

Optimal Monetary Policy during a Cost-of-Living Crisis

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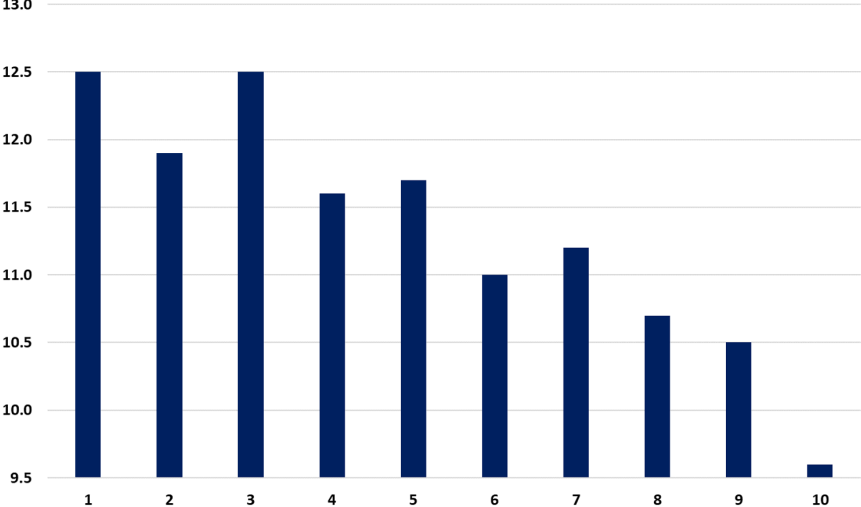
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Cost-of-Living Crisis

CPI inflation rate by income decile - UK, October 2022



Source: ONS.

This paper

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Literature

Households

Unit mass of households, indexed by j . Die with probability δ .
Born with some initial level of wealth, $b(j)$.

K goods sectors, indexed by $k = 1, 2, \dots$. Continuum of symmetric varieties within each sector, indexed by i .

Utility:

$$\mathbb{E}_t \sum_{s=0}^{\infty} (\beta(1 - \delta))^{t+s} (U(\mathbf{c}_{t+s}) - \chi(n_{t+s}))$$

where

$$U(\mathbf{c}) = U(\mathcal{U}_1(\mathbf{c}^1), \dots, \mathcal{U}_K(\mathbf{c}^K))$$

- ▶ *Outer* utility function U is weakly separable in products produced in different sectors, and twice differentiable.
- ▶ *Inner* utility function \mathcal{U}_k is concave, symmetric and twice Fréchet differentiable.

Households

- ▶ Households have an idiosyncratic productivity level $\theta(j)$.
- ▶ Households decide on consumption, labour supply and bond holdings.
- ▶ Budget constraint household j :

$$e_t(j) + b_t(j) = R_{t-1}b_{t-1}(j) + \theta(j)n_t(j)W_t + \sum_k \zeta(j) \text{div}_{k,t},$$

where $e_t(j) = \sum_k e_{k,t}(j) = \sum_k \int_0^1 p_{k,t}(i) c_{k,t}(i, j) di$.

Extension 1: HtM households

Households

Key objects (at steady state)

Budget share: $s_k(j) = \frac{c_k(j)}{c(j)}$

Marginal budget share: $\tilde{\zeta}_k(j) = \frac{\partial c_k(j)}{\partial c(j)}$

Cross-price elasticity: $\rho_{k,l}(j) = \frac{\partial c_k(j)}{\partial P_l} \frac{P_l}{c_k(j)}$

Demand elasticity: $\epsilon_k(j) = \frac{\partial c_k(i,j)}{\partial p_k(i)} \frac{p_k(i)}{c_k(i,j)}$

Super-elasticity: $\epsilon_k^s(j) = \frac{\partial \epsilon_k(j)}{\partial p_k(i)} \frac{p_k(i)}{\epsilon_k(j)}$

Markup elasticity: $\gamma_{e,k}(j) = D\mu_k(e_k(j)) \frac{E_k}{\mu_k}$

Firms

- ▶ Monopolistically competitive. Maximize expected PV of profits.
- ▶ Can adjust their price only with probability $1 - \theta_k$.
- ▶ Production function:

$$y_{k,t}(i) = A_{k,t} l_{k,t}(i).$$

- ▶ aggregate + sectoral productivity shocks
- ▶ Demand constraint:

$$y_{k,t}(i) = \int_0^1 c_k(p_{k,t}(i), \mathbf{p}_{k,t}, e_{k,t}(j)) dj.$$

Extension 2: Input-output linkages

Government & Market clearing

- ▶ Fiscal policy eliminates steady-state markups.
- ▶ Monetary policy rule:

$$\hat{R}_t = \phi \pi_{cpi,t} + u_t^R.$$

alternatively: optimal policy

- ▶ Markets for goods, bonds and labor clear.
- ▶ Deceased households are replaced by their steady-state versions

New Keynesian Phillips Curve: 2 new wedges

case with homogeneous slope across sectors

$$\pi_{cpi,t} = \lambda (\tilde{Y}_t + \mathcal{N}\mathcal{H}_t + \mathcal{M}_t) + \beta \mathbb{E}_t \pi_{cpi,t+1},$$

Policy implications: analytical results

Two simplifying assumptions:

A1. The NKPC slope $\lambda_k(1 + \Gamma_k)$ is common across sectors.

A2. There is no s.s. wealth heterogeneity ($b(j) = 0$).

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When $\mathcal{M}_t = 0$, fluctuations in the output gap can be eliminated by stabilising the Marginal CPI index $\pi_{mpci,t} \equiv \sum_k \bar{\xi}_k \pi_{k,t}$.

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Result 3 (breakdown coincidence under non-CES preferences)

When $\mathcal{M}_t \neq 0$ there does not exist any inflation index which can be fully stabilised together with the output gap.

Cyclicalilty of the wedges: analytical results

Result 4 (movements in the \mathcal{NH} wedge)

When $\mathcal{M}_t = 0$, then \mathcal{NH} rises (falls) on impact, in response to a negative productivity shock to a necessity (luxury) sector.

Cyclicalilty of the wedges: analytical results

Result 4 (movements in the \mathcal{NH} wedge)

When $\mathcal{M}_t = 0$, then \mathcal{NH} rises (falls) on impact, in response to a negative productivity shock to a necessity (luxury) sector.

Can write the NKPC as:

$$\pi_{cpi,t} = \lambda \left((1 + \Gamma) \tilde{\mathcal{Y}}_t + \mathcal{NH}_t + \overline{\mathcal{M}}_t \right) + \beta \mathbb{E}_t \pi_{cpi,t+1},$$

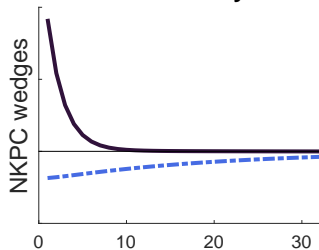
where $\overline{\mathcal{M}}_t = \Gamma \hat{\mathcal{Y}}_t^* + \mathcal{M}_t^D + \mathcal{M}_t^P$.

Result 5 (movements in the $\overline{\mathcal{M}}$ wedge)

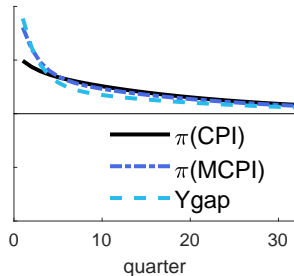
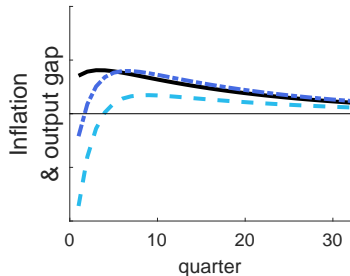
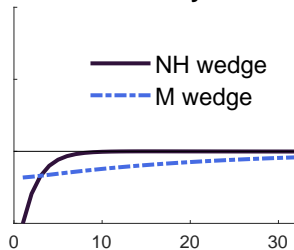
$\overline{\mathcal{M}}_t$ declines following a negative aggregate productivity shock.

Illustration

shock to necessity sector



shock to luxury sector



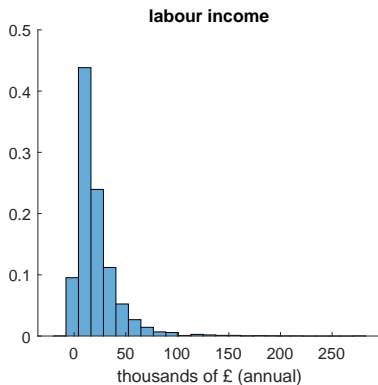
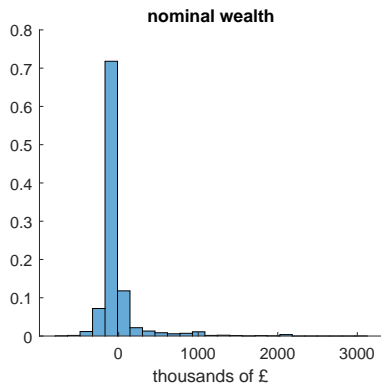
Model solution

The full model has a block-recursive structure:

- ▶ Can write as block of $4K + 3$ core equations
 - ▶ keeps track of relevant distributional objects.
- ▶ Straightforward to solve for dynamics distributions and aggregates.
- ▶ Quantitative implementation: discipline with data from the Living Costs and Food (LCF) survey.

Calibration: steady state

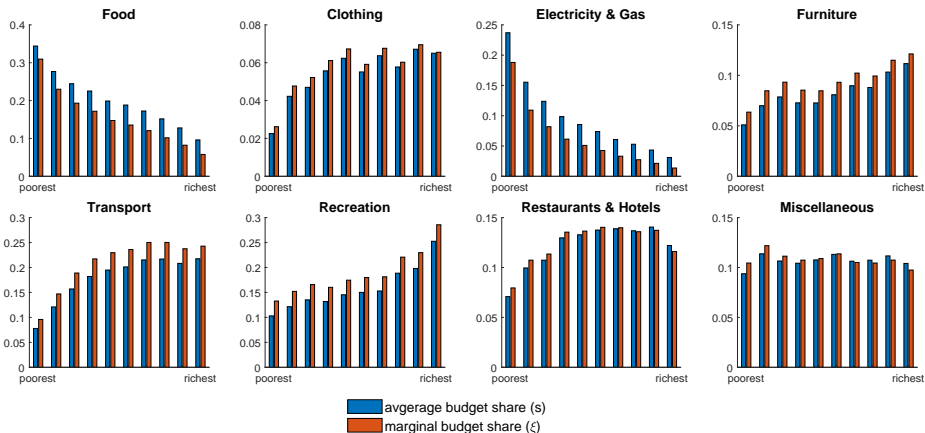
Feed in empirical distributions



Source: LCF.

Calibration outer utility: non-homothetic CES

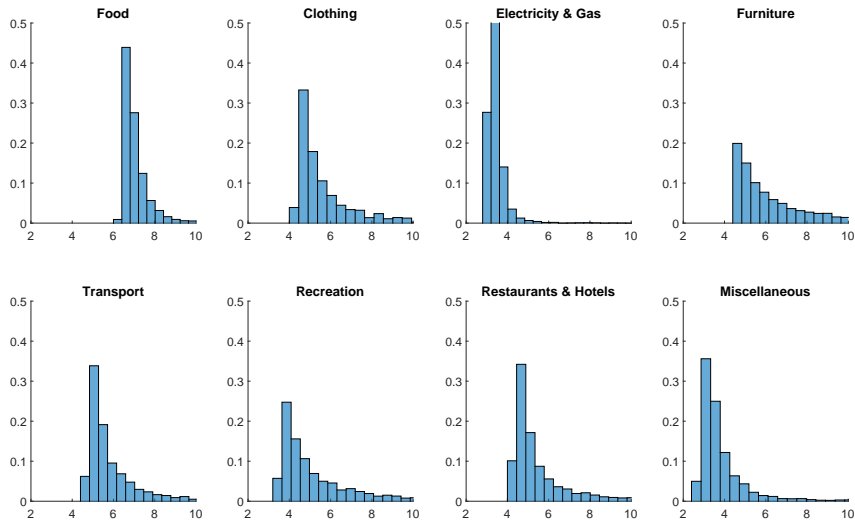
Follow Comin et al. (2021).



Estimate using LCF data; back out marginal budget shares, allowing for preference shifters.

Inner utility: HARA

Target empirical evidence on markups (ONS) and pass-through (Amiti et al. (2019)).



Note: histograms plotting the s.s. distribution of demand elasticities

Parameter values

Parameter	description	value
β	subjective discount factor	0.99
ψ	Frisch elasticity	1
σ	elasticity of intertemporal substitution	1
δ	death probability	0.0167
ϕ	Taylor rule coefficient	1.5
μ	cross-sector elasticity of substitution	1.774
ρ_R	persistence monetary policy shock	0.25
ρ_A	persistence productivity shocks	0.95

Notes: Model period: 1 quarter.

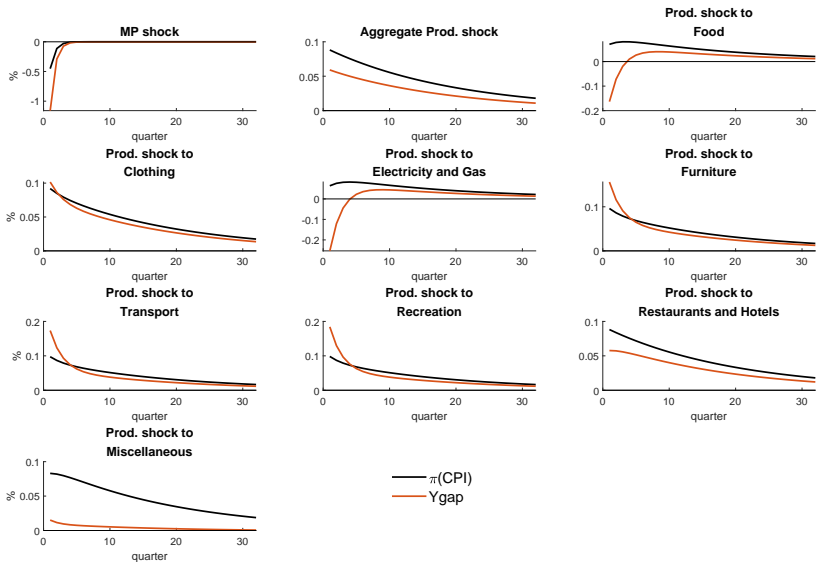
Parameter values

Sector	$\bar{\epsilon}_k$	$\bar{\eta}_k$	\bar{s}_k	$\bar{\zeta}_k$	$\bar{\gamma}_{c,k}$	θ_k	λ_k
Food	6.7832	3.8555	0.1551	0.1240	0.0145	0.4100	0.5130
Clothing	4.8346	2.5564	0.0597	0.0245	0.0285	0.3900	0.5761
Electr. & Gas	3.2852	1.5235	0.0619	0.0608	0.0618	0.6400	0.1237
Furniture	4.9751	2.6500	0.0949	0.0773	0.0269	0.4600	0.3836
Transport	5.1674	2.7783	0.2027	0.2496	0.0250	0.2600	1.2681
Recreation	4.0184	2.0123	0.1905	0.1625	0.0413	0.5100	0.2854
Rest. & Hotels	4.7313	2.4876	0.1281	0.1757	0.0298	0.7200	0.0670
Miscellaneous	3.1713	1.4475	0.1071	0.1255	0.0663	0.6700	0.0995

Notes: Calvo parameters target price durations reported by Dixon and Tian (2017).

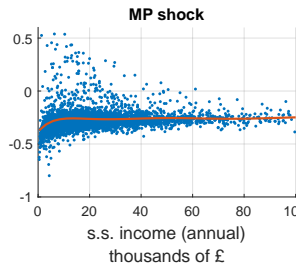
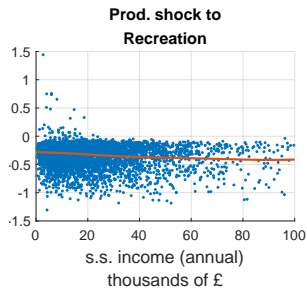
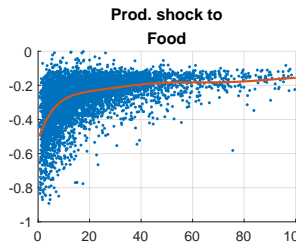
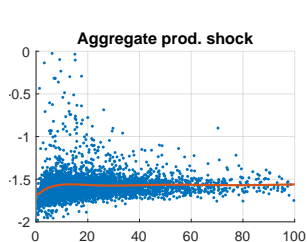
Responses to aggregate and sectoral shocks

Full model with IO linkages and HtM households



Effects across the distribution

Response of consumption in first year following the shock



Effects on different groups

Table 1. Heterogeneous consumption responses & household characteristics.

	<i>MP shock</i>		<i>Aggregate A shock</i>		<i>A shock to: Food</i>	
	coef.	std.err.	coef.	std.err.	coef.	std.err.
constant	-0.2804***	(0.0236)	-1.6058***	(0.0319)	-0.4259***	(0.011)
<i>region</i>						
London	-0.0254**	(0.0149)	0.0028	(0.02)	-0.0234***	(0.0069)
Northern Ireland	-0.0124	(0.0148)	-0.0016	(0.02)	-0.0158**	(0.0069)
Scotland	-0.0272**	(0.0125)	-0.0234	(0.0169)	-0.0211***	(0.0058)
Wales	-0.0218	(0.0167)	-0.0161	(0.0225)	-0.002	(0.0078)
<i>housing</i>						
owner - mortgage	-0.0651***	(0.0088)	-0.0939***	(0.0119)	-0.0184***	(0.0041)
renter - private	-0.0335***	(0.0091)	-0.0527***	(0.0123)	-0.0175***	(0.0043)
renter - social	-0.0501***	(0.0123)	-0.0609***	(0.0166)	-0.0534***	(0.0057)
<i>family</i>						
couple without child	-0.0009	(0.0086)	0.0035	(0.0116)	0.0334***	(0.004)
single with child	-0.0116	(0.0146)	-0.0109	(0.0197)	0.0306***	(0.0068)
single without child	-0.0072	(0.0105)	-0.0137	(0.0141)	0.0863***	(0.0049)
<i>age</i>						
38-50	0.006	(0.0089)	0.0095	(0.012)	-0.0267***	(0.0042)
51-64	0.0259***	(0.0096)	0.026**	(0.013)	-0.0311***	(0.0045)
>=65	0.019**	(0.0108)	0.0236	(0.0145)	-0.0558***	(0.005)
<i>race</i>						
black	-0.0069	(0.0265)	-0.0235	(0.0358)	0.0144	(0.0123)
mixed race	-0.0068	(0.031)	-0.0094	(0.0419)	0.0246*	(0.0144)
white	-0.0106	(0.0161)	0.0106	(0.0217)	0.0081	(0.0075)
other	-0.006	(0.0371)	-0.0081	(0.05)	-0.0285*	(0.0173)

Notes: Regression coefficients of the consumption response in the model, averaged over the first four quarters, on household characteristics. The omitted category is Eastern/owner-outright/couple with child/lowest income decile/age<38/asian. Standard errors between brackets. ***: p<0.01, **:p<0.05, *:p<0.1.

Optimal monetary policy

Replace the rule for R_t by an optimizing CB who maximizes:

$$\mathcal{W} = \mathbb{E} (1 - \delta) \int G(V^0(j), j) dj + \delta \sum_{t_0=0}^{\infty} \beta^{t_0} \int G(V^{t_0}(j), j) dj,$$

subject to all remaining model equations, where

$$V^{t_0}(j) = \sum_{s=0}^{\infty} ((1 - \delta) \beta)^s (v(e_{t_0+s}^{t_0}(j), P_{1,t_0+s}, \dots, P_{K,t_0+s}) - \chi(n_{t_0+s}^{t_0}(j)))$$

Social welfare function

Two assumptions:

i) CB treats steady-state inequality as efficient:

$$G' (V^{t_0}(j), j) \partial_e v (e(j), \hat{P}_1, \dots, \hat{P}_K) = 1.$$

ii) CB weighs households' utility fluctuations equally:

$$G'' (V_{ss}^{t_0}(j), j) = 0.$$

$$\Rightarrow \text{Pareto weight: } g(j) = \frac{E}{\psi^{\theta(j)} W n(j) + \sigma e(j)}$$

Welfare loss

Quadratic approximation

$$\mathcal{L} = (1 - \beta) \sum_s \beta^s \left(\mathcal{L}_s^l + \mathcal{L}_s^\pi + \mathcal{L}_s^s + \mathcal{L}_s^r \right) + \mathcal{L}^d$$

where

- ▶ $\mathcal{L}_s^{\tilde{y}}$: output gap (labor market) distortions
- ▶ \mathcal{L}_s^π : inflation distortions (within sector)
- ▶ \mathcal{L}_s^s : intratemporal misallocation (between sectors)
- ▶ \mathcal{L}_s^r : intertemporal misallocation of consumption
- ▶ \mathcal{L}^d : distributional motive

Equations

Analytical results - simple case

Common NKPC slope, no wealth heterogeneity

- ▶ Monetary policy only trades off output gap versus CPI inflation; cannot affect other welfare components.

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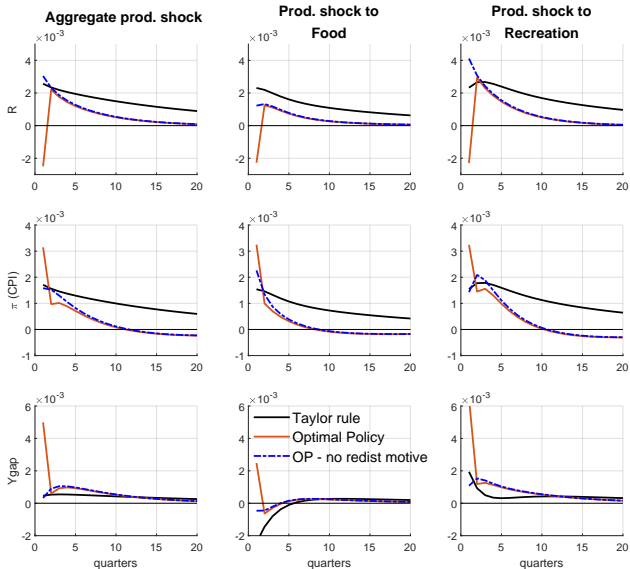
Common NKPC slope, no wealth heterogeneity

- ▶ Monetary policy only trades off output gap versus CPI inflation; cannot affect other welfare components.
- ▶ Equivalence to a Homothetic-CES RANK model with markup shocks.
- ▶ Under optimal policy, both CPI inflation and the output gap fluctuate. Can show that when $\mathcal{M} = 0$:

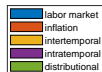
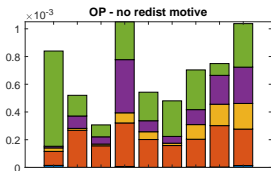
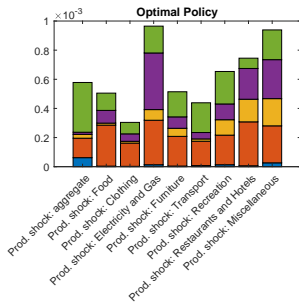
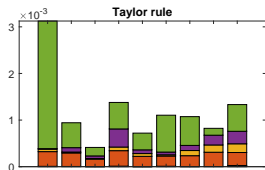
	Y gap	CPI	MCPI	NH wedge
Necessity shock (short run)	-	+	-	+
Necessity shock (medium run)	+	-	+	-
Luxury shock (short run)	+	-	+	-
Luxury shock (medium run)	-	+	-	+

Optimal policy - full model

Front-loaded redistribution



Welfare loss decomposition



Conclusion

- ▶ Tractable multi-sector HANK model with generalized preferences
 - ▶ *realistic heterogeneity in income, wealth and expenditures*
- ▶ Productivity shocks turn into markup shocks
 - ▶ *but with rich dynamics governed by inequality*
- ▶ Specific optimal policy response to cost-of-living crisis
 - ▶ *output gap overshooting*

Non-homothetic preferences +

- ▶ **Growth:** Herrendorf, Rogerson and Valentinyi (2014); Boppart (2014); Comin, Laskhari and Mestieri (2021), etc.
- ▶ **Inequality:** Engel (1857); Houthakker (1957); Hamilton (2001); Almås (2012); Argente and Lee (2021), etc.
- ▶ **Taxation:** Jaravel and Olivi (2021); Xhani (2021), etc.

Back

Extension 1 : Hand-to-Mouth households

- ▶ Within each household “type”, a fraction of households lives hand-to-mouth, setting $b_t(j) = b_{t-1}(j)$.
- ▶ Presence of HtM households may vary across distribution, can be flexibly calibrated.

Back

Extension 2 : Input-Output linkages

Generalized production function:

$$y_{k,t}(i) = A_{k,t} F_k(l_{k,t}(i), \tilde{Y}_{1,k,t}(i), \dots, \tilde{Y}_{K,k,t}(i))$$

The NKPC becomes:

$$\pi_{k,t} = \lambda_k \left(\omega_k \tilde{\mathcal{Y}}_t - \omega_k \mathcal{P}_{k,t} + \omega_k \mathcal{N} \mathcal{H}_t + s_k^c \mathcal{M}_{k,t} + \mathcal{I}_{k,t} \right) + \beta \mathbb{E}_t \pi_{k,t+1}.$$

where

$$\mathcal{I}_{k,t} = \sum_l \frac{P_l \tilde{Y}_{l,k}}{P_k \tilde{Y}_k} (\mathcal{P}_{l,t} - \mathcal{P}_{k,t}),$$

Back

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Definitions

$$\lambda_k = \frac{(1-\theta_k)(1-\beta\theta_k)}{\theta_k} \frac{\bar{\epsilon}_k - 1}{\bar{\epsilon}_k - 1 + \bar{\eta}_k}$$

$$\gamma_{e,k}(j) = \left(1 - \frac{\epsilon_k(j)}{\bar{\epsilon}_k} (1 + \epsilon_k^s(j)) \right) \frac{1}{\bar{\epsilon}_k - 1}$$

$$\bar{\epsilon}_k = \int \frac{e_k(j)}{E_k} \epsilon_k(j) dj$$

$$\bar{\eta}_k = \left(- \int (\epsilon_k(j) - \bar{\epsilon}_k)^2 \frac{e_k(j)}{E_k} dj + \int \frac{\epsilon_k^s(j)}{\epsilon_k(j)} \frac{e_k(j)}{E_k} dj \right) / \bar{\epsilon}_k$$

$$\bar{s}_k = E_k / E$$

$$\bar{\zeta}_k = \int_j \frac{\vartheta(j) W_n(j)}{\int_j \vartheta(j) W_n(j)} \zeta_k(j) dj$$

$$\Gamma = \sum_k \bar{s}_k \int \gamma_{e,k}(j) \bar{\zeta}_k(j) \frac{e(j)}{E_k} dj$$

$$\mathcal{M}_t^D = \bar{s}_k \sum_k \mathcal{M}_{k,t}^D$$

$$\mathcal{M}_t^P = \sum_k \bar{s}_k \sum_l \int_j \frac{e_k(j)}{E_k} \gamma_{e,k}(j) \rho_{k,l}(j) dj \cdot (\hat{P}_{l,t} - \hat{P}_{k,t})$$

Endogenous markup wedge

Tractable distributional dynamics

$$\mathcal{M}_{k,t} = \mathcal{M}_{k,t}^P + \mathcal{M}_{k,t}^E$$

$$\mathcal{M}_{k,t}^E = \Gamma \hat{Y}_t + \mathcal{M}_{k,t}^D$$

$$\mathcal{M}_{k,t}^P = \sum_l \mathcal{S}_{k,l} (\hat{P}_{l,t} - \hat{P}_{k,t})$$

$$\mathcal{M}_{k,t}^E = \mathbb{E}_t \mathcal{M}_{k,t+1}^E - \bar{\gamma}_{e,k} \bar{\sigma}_k^M \hat{R}_t + \sum_l \bar{\gamma}_{e,k} \bar{\sigma}_{k,l}^M \mathbb{E}_t \pi_{l,t+1} - \frac{\delta}{1-\delta} \mathbb{E}_t \mathcal{M}_{k,t}^0$$

Endogenous markup wedge

Tractable distributional dynamics

$$\mathcal{M}_{k,t} = \mathcal{M}_{k,t}^P + \mathcal{M}_{k,t}^E$$

$$\mathcal{M}_{k,t}^E = \Gamma \hat{\mathcal{Y}}_t + \mathcal{M}_{k,t}^D$$

$$\mathcal{M}_{k,t}^P = \sum_l \mathcal{S}_{k,l} (\hat{P}_{l,t} - \hat{P}_{k,t})$$

$$\mathcal{M}_{k,t}^E = \mathbb{E}_t \mathcal{M}_{k,t+1}^E - \bar{\gamma}_{e,k} \bar{\sigma}_k^M \hat{R}_t + \sum_l \bar{\gamma}_{e,k} \bar{\sigma}_{k,l}^M \mathbb{E}_t \pi_{l,t+1} - \frac{\delta}{1-\delta} \mathbb{E}_t \mathcal{M}_{k,t}^0$$

$$\begin{aligned} \frac{1}{(1-\delta)R} \hat{\mathcal{M}}_{k,t}^0 &= \hat{\mathcal{M}}_{k,t-1}^0 - \int \gamma_{b,k}(j) \frac{b(j)}{RE} dj \left(\hat{R}_t - \sum_l \bar{s}_l \pi_{l,t+1} \right) \\ &- \left(1 + \frac{\bar{\psi}}{\bar{\sigma}} \right) \int \gamma_{b,k}(j) \frac{wn(j)}{WL} dj \hat{\mathcal{Y}}_t + \frac{R-1}{R} \hat{\mathcal{M}}_{k,t}^E \\ &- \sum_l \int \gamma_{b,k}(j) \left(\frac{e(j)}{E} (\bar{s}_l - s_l(j)) + \frac{wn(j)}{WL} (\bar{\psi}_l - \psi_l(j)) \right) dj \hat{P}_{l,t} \end{aligned}$$

Output gap

Output gap:

$$\tilde{\mathcal{Y}}_t = \left(\frac{1}{\bar{\sigma}} + \frac{1}{\psi} \right) (\hat{\mathcal{Y}}_t - \hat{\mathcal{Y}}_t^*)$$

Aggregate demand index:

$$\hat{\mathcal{Y}}_t = \mathbb{E}_t \hat{\mathcal{Y}}_{t+1} - \bar{\sigma} \left(\hat{R}_t - \mathbb{E}_t \pi_{cpi,t+1} - \mathbb{E}_t \tilde{\pi}_{\mathcal{NH},t+1} \right),$$

where

$$\tilde{\pi}_{\mathcal{NH},t} = \sum_{k=1}^K \left(\frac{\bar{\sigma}_k + \psi \bar{\xi}_k}{\bar{\sigma} + \psi} - \bar{s}_k \right) \pi_{k,t}$$

Flex-price agg. demand index:

$$\hat{\mathcal{Y}}_t^* = \sum_k \frac{\psi \bar{\xi}_k + \bar{s}_k}{1 + \bar{\sigma}} \hat{A}_{k,t}$$

Welfare loss

Assumptions A1-A2 and $\mathcal{M} = 0$

$$\mathcal{L}_s^{\tilde{y}} = \frac{\bar{\psi}}{1 + \frac{\bar{\psi}}{\bar{\sigma}}} \left\{ \tilde{y}_s^2 - c_s^{\tilde{y}} \right\}$$

$$\mathcal{L}_s^{\pi} = \sum_k \vartheta \bar{s}_k \cdot \pi_{k,s}^2$$

$$\begin{aligned} \mathcal{L}_s^d = & \mathbb{E}_\delta \int g(j) \left(\tau_{t_0}(j) + \sum_k \sum_{s \geq t_0} \frac{R-1}{R^{s+1-t_0}} \frac{e(j)}{E} s_k(j) A_{k,s} \right)^2 dj \\ & - 2\mathbb{E}_\delta \int \frac{\xi(j)}{1 + \frac{\theta(j)Wn(j)\psi}{e(j)\sigma}} \tau_{t_0}(j) \sum_k \sum_{s \geq t_0} \frac{R-1}{R^{s+1-t_0}} A_{k,s} dj \end{aligned}$$

Welfare loss

Assumptions A1-A2 and $\mathcal{M} = 0$

$$\mathcal{L}_s^{\tilde{y}} = \frac{\bar{\psi}}{1 + \frac{\bar{\psi}}{\bar{\sigma}}} \left\{ \tilde{y}_s^2 - c_s^{\tilde{y}} \right\}$$

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where

$$\tau_{t_0}(j) = \left(1 - \frac{1}{R} \right) \sum_{s \geq t_0} \frac{1}{R^{s-t_0}} \left(\frac{b(j)}{RE} (R_s - \pi_{cpi,s+1}) - \sum_k \frac{e(j)}{E} (s_k(j) - \bar{s}_k) \right)$$

$$c_s^{\tilde{y}} = \mathbb{E}_\delta \int \frac{(1 - \frac{1}{R}) \frac{b(j)}{E}}{1 + \frac{\theta(j)Wn(j)\psi}{e(j)\sigma}} \left(\sum_{s \geq t_0} \frac{R-1}{R^{s+1-t_0}} \left(\tilde{y}_s - \sum_k (\bar{\xi}(j) - \bar{\xi}) (P_{k,s} - P_{k,s}^*) \right) \right)^2 dj$$

Welfare loss

general

$$\mathcal{L}_s^{\tilde{y}} = \frac{\bar{\psi}}{1 + \frac{\bar{\psi}}{\bar{\sigma}}} \int \frac{e(j)}{E} \left(\hat{W}_s - \sum_k \xi_k(j) (\hat{P}_{k,s} + \hat{A}_{k,s}) \right)^2 dj + \mathcal{C}_s^{\tilde{y}}$$

$$\mathcal{L}_s^{\pi} = \sum_k \bar{s}_k \vartheta_k \pi_{k,s}^2$$

$$\mathcal{L}_s^s = - \sum_k \bar{s}_k \sum_l \mathcal{S}_{k,l} (\hat{P}_{l,s} + \hat{A}_{l,s}) (\hat{P}_{k,s} + \hat{A}_{k,s})$$

$$\mathcal{L}_s^r = \frac{\bar{\sigma}}{1 + \frac{\bar{\psi}}{\bar{\sigma}}} \sum_{k,l} \mathcal{E}_{k,l} \{ (\hat{P}_{k,s} + \hat{A}_{k,s}) (\hat{P}_{l,s} + \hat{A}_{l,s}) - \mathcal{C}_{k,l}^r \}$$

$$\mathcal{L}^d = \mathbb{E}_\delta \int g(j) \hat{\tau}_{t_0}(j)^2 dj + \mathcal{C}^d$$

Back