

Price Setting with Customer Capital: Sales, Teasers, and Rigidity*

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Abstract

An equilibrium search model of frictional product markets with long-term customer relationships gives rise to temporary sales when pricing is anonymous across a firm's customer base. Equilibrium prices are inefficiently high, leading to excess selling effort and trade, and the emergence of sale pricing can improve allocations by limiting overselling. Pricing is also characterized by an asymmetry involving a stable regular price and variable sale price when firms face idiosyncratic shocks. Absent anonymous pricing, the theory gives rise to teaser pricing, which attains efficient allocations, yet also features a stable regular price and variable teaser price.

JEL Codes: E30; D83; L11; L81.

Keywords: Product Market Search; Customer Base; Sales; Teasers; Price Rigidity.

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1 Introduction

This paper studies price setting in a model where customers are capital for firms: an equilibrium search model of frictional product markets with long-term customer relationships (Gourio and Rudanko 2014). I show that the theory gives rise to temporary sales as an equilibrium outcome when pricing is constrained to be anonymous across a firm’s customer base. Equilibrium prices are inefficiently high, leading to excess selling effort and trade in the product market, and the emergence of sale pricing can improve allocations by limiting this overselling. Equilibrium pricing is also characterized by an asymmetry involving a stable regular price and a variable sale price when firms face idiosyncratic shocks. Absent anonymous pricing, the theory gives rise to teaser pricing, with new customers facing a lower price than existing customers. Teaser pricing is also characterized by a stable regular price and variable teaser price, but in this case allocations are efficient and the seeming price rigidity not allocative.

A literature in macroeconomics has highlighted the prevalence of temporary sales in consumer pricing. Figure 1 illustrates this characteristic of consumer pricing, where a product’s price undergoes repeated temporary downward shifts over time. The broadest evidence on consumer pricing draws on the micro data underlying the consumer price index, and research studying that data has documented the prevalence of temporary sales across a wide range of consumer product markets.¹ This evidence indicates that there are also product markets where temporary sales do not appear to play a role, however, notably in markets for services. Temporary sales do not appear to be a characteristic of producer pricing either, as research studying the micro data underlying the producer price index has shown (Nakamura and Steinsson 2008).

How should one think about the emergence of temporary sales in consumer product markets and why they affect some markets and not others? This paper proposes a theory of temporary sales that builds on the premise that customer base concerns play a role in price setting in most markets. The theory distinguishes between markets where pricing is constrained to be anonymous—with the firm setting a common price across its customers at each point in time—and ones where pricing can reflect the long-term customer relationship in a more flexible way. Anonymous pricing is a natural feature of many retail markets with repeat customers, where the long-term customer relationship is implicit rather than explicit. In this case the theory predicts the emergence of temporary sales in pricing—as often observed. By contrast, in markets where long-term customer relationships are explicit

¹See Appendix A.

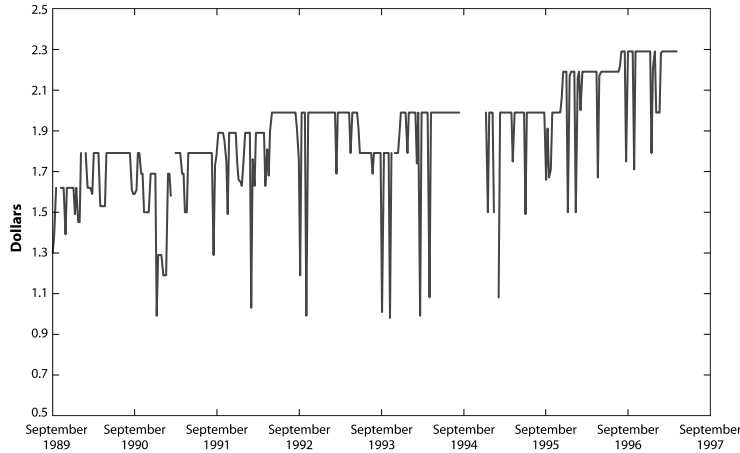


Figure 1: Illustrating Temporary Sales in Consumer Prices

Notes: The figure displays the evolution of the price of a package of Nabisco saltine crackers in a grocery store in Chicago 1989–1997. Source: Nakamura and Steinsson (2013).

and allow pricing accordingly, the theory predicts the emergence of teaser pricing instead—as also often observed.

The paper builds on the customer capital framework of Gourio and Rudanko (2014). In that model, firms produce heterogeneous products and buyers differ in their preferences over these products. Firms take on costly sales activities to inform potential buyers of their products and buyers search for products that fit their tastes. Bringing the two together, to allow buyers to assess whether a firm’s product fits their tastes, involves coordination frictions. In addition to sales activities, firms use prices to influence customer acquisition: setting a low price attracts more searching buyers, leading to more new customers. And due to the search frictions, customers remain with the firm for a period of time.

Anonymous pricing creates a tension in the firm pricing problem between the firm’s incentive to attract new customers via low prices versus its incentive to profit from existing customers via high prices. Attracting new customers requires a competitive price and costly sales activities to inform searching buyers of the firm’s product, while existing customers that have already found the firm’s product acceptable are willing to pay more and do not require the latter. A firm may find it optimal to set a competitive price and take on costly sales activities to attract new customers, or it may find it optimal to set a higher price without taking on sales activities, to focus on making profit on its existing customers instead.² In

²Evidence from retail pricing supports the idea that firms spend resources on costly sales activities in conjunction with sale pricing (Hitsch, Hortacsu, and Lin 2021).

particular, equilibrium pricing can involve firms randomizing between a low “sale” price and a higher “regular” price across firms and over time—reminiscent of the fluctuating prices of temporary sale pricing.

Equilibrium outcomes depend on the prevalence of existing customer relationships in the market. Sale pricing emerges as a unique equilibrium outcome when the share of buyers in existing customer relationships is sufficiently large. When few buyers are in existing customer relationships, and the pool of searching buyers is consequently large, firms find it profitable to focus on attracting new customers. As the share of buyers in existing customer relationships increases, the profitability of doing so falls, eventually leading to firms switching to randomizing between seeking to attract new customers versus focusing on profiting from existing customers instead. Firms dropping out of the market for new customers in turn serves to sustain that market, with the equilibrium featuring firms randomizing in pricing as long as the share of buyers in existing customer relationships is sufficient.

I show that equilibrium outcomes are inefficient, comparing to a corresponding planner problem. Absent sale pricing, firms price too high due to their incentive to profit from existing customers, with the high prices resulting in excess selling effort and trade in the product market. Relative to this starting point, the emergence of sale pricing can be good for efficiency of resource allocation (in a second best sense) by limiting the excess selling taking place. Equilibrium pricing also becomes too focused on buyer valuation for the product and too unresponsive to firm cost. The emergence of sale pricing only makes this rigidity with respect to cost more pronounced in the sense that regular prices respond to cost even less, even if sale pricing does also introduce discontinuous jumps between sale price and regular price.

When firms face transitory idiosyncratic shocks, the theory gives rise to asymmetric pricing akin to that in Figure 1, with a relatively stable regular price that undergoes repeated, temporary, downward shifts of varying magnitude over time. In an equilibrium featuring sale pricing, a decline in firm cost leads to the firm setting a sale price that reflects firm cost, but an increase in cost to the firm setting the regular price that is independent of firm cost. In a setting where firms face such shocks, sale pricing is thus triggered by sufficiently low cost realizations, with remaining firms setting the regular price.³ In a setting where firms face

³A literature in macroeconomics studies price setting with menu costs in settings where firms face idiosyncratic shocks, typically to cost or both cost and demand (e.g., Golosov and Lucas (2007), Gertler and Leahy (2008), Midrigan (2011), Klenow and Willis (2016)). Eichenbaum, Jaimovich, and Rebelo (2011) provide evidence that retail firms face frequent/weekly changes in cost, that prices do not always change with costs, but that price changes tend to be associated with cost changes. Anderson, Malin, Nakamura, Simester, and Steinsson (2017) also consider the relationship between retail price and cost, based on a different measure of cost. Note that the discussion above restricts attention to transitory firm-specific shocks, without implying

similar shocks to buyer valuation for their product also, sales can be triggered by sufficiently high buyer valuation as well.

When firms are able to keep track of individual customers and price accordingly—relaxing anonymous pricing—the theory predicts the emergence of teaser pricing instead. To profit from their existing customers, firms charge existing customers their full willingness to pay for the product. To attract new customers, firms simultaneously set a competitive price for them. Pricing thus involves an initial discount and a higher price in subsequent periods of the customer relationship. This added flexibility in pricing allows attaining efficient allocations in equilibrium. Despite efficiency of allocations, teaser pricing continues to feature high prices that are unresponsive to cost for existing customers, however. In this case the seemingly high and rigid pricing is thus not allocative.

I begin with a static model to illustrate ideas, before extending the analysis to a dynamic infinite horizon setting. The dynamic model demonstrates how ideas carry over to a setting where prices explicitly fluctuate over time, but the extension brings with it technical challenges, as long-term customer relationships imply that buyer behavior depends also on future prices. The dynamic firm problem is characterized by a time-inconsistency, as firms have an incentive to promise low prices for the future to attract new customers, yet also an incentive to charge high prices today to profit from existing customers. The analysis focuses on Markov perfect equilibria where firms are free to reoptimize prices each period. Analyzing Markov perfect equilibria in an environment with a time-inconsistency can be challenging because the decision-maker’s objective does not coincide with maximizing their value function, implying that standard dynamic programming arguments cannot be directly applied.⁴ In this case equilibrium outcomes remain transparent to analyze nevertheless, due to the structure of the model, allowing analytical characterizations of prices also in the dynamic setting.

Related literature Understanding the nature of price setting is a fundamental question in macroeconomics that has been considered by a significant body of research.

Within this body of research, this paper is, in particular, related to work studying firm behavior and equilibrium outcomes when customer base concerns play a role in firm decision making. This literature goes back to Phelps and Winter (1970), with early contributions including Bills (1989) and Klemperer (1995). More recent studies formalizing the idea of

that regular prices do not respond to broader/persistent changes in cost.

⁴Time-inconsistencies appear in multiple contexts, arising due to preferences or the economic environment. See Klein, Krusell, and Rios-Rull (2008) for a discussion on characterizing Markov perfect equilibria in problems with time-inconsistency.

customer base concerns in macroeconomic models with frictional product markets include Kleshchelski and Vincent (2009), Dinlersoz and Yorukoglu (2012), Drozd and Nosal (2012), Gourio and Rudanko (2014), Petrosky-Nadeau and Wasmer (2015), Perla (2016), Paciello, Pozzi, and Trachter (2019), Chiavari (2020), Kaas and Kimasa (2021), Roldan and Gilbukh (2021) and Eaton, Jinkins, Tybout, and Xu (2022). Many of these papers consider the implications for price setting, but without connecting to sale or teaser pricing per se, while others focus on firm growth/dynamics or international trade relationships.⁵

The present paper studies price setting within the search theoretic framework of Gourio and Rudanko (2014). That paper showed—focusing on the intangible capital embodied in the customer base, as well as how customer base concerns affect how firm investment and sales respond to shocks—that such a theory helps explain firms’ market valuations and behavior. The present paper demonstrates that the theory can also give rise to fluctuating prices akin to temporary sales when firms set a common price across their customer base—a natural feature of many consumer product markets—in addition to giving rise to teaser pricing absent such constraints. The firm price-setting problem considered here is more challenging than in that earlier paper, because with anonymous pricing the time-inconsistency inherent in the firm problem affects allocations directly. To make progress, the paper adopts a homogenous specification for the firm problem that admits a transparent analysis nevertheless. The paper then proceeds to demonstrate the emergence of sale pricing in this setting, as well as discussing the implications for resource allocation and the dynamics of prices further.

Relatedly, the paper also connects with Nakamura and Steinsson (2011), who study firm price setting when consumers have goods-level habit preferences (as in Ravn, Schmitt-Grohe and Uribe 2006) that also imply that a firm’s current demand matters for future demand. They discuss the time-inconsistency of the firm pricing problem in this setting, and consider the implications for price rigidities under alternative assumptions about firm commitment. The present framework shares the time-inconsistency, but differs in considering an equilibrium search model where multi-customer firms set prices facing turnover in their customer base, with or without anonymous pricing, and accommodating also idiosyncratic shocks across firms.

In considering temporary sales, the paper connects with an earlier literature in theoretical industrial organization proposing models of sale pricing, such as Varian (1980), Salop and Stiglitz (1982) and Sobel (1984). The present paper demonstrates that customer base

⁵A related paper that connects customer base concerns with sale pricing is Shi (2016), who considers the endogeneity of long-term customer relationships in frictional product markets. His theory does not distinguish between sale and teaser pricing, however, in focusing on single-customer firms.

concerns can give rise to pricing that fluctuates between sale and regular price, as well as giving rise to teaser pricing, in settings that naturally coincide with where these forms of pricing are observed, offering a macro perspective.

In discussing price rigidities, the paper connects with a macroeconomic literature on price setting considering whether sale pricing may be viewed as restoring price flexibility when regular prices appear more rigid (Guimaraes and Sheedy 2011, Kehoe and Midrigan 2015, Kryvtsov and Vincent 2021). With respect to this work, the present paper offers a perspective reflecting the idea that with long-term customer relationships, gauging allocative price rigidities requires considering longer-term measures of prices.⁶

2 Static Model

Consider a competitive product market where buyers and sellers face search frictions in coming together to trade. The market has measure one buyers and a large number of firms. The firms produce heterogeneous products, at unit cost c . Buyers have unit demand, but heterogeneous tastes across these products, valuing a firm’s product at either $u(>0)$ or zero.

Firms begin the period with some existing customers, of measure $n_i(>0)$, that have already found the firm’s product acceptable. This leaves $1 - \sum_i n_i(>0)$ buyers unmatched and searching in the product market. Acquiring additional customers requires costly sales activities on the part of firms, to inform searching buyers of the firm’s product. Selling effort s is subject to a cost $\kappa(s, n) = \hat{\kappa}(s/n)n$, where $\hat{\kappa}$ is increasing and convex. The convex cost reflects a firm’s limited ability to increase selling effort locally, and the homothetic form that larger firms are less affected by this due to operating in multiple locations.

The process of bringing firms and searching buyers together involves coordination frictions, captured with a matching function. If a firm exerts selling effort s and attracts b searching buyers, the measure of new customer relationships created is given by the constant returns to scale function $m(b, s)$. With this, the probability that a unit of selling effort leads to a new customer, $\eta(\theta) = m(b, s)/s$, becomes an increasing and concave function of the queue of searching buyers per unit of selling effort at the firm, $\theta = b/s$. The probability that

⁶When it comes to broader market-wide shifts, the present theory is consistent with an increased frequency of sales during contractions driven by market-wide increases in costs or declines in buyer valuation, in the spirit of evidence in Kryvtsov and Vincent (2021). The theory is also consistent with an increased frequency of sales during expansions driven by greater numbers of buyers in the market, in the spirit of evidence in Chevalier, Kashyap, and Rossi (2003). The overall change in the frequency of sales across expansions and contractions is ambiguous, accommodating observations in the spirit of Coibion, Gorodnichenko, and Hong (2015).

a searching buyer becomes the firm's customer, $\mu(\theta) = m(b, s)/b$, becomes a decreasing and convex function of the same. The elasticity of the matching function, $\varepsilon(\theta) = \theta\eta'(\theta)/\eta(\theta)$, is assumed to be weakly decreasing, reflecting that customer acquisition becomes less responsive to queues when queues become longer.

In setting its price a firm faces a tradeoff between profit per customer and number of customers served. A higher price raises profit per customer, but also attracts fewer searching buyers and hence results in fewer new customers. A firm expects the queue of searching buyers it attracts to be such that searching buyers are left indifferent between choosing this firm versus any other in the market. Denoting the equilibrium buyer value of search by S , the queue of searching buyers θ_i a firm expects to attract with price p_i satisfies

$$S = \mu(\theta_i)(u - p_i), \quad (1)$$

where a searching buyer becomes a customer with probability $\mu(\theta_i)$, getting the good at valuation u and paying the price p_i . The relationship (1) determines the queue of buyers as a decreasing function of the firm's price, with firms taking the equilibrium value of search as given. I refer to this relationship by $\theta = g(p; S)$ in what follows.

Each firm chooses its price and selling effort to maximize its profits:

$$\begin{aligned} \max_{p_i, s_i} & (n_i + \eta(\theta_i)s_i)(p_i - c) - \kappa(s_i, n_i) \\ \text{s.t.} & S = \mu(\theta_i)(u - p_i) \text{ if } s_i > 0, \\ & p_i \leq u, \end{aligned}$$

taking its existing customer base and the equilibrium value of search as given. The profits reflect sales to existing and new customers at price p_i and production cost c , net of selling costs. If the firm is seeking to attract new customers, it faces constraint (1) characterizing the queue of searching buyers attracted by the firm's price. Either way, the price cannot exceed buyer valuation for the product, or buyers would not agree to trade.

Note that the problem is effectively independent of scale. Dividing by n_i and defining selling effort per existing customer as $\hat{s}_i = s_i/n_i$, the problem may be rewritten as

$$\begin{aligned} \max_{p_i, \hat{s}_i} & (1 + \eta(\theta_i)\hat{s}_i)(p_i - c) - \hat{\kappa}(\hat{s}_i) \\ \text{s.t.} & S = \mu(\theta_i)(u - p_i) \text{ if } \hat{s}_i > 0, \\ & p_i \leq u, \end{aligned} \quad (2)$$

with the equilibrium value of search taken as given. Firm decisions are thus independent of the size of its existing customer base n_i .

The firm has two distinct options here. It can seek to attract new customers, which requires a price that is competitive in the market for searching buyers, or hold off on doing so to focus on making profit on its existing customers instead.

Option 1: Active selling to attract new customers If the firm seeks to attract new customers, it chooses a positive selling intensity $\hat{s}_i > 0$ together with a price that is competitive in the market for searching buyers. This price will generally be strictly below buyer valuation for the product, as firms use prices to compete for buyers, implying that the second condition in the firm problem (2) becomes superfluous.

An interior optimum with active selling must satisfy first order conditions.⁷ The first order condition for the selling intensity,

$$\kappa_s(\hat{s}_i) = \eta(\theta_i)(p_i^s - c), \quad (3)$$

states that the firm chooses a selling intensity where the marginal cost of selling, on the left, is equated to the profits from sales to the additional customers acquired, on the right.

The first order condition for the price,

$$1 + \eta(\theta_i)\hat{s}_i = -\eta'(\theta_i)g_p(p_i^s; S)\hat{s}_i(p_i^s - c), \quad (4)$$

states that the firm raises the price to a point where the increase in profits due to greater profit margins per customer, on the left, equals the decrease in profits due to reduced customer acquisition, on the right. Here the increase in price reduces the queue of searching buyers according to $g_p(p_i^s; S) = \mu(\theta_i)/(\mu'(\theta_i)(u - p_i^s)) < 0$.

The optimality condition (4) implies that the price may be written as a weighted average of firm cost and buyer valuation:

$$p_i^s = \gamma_i c + (1 - \gamma_i)u \quad \text{with} \quad \gamma_i = \varepsilon_i \Delta_i / (1 - \varepsilon_i + \Delta_i),$$

where $\varepsilon_i \in (0, 1)$ is the matching function elasticity and $\Delta_i = \eta(\theta_i)\hat{s}_i > 0$ the share of new versus existing customers in firm sales. This price reflects, in addition to firm cost and buyer valuation, how effective pricing is at attracting new customers and how important new customers are in firm sales. If longer queues of searching buyers increase customer acquisition effectively (elasticity ε is large), the firm sets a lower price to attract more of them. If new customers make up a large share of firm sales (share Δ is large), the price is similarly lower to attract more of them.

Option 2: Profit from existing customers Alternatively, the firm can forgo attracting new customers to focus on making profits on its existing customers instead.

⁷I restrict attention to circumstances where first order conditions are also sufficient for interior optimum.

If the firm does not participate in the market for searching buyers, its selling intensity is zero and its pricing problem reduces to:

$$\begin{aligned} \max_{p_i} p_i - c & \quad (5) \\ \text{s.t. } p_i & \leq u. \end{aligned}$$

The optimal price becomes $p_i^r = u$, with the firm taking the full gains from the relationship. This price is too high to attract searching buyers, but existing customers are willing to pay it.⁸ Existing customers have already determined that they find the firm's product acceptable, whereas searching buyers must be enticed to find out, in the face of competition from other firms.

Which of the two options dominates for the firm depends on the equilibrium value of search S . The profits from seeking to attract new customers are decreasing in S , because a higher S means it is more costly for the firm to attract searching buyers. Meanwhile, the profits from focusing on profiting from existing customers are independent of S , at $p^r - c = u - c$. Thus, for sufficiently low values of S the firm prefers to seek to attract new customers and from some S upward the firm focuses on profiting from existing customers. Of course, in between there will generally be a value of search where firms are indifferent between the two alternatives, and might randomize between active selling at a lower price versus profiting from existing customers with a higher price.

Equilibrium firm behavior must maximize profits, as well as be consistent with a market clearing condition for searching buyers. From the optimality conditions, it is clear that if all firms are identical (aside from possible differences in size), their choices are also identical in that all actively selling firms choose the same selling intensity and price (resulting in the same queue length), while all remaining firms choose the same price. Denoting the total measure of existing customers across firms in the beginning of the period by $N = \sum_i n_i$ and the probability of a firm actively selling by α , the market clearing condition requires that the total measure of searching buyers across firms, $\theta \hat{s} \alpha N$, equals the total measure of searching buyers in the market $1 - N$.⁹ This condition determines the equilibrium value of search.

Definition 1. *A competitive search equilibrium with anonymous pricing specifies a probability of a firm actively selling α , corresponding sale price p^s , queue θ and selling intensity \hat{s} , as well as regular price p^r and value of search S such that: i) $\{p^s, \theta, \hat{s}\}$ solve the firm problem*

⁸In the dynamic model of Section 3 the price reflects also the fact that existing customers can return to search for a new firm.

⁹Each actively selling firm attracts $\theta \hat{s} n_i$ buyers. If the probability that a firm (of any size) actively sells is α , adding these buyers up across firms yields a total measure of searching buyers $\theta \hat{s} \alpha N$, to equal $1 - N$.

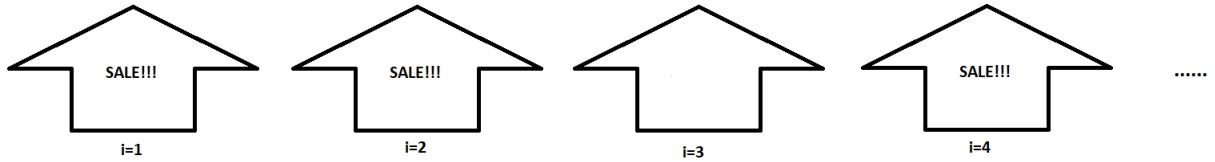


Figure 2: In the Sale Equilibrium Some Sellers Advertise a Reduced Price

(2) with $\hat{s} > 0$, and p^r solves the firm problem (5), ii) if $0 < \alpha < 1$, firms are indifferent between active selling and not, and if $\alpha = 1$, firms weakly prefer active selling, and iii) the market for searching buyers clears: $1 - N = \theta \hat{s} \alpha N$.

I refer to the equilibrium as featuring sale pricing when each firm has a positive probability of charging both a lower “sale” price and a higher “regular” price, in which case the equilibrium features firms pursuing both pricing strategies.¹⁰

If firms are randomizing across these strategies in equilibrium, then they must be indifferent between them. The profits from active selling consist of the profits from existing customers $n[p^s - c]$ and the profits from new customers $n[\eta(\theta)\hat{s}(p^s - c) - \hat{\kappa}(\hat{s})] = -n\kappa_n(\hat{s})$, both positive.¹¹ Holding off on active selling yields greater profits on existing customers $n[u - c]$, but none on new customers. In such an equilibrium the profits from new customer acquisition must thus just make up for charging existing customers less: $p^s - c - \kappa_n(\hat{s}) = u - c$. By contrast, if all firms are actively selling, it must be that $p^s - c - \kappa_n(\hat{s}) \geq u - c$.

The following result characterizes equilibrium outcomes assuming a standard form for the selling cost and that the matching function elasticity declines from one toward zero as queues grow:

Proposition 1. *Let $\hat{\kappa}(\hat{s}) = \hat{s}^\varphi/\varphi$ such that $\varphi > 1$, $\lim_{\theta \rightarrow 0} \varepsilon(\theta) = 1$, and $\lim_{\theta \rightarrow \infty} \varepsilon(\theta) = 0$. There exists an $N^* \in (0, 1)$ such that when $N > N^*$, the competitive search equilibrium with anonymous pricing is unique and features sale pricing, and when $N \leq N^*$, the competitive search equilibrium with anonymous pricing is unique and does not feature sale pricing.*

The prevalence of existing customer relationships among buyers matters for outcomes in the market. When few buyers are in existing customer relationships to begin with, and the pool of searching buyers is consequently large, firms find it profitable to focus on attracting

¹⁰Note that while the static model focuses on cross-sectional differences in price, the dynamic model will demonstrate how this gives rise to inter-temporal variation in prices in a dynamic setting.

¹¹This uses the first order condition for the selling intensity (3) and that $\kappa_n(\hat{s}) = \hat{\kappa}(\hat{s}) - \hat{s}\kappa_s(\hat{s})$.

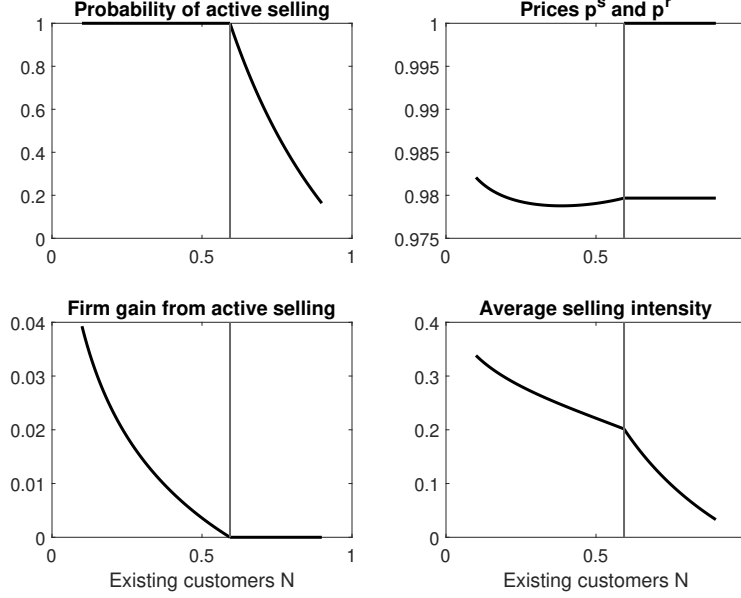


Figure 3: Equilibrium Outcomes as a Function of Existing Customer Relationships

Notes: The figure illustrates equilibrium outcomes as a function of the share of existing customer relationships in the market. The top left panel plots the probability α , the top right the prices p^s, p^r , the bottom left a firm's net gain from active selling relative to forgoing doing so, and the bottom right the marketwide average selling intensity $\alpha \hat{s}$. Here $u = 1$, $c = 1/2$, $\hat{\kappa}(\hat{s}) = \hat{s}^2/2$ and $\eta(\theta) = \theta/(1 + \sqrt{\theta})^2$.

new customers. As the share of buyers in existing customer relationships increases, the profitability of doing so falls, eventually leading to firms switching to randomizing between seeking to attract new customers versus focusing on profiting from existing customers instead. Firms dropping out of the market for new customers in turn serves to sustain that market, with a greater share of buyers in existing customer relationships implying a lower equilibrium probability of an individual firm seeking to attract new customers.

Figure 3 illustrates these patterns in the context of a parameterized example. On the left side of each panel, few buyers are in existing customer relationships, and the equilibrium features all firms actively selling at a price that attracts searching buyers. Moving right, the share of buyers in existing customer relationships increases, leading to the profitability of active selling declining, until firms become indifferent between the two pricing strategies at the vertical line. Moving further right, firms randomize between the two pricing strategies, remaining indifferent between them due to an increasing share of firms dropping out of active selling. When nearly all buyers are in existing customer relationships, the probability of active selling approaches zero.

Efficient allocations How do equilibrium outcomes in this market compare to efficient allocations? To shed light on this question, this section turns to a planner problem.

A benevolent planner maximizes the value of output net of the costs of production and selling, facing the same frictions in creating customer relationships as market participants. The planner takes as given the existing customers at each firm, and decides how much selling effort each firm should take on as well as how to allocate searching buyers among firms:

$$\begin{aligned} \max_{\{s_i, \theta_i\}_{i \in I}} \quad & \sum_{i \in I} (n_i + \eta(\theta_i) s_i) (u - c) - \sum_{i \in I} \kappa(s_i, n_i) \\ \text{s.t.} \quad & \sum_{i \in I} \theta_i s_i = 1 - \sum_{i \in I} n_i. \end{aligned}$$

The planner does so subject to the constraint that the total measure of buyers allocated among firms equals the total measure of searching buyers.

The planner allocates searching buyers such that the gains from additional customer relationships created equal the shadow value of additional buyers, $\eta'(\theta_i)(u - c) = \lambda$, across all firms i . This condition may also be expressed as $\lambda = \mu(\theta_i) \varepsilon_i (u - c)$, stating that the efficient value of searching buyers equals the product of the buyer's probability of entering into a customer relationship and share ε of the gains from the relationship.

Correspondingly, the planner allocates selling effort such that the gains from the additional customer relationships created equal the costs of additional selling, $\eta(\theta_i)(u - c) = \kappa_s(\hat{s}_i) + \lambda \theta_i$, across all firms i . This condition may also be written as $\kappa_s(\hat{s}_i) = \eta(\theta_i)(1 - \varepsilon_i)(u - c)$, stating that the marginal selling cost is equated to the product of the sellers probability of entering into a customer relationship and share $1 - \varepsilon$ of the gains from the relationship.

To relate equilibrium pricing and allocations to their efficient counterparts, it is convenient to note that efficient allocations may be decentralized by firms setting the price $p_i^p = \varepsilon_i c + (1 - \varepsilon_i)u$, with the corresponding privately optimal selling effort satisfying the firm's optimality condition (3) with this price.¹² Efficient pricing may thus also be expressed as a weighted average of firm cost and buyer valuation, but with a different weight than in equilibrium. Equilibrium prices p^s and p^r both exceed the efficient price, as equilibrium pricing is influenced by the firms' incentive to profit from existing customers, pushing prices up.

¹²The equilibrium value of search then satisfies $S = \mu(\theta_i) \varepsilon_i (u - c)$ and equilibrium selling intensity $\kappa_s(\hat{s}_i) = \eta(\theta_i)(1 - \varepsilon_i)(u - c)$. These conditions coincide with the planner's optimality conditions in the text, with the equilibrium value of search coinciding with the planner's shadow value of searching buyers.

Proposition 2. *The competitive search equilibrium with anonymous pricing without sale pricing has a strictly higher price, selling intensity, volume of trade and firm profit than efficient.*

The equilibrium without sale pricing is inefficient, because too-high prices lead to firms taking on excess selling effort, resulting in excess trade in the product market. Even though the equilibrium with sale pricing is also inefficient, relative to this starting point the emergence of sale pricing may thus nevertheless be viewed as beneficial for resource allocation, in reducing the overselling taking place as a share of firms withdraw from active selling.

Comparing the expressions for equilibrium versus efficient prices also shows that equilibrium pricing becomes too focused on buyer valuation for the product and too unresponsive to cost. The emergence of sale pricing only makes this rigidity with respect to cost more pronounced in the sense that regular prices respond to cost even less than the sale prices used to attract searching buyers.

Asymmetric pricing The price paths used to illustrate sale pricing (as in Figure 1) are typically characterized by an asymmetry, featuring a relatively stable regular price that undergoes repeated temporary downward shifts of varying magnitude over time. A similar pattern emerges in the model when firms face idiosyncratic shocks.

To see this, consider the equilibrium with sale pricing, where firms are indifferent between charging a lower sale price and a higher regular price. Suppose then that a single firm faces a slightly lower production cost in this period than other firms. Instead of being indifferent between the two pricing strategies, the firm will strictly prefer to seek to attract new customers. Its profits from both strategies exceed other firms' due to its lower cost, but the profits from attracting new customers increase more because the firm benefits from lower costs on new customers as well.¹³ The firm thus responds to the lower cost by setting a sale price and taking on sales activities, and with both price and selling intensity depending on the realized cost.¹⁴

On the other hand, if the firm faces a slightly higher production cost in this period than other firms, then the firm will strictly prefer to hold off on seeking to attract new customers, focusing on making profit on its existing customers instead. The firm thus responds to the higher cost by setting the regular price, which is independent of firm cost. Among the firms

¹³The difference in profits between seeking to attract new customers versus not, $[(1 + \eta(\theta)\hat{s})(p^s - c) - \kappa(\hat{s}) - (u - c)]n$ with p^s, θ, \hat{s} satisfying the corresponding first order conditions, decreases in cost c .

¹⁴The firm's sale price, queue of buyers, and selling intensity satisfy the same expressions as equilibrium firms' but with a different cost: $p^s = \gamma c + (1 - \gamma)u$, $S = \mu(\theta)(u - p^s)$, $\kappa_s(\hat{s}) = \eta(\theta)(p^s - c)$ with S unchanged.

in the market, some charging the regular price and some the sale price, an individual firm's responses to increases and decreases in cost are thus asymmetric.

One can extend the logic to consider a setting where different firms face somewhat different costs this period, and anticipate the equilibrium to feature sufficiently low cost firms seeking to attract new customers—charging a sale price that reflects their cost while taking on sales activities—and sufficiently high cost firms focusing on making profit on existing customers—charging the regular price that is independent of their cost. In this setting the model generates sale pricing where sales are triggered by sufficiently low cost realizations, with sale prices reflecting cost and regular prices independent of cost.

One can also consider demand side factors such as buyer valuation for the firm product, enriching pricing further. If a single firm faces a slightly higher buyer valuation for their product in this period, the firm will strictly prefer to seek to attract new customers, because it can benefit more from a larger customer base. On the other hand, if the firm faces a slightly lower buyer willingness to pay in this period than other firms, then the firm will strictly prefer to hold off on seeking to attract new customers, focusing on making profit on its existing customers instead. In this setting sales are triggered by sufficiently high buyer valuation. Both sale prices and regular prices reflect buyer valuation, but the switch to sale pricing limits increases in regular price in response to increased buyer valuation.

In a market where firms face both differing costs and differing buyer valuation in this period, sale pricing may be triggered by low costs or high buyer valuation for the firm's product. Sale prices reflect both cost and buyer valuation, but also regular prices can shift with buyer valuation.

Teaser pricing What happens if firms can keep track of individual customers and differentiate accordingly in pricing, instead of being constrained by anonymous pricing? In such settings the theory predicts the emergence of teaser pricing, as also often observed in consumer markets with more explicit long-term customer relationships.

In thinking about price setting when firms are able to differentiate among customers, the first thing to note is that firms optimally charge their existing customers their full valuation for the product, $p^e = u$. This is the most the firm can charge an existing customer while still retaining them, and is hence what a profit maximizing firm should charge its existing customers.¹⁵

¹⁵In the dynamic model of Section 3 the price reflects also the fact that existing customers can return to search for a new firm.

With this, the firm's pricing problem reduces to a question of how to set the price p^n for new customers:

$$\begin{aligned} \max_{p_i^n, s_i} \quad & \eta(\theta_i) s_i (p_i^n - c) - \kappa(s_i, n_i) \\ \text{s.t.} \quad & S = \mu(\theta_i)(u - p_i^n) \text{ if } s_i > 0, \\ & p_i^n \leq u, \end{aligned}$$

taking its existing customer base and the equilibrium value of search as given. Here the profits reflect sales to new customers at price p_i^n and production cost c , net of selling costs. If the firm is seeking to attract new customers, it faces constraint (1) characterizing the queue of searching buyers attracted by the firm's price. Either way, the price cannot exceed buyer valuation for the product, or buyers would not agree to trade.

The firm problem is again effectively independent of scale. Dividing by n_i and defining selling effort per existing customer as $\hat{s}_i = s_i/n_i$, the problem may be written as

$$\begin{aligned} \max_{p_i^n, \hat{s}_i} \quad & \eta(\theta_i) \hat{s}_i (p_i^n - c) - \hat{\kappa}(\hat{s}_i) \\ \text{s.t.} \quad & S = \mu(\theta_i)(u - p_i^n) \text{ if } \hat{s}_i > 0, \\ & p_i^n \leq u, \end{aligned}$$

taking the equilibrium value of search as given.

The optimal selling intensity and price satisfy the corresponding first order conditions. The first order condition for selling,

$$\kappa_s(\hat{s}_i) = \eta(\theta_i)(p_i^n - c), \tag{6}$$

states that the firm chooses a selling intensity where the marginal cost of selling is equated to the profits from sales to the additional customers acquired.

The first order condition for the price,

$$\eta(\theta_i) \hat{s}_i = -\eta'(\theta_i) g_p(p_i^n; S) \hat{s}_i (p_i^n - c), \tag{7}$$

states that the firm raises the price to point where the increase in profits due to higher profit margins per customer equals the decrease in profits due to reduced customer acquisition. Here the increase in price again reduces the queue of searching buyers according to $g_p(p_i^n; S)$.

The optimality condition (7) implies that the price may be written as a weighted average of firm cost and buyer valuation, $p_i^n = \varepsilon_i c + (1 - \varepsilon_i)u$, with the weight given by the matching function elasticity. Relaxing anonymous pricing thus leads to teaser pricing, where new customers pay a lower price than existing customers: $p^n < p^e$.

Proposition 3. *The competitive search equilibrium without anonymous pricing is unique and equilibrium allocations efficient.*

Relaxing anonymous pricing also leads to equilibrium allocations becoming efficient. To profit on existing customers, firms charge existing customers their full valuation for the product. Yet to attract new customers, firms simultaneously set a competitive price for them. This added flexibility in pricing is enough to attain efficient allocations. Anonymous pricing rules out teaser pricing, but in a sense the sale pricing that emerges with anonymous pricing could be viewed as a proxy for teaser pricing—that can also serve to improve on efficiency of resource allocation—when the latter is not feasible.

Note that teaser pricing also continues to feature high prices that respond little to firm cost for existing customers, but with the initial discounts allowing attaining efficient allocations nevertheless. In this case the seemingly high and rigid pricing (for existing customers) is thus not allocative.

In all, this section has discussed the emergence and features of sale and teaser pricing focusing on a static model where a firm’s existing customer base nevertheless matters for firm behavior. To extend the discussion to a dynamic setting, the next section extends the analysis to an infinite horizon model. The extension raises technical challenges, as long-term customer relationships imply that buyer behavior depends also on future prices, with the firm problem featuring a related time-inconsistency. For simplicity, I seek to keep the exposition in line with the static case as much as possible.

3 Dynamic Model

Consider a competitive product market where buyers and sellers face search frictions in coming together to trade, with the frictions giving rise to long-term customer relationships.

Time is discrete and the horizon infinite. The market has measure one buyers and a large number of firms. All agents are risk neutral and discount the future at rate β . The firms produce heterogeneous products, at unit cost $c(z)$, where productivity z follows a Markovian process. Buyers have unit demand per period, but heterogeneous tastes across products, valuing a firm’s product at either $u(> 0)$ or zero.

Firms begin each period with some existing customers, of measure $n_{it}(> 0)$, that have already found the firm’s product acceptable. This leaves $1 - \sum_i n_{it}(> 0)$ buyers unmatched and searching in the product market. Acquiring additional customers requires costly sales

activities on the part of firms, to inform searching buyers of the firm's product. Selling effort s_t is subject to a cost $\kappa(s_t, n_t) = \hat{\kappa}(s_t/n_t)n_t$, where $\hat{\kappa}$ is increasing and convex.

The process of bringing firms and searching buyers together involves coordination frictions, captured with a matching function. If a firm exerts selling effort s_t and attracts b_t searching buyers, the measure of new customer relationships created is given by the constant returns to scale function $m(b_t, s_t)$. With this, the probability that a unit of selling effort leads to a new customer, $\eta(\theta_t) = m(b_t, s_t)/s_t$, becomes an increasing and concave function of the queue of searching buyers per unit of selling effort at the firm, $\theta_t = b_t/s_t$. The probability that a searching buyer becomes the firm's customer, $\mu(\theta_t) = m(b_t, s_t)/b_t$, becomes a decreasing and convex function of the same. The elasticity of the matching function, $\varepsilon(\theta) = \theta\eta'(\theta)/\eta(\theta)$, is assumed to be weakly decreasing.

Customer relationships last until severed for exogenous, idiosyncratic reasons with probability $\delta \in (0, 1)$ each period, or until the buyer or firm prefers to end the relationship. During the customer relationship, the firm supplies the buyer with a unit of the product each period, with the buyer paying the corresponding price.

In setting its prices a firm faces a tradeoff between profit per customer and number of customers served. A firm expects the queue of searching buyers it attracts in any period to be such that searching buyers are left indifferent between choosing this firm versus any other in the market. Denoting the equilibrium buyer value of search by S_t , the queue of searching buyers θ_t a firm expects to attract with prices $\{p_{t+k}\}_{k=0}^{\infty}$ satisfies

$$S_t = \mu(\theta_t)E_t \sum_{k=0}^{\infty} \beta^k (1 - \delta)^k (u - p_{t+k} + \beta\delta S_{t+1+k}) + (1 - \mu(\theta_t))\beta E_t S_{t+1}. \quad (8)$$

Here a searching buyer becomes a customer with probability $\mu(\theta_{it})$, getting the good at valuation u and paying the price p_t until the relationship ends and the buyer returns to search. If the searching buyer does not become a customer, they continue to search next period. The relationship (8) determines the queue of buyers as a decreasing function of the firm's prices, with firms taking the equilibrium value of search as given.

Given a sequence of prices and selling effort $\{p_t, s_t\}_{t=0}^{\infty}$, the present value of firm profits reads:

$$E_0 \sum_{t=0}^{\infty} \beta^t [(n_t + \eta(\theta_t)s_t)(p_t - c_t) - \kappa(s_t, n_t)], \quad (9)$$

where per-period profits reflect sales to existing and new customers at price p_t and production cost c_t , net of selling costs. Here the firm's customer base follows the law of motion: $n_{t+1} =$

$(1 - \delta)(n_t + \eta(\theta_t)s_t)$, for all t , and if the firm is seeking to attract new customers in a given period, it faces constraint (8) characterizing the queue of buyers attracted by the firm's prices. Whether the firm is seeking to attract new customers in a given period or not, for existing customers to remain with the firm, pricing must also respect the buyer option to return to search for a new firm if preferred.

Equation (8) makes explicit that buyer behavior reflects pricing during the entire customer relationship. To express this relationship more compactly, define the present values of prices and products received as $P_t = E_t \sum_{k=0}^{\infty} \beta^k (1 - \delta)^k p_{t+k}$ and $U = E_t \sum_{k=0}^{\infty} \beta^k (1 - \delta)^k u$, respectively. Further, define the flow value of search as $x_t = S_t - \beta E_t S_{t+1}$, and the present value of these flows that a buyer forgoes during a customer relationship as $X_t = E_t \sum_{k=0}^{\infty} \beta^k (1 - \delta)^k x_{t+k}$. Equation (8) may then be rewritten as

$$x_t = \mu(\theta_t)(U - P_t - \beta(1 - \delta)E_t X_{t+1}), \quad (10)$$

expressing the flow value of search as a product of the probability that a searching buyer becomes a customer and the resulting present value of products net of the present values of price and forgone search while a customer.

Relationship (10) determines the queue of buyers as a decreasing function of the firm's present value-price, with firms taking the equilibrium value of search as given. Even if the firm is not seeking new customers within a period, however, for existing customers to remain with the firm it must be that $P_t \leq U - \beta(1 - \delta)E_t X_{t+1}$.

Firm value may also be expressed in terms of present value prices as:

$$n_0[P_0 - C_0] + E_0 \sum_{t=0}^{\infty} \beta^t [\eta(\theta_t)s_t(P_t - C_t) - \kappa(s_t, n_t)], \quad (11)$$

where I denote the present value of costs by $C_t = E_t \sum_{k=0}^{\infty} \beta^k (1 - \delta)^k c_{t+k}$. Firm value derives from the present-value profits on the initial cohort of customers of measure n_0 together with present-value profits on subsequent cohorts of customers of measure $\eta(\theta_t)s_t$. Present-value prices P_t determine present-value profits per customer, but also influence customer acquisition via (10). Note that, with this, the firm may equivalently be thought of as choosing a sequence of present-value prices $\{P_t\}_{t=0}^{\infty}$ instead of per-period prices.

Writing firm value in these terms makes explicit a time-inconsistency affecting price setting, as the tradeoffs the firm faces in the initial period differ from later periods. In setting P_0 , the firm takes into account that raising this present-value price raises present-value profits on both existing and new customers, at the cost of reduced customer acquisition. In setting P_t for subsequent periods, the firm only considers the impact on new customers,

however. The firm's preferences regarding pricing for the initial period thus differ from subsequent periods—and this is true in any period in which the firm reoptimizes.

I proceed by considering price setting assuming that firms are free to reoptimize prices each period, in a Markov perfect equilibrium. To that end, I denote the market-wide state as $\Omega = (N, z)$. The firm problem may then be written recursively as:

$$\begin{aligned} \max_{P,s} & (n + \eta(\theta)s)(P - C(z)) - \kappa(s, n) + \beta E_{\Omega} V(n', \Omega') \\ \text{s.t. } & n' = (1 - \delta)(n + \eta(\theta)s), \\ & x(\Omega) = \mu(\theta)(U - P - \beta(1 - \delta)E_{\Omega} X(\Omega')) \text{ if } s > 0, \\ & P \leq U - \beta(1 - \delta)E_{\Omega} X(\Omega'), \end{aligned}$$

where the continuation value follows the accounting equation $V(n, \Omega) = \eta(\theta)s(P - C(z)) - \kappa(s, n) + \beta E_{\Omega} V(n', \Omega')$ and the firm takes its existing customer base and the equilibrium value of search as given. Note that here the firm objective reflects the terms appearing in (11) over time, while decision-making reflects reoptimization each period.

A Markov perfect equilibrium generally prescribes a firm's choices as functions of all payoff-relevant state variables, here (n, Ω) . Due to the scale-independence of the firm problem, it becomes natural and convenient to restrict attention to equilibria where firm choices are independent of scale as well. To see the scale-independence, I first define the scaled variables $\hat{s} = s/n$, $\hat{V}(\Omega) = V(n, \Omega)/n$. Scaling the recursive firm problem by n then yields a firm problem that is independent of n :¹⁶

$$\begin{aligned} \max_{P,\hat{s}} & (1 + \eta(\theta)\hat{s})(P - C(z)) - \hat{\kappa}(\hat{s}) + \beta(1 - \delta)(1 + \eta(\theta)\hat{s})E_{\Omega}\hat{V}(\Omega') \quad (12) \\ \text{s.t. } & x(\Omega) = \mu(\theta)(U - P - \beta(1 - \delta)E_{\Omega} X(\Omega')) \text{ if } s > 0, \\ & P \leq U - \beta(1 - \delta)E_{\Omega} X(\Omega'), \end{aligned}$$

where $\hat{V}(\Omega) = \eta(\theta)\hat{s}(P - C(z)) - \hat{\kappa}(\hat{s}) + \beta(1 - \delta)(1 + \eta(\theta)\hat{s})E_{\Omega}\hat{V}(\Omega')$ and the firm takes the equilibrium value of search as given. Due to the scale-independence of the firm problem, I restrict attention to firm choices that are also functions of the market-wide state Ω alone.

As in the static problem, the firm has two distinct options here. It can seek to attract new customers, which requires a present-value price that is competitive in the market for searching buyers, or focus on making profit on existing customers instead. Note that this problem remains relatively similar to the static case, despite the dynamics.

¹⁶Note that in general one would expect that $P(n), s(n)$ depend on size. Differentiating value function gives $V'(n_t) = -\kappa_n(s_t, n_t) + \beta(1 - \delta)E_t V'(n_{t+1}) + [\eta(\theta_t)(P_t - C_t + \beta(1 - \delta)E_t V(n_{t+1})) - \kappa_s(s_t, n_t)]s'(n_t) + [\eta(\theta_t)s_t + \eta'(\theta_t)f'(P_t)s_t(P_t - C_t + \beta(1 - \delta)E_t V(n_{t+1}))]P'(n_t)$ and then using the FOC yields $V'(n_t) = -\kappa_n(s_t, n_t) + \beta(1 - \delta)E_t V'(n_{t+1}) - n_t P'(n_t)$. In general the marginal value thus depends on the derivative of the decision rule. The final term vanishes if P is independent of size.

Option 1: Active selling to attract new customers If the firm seeks to attract new customers, it chooses a positive selling intensity $\hat{s} > 0$ together with a present-value price that is competitive in the market for searching buyers. This price will generally be strictly below the buyers' willingness to pay, as firms use prices to compete for buyers, implying that the second condition in the firm problem (12) becomes superfluous.

An interior optimum with active selling must satisfy first order conditions.¹⁷ The first order condition for the selling intensity,

$$\kappa_s(\hat{s}) = \eta(\theta)(P^s - C(z) + \beta(1 - \delta)E_\Omega \hat{V}(\Omega')), \quad (13)$$

states that the firm chooses a selling intensity where the marginal cost of selling is equated to the present-value profits from sales to the additional customers acquired. Additional customers yield a present value of prices P^s net of costs C , as well as reducing the costs of sales activities in future periods, as reflected in the continuation value \hat{V} (discussed below).

The first order condition for the present-value price,

$$1 + \eta(\theta)\hat{s} = -\eta'(\theta)g_P(P^s; \Omega)\hat{s}(P^s - C(z) + \beta(1 - \delta)E_\Omega \hat{V}(\Omega')), \quad (14)$$

states that the firm raises the present-value price to a point where the increase in firm value due to greater profit margins per customer equals the decrease in firm value due to reduced customer acquisition. Here the increase in present-value price reduces the queue of searching buyers according to $g_P(P^s; \Omega) = \mu(\theta)/(\mu'(\theta)(U - P^s - \beta(1 - \delta)E_\Omega X(\Omega'))) < 0$.

The optimality condition (14) implies that the present-value price may be written as a weighted average of firm cost and buyer willingness to pay:

$$P^s = \gamma(C(z) - \beta(1 - \delta)E_\Omega \hat{V}(\Omega')) + (1 - \gamma)(U - \beta(1 - \delta)E_\Omega X(\Omega')) \\ \text{with } \gamma = \varepsilon\Delta/(1 - \varepsilon + \Delta), \quad (15)$$

where $\varepsilon \in (0, 1)$ is the matching function elasticity and $\Delta = \eta(\theta)\hat{s} > 0$ the share of new versus existing customers in firm sales. This price reflects, in addition to firm cost and buyer willingness to pay, how effective pricing is at attracting new customers and how important new customers are in firm sales. In terms of firm costs, additional customers imply both added costs of production as well as reducing the costs of customer acquisition going forward. Buyer willingness to pay, on the other hand, takes into account both the buyer valuation for the products and the forgone value of search during the relationship. Note that the expression for the present-value price aligns relatively closely with the static case.

¹⁷I restrict attention to circumstances where first order conditions are also sufficient for interior optimum.

Option 2: Profit from existing customers Alternatively, the firm can forgo attracting new customers to focus on making profits on its existing customers instead.

If the firm does not participate in the market for searching buyers, its selling intensity is zero and its pricing problem reduces to:

$$\begin{aligned} \max_P P - C(z) + \beta(1 - \delta)E_\Omega \hat{V}(\Omega') \\ P \leq U - \beta(1 - \delta)E_\Omega X(\Omega'), \end{aligned} \tag{16}$$

taking the equilibrium value of search as given. The optimal present-value price becomes $P^r = U - \beta(1 - \delta)E_\Omega X(\Omega')$, reflecting both the buyer valuation for the products and the forgone value of search during the relationship, with the firm thus taking the full gains from the relationship. This price is too high to attract searching buyers, but existing customers are willing to pay it.

Which of the two options dominates for the firm in a given period becomes more subtle in the dynamic model than the static one. The firm problem retains essentially the same form as in the static model, however, with firm behavior continuing to depend on the prevailing value of search S_t in much the same way as before when holding other things equal.¹⁸ The present-value profits from seeking to attract new customers are decreasing in S_t , because a higher value of S_t means it is more costly for the firm to attract searching buyers. Meanwhile, the present-value profits from focusing on profiting from existing customers are independent of S_t . Thus, for sufficiently low values of S_t the firm prefers to seek to attract new customers, and from some S_t up it focuses on profiting from existing customers instead. In between there will generally be a value of search where firms are indifferent between the two alternatives, and might randomize between active selling at a lower price versus profiting from existing customers with a higher price.

Equilibrium firm behavior must maximize firm value, as well as be consistent with a market clearing condition for searching buyers, each period. From the optimality conditions, it is clear that if all firms are identical (aside from possible differences in size), their choices are also identical in that all actively selling firms choose the same selling intensity and present-value price (with the same queue length), while all remaining firms choose the same present-value price. Denoting the total measure of existing customers across firms in the beginning of period t by $N_t = \sum_i n_{it}$, and the probability of a firm actively selling by α_t , the market clearing condition requires that the total measure of searching buyers across firms, $\theta_t \hat{s}_t \alpha_t N_t$, equals the total measure of searching buyers in the market, $1 - N_t$, for all t .¹⁹

¹⁸A higher current value of search S_t implies a higher x_t , whereas X_{t+1} depends on future values of search.

¹⁹Each actively selling firm attracts $\theta_t \hat{s}_t n_{it}$ buyers. If the probability that a firm (of any size) actively sells

These conditions determine the equilibrium values of search.

Definition 2. *A competitive search equilibrium with anonymous pricing specifies probabilities of a firm actively selling α_t , corresponding sale prices p_t^s , queues θ_t and selling intensities \hat{s}_t , as well as regular prices p_t^r and values of search S_t for $t \geq 0$ such that: i) $\{P_t^s, \theta_t, \hat{s}_t\}$ solve the firm problem (12) with $\hat{s}_t > 0$, and P_t^r solves the firm problem (16), ii) if $0 < \alpha_t < 1$, firms are indifferent between active selling and not, and if $\alpha_t = 1$, firms weakly prefer active selling, iii) the market for searching buyers clears: $1 - N_t = \theta_t \hat{s}_t \alpha_t N_t$, and iv) the law of motion $N_{t+1} = (1 - \delta)(N_t + \mu(\theta_t)(1 - N_t))$ for all $t \geq 0$.*

I refer to the equilibrium as featuring sale pricing within a period when each firm has a positive probability of charging both a lower “sale” price and a higher “regular” price in that period. The equilibrium then features firms pursuing both pricing strategies within the period as well.

If firms are randomizing across these strategies in period t , then they must be indifferent between them. The present-value profits from active selling consist of those from existing customers $n_t[P_t^s - C_t + \beta(1 - \delta)E_t\hat{V}_{t+1}]$ and those from new customers $n_t[\eta(\theta_t)\hat{s}_t(P_t^s - C_t + \beta(1 - \delta)E_t\hat{V}_{t+1}) - \hat{\kappa}(\hat{s}_t)] = -n_t\kappa_n(\hat{s}_t)$, both positive.²⁰ Holding off on active selling yields higher present-value profits on existing customers, $n_t[U - \beta(1 - \delta)E_tX_{t+1} - C_t + \beta(1 - \delta)E_t\hat{V}_{t+1}]$, but none on new customers. In such an equilibrium the present-value profits from new customer acquisition must thus just make up for charging existing customers less: $P_t^s - \kappa_n(\hat{s}_t) = U - \beta(1 - \delta)E_tX_{t+1}$. By contrast, if all firms are actively selling, it must be that $P_t^s - \kappa_n(\hat{s}_t) \geq U - \beta(1 - \delta)E_tX_{t+1}$.

Firms that are actively selling in period t have selling intensity and present-value price satisfying:

$$\begin{aligned} \kappa_s(\hat{s}_t) &= \eta(\theta_t)(P_t^s - C_t + E_t \sum_{k=1}^{\infty} \beta^k (1 - \delta)^k \alpha_{t+k} \kappa_n(\hat{s}_{t+k})) \text{ and} & (17) \\ P_t^s &= \gamma_t(C_t + E_t \sum_{k=1}^{\infty} \beta^k (1 - \delta)^k \alpha_{t+k} \kappa_n(\hat{s}_{t+k})) + (1 - \gamma_t)(U - \beta(1 - \delta)E_tX_{t+1}). \end{aligned}$$

These conditions correspond to (13) and (15), but with the effect of existing customers to reduce future selling costs made explicit. Acquiring additional customers implies reduced costs of selling in future periods where the firm is actively selling.²¹

is α , adding these buyers up across firms yields a measure of searching buyers $\theta_t \hat{s}_t \alpha_t N_t$, to equal $1 - N_t$.

²⁰This uses the first order condition for selling intensity (13) and that $\kappa_n(\hat{s}) = \hat{\kappa}(\hat{s}) - \hat{s}\kappa_s(\hat{s})$.

²¹The continuation values reflect the value of existing customers to reduce the costs of selling in periods in

The remaining firms charge a higher present-value price that does not attract new customers:

$$P_t^r = U - \beta(1 - \delta)E_t X_{t+1}.$$

How are present value prices related to per-period prices? With each firm setting the lower price with probability α_t , independently across firms and over time, the per-period prices p_t^s and p_t^r satisfy:

$$\begin{aligned} P_t^s &= p_t^s + \beta(1 - \delta)E_t(\alpha_{t+1}P_{t+1}^s + (1 - \alpha_{t+1})P_{t+1}^r), \\ P_t^r &= p_t^r + \beta(1 - \delta)E_t(\alpha_{t+1}P_{t+1}^s + (1 - \alpha_{t+1})P_{t+1}^r). \end{aligned}$$

These relationships imply that the per-period regular price exceeds the per-period sale price, as firms randomize between the two pricing strategies and hence $p_t^r - p_t^s = P_t^r - P_t^s > 0$. The per-period sale price can also be lower than the per-period cost when the probability of a sale price is relatively low. If the probability of a sale price is low, the firm must implement any desired lower present-value sale price today with a low per-period sale price today, as prices are expected to be high in subsequent periods. (See Figure 4 for an illustration.)

Reflecting the static model, it is natural to expect that the prevalence of existing customer relationships among buyers matters for outcomes in this market. In the dynamic model, the prevailing measure of existing customer relationships not only affects market outcomes, but also the evolution of existing customer relationships going forward. Whatever the initial level, the market thus evolves over time, and it is natural to expect it to evolve toward a steady state that reflects prevailing conditions. The steady state may feature all firms focusing on attracting new customers, or firms randomizing between seeking to attract new customers versus focusing on profiting from existing customers. A low initial level of existing customer relationships may also imply an initial period of growth where all firms focus on attracting new customers, toward a longer run outcome where firms randomize in pricing.

Longer run outcomes reflect the prevalence of long-term customer relationships in the market. If customer turnover is high, then the market will tend to feature fewer existing customer relationships at any point in time. On the other hand, if customer turnover is low, then the market will tend to feature more existing customer relationships at any point in time. As such, a market with high turnover will tend to feature all firms focusing on

which the firm is actively selling. To see this, note that when a firm is actively selling today, its continuation value may be expressed as $\eta(\theta)\hat{s}(P^s - C(z)) - \hat{\kappa}(\hat{s}) + \beta(1 - \delta)(1 + \eta(\theta)\hat{s})E_\Omega \hat{V}(\Omega') = -\kappa_n(\hat{s}) + \beta(1 - \delta)E_\Omega \hat{V}(\Omega')$, using the first order condition and $\kappa_n(\hat{s}) = \hat{\kappa}(\hat{s}) - \hat{s}\kappa_s(\hat{s})$. When a firm is not actively selling, the value reduces to $\beta(1 - \delta)E_\Omega \hat{V}(\Omega')$ instead. If the firm randomizes with probability α , the expected continuation value becomes $-\alpha\kappa_n(\hat{s}) + \beta(1 - \delta)E_\Omega \hat{V}(\Omega')$, reflecting the fact that actively selling firms benefit from existing customers via lower selling costs whereas the remainder of firms do not.

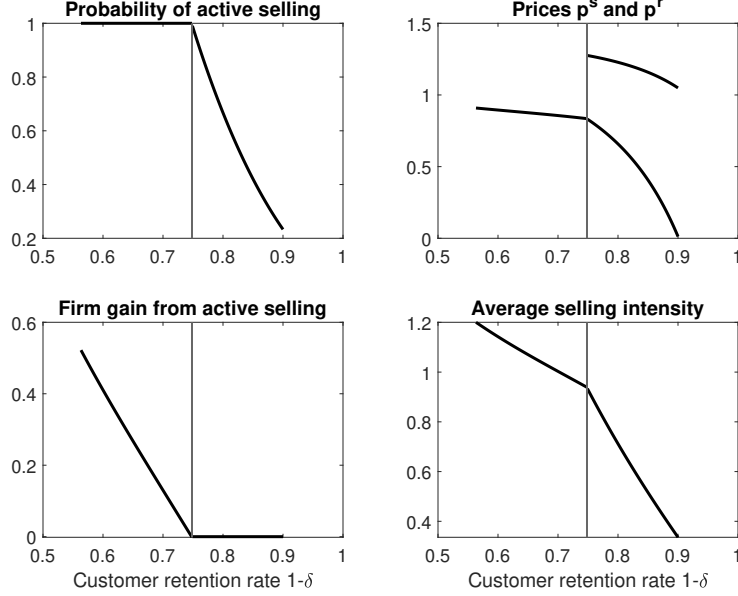


Figure 4: Steady State as a Function of the Customer Retention Rate $1 - \delta$

Notes: The figure illustrates steady-state equilibrium outcomes as a function of the share of existing customer relationships in the market. The top left panel plots the probability α , the top right the prices p^s, p^r , the bottom left a firm's net gain from active selling relative to forgoing doing so, and the bottom right the average selling intensity $\alpha \hat{s}$. Here $u = 1$, $c = 1/2$, $\hat{\kappa}(\hat{s}) = \hat{s}^2/2$, $\eta(\theta) = \theta/(1 + \sqrt{\theta})^2$ and $\beta = 1.05^{-1/24}$.

attracting new customers and a market with sufficiently low customer turnover feature firms randomizing in pricing.

The following result characterizes steady state outcomes assuming a standard form for the selling cost and that the matching function elasticity declines from one toward zero as queues grow:

Proposition 4. *Let $\hat{\kappa}(\hat{s}) = \hat{s}^\varphi/\varphi$ such that $\varphi > 1$, $\lim_{\theta \rightarrow 0} \varepsilon(\theta) = 1$, and $\lim_{\theta \rightarrow \infty} \varepsilon(\theta) = 0$. There exists a $\delta^* \in (0, 1)$ such that when $\delta < \delta^*$, the competitive search equilibrium with anonymous pricing has a unique steady state and this steady state features sale pricing, and when $\delta \geq \delta^*$, the competitive search equilibrium with anonymous pricing has a unique steady state and this steady state does not feature sale pricing.*

Figure 4 illustrates these long-run outcomes in the context of a parameterized example. On the left side of each panel, customer turnover is high and few buyers consequently in existing customer relationships. In this case the steady state features all firms actively selling at a price that attracts searching buyers. Moving right, turnover declines and the share of buyers in existing customer relationships increases, leading to the profitability of active selling declining, until firms become indifferent between the two pricing strategies at

the vertical line. Moving further right, firms randomize between the two pricing strategies, remaining indifferent between them due to firms dropping out of active selling.

Efficient allocations How do equilibrium outcomes in this dynamic market compare to efficient allocations?

A benevolent planner maximizes the present value of output net of the costs of production and selling, facing the same frictions in creating customer relationships as market participants. The planner takes as given the existing customers at each firm and decides, for each period, how much selling effort each firm should take on as well as how to allocate searching buyers among firms:

$$\begin{aligned} \max_{\{\theta_{it}, s_{it}\}_{t=0, i \in I}^{\infty}} \quad & E_0 \sum_{t=0}^{\infty} \beta^t \sum_{i \in I} [(n_{it} + \eta(\theta_{it})s_{it})(u - c_t) - \kappa(s_{it}, n_{it})] \\ \text{s.t.} \quad & n_{it+1} = (1 - \delta)(n_{it} + \eta(\theta_{it})s_{it}), \quad \forall i \in I, t \geq 0, \\ & \sum_{i \in I} \theta_{it}s_{it} = 1 - \sum_{i \in I} n_{it}, \quad \forall t \geq 0. \end{aligned}$$

The planner does so subject to the law of motion for the customer base of each firm and the constraint that, each period, the total measure of buyers allocated among firms equals the total measure of searching buyers.

The planner allocates searching buyers such that the gains from the additional customer relationships created equal the shadow value of additional buyers,

$$\eta'(\theta_{it})[u - c_t + E_t \sum_{k=1}^{\infty} \beta^k (1 - \delta)^k (u - c_{t+k} - \kappa_n(\hat{s}_{it+k}) - \lambda_{t+k})] = \lambda_t, \quad (18)$$

across firms and over time. The gains accrue for as long as the relationship lasts, with existing customers forgoing search during that time.

Correspondingly, the planner allocates selling effort such that the gains from the additional customer relationships created equal the costs of additional selling,

$$\eta(\theta_{it})[u - c_t + E_t \sum_{k=1}^{\infty} \beta^k (1 - \delta)^k (u - c_{t+k} - \kappa_n(\hat{s}_{it+k}) - \lambda_{t+k})] = \kappa_s(\hat{s}_{it}) + \lambda_t \theta_{it}, \quad (19)$$

across firms and over time.

To relate equilibrium pricing and allocations to their efficient counterparts, it is convenient to note that the efficient allocations may be decentralized by firms setting the present-value price:

$$P_t^p = \varepsilon_t (C_t + E_t \sum_{k=1}^{\infty} \beta^k (1 - \delta)^k \kappa_n(\hat{s}_{it+k})) + (1 - \varepsilon_t)(U - \beta(1 - \delta)E_t X_{t+1}), \quad (20)$$

with the privately optimal selling effort satisfying the firm's optimality condition (17) with this present-value price.²² Efficient pricing may thus also be expressed as a weighted average of firm cost and buyer willingness to pay, but with a different weight than in equilibrium.

What the dynamic planner problem does not pin down are the efficient per-period prices, short of additional assumptions, leaving the efficient dynamics of per-period prices undetermined. If efficient pricing must satisfy the anonymity property discussed—all customers facing a common per-period price each period—then efficient per-period prices are determined via the relationship $P_t^p = p_t^p + \beta(1 - \delta)E_t P_{t+1}^p$, with efficient per-period prices reflecting efficient present-value prices somewhat directly.

The preceding discussion shows that the expressions for equilibrium and efficient present-value prices and allocations in the dynamic setting are closely related to their counterparts in the static model. These relationships allow extending results from the static model to the dynamic setting as well.

Most immediately, equilibrium present-value prices P^s and P^r both exceed the efficient present-value price, as equilibrium pricing is influenced by the firms' incentive to profit from existing customers. The too-high prices lead to firms taking on excess selling effort, resulting in excess trade in the product market. The emergence of sale pricing may thus be viewed as beneficial for resource allocation, in reducing overselling.

Proposition 5. *The steady state of the competitive search equilibrium with anonymous pricing that does not feature sale pricing has strictly higher present-value price, selling intensity, volume of trade and firm profit than efficient.*

Comparing the expressions for equilibrium versus efficient present-value prices also shows that equilibrium pricing continues to be too focused on buyer willingness to pay for the product and too unresponsive to cost. The emergence of sale pricing only makes the rigidity with respect to cost more pronounced in the sense that regular present-value prices respond to cost even less than the sale present-value prices used to attract searching buyers.²³

²²To see this, note that (18) may be rewritten as $\mu(\theta_{it})\varepsilon_{it}[u - c_t + E_t \sum_{k=1}^{\infty} \beta^k (1 - \delta)^k (u - c_{t+k} - \kappa_n(\hat{s}_{it+k}) - \lambda_{t+k})] = \lambda_t$, which indicates that the efficient value of search is equated to the product of the probability of entering into a customer relationship and share ε_{it} of the gains from the relationship. Further, (19) may be rewritten as $\eta(\theta_{it})(1 - \varepsilon_{it})[u - c_t + E_t \sum_{k=1}^{\infty} \beta^k (1 - \delta)^k (u - c_{t+k} - \kappa_n(\hat{s}_{it+k}) - \lambda_{t+k})] = \kappa_s(\hat{s}_{it})$, which indicates that the marginal selling cost is equated to the product of the probability of entering into a customer relationship and share $1 - \varepsilon_{it}$ of the gains from the relationship. By contrast, (20) implies that the equilibrium value of search satisfies $x_t = \mu(\theta_{it})\varepsilon_{it}[U_t - C_t - E_t \sum_{k=1}^{\infty} \beta^k (1 - \delta)^k (\kappa_n(\hat{s}_{it+k}) + x_{t+k})]$ and equilibrium selling and equilibrium selling effort $\kappa_s(\hat{s}_{it}) = \eta(\theta_{it})(1 - \varepsilon_{it})[U - C_t - E_t \sum_{k=1}^{\infty} \beta^k (1 - \delta)^k (\kappa_n(\hat{s}_{it+k}) + x_{t+k})]$. These conditions coincide with the planner's optimality conditions with the value of search coinciding with the planner's shadow value of searching buyers.

²³Note that the present value prices are the price measures that matter for allocations here.

Asymmetric pricing The dynamic model continues to give rise to asymmetric price paths—a relatively stable regular price that undergoes repeated temporary downward shifts of varying magnitude over time—when firms face transitory idiosyncratic shocks.

To see this, consider a steady state with sale pricing, where firms are indifferent between charging a lower sale price and a higher regular price. Suppose then that a single firm faces a slightly lower production cost this period than the other firms, with costs in future periods unaffected. Instead of being indifferent between the two pricing strategies, the firm will strictly prefer to seek to attract new customers. Its present-value profits from both strategies exceed other firms’ due to its lower cost, but the profits from attracting new customers increase more because the firm benefits from lower costs on new customers as well.²⁴ The firm thus responds to the lower cost by setting a sale price and taking on sales activities, and with both price and selling intensity depending on realized cost.

On the other hand, if the firm faces a slightly higher production cost this period than other firms, then the firm will strictly prefer to hold off on seeking to attract new customers, focusing on making profit on its existing customers instead. The firm thus responds to the higher cost by setting the regular price, which is independent of cost. Among the firms in the market, some charging the regular price and some the sale price, an individual firm’s responses to increases and decreases in cost are asymmetric.

One can extend the logic to consider a setting where different firms face somewhat different costs this period, and anticipate the equilibrium to feature sufficiently low cost firms seeking to attract new customers—charging a sale price that reflects their cost while taking on sales activities—and sufficiently high cost firms focusing on making profit on existing customers—charging the regular price that is independent of their cost. In this setting the model then generates sale pricing where sales are triggered by sufficiently low cost realizations, with sale prices reflecting cost and the regular price independent of cost.

Teaser pricing If firms can keep track of individual customers and differentiate accordingly in pricing, the theory predicts the emergence of teaser pricing, with new customers facing a lower price than existing customers.

In thinking about price setting when firms are able to differentiate among customers, the first thing to note is that firms optimally charge their existing customers their full willingness

²⁴The difference in present-value profits between seeking to attract new customers versus not decreases in production cost. This difference reads $n_t[(1 + \eta(\theta_t)\hat{s}_t)(P_t^s - C_t + \beta(1 - \delta)E_t\hat{V}_{t+1}) - \kappa(\hat{s}_t) - (U - \beta(1 - \delta)E_tX_{t+1} - C_t + \beta(1 - \delta)E_t\hat{V}_{t+1})]$ where $P_t^s, \theta_t, \hat{s}_t$ satisfy the corresponding first order conditions, which decreases in C_t .

to pay for the product, in present value terms $P_t^e = U - \beta(1 - \delta)E_t X_{t+1}$.²⁵ This is the most that the firm can charge an existing customer while still retaining them, and is hence what a profit maximizing firm should charge existing customers.

With this, the firm's pricing problem reduces to a question of how to set the prices P_t^n for new customers:

$$\begin{aligned} \max_{\{P_t^n, s_t\}_{t=0}^{\infty}} \quad & E_0 \sum_{t=0}^{\infty} \beta^t [\eta(\theta_t) s_t (P_t^n - C(z_t)) - \kappa(s_t, n_t)] \\ \text{s.t.} \quad & n_{t+1} = (1 - \delta)(n_t + \eta(\theta_t) s_t), \quad \forall t \geq 0, \\ & x_t = \mu(\theta_t)(U - P_t^n - \beta(1 - \delta)E_t X_{t+1}) \text{ if } s_t > 0, \quad \forall t \geq 0, \\ & P_t^n \leq U - \beta(1 - \delta)E_t X_{t+1}, \quad \forall t \geq 0, \end{aligned}$$

taking its existing customer base and the equilibrium value of search as given. The objective reflects firm value derived from customer acquisition over time: the present-value profits from cohorts of new customers of measure $\eta(\theta_t) s_t$, with sales at present-value price P_t^n and production cost C_t , net of selling costs. The firm is constrained by the law of motion for the customer base, and if the firm is seeking to attract new customers, it also faces constraint (10) characterizing the queue of searching buyers attracted by the firm's prices. Either way, present-value prices cannot exceed buyer willingness to pay for the product, or buyers would not agree to trade.

The optimal selling intensity and price satisfy the corresponding first order conditions. The first order condition for selling,

$$\kappa_s(\hat{s}_t) = \eta(\theta_t)(P_t^n - C_t - E_t \sum_{k=1}^{\infty} \beta^k (1 - \delta)^k \kappa_n(\hat{s}_{t+k})),$$

states that the firm chooses a selling intensity where the marginal cost of selling is equated to the profits from sales to the additional customers acquired. The first order condition for the price,

$$\eta(\theta_t) \hat{s}_t = -\eta'(\theta_t) g_P^t(P_t^n) \hat{s}_t (P_t^n - C_t - E_t \sum_{k=1}^{\infty} \beta^k (1 - \delta)^k \kappa_n(\hat{s}_{t+k})), \quad (21)$$

states that the firm raises the price to a point where the increase in profits due to higher profit margins per customer equals the decrease in profits due to reduced customer acquisition. The increase in price again reduces the queue of searching buyers according to $g_P(P_t^n)$.

²⁵An existing customer is willing to remain with the firm as long as $E_t \sum_{k=0}^{\infty} \beta^k (1 - \delta)^k (u - p_{t+k}^e + \beta \delta S_{t+k+1}) \geq \beta E_t S_{t+1}$, meaning if the present value of remaining weakly exceeds the present value of returning to search. In the notation introduced, this inequality reads $U - P_t^e - \beta(1 - \delta)E_t X_{t+1} \geq 0$.

The optimality condition (21) implies that the present-value price may be written as a weighted average of firm cost and buyer willingness to pay:

$$P_t^n = \varepsilon_t(C_t + E_t \sum_{k=1}^{\infty} \beta^k (1 - \delta)^k \kappa_n(\hat{s}_{t+k})) + (1 - \varepsilon_t)(U - \beta(1 - \delta)E_t X_{t+1}),$$

with the weight given by the matching function elasticity.

As the present-value price for new customers is lower than that for existing customers, it follows that new customers also face a lower per-period price than existing customers—teaser pricing during the customer relationship.²⁶

Proposition 6. *The competitive search equilibrium without anonymous pricing is unique and equilibrium allocations efficient.*

Relaxing anonymous pricing also leads to equilibrium allocations becoming efficient.²⁷ To profit on existing customers, firms charge existing customers their full willingness to pay for the product. Yet to attract new customers, firms simultaneously set a competitive price for them. This added flexibility in pricing is enough to attain efficient allocations. Anonymous pricing rules out teaser pricing, but in a sense the sale pricing that emerges with anonymous pricing could be viewed as a proxy for teaser pricing—that can also serve to improve on efficiency of resource allocation—when the latter is not feasible.

Finally, note that teaser pricing continues to feature high prices that respond little to firm cost for existing customers, but with initial discounts allowing attaining efficient allocations nevertheless.²⁸ In this case the seemingly high and rigid pricing (for existing customers) is thus not allocative.

In all, this section has demonstrated how the analysis of the static model extends to give rise to sale and teaser pricing in a dynamic, infinite-horizon setting. The analysis becomes more involved, but yields relatively parsimonious characterizations of prices that allow extending results from the static case nevertheless.

4 Conclusions

This paper has studied price setting in an equilibrium search model of frictional product markets with long-term customer relationships (Gourio and Rudanko 2014). I showed that

²⁶Recall that $P_t^n = p_t^n + \beta E_t P_{t+1}^e$ and $P_t^e = p_t^e + \beta E_t P_{t+1}^e$ imply that $p_t^e - p_t^n = P_t^e - P_t^n > 0$.

²⁷See footnote 22.

²⁸The per-period price of existing customers p_t^e satisfies $P_t^e = p_t^e + \beta(1 - \delta)E_t P_{t+1}^e$, where $P_t^e = U - \beta(1 - \delta)E_t X_{t+1}$. Transitory idiosyncratic shocks to firm cost do not affect either one of these prices.

the theory gives rise to temporary sales as an equilibrium outcome when pricing is constrained to be anonymous across a firm’s customer base. Equilibrium prices are inefficiently high, leading to excess selling effort and trade in the product market, and the emergence of sale pricing can improve allocations by limiting this overselling. Equilibrium pricing is also characterized by an asymmetry involving a rigid regular price and variable sale price when firms face idiosyncratic shocks. Absent anonymous pricing, the theory gives rise to teaser pricing, which attains efficient allocations. Teaser pricing is also characterized by a stable regular price and a variable teaser price, but in this case the seeming rigidity is thus not allocative.

From the perspective of this theory, sale pricing may thus be viewed as an efficiency-enhancing feature of product markets with repeat customers where firms set a common price for their customers. Where feasible, teaser pricing does better still, however, in avoiding distortions in customer acquisition due to the firms’ price-setting power over existing customers. That both pricing schemes feature a relatively stable regular price and a variable sale/teaser price highlights the fact that per-period prices can be misleading for gauging allocative price rigidities in the context of long-term customer relationships.

References

- ANDERSON, E., B. MALIN, E. NAKAMURA, D. SIMESTER, AND J. STEINSSON (2017): “Informational Rigidities and the Stickiness of Temporary Sales,” *Journal of Monetary Economics*, 90, 64–83, doi: 10.1016/j.jmoneco.2017.06.003.
- BILS, M. (1989): “Pricing in a Customer Market,” *Quarterly Journal of Economics*, 104(4), 699–718, doi: 10.2307/2937863.
- CHEVALIER, J. A., A. K. KASHYAP, AND P. E. ROSSI (2003): “Why Don’t Prices Rise during Periods of Peak Demand? Evidence from Scanner Data,” *American Economic Review*, 93, 15–37, doi: 10.1257/000282803321455142.
- CHIAVARI, A. (2020): “Customer Accumulation, Returns to Scale and Secular Trends,” Unpublished manuscript, Oxford University.
- COIBION, O., Y. GORODNICHENKO, AND G. H. HONG (2015): “The Cyclicity of Sales, Regular and Effective Prices: Business Cycle and Policy Implications,” *American Economic Review*, 105, 993–1029, doi: 10.1257/aer.20121546.

- DINLERSOZ, E. M., AND M. YORUKOGLU (2012): “Information and Industry Dynamics,” *American Economic Review*, 102, 884–913, doi: 10.1257/aer.102.2.884.
- DROZD, L., AND J. NOSAL (2012): “Understanding International Prices: Customers as Capital,” *American Economic Review*, 102, 364–95, doi: 10.1257/aer.102.1.364.
- EATON, J., D. JINKINS, J. R. TYBOUT, AND D. XU (2022): “Two-Sided Search in International Markets,” NBER working paper 29684.
- EICHENBAUM, M., N. JAIMOVICH, AND S. REBELO (2011): “Reference Prices, Costs, and Nominal Rigidities,” *American Economic Review*, 101(1), 234–262, doi: 10.1257/aer.101.1.234.
- GERTLER, M., AND J. LEAHY (2008): “A Phillips Curve with an Ss Foundation,” *Journal of Political Economy*, 116(3), 533–572, doi: 10.1086/589522.
- GOLOSOV, M., AND R. E. LUCAS, JR. (2007): “Menu Costs and Phillips Curves,” *Journal of Political Economy*, 115(2), 171–199, doi: 10.1086/512625.
- GOURIO, F., AND L. RUDANKO (2014): “Customer Capital,” *Review of Economic Studies*, 81(3), 1102–36, doi: 10.1093/restud/rdu007.
- GUIMARAES, B., AND K. D. SHEEDY (2011): “Sales and Monetary Policy,” *American Economic Review*, 101, 844–76, doi: 10.1257/aer.101.2.844.
- HITSCH, G. J., A. HORTACSU, AND X. LIN (2021): “Prices and Promotions in U.S. Retail Markets,” *Quantitative Marketing and Economics*, 19(3), 289–368, doi: 10.1007/s11129-021-09238-x.
- KAAS, L., AND B. KIMASA (2021): “Firm Dynamics with Frictional Product and Labor Markets,” *International Economic Review*, 62(3), 1281–1317, doi: doi.org/10.1111/iere.12510.
- KEHOE, P., AND V. MIDRIGAN (2015): “Prices Are Sticky After All,” *Journal of Monetary Economics*, 75, 35–53, doi: 10.1016/j.jmoneco.2014.12.004.
- KLEIN, P., P. KRUSELL, AND V. RIOS-RULL (2008): “Time-Consistent Public Policy,” *Review of Economic Studies*, 75, 789–808, doi: 10.1111/j.1467-937X.2008.00491.x.
- KLEMPERER, P. (1995): “Competition When Consumers Have Switching Costs,” *Review of Economic Studies*, 62, 515–539, doi: 10.2307/2298075.

- KLENOW, P., AND J. WILLIS (2016): “Real Rigidities and Nominal Price Changes,” *Economica*, 83, 443–472, doi: 10.1111/ecca.12191.
- KLESHCHELSKI, I., AND N. VINCENT (2009): “Market Share and Price Rigidity,” *Journal of Monetary Economics*, 56(3), 344–352, doi: 10.1016/j.jmoneco.2009.02.006.
- KRYVTSOV, O., AND N. VINCENT (2021): “The Cyclicalities of Sales and Aggregate Price Flexibility,” *Review of Economic Studies*, 88, 334–377, doi: 10.1093/restud/rdaa050.
- MIDRIGAN, V. (2011): “Menu Costs, Multi-Product Firms and Aggregate Fluctuations,” *Econometrica*, 79, 1139–1180, doi: 10.3982/ECTA6735.
- NAKAMURA, E., AND J. STEINSSON (2008): “Five Facts about Prices: A Re-evaluation of Menu Cost Models,” *Quarterly Journal of Economics*, 123, 1415–1464, doi: 10.1162/qjec.2008.123.4.1415.
- (2011): “Price Setting in Forward-Looking Customer Markets,” *Journal of Monetary Economics*, 58, 220–233, doi: 10.1016/j.jmoneco.2011.06.004.
- (2013): “Price Rigidity: Microeconomic Evidence and Macroeconomic Implications,” *Annual Review of Economics*, 5, 133–63, doi: 10.3386/w18705.
- PACIELLO, L., A. POZZI, AND N. TRACHTER (2019): “Price Dynamics with Customer Markets,” *International Economic Review*, 60(1), 413–446, doi: 10.1111/iere.12358.
- PERLA, J. (2016): “Product Awareness, Industry Life Cycles, and Aggregate Profits,” Unpublished manuscript, University of British Columbia.
- PETROSKY-NADEAU, N., AND E. WASMER (2015): “Macroeconomic dynamics in a model of goods, labor, and credit market frictions,” *Journal of Monetary Economics*, 72, 97–113, doi: 10.1016/j.jmoneco.2015.01.006.
- PHELPS, E., AND S. WINTER (1970): “Optimal Price Policy under Atomistic Competition,” in *Microeconomic Foundations of Employment and Inflation Theory*, pp. 309–337. W. W. Norton & Company.
- RAVN, M., S. SCHMITT-GROHE, AND M. URIBE (2006): “Deep Habits,” *Review of Economic Studies*, 73(1), 195–218, doi: 10.1111/j.1467-937X.2006.00374.x.
- ROLDAN, P., AND S. GILBUKH (2021): “Firm Dynamics and Pricing under Customer Capital Accumulation,” *Journal of Monetary Economics*, 118, 99–119, doi: 10.1016/j.jmoneco.2020.01.011.

- SALOP, S., AND J. E. STIGLITZ (1982): “The Theory of Sales: A Simple Theory of Equilibrium Price Dispersion with Identical Agents,” *American Economic Review*, 72, 1121–1130.
- SHI, S. (2016): “Customer Relationship and Sales,” *Journal of Economic Theory*, 166, 483–516, doi: 10.1016/j.jet.2016.09.012.
- SOBEL, J. (1984): “The Timing of Sales,” *Review of Economic Studies*, 51, 353–368, doi: 10.2307/2297428.
- VARIAN, H. R. (1980): “A Model of Sales,” *American Economic Review*, 70, 651–659.

A Appendix

Table 1: Frequency of Sales across Consumer Spending Categories

	Processed food	Unprocessed food	Household furnishings	Apparel	Recreation	Other	Vehicle fuel	Travel	Utilities	Services	All
Fraction observations	16.6	17.1	21.2	34.5	10.9	15.3	0.3	2.1	0.0	0.5	7.4
Fraction price changes	57.9	37.9	66.8	87.1	49.1	32.6	0.0	1.5	0.0	3.1	21.5
Expenditure weight	8.2	5.9	5.0	6.5	3.6	5.4	5.1	5.5	5.3	38.5	100

Notes: The table reports the share of price observations corresponding to a sale price and share of price changes associated with a sale within the data underlying the consumer price index during 1998-2005 by product category, in percent, and expenditure weighted. Note the higher prevalence of sales in the retail categories on the left as opposed to services on the right. Vehicle fuel is a homogenous product where the theory would hence not predict sale pricing, and travel somewhat akin to a service albeit not always dealing with repeat relationships. Source: Nakamura and Steinsson (2008), Table 2.

Proof of Proposition 1 An equilibrium without sale pricing is characterized by $\{\theta, p^s, \hat{s}\}$ that satisfy first order conditions for optimal selling intensity (3) and price (4) together with the market clearing condition $1 - N = \theta \hat{s} N$, such that active selling dominates, $p^s - \kappa_n(\hat{s}) \geq u$. The first order condition for price yields, using the other two equations to substitute out price and selling intensity, the following equation for equilibrium θ :

$$\theta + \eta(\theta) \frac{1 - N}{N} = \frac{\varepsilon(\theta)}{1 - \varepsilon(\theta)} \frac{1 - N}{N} \frac{\kappa_s(\frac{1-N}{N\theta})}{u - c - \frac{\kappa_s(\frac{1-N}{N\theta})}{\eta(\theta)}}. \quad (22)$$

The left hand side of (22) is positive and strictly increasing in θ , increasing from zero when $\theta = 0$ toward infinity when θ approaches infinity. The right hand side of (22) is positive when $\theta > \underline{\theta}$, where $\underline{\theta} > 0$ is such that $u - c - \kappa_s(\frac{1-N}{N\underline{\theta}}) = 0$. In this region, the right hand side is strictly decreasing, approaching infinity when θ approaches $\underline{\theta}$ (from above) and approaching zero when θ approaches infinity. Equation (22) thus determines a unique θ . The remaining equations then determine unique values of p^s, \hat{s}, S based on this θ .

Both θ and \hat{s} decrease in N . To see this, note that combining (3) and (4) implies $\frac{1}{\hat{s}} + \eta(\theta) = \frac{\varepsilon(\theta)}{1 - \varepsilon(\theta)} \frac{\kappa_s(\hat{s})}{u - c - \kappa_s(\hat{s})/\eta(\theta)}$, which implicitly determines \hat{s} as a strictly increasing function of θ when

$\theta > \underline{\theta}$. With this, the market clearing condition implies that both θ and \hat{s} are strictly decreasing in N .

The profitability condition may be written, using (3) and (4), as:

$$\begin{aligned} p^s - \kappa_n(\hat{s}) - u &= -\frac{\eta(\theta)\hat{s}}{1 + \eta(\theta)\hat{s}}(p^s - c)\frac{\varepsilon(\theta)}{1 - \varepsilon(\theta)} - \kappa_n(\hat{s}) = -\frac{\kappa_s(\hat{s})\hat{s}}{1 + \eta(\theta)\hat{s}}\frac{\varepsilon(\theta)}{1 - \varepsilon(\theta)} - \kappa_n(\hat{s}) \\ &= -\kappa_n(\hat{s})\left[1 - \frac{\varphi}{\varphi - 1}\frac{1}{1 + \eta(\theta)\hat{s}}\frac{\varepsilon(\theta)}{1 - \varepsilon(\theta)}\right] \geq 0. \end{aligned}$$

The first term is positive but the term in brackets is strictly decreasing in N , as both θ and \hat{s} are strictly decreasing in N . The term in brackets approaches a positive value when N approaches zero (from above) and turns negative at some threshold N^* (where the profitability condition thus holds as an equality). When $N \leq N^*$, the profitability condition thus holds and there exists a unique equilibrium without sale pricing. When $N > N^*$, such an equilibrium does not exist.

An equilibrium with sale pricing is characterized by $\{\theta, p^s, \hat{s}\}$ that satisfy first order conditions for optimal selling intensity (3) and price (4), together with the profitability condition $p^s - \kappa_n(\hat{s}) = u$. The probability of a sale, $\alpha = (1 - N)/(N\theta\hat{s})$, must satisfy $\alpha < 1$.

The first order condition for price yields, using the other two equations, selling intensity \hat{s} as a function of θ : $\hat{s} = (\frac{\varepsilon(\theta)}{1 - \varepsilon(\theta)}\frac{\hat{s}\kappa_s(\hat{s})}{\kappa_n(\hat{s})} - 1)/\eta(\theta) = (\frac{\varepsilon(\theta)}{1 - \varepsilon(\theta)}\frac{\varphi}{\varphi - 1} - 1)/\eta(\theta)$. This selling intensity is positive when $\theta \leq \bar{\theta}$, where $\bar{\theta} > 0$ is such that $\frac{\varepsilon(\bar{\theta})}{1 - \varepsilon(\bar{\theta})}\frac{\varphi}{\varphi - 1} = 1$. In this region, \hat{s} is strictly decreasing in θ , approaching infinity as θ approaches zero (from above) and falling to zero when θ approaches $\bar{\theta}$ (from below).

At the same time, combining the first order condition for selling with the profitability condition also implies that $\theta = \eta^{-1}(\kappa_s(\hat{s})/(u - c + \kappa_n(\hat{s})))$, which implicitly determines \hat{s} as a strictly increasing function of θ , increasing from zero when $\theta = 0$ toward a positive upper bound $\bar{\hat{s}}$ as θ approaches infinity.

The intersection of these two curves determines unique values for θ and \hat{s} , and the remaining equations then determine unique p^s, S based on these. This characterizes an equilibrium with sale pricing iff $\alpha = (1 - N)/(N\theta\hat{s}) < 1$. An equilibrium with sale pricing thus only exists if $N > 1/(1 + \theta\hat{s})$.

Note that the threshold for N is the same across the two cases, with the equilibrium conditions coinciding with $\alpha = 1$.

Proof of Proposition 2 For the equilibrium without sale pricing, equilibrium θ is characterized by equation (22). The corresponding equation for efficient allocations is derived similarly based on planner first order conditions. This efficient counterpart is otherwise iden-

tical but omits the θ on the left hand side, implying that the efficient θ is strictly greater than in equilibrium. It follows that the efficient volume of trade $N + \mu(\theta)(1 - N)$, selling intensity $\hat{s} = (1 - N)/(N\theta)$, and total selling $\hat{s}N$ are all lower than in equilibrium.

Firm profits from customer acquisition may be written (using the first order condition for selling) as $\eta(\theta)\hat{s}(p^s - c) - \hat{\kappa}(\hat{s}) = -\kappa_n(\hat{s})$, which is increasing in \hat{s} , implying these profits are greater in equilibrium than efficient.

Proof of Proposition 3 The first order conditions and market clearing condition reduce to the same equations for allocations as in the planner problem.

Proof of Proposition 4 For a steady state without sale pricing, combining the first order conditions for selling intensity (13) and price (14) implies the relationship:

$$\frac{x}{\mu(\theta)} = \frac{\kappa_s(\hat{s})}{\eta(\theta)} \frac{\varepsilon}{1 - \varepsilon} \frac{\eta(\theta)\hat{s}}{1 + \eta(\theta)\hat{s}}.$$

At the same time, the product market constraint $x = \mu(\theta)(U - P^s - \beta(1 - \delta)X)$ together with the expression for the steady state present value $X = \frac{x}{1 - \beta(1 - \delta)}$ implies

$$\frac{x}{\mu(\theta)} = \frac{U - P^s}{1 + \frac{\beta(1 - \delta)\mu(\theta)}{1 - \beta(1 - \delta)}}.$$

Here the present-value price may be expressed, using the first order condition for selling intensity, as

$$P^s = C + \frac{\beta(1 - \delta)\kappa_n(\hat{s})}{1 - \beta(1 - \delta)} + \frac{\kappa_s(\hat{s})}{\eta(\theta)},$$

or (using that $\kappa_n(\hat{s}) = \hat{\kappa}(\hat{s}) - \hat{s}\kappa_s(\hat{s})$) as

$$P^s = C + \frac{\beta(1 - \delta)\hat{\kappa}(\hat{s})}{1 - \beta(1 - \delta)} + \frac{(1 - \delta)(1 - \beta)\hat{s}\kappa_s(\hat{s})}{\delta(1 - \beta(1 - \delta))}.$$

Equating the two expressions for $\frac{x}{\mu(\theta)}$ then yields the equation:

$$\frac{\kappa_s(\hat{s})}{\eta(\theta)} \frac{\varepsilon}{1 - \varepsilon} \frac{\eta(\theta)\hat{s}}{1 + \eta(\theta)\hat{s}} = \frac{U - C - \frac{\beta(1 - \delta)\hat{\kappa}(\hat{s})}{1 - \beta(1 - \delta)} - \frac{(1 - \delta)(1 - \beta)\hat{s}\kappa_s(\hat{s})}{\delta(1 - \beta(1 - \delta))}}{1 + \frac{\beta(1 - \delta)\mu(\theta)}{1 - \beta(1 - \delta)}},$$

which may also be written (using $\eta(\theta)\hat{s} = \delta/(1 - \delta)$) as

$$1 = \frac{1 - \varepsilon u - c - \beta(1 - \delta)\hat{\kappa}(\hat{s}) - \hat{s}\kappa_s(\hat{s})(1 - \delta)(1 - \beta)/\delta}{\varepsilon \hat{s}\kappa_s(\hat{s})(1 - \delta)(1 - \beta(1 - \delta)(1 - \mu(\theta)))}. \quad (23)$$

With $\hat{s} = \delta/((1 - \delta)\eta(\theta))$, equation (23) determines a unique θ . To see this, note that the right hand side is positive for $\theta \geq \underline{\theta}$, where $\underline{\theta}$ is such that the second term in the numerator

equals zero. In this region, the right hand side is strictly increasing in θ , increasing from zero at $\underline{\theta}$ toward infinity as θ approaches infinity. The equation thus has a unique solution.

This θ is strictly increasing in δ , as the right hand side of (23) is strictly decreasing in δ .

The profitability condition may be written as

$$P^s - U + \beta(1 - \delta)X - \kappa_n(\hat{s}) = -\kappa_n(\hat{s})\left[1 - \frac{\varepsilon}{1 - \varepsilon}(1 - \delta)\frac{\varphi}{\varphi - 1}\right] \geq 0.$$

The first term is positive but the term in brackets is strictly increasing in δ , with θ strictly increasing in δ . The term in brackets approaches a positive value when δ approaches one and turns negative at some threshold δ^* (where the profitability condition holds as equality). When $\delta \geq \delta^*$, the profitability condition thus holds and there exists a unique steady state without sale pricing. When $\delta < \delta^*$, such a steady state does not exist.

For a steady state with sale pricing, the product market constraint $x = \mu(\theta)(U - P^s - \beta(1 - \delta)X)$ together with the profitability condition implies that

$$\frac{x}{\mu(\theta)} = -\kappa_n(\hat{s}).$$

At the same time, the product market constraint $x = \mu(\theta)(U - P^s - \beta(1 - \delta)X)$ together with the expression for the steady state present value $X = \frac{x}{1 - \beta(1 - \delta)}$ again implies

$$\frac{x}{\mu(\theta)} = \frac{U - P^s}{1 + \frac{\beta(1 - \delta)\mu(\theta)}{1 - \beta(1 - \delta)}}.$$

The first order condition for selling yields a slightly different expression for the present-value price here (as the firm is not always actively selling):

$$P^s = C + \frac{\beta(1 - \delta)\alpha\kappa_n(\hat{s})}{1 - \beta(1 - \delta)} + \frac{\kappa_s(\hat{s})}{\eta(\theta)} = C + \frac{\beta\delta\kappa_n(\hat{s})}{\hat{s}\eta(\theta)(1 - \beta(1 - \delta))} + \frac{\kappa_s(\hat{s})}{\eta(\theta)},$$

or (using $\kappa_n(\hat{s}) = \hat{\kappa}(\hat{s}) - \hat{s}\kappa_s(\hat{s})$),

$$P^s = C + \frac{\beta\delta\hat{\kappa}(\hat{s})}{\eta(\theta)\hat{s}(1 - \beta(1 - \delta))} + \frac{(1 - \beta)\kappa_s(\hat{s})}{\eta(\theta)(1 - \beta(1 - \delta))}.$$

Equating the two expressions for $\frac{x}{\mu(\theta)}$ yields the equation:

$$-\kappa_n(\hat{s}) = \frac{U - C - \frac{\beta\delta\hat{\kappa}(\hat{s})}{\eta(\theta)\hat{s}(1 - \beta(1 - \delta))} - \frac{(1 - \beta)\kappa_s(\hat{s})}{\eta(\theta)(1 - \beta(1 - \delta))}}{1 + \frac{\beta(1 - \delta)\mu(\theta)}{1 - \beta(1 - \delta)}},$$

or

$$1 = \frac{u - c - \frac{\beta\delta\hat{\kappa}(\hat{s})}{\eta(\theta)\hat{s}} - \frac{(1 - \beta)\kappa_s(\hat{s})}{\eta(\theta)}}{-\kappa_n(\hat{s})(1 - \beta(1 - \delta)(1 - \mu(\theta)))}. \quad (24)$$

The first order condition for price combined with the profitability condition, $1 + \eta(\theta)\hat{s} = \frac{\varepsilon}{1-\varepsilon} \frac{\hat{s}\kappa_s(\hat{s})}{-\kappa_n(\hat{s})}$, further determine \hat{s} as a function of θ : $\hat{s} = (\frac{\varepsilon}{1-\varepsilon} \frac{\varphi}{\varphi-1} - 1)/\eta(\theta)$. This selling intensity is positive when $\theta \leq \bar{\theta}$, where $\bar{\theta} > 0$ is such that $\frac{\varepsilon(\bar{\theta})}{1-\varepsilon(\bar{\theta})} \frac{\varphi}{\varphi-1} = 1$. In this region, \hat{s} is strictly decreasing in θ , approaching infinity as θ approaches zero (from above) and falling to zero when θ approaches $\bar{\theta}$ (from below).

With this selling intensity, equation (24) determines a unique θ . To see this, note that in this region the numerator is positive when $\theta > \underline{\theta}$ for some $0 < \underline{\theta} < \bar{\theta}$. From $\underline{\theta}$ to $\bar{\theta}$, the right hand side of (24) is strictly increasing from zero at $\underline{\theta}$ toward infinity as θ approaches $\bar{\theta}$. The equation thus determines a unique θ , implying also a unique \hat{s} .

These values characterize a unique steady state with sale pricing iff $\alpha = \frac{\delta}{(1-\delta)\eta(\theta)\hat{s}} = \frac{\delta}{(1-\delta)(\frac{\varepsilon}{1-\varepsilon} \frac{\varphi}{\varphi-1} - 1)} < 1$. The θ increases and \hat{s} decreases in δ , as the right hand side of (24) is a strictly decreasing function of δ . The probability of sale pricing thus increases in δ , and in order for this probability to not exceed one, δ must thus be below some threshold. When δ is strictly below this threshold, there exists a unique steady state with sale pricing. When δ is above the threshold, such a steady state does not exist.

Note that the threshold for δ , is the same across the two cases, with the equilibrium conditions coinciding with $\alpha = 1$.

Proof of Proposition 5 A steady state without sale pricing is characterized by (see proof of Proposition 4)

$$\frac{\kappa_s(\hat{s})}{\eta(\theta)} \frac{\varepsilon}{1-\varepsilon} \frac{\eta(\theta)\hat{s}}{1 + \eta(\theta)\hat{s}} = \frac{U - C - \frac{\beta(1-\delta)\hat{\kappa}(\hat{s})}{1-\beta(1-\delta)} - \frac{(1-\delta)(1-\beta)\hat{s}\kappa_s(\hat{s})}{\delta(1-\beta(1-\delta))}}{1 + \frac{\beta(1-\delta)\mu(\theta)}{1-\beta(1-\delta)}}. \quad (25)$$

The efficient steady state is characterized by the same equation but with the term $\eta(\theta)\hat{s}/(1 + \eta(\theta)\hat{s})$ replaced by one. With $\hat{s} = \delta/((1-\delta)\eta(\theta))$, equation (25) and its efficient counterpart determine a unique θ for both equilibrium and efficient case. (The left hand side is positive and strictly decreasing in θ , approaching infinity as θ approaches zero from above, and approaching zero as θ approaches infinity. The right hand side is positive and strictly increasing when $\theta \geq \bar{\theta}$, where $\bar{\theta}$ is such that the numerator is zero.) From the difference between the cases, it further follows that θ is strictly higher in equilibrium than efficient.

Proof of Proposition 6 This follows from the first order conditions and other equilibrium conditions reducing to the same equations for allocations as in the planner problem.