Asset Pricing and Risk Sharing Implications of Alternative Pension Plan Systems

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Abstract

We show that incorporating defined benefit pension funds in an asset pricing model with incomplete markets improves its ability to jointly match the historical equity premium and riskless rate, and has important implications for risk sharing. We emphasize the importance of the pension fund's size and asset demands in determining equilibrium asset prices and discuss a new risk channel arising from fluctuations in the fund's endowment. We use our calibrated model to study the implications of a shift from an economy with defined benefit pension schemes to one with defined contribution plans. We find that the new steady-state is characterized by a higher riskless rate and a lower equity premium. Consumption volatility increases for retirees but decreases for workers.

JEL Classification: E21, E44, G11, G12, G50.

Key Words: Equity Premium, Pension Funds, Defined Contribution and Defined Benefit Pension Plans, Intermediary-Based Asset Pricing, Incomplete Risk Sharing.

1 Introduction

The most important savings motive for most individuals is financing consumption during retirement. In most countries, the majority of those savings are made automatically through defined benefit pension plans. This accumulated wealth is managed by pension funds which are responsible for making pension payments during retirement. In general equilibrium asset pricing models, it is common practice to abstract from these pension arrangements. However, the total asset size of these defined benefit pension funds is quite significant, averaging to about 67% of U.S. GDP between 1975 and 2021.[1]

Assuming that the wealth of defined benefits pension funds is invested according to household first-order conditions abstracts from the important institutional considerations and restrictions that apply to these funds. Pension funds have historically invested in relatively conservative portfolios, namely by holding a significant fraction of the less risky assets in the economy.[2] Furthermore, the (stochastic) return on their endowment of wealth is an important source of funding of pension benefits, and this has important implications for both household-level and firm-level risk.[3] Negative shocks to pension fund wealth are reflected either in increases to the contribution rates of workers and/or in increases to the contributions made by firms.[4]

In this paper we consider an asset pricing model with an explicit defined benefit pension fund (DBPF). We first show how that this improves the fit to standard asset pricing moments. We then use the model to study the implications of shifting from an economy where retirement savings are managed by defined benefit pension schemes, to one where those schemes have been replaced by defined contribution plans, a trend that we currently observe in several countries. Models that abstract from pension arrangements are silent on the potential asset pricing and macro-economic implications of this seismic shift in the retirement savings landscape.

We first present the main results in the context of a simplified model that captures the essential

[1] This has implications for the overall development of the financial system across different economies, as emphasized by Scharfstein (2018).

[2] Greenwood and Vissing-Jorgensen (2018) and Klingler and Sundaresan (2019) document a substantial price impact of defined benefit pension fund asset size, in the treasury bond market and the interest rate swap market, respectively.

[3] Munnell and Soto (2007) document that investment returns represented about 70% of the total income of state and local DB pension funds between 1996 and 2006.

[4] In the case of pension funds for public employees, potentially also higher taxes.

ingredients for our analysis. We then proceed to discuss our baseline model, where we introduce additional features which are important for matching asset prices and/or for our analysis of a DC-only economy at the end. Our baseline economy is a production economy with incomplete markets and overlapping generations of households. During working life households contribute to a defined benefit pension plan and make social security contributions. Once retired, households collect both social security benefits and defined benefit pension payments. In addition, they can also use their accumulated private savings to finance consumption. The model includes borrowing constraints and uninsurable labor income risk, which induce precautionary savings (as in Deaton (1991), Hubbard et al. (1995), Carroll (1997), Gourinchas and Parker (2002) and Cocco et al. (2005), for example), so retirement is not the only savings motive.

Pension payments during retirement are financed by contributions from both workers and firms, and from the stochastic return on the endowment of the defined benefit pension fund. Those returns represent a source of fluctuations in the funding position of the DBPF. Since the payments to retirees are fixed, these fluctuations imply adjustments in the contributions of workers and/or firms. Through this channel, our model captures an important source of (indirect) exposure to stock returns for households, arising from the investment decisions of the pension fund. The two alternative adjustment mechanisms, contributions from workers or from firms, have different implications for consumption volatility and risk sharing. In the second case the risk is directly absorbed by firms and consequently by shareholders. On the other hand, increases in workers' contribution rates increase the volatility of consumption growth for all households.

In the U.S about half of the population does not invest in equities. The incentives to become a stock market participant are likely to change if households are enrolled in a defined contribution pension plan versus a defined benefit pension plan. Therefore, we incorporate limited stock market participation in the model and crucially treat it as an endogenous decision (as in Vissing-Jørgensen (2002), Gomes and Michaelides (2005), and Fagereng et al. (2017)), allowed to change both over time for a given pension system, and across pension systems.[5] We consider two types of households with heterogeneous preferences. One group has a strong preference for savings, while the other

[5]The asset pricing implications of limited stock market participation are investigated by Mankiw and Zeldes (1991), Basak and Cuoco (1998), Luttmer (1999), Cao et al. (2005), Gomes and Michaelides (2008), Guvenen (2009) and Favilukis (2013), among others.

saves very little but, crucially they are still optimizing agents with some wealth. This is particularly important when we consider different comparative statics and counterfactuals, as we want to allow those agents to re-optimize their behavior.

We first show that the baseline model is able to match the historical average riskless rate and closely replicate the historical equity premium: 6.61% in the model versus 7.55% in the data. Our economy also delivers a stable risk-free rate and limited stock market participation consistent with the data.

We then compare our results with those obtained in an otherwise identical economy where the pension scheme is a simple pass-through entity, collecting pension contributions from current workers and paying pension benefits to current retirees. This is the standard assumption in asset pricing models with an explicit retirement. We refer to such a model as Pure-Pay-as-you-Go model (PPG). When both economies are calibrated to deliver the same riskless rate, our baseline economy has a substantially higher Sharpe ratio (0.33 versus 0.22) and equity premium (6.61% versus 3.97%)

Two channels explain the asset pricing results. The first is the more conservative demand of the defined benefit pension fund. As a result, including a DBPF in the model decreases the riskless rate and increases the equity premium. Households are unable to undo this with their private savings, both because of the size of the pension fund's endowment, and because the pension fund is forcing several of them to save more than their private optimum, particularly early in life.

In addition there is also a risk channel at work. Intuitively, in both models agents have a stable income stream at retirement. However, when the DBPF exists, retirement income is largely funded by (risky) returns on its endowment. Therefore, this retirement income can only be riskless if workers and firms absorb the shocks to the endowment's valuation. In equilibrium, the standard deviation of consumption growth for workers and the standard deviation of profits for firms are both higher. The additional cross-sectional volatility of consumption growth increases the risk premium in the model, as in Constantinides and Duffie (1996). This second channel also helps to deliver limited stock market participation in a model with moderate participation costs, since it makes non-stockholders are already (indirectly) exposed to stock market risk.

Pension fund contributions are infrequently adjusted, only as the financial position of the

pension fund reaches a certain threshold, while in our model, they are adjusted every year. This is done mainly for tractability reasons, to avoid having the funding position of the DBPF as an additional state variable.[6] Arguably, if we considered a model with infrequent adjustments, the risk premium implications might even be larger than the ones we capture, as suggested by the work of Constantinides and Ghosh (2017) and Schmidt (2016).

Given the current funding problems of most defined benefit pension schemes, we observe an important shift towards DC pension plans in several countries. In the second part of the paper we compare the equilibrium in our baseline economy with the one obtained in an otherwise identical economy where the DBPF has been shut down, and households are now saving for retirement fully in their own private pension accounts ("DC-only economy").[7]

In the "DC-only economy" total *private* wealth accumulation is naturally higher, since households must save more for retirement. But it is important to realize that they don't have an incentive to fully replace the wealth accumulation of the DBPF. The Pension Fund was forcing households with a low discount factor to save more for retirement than their optimal savings decision would imply. Furthermore, there is a reduction in precautionary savings, since the volatility of disposable income is now lower, as households are no longer exposed to fluctuations in DB contribution rates. Therefore, total capital accumulation in the new economy is actually lower.

In the absence of the DBPF, the demand for bonds is reduced, implying a higher equilibrium risk-free rate, while the lower standard deviation of consumption growth leads to a smaller equity premium and Sharpe ratio. The lower equity premium reduces stock market participation among households with a high savings motive, but the percentage of stockholders among those with a low savings motive increases, since they must now save more for retirement. On net, stock market participation remains essentially unchanged.

It is sometimes argued that DC pension schemes have one important drawback. By not providing a guaranteed income stream during retirement, they significantly increase the level of

[6] In addition, it avoids having to set arbitrary rules for the threshold levels of the fund's position that would trigger an adjustment in the contributions.

[7] Different dimensions of the costs and benefits associated with this change have been studied by İmrohoroğlu et al. (1998), Conesa and Krueger (1999) and Nishiyama and Smetters (2007), for example, but they have not considered the joint macroeconomic and asset pricing implications that we are studying here. This part of our paper is also related to the analysis in Abel (2001), who studies the implications of shifting social security investments to the stock market.

risk faced by retirees. We find that this is indeed the case in our model, as the volatility of consumption growth for those agents increases. However, as previously discussed, pension income in the DB system was only riskless because workers (and firms) were absorbing the risk. As a result, under the DC system, both the standard deviation of consumption growth for workers and the standard deviation of aggregate consumption growth are now lower.

Our paper is part of the literature on asset pricing with incomplete risk sharing, such as the production economy models of Storesletten et al. (2007), Gomes and Michaelides (2008), Guvenen (2009), Croce et al. (2012), Favilukis (2013), Gomes et al. (2013), Kung and Schmid (2015) Favilukis et al. (2017) and Elenev et al. (2021), and the exchange economy models of Telmer (1993), Lucas (1994), Constantinides and Duffie (1996), Heaton and Lucas (1996), Constantinides et al. (2002), Schmidt (2016), and Constantinides and Ghosh (2017). In these papers, either there is no retirement period or retirement income is typically modelled as the outcome of a pure-pay-as-you-go system, and there is no explicit role for pension funds.

Our paper is also related to a growing literature studying the importance of financial intermediaries for macroeconomic activity and asset pricing. Previous work has mostly focused on the role of banks and asset managers such as mutual funds and hedge funds (e.g. Adrian and Shin (2010), He and Krishnamurthy (2013), Adrian and Shin (2014), Adrian et al. (2014), Brunnermeier and Sannikov (2014), He et al. (2017), He and Krishnamurthy (2018), Coimbra and Rey (2020), Elenev et al. (2021) and Khorrami (2021)). Here we consider the importance of asset owners: defined benefit pension funds. Furthermore, our paper highlights a new mechanism that operates through changes in the stochastic discount factor of households. Finally, while in other models of financial intermediation the equity premium typically increases as the wealth of intermediaries decreases and they become more constrained, here it is the opposite. If the size of the pension fund increases, then the equity premium also increases.

Our paper is also related to the literature on delegated portfolio management where institutional investors operate under constraints. Rauh (2009), Lucas and Zeldes (2009), Chuk (2013), Andonov et al. (2017)) and Greenwood and Vissing-Jorgensen (2018) discuss how the regulations and/or political constraints faced by pension funds affect their asset allocation decisions. Corporate pension plans might also be subject to performance constraints faced by other institu-

tional investors (Basak and Pavlova (2013)), or have mandates that determine how to respond to changing asset prices (Gabaix and Koijen (2021)).

The paper is organized as follows. Section 2 presents a first model that captures the essential ingredients for our analysis and thus highlights the main economic mechanisms. In section 3 we present our baseline model and its calibration, while in Section 4 we discuss the equilibrium results. In Section 5, we compare the results of the baseline model with the ones obtained when we ignore the defined benefit pension fund. In Section 6, we compare our current equilibrium with one where DB plans have been fully phased out and we have reached a new steady-state where only DC pension plans exist. We provide concluding remarks in Section 7.

2 A first model

2.1 Outline

Our baseline model includes several features that are important for matching asset prices and for capturing household heterogeneity, which is particularly relevant for some of our comparative statics, namely the comparison with the DC economy. However, to facilitate the understanding of the main economic mechanisms resulting from the modelling of the DB pension fund, we start with a simpler model that includes a minimal set of features.

We consider an asset pricing production economy with incomplete markets. Markets are incomplete because households face uninsurable labor income risk with borrowing constraints, and because they have a finite horizon. From ages 20 to 65 (working life), households supply labor inelastically and face countercyclical earnings risks as in Guvenen et al. (2014). During retirement (after age 65), they receive income from both social security and a defined benefit pension. The social security payments are financed by taxes on current workers' wages. The defined benefit pension is financed both by contributions made by current workers and/or firms, and by the return on the accumulated wealth of the pension fund.

The production side of the model is fairly standard. All firms are identical and perfectly competitive, and use capital and labor in a constant return to scale technology. In this first model, financial markets are also quite simple. Households can invest in two assets, a claim to the

risky capital stock (equity) and a zero-net supply riskless bond.

2.2 Firms

Since the focus of the paper is on the role of pension plans, retirement savings and household risk, the production side of the economy is quite standard, except for the inclusion of the contribution rates to the pension fund.

2.2.1 Production technology

Firms produce a single non-durable consumption good using a standard Cobb-Douglas production function, with total output at time t given by:

$$Y_t = Z_t K_t^{\alpha} L_t^{1-\alpha} \tag{1}$$

where K is the total capital stock in the economy, L is the total labor supply, and Z is a stochastic productivity shock, which follows the process:

$$Z_t = G_t U_t \tag{2}$$

$$G_t = (1+g)^t \tag{3}$$

where g captures aggregate growth, and the productivity shocks (U_t) follow a two-state Markov chain capturing business cycle fluctuations.

Standard frictionless production economies cannot generate sufficient return volatility, since agents can adjust their investment plans to smooth consumption over time (see Jermann (1998) or Boldrin et al. (2001)), but introducing adjustment costs of capital in a model with incomplete markets is conceptually challenging.[8] We address this problem by following a common approach

[8] Adjustment costs add an intertemporal dimension to the firm's problem, and the solution to such problem is not well defined under incomplete markets (see Grossman and Hart (1979)). Favilukis et al. (2017) offer a practical solution by considering a sensible stochastic discount factor in the firm's optimization problem.

in this literature and modelling the depreciation rate as stochastic:[9]

$$\delta_t = \bar{\delta}(U_t) + \sigma^{\delta}(U_t)\eta_t \tag{4}$$

where η_t is an i.i.d. standard normal shock. Therefore, δ_t is interpreted as a more general measure of economic depreciation, combining physical depreciation, adjustment costs, capital utilization, and investment-specific productivity shocks.[10] Both the conditional mean ($\bar{\delta}$) and standard deviation (σ^{δ}) of depreciation are correlated with aggregate productivity shocks (U_t) as discussed in the calibration section.

2.2.2 Pension Contributions

Firms might also have to make contributions to the defined benefit pension fund. These contributions reduce corporate profits and therefore lower the gross returns to capital. We let τ_t^{kdb} denote the pension fund contributions as a proportion of capital at time t, so that $\tau_t^{kdb}K_t$ is the total value of employer contributions to the pension fund.[11]

2.2.3 Maximization problem

Firms are perfectly competitive, so they take wages (W_t) and return on capital (R_t^K), as given. They face no frictions (e.g. no adjustment costs of capital) and make their decisions after observing the aggregate shocks. Therefore, they solve a sequence of static maximization problems:

$$\max_{K_t, L_t} Z_t K_t^{\alpha} L_t^{1-\alpha} - W_t L_t - (R_t^K + \tau_t^{kdb})K_t \tag{5}$$

where the capital stock evolves according to

$$K_{t+1} = (1 - \delta_t)K_t + I_t. \tag{6}$$

[9] See, for example, Krueger and Kubler (2006), Storesletten et al. (2007), Gomes and Michaelides (2008), Gottardi and Kubler (2011), and Gomes et al. (2013).

[10] Greenwood et al. (1988) use the same approach to model fluctuations in capital utilization.

[11] Alternatively we can write down the contribution rate as applying to the return on capital. The two formulations are equivalent as there is a one-to-one mapping between the two.

The first-order conditions to this optimization problem are

$$W_t = (1-\alpha) Z_t (K_t/L_t)^{\alpha} \tag{7}$$

$$R_t^K = \alpha Z_t (L_t/K_t)^{1-\alpha} + 1 - \delta_t - \tau_t^{kdb} \tag{8}$$

Since the DB fund will require higher contributions when (pre-contribution) returns are low, adjustments in τ_t^{kdb} lead to an increase in the volatility of (net) equity returns.

2.3 Households and financial markets

2.3.1 Life-cycle and preferences

We follow the convention in life-cycle models and let adult age (a) correspond to effective age minus 19. Each period corresponds to one year and agents live for a maximum of 81 periods (age 100). The probability of being alive at age $(a+1)$, conditional on being alive at age a, is denoted by p_a (with $p_0 = 1$). At each point in time there is a stationary age distribution of households in the economy, with no population growth.

Households have Epstein-Zin-Weil preferences (Epstein and Zin (1989) and Weil (1990)) defined over consumption of a single non-durable good (C_a^i):

$$V_a^i = \left\{ (1-\beta) C_a^{1-1/\psi} + \beta (E_a (p_a V_{a+1}^{1-\gamma}))^{\frac{1-1/\psi}{1-\gamma}} \right\}^{\frac{1}{1-1/\psi}} \tag{9}$$

where β is the discount factor, γ is the coefficient of relative risk aversion and ψ is the elasticity of intertemporal substitution.[12]

[12] We do not include a bequest motive in the model because, in general equilibrium with overlapping generations and stochastic mortality, this would require (young) agents to form expectations about the future bequest that they might receive. Instead, we assume that accidental bequests in the model are fully taxed and used to finance government expenditures that are not productive and do not enter the utility function of agents. Another alternative would be to assume perfect annuity markets after retirement, but that would eliminate longevity risk from the model.

2.3.2 Financial markets

There are two financial assets in the model, so markets are incomplete. The first asset (equity) is a claim on the capital stock of firms and has a risky return (R_t^K). The second asset is the one-period riskless bond which exists in zero net supply. The rate of return the riskless asset can be written as

$$R_t^B = \frac{1}{P_{t-1}^B} \tag{10}$$

where P^B denotes the bond price. Households cannot borrow against their future labor income to finance consumption but, as standard in models where bonds exist in zero net supply, they can short them to invest in stocks.

2.3.3 Labor income

Before retirement all households supply labor inelastically, and face individual-specific productivity shocks. Individual labor income (H_{at}^i) is the product of individual productivity (L_{at}^i) and the aggregate wage per unit of productivity (W_t):

$$H_{at}^i = W_t L_{at}^i \tag{11}$$

The aggregate wage is determined in equilibrium by equation (7), while the stochastic process for individual productivity is given by a permanent component P_{at}^i and a transitory shock ε_t^i.:

$$L_{at}^i = P_{at}^i \varepsilon_t^i \tag{12}$$

$$P_{at}^i = \exp(f(a)) P_{a-1,t-1}^i \xi_t^i \tag{13}$$

where $f(a)$ is a deterministic function of age, capturing the typical hump-shape profile in life-cycle earnings and ξ_t^i are shocks to the permanent component. We assume that $\ln \varepsilon^i$ is independent and identically distributed with mean $\{-.5 * \sigma_\varepsilon^2\}$, and variances σ_ε^2.

Following Guvenen, Ozkan and Song (2014), we model countercyclical earnings risk by assuming $\ln \xi_t^i$ is a mixture of normal distributions.[13] Conditional on the state of the economy U_t the

[13]Catherine (2022) and Shen (2020) study the portfolio choice implications of this earnings process, while

innovation $\ln \xi_t^i$ is drawn from one distribution with probability q_1, and with probability $(1 - q_1)$ from a second distribution:

$$\ln \xi_t^i = \begin{cases} \ln \xi_{t,1}^i \sim N(\mu_{1,U_t}, \sigma_{1,U_t}^2) & \text{with prob } q_1 \\ \ln \xi_{t,2}^i \sim N(\mu_{2,U_t}, \sigma_{2,U_t}^2) & \text{with prob } 1 - q_1 \end{cases} \tag{14}$$

Both the expected growth rates and the conditional volatility are functions of aggregate productivity state (U_t), thus allowing for countercyclical earnings risk as emphasized by Guvenen et al. (2014).

2.3.4 Retirement income

Retired households receive income in the form of both social security and defined benefit pension payments. Both of these are proportional to the product of permanent income and aggregate wage. Total retirement income is then given by:

$$H_{at}^i = (\lambda^{ss} + \lambda^{db}) P_{a^R t^R}^i W_t \ , \ a > a^R \tag{15}$$

where λ^{ss} and λ^{db} are the (exogenous) replacement ratios for (respectively) social security and the defined benefits scheme. The notation t^R refers to the calendar year in which the individual has retired (i.e. the year with age $a = a^R$).

Consumption at retirement is also financed by private savings, which also reflect defined contribution pension schemes in the data. Our baseline economy should therefore be viewed as an economy where both systems (DB and DC) co-exist, as is currently the case in the U.S.. Currently, employees in the U.S. are enrolled in DC plans, or DB plans, or hybrid DB-DC systems, or neither. In addition, several of those enrolled in DC or in hybrid plans also have a legacy DB plan. We capture this large heterogeneity in a simplified form by considering that our households are all enrolled in a hybrid system, combining both DB and DC.

Constantinides and Ghosh (2017) and Schmidt (2016) considers its asset pricing implications.

2.3.5 Social security

Social security is modelled as a fully funded pay-as-you-go system. In the U.S., social security also has a trust fund, but this is being depleted and is projected to disappear in the near future. The potential implications of this are certainly interesting to explore. However, we are already extending the standard framework by modelling pension funds, hence we leave this question for other research.[14]

Social security benefits are financed by a proportional tax rate on labor income (τ^{ss}), which is determined endogenously by the relative demographic weights of workers and retirees, so that the system is balanced at all times.

$$\sum_{a=20}^{65} \int_{i \in I^a} \tau^{ss} L_{at}^i w_t di = \sum_{a=66}^{100} \int_{i \in I^a} [\lambda^{ss} \exp(f(a^R)) w_t P_{a^R t^R}^i] di \ , \tag{16}$$

where the notation I^a refers to the set of individuals with age a. This equation determines the value of the social security tax/contribution (τ^{ss}) for a given value of the social security retirement replacement ratio (λ^{ss}).

2.4 Pension system

The defined benefit pension is managed by a pension fund that collects contributions from both employees and employers and pays the pensions of current retirees. In addition, the pension fund has an accumulated stock of wealth, and therefore the return on this endowment therefore constitutes another source of income that can be used to finance pension payments. Munnell and Soto (2007) document that this was by far the most important source of financing for state and local DB pension funds between 1996 and 2006 (70% of their total income).

2.4.1 Endowment

We model the pension fund as having an endowment of wealth (W^P) such that it is fully funded if future return realizations (for stocks and bonds) are equal to their unconditional means. That

[14] In addition, studying the implications of the dynamics of the social security trust fund would require solving for a dynamic transition path along this dimension.

is, suppose the system is closed such that no new generations/employers are enrolled. In that case, the current endowment (plus its return) is exactly enough to pay off (net) liabilities in expectation.[15]

This is an endogenous quantity in the model since it will depend on the equilibrium levels of both returns (on capital and on bonds) and wages. We assume that the pension fund keeps its endowment of wealth fixed at a level $\bar{W}^P$, so that it adjusts contribution rates to compensate for realized returns, as discussed in subsection 2.4.3.

2.4.2 Asset allocation

The Pension Fund's endowment (W^P) is invested in a combination of equities and (one-period) risk-free bonds. The fund's portfolio return (R_t^P) is given by:

$$R_t^P = \alpha_t^P R_t^K + (1 - \alpha_t^P) R_t^B, \tag{17}$$

where (α^P) is the risky share of the pension fund. We model this risky share with the following reduced-form equation:[16]

$$\alpha_t^P = a^P + b^P \ R_t^B. \tag{18}$$

This formulation allows the asset allocation to respond to interest rates movements. In particular, if b^P is negative (as we will consider) this captures a reach-for-yield behavior, with the risky share of the pension fund falling when the riskless rate rises.

2.4.3 Contributions

Private DB pension funds are financed by contributions from both employers and employees, and we consider both of these in the model (respectively, τ^{kdb} and τ^{db}). In the US it is mostly the former, while most other countries have a mix of the two. Public DB pension funds can be financed by either labor taxes or corporate taxes.[17] Labor taxes have direct impact on workers' net wages,

[15] Details on the exact calculation are provided in Appendix A.

[16] We ignore the issue of the optimal design of a pension fund asset allocation (see Lucas and Zeldes (2009) and Dahlquist et al. (2018)).

[17] For simplicity we abstract from other forms of taxation in the model, such as consumption/sales taxes.

while corporate taxes decrease firms' after-tax profits. Therefore, in the context of our model, they are equivalent to the employer and employee contributions for private funds.[18]

When the pension fund runs a deficit/surplus this prompts an increase/decrease in contributions.[19] To facilitate the exposition, we first discuss the two extreme cases, where only one margin of adjustment is used, and present the general case at the end.

Adjustment through changes in employee contributions

Consider the polar case in which fluctuations in pension wealth are compensated by changes to employee contribution rates only. Then, in periods of high returns, contribution rates out of labour income will fall (and vice-versa) to stabilize pension fund wealth. At any point in time, the contribution rate τ_t^{db} can therefore be calculated according to the following equation

$$\sum_{a=20}^{65} \int_{i \in I^a} \tau_t^{db} L_{at}^i w_t di + (R_t^P - 1)\bar{W}^P = \sum_{a=66}^{100} \int_{i \in I^a} [\lambda^{db} \exp(f(a^R)) w_t P_{a^R t^R}^i] di. \tag{19}$$

where the left-hand-side of the equation captures the yearly income of the fund (contributions plus the net return on its endowment), while the right-hand-side measures its corresponding liabilities.

Given Equation (19), we can also solve for the steady-state contribution rates $\bar{\tau}^{db}$ as a function of the steady-state endowment $\bar{W}^P$, and unconditional expected returns and wages.

Adjustment through changes in employer contributions

Now we consider the other extreme case, in which fluctuations in pension fund's wealth are fully offset by changes in employer contributions (τ_t^{kdb}), which are described in Section (2.2).

The value of τ_t^{kdb} such that pension fund wealth remains fixed over time can be obtain from:

$$\sum_{a=20}^{65} \int_{i \in I^a} \bar{\tau}^{db} L_{at}^i w_t di + (R_t^P - 1)\bar{W}^P + \tau_t^{kdb} K_t = \sum_{a=66}^{100} \int_{i \in I^a} [\lambda^{db} \exp(f(a^R)) w_t P_{a^R t^R}^i] di. \tag{20}$$

Note that, while capital contribution rates apply to the entire capital stock, contributions on capital held by the fund itself cancel out since they are subtracted in the term $(R_t^P - 1)\bar{W}^P$. It

[18] In our model all households and firms are enrolled in the Pension Plan. Therefore we abstract from distributional implications between taxpayers who are not public employees, and those who are. Capturing this would require introducing additional household heterogeneity in the model.

[19] Equivalently, there could be a reduction in the accrual of new retirement benefits going forward. Defaults on promised payments are rare events and therefore we do not consider them in the model.

follows that, given a level of capital, the larger the share of this capital that is being held by the fund, the more volatile these contribution rates will have to be.

General Case

More generally, we consider adjustments in both contributions. Let θ^P be the share of adjustment covered by employer contributions and define $\tilde{W}_t^P$ as the wealth of the fund before adjusting contributions (i.e. $\tau^{kdb} = 0$ and $\tau_t^{db} = \bar{\tau}^{db}$):

$$\tilde{W}_t^P = R_t^P \bar{W}^P + \sum_{a=20}^{65} \int_{i \in I^a} \bar{\tau}^{db} L_{at}^i w_t di - \sum_{a=66}^{100} \int_{i \in I^a} [\lambda^{db} \exp(f(a^R)) w_t P_{a^R t^R}^i] di. \tag{21}$$

If the contributions don't adjust, then each year the endowment value increases by its return (first term), and by the average value of annual contributions (second-term), and decreases by the value of its yearly liabilities (third term). $\bar{W}^P - \tilde{W}_t^P$ is then the shortfall (or surplus) before adjusting contributions.

The required contribution rate for employers τ_t^{kdb} is then:

$$\tau_t^{kdb} = \theta^P \frac{\bar{W}^P - \tilde{W}_t^P}{K_t - \alpha^P \bar{W}^P} \tag{22}$$

The contribution rate for employers is equal to the required adjustment (i.e. shortfall multiplied by θ^P), divided by the effective contribution base, which is the privately held capital.

Similarly, the contribution rate of employees can be calculated as the required adjustment divided by its contribution base:

$$\tau_t^{db} - \bar{\tau}^{db} = (1 - \theta^P) \frac{\bar{W}^P - \tilde{W}_t^P}{\sum_{a=20}^{65} \int_{i \in I^a} L_{at}^i w_t di} \tag{23}$$

This general case nests the two polar cases described before: $\theta^P = 0$ is the case with full adjustment with employee contributions, and $\theta^P = 1$ is the case with full adjustment with employer contributions.

2.4.4 Discussion

In our model, for tractability, fluctuations in the financial situation of the pension fund are translated each year into changes in the contribution rates, such that its endowment is constant over time. In reality, DB pension funds adjust those rates infrequently, typically in response to significant changes in their funding ratio. Modelling this would require adding the funding ratio of the DBPF as an additional state variable, and imposing arbitrary assumptions with regards to the rules for adjusting the contribution rates. Regardless, this would not change the fact that workers and firms would ultimately be facing this risk. Instead of it being reflected in one-for-one yearly adjustments, it would lead to large discrete jumps in some years, as eventually the adjustment mechanisms would be triggered. It is possible that the alternative formulation (with less frequent but more significant shocks) could have even larger quantitative implications for asset prices, in line with the results in Constantinides and Ghosh (2017) and Schmidt (2016).

2.5 The individual optimization problem

Each period (t) agents earn returns on their wealth invested in bonds (B^i_{at}) and stocks (K^i_{at}), and earn labor income ($L^i_{at}W_t$), which is subject to the social security tax (τ^{ss}) and the pension contribution (τ^{db}_t). Wealth (cash-on-hand) at time t is given by:[20]

$$X^i_{at} = K^i_{at}(1 + r^K_t) + B^i_{at}(1 + r^B_t) + L^i_{at}(1 - \tau^{ss} - \tau^{db}_t)W_t \tag{24}$$

before retirement ($a < a^R$), and by:

$$X^i_{at} = K^i_{at}(1 + r^K_t) + B^i_{at}(1 + r^B_t) + (\lambda^{db} + \lambda^{ss})P^i_{a^R t^R}W_t \tag{25}$$

during retirement ($a \geqslant a^R$).

Households can borrow to finance investments in stocks, but not to finance consumption.[21]

[20] Below we use lower case letters to denote simple returns instead of gross returns. So $r^K_t = R^K_t - 1$ and $r^B_t = R^B_t - 1$.

[21] Allowing for levered positions in stocks is a requirement to obtain market clearing in a model with heterogeneous agents and a riskless asset in zero net supply. In the extended model this will not be required and both types of borrowing are ruled out.

Households maximize utility given their expectations about future asset returns and aggregate wages. Under rational expectations, the latter are given by equations (7) and (8), and are therefore determined by the equilibrium level of the capital stock, and the exogenous aggregate shocks.

As standard in the literature we follow the approach proposed by Krusell and Smith (1998): conditional on the shocks (productivity, U_t , and stochastic depreciation, η_t), the future aggregate capital can be very well predicted by its current value:

$$K_{t+1} = \Gamma_K(K_t, U_t, \eta_t) \ . \tag{26}$$

Since bonds are only riskless over one period, households must also forecast future bond prices (P^B_{t+1}). Following the literature (e.g. Storesletten et al. (2007) and Gomes and Michaelides (2008)) we consider a similar forecasting model, augmented to include the current bond price:

$$P^B_{t+1} = \Gamma_P(P^B_t, K_t, U_t, \eta_t) \ . \tag{27}$$

Details are given in Online Appendix A. This procedure introduces four aggregate state variables in the individual's maximization problem (P^B_t, K_t, U_t, and η_t).

The full dynamic programming problem and the equilibrium equations for this version of the model are presented in Online Appendix B, to avoid excessive repetition relative to the presentation of the full model.

2.6 Calibration

We set the coefficient of relative risk aversion (γ) to 6 and the EIS (ψ) to 0.5.[22] We then choose the discount factor (β) to deliver a low risk-free rate. More precisely, we fix β=0.932.

The parameters of the income process are taken from Cocco et al. (2005) and Guvenen et al. (2014). In the spirit of keeping this model as simple as possible, we set b^P equal to zero, and leave the more general specification of the Pension Fund's portfolio rule for the extended model. We then calibrate a^P to match the average risky share of DB Pension Funds in the data (52%). A

[22]These are the values that we consider in the extended model below, so we use them here as well to facilitate both the exposition and potential comparisons across the two models.

more detailed discussion of the calibration is provided when presenting the baseline model.[23]

2.7 Results

2.7.1 Comparison with Pure-Pay-as-you-Go Model

Table 1 reports the equilibrium moments for the model with the Defined Benefit Pension Fund (hereafter DBPF model), and for an alternative model where the pension scheme is a simple pass-through entity, i.e. $\bar{W}^P$=0 at all times. We refer to the alternative model as the Pure-Pay-as-you-Go (PPG) model, and it captures the standard formulation in OLG models with retirement (see, for example, Storesletten et al. (2007), Gomes and Michaelides (2008), Favilukis (2013), Gomes et al. (2009), or Favilukis et al. (2017)).

The DBPF model is calibrated to deliver a low risk-free rate. In the extended model below we target additional moments, namely the standard deviation of consumption growth. For the PPG model we report results both with the same calibration as in the DBPF model, and for two alternative re-calibrated economies (rPPG1 and rPPG2).

[INSERT TABLE 1 HERE]

We start by comparing the two models for the same parameter values in columns (i) and (ii). In the PPG model there is no DBPF with an accumulated stock of wealth. This is not compensated by higher household wealth accumulation since households receive the same level of retirement benefits in both economies.[24] Therefore, the total capital stock in the economy is lower, 4.53 versus 5.31. Equilibrium asset prices naturally reflect the lower wealth accumulation, leading to a higher average riskless rate (3.26%), and a higher average return on capital. However, the riskless rate increases by more for two reasons. First, unlike the demand for capital, the supply of bonds is inelastic. Second, the pension fund's portfolio allocation is relatively more conservative, due to their higher demand for bonds. As a result, for the same preference parameters, the PPG economy delivers a lower equity premium (3.02%) than the DBPF economy (4.55%).

[23]The values of the parameters in the technology process, including the stochastic depreciation, are the same in both models.

[24]Later we consider an experiment where we shut down the DBPF fund, in which case retirement benefits are affected. Here we are merely comparing the implications of the two modelling frameworks: DBPF versus PPG.

In column (iii) we report results for a re-calibrated PPG economy (rPPG1) that delivers the same riskless rate as the DBPF economy. This is achieved by increasing the discount factor of the agents (from 0.932 to 0.955), which increases wealth accumulation. However, since pension funds have a relatively higher demand for bonds than households, to obtain the same risk-free rate in the two models we need more total wealth in the rPPG1 economy than in the DBPF one. For the same wealth accumulation, given that households have a higher average risky share than DB pension funds, the demand for bonds would be too low, i.e. it would imply a counterfactually high risk-free rate.

Higher wealth accumulation is reflected in a total capital stock of 5.95 in the rPPG1 economy versus 5.31 in the DPBF one, respectively. As a result, the average equity return, the equity premium and the Sharpe ratio are all lower in the rPPG1 economy. The rPPG1 economy has a lower volatility of consumption growth, but this only partially explains the lower equity premium, as shown in column (iv) of Table 1. Column (iv) reports results for a second recalibration of PPG economy (rPPG2), which delivers both the same riskless rate and the same standard deviation of consumption growth as the DBPF economy.[25] The equity premium and the sharpe ratio in rPPG2 calibration are both naturally higher than in the rPPG1 case, but still significantly below the ones in the DBPF model.

We can obtain additional insights into the differences between the two models from Panel B of Table 1. Here we report the cross-sectional standard deviations of consumption growth for different age groups: 20-35, when agents are mostly saving for precautionary reasons, 36-65, when the retirement savings motive dominates, and after age 66, the group of retirees. In both models agents have a stable income stream at retirement, hence the low volatilities of consumption growth during this period.[26]

However, in a model where retirement income is partially funded by (risky) returns on the Pension Fund's endowment (the DBPF model), the only way to guarantee a riskless income for retirees is if workers and firms absorb the shocks to the value of the pension fund. As a result,

[25]For rPPG2, in addition to lowering subjective discount rates to match risk free rates, we also lower the volatility of the depreciation shock to match the volatility of consumption growth.

[26]The standard deviation is slightly higher in the rPPG economy because, as shown in Panel A and as discussed earlier, households accumulate more wealth in this model. Consequently a higher fraction of their retirement savings is subject to stock return volatility.

the standard deviation of consumption growth for workers is higher: 10.1% (7.6%) versus 9.2% (7.0%) in the rPPG economy, for agents in the age group 36-65 (20-35). This increase in the cross-sectional volatility of consumption growth generates an additional risk premium in the model (as in the framework of Constantinides and Duffie (1996)).

Having documented important differences in the asset pricing implications of the two models, the next set of experiments helps us understand the economic mechanisms that generate these differences.

2.7.2 Results for different pension fund adjustment rules

When the DB pension fund is running a deficit or a surplus, it can adjust either the contributions received from the sponsor company or from the employees. As discussed in section 2.4.4, we consider both margins of adjustment in our model, with a parameter (θ^P) to determine the relative weight of the former. The results presented in Table 1 refer to our baseline specification, where we set θ^P=0.5. In Table 2, we illustrate the impact of each of the two channels by presenting results for θ^P=0.15 and θ^P=0.85.

[INSERT TABLE 2 HERE]

Under a higher θ^P the contributions from firms to the Pension Fund have to increase more when its endowment registers a loss, i.e. when returns are low. This additional contribution further decreases firm profits, leading to even lower returns (and vice-versa when returns are high). Therefore, for higher values of θ^P we have higher return volatility. This explains why in column (ii) of Table 2 the standard deviation of returns is only 18.39% for θ^P=0.15, while for the baseline calibration of column (i) it is 19.69% and it increases further in column (iii) to 21.03% when θ^P=0.85.

When θ^P=0.15, the pension fund's endowment remains balanced largely due to fluctuations in the pension contributions of employees. Although their disposable income becomes more volatile, their total consumption volatility actually falls because the volatility of stock returns is now lower, as explained above. Relative to the baseline calibration (θ^P=0.5, column i), the higher level of background risk in the θ^P=0.15 economy (column ii) reduces the demand for equity, but this is

again counteracted by the lower standard deviation of returns, leading to a very small (0.6%) net decrease in the capital stock. The lower equity volatility delivers a lower return on equity and a lower equity premium. On the flip side, this increase in the demand for risky assets reduces the demand for bonds leading to a higher equilibrium risk-free rate. As expected, in the θ^P=0.85 economy (column iii) we observe the opposite patterns: lower capital accumulation, higher equity premium and lower risk-free rate.

Despite the differences in equity premium and riskless rate, the three cases deliver essentially the same Sharpe ratio. In this first model, all agents are stockholders and have the same preference parameters, so they are (in expectation) all equally affected by these two channels. Later on, we consider a model with limited stock market participation and preference heterogeneity, where the two adjustment mechanisms have important cross-sectional implications.

2.7.3 Results for different asset allocations of the pension fund

In our final comparison we report results for different values of α^P, the risky share of the defined benefit pension fund. In our previous results this value was set to its empirical counterpart of 0.52. In Table 3 we contrast those with the results obtained when we set $\alpha^P = 0.8$.

[INSERT TABLE 3 HERE]

As we increase the pension fund's equity share, the total allocation to capital in the economy increases and, as a result, the equity premium and the Sharpe ratio both fall (to 4.30% and 0.21, respectively). In contrast, the demand for bonds decreases leading to a higher equilibrium risk-free rate (1.37%). These results illustrate the other important channel behind the higher risk premium in the DBPF model: the portfolio demand of the pension fund. To the extent that DBPFs have a lower equity allocation than the other agents in the economy, including them in (excluding them from) the model will lead to a higher (lower) equilibrium risk premium. The intuition is similar to the one from a two-agent model, where the agents have different optimal portfolios.[27] For this

[27] A classic comparison would be a model where we have stockholders with high and low risk aversion. If we compare that equilibrium with one where we only have those with low risk aversion, the equity premium will be higher in the former than in the latter (e.g. Dumas (1989)).

result it is important that the equity share of DB pension funds is indeed lower than the average equity share in the economy. We discuss this in the calibration section.[28]

As the equity share of the pension fund increases, its endowment becomes more correlated with the stock market and therefore households now have a higher indirect exposure to equity returns. Combined with the lower equity premium this leads to a significant reduction their demand for stocks: the private capital stock falls from 4.44 to 3.98. As a result, despite pension funds having increased their allocation to stocks, the total capital stock is actually lower (5.16, compared with 5.31 in the baseline economy). The impact on the return on capital depends on which of the two supply curves for the capital shifts by more: the one for households (which decreases) or the one from the pension fund (which increases). Under our calibration the first effect dominates leading to a modest increase in the return on capital.

As the previous results illustrate, households are trying to offset the asset allocation of the DB pension fund in their own private decisions: they invest less in equities when the pension fund increases its risky share. However, they are not fully able to offset the presence of the pension fund because, several of them, have limited wealth and face borrowing constraints. In this version of the model this is particularly the case for young households, but later on it will also apply to households with low savings incentives.

3 Baseline Model

3.1 Outline

As before, we consider an overlapping generations production economy with incomplete markets. However, we extend the previous model along several dimensions.

First, investing in equities now requires paying participation costs, both a first-time entry cost and a per-period cost. Second, we consider two groups of households with heterogeneous preferences. This heterogeneity is important for obtaining significant cross-sectional wealth inequality and for generating endogenous limited stock market participation with realistic participation costs

[28] In our model the only other agents in the economy are households, but they also represent the financial positions of banks and asset management companies which are ultimately intermediaries investing household wealth.

(see, for example, Gomes and Michaelides (2005) and Gomes and Michaelides (2008)).

Third, the riskless asset is now issued by the government, and exists in positive net supply. If the riskless asset is in zero net supply, then the average risky portfolio share in the model would be 100%, which is inconsistent with the data on household portfolios. Furthermore, the quantitative impact of the differential asset demands of the pension fund is related to the relative supply of the two assets. Therefore, a correct calibration of the bond supply becomes important. Fourth, and related to the previous extension, we now model a government sector. The interest payments on public debt are financed by taxes on capital gains, bequests and wages.

In the baseline version of the model retired households receive income from both social security and a defined benefit pension, as before. The production side of the model is identical to the one in the simplified model, and therefore we refer the reader to that particular subsection to avoid repetition. Likewise, the features of the DBPF are identical to the ones described in section 2.4.

3.2 The government sector

The government issues one-period riskless bonds, which therefore exist in positive net supply. The government's budget constraint is

$$C_t^G + R_t^B B_t = B_{t+1} + T_t \tag{28}$$

where C^G is government consumption, B is public debt, R^B is the gross interest rate on government bonds, and T denotes the tax revenues.

Tax revenues are collected from proportional taxes on capital income (tax rate τ^K), on bond interest payments (tax rate τ^B), wages (tax rate τ^W) and bequests (tax rate τ^E). Government expenditures do not enter the agents' utility functions, and are determined as the residual from Equation (28), given the (exogenous) level of debt, the (exogenous) tax rates, and the (endogenous) interest rate on bonds.

3.3 Households and financial markets

3.3.1 Household Heterogeneity

Households have Epstein-Zin preferences as given by equation (9). There are two types of households in the model (A and B). They are ex-ante different because type-B households have preferences that imply high wealth accumulation, while type-A households have preferences that lead them to consume most (but not all) of their labor income. Both types of households have the same degree of risk aversion ($\gamma^A = \gamma^B$), but they have a different EIS and different discount factors, as discussed in the calibration section. In equilibrium, most type-B households will find it optimal to pay the participation costs and invest in stocks, while most type-A households will invest only in government bonds.

3.3.2 Financial markets

As before, there are two financial assets in the model: a claim on the capital stock of firms (with risky return R_t^K) and a one-period riskless bond. This riskless bond now exists in positive supply and it is issued by the government.

Before investing in stocks for the first time, households must pay a one-time fixed cost: $F^0 P_{at}^i W_t$. This entry fee captures both explicit pecuniary costs (e.g. transaction cost from opening a brokerage account and/or hiring a financial advisor), and the (opportunity) cost of acquiring information about the stock market. In addition, every period in which they have positive stockholdings, households must pay a (lower) per-period participation cost, $F^1 P_{at}^i W_t$, which reflects the (opportunity) cost of managing the portfolio and (again) acquiring information about the stock market. The participation costs are scaled by the current value of the permanent component of labor income (P_{at}^i) and by the aggregate wage (W_t), both because it significantly simplifies the solution of the model, and because this is consistent with the opportunity cost interpretation.

Households cannot borrow against their future labor income, and cannot short either asset, so both their bond holdings, B_{at}^i, and their stock holdings, K_{at}^i, must be non-negative:

$$B_{at}^i \geq 0 \tag{29}$$

$$K^i_{at} \geq 0 \ . \tag{30}$$

3.4 The individual optimization problem

3.4.1 Household wealth accumulation

Relative to the previous model, households now face stock market participation costs and taxes. We define the dummy variable I^i_E as equal to one in the period in which the entry cost is paid, and zero otherwise, and the dummy variable I^i_S as equal to one if the household has a positive holding of stocks, and zero otherwise. We then capture the total participation costs paid by agent i at time t with the notation

$$PC^i_{at} = I^i_E F^0 P^i_{at} W_t + I^i_S F^1 P^i_{at} W_t \tag{31}$$

Wealth (cash-on-hand) at time t is now given by:

$$X^i_{at} = K^i_{at}(1 + (1-\tau^K) r^K_t) + B^i_{at}(1 + (1-\tau^B) r^B_t) + L^i_{at}(1 - \tau^{ss} - \tau^W - \tau^{db}_t) W_t - PC^i_{at} \tag{32}$$

before retirement ($a < a^R$), and by:

$$X^i_{at} = K^i_{at}(1 + (1-\tau^K) r^K_t) + B^i_{at}(1 + (1-\tau^B) r^B_t) + (\lambda^{db} + \lambda^{ss}) P^i_{a^R t^R}(1-\tau^W) W_t - PC^i_{at} \tag{33}$$

during retirement ($a \geqslant a^R$). Naturally if the household chooses not to pay the participation cost then $K^i_{at} = 0$ in these equations.

Households form expectations about future asset returns as before, using equations (26) and (27).

3.4.2 The dynamic programming problem

We write the model in stationary form, scaling all variables by aggregate productivity growth ($G_t^{\frac{1}{1-\alpha}}$). We further normalize the individual variables by the current level of permanent labor income (P^i_{at}), to reduce the dimensionality of the state vector by one. Normalized variables are

denoted by lower-case letters.[29]

After the normalizations, the individual maximization problem has seven state variables. Age (a), normalized cash on hand (x^i_{at}), a zero-one variable indicating whether the entry cost has been paid or not (E^i_a), and the four aggregate variables from the forecasting equations ((26) and (27)). The full optimization problem is written as:

$$V_a(x^i_{at}, E^i_a; k_t, U_t, \eta_t, P^B_t) = \underset{\{k^i_{a+1,t+1}, b^i_{a+1,t+1}\}_{a=1}^{A}}{Max} \{(1-\beta)(c^i_{at})^{1-1/\psi} \tag{34}$$
$$+\beta(E_t[(\frac{P^i_{a+1,t+1}}{P^i_{at}}(1+g)^{\frac{1}{1-\alpha}})^{1-\rho} p_a V^{1-\rho}_{a+1}(x^i_{a+1,t+1}, E^i_{a+1}; k_{t+1}, U_{t+1}, \eta_{t+1}, P^B_{t+1})])^{\frac{1-1/\psi}{1-\rho}}\}^{\frac{1}{1-1/\psi}},$$

subject to the constraints:

$$k^i_{a+1,t+1} \geq 0 \quad , \quad b^i_{a+1,t+1} \geq 0 \tag{35}$$

$$c^i_{at} + b^i_{a+1,t+1} + k^i_{a+1,t+1} = x^i_{at} \tag{36}$$

and

$$x^i_{a+1,t+1} = \begin{cases} \frac{\left[k^i_{a+1,t+1}(1+(1-\tau^K)r^K_{t+1}) + b^i_{a+1,t+1}(1+(1-\tau^K)r^B_{t+1})\right]}{[(P^i_{a+1,t+1}/P^i_{at})(1+g)^{\frac{1}{1-\alpha}}]} \\ +\varepsilon^i(1-\tau^{ss}-\tau^W-\tau^{db}_t)w_{t+1} - I^i_E F^0 w_{t+1} - I^i_S F^1 w_{t+1} \quad a < a^R \\ \frac{\left[k^i_{a+1,t+1}(1+(1-\tau^K)r^K_{t+1}) + b^i_{a+1,t+1}(1+(1-\tau^K)r^B_{t+1})\right]}{[(P^i_{a+1,t+1}/P^i_{at})(1+g)^{\frac{1}{1-\alpha}}]} \\ +(\lambda^{db}+\lambda^{ss})w_{t+1} - I^i_E F^0 w_{t+1} - I^i_S F^1 w_{t+1} \quad a > a^R \end{cases}, \tag{37}$$

the stochastic process for individual labor productivity (equations (11) to (14)), and the forecasting equations (26) and (27).

The individual takes as given all aggregate variables, i.e. capital stock, returns, bond price, wages, tax rates and the other government variables.

3.5 Equilibrium

Equilibrium prices and quantities and determined by the following set of conditions:

1. Firms hire capital and labor to maximize profits (equations (7) and (8)).

[29] Specifically, household-specific variables are normalized as $x^i_{at} \equiv \frac{X^i_a}{P^i_{at} G_t^{\frac{1}{1-\alpha}}}$, $c^i_{at} \equiv \frac{C^i_a}{P^i_{at} G_t^{\frac{1}{1-\alpha}}}$, $b^i_{a+1,t+1} \equiv \frac{B^i_{a+1,t+1}}{P^i_{at} G_t^{\frac{1}{1-\alpha}}}$, $k^i_{a+1,t+1} \equiv \frac{K^i_{a+1,t+1}}{P^i_{at} G_t^{\frac{1}{1-\alpha}}}$ while aggregate variables are normalized as $k_t \equiv \frac{K_t}{G_t^{\frac{1}{1-\alpha}}}$, and $w_t \equiv \frac{W_t}{G_t^{\frac{1}{1-\alpha}}}$.

2. Individuals choose their consumption and asset allocation to maximize their expected lifetime utility, i.e. maximize equation (34) subject to the constraints described in the previous subsection.

3. The social security system is balanced at all times, as given by equation (16)

4. The defined benefit pension fund is in a balanced path with a constant endowment ($\bar{W}^P$), and endogenous contribution rates (τ_t^{db} and τ_t^{kdb}) as given by Equations (23) and (22).

5. The government budget (equation (28)) is balanced every period for a given ratio of government debt to GDP.

6. All markets clear, specifically the markets for capital, bonds and the consumption good:[30]

$$k_t = \int_i \int_a P^i_{a-1,t-1} k^i_{at} da di \tag{38}$$

$$b_t = \int_i \int_a P^i_{a-1,t-1} b^i_{at} da di \tag{39}$$

$$U_t k_t^{\alpha} L_t^{1-\alpha} = \frac{C_t^G}{G_t^{\frac{1}{1-\alpha}}} + (1+g)^{\frac{1}{1-\alpha}} k_{t+1} - (1-\delta_t) k_t + \int_i \int_a P^i_{at} c^i_{at} da di \tag{40}$$

By Walras' law, once two of these equations are verified, the third is also automatically satisfied.

7. Household expectations for market prices (equations (26) and (27)) are verified in equilibrium.

We describe the numerical solution of the model in Online Appendix A.

3.6 Calibration

In this section, we discuss the calibration of the model. The baseline parameter values are reported in Table 4.

[INSERT TABLE 4 HERE]

3.6.1 Aggregate variables

The productivity shock follows a first-order Markov process with two values, corresponding to expansions and recessions. We calibrate the transition matrix to fit NBER data. The probability

[30] The market for labor is trivial since there is no labor-leisure choice.

of remaining in recessions (π_r) is 16/37, and the probability of remaining in expansions (π_e) is 60/81, yielding an average business cycle duration of six years. We also fit the expansion and recession values of productivity to match the conditional growth rate of labour income as in Guvenen et al. (2014), as described below. This implies a standard deviation of TFP of 2.6%. The capital's share of output (α) is set to 34%, while the average annual depreciation rate (δ) is set to 10%, with a standard deviation of 10%.

The aggregate supply of bonds is calibrated to deliver an endogenous ratio to GDP of 42%, the average value of U.S. Treasury securities held by the U.S. public (data from the Congressional Budget Office).[31] The tax rate on bond interest payments (τ^B) is set at 20%, while tax rate on stock returns (τ^K) is 40%. This is meant to capture a 20% personal tax rate on both sources of equity income (dividends and capital gains), and a 20% corporate tax rate on firm profits. Bequests are fully taxed ($\tau^E = 100\%$), but total bequests are a very small fraction of total government revenues (both in the model and in the data), so this assumption is only made for simplicity.[32]

Since the firms in the model are unlevered, the return on capital is a return on unlevered equity. We obtain the implied levered equity return by assuming a leverage ratio of 0.44, from Rajan and Zingales (1995).

3.6.2 Household variables

We calibrate the conditional survival probabilities ($\{p_a\}_{a=1}^{81}$) from the mortality tables of the National Center for Health Statistics. Both types of households (A and B) have a risk aversion coefficient of 6 ($\gamma^A = \gamma^B = 6$), a standard value in asset pricing models which target a realistic value for the equity premium. The other preference parameters (discount factor and EIS) are calibrated to match the standard deviation of consumption growth, the level and standard deviation of the risk-free rate. In addition, the preference parameters of the type-A households are also chosen to deliver low wealth accumulation, and consequently endogenous low stock market participation (for reasonable values of the participation costs). More precisely, we set β^A equal to

[31] We specifically consider U.S. debt held by the U.S. public only, since ours is a closed-economy model.

[32] Otherwise we would have to re-distribute this wealth to the surviving generations, and households would need to form expectations over these transfers and consider those expectations in their optimization problem.

0.83 and ψ^A equal to 0.25. For the type-B households we set β^B at 0.965 and ψ^B at 0.5.

The stock market participation costs are based on the values previously considered in the literature.[33] We calibrate the entry cost of participation (F^0) to 6% and the per-period cost of participation (F^1) to 1%.

We take the deterministic labor income profile from Cocco et al. (2005). The variance of the transitory shocks (σ_ε) is set to 10%, while for the permanent income shocks, we follow Guvenen et al. (2014) who estimate countercyclical expected growth rates (0.045 during booms and -0.002 during recessions), as well as countercyclical left-skewness (0.8 in expansions and -1.02 in recessions). As them, we implement this with a mixture of normal distributions for each of the aggregate productivity states and set the probability of the mixture q_1 to 0.49. We then discretize the underlying normal distributions and use eight additional parameters (see equation (14)) so that the mixture distributions exactly match the first four moments during expansions and recessions.

3.6.3 Pension fund

The total replacement ratio of age-65 income ($\lambda^{db} + \lambda^{ss}$) is set to 0.68212, from Cocco et al. (2005). We decompose the two separate components using data on the relative payoffs of DB pension schemes and Social Security based on data from the Social Security Administration, Fred and Public Plans Data thus giving us values of λ^{ss} and λ^{db} equal to 0.4596 and 0.2225, respectively.

We use the Flow of Funds data in the U.S. from 1945 to 2021 to calibrate the portfolio allocation of the representative Defined Benefit Pension Fund's endowment (as a fraction of GDP), i.e. α^P. We consider all three different categories of DB plans (federal, state and local government, and private).[34] To compute the average risky share we assign a risky weight of one to corporate equities and mutual fund shares, and a weight of 0.15 to debt securities like Treasuries municipal securities. repurchase agreements, commercial paper, mortgages, and foreign corporate debt.[35] With these assumptions, we obtain an average risky share of 50.9% (52.7%) for the period 1970 (1980) to

[33] See, for example, Gomes and Michaelides (2008), Favilukis (2013), and Fagereng et al. (2017).

[34] The data reports both funded and total financial assets, where the former exclude unfunded liabilities. We use total financial assets since there are no unfunded assets in our model.

[35] Additional details can be found in Appendix B.

2021.[36]

In the baseline model we consider two calibrations of the portfolio allocation rule (α^P), given by equation (18). In the first case we set b^P equal to zero and therefore set a^P to match the average risky share in the data (52%). In an alternative version we set $b^P = -2$. This implies that the risky share of the Pension fund (α^P) increases (decreases) to 57.23% (46.77%) when the risk-free rate is 2 standard deviations below (above) its unconditional mean. We then re-calibrate a^P so that we again obtain an average risky share of 52%.

4 Baseline results

The results for our baseline model are shown in Table 5, where we also report the corresponding moments in the data. The asset pricing data is taken from CRSP, while stock market participation is computed as the historical average from the Survey of Consumer Finances. Since ours is a real model, we take the mean and standard deviation of the real risk-free from Croce et al. (2012), who adjust the nominal rate for inflation expectations. The consumption data is taken from the NIPA tables provided by the Federal Reserve Bank of St. Louis, and we use the full annual sample from 1930 to 2018.

[INSERT TABLE 5 HERE]

As discussed in Section 2.5, our model incorporates the two margins of adjustment that can be used in the real world to respond to fluctuations in the endowment of the pension fund: the contribution rate of employees and the contributions of the employers. To understand the implications of each of these, in Table 5 we report results for different values of θ^p, which determines the relative weight of the these two adjustment mechanisms. In addition, as discussed in the calibration section, we also consider two alternative parametrizations of the portfolio share equation (18) for the Defined Benefit Pension Fund. In the first one we set α^P equal to a constant, while in the second we allow the asset allocation of the fund to respond to interest rates, capturing a reach-for-yield behavior. The results in columns (i) to (iii) consider the first formulation, while

[36]If we restrict the data to more recent periods the average risky share increases, consistent with the evidence in Ivashina and Lerner (2018) and Munnell and Soto (2007). We use the longer sample since it is more consistent with the data that we use for both returns and consumption.

those in column (iv) consider the second one (for the general case of the adjustment rule). Since our objective here is to understand the underlying economic mechanisms, we report results for the all versions of the model using the same parameter values as for the baseline specification.

4.1 Baseline model

Figure 1 plots the life-cycle wealth accumulation of the two types of agents, in the baseline version of the model (θ^P=0.5). With their high discount factor ($\beta^B = 0.965$), type-B agents have a high savings rate from early on. On the other hand, type-A agents, have a much lower discount factor ($\beta^A = 0.83$), and only accumulate significant wealth closer to retirement.

The asset pricing moments implied by this version of the model are presented in column (i) of Table 5. The low discount factor of the type-A households drives up the risk-free rate in the economy.[37] Nevertheless, we still obtain a low average value (0.93%) because type-A agents accumulate limited wealth, and therefore bond prices are primarily determined by the discount factor of type-B agents. The model also matches extremely well the standard deviation of the real riskless rate: 1.26% versus 1.35% in the data. The baseline economy also matches stock return volatility very well (19.64% versus 19.81% in the data) and delivers an equity premium that is very close to its empirical counterpart (7.08% versus 7.55%). In addition the Sharpe ratio matches very losely the one in the data: 0.37 versus 0.36.

The capital-output ratio in the model is 2.30 which compares with 2.31 from the NIPA tables, or 2.40 if we also include durables.[38] The average stock market participation is 59.5%, which is also close to the 51.1% historical average. The type-B agents, given their high discount factor, have a strong incentive to pay the stock market participation cost early in life, and therefore they quickly become stockholders. The average participation rate among these households is 93.9%. On the other hand, Type-A agents have a low discount factor and, for most of their lives, only accumulate limited savings for precautionary reasons, as shown in Figure 1. Therefore they have a limited incentive to pay the entry cost. Only as they approach retirement do their savings become

[37]The impact of β^A on the risk-free rate is conditional on the stock market participation rate for these households remaining relatively low. If we increase β^A such that the majority of these agents decide to pay the participation cost then, over a certain range of the discount factor, their overall demand for bonds might actually fall.

[38]See Rios-Rull and Santaeulalia-Llopis (2010).

more significant and, as a result, some of them become stockholders temporarily.

4.2 Baseline model with different adjustment rules

Table 5 also reports results for the alternative versions of the baseline model, where the pension fund's endowment is kept constant primarily either through adjustments in the contributions made by employees/workers (column (ii)) or in the contributions made by employers/firms (column (iii)).[39] We discussed the importance of these two channels in the simplified model (Section 3), but it is important to highlight how they now interact with preference heterogeneity and limited stock market participation.

When the adjustment is made primarily by changes in the contributions of workers (θ^P=0.15, column (ii)), the standard deviation of returns is reduced since there is no impact on firm's earnings. On the other hand, the volatility of household disposable income increases, and consequently the standard deviation of total consumption growth is higher in this economy (3.06%). Interestingly in the simpler economy, without preference heterogeneity or limited stock market participation, we had the opposite result (see Table 2): lower consumption growth volatility. This was case since, in that model, all agents were stockholders with a high β they were able to smooth income shocks relatively well. This is indeed still the case here, for the type-B agents in our baseline economy, for whom the volatility of consumption growth is lower when θ^P=0.15 than when θ^P=0.5. However, the type-A agents, with a low β and most of them non-stockholders, are much less able to smooth these shocks. As such, the standard deviation of their consumption growth is now substantially higher (3.19% versus 2.63%). This intuition also explains why the changes in the volatility of consumption growth for type-A households are much larger across the different specifications than those for type-B households.

The increased volatility of disposable income makes agents less willing to become stockholders, but this is counteracted by a higher equity sharpe ratio. The overall participation rate is therefore almost unchanged, and the private capital stock is essentially unchanged. Naturally, the results in column (iii), with θ^P=0.85, provide exactly the reverse conclusions: lower standard deviation of consumption growth, particularly for the type-A agents, higher equity premium, and slightly

[39] As previously, for the case of public DB pension funds, the first alternative captures changes in labor taxes, and the second captures changes in corporate taxes.

lower aggregate capital stock.

4.3 Baseline model with reaching-for-yield behavior by DBPF

In our most general formulation, we express the portfolio rule of the pension as a function of the bond price (equation (18)). More precisely, we set $b^P = -2$, such that the risky share of the Pension fund (α^P) increases (decreases) to 57.23% (46.77%) when the risk-free rate is 2 standard deviations below (above) its unconditional mean. We then re-calibrate α^P such that the average allocation of the pension fund matches the average risky share in the data (52%), as before.

The results for this specification are reported in column (iv) of Table 5. We consider the case where the pension fund's endowment is kept constant through adjustments in both employer and employee contributions as in our baseline model, so these results should be compared to those in column 3. We find that the two formulations in columns (i) and (iv) yield very similar conclusions. The volatility of consumption, stock market participation, equity return, risk-free rate and Sharpe ratio are all extremely similar in both cases. Since these two versions of the model deliver almost identical results, going forward we only consider the specification with $b^P = 0$, thus eliminating one parameter from the model.

In Online Appendix C we discuss additional comparative statics for the baseline model. In particular, we report results obtained with a higher average value of α^P, a comparative statics that was already discussed in the context of the simpler model. In addition, we also report results obtained for a lower value of the discount factor.

5 Comparison with Pure-Pay-as-you-Go model

In this section, we present an alternative model where the pension scheme is a simple pass-through entity, as typically done in asset pricing models with retirement. More precisely, we now have $\bar{W}^P$=0 at all times. As before, we refer to this framework as the Pure-Pay-as-you-Go (PPG) model. The results are shown in Table 6, which also includes those obtained for the baseline model in column (i), and the values of the data in column (iv). To facilitate the comparisons, we report results for the PPG obtained both with the baseline calibration (column ii) and for a

re-calibrated version of model (column iii).

5.1 Results for PPG model with the same calibration

To understand the economic differences between the two models, we first compare the two economies for the same calibration of the structural parameters.

[INSERT TABLE 6 HERE]

Since the PPG model does not feature a pension fund with an accumulated stock of wealth, it does not benefit from the return on those assets as an additional source of income. This implies that the average (defined-benefit) contribution rate for households increases from 4.15% in the baseline economy to 7.14% in this alternative formulation. Since these contributions are no longer subject to fluctuations induced by changes in the wealth of the DBPF, the volatility of consumption growth falls to 2.14%, versus 2.83% in the baseline economy This channel is particularly important for households financing their consumption mostly out of their current disposable income, namely the type-A agents, for whom the standard deviation of consumption growth falls from 2.63% to 1.79%.

Another first-order implication of ignoring the DBPF is that there is less total financial wealth in the economy. It is important to remember that households do not have an incentive to compensate for this since they have the same level of retirement income as before. This is not a model where we have closed down the defined benefit fund (we will consider that experiment later). It is a model where we have failed to account for its endowment.[40] Moreover, the lower volatility of income also decreases precautionary savings, so that households actually have an even lower incentive to accumulate wealth than in our baseline model. As a result, for the same parameter values, capital accumulation by households falls from 3.76 to 3.52 and total capital accumulation is 22% lower (3.52 versus 4.52).

The lower capital stock leads to a higher equity return (10.02% versus 8.02%), and likewise the demand for bonds is also reduced, leading to a higher risk-free rate (4.37% versus 0.93%). The net effect is a lower Sharpe ratio (0.32) and a lower equity premium (5.65%).

[40] In models that do not include that endowment pension payments are fully financed by the contributions of current workers. Therefore, those are on average higher than in our baseline economy, as mentioned above.

5.2 Re-calibrated PPG model

We now re-calibrate the PPG model to deliver the same risk-free rate and stock market participation as the baseline model. For simplicity, we will refer to this as the "rPPG economy", and the results are shown in column (iii) of Table 6. In the new calibration we increase the discount factor of the type-B and type-A households to 0.996 and 0.86, respectively.

5.2.1 Results

With the new preference parameters agents have a stronger preference for savings, and this is reflected in a higher total capital stock (5.66). Likewise, the demand for bonds also increases, and the riskless rate in the re-calibrated model is now identical to the one in the baseline economy. However, the increase in wealth accumulation, particularly in the demand for capital, drives down the return on equity to 4.32%, corresponding to an equity premium of 3.40%, which compares with 7.08% in the baseline economy. In terms of the market price of risk, the rPPG economy delivers a Sharpe ratio of 0.19 versus 0.37 in the baseline economy.

It is important to note that, despite the differences in equity premium and equity Sharpe ratio, the stock market participation rate is not very different in the baseline and rPPG economies. In the baseline economy even non-stockholders are already (indirectly) exposed to stock return volatility through the fluctuations in their contribution rates, thus making them less willing to pay the participation costs.

In the re-calibrated PPG economy, the standard deviation of aggregate consumption growth is lower (2.12% compared with 2.83% in the baseline model). This occurs for two reasons. First because stock return volatility is lower, and second because households do not face the risk of changes in their pension contributions. The lower volatility of consumption growth partially explains the lower equity premium, but the Sharpe ratio comparison makes it clear that this is only part of the story: the ratio of the equity premium in the two model is 2.08, while the ratio of the standard deviation of consumption growth in the two models is only 1.33.[41]

[41] The ratio of the volatility of consumption growth of type-B agents (the majority of stockholders) is even smaller: 1.15.

6 Economy with defined contribution pension scheme only

Defined benefit pension schemes face increasing funding problems due to a wide range of factors, namely increases in longevity not accompanied by increases in retirement age. As a result, in several countries these plans are being progressively replaced with defined contribution schemes, in which individuals are saving into their own private retirement accounts.[42]

6.1 The set-up

In the final section of the paper, we explore the implications of the eventual/potential phasing out of DB pension plans for both asset prices and risk sharing. More precisely, we consider an alternative economy where the defined benefit pension fund does not exist, so

$$\lambda^{db} = 0 \text{ and } W^P = 0. \tag{41}$$

As a result, households must finance their retirement consumption using "only" their own personal savings and social security income.

Although the model is calibrated to the US economy, the results in this section are aimed at making a point about shifts from DB to DC systems, in general. In fact, in the US, the last decades have been characterized by an increase in both DB and DC assets, rather a substitution from one to the other as we consider here.

Note that the analysis in this section differs from the one considered in Section 5. Before we considered an alternative equilibrium where the DB pension scheme still exists (so λ^{db} remained at its baseline value), but there was no pension fund with an endowment of wealth (W^P was equal to zero). In such a model, household retirement income was unchanged relative to the baseline economy. The only difference was how that income was being financed. By contrast, in the current exercise, retirement income is now limited to social security transfers, and therefore, if households wish to keep their retirement consumption unchanged, they must save more during their working lives.

[42] In some countries, the contribution amount is determined by the rules of the pension plan, but in the U.S., this is typically at the discretion of the employee, subject to a cap.

It would be interesting to also solve for the transition dynamics, but this would be computationally challenging. We would have to introduce at least one additional state variable to capture the wealth/size of the DB pension fund, and potentially a second one to capture the fraction of households enrolled in the DB pension plan, or equivalently the fraction of their retirement savings that are being allocated to the DB pension plan. In addition, we would have to make arbitrary assumptions both about how the DB plans would be phased out over time, and about households' expectations regarding this process.

6.2 Capturing defined contribution accounts in the model

Wealth accumulation in defined contribution (DC) accounts (such as those in 401k plans) is subject to important differences relative to wealth accumulation in non-retirement accounts. The former savings have tax benefits, withdrawal penalties and often also benefit from additional employer contributions. In addition, they are likely subject to lower stock market participation costs.

In the model, we do not distinguish between DC wealth and non DC-wealth at the household level, as this would require adding one additional state variable and two additional choice variables.[43] Therefore, we capture the previous features in a reduced form, as described below.

6.2.1 Taxes and illiquidity

In our baseline economy, we do not tax the returns of the DBPF, consistent with current tax regulations. Likewise, wealth accumulation in DC pension accounts is also tax-free.[44] Furthermore, individuals are not supposed to withdraw funds from their DC accounts before retiring, and doing so, incurs a 10% penalty, except under special circumstances.[45]

We incorporate the tax benefits by decreasing the tax rate on total household wealth accumulation by the same ratio as the percentage increase in their wealth relative to the baseline economy. Implicitly we are assuming that the new savings are being made in a DC account, since

[43] The additional state variable would be the balance in the DC account relative to total wealth, or relative to non-DC wealth. The additional choice variables would be the contribution to the DC account and the portfolio allocation in the DC account. Gomes et al. (2009) solve such a model in partial equilibrium.

[44] An additional potential tax benefit of DC accounts is that households pay income taxes at their marginal tax rate during retirement, which might be a lower number. However, this is also the case with the DB system, and in our model income taxes are linear, so that effect is not present anyway.

[45] Special circumstances include facing a hardship event.

they are substituting for the (missing) retirement income from the DB scheme. So, for example, if household wealth in the new economy is 10% higher, we decrease the tax rate on capital gains by 10%, because those new savings are accumulating at a tax-free rate of return. Likewise, we apply the 10% withdrawal penalty to that fraction of household dis-savings only.

Implementing the previous approach requires introducing an iterative loop in our numerical solution, to find the corresponding fixed point. We start with a guess for the percentage increase in private wealth accumulation which determines the adjustments to the capital gains tax rate and the withdrawal penalty. We then solve the model and obtain the implied value of private wealth. Based on this we reset the capital gains tax rate and the withdrawal penalty, and solve the model again until convergence.

6.2.2 Employer contributions

As with defined benefit schemes, employee contributions to their DC pension account are typically accompanied by additional contributions from the employer, which can be linked to the size of the employee's contribution (i.e. "matching contributions"), depending on the specific features of the pension plan. Given our set-up, the contribution rate captures both the direct salary deductions taken from employee wages, and the top-up contributions made by firms since both represent a payment that is proportional to total wage compensation.

6.2.3 Stock market participation costs

It has been shown that DC accounts significantly reduced stock market participation costs by making it much easier for households to both initiate and maintain an equity position (e.g. Choukhmane and de Silva (2022)).[46] To explore this channel we consider two alternative versions of the DC-only economy. In the first version we keep the participation costs unchanged, while in the second we reduce the stock market entry (per-period) costs from our baseline values of 6% (1%) to 3% (0.1%).

[46]In several 401K plans the default investment option is a fund with (at least some) equity exposure. Households are therefore automatically assigned to such investments unless they explicitly make a different choice.

6.3 Results

6.3.1 Household wealth accumulation

When the DBPF is shut down, households must increase their personal saving to finance retirement. On the other hand, since they no longer face fluctuations in their DB contribution rates (which are now zero), the volatility of their disposable labor income is smaller, and therefore precautionary savings should decrease. Consequently, the net effect on household wealth accumulation is theoretically ambiguous.

Figure 2*a* plots wealth accumulation over the life-cycle for the two groups of agents, both in the baseline economy and in the DC-only economy with the same stock market participation costs. From mid-life onward, wealth accumulation is substantially higher for both type-A and type-B agents. The absence of a defined benefit pension leads them to increase their private savings to finance retirement. The savings behavior early is life, however, reflects both the increased retirement savings motive and the decreased precautionary savings motive.

The trade-off is visible in Figure 2*b*, which plots the ratio of wealth accumulation in the two economies for the pre-retirement period. Type-A agents, with their low discount factor, save early in life for precautionary reasons. Therefore, in the DC-only economy, their savings remain fairly constant at this stage of the life-cycle. Only from age 30 onward is their average wealth higher in the DC-only economy than in the baseline economy. By contrast, the type-B agents save for retirement from early on, and therefore their average wealth accumulation exceeds the one in the baseline economy from the start.

6.3.2 Asset pricing and macro moments

In Table 7 we compare several aggregate quantities in the baseline economy and the DC-only economy, namely asset prices, consumption volatility, capital accumulation and stock market participation. As previously discussed, for the DC-only economy we consider two scenarios for the stock market participation costs. To facilitate the exposition, we first compare our baseline economy of column (i) with the results in column (ii), where the participation costs are held constant. We discuss the results in column (iii) later in the paper.

[INSERT TABLE 7 HERE]

As shown in Figures 2*a* and 2*b*, private household wealth accumulation increases in the DC-only economy, as households must now save more for retirement, and this dominates the reduction in precautionary savings. As a result, the private capital stock increases. However, households don't have an incentive to fully compensate for the (missing) wealth accumulation of the pension fund. If they wanted to have saved more when the pension fund existed, they could have done so. By contrast the pension fund was forcing them to save more for retirement than they wished, particularly for the type-A households with their low discount factor.[47] Therefore, the actual increase in private capital stock is modest (from 3.76 to 3.88), and the total capital stock in the economy in now lower (3.88 versus 4.52).

The impact on asset prices is more complicated because the relative demand for bonds and stocks has changed for two reasons. First, the asset demand curves of households are different from those of the DB pension fund. Second, the demand curves of households have shifted in the new equilibrium. Therefore, we require a calibrated quantitative model to understand the impact on equilibrium returns.

A lower capital in the economy implies a slightly higher average return on equity (8.68%). From the household side, there are two counteracting effects on their optimal risky share. They now save more, and the risky share is a decreasing function of wealth, but they also face less background risk. Therefore, the bond demand from households doesn't shift enough to compensate for the absence of the pension fund, which had a relatively higher demand for bonds. Therefore, the riskless rate is now higher (3.62%) and as a result, the equity premium falls to 5.06%, and the Sharpe ratio is also lower (0.29).

As households increase their private savings and face less background risk they have a stronger incentive to pay the participation costs and invest in stocks, but this is counteracted by the lower average equity premium. These two effects are visible when we look at the behavior of the two types of agents separately. Among type-A households, that relied more on the pension fund to

[47] Figure 2b shows that, with the shift to the DC-only economy, type-A households increase their savings by more, when this is computed as a fraction. But this fraction is relative to their own savings, and merely reflects the fact that their private savings were negligible as shown in Figures 1 and 2a. Figure 2a reveals that it is the type-B households that increase their wealth by more, in absolute terms.

finance their retirement, the first effect dominates and stock market participation increases from 25.1% to 33.5%. On the other hand, type-B households already had high savings, and therefore pension wealth represented a small fraction of their total retirement resources. As a result, the lower equity premium channel dominates, and stock market participation for this group actually falls, from 93.9% to 89.4%. Combining the two groups, we find that the percentage of stockholders in the economy is only slightly increased.

6.3.3 Consumption smoothing

Interestingly, total consumption volatility decreases in the DC-only economy. Firms now face less risk, since they don't have to provide the defined benefit pension to households, and therefore the volatility of returns falls. In equilibrium this decreases the standard deviation of consumption growth. In addition, in the absence of the pension fund, households have a less volatile disposable income, which further reduces their consumption volatility. This second effect is particularly important for the type-A agents, who are also the ones less able to smooth consumption shocks. As a result the decrease in the standard deviation of consumption growth is more pronounced for this group.

It has been argued that a transition to DC pension schemes will increase household consumption volatility since they don't have a (largely) guaranteed income stream at retirement anymore. We see this in the last column of Table 7: the standard deviation of consumption growth for retirees increases from 1.7% in the baseline economy to 2.4% in the DC-only economy. However, this is not the full story, because for workers this volatility actually decreases.

As previously discussed, in a DB system the volatility of income for retirees is only low because workers and firms are buffering the shocks to the pension fund's endowment. In the absence of the DB pension fund they are no longer required to provide this insurance to retirees. As a result, the standard deviation of both stock returns and disposable income before retirement, decrease. This is reflected in a lower standard deviation of consumption growth for workers: from 11.4% (8.4%) to 10.5% (7.8%) for those in the age group 20-35 ($36-65$). This explains why, the volatility aggregate consumption growth is lower in the DC-only economy.

6.3.4 DC-only economy with lower participation costs

The results in column (iii) of Table 7 consider a DC-only economy with lower stock market participation costs, as suggested by the evidence in Choukhmane and de Silva (2022). More precisely, we set the entry cost (F^0) to 3% and the per-period cost (F^1) to 0.1%.

Interestingly the equilibrium in the DC-only economy is very similar in both participation costs scenarios. Allowing for a substantial reduction in stock market participation costs (column ii), naturally increases stock market participation (to 83%), but has a modest impact on aggregate quantities and prices: wealthier households were already investing in equities, and the new stockholders have much less wealth.[48]

The higher stock market participation further decreases the demand for bonds and increases the demand for equity, leading to an even higher riskless rate and lower equity premium, although in both cases the differences are small. As more households become stockholders the capital stock increases from 3.88 to 3.97.

7 Conclusion

Our paper is part of the growing literature emphasizing the importance of financial intermediaries in determining equilibrium asset prices. We solve a general equilibrium asset pricing model with an explicit defined benefit pension plan, and show that the augmented model is better able to jointly match the riskless rate, equity Sharpe ratio and volatility of consumption growth. The results highlight the role of the asset demands of the defined benefit pension fund and identify a new risk channel, coming from fluctuations in pension contributions.

Our results have implications for the role of DB pension wealth in determining asset prices. Greenwood and Vissing-Jorgensen (2018) find strong evidence for this in the pricing of long maturity government bond yields, by identify exogenous shocks to DB wealth in a cross-section of countries. Klingler and Sundaresan (2019) find similar evidence for the pricing of long maturity interest rate swaps.

We further use the calibrated model to solve for an equilibrium where the DB pension plan

[48] This is similar to the point made by Gomes and Michaelides (2008) about the impact of participation costs on asset prices.

has been replaced by a DC pension plan, a trend that we are observing in several countries, due to the funding problems of most existing DB plans. The new economy is characterized by less precautionary savings and more retirement savings, which differentially affect wealth accumulation at different stages of the life cycle. In the new steady-state, the riskless rate is higher and the Sharpe ratio on equities is lower. Risk sharing in the economy is also affected, with the volatility of consumption growth increasing for retirees but decreasing for workers.

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Table 1: Comparison: DBPF Model and PPG Model.

Panel A: Returns and Macro Moments					
Variable	Moment	Models			
		(i) DBPF	(ii) PPG	(iii) rPPG	(iv) rPPG2
r^B	Mean	0.97%	3.26%	0.96%	0.93%
r^B	St. Dev.	1.60%	1.81%	1.67%	1.94%
r^m	Mean	5.52%	6.28%	3.75%	4.47%
r^m	St. Dev.	19.69%	17.93%	17.84%	20.22%
$r^m - r^B$	Mean	4.55%	3.02%	2.79%	3.53%
$\frac{Mean(r^m - r^B)}{Std.Dev.(r^m - r^B)}$		0.23	0.17	0.16	0.18
Cons. growth	St. Dev.	5.09%	4.47%	4.41%	4.98
K	Mean	5.31	4.53	5.95	5.72
$Kprivate$	Mean	4.44	4.53	5.95	5.72
K/Y	Mean	2.56	2.30	2.75	2.68

Panel A of Table 1 reports asset pricing moments (where r^m denotes levered r^K), aggregate consumption volatility, capital and the capital/output ratio for the DBPF model (column (i)), the PPG model for the same parameter values (column (ii)) and the two versions of the re-calibrated PPG model: "rPPG" (column (iii)) and "rPPG2" (column (iv)).

Panel B: Cross-Sectional Cons. Growth Volatility			
Age Groups	20-35	36-65	>= 66
DBPF	10.1%	7.6%	1.0%
rPPG	9.2%	7.0%	1.2%

Panel B of Table 1 reports the consumption-growth volatility in the DBPF and rPPG models (respectively, rows 2 and 3), for different age groups.

Table 2: DBPF Model for alternative pension fund adjustment rules.

Variable	Moment	DBPF Model		
		(i) (θ^P=0.5)	(ii) (θ^P=0.15)	(iii) (θ^P=0.85)
r^B	Mean	0.97%	1.11%	0.82%
r^B	St. Dev.	1.60%	1.43%	1.78%
r^m	Mean	5.52%	5.43%	5.64%
r^m	St. Dev.	19.69%	18.39%	21.03%
$r^m - r^B$	Mean	4.55%	4.32%	4.82%
$\frac{Mean(r^m - r^B)}{Std.Dev.(r^m - r^B)}$		0.23	0.24	0.23
Cons. growth	St. Dev.	5.09%	4.91%	5.31%
K	Mean	5.31	5.28	5.33
$Kprivate$	Mean	4.44	4.42	4.45
K/Y	Mean	2.56	2.55	2.56

Table 2 reports asset pricing moments (where r^m denotes levered r^K), consumption volatility, capital and capital/output ratio for different versions of the DBPF model. The results in column (i) refer to a version of the model where the pension fund's wealth is kept balanced by changes in both the contribution rate of employees and the contributions of employers. The results in column (ii) (resp. iii) consider a formulation where 85% of the adjustment is done by varying employee (resp. employer) contributions.

Table 3: DBPF Model for Different Portfolio Allocations of the Pension Fund.

Variable	Moment	DBPF Model	
		(i)	(ii)
		Baseline	α^P=0.8
r^B	Mean	0.97%	1.37%
r^B	St. Dev.	1.60%	1.75%
r^m	Mean	5.52%	5.67%
r^m	St. Dev.	19.69%	20.67%
$r^m - r^B$	Mean	4.55%	4.30%
$\frac{Mean(r^m - r^B)}{Std.Dev.(r^m - r^B)}$		0.23	0.21
Cons. growth	St. Dev.	5.09%	5.06%
K	Mean	5.31	5.16
$Kprivate$	Mean	4.44	3.98
K/Y	Mean	2.56	2.51

Table 3 reports asset pricing moments (where r^m denotes levered r^K), consumption volatility, capital and capital/output ratio for the DBPF model under different calibrations of the portfolio allocation of the DBPF. The results in column (i) report the baseline calibration with a risky share of 0.52, while the results in column (ii) correspond to a calibration where the pension fund's risky share is increased to 0.8.

Table 4: Calibrated Parameters for the Baseline Models

Aggregate Variables		Household Variables	
Technology		Preferences	
π_r	16/37	γ^A	6
α	34%	γ^B	6
Mean(δ)	10%	β^A	0.83
Vol(δ)	10%	β^B	0.965
		ψ^A	0.25
Debt and Taxes		ψ^B	0.5
B/Y	42%		
τ^B	20%	Participation Costs	
τ^K	40%	F^0	6%
τ^E	100%	F^1	1%
Pension Fund		Retirement Income	
α^P	52%	λ^{ss}	0.4596
θ^P	0.5	λ^{db}	0.2225

Table 4 reports the baseline calibration of the different parameters of the model. The household-level income processes are given by a combination of values from Cocco et al. (2005), Guvenen et al. (2014) and own estimations, as described in the calibration section.

Table 5: Baseline Results.

Variable	Moment	Baseline Model				Data
		(i) (θ^P=0.5) ($b^P = 0$)	(ii) (θ^P=0.15) ($b^P = 0$)	(iii) (θ^P=0.85) ($b^P = 0$)	(iv) (θ^P=0.5) ($b^P = -2$)	(v)
r^B	Mean	0.93%	0.82%	0.95%	0.96%	0.86%
r^B	St. Dev.	1.26%	1.22%	1.29%	1.26%	1.35%
r^m	Mean	8.02%	8.05%	8.06%	8.00%	8.17%
r^m	St. Dev.	19.64%	18.34%	20.94%	19.64%	19.81%
$r^m - r^B$	Mean	7.08%	7.22%	7.11%	7.04%	7.55%
$\frac{Mean(r^m - r^B)}{Std.Dev.(r^m - r^B)}$		0.37	0.40	0.34	0.36	0.36
Cons. growth (all)	St. Dev.	2.83%	3.06%	2.60%	2.83%	2.90%
Cons. growth (A)	St. Dev.	2.63%	3.19%	2.13%	2.62%	(-)
Cons. growth (B)	St. Dev.	3.16%	3.05%	3.25%	3.16%	(-)
K	Mean	4.52	4.52	4.51	4.51	(-)
$Kprivate$	Mean	3.76	3.76	3.76	3.76	(-)
W^P/Y	Mean	0.75	0.75	0.75	0.74	0.67
K/Y	Mean	2.30	2.30	2.30	2.30	2.31
Participation (all)	Mean	59.5%	59.2%	60.3%	59.4%	51.1%
Participation (A)	Mean	25.1%	24.9%	26.5%	25.0%	(-)
Participation (B)	Mean	93.9%	93.5%	94.2%	93.9%	(-)

Table 5 reports asset pricing moments (where r^m denotes levered r^K), consumption volatility, stock market participation, capital, capital/output ratio and pension wealth to GDP for the baseline model, columns (i) to (iv), and the data in column (v). The results in columns (i) and (iv) refer to a version of the model where the pension fund's wealth is kept balanced by both changes in the contribution rates of employees and employers. The results in column (ii) (resp. iii) consider a formulation where 85% of the adjustment is done by varying employee (resp. employer) contributions. In columns (i) to (iii) the risky share of the pension fund is constant, while in column (iv) it is a function of the current interest rate. The asset pricing data is taken from CRSP, with the risk-free data from Croce et al. (2012). Stock market participation is from the Survey of Consumer Finances, while consumption and capital-output data is from the NIPA tables provided by the Federal Reserve Bank of St. Louis. Pension wealth data is also from the Federal Reserve Bank of St. Louis.

Table 6: Comparison of Baseline Model and PPG Model.

Variable	Moment	Models			Data
		(i) Baseline	(ii) PPG	(iii) rPPG	(iv)
r^B	Mean	0.93%	4.37%	0.93%	0.86%
r^B	St. Dev.	1.26%	1.22%	1.01%	1.35%
r^m	Mean	8.02%	10.02%	4.32%	8.17%
r^m	St. Dev.	19.64%	17.89%	17.74%	19.81%
$r^m - r^B$	Mean	7.08%	5.65%	3.40%	7.55%
$\frac{Mean(r^m - r^B)}{Std.Dev.(r^m - r^B)}$		0.37	0.32	0.19	0.36
Cons. growth (all)	St. Dev.	2.83%	2.14%	2.12%	2.90%
Cons. growth (A)	St. Dev.	2.63%	1.79%	1.69%	(-)
Cons. growth (B)	St. Dev.	3.16%	2.61%	2.75%	(-)
K	Mean	4.52	3.52	5.66	(-)
$Kprivate$	Mean	3.76	3.52	5.66	(-)
W^P/Y	Mean	0.75	0	0	0.67
K/Y	Mean	2.30	1.95	2.67	2.31
Participation (all)	Mean	59.5%	60.1%	52.0%	51.1%
Participation (A)	Mean	25.1%	30.1%	12.3%	(-)
Participation (B)	Mean	93.9%	90.2%	91.8%	(-)

Table 6 reports asset pricing moments (where r^m denotes levered r^K), consumption volatility, stock market participation, capital and the capital/output ratio for the baseline model (column (i)), the PPG model for the same parameter values (column (ii)), the re-calibrated PPG ("rPPG") model (column (iii)) and in the data (column (iv)). The asset pricing data is taken from CRSP. The mean and volatility of the real risk free is taken from Croce et al. (2012). Stock market participation is computed from the Survey of Consumer Finances. The consumption and capital-output data is taken from the NIPA tables provided by the Federal Reserve Bank of St. Louis. Pension wealth data is also from the Federal Reserve Bank of St. Louis.

Table 7: Baseline Economy versus DC-only Economy

Variable	Moment	(i) Baseline	(ii) DC-Only	(iii) DC-Only (Lower Costs)
r^B	Mean	0.93%	3.62%	3.64%
r^B	St. Dev.	1.26%	1.22%	1.21%
r^m	Mean	8.02%	8.68%	8.18%
r^m	St. Dev.	19.64%	17.87%	17.85%
$r^m - r^B$	Mean	7.08%	5.06%	4.54%
$\frac{Mean(r^m - r^B)}{Std.Dev.(r^m - r^B)}$		0.37	0.29	0.26
Cons. growth (all)	St. Dev.	2.83%	2.01%	2.09%
Cons. growth (A)	St. Dev.	2.63%	1.78%	1.86%
Cons. growth (B)	St. Dev.	3.16%	2.37%	2.41%
K	Mean	4.52	3.88	3.97
$Kprivate$	Mean	3.76	3.88	3.97
W^P/Y	Mean	0.75	0.0	0.0
K/Y	Mean	2.30	2.08	2.11
Participation (all)	Mean	59.5%	61.5%	82.7%
Participation (A)	Mean	25.1%	33.5%	69.7%
Participation (B)	Mean	93.9%	89.4%	95.8%
Cons. growth (20-35)	St. Dev.	11.4%	10.5%	10.5%
Cons. growth (36-65)	St. Dev.	8.4%	7.8%	7.7%
Cons. growth (>= 66)	St. Dev.	1.7%	2.4%	2.3%

Table 7 reports asset pricing moments (r^m denotes levered r^K), consumption volatility, stock market participation, capital and the capital/output ratio for the baseline economy (column (i)), and for an alternative economy where the defined-benefit pension fund is closed and a new equilibrium steady-state has been reached ("DC-only" economy, column ii). Column (iii) reports results for an alternative version of the "DC-only" economy where the stock market participation costs have been reduced to $F^0 = 3\%$ and $F^1 = 0.1\%$.

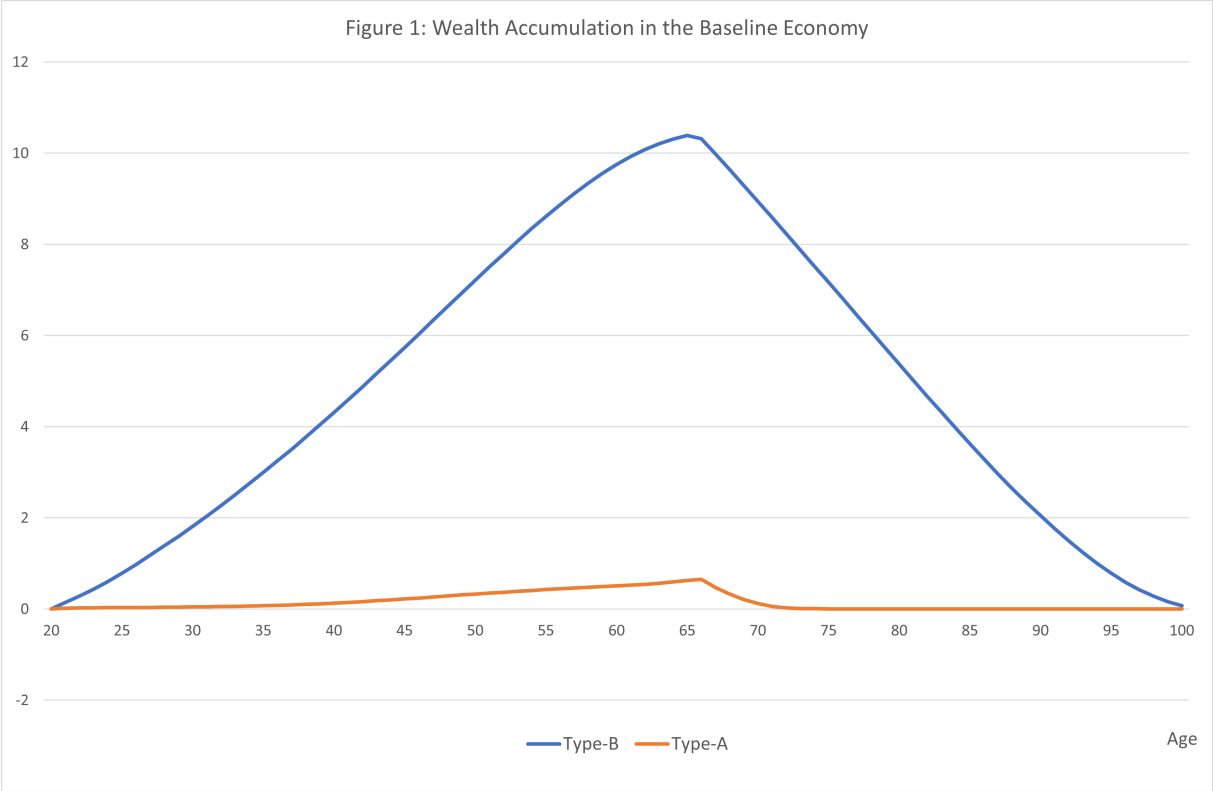


Figure 1 reports the average wealth accumulation over the life-cycle (ages 20 to 100) for both Type-A households and Type-B households, in the baseline economy.

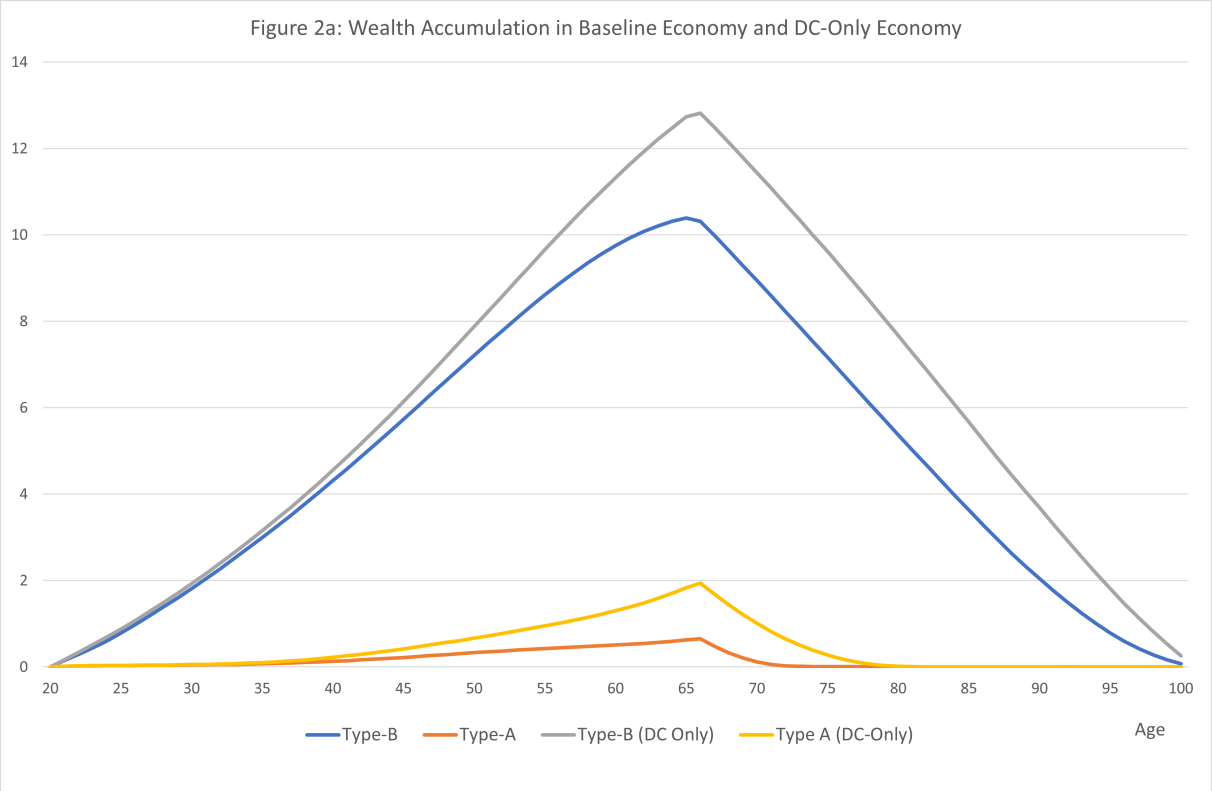


Figure 2a reports the average wealth accumulation over the life-cycle (ages 20 to 100) for both Type-A households and Type-B households, in the baseline economy and in the DC-only economy.

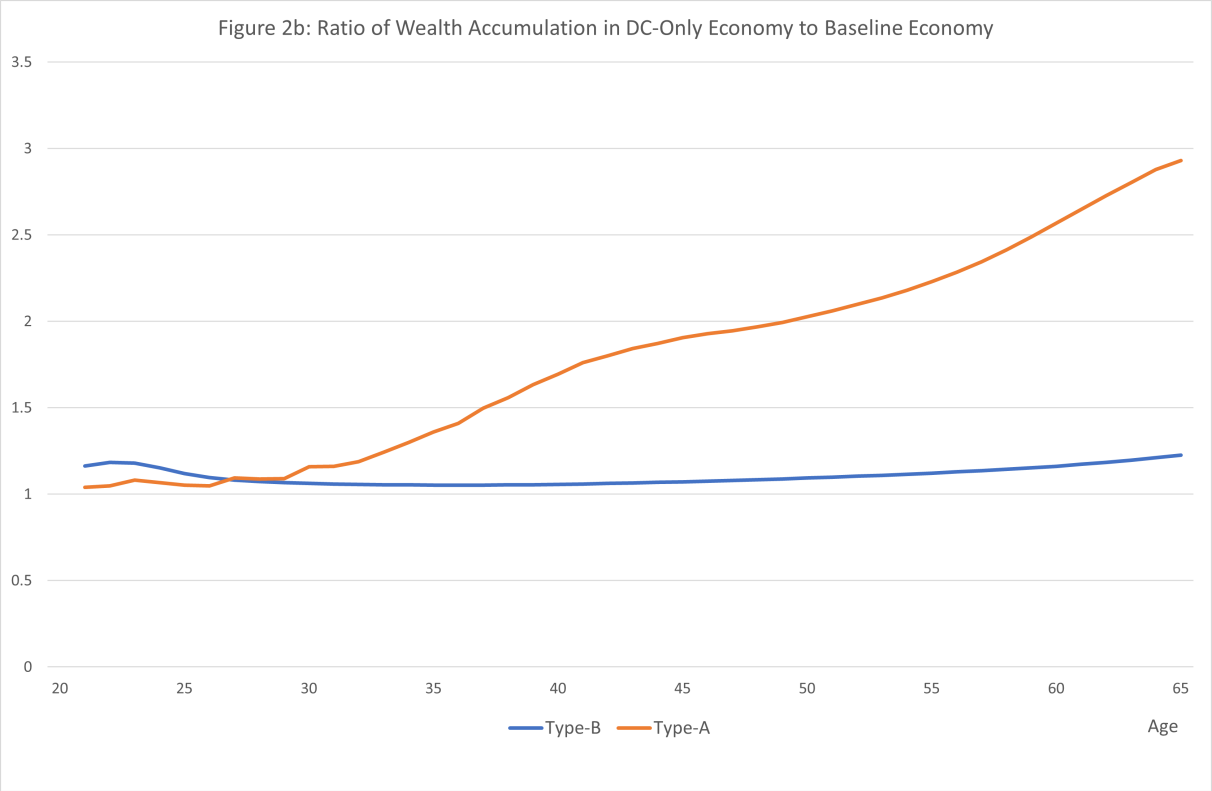


Figure 2b reports the ratio of average wealth accumulation over the life-cycle in the DC-only economy relative to the baseline economy before retirement (ages 20 to 65). Results are shown separately for the two types of agents, type-A and type-B.

Appendix:

Appendix A: Pension Fund Endowment

We set endowment of the pension fund (W^P) such that the fund is fully funded if future return realizations (for stocks and bonds) are equal to their unconditional means. That is, suppose the system is shut down and no new generations/employers are enrolled. In that case, the current endowment (plus its return) is exactly enough to pay off its (net) liabilities, in expectation.

More specifically, assume that the fund is closed to new members, in an arbitrary year t^*. Then it must still pay retirement benefits to current retirees. It will also continue to receive contributions from current workers (the last cohorts to do so), and will have to pay them pension benefits once they retire. In this scenario, the pension payments in year t ($>= t^*$) are given by:

$$Payments_t = \begin{cases} \sum_{a=66}^{100} \int_{i \in I^a} [\lambda^{db} \exp(f(a^R)) w_t P^i_{a^R t^R}] di & \text{t} \in [t^*, t^* + 45] \\ \sum_{a=66+t-(t^*+45)}^{100} \int_{i \in I^a} [\lambda^{db} \exp(f(a^R)) w_t P^i_{a^R t^R}] di & \text{t} \in]t^* + 45, t^* + 80] \\ 0 & t > t^* + 80 \end{cases} \tag{42}$$

Until year $t = t^* + 45$ the pension payments remain as before: the fund must make payments to all agents that are retired, i.e. all those between ages 66 and 100. At $t = t^* + 45$, the last cohort of DB members retires. So, for each following year, the pension payments shrink, as there is one less cohort of individuals to whom they are due. And naturally these payments become zero for t greater than $t^* + 80$, when there are no more remaining members of the plan still alive.

We denote by $Contributions_t$ the value of the contributions at time t, under the scenario in which the fund is closed at time t^*.[49]. The required endowment of the fund is then given by

$$\bar{W}^P = \sum_{t=1}^{80} (Payments_t - Contributions_t)/(1 + E[R^P])^t \tag{43}$$

where $E[R^P]$ is the expected return on the pension fund's endowment.[50]

[49] Each year there is one less cohort contributing until year $t = t^* + 45$, when the last cohort of DB members retires, and the contributions become zero. The contribution rules are described in subsection 2.4.3)

[50] This is not a fair value/economic calculation of the fund's funding ratio. Such valuation would assign appropriate discount rates to future contributions and liabilities. This formula simply equates the expect future payoffs, without any risk adjustment.

Appendix B: Pension Fund Data

We use Flow of Funds data to calculate the size of total funded and unfunded DB plans in the U.S., from 1945 to 2021. There are three different categories of DB plans; federal, state & local government and private ones. For each of these three categories we calculate the ratio of total financial assets to GDP and then add them up to calibrate the size of the DB pension plan assets relative to GDP in the model. In the data there are both funded and total financial assets (that include unfunded liabilities). We use total financial assets in calibrating the size of the DB plan.

The specific series are as follows. For the federal government, total retirement financial assets are given by series FL344090005.A. For state and local governments the relevant series are FL224090045.A (funded and unfunded) and FL223073045.A (unfunded). For the private plans, total financial assets are given by FL574090045.A.

To calibrate the average risky share we compute the share of funded assets invested in risky assets for each category separately, and take an asset size-weighted average of the three. For private plans risky asset holdings includes corporate equities (FL573064143.A) and mutual fund shares (FL573064243.A). We also assign a 0.15 risky weight to Treasury securities (FL573061143.A), agency- and GSE-backed securities (FL573061743.A), corporate and foreign bonds (FL573063043.A), other mortgage assets (FL573065043.A), unallocated insurance contracts (FL573095405.A), commercial paper (FL573069143.A) and pension fund contributions receivable (FL573074043.A). Finally, we divide this number by their total funded financial assets to obtain a risky share.

For the state and local government DB plans we follow a similar weighting scheme with corporate equities (FL223064145.A) and mutual fund shares (FL223064243.A) getting a weight of one, and a 0.15 weight to debt securities (FL224022045.A), mortgages (FL223065043.A) and repurchase agreements (FL222051043.A).

For the federal government DB plans (that have by far the most conservative allocations of the three groups) we use the same approach. Debt securities (LM344022005.A) have a 0.15 weight, as well as agency- and GSE-backed securities (LM343061705.A), municipal securities held by the National Railroad Retirement Investment Trust (LM343062033.A) and corporate and foreign bonds (LM343063005.A).

ONLINE APPENDIX

[NOT FOR PUBLICATION]

Online Appendix for "Asset Pricing and Risk Sharing Implications of Alternative Pension Plan Systems"

Online Appendix A: Solution Method

We follow a variant of the Krusell and Smith (1998) approach where households predict the next period capital stock using the first moment of the endogenously evolving wealth distribution. In our case we have two assets and we need to forecast the evolution of the bond price as well; the bond price is in turn clearing the government bond market every period.

There are seven state variables in this system; age (a), normalized cash on hand (x^i_{at}), the stock market participation status (E^i_a, a zero-one variable indicating whether the entry cost has been paid or not), and the four aggregate variables from the forecasting equations. The guess-and-verify equation for the log-capital stock is given by

$$log(k_{t+1}) = a_k + b_k \cdot log(k_t) \tag{44}$$

where each coefficient in the equation depends on the current realization of the aggregate productivity shock and the stochastic depreciation shock. The guess-and-verify equation for the log-bond price is similarly given by

$$log(P^B_{t+1}) = a_P + b_P \cdot log(k_t) + c_P \cdot log(P^B_t) \tag{45}$$

where again each coefficient in the bond pricing function depends on the current realization of the aggregate productivity shock and the stochastic depreciation shock.

The full optimization problem is written as:

$$V_a(x^i_{at}, E^i_a; k_t, U_t, \eta_t, P^B_t) = \underset{\{k^i_{a+1,t+1}, b^i_{a+1,t+1}\}_{a=1}^{A}}{Max} \{(1-\beta)(c^i_{at})^{1-1/\psi} \tag{46}$$

$$+\beta(E_t[(\frac{P^i_{a+1,t+1}}{P^i_{at}}(1+g)^{\frac{1}{1-\alpha}})^{1-\rho} p_a V^{1-\rho}_{a+1}(x^i_{a+1,t+1}, E^i_{a+1}; k_{t+1}, U_{t+1}, \eta_{t+1}, P^B_{t+1})])^{\frac{1-1/\psi}{1-\rho}}\}^{\frac{1}{1-1/\psi}},$$

subject to the constraints:

$$k^i_{a+1,t+1} \geq 0 \quad , \quad b^i_{a+1,t+1} \geq 0 \tag{47}$$

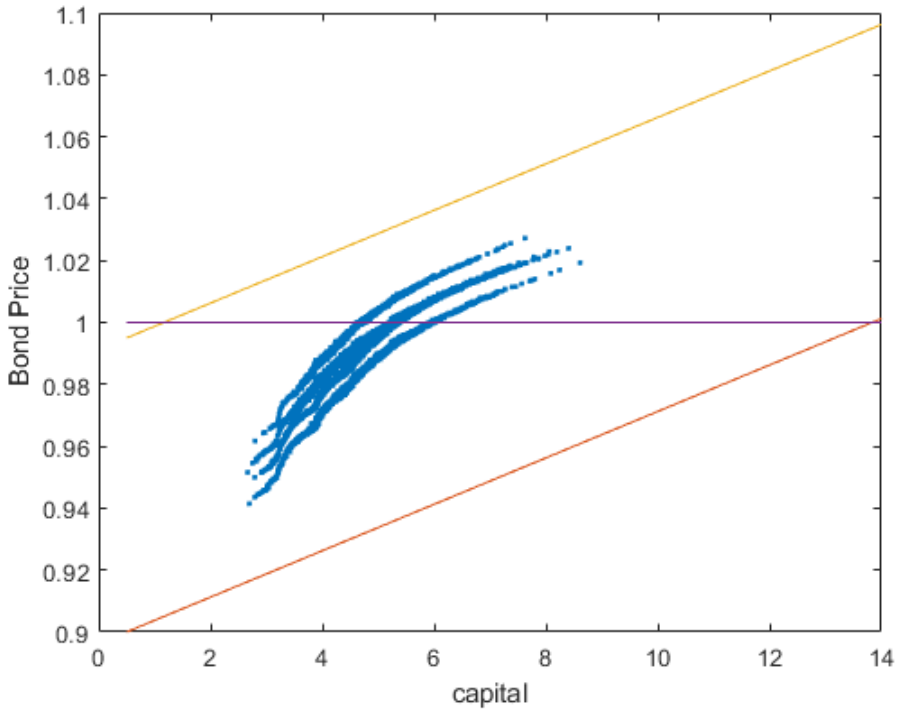
$$c^i_{at} + b^i_{a+1,t+1} + k^i_{a+1,t+1} = x^i_{at} \tag{48}$$

and

$$x^i_{a+1,t+1} = \begin{cases} \frac{\left[k^i_{a+1,t+1}(1+(1-\tau_K)r^K_{t+1})+b^i_{a+1,t+1}(1+(1-\tau^K)r^B_{t+1})\right]}{[(P^i_{a+1,t+1}/P^i_{at})(1+g)^{\frac{1}{1-\alpha}}]} \\ +\varepsilon^i(1-\tau^s)w_{t+1} - I^i_E F^0 w_{t+1} - I^i_S F^1 w_{t+1} \quad a < a^R \\ \frac{\left[k^i_{a+1,t+1}(1+(1-\tau_K)r^K_{t+1})+b^i_{a+1,t+1}(1+(1-\tau^K)r^B_{t+1})\right]}{[(P^i_{a+1,t+1}/P^i_{at})(1+g)^{\frac{1}{1-\alpha}}]} \\ +(\lambda^{db}+\lambda^{ss})w_{t+1} - I^i_E F^0 w_{t+1} - I^i_S F^1 w_{t+1} \quad a > a^R \end{cases}, \tag{49}$$

the stochastic process for individual labor productivity, and the forecasting equations.

We use cubic spline interpolations for wealth and linear interpolation for aggregate states (the next period capital stock and the bond price). Once the coefficients in the forecasting equations have converged in an outer loop, we check (i) that the relevant conditional R-2 results are very high (typically above 99.99 percent) and (ii) that all simulated individual and state variables are within the range assumed when setting the parameters of the model. We use a parallel grid (rather than a rectangular grid) for the bond pricing function as a function of the capital stock, as shown in the graph below, together with the simulation results:



Online Appendix B: Dynamic Programming Problem and Equilibrium Conditions for the DBPF Model

We write the model in a stationary form, by scaling all variables by aggregate productivity growth ($G_t^{\frac{1}{1-\alpha}}$). We further normalize the individual variables by the current level of permanent labor income (P_{at}^i), to reduce the dimensionality of the state vector by one variable.

After the normalizations, the individual maximization problem has six state variables. Age (a), normalized cash on hand (x_{at}^i) and the four aggregate variables from the forecasting equations ((26) and (27)). The full optimization problem is written as:

$$V_a(x_{at}^i; k_t, U_t, \eta_t, P_t^B) = \underset{\{k_{a+1,t+1}^i, b_{a+1,t+1}^i\}_{a=1}^A}{Max} \{(1-\beta)(c_{at}^i)^{1-1/\psi} \tag{50}$$
$$+\beta(E_t[(\frac{P_{a+1,t+1}^i}{P_{at}^i}(1+g)^{\frac{1}{1-\alpha}})^{1-\rho} p_a V_{a+1}^{1-\rho}(x_{a+1,t+1}^i; k_{t+1}, U_{t+1}, \eta_{t+1}, P_{t+1}^B)])^{\frac{1-1/\psi}{1-\rho}}\}^{\frac{1}{1-1/\psi}},$$

subject to the constraints:

$$k_{a+1,t+1}^i \geq 0 \tag{51}$$

$$b_{a+1,t+1}^i + k_{a+1,t+1}^i \geq 0 \tag{52}$$

$$c_{at}^i + b_{a+1,t+1}^i + k_{a+1,t+1}^i = x_{at}^i \tag{53}$$

and

$$x_{a+1,t+1}^i = \begin{cases} \frac{[k_{a+1,t+1}^i r_{t+1}^K) + b_{a+1,t+1}^i (1+r_{t+1}^B)]}{[(P_{a+1,t+1}^i / P_{at}^i)(1+g)^{\frac{1}{1-\alpha}}]} + \varepsilon^i (1 - \tau^{ss} - \tau_t^{db}) w_{t+1} & a < a^R \\ \frac{[k_{a+1,t+1}^i r_{t+1}^K) + b_{a+1,t+1}^i (1+r_{t+1}^B)]}{[(P_{a+1,t+1}^i / P_{at}^i)(1+g)^{\frac{1}{1-\alpha}}]} + (\lambda^{db} + \lambda^{ss}) w_{t+1} & a > a^R \end{cases}, \tag{54}$$

the stochastic process for individual labor productivity (equations (11) to (14)), and the forecasting equations (26) and (27).

The individual takes as given all aggregate variables, i.e. capital stock, returns, bond price and wages. Equilibrium prices and quantities and determined by the following set of conditions:

1. Firms hire capital and labor to maximize profits (equations (7) and (8)).

2. Individuals choose their consumption and asset allocation to maximize their expected life-time utility, i.e. maximize equation (50) subject to the constraints described above.

3. The social security system is balanced at all times:

$$\int_i \int_{a \in I_W} \tau^{ss} L^i_{at} w_t da di = \int_i \int_{a \in I_R} [\lambda^{ss} \exp(f(a^R)) w_t P^i_{at^R}] da di \ , \tag{55}$$

where the left-hand side is integrated over all workers ($a \in I_W$), while the right-hand side is integrated over retirees ($a \in I_R$). This equation determines the value of the social security tax/contribution (τ^{ss}) for a given value of the social security retirement replacement ratio (λ^{ss}).

4. The defined benefit pension fund is in a balanced path with a constant endowment (W^P), and an endogenous contribution rate (τ^{db}_t) as given by Equation (19).

5. Bequests are fully taxed and the proceeds are used to finance government expenditures that have no productive value and do no enter the households' utility functions.

6. All markets clear, specifically the markets for capital, bonds and the consumption good:[51]

$$k_t = \int_i \int_a P^i_{a-1,t-1} k^i_{at} da di \tag{56}$$

$$b_t = \int_i \int_a P^i_{a-1,t-1} b^i_{at} da di \tag{57}$$

$$U_t k^{\alpha}_t L^{1-\alpha}_t = \frac{C^G_t}{G^{\frac{1}{1-\alpha}}_t} + \ (1+g)^{\frac{1}{1-\alpha}} k_{t+1} - (1-\delta_t) k_t + \int_i \int_a P^i_{at} c^i_{at} da di \tag{58}$$

By Walras' law, once two of these equations are verified, the third is also automatically satisfied.

7. Household expectations for market prices (equations (26) and (27)) are verified in equilibrium.

[51] The market for labor is trivial since there is no labor-leisure choice.

Online Appendix C: Additional Comparative Statics for the Baseline Model

The DB pension fund affects the equilibrium in our economy because of its portfolio allocation, and because it forces households to save for retirement. Here, we discuss additional comparative statics which explore those two different mechanisms. The results are shown in Table A1. In the first comparative statics, in column 4, we consider a different portfolio allocation for the pension fund. More precisely, we increase its risky asset share (α^{db}) from our baseline value of 0.52 to 0.80. In column 5, we report results for a re-calibration of the model where we have increased the discount factors of the two types of agents are increased by 0.005 (hence we have β^A=0.838 and $\beta^B = 0.967$).

[INSERT TABLE A1 HERE]

When we increase the risky share of the DB pension fund (column 4 of Table A1), the riskless rate increases to 2.52%, as the overall demand for bonds in the economy falls. The return on equity remains largely unchanged, and therefore the equity premium falls to 4.13% while the Sharpe ratio decreases to 0.26. With DBPF investing more in equities, the return on equity would have been expected to fall. However, we observe that the total capital stock is actually lower in the new economy. This is explained by a reduction in the demand for equities by households, which is a response to the higher riskless rate, lower sharpe ratio, and the increase in background risk arising from the higher risky share of the pension fund. In the new equilibrium, the capital held by households falls from 3.55 to 3.19, and the average stock market participation rate falls from 57.6% to 48.3%.

Increasing the discount factor of households (column 5 of Table A1) naturally delivers higher aggregate savings and, in equilibrium, the rates of return on both assets fall. However, since the supply of bonds is perfectly inelastic the riskless rate falls substantially more, to -1.78%, while the average return on equity is still 6.49%. This results in both higher equity premium and sharpe ratio. Combined with the increased demand for savings we have a significantly higher stock market participation rate (65.8%), total capital (4.60) and capital held by households (3.66). Although this calibration can deliver a higher market price of risk, we reject it because it fails to match the riskless rate, in addition to increasing the volatility of consumption growth and the stock market participation rate.

Table A1: Additional Comparative Statics for the Baseline Model.

Variable	Moment	Baseline Model: Calibrations			
		(i) Baseline	(ii) α^P=0.8	(iii) Lower βs	(iv) Data
r^B	Mean	0.93%	2.40%	2.46%	0.86%
r^B	St. Dev.	1.26%	1.17%	1.16%	1.35%
r^m	Mean	8.02%	7.57%	8.58%	8.17%
r^m	St. Dev.	19.64%	20.37%	19.52%	19.81%
$r^m - r^B$	Mean	7.08%	5.18%	6.11%	7.55%
$\frac{Mean(r^m - r^B)}{Std.Dev.(r^m - r^B)}$		0.37	0.26	0.32	0.34
Cons. growth (all)	St. Dev.	2.83%	2.86%	2.70%	2.90%
Cons. growth (A)	St. Dev.	2.63%	2.74%	2.48%	(-)
Cons. growth (B)	St. Dev.	3.16%	3.07%	3.04%	(-)
K	Mean	4.52	4.35	4.10	(-)
$Kprivate$	Mean	3.76	3.40	3.46	(-)
W^P/Y	Mean	0.75	0.61	0.65	0.67
K/Y	Mean	2.30	2.24	2.16	(-)
Participation (all)	Mean	59.5%	53.6%	56.0%	51.1%
Participation (A)	Mean	25.1%	16.8%	20.4%	(-)
Participation (B)	Mean	93.9%	90.4%	91.5%	(-)

Table A1 reports asset pricing moments, consumption volatility, stock market participation, capital and the capital/output ratio for the baseline model for different parameter calibrations. Column (i) reports results for the baseline calibration, and column (ii) for when the pension fund risky share (α^P) is 0.8. In column (iii) the discount factors of the two types of agents are decreased by 0.005 (hence we have β^A=0.825 and $\beta^B = 0.960$). The asset pricing data is taken from CRSP. The mean and volatility of the real risk free is taken from Croce et al. (2012). Stock market participation is computed from the Survey of Consumer Finances. The consumption data is taken from the NIPA tables provided by the Federal Reserve Bank of St. Louis. Pension wealth data is also from the Federal Reserve Bank of St. Louis. r^m denotes levered r^K.