Factor-Biased Outsourcing: Implications for Substitution between Capital and Labor*

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Abstract

Outsourcing can be factor biased, displacing disproportionately either capital or labor inside the firm. Relying solely on the assumption of homogeneous input demand, this paper shows that factor-biased outsourcing leads the capital–labor ratio to respond differently to a change in the price of labor and to a change in the price of capital. Indirect estimates based on a meta analysis of studies since the 1960s, as well as direct estimates using U.S. data for 1963–2016, indicate that outsourcing principally displaces labor and that, consequently, the capital-labor ratio responds 30-80% more strongly to the price of labor than to the price of capital. These findings also help reframe historical disagreements regarding the extent of capital–labor substitution, and they have direct implications for modeling production and cost, implying that capital and labor are not separable from outsourced inputs.

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1 Introduction

The extent of substitution between capital and labor is fundamental to a host of economic questions. The impact of automation on wages and welfare, the consequences of a change in the minimum wage or a reform in capital taxation all depend on the extent of this substitution. Yet, estimates of this substitutability remain widely dispersed and debated. Existing studies provide support for nearly any conclusion about how capital and labor could be used in production: in fixed proportion, as substitutes or as complements.[1]

This paper argues that factor-biased outsourcing—the extent to which outsourced intermediate inputs displace capital versus labor—is key to understanding the substitution between the capital and the labor inside the firm. In short, outsourcing might disproportionately displace labor: it might look like firing the janitor and renting cleaning services from a company like Sodexo. Alternatively, outsourcing might disproportionately displace capital: it might look like foregoing capital investments into hardware and instead renting cloud storage from a company like Dropbox. Whether outsourcing looks more like the janitor or the Dropbox story can shape the extent of substitutability between the capital and the labor left inside the firm.

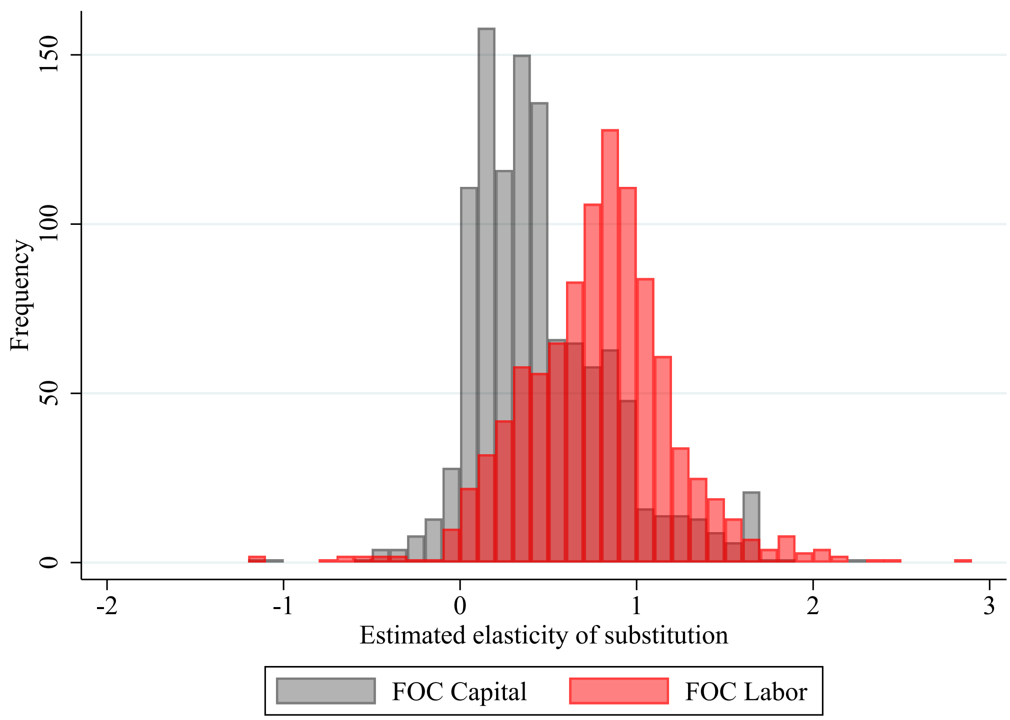
Relying solely on the assumption of homogeneous input demand—consistent with standard cost-minimizing input demand functions—this paper shows that factor-biased outsourcing leads to an asymmetry: it leads the capital-labor ratio to respond differently to a change in the price of labor and to a change in the price of capital. Under homogeneity, the capital-labor ratio is subject to a zero-sum constraint. Specifically, the elasticities of the capital-labor ratio with respect to the full set of input prices must jointly sum to zero. In a two-factor world, the elasticities of the capital-labor ratio with respect to the prices of labor and capital are symmetric: they are equal in magnitude but opposite in sign. Introducing additional, outsourced factors of production into the zero-sum constraint can lead to an asymmetry: the capital-labor ratio can respond differently to a change in the price of labor and to a change in the price of capital.

Factor-biased outsourcing can help rationalize a striking pattern in the meta analysis of capital-labor substitution: existing estimates of capital-labor substitution tend to be higher when estimated using labor-price variation and tend to be lower when estimated using capital-price variation. Figure 1 displays this pattern using meta analysis data from Gechert, Havranek, Irsova and Kolcunova (2022), covering 134 published studies dating back to the early 1960s. The red bars in the figure—with a mean of 0.85 and a median of 0.81—show the distribution of elasticities from estimating equations based on the first-

[1] Gechert, Havranek, Irsova and Kolcunova (2022) overview 134 such studies dating back to the 1960s.

order condition for labor, which leverages variation solely in labor prices. The gray bars—with a mean of 0.45 and a median of 0.38—show the distribution of elasticities estimated using the first order condition for capital, leveraging variation solely in capital prices. Comparing the two distributions suggests that the capital-labor ratio is about 40% more responsive to changes in labor prices than to changes in capital prices.

Figure 1: Estimates of Capital-Labor Substitution with Different Price Variation



Note: Data from Gechert, Havranek, Irsova and Kolcunova (2022). FOC capital estimates use capital-price variation. FOC labor estimates use labor-price variation.

Viewed through the zero-sum constraint imposed by homogeneous input demand, the difference between these meta-analysis estimates suggests that outsourcing displaces labor more than it displaces capital. Intuitively, if outsourced inputs are a closer substitute for in-house labor than for in-house capital, when in-house labor becomes more expensive the firm has more options for substituting away from it. The capital-labor ratio is then more responsive to changes in labor prices than to changes in capital prices. Furthermore, the capital-labor ratio should then respond inversely to changes in the price of outsourcing. As outsourcing becomes less expensive—if outsourcing is biased towards displacing labor—then the capital-labor ratio should increase.

Controlling for differences across and within existing studies yields three takeaways in the form of three elasticities. Factor biased outsourcing—formally defined as the elasticity of the capital-labor ratio to the price of outsourced inputs—can be measured under homogeneity as the asymmetry in figure 1. This outsourcing elasticity is in the range of –0.3 to –0.4, suggesting that a fall in the price of outsourcing disproportionately displaces labor. In addition, the capital-labor ratio responds more strongly to changes in the price of labor (with an elasticity of 0.7 to 0.8) and less strongly to changes in the price of capital (with an elasticity of –0.3 to –0.4). These patterns are also robust within different subsets of studies, which can help rule out some alternative explanations for the asymmetry in figure 1, such as those coming from differential adjustment of capital and labor in production.

Using aggregated data for the U.S. economy that separates prices from quantities of inputs, I then provide complementary evidence that outsourcing indeed displaces labor disproportionately more than capital. Specifically, I use data from the Bureau of Economic Analysis and the Bureau of Labor Statistics for 1960 to 2016 to estimate how changes in the capital-labor ratio respond to changes in the price of intermediates purchased from other firms. The estimated elasticity of the capital-labor ratio to the price of these outsourced inputs ranges in value from –0.5 (when using ordinary least squares) to –0.8 (when instrumenting the price of outsourced inputs using plausibly exogenous variation in global commodity prices). These estimates suggest that as the price of outsourced inputs declines, firms disproportionately displace labor, leading the capital-labor ratio to increase. Viewed through the lens of the zero-sum constraint, these estimates suggest that the capital-labor ratio is some 50–80% more responsive to changes in labor prices than it is to changes in capital prices, broadly in line with the meta-analysis implications for the same comparison.

The differential responses of the capital-labor ratio to different factor prices are directly policy relevant: for instance, Chirinko (2002) finds that moving from an elasticity of 1 to 0.5 decreases the welfare losses associated with a range of capital taxation policies by 50–79 percent. This change in elasticity is of a similar magnitude to that implied by the meta analysis in figure 1: it corresponds roughly to switching from using the estimated response of the capital-labor ratio to the price of labor to the—contextually more appropriate—response to the price of capital. These differential responses also suggest, for instance, that a change of the minimum wage is mediated into the rest of the economy through a larger elasticity than is, say, a change in central-bank interest rate policy. This notion of using different elasticities—rather than a common one—is important because the estimated asymmetry is substantially different from zero, and even modest changes in the extent of substitution could lead to large changes in policy outcomes.

Moreover, the factor biased nature of outsourcing has direct implications for modeling production and cost, implying that capital and labor are not separable from outsourced inputs. In a world of factor-biased outsourcing, value added—comprising capital and labor—cannot be modeled as separable from outsourced intermediate inputs in the production of gross output. That implication, recast in section 4 in terms of the separability conditions of Goldman and Uzawa (1964), contributes to a large literature dating back to Diewert (1971) and Christensen, Jorgenson and Lau (1973), who emphasized generalized Leontief and translog specifications as more comprehensively capturing the interactions of different factors in modeling production and cost.

More generally, factor-biased outsourcing offers a way to understand historical disagreements regarding the measurement of relative-price elasticities of substitution. Building on the pioneering work of Hicks (1932) and Robinson (1933), relative-price elasiticities measure the response of the capital-labor ratio to the price of labor relative to the price of capital. The key implication of the current paper is that—under factor biased outsourcing—studies using different sources of price variation will estimate different relative-price elasticities *even if they are studying the same firm*. Consider, for instance, two unimpeachable instruments generating identical variation in the relative price: one shifts around primarily the price of capital and the other primarily the price of labor. In a world of factor-biased outsourcing these two instruments would estimate different relative-price elasticities. One estimate would reflect more closely the capital-labor response to the price of labor while the other would reflect more strongly its response to the price of capital.

By combining estimates of factor-biased outsourcing with data on relative-price variation, I show that these two forces can drive dispersion of estimated relative-price elasticities that is comparable to the dispersion of estimates in meta analyses. Formally, a relative-price elasticity can be written as a weighted sum of underlying single-price elasticities. The weights in question capture how much of the relative-price variation comes from changes in labor versus changes in capital prices. These weights, while they do sum to one, can take both positive and negative values, and can be arbitrarily large. The interaction of factor-biased outsourcing and this price variation can render estimated relative-price elasticities uninformative about the substitution between capital and labor.

Through its emphasis on factor bias in outsourcing, this paper contributes to a larger literature on the measurement of capital-labor substitution. Within this literature, other papers have also touched upon the tendency for first-order conditions for capital and labor to provide systematically different estimates. Antras (2004) points out that the relationship has been noted since at least Berndt (1976), and Leon-Ledesma, McAdam and Willman (2010) discuss differential measurement error as one potential source of these patterns. The current paper offers an outsourcing-driven rationale to help explain these historical patterns and supports that interpretation with both direct and indirect measurement.

While this paper considers outsourcing broadly defined to include any inputs from outside the firm, different subsets of this broad definition have been studied by different literatures. In labor economics, the study of outsourcing has often focused on the notion of outsourcing specific individuals or occupations and then buying back their services, generally without considering the possibility of outsourcing capital tasks and services (e.g., Abraham and Taylor, 1996; Katz and Krueger, 2018). A large literature in international economics has focused on the outsourcing production not only outside the boundary of the firm but also outside the boundary of a home country in the form of offshoring (e.g., Feenstra and Hanson, 1996; Grossman and Helpman, 2005). And a literature in energy and environmental economics has studied the relationship specifically of electricity with the labor and capital within the firm (e.g., Berndt and Wood, 1975, 1981).

The rest of the paper proceeds as follows. Section 2 provides the theoretical structure for the paper, showing how homogeneous input demand constrains the behavior of the capital-labor ratio with respect to input prices and thereby helps formalize the notion of factor-biased outsourcing. Section 3 uses two approaches—one indirect and one direct—to quantify the extent of factor-biased outsourcing for the aggregate U.S. economy since the 1960s. Section 4 draws out implications of the estimated factor bias for policy, for traditional measures of capital-labor substitution, and for the separability of capital and labor in production and costs. Section 5 concludes.

2 Theory: How Factor-Biased Outsourcing Shapes a Firm's Capital-Labor Ratio

To motivate the notion of factor-biased outsourcing and its role in capital-labor substitution, this section starts with two simple parametric examples. I then generalize the results relying solely on the assumption of homogeneous demand for capital and labor.

The first example focuses on a world without outsourcing. I model output produced using solely capital and labor through a Constant-Elasticity-of-Substitution (CES) production function. Within this example, the elasticities of the capital-labor ratio with respect to the prices of labor and capital sum to zero; they are equal in magnitude but opposite in sign. This symmetry allows us to identify the same measured elasticity in three different ways: by looking at how the capital-labor ratio responds to an increase in the price of labor, a decrease in the price of capital, or an increase in the relative price of labor versus capital.

The second example uses a nested CES production function to introduce outsourced inputs as a third factor, and to allow for the possibility that outsourcing is differently substitutable with capital and with labor inside the firm. With this example, I highlight two results. First, the elasticities of the capital-labor ratio with respect to the prices of labor, capital and outsourcing jointly sum to zero. Second, when outsourcing is differently substitutable with labor and with capital, the symmetry from the first example is broken: the capital-labor ratio now responds differently to changes in the price of labor and to changes in the price of capital. The extent of asymmetry between these two responses is equal to the elasticity of the capital-labor ratio with respect to the price of outsourcing

To show that the results from the parametric examples hold more broadly, this section then models a firm's demand for capital and labor in more general terms. Specifically, I assume only that demand for capital and labor is homogeneous of some arbitrary degree h in factor prices. This assumption is consistent with a broad array of production and cost functions, and is more general than those underpinning standard results on cost minimization or profit maximization. Despite being quite general, the assumption of homogeneity yields a sharp result: it imposes a zero-sum constraint on how the capital-labor ratio responds to factor prices.

The theory in this section highlights an understudied elasticity: the response of the capital-labor ratio to the price of outsourcing. The sign of the outsourcing elasticity reveals the direction of factor bias in outsourcing. When the price of outsourcing goes down, whether the capital-labor ratio falls or rises tells us which factor of production (capital or labor) is being disproportionally displaced by outsourcing. The magnitude of the outsourcing elasticity—viewed through the lens of the zero-sum constraint—shapes an asymmetry: when outsourcing is factor biased, the capital-labor ratio responds differently to labor and to capital prices. The more that outsourcing is factor biased, the greater the asymmetry between the response to labor and to capital prices.

2.1 Parametric Example without Factor-Biased Outsourcing

In this subsection, I model output produced with CES technology and only two factors of production—capital K and labor L—to highlight the idea that this production structure leads the capital-labor ratio to respond symmetrically to changes in labor and in capital prices. Specifically, I assume that a firm produces output Y with the following technology:

$$Y = \left(K^{\frac{\sigma-1}{\sigma}} + L^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}, \tag{1}$$

where σ governs the substitution between the two factor of production, and where I take the price of output Y to be the numeraire. Note that this simple specification abstracts from technological change without compromising the key conclusions.

Combining this production function with price-taking and cost-minimizing behavior yields elasticities of the capital-labor ratio with respect to both the price of labor w_L and the price of capital w_K:

$$\varepsilon_{\frac{K}{L},w_L} = \frac{\partial \ln \frac{K}{L}}{\partial \ln w_L} = \sigma \text{ and } \varepsilon_{\frac{K}{L},w_K} = \frac{\partial \ln \frac{K}{L}}{\partial \ln w_K} = -\sigma.$$

The two elasticities are equal in magnitude and opposite in sign. As a result, the same parameter σ can also be identified from a relative-price elasticity that examines how the capital-labor ratio responds to a change in the relative prices of labor and capital, w_L/w_K:

$$\varepsilon_{\frac{K}{L},\frac{w_L}{w_K}} = \frac{\partial \ln \frac{K}{L}}{\partial \ln \frac{w_L}{w_K}} = \sigma.$$

In this two-factor CES setting, the capital-labor ratio responds identically to an increase in the price of labor, a decrease in the price of capital, and an increase in the price of labor relative to the price of capital.

2.2 Parametric Example with Factor-Biased Outsourcing

In this subsection, I introduce outsourced inputs that are potentially differently substitutable with capital and with labor. I then show how this factor bias in outsourcing can break the symmetry from the two-factor setting. Consider a firm that produces output Y by combining capital K, labor L, and outsourced inputs O using a nested CES technology:

$$Y = \left[K^{\frac{\sigma_{out}-1}{\sigma_{out}}} + \left(L^{\frac{\sigma_{in}-1}{\sigma_{in}}} + O^{\frac{\sigma_{in}-1}{\sigma_{in}}} \right)^{\frac{\sigma_{in}}{\sigma_{in}-1}\frac{\sigma_{out}-1}{\sigma_{out}}} \right]^{\frac{\sigma_{out}}{\sigma_{out}-1}}, \tag{2}$$

where the elasticity σ_{out} governs the extent of substitution between K and the two factors in the inner nest, and the elasticity σ_{in} governs the extent of substitution between L and O. I take the price of output to be the numeraire. Key to the example is that outsourced inputs are in the inner nest, so that they can potentially be differently substitutable with capital and with labor. For the main results, it does not matter whether capital or labor accompanies outsourcing in the inner nest.

Combining this production function with price-taking and cost-minimizing behavior by the firm allows me to derive elasticities of relative-factor usage with respect to an individual factor price w_f. Those elasticities are presented in table 1, with each row corresponding to a relative factor demand and each column presenting the elasticities with respect to a given factor price w_L, w_K, or w_O. For instance, the three elements of the first row describe the elasticities of the capital-labor ratio with respect to the price of labor $\varepsilon_{\frac{K}{L},w_L}$, the price of capital $\varepsilon_{\frac{K}{L},w_K}$ and the price of outsourcing $\varepsilon_{\frac{K}{L},w_O}$. I further denote by α the outsourcing expenditures $w_O O$ as a share of combined expenditures $w_O O + w_L L$.

Table 1: Elasticities of relative-factor demand with respect to input prices

	w_L	w_K	w_O	Row Sum
$\varepsilon_{\frac{K}{L},w_f}$	$\alpha\sigma_{in} + (1-\alpha)\sigma_{out}$	$-\sigma_{out}$	$-(\sigma_{in} - \sigma_{out})\alpha$	0
$\varepsilon_{\frac{L}{O},w_f}$	$-\sigma_{in}$	0	σ_{in}	0
$\varepsilon_{\frac{K}{O},w_f}$	$-(\sigma_{in}-\sigma_{out})(1-\alpha)$	$-\sigma_{out}$	$(1-\alpha)\sigma_{in} + \alpha\sigma_{out}$	0

Note: The table presents elasticities of relative-factor demand for the nested CES production function specified by equation (2), with α denoting the share of outsourcing expenditures in the sum of outsourcing and labor expenditures $w_O O/(w_O O + w_L L)$.

The terms of each row in table 1 sum to zero, suggesting that there is a zero-sum constraint on the behavior of any relative factor demand with respect to the full set of input prices. Focusing on the capital-labor ratio in the first row, we therefore have that the elasticities $\varepsilon_{\frac{K}{L},w_L}$, $\varepsilon_{\frac{K}{L},w_K}$ and $\varepsilon_{\frac{K}{L},w_O}$ jointly sum to zero. The same zero-sum constraint holds for the relative demand between labor and capital in the second row and the relative demand between capital and outsourcing in the bottom row.

When outsourcing is equally substitutable with capital and with labor (i.e., when $\sigma_{in} = \sigma_{out} = \sigma$), then the capital-labor ratio responds symmetrically to a change in the price of labor and a change in the price of capital, as in section 2.1. We can see this symmetry in the first row of table 1. When $\sigma_{in} = \sigma_{out}$, the elasticities $\varepsilon_{\frac{K}{L},w_L}$ and $\varepsilon_{\frac{K}{L},w_K}$ are once again equal in magnitude and opposite in sign. Moreover, the elasticity of the capital-labor ratio with respect to the price of outsourcing $\varepsilon_{\frac{K}{L},w_O}$ is zero: a change in the price of outsourcing does not lead the firm to alter its capital-labor ratio. These results suggest that key to the symmetry from section 2.1 is not the restriction to two factors, but the restriction to outsourcing being unbiased in the extent to which it displaces capital versus labor.

When outsourcing is differentially substitutable with capital and with labor (i.e, when $\sigma_{in} \neq \sigma_{out}$), the capital-labor ratio responds differently to a change in the price of labor and to a change in the price of capital. For instance, if σ_{in} is larger than σ_{out}—so that outsourcing is more substitutable with labor in the inner CES nest than it is with capital in the outer CES nest—then $\varepsilon_{\frac{K}{L},w_L}$ is greater than $\varepsilon_{\frac{K}{L},w_K}$ in absolute value.

Moreover, when outsourcing is differentially substitutable with capital and with labor, the outsourcing elasticity $\varepsilon_{\frac{K}{L},w_O}$ is different from zero and it concisely summarizes the difference between the capital-labor response to changes in the price of labor and changes in the price of capital. If σ_{in} is larger than σ_{out}—so that outsourcing is more substitutable with labor than it is with capital—then $\varepsilon_{\frac{K}{L},w_O}$ is negative. A reduction in the price of outsourcing would lead the firm to displace more labor than capital, and the capital-labor ratio would increase. The magnitude of this increase corresponds precisely to the extent to which $\varepsilon_{\frac{K}{L},w_L}$ is greater than $\varepsilon_{\frac{K}{L},w_K}$ in absolute value.

2.3 Zero-Sum Constraint on the Capital-Labor Ratio

In this subsection, I show that the zero-sum constraint on the behavior of the capital-labor ratio from the parametric example is a more general result. To that effect, I derive the zero-sum constraint using only the assumption that capital and labor demand are homogeneous of the same arbitrary degree h in the vector of factor prices $\mathbf{w}$. For a factor of production f facing factor prices $\mathbf{w}$, factor demand $f(\mathbf{w})$ is homogeneous of degree h if and only if:

$$f(\lambda\mathbf{w}) = \lambda^h f(\mathbf{w}). \tag{3}$$

This homogeneity assumption requires that an increase in factor prices by λ would increase factor demand $f(\mathbf{w})$ by λ^h, where h is the degree of homogeneity. To derive the zero-sum constraint, I make no additional economic assumptions; instead, I simply leverage two well-known properties of homogeneous functions.

From a first property of homogeneous functions—under the assumption that capital and labor demand are both homogeneous of the same arbitrary degree *h*—we have that the capital-labor ratio is homogeneous of degree zero:

$$\frac{K}{L}(\lambda\mathbf{w}) = \frac{K(\lambda\mathbf{w})}{L(\lambda\mathbf{w})} = \frac{\lambda^h K(\lambda\mathbf{w})}{\lambda^h L(\lambda\mathbf{w})} = \lambda^{h-h}\frac{K(\mathbf{w})}{L(\mathbf{w})} = \lambda^0 \frac{K}{L}(\mathbf{w}). \tag{4}$$

This degree-zero homogeneity relies simply on the property that the ratio of any two homogeneous functions is itself homogeneous, and that it is homogeneous of degree zero when the degrees of homogeneity are the same in the numerator and the denominator.

From a second property of homogeneous functions—known as the Euler theorem for homogeneous functions—a capital-labor ratio that is homogeneous of degree zero is subject to a zero-sum constraint on its behavior in response to factor prices. Euler's theorem for homogeneous functions specifies that a function $f(\mathbf{w})$ that is homogeneous of degree

h must satisfy the property that

$$\nabla f(\mathbf{w})\mathbf{w} = hf(\mathbf{w}).$$

In short, the gradient $\nabla f(\mathbf{w})$, multiplied by the factor price vector $\mathbf{w}$, must be equal to the degree of homogeneity h times the function $f(\mathbf{w})$ itself. Applying the theorem to the capital-labor ratio whose degree of homogeneity h is zero yields the zero-sum constraint:

$$\nabla \frac{K}{L}(\mathbf{w})\mathbf{w} = 0.$$

This zero-sum constraint becomes easier to interpret when we unpack the gradient of the capital-labor ratio and emphasize that it comprises the vector of derivatives of the capital-labor ratio with respect to all $f \in F$ factor prices w_f:

$$\sum_{f \in F} \frac{\partial \frac{K}{L}}{\partial w_f} w_f = 0.$$

To help interpret the zero-sum constraint, I assume—without loss of generality—that the capital-labor ratio is not zero; under this additional assumption, the zero-sum constraint can be stated as the requirement that elasticities of the capital-labor ratio with respect to all factor prices must collectively sum to zero:

$$\sum_{f \in F} \varepsilon_{\frac{K}{L}, w_f} = 0 \text{ with elasticities } \varepsilon_{\frac{K}{L}, w_f} = \frac{\partial \frac{K}{L}}{\partial w_f} \frac{w_f}{\frac{K}{L}}. \tag{5}$$

This zero-sum constraint on elasticities is the key organizing expression of this paper and the homogeneity assumption behind that constraint is implied by elementary proofs of cost minimization. For instance, cost or production functions consistent with elementary proofs of cost minimization or profit maximization (e.g., Mas-Colell, Whinston and Green, 1995) have as a necessary (but not sufficient) condition that input demands are all homogeneous of degree $h = 0$. If both capital and labor demand are individually homogeneous of degree zero, then by equation (4) the capital-labor ratio is also homogeneous of degree zero. This degree-zero homogeneity then leads to the zero-sum constraint in equation (5).

More generally, the homogeneity assumption made here can accommodate a broad range of commonly-specified factor demands, cost functions and production functions, even those in settings more general than what is commonly assumed by cost minimization. For instance, the homogeneity assumption in this paper can be satisfied by capital and labor demands that are homogeneous of any shared degree $h \neq 0$. Furthermore, degree-zero homogeneity of the capital-labor ratio is also consistent with departures from homogeneity in the overall cost or production function, as long as the source of the non-homogeneity comes from some non-capital, non-labor input.[2] In short, the zero-sum constraint at the center of this paper encompasses the common assumption of cost minimization and can also accommodate a range of deviations away from that common assumption.

2.4 Factor Bias and Asymmetry in Capital-Labor Dynamics

To highlight why the zero-sum constraint is helpful for bringing together factor-biased outsourcing and capital-labor substitution, I proceed in two steps: first I discuss the constraint in a world where firms produce using only capital and labor, and then second I introduce outsourcing as an additional factor of production. For completeness, I end this section with a generalization to any arbitrary number of factors.

In a world without outsourcing—where firms produce using only capital and labor—the zero-sum constraint implies that the capital-labor ratio responds symmetrically to changes in the price of labor and to changes in the price of capital. We can see this symmetric response by restricting the set of inputs in equation (5) to only capital and labor:

$$\varepsilon_{\frac{K}{L},w_L} + \varepsilon_{\frac{K}{L},w_K} = 0. \tag{6}$$

With only two factors of production, the elasticities of the capital-labor ratio with respect to the price of labor $\varepsilon_{\frac{K}{L},w_L}$ and with respect to the price of capital $\varepsilon_{\frac{K}{L},w_K}$ must be equal in magnitude and yet opposite in sign.

This symmetric response to changes in the price of labor and the price of capital in a two-factor world underpins the motivation for the relative-price elasticities of substitution in the spirit of Hicks (1932) and Robinson (1933). While the elasticity of substitution in a two-factor world is often called the Hicksian elasticity of substitution, Robinson popularized its specification as a relative-price elasticity $\varepsilon_{\frac{K}{L},\frac{w_L}{w_K}}$. Equation (6) shows the appeal of this relative-price elasticity: it does not matter whether the relative price w_L/w_K changes because of changes in the price of labor w_L or because of changes in the price of capital w_K. Under the assumption of homogeneous demand with only two factors of production, the underlying single-price elasticities $\varepsilon_{\frac{K}{L},w_L}$ and $\varepsilon_{\frac{K}{L},w_K}$ are equal in magnitude and opposite in sign. In other words, the capital-labor ratio responds identically to a one percent increase in a price of labor and to a one-percent decrease in the price of capital.

In a world where firms have access to outsourced inputs as a third factor of production,

[2]E.g., a translog function of three inputs could be non-homogeneous overall while yielding degree-h homogeneous demand for capital and labor.

the zero-sum condition implies that outsourcing comes in two forms: factor neutral and factor biased. With three factors of production, equation (5) yields the zero-sum constraint

$$\varepsilon_{\frac{K}{L},w_L} + \varepsilon_{\frac{K}{L},w_K} + \varepsilon_{\frac{K}{L},w_O} = 0, \tag{7}$$

where the elasticity $\varepsilon_{\frac{K}{L},w_O}$ captures how the capital-labor ratio responds to changes in the price of outsourcing w_O. When $\varepsilon_{\frac{K}{L},w_O}$ is zero, then outsourcing is factor neutral: a change in the price of outsourcing might change the amount of outsourced inputs that a firm purchases, but it does not change the firm's capital-labor ratio. By contrast, when outsourcing changes the firm's capital-labor ratio, then the elasticity $\varepsilon_{\frac{K}{L},w_O}$ is not zero and outsourcing is factor biased.

Factor-biased outsourcing ($\varepsilon_{\frac{K}{L},w_O} \neq 0$) leads to an asymmetry, driving apart the response of the capital-labor ratio to the price of labor $\varepsilon_{\frac{K}{L},w_L}$ and its response to the price of capital $\varepsilon_{\frac{K}{L},w_K}$. When the outsourcing elasticity $\varepsilon_{\frac{K}{L},w_O}$ is negative, a decline in the price of outsourcing displaces labor disproportionately, leading to an increase in the capital-labor ratio. In this context of labor-displacing outsourcing—like with the notion of outsourcing displacing the in-house janitor—the capital-labor ratio responds more strongly to changes in the price of labor than it responds to changes in the price of capital. When the outsourcing elasticity $\varepsilon_{\frac{K}{L},w_O}$ is positive, a decline in the price of outsourcing displaces capital disproportionately, leading to an decrease in the capital-labor ratio. In this context of capital-displacing outsourcing—like with the notion of a Dropbox subscription displacing in-house capital-intensive data storage—the capital-labor ratio responds more strongly to changes in the price of capital than it responds to changes in the price of labor.

Formally, these single-price elasticities $\varepsilon_{\frac{K}{L},w_L}$ and $\varepsilon_{\frac{K}{L},w_K}$ have often been termed Morishima elasticities of substitution (see Morishima, 1967; Blackorby and Russell, 1989), and the focus of the current paper is to emphasize that—under the assumption of homogeneous demand for capital and labor—the difference in these Morishima elasticities speaks to the extent of factor bias in outsourcing.[3]

Both for expositional simplicity and out of data limitations, the three-factor model is the baseline setting in this paper; nonetheless, the key result generalizes to any number of factors of production. Specifically, we can generalize the zero-sum constraint from equation (7) to include any number of non-capital, non-labor factors f as follows:

$$\varepsilon_{\frac{K}{L},w_L} + \varepsilon_{\frac{K}{L},w_K} + \sum_{f \notin \{K,L\}} \varepsilon_{\frac{K}{L},w_f} = 0. \tag{8}$$

[3]Preceding Morishima (1967), work on elasticities of substitution with more than two factors yielded measures of substitution that were symmetric (e.g., Hicks and Allen, 1934; Allen, 1938; Uzawa, 1962).

The asymmetry between the capital and labor price responses is then the *net* effect of those f factor prices w_f on the capital-labor ratio.

2.5 Factor-Biased Outsourcing Confounds the Measurement of Relative-Price Elasticities of Substitution

The discussion in the paper thus far has centered on the role of single-price elasticities, asking the question how the capital-labor ratio responds to the change in an individual factor price keeping other factor prices constant. In that spirit, the response of the capital-labor ratio to the price of labor $\varepsilon_{\frac{K}{L},w_L}$ could potentially be different not only in sign but also in magnitude to its response to the price of capital $\varepsilon_{\frac{K}{L},w_K}$. This discussion stands in contrast to the frequent focus on—and estimation of—a relative-price elasticity $\varepsilon_{\frac{K}{L},\frac{w_L}{w_K}}$, studying how the capital-labor ratio responds to a change in the relative prices of labor and capital w_L/w_K.

Building on the notion of distinct single-price elasticities, I next show that the relative-price elasticity of substitution $\varepsilon_{\frac{K}{L},\frac{w_L}{w_K}}$ is subject to two confounding factors in estimation: the extent of factor bias in outsourcing and the extent to which relative-price variation comes from changes in labor versus capital prices. The key implication is that—under factor biased outsourcing—studies using different sources of price variation will estimate different relative-price elasticities *even if they are studying the same firm*. This notion could potentially help explain some of the historical disagreements in estimates of relative-price elasticities of capital-labor substitution. Despite the noise introduced by relative prices, I also show how certain special cases in the estimation of the relative-price elasticity could be used to infer indirectly the extent of factor bias in outsourcing.

We can express the estimated relative-price elasticity $\widehat{\varepsilon_{\frac{K}{L},\frac{w_L}{w_K}}}$ as a weighted sum of the underlying single price elasticities $\varepsilon_{\frac{K}{L},w_L}$ and $\varepsilon_{\frac{K}{L},w_K}$,

$$\widehat{\varepsilon_{\frac{K}{L},\frac{w_L}{w_K}}} = \phi_{w_L} \times \varepsilon_{\frac{K}{L},w_L} + \phi_{w_K} \times (-\varepsilon_{\frac{K}{L},w_K}). \tag{9}$$

The weights ϕ_{w_L} and ϕ_{w_K} capture the extent to which the relative price variation comes from changes in the price of labor w_L and in the price of capital w_K, respectively:

$$\phi_{w_L} = \frac{\Delta \ln w_L}{\Delta \ln(w_L/w_K)} \text{ and } \phi_{w_K} = 1 - \phi_{w_L} = \frac{-\Delta \ln w_K}{\Delta \ln(w_L/w_K)}.$$

The single-price elasticities enter the expression with opposite signs because an *increase* in w_L and a *decrease* in w_K move the capital-labor ratio in the same direction.

This weighted-sum reformulation of $\widehat{\varepsilon_{\frac{K}{L},\frac{w_L}{w_K}}}$ is notable because the weights in question, while they do sum to one, can take both positive and negative values and can be arbitrarily large. To see the possibility of a negative weight, note that the relative-price change in the denominators of both ϕ_{w_L} and ϕ_{w_K} can move in the opposite direction of the factor-price in the numerator. For instance, if both capital and labor increase in price but the price of capital goes up more, then the relative price would decline and ϕ_{w_L} would be negative. Similarly, if both capital and labor prices increase substantially and by similar amounts, the relative price change in the denominator could be quite small relative to the individual price changes in the numerator; each weight would then be quite large—well in excess of unity—in absolute value. In an extreme scenario, as the relative price change approaches zero, the two weights approach (plus and minus) infinity.

By substituting the zero-sum constraint on the capital-labor ratio into the weighted sum above, we can highlight the simultaneous importance of factor-biased outsourcing and relative-price variation for the measurement of the relative-price elasticity $\widehat{\varepsilon_{\frac{K}{L},\frac{w_L}{w_K}}}$. There are two equivalent ways to see this relationship. First, if we substitute out $\varepsilon_{\frac{K}{L},w_K}$ from equation (9) with the zero-sum constraint from equation (7), we can express $\widehat{\varepsilon_{\frac{K}{L},\frac{w_L}{w_K}}}$ as:

$$\widehat{\varepsilon_{\frac{K}{L},\frac{w_L}{w_K}}} = \varepsilon_{\frac{K}{L},w_L} + \phi_{w_K} \times \varepsilon_{\frac{K}{L},w_O}. \tag{10}$$

Alternatively, substituting out $\varepsilon_{\frac{K}{L},w_L}$ from equation (9) results in a similarly informative:

$$\widehat{\varepsilon_{\frac{K}{L},\frac{w_L}{w_K}}} = (-\varepsilon_{\frac{K}{L},w_K}) - \phi_{w_L} \times \varepsilon_{\frac{K}{L},w_O}. \tag{11}$$

Both formulations makes explicit that the relative-price elasticity is shaped by three forces: the single-price elasticities ($\varepsilon_{\frac{K}{L},w_L}$ or $\varepsilon_{\frac{K}{L},w_K}$), the extent of factor bias in outsourcing ($\varepsilon_{\frac{K}{L},w_O}$), and the extent to which the relative price variation w_L/w_K comes from changes in the price of capital versus the price of labor (ϕ_{w_K}, ϕ_{w_L}).

When outsourcing is factor neutral, the relative-price elasticity $\widehat{\varepsilon_{\frac{K}{L},\frac{w_L}{w_K}}}$ is highly informative about the underlying single-price elasticities $\varepsilon_{\frac{K}{L},w_L}$ and $\varepsilon_{\frac{K}{L},w_K}$. Recall that under factor-neutral outsourcing $\varepsilon_{\frac{K}{L},w_O}$ is zero and the single-price elasticities $\varepsilon_{\frac{K}{L},w_L}$ and $\varepsilon_{\frac{K}{L},w_K}$ are equal in magnitude and opposite in sign. Therefore, although the weights ϕ may be arbitrarily large, from equation (10) we see that $\widehat{\varepsilon_{\frac{K}{L},\frac{w_L}{w_K}}}$ will nonetheless fully reflect the magnitude of the underlying single-price elasticities when $\varepsilon_{\frac{K}{L},w_O}$ is zero.

When outsourcing is factor biased, the potentially arbitrary nature of the relative price weights ϕ can lead the estimated relative-price elasticity $\widehat{\varepsilon_{\frac{K}{L},\frac{w_L}{w_K}}}$ to be similarly arbitrary in value and divorced from the underlying single-price elasticities. We can see from equation

(8) that when $\varepsilon_{\frac{K}{L},w_O}$ is not zero, then the price variation in ϕ shapes the estimated $\widehat{\varepsilon_{\frac{K}{L},\frac{w_L}{w_K}}}$. In other words, even if the elasticities $\varepsilon_{\frac{K}{L},w_L}$, $\varepsilon_{\frac{K}{L},w_K}$, and $\varepsilon_{\frac{K}{L},w_O}$ were known and constant, the estimated $\widehat{\varepsilon_{\frac{K}{L},\frac{w_L}{w_K}}}$ will vary with relative prices in a factor-biased world.

While the main takeaway of this subsection is that relative-price elasticities combine information on single-price elasticities with noise from relative prices, two special cases can help indirectly identify the extent of factor bias in outsourcing. The two special cases are those where the relative-price variation comes just from changes in the price labor ($\phi_{w_L} = 1$) or just from changes in the price of capital ($\phi_{w_K} = 1$). In the former case of $\phi_{w_L} = 1$, the relative price elasticity identifies the elasticity of the capital-labor ratio with respect to the price of labor $\varepsilon_{\frac{K}{L},w_L}$:

$$\left.\widehat{\varepsilon_{\frac{K}{L},\frac{w_L}{w_K}}}\right|_{\phi_{w_L}=1} = \varepsilon_{\frac{K}{L},w_L}.$$

In the latter case of $\phi_{w_K} = 1$, it identifies the elasticity of the capital-labor ratio with respect to the price of capital $\varepsilon_{\frac{K}{L},w_K}$:

$$\left.\widehat{\varepsilon_{\frac{K}{L},\frac{w_L}{w_K}}}\right|_{\phi_{w_K}=1} = -\varepsilon_{\frac{K}{L},w_K}.$$

Through the lens of the zero-sum constraint, the difference in the estimates from these two special cases is informative about the extent of factor-biased outsourcing $\varepsilon_{\frac{K}{L},w_O}$:

$$\varepsilon_{\frac{K}{L},w_O} = \left.\widehat{\varepsilon_{\frac{K}{L},\frac{w_L}{w_K}}}\right|_{\phi_{w_K}=1} - \left.\widehat{\varepsilon_{\frac{K}{L},\frac{w_L}{w_K}}}\right|_{\phi_{w_L}=1}. \tag{12}$$

I use this idea in the next section to look back at the history of relative-price estimates in published papers dating back to the 1960s and indirectly infer factor-biased outsourcing.

3 U.S. Outsourcing Displaces Labor More than Capital

In this section, I show that outsourcing in the United States is factor biased and that it displaces labor disproportionately relative to capital. I first document factor bias indirectly via a meta analysis of studies dating back to the 1960s. Specifically, I compare two types of estimates of capital-labor substitution: those estimates generated using only labor-price variation and those estimates generated using only capital-price variation. Under the assumption of homogeneous input demand, an asymmetry between these estimates is informative about the extent and direction in which outsourcing is factor biased. I then document this factor bias directly by examining how the capital-labor ratio for the U.S. economy responds to changes in the price of outsourced inputs. Both approaches imply the same factor bias in outsourcing: a fall in the price of outsourcing disproportionately displaces labor and causes the capital-labor ratio to increase.

3.1 Indirect Measurement Using Meta Analysis of Existing Studies

To quantify factor bias in outsourcing, I first rely on a meta-analysis of historical studies of capital-labor substitution. Specifically, I use meta analysis data compiled by Gechert, Havranek, Irsova and Kolcunova (2022) to study publication bias, and I emphasize that estimates of capital-labor substitution have been smaller when estimated using capital-price variation relative to those estimated with labor-price variation. Viewed through the lens of the homogeneity assumption in Section 2, the asymmetry between these estimates suggests that outsourcing displaces labor more than capital and is consistent with an inverse relationship between the capital-labor ratio and the price of outsourcing.

3.1.1 Data: Gechert, Havranek, Irsova & Kolcunova (2022)

A key aspect of the compiled meta-analysis data is that it includes a range of descriptive variables capturing the context and specification behind each parameter estimate. The compiled data cover 3,186 estimates of capital-labor substitution from 121 published studies dating back to the early 1960s.

Specifically of interest to the indirect inference here, the data highlight whether a given estimate comes from either the first-order condition for labor (ε_{FOC_L}) or the first-order condition for capital (ε_{FOC_K}). Optimization leads these first-order conditions to contain only a single factor price: the price of labor services w_L in the case of ε_{FOC_L} and the price of capital services w_K in the case of ε_{FOC_K}. In the former case, the elasticity ε_{FOC_L} would identify the elasticity of the capital-labor ratio with respect to the price of labor $\varepsilon_{\frac{K}{L},w_L}$. In the latter case, the elasticity ε_{FOC_K} would identify (the negative of) $\varepsilon_{\frac{K}{L},w_K}$, since the prices of capital and labor move the capital-labor ratio in opposite directions.

Commonly-used assumptions in the literature impose symmetry between ε_{FOC_L} and ε_{FOC_K} estimates of substitution. Specifically, these estimating equations are generally derived in settings where not only is input demand homogeneous, but where we additionally tend to assume either (1) that outsourced inputs are absent (i.e., a two-factor model of production) or (2) that outsourcing is factor unbiased (e.g., a Cobb Douglas or a CES production function). These additional assumptions impose that the outsourcing elasticity $\varepsilon_{\frac{K}{L},w_O}$ is zero. Consequently–through the zero-sum constraint of section 2—ε_{FOC_L} and

ε_{FOC_K} should yield identical estimates of substitution.

However, relaxing those commonly-used assumptions and relying on homogeneity alone allows ε_{FOC_L} and ε_{FOC_K} to yield different estimates of substitution, and allows us to interpret the asymmetry between the two sets of estimates as the extent of factor bias in outsourcing, $\varepsilon_{\frac{K}{L},w_O}$. Using the notation from this section, we can rewrite the indirect inference from equation (12) as:

$$\varepsilon_{\frac{K}{L},w_O} = \widehat{\varepsilon_{\frac{K}{L},\frac{w_L}{w_K}}}\Bigg|_{\phi_{w_K}=1} - \widehat{\varepsilon_{\frac{K}{L},\frac{w_L}{w_K}}}\Bigg|_{\phi_{w_L}=1} = \varepsilon_{FOC_K} - \varepsilon_{FOC_L}. \tag{13}$$

3.1.2 Empirical Strategy

To implement this indirect inference of factor bias, I compare historical measures of capital-labor substitution estimated as ε_{FOC_K} to those estimated as ε_{FOC_L}. In the specification that follows, $\widehat{\varepsilon}_{i,s}$ is an elasticity estimate i from study s:

$$\widehat{\varepsilon}_{is} = \beta \, \mathbb{1}(\varepsilon_{FOC_K})_{is} + \delta_s + \text{Constant} + e_{is} \tag{14}$$

where $\mathbb{1}(\varepsilon_{FOC_K})_{is}$ is an indicator variable taking value one when the specification relies on ε_{FOC_K} and taking value 0 when the specification relies on ε_{FOC_L}, δ_s is a potentially-included study fixed effect, and e_{is} is an error term.

By restricting the sample to only those estimates generated using ε_{FOC_K} or ε_{FOC_L}, the estimated coefficient $\widehat{\beta}$ (indirectly) measures the extent of factor-biased outsourcing. We can see this inference in two steps. First note that putting aside the fixed effect δ_s, the Constant measures the average elasticity when the indicator variable $\mathbb{1}(\varepsilon_{FOC_K})_{is}$ takes the value zero and thus excludes all estimates of ε_{FOC_K}. In this way—given the sample restriction to only ε_{FOC_K} and ε_{FOC_L} specifications—the Constant captures the average elasticity estimated using labor-price variation, i.e., the average ε_{FOC_L}. Second, the estimated coefficient $\widehat{\beta}$ on the indicator variable $\mathbb{1}(\varepsilon_{FOC_K})_{is}$ captures the asymmetry–the difference–between the average ε_{FOC_K} and the average ε_{FOC_L} estimate. Under the assumption of homogeneous demand for capital and labor, as in equation (13), this asymmetric response identifies the extent of factor bias in outsourcing, $\varepsilon_{\frac{K}{L},w_O}$.

3.1.3 Results

The results in table 2 suggest—across a variety of econometric specifications and subsamples from the meta-analysis data—that outsourcing is factor biased and that it displaces labor disproportionately relative to capital. Through the lens of the zero-sum constraint from section 2, the estimate in column (1) suggests that a 1% decrease in the real price of outsourcing disproportionately displaces labor and increases the aggregate capital-labor ratio by 0.484%. The notion that outsourcing is primarily labor-displacing persists even when we control for differences in the statistical precision of different estimates. Specifically, the regression in column (2) weights the meta-analysis observations by the inverse of each observation's standard error, giving greater weight to more precisely estimated elasticities; the baseline conclusion remains unchanged, with an implied outsourcing elasticity of –0.433 remaining very close compared to the earlier –0.484.

Table 2: Indirect Estimates of Factor-Biased Outsourcing

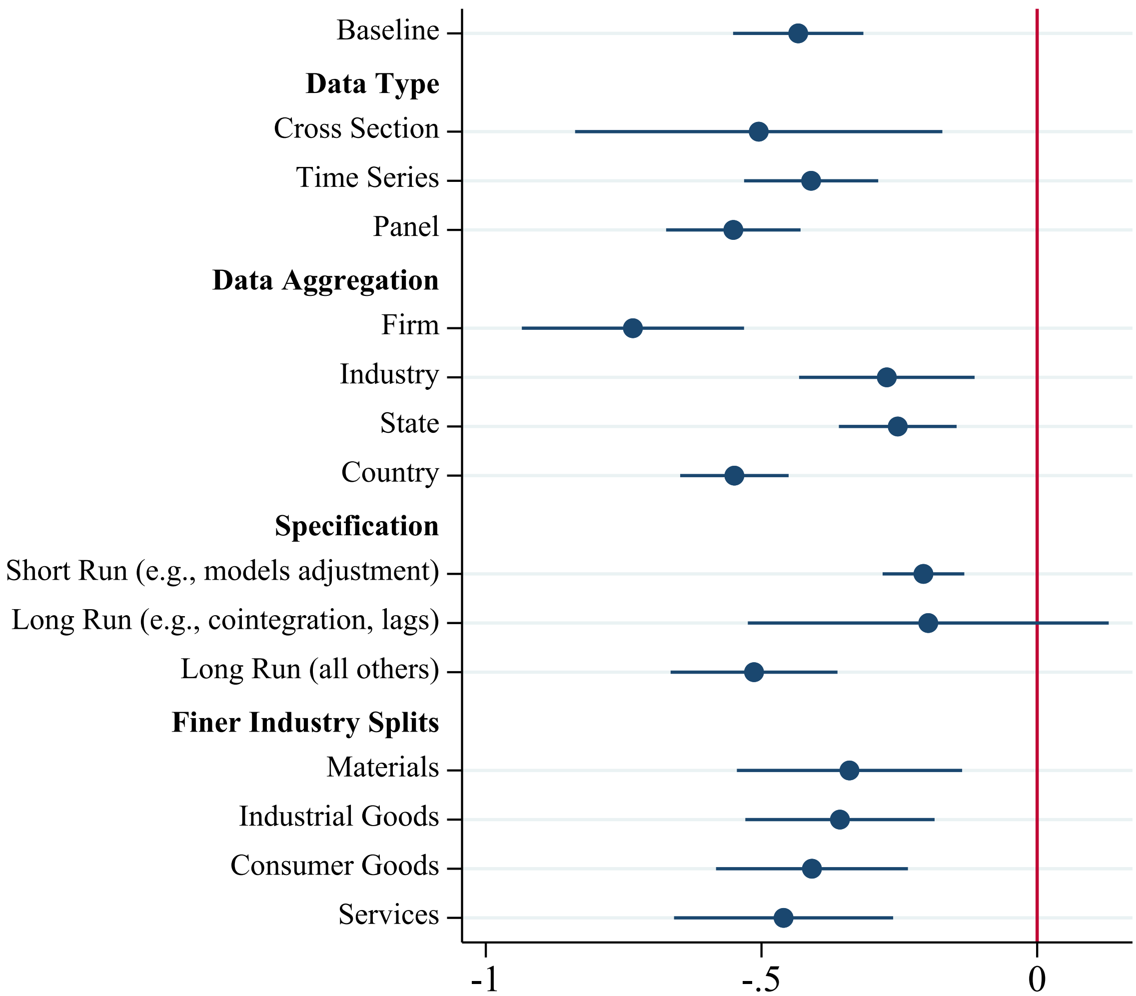
Dependent Variable	Elasticity estimate from meta study (ε_{si})				
	(1)	(2)	(3)	(4)	(5)
Indicator for ε_{FOC_K}	–0.484***	–0.433***	–0.343***	–0.318***	–0.372***
($\varepsilon_{\frac{K}{L},w_O} = \varepsilon_{FOC_K} - \varepsilon_{FOC_L}$)	(0.133)	(0.072)	(0.084)	(0.109)	(0.068)
Constant	0.892***	0.778***	0.714***	0.714***	0.710***
(ε_{FOC_L})	(0.127)	(0.062)	(0.058)	(0.080)	(0.031)
Study Sample	All	All	All	U.S.	Manuf.
Study FEs	No	No	Yes	Yes	Yes
Precision Weighted	No	Yes	Yes	Yes	Yes
R^2	0.020	0.248	0.490	0.799	0.740
N	1764	1764	1760	952	967

Note: An observation ε_{is} is an individual elasticity estimate i from a published study s. The data come from Gechert, Havranek, Irsova and Kolcunova (2022). Standard errors are clustered at the study level. The sample includes only estimates based either on the first-order condition for labor or the first-order condition for capital. Weights are defined as the inverse of the standard error of the estimate.

The implication that outsourcing is principally labor displacing is robust to including controls for cross-study differences in estimation approaches and contexts. To that effect, columns (3)–(5) introduce study-level fixed effects so that the estimation now leverages within-study variation from studies that have estimated both ε_{FOC_L} and ε_{FOC_K}. In column (3) the resulting outsourcing elasticity remains strongly negative—maintaining the inference that outsourcing is labor displacing—with the point estimate declining from –0.433 to –0.343. Columns (4) and (5) repeat the same exercise while restricting the sample, respectively, to only U.S. data and to only manufacturing data. Column (4) suggests that a 1% decrease in the real price of outsourcing increases the U.S. capital-labor ratio by 0.318%, while column (5) suggests an increase of 0.372% for the manufacturing sector.

While the inclusion of study-level fixed effects addresses many potential differences across studies simultaneously, figure 2 shows that the evidence for factor-biased outsourcing persists even within specific subsamples of the meta-analysis data. Specifically, the figure plots the baseline estimate of factor bias from column (2) of table 2, as well as for the enumerated subsamples when using the same specification. The subsamples are constructed courtesy of indicator variables that classify estimates based on the type of data used (cross section, time series or panel), the level of data aggregations (firm, industry, state or country), a few coarse industry groups (materials, industrial goods, consumer goods, and services), as well as information on the extent to which the specification con-

Figure 2: Estimates of Factor-Biased Outsourcing by Meta-Analysis Subsample



Note: The baseline estimate of factor-biased outsourcing corresponds to the coefficient estimate from column (2) of table 2, where meta-analysis observations are precision weighted by the inverse of their standard errors. Subsequent estimates in the figure correspond to the same specification but on the indicated subset of metal-analysis estimates.

trolled for short-run or long-run forces in estimation. Across the different subsamples there is evidence for outsourcing that principally displaces labor relative to capital.

Notably, figure 2 suggests that the factor-biased nature of outsourcing in the meta analysis is unlikely to be driven by the differential adjustment of capital and labor in production. To that effect, the subsample labeled by the meta analysis as Short Run focuses on studies that account for temporal dynamics by, for instance, modeling the impact of frictions that affect capital adjustment. The resulting outsourcing elasticity is roughly half of the baseline –0.433, suggesting that, indeed, some of the asymmetry in estimates coming from the first-order condition for labor ε_{FOC_L} versus those for capital ε_{FOC_K} might be driven by differential adjustment. However, the estimate of the outsourcing elasticity in this subsample is still strongly negative, consistent with the baseline results of outsourcing disproportionately displacing labor. Moreover, studies that address the question of factor adjustment through other means—relying, for instance, on cointegration relationship, leads and lags, and similar adjustments—are grouped into two Long Run subsamples both of which tend to reinforce the same conclusion as in the baseline estimates.

In addition to characterizing the factor bias of outsourcing, the results in this section also quantify the extent to which the capital-labor ratio responds differently to changes in the price of labor and to changes in the price of capital: they imply that the elasticity of the capital-labor ratio with respect to the price of labor $\varepsilon_{\frac{K}{L},w_L}$ is around 0.7–0.8, while the elasticity of the capital-labor ratio with respect to the price of capital $\varepsilon_{\frac{K}{L},w_K}$ is around 0.3–0.4. We can read the estimates of $\varepsilon_{\frac{K}{L},w_L}$ from table 2 as the values of the regression Constant. The Constant provides the conditional average of meta-analysis estimates when the indicator variable for ε_{FOC_K} takes the value zero, and hence it provides the average across estimates using ε_{FOC_L}. The average $\varepsilon_{\frac{K}{L},w_K}$ is then simply the difference of the coefficient on the Constant and the coefficient on the indicator variable.

3.2 Direct Measurement Using Input Quantities and Prices

Using data that separates input prices from input quantities for the U.S. economy, I next corroborate the indirect inference in the previous subsection by showing that changes in the capital-labor ratio are inversely related to changes in the price of outsourced inputs. I begin by describing the data and by emphasizing that purchases of outsourced inputs are often the largest cost that firms incur, exceeding firms' expenditures on the labor and the capital services that they hire directly. I then introduce the main estimating equation, discuss its implementation, and present results suggesting that U.S. outsourcing disproportionately displaces labor.

3.2.1 Data: BEA & BLS Integrated Production Accounts

The Integrated Production Accounts from the U.S. Bureau of Economic Analysis and the U.S. Bureau of Labor Statistics cover 1960 to 2016 and distinguish price from quantity for different factors of production; in doing so, the production accounts make possible the direct measurement of factor-biased outsourcing as the response of the capital-labor ratio to the price of outsourcing. These production accounts are a KLEMS data set in the spirit of Jorgenson, Gollop and Fraumeni (1987) that, in theory, separates factors of production into capital, labor, energy, materials and services. In practice, the data contain information on three factors of production: capital, labor, and intermediates, the last of which combines energy, materials and services, and which I treat as the outsourced inputs.

Outsourcing—defined here as the expenditure on all inputs purchased from others—is not a minor consideration; in fact, it has historically been the largest factor expenditure in the U.S. economy, exceeding 40% of U.S. gross output since the 1960s. In panel (a), figure 3 plots the time series of these outsourcing expenditures along side those of capital and labor, all expressed as a share of the economy's gross output. Panel (b) shows that, although there is substantial heterogeneity, outsourcing is ubiquitous across sectors. In manufacturing, for instance, expenditures on outsourcing in 2005 were more than 60% of manufacturing's gross output, making outsourcing a larger expenditure for manufacturing firms than expenditures on capital and labor combined.

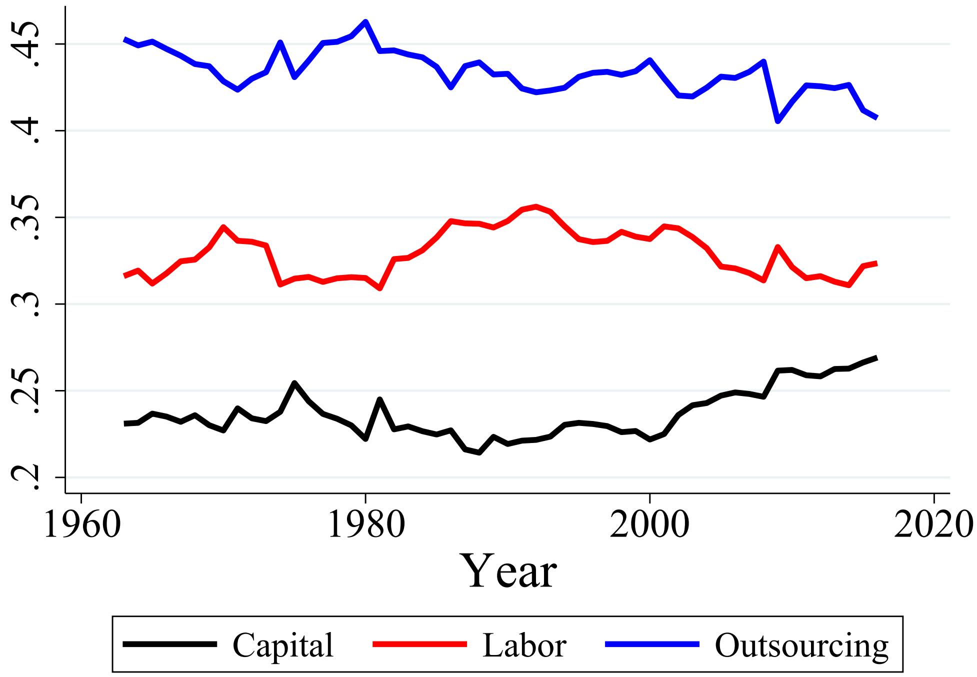
Key to the direct measurement of factor bias in this section is the price index for intermediate inputs, used here as the price of outsourcing. These price indexes are constructed by the BEA based on annual input-output use tables for U.S. economy. Weights from these tables are combined with the price indexes of the input industries/commodities to construct chain-type indexes for the overall bundle of intermediate inputs. Domestic and imported portions of intermediate inputs are deflated separately to account for the commodities purchased as inputs from domestic and from foreign sources.

The quantity indexes of capital and labor reflect efforts by the BEA and the BLS to create measures of capital and labor services that are adjusted for underlying heterogeneity and quality. Capital services are calculated using perpetual inventory methods and cover physical capital assets—equipment, structures, inventories and land—taking into account differences in nominal returns, rates of economic depreciation, and tax treatment. Labor services measures start with hours worked and make adjustments for the composition of the labor force by education and experience.

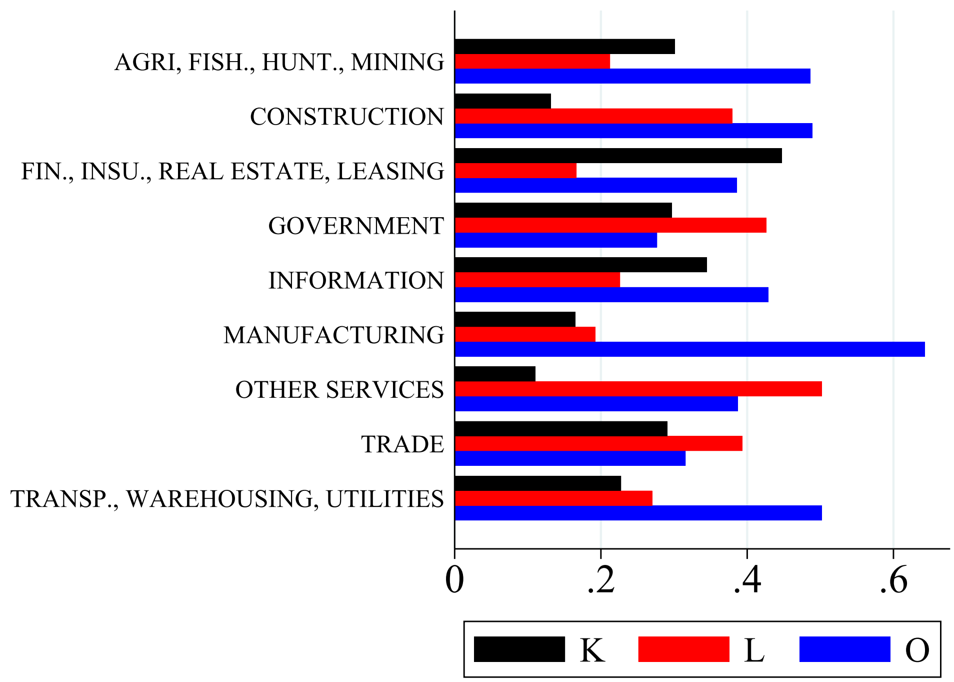
The focus of the empirical work is quantifying the extent of factor-biased outsourcing for the aggregate U.S. economy. The aggregate price and quantity indexes are constructed as Tornqvist indexes of the underlying industry data. Panels of broad sectors—such as

Figure 3: U.S. Factor Shares of Gross Output

(a) Aggregate Economy, 1963-2016



(b) Sectors, 2005



those in panel (b) of figure 3—can be helpful to decompose the aggregate factor bias into a component driven by reallocation across sectors and a component driven by factor bias within sectors.

3.2.2 Empirical Strategy

To estimate directly the factor bias of outsourcing $\varepsilon_{\frac{K}{L},w_O}$ we need to measure how the capital-labor ratio responds to the price of outsourcing; as we observe indexes rather than levels of price and quantities, I estimate the factor bias of outsourcing in differences:

$$\Delta \ln \frac{K}{L}_{it} = \varepsilon_{\frac{K}{L},w_O} \Delta \ln \frac{W_O}{P}_{it} + \gamma\, \Delta\text{Controls}_{it} + \Delta e_{it}. \tag{15}$$

In this specification, the price of outsourcing w_O is normalized by the price of gross output P, focusing attention on real rather than nominal factor prices. The controls in question can include a variety of fixed effects or other factor prices, depending on the specification.

In addition to baseline estimates using ordinary least squares, I also present results where an index of global commodity prices acts as an instrument to isolate plausibly-exogenous variation in the U.S. cost of outsourcing. The motivation for the instrument is that the price of many commodities is globally determined and could therefore affect prices faced by U.S. firms in ways that have little to do with U.S. economic conditions. In step with this idea, I focus on commodities for which the U.S. has not been the major global producer, which effectively restricts the sample to a subset of non-energy commodities. Drawing on a commodity database from the World Bank, I construct an index of the following form:

$$\Delta \ln P_t^{com} = \sum_j \omega_{j,t} \times \Delta \ln P_{j,t}^{com}, \tag{16}$$

where $P_{j,t}^{com}$ is the real price of commodity j at time t and $\omega_{j,t}$ are commodity weights based on the value of commodity j imports in overall commodity imports. The weights in question are either time-invariant or lagged three-year rolling averages.

3.2.3 Results

Much like the indirect inference from the meta analysis, the results in table 3 suggest that outsourcing is factor biased and that it displaces labor disproportionately relative to capital. Column (1)'s estimated elasticity of –0.529 implies that a 1% decrease in the real price of outsourcing increases the aggregate capital-labor ratio by 0.529%, controlling for changes in the price of capital and labor. As the specification in column (1) is estimated in differences, it is consistent both with models of Hicks-neutral technological change and

Table 3: Direct Estimates of Factor-Biased Outsourcing for the United States

Dep. Variable	$\Delta \ln K/L$	$\Delta \ln K/L$	$\Delta \ln K_i/L$	$\Delta \ln K/L$	$\Delta \ln K/L$	$\Delta \ln K_i/L$
	(1) OLS	(2) OLS	(3) OLS	(4) IV	(5) IV	(6) IV
$\Delta \ln W_O/P$	–0.529*	–0.750***	–0.226***	–0.889*	–0.881*	–0.932**
	(0.279)	(0.270)	(0.064)	(0.471)	(0.441)	(0.420)
Data	Aggregate	Aggregate	Sectoral	Aggregate	Aggregate	Sectoral
Controls	Yes	Yes	Yes	Yes	Yes	Yes
Time Trend	No	Yes	No	No	Yes	No
R^2	0.240	0.264	0.121			
N	53	53	477	53	53	477
1st stage F-stat				22.94	25.16	23.99

Note: An observation is at the country-year level for aggregate data and at the sector-year level otherwise. The data span 1963-2016, and controls include changes in the cost of labor and capital, as well as sector fixed effects for sectoral regressions. Sectoral regressions are weighted by the sector's share in aggregate capital. Standard errors are robust to autocorrelation, and sectoral regressions further allow for arbitrary clustering following Driscoll-Kraay.

with standard implementations of factor-biased technological change.[4] Nonetheless, to accommodate more flexibly the possibility of differential growth trends in factor-biased technological change, the specification in column (2) additionally includes time trends in the differences specification. The resulting estimate suggests a greater extent of factor bias with the key outsourcing elasticity taking a value of –0.750. Columns (4) and (5) provide estimates from parallel specifications using the change in global commodity prices as an instrument; the associated elasticities of –0.889 and –0.881 tell a similar story of factor-biased outsourcing as do the OLS baselines.

The notion that outsourcing is factor biased and displaces labor disproportionately is further reinforced by the weighted sectoral regressions in columns (3) and (6). These weighted regressions use a panel of sectors to estimate the aggregate impact of outsourcing on the capital-labor ratio. The OLS estimate in column (3) of –0.226 and the IV estimate in column (6) of –0.932 comprise both within-sector factor bias and the outsourcing-induced reallocation of capital and labor demand across sectors. In section 5 I present some aggregation results and a formal decomposition of the estimated aggregate factor bias into reallocation and within-sector substitution.

[4] A common assumption taken to the data (e.g., Diamond, McFadden and Rodriguez, 1978; Antras, 2004) is that factor-biased technological change grows at constant rates. Estimation in levels would then entail including a time trend. In differences, however, the time trend would be absorbed by the constant.

4 Implications of Factor-Biased Outsourcing

This section highlights three implications of the estimated factor bias in outsourcing. First, that the capital-labor ratio responds more strongly to changes in the price of labor than to changes in the price of capital is directly policy relevant. Second, factor bias and the varied sources of relative-price variation (capital versus labor) cloud the interpretation of relative-price elasticities. Third, factor-biased outsourcing violates the assumptions behind commonly-used production and cost functions that make capital and labor separable from outsourced inputs, presenting a challenge for modeling the production of value added.

4.1 Different Elasticities for Different Policy Questions

When outsourcing is factor biased it leads to an asymmetry: the capital-labor ratio to respond differently to a change in the price of labor and to a change in the price of capital. This asymmetry suggests that different policy questions could be mediated through different elasticities, $\varepsilon_{\frac{K}{L},w_L}$ and $\varepsilon_{\frac{K}{L},w_K}$. In particular, policies affecting the cost of capital (e.g., a change to capital taxation or central bank interest rate policy) would be transmitted to the rest of the economy through a different elasticity than policies affecting the cost of labor (e.g. a change in the minimum wage).

The estimated factor bias in this paper—viewed through the lens of the meta analysis in table 2—implies that the elasticity of the capital-labor ratio with respect to the price of labor $\varepsilon_{\frac{K}{L},w_L}$ is around 0.7–0.8, while the elasticity of the capital-labor ratio with respect to the price of capital $\varepsilon_{\frac{K}{L},w_K}$ is around 0.3–0.4. As a result, U.S. policies affecting the cost of capital would be transmitted to the rest of the economy through a smaller elasticity than policies affecting the cost of labor. This notion of using different elasticities—rather than a common one—is important because the estimated factor bias is substantial, and even a modest change in the extent of substitution can lead to large changes in policy outcomes. For instance, Chirinko (2002) finds that moving from an elasticity of 0.5 to 1 increases the welfare losses associated with a range of capital taxation policies by 50-79 percent, depending on the details of the study and comparison.

4.2 Uncertain Interpretation of Relative-Price Elasticities

Factor biased outsourcing can also help explain why economists could estimate different relative-price elasticities even when studying the same firm, industry or country. Consider two studies looking at the same firm and estimating the relative-price elasticity $\varepsilon_{\frac{K}{L},\frac{w_L}{w_K}}$ using different sources of relative price variation. One study has an unimpeachable instrument generating variation in the relative price w_L/w_K primarily by varying the price of labor w_L. By contrast, the other study has a similarly unimpeachable instrument generating variation in the relative price w_L/w_K primarily by varying the price of capital w_K. In a world of factor-biased outsourcing, these two studies would estimate different relative-price elasticities despite studying the same firm.

The results in section 2.3—by incorporating the zero-sum constraint into the measurement of the relative-price elasticity—emphasized that both the extent of factor-biased outsourcing and the extent of relative-price variation shape the estimated $\varepsilon_{\frac{K}{L},\frac{w_L}{w_K}}$. From the earlier equation (10), we can see that—even if the elasticities $\varepsilon_{\frac{K}{L},w_L}$, $\varepsilon_{\frac{K}{L},w_K}$, and $\varepsilon_{\frac{K}{L},w_O}$ are known and constant—there could be dispersion in the estimated relative-price elasticities:

$$Variance(\widehat{\varepsilon_{\frac{K}{L},\frac{w_L}{w_K}}}) = \underbrace{(\varepsilon_{\frac{K}{L},w_O})^2 \times Variance(\phi_{w_K})}_{\text{Factor-Biased Outsourcing} \times \text{Price Variation}} .$$

The extent of this dispersion is increasing both in the extent of factor bias $\varepsilon_{\frac{K}{L},w_O}$ and in the extent to which the different studies use different sources of relative price variation, ϕ_{w_K}.

The price variation in the U.S. data and the estimated factor bias combine to generate a broad range of potentially-estimable relative-price elasticities. In aggregate U.S. data for 1963 to 2016, the price variation term ϕ_{w_K}—quantifying the ratio of the growth rate of w_K relative to the growth rate of the relative price w_K/w_L—is highly volatile. It has a mean of 0.29 and a variance of 5.77. Using equation (18) we can get a sense of the expected dispersion of estimated relative-price elasticities $\widehat{\varepsilon_{\frac{K}{L},\frac{w_L}{w_K}}}$; to do so, we can combine the price dispersion ϕ_{w_K} from the data with estimates of factor bias, $\varepsilon_{\frac{K}{L},w_O}$. At the more conservative end for estimates of factor bias, say with an estimate of $\varepsilon_{\frac{K}{L},w_O}$ around –0.3, the expected dispersion of $\widehat{\varepsilon_{\frac{K}{L},\frac{w_L}{w_K}}}$ is around 0.52. With an estimate of factor bias around –0.8, the expected dispersion would be 3.69. Given that estimates of $\widehat{\varepsilon_{\frac{K}{L},\frac{w_L}{w_K}}}$ from meta studies often average below unity, this dispersion and uncertainty caused by factor bias is substantial.

4.3 Capital and Labor Are Not Separable from Other Inputs

In addition to its implications for policy and measurement, factor-biased outsourcing is also informative about the theoretical modeling of cost and production, in particular for the question of whether capital and labor are separable from other inputs.

A common practice is to model the cost (or production) of gross output $C(K, L, O)$ as separable in value added, comprised of capital K and labor L, and in outsourced inputs inputs O. Goldman and Uzawa (1964) conditions for separability imply that the separability of capital and labor from a third outsourced factor could be summarized as the following requirement on the cost function C:

$$C_{w_L} C_{w_K w_O} - C_{w_K} C_{w_L w_O} = 0,$$

with C_{w_F} a partial derivative with respect to factor price w_F, and $C_{w_F w_{F'}}$ the second partial derivative with respect to factor prices w_F and $w_{F'}$.

Factor-biased outsourcing violates the key assumption needed to model capital and labor as separable from outsourced inputs. To see the link between factor-biased outsourcing and separability, note that Shepherd's Lemma implies that the partial derivative of the cost function C_{w_F} is the factor demand for F, and the second derivative of the cost function $C_{w_F w_{F'}}$ is the derivative $\frac{\partial F}{\partial w_{F'}}$ of demand for F with respect to factor price $w_{F'}$. With a bit of manipulation, we can therefore rewrite the separability condition as:[5]

$$L^2 \varepsilon_{\frac{K}{L}, w_O} = 0.$$

Separability of capital and labor from outsourced inputs therefore requires that outsourcing be factor neutral, $\varepsilon_{\frac{K}{L}, w_O} = 0$. If outsourcing is factor biased, then the key assumption for separability is violated.

The finding in this paper that outsourcing is factor biased, so that $\varepsilon_{\frac{K}{L}, w_O} \neq 0$, complements a larger literature questioning the empirical realism of specifying value-added production and costs that treats capital and labor as separable from outsourced inputs.

5 Aggregate Factor Bias: Reallocation versus Substitution

Next, I emphasize that the results in this paper apply in any setting and at any level of aggregation at which capital and labor demand are homogeneous of the same degree, and I provide an aggregation result showing that aggregate factor bias is a combination of across-firm reallocation and within-firm factor bias.

To think about the aggregation of factor bias, I leverage here a third property of homogeneous functions to show that aggregate capital and labor demand can inherit the homogeneity of the underlying firm-level input demands. Specifically, if firm-level input demands $K_i(w)$ and $L_i(w)$ are homogeneous of degree h for all firms i, then aggregate input demands $K(w)$ and $L(w)$ are homogeneous of the same degree h. In the context of

[5] $C_{w_L} C_{w_K w_O} - C_{w_K} C_{w_L w_O} = L \frac{\partial K}{\partial w_O} - K \frac{\partial L}{\partial w_O} = L^2 \left[\frac{1}{L} \frac{\partial K}{\partial w_O} - \frac{K}{L^2} \frac{\partial L}{\partial w_O} \right] = L^2 \frac{\partial \frac{K}{L}}{\partial w_O} = L^2 \varepsilon_{\frac{K}{L}, w_O}$

capital, we have that

$$K(\lambda\mathbf{w}) = \sum_i K_i(\lambda\mathbf{w}) = \sum_i \lambda^h K_i(\mathbf{w}) = \lambda^h \sum_i K_i(\mathbf{w}) = \lambda^h K(\mathbf{w}), \tag{17}$$

with the same relationship holding for aggregate labor demand $L(w)$. As a result of capital and labor demand being homogeneous of the same degree h at both the aggregate and the firm level, the zero-sum constraint on the behavior of the capital-labor ratio from section 2 applies both to the aggregate economy and to the underlying firms.

We can use the above aggregation structure to decompose the response of the aggregate capital-labor ratio to a change in any factor price into two forces: reallocation across firms and substitution within firms. To that effect, note first that the aggregate K/L can be written as a weighted average of firms' capital-labor ratios K_i/L_i,

$$\frac{K}{L} = \sum_i \frac{L_i}{L}\frac{K_i}{L_i},$$

with the weights being the firm's relative size in terms of labor, L_i/L. We can then differentiate both sides of the expression with respect to a factor price w_f and express the impact on the aggregate K/L ratio as a sum of w_f's impacts on the relative size of firms L_i/L and on the within-firm capital-labor ratio K_i/L_i:

$$\varepsilon_{\frac{K}{L},w_f} = \sum_i \frac{K_i}{K} \times \varepsilon_{\frac{L_i}{L},w_f} + \sum_i \frac{K_i}{K} \times \varepsilon_{\frac{K_i}{L_i},w_f}. \tag{18}$$

The aggregate elasticity $\varepsilon_{\frac{K}{L},w_f}$ is then comprised of two capital-weighted average elasticities, the elasticity of relative firm size $\varepsilon_{\frac{L_i}{L},w_f}$ (reallocation across firms) and the elasticity of the capital-labor ratio $\varepsilon_{\frac{K_i}{L_i},w_f}$ (substitution across firms).

Applied to outsourcing, the decomposition suggests that factor bias can potentially look different at the aggregate and at the firm level because of reallocation across firms. For instance, we could have aggregate factor bias driven entirely by reallocation. Consider a world where outsourcing is factor neutral for individual firms and where most outsourcing comprises labor-intensive firms making purchases from capital-intensive firm. When the price of outsourcing falls, demand will increase for the services of the capital-intensive firms, leading to an increase aggregate demand for capital relative to labor. As aggregate K/L rises following a fall in w_O, we then have aggregate factor bias even though none exists at the level of a firm.

Using sectoral U.S. data, I decompose the aggregate factor bias estimated in section 3 that suggests U.S. outsourcing disproportionaly displaces labor relative to capital. Specifically, I use the panel of sectors to estimate both sides of equation (18), highlighting the extent to which labor-displacing outsourcing comes from outsourcing-induced reallocation across sectors versus within-sector factor bias.

The results in table 4 suggest that both reallocation across sectors and substitution within sectors contribute to the aggregate bias, pushing it in the same direction. Columns (1) and (4) in table 4 are copies of columns (3) and (6) from table 3, and they present the estimates of aggregate factor bias based on capital-weighted sectoral regressions. These two columns are direct estimates of the left-hand side of equation (18); both suggest that a decline in the price of outsourcing displaces labor disproportionately, increasing the capital-labor ratio. Columns (2) and (3) decompose the aggregate factor bias of column (1), suggesting that both across-sector reallocation and within-sector substitution push towards a labor-displacing vision of outsourcing, with within-sector factor bias playing a proportionally larger role. The decomposition in columns (5) and (6) tells a similar story.

Table 4: Decomposing Factor Bias: Across Sectors vs. Within Sectors

Dep. Variable	Total $\Delta \ln K_i/L$	Across $\Delta \ln L_i/L$	Within $\Delta \ln K_i/L_i$	Total $\Delta \ln K_i/L$	Across $\Delta \ln L_i/L$	Within $\Delta \ln K_i/L_i$
	(1) OLS	(2) OLS	(3) OLS	(4) IV	(5) IV	(6) IV
$\Delta \ln W_O/P$	–0.226*** (0.064)	–0.051 (0.067)	–0.175** (0.084)	–0.932** (0.420)	–0.362*** (0.132)	–0.570 (0.363)
Controls	Yes	Yes	Yes	Yes	Yes	Yes
Sector FEs	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.121	0.015	0.087			
N	477	477	477	477	477	477
1^{st}stage F-stat				23.99	23.99	23.99

Note: An observation is at the sector-year level. The data span 1963-2016, and controls include changes in the cost of labor and capital. The regressions are weighted by the sector's share in aggregate capital. Standard errors are robust to autocorrelation and arbitrary clustering following Driscoll-Kraay.

6 Conclusion

Outsourcing can be factor biased. It might disproportionately displace labor, as in the case of a janitor who is fired and whose services are then rented from an outside cleaning company. Or, it might disproportionately displace capital, as in the case of a firm phasing out its internal data storage with the cloud storage services of a company like Dropbox. By disproportionately displacing one factor of production, outsourcing can potentially reshape the extent of substitutability between the capital and that labor that remain inside the firm.

This paper leverages the commonly-used assumption of homogeneous input demand to sharpen measurement and to derive implications of factor-biased outsourcing. The notion of outsourcing considered here is quite broad: it encompasses all goods and services used as inputs that come from outside the firm. These outsourced inputs are a large share of economic activity, comprising over 40% of U.S. gross output since the 1960s, outstripping the costs associated with compensating either capital or labor services directly.

Both indirect and direct measures of factor bias suggest that outsourcing in the United States displaces labor relatively more than it displaces capital. A direct implication of labor-displacing outsourcing is that the capital-labor ratio responds more strongly to changes in labor prices than it responds to changes in capital prices. These differential responses are directly policy relevant. They suggest that changes in the price of labor (e.g., a change in the minimum wage or unionization) would be mediated to the rest of the economy through a larger elasticity than an equivalent change in the price of capital (e.g., from changes in central-bank interest rate policy or a reform of capital taxation).

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