currencies. The empirical specification follows that of Engel and Wu (2018), which includes a role for the convenience yield in driving exchange rates and controls for standard determinants as well, but there are two important differences relative to that analysis. First, we examine daily changes in exchange rates (rather than monthly as in Engel and Wu (2018), or quarterly, as in Jiang et al. (2021a)) from 10-Jan-2001 to 29-Jul-2014. Second, and more importantly, we use plausible instruments for the convenience yield that link movement in the convenience yield to our model. The regression specification is from Engel and Wu (2018):

\[
\Delta s_{j,t} = \alpha_j + \beta_1 s_{j,t-1} + \beta_2 \Delta \eta_{j,t} + \beta_3 \Delta (i - i^*)_{j,t} + \beta_4 \eta_{j,t-1} + \beta_5 (i - i^*)_{j,t-1} + u_{j,t}
\]

where \(s\) is the nominal exchange rate of the dollar price of a unit of foreign currency, \(\eta\) is a measure of the convenience yield and \((i - i^*)\) is the US minus foreign government bond interest rate differential. We consider a panel exchange rate series of the U.S. dollar vs rest of the G10 currencies obtained from FRED. The convenience yield and interest rate data are obtained from Du et al. (2018b) and we use the shortest tenor available, which is 3-month. The convenience yield is measured as the payoff of a synthetic dollar government bond that is constructed by buying the foreign government bond and hedging the exchange rate exposure by entering a forward contract. Since the U.S. government bond and the synthetic dollar government bond both pay dollar returns, the difference between the two gives a measure of the relative difference in liquidity services of the U.S. and foreign government bonds. The specification here differs slightly from that of Engel and Wu (2018) in that the error-correction term - the lagged exchange rate - is nominal rather than real. That change is necessitated by the use of daily data, but during this period at all measurable frequencies real exchange rate movements and nominal exchange rate movements are highly correlated for all of the currencies.

Our particular interest is the effect of financial shocks on the exchange rate and the global economy. Focusing on daily frequency enables us to make use of a high-frequency identification strategy to identify changes in the convenience yield that are caused by financial shocks. In general, the convenience yield could be correlated with other macroeconomic variables, as Cerutti et al. (2021) have argued, and these may also influence the exchange rate. To provide direct evidence of the mechanism from banking to exchange rates, we employ an instrumental variable regression. We instrument the change of convenience yield \(\Delta \eta_{j,t}\) using financial shocks constructed by Ottonello and Song (2022), which are high-frequency changes in the market value of US intermediaries’ net

\(^9\)The other G10 currencies are the euro, Australian dollar, Canadian dollar, Japanese yen, New Zealand dollar, Norwegian krone, Swedish krona, Swiss franc, and U.K. pound.

\(^{10}\)Our sample size is limited by the data we use as instruments described below, which was kindly provided to us by the authors of Ottonello and Song (2022).
worth in a narrow 60-min window around their earnings announcements. In our model, a financial shock changes the U.S. banks’ net worth, which changes their demand for liquid and convenient assets, therefore creates a shift in convenience yield. To be specific, we make use of all four measures of financial shock in Ottonello and Song (2022) as our first stage instruments. They are the market value change of the earnings release bank around announcements, market value change of the all sample banks around announcements, and broader measures of these two which include earnings releases announced after market closed.

The first-stage regression is promising. The F-statistic for the joint significance of the instruments is 14.60. The Hansen J-statistic for correlation of the instruments with the error term in the second-stage regression is 7.21, and we fail to reject the null of no correlation.

The panel exchange-rate regression estimates are reported in Table 1. The coefficient on \( \Delta \hat{\eta}_{j,t} \) is significantly negative. The coefficient of -9.07 indicates a 1% increase in convenience yield is associated with 9.07% appreciation of the U.S. dollar. Similar to Engel and Wu (2018), we find that both the change of the convenience yield and the change of the interest rate differential are significant explanatory variables for exchange rate movements. More importantly, the instrumental variable regression highlights the convenience yield movement that is driven by change in banking net worth. We interpret this as evidence that banking demand for liquidity plays an important role in determining exchange rates. In the model section, we provide a full NK-DSGE model to study exchange rate determination with endogenous convenience yields.

Table 1: Daily exchange rate regression with financial shocks as instrumental variables

<table>
<thead>
<tr>
<th>LHS: ( \Delta s_{j,t} )</th>
<th>( \Delta \hat{\eta}_{j,t} )</th>
<th>-9.07**</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(4.53)</td>
<td></td>
</tr>
<tr>
<td>( \Delta i - i_{j,t}^* )</td>
<td>-8.58**</td>
<td>(3.86)</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td></td>
</tr>
<tr>
<td>( \eta_{j,t-1} )</td>
<td>-0.002</td>
<td>(0.03)</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td></td>
</tr>
<tr>
<td>( i - i_{j,t-1}^* )</td>
<td>-0.040</td>
<td>(0.03)</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td></td>
</tr>
<tr>
<td>( s_{j,t-1} )</td>
<td>-0.003</td>
<td>(0.002)</td>
</tr>
</tbody>
</table>

Standard errors cluster by time in parentheses* p<0.1, ** p<0.05, *** p<0.01. The first stage using four financial shock measures in Ottonello and Song (2022) as the instruments for the change of convenience yield. The first stage F-statistics is 14.60.

3 Model

We describe a two country model, denoted Home and Foreign. The countries are symmetric in all dimension except for the pledgeability of government bonds. Agents supply labor and consume goods from both countries. The world is populated with a unit mass of agents and Home has share
of these, with Foreign share 1−n. We assume that firms set prices in domestic currency (PCP) for home sales and foreign currency (LCP) for exports, and adjust prices constrained by Rotemberg-style price adjustment costs.

3.1 Households

There are two types of households, the Ricardian type (R) who has access to financial market and the Keynesian (K) type who is hand-to-mouth. The Keynesian type has a mass of m and the Ricardian has a mass of 1−m. Ricardian agents in the Home country have preferences over consumption and hours given by

\[
U = \frac{(C^R_i t)^{1-\sigma} - 1}{1 - \sigma} - \frac{\chi}{1 + \psi} (H^R_i t)^{1+\psi}
\]

(2)

Financial markets are restricted for households. Households can interact only with domestic banks in the form of non-contingent home currency denominated bonds. Banks in turn hold domestic and foreign currency denominated bonds as well as domestic equity. The environment for banks is described further below.

The Ricardian agents budget constraint is

\[
P_i C^R_i + B_i = W_i H^R_i + R_i B_{i-1} + \Pi_i + TR_i - T_{s,f}
\]

(3)

where \(P_i\) is the CPI in Home currency, \(B_i\) represents households deposits of domestic currency in the home banking system, \(\Pi_i\) represents the net receipts that households receive from production firms and banks, and \(TR_i\) is a transfer made from the home government to the households. Here, \(R_i\) is the domestic home currency interest rate received by households on bank deposits. Finally, \(T_{s,f}\) is the startup capital transferred to new banks at time t.

Keynesian agents in the Home country have GHH preferences over consumption and hours given by

\[
U = \frac{(C^K_i t - \chi (H^K_i t)^{1+\psi})^{1-\sigma} - 1}{1 - \sigma}
\]

(4)

The Keynesian agents budget constraint is

\[
P_i C^K_i = W_i H^K_i
\]

(5)

In aggregate, we have

\[
C_i = (1 - m) \times C^R_i + (m) \times C^K_i
\]

\[
H_i = (1 - m) \times H^R_i + (m) \times H^K_i
\]

It is assumed that

\[
C_i = \left( \omega^\frac{1}{\lambda} C_{h,i}^{\frac{1}{\lambda}} + (1 - \omega)^{\frac{1}{\lambda}} C_{f,i}^{\frac{1}{\lambda}} \right)^{-\frac{1}{1-\lambda}}
\]

(6)
where $\omega \geq n$, representing the possibility of home bias in preferences. Given this assumption, the home CPI is written as

$$P_t = \left( \omega P_{h,t}^{1-\lambda} + (1 - \omega)P_{f,t}^{1-\lambda} \right)^{1-x}$$

(7)

where $P_{h,t}$ ($P_{f,t}$) represents the home currency price of Home (Foreign) goods.

11 Optimal consumption of Home and Foreign goods for the Home consumer is

$$C_{h,t} = \omega \left( \frac{P_{h,t}}{P_t} \right)^{-\lambda} C_t$$

(8)

$$C_{f,t} = (1 - \omega) \left( \frac{P_{f,t}}{P_t} \right)^{-\lambda} C_t$$

(9)

Optimal labor supply is described by

$$W_t = \chi P_t (C_t)^{\sigma} (H_t)^{\psi}$$

(10)

$$W_t = \chi P_t (H_t^R)^{\psi}$$

(11)

Given $R_{t+1}$ and $\beta_{t+1}$ (known at time $t$), the return on deposits, Home household’s Euler equation is

$$1 = E_t R_{t+1} \beta_{t+1} (C_{t+1})^{-\sigma} P_t (C_t)^{-\sigma} P_t^{-1} \equiv E_t R_{t+1} \Omega_{t+1}$$

(12)

The preferences, budget constraints, and optimal choices for the Foreign economy are analogous. The presence of home bias in Foreign preferences then implies that the price index for the Foreign economy is

$$P_t^* = \left( \omega^* P_{f,t}^{1-\lambda} + (1 - \omega^*)P_{h,t}^{1-\lambda} \right)^{1-x}$$

(13)

3.2 Firms

A measure $n$ of firms in the Home economy produce differentiated goods. The aggregate Home good is a composite of these differentiated goods, where the elasticity of substitution between individual goods is denoted as $\epsilon > 1$. The production function for firm $i$ in the Home country is

$$Y_{i,t} = A_t \left( L_{i,t}^{1-\alpha} K_{i,t}^{\alpha} \right)$$

(14)

11 Letting $0 \leq x \leq 1$ represent the degree of home bias in preferences, where $x = 0$ ($x = 1$) represents zero (full) home bias, we can define $\omega = n + x(1-n)$.

12 We allow for the possibility of a preference shock as a simple way to model demand shock in our model.
where \( A_t \) is an aggregate productivity term, \( K_{i,t} \) is the firm’s use of capital, and \( L_{i,t} \) the use of labor.

We assume that the firm in each country sets two prices, one for sales in the domestic market in the domestic currency, and one for sales in the export market in the local currency of the importer. Thus, both countries engage in ‘local currency pricing’ (LCP).

The profits of the Home firm \( i \) are then represented as

\[
\Pi_{i,t} = \left( (1 + s_{i,t}) (P_{i,h,t} Y_{i,h,t} + S_t P^*_h Y^*_h) - MC_t (Y_{i,h,t} + Y^*_h) \right)
\]

(15)

where \( P_{i,h,t} \) is the price set in domestic currency for Home sales, and \( P^*_h \) is the Foreign currency price, with \( S_t \) being the exchange rate (Home price of Foreign Currency). Also, \( MC_t \) denotes the firm’s marginal cost, and \( s_{i,t} \) represents a subsidy that may be given to the firm to offset the monopoly distortion in pricing. Cost minimization by the firm implies:

\[
A_t (1 - \alpha) (L_{i,t}^{1-\alpha} K_{i,t}^{\alpha}) MC_t = W_t L_{i,t}
\]

(16)

\[
A_t \alpha (L_{i,t}^{1-\alpha} K_{i,t}^{\alpha}) MC_t = R_{K,t} K_{i,t}
\]

(17)

The firm chooses its Home and Foreign price to maximize the present value of expected profits, net of price adjustment costs

\[
E_t \sum_{j=0}^{\infty} \Omega_t \left( \Pi_{i,t} - \xi_t \left( \frac{P_{i,h,t}}{P_{i,h,t-1}} \right) P_h Y_{h,t} - \xi_t \left( \frac{P^*_h}{P^*_{h,t-1}} \right) S_t P^*_h Y^*_h \right)
\]

(18)

where \( \Omega_{t+1} = \beta_{t+1} \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} P_{t+1} \) is the firm’s nominal stochastic discount factor, and \( \xi_t(\cdot) \) represents a price adjustment cost function for the firm. We assume that \( \xi_t(\cdot) > 0 \), and \( \xi_t''(\cdot) > 0 \).

Price adjustment costs are proportional to the nominal value of Home sales to each of the Home and Foreign markets, to be consistent with the nominal profit objective function of the firm.

The first order conditions for profit maximization for the Home firm \( i \) can be described as

\[
(1 + s_{i,t}) Y_{i,h,t} = \epsilon (P_{i,h,t} (1 + s_{i,t}) - MC_t) \frac{Y_{i,h,t}}{P_{i,h,t}} + \xi'(P_{i,h,t}) \frac{1}{P_{i,h,t-1}} P_{i,h,t} Y_{i,h,t}
\]

\[
-E_t \Omega_{t+1} \xi'(P_{i,h,t+1}) \frac{P_{i,h,t+1}}{P_{i,h,t}} P_{i,h,t+1} Y_{h,t+1}
\]

(19)

\[
(1 + s_{i,t}) S_t Y^*_h = \epsilon (S_t P^*_h (1 + s_{i,t}) - MC_t) \frac{Y^*_h}{P^*_h} + \xi'(P^*_h) \frac{1}{P^*_h} S_t P^*_h Y^*_h
\]

\[
-E_t \Omega_{t+1} \xi'(P^*_h) \frac{P^*_h}{P^*_h} S_t P^*_h Y^*_h
\]

(20)

### 3.3 Banks

Banks are modeled as in Gertler and Karadi (2011). A fraction of household members \( 1 - \theta \) become bankers in any period, continue as bankers with probability \( \theta \), and revert to being consumers

\( \text{In the calibration, } \xi_t = \frac{\sigma}{2} (\frac{P_{i,h,t}}{P_{i,h,t-1}} - 1)^2. \)
with probability $1 - \theta$. When starting up, a bank receives some start up capital from households to establish its net worth, and borrows from households at fixed rates to invest in claims to capital, home and foreign currency denominated government bonds. Besides government, only banks operate in international financial markets. A Home banker $i$’s balance sheet in period $t$ is

$$B_{i,t} + N_{i,t} = Q_t K_{h,i,t+1} + S_t Q_t^* K_{f,i,t+1} + D_{h,i,t} + S_t D_{f,i,t}$$

(21)

where $B_{i,t}$ represents domestic currency deposits from households, $N_{i,t}$ is beginning net worth, $Q_t$ and $Q_t^*$ are the price of a unit of domestic and foreign capital, $K_{h,i,t+1}$ and $K_{f,i,t+1}$ are the holding of domestic and foreign capital, $D_{h,i,t}$ is the bank’s purchase of home denominated government bonds, and $D_{f,i,t}$ is it’s purchase of foreign currency denominated government bonds. This says that bankers use their net worth and new debt to invest in the home and foreign capital and home and foreign bonds. We calibrate so that banks will hold a positive position in all assets.\(^{14}\)

Banker $i$ chooses $K_{h,i,t+1}$, $K_{f,i,t+1}$, $D_{h,i,t}$, and $D_{f,i,t}$ to maximize her value evaluated using the SDF of home Ricardian households, $\Omega_{t+1} = \beta_{t+1} \frac{(Q_t^*)^{-\sigma}}{(Q_t)^{-\sigma}} P_t$. Following Gertler and Karadi, conjecture that the value function of the bank is a time varying linear function of her net worth, so that $V_{i,t} = \nu_t N_{i,t}$.

The banker’s value function then satisfies:

$$V_{i,t} = E_t \Omega_{t+1} \left((1 - \theta)N_{i,t+1} + \theta V_{i,t+1}\right)$$

(22)

This captures the fact that the banker will revert to being a consumer with probability $1 - \theta$ and consume it’s net worth, and continue to be a banker with probability $\theta$. The net worth dynamics must satisfy

$$N_{i,t+1} = \tilde{R}_{K,i+1} Q_t K_{h,i,t+1} + \tilde{R}_{Q,i+1}^* Q_t^* S_{t+1} K_{f,i,t+1} + R_{h,i,t+1} D_{h,i,t} + S_{t+1} R_{f,i,t+1} D_{f,i,t} - R_{t+1} B_{i,t}$$

(23)

$$= (\tilde{R}_{K,i+1} - R_{t+1}) Q_t K_{h,i,t+1} + (S_{t+1} \tilde{R}_{K,i+1} - R_{t+1}) Q_t^* S_{t+1} K_{f,i,t+1} + (R_{h,i,t+1} - R_{t+1}) D_{h,i,t} + (S_{t+1} R_{f,i,t+1} - R_{t+1}) S_{t} D_{f,i,t} + R_{t+1} N_{i,t}$$

Here, $\tilde{R}_{K,i+1} = \frac{R_{K,i+1} + (1 - \delta) Q_{t+1}}{Q_t}$ and $\tilde{R}_{Q,i+1} = \frac{R_{Q,i+1} + (1 - \delta) Q_{t+1}}{Q_t}$ are the net return on the home capital where $\delta$ is the depreciation rate on capital, $R_{h,i+1}$ is the return on the domestic currency government bond and $R_{f,i+1}$ is the analogous return on the foreign government bond.

Banks maximize this subject to (21) and subject to the participation constraint:

$$V_{i,t} \geq \partial_t \left( (\kappa_{h1,i,t} + \kappa_{h2,i,t} \tilde{D}_{h,i,t}) D_{h,i,t} + (\kappa_{f1,i,t} + \kappa_{f2,i,t} S_i \tilde{D}_{f,i,t}) S_i D_{f,i,t} \right)$$

$$+ \partial_t \left( (\kappa_{Kh1,i,t} + \kappa_{Kh2,i,t} \tilde{Q}_t \tilde{K}_{h,i,t+1} + (\kappa_{Kf1,i,t} + \kappa_{Kf2,i,t} S_i \tilde{Q}_t \tilde{K}_{f,i,t+1}) S_i Q^*_t K_{f,i,t+1} \right)$$

(24)

We introduce a set of asset-specific constraint parameters to allow for differential pledgeability.

\(^{14}\)In order to narrow the focus of the model, we abstract from foreign currency funding of banks. This important channel has been explored by Bruno and Shin (2015) and Avdjiev et al. (2019) among many others.
as a collateral across assets. Specifically, we have \( \kappa_{kh1,t}, \kappa_{kh2,t}, \kappa_{h1,t} \) and \( \kappa_{f1,t} \) as the constraint parameters for home capital, foreign capital, home bonds and foreign bonds respectively.\(^{15}\) As we discuss at length below, we posit that holdings of capital are more constrained than government bonds, and in turn, foreign government bonds holdings are more constrained than home bonds. The key idea is that government bonds are more pledgeable than equity or capital because of its safety and liquidity. In particular, the US Treasury is the most pledgeable assets in the world because of the absolute safety and deep market.

The bank’s first order conditions for, respectively, capital, home and foreign government bonds, are given by

\[
E_t \Lambda_{i,t+1} \left( \tilde{R}_{k,t+1} - R_{t+1} \right) = \eta_{i,t} \partial_t (\kappa_{kh1,t} + \kappa_{kh2,t} Q_t \tilde{K}_{h,i,t+1}) \tag{25}
\]

\[
E_t \Lambda_{i,t+1} \left( \frac{S_{t+1}}{S_t} \tilde{R}_{f,t+1} - R_{t+1} \right) = \eta_{i,t} \partial_t (\kappa_{f1,t} + \kappa_{f2,t} S_t Q_t \tilde{K}_{f,i,t+1}) \tag{26}
\]

\[
E_t \Lambda_{i,t+1} \left( R_{h,t+1} - R_{t+1} \right) = \eta_{i,t} \partial_t (\kappa_{h1,t} + \kappa_{h2,t} \tilde{D}_{h,i,t}) \tag{27}
\]

\[
E_t \Lambda_{i,t+1} \left( \frac{S_{t+1}}{S_t} R_{f,t+1} - R_{t+1} \right) = \eta_{i,t} \partial_t (\kappa_{f1,t} + \kappa_{f2,t} S_t \tilde{D}_{f,i,t}) \tag{28}
\]

Here, \( \eta_t \) is the Lagrange multiplier on the bank’s participation constraint (24) and

\[
\Lambda_{i,t+1} = \Omega_{i+1} \left( (1 - \theta) + \theta v_{i,t+1} \right)
\]

is the banker’s effective SDF. The banks value function can be retrieved from the envelope condition:

\[
v_{i,t} = \frac{E_t \Lambda_{i,t+1} R_{t+1}}{1 - \eta_{i,t}} \tag{29}
\]

Now we can use the fact that banks are homogeneous, and aggregate across all Home banks, adding the start up capital that is given to new banks, which we assume is \( \varphi \left( Q_t K_{h,t+1} + S_t Q_t^* K_{f,t+1} + D_{h,t} + S_t D_{f,t} \right) \), to get the dynamics of total net worth for the domestic banking sector as:

\[
N_{t+1} = \varphi \left( \tilde{R}_{k,t+1} - R_{t+1} \right) Q_t K_{h,t+1} + \left( \tilde{R}_{f,t+1} - R_{t+1} \right) S_t Q_t^* K_{f,t+1} +
(R_{h,t+1} - R_{t+1}) D_{h,t} + \left( \frac{S_{t+1}}{S_t} R_{f,t+1} - R_{t+1} \right) S_t D_{f,t} + R_{t+1} N_{t+1} \right)
\]

\[
+ \varphi \left( Q_t K_{h,t+1} + S_t Q_t^* K_{f,t+1} + D_{h,t} + S_t D_{f,t} \right) \tag{30}
\]

3.4 Capital Goods Producers

Capital goods producers buy the unused capital from banks, and engage in new investment, and sell the new capital to banks at price \( Q_t \). The representative capital goods producer has the profit

\[^{15}\)We also introduce \( \kappa_{kh2,t}, \kappa_{f2,t}, \kappa_{h2,t} \) and \( \kappa_{f2,t} \) so the constraint depends on the aggregate bank holding of the assets (denoted with \( D_{h,t}, D_{f,t}, \tilde{K}_{h,i,t+1} \) and \( \tilde{K}_{f,i,t+1} \)). The idea is that the monitoring cost is increasing with the asset size. We set these parameter values very small (0.025) and the main purpose of these parameters is to introduce stationarity in the linearized model.
function
\[ Q_t I_t - P_t (I_t + K_t \psi(I_t/K_t)) \] (31)
where \( \psi(.) \) is an adjustment cost function, satisfying \( \psi'(.) > 0 \) and \( \psi''(.) > 0 \), with \( \psi(\delta) = 0 \). This implies that the price of capital is
\[ Q_t = P_t (1 + \psi'(I_t/K_t)) \] (32)

3.5 Monetary policy

We assume a Taylor rule where the Central Bank targets the CPI inflation rate and uses the government interest rate as an instrument:
\[ R_{h,t+1} = \frac{1}{R_{h,ss}} \left( \frac{P_t}{P_{t-1}} \right)^{\eta^R} (R_{h,t})^{1+\pi} \] (33)

3.6 Fiscal policy

The Home and Foreign governments make transfers to the households and subsidize firms by issuing government debt. For the Home country we have
\[ \bar{D}_{h,t} = R_{h,t}\bar{D}_{h,t-1} + s_t(P_{h,t}Y_{h,t} + S_tP^*_hY^*_h) + TR_t \] (34)
where \( \bar{D}_{h,t} \) is the total outstanding debt of the Home government and is assumed to be exogenous.

3.7 Balance of Payments

The profit of the Home production firms is
\[ \Pi^p_t = \left( (1 + s_t)(P_{h,t}Y_{h,t} + S_tP^*_hY^*_h) - MC_{h,t}(Y_{h,t} + Y^*_h) \right. \\
\left. - \xi_t \left( \frac{P_{h,t}}{P_{h,t-1}} \right) P_{h,t}Y_{h,t} - \xi_t \left( \frac{P^*_h}{P^*_{h,t-1}} \right) S_tP^*_hY^*_h \right) \] (35)
Given constant returns to scale, we can write \( MC_{h,t}(Y_{h,t} + Y^*_h) \) as
\[ W_tL_{h,t} + r_{K,t}K_t \]
In equilibrium, labor supply must equal labor demand, so that
\[ H_t = L_t \]
The profit of Home capital producing firms is:
\[ \Pi^K_t = Q_t I_t - P_t (I_t + K_t \psi(I_t/K_t)) \] (36)
In addition, the capital stock accumulation equation must satisfy

\[ K_{t+1} = I_t + (1 - \delta)K_t \]

where \( \delta \) is the depreciation rate on capital.

Total profits from the corporate non-financial sector are then

\[ \Pi^P_t + \Pi^K_t \]

In addition, the return on deposits to Home households may be expressed as

\[ R_tB_{t-1} = r_{K,t}K_{h,t} + (1 - \delta)Q_tK_{h,t} + r_{K,t}^*K_{f,t} + (1 - \delta)S_tQ_t^*K_{f,t} + R_{h,t}D_{h,t-1} + S_tR_{f,t}D_{f,t-1} - N_t^e \]

where \( N_t^e \) represents the net worth of existing banks. The startup capital transferred from households to banks is

\[ T_{s,t} = \varphi(Q_{t+1} + D_{h,t} + S_tD_{f,t}) \]

So total net worth of the banking sector at time \( t \) is \( N_t = N_t^e + T_{s,t} \).

Finally, government transfers are:

\[ TR_t = \tilde{D}_{h,t} - R_{h,t} \tilde{D}_{h,t-1} - s_t(P_{h,t}Y_{h,t} + S_tP_{h,t}^*Y_{h,t}^*) \]

Note that the net deposits from households to financiers can be defined as

\[ B_t = Q_tK_{h,t+1} + S_tQ_t^*K_{f,t+1} + D_{h,t} + S_tD_{f,t} - N_t \]

Now putting together (35), (36), (37), (38), (39), into the home budget constraint (3), we get the balance of payments condition:

\[ P_t(C_t + X_t + I_t + K_t\psi\left(\frac{K_t}{K}\right)) + D_{h,t} - \tilde{D}_{h,t} + S_tD_{f,t} + Q_t(K_{h,t} - K_t) + S_tQ_t^*K_{f,t} = \]

\[ P_{h,t}Y_{h,t} - \xi\left(\frac{P_{h,t}}{P_{h,t-1}}\right)P_{h,t}Y_{h,t} + S_tP_{h,t}^*Y_{h,t}^* - \xi\left(\frac{P_{h,t}}{P_{h,t-1}}\right)S_tP_{h,t}^*Y_{h,t}^* \]

\[ + R_{h,t}(D_{h,t-1} - \tilde{D}_{h,t-1}) + S_tR_{f,t}D_{f,t-1} + \tilde{R}_{k,t+1}(K_{h,t-1} - K_{t-1}) + S_t\tilde{R}_{k,t+1}^*K_{f,t-1} \]

where \( \tilde{D}_{h,t} \) is the total outstanding debt of the Home government.

3.8 Adjusted UIP condition

By combining equation 27 and 28 we get:

\[ E_t\Lambda_{t+1}\left(\frac{S_{t+1}}{S_t}R_{f,t+1} - R_{h,t+1}\right) = \eta_t \vartheta(\kappa_f - \kappa_h) \]
Log linearizing the equation, we have a UIP condition adjusted for the balance sheet friction:

\[ E_t s_{t+1} - s_t = (r_{h,t+1} - r_{f,t+1}) - \tilde{\eta}_t \]  

(42)

Our emphasis is on demand for liquidity. Linearizing the equation eliminates the role of a time-varying risk premium, which also may play a role in driving exchange rates. \(^{16}\) Forward iterating the equation gives:

\[ s_t = -E_t \sum_{j=1}^{\infty} (r_{h,t+j} - r_{f,t+j} - (r_h - r_f)) - E_t \sum_{j=0}^{\infty} (\tilde{\eta}_{t+j} - (\tilde{\eta})) + \lim_{k \to \infty} (E_t s_{t+k} - k(s_{t+1} - s)) \]  

(43)

This shows that the transitory component of the exchange rate appreciates in response to the sum of expected future (transitory) interest rate differentials, as is usual, but also to the sum of expected future (transitory) convenience yields.

### 4 Calibration

The model frequency is quarterly. We calibrate the model to match U.S. moments. We think of Home as the U.S. and Foreign as the rest of the World. To highlight the important role of the asymmetric collateral value, we set all other parameters the same across the Home and the Foreign. The parameter values are summarized in Table 2.

The parameters can be partitioned into two blocs. The first bloc is externally set and mostly set to values that are in line with standard literature values. The second bloc is calibrated to match some long-term averages in the data.

On the household side, we set the discount factor (\(\beta\)) to be 0.99 and the CRRA coefficient (\(\sigma\)) to be 5. Home bias (\(\omega, \omega^*\)), the cross-country elasticity of substitution of goods (\(\lambda\)) and the within-country elasticity of substitution of goods (\(\varepsilon\)) are set at 3.8 and 6 respectively. The inverse of the Frisch elasticity (\(\psi\)) is set at 1 and we calibrate the disutility of labor \(\chi\) to match a steady-state \(H^*\) of 0.33.

On the production side, the country mass (\(n\)) is set at 0.5 to preserve symmetry. The Rotemberg price adjustment cost (\(\phi_{\pi}\)) is set at 155.88, matching an annual probability of price change of 0.84 in a Calvo-type model of sticky prices. The capital adjustment cost (\(\phi_k\)) is set at 5. The capital share (\(\alpha\)) is set at 0.33 and depreciation rate (\(\delta\)) is set at 0.04.

On the government side, the Taylor coefficient (\(\eta_\pi\)) is set at 1.5 with a smoothing parameter (\(\rho^R\)) equal to 0.85 (taken from Justiniano et al. (2010)). Government debt is fixed (\(\tilde{D}_h\) and \(\tilde{D}_f\)) at a constant value of 2.7, resulting in a steady state home debt to GDP ratio (\(\tilde{D}_h / GDP\)) of 83%, matching the long-run average of the U.S.. The monopoly subsidy is set at 0.2 (\(s = \frac{1}{\varepsilon - 1}\)) to eliminate the steady state mark-up distortion. For simplicity, gross steady state inflation (\(\tilde{\pi}\)) is set to zero.

On the banking side, we calibrate the bond constraint parameters to match the U.S. external

\(^{16}\)See, for example, Kalemli-Özcan and Varela (2021); Akinci et al. (2022); Obstfeld and Zhou (2023) for empirical evidence linking risk, UIP deviations, and exchange rates.
positions. This is the main source of asymmetry between the two countries. We calibrate the four bond constraint parameters \((\kappa_{h1}, \kappa_{f1}, \kappa^*_h, \kappa^*_f)\) to match a steady state convenience yield of 1% (which is the same as steady state home minus foreign interest rate differential of -1%), a steady state positive income account of +0.0013 ((current account - trade balance)/GDP of US from 1990-2019), a steady state -18.5% NFA position and a steady state foreign holding of US government bond \((D^*_h/\bar{D}_h)\) of 45% (average from Tabova and Warnock (2021)). While these are calibrated jointly, lowering the home bond constraint parameter \((\kappa_{h1}\text{ and } \kappa^*_h)\) relative to the foreign bond constraint parameter \((\kappa_{f1}\text{ and } \kappa^*_f)\) generates a positive convenience yield. Since the net foreign bond position is defined as \(-D^*_h + S_t D^*_f, t\), a relatively low value of \(\kappa^*_h\) to \(\kappa^*_f\) is useful to generate a negative NFA position. A higher \(\kappa^*_h\) (relative to \(\kappa_{h1}\)) helps to match the 45% share of foreign holdings of U.S. Treasury obligations. Finally, the relative values of \(\kappa_{h1}\) and \(\kappa_{f1}\) are useful for pinning down the positive income account. On the capital side, we maintain an agnostic view about the relative collateral values in generating the asymmetry of external positions. Instead, we calibrate the model such that \(\kappa_{Kh} = \kappa_{Kh}^* < \kappa_{Kf} = \kappa_{Kf}^*\). That is, the Home capital serves as a better collateral than Foreign capital for Home banks and the opposite is true for Foreign banks. We set \(\kappa_{Kf} = \kappa_{Kh}^* = 0.49\) and \(\kappa_{Kh} = \kappa_{Kf}^* = 0.41\) to match a net equity return of 5.7% and equity home share of 70% (average of 1990-2016 from Hnatkovska (2019)), which are in the range of estimates from the equity premium puzzle and home equity bias literatures. We normalize the country specific collateral constraint parameters \((\vartheta, \vartheta^*)\) to be 1. Finally, we set the bank survival probability \((\theta)\) and capital injection rate \((\phi)\) to be at standard values of 0.95 and of 0.01 respectively, resulting in a steady state leverage of around 3.

The bond constraints also include the parameters \((\kappa_{h2}, \kappa_{f2}, \kappa^*_h, \kappa^*_f)\). Our solution method is linearization around the non-stochastic steady state, but the equilibrium portfolio is indeterminate without these parameters, as investors would be indifferent between assets that paid the same expected return inclusive of the liquidity yield. The problem is similar to the indeterminacy of portfolios in the non-stochastic steady state in the mean-variance framework (see Devereux and Sutherland (2010, 2011).) While it is necessary to introduce these terms, we set all four at very small values (0.005) so as to have effectively no influence on the dynamic adjustment process. This is analogous to the fix proposed by Schmitt-Grohé and Uribe (2003) of introducing a debt-elastic interest rate to solve the problem of the absence of a steady state in small-open economy models with incomplete markets. An alternative approach would be to solve the model using some sort of higher-order approximation, such as the “non-stochastic steady state” used in the closed economy model of Gertler et al. (2012). The steady-state portfolios in such a set-up would be chosen as the solution to a mean-variance criterion. But this type of model has proven to be notably unsuccessful in capturing important features of international portfolio holdings, specifically the high degree of home bias in equity holdings and the relatively higher return that the U.S. earns on its foreign portfolio compared to other countries. Our choices of \((\kappa_{h1}, \kappa_{f1}, \kappa^*_h, \kappa^*_f)\), as described above, are chosen to more nearly approximate those features of the data. This makes our calibration of the steady state closer in spirit to the models that introduce a makeshift “preferred habitat” into port-
folio choice (such as in Greenwood et al. (2020) and Gourinchas et al. (2022)), or the empirical models in which the factors affecting portfolio choice are data-driven (Koijen and Yogo (2020); Jiang et al. (2022).)

We solve the model assuming right shocks; Home and Foreign productivity shocks $A_t, A^*_t$, Home and Foreign balance sheet constraint values, $\vartheta_t, \vartheta^*_t$, Home and Foreign monetary shocks and Home and Foreign preference shocks ($\beta_t, \beta^*_t$). All shocks are assumed to be AR 1.

$$\log(A_t) = \rho^A \log(A_{t-1}) + \sigma^A \epsilon^A_t, \log(A^*_t) = \rho^{A*} \log(A^*_{t-1}) + \sigma^{A*} \epsilon^{A*}_t$$

$$\log(\vartheta_t) = \rho^\vartheta \log(\vartheta_{t-1}) + \sigma^\vartheta \epsilon^\vartheta_t, \log(\vartheta^*_t) = \rho^{\vartheta*} \log(\vartheta^*_{t-1}) + \sigma^{\vartheta*} \epsilon^{\vartheta*}_t$$

$$\log(M_t) = \rho^M \log(M_{t-1}) + \sigma^M \epsilon^M_t, \log(M^*_t) = \rho^{M*} \log(M^*_{t-1}) + \sigma^{M*} \epsilon^{M*}_t$$

$$\log(\beta_t) = \rho^\beta \log(\beta_{t-1}) + \sigma^\beta \epsilon^\beta_t, \log(\beta^*_t) = \rho^{\beta*} \log(\beta^*_{t-1}) + \sigma^{\beta*} \epsilon^{\beta*}_t$$

We set the persistence $(\rho^A, \rho^{A*} = 0.97)$ and standard deviation $(\sigma^A, \sigma^{A*} = 0.008)$ of productivity and persistence $(\rho^M, \rho^{M*} = 0.25)$ and standard deviation $(\sigma^M, \sigma^{M*} = 0.002)$ monetary shocks at standard values. The balance sheet constraint (financial) shock are calibrated to match 90% of $\Delta s$ to be explained by financial shock (Itskhoki and Mukhin (2021a) and Miyamoto et al. (2022)) and $\text{corr}(\Delta \text{net export}, \Delta q) \approx 0$, resulting in $\rho^\vartheta, \rho^{\vartheta*} = 0.98$ and $\sigma^\vartheta, \sigma^{\vartheta*} = 0.27$. We set the preference shock persistence at 0.97 and standard deviation at 0.01 so that preference shock explains 75% of relative consumption change, consistent with the estimation result by Smets and Wouters (2003). We allow the Home and Foreign shocks to be correlated among each type of shocks. We set the $\text{corr}(\epsilon^A_t, \epsilon^{A*}_t)$ to be 0.85 to match the correlation of Home and Foreign output of 0.88. We set $\text{corr}(\epsilon^A_t, \epsilon^{A*}_t)$ to be 0.56 as in Itskhoki and Mukhin (2021a). We set $\text{corr}(\epsilon^\vartheta_t, \epsilon^{\vartheta*}_t)$ and $\text{corr}(\epsilon^\beta_t, \epsilon^{\beta*}_t)$ to be 0.05 so they are weakly correlated but allow for the possibility of global shocks.
### Table 2: Parameter values

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Meaning / description</th>
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### 5 Steady state values

The model is solved by linearizing around the non-stochastic steady state. The table below present some steady state values using the parameterization listed above.\(^{17}\)

The only asymmetry of the model is the bond constraint parameters. As we noted above, we set the home bond to be better collateral for both Home and Foreign banks ($\kappa_{h1}, \kappa_{h1}^* < \kappa_{f1}, \kappa_{f1}^*$). With only this asymmetry, we are able to generate three important features of the US external position. First, the US has a government rate that is lower than the Foreign government rate by 1\%.

\(^{17}\)A more comprehensive list of the steady state values is provided in the Appendix.
This can be understood by looking at steady state version of equation 42. Since $E_t s_{t+1} - s_t = 0$ at the steady state, the excess monetary return of foreign bond reflects the additional non-pecuniary balance sheet cost of holding it. This demonstrates the convenience yield arising from the better pledgeability of US debt.

Second, the US has a negative NFA position, meaning that it has a net liability to the rest of the world. Third, despite the net liability, the US has a steady state trade balance deficit and positive income account. This is because while it owes the rest of the world repayment, it pays a lower interest rate on its liability to the rest of the world than the rest of the world pays to the US. Overall, this characterizes an exorbitant privilege that follows from the convenience yield.

The asymmetry in pledgeability or acceptability as collateral has macroeconomic implications. The U.S. has a higher steady-state consumption than the rest of the world (0.6118 vs 0.6113). While the steady state capital is the same, the U.S. has lower steady state employment and a higher steady state wage than the rest of the world. Taken together, this implies that while the U.S. produces less than the rest of the world, it lives with a higher consumption and wage levels purely due to the “seigniorage” from the exorbitant privilege. On the financial side, we observe that the U.S. banks have a higher leverage than the rest of the world (3.01 vs 2.99) but less tight constraint, reflected by a lower $\eta$ in the steady state (0.0093 vs 0.0094).

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Steady state value</th>
<th>Symbol</th>
<th>Steady state value</th>
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</thead>
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<td>$RER$</td>
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Table 3: Steady state values
6  Impulse responses

6.1  Global Financial Shock

In this section, we look at the impulse responses from the simulated model to understand the model mechanism and also ask how it could explain the dynamics around the global financial crisis. As a baseline, in Figure 2 we look at a case where there is a uniform increase of one percent in \( \vartheta \) and \( \vartheta^* \), the tightness parameter on the capital market constraint (see equation 24). This represents a negative shock to the banking system in each country, requiring a higher bank value for all types of collateral and forcing the banks to de-lever as well as adjust their investment across asset classes.

This baseline case (in Figure 2) represents a global tightening of the financial constraint faced by banks, which may be precipitated by a sudden loss in confidence in the value of banks’ collateral. Bernanke (2018) and Gorton and Metrick (2012), for example, make the case that in the early stages of the Global Financial Crisis, financial intermediaries began to mistrust the value of collateral put up by other financial intermediaries, firms, and households as the housing bubble in the U.S. began to burst. As the downturn began, credit conditions tightened even more.

The shock to the financial constraint leads to an immediate appreciation of the US real exchange rate, followed by an expected depreciation. The top right panel of Figure 2 shows that this is associated with a rise in convenience yield (defined as negative of \( \tilde{r}_{ht} - \tilde{r}_{ft} - (rer_{t+1} - rer_t) \), the blue line in the top right panel), where \( \tilde{r}_{ht} \) represents the home CPI real interest rate, and analogously for the foreign country. Thus, there is an increase in the deviation from UIP; the excess expected return on foreign relative to US government bonds rises in response to the global financial shock. To understand the mechanics of this response, take the difference between equation (27) and equation (28) for the home bank, which gives the condition

\[
E_t \Lambda_{i,t+1} \left( R_{h,t+1} - R_{f,t+1} \frac{S_{t+1}}{S_t} \right) = \eta_{i,t} \vartheta_t (\kappa_{h,1,t} - \kappa_{f,1,t}) \tag{44}
\]

The term inside the parenthesis on the right hand side is negative, given that the home bond is better collateral than the foreign bond. Then, abstracting from second-order terms, the expected return on the home bond is less than that of the foreign bond, so that on average, UIP will not hold, as in the data. But then it follows that given a negative financial shock (a rise in \( \vartheta_t \)), the impact on the left-hand side of this equation must be negative, so that the the expected return on the home bond must fall relative to that on the foreign bond - hence the convenience yield on US government bonds rises further given a global negative financial shock. While this explanation uses the first-order conditions for the home bank alone, a similar explanation holds for the foreign bank. We see therefore that a direct implication of the asymmetric collateral value of US relative to foreign government debt is that a uniform tightening of global financial conditions leads to an increase in the convenience yield on US bonds. In fact, given the monetary rule, the interest rates on home

\[\text{footnote}{We could think of the leverage shock as being induced by pessimistic expectations, as in Perri and Quadrini (2018).}\]

\[\text{footnote}{In this description, we abstract from the quadratic terms in the collateral constraint and the first order conditions 27 and 28, since in practice these terms are very small, and do not affect the argument.}\]
and foreign bonds do not change very much, as can be seen in Figure 2 top right panel. Instead, the increase in the convenience yield on US bonds is achieved by an immediate appreciation and therefore an expected depreciation of the home real exchange rate.

We can examine Figure 2 in more detail to track the macroeconomic responses to the negative financial shock. The response of $\eta$ and $\eta^*$ represent the endogenous increase in the Lagrange multipliers on the bank’s collateral constraints. While this increases for both countries, it increases more for the home bank than the foreign bank. This is because the home bank disinvests in physical capital more than the foreign bank, given the advantage of home bonds over foreign bonds, and the fact that the relative balance sheet cost of home bonds to capital for the home bank is less than the equivalent relative cost for the foreign bank. Again, we can see the intuition behind this response by focusing on equations (25) and (27) above, and the equivalent conditions for the foreign bank. This gives us the conditions:

$$E_t A_{t,t+1} \left( \tilde{R}_{k,t+1} - R_{ht+1} \right) = \eta_{t,t} \vartheta_t (\kappa_{Kh1,t} - \kappa_{h1,t})$$  (45)

$$E_t A_{t,t+1}^* \left( \tilde{R}_{k,t+1}^* - R_{ft+1} \right) = \eta_{t,t} \vartheta_t (\kappa_{Kh1,t}^* - \kappa_{f1,t}^*)$$  (46)

Equation (45) describes the home bank’s trade off between home government bonds and home capital (or equity), while equation (46) describes the analogous trade-off for the foreign bank between foreign government bonds and foreign capital. The negative financial shock reduces investment in both countries, but investment falls by more in the home country. The term in parentheses on the right hand side of (45) is larger than the equivalent term in (46) precisely because home government bonds represent better collateral, relative to home capital, than foreign government bonds relative to foreign capital. Hence the expected excess return on home capital relative to home bonds must rise more than that on foreign capital relative to foreign bonds, and the end result is that home investment must fall relative to foreign investment, translating into a greater fall in home output relative to foreign output.\(^{20}\)

Since home investment and consumption fall relative to the foreign country, Figure 2 (row 2 column 4, and row 4 column 3, respectively) shows that the home trade balance improves after the financial shock. We can interpret this as the US effectively making a transfer to the rest of the world as part of the 'exorbitant duty' during a financial crisis. In addition, the real exchange rate appreciation and the fall in the value of home capital translates into a greater fall in the net worth of home banks relative to foreign banks, which can be interpreted as another way in which the US acts as an implicit 'insurer' to the rest of the world in times of global crisis, given that banks are owned by domestic residents. The asymmetric response of home and foreign banks also has a striking implication for gross asset holding positions. Given that the home bank increases its demand for

\(^{20}\)While this argument seems incomplete because both home and foreign banks invest in bonds and capital in each others country, the fact that in both equity capital and bond holdings banks have relative larger positions in their home market ensures that the financial shock hits home investment more than it does foreign investment. Abbassi and Bräuning (2023) and Agarwal (2019) have recently empirically investigated how credit contractions in the financial sector influence investment in an international setting.
home bonds as it substitutes away from capital, there is an external portfolio retrenchment - home banks increase their holding of home bonds and decrease their holdings of foreign bonds, and the opposite applies to the foreign banks. As noted in the introduction, this retrenchment is a common feature of financial crises and was clearly a feature of the Global Financial Crisis. The detailed aspects of this retrenchment are discussed further below.

Note also that if we measure the relative convenience yield as in Engel and Wu (2018), the difference between the deposit rate in the U.S. and the government bond interest rate, relative to the same interest rate differential in the foreign country, that also increases at the time of the financial shock (row 1, column 4.) This is in agreement with the empirical regularities plotted in 1.

Another striking feature of the response of the model is that the real exchange rate appreciation is almost wholly driven by deviations from the law of one price. As shown in bottom right panel of Figure 2, there is almost no response of the terms of trade to the financial shock. This offers a potential answer to the Maggiori (2017) ‘reserve currency paradox’, which questions how the US dollar country can appreciate during times when US relative wealth falls, and US consumers are biased towards home produced goods (so the terms of trade should deteriorate). In our model, this does not present a puzzle, since the movement in the real exchange rate can be entirely divorced from the terms of trade, when prices are sticky firms set prices according to LCP.

The response of variables to the financial shock replicates the empirical findings of Davis and van Wincoop (2022), which estimates the response of asset prices and capital flows in response to a global financial shock (their “GFC factor.”) Specifically, that paper finds empirically, and our model predicts: 1) a global fall in equity prices; 2) a global drop in real interest rates; and, 3) retrenchment. Moreover, Davis and van Wincoop (2022) find asymmetric effects for countries that are net debtors of safe assets, which our model also predicts for the U.S.: 4) the current account rises; 5) saving increases in the U.S. relative to the rest of the world; 6) investment declines in the U.S. relative to the rest of the world; 7) the U.S. sells fewer Treasury bonds abroad; and, 8) the U.S. sheds holdings of foreign risky assets (i.e., equities).

To provide a clean illustration, we also look at the response to the same financial shocks, except in the case when there is no capital trade across countries in figure 3. The qualitative and quantitative results of the financial shock are very similar to those in figure 2, although the expected depreciation of the real exchange rate (and the consequent rise in the U.S. convenience yield) is somewhat less in this case.

Finally, figure 4 illustrates the case where $\kappa_{h1} = \kappa_{h1}^*$ and $\kappa_{f1} = \kappa_{f1}^*$. In this case, while the US government bond still has a collateral advantage, the collateral constraints are identical for both home and foreign banks, so there is no home bias in bond holdings among banks. This figure helps us to understand the forces driving retrenchment. In this case, because the banks in each country face identical constraints on U.S. and foreign bonds, in essence at steady state their portfolios of U.S. bonds and foreign bonds are nearly identical.21 The tightening of the credit constraint raises demand for U.S. bonds relative to foreign bonds because U.S. bonds are preferred collateral. If all

21There is a slight difference arising from the fact that U.S. wealth is somewhat higher in steady state because of the seignorage it earns from its exorbitant privilege.
investors were identical and evaluated assets in the same currency, given that the supplies of bonds from both countries are fixed in the short run, the increase in demand would lower the expected return on U.S. bonds relative to foreign bonds until investors were satisfied to hold the existing stock of both bonds. However, in reality, the appreciation of the dollar raises the foreign currency value of U.S. bonds, and by itself increases the share of those bonds in the foreign bank’s portfolio. Given that the foreign bank has increased its share of U.S. bonds in this way, the U.S. bank can, in equilibrium, increase the share of U.S. bonds in its portfolio by buying some from the foreign bank. The fact that the U.S. bank has a stronger incentive to switch its portfolio out of capital and into home bonds further reinforces the retrenchment. The relative reduction in demand for foreign bonds works in the opposite way. The depreciation of the foreign currency automatically reduces the share of foreign bonds in the U.S. bank’s portfolio, so the foreign bank balances the market by buying some foreign bonds from U.S. banks.

The contribution here is two-fold. First, we see an explanation for the appreciation of the dollar during times of financial contraction. If there were only risk motives for holding bonds, the reserve currency paradox arises. If the dollar is a good asset to hold because it appreciates during times of global stress, what factors lead to its appreciation? In the first place, it cannot be that investors buy more insurance at that point (that is, buy more U.S. bonds) because once the downturn happens, it is too late to buy insurance. The paradox is that the U.S. must make its insurance payment to foreigners during these global financial recessions, but that implies a reduction in U.S. wealth. In a standard model in which the real exchange rate is linked to the terms of trade, and there is home bias in preferences, this relative reduction in U.S. wealth ought to lead to a reduction in demand for U.S. goods, and therefore a real depreciation. In our model, these difficulties are resolved. The demand for dollar assets increases in bad times because of the increase in demand for the asset that provides better collateral when financial constraints are tightened. That indeed may mean the U.S. is making an insurance payment to foreigners, though our linear approximation has simplified away the risk motives for holding assets (which is not to say they are unimportant.) But with local-currency pricing, the link between the terms of trade and the real exchange rate is broken, and so even if U.S. relative wealth falls, a real appreciation is still possible. Of course, the exchange rate must satisfy both goods and asset market equilibrium, but in a sense, because prices are sticky, in the short run it is the asset markets that play the dominant role in exchange rate determination, and goods markets adjust as in Keynesian models through demand-determined changes in output.

The second contribution is a better understanding of asset flows. The literature has puzzled over how to reconcile the empirical finding of global retrenchment during times of financial stress with the observation that the dollar appreciates. Retrenchment seems to imply that while the U.S. is shifting its demand away from foreign assets, the rest of the world is shifting its demand away from dollar assets. But as we have seen, that understanding of retrenchment mixes the equilibrium change in asset holdings with the change in demand. We see that foreign banks do increase their demand for U.S. assets, as do U.S. financial intermediaries, and that contributes to the appreciation of the dollar. But it is the change in the exchange rate itself as well as the influence of changes
in demand for other assets that leads to the equilibrium portfolio adjustment - with foreign banks reducing their holdings of U.S. bonds, while U.S. banks increase theirs.\footnote{In a panel study of 188 countries, Kim and Min (2022) find that relative valuation effects among countries are mostly associated with exchange-rate changes (rather than asset price changes), and that there tends to be a negative correlation between real exchange rates and trade balances (that is, the trade balance tends to rise into surplus when the currency appreciates, as above.)}

### 6.2 Global Productivity Shock

Figure 5 presents the effects of a 1% global decline in TFP, which illustrates how a global slowdown arising from a productivity slowdown may have similar effects on the value of the dollar and the dollar liquidity premium. The effects on exchange rates, asset prices, and capital flows is mostly very similar to the effects of a tightening of the financial constraint. That is because the drop in productivity reduces the profitability of the bank, and so endogenously tightens the lending constraint. As in the literature on the “financial accelerator” (e.g., Bernanke and Gertler (1989), Kiyotaki and Moore (1997), Bernanke et al. (1999), Gertler and Kiyotaki (2010), Gertler and Karadi (2011)), the financial squeeze reduces investment and exacerbates the effects of the original drop in productivity on real output and employment. As in the case of an increase in $\vartheta$ and $\vartheta^*$, the drop in global productivity (a 1% initial decline in $A_t$ and $A_t^*$) leads to a tightening of the lending constraint as can be seen by the increase in the multipliers, $\eta_t$ and $\eta_t^*$.

Because the U.S. bond is relatively less constrained, demand for it increases, which lowers the expected return on the bond. This is mainly accomplished through an appreciation of the dollar, leading to an expectation of a depreciation. There is a larger decline in investment in the U.S. for the same reason described above for a financial shock, leading to a larger decline in U.S. output.

### 6.3 U.S. Monetary Contraction

Figure 6 shows the response to a surprise monetary tightening in the U.S. Rey (2015), Rey (2016), Kalemli-Özcan (2019), Curcuru et al. (2018), and Miranda-Agrippino and Rey (2020) have emphasized the importance of spillovers from U.S. monetary policy in driving the business cycle and monetary policy choices of other countries. In this figure, we see the usual effects on the U.S. of a monetary contraction - increased real interest rates, a drop in investment, consumption and output. The monetary contraction leads to an appreciation through the usual channel of tight money, but also there is an increase in the convenience yield on U.S. Treasury bonds. As investment in capital becomes less attractive, U.S. banks switch their demand toward domestic bonds, which lowers their expected return relative to foreign bonds - that is, an increase in the liquidity yield on U.S. bonds. We can also see that the gap between deposit rates and liquid bond rates in the U.S. widens relative to that in the rest of the world. There is an increase in the U.S. trade balance, though that is not driven primarily by an expenditure switching response (because of local-currency pricing) but rather through the expenditure reducing effects of lower investment and consumption. The contraction in the U.S. spills over to the foreign country through conventional channels, but also through financial channels that are unconventional. As U.S. banks demand switches from equities
to U.S. Treasury bonds, lowering the return on those bonds, they acquire U.S bonds from foreign banks. There is a tightening of financial constraints in the foreign country, which leads to a drop in investment demand there, and an increase in demand for local government bonds. However, the effects on the real economy are smaller in the foreign country than in the U.S.

6.4 **Global Monetary Contraction**

Figure 7 illustrates an equal unexpected monetary contraction in the U.S. and Foreign country. Even though the direct effect of this shock is for both countries to raise their interest rates equally, the effects on the exchange rate, expected returns, and the policy instrument itself are not equal in equilibrium. The tightening monetary policy reduces aggregate demand and lowers the value of capital. It also lowers the value of the banks that own capital, tightening the balance sheet constraint. As with the global financial shock, the effect of the more stringent constraint is to raise the demand for U.S. dollar bonds globally. The liquidity return on these bonds increases, and the dollar appreciates. The disinflationary effect is stronger in the U.S. because banks there are more incentivized to switch out of equities investments and into U.S. bonds, thus lowering aggregate demand to a greater extent. As a result, monetary policy must tighten more in the Foreign country - its policy rate must rise relative to the policy rate in the U.S.

6.5 **Alternative Assumptions on Price Setting**

Figure 8 shows how the results are dependent on the assumption that retail prices are set in the currency of the buyer, i.e. LCP. The figure illustrates the impact of the same financial shock as in Figure 2 but now assuming that all goods prices are set in the seller’s currency, (PCP), so that the firm in each country sets only one price in the domestic currency. The most notable difference from the baseline case is the much smaller response of the real exchange rate. With PCP, real exchange rate appreciation can occur only due to terms of trade appreciation, in combination with home bias in consumer preferences. In Figure 8, despite that home consumption falls relative to foreign consumption, we do observe terms of trade appreciation in response to the financial shock. This is due to the larger drop in U.S. output. But the resultant real exchange rate appreciation is much smaller than in the baseline case. The Figure also shows that the real convenience yield is mostly driven by the differential in real interest rates across countries rather than expected real exchange rate depreciation, as in the baseline case. Of course, in this case, there is no deviation from the law of one price in traded goods.

Figure 9 looks at the case with purely flexible prices. While the financial shock still raises the convenience yield, which is driven by the banking block of the system, the real exchange rate now moves in the wrong direction. The Figure shows a very small real exchange rate depreciation. This is driven by a small terms of trade deterioration. This case speaks directly to the role of LCP in resolving the reserve currency paradox introduced by Maggiori (2017). The nominal appreciation of the dollar at the time of the shock results in a gain in income on dollar assets held by foreign banks and loss in income on foreign assets held by U.S. banks. There is a wealth “transfer” from
the U.S. to the rest of the world, as Maggiori explains (though in our setting this is partly offset by the gain in wealth for the U.S. from the value of higher seignorage, as in Jiang et al. (2020).) There is a real depreciation of the dollar resulting from the wealth transfer, even though there is a nominal appreciation. This model result under flexible prices is strongly counterfactual, and argues for adopting the LCP formulation.

We can conclude from these two Figures that a complete analysis of the impact of a global financial shock that can account for the response of the U.S. convenience yield, the real appreciation of the U.S. dollar, and the feature of global portfolio retrenchment requires the combination of capital constrained banks, an advantage in collateral value for U.S. bonds, as well as sticky prices with limited exchange rate pass-through, captured by a local currency pricing rule for exporting firms.

6.6 Quantitative tightening and exchange rate intervention

We next examine the implications of quantitative tightening (QT) and exchange rate (FX) intervention in our model. We postulate the Home country conducts quantitative tightening and the Foreign country conducts FX intervention. To do so, we define a new variable for the central bank balance sheets, where $d_{CBh,t}$ and $d_{CBh}*t$ are the Home and Foreign central bank holding of home bonds, $d_{CBf}*t$ is the Foreign central bank holding of foreign bonds, and $K_{CBh,t}$ is the Home central bank holding of home capital. We assume $d_{CBh,t}$ and $d_{CBh}*t$ follow an log AR 1 process with a persistence parameter $\rho$ of 0.7:

$$\log(d_{CBh,t}) = \rho \log(d_{CBh,t-1}) + \epsilon_{d_{CBh}}$$

$$\log(d_{CBf}*t) = \rho \log(d_{CBf}*t-1) + \epsilon_{d_{CBf}}$$

The modified market clearing conditions are:

Home bond market clear: $\bar{d}_h = d_{h,t} + d_{h}*t + d_{CBh}*t$

Foreign bond market clear: $\bar{d}_f = d_{f,t} + d_{f}*t + d_{CBf}*t$

Home capital market clear: $K = K_{h,t} + K_{h}*t + K_{CBh}$

The central banks’ balance sheet are:

Home central bank: $d_{CBh,t} + q_tK_{CBh,t} = 0$

Foreign central bank: $d_{CBf}*t + RER_t d_{CBf}*t = 0$

A quantitative tightening, or large-scale asset sale, in this model will be an increase in $d_{CBh,t}$ and the equivalent sales of $q_tK_{CBh,t}$. Normally, one specifies quantitative intervention by the central bank as altering the amount of reserves held in the system, but our set-up does not specifically
include central bank reserves as an asset. However, we subscribe here to the argument of Rogoff (2017) that for the consolidated government budget, reserves held at the central bank are essentially identical to short-term bonds issued by the government. When reserves pay interest, they are a debt obligation of the central bank, which then reduces the amount of “profit” from its portfolio that the central bank remits to the general government revenue each year. In other words, the implications for the overall budget are identical to those from issuing short-term bonds. The fact that in the U.S. there have been only very slight differences between interest paid on reserves and interest on the shortest-term Treasury bonds suggests that these two assets are considered to be close substitutes. In our framework, when \( d_{t}^{CB} \) increases, it is equivalent to the central bank reducing reserves held by the banking system. The central bank contracts its balance sheet by selling off some of its less liquid assets, which in our model are represented by the government holdings of equities.

Since the convenience of the Home bond is higher than capital, a QT operation is lowering the aggregate liquidity of assets in the private sector. On the other hand, a foreign exchange accumulation is an increase in \( d_{f,t}^{*} \) and equivalent sales of \( RER_{t}d_{t}^{CB} \). Figure 11 reports the IRFs of a 1\% QT shock. As mentioned, the QT shock reduces aggregate liquidity. This results in a rise in the excess demand for the most pledgeable asset, therefore a rise of home convenience yield and an appreciation of the U.S. dollar. The drain of liquidity is also associated with a drop in world output due to a fall in capital price that lowers the banking net worth.

The interesting insight of this exercise is that it demonstrates that the central bank effectively has two separate instruments at its disposal, which do not have identical influences on the domestic or global economy. For example, in 2021-2022, the U.S. Federal Reserve responded to rising inflation by raising its policy rate, as our model for the monetary policy rule captures. At the same time, it also embarked on quantitative tightening. One effect of combining these two policies is a strong appreciation of the dollar because both the higher interest rate and the reduction in liquidity work to raise the value of the dollar. In fact, the dollar appreciated so strongly during this period, that the exchange rate became a major concern of the rest of the world, and then unavoidably for U.S. policymakers. Our analysis suggests that a different menu of interest-rates and balance sheet polices may have improved outcomes.

Figure 12 reports the IRFs of a 1\% FXI shock where the Foreign central bank buys more bonds (accumulation of reserves). Similar to a QT shock, an FXI accumulation also reduces aggregate liquidity. Therefore there are a rise of home convenience yield and an appreciation of the U.S. dollar. The drain of liquidity is also associated with a drop in world output because the banking sector are more constrained than before. There are some similarities of this analysis and that of Fanelli and Straub (2021). In both models, the central bank gains an additional instrument that allows it

---

23Note also that in the U.S. QT is normally characterized by the central bank selling some of its holdings of Treasury bonds and reducing its reserve liabilities to private banks, while here we assume the central bank is selling equity and buying government bonds. But our interpretation is consistent with actual QT in the sense that the central bank is swapping a less liquid asset (equity) for the more liquid asset (government bonds). See Dedola et al. (2021) for an empirical study of quantitative easing and its effects on exchange rates and international spillovers.

24The potential for quantitative easing policies to operate separately from interest rate policy in the presence of financial frictions has been widely recognized, e.g. Gertler and Karadi, 2010, Bernanke, 2020.
to conduct sterilized intervention because of balance sheet constraints facing the banking sector. However, in our set-up, the constraints take a different aspect because of the special role of the dollar, and our analysis is embedded in a two-country set-up that then permits us to see explicitly the spillover effects of sterilized intervention. Our analysis also bears some resemblance to that of Bianchi et al. (2021), who emphasize the liquidity demand for dollars from the financial intermediation sector. As we have mentioned above, that study lays down different microfoundations for liquidity demand and the general equilibrium model is very simplified, not allowing for in-depth analysis of the potential real effects of conventional and unconventional monetary policy.

7 Model simulation

In this section, we use the model simulated moments to compare with data moments to validate the model relevance.

In the first exercise, we conduct the regression specification as in equation (1) from Engel and Wu (2018). The quarterly model is simulated for 15,000 quarters and the first 100-period is eliminated. We also estimate the same regression in panel setting in quarterly frequency using data of the US relative to all other G10 currencies. The empirical data series are 3-month interest rate and convenience yield measure from Du et al. (2018b) dataset and starts from Jan 1999 to Jan 2018.

Table 4 reports the coefficient estimates. In column (1), the empirical estimate for the change of convenience and change of interest rate differential are -1.65 and -2.61. This indicates a 1% increase in convenience yield or interest rate differential (annualized) are associated 1.65% or 2.61% increase in exchange rate compared to the previous quarter. In column (2), the model implied coefficients for the change of convenience and change of interest rate differential are -1.62 and -1.57. these coefficients are reasonably close to and within one standard deviation of the empirical counterpart. These untargeted moments provide support to our model and quantitative calibration.

<table>
<thead>
<tr>
<th></th>
<th>Panel regression G10 currenices</th>
<th>Model implied regression</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>$\Delta \eta_{j,t}$</td>
<td>-1.652** (0.760)</td>
<td>-1.62</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta (i - i^*)_{j,t}$</td>
<td>-2.605*** (0.972)</td>
<td>-1.57</td>
</tr>
<tr>
<td>$\eta_{j,t-1}$</td>
<td>-2.076** (0.869)</td>
<td>-0.04</td>
</tr>
<tr>
<td>$(i - i^*)_{j,t-1}$</td>
<td>-0.440** (0.216)</td>
<td>-0.03</td>
</tr>
<tr>
<td>$s_{t-1}$</td>
<td>-0.057** (0.024)</td>
<td>-0.02</td>
</tr>
<tr>
<td>Within $R^2$</td>
<td>0.091</td>
<td>14.900</td>
</tr>
<tr>
<td>Observations</td>
<td>739</td>
<td>14,900</td>
</tr>
</tbody>
</table>

Table 4: Model implied regression and the empirical counterpart

Standard errors in parentheses are clustered by time. * p<0.1, ** p<0.05, *** p<0.01. Model implied regression is performed with 15,000-quarter observations and burning the first 100 quarters.
In the second exercise, we report long-run moments of the model in Table 5. We compute the data long run moments using the US and the Eurozone data from 1999Q1 to 2023Q1. As discussed above, we use the persistence of financial shock to target a close to zero $\text{corr}(\Delta nx, \Delta q)$. The rest of the moments in the Table is untargeted. The model can successfully generate an empirically close $\sigma(\Delta nx)/\sigma(\Delta q)$ so it performs reasonably well on the relationship of trade balance and real exchange rate. The model can also produce a fairly persistent real exchange rate, equal volatility of real and nominal exchange rate and a weakly positive Backus-Smith correlation of consumption growth and real exchange rate growth. The model replicate a fairly volatile exchange rate relative to consumption growth but underperform in matching the volatility relative to GDP growth. The model gives a negative Fama coefficient and volatile exchange rate relative to interest rate ($\sigma(i - i^*)/\sigma(\Delta s)$). Finally, the model can replicate reasonably well business cycle moments within the country and across the country.

Table 5: Long-run moments

<table>
<thead>
<tr>
<th></th>
<th>Data moment of Eurozone vs US</th>
<th>Model implied moment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trade balance</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\text{corr}(\Delta nx, \Delta q)$</td>
<td>-0.06</td>
<td>0.07</td>
</tr>
<tr>
<td>$\sigma(\Delta nx)/\sigma(\Delta q)$</td>
<td>0.07</td>
<td>0.14</td>
</tr>
<tr>
<td>Exchange rates</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho(\Delta s)$ (NER)</td>
<td>-0.03</td>
<td>-0.13</td>
</tr>
<tr>
<td>$\rho(q)$ (RER)</td>
<td>0.93</td>
<td>0.89</td>
</tr>
<tr>
<td>$\sigma(\Delta q)/\sigma(\Delta s)$</td>
<td>0.99</td>
<td>0.90</td>
</tr>
<tr>
<td>$\rho(\Delta q, \Delta c - \Delta c^*)$</td>
<td>0.05</td>
<td>0.3</td>
</tr>
<tr>
<td>$\sigma(\Delta s)/\sigma(\Delta c)$</td>
<td>3.26</td>
<td>2.22</td>
</tr>
<tr>
<td>$\sigma(\Delta s)/\sigma(\Delta GDP)$</td>
<td>3.57</td>
<td>0.85</td>
</tr>
<tr>
<td>Fama $\beta$</td>
<td>-0.18</td>
<td>-1</td>
</tr>
<tr>
<td>$\sigma(i - i^*)/\sigma(\Delta s)$</td>
<td>0.07</td>
<td>0.21</td>
</tr>
<tr>
<td>$\rho(i - i^*)$</td>
<td>0.95</td>
<td>0.97</td>
</tr>
<tr>
<td>$\rho(i)$</td>
<td>0.97</td>
<td>0.99</td>
</tr>
<tr>
<td>Business cycle</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma(\Delta c)/\sigma(\Delta GDP)$</td>
<td>1.08</td>
<td>0.38</td>
</tr>
<tr>
<td>$\rho(\Delta c, \Delta GDP)$</td>
<td>0.94</td>
<td>0.71</td>
</tr>
<tr>
<td>$\rho(\Delta I, \Delta GDP)$</td>
<td>0.81</td>
<td>0.97</td>
</tr>
<tr>
<td>$\rho(\Delta GDP, \Delta GDP^*)$</td>
<td>0.88</td>
<td>0.87</td>
</tr>
<tr>
<td>$\rho(\Delta c, \Delta c^*)$</td>
<td>0.90</td>
<td>0.22</td>
</tr>
<tr>
<td>$\rho(\Delta I, \Delta I^*)$</td>
<td>0.55</td>
<td>0.80</td>
</tr>
</tbody>
</table>

8 Conclusions

The special features of the US dollar, and US financial assets in the world financial system has generated enormous academic interest over the last decade. The literature has established that the US benefits from an "exorbitant privilege", with US dollar denominated liabilities offering low returns to foreign investors in normal times, but on the flip side there is an “exorbitant duty” associated with a large US dollar appreciation during global crises, particularly global financial crises. We
take a very standard New Keynesian open economy model with balance sheet constrained banks and make a minimal additional assumption by letting US government assets have a higher collateral value than those of the rest of the world. We believe this is a highly realistic assumption quite accurately characterizing the special liquidity features of US treasuries in the global financial system. In a steady state, this model captures the “exorbitant privilege” in the sense that returns on US treasuries are below those of foreign governments, the US is a net debtor, but has a negative trade balance due to a excess income flows on its foreign assets relative to liabilities. In response to a global financial shock coming from a sudden tightening of balance sheet constraints for all banks, we show that the model accurately captures the empirical observations discussed in the introduction. Notably, the US dollar appreciates strongly, and the appreciation is associated with a spike in the convenience yield on US treasuries relative to foreign government assets. Moreover, this appreciation is achieved even though the US trade balance improves during a crisis, and US households have a diminished share of world wealth. The appreciation of the US dollar and the improvement in the trade balance gives the “exorbitant duty”. In addition, although the US government bond represents better collateral, and a global financial crisis leads to an increase in demand for the ‘safe asset’, in equilibrium we see an external retrenchment in capital flows, as in the data. We show that these results are robust to alternative calibrations of the model, and most results carry over to a case where a global downturn is precipitated by a uniform negative shock to all country’s productivity.
Figure 2: Baseline with 1% global financial shock ($\vartheta$, $\vartheta^*$)
Figure 3: 1% global financial shock ($\vartheta$, $\vartheta^*$) with no capital trade
Figure 4: 1% global financial shock ($\vartheta$, $\vartheta^*$) with $\kappa_{h1} = \kappa_{f1}^{*} = 0.05 < \kappa_{f1} = \kappa_{f1}^{*} = 0.401$
Figure 5: 1% TFP shock ($A, A^*$) with baseline calibration
Figure 6: 25 basis point Home monetary shock with baseline calibration
Figure 7: 25 basis point both Home and Foreign monetary shock with baseline calibration
Figure 8: 1% global financial shock ($\vartheta$, $\vartheta^*$) with baseline calibration in PCP case
Figure 9: 1% global financial shock ($\vartheta$, $\vartheta^*$) with baseline calibration in flexible price case
Figure 10: Portfolio dynamics under baseline case with 1% global financial shock ($\vartheta$, $\vartheta^*$)
Figure 11: 1% QT shock with purchase of Home bonds and sales of Home capital by Home central bank
Figure 12: 1% FXI shock with purchase of Home bonds and sales of foreign bonds by Foreign central bank


Miyamoto, W., Nguyen, T. L., and Oh, H. (2022). In search of dominant drivers of the real exchange rate. *Available at SSRN 4251909*.


Appendix

A Collecting all the equations and rewriting the system in real terms

Define real debt holdings: $d_{h,t} = \frac{D_{h,t}}{P_t}$, $d_{f,t} = \frac{D_{f,t}}{P_t}$.

Terms of Trade: $\mathcal{J}_t = \frac{P_{f,t}}{P_{h,t}}$, $\mathcal{J}_t^* = \frac{P_{f,t}^*}{P_{h,t}}$.

Deviations from LOOP: $\mathcal{D}_t = \frac{S_{h,t}^P}{P_{h,t}}$, $\mathcal{D}_t^* = \frac{S_{f,t}^P}{P_{f,t}}$.

Real Returns: $r_{h,t} = \frac{R_{h,t} P_{h,t}}{P_t}$, $r_{f,t} = \frac{R_{f,t} P_{f,t}}{P_t}$.

Real marginal product of capital: $r_{K,t} = \frac{R_{K,t} P_{K,t}}{P_t}$.

Real net worth: $n_t = \frac{N_t^C}{P_t}$, $n_t^* = \frac{N_t^{C^*}}{P_t}$.

Real equity prices: $q_t = \frac{Q_t}{P_t}$, $q_t^* = \frac{Q_t^{C^*}}{P_t}$.

Real marginal costs: $mc_t = \frac{MC_t}{P_t}$, $mc_t^{C^*} = \frac{MC_t^{C^*}}{P_t}$.

Price indices: $P(1.\mathcal{J}_t) = \frac{P_t}{P_{h,t}}$, $P^*(1.\mathcal{J}_t^*) = \frac{P^*}{P_{h,t}}$.

PPI inflation rates: $\pi_{h,t} = \frac{P_{h,t}}{P_{h,t-1}}$, $\pi_{h,t}^* = \frac{P_{h,t}^*}{P_{h,t-1}}$, $\pi_{f,t} = \frac{P_{f,t}}{P_{f,t-1}}$, $\pi_{f,t}^* = \frac{P_{f,t}^*}{P_{f,t-1}}$.

Also impose bond market clearing: $d_{h,t}^* = d_{h,t} - d_{h,t}$, $d_{f,t} = d_{f,t} - d_{f,t}^*$.

labor market clearing: $L_d = H_d$, $L_d^* = H_d^*$.

Real wage: $w_t = \frac{W_t}{P_t}$, $w_t^* = \frac{W_t^{C^*}}{P_t}$.

Real deposit (policy) rate: $r_{t+1} = \frac{R_{t+1} P_t}{P_{t+1}}$, $r_{t+1}^* = \frac{R_{t+1}^{C^*} P_t}{P_{t+1}}$.

Note we don’t explicitly define the real exchange rate, but it is implied by $\frac{S_{h,t}^P}{P_t} = \frac{\mathcal{D}_t P(1.\mathcal{J}_t)}{P(1.\mathcal{J}_t)}$. That is, the real exchange rate depends on both deviations from LOOP as well as movements in the terms of trade. Also, set $\bar{\pi} = 0$.

Balance of payments:

\[
(C_t + I_t + K_t \phi \left( \frac{K_t}{Y_t} \right)) + d_{h,t} - \bar{d}_{h,t} + \frac{\mathcal{D}_t P(1.\mathcal{J}_t)}{P(1.\mathcal{J}_t)} d_{f,t} = \frac{Y_{h,t}}{P(1.\mathcal{J}_t)} (1 - \xi (\pi_{h,t})) + \frac{\mathcal{D}_t}{P(1.\mathcal{J}_t)} Y_{h,t}^* (1 - \xi (\pi_{h,t}^*)) + r_{h,t} (d_{h,t-1} - \bar{d}_{h,t-1}) + \frac{\mathcal{D}_t}{P(1.\mathcal{J}_w)} r_{f,t} d_{f,t-1} \tag{47}
\]

Home Euler equation:

\[
1 = E_t \beta r_{t+1} \frac{C_t^{R-\sigma}}{C_t^{K-\sigma}} \tag{48}
\]

Foreign Euler equation:

\[
1 = E_t \beta r_{t+1}^* \frac{C_t^{R_s-\sigma}}{C_t^{K_s-\sigma}} \tag{49}
\]

\[
C_t^K = w_t H_t^K \tag{50}
\]
\[ C^K_t = w_t^* H_t^K \]

In aggregate, we have
\[ C_t = (1 - m) \times C^R_t + (m) \times C^K_t \]

\[ H_t = (1 - m) \times H^R_t + (m) \times H^K_t \]

Profit max home:
\[ (1 + s_t)Y_{h,t} - \varepsilon((1 + s_t) - mc_t)Y_{h,t} - \xi'(\pi_{h,t})\pi_{h,t} Y_{h,t} + E_t \beta \frac{C_t^{\gamma}}{C_t^{\gamma-\sigma}} \frac{p(1,\mathcal{J}_t)}{p(1,\mathcal{J}_{t+1})} \xi'(\pi_{h,t+1})\pi_{h,t+1} Y_{h,t+1} = 0 \]

Factor markets home:
\[ A_t(1 - \alpha)\zeta(L_t^{1-\alpha} K_t^{\alpha})mc_t = w_t P(1,\mathcal{J}_t) L_t \]

\[ A_t \alpha \zeta(L_t^{1-\alpha} K_t^{\alpha})mc_t = r_{K,t} P(1,\mathcal{J}_t) K_t \]

\[ w_t = \chi C_t^{R\sigma} L_t^{R\psi} \]

\[ w_t = \chi L_t^{K\psi} \]

Capital and Price of capital at Home:
\[ K_{t+1} = I_t + (1 - \delta)K_t \]

\[ q_t = (1 + \psi'\left(\frac{I_t}{K_t}\right)) \]

\[ Y_{h,t} + Y^*_h = A_t(L_t^\alpha K_t^{1-\alpha}) \]

Profit Max Foreign:
\[ (1 + s^*_t)Y^*_{f,t} - \varepsilon((1 + s_t) - mc_t^*)Y^*_{f,t} - \xi'(\pi^*_{f,t})\pi^*_{f,t} Y^*_{f,t} + E_t \beta \frac{C_t^{\gamma}}{C_t^{\gamma-\sigma}} \frac{p(1,\mathcal{J}_t)}{p(1,\mathcal{J}_{t+1})} \xi'(\pi^*_{f,t+1})\pi^*_{f,t+1} Y^*_{f,t+1} = 0 \]
\((1 + s_t^*) Y_{f,t} - \varepsilon ((1 + s_t^*) - \mathcal{D}_t^* mc_t^*) Y_{f,t} = 0\) (64)

Foreign factor markets:

\[ A_t^* (1 - \alpha) \xi (L_t^*(1-\alpha) K_t^*) mc_t^* = \frac{P^*(1, \mathcal{I}_t^*)}{\mathcal{I}_t^*} w_t^* L_t^* \] (65)

\[ A_t^* \alpha \xi (L_t^*(1-\alpha) K_t^*) mc_t^* = K_t^* \frac{P^*(1, \mathcal{I}_t^*)}{\mathcal{I}_t^*} r_{K,t}^* \] (66)

\[ w_t^* = \chi C_t^{R^*} \sigma L_t^{R*} \] (67)

\[ w_t^* = \chi L_t^{K*} \] (68)

Capital and Price of capital Foreign:

\[ K_{t+1}^* = I_t^* + (1 - \delta) K_t^* \] (69)

\[ q_t^* = (1 + \psi' \left( \frac{I_t}{K_t^*} \right)) \] (70)

\[ Y_t^* + Y_{f,t} = A_t^* (I_t^* \alpha K_t^*(1-\alpha)) \] (71)

Market Clearing:

Bond market clearing:

\[ d_{h,t}^* = \tilde{d}_{h,t} - d_{h,t}, \quad d_{f,t} = \tilde{d}_{f,t} - d_{f,t}^* \]

Home good:

\[ Y_{h,t}^* (1 - \xi (\pi_{h,t})) = \omega \left( \frac{1}{P(1, \mathcal{I}_t^*)} \right)^{-\lambda} \left( C_t + I_t + K_t^* \phi \left( \frac{I_t}{K_t^*} \right) \right) \] (72)

\[ Y_{h,t}^* (1 - \xi (\pi_{h,t}^*)) = \frac{(1-n)}{n} (1 - \omega^*) \left( \frac{1}{P^*(1, \mathcal{I}_t^*)} \right)^{-\lambda} \left( C_t^* + I_t^* + K_t^* \phi \left( \frac{I_t^*}{K_t^*} \right) \right) \] (73)

Foreign good:

\[ Y_{f,t}^* (1 - \xi (\pi_{f,t})) = \frac{n}{1-n} (1 - \omega) \left( \frac{\mathcal{I}_t}{P(1, \mathcal{I}_t^*)} \right)^{-\lambda} \left( C_t + I_t + K_t^* \phi \left( \frac{I_t}{K_t^*} \right) \right) \] (74)

\[ Y_{f,t}^* (1 - \xi (\pi_{f,t}^*)) = \omega^* \left( \frac{\mathcal{I}_t^*}{P^*(1, \mathcal{I}_t^*)} \right)^{-\lambda} \left( C_t^* + I_t^* + K_t^* \phi \left( \frac{I_t^*}{K_t^*} \right) \right) \] (75)

Home Bank:

\[ E_t \tilde{\Lambda}_{t+1} (\tilde{r}_{k,t+1} - r_{t+1}) = \eta_t \partial_t (\kappa_{Kt1,t} + \kappa_{Kt2,t} K_{h,t}) \] (76)
\begin{equation}
E_t \tilde{\Lambda}_{t+1} \left( \frac{RER_t+1}{RER_t} \tilde{r}_{k,t+1} - r_{t+1} \right) = \eta_t \partial_t (\kappa_{Kf1,t} + \kappa_{Kf2,t} RER_t K_{f,t}) \tag{77}
\end{equation}

\begin{equation}
E_t \tilde{\Lambda}_{t+1} \left( r_{h,t+1} - r_{t+1} \right) = \eta_t \partial_t (\kappa_{h1,t} + \kappa_{h2,t} d_{h,t}) \tag{78}
\end{equation}

\begin{equation}
E_t \tilde{\Lambda}_{t+1} \left( \frac{RER_t+1}{RER_t} r_{f,t+1} - r_{t+1} \right) = \eta_t \partial_t (\kappa_{f1,t} + \kappa_{f2,t} RER_t d_{f,t}) \tag{79}
\end{equation}

\[ \tilde{\Lambda}_{t+1} = \Omega_{t+1} \frac{\pi_{h,t+1} P(1, q_{t+1})}{P(1, \pi_{t})} ((1 - \theta) + \theta v_{t+1}) \]

\[ \tilde{r}_{K,t+1} = \frac{r_{K,t+1} + (1 - \delta) q_{t+1}}{q_t} \]

Home envelope condition:

\[ v_t = \frac{E_t \Omega_{t+1} \frac{\pi_{h,t+1} P(1, q_{t+1})}{P(1, \pi_{t})} ((1 - \theta) + \theta v_{t+1}) r_{t+1}}{1 - \eta_t} \tag{80} \]

Home participation constraint:

\[ v_t n_t = \partial_t (\kappa_{Kh1,t} + \kappa_{Kh2,t} q_t K_{h,t+1}) q_t K_{h,t+1} + (\kappa_{Kf1,t} + \kappa_{Kf2,t} RER_t q_t^* K_{h,t+1}^*) RER_t q_t^* K_{h,t+1}^* + (\kappa_{h1,t} + \kappa_{h2,t} d_{h,t}) d_{h,t} + (\kappa_{f1,t} + \kappa_{f2,t} RER_t d_{f,t}) RER_t d_{f,t} \tag{81} \]

Home net worth dynamics:

\[ n_{t+1} = \theta ((\tilde{r}_{k,t+1} - r_{t+1}) q_t K_{t+1} + (r_{h,t+1} - r_{t+1}) d_{h,t} + (r_{f,t+1} - r_{t+1}) \frac{P(1, q_{t+1})}{P(1, \pi_{t})} d_{f,t} + r_{t+1} n_t) \]

\[ + \varphi \frac{\pi_{h,t+1} P(1, q_{t+1})}{P(1, \pi_{t})} (q_t K_{t+1} + d_{h,t} + \frac{P(1, q_{t+1})}{P(1, \pi_{t})} d_{f,t}) \tag{82} \]

Foreign Bank:

\[ E_t \tilde{\Lambda}_{t+1}^* \left( \frac{RER_t}{RER_{t+1}} \tilde{r}_{k,t+1}^* - r_{t+1}^* \right) = \eta_t^* \partial_t^* (\kappa_{Kh1,t}^* + \kappa_{Kh2,t}^* K_{h,t}^*/RER_t) \tag{83} \]

\[ E_t \tilde{\Lambda}_{t+1}^* \left( \tilde{r}_{k,t+1}^* - r_{t+1}^* \right) = \eta_t^* \partial_t^* (\kappa_{Kf1,t}^* + \kappa_{Kf2,t}^* K_{f,t}^*) \tag{84} \]

\[ E_t \tilde{\Lambda}_{t+1}^* \left( \frac{RER_t}{RER_{t+1}} r_{h,t+1}^* - r_{t+1}^* \right) = \eta_t^* \partial_t^* (\kappa_{h1,t}^* + \kappa_{h2,t}^* d_{h,t}^*/RER_t) \tag{85} \]

\[ E_t \tilde{\Lambda}_{t+1}^* \left( r_{f,t+1}^* - r_{t+1}^* \right) = \eta_t^* \partial_t^* (\kappa_{f1,t}^* + \kappa_{f2,t}^* d_{f,t}^*) \tag{86} \]

\[ \tilde{\Lambda}_{t+1}^* = \Omega_{t+1}^* \pi_{h,t+1}^* \frac{P(1, q_{t+1})}{P(1, \pi_{t})} ((1 - \theta) + \theta v_{t+1}^*) \]
Foreign participation constraint:

\[
\bar{r}_{K,t+1}^* = \frac{r_{K,t+1}^* + (1 - \delta)q_{t+1}^*}{q_t^*}
\]

Foreign envelope condition:

\[
\nu_t^* = \frac{E_t\Omega_{t+1}^*\pi_{h,t+1}^* P(1,\mathcal{L}_{t+1}^*)^*}{P(1,\mathcal{L}_{t}^*)^*} \left( (1 - \theta) + \theta \nu_{t+1}^* \right) r_{t+1}^* \]

(87)

Foreign participation constraint:

\[
u_t^* n_t^* = \nu_t^* \left( (\kappa_{f1,t}^* + \kappa_{f2,t}^* q_t^* K_{f,t+1}^*) q_t^* K_{f,t+1}^* + (\kappa_{h1,t}^* + \kappa_{h2,t}^* \frac{1}{RER_t} q_t K_{f,t+1}^*) \frac{1}{RER_t} q_t K_{f,t+1}^* 
+ (\kappa_{h1,t}^* + \kappa_{h2,t}^* \frac{1}{RER_t} d_{h,t}^*) \frac{1}{RER_t} d_{h,t}^* + (\kappa_{f1,t}^* + \kappa_{f2,t}^* d_{f,t}^*) d_{f,t}^* \right)
\]

(88)

Foreign net worth dynamics:

\[
n_{t+1}^* = \theta \left( (\bar{r}_{t+1}^* - r_{t+1}^*) q_t^* K_{t+1}^* + (\frac{\mathcal{D}_t}{\mathcal{D}_{t+1}^*} P(1,\mathcal{L}_{t+1}^*) \frac{P(1,\mathcal{L}_{t}^*)^*}{P(1,\mathcal{L}_{t}^*)^*} r_{t+1}^* - r_{t+1}^* \right) \frac{P(1,\mathcal{L}_{t}^*)^*}{\mathcal{D}_t^* \mathcal{P}(1,\mathcal{L}_{t}^*)^*} d_{h,t}^*
+ (r_{f,t+1}^* - r_{t+1}^* d_{f,t}^* + r_{t+1}^* n_t^*)
+ \phi \left( \frac{\pi_{h,t+1}^* P(1,\mathcal{L}_{t+1}^*)^*}{\mathcal{D}_t^* P(1,\mathcal{L}_{t}^*)^*} q_t^* K_{t+1}^* + \frac{P(1,\mathcal{L}_{t}^*)^*}{\mathcal{D}_t^* P(1,\mathcal{L}_{t}^*)^*} d_{h,t}^* + d_{f,t}^* \right)
\]

(89)

Home monetary Rule:

\[
r_{t+1}^* \frac{P(1,\mathcal{L}_{t}^*)^*}{P(1,\mathcal{L}_{t-1}^*)^*} = \frac{1}{\beta} \left( \frac{P(1,\mathcal{L}_{t}^*)^*}{P(1,\mathcal{L}_{t-1}^*)^*} \right)^{\eta}
\]

(90)

Foreign monetary Rule:

\[
r_{t+1}^* \frac{\mathcal{I}_{t-1}^* P^*(1,\mathcal{L}_{t-1}^*)^* \pi_{f,t}^*}{\mathcal{I}_t^* P^*(1,\mathcal{L}_{t-1}^*)^*} = \frac{1}{\beta} \left( \frac{\mathcal{I}_{t-1}^* P^*(1,\mathcal{L}_{t-1}^*)^* \pi_{f,t}^*}{\mathcal{I}_t^* P^*(1,\mathcal{L}_{t-1}^*)^*} \right)^{\eta}
\]

(91)

Definitions:

\[
\pi_{h,t} = \pi_{f,t} \frac{\mathcal{I}_{t-1}^*}{\mathcal{I}_t^*}
\]

(92)

\[
\pi_{f,t}^* = \pi_{h,t} \frac{\mathcal{I}_t^*}{\mathcal{I}_{t-1}^*}
\]

(93)

\[
\mathcal{D}_t = \mathcal{D}_t^* \frac{\mathcal{I}_t^*}{\mathcal{I}_{t-1}^*}
\]

(94)

Equations (47) - (94) give 42 equations in

\[
C_t, C_t^*, L_t, L_t^*, I_t, I_t^*, K_t, K_t^*, X_t, X_t^*,
\]

58
\[ \pi_{h,t}, \pi_{h,t}^*, \pi_{f,t}, \pi_{f,t}^*, w_t, w_t^*, q_t, q_t^*, mc_t, mc_t^* \]

\[ r_{K,t}, r_{K,t}^*, r_t, r_t^*, r_{h,t}, r_{f,t}, Y_{h,t}, Y_{h,t}^* \]

\[ Y_{f,t}, Y_{f,t}^*, \nu_t, \nu_t^*, \eta_t, \eta_t^* \]

\[ d_{h,t}, d_{f,t}, \mathcal{I}_t, \mathcal{I}_t^*, \mathcal{D}_t, \mathcal{D}_t^*, n_t, n_t^* \]

Endogenous state variables:
\[ n_t, n_t^*, K_t, K_t^*, d_{h,t}, d_{f,t}, \mathcal{I}_{t-1}, \mathcal{I}_{t-1}^* \]

### B Steady state equations

Define real debt holdings: \(d_h, d_f\),
- Terms of Trade: \(\mathcal{I} = \mathcal{I}^*\),
- Deviations from LOOP: \(\mathcal{D} = 1, \mathcal{D}^* = 1\)
- Real Returns: \(r_h, r_f\)
- Real equity prices: \(q = 1, q^* = 1\)
- Real marginal product of capital: \(r_K = R_K, r_K^* = R_K^*\)
- Real net worth \(n, n^*\)
- Real marginal costs: \(mc = mc^* = 1\) (Setting \(A = A^* = 1\))
- Price indices: \(P(1, \mathcal{I}), P^*(1, \mathcal{I})\)
- PPI inflation rates: \(\pi_h = 1, \pi_h^* = 1, \pi_f = 1, \pi_f^* = 1\)
- Also impose bond market clearing: \(d_h^* = \bar{d}_h - d_h, d_f^* = \bar{d}_f - d_f^*\)
- Labor market clearing: \(L = H, L^* = H^*\)
- Real wage \(w, w^*\)
- Real deposit (policy) rate \(r, r^*\)
- Subsidy rate \(s = \frac{1}{\epsilon - 1}\)

Define the steady state real exchange rate as \(\mathcal{Q} = \frac{P^*(1, \mathcal{I})}{P(1, \mathcal{I})}\). So it depends only on the terms of trade and home bias in consumption aggregators.

Balance of payments:
\[ C + X + \delta K + d_h - \bar{d}_h + \mathcal{Q} d_f = \frac{Y_h + Y_h^*}{P(1, \mathcal{I})} + r_h (d_h - \bar{d}_h) + \mathcal{Q} r_f d_f \]

Home Euler equation:
\[ 1 = \beta r \]
Foreign Euler equation:

\[ 1 = \beta r \] (97)

Profit max home:

\[ 1 = mc \] (98)

\[ 1 = mc \] (99)

Factor markets home:

\[ (1 - \alpha) \zeta (L^{1-\alpha}K^\alpha)^\zeta X^{1-\zeta} = \chi C^\sigma L^{1+\psi} P(1, \mathcal{J}) \] (100)

\[ \alpha \zeta (L^{1-\alpha}K^\alpha)^\zeta X^{1-\zeta} = r_K P(1, \mathcal{J}) K \] (101)

\[ (1 - \zeta) (L^{1-\alpha}K^\alpha)^\zeta X^{1-\zeta} = P(1, \mathcal{J}) X \] (102)

Capital and Price of capital at Home:

\[ K \delta = I \] (103)

\[ q = 1 \] (104)

\[ Y_h + Y_h^* = (L^\alpha K^{1-\alpha})^\zeta X^{1-\zeta} \] (105)

Profit Max Foreign:

\[ 1 = mc^* \] (106)

\[ 1 = mc^* \] (107)

Foreign factor markets:

\[ (1 - \alpha) \zeta (L^*(1-\alpha)K^*\alpha)^\zeta X^*(1-\zeta) = \frac{P^*(1, \mathcal{J})}{\mathcal{J}} \chi C^*\sigma L^{1+\psi} \] (108)

\[ \alpha \zeta (L^*(1-\alpha)K^*\alpha)^\zeta X^*(1-\zeta) = \frac{P^*(1, \mathcal{J})}{\mathcal{J}} r_K^* \] (109)

\[ (1 - \zeta) (L^*(1-\alpha)K^*\alpha)^\zeta X^*(1-\zeta) = \frac{P^*(1, \mathcal{J})}{\mathcal{J}} X^* \] (110)

Capital and Price of capital Foreign:

\[ K \delta^* = I^* \] (111)

\[ q^* = 1 \] (112)
\[ Y_f^* + Y_f = (L^* \alpha K^* (1-\alpha)) \xi X^{*(1-\xi)} \]  

(113)

Market Clearing:

Home good:

\[ Y_h + Y_f = \omega \left( \frac{1}{P(1,\mathcal{F})} \right)^{-\lambda} (C + X + \delta K) + \frac{(1-n)}{n} (1-\omega^*) \left( \frac{1}{P^*(1,\mathcal{F})} \right)^{-\lambda} (C^* + X^* + \delta K^*) \]  

(114)

Foreign good:

\[ Y_f + Y_f^* = \frac{n}{1-n} (1-\omega) \left( \frac{\mathcal{F}}{P(1,\mathcal{F})} \right)^{-\lambda} (C + X + \delta K) + \omega^* \left( \frac{\mathcal{F}}{P^*(1,\mathcal{F})} \right)^{-\lambda} (C^* + X^* + \delta K) \]  

(115)

Home Bank:

\[ \tilde{\Lambda} (\tilde{r}_k - r) = \eta \vartheta (\kappa_{K,h1} + \kappa_{K,h2} K_h) \]  

(116)

\[ \tilde{\Lambda} (\tilde{r}_k^* - r) = \eta \vartheta (\kappa_{K,f1} + \kappa_{K,f2} K_f) \]  

(117)

\[ \tilde{\Lambda} (r_h - r) = \eta \vartheta (\kappa_{h1} + \kappa_{h2} d_h) \]  

(118)

\[ \tilde{\Lambda} (r_f - r) = \eta \vartheta (\kappa_{f1} + \kappa_{f2} d_f) \]  

(119)

\[ \tilde{\Lambda} = \beta ((1-\theta) + \theta v) \]  

(120)

\[ \tilde{r}_K = r_K + (1-\delta) \]  

(121)

Home envelope condition:

\[ v = \frac{((1-\theta) + \theta v)}{1-\eta} \]  

(122)

Home participation constraint:

\[ vn = \vartheta (\kappa_K K + (\kappa_{h1} + \kappa_{h2} d_h) d_h + (\kappa_{f1} + \kappa_{f2} d_f) d_f) \]  

(123)

Home net worth dynamics:

\[ n = \theta ((\tilde{r}_k - r) K + (r_h - r) d_h + (r_f - r) d_f + r n) \]  

\[ + \varphi (K + d_h + d_f) \]  

(124)

Foreign Bank:

\[ \tilde{\Lambda}^* (\tilde{r}_k - r^*) = \eta^* \vartheta^*(\kappa_{K,h1}^* + \kappa_{K,h2}^* K_h^*) \]  

(125)
\[ \tilde{\Lambda}^* (\tilde{r}_k^* - r^*) = \eta^* \vartheta^* (\kappa_{Kf1}^* + \kappa_{Kf1}^* K_f^*) \]  
(126)

\[ \tilde{\Lambda}^* (r_h^* - r^*) = \eta^* \vartheta^* (\kappa_{h1}^* + \kappa_{h2}^*) d_h^* \]  
(127)

\[ \tilde{\Lambda}^* (r_f^* - r^*) = \eta^* \vartheta^* (\kappa_{f1}^* + \kappa_{f2}^*) d_f^* \]  
(128)

\[ \tilde{\Lambda}^* = \beta ((1 - \theta) + \theta \upsilon^*) \]  
(129)

\[ \tilde{r}_K^* = r_K^* + (1 - \delta) \]  
(130)

Foreign envelope condition:

\[ \upsilon^* = \frac{((1 - \theta) + \theta \upsilon^*)}{1 - \eta^*} \]  
(131)

Foreign participation constraint:

\[ \upsilon^* n^* = \vartheta \left( \kappa_{K}^* K^* + \kappa_{h} d_h^* + \kappa_{f} d_f^* \right) \]  
(132)

Foreign net worth dynamics:

\[ n^* = \theta ((\tilde{r}_K^* - r^*) K^* + (r_h^* - r^*) d_h^* + (r_f^* - r^*) d_f^*) + \varphi \left( K^* + \frac{d_h^*}{\vartheta} + d_f^* \right) \]  
(133)

Home monetary Rule:

\[ R_t = \frac{1}{\beta} \]  
(134)

Foreign monetary Rule:

\[ R_t^* = \frac{1}{\beta} \]  
(135)

Take mc, r, q, I, \tilde{\Lambda} as given (and same for foreign). Then we have (95), (100), (101), (102), (105), (108), (109), (110), (113), (114), (115), (116), (118), (119), (122), (123), (124), (126), (127), (128), (131), (132), (133),


(Note just add \( Y_h + Y_h^* = Y \) together)

\( I, d_h, d_f, n, n^*, \upsilon, \upsilon^* \eta, \eta^*, r_k, r_k^*, r_h, r_f \)