Can Trade Policy Mitigate Climate Change?

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September 2021

Abstract

Trade policy is often cast as a solution to the *free-riding* problem in international climate agreements. This paper uncovers the extent to which trade policy can deliver on this promise. We introduce abatement technology and carbon externality into a multi-country, multi-industry quantitative trade model. Our framework accommodates a rich set of policy considerations, including firm delocation, multilateral carbon leakage, and returns to scale in production and abatement. By deriving simple analytical formulas for optimal carbon, production, and border taxes, we are able to quantify the reduction in CO_2 emissions under two prominent proposals that combine carbon pricing with trade policy. First, we show that *carbon border taxes* can replicate at most 1% of the CO_2 reduction attainable under global climate cooperation. By comparison, Nordhaus's (2015) *climate club* proposal can foster global climate cooperation and reduce global CO_2 emissions by up to 61%. This successful outcome hinges on both the US and EU committing to the climate club as core members, using their collective trade penalties to enforce climate cooperation by reluctant governments.

*This paper has previously circulated as "Trade, Firm Delocation, and Optimal Climate Policy." We are grateful to Jonathan Eaton, William Nordhaus, Ralph Ossa, Robert Staiger, and Volodymyr Lugovskyy for their valuable comments, and participants in seminars at Indiana University, Penn State, Purdue, WIIW Seminar in International Economics, NBER Future of Globalization Conference, Virtual International Trade and Macro Seminar, Workshop on International Trade and Environmental Policy, WEST & FEST Conference, and SED for helpful discussions and feedback.
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1 Introduction

Climate change is accelerating at a worrying pace. Yet governments have failed in their attempts to forge an agreement that can effectively combat climate change. Major climate agreements, like the 1997 KYOTO PROTOCOL and the 2015 PARIS CLIMATE ACCORD, have failed to deliver a meaningful reduction in carbon emissions. This failure is often attributed to the *free-riding* problem: Countries have an incentive to free-ride on the rest of the world's reduction in carbon emissions without undertaking proportionate abatement themselves.

The failure of existing climate agreements has urged experts to devise alternative remedies that are immune to the free-riding problem. Two proposals, in particular, have gained traction:

Proposal 1: Climate-conscious governments use *carbon border taxes* as a second-best policy to curb carbon emissions in the rest of the world.[1]

Proposal 2: Climate-conscious governments form a *climate club* and incentivize climate cooperation by reluctant governments via collective trade penalties (Nordhaus 2015).

Both proposals combine carbon pricing with trade policy but differ starkly in their approach. Proposal 1 is grounded in unilateralism. It presumes that global climate cooperation is unlikely to materialize, but unilateral policies can serve as an effective second best. Proposal 2 relies on the presumption that unilateralism is ineffective and that the failure of recent global agreements does not stem from the nature of multilateralism but a flawed architecture.

The tension between these two approaches remains unresolved, in part because measuring the efficacy of Proposals 1 and 2 is challenging. To uncover the full potential of these proposals, one must determine optimal carbon border taxes and optimal trade penalties under the climate club model. This task is daunting even with state-of-the-art quantitative techniques. Rich theoretical results can facilitate this task, but traditional theories of optimal climate policy in open economies are based on stylized models and are not amenable to rich quantitative analysis. Given these challenges, existing analyses of Proposal 1 and 2 often focus on easy-to-implement but sub-optimal carbon border taxes or trade penalties. Such analyses, for all their merits, only uncover the partial efficacy of carbon border taxes or the climate club. Furthermore, they generally do not account for scale economies in production and abatement or firm delocation in response to policy. Recent micro-level evidence suggests that these previously overlooked margins are key to how firms cut emissions in response to policy (Shapiro and Walker 2018).

In this paper, we overcome these challenges to uncover the full potential of *carbon border taxes* and the *climate club* proposal. To this end, we introduce carbon externalities, abatement decisions, and scale economies into a multi-country, multi-industry general equilibrium trade model. We

[1] Carbon border taxes (or carbon border adjustments) are a prominent component of the EUROPEAN GREEN DEAL, and are currently explored as a likely component of president Biden's CLIMATE PLAN.

derive simple analytical formulas for unilaterally and globally optimal taxes that internalize climate damage from carbon emissions, misallocation due to scale economies, and terms-of-trade effects. We map these formulas to data on international trade, production, and carbon emissions to evaluate the full effectiveness of carbon border taxes and the climate club proposal.

Section 2 presents our theoretical model, which is a general equilibrium semi-parametric Krugman model with many countries and industries. Our framework incorporates international carbon externalities and firm-level abatement decisions, employing a generalized version of Copeland and Taylor's (2004) abatement technology. The resulting framework is particularly attractive as it combines the carbon externality, misallocation-correcting, and terms-of-trade rationales for policy intervention in a tractable and transparent fashion. Our theoretical model exhibits several features that distinguish it from prior theories of climate policy in open economies. First, our framework accommodates firm entry and multilateral carbon leakage in response to policy. These effects are often absent in prior theories that employ partial equilibrium two-by-two models. Second, firm entry, in our framework, creates economies of scale in both production and abatement. As it turns out, these previously overlooked margins have basic implications for the ability of trade policy to curb carbon emissions.

Sections 3 and 4 derive simple analytical formulas for optimal carbon, production, and border taxes in our multi-country, multi-industry general equilibrium framework. Our formulas for optimal carbon border taxes and domestic carbon taxes, in particular, present a notable advance over traditional theories. Aside from internalizing effects of multilateral leakage and economies of scale, our formulas are amenable to the state-of-the-art quantitative analysis spanning many countries and industries.

Our derivation of optimal policy is grounded in an envelope result that transforms our general equilibrium optimal policy problem into a simpler quasi-partial equilibrium problem. Our envelope result consists of two propositions: First, we show that general equilibrium wage effects are welfare-neutral, at the optimum, when the government has access to a complete set of tax instruments. Second, general equilibrium income effects are also welfare-neutral, at the optimum, when the government can tax all goods consumed in the domestic economy. Based on this result, one can derive optimal tax formulas while treating wages as invariant to policy and demand schedules as indifferent to general equlibrium income changes.

Our analytical formulas indicate that the *unilaterally* optimal domestic carbon tax equals the CPI-adjusted domestic disutility from carbon. This choice is sub-optimal from a global standpoint as it fails to internalize home's carbon externality on the rest of the world. Optimal domestic production subsidies are, in turn, carbon-blind and solely restore marginal-cost pricing. Optimal import tariffs and export subsidies are composed of a standard terms-of-trade improving component as well as carbon border adjustments that penalize high-carbon imports and promote low-carbon exports in each industry. These carbon border adjustments are smaller the higher the degree of scale economies in the targeted industry.

To put the above results in perspective, we also characterize optimal policy under global cooperation on trade and climate. To attain the first-best cooperative outcome, border taxes/subsidies should be set to zero as they are inefficient from a global standpoint. Globally optimal carbon taxes are, meanwhile, higher than the unilaterally optimal rate and equal to the CPI-adjusted *global* disutility from carbon emissions. In other words, they internalize not only each country's carbon externality on its domestic consumers but also consumers all over the world. We complement these results with a characterization of a second-best cooperative outcome in which governments can only use border taxes to pursue international climate objectives. This situation is relevant when political pressures prevent governments from elevating domestic carbon taxes beyond their unilaterally optimal level.

Sections 5 and 6 employ our analytical tax formulas together with required sufficient statistics to uncover the efficacy of *carbon border taxes* and the *climate club* proposal at reducing carbon emissions. As noted earlier, this task can be computationally infeasible without the aid of our optimal tax formulas. Indeed, traditional analyses have often relied on easy-to-implement—but sub-optimal—trade and carbon taxes.[2] This approach, while fruitful, cannot uncover the *full* potential of either carbon border taxes or the climate club.

With the aid of our theory, the efficacy of Proposals 1 and 2 can be computed as a function of the following sufficient statistics: First, observable shares that can be constructed from national and environmental accounts data. Second, the governments' perceived disutility from climate change, which can be inferred from their applied environmentally-related taxes. Third, structural parameters consisting of the industry-level trade and scale elasticities as well as the carbon input demand elasticity—all of which we estimate using existing techniques in the literature. To construct these sufficient statistics, we merge trade, production, carbon emissions, and tax data from multiple sources. Our compiled database covers 19 broadly defined industries which constitute the entire vector of production across 13 major countries, the European Union, and an aggregate of the rest of the world.

Our quantitative analysis indicates that carbon border taxes have limited efficacy—even when set optimally by all countries. If all countries simultaneously adopt non-cooperative border taxes, global carbon emissions will decline by a mere 0.6%. This is a modest reduction, which amounts to 1% of the carbon reduction attainable under the globally-first-best carbon taxes. The inefficacy of border taxes has less to do with their non-cooperative nature. We find that carbon border taxes remain almost equally ineffective when set *cooperatively* to their globally optimal level. In addition, if we were to overlook scale economies in production and abatement, non-cooperative border taxes would deliver a 1.3% reduction in carbon emissions, which replicates only 2.2% of the first-best outcome. While scale economies play a role, the reason behind the inefficacy of

[2] Importantly, our analytical tax formulas indicate that optimal carbon border taxes have a simple and transparent structure. As such, the policy-maker need not to sacrifice simplicity in the tax code to attain optimality.

carbon border taxes appears more fundamental.

The inefficacy of carbon border taxes is driven by two main factors. First, carbon border taxes are not granular enough to induce firm-level abatement. They penalize carbon emissions based on the average carbon intensity of national industries rather than individual firms. Since an individual firm takes the industry-level carbon intensity as given, carbon border taxes have limited ability at regulating firm-level abatement in foreign locations. Second, carbon border taxes are unable to target goods that do not cross international borders but are responsible for a significant fraction of carbon emissions. In fact, a great share of carbon emissions is generated by less tradable industries. Two-third of global carbon emissions are, for instance, generated by industries with a trade-to-GDP ratio of less than 10%.

Turning to the climate club proposal, we specify the climate club game as one where the EU and possibly the US are core members of the club and other countries play strategically. Core members commit to the rules of membership: they impose optimal trade penalties against non-members while setting zero trade taxes against member countries. They also adopt globally optimal carbon taxes that correct their global carbon externality. A non-member country can retaliate by applying non-cooperative trade taxes against members while keeping its other taxes at the status quo. When joining the club, a country evaluates the trade-off between adopting higher-than-unilaterally optimal carbon taxes versus exposure to trade penalties by club members.

Quantifying the effectiveness of the climate club proposal is challenging for two reasons. First, without analytical formulas, the computation of optimal trade penalties is practically infeasible with standard numerical optimization techniques. We overcome this barrier by appealing to our analytical formulas for optimal trade penalties. Second, solving the climate club game is plagued with the curse of dimensionality, as one has to search over an excessively large number of possible climate club combinations. To overcome this challenge, we shrink the space of possible equilibria via the iterative elimination of dominated strategies. This approach exploits a key property of the climate club game, wherein net gains from joining the club rise with the size of the club.

We find that the climate club proposal—with optimal trade penalties—can be remarkably effective at reducing carbon emissions. The climate club's success, however, depends crucially on the makeup of its core members. If the EU alone initiates a climate club as a core member, no other country will find it optimal to join the climate club. However, if the climate club is initiated by the US and EU as core members, all other countries will join the club in succession. As a result, global climate cooperation is achieved under which global carbon emissions decline by 61%.[3] The intuition is that the EU, alone, does not possess sufficient market power to maintain a climate club with unilateral trade penalties. The US and EU, however, possess enough collective market power to enforce climate cooperation by reluctant governments. These results indicate

[3] We report a range of values for the reduction in carbon emissions under global cooperation, with the 61% reduction corresponding to our baseline specification. The reduction in carbon emissions can be lower depending on the values assigned to the elasticity of carbon demand and the social cost of carbon.

that trade policy tools are more effective when used as a penalty device to deter free-riding. The commitment of the US is, however, crucial for these penalties to be effective.

Related Literature

Our work contributes to several strands of literature. First, we contribute to the theoretical literature on optimal trade and environmental taxes in open economies. A central insight from this literature is that unilaterally optimal tariffs include a tax on the pollution content of imports (e.g., Markusen (1975); Copeland (1996); Hoel (1996)). This insight is typically derived within partial equilibrium or two-country models that have been rarely used to guide general equilibrium quantitative policy analysis. We complement this literature by characterizing optimal policy in a multi-country multi-industry general equilibrium trade model that accommodates salient features of the global economy and is designed to map to data.[4]

A notable exception is Kortum and Weisbach (2020) who characterize optimal trade, production, and carbon taxes in a general equilibrium two-country Ricardian model à la Dornbusch et al. (1977). Our paper complements Kortum and Weisbach (2020) in three ways. First, their analysis explicitly specifies markets for energy, whereas our model extends Copeland and Taylor (2004) in which energy markets are implicitly defined. Second, Kortum and Weisbach (2020) extend and apply the primal approach formalized by Costinot et al. (2015) while we develop our methodology based on the dual approach.[5] Third, our theory accommodates arbitrarily many countries, which is essential for evaluating the climate club proposal and is tightly connected to the quantitative trade literature emphasizing the sufficient statistic approach.

Second, our analysis is related to an emerging body of *quantitative* work that analyzes the efficacy of trade policy at tackling climate objectives, e.g., Babiker (2005), Elliott et al. (2010), Nordhaus (2015), Böhringer et al. (2016), Larch and Wanner (2017). Despite their rich structure, existing analyses have mostly quantified the efficacy of easy-to-implement but sub-optimal trade policy initiatives. This approach allows researchers to circumvent the computational difficulties associated with optimal policy analysis. However, it does not uncover the full potential of border taxes in tackling carbon emissions. In comparison, we derive analytical formulas for optimal policy, which help us bypass these computational difficulties and uncover the full potential of trade policy at curbing carbon emissions.

Third, our intermediate envelope result speaks to an emerging literature that studies optimal policy in modern quantitative trade models, e.g., Bartelme et al. (2019), Lashkaripour and Lugovskyy (2020), Beshkar and Lashkaripour (2020), Lashkaripour (2021), Caliendo et al. (2021). These

[4] For a recent review of the literature on trade and the environment, see Copeland et al. (2021). For a recent discussion of how the existing world trade system can incorporate policies to address climate change, see Staiger (2021).

[5] Each of these approaches have their advantages. Under weak separability, the primal approach offers a powerful tool to break down a high-dimensional optimal policy problem into low-dimensional sub-problems (Costinot et al. (2015, 2020)). In comparison, we employ the dual approach and show that under this approach, a typical general equilibrium optimal policy problem can be transformed into a simple quasi-partial equilibrium problem.

studies have bridged a longstanding divide between classic partial equilibrium trade policy models and modern general equilibrium trade theories. This divide is partly driven by classic trade policy models assuming away general equilibrium wage and income effects. Our envelope result is a step forward in filling this divide. It shows that the simplifying assumptions that eliminate wage and income effects can be relaxed without sacrificing tractability. This result, in particular, is derived by borrowing and refining the dual approach in Lashkaripour and Lugovskyy (2020), and extending it to environments with international externalities such as climate damage that go beyond basic terms-of-trade effects.

Finally, we contribute to ongoing efforts to enhance quantitative trade theories. Over the past two decades, quantitative trade models have been enriched to account for firm selection, scale economies, input-output linkages, multinational production, and more (Costinot and Rodríguez-Clare (2014)). But less attention has been paid to embedding environmental externalities into the state-of-the-art quantitative trade models (Cherniwchan et al. (2017), Shapiro and Walker (2018)). Our conceptual framework and optimal policy results take a step in this direction.

This paper is organized as follows: In Section 2 we present our theoretical framework. In Section 3 we present our intermediate envelope result which we use to derive formulas for optimal unilateral policy. In Section 4 we discuss international cooperative and non-cooperative Nash outcomes. In Section 5 we map our theory with our optimal policy formulas to data, which we use in Section 6 to quantify the efficacy of trade policy via the above-mentioned Proposals 1 and 2 at reducing global carbon emissions.

2 Theoretical Setup

The global economy consists of multiple countries indexed by $i, j, n \in \mathbb{C}$ and multiple industries indexed by $k, g \in \mathbb{K}$. Each country i is endowed by $\bar{L}_i$ workers who are perfectly mobile across industries but immobile across countries. Each worker supplies one unit of labor inelastically.

2.1 Demand

Subscript ji, k indexes a composite variety corresponding to *origin j—destination i–industry k*. The representative consumer in country i maximizes a non-parametric utility function $U_i(\mathbf{Q}_i)$ by choosing the vector of quantities, $\mathbf{Q}_i = \{Q_{ji,k}\}_{j\in\mathbb{C},k\in\mathbb{K}}$ subject to the budget constraint, $Y_i = \sum_j \sum_k \tilde{P}_{ji,k} Q_{ji,k}$, where Y_i denotes national income, and $\tilde{P}_{ji,k}$ denotes the consumer price index of composite variety ji, k. The tilde notation on price distinguishes between after-tax consumer prices ($\tilde{P}_{ji,k}$) and before-tax producer prices ($P_{ji,k}$). Let $\tilde{\mathbf{P}}_i = \{\tilde{P}_{ji,k}\}_{j\in\mathbb{C},k\in\mathbb{K}}$ denote the entire vector of after-tax consumer prices in country i.

The consumer's problem implies an indirect utility function, $V_i(Y_i, \tilde{\mathbf{P}}_i)$, and a Marshallian demand function, $Q_{ji,k} = \mathcal{D}_{ji,k}(Y_i, \tilde{\mathbf{P}}_i)$, for each variety ji, k. The Marshallian demand function is characterized by a set of demand elasticities. First, the elasticity of demand for variety (ji, k) with respect to the price of variety (ni, g) is,

$$\varepsilon_{ji,k}^{(ni,g)} \equiv \frac{\partial \ln \mathcal{D}_{ji,k}(Y_i, \tilde{\mathbf{P}}_i)}{\partial \ln \tilde{P}_{ni,g}}.$$

Second, the elasticity of demand for ji, k with respect to income is:

$$\eta_{ji,k} \equiv \frac{\partial \ln \mathcal{D}_{ji,k}(Y_i, \tilde{\mathbf{P}}_i)}{\partial \ln Y_i}.$$

While we impose no parametric restrictions on the demand function, we require that it satisfies standard properties of Marshallian demand. We denote the own price elasticity of demand by $\varepsilon_{ji,k} \equiv \varepsilon_{ji,k}^{(ji,k)} \leq -1$. Lastly, since every individual consumer is infinitesimally small, they do not internalize the impact of their consumption choice on CO_2 emissions.[6]

2.2 Supply and Abatement

Firms and Market Structure. Production in each *origin j–industry k* is conducted by monopolistically competitive firms indexed by $\omega \in \Omega_{j,k}$. A large pool of ex-ante identical firms can pay an entry cost $w_j \bar{f}_{j,k}$ to supply their differentiated variety to various destinations, with w_j denoting the labor wage rate in origin j and $\bar{f}_{j,k}$ denoting the labor requirement for entry. After paying the entry cost, each firm operates with a CES production technology that combines labor and carbon inputs. The gross production quantity of variety ji, k by firm ω is given by

$$\bar{d}_{ji,k} q_{ji,k}(\omega) = \bar{\varphi}_{j,k} \left[(1 - \bar{\kappa}_{j,k}) \left(l_{ji,k}(\omega) \right)^{\frac{\varsigma - 1}{\varsigma}} + \bar{\kappa}_{j,k} \left(z_{ji,k}(\omega) \right)^{\frac{\varsigma - 1}{\varsigma}} \right]^{\frac{\varsigma}{\varsigma - 1}},$$

where $\bar{d}_{ji,k} \geq 1$ denotes for iceberg trade cost associated with variety ji, k, with $\bar{d}_{jj,k} = 1$. On the right-hand side, $l_{jik}(\omega)$ and $z_{ji,k}(\omega)$ respectively denote the quantity of labor and carbon inputs, while $\bar{\kappa}_{j,k} \in [0, 1]$ and $\bar{\varphi}_{j,k} > 0$ are exogenously given factor intensities and total factor productivity in origin country j–industry k. Parameter $\varsigma > 0$ is the elasticity of substitution between labor and carbon inputs. The above production function collapses to a Leontief function if $\varsigma \to 0$, a Cobb-Douglas function if $\varsigma \to 1$, and a linear function if $\varsigma \to \infty$. Labor and carbon are gross substitute if $\varsigma \in (1, \infty)$, and gross complements if $\varsigma \in (0, 1)$.

A firm's optimal labor employment $l_{ji,k}(\omega)$ and carbon input choice $z_{ji,k}(\omega)$ depend on the wage rate w_j in the origin country and the industry-specific carbon tax, $\tau_{j,k}$. Since firms originating from the same country can be treated as symmetric, we henceforth drop the firm index ω. All firms located in origin j–industry k employ the same amount of labor and carbon inputs, $l_{ji,k} \equiv$

[6] Throughout the paper we use carbon emissions interchangeably with CO_2 emissions—both as a shorthand for greenhouse gas emissions. We explain how disutility from emissions enters the government's objective function in Section 2.5, and how emissions are measured in data in Section 5.3.

$l_{ji,k}(\omega)$ and $z_{ji,k} \equiv z_{ji,k}(\omega)$, and supply a common net quantity $q_{jik} \equiv q_{ji,k}(\omega)$ to destination i.

Formulating Production in terms of Abatement. The firms' cost minimization problem can be equivalently expressed as a tradeoff between production and abatement. In this equivalent specification, every firm devotes a fraction $a_{j,k} \in [0,1]$ of its labor input to abatement activities and the rest to production:

$$\bar{d}_{ji,k} q_{ji,k} = \bar{\varphi}_{j,k} \left(1 - a_{j,k}\right) l_{ji,k}.$$

Under this reformulation, the choice of abatement by cost-minimizing firms in origin j equals

$$\left(1 - a_{j,k}\right) = \left(1 - \bar{\kappa}_{j,k}\right)^{-\varsigma} \left[\left(1 - \bar{\kappa}_{j,k}\right)^{\varsigma} + \left(\bar{\kappa}_{j,k}\right)^{\varsigma} \left(\frac{\tau_{j,k}}{w_j}\right)^{1-\varsigma}\right]^{\frac{\varsigma}{\varsigma-1}}. \tag{1}$$

This equation indicates that a firm's choice of abatement is a decreasing function of the carbon-tax-to-wage ratio, with the extent of the relationship governed by the industry-specific carbon intensity $\bar{\kappa}_{j,k}$ and the elasticity of substitution between labor and carbon inputs, ς. Accordingly, the input cost share of carbon, which we denote by $\alpha_{j,k}$, can be expressed as a function of abatement:

$$\alpha_{j,k} = 1 - \left(1 - \bar{\kappa}_{j,k}\right)\left(1 - a_{j,k}\right)^{-\frac{\varsigma-1}{\varsigma}}. \tag{2}$$

The firm-level CO_2 emission per unit of output is then given by:

$$\frac{z_{ji,k}}{\bar{d}_{ji,k} q_{ji,k}} = \left[\frac{1}{\bar{\kappa}_{j,k}} - \frac{1 - \bar{\kappa}_{j,k}}{\bar{\kappa}_{j,k}} \left(1 - a_{j,k}\right)^{-\frac{\varsigma-1}{\varsigma}}\right]^{\frac{\varsigma}{\varsigma-1}} = \left(\frac{\alpha_{j,k}}{\bar{\kappa}_{j,k}}\right)^{\frac{\varsigma}{\varsigma-1}}, \qquad \text{(Firm-Level Emission)}$$

and the firm-level marginal cost of production equals:[7]

$$c_{ji,k} = \frac{\bar{d}_{ji,k}}{\bar{\varphi}_{j,k}\left(1 - \bar{\kappa}_{j,k}\right)} \left(1 - a_{j,k}\right)^{-\frac{1}{\varsigma}} w_j. \qquad \text{(Marginal Cost).}$$

A typical firm faces the following trade-off: With a higher choice of abatement, $a_{j,k}$, the firm incurs a higher marginal cost of production, $c_{ji,k}$, but generates less CO_2 emissions per unit of output, as indicated by the equation labeled "(Firm-Level Emission)."[8]

Industry-Level Aggregates. The composite output of good ji,k (which corresponds to origin

[7] Equivalently, we can express the cost share of carbon by $\alpha_{j,k} = (\bar{\kappa}_{j,k})^{\varsigma}(\tau_{j,k})^{1-\varsigma}/(c_{j,k})^{1-\varsigma}$, CO_2 emissions by $z_{ji,k}/(\bar{d}_{ji,k}q_{ji,k}) = \alpha_{j,k}c_{ji,k}/\tau_{j,k}$, and the marginal cost by $c_{ji,k} = \bar{d}_{ji,k}\left[(1-\bar{\kappa}_{j,k})^{\varsigma}w_j^{1-\varsigma} + (\bar{\kappa}_{j,k})^{\varsigma}\tau_{j,k}^{1-\varsigma}\right]^{1/(1-\varsigma)}$. To see the equivalence, consider that the definition of abatement yields $1 - a_{j,k} \equiv \bar{d}_{ji,k}q_{ji,k}/\bar{\varphi}_{j,k}l_{ji,k} = \left[(1-\bar{\kappa}_{j,k}) + \bar{\kappa}_{j,k}(z_{ji,k}/l_{ji,k})^{(\varsigma-1)/\varsigma}\right]^{\varsigma/(\varsigma-1)}$. Using the equation for $\alpha_{j,k}$, we can obtain $z_{ji,k}/l_{ji,k} = \frac{\alpha_{j,k}w_j}{(1-\alpha_{j,k})\tau_{j,k}} = \frac{\bar{\kappa}_{j,k}^{\varsigma}\tau_{j,k}^{-\varsigma}}{(1-\bar{\kappa}_{j,k})^{\varsigma}w_j^{-\varsigma}}$. Replacing for $z_{ji,k}/l_{ji,k}$ in the expression for $(1-a_{j,k})$ delivers Equation (1).

[8] In the special case of $\varsigma \to 1$, our specification nests Copeland and Taylor (2004). In this special case, the CES

j–destination i–industry k) aggregates over firm-level quantities, $q_{ji,k}(\omega)$. Namely,

$$Q_{ji,k} = \left(\int_{\omega \in \Omega_{j,k}} q_{ji,k}(\omega)^{\frac{\gamma_k - 1}{\gamma_k}} d\omega \right)^{\frac{\gamma_k}{\gamma_k - 1}},$$

where $\gamma_k > 1$ denotes the elasticity of substitution between firm-level varieties from the same origin country and industry. Faced with a substitution elasticity, γ_k, firms charge a constant markup over marginal cost, which implies the following producer price index for composite variety ji, k:

$$P_{ji,k} = M_{j,k}^{\frac{1}{1-\gamma_k}} \frac{\gamma_k}{\gamma_k - 1} \frac{\bar{d}_{ji,k}}{\bar{\varphi}_{j,k}(1 - \bar{\kappa}_{j,k})} (1 - a_{j,k})^{-\frac{1}{\varsigma}} w_j. \qquad \text{(Price)}$$

In the above expression, $M_{j,k} \equiv \left|\Omega_{j,k}\right|$ denotes the mass of firms in origin j–industry k. It is pinned down by the free entry condition that requires entry costs, $M_{j,k} w_j \bar{f}_{j,k}$, be equal to gross profits across all destinations, $\sum_i \frac{1}{\gamma_k} P_{ji,k} Q_{ji,k}$. Noting that $P_{ji,k} = \bar{d}_{ji,k} P_{jj,k}$ and defining industry-level output quantity as, $Q_{j,k} \equiv \sum_i \bar{d}_{ji,k} Q_{ji,k}$, the free-entry condition yields the following expression for the mass of firms:

$$M_{j,k} = \frac{P_{jj,k} Q_{j,k}}{\gamma_k \bar{f}_{j,k} w_j}. \qquad \text{(Free Entry)}$$

Using Equations "(Price)" and "(Free Entry)", we can express aggregate price indexes and CO_2 emissions as functions of abatement and output in each origin-industry:

$$P_{ji,k} = \bar{d}_{ji,k} \bar{p}_{jj,k} w_j (1 - a_{j,k})^{\frac{1}{\varsigma \gamma_k} - \frac{1}{\varsigma}} Q_{j,k}^{-\frac{1}{\gamma_k}} \tag{3}$$

$$Z_{j,k} = \bar{z}_{j,k} \left(\frac{\alpha_{j,k}}{\bar{\kappa}_{j,k}} \right)^{\frac{\varsigma}{\varsigma - 1}} (1 - a_{j,k})^{\frac{1}{\varsigma \gamma_k}} Q_{j,k}^{1 - \frac{1}{\gamma_k}} \tag{4}$$

In the above expressions, the input cost share of carbon, $\alpha_{j,k}$, is given by Equation (2), and $\bar{p}_{jj,k}$ and $\bar{z}_{j,k}$ are exogenous shifters of price and CO_2 emissions.[9] Internal economies of scale are operative through both production and abatement, and are driven by firm entry. The resulting scale

production takes a Cobb-Douglas form, $\bar{d}_{ji,k} q_{ji,k} = \bar{\varphi}_{j,k} \left(z_{ji,k}\right)^{\alpha_{j,k}} \left(l_{ji,k}\right)^{1-\alpha_{j,k}}$, in which the input cost share of carbon $\alpha_{j,k}$ is constant and equal to $\bar{\kappa}_{j,k}$. Accordingly, Equation "(Firm-level Emission)" collapses to $z_{ji,k}/(\bar{d}_{ji,k} q_{ji,k}) = (1 - a_{j,k})^{1/\alpha_{j,k} - 1}$, with the corresponding marginal cost obtained by setting $\varsigma = 1$ in Equation "(Marginal Cost)." We depart from this special case since it imposes a restriction on the extent to which firms cut their CO_2 emission in response to policy. To see this, note that carbon inputs per dollar of production cost, $z_{ji,k}/(c_{ji,k} q_{ji,k})$, decreas with the carbon tax rate $\tau_{j,k}$ at rate

$$\frac{\partial \ln \left(z_{ji,k}/(c_{ji,k} q_{ji,k}) \right)}{\partial \ln \tau_{j,k}} = -1 + (1 - \varsigma)(1 - \alpha_{j,k}).$$

The above elasticity approximately equals $-\varsigma$ if the cost share of carbon, $\alpha_{j,k}$, is sufficiently low. However, in the special case of $\varsigma \to 1$, the above elasticity equals minus one, which may restrict the ability of the model to generate an admissible magnitude of carbon reductions in response to climate policies. As such, we let this elasticity depend on a free parameter, namely ς, which we later esimate.

[9] Specifically, $\bar{p}_{jj,k} \equiv \left(\gamma_k \bar{f}_{j,k}\right)^{1/\gamma_k} \left(\frac{\gamma_k}{\gamma_k - 1} \frac{1}{\bar{\varphi}_{j,k}(1 - \bar{\kappa}_{j,k})}\right)^{(\gamma_k - 1)/\gamma_k}$, $\bar{z}_{j,k} \equiv \left(\gamma_k \bar{f}_{j,k} / \bar{p}_{jj,k}\right)^{1/(\gamma_k - 1)}$. Both equation use the fact that the mass of firms, $M_{j,k}$, can be expressed as a function of abatement and output, $M_{j,k} = \bar{m}_{j,k}(1 - a_{j,k})^{\frac{1}{\varsigma \gamma_k} - \frac{1}{\varsigma}} Q_{j,k}^{1 - \frac{1}{\gamma_k}}$ where $\bar{m}_{j,k} \equiv \bar{p}_{jj,k} / (\gamma_k \bar{f}_{j,k})$.

effects impact both industry-level prices, $P_{ji,k}$, and CO_2 emissions, $Z_{j,k}$, via the common term $\left(Q_{j,k}/(1-a_{j,k})^{1/\varsigma}\right)^{-1/\gamma_k}$ in Equations (3) and (4). This formulation nests constant-returns to scale technologies as a special case where $1/\gamma_k \to 0$.

2.3 Tax Instruments & Revenues

The government in country i has access to a full set of border, production, and carbon tax instruments necessary to replicate the *unilaterally* first-best outcome. These tax instruments include:[10]

1. Import tax, $t_{ji,k}$, applied to imported variety ji,k ($t_{ii,k}=0$ by design);
2. Export subsidy, $x_{ij,k}$, applied to exported variety ij,k ($x_{ii,k}=0$ by design);
3. Production subsidy, $s_{i,k}$, applied to all domestic output from industry k;
4. Carbon tax, $\tau_{i,k}$, applied to the carbon content of domestic output from industry k.

Note that production and carbon taxes are applied irrespective of the location of final sales. The first three tax instruments create a wedge between the after-tax consumer price, $\tilde{P}_{ji,k}$, and before-tax producer price, $P_{ji,k}$, of a given variety (ji,k). Specifically, after-tax consumer prices are related to before-tax producer prices according to:[11]

$$\tilde{P}_{ji,k} = \frac{(1+t_{ji,k})}{(1+s_{i,k})(1+x_{ij,k})} P_{ji,k}$$

In the case where only country i sets taxes, the following one-to-one mapping holds between the set of tax instruments $\{t_{ji,k}, x_{ij,k}, s_{i,k}\}_{j,k}$ and the set of after-tax prices $\{\tilde{P}_{ji,k}, \tilde{P}_{ij,k}, \tilde{P}_{ii,k}\}_{j\neq i,k}$ associated with economy i:

$$(1+t_{ji,k}) = \frac{\tilde{P}_{ji,k}}{P_{ji,k}}, \qquad (1+x_{ij,k}) = \frac{P_{ij,k}}{\tilde{P}_{ij,k}}\frac{P_{ii,k}}{\tilde{P}_{ii,k}}, \qquad (1+s_{i,k}) = \frac{P_{ii,k}}{\tilde{P}_{ii,k}}. \tag{5}$$

The government can, thus, replicate any choice of border and production tax-cum-subsidies with the right choice of consumer-to-producer price wedges.

Carbon taxes, $\tau_{i,k}$ regulate firm-level abatement in origin i–industry k. Following Equation (1), a higher carbon tax-to-wage ratio induces more abatement, $a_{i,k}$, by cost-minimizing firms.[12]

Tax Revenues. Production, carbon, and border tax revenues are rebated to consumers in a

[10] Consumption and abatement taxes are redundant as their effects can be perfectly mimicked with the appropriate choice of the existing instruments, t, x, s, and τ.

[11] An alternative way of representing this relationship is to replace subsidies with their tax equivalent: $\tilde{P}_{ji,k} = (1+t_{ji,k})(1+s^{\text{a}}_{i,k})(1+x^{\text{a}}_{ij,k})P_{ji,k}$. Since the policy tools related to production and exports are typically applied in the form of *subsidies*, we have replaced $(1+s^{\text{a}}_{i,k}) = 1/(1+s_{i,k})$ and $(1+x^{\text{a}}_{ij,k}) = 1/(1+x_{ij,k})$.

[12] Notice that $\{t_{ji,k}, x_{ji,k}, s_{i,k}\}$ are ad valorem wedges free of monetary units, whereas carbon taxes $\{\tau_{i,k}\}$ must be applied in units of say dollars, which explains why firms' response to them is based on the carbon tax-to-wage ratio.

lump-sum fashion. We use T_i to denote the total tax revenues rebated to consumers in country i:

$$T_i = \overbrace{\sum_{k\in\mathbb{K}}\sum_{j\in\mathbb{C}}\left(\alpha_{j,k}\frac{\gamma_k-1}{\gamma_k}P_{ij,k}Q_{ij,k}\right)}^{\text{carbon taxes}} + \overbrace{\sum_{k\in\mathbb{K}}\left[\left(\tilde{P}_{ii,k}-P_{ii,k}\right)Q_{ii,k}\right]}^{\text{production subsidies}} \\ + \underbrace{\sum_{k\in\mathbb{K}}\sum_{j\in\mathbb{C},j\neq i}\left[\left(\tilde{P}_{ji,k}-P_{ji,k}\right)Q_{ji,k}\right]}_{\text{imports taxes}} + \underbrace{\sum_{k\in\mathbb{K}}\sum_{j\in\mathbb{C},j\neq i}\left[\left(\tilde{P}_{ij,k}-P_{ij,k}\right)Q_{ij,k}\right]}_{\text{exports subsidies}}. \tag{6}$$

Note that T_i can be positive or negative depending on the makeup of country i's taxes and subsidies. If country i offers a sufficiently large amount of production and export subsides that exhaust revenues, then T_i operates as a negative lump-sum tax on the consumers.

2.4 General Equilibrium

To streamline the presentation of our optimal policy results, we use a slightly unorthodox presentation of general equilibrium. First, we characterize all equilibrium outcomes (aside from wages and income levels) as a function of taxes, nationals-level wages, and income levels. We refer to this outcome as a *semi-equilibrium*. Then, we define general equilibrium as a semi-equilibrium in which wages and and income levels satisfy the market clearing and balanced budget conditions, given the vector of taxes.

Definition. [Semi-equilibrium.] *For any combination* $\mathbb{X} = (\mathbb{I}; \boldsymbol{Y}, \boldsymbol{w})$ *of taxes* $\mathbb{I} = \{t_{ji,k}, x_{ij,k}, s_{i,k}, \tau_{i,k}\}_{i,j\neq i,k}$, *income levels,* $\boldsymbol{Y} = \{Y_i\}_i$, *and wages,* $\boldsymbol{w} = \{w_i\}_i$, *a semi-equilibrium, labeled as* $\mathscr{S}(\mathbb{X})$, *consists of:*

- *firm-level abatement,* $\{a_{j,k}(\mathbb{X})\}_{j,k}$, *and carbon input cost share,* $\{\alpha_{j,k}(\mathbb{X})\}_{j,k}$, *according to (1)–(2); aggregate producer prices,* $\{P_{ji,k}(\mathbb{X})\}_{j,i,k}$, *and* CO_2 *emissions,* $\{Z_{j,k}(\mathbb{X})\}_{j,k}$, *according to (3)–(4);*
- *aggregate demand quantities characterized by the non-parametric Marshallian demand function,* $Q_{ji,k}(\mathbb{X}) = \mathcal{D}_{ji,k}(Y_i, \tilde{\mathbf{P}}_i(\mathbb{X}))$, *for all* ji, k, *where after-tax prices* $\tilde{\mathbf{P}}_i(\mathbb{X}) \equiv \{\tilde{P}_{ji,k}(\mathbb{X})\}_{j,k}$ *follow (5);*
- *national-level tax revenues* $\{T_i(\mathbb{X})\}_i$ *that are described by (6).*

Definition. [General equilibrium.] *Given taxes* $\mathbb{I}$, *general equilibrium is a combination* $\mathbb{X} = (\mathbb{I}; \boldsymbol{Y}, \boldsymbol{w})$ *such that the system of semi-equilibrium* $\mathscr{S}(\mathbb{X})$ *holds, and income levels,* $\mathbf{Y}$, *and wages,* $\mathbf{w}$, *satisfy the market clearing conditions:*[13]

$$w_i\bar{L}_i - \sum_{k\in\mathbb{K}}\sum_{j\in\mathbb{C}}\left(1-\alpha_{i,k}(\mathbb{X})\frac{\gamma_k-1}{\gamma_k}\right)P_{ij,k}(\mathbb{X})Q_{ij,k}(\mathbb{X}) = 0, \qquad \textit{for all } i\in\mathbb{C} \tag{7}$$

[13] The labor market clearing condition (LMC) is equivalent to the balance trade condition (BTC), $\sum_{k\in\mathbb{K}}\sum_{j\neq i\in\mathbb{C}}\left(\frac{1}{1+t_{ji,k}}P_{ji,k}Q_{ji,k} - \frac{1}{1+t_{ij,k}}P_{ij,k}Q_{ij,k}\right) = 0$, where exports and imports of every country i are measured in values outside the border of i (that are, exports are after-tax, but imports are before-tax). In our policy analysis, we sometimes use (BTC) instead of (LMC).

as well as the balanced budget conditions:

$$Y_i = \sum_j \sum_k \tilde{P}_{ji,k}(\mathbb{X}) Q_{ji,k}(\mathbb{X}) = w_i \bar{L}_i + T_i(\mathbb{X}), \qquad \textit{for all } i \in \mathbb{C} \tag{8}$$

Hereafter, when we express $\mathbb{X} = (\mathbb{I}; \mathbf{Y}, \mathbf{w}) \in \mathcal{E}$, we mean to indicate that $\mathbb{X}$ is in the set of feasible general equilibrium outcomes—i.e., it satisfies Equations (1)–(8).

Expenditure Shares and Carbon Intensities. To compress the notation, we define the following variables. We use e and λ to denote expenditure shares. The within-industry expenditure share of country i on variety ji, k (origin j–destination i–industry k) is denoted by $\lambda_{ji,k}$,[14] and the overall expenditure share of country i on industry k is denoted by $e_{i,k}$:

$$\lambda_{ji,k} \equiv \frac{\tilde{P}_{ji,k} Q_{ji,k}}{\sum_{n \in \mathbb{C}} \tilde{P}_{ni,k} Q_{ni,k}}; \qquad e_{i,k} = \frac{\sum_{n \in \mathbb{C}} \tilde{P}_{ni,k} Q_{ni,k}}{\sum_{n \in \mathbb{C}} \sum_{g \in \mathbb{K}} \tilde{P}_{ni,g} Q_{ni,g}} = \frac{\sum_{n \in \mathbb{C}} \tilde{P}_{ni,k} Q_{ni,k}}{Y_i}. \tag{9}$$

Country i's overall share of expenditure on variety ji, k is denoted by $e_{ji,k}$,

$$e_{ji,k} \equiv \frac{\tilde{P}_{ji,k} Q_{ji,k}}{\sum_{n \in \mathbb{C}} \sum_{g \in \mathbb{K}} \tilde{P}_{ni,g} Q_{ni,g}} = \frac{\tilde{P}_{ji,k} Q_{ji,k}}{Y_i} = \lambda_{ji,k} e_{i,k}. \tag{10}$$

We use r to denote revenue shares. The share of variety ji, k from *origin j–industry k*'s total sales revenues is denoted by $r_{ji,k}$, and the share of industry k from origin j's total sales revenues is denoted by $r_{j,k}$:

$$r_{ji,k} \equiv \frac{P_{ji,k} Q_{ji,k}}{\sum_{n \in \mathbb{C}} P_{jn,k} Q_{jn,k}}; \qquad r_{j,k} = \frac{\sum_{n \in \mathbb{C}} P_{jn,k} Q_{jn,k}}{\sum_{n \in \mathbb{C}} \sum_{g \in \mathbb{K}} P_{jn,g} Q_{jn,g}}. \tag{11}$$

Lastly, $v_{j,k}$ denotes the carbon intensity per unit value of output in *origin j–industry k*. Namely,

$$v_{j,k} \equiv \frac{Z_{j,k}}{P_{jj,k} Q_{j,k}} = \frac{\gamma_k - 1}{\gamma_k} \frac{\alpha_{j,k}}{\tau_{j,k}} \sim \text{carbon intensity}, \tag{12}$$

where $\alpha_{j,k}$ denotes the input cost share of carbon as given by Equation (2).

[14] In our framework, abatement choices and scale economies alter comparative advantage. Under the assumption that industry-level bundle of consumption is a CES aggregator across varieties of different origin countries with σ_k as the elasticity of substitution between varieties in industry k, trade shares are:

$$\lambda_{ji,k} = \frac{\left(\bar{d}_{ji,k} \tilde{w}_{j,k}\right)^{1-\sigma_k}}{\sum_{j \in \mathbb{C}} \left(\bar{d}_{ji,k} \tilde{w}_{j,k}\right)^{1-\sigma_k}}, \qquad \text{where} \qquad \tilde{w}_{j,k} \equiv \bar{c}_{j,k} \times \left((1 - a_{j,k})^{\frac{1}{\varsigma \gamma_k} - \frac{1}{\varsigma}}\right) \times \left(Q_{j,k}^{-\frac{1}{\gamma_k}}\right) \times \left(w_j\right),$$

where $\bar{c}_{j,k}$ is a constant that is inversely proportional to $\bar{\varphi}_{j,k}$ —i.e., total factor productivity of in the supplying country-industry. Additionally, the potential to export, reflected by $\tilde{w}_{j,k}$, is proportional to wage, altered by the cost of abatement and scale economies. In a special case where production technology is constant returns to scale (i.e., $1/\gamma_k \to 0$), and labor and carbon are combined in a Cobb-Douglas fashion (i.e., $\varsigma \to 1$), $\tilde{w}_{j,k} = \bar{c}_{j,k}(1 - a_{j,k})^{-1} w_j$.

2.5 The Objective Function of Non-Cooperative Governments

To present a non-cooperative government's objective function, we first specify the perceived cost of CO_2 emissions. We assume that the cost associated with CO_2 emissions has two components:

1. A climate cost, $\phi_i \left(\sum_n \sum_k Z_{n,k}\right)$, where ϕ_i reflects the climate-related disutility per unit of CO_2 emissions—as perceived by country i's government.[15]

2. A pollution cost associated with the local pollutants (like oxides of nitrogen or sulfur) that accompany CO_2 emissions in the production process. We assume that production in i,k generates $\bar{\zeta}_{i,k}$ units of local pollutants for every unit of CO_2. Based on this assumption, the disutility from local pollutants can be expressed as $\phi_i^0 \left(\sum_k Z_{i,k}^0\right) = \phi_i^0 \left(\sum_k \bar{\zeta}_{i,k} Z_{i,k}\right)$, where $Z_{i,k}^0 = \bar{\zeta}_{i,k} Z_{i,k}$ denotes the bundle of local pollutants from origin i-industry k.

Let $\mathbf{Z} = [Z_{n,k}]_{n\in\mathbb{C},k\in\mathbb{K}}$ be the long vector of CO_2 emissions from all country-industry pairs and define $\phi_{ik} \equiv \phi_i^0 \bar{\zeta}_{i,k}$. We can write country i's *disutility from CO_2 emissions*, $\Delta_i(\mathbf{Z})$, as the sum of the explicit climate cost and the implicit pollution cost:

$$
\begin{aligned}
\Delta_i(\mathbf{Z}) &\equiv \phi_i \left(\textstyle\sum_n \sum_k Z_{n,k}\right) + \phi_i^0 \left(\textstyle\sum_k Z_{i,k}^0\right) \\
&= \textstyle\sum_n \sum_k \left(\delta_{ni,k} Z_{n,k}\right) \qquad\qquad \text{where} \quad \delta_{ni,k} = \phi_i + \phi_{i,k}\mathbf{1}(n=i).
\end{aligned} \tag{13}
$$

The government's objective is to maximize the utility from consumption net of the perceived disutility from CO_2 emissions. Let $\mathbb{I}_i$ stack the instruments of policy available to country i's government, $\mathbb{I}_i \equiv \{t_{ji,k}, x_{ij,k}, s_{i,k}, \tau_{i,k}\}_{j,k}$, and $\mathbb{I} \equiv \{\mathbb{I}_i\}_{i\in\mathbb{C}}$ be the set of worldwide tax instruments. The objective function of the government in country i can be expressed as follows for each combination $\mathbb{X} = (\mathbb{I}; \mathbf{w}, \mathbf{Y})$ of taxes, national wages, and income levels:

$$
W_i(\mathbb{X}) = V_i(w_i \bar{L}_i + T_i(\mathbb{X}), \tilde{\mathbf{P}}_i(\mathbb{X})) - \Delta_i(\mathbf{Z}(\mathbb{X})) \tag{14}
$$

The first term on the right-hand side represents the indirect utility from consumption, which depends on total income, $Y_i = w_i \bar{L}_i + T_i$, and consumer prices, $\tilde{\mathbf{P}}_i$. The second term is the disutility from CO_2 emissions, as described by Equation (13). With the government's objective function at hand, we now define the unilaterally optimal policy.

Definition. The *Unilaterally Optimal Policy* for country i consists of tax instruments, $\mathbb{I}_i^\star$, that maximize country i's welfare subject to general equilibrium conditions (1)–(8). Namely,

$$
\mathbb{I}_i^\star = \arg\max \;\; W_i(\mathbb{I}_i, \bar{\mathbb{I}}_{-i}; \mathbf{w}, \mathbf{Y}) \quad \text{subject to} \quad (\mathbb{I}_i, \bar{\mathbb{I}}_{-i}; \mathbf{w}, \mathbf{Y}) \in \mathcal{E},
$$

where W_i is described by Equation (14) and $\bar{\mathbb{I}}_{-i}$ denotes policy choices in the rest of the world.

[15] Since CO_2 emission is a *global bad*, the government does not care from where a unit of CO_2 is emitted. Moreover, ϕ_i reflects the disutility experienced by the residents of country i. Since the government in country i act non-cooperatively, it does not care about the climate cost experienced by individuals in the rest of the world.

To elucidate the above definition, Appendix A.1 presents an expanded version of the unilaterally optimal policy problem. Lastly, for the analysis that follows, it is instructive to define $\tilde{\phi}_i \equiv \tilde{P}_i \phi_i$, $\tilde{\phi}_{i,k} \equiv \tilde{P}_i \phi_{i,k}$, and $\tilde{\delta}_{ni,k} \equiv \tilde{P}_i \delta_{ni,k} = \tilde{\phi}_{i,k} \mathbf{1}(n = i) + \tilde{\phi}_i$, as the CPI-adjusted welfare cost per unit of CO_2 emissions, where $\tilde{P}_i \equiv (\partial V_i(.)/\partial Y_i)^{-1}$ is the consumer price index in country i.

3 Unilaterally Optimal Carbon, Production and Border Taxes

In this section, we characterize a non-cooperative government's unilaterally optimal policy that consists of carbon, border, and production taxes. As we will discuss shortly, these taxes are *free-riding-proof* but inefficient from a global standpoint. The unilaterally optimal policy corrects three margins of inefficiency from a non-cooperative government's viewpoint:

1. The local and transboundary carbon externality imposed on domestic consumers.

2. Misallocation caused by markup differences across industries.

3. Unexploited terms-of-trade gains in relation to the rest of the world.

Deriving analytical formulas for optimal policy is challenging when general equilibrium considerations are taken into account. Traditional characterizations of optimal border and carbon taxes circumvent such challenges by focusing on two-country or partial equilibrium setups. Also, typically, they impose other simplifying assumptions like perfect competition, fixed location of firms, fixed set of products, a constant share of carbon in production, and constant returns to scale.

As an intermediate step, we develop a new methodological approach that overcomes the challenges facing general equilibrium optimal policy analysis. Our goal is to present a simple and systematic way of characterizing optimal policy in quantitative trade models with applications beyond this particular work. We present our theoretical proposition via an *intermediate envelope* result, which forms the basis of our subsequent optimal policy formulas. Throughout the paper, if not reported in the main text, our derivations and proofs are presented in the appendix.

3.1 Intermediate Envelope Result

The intermediate envelope result presented in this section converts our general equilibrium optimal policy problem into a simpler problem characterized by a set of partial equilibrium derivatives that treat wage and income as given. We establish this result in three steps.

Step 1: Reformulate the optimal policy problem in terms of consumer prices and abatement

The government in i can choose consumer prices $\tilde{\mathbf{P}}_{ji} = \{\tilde{P}_{ji,k}\}_{j \neq i, k}$, $\tilde{\mathbf{P}}_{ij} = \{\tilde{P}_{ij,k}\}_{j \neq i, k}$, $\tilde{\mathbf{P}}_{ii} = \{\tilde{P}_{ii,k}\}_k$ to replicate any set of border and production tax/subsidies $\mathbf{t}_{ji} \equiv \{t_{ji,k}\}_{j \neq i, k}$, $\mathbf{x}_{i,j} \equiv \{x_{ij,k}\}_{j \neq i, k}$, $\mathbf{s}_{i,k} \equiv$

$\{s_{i,k}\}_k$ according to Equation (5), and can choose abatement levels $\{a_{i,k}\}_k$ to replicate any set of carbon taxes $\{\tau_{i,k}\}$ according to Equation (1). Shifting the focus from the vector of taxes $\mathbb{I}_i \equiv \{t_{ji,k}, x_{ij,k}, s_{i,k}, \tau_{i,k}\}_{j\neq i,k}$ to their target variables $\mathbb{P}_i \equiv \{\tilde{P}_{ji,k}, \tilde{P}_{ij,k}, \tilde{P}_{ii,k}, a_{i,k}\}_{j\neq i,k}$ is useful, as it emphasizes the distortion each tax instrument intends to correct. We invoke this observation to convert our original optimal policy problem into a *reformulated* problem in which the government chooses consumer prices and abatement levels, $\mathbb{P}_i$, rather than tax instruments, $\mathbb{I}_i$. The following lemma presents that basic logic behind this reformulation.

Lemma 1. *Given the optimal choice of prices and abatement levels, $\mathbb{P}_i^\star = \{\tilde{P}^\star_{ji,k}, \tilde{P}^\star_{ij,k}, \tilde{P}^\star_{ii,k}, a^\star_{i,k}\}_{j\neq i,k}$, optimal taxes/subsidies $\mathbb{I}_i^\star = \{t^\star_{ji,k}, x^\star_{ij,k}, s^\star_{i,k}, \tau^\star_{i,k}\}_{j,k}$ can be recovered according to:*

$$1+t^\star_{ji,k} = \frac{\tilde{P}^\star_{ji,k}}{P_{ji,k}}, \qquad 1+x^\star_{ij,k} = \frac{P_{ij,k}}{\tilde{P}^\star_{ij,k}}\frac{P_{ii,k}}{\tilde{P}^\star_{ii,k}}, \qquad 1+s^\star_{i,k} = \frac{P_{ii,k}}{\tilde{P}^\star_{ii,k}}, \qquad \tau^\star_{i,k} = \frac{\gamma_k-1}{\gamma_k}\frac{\alpha_{j,k}(a^\star_{j,k})}{v_{j,k}(a^\star_{j,k})}$$

Considering the above Lemma, we henceforth direct attention to a reformulated problem in which $\mathbb{P}_i$ is chosen to maximize welfare, W_i, subject to equilibrium constraints. Since the rest of the world is passive (e.g., $\mathbb{P}_{-i} = \mathbf{0}$) we herein drop $\mathbb{P}_{-i}$ from the global policy vector. Altogether, country i's reformulated optimal policy problem can be specified as:

$$\mathbb{P}_i^\star = \arg\max \;\; W_i(\mathbb{P}_i; \mathbf{w}, \mathbf{Y}) \quad \text{subject to} \quad (\mathbb{P}_i, \mathbf{w}, \mathbf{Y}) \in \mathcal{E}^r,$$

where $\mathcal{E}^r$ denotes the set of policy-wage-income combinations that constitute a feasible general equilibrium outcome. This set is defined analogous to $\mathcal{E}$ with details provided in Appendix A.2. The optimal policy, based on the above problem, must satisfy the following first-order condition (F.O.C.) with respect to each policy instrument $\mathrm{P} \in \mathbb{P}_i$:

$$\frac{\mathrm{d}W_i(\mathbb{P}_i^\star; \mathbf{w}^\star, \mathbf{Y}^\star)}{\mathrm{d}\ln \mathrm{P}} = \frac{\partial W_i(\mathbb{P}_i^\star; \mathbf{w}^\star, \mathbf{Y}^\star)}{\partial \ln \mathrm{P}} + \underbrace{\frac{\partial W_i(\mathbb{P}_i^\star; \mathbf{w}^\star, \mathbf{Y}^\star)}{\partial \mathbf{w}} \cdot \frac{\mathrm{d}\mathbf{w}}{\mathrm{d}\ln \mathrm{P}}}_{\text{GE wage effects}} + \underbrace{\frac{\partial W_i(\mathbb{P}_i^\star; \mathbf{w}^\star, \mathbf{Y}^\star)}{\partial \mathbf{Y}} \cdot \frac{\mathrm{d}\mathbf{Y}}{\mathrm{d}\ln \mathrm{P}}}_{\text{GE income effects}} = 0. \tag{15}$$

The first term on the right-hand side corresponds to the direct effect of $\mathrm{P} \in \mathbb{P}_i$ on welfare, holding income and wages fixed. The second two terms capture general equilibrium (GE) effects that channel through changes in wage and income levels.[16] We appeal to the above decomposition of F.O.C.s to establish the welfare neutrality of wage and income effects at the optimum.

Step 2: Welfare-neutrality of wage effects at the optimum

Wage effects correspond to the welfare impacts of policy that channel through general equilibrium changes in wages $\mathbf{w} \equiv \{w_i\}_{i\in\mathbb{C}}$. We show that these general equilibrium wage effects are welfare-neutral at the optimum. That is, fixing $\mathbb{P}_i$ and $\mathbf{Y}$ at their unilaterally optimal level, a change in $\mathbf{w}$

[16] As detailed in Appendix A.4, $\mathrm{d}\mathbf{w}/\mathrm{d}\ln \mathrm{P}$ and $\mathrm{d}\mathbf{Y}/\mathrm{d}\ln \mathrm{P}$ can be obtained by applying the Implicit Function Theorem to the national-level labor market clearing and balanced budget conditionsEquations (7) and (8).

has no first-order effect on country i's welfare.

Lemma 2. *General equilibrium wage effects are welfare neutral at the optimum,* $\mathbb{P}_i^\star$*,*

$$\frac{\partial W_i(\mathbb{P}_i^\star; \mathbf{w}^\star, \mathbf{Y}^\star)}{\partial \mathbf{w}} \cdot \frac{\mathrm{d}\mathbf{w}}{\mathrm{d}\ln \mathsf{P}} = 0, \qquad \forall \mathsf{P} \in \mathbb{P}_i.$$

The above lemma is formally proven in Appendix B.1. Below, we provide a verbal summary. The global vector of wages is composed of the local wage w_i and foreign wages $\mathbf{w}_{-i} \equiv \{w_n\}_{n \neq i}$, each of which is welfare-neutral for different reasons. The welfare neutrality of w_i follows from the fact that the government in i can tax every unit of output produced in country i irrespective of industry and location of final sale. Since the choice vis-à-vis $\mathbb{P}_i$ fully determines consumer prices in country i, w_i can affect country i's welfare only via its impact on national income, $w_i L_i + T_i(\mathbb{P}_i; \mathbf{w}, \mathbf{Y})$.[17] Holding $\mathbb{P}_i$ fixed, the effect of w_i on national income consists of two countervailing elements: a positive effect on wage income and a negative effect on tax revenues. For any policy choice $\mathbb{P}_i$—and insofar as the labor market clearing condition holds—these two countervailing effects exactly offset each other.

The welfare neutrality of $\mathbf{w}_{-i}$ in the *two-country* case follows trivially from Walras' law—since one element of $\mathbf{w}_{-i}$ can be always normalized to one. Beyond the two-country case, the neutrality result is more subtle and derives from a combination of tax neutrality (in the spirit of the Lerner symmetry) and the *targeting principle*—see Appendix B.1 for details. Consider, for instance, $w_j \in \mathbf{w}_{-i}$. By the Lerner symmetry, any change in w_j can be perfectly mimicked with a uniform shift in country j's import taxes and export subsidies. Setting $\mathbb{P}_i$ to its unilaterally optimal value, these extraterritorial uniform tax changes have no first-order effect on $W_i(\mathbb{P}_i; \mathbf{w}, \mathbf{Y})$. In other words, in accordance with the targeting principle, any first-order gains from such extraterritorial uniform tax changes will be already internalized by the optimum choice, $\mathbb{P}_i^\star$.

Step 3: Welfare-neutrality of income effects at the optimum

Income effects corresponds to welfare impacts of policy that channel through general equilibrium changes in national income, $\mathbf{Y} \equiv \{Y_i\}_{i \in \mathbb{C}}$. Following Lemma 2, we can evaluate and solve the F.O.C. (15) while treating wages, $\mathbf{w} = \bar{\mathbf{w}}$, as fixed. In addition, since foreign countries $j \neq i$ are passive, their income is tied to their wage rates: $\bar{\mathbf{Y}}_{-i} = \{\bar{w}_j \bar{L}_j\}_{j \neq i}$, and can be also treated as fixed. As such, we henceforth drop $\bar{\mathbf{w}}$ and $\bar{\mathbf{Y}}_{-i}$ from our notation, and express the combination $(\mathbb{P}_i; \mathbf{w}, \mathbf{Y})$ as simply $(\mathbb{P}_i; Y_i)$. Accordingly, the F.O.C. with respect to $\mathsf{P} \in \mathbb{P}_i$ reduces from Equation (15) to

$$\frac{\mathrm{d}W_i(\mathbb{P}_i; Y_i)}{\mathrm{d}\ln \mathsf{P}} = \frac{\partial W_i(\mathbb{P}_i; Y_i)}{\partial \ln \mathsf{P}} + \frac{\partial W_i(\mathbb{P}_i; Y_i)}{\partial Y_i} \frac{\mathrm{d}Y_i}{\mathrm{d}\ln \mathsf{P}} = 0. \tag{16}$$

[17] This claim rests on the implicit assumption that $\mathbf{w}_{-i}$ is held constant, as $\mathbf{w}_{-i}$ is itself welfare-neutral at the optimum.

The way the above problem is set up, income effects channel *exclusively* through income-driven changes in demand quantities.[18] That is, after fixing $\mathbb{P}_i$, any general equilibrium change in Y_i affects producer prices, emissions, and tax revenues only through shifts in demand schedules. We can highlight this point by expressing these variables as $P_{ji,k}(\mathbb{P}_i;Y_i) = P_{ji,k}(\mathbb{P}_i, \mathbf{Q}_i(\mathbb{P}_i, Y_i))$, $Z_{j,k}(\mathbb{P}_i;Y_i) = Z_{j,k}(\mathbb{P}_i, \mathbf{Q}_i(\mathbb{P}_i, Y_i))$, and $T_i(\mathbb{P}_i;Y_i) = T_i(\mathbb{P}_i, \mathbf{Q}_i(\mathbb{P}_i, Y_i))$, where $\mathbf{Q}_i \equiv \{Q_{ji,k}, Q_{ij,k}\}_{j\in\mathbb{C}, k\in\mathbb{K}}$ denotes the entire vector of country i's output and consumption quantities, which is fully determined by country i's policy choice, $\mathbb{P}_i$, and income Y_i.[19] With this slight change in notation, country i's objective function can be expressed as:

$$W_i(\mathbb{P}_i;Y_i) \sim W_i(\mathbb{P}_i, \mathbf{Q}_i(\mathbb{P}_i;Y_i)) = V_i(\underbrace{\bar{w}_i\bar{L}_i + T_i(\mathbb{P}_i, \mathbf{Q}_i(\mathbb{P}_i;Y_i))}_{Y_i}, \tilde{\mathbf{P}}_i) - \Delta_i\left(\mathbf{Z}(\mathbb{P}_i, \mathbf{Q}_i(\mathbb{P}_i;Y_i))\right). \quad (17)$$

We can, accordingly, elucidate income effects by rewriting the F.O.C. (16) in the following form

$$\frac{\mathrm{d}W_i(\mathbb{P}_i, \mathbf{Q}_i)}{\mathrm{d}\ln \mathsf{P}} = \frac{\partial W_i(\mathbb{P}_i, \mathbf{Q}_i)}{\partial \ln \mathsf{P}} + \frac{\partial W_i(\mathbb{P}_i, \mathbf{Q}_i)}{\partial \ln \mathbf{Q}_i} \cdot \frac{\partial \ln \mathbf{Q}_i}{\partial \ln \mathsf{P}} + \underbrace{\frac{\partial W_i(\mathbb{P}_i, \mathbf{Q}_i)}{\partial \mathbf{Q}_i} \cdot \frac{\partial \mathbf{Q}_i(\mathbb{P}_i;Y_i)}{\partial Y_i} \frac{\mathrm{d}Y_i}{\mathrm{d}\ln \mathsf{P}}}_{\text{GE income effects}} = 0. \quad (18)$$

Using Equation (17) and noting that $\tilde{P}_i \equiv (\partial V_i(.)/\partial Y_i)^{-1}$, the revised F.O.C. can be unpacked as

$$\overbrace{\left[\frac{\partial T_i(\mathbb{P}_i, \mathbf{Q}_i)}{\partial \ln \mathsf{P}} + \tilde{P}_i \frac{\partial V_i(Y_i, \tilde{\mathbf{P}}_i)}{\partial \ln \mathsf{P}} - \tilde{P}_i \frac{\partial \Delta_i\left(\mathbf{Z}(\mathbb{P}_i, \mathbf{Q}_i)\right)}{\partial \ln \mathsf{P}}\right]}^{\frac{\partial W_i(\mathbb{P}_i, \mathbf{Q}_i)}{\partial \ln \mathsf{P}}} + \underbrace{\left[\frac{\partial T_i(\mathbb{P}_i, \mathbf{Q}_i)}{\partial \mathbf{Q}_i} - \tilde{P}_i \frac{\partial \Delta_i\left(\mathbf{Z}(\mathbb{P}_i, \mathbf{Q}_i)\right)}{\partial \mathbf{Q}_i}\right]}_{\frac{\partial W_i(\mathbb{P}_i, \mathbf{Q}_i)}{\partial \mathbf{Q}_i}} \cdot \frac{\partial \mathbf{Q}_i(\mathbb{P}_i, Y_i)}{\partial \ln \mathsf{P}}$$
$$+ \overbrace{\left[\frac{\partial T_i(\mathbb{P}_i, \mathbf{Q}_i)}{\partial \mathbf{Q}_i} - \tilde{P}_i \frac{\partial \Delta_i\left(\mathbf{Z}(\mathbb{P}_i, \mathbf{Q}_i)\right)}{\partial \mathbf{Q}_i}\right]} \cdot \frac{\partial \mathbf{Q}_i(\mathbb{P}_i, Y_i)}{\partial Y_i} \frac{\mathrm{d}Y_i}{\mathrm{d}\ln \mathsf{P}} = 0. \quad (19)$$

The first line expands the welfare effect of policy, $\mathsf{P} \in \mathbb{P}_i$, holding income fixed and the second line expands general equilibrium income effect, as specified under Equation (18). We prove that income effects are welfare-neutral at the optimum by establishing that $\frac{\partial W_i(\mathbb{P}_i^\star, \mathbf{Q}_i^\star)}{\partial \mathbf{Q}_i} = \mathbf{0}$. To this end, consider the elements of $\mathbb{P}_i$ that correspond to a consumer price variable in country i—i.e., any $\mathsf{P} \in \tilde{\mathbf{P}}_i$, where $\tilde{\mathbf{P}}_i \subset \mathbb{P}_i$ is the vector of prices facing a consumers in country i. For this subset of policy instruments, the first component of Equation (19) collapses to zero:

$$\mathsf{P} \in \tilde{\mathbf{P}}_i \implies \frac{\partial W_i(\mathbb{P}_i, \mathbf{Q}_i)}{\partial \ln \mathsf{P}} = 0. \quad (20)$$

[18] To be clear, we are focusing on policy-income combinations that constitute a feasible general equilibrium outcomes, i.e., $(\mathbb{P}_i;Y_i) \in \mathcal{E}^r$. See Appendix A.3 for a formal definition of general equilibrium in terms of $(\mathbb{P}_i;Y_i)$.

[19] Specifically, equilibrium values of $\mathbf{Q}_i$ are given by demand schedules in country i, $Q_{ji,k} = \mathcal{D}_{jik}(Y_i, \{\tilde{\mathbf{P}}_{ji}, \tilde{\mathbf{P}}_{ii}\}_{j\neq i})$, for the case where consumers in i purchase domestic or imported varieties, and demand schedules of foreign countries $j \neq i$, $Q_{ij,k} = \mathcal{D}_{ijk}(\bar{Y}_j = \bar{w}_j\bar{L}_j, \{\tilde{\mathbf{P}}_{ij}, \tilde{\mathbf{P}}_{-i,j}(\bar{\mathbf{w}}_{-i})\}_{i\neq j})$, for the case where producers in i sell abroad..

This result follows from Roy's identity and indicates that raising domestic consumer prices (for a fixed demand schedule, $\mathbf{Q}_i$) has no net effect on country i's welfare—the revenue gains from this change are nullified by the loss to consumer surplus.[20] Substituting Equation (20) into the F.O.C. (Equation 18), one immediately notices that a necessary condition for optimality is that $\frac{\partial W_i(\mathbb{P}_i^\star, \mathbf{Q}_i^\star)}{\partial \mathbf{Q}_i} = 0$. Hence, general equilibrium income effect (as specified in Equation 18) must equal zero at the optimum. It should be emphasized that this result holds only if all elements of $\tilde{\mathbf{P}}_i^\star = \left\{\tilde{\mathbf{P}}_{ji}^\star, \tilde{\mathbf{P}}_{ii}^\star\right\}_{j\neq i} \subset \mathbb{P}_i$ are set optimally. The following lemma summarizes this result.

Lemma 3. *If $\tilde{\boldsymbol{P}}_i \subset \mathbb{P}_i$ is chosen optimally, then general equilibrium income effects are welfare-neutral:*

$$\frac{\partial W_i(\mathbb{P}_i^\star; \boldsymbol{w}^\star, \boldsymbol{Y}^\star)}{\partial \boldsymbol{Y}} \sim \frac{\partial W_i(\mathbb{P}_i^\star, \boldsymbol{Q}_i(\mathbb{P}_i^\star; Y_i^\star))}{\partial Y_i} = 0.$$

Putting the Three Steps Together. We now combine Lemmas 1, 2, and 3 to establish an intermediate envelope result that simplifies our general equilibrium optimal policy problem.

Theorem 1. [***Intermediate Envelope Result***] *The system of F.O.C.s that determines country i's unilaterally optimal policy, $\mathbb{P}_i^\star$, can be derived and solved by treating wages as invariant to policy and demand schedules as invariant to changes in income.*

We refer to Theorem 1 as an "envelope" result because it demonstrates that, when all tax instruments are set optimally, changes in wage and consumer income have no first-order effect on welfare. Theorem 1 has implications well beyond this particular paper. It paves the way for future analyses of optimal policy in quantitative general equilibrium trade models—a task that has proven prohibitively challenging in the past. Indeed, traditional theories of optimal trade policy are often disconnected from modern quantitative trade models in two assumptions. First, traditional models assume that wages are fixed due to the presence of a sufficiently large and traded homogeneous sector. Second, they assume that preferences are quasi-linear which suppresses general equilibrium income effects. Theorem 1 indicates that neither assumption is necessary to achieve tractability, provided that the government is granted sufficient instruments in its policy set.In other words, our general equilibrium optimal policy problem can be solved as if it were a partial equilibrium problem.

[20] Specifically, Roy's identity implies that $\tilde{P}_i \frac{\partial V_i(\tilde{\mathbf{P}}_i, Y_i)}{\partial \ln \tilde{P}_{ji,k}} = -\tilde{P}_{ji,k} Q_{ji,k}$. Moreover, since $\tilde{P}_{ji,k} \in \mathbb{P}_i$, it follows immediately that $\frac{\partial \Delta_i(\mathbf{Z}(\mathbb{P}_i, \mathbf{Q}_i))}{\partial \tilde{P}_{ji,k}} = 0$ and $\frac{\partial T_i(\mathbb{P}_i, \mathbf{Q}_i)}{\partial \ln \tilde{P}_{ji,k}} = \tilde{P}_{ji,k} Q_{ji,k}$. The first equality holds because emission is fully determined by abatement levels and quantities. The second reflects the effect of an after-tax price increase on tax revenues holding the demand schedule fixed. Together, for $\mathsf{P} \in \tilde{\mathbf{P}}_i$,

$$\frac{\partial W_i(\mathbb{P}_i, \mathbf{Q}_i)}{\partial \ln \mathsf{P}} = \tilde{P}_i \frac{\partial V_i(\tilde{\mathbf{P}}_i, Y_i)}{\partial \ln \mathsf{P}} + \frac{\partial T_i(\mathbb{P}_i, \mathbf{Q}_i)}{\partial \ln \mathsf{P}} - \tilde{P}_i \frac{\partial \Delta_i\left(\mathbf{Z}(\mathbb{P}_i, \mathbf{Q}_i)\right)}{\partial \ln \mathsf{P}} = -\tilde{P}_{ji,k} Q_{ji,k} + \tilde{P}_{ji,k} Q_{ji,k} + 0 = 0.$$

3.2 Characterizing the Unilaterally Optimal Tax Schedule

Theorem 1 allows us to derive simple expressions for the F.O.C.s that determine the optimal policy in our general equilibrium model. Since we assume a non-parametric demand function, we present the resulting system of F.O.C.s using the own- and cross-price elasticities of demand defined in Section 2.1. The following lemma summarizes this step, with detailed derivations provided in Appendix B.2.

Lemma 4. *Country i's optimal policy, $\mathbb{P}_i^\star$, consisting of domestic & import prices $\{\tilde{P}_{ni,k}\}_{n,k}$, export prices $\{\tilde{P}_{ij,k}\}_{j\neq i,k}$, and abatement levels $\{a_{i,k}\}_k$, solves the following system of F.O.C.s:*

$$\begin{array}{ll}
\{\tilde{P}_{ni,k}\}_{n,k} & \sum_g \left[\left(\frac{\tilde{P}^\star_{ii,g}}{P_{ii,g}} - \frac{1}{\mu_g}\left(1 - \frac{\alpha_{i,g}}{\mu_g} + \tilde{\delta}_{ii,g} v_{i,g}\right)\right) \tilde{\varepsilon}^{(ni,k)}_{ii,g} + \sum_{m\neq i} \left(\frac{\tilde{P}^\star_{mi,g}}{P_{mi,g}} - \left(1 + \omega_{mi,g} + \frac{\tilde{\delta}_{mi,g} v_{m,g}}{\mu_g}\right)\right) \tilde{\varepsilon}^{(ni,k)}_{mi,g} \right] = 0; \\
\{\tilde{P}_{ij,k}\}_{j\neq i,k} & e_{ij,k} + \sum_g \left[\left(1 - \frac{1}{\mu_g}\left(1 - \frac{\alpha_{i,g}}{\mu_g} + \tilde{\delta}_{ii,g} v_{i,g}\right) \frac{P^\star_{ij,g}}{\tilde{P}_{ij,g}}\right) \tilde{\varepsilon}^{(ij,k)}_{ij,g} - \sum_{m\neq i} \left(\omega_{mi,g} + \frac{\tilde{\delta}_{mi,g} v_{m,g}}{\mu_g}\right) \tilde{\varepsilon}^{(ij,k)}_{mj,g} \right] = 0; \\
\{a_{i,k}\}_k & \tilde{\delta}_{ii,k} v_{i,k}(a^\star_{i,k}) - \frac{\alpha_{i,k}}{\mu_k} = 0;
\end{array}$$

where $\mu_k \equiv \frac{\gamma_k}{\gamma_k - 1}$ denotes the industry-specific markup, $\tilde{\varepsilon}^{(ni,g)}_{ji,k} \equiv e_{ji,k}\varepsilon^{(ni,g)}_{ji,k}$ is the expenditure-adjusted demand elasticity, and $\omega_{ji,k}$ denotes the inverse of good ji,k's general equilibrium supply elasticity.[21]

The optimality condition w.r.t. $a_{i,k}$ equalizes the marginal disutility from raising the cost of production with the marginal utility from lowering CO_2 emissions. Combining this condition with Equation (12)—that relates the equilibrium carbon intensity to the carbon tax—yields the following formula for the optimal carbon tax:

$$\tau^\star_{i,k} = \tilde{\delta}_{ii,k} = \tilde{\phi}_i + \tilde{\phi}_{i,k}, \tag{21}$$

where $\tilde{\phi}_i \equiv \tilde{P}_i \phi_i$, $\tilde{\phi}_{i,k} \equiv \tilde{P}_i \phi_{i,k}$, and $\tilde{\delta}_{ii,k} \equiv \tilde{P}_i \delta_{ii,k}$ denote the CPI-adjusted disutility from CO_2 emissions. The uniform term $\tilde{\phi}_i$ corrects the climate externality associated with CO_2 emissions, while the industry-specific term, $\tilde{\phi}_{i,k}$, targets the local pollution that accompanies CO_2 emissions. The unilaterally optimal carbon tax, specified above, solely corrects economy i's carbon externality on domestic consumers. It is inefficient from a global standpoint as it fails to internalize economy i's carbon externality on consumers in the rest of the world.

The F.O.C.s in lemma 4 with respect to $\{\tilde{P}_{ni,k}\}_{n,k}$ and $\{\tilde{P}_{ij,k}\}_{j\neq i,k}$ constitute a system of $2K(N -$

[21] As explained in Appendix A.6, $\omega_{ji,k}$ summarizes how a contraction in good ji,k's export supply affects the entire vector of producer prices associated with country i's economy. The same appendix provides an exact characterization of $\omega_{ji,k}$ as well as a first-order approximation,

$$\omega_{ji,k} \approx \frac{-\frac{1}{\gamma_k} r_{ji,k}}{1 - \frac{1}{\gamma_k}\sum_{\iota\neq i} r_{j\iota,k}\varepsilon_{j\iota,k}} \left[1 - \frac{1}{\gamma_k}\frac{w_i L_i}{w_j L_j}\sum_{n\neq i}\frac{\rho_{i,k} r_{in,k}}{\rho_{j,k} r_{jn,k}}\varepsilon^{(jn,k)}_{in,k}\right],$$

which depends on the industry-level degree pod scale economies, γ_k, revenues shares, $\left\{r_{ji,k}\right\}$, and $\{\rho_{i,k}\}$, and reduced-form demand elasticities, $\left\{\varepsilon^{(jn,k)}_{in,k}\right\}$.

$1) + K$ independent equations and unknowns. In Appendix B.3, we show that this system has a unique solution, which determines the optimal vector of price wedges, $\left\{\frac{\tilde{P}^{\star}_{ii,k}}{P_{ii,k}}, \frac{\tilde{P}^{\star}_{ji,k}}{P_{ji,k}}, \frac{P^{\star}_{ij,k}}{\tilde{P}_{ij,k}}\right\}_{j\neq i,k}$. Based on Lemma 1, we can convert these optimal price wedges into optimal tax and subsidy rates, as presented under the following theorem.[22]

Theorem 2. *The unilaterally optimal tax schedule for country i is given by*

$$\begin{aligned}
&[\textit{import tax}] && 1+t^{\star}_{ji,k} = 1+\omega_{ji,k}+\tilde{\phi}_i v_{j,k}\frac{\gamma_k-1}{\gamma_k} && \forall j,k \\
&[\textit{export subsidy}] && 1+x^{\star}_{ij,k} = \left(1+\frac{1}{\varepsilon_{ij,k}}\right)\chi_{ij,k} && \forall j,k \\
&[\textit{domestic subsidy}] && 1+s^{\star}_{i,k} = \frac{\gamma_k}{\gamma_k-1} && \forall k \\
&[\textit{carbon tax}] && \tau^{\star}_{i,k} = \tilde{\phi}_i+\tilde{\phi}_{i,k} && \forall k
\end{aligned} \tag{22}$$

where $\chi_{ij} = \left[\frac{e_{ij,g}\varepsilon^{(ij,k)}_{ij,g}}{e_{ij,k}\varepsilon_{ij,k}}\right]^{-1}_{k,g}\left(\boldsymbol{I}_K+\left[\frac{\sum_{n\neq i} t^{\star}_{ni,g}e_{nj,g}\varepsilon^{(ij,k)}_{nj,g}}{\sum_{\hat{n}\neq i,\hat{g}} e_{\hat{n}j,\hat{g}}\varepsilon^{(ij,k)}_{\hat{n}j,\hat{g}}}\right]_{k,g}\right)\mathbf{1}_K$ *is an export subsidy intended at lowering CO_2 emissions in foreign industries whose output competes with ij,k.*

To put it verbally, country i's unilaterally optimal policy consists of (i) a uniform Pigouvian carbon tax , $\tilde{\phi}_i$, adjusted for the local pollution externality that accompanies CO_2 emissions, $\tilde{\phi}_{i,k}$, (ii) industry-specific Pigouvian production subsidies, $s^{\star}_{i,k}$, that eliminate cross-industry markup heterogeneity, (iii) import taxes, $t^{\star}_{ji,k}$ that penalize high-carbon imports and capitalize on unexploited import market power in relation to the rest of world, and (iv) export subsidies, $x^{\star}_{ij,k}$, that promote low-carbon exports and capitalize on unexploited export market power in relation to the rest of the world. The fact that production subsides are *carbon-blind* is a manifestation of the *targeting principle*. Carbon taxes, based on this principle, are the optimal instrument to curb the carbon externality associated with domestic production, because they target the externality at its source.[23]

Optimal border (i.e., import and export) taxes are designed to both improve the terms-of-trade (ToT) and curb transboundary CO_2 emissions. Below, we clarify this point by dissecting the optimal import and export taxes/subsidies into (1) a terms-of-trade (ToT) driven component, and (2) a carbon border tax/subsidy component. First, consider the optimal import tax on variety ji,k. Following Theorem 2, this tax can be decomposed as

$$1+t^{\star}_{ji,k} = \underbrace{1+\omega_{ji,k}}_{\text{ToT driven}} + \underbrace{\tilde{\phi}_i v_{j,k}\frac{\gamma_k-1}{\gamma_k}}_{\text{carbon border tax}}. \tag{23}$$

[22] In Appendix C, we also characterize optimal policy under second-best scenarios in which the government is unable to use a subset of policy instruments in $\mathbb{P}_i$. We look at three second-best scenarios: First, a scenario where carbon taxes are inapplicable. Second, a scenario where export subsidies are inapplicable. Third, a scenario where the government can use carbon taxes but is unable to use any other tax instrument in $\mathbb{P}_i$.

[23] Recall that that we assume a one-to-one correspondence between local pollutants and CO_2 emissions. As a result, carbon taxes directly correct both the climate externality from CO_2 emissions and the local pollution externality from non-CO_2 emissions.

The ToT-driven component is motivated by country i's collective import market power vis-á-vis partner j. This tax component creates a mark-down on the producer price of goods imported from origin j–industry k, the optimal rate of which coincides with the inverse of good ji,k's *export supply elasticity*, $\omega_{ji,k}$. The CO_2 border tax curbs the transboundary CO_2 externality associated with goods produced by origin j–industry k. These border tax measures are rationalized by the fact that origin j's non-cooperative carbon tax is sub-optimal—it fails to internalize origin j's carbon externality on the home country $i \neq j$.

Likewise, optimal export subsides are chosen to both improve the terms-of-trade (ToT) and curb transboundary CO_2 emissions. The export subsidy on good ij,k can be decomposed as

$$1 + x^*_{ij,k} = \underbrace{\left(1 + \frac{1}{\varepsilon_{ij,k}}\right)}_{\text{ToT driven}} \times \underbrace{\chi_{ij,k}}_{\text{carbon border subsidy}} . \tag{24}$$

The ToT-driven term is analogous to an optimal markup, as if country i were pricing its composite export good as a single representative monopolist. The CO_2 border subsidy promotes export varieties that compete with high-carbon (high-v) foreign varieties in market j. The logic is that subsidizing exports to market j lowers the output and CO_2 emissions of high-carbon foreign industries by shrinking their sales to market j.[24]

3.3 Optimal Policy Formulas in Special Cases

Special Case: *Ricardian Model*s. In the limit where $\gamma_k \to \infty$ and $\bar{f}_{j,k} \to 0$, firms can be viewed as perfectly competitive and our framework reduces to a Ricardian trade model. This special case is isomorphic to the multi-industry Eaton and Kortum (2002) model. The optimal tax formulas in the Ricardian case can be attained by plugging the following values into Theorem 2:

$$\frac{\gamma_k}{\gamma_k - 1} \to 1; \quad \omega_{ji,k} \to 0. \qquad \text{(Ricardian Model)}$$

An immediate implication is that optimal import taxes are uniform *net* of the carbon border tax. Also, note that—in principle—Theorem 1 applies to a model with a continuum of industries. As a result, in the limit where $\varepsilon_{ij,k} \to \infty$, our optimal tax formulas nest the optimal policy formulas in Costinot et al. (2015), which is based on the Dornbusch et al. (1977) model.

Special Case: *Cobb-Douglas-CES Preferences*. To gain further intuition about the optimal policy formulas presented under Theorem 2, consider a parametric case of the model where preferences have a Cobb-Douglas-CES formulation. In particular,

[24] To be clear, $\chi_{ij,k}$ internalizes a second effect in addition to carbon reduction. Subsidizing exports to market j lowers the scale of production in foreign industries that sell to both markets j and i. This reduction in scale, indirectly impacts the producer price of goods imported by country i, and thus country i's terms-of-trade. $\chi_{ij,k}$ encompasses an adjustment to account for such effects.

$$U_i(\boldsymbol{Q}_i) = \prod_k \left(\sum_j b_{ji,k}^{1/\sigma_k} Q_{ji,k}^{\frac{\sigma_k - 1}{\sigma_k}} \right)^{\bar{e}_{i,k} \frac{\sigma_k}{\sigma_k - 1}}, \tag{25}$$

where $\bar{e}_{i,k}$ denotes country i's constant share of expenditure share on industry k, and σ_k denotes the Armington elasticity of substitution between origin countries. The price elasticities of demand in this special case are given by the following:

$$\varepsilon_{ji,k} \equiv \varepsilon_{ji,k}^{(ji,k)} = -1 - (\sigma_k - 1)(1 - \lambda_{ji,k}), \qquad \varepsilon_{ni,k}^{(ji,k)} = (\sigma_k - 1)\lambda_{ji,k} \ \ (n \neq j); \qquad \varepsilon_{ni,g}^{(ji,k)} = 0 \ \ (g \neq k).$$

Plugging the above values into Theorem 2 yields the following formula for optimal import tax and export subsidies in the Cobb-Douglas-CES case,

$$\begin{aligned} 1 + t_{ji,k}^{\star} &= 1 + \tilde{\phi}_i v_{j,k} \frac{\gamma_k - 1}{\gamma_k} + \frac{-\frac{1}{\gamma_k} r_{ji,k} \left[1 - \frac{\sigma_k - 1}{\gamma_k} \frac{w_i \bar{L}_i}{w_j^{\star} \bar{L}_j} \sum_{\ell \neq i} \frac{\rho_{i,k} r_{i\ell,k}}{\rho_{j,k} r_{j\ell,k}} \lambda_{j\ell,k} \right]}{1 + \frac{1}{\gamma_k} \sum_{\ell \neq i} r_{j\ell,k} \left[1 + (\sigma_k - 1)(1 - \lambda_{j\ell,k}) \right]} \\ 1 + x_{ij,k}^{\star} &= \left[1 + \sum_{n \neq i} t_{ni,k} \frac{\lambda_{nj,k}}{1 - \lambda_{ij,k}} \right] \left(1 + \frac{1}{(\sigma_k - 1)\left(1 - \lambda_{ij,k}\right)} \right)^{-1}. \end{aligned} \tag{26}$$

The last term in the first line corresponds to $\omega_{ji,k}$. The term inside the brackets in the second line corresponds to $\chi_{ij,k}$ in Theorem 2. Absent carbon externalities (i.e., set $v_{n,k} = 0$ for all n, k), the above formulas collapse to the familiar optimal import/export tax formulas in multi-industry quantitative trade models (see Lashkaripour and Lugovskyy (2020)). An illuminating special case of the above formula is the small open economy case, for which $r_{ji,k} \approx \lambda_{ij,k} \approx 0$. For a ***small open economy***, the optimal import tax and export subsidy formulas reduce to

$$\begin{aligned} 1 + t_{ji,k}^{\star} &= 1 + \tilde{\phi}_i v_{j,k} \frac{\gamma_k - 1}{\gamma_k} \\ 1 + x_{ij,k}^{\star} &= \left[1 + \tilde{\phi}_i \frac{\gamma_k - 1}{\gamma_k} \mathcal{V}_{-ij,k} \right] \frac{\sigma_k - 1}{\sigma_k}, \end{aligned}$$

where $\mathcal{V}_{-ij,k} \equiv \sum_{n \neq i} \left[v_{n,k} \frac{\lambda_{nj,k}}{1 - \lambda_{ij,k}} \right]$ is the average carbon intensity of foreign suppliers whose output competes with good ij, k. This formula elucidates our earlier claim that optimal carbon border subsidies promote exports to markets that are served heavily by high-carbon foreign suppliers.

3.4 Non-Cooperative Nash Equilibrium

Equipped with Theorem 2, we analyze a non-cooperative Nash equilibrium in which all countries simultaneously apply their unilaterally optimal policy. Such an equilibrium is inefficient from a global standpoint because all countries act non-cooperatively, and as such, they fail to internalize their terms-of-trade and carbon externality on the rest of the world. To fix ideas, we present a formal definition of the non-cooperative Nash equilibrium.

Definition. The *Non-Cooperative Nash Equilibrium* corresponds to a one-shot game where non-cooperative countries simultaneously choose their unilaterally optimal policy, taking policy choices in other countries as given.

Since we consider a game in which non-cooperative governments choose their taxes unilaterally and simultaneously, the optimal policy of each country is still characterized by Theorem 1. One should, however, note that optimal policy choices are interdependent between countries. More specifically, the optimal policy formulas under Theorem 1 depend on endogenous variables such as trade shares, $\lambda_{nj,k}$, and carbon intensities, $v_{j,k}$, which in turn depend on the policy choices of all countries. The optimal tax and subsidy rates under the Nash equilibrium are, therefore, different from those in an equilibrium in which a single country sets its policy unilaterally.

To highlight these interdependencies, consider country i's optimal export subsidies and import taxes. They depend on transboundary carbon intensities, $\{v_{j,k}\}_{j\neq i}$, which are regulated by applied carbon taxes in other countries ($j \neq i$). Specifically, given $\tau^{\star}_{j,k} = \tilde{\delta}_{jj,k} = \tilde{\phi}_i + \tilde{\phi}_{i,k}$ for all $j \in \mathbb{C}$, Equation (12) implies the following carbon intensity in origin j:

$$v^{\star}_{j,k} = \alpha_{j,k}\frac{\gamma_k - 1}{\gamma_k}\tilde{\delta}^{-1}_{jj,k}.$$

Supposing that preferences are Cobb-Douglas-CES, we can plug the above expression into Equation (26) to arrive at the following formulations for Nash carbon, production, and border tax rates.

Corollary 1. *The non-cooperative Nash equilibrium under CD-CES demand is characterized by each country $i \in \mathbb{C}$ applying the following taxes—which implicitly depend on applied taxes in all other countries:*

$$\begin{aligned}
&[\textit{import tax}] && 1 + t^{\star}_{ji,k} = 1 + \omega_{ji,k} + \tilde{\phi}_i \left(\frac{\gamma_k - 1}{\gamma_k}\right)^2 \frac{\alpha_{j,k}}{\tau^{\star}_{j,k}} \\
&[\textit{export subsidy}] && 1 + x^{\star}_{ij,k} = \left[1 + \textstyle\sum_{n\neq i} t^{*}_{ni,k}\frac{\lambda_{nj,k}}{1-\lambda_{ij,k}}\right]\left(1 + \frac{1}{(\sigma_k - 1)(1-\lambda_{ij,k})}\right)^{-1} \\
&[\textit{domestic subsidy}] && 1 + s^{\star}_{i,k} = \frac{\gamma_k}{\gamma_k - 1} \\
&[\textit{carbon tax}] && \tau^{\star}_{i,k} = \tilde{\phi}_{i,k} + \tilde{\phi}_i
\end{aligned}$$

As noted earlier, the above policy schedule differs from the unilaterally optimal policy schedule, i.e., Equation (22), in important ways. Above all, Nash carbon border taxes depend explicitly on applied carbon taxes in other countries. The extent of carbon border taxation is, correspondingly, regulated by cross-national differences in the perceived cost of CO_2 emissions. To see this clearly, suppose the perceived disutility from emissions is exclusively climate-related, i.e., $\tilde{\phi}_{i,k} = 0$. The carbon tax in this case collapses to $\tau^{\star}_{i,k} = \tilde{\phi}_i$ and the carbon border tax applied by country i on country j simplifies to $(\tilde{\phi}_i/\tilde{\phi}_j)\,((\gamma_k - 1)/\gamma_k)^2\,\alpha_{j,k}$. If the government in country i cares more about CO_2 emissions than its counterpart in country j, i.e., $\tilde{\phi}_i > \tilde{\phi}_i$, then country i imposes a relatively higher carbon border tax—after adjusting for the carbon input cost share, $\alpha_{j,k}$, and the degree of scale economies in production, γ_k.[25]

4 Globally Optimal Carbon-and-Border Taxes

We now analyze scenarios where governments act cooperatively. That is, they cooperate to maximize global welfare rather than act unilaterally in their self-interest. Subsection 4.1 outlines the *first-best* cooperative outcome under which cooperative governments coordinate their carbon tax policies. Subsection 4.2 analyzes a *second-best* cooperative scenario under which governments coordinate their carbon border taxes. Later in Section 6, we utilize these theoretical results to quantify the consequences of global cooperation versus non-cooperation on climate issues.

4.1 Globally Optimal Carbon Taxes

The globally optimum (or first-best) outcome is attainable when all countries coordinate their carbon taxes and correct their carbon externality on the rest of the world. Such a scenario is akin to a deep multilateral agreement on trade and climate. Below, we formally define this scenario, which we label global climate cooperation hereafter.

Definition. *Global Climate Cooperation* corresponds to an equilibrium wherein all governments set their policy instruments cooperatively in order to maximize global welfare, $\sum_i W_i$, subject to equilibrium conditions (1)-(8).

Under global climate cooperation, all countries apply zero border taxes or subsidies, as these policy measures are inefficient from a global standpoint. Globally optimal production subsidies solely correct markup distortions, i.e., they restore marginal cost pricing in each industry. Globally optimal carbon taxes have Pigouvian underpinning and correct each origin's local and transboundary carbon externality. Put formally, the globally optimal policy has the following structure in each country i:

$$\mathbf{x}_i^\star = \mathbf{t}_i^\star = \mathbf{0}; \qquad 1 + s_{i,k}^\star = \frac{\gamma_k}{\gamma_k - 1}; \qquad \tau_{i,k}^\star = \tilde{\phi}_{i,k} + \underbrace{\sum_{j \in \mathbb{C}} \tilde{\phi}_j}_{\tilde{\phi}^W}. \tag{27}$$

Globally optimal carbon taxes in country-industry (i,k) consists of two components: (ii) one that corrects the global climate externality associated with origin i's CO_2 emissions, $\tilde{\phi}^W = \sum_{j \in \mathbb{C}} \tilde{\phi}_j$, and (i) one that corrects the local pollution externality accompanying CO_2 emissions, $\tilde{\phi}_{i,k}$. The above

[25] A similar logic explains why the square of the inverse markup, $\left(\frac{\gamma_k - 1}{\gamma_k}\right)^2$, appears in formulas specified under Theorem 1. According to Equation (4), carbon intensity per unit of production, $Z_{n,k}/Q_{n,k}$ is proportional to $\left(Q_{n,k}/(1-a_{n,k})\right)^{-1/\gamma_k}$. That is, carbon intensity is affected by scale economies in both production and abatement, governed by a common parameter γ_k. In the formula for optimal import taxes $t_{ji,k}^\star$, the first $(\gamma_k - 1)/\gamma_k$ reflects the importing country i's desire to dampen the CO_2-reducing tariff given scale economies in "production". The second $(\gamma_k - 1)/\gamma_k$ is due to the origin country j's carbon taxes interacting with scale economies in "abatement".

formulas indicate that country i must raise its carbon tax when transitioning from the unilaterally optimal to the globally optimal tax schedule. Assuming away the equilibrium change to $\tilde{P}_i$, the raise in the carbon tax when committing to cooperation equals:

$$\tau_{i,k}^{\text{Global}} - \tau_{i,k}^{\text{Unilateral}} \approx \tilde{\phi}^W - \tilde{\phi}_i \tag{28}$$

The *free-riding* problem that impedes global cooperation is clearly manifested in Equation (28). When acting non-cooperatively, a country has no incentive to curb its carbon externality on residents of other countries. Accordingly, compensation schemes (such as international transfers that incentivize cooperation) or penalty devices (such as border taxes that punish non-cooperation) will lead to global climate cooperation if they imply national-level benefits that exceed the unilateral cost of cooperation, $(\tilde{\phi}^W - \tilde{\phi}_i)$.

4.2 Globally Optimal Carbon Border Taxes

As demonstrated above, the globally first-best policy for curbing CO_2 emissions requires that each country applies their *globally* optimal carbon tax. Suppose that—for political reasons—the first-best policy choice is infeasible. In that case, carbon border taxes can be used as a second-best cooperative policy measure to curb CO_2 emissions. Globally optimal carbon border taxes differ from their unilaterally optimal counterparts (characterized under Theorem 1) in that they are chosen to maximize global rather than unilateral welfare.

To present our results succinctly, we suppose $\gamma_k \to \infty$ which ensures that transboundary carbon externalities are the only distortion carbon border taxes are targeting. Appendix D.1 shows that (with international transfers) the globally optimal carbon border tax can be implemented using the following import tax schedule,[26]

$$1 + t^*_{ji,k} = \frac{1 + (\sigma_k - 1)\lambda_{ii,k}}{1 + \left[1 + \tilde{\phi}_{-i}\nu_{i,k}\right](\sigma_k - 1)\lambda_{ii,k}} \left(1 + \tilde{\phi}_{-j}\nu_{j,k}\right), \tag{29}$$

where $\tilde{\phi}_{-j} = \sum[\tilde{\phi}_n] - \tilde{\phi}_j$ measures economy j's transboundary climate externality, which has been overlooked by domestic carbon taxes. The globally optimal carbon border tax exhibits two key differences from the unilaterally optimal carbon border tax presented under Theorem 1. First, it penalizes import transactions based on their *global* climate externality. Second it includes an adjustment for cross-demand effects, because the application of carbon border taxes raises demand for domestic varieties whose global carbon externality cannot been appropriately taxed. Carbon border taxes should, accordingly, be set at a lower rate in markets where domestic suppliers are highly competitive (as measured $(\sigma_k - 1)\lambda_{ii,k}$) and high carbon-intensive (as measured by $\nu_{i,k}$).[27]

Globally optimal carbon border taxes are larger and should be more effective than their uni-

[26] The above formula derives from Cobb-Douglas-CES preferences. See Appendix D.1 for a more general formula.

[27] Globally optimal carbon border tax rates concern the optimal choice of a global planner that can manipulate the

laterally optimal counterparts. They, however, suffer from the same *free-riding* problem that has paralyzed existing climate agreements. That is, individual countries will find it optimal to free-ride and lower their carbon border tax from its globally optimal rate identified by Equation (29). Doing so will benefit the tax-imposing country but will deteriorate global welfare. Considering this limitation, an analysis based on globally optimal carbon border taxes may overstate the efficacy of border measures. In Appendix D.3, we analyze a third type of carbon border adjustment that is intended to level the playing field for domestic firms who are subjected to higher-than-unilaterally optimal carbon taxes. As we note, these border adjustments may have limited efficacy because restoring firm-level competitiveness can deteriorate the national-level terms of trade.

5 Mapping Theory to Data

In this section, we describe how the equilibrium relationships, including our optimal policy formulas, map to data. Our objective is to use this mapping to quantify the full effectiveness of trade policy at cutting global CO_2 emissions. For our quantitative analysis, we consider the Cobb-Douglas-CES case of our model and suppose that applied taxes in the baseline equilibrium (i.e, the status-quo) are as follows: (i) Tariffs, $\{t_{ji,k}\}$, are given by observed applied tariffs; (ii) domestic carbon taxes, $\{\tau_{i,k}\}$, are at their unilaterally optimal and inferred from applied emission taxes; (iii) production subsidies, $\{s_{i,k}\}$, and export subsidies, $\{x_{ij,k}\}$, are zero.

5.1 Non-cooperative Nash Equilibrium

With the aid of our unilaterally optimal tax formulas, we characterize the change in equilibrium values when moving from the baseline equilibrium to the counterfactual *non-cooperative* Nash outcome. This exercise requires information on expenditure shares, $\{\lambda_{ni,k}, e_{n,k}\}_{ni,k}$, revenue shares, $\{r_{ni,k}\}_{ni,k}$, labor shares, $\{\ell_{n,k}\}_{n,k}$, where $\ell_{n,k} \equiv L_{n,k}/\bar{L}_n$ is country n's share of employment in industry k, carbon intensities and input cost shares, $\{v_{n,k}, \alpha_{n,k}\}_{n,k}$, and national wage and income levels $\{w_n\bar{L}_n, Y_n\}_n$. Let $\mathcal{B}_v$ stack the *baseline data*. Aside from $\mathcal{B}_v$, we need information on applied taxes and subsidies $\mathcal{B}_t \equiv \{s_{n,k}, x_{in,k}, t_{ni,k}\}_{ni,k}$, carbon disutility parameters $\mathcal{B}_\phi = \{\tilde{\phi}_n, \tilde{\phi}_{n,k}\}_{n,k}$ and structural elasticities $\mathcal{B}_e = \{\gamma_k, \sigma_k, \varsigma\}_k$. Going forward, we let $\mathcal{B} \equiv \{\mathcal{B}_v, \mathcal{B}_t, \mathcal{B}_\phi, \mathcal{B}_e\}$ denote the set of sufficient statistics needed to conduct our counterfactual analysis.

For any generic variable z, we denote its counterfactual value in the non-cooperative Nash equilibrium as $z^\star$. Using the exact hat-algebra notation, we denote the change in variable z as $\hat{z} \equiv z^\star/z$. To determine counterfactual outcomes under the Nash equilibrium, we solve a system of equations consisting of equilibrium conditions and optimal tax formulas. The solution to this system determines the unilaterally optimal tax and subsidy rates, $\mathcal{R}_t \equiv \{s^\star_{n,k}, \tau^\star_{n,k}, x^\star_{in,k}, t^\star_{ni,k}\}_{ni,k}$,

price of all traded goods universally. Hence, they need not to be paired with export subsides, as import tariffs can adequately target and tax each export transaction from the global planner's perspective. In fact, compounding import tariffs with export subsides is inefficient from a global standpoint because it causes double marginalization.

changes to national wage and income $\{\hat{w}_n, \hat{Y}_n\}_n$, changes to labor employment shares and output quantities $\{\hat{\ell}_{n,k}, \hat{Q}_{n,k}\}_{n,k}$, changes to producer and consumer price indices $\{\hat{P}_{ni,k}, \hat{\tilde{P}}_{ni,k}, \hat{\tilde{P}}_{i,k}, \hat{\tilde{P}}_n\}_{ni,k}$, changes to expenditure and revenue shares $\{\hat{\lambda}_{ni,k}, \hat{r}_{ni,k}\}_{ni,k}$, changes to carbon input cost shares, abatement, CO_2 emissions, and carbon intensities $\{\hat{\alpha}_{n,k}, \widehat{1-a_{n,k}}, \hat{Z}_{n,k}, \hat{v}_{n,k}\}_{n,k}$, and changes to real consumption and welfare $\{\hat{V}_n, \hat{W}_n\}_n$. We let $\mathcal{R}_v$ denote the set of non-tax-related outcomes and $\mathcal{R} \equiv \{\mathcal{R}_t, \mathcal{R}_v\}$ to denote the full set of equilibrium outcomes we seek to solve.

In what follows we describe an approach that determines counterfactual outcomes, $\mathcal{R}$, given a minimal set of sufficient statistics, $\mathcal{B} \equiv \{\mathcal{B}_v, \mathcal{B}_t, \mathcal{B}_\phi, \mathcal{B}_e\}$. For a clearer exposition, we herein express all elements of $\mathcal{B}$ in blue. First, we present the set of equations that describe optimal tax/subsidy formulas, which following Section 3.2 are given by:[28]

$$\begin{cases} 1+t^\star_{ni,k} = 1+\omega^\star_{ni,k} + \tilde{\phi}_i v_{n,k}\hat{v}_{n,k}\,\hat{\tilde{P}}_i\frac{\gamma_k-1}{\gamma_k} & \text{a) optimal imp tax } (ni,k) \\ 1+x^\star_{in,k} \;\; = \left[1+\frac{1}{(\sigma_k-1)(1-\lambda_{in,k}\hat{\lambda}_{in,k})}\right]^{-1} & \\ \qquad\qquad \times\left[1+\sum_{\ell\neq i}\left(\omega^\star_{\ell i,k}+\tilde{\phi}_i\hat{\tilde{P}}_i v_{\ell,k}\hat{v}_{\ell,k}\frac{\gamma_k-1}{\gamma_k}\right)\frac{\lambda_{\ell n,k}\hat{\lambda}_{\ell n,k}}{1-\lambda_{in,k}\hat{\lambda}_{in,k}}\right] & \text{b) optimal exp tax } (in,k) \\ \widehat{1+t_{ni,k}} = \frac{1+t^\star_{ni,k}}{1+t_{ni,k}};\quad \widehat{1+x_{in,k}} = 1+x^\star_{in,k} & \\ \quad\widehat{1+s_{n,k}} = \gamma_k/(\gamma_k-1);\quad \hat{\tau}_{n,k} = \hat{\tilde{P}}_n & \text{d) all taxes in changes} \end{cases} \tag{30}$$

Next, we present the set of equations that represent equilibrium conditions (e.g., optimal demand, supply, and market clearing conditions). To formulate the change to equilibrium output quantities, we invoke the labor market clearing condition in country-industry (n,k): $w_n\ell_{n,k}\bar{L}_n = (1-\alpha_{n,k}\frac{\gamma_k-1}{\gamma_k})P_{nn,k}Q_{n,k}$. Replacing for $P_{nn,k}$ from Equation (3) and applying the hat algebra notation delivers:

$$\hat{Q}_{n,k} = \left[\tilde{\alpha}_{n,k} + (1-\tilde{\alpha}_{n,k})\hat{\alpha}_{n,k}\right]^{\frac{\gamma_k}{1-\gamma_k}}\left(\hat{\ell}_{n,k}\right)^{\frac{\gamma_k}{\gamma_k-1}}\left(\widehat{1-a_{n,k}}\right)^{\frac{1}{\varsigma}}, \tag{31}$$

where $\tilde{\alpha}_{n,k} \equiv \left(1-\alpha_{n,k}\frac{\gamma_k-1}{\gamma_k}\right)^{-1}$ is observable given the information in $\mathcal{B}$. The change to variety-specific producer prices and the corresponding change to the CES and Cobb-Douglas consumer price indices are, accordingly, given by

$$\begin{cases} \hat{P}_{ni,k} = \hat{w}_n\left(\widehat{1-a_{n,k}}\right)^{\frac{1-\gamma_k}{\varsigma\gamma_k}}\left(\hat{Q}_{n,k}\right)^{-\frac{1}{\gamma_k}} & \text{a) producer price } (ni,k) \\ \hat{\tilde{P}}_{ni,k} = \frac{\widehat{(1+t_{ni,k})}}{\widehat{(1+x_{ni,k})}\widehat{(1+s_{n,k})}}\hat{P}_{ni,k} & \text{b) consumer price } (ni,k) \\ \hat{\tilde{P}}_{i,k} = \left[\sum_{n=1}^N \lambda_{ni,k}(\hat{\tilde{P}}_{ni,k})^{1-\sigma_k}\right]^{\frac{1}{1-\sigma_k}} & \text{c) consumer price } (i,k) \\ \hat{\tilde{P}}_i = \prod_k(\hat{\tilde{P}}_{i,k})^{e_{i,k}} & \text{d) consumer price } (i) \end{cases} \tag{32}$$

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[28] In the first and second lines, $\omega^\star_{ni,k}$ denotes the inverse of the export supply elasticity in the counterfactual equilibrium:

$$\omega^\star_{ni,k} = \frac{-(1/\gamma_k)r_{ni,k}\hat{r}_{ni,k}\left[1-\frac{\sigma_k-1}{\gamma_k}\frac{w_i\hat{w}_i\bar{L}_i}{w_n\hat{w}_n\bar{L}_n}\sum_{\ell\neq i}\frac{\rho_{i,k}\hat{\rho}_{i,k}r_{i\ell,k}\hat{r}_{i\ell,k}}{\rho_{n,k}\hat{\rho}_{n,k}r_{n\ell,k}\hat{r}_{n\ell,k}}\lambda_{n\ell,k}\hat{\lambda}_{n\ell,k}\right]}{1+(1/\gamma_k)\sum_{\ell\neq i}r_{n\ell,k}\hat{r}_{n\ell,k}\left(1+(\sigma_k-1)(1-\lambda_{n\ell,k}\hat{\lambda}_{n\ell,k})\right)}$$

Note that $\hat{P}_{ni,k}$, in the above equation, is regulated by changes to the wage rate, $\hat{w}_n$, scale of production, $\hat{Q}_{n,k}$, and abatement, $(\widehat{1-a_{n,k}})$, all of which are core elements in the outcome set, $\mathcal{R}$. Given the change in consumer prices, the change in within-industry expenditure and revenue shares can be calculated as

$$\begin{cases} \hat{\lambda}_{ni,k} = \left(\hat{\tilde{P}}_{ni,k}/\hat{\tilde{P}}_{i,k}\right)^{1-\sigma_k} & \text{a) within-ind exp share } (ni,k) \\ \hat{r}_{ni,k} = \frac{(\widehat{1+t_{ni,k}})^{-1}(\widehat{1+x_{ni,k}})\hat{\lambda}_{ni,k}\hat{Y}_i}{\sum_\ell r_{j\ell,k}(\widehat{1+t_{n\ell,k}})^{-1}(\widehat{1+x_{n\ell,k}})\hat{\lambda}_{n\ell,k}\hat{Y}_\ell} & \text{b) within-ind rev share } (ni,k) \end{cases} \tag{33}$$

The change in industry-level carbon input cost shares, CO_2 emissions, abatement, and carbon intensities are given by:

$$\begin{cases} \widehat{1-a_{n,k}} = \left[(1-\alpha_{n,k}) + (\alpha_{n,k})\left(\hat{\tau}_{n,k}/\hat{w}_n\right)^{1-\varsigma}\right]^{\varsigma/(\varsigma-1)} & \text{a) abatement in country-industry } (n,k) \\ \hat{\alpha}_{n,k} = \frac{1}{\alpha_{n,k}} - \frac{1-\alpha_{n,k}}{\alpha_{n,k}}(\widehat{1-a_{n,k}})^{(1-\varsigma)/\varsigma} & \text{b) carbon cost share in country-industry } (n,k) \\ \hat{Z}_{n,k} = (\hat{\alpha}_{n,k})^{\varsigma/(\varsigma-1)}\left(\widehat{1-a_{n,k}}\right)^{\frac{1}{\varsigma\gamma_k}}\hat{Q}_{n,k}^{1-\frac{1}{\gamma_k}} & \text{c) emission from country-industry } (n,k) \\ \hat{v}_{n,k} = \hat{\alpha}_{n,k}/\hat{\tau}_{n,k} & \text{d) emission intensity of country-industry } (n,k) \end{cases} \tag{34}$$

The expressions for $\widehat{1-a_{n,k}}$, $\hat{\alpha}_{n,k}$, $\hat{Z}_{n,k}$ and $\hat{v}_{n,k}$ respectively derive from applying the hat-algebra notation to Equations (1), (2), (4), and (12). The change in wages and industry-level labor shares are governed by the labor market clearing condition expressed in changes:

$$\begin{cases} \hat{w}_n\hat{\ell}_{n,k}w_n\ell_{n,k}\bar{L}_n = \sum_j\left[\frac{(1-\hat{\alpha}_{n,k}\alpha_{n,k}\frac{\gamma_k-1}{\gamma_k})(1+s^\star_{n,k})(1+x^\star_{nj,k})}{(1+t^\star_{nj,k})}\hat{\lambda}_{nj,k}\lambda_{nj,k}e_{j,k}\hat{Y}_jY_j\right] & \text{a) LMC } (n,k) \\ \sum_k\hat{\ell}_{n,k}\ell_{n,k} = 1 & \text{b) sum of shares=1 } (n) \end{cases} \tag{35}$$

The first line in the above equation ensures that the industry-level wage bill equals total sales net of taxes/subsidies. The second line ensures that labor markets clear at the national level.[29] Finally, the change in national income, $\hat{Y}_n$, is governed by the representative consumer's budget constraint (BC), which can be expressed in changes as follows:

$$\hat{Y}_nY_n = \hat{w}_nw_n\bar{L}_n + \sum_k\sum_j\left[\frac{(\hat{\alpha}_{n,k}\alpha_{n,k}\frac{\gamma_k-1}{\gamma_k})(1+s^\star_{n,k})(1+x^\star_{nj,k})}{(1+t^\star_{nj,k})}\hat{\lambda}_{nj,k}\lambda_{nj,k}e_{j,k}\hat{Y}_jY_j\right]$$
$$+\sum_k\sum_j\left[\frac{\left[1-(1+s^\star_{n,k})(1+x^\star_{nj,k})\right]}{(1+t^\star_{nj,k})}\hat{\lambda}_{nj,k}\lambda_{nj,k}e_{j,k}\hat{Y}_jY_j + \frac{t^\star_{nj,k}}{1+t^\star_{jn,k}}\hat{\lambda}_{jn,k}\lambda_{jn,k}e_{n,k}\hat{Y}_nY_n\right]. \quad \text{BC } (n) \tag{36}$$

The above equation ensures that total income equals the wage bill plus tax revenues. The first

[29] The second line sums over industry-level labor market clearing conditions, which can be described more compactly as industry-level labor shares add up to one in the counterfactual equilibrium (i.e., $\sum_k \ell^\star_{n,k} = 1$).

and second sums respectively denote carbon tax revenues and non-carbon tax revenues.[30]

Solving Equations (30)-(36) determines counterfactual Nash taxes and equilibrium outcomes, $\mathcal{R} \equiv \{\mathcal{R}_t, \mathcal{R}_v\}$, with information on a sufficient set of observable or estimable statistics, $\mathcal{B} \equiv \{\mathcal{B}_v, \mathcal{B}_t, \mathcal{B}_\phi, \mathcal{B}_e\}$. Lastly, given $\mathcal{R}$ and $\mathcal{B}$, the change to real consumption and welfare can be calculated as:

$$\hat{V}_i = \left(\frac{\hat{Y}_i}{\hat{\tilde{P}}_i}\right), \qquad \hat{W}_i = \underbrace{\frac{Y_i}{Y_i - \sum_{n,k} \tilde{\delta}_{ni,k} Z_{n,k}} \hat{V}_i}_{\Delta \text{ consumption}} - \underbrace{\sum_{n,k} \frac{\tilde{\delta}_{ni,k} Z_{n,k}}{Y_i - \sum_{n,k} \tilde{\delta}_{ni,k} Z_{n,k}} \hat{Z}_{n,k}}_{\Delta \text{ disutility from } CO_2}. \tag{37}$$

5.2 Global Climate Cooperation

Taking the same approach, we can solve for equilibrium outcomes under the cooperative equilibrium wherein every country applies their globally optimal carbon tax—as specified by Equation (27). The change in taxes from the baseline rate to the globally optimal rate are given by:

$$\begin{cases} \widehat{1+t_{ji,k}} = \frac{1}{1+t_{ji,k}}, \quad \widehat{1+x_{ji,k}} = 1 & \text{a) border tax/subs} \\ \widehat{1+s_{j,k}} = \gamma_k / (\gamma_k - 1) & \text{b) production subs} \\ \hat{\tau}_{i,k} = \hat{\tilde{P}}_i + \sum_{j \neq i} \left(\frac{\tilde{\phi}_j}{\tilde{\phi}_{i,k} + \tilde{\phi}_i} \hat{\tilde{P}}_j \right) & \text{c) emission tax} \end{cases} \tag{38}$$

Solving the optimal tax equations given by (38) along-side equilibrium conditions (31)–(37), determines the change to equilibrium variables $\mathcal{R} \equiv \{\mathcal{R}_v, \mathcal{R}_t\}$ as a function of baseline data and parameters $\mathcal{B} \equiv \{\mathcal{B}_v, \mathcal{B}_t, \mathcal{B}_\phi, \mathcal{B}_e\}$. Note that the difference between non-cooperative and cooperative equilibrium outcomes is driven merely by differences in tax schedules between the two scenarios—as indicated by the set of equations (30) and (38).

5.3 Baseline Data ($\mathcal{B}_v$, $\mathcal{B}_t$)

In this section, we describe the baseline data on trade, production, and emissions (labeled as $\mathcal{B}_v$) as well as data on applied taxes (labeled as $\mathcal{B}_t$).

Data on Trade, Production, and Emissions. Data on international emissions and expenditure levels are taken from the 2009 WIOD database on Input-Output Tables and Environmental Accounts (Timmer et al. 2012).[31] The WIOD reports the full matrix of international expenditure levels across 41 major countries and 35 ISIC-level industries. Since the European Union (EU)

[30] We solve the general equilibrium system specified by Equations (31)-(36) as a nested fixed point with two tiers. In the inner tier, for a given schedule of taxes, all variables are solved to be in general equilibrium as required by Equations (31)-(36). In the outer tier, given the fixed point of the inner tier (i.e., general equilibrium variables conditional on a choice of taxes), we solve for optimal taxes according to Equation (30).

[31] Our baseline year is 2009 as the most recent year with available information on trade and production, emission, and environmentally related taxes. Specifically, 2009 is the last year reported in the WIOD Environmental Account and the first year with a large coverage in environmentally-related tax data.

often acts as one tax-imposing authority, we aggregate all EU members into one tax-imposing region. To merge the WIOD data with our other data sets, we aggregate our sample into 19 industries, the details of which are listed in Table 1. After applying these aggregations, we are left with 15 regions ($N = 15$) and 19 industries ($K = 19$, covering tradeables and nontradeables), which results in a $15 \times 15 \times 19$ matrix describing expenditure levels, $\tilde{P}_{ji,k}Q_{ji,k}$, per origin j–destination i–industry k.

The WIOD Environmental Accounts report emissions of several air pollutants by origin country and industry. First, we use these data to calculate CO_2 equivalent (CO_2e) emissions based on global warming potential (GWP-100) from the IPCC (2014) report. The WGP-100 measures how much emissions of one tonne of a gas will be absorbed in the atmosphere in a period of 100 years relative to the emissions of one tonne of CO_2. Using emission data of CO_2 (carbon dioxide), CH4 (methane), and N2O (nitrous oxide), we calculate CO_2e as $Z_{i,k} = Z_{i,k}^{\text{CO2}} + 28 \times Z_{i,k}^{\text{CH4}} + 265 \times Z_{i,k}^{\text{N2O}}$ for every pair of origin country and industry. According to the Environmental Protection Agency, emissions of CO_2, CH4, and N2O account for 97% of greenhouse gas emissions worldwide. Accordingly, we construct carbon intensity of *origin i–industry k* as:

$$v_{i,k} = \frac{Z_{i,k}}{P_{ii,k}Q_{i,k}} = \frac{(\text{CO}_2\text{e Emission})_{i,k}}{(\text{Gross Output})_{i,k}},$$

where the numerator is measured in tonnes of CO_2e, and the gross output is measured in US dollars. Throughout the paper, we use carbon emission or CO_2 emission as a shorthand for greenhouse gases or CO_2e.

Regarding non-CO_2 local pollution, the WIOD Environmental Accounts reports (under the category of acidification) the emissions of nitrogen oxides (NOx), sulfur oxides (SOx), and carbon monoxide (CO) for every origin–industry pair. We consider the aggregate of these local emissions as $Z_{i,k}^0$, and define $\bar{\zeta}_{i,k} \equiv Z_{i,k}^0 / Z_{i,k}$, as the rate at which origin i-industry k generates non-CO_2 local pollutants per tonne of CO_2 emissions.

Key Takeaways from Carbon Intensities across Industries. We report some key industry-level statistics that will help us interpret the results emerging from our quantitative analysis in Section 6. Table 1 reports the list of industries together with their main characteristics. We draw two immediate conclusions: First, *non-manufacturing* industries such as Agriculture, Electricity, and Transportation account for a huge share of global CO_2 emissions. In contrast, *manufacturing* as a whole (industries 3–14 as labeled in the table) account only for one-fifth of global CO_2 emissions. Second, there is an overall negative correlation between tradeablity and CO_2 emission share across industries (See Figure A.2 for the scatter plot of industries' trade-to-GDP ratio against their share from world CO_2 emissions). For example, the most tradeable industries, namely, Machinery & Electronics and Textiles & Leather, together account for less than 1% of global CO_2 emissions, while Electricity, Gas & Water and Agriculture that are among less tradeable industries account for more than half of global CO_2 emissions. In fact, industries with a trade-to-GDP ratio higher than 0.10 account for only one-third of global CO_2 emissions. In other words, low-tradeable industries account for the majority of global CO_2 emissions.

For detailed information on countries/regions in our sample, and their key characteristics, see Table A.2. In our baseline year of 2009, the EU has the largest share of world GDP, at 27.1%, while accounting for only 12.1% of world CO_2 emissions. In contrast, China has the largest share of CO_2 emissions, at 23.9%, while accounting for 13.7% of world GDP. Across countries, the ratio of Emission Share to GDP Share is the lowest for Japan (0.34) and the EU (0.45), and the highest for India (2.90) and Russia (2.96).

Data on Applied Trade and Carbon Taxes. We compile data on applied tariffs in year 2009 from the United Nations Statistical Division, Trade Analysis and Information System (UNCTAD-TRAINS). The 2009 version of UNCTAD-TRAINS covers 31 two-digit (in ISIC rev. 3) sectors, which are aggregated into the 19 industries for which we have compiled international expenditure and emissions data. The UNCTAD-TRAINS database reports multiple measures for applied tariffs. Following the earlier quantitative trade literature, we use the "simple tariff line average" of the "effectively applied tariff" (AHS). The applied tariff rates are reported for each *origin–destination–industry* combination, except for instances where the origin country is a member of the European Union (EU). In such cases we assign applied tariffs based on the fact that EU tariffs are reported, intra-EU trade is subject to no tariffs, and all EU members impose a common tariff on non-members. Finally, in accordance with the World Trade Organization rules, we assume that applied export and domestic subsidies are negligible, setting $s_{i,k} = x_{ij,k} = 0$ in all cases.

We make use of data on Environmental Taxes by Economic Activity from EUROSTAT as well as Environmentally-related Taxes from OECD-PINE. The data from EUROSTAT report environmentally-related taxes at the level of country-industry pairs, covering all European countries, based on NACE rev. 2 industries, which we map to the 19 ISIC industries in our sample. The data from OECD-PINE report environmentally-related tax data in every country as a percentage of that country's GDP. Our quantitative analysis treats these environmentally-related taxes as "carbon taxes" under the status quo, noting that carbon taxes in our framework are adjusted to account for local pollution. See Appendix E for more details on our data construction.

Carbon Cost Shares. We recover carbon cost shares from data on carbon taxes, carbon intensities, and markups. Following Equation (12), the cost share of carbon can be calculated as:

$$\alpha_{n,k} = \frac{\gamma_k}{\gamma_k - 1} \frac{\tau_{nk} Z_{n,k}}{P_{nn,k} Q_{n,k}} = \mu_{n,k} \tau_{n,k} v_{n,k}. \tag{39}$$

On the right hand side of Equation (39), $\tau_{nk} Z_{n,k}$ is total emission tax paid by firms in origin $n-$industry k, and the denominator, $P_{nn,k} Q_{n,k}$, is gross output. Together, with our markup parameters, which we estimate below, we obtain $\alpha_{n,k}$ for every country n-industry k. Table 1 reports the average value for $\alpha_{n,k}$ across countries for every industry k.

5.4 Estimation & Calibration of Model Parameters ($\mathcal{B}_\phi$, $\mathcal{B}_e$)

To evaluate policy outcomes, we need a set of parameters, $\mathcal{B}_e = \{\gamma_k, \sigma_k, \varsigma\}_k$, including the trade elasticity, $(\sigma_k - 1)$; the degree of firm-level market power, γ_k, which is tied to the markup, $\mu_k \equiv \gamma_k/(\gamma_k - 1)$; and the elasticity of input demand for carbon, ς. In addition, we require information on the emission disutility parameters $\mathcal{B}_\phi = \{\tilde{\phi}_n, \tilde{\phi}_{n,k}\}_{n,k}$. We estimate trade elasticities following Caliendo and Parro (2015), estimate markups following the cost-based approach in De Loecker and Warzynski (2012), and estimate the elasticity of input demand for carbon and the carbon disutility parameters using environmentally-related tax data.

Markups. To estimate markups, we use firm-level data from COMPUSTAT data and closely follow De Loecker et al. (2020) and Baqaee and Farhi (2020). To map COMPUSTAT data to industries as defined in WIOD, we map disaggregated NAICS-level industries from COMPUSTAT to the 19 aggregate 2-digit ISIC-level industries ($k = 1, ..., K = 19$) as well as disaggregated 3-digit ISIC industries. For every industry-year, we first estimate the output elasticity with respect to variable input, based on Olley-Pakes procedure in which, expressed in firm-level logs of real values, our dependent variable is sale and the variable input is COGS (Cost of Goods Sold). We use capital expenditure as the proxy variable and gross capital stock as the state variable, and following the usual practice in the literature, we control for a firm's share of sales within disaggregated industries (which are 3-digit ISIC in our data). The resulting estimated coefficient of log variable input gives the output elasticity $\theta_{k,t}$, for every industry-year k, t. For every firm ω in industry k at year t, the variable input share is the ratio of variable input (COGS) to sales, $\beta_{k,t}(\omega)$. Using the first order condition of the firm's cost minimization, markups are then given by:

$$\mu_{k,t}(\omega) = \theta_{kt}/\beta_{k,t}(\omega). \tag{40}$$

To obtain markups at the level of industries, we aggregate firm-level markups to the level of 19 industries with the weight assigned to a firm given by within-industry firm's sales share. We report our markup estimates at the level of industries in Table 1.[32]

Trade Elasticity. We estimate trade elasticities, $(\sigma_k - 1)$ by applying Caliendo and Parro's (2015) estimation technique to our 2009 data on trade values and applied tariffs. This approach recovers $(\sigma_k - 1)$ under the identifying assumption that applied tariffs are orthogonal to idiosyncratic

[32] For the weight assigned to a firm we consider the three-year period of 2008, 2009, and 2010 to make our estimates not sensitive to potential industry-level fluctuations in our baseline year of 2009. The resulting firm-level markup estimates are on average 1.58, with 1.07 at 25th and 1.84 at 75th percentile.

Table 1: Industry-Level Statistics and Elasticities

	Industry	CO2 Emissions (% of total)	$\frac{\text{Trade}}{\text{GDP}}$	Carbon Intensity (ν)	Carbon Input Share (α)	Trade Elasticity ($\sigma - 1$)	Markup ($\frac{\gamma}{\gamma-1}$)
1	Agriculture	19.9%	6.8%	100.0	0.020	2.05	1.46
2	Mining	8.0%	27.6%	40.4	0.019	1.80	1.53
3	Food	1.1%	9.0%	4.2	0.004	1.36	1.70
4	Textiles and Leather	0.4%	27.1%	4.2	0.005	0.86	2.11
5	Wood	0.2%	8.4%	5.4	0.010	3.42	1.28
6	Pulp and Paper	0.6%	8.9%	6.8	0.004	3.21	1.30
7	Coke and Petroleum	2.7%	17.9%	23.2	0.006	3.31	1.18
8	Chemicals	3.4%	24.6%	19.5	0.017	0.89	2.06
9	Rubber and Plastics	1.0%	14.0%	15.2	0.006	1.55	1.27
10	Non-Metallic Mineral	9.6%	13.1%	31.5	0.006	1.95	1.49
11	Metals	0.3%	25.9%	2.1	0.001	3.97	1.24
12	Machinery and Electronics	0.4%	37.1%	1.8	0.004	1.90	1.50
13	Transport Equipment	0.3%	23.3%	1.6	0.002	0.59	1.21
14	Manufacturing, Nec	0.4%	32.8%	10.1	0.005	0.59	1.91
15	Electricity, Gas and Water	32.0%	1.0%	205.5	0.018	7.14	1.12
16	Construction	0.9%	0.3%	2.1	0.008	7.14	1.10
17	Retail and Wholesale	1.8%	3.7%	2.6	0.009	6.93	1.14
18	Transportation	8.1%	10.9%	30.2	0.033	7.14	1.01
19	Other Services	9.0%	2.6%	4.1	0.007	1.59	1.60

Note: This table shows for every of the 19 industries the share from world CO2 emission, world-level trade-to-GDP ratio, global average carbon intensity (tonnes of CO_2 per dollar of output) normalized by that of agriculture, calibrated carbon cost shares reported as unweighted mean across countries within every industry, estimated trade elasticities, and markups. All CO_2 measures are CO_2 equivalent.

variations in bilateral trade costs. The estimated elasticity values are reported in Table 1.[33]

Carbon Input Demand Elasticity. We estimate the elasticity of input demand for carbon using the following equation that describes the relative expenditure on carbon versus labor inputs as a function of relative input prices:

$$\ln\left(\frac{\alpha_{i,k}}{1-\alpha_{i,k}}\right) = (1-\varsigma)\ln\left(\frac{\tau_{i,k}}{w_i}\right) + \varsigma\ln\left(\frac{\bar{\kappa}_{i,k}}{1-\bar{\kappa}_{i,k}}\right). \tag{41}$$

The term $\varsigma\ln[\bar{\kappa}_{i,k}/(1-\bar{\kappa}_{i,k})]$ is an exogenous demand residual specific to origin i and industry k. We assume that $\varsigma\ln[\bar{\kappa}_{i,k}/(1-\bar{\kappa}_{i,k})] = \Phi_k + \varepsilon_{i,k}$, where Φ_k is as an industry fixed effect capturing systematic cross-industry differences in carbon input demand, and $\varepsilon_{i,k}$ accounts for measurement errors and origin-specific variations in carbon input demand. We estimate the above equation with origin and industry-level data on inferred carbon prices, $\tau_{i,k}$, and carbon input cost shares, $\alpha_{i,k}$. Consistent with our model, use value added per capita to proxy for national-level wages.

[33] A necessary condition to ensure a unique equilibrium is $(1-\sigma_k)/(1-\gamma_k) \leq 1$. We adjust our estimates of trade elasticity for a few industries at a corner value to ensure uniqueness.

Table 2: Estimation: input demand elasticity for carbon

	OLS	IV
1 - ς	0.330	0.374
	(0.059)	(0.126)
R-squared	0.57	–
1st stage F-stat (CD)	–	41.7
Observations	266	266

Note: This table reports estimated values of ς, based on Equation (41). Each observation is a pair of country-industry, and observations of RoW are dropped from both regressions. The first and second rows, respectively, report estimates obtained from the OLS and IV (2SLS) estimators. Robust standard errors are reported in parentheses.

Estimating Equation (41) with ordinary least squares (OLS) can be problematic, as one suspects that $\ln(\tau_{i,k}/w_i)$ and $\varepsilon_{i,k}$ are correlated—due to either reverse causality which is a standard issue facing demand estimation, omitted variables, or measurement error. We address this problem by adopting an instrumental variable (IV) approach. We need an instrument that is correlated with relative carbon price but uncorrelated with the error term. We use energy reserves in every country i as an instrument for that location's relative carbon price, $\tau_{i,k}/w_i$. Energy reserves correspond to the sum of proven reserves of crude oil, natural gas, and coal, obtained from the BP Statistical Review of World Energy for year 2010, aggregated in common units of Btu.[34] The identifying assumption is that a country's energy reserves affect carbon prices exclusively through the supply channel. That is, local energy reserves are uncorrelated with each location's unobserved carbon demand shifter, implying that any effect of reserves on relative carbon input expenditure, $\ln[\alpha_{i,k}/(1-\alpha_{i,k})]$, channels through the price variable, $\ln(\tau_{i,k}/w_i)$. Table 2 reports the estimation results. Our IV estimation delivers an estimated value of $\varsigma = 0.624$, with an encouragingly high first-stage F-statistics.

Perceived Disutility from Emissions. We recover the perceived disutility from emissions based on two assumptions: (a) a country's applied environmentally-related taxes correspond to its unilaterally optimal domestic tax on CO_2 emissions and their implied non-CO_2 local pollution; and (b) the globally optimal carbon tax equals the global disutility from CO_2 emissions. Namely,

$$\begin{cases} T_i^E = \sum_k (\tilde{\varphi}_{i,k} + \tilde{\phi}_i) Z_{i,k} & \text{(a)} \\ SCC = \sum_i \tilde{\phi}_i & \text{(b)} \end{cases} \tag{42}$$

Here, T_i^E denotes total environmentally-related taxes collected by country i, and SCC denotes the Social Cost of Carbon. Estimating SCC is beyond the scope of our paper. Instead, we borrow the SSC estimated by United States Government's Interagency Working Group on Social Cost of Greenhouse Gases which reports a cost of \$31 per tonne of CO_2 in 2010.

[34] Because "the rest of the world" (RoW) consists of various countries with different taxes, wages, and carbon demand, we drop RoW from the sample used in this regression.

We recover the relative values of $\tilde{\phi}_i$ across countries by considering two key aspects of countries' attitude toward climate policy. First, if all individuals (irrespective of nationality) cared equally about climate change, then the disutility from CO_2 emissions would be proportional to country size. To account for the size effect, we impose the proportionality assumption that $\frac{\tilde{\phi}_i}{\tilde{\phi}_j} \propto \frac{Y_i}{Y_j}$. Second, governments around the world exhibit starkly different levels of care for climate change—even after we control for size. We do not intend to explain these differences, but we suppose that governments' care for CO_2 emissions can be to some extent inferred from their attitude toward environmental issues. Under this presupposition, we can infer a country's relative care for CO_2 emissions from their environmentally-related taxes in the spirit of revealed preferences. In particular, we let the relative care for CO_2 emissions across countries be proportional to their applied environmentally-related tax rates, i.e., $\frac{\tilde{\phi}_i}{\tilde{\phi}_j} \propto \frac{\left(T_i^E/Z_i\right)}{\left(T_j^E/Z_j\right)}$. Putting the pieces together, and defining $y_i \equiv Y_i/Y_W$ as country i's share in world GDP, we specify country i's perceived disutility from CO_2 emissions as $\tilde{\phi}_i = \bar{h} y_i (T_i^E/Z_i)$. This specification leaves us with a single parameter, $\bar{h}$, which we calibrate using Equation 42-(b). Then, given the recovered values for $\{\tilde{\phi}_i\}$, we recover $\{\tilde{\phi}_{i,k}\}$ according to Equation 42-(a). Further details about our calibration strategy are presented in Appendix E. Table A.2 reports our calibrated values for $\tilde{\phi}_i$ for each country in our sample.[35]

Magnitudes of Optimal Border Taxes. To set the stage for our quantitative analysis, we discuss the magnitude of unilaterally optimal border taxes as implied by our calibrated model. Recall that unilaterally optimal border policies involve both import tariffs and export subsidies. By the Lerner symmetry, however, only the combined rate of optimal import tariff and export subsidy is determinate. Based on our calibrated model, the median combined optimal border tax rate equals 52%—i.e., Median $\left(\frac{1+\text{optimal import tariff}}{1+\text{optimal export subsidy}}\right) = 1.52$. The combined optimal border tax rate varies notably across industries, and to a lesser extent across countries—with corresponding 25th and 75th percentiles at 1.26 and 1.74.[36] These values are broadly consistent with existing estimates obtained from models without carbon externalities (e.g., Ossa (2014)).

Optimal *carbon* border taxes (or subsidies) typically constitute a small portion of the optimal import tariffs (or export subsidies). Each country's optimal carbon border taxes vary considerably across industries as they are larger in high-carbon industries and less punitive in high-return to scale industries. Figure A.3 in Appendix G illuminates this point by plotting EU's unilaterally optimal carbon border taxes across various industries.[37]

[35] Notice, we recover CPI-adjusted disutility parameters, $\tilde{\phi}_{i,k} = \tilde{P}_i \phi_{i,k}$ and $\tilde{\phi}_i = \tilde{P}_i \phi_i$ rather than $\phi_{i,k}$ and ϕ_i. This is sufficient for performing our counterfactual equilibrium analyses, as explained in Section 5.

[36] The variations in terms-of-trade component of optimal border taxes largely depend on the industry-level trade elasticity, $(\sigma_k - 1)$. As noted earlier, these elasticities govern the degree of national-level market power.

[37] To give a broader sense of their potential magnitudes, Figure A.3 in Appendix G reports the optimal carbon border tax rates both at our benchmark SCC of 31 \$/tC, and for higher values of SCC.

6 Quantitative Assessment of Climate Proposals 1 and 2

In this section, we use our theory and parameter estimates to evaluate two popular climate proposals that combine carbon taxes with border measures.[38] These proposals are cast as a remedy for the *free-riding* problem in climate agreements. To fix ideas, we first provide a brief description of each proposal:

Proposal 1. *Non-cooperative carbon border taxes:* non-cooperative governments use carbon border taxes as a second-best policy to curb transboundary CO_2 emissions.

Proposal 2. *Climate Club:* climate-conscious governments form a climate club wherein they commit to globally optimum carbon taxes and use their collective trade penalties to incentivize climate cooperation by non-members (Nordhaus 2015).

Below, we combine our analytical formulas for optimal policy from Section 3 with the quantitative approach described in Section 5 to examine the efficacy of Proposals 1 and 2. Our analysis differs from the prior literature since it considers optimal carbon border taxes and trade penalties. This feature allows us to uncover the full (rather than partial) efficacy of the above proposals.

6.1 Proposal 1: The Efficacy of Non-Cooperative Border Taxes

Table 3 reports the change in CO_2 emissions and welfare when non-cooperative governments adopt their unilaterally optimal border and carbon taxes. We solve for the non-cooperative Nash outcome using Corollary 1 and its quantitative implementation which was described in Section 5.[39] Recall that carbon border taxes, in this scenario, are used as a second-best policy to curb transboundary CO_2 emissions. To put their efficacy in perspective, Table 3 also reports the consequences of globally optimal (or first-best) carbon taxes.

Under the non-cooperative Nash equilibrium, where governments use border taxes to pursue transboundary climate objectives, global CO_2 emissions decline by a mere 0.6%. This number corresponds to only 1% of the total CO_2 reductions attainable under globally optimal carbon taxes (i.e., $^{0.6}/_{60.9} \approx 1\%$). This result indicates that carbon border taxes are largely ineffective at curbing transboundary CO_2 emissions.

[38] The policy characterization under Theorem 2 can be used in at least two ways: First, to evaluate a non-cooperative outcome in which every government adopts its unilaterally optimal policy. Second, to calculate optimal trade penalties on trading partners. That is, penalties that are capable of inflicting the maximum terms-of-trade externality on other countries. Our analysis of carbon border taxes uses Theorem 2 in the former vein, while our analysis of the climate club use this theorem in the latter vein.

[39] Here, the baseline equilibrium is the one in which every country restores the marginal-cost-pricing by setting production subsidy at $1 + s_{i,k} = \gamma_k/(\gamma_k - 1)$. This is a sensible baseline for this exercise because moving from no production subsidies (which we assume to be the case in the observed data) to optimal ones at $1 + s_{i,k} = \gamma_k/(\gamma_k - 1)$ can alone increase production everywhere in the world, hence contributing to increases in CO_2 emissions. For comparison, we report in Table A.3 results we obtain for this exercise when the baseline equilibrium is given by the observed data.

Table 3: The Impact of Non-cooperative and Cooperative Tax Policies

	Non-Cooperative			Global Cooperation		
Country	ΔCO_2	ΔV	ΔW	ΔCO_2	ΔV	ΔW
AUS	-6.6%	-1.9%	-1.9%	-46.4%	-0.7%	0.3%
EU	0.7%	-1.2%	-1.3%	-9.2%	0.0%	2.0%
BRA	-6.0%	-1.3%	-1.3%	-70.7%	-1.3%	-0.8%
CAN	-12.6%	-4.3%	-4.3%	-47.1%	-0.6%	0.2%
CHN	3.0%	-1.0%	-1.0%	-69.3%	-1.3%	-0.9%
IDN	-3.1%	-2.0%	-2.0%	-67.0%	-1.7%	-1.2%
IND	1.1%	-4.4%	-4.4%	-76.0%	-2.6%	-2.1%
JPN	3.4%	-0.9%	-0.9%	-23.1%	-0.2%	1.5%
KOR	19.2%	-2.2%	-2.3%	-47.5%	-1.1%	-0.1%
MEX	-1.6%	-3.2%	-3.2%	-79.5%	-0.6%	-0.4%
RUS	-7.8%	-3.8%	-3.8%	-80.2%	-1.3%	-0.9%
TUR	-5.4%	-4.1%	-4.2%	-40.1%	-1.4%	0.1%
TWN	42.5%	-2.5%	-2.7%	-64.1%	-1.7%	-1.0%
USA	1.3%	-1.7%	-1.7%	-48.2%	-0.3%	0.3%
RoW	-6.0%	-2.3%	-2.3%	-84.7%	-1.0%	-0.8%
Global	-0.6%	-1.7%	-1.7%	-61.0%	-0.6%	0.4%

Note: This table shows for every country the change to CO2 emission, real consumption, and welfare from the baseline to non-cooperative and cooperative equilibrium. The baseline is one in which production subsidies restore the marginal-cost pricing in every industry and country. Optimal policy formulas for the non-cooperative outcome is detailed in Section 5.1 and for the cooperative outcome in Section 5.2.

The limited efficacy of carbon border taxes can be attributed to two main factors. First, carbon border taxes are not granular enough to induce firm-level abatement in the rest of the world. Carbon border taxes penalize CO_2 emissions based on the *origin country–industry*'s average carbon intensity. Individual firms, however, are incapable of lowering the origin and industry-wide carbon intensity. Carbon border taxes, as a result, cannot induce individual firms in foreign countries to undertake additional abatement.

Second, a large portion of CO_2 emissions are associated with goods and services that never cross international borders. The "Agriculture" and "Electricity, Gas, and Water" sectors, for example, account for 52% of global CO_2 emissions but have trade-to-GDP ratios of only 0.07 and 0.01, respectively (see Table 1). Figure (A.2) (in the appendix) plots industries' tradeability against their share from global emission. A basic takeaway is that the majority of CO_2 emissions are associated with industries that exhibit low levels of tradeability.[40]

The modest CO_2 reduction obtained with non-cooperative border taxes is accompanied by possibly strong and negative consumption effects. The average country loses 1.7% of its real consumption under the non-cooperative Nash equilibrium while gaining negligibly from the reductions in CO_2 emissions. By comparison, globally optimal carbon taxes deliver 60.9% reduction in global CO_2 emissions, accompanied by an only 0.6% loss to real global consumption. Together,

[40] For instance, 84% of global CO_2 emissions are associated with industries that exhibit a trade-to-GDP ratio lower than 0.15. See Section 5.3 and Figure A.2 for more details.

these figures translate to a 0.3% increase in *climate-adjusted* welfare under globally optimal carbon taxes.

Increasing vs Constant Returns to Scale. To elucidate the role of scale economies, we redo the above exercise under the case of constant returns to scale (CRS). Detailed results corresponding to this exercise are reported in Table A.4 of Appendix G. We find that global adoption of non-cooperative carbon/border taxes leads to a 1.3% reduction in global CO_2 emissions under CRS, compared to 0.6% under IRS. That is, overlooking scale economies leads to overstating the efficacy of carbon border taxes by a factor of two. Even then, non-cooperative border taxes can reproduce only 2.2% of the CO_2 reduction attainable under globally optimal carbon taxes.

Comparing outcomes under the IRS and CRS cases highlights the role of *firm delocation*. Under IRS, non-cooperative border taxes (which are meant to curb transboundary emissions) lead to an increase in CO_2 emissions in several countries. This outcome is reversed or is notably less pronounced under CRS. The difference derives from firm delocation effects under IRS. When firms are subjected to border tax hikes, they relocate to larger markets to evade such taxes. These delocation effects can raise the scale of production and CO_2 emissions even in climate-conscious regions like the EU that charge a relatively high carbon tax.

Alternative Specifications & Sensitivity Analysis. We consider several alternative specifications to examine the robustness of our claims about the efficacy of carbon border taxes. These auxiliary results are reported in Table A.5 in Appendix G. First, we redo our analysis by assigning an alternative value to the social cost of carbon (SCC). Following Cai and Lontzek (2019) we set SCC at 68 \$/tC, compared to 31 \$/tC in our main analysis.[41] Second, we consider a special case of our model that features the Cobb-Douglas abatement technology used by Copeland and Taylor (2004). In this case, the elasticity of substitution between labor and carbon, ς, is set to one, compared to $\varsigma = 0.624$ in our main analysis. We repeat our analysis under these alternative specifications for both the IRS and CRS cases of our model. Third, we consider an alternative policy scenario in which governments do not pursue terms-of-trade objectives. In this case, border taxes are chosen solely to reduce transboundary CO_2 emissions.[42] We compare the outcome arising from this counterfactual policy scenario to a baseline with zero tariffs and another with currently applied tariffs.

Lastly, we report the consequences of *globally* optimal carbon border taxes in Appendix F. As noted in Section 4.2, globally optimal carbon border taxes are relevant when governments are cooperative on climate issues but can only use border tax measures to curb global CO_2 emissions.

[41] They report 61 \$/tC for 2005 and 87 \$/tC for 2020. Making a linear assumption, we arrive at 68 \$/tC for 2009, which is our baseline year.

[42] We conduct this exercise by assigning a zero weight to the terms-of-trade component of the optimal border tax. Such a tax schedule can be rationalized as the optimal choice of a government that assigns zero weight to terms-of-trade gains in their objective function, but values CO_2 reduction.

In Appendix F we use our analytical formula for globally optimal carbon border taxes (Equation 29) to compute the efficacy of these taxes. The results reported in Table A.1 of Appendix F indicate that globally optimal carbon border taxes can reduce global CO_2 emissions by only 0.3%, as they suffer from the same limitations as non-cooperative carbon border taxes.

Under each alternative specification and policy scenario, carbon border taxes deliver a limited reduction in global CO_2 emissions. Our main specification, recall, indicated that carbon border taxes can replicate 1% of the globally first-best carbon reduction. Across all the alternative specifications and scenarios we consider, the corresponding number ranges between 0.5% and 3.7%.[43] To illustrate this point clearly, Table A.5 in the appendix reports the extent of CO_2 reduction in each scenario relative to the globally first-best outcome.

6.2 Proposal 2: The Efficacy of a Climate Club with Trade Penalties

Our previous findings indicated that non-cooperative border taxes are a poor substitute for globally optimal carbon taxes. The latter is, however, difficult to implement due to the *free-riding* problem. All countries—even those with a high disutility from CO_2 emissions—have an incentive to free-ride on the rest of the world's carbon abatement without undertaking proportionate abatement themselves. This problem is exacerbated by international misalignment in climate objectives, as some governments may find the burden of globally optimal carbon taxes disproportionately large to justify commitment to international climate agreements.

Seeking a solution to this problem, Nordhaus (2015) proposes that climate-conscience governments form a *climate club*, and enforce climate cooperation via collective trade penalties against non-members. Quantifying the full effectiveness of the climate club has proven prohibitively challenging. It involves solving a high-dimensional strategic game in which countries apply optimal trade penalties on each other—a task is impractical with standard numerical optimization techniques. Our optimal import and export tax formulas, by design, characterize the maximum terms-of-trade penalty countries can inflict on their trading partners. In this sense, they determine the optimal trade tax penalty on non-cooperative countries in the climate club model. With the aid of these formulas, we can bypass the computational challenges that have impeded the past literature, and uncover the full efficacy of the climate club.

To set the stage for our analysis, we formally specify the *climate club* game. We let a subset of countries be core members of the club and other countries play strategically. Table 4 details the structure of the *one-shot* climate club game. To provide a verbal description, core members of the climate club commit to the rules of membership in the club and other governments choose their strategies (i.e., whether or not to join the club and what taxes to adopt in each case) simultaneously. The outcome is a Nash equilibrium if no country has an incentive to deviate:

[43] These findings have little precedent as they uncover the consequences of *optimal* carbon border adjustments. They, nonetheless, echo previous findings on this topic. Larch and Wanner (2017), for instance, find that equalizing implied international carbon taxes via border adjustments, reduces global CO_2 emissions by only 0.5%.

Table 4: Climate Club Game - Main Specification

	Border taxes/subsidies set by	
	Members	*Non-members*
Against Members	zero	unilaterally optimal
Against Non-members	unilaterally optimal	status quo (i.e., applied tariffs)

Carbon taxes set by	
Members	*Non-members*
globally optimal	status quo (i.e., unilaterally optimal)

– *Rules of Membership*—A member country must set zero border taxes against other members while imposing unilaterally optimal trade penalties (in the form of border taxes) against non-members. A member must adopt a globally optimal carbon tax that corrects the global externality associated with its CO_2 emissions. By design, core members commit to these rules, and others adopt them only if they decide to join the club.

– *Non-members' Response*—A non-member country can retaliate against member countries by applying its unilaterally optimal import taxes and export subsidies against them. Other than this, non-member countries retain their status quo tax policies: They keep their existing applied tariffs against other non-members and maintain their existing domestic carbon tax (that is suboptimal from a global standpoint).[44]

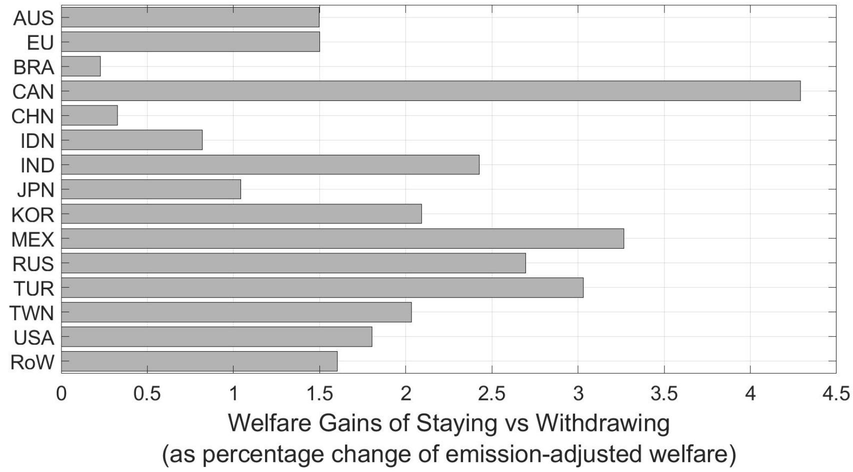
A non-member's decision to join the club is governed by the following trade-off: By joining the club the country incurs an efficiency loss due to adopting a higher-than-unilaterally-optimal carbon tax, but it escapes trade penalties imposed by climate club members. Another cost is that upon joining the climate, a country will find itself in a trade war with non-member countries. Overall, for a country to join the club, the cost of trade penalties imposed by club members must exceed the implied cost of joining the club. These costs evolve with the size of the climate club: A larger club posses greater collective market power and can impose more effective trade penalties to enforce further membership.

As an intermediate step, we find that the club-of-all nations is a Nash equilibrium no matter who the core members are. This point is illustrated in Figure 1, where every bar reports the national-level welfare gains from staying in the club-of-all-nations relative to withdrawing unilaterally. Larger countries with relatively low climate concerns, such as China or Brazil, exhibit the lowest net gains from climate club membership. This intermediate finding is, however, informative only if the club-of-all nations is the unique Nash equilibrium.

Next, we wish to determine if the club-of-all-nations is the *unique* Nash equilibrium. This task

[44] We are here adopting a conservative approach in which the motivation of non-members to join the club is mainly driven by members' penalty taxes. To see this clearly, suppose we were to allow non-member countries to set their unilaterally optimal border taxes against other non-members. In that case, non-members would have an extra motive beyond climate-related concerns to join the club, as there would be a trade war between non-member countries. Then, joining the club would be a way to escape from that trade war. In contrast, in our specification the motivation to join the club stems merely from members' trade penalties.

Figure 1: Welfare Gains of Staying vs Leaving the Club-of-All-Nations



Note: This figure shows for every country the percentage change to welfare of staying in the club-of-all-nations relative to leaving the club-of-all-nations unilaterally.

is complicated by the curse of dimensionality: with m core members, we have to check 2^{N-m} combinations of national-level strategies. Without a systemic way to shrink the outcome space, the aforementioned task will be virtually infeasible with standard computational techniques. We overcome this computational barrier by way of iterative elimination of dominated strategies, using our optimal tax formulas to expedite each iteration. In the first round, we identify the set of countries for which joining the club is a dominant strategy. Given the set of countries that join the club in the first round, we identify the set of countries for which joining the club is a dominant strategy in the second round and so forth.[45]

We confirm that when the EU is the only core climate club member, the club-of-only-EU is also a Nash equilibrium. That is, the net gains from joining the club-of-only-EU is negative for every country when all other countries also stay out. Accordingly, the EU alone cannot overcome the free-riding problem that has impeded past climate agreements. To overcome the free-riding problem, the set of core members must be larger than only the EU.

We next consider the case where the EU and the US commit to the climate club as core members. This coalition creates sufficient incentives for some of the US and EU's major trading partners to join the club in the first and second rounds. The accession of these new members incentivizes other countries—who were previously reluctant—to join the club in subsequent rounds, leading to the club-of-all-nations as the unique Nash equilibrium. Chart 5 shows the sequence by which joining the climate club becomes a dominant strategy for various countries.[46]

These results indicate that, with sufficient commitment, the climate club proposal can successfully overcome the free-riding problem and induce global climate cooperation. As reported earlier, global climate cooperation will bring along a 61% reduction in global CO_2 emissions. Out of this 61% reduction, 8.3% is accounted for by the core members, the EU and US, and the remain-

[45] A key property of the *climate club* game that makes this approach clinical is that the net gains from joining the club

Table 5: Climate Club Game - Successive Dominant Strategies

Core Members	*1st Round*	*2nd Round*	*3rd Round*	*4th Round*
EU, USA	CAN, RoW	AUS, IND, JPN, KOR, MEX, RUS, TUR, TWN	CHN	BRA, IDN

ing 52.7% derives from non-core member's abatement. Overall, these results indicate that the climate club's success hinges on the make-up of its core members. The EU alone lacks sufficient market power to induce global climate cooperation. The EU and US together, however, possess enough collective market power to incentivize climate cooperation across the globe.

Sensitivity Analysis & Tradeoff between Carbon Tax Target and Participation. Our baseline analysis found that a US-EU climate club can induce global climate cooperation and deliver the globally first-best outcome. Below, we examine the sensitivity of this result along two dimensions. We recalculate (1) the reduction in CO_2 emissions under the globally first-best outcome, and (2) the performance of the climate club at reaching this outcome under alternative values for the carbon demand elasticity, ς, and the social cost of carbon (SCC).

Figure A.5 in Appendix G plots the reduction in global CO_2 emissions under the globally first-best outcome for all $\varsigma \in (0, 1)$, under two values of SCC: namely, 31 $/tC, as in our main specification, and 68$/tC, which is borrowed from Cai and Lontzek (2019). As expected, the estimated reduction in global CO_2 emissions is lower the smaller the carbon demand elasticity. The intuition is that firms cut their CO_2 emissions less aggressively when it is harder to substitute carbon inputs with labor in response to a carbon tax hike. The least amount of globally-first-best CO_2 reduction occurs under the Leontief production function, where $\varsigma \to 0$. In this limiting case, global CO_2 emissions decline by 10% and 15% for SCC values of 31 $/tC and 68 $/tC.

Tables A.7-A.9 in Appendix G depict the performance of the climate club when the SCC equals 68$/tC instead of 31$/tC, and when ς is set at 0.25 or 0.99 instead of 0.63. Each table shows the rounds of successive membership, maintaining the assumption that the EU and US are core members. In the case where $\varsigma = 0.99$, we obtain similar results as in our main specification. When SCC is raised to 68$/tC or the carbon demand elasticity, ς, is set at 0.25, the club-of-all-nations is no longer a Nash equilibrium, since Brazil, China, India, and Indonesia choose to stay out of the club. All the other countries, however, join the club in successive rounds.

These findings highlight a key trade-off facing the design of the climate club. If the SSC is high,

rise with the size of the club. For example, a country that stays outside of the club in the first round may find it optimal to join the club in the second round provided that one or two additional countries join the club in the first round. While our game is a one-shot static game, this procedure can provide a rare glimpse into the possible expansion path of the climate club.

[46] In addition, we have performed this analysis for the constant-returns-to-scale version of our model. Our main takeaways remain the same: the club-of-all-nations is a Nash equilibrium; and, both the EU and US are required to be core members to make the club-of-all-nations the unique outcome. Figure A.4 and Chart A.6 show these results under the CRS case. Under CRS, the net gains from joining the club are somewhat larger than IRS, leading to only three rounds to reach the club-of-all-nations as the unique outcome, with a bit larger net gains of staying there compared to those under IRS.

the globally optimal carbon tax rate can be too high to justify membership by some countries. In such cases, core members can incentivize participation by lowering the carbon tax requirement below its globally optimal level. Indeed, for any set of model parameters, the club-of-all-nations can be formed via a carbon tax target that is higher than the unilaterally optimal rate but possibly lower than the globally optimal rate implied by larger values of the SCC. The trade-off is that each club member cuts their emissions to a lesser extent but the club elicits more participation.[47]

7 Conclusion

The realization that we are approaching a tipping point in climate change has raised public awareness of climate policy. Experts are advocating for the use of border tax measures to mitigate global CO_2 emissions and climate change. This paper examined two popular climate proposals that leverage border tax measures, but in fundamentally different ways: The first proposal advocates for carbon border taxes as a second-best device to curb transboundary CO_2 emissions. The second proposal (namely, the *climate club*) advocates for border taxes as a penalty device to incentivize climate cooperation by reluctant governments.

To evaluate these two climate proposals, we characterized optimal border and carbon tax policies in a multi-country, multi-industry, general equilibrium trade model featuring abatement technology, scale economies, and transboundary carbon externality. Our analysis delivered simple analytical formulas for unilaterally and globally optimal carbon and border taxes. Capitalizing on our theoretical framework and optimal policy formulas, we computed the *full* efficacy of carbon border taxes and the climate club proposal—a task that has eluded the past literature.

Our findings indicated that *carbon border taxes* can replicate only 1% of the CO_2 reduction attainable under globally first-best carbon taxes. In contrast, optimal trade penalties under the *climate club* model can deliver the globally first-best outcome and reduce global CO_2 emissions by 61%. This successful outcome, however, hinges on the initial makeup of the climate club's core members—it is possible only if, in addition to the European Union, the United States commits to the climate club as a core member.

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[47] The following comparison elucidates this trade-off: In our main specification, where the climate club requires a carbon tax rate of 31\$/tC, all countries join the club and collectively reduce CO_2 emissions by 61%. If the targeted carbon tax rate is raised from 31\$/tC to 68\$/tC, Brazil, China, India, and Indonesia—which account for more than one-third of global CO_2 emissions—would stay out of the club. In this case, the higher emission reduction by the remaining members does not compensate for loss in participation: With a carbon tax target of 68\$/tC, the climate club reduces CO_2 emissions by 35% compared to 61% under the lower target of 31\$/tC.

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Appendices (for online publication)

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A Theoretical Preliminaries

A.1 Detailed Statement of the Unilaterally Optimal Policy Problem

We layout the unilaterally optimal policy for the government in country i, with minimal set of equations that constrain the problem with general equilibrium relationships. We do this for the reformulated optimal policy problem, as discussed under Lemma 1. We denote by $\mathbb{P}_i \equiv \{\tilde{P}_{ii,k}, \tilde{P}_{ji,k}, \tilde{P}_{ij,k}, a_{i,k}\}_{j\neq i, j\in\mathbb{C}, k\in\mathbb{K}}$ the policy instruments in country i, by $\tilde{\mathbf{P}}_i \equiv \{\tilde{P}_{ji,k}\}_{j\in\mathbb{C}, k\in\mathbb{K}} \in \mathbb{P}_i$ the vector of consumer prices in country i, by $\mathbf{w} \equiv \{w_j\}_{j\in\mathbb{C}}$ the vector of wages, and by $\mathbf{Z} \equiv \{Z_{n,g}\}_{n\in\mathbb{C}, g\in\mathbb{K}}$ the long vector of CO_2 emissions. The problem of the government in country i is:

$$\max_{\mathbb{P}_i} \quad V_i(Y_i, \tilde{\mathbf{P}}_i) - \Delta_i(\mathbf{Z}),$$

subject to the general equilibrium relationships, characterized by the following: for all $i, j \in \mathbb{C}$,and $k \in \mathbb{K}$,

$$\begin{aligned}
&\text{(Optimal Demand)} && Q_{ji,k} = \mathcal{D}_{ji,k}(Y_i, \tilde{\mathbf{P}}_i)\\
&\text{(Aggregate Output)} && Q_{j,k} = \sum_i \bar{d}_{ji,k} Q_{ji,k}\\
&\text{(Producer Price)} && P_{ji,k}(w_j, a_{j,k}; Q_{j,k}) = \bar{d}_{ji,k}\bar{p}_{jj,k} w_j (1-a_{j,k})^{\frac{1}{\varsigma\gamma_k}-\frac{1}{\varsigma}} Q_{j,k}^{-\frac{1}{\gamma_k}}\\
&\text{(Carbon Cost Share)} && \alpha_{j,k} = 1-(1-\bar{\kappa}_{j,k})(1-a_{j,k})^{\frac{1-\varsigma}{\varsigma}}\\
&\text{(Carbon Emission)} && Z_{j,k} = \bar{z}_{j,k}\left(\frac{\alpha_{j,k}}{\bar{\kappa}_{j,k}}\right)^{\frac{\varsigma}{\varsigma-1}} (1-a_{j,k})^{\frac{1}{\varsigma\gamma_k}} Q_{j,k}^{1-\frac{1}{\gamma_k}}\\
&\text{(Income = Revneue)} && Y_i = w_i\bar{L}_i + \sum_{k,\, j\neq i}\left[\left(\tilde{P}_{ji,k} - P_{ji,k}\right)Q_{ji,k}\right] + \sum_{k,\, j}\left[\left(\tilde{P}_{ij,k} - (1-\alpha_{i,k}\frac{\gamma_k-1}{\gamma_k})P_{ij,k}\right)Q_{ij,k}\right]\\
&\text{(Balanced Trade)} && B_i \equiv \sum_{j\neq i}\sum_k P_{ji,k}Q_{ji,k} - \sum_{j\neq i}\sum_k \tilde{P}_{ij,k}Q_{ij,k} = 0
\end{aligned}$$

The demand function $\mathcal{D}_{ji,k}(Y_i, \tilde{\mathbf{P}}_i)$ is characterized by the set of price and income elasticities of demand, defined in Section 2.1. (Aggregate Output) indicates the output quantity in country $j-$industry k. Equations (Producer Price), (Carbon Cost Share), and (Carbon Emission) reproduce (3), (2), and (4). Equation (Income = Revenue) reproduces (8) by replacing for taxes from (6), and (Balanced Trade) is equivalent to labor market clearing condition (7) —See footnote 13). Throughput our proof, we assign the labor in one foreign country as the numeraire.

A.2 Expressing General Equilibrium as a Function of $(\mathbb{P}_i; \mathbf{w}, \mathbf{Y})$

Consider system $\mathscr{S}(\mathbb{P}_i, \mathbf{w}, \mathbf{Y})$ that defines a semi-equilibrium (similar to that in Section 2.4) by encompassing all equilibrium conditions excluding the national-level labor-market clearing and balanced budget conditions. This system determines equilibrium outcomes (other than $\mathbf{w}$ and $\mathbf{Y}$) conditional on choices of policy, wages, and income levels.

In the main text, we defined general equilibrium by using the combination $\mathbb{X} = (\mathbb{I}; \mathbf{w}, \mathbf{Y})$. Here, we define

$$\mathbb{X}^r = (\mathbb{P}_i; \mathbf{w}, \mathbf{Y}),$$

where the tilde notation on $\mathbb{X}^r$ indicates that we are considering the reformulated problem, in which the government uses $\mathbb{P}$, rather than $\mathbb{I}$, as the set of policy instruments. In addition, since the unilaterally optimal policy of government i takes policy choices in other countries as given, here we simplify the notation by setting taxes in the rest of the world to zero and exclude $\mathbb{P}_{-i}$ from $\mathbb{X}^r$.

Definition. [Semi-equilibrium expressed in terms of $\mathbb{X}^r = (\mathbb{P}_i; \mathbf{w}, \mathbf{Y})$] A semi-equilibrium, labeled as $\mathscr{S}(\mathbb{P}_i, \mathbf{w}, \mathbf{Y})$, consists of the following set of equations: For all $n, j \in \mathbb{C}$,and $k \in \mathbb{K}$:

$$\begin{aligned}
&\text{(Optimal Demand)} && Q_{nj,k}(\mathbb{X}^r) = \begin{cases} \mathcal{D}_{ni,k}(\tilde{\mathbf{P}}_i, Y_i) & \text{if } j = i \\ \mathcal{D}_{nj,k}(\tilde{\mathbf{P}}_{ij}, \{\tilde{\mathbf{P}}_{nj}(\mathbb{X}^r)\}_{n \neq i}, Y_j) & \text{if } j \neq i \end{cases} \\
&\text{(Indusry Output)} && Q_{n,k}(\mathbb{X}^r) = \textstyle\sum_{j \in \mathbb{C}} \bar{d}_{nj,k} Q_{nj,k}(\mathbb{X}^r) \\
&\text{(Producer Price)} && P_{nj,k}(\mathbb{X}^r) = \bar{d}_{nj,k} \bar{p}_{nn,k} w_n \left(1 - a_{n,k}\right)^{\frac{1}{\varsigma \gamma_k} - \frac{1}{\varsigma}} \left(Q_{n,k}(\mathbb{X}^r)\right)^{-\frac{1}{\gamma_k}} \\
&\text{(Carbon Cost Share)} && \alpha_{n,k}(\mathbb{X}^r) = 1 - \left(1 - \bar{\kappa}_{n,k}\right)\left(1 - a_{n,k}\right)^{\frac{1-\varsigma}{\varsigma}} \\
&\text{(Carbon Emission)} && Z_{n,k}(\mathbb{X}^r) = \bar{z}_{n,k} \left(\frac{\alpha_{n,k}(\mathbb{X}^r)}{\bar{\kappa}_{n,k}}\right)^{\frac{\varsigma}{\varsigma - 1}} \left(1 - a_{n,k}\right)^{\frac{1}{\varsigma \gamma_k}} \left(Q_{n,k}(\mathbb{X}^r)\right)^{1 - \frac{1}{\gamma_k}} \\
&\text{(Tax Revenues)} && T_n(\mathbb{X}^r) = \begin{cases} \sum_{k,\, j \neq i} \left[\left(\tilde{P}_{ji,k} - P_{ji,k}(\mathbb{X}^r)\right) Q_{ji,k}(\mathbb{X}^r)\right] & \text{if } n = i \\ + \sum_{k,\, j} \left[\left(\tilde{P}_{ij,k} - (1 - \alpha_{n,k}(\mathbb{X}^r) \frac{\gamma_k - 1}{\gamma_k}) P_{ij,k}(\mathbb{X}^r)\right) Q_{ij,k}(\mathbb{X}^r)\right] & \\ 0 & \text{if } n \neq i \end{cases}
\end{aligned}$$

Here, $\tilde{\mathbf{P}}_i \subset \mathbb{P}_i$ is the vector of consumer prices in home country i, $\tilde{\mathbf{P}}_{ij} \subset \mathbb{P}_i$ is the vector of consumer prices in foreign country j of varieties produced in home, and $a_{i,k} \in \mathbb{P}_i$ is the abatement in home. All these are instruments of policy to be chosen by the home government that is indexed by i. In contrast, every foreign country $n \neq i$ has some fixed abatement level $a_{n,k} = \bar{a}_{n,k}$ and no tax revenues $T_n = 0$. As noted earlier, the semi-equilibrium $\mathscr{S}(\mathbb{P}_i, \mathbf{w}, \mathbf{Y})$ characterizes quantities, producer prices, carbon cost shares, carbon emissions, and tax revenues in all economies as a function of $\mathbb{X}^r = (\mathbb{P}_i; \mathbf{w}, \mathbf{Y})$. Correspondingly, welfare in country i can be expressed as,

$$W_i(\mathbb{X}^r) = V_i(w_i \bar{L}_i + T_i(\mathbb{X}^r), \tilde{\mathbf{P}}_i) - \Delta_i(\mathbf{Z}(\mathbb{X}^r)),$$

where note that $\tilde{\mathbf{P}}_i \subset \mathbb{X}^r$. By design, semi-equilibrium $\mathscr{S}(\mathbb{P}_i, \mathbf{w}, \mathbf{Y})$ excludes the national-level labor market clearing and balanced budget conditions. Instead, it takes $\mathbf{w}$ and $\mathbf{Y}$ as given, and characterizes equilibrium outcomes as a function of $\mathbb{P}_i$, $\mathbf{w}$, and $\mathbf{Y}$. All combinations $\mathbb{X}^r = (\mathbb{P}_i; \mathbf{w}, \mathbf{Y})$, however, are not necessarily consistent with the labor market clearing and balanced budget conditions. So, when characterizing optimal policy we must restrict attention to the feasible policy–wage–income combinations.

Definition. [General equilibrium expressed in terms of $\mathbb{X}^r = (\mathbb{P}_i; \mathbf{w}, \mathbf{Y})$] Given the policy instruments $\mathbb{P}_i$, a general equilibrium is a combination $\mathbb{X}^r = (\mathbb{P}_i; \mathbf{Y}, \mathbf{w})$ such that the system of semi-equilibrium $\mathscr{S}(\mathbb{P}_i, \mathbf{w}, \mathbf{Y})$ holds, and income levels and wages, $(\boldsymbol{Y}, \boldsymbol{w})$, satisfy the national-level labor market clearing and balanced budget conditions:

$$w_i \bar{L}_i - \sum_{k \in \mathbb{K}} \sum_{j \in \mathbb{C}} (1 - \alpha_{i,k}(\mathbb{X}^r) \frac{\gamma_k - 1}{\gamma_k}) P_{ij,k}(\tilde{\mathbb{X}}) Q_{ij,k}(\mathbb{X}^r) = 0, \qquad \text{(LMC)}$$

$$Y_i = w_i \bar{L}_i + T_i(\mathbb{X}^r), \qquad \text{(BB)}$$

A.3 Expressing General Equilibrium as a Function of $(\mathbb{P}_i; Y_i)$

Following Lemma 2, when needed, we can treat wages, $\bar{\mathbf{w}} = \{\bar{w}_n\}_{n \in \mathbb{C}}$, as fixed; and since each foreign country $j \neq i$ is passive and does not impose taxes, its income, $\bar{Y}_j = \bar{w}_j \bar{L}_j$, can be also treated as fixed. Hence, we can reduce the combination $(\mathbb{P}_i; \mathbf{w}, \mathbf{Y})$ to $(\mathbb{P}_i; Y_i)$, and redefine the general equilibrium accordingly. Consider a system in which wages and income levels in foreign countries are constant, and all equilibrium conditions hold excluding the balanced budget condition. We refer to this system of equations as a semi-equilibrium defined in terms of $(\mathbb{P}_i; Y_i)$, which we label as $\mathscr{S}(\mathbb{P}_i, Y_i)$.

Definition. [Semi-equilibrium expressed in terms of $(\mathbb{P}_i; Y_i)$] A semi-equilibrium, labeled as $\mathscr{S}(\mathbb{P}_i, Y_i)$, consists of the following set of equations: For all $n, j \in \mathbb{C}$,and $k \in \mathbb{K}$,

$$
\begin{array}{rl}
\text{(Optimal Demand)} & Q_{nj,k}(\mathbb{P}_i; Y_i) = \begin{cases} \mathcal{D}_{ni,k}(\tilde{\mathbf{P}}_i, Y_i) & \text{if } j = i \\ \mathcal{D}_{nj,k}(\tilde{\mathbf{P}}_{ij}, \{\tilde{\mathbf{P}}_{nj}(\mathbb{P}_i; Y_i)\}_{n \neq i}, \bar{Y}_j) & \text{if } j \neq i \end{cases} \\
\text{(Indusry Output)} & Q_{n,k}(\mathbb{P}_i; Y_i) = \sum_{j \in \mathbb{C}} \bar{d}_{nj,k} Q_{nj,k}(\mathbb{P}_i; Y_i) \\
\text{(Producer Price)} & P_{nj,k}(\mathbb{P}_i; Y_i) = \bar{d}_{nj,k} \bar{p}_{nn,k} \bar{w}_n \left(1 - a_{n,k}\right)^{\frac{1}{\varsigma \gamma_k} - \frac{1}{\varsigma}} \left(Q_{n,k}(\mathbb{P}_i; Y_i)\right)^{-\frac{1}{\gamma_k}} \\
\text{(Carbon Cost Share)} & \alpha_{n,k}(\mathbb{P}_i; Y_i) = 1 - \left(1 - \bar{\kappa}_{n,k}\right) \left(1 - a_{n,k}\right)^{\frac{1-\varsigma}{\varsigma}} \\
\text{(Carbon Emission)} & Z_{n,k}(\mathbb{P}_i; Y_i) = \bar{z}_{n,k} \left(\frac{\alpha_{n,k}(\mathbb{P}_i; Y_i)}{\bar{\kappa}_{n,k}}\right)^{\frac{\varsigma}{\varsigma - 1}} \left(1 - a_{n,k}\right)^{\frac{1}{\varsigma \gamma_k}} \left(Q_{n,k}(\mathbb{P}_i; Y_i)\right)^{1 - \frac{1}{\gamma_k}} \\
\text{(Taxes)} & T_n(\mathbb{P}_i; Y_i) = \begin{cases} \sum_{k,\, j \neq i} \left[\left(\tilde{P}_{ji,k} - P_{ji,k}(\mathbb{P}_i; Y_i)\right) Q_{ji,k}(\mathbb{P}_i; Y_i) \right] & \text{if } n = i \\ + \sum_{k,\, j} \left[\left(\tilde{P}_{ij,k} - (1 - \alpha_{i,k}(\mathbb{P}_i; Y_i) \frac{\gamma_k - 1}{\gamma_k}) P_{ij,k}(\mathbb{P}_i; Y_i)\right) Q_{ij,k}(\mathbb{P}_i; Y_i) \right] & \\ 0 & \text{if } n \neq i \end{cases} \\
\text{(LMC)} & \bar{w}_i \bar{L}_i - \sum_{k \in \mathbb{K}} \sum_{j \in \mathbb{C}} (1 - \alpha_{i,k}(\mathbb{P}_i; Y_i) \frac{\gamma_k - 1}{\gamma_k}) P_{ij,k}(\mathbb{P}_i; Y_i) Q_{ij,k}(\mathbb{P}_i; Y_i) = 0
\end{array}
$$

In the system of semi-equilibrium $\mathscr{S}(\mathbb{P}_i, Y_i)$ compared to $\mathscr{S}(\mathbb{P}_i, \mathbf{w}, \mathbf{Y})$, we drop $\mathbf{w}$ and $\mathbf{Y}_{-i} \equiv \{Y_j\}_{j \neq i}$, while we add the labor market clearing condition (LMC). That is, we are considering fixed wages that satisfy LMC. We can correspondingly express welfare in country i as,

$$W_i(\mathbb{P}_i; Y_i) = V_i(\bar{w}_i \bar{L}_i + T_i(\mathbb{P}_i; Y_i), \tilde{\mathbf{P}}_i) - \Delta_i(\mathbf{Z}(\mathbb{P}_i; Y_i)).$$

where $\tilde{\mathbf{P}}_i$ is itself a subset of $\mathbb{P}_i$.

Definition. [General equilibrium expressed in terms of $(\mathbb{P}_i; Y_i)$] Given the policy instrument, $\mathbb{P}_i$, a general equilibrium is a combination $(\mathbb{P}_i; Y_i)$ such that the system of semi-equilibrium $\mathscr{S}(\mathbb{P}_i, Y_i)$ holds, and country i's income, Y_i, satisfies the balanced budget condition:

$$Y_i = \bar{w}_i \bar{L}_i + T_i(\mathbb{P}_i; Y_i), \qquad \text{(BB)}$$

A.4 Characterizing Equilibrium Wage and Income Effects

Suppose we formulate all equilibrium variables as a function of $\mathbb{P}_i$, $\mathbf{w}$, and $\mathbf{Y}$ as detailed in Appendix A.2. The feasible vector of wages, $\mathbf{w}$, and income levels, $\mathbf{Y}$, solve the system of labor market clearing conditions:

$$
\begin{cases}
f_1(\mathbb{P}_i; \mathbf{w}, \mathbf{Y}) \equiv w_1 L_1 - \sum_{j \in \mathbb{C}} \sum_{k \in \mathbb{K}} \left[(1 - \alpha_{1,k}(\mathbb{P}_i; \mathbf{w}, \mathbf{Y}) \frac{\gamma_k - 1}{\gamma_k}) P_{1j,k}(\mathbb{P}_i; \mathbf{w}, \mathbf{Y}) Q_{1j,k}(\mathbb{P}_i; \mathbf{w}, \mathbf{Y}) \right] = 0 \\
\vdots \\
f_N(\mathbb{P}_i; \mathbf{w}, \mathbf{Y}) \equiv w_N L_N - \sum_{j \in \mathbb{C}} \sum_{k \in \mathbb{K}} \left[(1 - \alpha_{N,k}(\mathbb{P}_i; \mathbf{w}, \mathbf{Y}) \frac{\gamma_k - 1}{\gamma_k}) P_{Nj,k}(\mathbb{P}_i; \mathbf{w}, \mathbf{Y}) Q_{Nj,k}(\mathbb{P}_i; \mathbf{w}, \mathbf{Y}) \right] = 0
\end{cases}
\tag{A.1}
$$

and the national-level balanced budget conditions:

$$
\begin{cases}
g_1(\mathbb{P}_i; \mathbf{w}, \mathbf{Y}) \equiv Y_1 - w_1 L_1 - T_1(\mathbb{P}_i; \mathbf{w}, \mathbf{Y}) = 0 \\
\vdots \\
g_N(\mathbb{P}_i; \mathbf{w}, \mathbf{Y}) \equiv Y_N - w_N L_N - T_N(\mathbb{P}_i; \mathbf{w}, \mathbf{Y}) = 0
\end{cases}.
$$

Note that by Walras' law one equation is redundant so we can assign one element of $\mathbf{w}$ as the numeraire:

$$\sum_n f_n(\mathbb{P}_i; \mathbf{w}, \mathbf{Y}) = 0. \qquad \text{[Walras' Law]}.$$

Recall that the F.O.C. with respect to any $\mathsf{P} \in \mathbb{P}_i$, can be stated as follows:

$$\frac{\mathrm{d}W_i(\mathbb{P}_i^\star; \mathbf{w}^\star, \mathbf{Y}^\star)}{\mathrm{d}\ln \mathsf{P}} = \frac{\partial W_i(\mathbb{P}_i^\star; \mathbf{w}^\star, \mathbf{Y}^\star)}{\partial \ln \mathsf{P}} + \underbrace{\frac{\partial W_i(\mathbb{P}_i^\star; \mathbf{w}^\star, \mathbf{Y}^\star)}{\partial \mathbf{w}} \cdot \frac{\mathrm{d}\mathbf{w}}{\mathrm{d}\ln \mathsf{P}}}_{\text{GE wage effects}} + \underbrace{\frac{\partial W_i(\mathbb{P}_i^\star; \mathbf{w}^\star, \mathbf{Y}^\star)}{\partial \mathbf{Y}} \cdot \frac{\mathrm{d}\mathbf{Y}}{\mathrm{d}\ln \mathsf{P}}}_{\text{GE income effects}} = 0$$

We can characterize the terms $\frac{\mathrm{d}\mathbf{w}}{\mathrm{d}\ln \mathsf{P}}$ and $\frac{\mathrm{d}\mathbf{Y}}{\mathrm{d}\ln \mathsf{P}}$in the F.O.C. by applying the Implicit Function Theorem to the above system of equations characterized by functions $\boldsymbol{f}(\mathbb{P}_i; \mathbf{w}, \mathbf{Y}) \equiv \{f_n(\mathbb{P}_i; \mathbf{w}, \mathbf{Y})\}_n$ and $\boldsymbol{g}(\mathbb{P}_i; \mathbf{w}, \mathbf{Y}) \equiv \{g_n(\mathbb{P}_i; \mathbf{w}, \mathbf{Y})\}_n$ as follows:

$$\begin{bmatrix} \frac{\mathrm{d}\mathbf{w}}{\mathrm{d}\ln \mathsf{P}} \\ \frac{\mathrm{d}\mathbf{Y}}{\mathrm{d}\ln \mathsf{P}} \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{\partial \boldsymbol{f}(\mathbb{P}_i;\mathbf{w},\mathbf{Y})}{\partial \mathbf{w}} & \frac{\partial \boldsymbol{f}(\mathbb{P}_i;\mathbf{w},\mathbf{Y})}{\partial \mathbf{Y}} \\ \frac{\partial \boldsymbol{g}(\mathbb{P}_i;\mathbf{w},\mathbf{Y})}{\partial \mathbf{w}} & \frac{\partial \boldsymbol{g}(\mathbb{P}_i;\mathbf{w},\mathbf{Y})}{\partial \mathbf{Y}} \end{bmatrix}^{-1}}_{\mathbf{A}_i^{-1}} \begin{bmatrix} \frac{\partial \boldsymbol{f}(\mathbb{P}_i;\mathbf{w},\mathbf{Y})}{\mathrm{d}\ln \mathsf{P}} \\ \frac{\partial \boldsymbol{g}(\mathbb{P}_i;\mathbf{w},\mathbf{Y})}{\mathrm{d}\ln \mathsf{P}} \end{bmatrix}. \tag{A.2}$$

Approximating $\frac{\mathrm{d}\mathbf{w}_{-i}}{\mathrm{d}\ln \mathsf{P}}$. Note that with only two countries, i.e., $\mathbb{C} = \{i, -i\}$, Walras' law implies that $\frac{\mathrm{d}\mathbf{w}_{-i}}{\mathrm{d}\ln \mathsf{P}} = 0$. Below, we show for all practical purposes, $\frac{\mathrm{d}\mathbf{w}_{-i}}{\mathrm{d}\ln \mathsf{P}} \approx \mathbf{0}$ up to a choice of numeraire. This result is especially helpful when characterizing optimal policy in second-best scenarios. Let us briefly focus on Cobb-Douglas abatement technologies and constant returns to scale production ($\gamma_k \to \infty$). This simplification helps us convey our main point succinctly; but does not imply it. We can characterize the top-left block in matrix $\mathbf{A}_i$:

$$\frac{\partial \boldsymbol{f}(\mathbb{P}_i; \mathbf{w}, \mathbf{Y})}{\partial \mathbf{w}} = \begin{bmatrix} \frac{\partial f_1(\mathbb{P}_i;\mathbf{w},\mathbf{Y})}{\partial w_1} & \frac{\partial f_1(\mathbb{P}_i;\mathbf{w},\mathbf{Y})}{\partial w_2} & \cdots & \frac{\partial f_1(\mathbb{P}_i;\mathbf{w},\mathbf{Y})}{\partial w_N} \\ \frac{\partial f_2(\mathbb{P}_i;\mathbf{w},\mathbf{Y})}{\partial w_1} & \frac{\partial f_2(\mathbb{P}_i;\mathbf{w},\mathbf{Y})}{\partial w_2} & \cdots & \frac{\partial f_2(\mathbb{P}_i;\mathbf{w},\mathbf{Y})}{\partial w_N} \\ \vdots & \ddots & \ddots & \vdots \\ \frac{\partial f_N(\mathbb{P}_i;\mathbf{w},\mathbf{Y})}{\partial w_1} & \frac{\partial f_N(\mathbb{P}_i;\mathbf{w},\mathbf{Y})}{\partial w_2} & \cdots & \frac{\partial f_N(\mathbb{P}_i;\mathbf{w},\mathbf{Y})}{\partial w_N} \end{bmatrix}$$

$$= \begin{bmatrix} -\sum_{n\neq i}\sum_{k,g}\left[\rho_{1n,k}\varepsilon_{1n,k}^{(1n,g)}\right] & \cdots & -\sum_{n\neq i}\sum_{k,g}\left[\rho_{1n,k}\varepsilon_{1n,k}^{(Nn,g)}\right] \\ -\sum_{n\neq i}\sum_{k,g}\left[\rho_{2n,k}\varepsilon_{2n,k}^{(1n,g)}\right] & \cdots & -\sum_{n\neq i}\sum_{k,g}\left[\rho_{2n,k}\varepsilon_{2n,k}^{(Nn,g)}\right] \\ \vdots & \ddots & \vdots \\ -\sum_{n\neq i}\sum_{k,g}\left[\rho_{Nn,k}\varepsilon_{Nn,k}^{(1n,g)}\right] & \cdots & -\sum_{n\neq i}\sum_{k,g}\left[\rho_{Nn,k}\varepsilon_{Nn,k}^{(Nn,g)}\right] \end{bmatrix},$$

where recall that $\rho_{jn,k}$ denotes the good-specific labor shares and $\varepsilon_{jn,k}^{(\ell n,g)}$ denotes the elasticity of good jn,k's demand w.r.t. the price of good $\ell n, g$. Likewise, the top-right block in matrix $\mathbf{A}_i$ can be expressed as follows:

$$\frac{\partial \boldsymbol{f}(\mathbb{P}_i; \mathbf{w}, \mathbf{Y})}{\partial \mathbf{Y}} = \begin{bmatrix} \frac{\partial f_1(.)}{\partial Y_1} & \cdots & \frac{\partial f_1(.)}{\partial Y_N} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_N(.)}{\partial Y_1} & \cdots & \frac{\partial f_N(.)}{\partial Y_N} \end{bmatrix} = \begin{bmatrix} -\sum_k\left[\rho_{11,k}\eta_{11,k}\right]\frac{w_1L_1}{Y_1} & \cdots & -\sum\left[\rho_{1N,k}\eta_{1N,k}\right]\frac{w_1L_1}{Y_1} \\ \vdots & \ddots & \vdots \\ -\sum_k\left[\rho_{N1,k}\eta_{N1,k}\right]\frac{w_NL_N}{Y_N} & \cdots & -\sum_k\left[\rho_{NN,k}\eta_{NN,k}\right]\frac{w_NL_N}{Y_N} \end{bmatrix}.$$

where $\eta_{jn,k}$ denotes the income elasticity of demand associated with good jn, k. Under Cobb-Douglas-CES preferences, $\eta_{jn,k} = 1$ for all jn, k. Also, the elements of $\frac{\partial \boldsymbol{f}(\mathbb{P}_i;\mathbf{w},\mathbf{Y})}{\partial \mathbf{w}}$ can be represented as follows:

$$\sum_{n\neq i}\sum_{k,g}\left[\rho_{\ell n,k}\varepsilon_{\ell n,k}^{(jn,g)}\right] = \begin{cases} \rho_{\ell i} + \sum_{n\neq i}\sum_k\left[(\sigma_k - 1)(1-\lambda_{\ell n,k})\rho_{\ell n,k}\right] & \text{if } \ell = j \\ -\sum_{n\neq i}\sum_k\left[(\sigma_k-1)\rho_{\ell n,k}\lambda_{jn,k}\right] & \text{if } \ell \neq j \end{cases}.$$

Actual data indicate that $\rho_{\ell j,k} \approx \sum_{n\neq i}\rho_{\ell n,k}\lambda_{jn,k} \approx 0$ if $j \neq \ell$. Appealing to this observation, the top-right and left blocks of $\mathbf{A}_i$ are near-diagonal matrixes:

$$\frac{\partial \boldsymbol{f}(\mathbb{P}_i; \mathbf{w}, \mathbf{Y})}{\partial \mathbf{Y}} \approx \operatorname{diag}\left([\rho_{nn}]_n\right); \qquad \frac{\partial \boldsymbol{f}(\mathbb{P}_i; \mathbf{w}, \mathbf{Y})}{\partial \mathbf{w}} \approx \operatorname{diag}\left(\left[\sum_k(\sigma_k - 1)\rho_{n,k}\right]_n\right).$$

Now, move on to the bottom-left and right block of matrix $\mathbf{A}_i$ in Equation A.2. Here, we appeal to the fact that the rest of world (outside of i) is passive, indicating that $T_n(\mathbb{P}_i; \mathbf{w}, \mathbf{Y}) = 0$ if $n \neq i$. Hence, the top-left and right blocks of $\mathbf{A}_i$ (aside from Row i) can written as

$$\frac{\partial g(\mathbb{P}_i; \mathbf{w}, \mathbf{Y})}{\partial \mathbf{w}} \approx \begin{bmatrix} -1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & -1 \end{bmatrix}; \qquad \frac{\partial g(\mathbb{P}_i; \mathbf{w}, \mathbf{Y})}{\partial \mathbf{Y}} \approx \begin{bmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \end{bmatrix}.$$

Note that both matrixes are different from the identity matrix, as Row i should account for the non-carbon tax revenues collected by country i. However, Row i is unimportant for our following line of argument. We only need that the above two matrixes be near-diagonal. Given the properties of block matrixes and Equation A.2, the vector $\frac{\mathrm{d}\mathbf{w}}{\mathrm{d}\ln \mathsf{P}}$ can be calculated as

$$\frac{\mathrm{d}\mathbf{w}}{\mathrm{d}\ln \mathsf{P}} = \left(\frac{\partial f}{\partial \mathbf{w}} - \frac{\partial f}{\partial \mathbf{Y}}\left(\frac{\partial g}{\partial \mathbf{Y}}\right)^{-1}\frac{\partial g}{\partial \mathbf{w}}\right)^{-1}\frac{\partial f}{\partial \ln \mathsf{P}} - \left(\frac{\partial f}{\partial \mathbf{w}} - \frac{\partial f}{\partial \mathbf{Y}}\left(\frac{\partial g}{\partial \mathbf{Y}}\right)^{-1}\frac{\partial g}{\partial \mathbf{w}}\right)^{-1}\frac{\partial f}{\partial \mathbf{Y}}\left(\frac{\partial g}{\partial \mathbf{Y}}\right)^{-1}\frac{\partial g}{\partial \ln \mathsf{P}}. \quad \text{(A.3)}$$

Note that the evaluation of the above equation is simplified by the fact that the elements of $\mathbf{A}_i$ are near-diagonal. Next, we need to characterize $\frac{\partial g(.)}{\partial \ln \mathsf{P}}$ and $\frac{\partial f(.)}{\partial \ln \mathsf{P}}$. To this end, we consider the case where $\mathsf{P} = \tilde{P}_{ji,k}$ and assign w_j as the numeraire. The derivative of $\boldsymbol{f}_{-j}$ (i.e., $\boldsymbol{f}$ excluding row j) w.r.t. $\tilde{P}_{ji,k}$ holding $\mathbf{w}$, $\mathbf{Y}$, and $\mathbb{P}_i - \tilde{P}_{ji,k}$ fixed is given by:

$$\frac{\partial \boldsymbol{f}_{-j}(\mathbb{P}_i; \mathbf{w}, \mathbf{Y})}{\partial \ln \tilde{P}_{ji,k}} = \begin{bmatrix} \frac{\partial f_1(\mathbb{P}_i;\mathbf{w},\mathbf{Y})}{\partial \ln \tilde{P}_{ji,k}} \\ \frac{\partial f_2(\mathbb{P}_i;\mathbf{w},\mathbf{Y})}{\partial \ln \tilde{P}_{ji,k}} \\ \vdots \\ \frac{\partial f_N(\mathbb{P}_i;\mathbf{w},\mathbf{Y})}{\partial \ln \tilde{P}_{ji,k}} \end{bmatrix} = \begin{bmatrix} \sum_g \rho_{1i,g}\varepsilon_{1i,g}^{(ji,k)} \\ \sum_g \rho_{2i,g}\varepsilon_{2i,g}^{(ji,k)} \\ \vdots \\ \sum_g \rho_{Ni,g}\varepsilon_{Ni,g}^{(ji,k)} \end{bmatrix} \xrightarrow{\text{Cobb-Douglas-CES}} = \begin{bmatrix} \rho_{1i,k} \\ \vdots \\ \rho_{j-1i,k} \\ \rho_{j+1i,k} \\ \vdots \\ \rho_{Ni,k} \end{bmatrix} (\sigma_k - 1)\lambda_{ji,k}$$

It is easy to check that the elements of $\frac{\partial g_{-j}(\mathbb{P}_i;\mathbf{w},\mathbf{Y})}{\partial \ln \tilde{P}_{ji,k}}$ are zero apart from the i'th element. Combining these two observations with Equation A.3 yields

$$\frac{\mathrm{d}\ln\mathbf{w}_{-i,j}}{\mathrm{d}\ln \tilde{P}_{ji,k}} \approx \left[\frac{\sigma_k - 1}{\sum_g(\sigma_g - 1)\rho_{n,g}}\rho_{ni,k}\lambda_{ji,k}\right]_{n\neq i,j} \approx 0,$$

where $\mathbf{w}_{-i,j} = \mathbf{w} - \{w_i, w_j\}$ and the last line follows from the empirical observation that $\rho_{ni,k}\lambda_{ji,k} \approx 0$ if $n \neq i$. The same steps can be taken with regards to any other price instrument in $\mathbb{P}_i$. Furthermore, the above argument goes through if we allow for a finite γ_k and a variable $\alpha_{n,k}$.

A.5 Emission-related Elasticities

Recall that total emission, as a function of abatement and output, is given by

$$Z_{j,k} = \bar{z}_{j,k}\left(\frac{\alpha_{j,k}}{\bar{\kappa}_{j,k}}\right)^{\frac{\varsigma}{\varsigma-1}}(1 - a_{j,k})^{\frac{1}{\varsigma\gamma_k}} Q_{j,k}^{1-\frac{1}{\gamma_k}}, \qquad \text{where} \qquad \alpha_{j,k} = 1 - (1 - \bar{\kappa}_{j,k})(1 - a_{j,k})^{-\frac{\varsigma-1}{\varsigma}}$$

To track the policy response of emission we use two following partial derivatives. The first one, accounts for scale effects in emission:

$$\frac{\partial \ln Z_{j,k}(a_{j,k}, Q_{j,k})}{\partial \ln Q_{j,k}} = 1 - \frac{1}{\gamma_k}, \quad \text{(A.4)}$$

and, the second one accounts for abatement effects in emission:

$$\frac{\partial \ln Z_{j,k}(a_{j,k}, Q_{j,k})}{\partial \ln(1 - a_{j,k})} = \frac{1}{\alpha_{j,k}} + \frac{1}{\varsigma \gamma_k} - 1. \tag{A.5}$$

Note that $a_{i,k}$ is directly chosen by the policy-maker in our reformulated optimal policy problem. $Q_{j,k} = Q_{j,k}(\mathbb{P}_i)$ is implicitly determined by the government's choice of policy instruments, $\mathbb{P}_i \equiv \{\tilde{P}_{ii,k}, \tilde{P}_{ji,k}, \tilde{P}_{ij,k}, a_{i,k}\}_{j \neq i, j \in \mathbb{C}, k \in \mathbb{K}}$.

In addition, recall that the cost share of carbon, $\alpha_{i,k}$, is endogenous, and its value depends on the choice of abatement. Using Equations (1) and (2), the partial derivative of $\alpha_{i,k}$ w.r.t. the choice of $(1 - a_{i,k})$ is given by:

$$\frac{\partial \ln \alpha_{i,k}}{\partial \ln(1 - a_{i,k})} = -\frac{1 - \varsigma}{\varsigma} \frac{1 - \alpha_{i,k}}{\alpha_{i,k}}$$

Notice, in the special case of $\varsigma \to 1$, i.e., when production technology features a Cobb-Douglas between carbon and labor, then the cost share of carbon is a constant parameter which does not change with the choice of abatement.

A.6 Export Supply Elasticities

Below, we define and characterize the export supply elasticity. To that end, we first introduce some intermediate partial derivatives that enter the export supply elasticity formula. These partial derivatives are also independently useful to our subsequent optimal analysis.

Following Lashkaripour and Lugovskyy (2020), the definition for the *general equilibrium* export supply elasticity can be expressed as follows

$$\omega_{ji,k} \equiv \frac{1}{r_{ji,k}\rho_{j,k}} \sum_g \left[\frac{w_i L_i}{w_j L_j} \rho_{i,g} \left(\frac{\partial \ln P_{ii,g}}{\partial \ln Q_{ji,k}} \right)_{\mathbb{P}_i, \mathbf{w}, \mathbf{Y}} + \sum_{n \neq i} \frac{w_n L_n}{w_j L_j} r_{ni,g} \rho_{n,g} \left(\frac{\partial \ln P_{ni,g}}{\partial \ln Q_{ji,k}} \right)_{\mathbb{P}_i, \mathbf{w}, \mathbf{Y}} \right], \tag{A.6}$$

where $r_{ni,g} = P_{ni,g} Q_{ni,g} / \sum_{\iota \in \mathbb{C}} \left(P_{n\iota,g} Q_{n\iota,g} \right)$ and $\rho_{n,g} = \sum_{\iota \in \mathbb{C}} \left(P_{n\iota,g} Q_{n\iota,g} \right) / w_n L_n$ respectively denote the good ni, g-specific and industry-wide sales shares associated with origin $n \in \mathbb{C}$. The above expression accounts for the fact that a change in the export supply of good ji, k will affect the producer price of goods supplied by *origin j–industry k* as well as other suppliers via cross-demand effects.

Before unpacking and simplifying Equation A.6, let us provide a brief description. Contracting the supply of good ji, k (i.e., $Q_{ji,k}$) increases the price of goods supplied by origin j–industry k through firm-entry (or scale) effects. Holding $\mathbb{P}_i$, $\mathbf{w}$, and $\mathbf{Y}$ constant, this change in price can affect the demand facing other suppliers via cross-demand effects in markets outside of i. Consumer prices in destination i are fixed by the government's choice vis-à-vis $\mathbb{P}_i$. So, once we fix $\mathbb{P}_i$, a change in $P_{in,g}$ has no bearing on the demand for other suppliers in market i. Outside of market i, however, a change in producer prices is completely passed on to consumer prices. Considering this, a change in $P_{jn,k}$ (which recall is triggered by a contraction in $Q_{ji,k}$) influences the demand for all suppliers serving market n. This change in demand, in turn, impacts the producer price of goods supplied by each international industry through scale effects. Equation A.6 measures how these changes impact country i's ToT.

We can follow the procedure in Lashkaripour and Lugovskyy (2020) to derive a simple first-order approximation for $\omega_{ji,k}$ in the absence of cross-industry demand effects. To this end, note that the producer price of good ni, g is given by $P_{ni,g} = \bar{d}_{ni,g} P_{nn,g}$, where $\bar{d}_{ni,g}$ denotes a constant iceberg trade cost and $P_{nn,g}$ denotes the price of goods supplied by *origin n–industry g* in the domestic market. As detailed in Section 2.2, $P_{nn,g}$ is an explicit function of *origin n–industry g*'s abatement, wage, and output schedule:

$$P_{nn,g}(a_{n,g}, w_n, Q_{n1,g}, ..., Q_{nN,g}) = \bar{p}_{nn,g} w_n (1 - a_{n,g})^{\frac{1}{\varsigma \gamma_g} - \frac{1}{\varsigma}} \mathcal{Q}_{n,g}(Q_{n1,g}, ... Q_{nN,g})^{-\frac{1}{\gamma_g}},$$

where $\mathcal{Q}_{n,g}(.)$ is total effective output in origin j–industry k, as given by

$$\mathcal{Q}_{j,k}(Q_{j1,k}, ... Q_{jN,k}) = \bar{d}_{j1,k} Q_{j1,k} + ... + \bar{d}_{jN,k} Q_{jN,k}.$$

Considering the above formulation, characterizing $\omega_{ji,k}$ requires that we first characterize $\left(\frac{\partial \ln P_{ni,g}}{\partial \ln Q_{ji,k}}\right)_{\mathbb{P}_i, \mathbf{w}, \mathbf{Y}} = \left(\frac{\partial \ln P_{ii,g}}{\partial \ln Q_{ji,k}}\right)_{\mathbb{P}_i, \mathbf{w}, \mathbf{Y}}$ for each *origin n–industry g*. For this purpose, define the following function for each *origin j–industry k* that treats $\mathbb{P}_i$, $\mathbf{w}$, and $\mathbf{Y}$ as given. In particular,

$$F_{j,g}(\mathbf{Q}_{i,g}, \mathbf{P}_g) \equiv P_{jj,g} - \bar{p}_{jj,g} w_j (1 - \bar{a}_{j,g})^{\frac{1}{\varsigma \gamma_g} - \frac{1}{\varsigma}} \left[\bar{d}_{ji,g} \bar{Q}_{ji,g} + \sum_{n \neq i} \bar{d}_{jn,g} Q_{jn,g}(\mathbf{d}_{-in,g} \odot \mathbf{P}_{-i,g}) \right]^{-\frac{1}{\gamma_g}} = 0.$$

In the above formulation, $\mathbf{Q}_{i,g} \equiv \{Q_{1i,g}, ..., Q_{Ni,g}\}$ denotes the vector of demand for industry g goods in destination i; vector $\mathbf{P}_g \equiv \{P_{\ell\ell,g}\}$ is the global vector of producer prices in industry g and $\mathbf{P}_{-i,g} = \mathbf{P}_g - \{P_{ii,g}\}$ encompasses the producer price for each origin aside from i. Correspondingly, $\tilde{\mathbf{P}}_{-in,g} \equiv \mathbf{d}_{-in,g} \odot \mathbf{P}_{-i,g}$ denotes the vector of consumer prices associated with non-i origins in destination $n \neq i$. Finally, the function $Q_{n\ell,g}(\tilde{\mathbf{P}}_{-in,g}) = \mathcal{D}_{ni,g}(\tilde{\mathbf{P}}_{-in,g}, \overline{\tilde{P}_{in,g}}, \overline{Y}_n)$ derives from the Marshallian demand function, treating $\tilde{P}_{in,g} \in \bar{\mathbb{P}}_i$, $w_n \in \mathbf{w}$, and $\overline{Y}_n \in \mathbf{Y}$ as given. For any given $\mathbb{P}_i$, $\mathbf{w}$, and $\mathbf{Y}$, the global vector of produce prices, $\mathbf{P}_g$, can be characterized as a function, $\mathbf{Q}_{i,g}$, based on the following system:

$$\begin{cases} F_{1,g}(Q_{1i,g}, ..., Q_{Ni,g}, P_{11,g}, ..., P_{NN,g}) = 0 \\ \vdots \\ F_{N,g}(Q_{1i,g}, ..., Q_{Ni,g}, P_{11,g}, ..., P_{NN,g}) = 0 \end{cases}.$$

Applying the Implicit Function Theorem to the above system of equations, yields the following matrix of inverse export supply elasticities:

$$\begin{bmatrix} \left(\frac{\partial \ln P_{11,k}}{\partial \ln Q_{1i,k}}\right)_{\mathbb{P}_i, \mathbf{w}, \mathbf{Y}} & \cdots & \left(\frac{\partial P_{11,k}}{\partial \ln Q_{Ni,k}}\right)_{\mathbb{P}_i, \mathbf{w}, \mathbf{Y}} \\ \vdots & \ddots & \vdots \\ \left(\frac{\partial \ln P_{NN,k}}{\partial \ln Q_{1i,k}}\right)_{\mathbb{P}_i, \mathbf{w}, \mathbf{Y}} & \cdots & \left(\frac{\partial P_{NN,k}}{\partial \ln Q_{Ni,k}}\right)_{\mathbb{P}_i, \mathbf{w}, \mathbf{Y}} \end{bmatrix} = - \underbrace{\begin{bmatrix} \frac{\partial F_{1,k}(.)}{\partial \ln P_{11,k}} & \cdots & \frac{\partial F_{1,k}(.)}{\partial \ln P_{NN,k}} \\ \vdots & \ddots & \vdots \\ \frac{\partial F_{N,k}(.)}{\partial \ln P_{11,k}} & \cdots & \frac{\partial F_{N,k}(.)}{\partial \ln P_{NN,k}} \end{bmatrix}^{-1}}_{\mathbf{A}_i} \begin{bmatrix} \frac{\partial F_{1,k}(.)}{\partial \ln Q_{1i,k}} & \cdots & \frac{\partial F_{1,k}(.)}{\partial \ln Q_{Ni,k}} \\ \vdots & \ddots & \vdots \\ \frac{\partial F_{N,k}(.)}{\partial \ln Q_{1i,k}} & \cdots & \frac{\partial F_{N,k}(.)}{\partial \ln Q_{Ni,k}} \end{bmatrix}, \tag{A.7}$$

Since function $F_{i,k}(.)$ treats $\mathbb{P}_i$, $\mathbf{w}$, and $\mathbf{Y}$ as given, each element of the matrices on the right-hand side can be specified as follows:

$$\frac{\partial F_{n,k}(.)}{\partial \ln P_{jj,k}} = \mathbb{1}_{j=n} + \mathbb{1}_{j \neq i} \frac{1}{\gamma_g} \sum_{\ell \neq i} \left[r_{n\ell,k} \varepsilon_{n\ell,k}^{(j\ell,k)} \right]; \qquad \frac{\partial F_{n,k}(.)}{\partial \ln Q_{ji,k}} = \mathbb{1}_{j=n} \frac{1}{\gamma_g} r_{ji,k}.$$

Considering the above expression for $\partial F_{n,k}(.)/\partial \ln P_{jj,k}$, it is straightforward to show that $\mathbf{A}_i$ is diagonally-dominant. Hence, following Lashkaripour and Lugovskyy (2020), we can produce a simple first-order approximation for $\mathbf{A}_i^{-1}$ around $r_{ji,k} \approx 0$ (for $j \neq i$), which yields the following:

$$\left(\frac{\partial \ln P_{nn,g}}{\partial \ln Q_{ji,g}}\right)_{\mathbb{P}_i,\mathbf{w},\mathbf{Y}} \approx \begin{cases} \frac{-\frac{1}{\gamma_g} r_{ni,g}}{1+\frac{1}{\gamma_g}\sum_{\iota\neq i} r_{n\iota,g}\varepsilon_{n\iota,g}^{(n\iota,g)}} & n = j \\ \frac{\frac{1}{\gamma_g} r_{ji,g}}{1+\frac{1}{\gamma_g}\sum_{\iota\neq i} r_{n\iota,g}\varepsilon_{n\iota,g}^{(n\iota,g)}} \left(\frac{1}{\gamma_g}\sum_{\iota\neq i} r_{n\iota,g}\varepsilon_{n\iota,g}^{(j\iota,g)}\right) & n \neq j \end{cases}.$$

We can then plug the above expression back into into Equation A.6 to produce the following approximation for the export supply elasticity—noting that $r_{ni,g} \times r_{ji,g} \approx 0$ if $j \neq i$ and $n \neq i$:

$$\omega_{ji,k} \approx \frac{-\frac{1}{\gamma_k} r_{ji,k}}{1+\frac{1}{\gamma_k}\sum_{\iota\neq i} r_{j\iota,k}\varepsilon_{j\iota,k}} \left[1 - \frac{1}{\gamma_k}\frac{w_i L_i}{w_j L_j}\sum_{n\neq i}\frac{\rho_{i,k} r_{in,k}}{\rho_{j,k} r_{ji,k}}\varepsilon_{in,k}^{(jn,k)}\right].$$

B Proofs and Derivations for Unilaterally Optimal Policy

B.1 Proof of Lemma 2

We prove the welfare neutrality of wage effects in two steps. First, we show that insofar as the system is in general equilibrium a change in the wage rate of the home country i, does not change the welfare there. Second, we show that if the system is in general equilibrium at the government's choice of policy is at the optimum, then a chnage in the wage rates of foreign countries does not have an effect on the welfare at home. Together, this means, provided that the policy is at the optimum, welfare at home remains unchanged in response to general equilibrium effects from changes in the worldwide vector of wages.

Step 1. First, we show that in general equilibrium, home's welfare is invariant to changes in home's wage. Consider any policy–wage-income combination that satisfies general equilibrium constraints, i.e., $\mathbb{X} = (\mathbb{P}_i; \mathbf{w}, \mathbf{Y}) \in \mathcal{E}^r$. Then,

$$\frac{\partial W_i(\mathbb{X})}{\partial w_i} = \left[\frac{\partial V_i(.)}{\partial Y_i} - \frac{1}{Y_i}\sum_j\sum_k\left(\delta_{ji,k} Z_{j,k}\frac{\partial \ln Z_{j,k}(.)}{\partial \ln Q_{j,k}}\frac{\partial \ln Q_{j,k}(.)}{\partial \ln Q_{ji,k}}\frac{\partial \ln \mathcal{D}_{ji,k}(.)}{\partial \ln Y_i}\right)\right]\frac{\partial \overbrace{(w_i L_i + T_i(\mathbb{X}))}^{Y_i}}{\partial \ln w_i} = 0$$

We establish the welfare neutrality of home's wage, w_i, by proving that:

$$\frac{\partial\left(w_i^\star L_i + T_i(\mathbb{P}_i^\star; \mathbf{w}^\star, \mathbf{Y}^\star)\right)}{\partial \ln w_i} = 0 \quad \Rightarrow \quad \frac{\partial W_i(\mathbb{X}^\star)}{\partial w_i} = 0.$$

To be clear, the above partial derivative w.r.t. w_i is applied holding the policy vector $\mathbb{P}_i$, wages in the rest of the world, $\mathbf{w}_{-i}$, and the income vector, $\mathbf{Y}$, as unchanged. Using the expression for tax revenues,

$$T_i(\mathbb{X}) = \overbrace{\sum_{k\in\mathbb{K}}\sum_{j\in\mathbb{C}}\left(\alpha_{j,k}(\mathbb{X})\frac{\gamma_k - 1}{\gamma_k}P_{ij,k}(\mathbb{X})Q_{ij,k}(\mathbb{X})\right)}^{\text{carbon taxes}} + \overbrace{\sum_{k\in\mathbb{K}}\left[\left(\tilde{P}_{ii,k} - P_{ii,k}(\mathbb{X})\right)Q_{ii,k}(\mathbb{X})\right]}^{\text{production subsidies}}$$
$$+ \underbrace{\sum_{k\in\mathbb{K}}\sum_{j\in\mathbb{C},j\neq i}\left[\left(\tilde{P}_{ji,k} - P_{ji,k}(\mathbb{X})\right)Q_{ji,k}(\mathbb{X})\right]}_{\text{imports taxes}} + \underbrace{\sum_{k\in\mathbb{K}}\sum_{j\in\mathbb{C},j\neq i}\left[\left(\tilde{P}_{ij,k} - P_{ij,k}(\mathbb{X})\right)Q_{ij,k}(\mathbb{X})\right]}_{\text{exports subsidies}},$$

we can express the partial derivative of country i's total income from wage payments and tax revenues $(w_i L_i + T_i(\mathbb{X}))$ w.r.t. w_i as

$$\frac{\partial\left(w_i L_i + T_i(\mathbb{X})\right)}{\partial \ln w_i} = w_i \bar{L}_i - \sum_k \sum_{j \neq i} \left(\left[\frac{\partial \ln P_{ji,k}(\mathbb{X})}{\partial \ln w_i} + \frac{\partial \ln Q_{ji,k}(\mathbb{X})}{\partial \ln w_i} \right] P_{ji,k} Q_{ji,k} \right)$$
$$- \sum_k \sum_j \left[\left((1 - \alpha_{i,k}(\mathbb{X}) \frac{\gamma_k - 1}{\gamma_k}) \left[\frac{\partial \ln P_{ij,k}(\mathbb{X})}{\partial \ln w_i} + \frac{\partial \ln Q_{ij,k}(\mathbb{X})}{\partial \ln w_i} \right] - \alpha_{i,k}(\mathbb{X}) \frac{\gamma_k - 1}{\gamma_k} \frac{\partial \ln \alpha_{i,k}(\mathbb{X})}{\partial \ln w_i} \right) P_{ij,k} Q_{ij,k} \right]. \tag{B.1}$$

Demand in market $j \neq i$ is given by the Marshallian demand function, $Q_{ij,k}(\mathbb{X}) = \mathcal{D}_{ij,k}(Y_j, \tilde{\mathbf{P}}_{ij}, \tilde{\mathbf{P}}_{-ij})$, and is fully determined by $Y_j \in \mathbf{Y}$, $\tilde{\mathbf{P}}_{ij} \in \mathbb{P}_i$, and $\tilde{\mathbf{P}}_{-ij}$. The price vector $\tilde{\mathbf{P}}_{-ij}$, which encompasses all consumer price variables unassociated with exporter i, is itself fully determined by $\mathbb{P}_i$ and $\mathbf{w}_{-i}$ (see Appendix A.2). Hence, holding all elements of $\mathbb{X}$ aside from w_i constant, $Q_{ij,k}(\mathbb{X})$ is invariant to any change in w_i. Likewise, demand in the home market i, $Q_{ji,k}(\mathbb{X}) = \mathcal{D}_{ji,k}(Y_i, \tilde{\mathbf{P}}_i)$, is fully determined by $Y_i \in \mathbf{Y}$ and $\tilde{\mathbf{P}}_i \in \mathbb{P}_i$. So, holding all elements of $\mathbb{X}$ aside from w_i constant, $Q_{ji,k}(\mathbb{X})$ is also invariant to any change in w_i. These observations entail that (for all k, and for $j \neq i$),

$$\frac{\partial \ln Q_{ij,k}(\mathbb{X})}{\partial \ln w_i} = \frac{\partial \ln Q_{ij,k}(\mathbb{X})}{\partial \ln w_i} = 0.$$

Producer prices associated with origin $j \neq i$ are given by $P_{ji,k} = \bar{p}_{jj,k} w_j (1 - a_{j,k})^{\frac{1}{\varsigma\gamma_k} - \frac{1}{\varsigma}} Q_{j,k}(Q_{j1,k}, ...Q_{jN,k})^{-\frac{1}{\gamma_k}}$. As such, they are fully determined by $w_j \in \mathbf{w}_{-i}$, abatement choices, $a_{j,k}$, and demand quantities, $\{Q_{jn,,k}\}_n$. Abatement and demand quantities associated with origin j are themselves fully determined by $\mathbf{w}_{-i}$, $\mathbb{P}_i$, and $\mathbf{Y}$. So, holding all elements of $\mathbb{X}$ aside from w_i constant, $P_{ji,k}(\mathbb{X})$ is invariant to any change in w_i. Likewise, producer prices associated with origin i are given by $P_{ij,k} = \bar{p}_{ii,k} w_i (1 - a_{i,k})^{\frac{1}{\varsigma\gamma_k} - \frac{1}{\varsigma}} Q_{i,k}(Q_{i1,k}, ...Q_{iN,k})^{-\frac{1}{\gamma_k}}$. Hence, $P_{ij,k}$ (net of w_i) is fully determined by abatement, $a_{i,k} \in \mathbb{P}_i$, and demand quantities, $\{Q_{in,,k}\}_n$, which are themselves fully determined by $\mathbb{P}_i$, $\mathbf{w}_{-i}$, and $\mathbf{Y}$. So, holding $\mathbf{w}_{-i}$, $\mathbb{P}_i$, and $\mathbf{Y}$ constant, any change in w_i is fully passed on to $P_{ij,k}(\mathbb{X})$. Combining these arguments yields the following partial derivatives:

$$\frac{\partial \ln P_{ij,k}(\mathbb{X})}{\partial \ln w_i} = 1, \qquad \frac{\partial \ln P_{ji,k}(\mathbb{X})}{\partial \ln w_i} = 0 \quad (\text{if } j \neq i).$$

Lastly, $\alpha_{i,k}(\mathbb{X}) = 1 - (1 - \bar{\kappa}_{i,k})(1 - a_{i,k})^{-\frac{\varsigma - 1}{\varsigma}}$ is fully determined by $a_{i,k} \in \mathbb{P}_i$. Hence, holding $\mathbb{P}_i$ constant, the change in w_i has no effect on the carbon cost share, i.e., $\partial \ln \alpha_{i,k}(\mathbb{X}) / \partial \ln w_i = 0$. Plugging these partial derivative values into Equation B.1 implies

$$\frac{\partial\left(w_i L_i + T_i(\mathbb{X})\right)}{\partial \ln w_i} = \underbrace{\left(w_i \bar{L}_i - \sum_{k,j} (1 - \alpha_{i,k} \frac{\gamma_k - 1}{\gamma_k}) P_{ij,k} Q_{ij,k} \right)}_{=0 \text{ (by labor market clearing)}} = 0,$$

where the right-hand side term in the parentheses equals zero since the system is in general equilibrium, i.e., $\mathbb{X} = (\mathbb{P}_i; w_i, \mathbf{w}_{-i}, \mathbf{Y}) \in \mathcal{E}^r$, which guarantees that the labor market clearing condition (7) holds. Notice, the above derivations hold whether or not the policy vector $\mathbb{P}_i$ is at the optimum. While this is a striking result, we remain to be interested in the above derivative when it is evaluated at the optimum —i.e., the policy–wage-income combination which we index by $(\star)$, $\mathbb{X}^\star = (\mathbb{P}_i^\star; \mathbf{w}^\star, \mathbf{Y}^\star) \in \mathcal{E}^r$. The reason, as we show in Step 2, is that the effects of wages foreign countries on home's welfare is neutral only at the optimum. Hence, the claim that the entire vector of wages, $\mathbf{w} = \{w_n\}_n$, is welfare neutral will be true only at the optimum.

Step 2. In the two country case, $\frac{dw_{-i}}{dP} = 0$ for all $P \in \mathbb{P}_i \equiv \{\tilde{P}_{ji,k}, \tilde{P}_{ij,k}, \tilde{P}_{ii,k}\}_{j\neq i,k}$ by choice of numeraire. Beyond the two country case, we can infer from actual data that $\frac{d\mathbf{w}_{-i}}{dP} \approx \mathbf{0}$ for all $P \in \mathbb{P}_i$—a point already established in Appendix A.4. These results indicate that wage effects in the rest of the world are, in general, welfare neutral—either exactly or to a first-order approximation. This result holds even in second-best scenarios where the government is banned from manipulating a subset of price instruments in $\mathbb{P}_i$. In the first-best scenario we can establish the welfare neutrality of international wage effects using the multiplicity of optimal policy choices and the *Targeting Principle*. We first establish the result regarding multiplicity of optimal policy choices. To present this result, we use $\mathbb{X} \equiv (\mathbb{I}_i, \mathbb{I}_{-i}, w_i, \mathbf{w}_{-i}, Y_i, \mathbf{Y}_{-i})$ to denote a global policy-wage-income combination, with $\mathbb{I}_i = (\mathbf{s}_i, \mathbf{t}_i, \mathbf{x}_i, \boldsymbol{\tau}_i)$. As before, we use $\mathcal{E}$ to denote the set of feasible policy-wage-income combinations that satisfy general equilibrium constraints, as defined in Section 2.4. Considering this choice of notation, we want to prove the following result, which is a basic extension of Lemma 1 in Lashkaripour and Lugovskyy (2020) to an economy with carbon externality and abatement.

Lemma 5. *For any a and $\tilde{a} \in \mathbb{R}_+$, the following two results hold: (1) if $\mathbb{X} = (\mathbf{1}+\boldsymbol{t}_i, \boldsymbol{t}_{-i}, \mathbf{1}+\boldsymbol{x}_i, \boldsymbol{x}_{-i}, \mathbf{1}+\boldsymbol{s}_i, \boldsymbol{s}_{-i}, \tau_i, \boldsymbol{\tau}_{-i}; w_i, \boldsymbol{w}_{-i}, Y_i, \mathbf{Y}_{-i}) \in \mathcal{E}$, then*

$$\mathbb{X}' = (a(\mathbf{1}+\mathbf{t}_i), \mathbf{t}_{-i}, a(\mathbf{1}+\mathbf{x}_i), \mathbf{x}_{-i}, \frac{1}{\tilde{a}}(\mathbf{1}+\mathbf{s}_i), \mathbf{s}_{-i}, \frac{a}{\tilde{a}}\tau_i, \boldsymbol{\tau}_{-i}; \frac{a}{\tilde{a}}w_i, \mathbf{w}_{-i}, aY_i, \mathbf{Y}_{-i}) \in \mathcal{E};$$

and (2) Welfare is preserved under policy-wage combination $\mathbb{X}$ and $\mathbb{X}'$: $W_n(\mathbb{X}') = W_n(\mathbb{X})$ for all $n \in \mathbb{C}$.

Proof. The proof closely follows the proof of Lemma 1 in Lashkaripour and Lugovskyy (2020). The only difference is that the labor market clearing condition must be adjusted to account for abatement activity. To restate the objective of the proof, consider two policy-wage combinations, $\mathbb{X} = (\mathbf{s}, \mathbf{t}, \mathbf{x}, \boldsymbol{\tau}; \mathbf{w})$, and $\mathbb{X}' = (\mathbf{s}', \mathbf{t}', \mathbf{x}', \boldsymbol{\tau}'; \mathbf{w}')$, that differ in uniform shifters a and $\tilde{a} \in \mathbb{R}_+$ with regards to country i's taxes:

$$\mathbf{1}+\mathbf{x}_i' = a\left(\mathbf{1}+\mathbf{x}_i\right); \quad \mathbf{1}+\mathbf{t}_i' = a\left(\mathbf{1}+\mathbf{t}_i\right); \quad ;\mathbf{1}+\mathbf{s}_i' = \left(\mathbf{1}+\mathbf{s}_i\right)/\tilde{a}; \quad ;w_i' = \left(a/\tilde{a}\right)w_i; \quad \tau_i' = \left(a/\tilde{a}\right)\tau_i;$$

but consist of the same tax levels in the rest world (namely, $-i$):

$$\mathbf{1}+\mathbf{x}_{-i}' = \mathbf{1}+\mathbf{x}_{-i} \quad \mathbf{1}+\mathbf{t}_{-i}' = \mathbf{1}+\mathbf{t}_{-i} \quad \mathbf{1}+\mathbf{s}_{-i}' = \mathbf{1}+\mathbf{s}_{-i} \quad \mathbf{w}_{-i}' = \mathbf{w}_{-i}; \quad \boldsymbol{\tau}_{-i}' = \boldsymbol{\tau}_{-i}.$$

Our goal is to prove that (i) if $\mathbb{X} \in \mathcal{E}$ then $\mathbb{X}' \in \mathcal{E}$, and (ii) $W_n(\mathbb{X}') = W_n(\mathbb{X})$ for all $n \in \mathbb{C}$.

Step 1. The first step proves that $\mathbf{Q}(\mathbb{X}') = \mathbf{Q}(\mathbb{X})$ using two intermediate claims. The first claim posits that if we suppose $\mathbf{Q}(\mathbb{X}') = \mathbf{Q}(\mathbb{X})$), then the implied nominal income and price levels under $\mathbb{X}$ and $\mathbb{X}'$ are the same up to a scale. Stated mathematically,

$$[\text{Claim 1}] \quad \mathbf{Q}(\mathbb{X}') = \mathbf{Q}(\mathbb{X}) \implies \begin{cases} \tilde{\mathbf{P}}_i\left(\mathbb{X}'\right) = a\tilde{\mathbf{P}}_i\left(\mathbb{X}\right); \\ \tilde{\mathbf{P}}_{-i}\left(\mathbb{X}'\right) = \tilde{\mathbf{P}}_{-i}\left(\mathbb{X}\right); \end{cases} .$$

In the above formulation $\tilde{\mathbf{P}}_i = \{\tilde{\mathbf{P}}_{1i}, ..., \tilde{\mathbf{P}}_{Ni}\}$ denotes the entire vector of consumer price indexes in destination i, and $\mathbf{Q} \equiv \{Q_{n\ell,g}\}_{n,\ell,g}$ is the entire vector of equilibrium quantities. To prove the above claim, we compute nominal income and consumer price indexes under $\mathbb{X}$ and $\mathbb{X}'$ starting from the assumption that $\mathbf{Q}(\mathbb{X}') = \mathbf{Q}(\mathbb{X})$. First, consider nominal price indexes. To simplify the notation for prices, define

$$\delta_{jn,k}(\mathbb{X}) \equiv \bar{\rho}_{jn,k}\left(1 - a_{j,k}(\mathbb{X})\right)^{\frac{1}{\varsigma\gamma_k} - \frac{1}{\varsigma}} \mathcal{Q}_{j,k}(\mathbb{X})^{-\frac{1}{\gamma_k}}.$$

By assumption, $a_{j,k}(\mathbb{X}') = a_{j,k}(\mathbb{X})$ and $\mathcal{Q}_{j,k}(\mathbb{X}') = \mathcal{Q}_{j,k}(\mathbb{X})$, indicating that$\delta_{jn,k}(\mathbb{X}) = \delta_{jn,k}(\mathbb{X}') = \bar{\delta}_{jn,k}$. Now, consider the price index of a generic good ji,k imported by i from origin $j \neq i$. Invoking Equations 3 and 5, the *consumer* price index of good ji,k under $\mathbb{X}'$ and $\mathbb{X}$ exhibit the following relationship:

$$\tilde{P}_{ji,k}(\mathbb{X}') = \bar{\delta}_{ji,k}\frac{1+t'_{ji,k}}{(1+x'_{ji,k})(1+s'_{j,k})}w'_j = \bar{\delta}_{ji,k}\frac{a(1+t_{ji,k})}{(1+x_{ji,k})(1+s_{j,k})}\,w_j = a\tilde{P}_{ji,k}(\mathbb{X}),$$

where the third equality follows from the fact that $1+t'_{ji,k} = a(1+t_{ji,k})$, while $w'_j = w_j$, $x'_{ji,k} = x_{ji,k}$, and $s'_{j,k} = s_{j,k}$ (since $w_j \in \mathbf{w}_{-i}$, $x_{ji,k} \in \mathbf{x}_{-i}$, and $s_{j,k} \in \mathbf{s}_{-i}$). Second, consider a typical good ii,k that is produced and consumed locally in country i. The consumer price of ii,k under combination $\mathbb{X}'$ can be related to its price under $\mathbb{X}$ as follows

$$\tilde{P}_{ii,k}(\mathbb{X}') = \bar{\delta}_{ii,k}\frac{1}{1+s'_{i,k}}w'_i = \bar{\delta}_{ii,k}\frac{1}{\frac{1}{\tilde{a}}(1+s_{i,k})}\times\frac{a}{\tilde{a}}w_i = a\tilde{P}_{ii,k}(\mathbb{X}),$$

where the third equality follows from the fact that $1+s'_{i,k} = (1+s_{i,k})/\tilde{a}$ and $w'_i = aw_i/\tilde{a}$. Third, consider the price of a typical good ij,k export by i to destination market $j \neq i$. The consumer price of ij,k under combination $\mathbb{X}'$ can be related to its price under $\mathbb{X}$ as follows:

$$\tilde{P}_{ij,k}(\mathbb{X}') = \bar{\delta}_{ij,k}\frac{1+t'_{ij,k}}{(1+x'_{ij,k})(1+s'_{i,k})}w'_i = \bar{\delta}_{ij,k}\frac{1+t_{ij,k}}{a(1+x'_{ij,k})\times\frac{1}{\tilde{a}}(1+s'_{i,k})}\times\frac{a}{\tilde{a}}w_i = \tilde{P}_{ij,k}(\mathbb{X}),$$

where the third equality follows from the fact that $1+x'_{ij,k} = a(1+x_{ij,k})$, $1+s_{i,k} = (1+s'_{i,k})/\tilde{a}$, and $w'_i = aw_i/\tilde{a}$; while $t'_{ij,k} = t_{ji,k}$ since $t_{ji,k} \in \mathbf{t}_{-i}$. Lastly, is follows trivially that $\tilde{P}_{jn,k}(\mathbb{X}') = \tilde{P}_{jn,k}(\mathbb{X})$ if j and $n \neq i$. Considering that $\tilde{\mathbf{P}}_i = \left\{\tilde{\mathbf{P}}_{ji}, \tilde{\mathbf{P}}_{ii}\right\}$, the above equations establish that

$$\tilde{\mathbf{P}}_i\left(\mathbb{X}'\right) = a\tilde{\mathbf{P}}_i\left(\mathbb{X}\right), \qquad \tilde{\mathbf{P}}_{-i}\left(\mathbb{X}'\right) = \tilde{\mathbf{P}}_{-i}\left(\mathbb{X}\right). \tag{B.2}$$

which amounts to Claim (1). Stepping back, Claim (1) starts from the assumption that $\mathbf{Q}(\mathbb{X}') = \mathbf{Q}(\mathbb{X})$. Our next claim indicates that this assumption is validated by the nominal income and price levels implied by $\mathbb{X}'$ and $\mathbb{X}$. Below, we state this lemma noting that it follows trivially from the Marshallian demand function, $Q_{ji,k} = \mathcal{D}_{ji,k}(Y_i, \tilde{\mathbf{P}}_i)$, being homogeneous of degree zero. In particular,

$$[\text{Claim 2}] \quad \forall a \in \mathbb{R}_+ : \begin{cases} \tilde{\mathbf{P}}_i\left(\mathbb{X}'\right) = a\tilde{\mathbf{P}}_i\left(\mathbb{X}\right) & Y'_i = aY_i \\ \tilde{\mathbf{P}}_{-i}\left(\mathbb{X}'\right) = \tilde{\mathbf{P}}_{-i}\left(\mathbb{X}\right) & \mathbf{Y}'_{-i} = \mathbf{Y}_{-i} \end{cases} \implies \mathbf{Q}(\mathbb{X}') = \mathbf{Q}(\mathbb{X}).$$

Together, Claims (1) and (2) establish that equilibrium quantities should be indeed identical under policy-wage combinations $\mathbb{X}$ and $\mathbb{X}'$. That is, $\mathbf{Q}(\mathbb{X}') = \mathbf{Q}(\mathbb{X})$.

Step 2. The second step uses the fact $\mathbf{Q}(\mathbb{X}') = \mathbf{Q}(\mathbb{X})$ to establish that if $\mathbb{X} \in \mathcal{E}$ then $\mathbb{X}' \in \mathcal{E}$. That is, $w'_i = \frac{a}{\tilde{a}}w_i$ and $Y'_i = aY_i$ satisfy the labor market clearing and balanced budget conditions given tax vector $\mathbb{I}'$, provided that w_i and Y_i satisfy the same conditions given tax vector $\mathbb{I}$. To simplify the presentation, we hereafter use $\mathcal{X} \equiv \mathcal{X}(\mathbb{X})$ and $\mathcal{X}' \equiv \mathcal{X}(\mathbb{X}')$ to denote the value of a generic variable $\mathcal{X}$ under policy-wage-income combinations $\mathbb{X}$ and $\mathbb{X}'$. Starting with the labor market clearing condition, our goal is to show that if $w_iL_i = \sum_k\sum_j\left[(1-\alpha_{i,k})\frac{\gamma_k-1}{\gamma_k}P_{ij,k}Q_{ij,k}\right]$ then $w'_iL_i = \sum_k\sum_j\left[(1-\alpha'_{i,k})\frac{\gamma_k-1}{\gamma_k}P'_{ij,k}Q'_{ij,k}\right]$. This result can be established as follows. First, starting from the labor market clearing condition under $\mathbb{X} = (\mathbb{I};\mathbf{w},\mathbf{Y})$ and the fact that $w'_i = \frac{a}{\tilde{a}}w_i$, we can produce the following equation:

$$\begin{aligned} w'_iL_i = \frac{a}{\tilde{a}}w_iL_i &= \frac{a}{\tilde{a}}\sum_k\sum_j\left[(1-\alpha_{i,k})\frac{\gamma_k-1}{\gamma_k}\frac{(1+x_{ij,k})(1+s_{i,k})}{1+t_{ij,k}}\tilde{P}_{ij,k}Q_{ij,k}\right] \\ &= \sum_k\sum_j\left[(1-\alpha_{i,k})\frac{\gamma_k-1}{\gamma_k}\frac{a(1+x_{ij,k})\frac{1}{\tilde{a}}(1+s_{i,k})}{1+t_{ij,k}}\tilde{P}_{ij,k}Q_{ij,k}\right]. \end{aligned}$$

Note that—by construction—$1 + t'_{ji,k} = (1 + t_{ji,k})/a$, $1 + x_{ij,k} = (1 + x'_{ij,k})/a$, $1 + s_{i,k} = (1 + s'_{i,k})/\tilde{a}$. Also, Step 1 of the proof already established that $Q'_{ij,k} = Q_{ij,k}$ and $\tilde{P}'_{ij,k} = \tilde{P}_{ij,k}$ for all ij,k. The carbon cost share is also the same under $\mathbb{X}$ and $\mathbb{X}'$ (i.e., $\alpha'_{i,k} = \alpha_{i,k}$) given that $\alpha_{i,k}$ is determined by the *wage-to-carbon tax* ratio and that $\tau'_{i,k}/w'_i = \tau_{i,k}/w_i$, by construction. Plugging these relationships into the last line of the above equation delivers the desired result that $\mathbb{X}'$is consistent with the labor market clearing condition:

$$w'_i L_i = \sum_k \sum_j \left[(1 - \alpha'_{i,k}) \frac{\gamma_k - 1}{\gamma_k} \frac{(1 + x'_{ij,k})(1 + s'_{i,k})}{1 + t'_{ij,k}} \tilde{P}'_{ij,k} Q'_{ij,k} \right] = \sum_k \sum_j \left[(1 - \alpha'_{i,k}) \frac{\gamma_k - 1}{\gamma_k} P'_{ij,k} Q'_{ij,k} \right].$$

Next, consider the balance budget condition our goal is to show that $Y'_i = w'_i L_i + T'_i$ if $Y_i = w_i L_i + T_i$, where T'_i and T_i are respectively the tax revenues under combinations $\mathbb{X}$ and $\mathbb{X}'$. Note that by construction $Y'_i = aY_i$, which implies:

$$\begin{aligned} Y_i = aY_i = a w_i L + a \sum_k \sum_j \left(\alpha_{i,k} \frac{\gamma_k - 1}{\gamma_k} P_{ij,k} Q_{ij,k} \right) + a \sum_k \left[\left(\frac{1}{1 + s_{i,k}} - 1 \right) P_{ii,k} Q_{ii,k} \right] \\ + a \sum_k \sum_{j \neq i} \left(\frac{t_{ji,k}}{(1 + x_{ji,k})(1 + s_{j,k})} P_{ji,k} Q_{ji,k} + \left[\frac{1}{(1 + x_{ij,k})(1 + s_{i,k})} - 1 \right] P_{ij,k} Q_{ij,k} \right). \end{aligned} \tag{B.3}$$

Noting that the labor market clearing condition, $w_i L_i - \sum_k \sum_j \left[\left(1 - \alpha_{i,k} \frac{\gamma_k - 1}{\gamma_k} \right) P_{ij,k} Q_{ij,k} \right] = 0$, is satisfied under combination $\mathbb{X}_i$, we can subtract the following (equal zero) term,

$$\left(\frac{a}{\tilde{a}} - a \right) w_i L_i + \left(\frac{a}{\tilde{a}} - a \right) \sum_k \sum_j \left((1 - \alpha_{i,k}) \frac{\gamma_k - 1}{\gamma_k} P_{ij,k} Q_{ij,k} \right) = 0,$$

from Equation B.3 to arrive at the following updated expression for Y'_i:

$$\begin{aligned} Y'_i = & \frac{a}{\tilde{a}} w_i L_i + a \sum_k \sum_j \left(\alpha_{i,k} \frac{\gamma_k - 1}{\gamma_k} P_{ij,k} Q_{ij,k} \right) + \sum_k \left[\left(\frac{a}{1 + s_{i,k}} - \frac{a}{\tilde{a}} \right) P_{ii,k} Q_{ii,k} \right] \\ & + \sum_k \sum_{j \neq i} \left(\frac{a t_{ji,k}}{(1 + x_{ji,k})(1 + s_{j,k})} P_{ji,k} Q_{ji,k} + \left[\frac{a}{(1 + x_{ij,k})(1 + s_{i,k})} - \frac{a}{\tilde{a}} \right] P_{ij,k} Q_{ij,k} \right) \\ = & \frac{a}{\tilde{a}} w_i L_i + \sum_k \sum_j \left(\alpha_{i,k} \frac{\gamma_k - 1}{\gamma_k} \frac{a(1 + x_{ij,k}) \frac{1}{\tilde{a}} (1 + s_{i,k})}{1 + t_{ij,k}} \tilde{P}_{ij,k} Q_{ij,k} \right) + \sum_k \left[\left(1 - \frac{1}{\tilde{a}} (1 + s_{i,k}) \right) a \tilde{P}_{ii,k} Q_{ii,k} \right] \\ & + \sum_{j,k} \left[\left(1 - \frac{1}{a(1 + t_{ji,k})} \right) a \tilde{P}_{ji,k} Q_{ji,k} + \left[\frac{1}{1 + t_{ij,k}} - \frac{a(1 + x_{ij,k}) \times \frac{1}{\tilde{a}} (1 + s_{i,k})}{1 + t_{ij,k}} \right] \tilde{P}_{ij,k} Q_{ij,k} \right]. \end{aligned} \tag{B.4}$$

Noting that the balanced trade condition, $\sum_k \sum_{j \neq i} \left(\frac{1}{1 + t_{ji,k}} \tilde{P}_{ji,k} Q_{ji,k} - \frac{1}{1 + t_{ij,k}} \tilde{P}_{ij,k} Q_{ij,k} \right) = 0$, is satisfied under combination $\mathbb{T}_i$, we can subtract the following (equal zero) term,

$$(1 - a) \sum_k \sum_{j \neq i} \left(\frac{1}{1 + t_{ji,k}} \tilde{P}_{ji,k} Q_{ji,k} - \frac{1}{1 + t_{ij,k}} \tilde{P}_{ij,k} Q_{ij,k} \right) = 0,$$

from Equation B.4 to update the expression for Y'_i as follows:

$$\begin{aligned} Y'_i = & \frac{a}{\tilde{a}} w_i L_i + \sum_k \sum_j \left(\alpha'_{i,k} \frac{\gamma_k - 1}{\gamma_k} \frac{a(1 + x_{ij,k}) \frac{1}{\tilde{a}} (1 + s_{i,k})}{1 + t_{ij,k}} \tilde{P}_{ij,k} Q_{ij,k} \right) + \sum_k \left[\left(1 - \frac{1}{\tilde{a}} (1 + s_{i,k}) \right) a \tilde{P}_{ii,k} Q_{ii,k} \right] \\ & + \sum_{j,k} \left[\left(1 - \frac{1}{a(1 + t_{ji,k})} \right) a \tilde{P}_{ji,k} Q_{ji,k} + \left[\frac{1}{1 + t_{ij,k}} - \frac{a(1 + x_{ij,k}) \times \frac{1}{\tilde{a}} (1 + s_{i,k})}{1 + t_{ij,k}} \right] \tilde{P}_{ij,k} Q_{ij,k} \right]. \end{aligned}$$

Observe that policy-wage-income combinations $\mathbb{X}$ and $\mathbb{X}'$ are constructed such that $1+t'_{ji,k} = a(1+t_{ji,k})$, $1+x'_{ij,k} = a(1+x_{ij,k})$, $1+s_{i,k} = (1+s'_{i,k})/\tilde{a}$, $w'_i = aw_i/\tilde{a}$, and $t'_{ij,k} = t_{ji,k}$. Also, Step 1 of the proof established that $Q'_{ii,k} = Q_{ii,k}$, $Q'_{ji,k} = Q_{ji,k}$, $Q'_{ij,k} = Q_{ij,k}$, $\tilde{P}_{ii,k} = aP_{ii,k}$, $P'_{ji,k} = a\tilde{P}_{ji,k}$, $\tilde{P}'_{ij,k} = \tilde{P}_{ij,k}$, and $\alpha'_{i,k} = \alpha_{i,k}$. Plugging these expressions into the above equation yields

$$Y'_i = w'_i L_i + \sum_k \sum_j \left(\alpha_{i,k} \frac{\gamma_k - 1}{\gamma_k} P'_{ij,k} Q'_{ij,k} \right) + \sum_k \left[\left(\frac{1}{1+s'_{i,k}} - 1 \right) P'_{ii,k} Q'_{ii,k} \right]$$
$$+ \sum_k \sum_{j \neq i} \left(\frac{t'_{ji,k}}{(1+x'_{ji,k})(1+s'_{j,k})} P'_{ji,k} Q'_{ji,k} + \left[\frac{1}{(1+x'_{ij,k})(1+s'_{i,k})} - 1 \right] P'_{ij,k} Q'_{ij,k} \right),$$

which ensures that combination $\mathbb{X}'$ is consistent with the balanced budget condition, $Y'_i = w'_i L_i + T_i$. Combing the above findings we can conclude that

$$\mathbb{X} = (\mathbb{I}, \mathbf{w}, \mathbf{Y}) \in \mathcal{E} \implies \mathbb{X}' = (\mathbb{I}', \mathbf{w}', \mathbf{Y}') \in \mathcal{E}$$

where $\mathbb{I}' = (a(\mathbf{1}+\mathbf{t}_i), \mathbf{t}_{-i}, a(\mathbf{1}+\mathbf{x}_i), \mathbf{x}_{-i}, \frac{1}{\tilde{a}}(\mathbf{1}+\mathbf{s}_i), \mathbf{s}_{-i}, \frac{a}{\tilde{a}}\tau_i, \boldsymbol{\tau}_{-i})$, $\mathbf{w} = (\frac{a}{\tilde{a}} w_i, w_{-i})$, and $\mathbf{Y} = (aY_i, Y_{-i})$. Moreover, since $\mathbf{Q}(\mathbb{X}') = \mathbf{Q}(\mathbb{X})$ it should be the case that $W_n(\mathbb{X}') = W_n(\mathbb{X})$ for all $n \in \mathbb{C}$, which establishes both claims under Lemma 5. $\square$

Now we present our international wage neutrality argument based on Lemma 5. Suppose the government in i solves a problem where it can set the entire vector of prices globally. This choice vector includes the extraterritorial price $\tilde{P}_{jn,k}$, where j and $n \neq i$. The revenue from extraterritorial price manipulations are rebated to foreign consumers. Lemma 5 ensures that the choice with respect to the global price vector can replicate any welfare gains from wage effects. That is, if $\tilde{P}^*_{jn,k} = P_{jn,k}$ for all j and $n \neq i$, it should be the case that $\partial W_i / \partial \mathbf{w}_{-i} = 0$. We can prove that this is the case as follows: Since revenues from extraterritorial taxes accrue to foreign economies, the gains from $\tilde{P}_{jn,k}$ channel exclusively through the effect on country i's export and import tax revenues. By the Targeting principle, however, these indirect gains will be internalized by the optimal choice $\mathbb{P}^\star_i$ that directly regulates country i's import/export tax revenues.

B.2 Solving the System of F.O.C.s: Proofs of Lemmas 3-4 & Theorems 1-2.

Recall that the set of policy instruments given to the government of country i is given by $\mathbb{P}_i \equiv \{\tilde{\mathbf{P}}_{ij}, \tilde{\mathbf{P}}_{ji}, \tilde{\mathbf{P}}_{ii}, \mathbf{a}_i\}_{j \neq i}$. Furthermore, we express variety-level demand quantities in country i by the Marshallian demand function, $Q_{jik} = \mathcal{D}_{ji,k}(Y_i, \tilde{\mathbf{P}}_i)$, that is a function of income Y_i and after-tax consumer prices in country i, $\tilde{\mathbf{P}}_i \equiv \{\tilde{\mathbf{P}}_{ji}, \tilde{\mathbf{P}}_{ii}\}_{j \neq i} \in \mathbb{P}_i$. Since all equilibrium values are a function of $(\mathbb{P}_i, Y_i)$, demand quantities in foreign countries $j \neq i$ for varieties made in home country i, can be expressed as $Q_{ij,k}(\mathbb{P}_i, Y_i)$. Together, demand quantities can be expressed as $\mathbf{Q}_i(\mathbb{P}_i, Y_i)$, where $\mathbf{Q}_i$ is the quantity of all varieties that home country i consumes (domestically purchased or imported) and exports, i.e., $\mathbf{Q}_i \equiv \{Q_{ji,k}, Q_{ij,k}\}_{j \in \mathbb{C}, k \in \mathbb{K}}$.

Next, let us reproduce the unilaterally optimal policy problem of country i,

$$\max_{\mathbb{P}_i} \quad W_i(\mathbb{P}_i; Y_i) \quad \text{subject to } (\mathbb{P}_i; Y_i) \in \mathcal{E}^r_Y,$$

where $\mathcal{E}^r_Y$ is the feasible set of policy-income pairs $(\mathbb{P}_i; Y_i)$ that satisfy the general equilibrium constraints, as formally defined in Appendix A.3. Recall that in this reformulation, vector of wages, $\bar{\mathbf{w}}$, can be treated as fixed. As detailed in Section 3.1, prices $\{P_{ji,k}\}_{j,k}$, emission levels $\{Z_{j,k}\}_{j,k}$, taxes $\{T_{i,k}\}_k$, etc., are functions of $(\mathbb{P}_i; Y_i)$ in a way that their dependence on income Y_i channels exclusively via demand quantities $\mathbf{Q}_i(\mathbb{P}_i; Y_i)$. As such, we can express prices as $P_{ji,k} = P_{ji,k}(\mathbb{P}_i; Y_i) = P_{ji,k}(\mathbb{P}_i; \mathbf{Q}_i(\mathbb{P}_i; Y_i))$, emission levels as $Z_{j,k} = Z_{ji,k}(\mathbb{P}_i; Y_i) = Z_{ji,k}(\mathbb{P}_i; \mathbf{Q}_i(\mathbb{P}_i; Y_i))$, taxes as $T_{i,k} = T_{i,k}(\mathbb{P}_i; Y_i) = T_{i,k}(\mathbb{P}_i; \mathbf{Q}_i(\mathbb{P}_i; Y_i))$, and so forth. Consequently, the objective function of country i can be written in the following way:

$$W_i(\mathbb{P}_i;\mathbf{Q}_i(\mathbb{P}_i;Y_i)) = V_i(\underbrace{\bar{w}_i\bar{L}_i + T_i(\mathbb{P}_i;\mathbf{Q}_i(\mathbb{P}_i;Y_i))}_{Y_i},\tilde{\mathbf{P}}_i) - \Delta_i\left(\mathbf{Z}(\mathbb{P}_i;\mathbf{Q}_i(\mathbb{P}_i;Y_i))\right),$$

The first order condition with respect to each of policy instruments $\mathsf{P} \in \mathbb{P}_i \equiv \{\tilde{P}_{ji,k}, \tilde{P}_{ij,k}, \tilde{P}_{ii,k}, a_{i,k}\}_{j\neq i,\, k}$ delivers the following equation:

$$\begin{aligned}&\tfrac{\partial T_i(\mathbb{P}_i,\mathbf{Q}_i)}{\partial \ln \mathsf{P}} + \tilde{P}_i\tfrac{\partial V_i(Y_i,\tilde{\mathbf{P}}_i)}{\partial \ln \mathsf{P}} - \tilde{P}_i\tfrac{\partial \Delta_i(\mathbf{Z}(\mathbb{P}_i,\mathbf{Q}_i))}{\partial \ln \mathsf{P}} \\ &+ \left[\tfrac{\partial T_i(\mathbb{P}_i,\mathbf{Q}_i)}{\partial \mathbf{Q}_i} - \tilde{P}_i\tfrac{\partial \Delta_i(\mathbf{Z}(\mathbb{P}_i,\mathbf{Q}_i))}{\partial \mathbf{Q}_i}\right]\cdot \tfrac{\partial \mathbf{Q}_i(\mathbb{P}_i,Y_i)}{\partial \ln \mathsf{P}} \\ &+ \left[\tfrac{\partial T_i(\mathbb{P}_i,\mathbf{Q}_i)}{\partial \mathbf{Q}_i} - \tilde{P}_i\tfrac{\partial \Delta_i(\mathbf{Z}(\mathbb{P}_i,\mathbf{Q}_i))}{\partial \mathbf{Q}_i}\right]\cdot \tfrac{\partial \mathbf{Q}_i(\mathbb{P}_i,Y_i)}{\partial Y_i}\tfrac{\mathrm{d}Y_i}{\mathrm{d}\ln \mathsf{P}} \quad = 0\end{aligned} \tag{B.5}$$

where $\tilde{P}_i \equiv \left(\frac{\partial V_i(.)}{\partial Y_i}\right)^{-1}$ is the consumer price index. In what follows, as a detailed analysis of unilaterally optimal policy, we undertake the task of solving for the system of F.O.C.s described by Equation (B.5). In doing so, we will provide proofs and derivations for Lemma 3, Theorem 1, Lemma 4, and Theorem 2.

Before moving forward, let us emphasize three important details:

1. To economize on notation, we occasionally express demand quantities simply as $\mathbf{Q}_i$ without explicitly stating its functional specification, $\mathbf{Q}_i = \mathbf{Q}_i(\mathbb{P}_i;Y_i)$.

2. Following Lemma 2, we are treating the vector of wages, $\mathbf{w} = \bar{\mathbf{w}}$, as constant throughout our proof. So, all partial derivatives should be interpreted as partial derivatives that hold $\mathbf{w}$ constant. In addition, every time we partially differentiate w.r.t. policy instrument $\mathsf{P} \in \mathbb{P}_i$, we are fixing the remaining elements of $\mathbb{P}_i$—because the government directly chooses every single element of $\mathbb{P}_i$. To connect this point to the forthcoming choice of notation, the partial derivative w.r.t. policy instrument $\mathsf{P} \in \mathbb{P}_i$ takes Y_i, $\mathbf{w}$, and $\mathbb{P}_i - \{\mathsf{P}\}$ as fixed, e.g., $\frac{\partial Q_{ji,k}(\mathbb{P}_i;Y_i)}{\partial \ln \mathsf{P}} \equiv \left(\frac{\partial Q_{ji,k}(\mathbb{P}_i;Y_i)}{\partial \ln \mathsf{P}}\right)_{Y_i,\mathbf{w},\mathbb{P}_i-\{\mathsf{P}\}}$.

3. The same point applies to our notation for partial derivatives when holding demand quantities fixed, e.g., $\frac{\partial T_i(\mathbb{P}_i;\mathbf{Q}_i)}{\partial \ln \mathsf{P}} \sim \left(\frac{\partial T_i(\mathbb{P}_i;\mathbf{Q}_i)}{\partial \ln \mathsf{P}}\right)_{\mathbf{Q}_i,\mathbf{w},\mathbb{P}_i-\{\mathsf{P}\}}$.

With these points in mind, we now proceed with the proof of Lemmas 3 and 4,. Our goal is to unpack Equation (B.5) with respect to all policy instruments, namely: (i) domestic and import prices $\tilde{\mathbf{P}}_i \equiv \{\tilde{\mathbf{P}}_{ii}, \tilde{\mathbf{P}}_{ji}\}_{j\neq i}$, (ii) abatement levels $\mathbf{a}_i$, (iii) export prices $\{\tilde{\mathbf{P}}_{ij}\}_{j\neq i}$. We establish the welfare-neutrality of income effect within step (i), which we use in our analysis of steps (ii) and (iii).

B.2.1 F.O.C.s for Domestic and Import Prices: $\tilde{\mathbf{P}}_i \equiv \{\tilde{\mathbf{P}}_{ii}, \tilde{\mathbf{P}}_{ji}\}_{j\neq i}$

Consider $\mathsf{P} = \tilde{P}_{ji,k} \in \tilde{\mathbf{P}}_i = \{\tilde{\mathbf{P}}_{ii}, \tilde{\mathbf{P}}_{ji}\}$, that is in the set of consumer prices in i—either domestic ($j = i$) or imported ($j \neq i$) varieties. We unpack the three terms in the first line of Equation (B.5):

– Holding the demand schedule $\mathbf{Q}_i$ fixed, an increase in consumer price $\tilde{P}_{ji,k}$ raises tax revenues mechanically in proportion to demand quantity. In particular,

$$\frac{\partial T_i(\mathbb{P}_i,\mathbf{Q}_i)}{\partial \ln \tilde{P}_{ji,k}} = \tilde{P}_{ji,k}Q_{ji,k}.$$

– Applying the Roy's identity, we can express the effect of raising consumer price $\tilde{P}_{ji,k}$ on consumer surplus (i.e., indirect utility) as

$$\tilde{P}_i \frac{\partial V_i(\tilde{\mathbf{P}}_i, Y_i)}{\partial \ln \tilde{P}_{ji,k}} = -\tilde{P}_{ji,k} Q_{ji,k}.$$

– Holding output quantities fixed, emission depends only on abatement which is pinned down by the choice set $\mathbb{P}_i$ (see equations 2 and 4). Hence, the change in $\tilde{P}_{ji,k}$ has no direct effect on CO_2 emissions given $\mathbf{Q}_i$ and $\mathbb{P}_i - \{\tilde{P}_{ji,k}\}$:

$$\frac{\partial \Delta_i \left(\mathbf{Z}(\mathbb{P}_i, \mathbf{Q}_i) \right)}{\partial \tilde{P}_{ji,k}} = 0.$$

Combining the three bullet points listed above, the first line of Equation (B.5) collapses to zero when $\mathsf{P} = \tilde{P}_{ji,k} \in \tilde{\mathbf{P}}_i$. Namely,

$$\frac{\partial T_i(\mathbb{P}_i, \mathbf{Q}_i)}{\partial \ln \tilde{P}_{ji,k}} + \tilde{P}_i \frac{\partial V_i(\tilde{\mathbf{P}}_i, Y_i)}{\partial \ln \tilde{P}_{ji,k}} - \frac{\partial \Delta_i \left(\mathbf{Z}(\mathbb{P}_i, \mathbf{Q}_i) \right)}{\partial \tilde{P}_{ji,k}} = 0$$

Hence, for $\mathsf{P} = \tilde{P}_{ji,k} \in \tilde{\mathbf{P}}_i$, Equation (B.5) reduces to

$$\begin{aligned} & \left[\tfrac{\partial T_i(\mathbb{P}_i, \mathbf{Q}_i)}{\partial \mathbf{Q}_i} - \tilde{P}_i \tfrac{\partial \Delta_i (\mathbf{Z}(\mathbb{P}_i, \mathbf{Q}_i))}{\partial \mathbf{Q}_i} \right] \cdot \tfrac{\partial \mathbf{Q}_i(\mathbb{P}_i, Y_i)}{\partial \ln \tilde{P}_{ji,k}} \\ + & \left[\tfrac{\partial T_i(\mathbb{P}_i, \mathbf{Q}_i)}{\partial \mathbf{Q}_i} - \tilde{P}_i \tfrac{\partial \Delta_i (\mathbf{Z}(\mathbb{P}_i, \mathbf{Q}_i))}{\partial \mathbf{Q}_i} \right] \cdot \tfrac{\partial \mathbf{Q}_i(\mathbb{P}_i, Y_i)}{\partial Y_i} \tfrac{\mathrm{d} Y_i}{\mathrm{d}\ln \tilde{P}_{ji,k}} \quad = 0. \end{aligned} \tag{B.6}$$

The first line of Equation (B.6) accounts for the general equilibrium welfare effects of raising $\tilde{P}_{ji,k} \in \tilde{\mathbf{P}}_i$ while holding income Y_i fixed—this is true because the first line of Equation (B.5) turned out to be zero for $\tilde{P}_{ji,k} \in \tilde{\mathbf{P}}_i$— and, the second line summarizes general equilibrium income effects. The trivial solution to Equation (B.6) ensures that

$$\frac{\partial W_i(\mathbb{P}_i, \mathbf{Q}_i)}{\partial \mathbf{Q}_i} = \frac{\partial T_i(\mathbb{P}_i, \mathbf{Q}_i)}{\partial \mathbf{Q}_i} - \tilde{P}_i \frac{\partial \Delta_i \left(\mathbf{Z}(\mathbb{P}_i, \mathbf{Q}_i) \right)}{\partial \mathbf{Q}_i} = 0. \tag{B.7}$$

It is important to show that this trivial solution is the unique solution to the system of equations described by Equation (B.6) —which constitutes $N \times K$ equations. We undertake this task in Appendix B.3 where we show that the trivial solution (from which Equation (B.7) derives) is the unique solution to the system of Equations (B.6). That is, Equation (B.7) outlines the necessary and sufficient conditions for optimality w.r.t. $\tilde{P}_{ji,k} \in \mathbb{P}_i$, starting from Equation (B.5).

Importantly, Equation (B.7) establishes the welfare-neutrality of income effects at the optimum—as stated under Lemma 3 of the main text. Specifically, the solution to Equation (B.5) requires that $\frac{\partial W_i(\mathbb{P}_i, \mathbf{Q}_i)}{\partial \mathbf{Q}_i} = 0$, from which it follows that for each $\mathsf{P} \in \mathbb{P}_i$, the general equilibrium income effects add up to zero:

$$\left[\frac{\partial T_i(\mathbb{P}_i, \mathbf{Q}_i)}{\partial \mathbf{Q}_i} - \tilde{P}_i \frac{\partial \Delta_i \left(\mathbf{Z}(\mathbb{P}_i, \mathbf{Q}_i) \right)}{\partial \mathbf{Q}_i} \right] \cdot \frac{\partial \mathbf{Q}_i(\mathbb{P}_i, Y_i)}{\partial Y_i} \frac{\mathrm{d} Y_i}{\mathrm{d}\ln \mathsf{P}} = 0 \tag{B.8}$$

The above equation states that at the optimal policy choice $\mathbb{P}_i^\star$, the last line of Equation (B.5) must equal zero for every $\mathsf{P} \in \mathbb{P}_i$. In other words, we can simplify the first-order conditions characterized by Equation (B.5) by the dropping the term corresponding to general equilibrium income effects. This result, which holds under very generic conditions, implies that in deriving the first-order conditions demand can be treated as income inelastic.

Next, we unpack and simplify Equation (B.7), which characterizes the optimal choice w.r.t. $\tilde{P}_{ji,k} \in \mathbb{P}_i$. As outlined in Appendix A.3 , taxes collected by country i's government, $T_i(.)$, and emission levels from all origin-industry locations, $\{Z_{n,g}(.)\}_{n,g}$, are formulated as follows:

$$\begin{cases} T_i(\mathbb{P}_i;Y_i) = \sum_g \left[\left(\tilde{P}_{ii,g} - (1-\alpha_{i,g}(\mathbb{P}_i;Y_i)\frac{\gamma_g-1}{\gamma_g}) P_{ii,g}(\mathbb{P}_i;Y_i) \right) Q_{ii,g}(\mathbb{P}_i;Y_i) \right] \\ \qquad + \sum_{n\neq i}\sum_g \left[\left(\tilde{P}_{in,g} - (1-\alpha_{i,g}(\mathbb{P}_i;Y_i)\frac{\gamma_g-1}{\gamma_g}) P_{in,g}(\mathbb{P}_i;Y_i) \right) Q_{in,g}(\mathbb{P}_i;Y_i) \right] \\ \qquad + \sum_{n\neq i}\sum_g \left[(\tilde{P}_{ni,g} - P_{ni,g}(\mathbb{P}_i;Y_i)) Q_{ni,g}(\mathbb{P}_i;Y_i) \right] \\ Z_{n,g}(\mathbb{P}_i;Y_i) = \bar{z}_{n,g} \left(\frac{\alpha_{n,g}(\mathbb{P}_i;Y_i)}{\bar{\kappa}_{n,g}} \right)^{\frac{\varsigma}{\varsigma-1}} (1-a_{n,g})^{\frac{1}{\varsigma\gamma_g}} \left(Q_{n,g}(\mathbb{P}_i;Y_i)\right)^{1-\frac{1}{\gamma_g}}; \\ \quad \text{where} \qquad \alpha_{n,g}(\mathbb{P}_i;Y_i) = 1-(1-\bar{\kappa}_{n,g})\,(1-a_{n,g})^{\frac{1-\varsigma}{\varsigma}}. \end{cases} \tag{B.9}$$

We continue to derive the partial derivative of taxes $T_i(\mathbb{P}_i;Y_i)$ and emissions $Z_{n,g}(\mathbb{P}_i;Y_i)$ w.r.t. $\mathsf{P} = \tilde{P}_{ji,k} \in \tilde{\mathbf{P}}_i$. Starting with tax revenues,

$$\begin{aligned} \frac{\partial T_i(\mathbb{P}_i,\mathbf{Q}_i)}{\partial \mathbf{Q}_i}\cdot\frac{\partial \mathbf{Q}_i(\mathbb{P}_i,Y_i)}{\partial \ln \tilde{P}_{ji,k}} = &\sum_{n\neq i}\sum_g \left[\left(\tilde{P}_{ni,g}-P_{ni,g}\right) Q_{ni,g}\left(\frac{\partial \ln Q_{ni,g}}{\partial \ln \tilde{P}_{ji,k}}\right)_{Y_i} - P_{ni,g}Q_{ni,g}\sum_\ell\sum_s\left[\left(\frac{\partial \ln P_{ni,g}}{\partial \ln Q_{\ell i,s}}\right)_{\mathbb{P}_i,Y_i}\left(\frac{\partial \ln Q_{\ell i,s}}{\partial \ln \tilde{P}_{ji,k}}\right)_{Y_i}\right]\right] \\ &+\sum_{n\neq i}\sum_g\left[-\psi_{i,g}P_{in,g}Q_{in,g}\left[\frac{\partial \ln P_{in,g}(...,Q_{ii,g})}{\partial \ln Q_{ii,g}}\left(\frac{\partial \ln Q_{ii,g}}{\partial \ln \tilde{P}_{ji,k}}\right)_{Y_i}+\sum_{\ell\neq i}\sum_s\left(\frac{\partial \ln P_{in,g}}{\partial \ln Q_{\ell i,s}}\right)_{\mathbb{P}_i,Y_i}\left(\frac{\partial \ln Q_{\ell i,s}}{\partial \ln \tilde{P}_{ji,k}}\right)_{Y_i}\right]\right] \\ &+\sum_g\left[\left[\tilde{P}_{ii,g}-\psi_{i,g}P_{ii,g}\right]Q_{ii,g}\left(\frac{\partial \ln Q_{ii,g}}{\partial \ln \tilde{P}_{ji,k}}\right)_{Y_i} - \psi_{i,g}P_{ii,g}Q_{ii,g}\left[\frac{\partial \ln P_{ii,g}(...,Q_{ii,g})}{\partial \ln Q_{ii,g}}\left(\frac{\partial \ln Q_{ii,g}}{\partial \ln \tilde{P}_{ji,k}}\right)_{Y_i}+\sum_{\ell\neq i}\sum_s\left(\frac{\partial \ln P_{in,g}}{\partial \ln Q_{\ell i,s}}\right)_{\mathbb{P}_i,Y_i}\left(\frac{\partial \ln Q_{\ell i,s}}{\partial \ln \tilde{P}_{ji,k}}\right)_{Y_i}\right]\right], \end{aligned} \tag{B.10}$$

where $\psi_{i,g} \equiv \left(1-\alpha_{i,g}\frac{\gamma_g-1}{\gamma_g}\right)$ and in the second line we set $\left(\frac{\partial \ln Q_{in,g}}{\partial \ln \tilde{P}_{ji,k}}\right)_{Y_i} = 0$ for $n \neq i$. To simplify the above equation, we use Marshallian elasticities to denote how demand quantities react to a change in consumer price $\tilde{P}_{ji,k} \in \tilde{\mathbf{P}}_i$. In particular,

$$\left(\frac{\partial \ln Q_{ni,g}}{\partial \ln \tilde{P}_{ji,k}}\right)_{Y_i} = \frac{\partial \ln \mathcal{D}_{ni,g}(Y_i,\tilde{\mathbf{P}}_i)}{\partial \ln \tilde{P}_{ji,k}} \equiv \varepsilon^{(ji,k)}_{ni,g}, \tag{B.11}$$

where $\varepsilon^{(ji,k)}_{ni,g}$ is the price elasticity of demand. To characterize the change in producer prices we invoke the free-entry condition and refer to the definitions and derivations in Appendix A.6. In particular, the partial derivative of producer prices in Equation (B.10) can be represented as follows:

$$\begin{cases} \sum_n \left[P_{in,g}Q_{in,g}\left(\frac{\partial \ln P_{in,g}}{\partial \ln Q_{ii,g}}\right)_{\mathbb{P}_i,Y_i}\left(\frac{\partial \ln Q_{ii,g}}{\partial \ln \tilde{P}_{ji,k}}\right)_{Y_i}\right] = -\frac{r_{ii,g}}{\gamma_g}P_{ii,g}Q_{i,g}\varepsilon^{(ji,k)}_{ii,g}, & \text{Producer price of home } i; \\ \sum_g\sum_{n\neq i}\left[P_{ni,g}Q_{ni,g}\left(\frac{\partial \ln P_{ni,g}}{\partial \ln Q_{ji,k}}\right)_{\mathbb{P}_i,Y_i}\right] + \sum_g\left[P_{ii,g}Q_{i,g}\left(\frac{\partial \ln P_{ii,g}}{\partial \ln Q_{ji,k}}\right)_{\mathbb{P}_i,Y_i}\right] = \omega_{ji,k}P_{ji,k}Q_{ji,k}, & \text{Producer price of foreign } n\neq i. \end{cases} \tag{B.12}$$

where $P_{ii,g}Q_{i,g} = \sum_n P_{in,g}Q_{in,g}$ denotes origin i–industry g's total sales and $r_{ii,g} = P_{ii,g}Q_{ii,g}/\sum_n P_{in,g}Q_{in,g}$ denotes the sales share associated with variety ii,g. The first line in the above equation, follows from (i) the free-entry condition, and (ii) the fact that all consumer prices associated with production in origin i are fully pinned down by the policy vector $\mathbb{P}_i$.[48] The second line follows from our previous definition for the general equilibrium export supply elasticity, $\omega_{ji,k}$, which was formally presented in Appendix A.6. Recall that $\omega_{ji,k}$ can itself be fully characterized in terms of Marshallian demand elasticities and expenditure/revenue shares.

[48] This explains the difference between the derivatives for producer price of home i, $P_{ii,g}$, and of foreign $n \neq i$, $P_{nn,g}$. The case of $n \neq i$ is more involved because prices charged by $n \neq i$ for itself and any third country $\ell \neq i$, $\tilde{P}_{n\ell,g} = P_{n\ell,g}$, is not a policy choice of country i's government, and so, we must track its change in general equilibrium—see Appendix A.6 for more details.

Replacing the partial derivative expressions (B.11) and (B.12) back into Equation (B.10) yields[49]

$$\frac{\partial T_i(\mathbb{P}_i,\mathbf{Q}_i)}{\partial \mathbf{Q}_i} \cdot \left(\frac{\partial \mathbf{Q}_i(\mathbb{P}_i,Y_i)}{\partial \ln \tilde{P}_{ji,k}}\right)_{Y_i} = \sum_{n\neq i}\sum_g \left[\tilde{P}_{ni,g}Q_{ni,g}\left(1-(\omega_{ni,g}+1)\frac{P_{ni,g}}{\tilde{P}_{ni,g}}\right)\varepsilon^{(ji,k)}_{ni,g}\right] \\ -\sum_g\left[P_{ii,g}Q_{i,g}(1-\alpha_{i,g}\frac{\gamma_g-1}{\gamma_g})\left(-\frac{r_{ii,g}}{\gamma_g}\right)\varepsilon^{(ji,k)}_{ii,g}\right]+\sum_g\left[\tilde{P}_{ii,g}Q_{ii,g}\left(1-(1-\alpha_{i,g}\frac{\gamma_g-1}{\gamma_g})\frac{P_{ii,g}}{\tilde{P}_{ii,g}}\right)\varepsilon^{(ji,k)}_{ii,g}\right], \tag{B.13}$$

Next, we characterize the partial derivative of emission levels w.r.t. $\tilde{P}_{ji,k}\in\mathbb{P}_i$. Before moving forward, note from Equation (2) that $\alpha_{i,k}$ is exclusively a function of $(1-a_{i,k})$, Hence, the value of $\alpha_{i,k}$ remains unchanged in response to a choice of $\tilde{P}_{ji,k}$ given that $\alpha_{i,k}$ is pined down by $a_{i,k}=\mathbb{P}_{i,k}$. Recalling that $\Delta_i(\mathbf{Z}(\mathbb{P}_i,Y_i))=\sum_{n,g}\delta_{ni,g}Z_{n,g}(\mathbb{P}_i;Y_i)$, we obtain

$$\frac{\partial(\Delta_i(\mathbf{Z}(\mathbb{P}_i,\mathbf{Q}_i)))}{\partial \ln \mathbf{Q}_i}\left(\frac{\partial \mathbf{Q}_i(\mathbb{P}_i,Y_i)}{\partial \ln \tilde{P}_{ji,k}}\right)_{Y_i} = \sum_g\sum_n\left[\delta_{ni,g}\frac{\partial Z_{n,g}(\ldots;Q_{n,g})}{\partial \ln Q_{n,g}}\frac{\partial \ln Q_{n,g}(Q_{n1,g},\ldots,Q_{nN,g})}{\partial \ln Q_{ni,g}}\left(\frac{\partial \ln Q_{ni,g}}{\partial \ln \tilde{P}_{ji,k}}\right)_{Y_i}\right] \\ =\sum_g\sum_n\left[\delta_{ni,g}v_{n,g}P_{ni,g}Q_{ni,g}(\frac{\gamma_g-1}{\gamma_g})\varepsilon^{(ji,k)}_{ni,g}\right] \tag{B.14}$$

where the second line follows from the fact that (i) $\frac{\partial Z_{n,g}(\ldots;Q_{j,g})}{\partial \ln Q_{n,g}}=\frac{\gamma_g-1}{\gamma_g}Z_{n,g}$, (ii) $\frac{\partial \ln Q_{n,g}(Q_{n1,k},\ldots,Q_{nN,k})}{\partial \ln Q_{ni,g}}=r_{ni,g}\equiv P_{ni,g}Q_{ni,g}/(P_{nn,g}Q_{n,g})$, and (iii) $v_{n,g}\equiv Z_{n,g}/P_{nn,g}Q_{n,g}$, which is defined to denote CO_2 emissions per unit value of output.

Now we can plug Equations (B.13) and (B.14) back into Equation (B.7). Recall that Equation (B.7) describes the necessary conditions for optimality w.r.t. $\tilde{P}_{ji,k}\in\mathbb{P}_i$. Doing so delivers the following expression, which is essentially the simplified first-order condition w.r.t. $\tilde{P}_{ji,k}\in\mathbb{P}_i$:

$$\sum_g\left[\Psi_{ii,g}e_{ii,g}\varepsilon^{(ii,k)}_{ii,g}\right]+\sum_{n\neq i}\sum_g\left[\Psi_{ni,g}e_{ni,g}\varepsilon^{(ji,k)}_{ni,g}\right]=0, \qquad \text{for all } \{ji,k\}_{j,k} \tag{B.15}$$

where $e_{ni,g}\equiv\tilde{P}_{ni,g}Q_{ni,g}/Y_i$ denotes gross expenditure shares and $\Psi_{ii,g}$ and $\Psi_{ni,g}$ are composite terms defines as follows:

$$\Psi_{ii,g}\equiv 1-\left(\frac{\gamma_g-1}{\gamma_g}\right)\left[(1-\alpha_{i,g}\frac{\gamma_g-1}{\gamma_g})+\tilde{\delta}_{ii,g}v_{i,g}\right]\frac{P_{ii,g}}{\tilde{P}_{ii,g}} \qquad \text{for all } \{i,g\}_g \tag{B.16}$$

$$\Psi_{ni,g}\equiv 1-\left((1+\omega_{ni,g})+\frac{\gamma_g-1}{\gamma_g}\tilde{\delta}_{ni,g}v_{n,g}\right)\frac{P_{ni,g}}{\tilde{P}_{ni,g}}, \qquad \text{for all } \{ni,g\}_{n\neq i,g} \tag{B.17}$$

To be clear, $\tilde{\delta}_{ni,g}\equiv\tilde{P}_i\delta_{ni,g}$ denotes the CPI-adjusted disutility for carbon. The trivial solution to the system of equations characterized by Equation (B.15) ensures that

$$\Psi_{ii,g}=\Psi_{ni,g}=0, \qquad \text{for all } n\neq i,\ g$$

As noted earlier Appendix B.3 proves that this trivial solution is the unique solution to the system of Equations (B.15) . From $\Psi_{ii,g}=\Psi_{ni,g}=0$ for all $n\neq i$ and all g, we can can immediately derive the following formulas for optimal import taxes and production subsidies:

$$\begin{cases} 1+t^\star_{ni,g}=\frac{\tilde{P}^\star_{ni,g}}{P_{ni,g}}=1+\omega_{ni,g}+\tilde{\delta}_{ni,g}v_{n,g}\frac{\gamma_g-1}{\gamma_g} \\ 1+s^\star_{i,g}=\frac{P^\star_{ii,g}}{\tilde{P}^\star_{ii,g}}=\frac{\gamma_g}{\gamma_g-1} \end{cases}. \tag{B.18}$$

B.2.2 Spelling out the Welfare Neutrality of Income Effects

Before characterizing the optimal abatement levels and export subsidies, let us elucidate the result about the welfare neutrality of income effects. Recall that third line of the original F.O.C. (namely,

[49] To be clear about notation, the second line in Equation B.13 uses $P_{ii,g}Q_{i,g}=\sum_n P_{in,g}Q_{in,g}$ to denote industry-wide sales.

Equation (B.5)) represents general equilibrium income effects. Suppose we included this line when solving the first-order condition w.r.t. $\tilde{P}_{ji,k}$. Then, we would arrive at the following system of $N \times K$ equations:

$$\begin{aligned}&\sum_g \left[\Psi_{ii,g} e_{ii,g} \varepsilon_{ii,g}^{(ii,k)} + \sum_{n \neq i} \Psi_{ni,g} e_{ni,g} \varepsilon_{ni,g}^{(ji,k)}\right] \\ &+ \left[\sum_g \Psi_{i,g} e_{ii,g} \eta_{ii,g} + \sum_{n \neq i} \sum_g \Psi_{ni,g} e_{ni,g} \eta_{ni,g}\right] \frac{\mathrm{d} \ln Y_i}{\mathrm{d} \ln \tilde{P}_{ji,k}} \quad = 0, \qquad \text{for all } \{ji, k\}_{j,k}\end{aligned} \tag{B.19}$$

where $\eta_{ni,g} \equiv \left(\frac{\partial \ln Q_{nig}}{\partial \ln Y_i}\right)_{\mathbb{P}_i} = \frac{\partial \ln \mathcal{D}_{ni,g}(Y_i, \tilde{\mathbf{P}}_i)}{\partial \ln Y_i}$ is the income elasticity of demand. The second line in Equation (B.15) corresponds to the same equation presented in its compact form under Equation (B.8). We could equivalently express the system characterized by Equation B.19 in matrix notation as

$$\boldsymbol{\Psi}_i \tilde{\mathbf{E}}_i^{(ji,k)} + \boldsymbol{\Psi}_i \tilde{\boldsymbol{\Gamma}}_i \left(\frac{d \ln Y_i}{d \ln \tilde{P}_{ji,k}}\right) = 0, \qquad \text{for all } \{ji, k\}_{j,k}$$

where $\boldsymbol{\Psi}_i$ is a $NK \times 1$ column vector, whereas $\tilde{\mathbf{E}}_i^{(ji,k)}$ and $\tilde{\boldsymbol{\Gamma}}_i$ are $1 \times NK$ row vectors consisting of expenditure-adjusted demand elasticities:

$$\boldsymbol{\Psi}_i \equiv \left[\begin{array}{ccccccccc} \Psi_{1i,1} & ... & \Psi_{Ni,1} & \Psi_{1i,2} & ... & \Psi_{Ni,2} & ... & \Psi_{1i,K} & ... & \Psi_{Ni,K} \end{array}\right];$$

$$\tilde{\mathbf{E}}_i^{(ji,k)} \equiv \left[\begin{array}{ccccccccc} e_{1i,1}\varepsilon_{1i,1}^{(ji,k)} & \cdots & e_{1i,K}\varepsilon_{1i,K}^{(ji,k)} & e_{2i,1}\varepsilon_{2i,1}^{(ji,k)} & \cdots & e_{2i,K}\varepsilon_{2i,K}^{(ji,k)} & ... & e_{Ni,1}\varepsilon_{Ni,1}^{(ji,k)} & \cdots & e_{Ni,K}\varepsilon_{Ni,K}^{(ji,k)} \end{array}\right]^T;$$

$$\tilde{\boldsymbol{\Gamma}}_i \equiv \left[\begin{array}{ccccccccc} e_{1i,1}\eta_{1i,1} & \cdots & e_{1i,K}\eta_{1i,K} & e_{2i,1}\eta_{2i,1} & \cdots & e_{2i,K}\varepsilon_{2i,K} & ... & e_{Ni,1}\eta_{Ni,1} & \cdots & e_{Ni,K}\eta_{Ni,K} \end{array}\right]^T.$$

Our result about the neutrality of income effects can be seen most clearly by inspecting the above equation. The trivial, unique solution to the system ensures that $\boldsymbol{\Psi}_i = \mathbf{0}_{NK\times 1}$. Hence, the general equilibrium income effect (i.e., the second term in the above equation, $\boldsymbol{\Psi}_i \tilde{\boldsymbol{\Gamma}}_i$) has to be equal to zero. Again, this result follows from setting $\boldsymbol{\Psi}_i = \mathbf{0}_{NK\times 1}$, which means that import tariffs and production subsidies are set to their optimum values, as characterized by Equation (B.18).

B.2.3 F.O.C.s for Abatement: $\mathbf{a}_i$

Next, we characterize and simplify the first-order condition w.r.t. $(1 - a_{i,k}) \in \mathbb{P}_i$ to characterize optimal abatement, $a_{i,k}^{\star}$. To this end, we appeal the general characterization of the first-order conditions under Equation (B.5), and set $\mathsf{P} = a_{i,k}$. The third line of Equation (B.5) equals zero given our earlier result about the neutrality of income effects (which holds if $\tilde{\mathbf{P}}_i \equiv \{\tilde{\mathbf{P}}_{ii}, \tilde{\mathbf{P}}_{ji}\}_{j\neq i}$ is chosen optimally). In addition, the second line of Equation (B.5) is trivially zero, because holding national income, $Y_i = w_i L_i + T_i$, and $\mathbb{P}_i - \{a_{i,k}\}$ fixed, demand quantities are invariant to the choice of abatement. Namely,

$$\frac{\partial Q_{ni,g}(\mathbb{P}_i, Y_i)}{\partial \ln(1 - a_{i,k})} = 0, \qquad \text{for all } n, g$$

Consequently, what all remains is the task of unpacking the first line in Equation (B.5). Since all instruments except for $a_{i,k}$ (i.e., $\mathbb{P}_i - \{a_{i,k}\}$) are held fixed, the choice of abatement has no direct effect on the indirect utility from consumption, V_i,

$$\left(\frac{\partial V_i(.)}{\partial \ln(1 - a_{i,k})}\right)_{Y_i} = 0.$$

The choice of abatement, however, affects the indirect utility from consumption through its effect on tax revenues. In particular,

$$\begin{aligned}
\frac{\partial T_i(\mathbb{P}_i,\mathbf{Q}_i)}{\partial \ln(1-a_{i,k})} &= -\sum_{j=1}^N \left((1-\alpha_{i,k}\tfrac{\gamma_k-1}{\gamma_k}) P_{ij,k}Q_{ij,k}\frac{\partial \ln P_{ii,k}(\mathbb{P}_i,\mathbf{Q}_i)}{\partial \ln(1-a_{i,k})}\right) + \sum_{j=1}^N \left(\alpha_{i,k}\tfrac{\gamma_k-1}{\gamma_k} P_{ij,k}Q_{ij,k}\frac{\partial \ln \alpha_{i,k}(\mathbb{P}_i,\mathbf{Q}_i)}{\partial \ln(1-a_{i,k})}\right)\\
&= -\sum_{j=1}^N \left[(1-\alpha_{i,k}\tfrac{\gamma_k-1}{\gamma_k})\tfrac{1}{\varsigma}(\tfrac{1-\gamma_k}{\gamma_k}) + \tfrac{\gamma_k-1}{\gamma_k}\tfrac{1-\varsigma}{\varsigma}(1-\alpha_{i,k})\right] P_{ij,k}Q_{ij,k}\\
&= \alpha_{i,k}\left(\tfrac{\gamma_k-1}{\gamma_k}\right)\left(\tfrac{1}{\varsigma\gamma_k}+\tfrac{1}{\alpha_{i,k}}-1\right) P_{ii,k}Q_{i,k}
\end{aligned}$$

where the second line uses $\frac{\partial \ln P_{ii,k}(\mathbb{P}_i,\mathbf{Q}_i)}{\partial \ln(1-a_{i,k})} = \frac{1}{\varsigma}(\frac{1-\gamma_k}{\gamma_k})$ according to Equation 3 and $\frac{\partial \ln \alpha_{i,k}(\mathbb{P}_i,\mathbf{Q}_i)}{\partial \ln(1-a_{i,k})} = -\frac{1-\varsigma}{\varsigma}\frac{1-\alpha_{i,k}}{\alpha_{i,k}}$ according to Equation 2. In the third line, $P_{ii,k}Q_{i,k} = \sum_{j=1}^N P_{ij,k}Q_{ij,k}$ total sales associated with origin i–industry k.

Lastly, the choice of abatement affects local carbon emissions and the corresponding disutility from carbon, $\Delta_i(\mathbf{Z}(\mathbb{P}_i,\mathbf{Q}_i)) = \sum_n \sum_i \left[\delta_{ni,g} Z_{n,g}(\mathbb{P}_i,\mathbf{Q}_i)\right]$. These effects are described by the following partial derivative:

$$\frac{\partial \Delta_i(\mathbf{Z}(\mathbb{P}_i,\mathbf{Q}_i))}{\partial \ln(1-a_{i,k})} = \left(\frac{1}{\varsigma\gamma_k}+\frac{1}{\alpha_{i,k}}-1\right)\delta_{ii,k}Z_{i,k}.$$

Next, we plug the expressions for $\frac{\partial T_i(.)}{\partial \ln(1-a_{i,k})}$ and $\frac{\partial \Delta_i(\mathbf{Z}(.))}{\partial \ln(1-a_{i,k})}$ back into Equation (B.5) for the case where $\mathsf{P} = (1-a_{i,k})$. In this process we not that the third line in Equation (B.5) equals zero when the remaining elements of $\mathbb{P}_i$ are chosen optimally. This step yields the following optimality condition w.r.t. abatement:

$$\begin{aligned}
\alpha_{i,k}\left(\frac{\gamma_k-1}{\gamma_k}\right)\left(\frac{1}{\varsigma\gamma_k}+\frac{1}{\alpha_{i,k}}-1\right) P_{ii,k}Q_{i,k} \;-\; \tilde{P}_i\left(\frac{1}{\varsigma\gamma_k}+\frac{1}{\alpha_{i,k}}-1\right)\delta_{ii,k}Z_{i,k} = 0 \qquad \text{(B.20)}\\
\Rightarrow \quad \frac{\alpha_{i,k}\left(\frac{\gamma_k-1}{\gamma_k}\right)}{\tilde{\delta}_{ii,k}} - \frac{Z_{i,k}}{P_{ii,k}Q_{i,k}} = 0
\end{aligned}$$

In the above expression, $\tilde{\delta}_{ii,k} = \tilde{P}_i\delta_{ii,k}$ denotes the CPI-adjusted disutility per unit of CO_2 emissions and $v_{i,k} \equiv \frac{Z_{i,k}}{P_{ii,k}Q_{i,k}}$ denotes carbon intensity (i.e., CO_2 emissions per unit value). Firm-level cost-minimization implies the following relationship between carbon intensity and carbon taxes: $\nu_{i,k} = \frac{\gamma_k-1}{\gamma_k}\frac{\alpha_{i,k}}{\tau_{i,k}}$. Appealing to this relationship, the last line in the above equation delivers the following formula for country i's unilaterally optimal carbon taxes:

$$\tau^\star_{i,k} = \tilde{\delta}_{ii,k}. \qquad \text{(B.21)}$$

B.2.4 F.O.C.s for Exported Prices: $\{\tilde{\mathbf{P}}_{ij}\}_{j\neq i}$

Finally, we characterize and simplify the first-order condition w.r.t. $\tilde{P}_{ij,k} \in \mathbb{P}_i$ to characterize optimal vector of export prices to destination j, $\tilde{\mathbf{P}}^\star_{ij}$. To this end, we appeal the general characterization of the first-order conditions under Equation (B.5), and set $\mathsf{P} = P_{ij,k}$ for all $j \neq i$ and all k. Following our earlier result about the neutrality of income effects, we can set the third line of Equation (B.5) to zero (given that $\tilde{\mathbf{P}}_i \equiv \{\tilde{\mathbf{P}}_{ii}, \tilde{\mathbf{P}}_{ji}\}_{j\neq i}$ is chosen optimally). The direct effect of $\tilde{P}_{ij,k}$ on the indirect utility from consumption is also zero in Equation (B.5). That is because consumer prices in a foreign location $j \neq i$ do not directly enter the indirect utility function, $V_i(Y_i, \tilde{\mathbf{P}}_i)$:

$$\tilde{P}_{ij,k} \notin \tilde{\mathbf{P}}_i \implies \left(\frac{\partial V_i(Y_i,\tilde{\mathbf{P}}_i)}{\partial \ln \tilde{P}_{ij,k}}\right)_{Y_i} = 0$$

Given the above observations, we can organize the remaining terms in Equation (B.5) to arrive at the following expression for optimality w.r.t. $\tilde{P}_{ij,k}$:

$$\underbrace{\frac{\partial T_i(\mathbb{P}_i,\mathbf{Q}_i)}{\partial \ln \tilde{P}_{ij,k}} + \frac{\partial T_i(\mathbb{P}_i,\mathbf{Q}_i)}{\partial \mathbf{Q}_i}\frac{\partial \mathbf{Q}_i(\mathbb{P}_i,Y_i)}{\partial \ln \tilde{P}_{ij,k}}}_{\frac{\partial T_i(\mathbb{P}_i,Y_i)}{\partial \ln \tilde{P}_{ij,k}}} - \tilde{P}_i \underbrace{\left[\frac{\partial \Delta_i\left(\mathbf{Z}(\mathbb{P}_i,\mathbf{Q}_i)\right)}{\partial \ln \tilde{P}_{ij,k}} + \frac{\partial \Delta_i\left(\mathbf{Z}(\mathbb{P}_i,\mathbf{Q}_i)\right)}{\partial \mathbf{Q}_i}\cdot\frac{\partial \mathbf{Q}_i(\mathbb{P}_i,Y_i)}{\partial \ln \tilde{P}_{ij,k}}\right]}_{\frac{\partial \Delta_i\left(\mathbf{Z}(\mathbb{P}_i,Y_i)\right)}{\partial \ln \tilde{P}_{ij,k}}} = 0 \tag{B.22}$$

We now unpack the non-zero terms in Equation (B.22), starting with the partial derivative of tax revenues. The mechanical effect of export prices on tax revenues, holding demand quantities fixed, is proportional to export quantity of the taxed variety. Specifically given that $\tilde{P}_{ij,k} \in \mathbb{P}_i$,

$$\frac{\partial T_i(\mathbb{P}_i,\mathbf{Q}_i)}{\partial \ln \tilde{P}_{ij,k}} = \tilde{P}_{ij,k}Q_{ij,k}. \tag{B.23}$$

The next step is to characterize the behavioral and general equilibrium effects of $\tilde{P}_{ij,k}$ on tax revenues. These former tracks the change in demand quantities in response to a change in export price $\tilde{P}_{ij,k}$, holding income levels fixed. Taking a partial derivative from the tax revenue function (B.9) and noting that $\tilde{P}_{ij,k} \in \mathbb{P}_i$, we obtain,

$$\begin{aligned}
\frac{\partial T_i(\mathbb{P}_i,\mathbf{Q}_i)}{\partial \mathbf{Q}_i}\cdot\frac{\partial \mathbf{Q}_i(\mathbb{P}_i,Y_i)}{\partial \ln \tilde{P}_{ij,k}} &= \sum_{n\neq i}\sum_g\left[-P_{ni,g}Q_{ni,g}\sum_\ell\sum_s\left[\left(\frac{\partial \ln P_{ni,g}}{\partial \ln Q_{\ell j,s}}\right)_{\mathbb{P}_i,Y_i}\left(\frac{\partial \ln Q_{\ell j,s}}{\partial \ln \tilde{P}_{ij,k}}\right)_{Y_i}\right]\right] \\
&+\sum_g\left[-\psi_{i,g}P_{ii,g}Q_{ii,g}\left[\frac{\partial \ln P_{ii,g}(...,Q_{ij,g})}{\partial \ln Q_{ij,g}}\left(\frac{\partial \ln Q_{ii,g}}{\partial \ln \tilde{P}_{ji,k}}\right)_{Y_i} + \sum_{\ell\neq i}\sum_s\left(\frac{\partial \ln P_{ii,g}}{\partial \ln Q_{\ell j,s}}\right)_{\mathbb{P}_i,Y_i}\left(\frac{\partial \ln Q_{\ell j,s}}{\partial \ln \tilde{P}_{ij,k}}\right)_{Y_i}\right]\right] \\
&+\sum_g\left[\left[\tilde{P}_{ij,g}-\psi_{i,g}P_{ij,g}\right]Q_{ij,g}\left(\frac{\partial \ln Q_{ij,g}}{\partial \ln \tilde{P}_{ij,k}}\right)_{Y_i} - \sum_{n\neq i}\psi_{i,g}P_{in,g}Q_{in,g}\left[\frac{\partial \ln P_{in,g}(...,Q_{ij,g})}{\partial \ln Q_{ij,g}}\left(\frac{\partial \ln Q_{ij,g}}{\partial \ln \tilde{P}_{ij,k}}\right)_{Y_i} + \sum_{\ell\neq i}\sum_s\left(\frac{\partial \ln P_{in,g}}{\partial \ln Q_{\ell j,s}}\right)_{\mathbb{P}_i,Y_i}\left(\frac{\partial \ln Q_{\ell j,s}}{\partial \ln \tilde{P}_{ij,k}}\right)_{Y_i}\right]\right]
\end{aligned} \tag{B.24}$$

where $\psi_{i,g} \equiv \left(1-\alpha_{i,g}\frac{\gamma_g-1}{\gamma_g}\right)$. Let us briefly elaborate on the derivation: Noticing that $j \neq i$ is a foreign destination, a change in prices in that destination has no *direct* effect on the demand schedule in market i: (1) $\left(\frac{\partial \ln Q_{ii,g}}{\partial \ln \tilde{P}_{ij,k}}\right)_{Y_i} = \left(\frac{\partial \ln Q_{ni,g}}{\partial \ln \tilde{P}_{ij,k}}\right)_{Y_i} = 0$, and (2) $\left(\frac{\partial \ln Q_{in,g}}{\partial \ln \tilde{P}_{ij,k}}\right)_{Y_i} = 0$ for $n \neq j$. As before, we characterize the change in producer prices by invoking the free-entry condition and referring to the definitions and derivations in Appendix A.6. In particular, the partial derivative of producers in Equation (B.24) can be represented as follows:

$$\begin{cases}
\sum_n\left[P_{in,g}Q_{in,g}\left(\frac{\partial \ln P_{in,g}}{\partial \ln Q_{ij,g}}\right)_{\mathbb{P}_i,Y_i}\left(\frac{\partial \ln Q_{ij,g}}{\partial \ln \tilde{P}_{ij,k}}\right)_{Y_i}\right] = -\frac{r_{ij,g}}{\gamma_g}P_{ii,g}Q_{i,g}\varepsilon^{(ij,k)}_{ij,g}, & \text{Producer price of home } i; \\
\sum_g\sum_{n\neq i}\left[P_{ni,g}Q_{ni,g}\left(\frac{\partial \ln P_{ni,g}}{\partial \ln Q_{ij,k}}\right)_{\mathbb{P}_i,Y_i}\right] + \sum_g\left[P_{ii,g}Q_{i,g}\left(\frac{\partial \ln P_{ii,g}}{\partial \ln Q_{ij,k}}\right)_{\mathbb{P}_i,Y_i}\right] = \omega_{ji,k}P_{ij,k}Q_{ij,k}, & \text{Producer price of foreign } n \neq i.
\end{cases}$$

where, as before, $P_{ii,g}Q_{i,g} = \sum_n P_{in,g}Q_{in,g}$ denotes origin i–industry g's total sales and $r_{ij,g}$ denotes the sales share associated with variety ij,g. To repeat ourselves, the first line in the above equation follows from (1) the free-entry condition, and (2) the fact that all consumer prices associated with production in origin i are fully pinned down by the policy vector $\mathbb{P}_i$.[50] The second line follows from our definition for the general equilibrium export supply elasticity ($\omega_{ji,g}$) in Appendix **??** and

[50] This explains the difference between the derivatives for producer price of home i, $P_{ii,g}$, and of foreign $n \neq i$, $P_{nn,g}$. The

the fact that $P_{ni,g}Q_{ni,g}\left(\frac{\partial \ln P_{ni,g}}{\partial \ln Q_{nj,k}}\right)_{\mathbb{P}_i,Y_i} = P_{nj,g}Q_{nj,g}\left(\frac{\partial \ln P_{ni,g}}{\partial \ln Q_{ni,k}}\right)_{\mathbb{P}_i,Y_i}$.[51]

Plugging the above expressions for partial derivatives into Equation B.24 and noting that $\left(\frac{\partial \ln Q_{ij,g}}{\partial \ln \tilde{P}_{ij,k}}\right)_{Y_i} = \varepsilon_{ij,g}^{(ij,k)}$, simplifies Equation B.24 as follows:

$$\begin{aligned}\frac{\partial T_i(\mathbb{P}_i,\mathbf{Q}_i)}{\partial \mathbf{Q}_i}\cdot\frac{\partial \mathbf{Q}_i(\mathbb{P}_i,Y_i)}{\partial \ln \tilde{P}_{ij,k}} = \; & \tilde{P}_{ij,k}Q_{ij,k} + \sum_g\left[\left(\tilde{P}_{ij,g} - (1-\alpha_{i,g}\tfrac{\gamma_g-1}{\gamma_g})P_{ij,g}\right)Q_{ij,g}\varepsilon_{ij,g}^{(ij,k)}\right] \\ & - \sum_g\left[-(1-\alpha_{i,g}\tfrac{\gamma_g-1}{\gamma_g})\tfrac{r_{ij,g}}{\gamma_g}\varepsilon_{ij,g}^{(ij,k)}\sum_n\left(P_{in,g}Q_{in,g}\right)\right] - \sum_{n\neq i}\sum_g\left[P_{ni,g}Q_{ni,g}\omega_{nj,g}\varepsilon_{nj,g}^{(ij,k)}\right]\end{aligned}$$

Replacing for $r_{ij,g}\times\sum_n\left(P_{in,g}Q_{in,g}\right) = P_{ij,g}Q_{ij,g}$ in the first line and combining the sums in the first and second lines further simplifies Equation B.24 as

$$\frac{\partial T_i(\mathbb{P}_i,\mathbf{Q}_i)}{\partial \mathbf{Q}_i}\cdot\frac{\partial \mathbf{Q}_i(\mathbb{P}_i,Y_i)}{\partial \ln \tilde{P}_{ij,k}} = \tilde{P}_{ij,k}Q_{ij,k} + \sum_g\left[\left(\tilde{P}_{ij,g} - (1-\alpha_{i,g}\tfrac{\gamma_g-1}{\gamma_g})\tfrac{\gamma_g-1}{\gamma_g}P_{ij,g}\right)Q_{ij,g}\varepsilon_{ij,g}^{(ij,k)}\right] - \sum_{n\neq i}\sum_g\left(P_{nj,g}Q_{nj,g}\omega_{ni,g}\varepsilon_{nj,g}^{(ij,k)}\right) \tag{B.25}$$

We now turn to the second line of Equation (B.22) that tracks the effect of a change in export price, $\tilde{P}_{ij,k}$, on the disutility from global CO_2 emissions, $\mathbf{Z}$. To characterize these effects, we use (B.9) that specifies CO_2 emissions as a function of $(\mathbb{P}_i, Y_i)$. Taking partial derivatives from $\mathbf{Z}(\mathbb{P}_i, Y_i)$ w.r.t. $\tilde{P}_{ij,k}\in\mathbb{P}_i$ delivers

$$\begin{aligned}\frac{\partial \Delta_i(\mathbf{Z}(\mathbb{P}_i;Y_i))}{\partial \ln \tilde{P}_{ij,k}} &= \underbrace{\frac{\partial \Delta_i\left(\mathbf{Z}(\mathbb{P}_i,\mathbf{Q}_i)\right)}{\partial \ln \tilde{P}_{ij,k}}}_{=0} + \frac{\partial \Delta_i(\mathbf{Z}(\mathbb{P}_i,\mathbf{Q}_i))}{\partial \mathbf{Q}_i}\cdot\frac{\partial \mathbf{Q}_i(\mathbb{P}_i,Y_i)}{\partial \ln \tilde{P}_{ij,k}} \\ &= \sum_{n,g}\left[\delta_{ni,g}\frac{\partial Z_{n,g}(\ldots,Q_{n,g})}{\partial \ln Q_{nj,g}}\frac{\partial \ln Q_{n,g}(Q_{n1,g},\ldots,Q_{nN,g})}{\partial \ln Q_{nj,g}}\left(\frac{\partial \ln Q_{nj,g}}{\partial \ln \tilde{P}_{ij,k}}\right)_{Y_i}\right] \\ &= \sum_n\sum_g\left[\delta_{ni,g}v_{n,g}\frac{\gamma_g-1}{\gamma_g}P_{nj,g}Q_{nj,g}\varepsilon_{nj,g}^{(ij,k)}\right],\end{aligned} \tag{B.26}$$

where the last line follows from (1) $\frac{\partial Z_{n,g}(\ldots;Q_{j,g})}{\partial \ln Q_{n,g}} = \frac{\gamma_g-1}{\gamma_g}Z_{n,g}$, (2) $\frac{\partial \ln Q_{n,g}(Q_{n1,k},\ldots,Q_{nN,k})}{\partial \ln Q_{nj,g}} = P_{nj,g}Q_{nj,g}/(P_{nn,g}Q_{n,g})$, and (3) $v_{n,g}\equiv Z_{n,g}/P_{nn,g}Q_{n,g}$. Plugging Equation (B.25) and the last line in Equation (B.26) back into (B.22), and dividing everything by $\tilde{P}_{ij,k}Q_{ij,k}$ delivers a simplified expression for the first-order condition w.r.t. $\tilde{P}_{ij,k}$:

$$\begin{aligned}1+ \; & \sum_g\left[\left(1-\left(1-\alpha_{i,g}\frac{\gamma_g-1}{\gamma_g}+\tilde{\delta}_{ii,g}v_{i,g}\right)\frac{\gamma_g-1}{\gamma_g}\frac{P_{ij,g}}{\tilde{P}_{ij,g}}\right)\frac{e_{ij,g}}{e_{ij,k}}\varepsilon_{ij,g}^{(ij,k)}\right] \\ & - \sum_{n\neq i}\sum_g\left[\left(\omega_{ni,g}+\tilde{\delta}_{ni,g}v_{n,g}\frac{\gamma_g-1}{\gamma_g}\right)\frac{P_{nj,g}}{\tilde{P}_{nj,g}}\frac{e_{nj,g}}{e_{ij,k}}\varepsilon_{nj,g}^{(ij,k)}\right] = 0\end{aligned} \tag{B.27}$$

As before, $\tilde{\delta}_{ni,g}\equiv\tilde{P}_i\times\delta_{ni,g}$ denotes the CPI-adjusted disutility from CO_2 emissions and $e_{nj,g}\equiv(\tilde{P}_{nj,g}Q_{nj,g})/Y_j$ denotes the gross expenditure share on variety nj,g. Following Equations (B.18) and (B.21), when import taxes, production subsidies, and carbon taxes are set optimally, we can simplify Equation B.27 by setting $\omega_{ni,g}+\tilde{\delta}_{ni,g}v_{n,g}\frac{\gamma_g-1}{\gamma_g} = t^\star_{ni,g}$, and $\tilde{\delta}_{ii,g}v_{i,g}(\tau^\star_{i,g}) - \alpha_{i,g}\frac{\gamma_g-1}{\gamma_g} = 0$. To recover the optimal export subsidy formula from Equation B.27, we guess the following formula-

case of $n\neq i$ is more involved because prices charged by $n\neq i$ for itself and any third country $\ell\neq i$, $\tilde{P}_{n\ell,g} = P_{n\ell,g}$, is not a policy choice of country i's government, and so, we must track its change in general equilibrium—see Appendix **??** for more details.

[51] More specifically, this relationship can be derived as follows:

$$\begin{aligned}P_{ni,g}Q_{ni,g}\left(\frac{\partial \ln P_{ni,g}}{\partial \ln Q_{nj,k}}\right)_{\mathbb{P}_i,Y_i} &= P_{ni,g}Q_{ni,g}\left(\frac{\partial \ln P_{nn,g}}{\partial \ln Q_{n,g}}\right)_{Y_i}\frac{\partial \ln Q_{n,g}(Q_{n1,g},\ldots,Q_{nN,g})}{\partial \ln Q_{nj,g}} = r_{nj,k}P_{ni,g}Q_{ni,g}\left(\frac{\partial \ln P_{nn,g}}{\partial \ln Q_{n,g}}\right)_{Y_i} \\ &= r_{ni,k}P_{nj,g}Q_{nj,g}\left(\frac{\partial \ln P_{nn,g}}{\partial \ln Q_{n,g}}\right)_{Y_i} = P_{nj,g}Q_{nj,g}\left(\frac{\partial \ln P_{nn,g}}{\partial \ln Q_{n,g}}\right)_{Y_i}\frac{\partial \ln Q_{n,g}(Q_{n1,g},\ldots,Q_{nN,g})}{\partial \ln Q_{ni,g}} = P_{nj,g}Q_{nj,g}\left(\frac{\partial \ln P_{ni,g}}{\partial \ln Q_{ni,k}}\right)_{\mathbb{P}_i,Y_i}.\end{aligned}$$

tion which features a composite shifter, $\chi_{ij,g}$:

$$(1+x_{ij,g}) \equiv \frac{P_{ij,g}}{\tilde{P}_{ij,g}}\frac{P_{ii,g}}{\tilde{P}_{ii,g}} = \frac{\gamma_g}{\gamma_g - 1}\frac{P_{ij,g}}{\tilde{P}_{ij,g}} = \left(1+\frac{1}{\varepsilon_{ij,g}}\right)\chi_{ij,g} \tag{B.28}$$

To be clear about the notation, $\varepsilon_{ij,k} \sim \varepsilon_{ij,k}^{(ij,k)}$ denotes the own the price elasticity of demand. Our objective is to determine $\chi_{ij,g}$ so that the above formula satisfies the first-order condition described by Equation B.27. Plugging the above guess back into the Equation B.27 yields the following equation in terms of $\mathcal{X}_{ij,k}$:

$$1+\sum_g\left[\left(1-\chi_{ij,k}\frac{1+\varepsilon_{ij,k}^{(ij,k)}}{\varepsilon_{ij,k}^{(ij,k)}}\right)\frac{e_{ij,g}\varepsilon_{ij,g}^{(ij,k)}}{e_{ij,k}}\right]-\sum_{n\neq i}\sum_g\left[t^{\star}_{ni,g}\frac{e_{nj,g}\varepsilon_{nj,g}^{(ij,k)}}{e_{ij,k}}\right]=0; \qquad [\tilde{P}_{ij,k}]$$

To simplify the above equation, we net out the zero-sum terms using *Cournot aggregation*,$1+\sum_g\left[\frac{e_{ij,g}}{e_{ij,k}}\varepsilon_{ij,g}^{(ij,k)}\right] = -\sum_{n\neq i}\sum_g\left[\frac{e_{nj,g}}{e_{ij,k}}\varepsilon_{nj,g}^{(ij,k)}\right]$, and divide everything by $1+\varepsilon_{ij,k}$. This step simplifies the above equation as follows:

$$-\sum_g\left[\chi_{ij,k}\frac{e_{ij,g}}{e_{ij,k}}\frac{\varepsilon_{ij,g}^{(ij,k)}}{\varepsilon_{ij,k}^{(ij,k)}}\right]-\sum_{n\neq i}\sum_g\left[\frac{(1+t^{\star}_{ni,g})e_{nj,g}\varepsilon_{nj,g}^{(ij,k)}}{e_{ij,k}\left(1+\varepsilon_{ij,k}^{(ij,k)}\right)}\right]=0.$$

Noting that $\left(1+\varepsilon_{ij,k}^{(ij,k)}\right)e_{ij,k} = -\sum_{n\neq i}\sum_g e_{nj,g}\varepsilon_{nj,g}^{(ij,k)}$, we can re-write the above equation in matrix notation as

$$\underbrace{\begin{bmatrix}\frac{e_{ij,1}}{e_{ij,1}}\frac{\varepsilon_{ij,1}^{(ij,1)}}{\varepsilon_{ij,1}^{(ij,1)}} & \cdots & \frac{e_{ij,K}}{e_{ij,1}}\frac{\varepsilon_{ij,K}^{(ij,1)}}{\varepsilon_{ij,1}^{(ij,1)}}\\ \vdots & \ddots & \vdots\\ \frac{e_{ij,1}}{e_{ij,K}}\frac{\varepsilon_{ij,1}^{(ij,K)}}{\varepsilon_{ij,K}^{(ij,K)}} & \cdots & \frac{e_{ij,K}}{e_{ij,K}}\frac{\varepsilon_{ij,K}^{(ij,K)}}{\varepsilon_{ij,K}^{(ij,K)}}\end{bmatrix}}_{\mathbf{E}_{ij}}\begin{bmatrix}\chi_{ij,1}\\ \vdots\\ \chi_{ij,K}\end{bmatrix}=\begin{bmatrix}1+\frac{\sum_{n\neq i}\sum_g t^{\star}_{ni,g}e_{nj,g}\varepsilon_{nj,g}^{(ij,1)}}{\sum_{n\neq i}\sum_g e_{nj,g}\varepsilon_{nj,g}^{(ij,1)}}\\ \vdots\\ 1+\frac{\sum_{n\neq i}\sum_g t^{\star}_{ni,g}e_{nj,g}\varepsilon_{nj,g}^{(ij,K)}}{\sum_{n\neq i}\sum_g e_{nj,g}\varepsilon_{nj,g}^{(ij,K)}}\end{bmatrix}.$$

Since $\mid e_{ij,k}\varepsilon_{ij,k}^{(ij,k)} \mid - \sum_{k\neq j} e_{ij,g}\varepsilon_{ij,g}^{(ij,k)} = e_{ij,k}+\sum_{n\neq i}\sum_g e_{ij,g}\varepsilon_{nj,g}^{(ij,k)} > 0$, then $\mathbf{E}_{ij} = \left[\frac{e_{ij,g}\varepsilon_{ij,g}^{(ij,k)}}{e_{ij,k}\varepsilon_{ij,k}^{(ij,k)}}\right]_{k,g}$ is strict diagonally dominant. Hence, given the Lèvy-Desplanques Theorem, $\mathbf{E}_{ij}$ is invertible (Horn and Johnson (2012)) and the above system recovers χ_{ij} as

$$\chi_{ij} = \left[\frac{e_{ij,g}\varepsilon_{ij,g}^{(ij,k)}}{e_{ij,k}\varepsilon_{ij,k}^{(ij,k)}}\right]_{k,g}^{-1}\left(\mathbf{1}_K+\left[\frac{\sum_{n\neq i}t^{\star}_{ni,g}e_{nj,g}\varepsilon_{nj,g}^{(ij,k)}}{\sum_{n\neq i}\sum_g e_{nj,g}\varepsilon_{nj,g}^{(ij,k)}}\right]_k\right). \tag{B.29}$$

Putting together Equations (B.18), (B.21), and (B.28), with $\chi_{ij,k}$ (which is given by Equation (B.29)), country i's unilaterally optimal policy schedule can be summarized as

$$\begin{cases}1+s^{\star}_{i,k}=\frac{\gamma_k}{\gamma_k-1}; \qquad \tau^{\star}_{i,k}=\tilde{\delta}_{ii,k}\\ 1+t^{\star}_{ji,k}=(1+\omega_{ji,k})+\tilde{\delta}_{ji,k}\left(\frac{\gamma_k-1}{\gamma_k}\right)v_{j,k}\\ 1+x^{\star}_{ij,k}=\left(1+\frac{1}{\varepsilon_{ij,k}}\right)\chi_{ij,k}\end{cases}. \tag{B.30}$$

B.3 Uniqueness of Solution to the F.O.C.s

The first-order conditions w.r.t. $\tilde{P}_{ji,k}$ and $\tilde{P}_{ii,k}$ (second line in Lemma 4) are inter-dependent. These condition, however, regulate optimal consumer-to-producer price ratios in market i, i.e., $\tilde{P}^{\star}_{ji,g}/P_{ji,g}$, and do not show up in the first-order conditions w.r.t. export prices, $\tilde{P}_{ij,k}$. Setting $\tau^{\star}_{i,k} = \tilde{\delta}_{ii,k}$, the interdependent first-order conditions w.r.t. $\tilde{P}_{ji,k}$ and $\tilde{P}_{ii,k}$ amount to NK equations and NK unknowns, which are summarized by the following matrix equation:

$$\begin{bmatrix} e_{1i,1}\varepsilon^{(1i,1)}_{1i,1} & \cdots & e_{Ni,}\varepsilon^{(1i,1)}_{Ni,1} & \cdots & e_{1i,K}\varepsilon^{(1i,1)}_{1i,K} & \cdots & e_{Ni,K}\varepsilon^{(1i,1)}_{Ni,K} \\ \vdots & \ddots & & \ddots & & \ddots & \vdots \\ e_{1i,1}\varepsilon^{(Ni,K)}_{1i,1} & \cdots & e_{Ni,}\varepsilon^{(Ni,K)}_{Ni,1} & \cdots & e_{1i,K}\varepsilon^{(Ni,K)}_{1i,K} & \cdots & e_{Ni,K}\varepsilon^{(Ni,K)}_{Ni,K} \end{bmatrix} \begin{bmatrix} \frac{\tilde{P}^{\star}_{1i,k}}{P_{1i,1}} - \left(1 + \omega_{1i,k} + \tilde{\delta}_{1i,k} v_{1,k} \frac{\gamma_k - 1}{\gamma_k}\right) \\ \vdots \\ \frac{\tilde{P}^{\star}_{ii,k}}{P_{ii,k}} - \frac{\gamma_k - 1}{\gamma_k} \\ \vdots \\ \frac{\tilde{P}^{\star}_{Ni,k}}{P_{Ni,k}} - \left(1 + \omega_{Ni,k} + \tilde{\delta}_{Ni,k} v_{N,k} \frac{\gamma_k - 1}{\gamma_k}\right) \end{bmatrix} = \mathbf{0}. \tag{B.31}$$

The first matrix is $NK \times NK$ and the second is $NK \times 1$. Inverting the above matrix equation identifies the optimal tariff, $1 + t^{\star}_{ji,k} = \tilde{P}^{\star}_{ji,k}/P_{ji,k}$, and production subsidy, $1 + s^{\star}_{i,k} = P_{ii,k}/\tilde{P}^{\star}_{ii,k}$ independent from the choice of export subsidies (or $P_{ij,k}/\tilde{P}^{\star}_{ij,k}$). To perform this step, we first prove that the system of equations specified by (B.31) is invertible.

Lemma 6. *The square matrix,* $\Xi = \left[e_{ji,k}\varepsilon^{(ni,g)}_{ji,k}\right]_{ng,jk}$, *is non-singular, with* $|\det(\Xi)| > \prod_{n,k} e_{ni,k} > 0$

The proof of this lemma can be put as follows. Following Proposition 2.E.2 in Mas-Colell et al. (1995) the Walrasian demand function satisfies $e_{ji,k} = | e_{ji,k}\varepsilon^{(ji,k)}_{ji,k} | - \sum_{n,g\neq j,k} | e_{ni,g}\varepsilon^{(ji,k)}_{ni,g} |$. Because there exists a ji,k such that $e_{ji,k} > 0$, the matrix Ξ is strictly diagonally dominant. The Lèvy-Desplanques Theorem (Horn and Johnson (2012)), therefore, ensures that Ξ is non-singular. The lower bound on $\det(\Xi)$ follows trivially from Gerschgorin's circle theorem. Specifically, following Ostrowski (1952),

$$|\det(\Xi)| \geq \prod_j \prod_k \left(| e_{ji,k}\varepsilon^{(ji,k)}_{ji,k} | - \sum_{n,g\neq j,k} | e_{ni,g}\varepsilon^{(ji,k)}_{ni,g} | \right) = \prod_j \prod_k e_{ji,k} > 0.$$

Given Lemma 6, the trivial solution to Equation B.31, which we presented by Equation (B.18), must be the unique solution.

C Proofs and Derivations for Second-best Policies

In this appendix, we analyze unilaterally optimal policy choices under second-best scenarios where the government is unable to use a subset of policy instruments in $\mathbb{P}_i$. To fix ideas, we formally define our notation of second-best.

Definition. The *Unilaterally Second-best Policy* for country i is achieved by choosing a *subset* of policy instruments in $\mathbb{P}_i$ to maximize carbon-adjusted welfare, W_i (equation 14), subject to equilibrium conditions (1)-(8).

We consider three cases: (1) Carbon taxes are unavailable, (2) Export subsidies are unavailable, (3) All policy instruments but carbon taxes are unavailable. For the sake of exposition, this appendix focuses on the special case where abatement technologies have a Cobb-Douglas specification. Namely, we assume a unitary elasticity of substitution between labor and carbon inputs, i.e., $\varsigma = 1$. Under this assumption, the carbon cost share, $\alpha_{i,k}$, is constant and can be interpreted as the CO_2 emission elasticity. Without loss of generality, we also assume that $\alpha_{i,k} = \alpha_k$ is common across origins.

C.1 The government is unable to use carbon taxes

As the emission elasticity approaches zero, i.e., $\alpha_k \to 0$, our model collapses to a model with exogenous emission intensity à la Markusen (1975). As such, carbon taxes are redundant and can be dropped from the model as firms do not undertake abatement. In this case, the optimal production subsidy will include the markup-correcting term $\frac{\gamma_k}{\gamma_k - 1}$ plus an extra term that subsidizes low-carbon-intensive (low-v) industries. Namely,

$$1 + s_{i,k}^{\star\star} = \frac{\gamma_k}{\gamma_k - 1}\left(1 + \tilde{\delta}_{ii,k} v_{i,k}\right)^{-1}$$

As before, the carbon-reducing term depends on $\frac{\gamma_k - 1}{\gamma_k}$ because there are scale economies in abatement. For instance, it may be optimal to subsidize a high-returns-to-scale industry that exhibits a high carbon intensity. That is because subsidizing such an industry may lower CO_2emission through scale effects, dominating the relocation effects that promote high-carbon-intensive firms.

Alternatively, maintaining the assumption that $\alpha_k \in (0,1)$, we could examine second-best production subsidies in cases where the government is not afforded the ability to directly tax carbon. Suppose $v_{i,k}$ is the emission intensity under some carbon tax that is different from the unilaterally optimal. This might be either because emission is unabated, or quite the opposite, because home country has set its carbon tax in line with international agreements at a higher level compared to the unilaterally first best. In either case, production taxes must correct emission externalities that are too little or too much from the unilateral point of view:

$$1 + s_{i,k}^{\star\star} = \frac{\gamma_k}{\gamma_k - 1}\left[1 + \tilde{\delta}_{ii,k}(v_{i,k} - v_{i,k}^{\star})\right]^{-1},$$

where $v_k^{\star}$ is the first-best carbon intensity attainable under the unilaterally optimal policy. Consider a country that emits a unilaterally sub-optimal amount of carbon, i.e., $v_{i,k} < v_{i,k}^{\star}$. This scenario would occur if the country is abiding by international climate agreements. It is optimal in that case to offer additional subsidies to possibly carbon-intensive industries to promote domestic production.

C.2 The government is unable to use export subsidies

In this case, the optimal carbon tax remains uniform and follows the same rule as in the first-best case. Derivations for this case are similar to those resulting in Theorem 2. There is, however, one key difference between this second-best case and the first-best. When export subsidies are restricted, Lemma 2 no longer holds. That is, wage effects are no longer welfare neutral. Instead, even when $\tilde{\mathbf{P}}_i$ is chosen optimally, country i can improve its terms-of-trade by inflating w_i relative to $\mathbf{w}_{-i}$. The resulting optimal tax schedule internalizes these gains by featuring an additional tariff shifter, $\bar{t}_i$, that is uniform and strictly positive. In particular,

$$\begin{cases} 1 + t_{ji,k}^{\star} = (1 + \bar{t}_i)(1 + \omega_{ji,k}) + \tilde{\delta}_{ji,k} v_{j,k} \frac{\gamma_k - 1}{\gamma_k} & \forall j, k \\ 1 + s_{i,k}^{\star} = \frac{\gamma_k}{\gamma_k - 1} & \forall k \\ \tau_{i,k}^{\star} = \tau_i^{\star} = \tilde{\delta}_{ii,k} & \forall k \end{cases} \tag{C.1}$$

More formally, $\bar{t}_i \equiv \left(\frac{\partial \ln(w_i L_i + T_i)}{\partial \ln w_i}\right)_{\hat{\mathbb{P}}_i, \mathbf{Y}} \Big/ \left(\frac{\partial \mathrm{B}_i}{\partial \ln w_i}\right)_{\hat{\mathbb{P}}_i, \mathbf{Y}}$ where $\mathrm{B}_i \equiv \sum_k \sum_{j \neq i} \left[P_{ji,k} Q_{ji,k} - \frac{1}{1+s_{i,k}} P_{ij,k} Q_{ij,k}\right]$ denotes country i's balanced trade condition and $\hat{\mathbb{P}}_i \equiv \mathbb{P}_i - \{\tilde{P}_{ij,k}\}_{j \neq i, k}$ is the second-best policy set that excludes export subsidies. Intuitively, $\bar{t}_i$ is zero under the first-best policy schedule—because when export subsidies are available, they can more-than-replicate the gains from shifting tariffs by $\bar{t}_i > 0$.

C.3 Carbon taxes are used as protection in disguise

Suppose the government is banned from using all tax instruments apart from carbon taxes. In that case, optimal carbon taxes are no longer uniform. It is instead optimal for country i to apply a higher carbon tax on industries where it possesses more export market power—using carbon taxes as protection in disguise. To make this point succinctly, consider a simplified version of our baseline model with the following set of assumptions:

1. Preferences are given by the Cobb-Douglas-CES specification under equation (25);
2. Country i is a small open economy with $\delta_{-ii,k} = 0$; and
3. All industries are perfectly competitive, i.e., $\gamma_k \to \infty$ and $\bar{f}_{i,k} = 0$ for all i and k.

Below, we present an implicit formula for optimal carbon taxes, which indicates that (when the government is banned from using trade policy measures) it is optimal to tax carbon above the first-best level in low-σ industries.

Proposition 1. *Suppose the government in i is unable to apply any tax instruments apart from carbon taxes. The unilaterally optimal carbon tax in this second-best scenario is given by*

$$\tau^*_{i,k} = \left(\frac{\alpha(1-\sigma_k)\left(1-\lambda_{ii,k}r_{ii,k}\right)+1}{\tilde{\alpha}_i(1-\sigma_k)\left(1-\lambda_{ii,k}r_{ii,k}\right)+r_{ii,k}} \right) \tilde{\delta}_{ii}, \tag{C.2}$$

where $\tilde{\alpha}_i > \alpha$ is a country-wide term that depends on the industry-composition of country i's production.

Proof. We can express global vector of CO_2emissions in terms of $\mathbf{Z}(\mathbf{Y}, \tilde{\mathbf{P}}) \equiv \mathbf{Z}(\mathbf{D}(\mathbf{Y}, \tilde{\mathbf{P}}))$ where $\mathbf{D}(\mathbf{Y}, \tilde{\mathbf{P}}) \equiv \left\{\mathcal{D}_{jn,k}(Y_n, \tilde{\mathbf{P}}_n)\right\}_{j,n,k}$ denotes the global vector of Marshallian demand functions. Appealing to this choice of notation, the first-order condition w.r.t. $1 - a_{i,k}$ can be expressed as,

$$\left(\frac{\partial V_i(Y_i, \tilde{\mathbf{P}}_i)}{\partial Y_i} - \frac{\partial \Delta_i(\mathbf{Z}(\mathbf{Y}, \tilde{\mathbf{P}}))}{\partial Y_i}\right)\left(\frac{\partial Y_i}{\partial \ln(1-a_{i,k})}\right)_{\mathbf{w}} + \left(\frac{\partial V_i(.)}{\partial \ln \tilde{\mathbf{P}}_i} - \frac{\partial \Delta_i(\mathbf{Z}(\mathbf{Y}, \tilde{\mathbf{P}}))}{\partial \ln \tilde{\mathbf{P}}_i}\right) \cdot \left(\frac{\partial \ln \tilde{\mathbf{P}}_i}{\partial \ln(1-a_{i,k})}\right)_{\mathbf{w}} + \left(\frac{\partial V_i(.)}{\partial \ln \mathbf{w}} - \frac{\partial \Delta_i(\mathbf{Z})}{\partial \mathbf{w}}\right) \cdot \frac{\mathrm{d} \ln \mathbf{w}}{\mathrm{d} \ln(1-a_{i,k})} = 0, \tag{C.3}$$

where $\Delta_i(\mathbf{Z}) \equiv \sum_{n,k}(\delta_{ni} Z_{n,k})$ and $Y_i = w_i L_i + \sum_k [\alpha_{i,k} P_{ii,k} Q_{i,k}]$. Under constant returns to scale, $\left(\frac{\partial \ln P_{in,k}}{\partial \ln(1-a_{i,k})}\right)_{\mathbf{w}} = -1$. Also, noting that $Z_{i,k} = v_{i,k} P_{ii,k} Q_{i,k}$ and $\boldsymbol{\delta}_{-ii} = \mathbf{0}$, we can write the partial derivative of CO_2emissions w.r.t. abatement as follows

$$\frac{\partial \Delta_i(\mathbf{Z}(.))}{\partial \ln \tilde{\mathbf{P}}_i} \cdot \left(\frac{\partial \ln \tilde{\mathbf{P}}_i}{\partial \ln(1-a_{i,k})}\right)_{\mathbf{w}} + \frac{\partial \Delta_i(\mathbf{Z}(.))}{\partial Y_i}\left(\frac{\partial Y_i}{\partial \ln(1-a_{i,k})}\right)_{\mathbf{w}}$$
$$= -\delta_{ii,k} v_{i,k} \sum_n \left[P_{in,k} Q_{ij,k} \varepsilon^{(in,k)}_{in,k}\right] + \left(\frac{1}{\alpha_{i,k}} - 1\right) \delta_{ii,k} v_{i,k} P_{ii,k} Q_{i,k} + \delta_{ii,k} v_{i,k} P_{ii,k} Q_{ii,k} \left(\frac{\partial Y_i}{\partial \ln(1-a_{i,k})}\right)_{\mathbf{w}}.$$

Following Appendix **??**, we can set $\frac{\mathrm{d} \ln \mathbf{w}_{-i}}{\mathrm{d} \ln(1-a_{i,k})} \approx \mathbf{0}$ by choice of numeriare—noting that the approximation holds exactly in the two-country case. General equilibrium wage effects relating to w_i can be characterized by applying the Implicit Function Theorem to the balanced trade condition, $\mathrm{B}_i(\mathbf{a}_i, w_i; \bar{\mathbf{w}}_{-i}) = \sum_{j \neq i} \sum_g \left(P_{ji,g}(\bar{w}_j) Q_{ji,g}(\mathbf{a}_i, w_i; \bar{\mathbf{w}}_{-i}) - P_{ij,g}(\mathbf{a}_i, w_i) Q_{ij,g}(\mathbf{a}_i, w_i; \bar{\mathbf{w}}_{-i})\right)$. This application yields the following:

$$\frac{\mathrm{d} \ln w_i}{\mathrm{d} \ln(1-a_{i,k})} = -\left(\sum_{j \neq i}\left[P_{ji,k} Q_{ji,k} \varepsilon^{(ii,k)}_{ji,k} - P_{ij,k} Q_{ij,k}\left(1 + \varepsilon^{(ij,k)}_{ij,k}\right)\right] + \sum_{j \neq i} \sum_g \left(P_{ji,g} Q_{ji,g}\right)\left(\frac{\partial Y_i}{\partial \ln(1-a_{i,k})}\right)_{\mathbf{w}}\right)\left(\frac{\partial \mathrm{B}_i}{\partial \ln w_i}\right)^{-1}$$

Plugging the above two expressions back into Equation C.3, invoking Roy's identity ($\frac{\partial V_i}{\partial \tilde{P}_{ni,k}} = -\frac{\partial V_i}{\partial Y_i} Q_{ni,k}$), and recalling that $Y_i = w_i L_i + \sum_k \alpha_{i,k} P_{ii,k} Q_{i,k}$, yields the following equation that describes the first-order condition w.r.t. $a_{i,k}$,

$$\begin{aligned} P_{ii,k}Q_{ii,k} &- \alpha_{i,k} \sum_j \left[P_{ij,k}Q_{ij,k} \left(1 + \varepsilon_{ij,k}^{(ij,k)}\right)\right] + \tilde{\delta}_{ii} v_{i,k} \sum_j \left[P_{ij,k}Q_{ij,k}\varepsilon_{ij,k}^{(ij,k)}\right] \\ &- \left(\frac{1}{\alpha_k} - 1\right) \tilde{\delta}_{ii,k} v_{i,k} P_{ii,k} Q_{i,k} - \sum_g \left(\left[\alpha_{i,g} - \tilde{\delta}_{ii,g} v_{i,g}\right] P_{ii,g}Q_{ii,g}\right) \left(\frac{\partial \ln Y_i}{\partial \ln(1-a_{i,k})}\right)_{\mathbf{w}} \\ &- \bar{\Delta}_i \left[\sum_{j\neq i} \left[P_{ji,k}Q_{ji,k}\varepsilon_{ji,k}^{ii} - P_{ij,k}Q_{ij,k}\left(1+\varepsilon_{ij,k}\right)\right] + \sum_{j\neq i}\sum_g \left(P_{ji,g}Q_{ji,g}\right) \left(\frac{\partial \ln Y_i}{\partial \ln(1-a_{i,k})}\right)_{\mathbf{w}} \right] = 0, \end{aligned} \tag{C.4}$$

where $\tilde{\delta}_{ii,k} = \tilde{P}_i \delta_{ii,k}$ denotes the CPI-adjusted disutility from carbon and $\bar{\Delta}_i \equiv \frac{\partial (V_i - \Delta_i(\mathbf{Z}))/\partial \ln w_i}{\partial B_i(\mathbf{a}, w_i; \bar{\mathbf{w}}_{-i})/\partial \ln w_i}$ is a uniform term without an industry subscript. Dividing Equation C.4 by total sales from origin i, $R_{i,k} = \sum_n P_{in,k} Q_{in,k}$, and defining $\mathcal{E}_{i,k} \equiv \sum_j \left[r_{ij,k} \left(1 + \varepsilon_{ij,k}^{(ij,k)}\right)\right] = -\epsilon_k \left(1 - r_{ii,k}\lambda_{ii,k}\right)$ allows us to express the above equation in more compact form as follows,

$$\begin{aligned} &r_{ii,k} - \alpha_{i,k}\mathcal{E}_{i,k} + \alpha_k \frac{\tilde{\delta}_{ii}}{\tau_{i,k}} \left(\mathcal{E}_{i,k} - 1\right) - \left(1 - \alpha_{i,k}\right) \frac{\tilde{\delta}_{ii,k}}{\tau_{i,k}} \\ &+ \sum_g \left(\alpha_{i,g} \left[1 - \frac{\tilde{\delta}_{ii}}{\tau_{i,g}}\right] r_{ii,g} r_{i,g}\right) \left(\frac{\partial \ln Y_i}{\partial \ln(1-a_{i,k})}\right)_{\mathbf{w}} r_{i,k}^{-1} - \bar{\Delta}_i \left[\mathcal{E}_{i,k} + \left(1 - \lambda_{ii}\right) \left(\frac{\partial \ln Y_i}{\partial \ln(1-a_{i,k})}\right)_{\mathbf{w}} r_{i,k}^{-1}\right] = 0. \end{aligned} \tag{C.5}$$

Next, we need to characterize $\left(\frac{\partial Y_i}{\partial \ln(1-a_{i,k})}\right)_{\mathbf{w}}$ .To this end, we apply the Implicit Function Theorem to $Y_i = w_i L_i + \sum_k \left[\alpha_{i,k} P_{ii,k} Q_{i,k}\right]$, treating $\mathbf{w}$ as given. In this process we also note that $\eta_{in,k} = 1$ given our parametric assumption with regards to preferences. Performing this step delivers the following expression for the partial derivative of income w.r.t. abatement,

$$\left(\frac{\partial Y_i}{\partial \ln(1-a_{i,k})}\right)_{\mathbf{w}} = \frac{-\alpha_{i,k} \sum_j \left[P_{ij,k} Q_{ij,k} \left(1 + \varepsilon_{ij,k}\right)\right]}{Y_i - \sum_g \alpha_{i,g} \eta_{ii,g} P_{ii,g} Q_{ii,g}} = \frac{-\alpha_{i,k}\mathcal{E}_{i,k}}{1 - \bar{\alpha}_i \lambda_{ii}} r_{i,k}.$$

where $\bar{\alpha}_i = \sum_k \alpha_{i,k} r_{i,k}$ is the weighted average carbon elasticity associated with economy i. Plugging the above equation back into the first-order condition characterized by Equation C.5 yields the following implicit formula for second-best carbon taxes:

$$\frac{\tilde{\delta}_{ii,k}}{\tau_{i,k}} - 1 = \frac{(\widetilde{\alpha}_{i,k} - \alpha_{i,k})\mathcal{E}_{i,k} + 1 - r_{ii,k}}{\alpha_{i,k}\mathcal{E}_{i,k} - 1} \implies \tau_{i,k} = \left(\frac{\alpha_{ik}\mathcal{E}_{i,k} - 1}{\widetilde{\alpha}_{i,k}\mathcal{E}_{i,k} - r_{ii,k}}\right) \tilde{\delta}_{ii,k},$$

where $\tilde{\alpha}_{i,k}$ is a composite term that is defined as follows relative to $\alpha_{i,k}$:

$$\widetilde{\alpha}_{i,k} - \alpha_{i,k} \equiv \bar{\Delta}_i \left[\frac{1 - \alpha_{i,k}}{1 - \bar{\alpha}_i \lambda_{ii}}\right] - \frac{\alpha_{i,k}}{1 - \bar{\alpha}_i \lambda_{ii}} \sum_g \left(\alpha_{i,g} \left[1 - \frac{\tilde{\delta}_{ii,g}}{\tau_{i,g}}\right] r_{ii,g} r_{i,g}\right).$$

To finalize the proof, we need to characterize $\bar{\Delta}_i$, which will in turn deliver a more specific characterization of $\widetilde{\alpha}_{i,k}$. For this purpose , we appeal to the definition, $\bar{\Delta}_i \equiv \frac{\partial (V_i - \Delta_i(\mathbf{Z}))/\partial \ln w_i}{\partial B_i(\mathbf{a}, w_i; \bar{\mathbf{w}}_{-i})/\partial \ln w_i}$, which implies that

$$\bar{\Delta}_i = \frac{(1-\bar{\alpha}_i) - \lambda_{ii} + \sum_k \left(\left[\alpha_{i,k}\mathcal{E}_{i,k} - \alpha_k \frac{\tilde{\delta}_{ii,k}}{\tau_{i,k}} \left(\mathcal{E}_{i,k} - 1 \right) \right] r_{i,k} \right) + \sum_k \left(\left[\alpha_{i,k} - \delta_{ii,k} v_{i,k} \right] r_{ii,k} r_{i,k} \right) \left(\frac{\partial Y_i}{\partial \ln w_i} \right)_{\mathbf{a}_i}}{(1-\lambda_{ii}) \left(\frac{\partial Y_i}{\partial \ln w_i} \right)_{\mathbf{a}_i} - \sum_k \left[\mathcal{E}_{i,k} r_{i,k} \right]}$$

We can replace for $\alpha_k \mathcal{E}_{i,k} - \alpha_{i,k} \frac{\tilde{\delta}_{ii}}{\tau_{i,k}} \left(\mathcal{E}_{i,k} - 1 \right)$ from the F.O.C. (Equation C.5), which implies

$$\begin{aligned}
\bar{\Delta}_i &= \frac{(1-\bar{\alpha}_i) - \lambda_{ii} + \sum_g \left(\left[r_{ii,g} - (1-\alpha_{i,g}) \frac{\tilde{\delta}_{ii,g}}{\tau_{i,g}} \right] r_{i,g} \right) + \sum_g \left(\alpha_{i,g} \left[1 - \frac{\tilde{\delta}_{ii,g}}{\tau_{i,g}} \right] r_{ii,g} r_{i,g} \right) \left[\left(\frac{\partial Y_i}{\partial \ln w_i} \right)_{\mathbf{a}_i} + \sum_g \left(\frac{\partial \ln Y_i}{\partial \ln (1-a_{i,g})} \right)_{\mathbf{w}} \right]}{(1-\lambda_{ii}) \left[\left(\frac{\partial Y_i}{\partial \ln w_i} \right)_{\mathbf{a}_i} + \sum_k \left(\frac{\partial Y_i}{\partial \ln (1-a_{i,k})} \right)_{\mathbf{w}} \right]} \\
&= \frac{\sum_g \left[(1-\alpha_{i,g}) \left(1 - \frac{\tilde{\delta}_{ii,g}}{\tau_{i,g}} \right) r_{i,g} \right] + \sum_g \left(\alpha_{i,g} \left[1 - \frac{\tilde{\delta}_{ii,g}}{\tau_{i,g}} \right] r_{ii,g} r_{i,g} \right) \left[\left(\frac{\partial Y_i}{\partial \ln w_i} \right)_{\mathbf{a}_i} + \sum_g \left(\frac{\partial \ln Y_i}{\partial \ln (1-a_{i,g})} \right)_{\mathbf{w}} \right]}{(1-\lambda_{ii}) \left[\left(\frac{\partial Y_i}{\partial \ln w_i} \right)_{\mathbf{a}_i} + \sum_k \left(\frac{\partial Y_i}{\partial \ln (1-a_{i,k})} \right)_{\mathbf{w}} \right]}
\end{aligned} \tag{C.6}$$

Considering the updated expression for $\bar{\Delta}_i$ we can reapply the Implicit Function Theorem to $Y_i = w_i L_i + \sum_k \left(\alpha_{i,k} P_{ii,k} Q_{i,k} \right)$ to characterize the term $\left(\frac{\partial Y_i}{\partial \ln w_i} \right)_{\mathbf{a}_i} + \sum_g \left(\frac{\partial \ln Y_i}{\partial \ln (1-a_{i,g})} \right)_{\mathbf{w}}$ in the numerator and denominator. In particular,

$$\left(\frac{\partial Y_i}{\partial \ln w_i} \right)_{\mathbf{a}_i} + \sum_g \left(\frac{\partial \ln Y_i}{\partial \ln (1-a_{i,g})} \right)_{\mathbf{w}} = \frac{1 - \sum_k \left(\alpha_{i,k} r_{i,k} \right) + \sum_k \left(\alpha_{i,k} \mathcal{E}_i r_{i,k} \right)}{1 - \bar{\alpha}_i \lambda_{ii}} - \sum_k \frac{\alpha_{i,k} \mathcal{E}_i r_{i,k}}{1 - \bar{\alpha}_{ii} \lambda_{ii}} = \frac{1 - \bar{\alpha}_i}{1 - \bar{\alpha}_{ii} \lambda_{ii}}.$$

Plugging the above expression back into Equation C.6 and assuming that $\alpha_{i,k} = \alpha$ for all k, yields the following equation that implicitly determines $\frac{1-\alpha}{1-\alpha\lambda_{ii}} \bar{\Delta}_i$:

$$(1-\lambda_{ii}) \frac{1-\alpha}{1-\alpha\lambda_{ii}} \bar{\Delta}_i = (1-\alpha) \sum_g \left[\left(1 - \frac{\tilde{\delta}_{ii}}{\tau_{i,g}} \right) r_{i,g} \right] + \frac{1-\alpha}{1-\alpha\lambda_{ii}} \sum_g \left(\alpha \left[1 - \frac{\tilde{\delta}_{ii}}{\tau_{i,g}} \right] r_{ii,g} r_{i,g} \right),$$

Finally, plugging the expression for $\frac{1-\alpha}{1-\alpha\lambda_{ii}} \bar{\Delta}_i$ implied by the above equation back into our earlier expression for $\tilde{\alpha}_{i,k} - \alpha$, updates the expression as follows:

$$\begin{aligned}
\tilde{\alpha}_{i,k} - \alpha &= \left[(1-\alpha) \sum_g \left[\left(1 - \frac{\tilde{\delta}_{ii}}{\tau_{i,g}} \right) r_{i,g} \right] + \alpha \sum_g \left(\left[1 - \frac{\tilde{\delta}_{ii}}{\tau_{i,g}} \right] r_{ii,g} r_{i,g} \right) \right] (1-\lambda_{ii})^{-1} \\
&= \sum \left[\left(1 - \frac{\tilde{\delta}_{ii}}{\tau_{i,k}} \right) \frac{1 - \alpha(1 - r_{ii,g})}{1 - \lambda_{ii}} r_{i,g} \right] = -\sum_g \left[\left(\frac{(\tilde{\alpha}_{i,g} - \alpha) \mathcal{E}_{i,g} + 1 - r_{ii,g}}{\alpha \mathcal{E}_{i,g} - 1} \right) \frac{1 - \alpha(1 - r_{ii,g})}{(1 - \lambda_{ii})} r_{i,g} \right].
\end{aligned}$$

The above system implies that $\tilde{\alpha}_{i,k} = \tilde{\alpha}_i$ is uniform, provided that the emission elasticity $\alpha_{i,k} = \alpha$, is also uniform. Capitalizing on this observation and replacing $\mathcal{E}_{i,g} = -\epsilon_g \left(1 - r_{ii,g} \lambda_{ii,g} \right)$, we can obtain the following expression for $\tilde{\alpha}_i$ in terms of α, which asserts that $\tilde{\alpha}_i > \alpha$:

$$\tilde{\alpha}_i - \alpha = \frac{\sum_g \left[\frac{1 - r_{ii,g}}{\epsilon_k \left(1 - r_{ii,g} \lambda_{ii,g} \right) + 1} \frac{1 - \alpha(1 - r_{ii,g})}{(1 - \lambda_{ii})} r_{i,g} \right]}{\sum_g \left[\left(1 + \frac{\epsilon_g \left(1 - r_{ii,g} \lambda_{ii,g} \right)}{\epsilon_g \left(1 - r_{ii,g} \lambda_{ii,g} \right) + 1} \frac{1 - \alpha(1 - r_{ii,g})}{(1 - \lambda_{ii})} \right) r_{i,g} \right]} > 0$$

□

D Proofs and Derivations for Cooperative Border and Production Taxes

D.1 Globally Optimal Carbon Border Taxes

We first solve for globally optimal carbon border taxes in the perfectly competitive case ($\gamma_k \to \infty$) where the market equilibrium is globally efficient barring climate externalities. To present the optimal policy problem, we formulate all equilibrium variables as a function of global tariffs, $\mathbf{t}$,global wages, $\mathbf{w}$, and global income levels, $\mathbf{Y}$, following the logic discussed in Section 3.1 and Appendix A.2. When working with this formulation of equilibrium variables, note that $\mathbf{w}$ and $\mathbf{Y}$ are implicit functions of $\mathbf{t}$. Accordingly, $(\mathbf{t};\mathbf{w},\mathbf{Y})$ is feasible if $\mathbf{w}$ and $\mathbf{Y}$ satisfy the labor market clearing and balanced budget conditions given $\mathbf{t}$. In line with our earlier choice of notation, we let $\mathcal{E}$ denote the set of feasible policy–wage–income combinations that satisfy the general equilibrium constraints. Under this notation, carbon-adjusted welfare in country i can be expressed as

$$W_i(\mathbf{t};\mathbf{w},\mathbf{Y}) = V_i(w_i L_i + T_i(\mathbf{t};\mathbf{w},\mathbf{Y}), \tilde{\mathbf{P}}_i(\mathbf{t};\mathbf{w})) - \Delta_i(\mathbf{Z}(\mathbf{t};\mathbf{w},\mathbf{Y})).$$

Globally optimum carbon border taxes solve the following problem, wherein a central planner chooses $\mathbf{t}$ to maximize global welfare subject to feasibility:

$$\mathbf{t}^\star = \arg\max_{\mathbf{t}} \sum_{i=1}^{N} W_i\left(\mathbf{t};\mathbf{w},\mathbf{Y}\right) \quad s.t.\ (\mathbf{t};\mathbf{w},\mathbf{Y}) \in \mathcal{E} \qquad \text{(P)}$$

Similar to the unilaterally optimal policy problem, we can simplify Problem (P) by capitalizing on a set of intermediate envelope results. First, we can appeal to a version of the Lerner symmetry to show that wage effects are welfare neutral. To present this result, it is easiest to temporarily switch notation and express welfare in terms of feasible tariff–wage combinations, $(\mathbf{t},\mathbf{w}) \in \mathcal{E}_w$. Here, $\mathcal{E}_w$ is defined with the same logic as the discussed earlier: $(\mathbf{t},\mathbf{w}) \in \mathcal{E}_w$ if $\mathbf{w}$ satisfies the labor market clearing conditions given $\mathbf{t}$. Let $\mathbf{t}_{ji}$ and $\mathbf{t}_{ij}$ respectively denote the tariffs collected and paid by country i. Relatedly, let $\mathbf{t}_{-i} \equiv \mathbf{t} - \left\{\mathbf{t}_{ji},\mathbf{t}_{ij}\right\}$ denote all tariff variables unrelated to economy i. Appealing to the Lerner symmetry (and noting our temporary switch of notation) we can conclude that for any $a \in \mathbb{R}_+$,

$$\begin{cases} (\mathbf{t}_{ji},\mathbf{t}_{ij},\mathbf{t}_{-i};w_i,\mathbf{w}_{-i}) \in \mathcal{E}_w \implies (a\mathbf{t}_{ji},\mathbf{t}_{ij}/a,\mathbf{t}_{-i};aw_i,\mathbf{w}_{-i}) \in \mathcal{E}_w \\ W_n(\mathbf{t}_{ji},\mathbf{t}_{ij},\mathbf{t}_{-i};w_i,\mathbf{w}_{-i}) = W_n(a\mathbf{t}_{ji},\mathbf{t}_{ij}/a,\mathbf{t}_{-i};aw_i,\mathbf{w}_{-i}) \qquad \forall n \end{cases}.$$

The above result immediately indicates that any possible welfare gains from changing $\mathbf{w}$, can be perfectly mimicked with an appropriate adjustment to the global tariff vector $\mathbf{t}$. Hence, any gains that channel through wage effects will be already internalized by the optimal choice $\mathbf{t}^*$:

$$\frac{\partial W(\mathbf{t}^*;\mathbf{w}^*)}{\partial \mathbf{w}} = 0.$$

Second, since the planner has access to lump-sum international income transfers, it effectively has full control over $\mathbf{Y}$. That is, conditional on the optimum choice of transfers, income effects are welfare neutral:

$$\frac{\partial W(\mathbf{t}^*;\mathbf{w}^*,\mathbf{Y}^*)}{\partial \mathbf{Y}} = 0.$$

Before using these two envelope results, it is useful to reformulate Problem (P) into a problem where the central planner chooses prices rather than tariffs. Note that—by assumption—the central planner cannot apply domestic subsidies but has full discretion vis-à-vis tariffs. That is to say, the central planner can set all consumer prices associated with goods that cross international borders—namely, $\left\{\tilde{\mathbf{P}}_{-nn}\right\}_{n\in\mathbb{C}}$where $\tilde{\mathbf{P}}_{-nn} \equiv \tilde{\mathbf{P}}_n - \tilde{\mathbf{P}}_{nn}$. Using this correspondence, we can reformulate all equilibrium variables as function of$\left\{\tilde{\mathbf{P}}_{-nn}\right\}_n$, and the vector of wages, $\mathbf{w}$, and income levels, $\mathbf{Y}$. After the reformulation, welfare in country i can be expressed as follows:

$$W_i\left(\mathbf{t};\mathbf{w},\mathbf{Y}\right) \sim W_i(\{\tilde{\mathbf{P}}_{-nn}\}_n;\mathbf{w},\mathbf{Y}) \equiv V_i(w_iL_i + T_i(\{\tilde{\mathbf{P}}_{-nn}\}_n;\mathbf{w},\mathbf{Y}),\tilde{\mathbf{P}}_{-ii},\tilde{\mathbf{P}}_{ii}(w_i)) - \Delta_i(\mathbf{Z}(\{\tilde{\mathbf{P}}_{-nn}\}_n;\mathbf{w},\mathbf{Y})).$$

Appealing to the above formulation and the previously-discussed envelope conditions, we can recast Problem (P) as one where global welfare is maximized by directly choosing the "consumer" price of all traded goods, treating $\mathbf{w}$ and $\mathbf{Y}$ as fixed vectors.

Lemma 7. *Problem (P) can be reformulated as one where a central planer chooses the "consumer" price of traded varieties while treating $\boldsymbol{w}$ as if it were constant and treating the import demand schedule as if it were invariant to$\mathbf{Y}$. To put it formally, the reformulated optimal policy problem can be expressed as*

$$\max_{\{\tilde{\boldsymbol{P}}_{-ii}\}_i} \sum_{i=1}^{N} W_i\left(\{\tilde{\boldsymbol{P}}_{-nn}\}_n;\overline{\boldsymbol{w}},\overline{\mathbf{Y}}\right) \qquad (P'),$$

The solution to which determines the globally optimal carbon border tariffs as the optimal wedge between consumer and producer prices:

$$1+t^{\star}_{ji,k} = \tilde{P}^{\star}_{ji,k}/P_{ji,k} \qquad \forall ji,k \neq ii,k.$$

Next, we derive the necessary F.O.C.s for optimality w.r.t. each price instrument. Using the notation introduced for partial derivatives in *Section A* of the main appendix (under *Notation A*), we can express the F.O.C. w.r.t. to $\tilde{P}_{ji,k}$ as

$$\sum_{n=1}^{N}\left(\frac{\partial W_n}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbf{Y}} = \frac{\partial V_i(.)}{\partial \ln \tilde{P}_{ji,k}} + \sum_{n=1}^{N}\left[\tilde{P}_n\left(\frac{\partial T_n}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbf{Y}} - \delta_i\cdot\left(\frac{\partial \mathbf{Z}}{\partial \ln \tilde{P}_{ji,k}}\right)_{\mathbf{w},\mathbf{Y}}\right] = 0. \qquad \text{(D.1)}$$

Capitalizing the same technique presented earlier, we can write the above F.O.C. in terms of Marshallian demand elasticities as follows:

$$\sum_g\sum_n\left[\alpha_{n,g}P_{ni,g}Q_{ni,g}\varepsilon^{(ji,k)}_{ni,g}\right] + \sum_g\sum_{n\neq i}\left[(\tilde{P}_{ni,g}-P_{ni,g})Q_{ni,g}\varepsilon^{(ji,k)}_{ni,g}\right] - \sum_\ell\sum_n\sum_g\left[\tilde{\delta}_{n\ell}\nu_{n,g}P_{ni,g}Q_{ni,g}\varepsilon^{(ji,k)}_{ni,g}\right] =$$
$$\sum_g\sum_{n\neq i}\left[\left(\tilde{P}_{ni,g}-[1+\tilde{\delta}_{-n}\nu_{n,g}]P_{ni,g}\right)Q_{ni,g}\varepsilon^{(ji,k)}_{ni,g}\right] - \tilde{\delta}_{-i}\sum_g\left[\nu_{i,g}P_{ii,g}Q_{ii,g}\varepsilon^{(ji,k)}_{ii,g}\right] = 0, \qquad \text{(D.2)}$$

where the second line derives from the following equilibrium relationship and definition for $\tilde{\delta}_{-i}$:

$$\sum_g \alpha_{n,g}P_{ni,g}Q_{ni,g} = \sum_g \tilde{\delta}_n\nu_{n,g}P_{ni,g}Q_{ni,g}; \qquad \tilde{\delta}_{-i} \equiv \sum_n\left[\tilde{\delta}_n\right] - \tilde{\delta}_i.$$

The F.O.C. described by the last line in Equation D.2 describes a system of equations, which can be expressed as a matrix equation—similar to we did in our baseline optimal policy problem. Before transitioning into matrix notation, it is useful to simplify Equation D.2 using some primitive properties of Marshallian demand functions. First, supposing preferences are homothetic, we can appeal to a corollary of the Slutsky equation—namely, $\tilde{P}_{ni,g}Q_{ni,g}\varepsilon^{(ji,k)}_{ni,g} = \tilde{P}_{ji,k}Q_{ji,k}\varepsilon^{(ni,g)}_{ji,k}$—to simplify Equation D.2 as:

$$\sum_g\sum_{n\neq i}\left[\left(1-[1+\tilde{\delta}_{-n}\nu_{n,g}]\frac{P_{ni,g}}{\tilde{P}_{ni,g}}\right)\varepsilon^{(ni,g)}_{ji,k}\right] - \tilde{\delta}_{-i}\sum_g\left[\nu_{i,g}\varepsilon^{(ii,g)}_{ji,k}\right] = 0. \qquad \text{(D.3)}$$

Since demand is homogeneous of degree zero, we can show that $\sum_{n\neq i}\sum_g \varepsilon^{(ni,g)}_{ji,k} = -\sum_g \varepsilon^{(ii,g)}_{ji,k}$.

Using this property, the first-order condition w.r.t. $\tilde{P}_{ji,k}$ (described by Equation D.3) reduces to

$$\sum_{g}\sum_{n\neq i}\left[\frac{P_{ni,g}}{\tilde{P}_{ni,g}}[1+\tilde{\delta}_{-n}\nu_{n,g}]\varepsilon_{ji,k}^{(ni,g)}\right]=\sum_{g}\left[[1+\tilde{\delta}_{-i}\nu_{i,g}]\varepsilon_{ji,k}^{(ii,g)}\right]. \tag{D.4}$$

We can write the system of F.O.C.s specified by the above equation in matrix algebra and solve for the l $N(K-1)\times 1$ vector of optima import price wedges $\mathbf{T}_{-ii}^{*}=\left[\frac{P_{ji,k}}{\tilde{P}_{ji,k}^{*}}\right]_{jk}$. In particular, Equation D.4 is analogous to the following matrix equation:

$$\mathbf{T}_{-ii}^{*}=\left(\tilde{\mathbf{E}}_{-ii}^{(-ii)}\right)^{-1}\tilde{\mathbf{E}}_{-ii}^{(ii)}\mathbf{1}_{K},$$

where$\mathbf{1}$ is a $K\times 1$ column vector of ones; and $\tilde{\mathbf{E}}_{-ii}^{(-ii)}$ and $\tilde{\mathbf{E}}_{-ii}^{(ii)}$ are respectively $(N-1)K\times N(K-1)$ and $N(K-1)\times K$ matrixes of CO_2-adjusted demand elasticities:

$$\tilde{\mathbf{E}}_{-ii}^{(-ii)}\equiv\left[\left(1+\tilde{\delta}_{-n}\nu_{n,g}\right)\varepsilon_{ji,k}^{(ni,g)}\right]_{jk,ng};\qquad\qquad \tilde{\mathbf{E}}_{-ii}^{(ii)}\equiv\left[\left(1+\tilde{\delta}_{-i}\nu_{i,g}\right)\varepsilon_{ji,k}^{(ii,g)}\right]_{jk,g}.$$

As in the baseline optimal policy problem, the optimal price wedges identified by $\mathbf{T}_{-ii}^{*}$ fully determine the optimal tariff w.r.t. to each traded goods as $1+t_{ji,k}^{*}=\frac{\tilde{P}_{ji,k}^{*}}{P_{ji,k}}$. Finally, note that the invertibility of $\tilde{\mathbf{E}}_{-ii}^{(-ii)}$ can be established using the same logic presented under Lemma 5.

Cobb-Douglas-CES Preferences. We can derive simple formulas for globally optimal carbon border taxes in the special case where preferences have a Cobb-Douglas-CES specification (see Section 3.3 for details about this specification). In that case, Marshallian demand elasticities assume the following formulations:

$$\varepsilon_{ji,k}^{ni,g}=0\ \text{ if } g\neq k;\qquad \varepsilon_{ji,k}^{(ni,k)}=(\sigma_k-1)\lambda_{ni,k}\ \text{ if } n\neq j;\qquad \varepsilon_{ji,k}^{(ji,k)}=-1-(\sigma_k-1)(1-\lambda_{ji,k}).$$

Plugging these elasticity values back into Equation D.4, delivers the following first-order condition w.r.t. the price of good ji,k:

$$\sum_{n\neq i}\left[\frac{P_{ni,k}}{\tilde{P}_{ni,k}}[1+\tilde{\delta}_{-n}\nu_{n,k}]\left[(\sigma_k-1)\,\lambda_{ni,k}-\mathbb{1}_{n=j}\,(1+(\sigma_k-1))\right]\right]=[1+\tilde{\delta}_{-i}\nu_{i,k}]\,(\sigma_k-1)\,\lambda_{ii,k}$$

One immediately observe from the above equation that the first-order condition w.r.t. to the price of good ji,k has no index specific to ji,k. Capitalizing on this observation, we define $\mathcal{T}_{ni,k}\equiv\frac{P_{ni,g}}{\tilde{P}_{ni,g}}[1+\tilde{\delta}_{-n}\nu_{n,g}]$ and rewrite the above equation as

$$\sum_{n\neq i}\left[\mathcal{T}_{ni,k}\left[(\sigma_k-1)\,\lambda_{ni,k}-\mathbb{1}_{n=j}\,(1+(\sigma_k-1))\right]\right]=[1+\tilde{\delta}_{-i}\nu_{i,k}]\,(\sigma_k-1)\,\lambda_{ii,k}.$$

Since the above equation is independent of ji,k, the optimal price vector should yields a $\mathcal{T}_{ni,k}$ that is independent of j and uniform across all export partners—i.e., $\mathcal{T}_{ji,k}^{*}=\mathcal{T}_{i,k}^{*}$. Invoking this observation about uniformity, it is straightforward to solve for $\mathcal{T}_{i,k}^{*}$, which yields the following formula for the globally optimal carbon border tariff on good ji,k:

$$1+t_{ji,k}^{*}=\frac{\tilde{P}_{ji,k}^{*}}{P_{ji,k}}=\frac{1+(\sigma_k-1)\lambda_{ii,k}}{1+[1+\tilde{\phi}_{-i}\nu_{i,k}]\,(\sigma_k-1)\lambda_{ii,k}}\left(1+\tilde{\phi}_{-j}\nu_{j,k}\right).$$

D.2 Globally Optimal Production Taxes

Globally optimal carbon border taxes are limited by their inability to tax domestic transactions. This is a serious limitation since most CO_2emissions are associated with goods that never cross international borders. Cooperative can overcome this limitation by applying global optimal production taxes. The optimal policy problem, in that case, choose the entire vector of consumer prices not just those that cross international borders:

$$\max_{\{\tilde{\mathbf{P}}_n\}_n} \sum_{i=1}^{N} W_i\left(\{\tilde{\mathbf{P}}_n\}_n ; \overline{\mathbf{w}}, \overline{\mathbf{Y}}\right) \qquad (\mathrm{P}'').$$

We can repeat the steps presented above to derive an analytical solution for the above problem. Without repeating all the derivation details, the optimal production subsidy to goods associated with origin i–industry k is given by

$$1 + s_{i,k}^* = \frac{P_{in,k}}{\tilde{P}_{in,k}^*} = \frac{1}{1 + \tilde{\phi}_{-i} v_{i,k}}.$$

The above tax schedule differs from the optimal border tax in two ways. First, it applies to all goods and services including those that are produced and sold within-national borders. Second, it does not feature the correction for cross-demand effects. The reason is that all goods can be taxed, so there is no need for taxes to internalize cross-substitution between taxable and non-taxable goods. Later, in Appendix F, we show that globally optimal production taxes are significantly more effective at cutting global CO_2 emissions that pure carbon border taxes.

D.3 Border Adjustments Intended to Level the Playing Field

Carbon border adjustments can be used to level the playing field for firms subjected to high carbon taxes in climate conscious countries. Suppose country i applies its a carbon tax rate in excess of its unilaterally. The high carbon tax prompts abatement by local firms, which raises their marginal cost and puts them at a competitive disadvantage against foreign firms. The level the playing field for individual domestic industries, the government in i can offer the following border tax adjustment:

$$1 + t_{ji,k}^* = \left(1 - a_{i,k}^*\right)^{\frac{1-\gamma_k}{\varsigma\gamma_k}}; \qquad 1 + x_{ij,k} = \left(1 - a_{i,k}^*\right)^{\frac{\gamma_k - 1}{\varsigma\gamma_k}},$$

where $a_{i,k}^*$ is the abatement choice implied by the globally optimum tax rate. The efficacy of the above policy is limited by a possible misalignment between firm-level profits and national-level welfare. Specifically, under a set of plausible assumptions, the above tariff and export subsidy schedule may very well worsen the national-level terms of trade. So, it appears to be motivated primarily by political economy rather than welfarist considerations.

E Data and Calibration

Expenditures, Revenues, and CO_2 Emissions. Given information on expenditure, CO_2 emission, and applied tariffs $\{\tilde{P}_{ji,k}Q_{ji,k}, Z_{i,k}, t_{ji,k}\}_{ji,k}$, and the estimated structural parameters, $\{\gamma_k, \sigma_k, \alpha_{i,k}\}_{i,k}$, we construct the baseline data, $\mathcal{B}_v \equiv \{\lambda_{ji,k}, e_{i,k}, r_{ji,k}, \rho_{j,k}, v_{i,k}, w_i\bar{L}_i, Y_i\}_{ji,k}$, needed to implement our quantitative analysis. Values for CO_2 emission intensities, $v_{i,k}$, are calculated as follows

$$v_{i,k} = \frac{Z_{i,k}}{\sum_{n=1}^{15} \frac{1}{1+t_{in,k}} \tilde{P}_{in,k} Q_{in,k}}$$

where by structure, $t_{ii,k} = 0$. Total national expenditure Y_i and expenditure share variables, $\lambda_{ji,k}$, and $e_{i,k}$ can be recovered from variety-level expenditure and tariff data as follows:

$$Y_i = \sum_{j=1}^{15}\sum_{k=1}^{19} \tilde{P}_{ji,k}Q_{ji,k}, \qquad \lambda_{ji,k} = \frac{\tilde{P}_{ji,k}Q_{ji,k}}{\sum_{n=1}^{15}\tilde{P}_{ni,k}Q_{ni,k}}, \qquad e_{i,k} = \frac{\sum_{n=1}^{15}\tilde{P}_{ni,k}Q_{ni,k}}{Y_i}.$$

Finally, the national wage bill, $w_i\bar{L}_i$, industry-level labor shares, $\rho_{i,k}$, and revenue shares, $r_{ji,k}$, can be constructed as follows, given variety-level expenditure and tariff data and the estimated structural parameters:

$$w_i\bar{L}_i = \sum_{j=1}^{15}\sum_{k=1}^{19}\left[\left(1-\alpha_{i,k}\frac{\gamma_k-1}{\gamma_k}\right)\frac{1}{1+t_{ij,k}}\tilde{P}_{ij,k}Q_{ij,k}\right];$$

$$\rho_{i,k} = \frac{\sum_{j=1}^{15}\left(1-\alpha_{i,k}\frac{\gamma_k-1}{\gamma_k}\right)\frac{\tilde{P}_{ij,k}Q_{ij,k}}{1+t_{ij,k}}}{w_i\bar{L}_i}; \qquad r_{ji,k} = \frac{\frac{1}{1+t_{in,k}}\tilde{P}_{in,k}Q_{in,k}}{\sum\frac{1}{1+t_{in,k}}\tilde{P}_{in,k}Q_{in,k}}.$$

Environmentally-related Taxes. We have collected environmentally-related tax data from EUROSTAT and OECD-PINE. Both of these data report environmentally-related taxes based on the same definition making them compatible in order for us to merge them. Specifically, in both data sets, environmentally-related taxes consist of taxes on energy, pollution, transports, and resources. The data from EUROSTAT cover only European countries, reported for every industry and also for the category of household consumption. The data from OECD-PINE cover more than a hundred countries, reported at the level of countries, meaning that in these data we observe neither disaggregated industry-level records nor the distinction between firms and households. We call these environmentally-related taxes as emission taxes.

We face two issues in mapping our model to these data. First, our model allows emission taxes only on production while in the data, a portion of them are paid by households. Second, we do not observe these taxes by industry disaggregation in non-European countries. We continue to explain how we re-calibrate the data to make our model quantification consistent with the accounting of taxes and emissions.

For a generic variable x, let $x_{i,k}$ be that variable in country-industry ik, and x_i be the country-level aggregate. In addition, let x^P be that variable from production side, and x^H from household consumption side, amounting to $x = x^P + x^H$. Specifically, we have: $T^E_{i,k} = \tau_{i,k}Z_{i,k}$, where $T^E_{i,k}$ refers to emission tax paid by country-industry ik, $Z_{i,k}$ measures tonnes of CO_2 emission in country-industry ik, and $\tau_{i,k}$ is the associated tax rate. First, we explain how we scale the data from production side to make them consistent with the national accounting of taxes and emissions. For European countries, we directly observe these variables by industry disaggregation, but for household consumption, we only observe variables at the country level. Starting with CO_2 emissions, let c^Z_i be an adjustment scalar that brings emission data into country-level aggregates. This adjustment requires: $Z_{i,k} = c^Z_i Z^P_{i,k}$ where $c^Z_i \equiv \frac{Z_i}{Z^P_i}$. Similarly, let c^τ_i be an adjustment to bring emission tax data into country-level aggregate: $\tau_{i,k} = c^\tau_i \tau^P_{i,k}$ where $c^\tau_i \equiv \frac{T^E_i}{\sum_k \tau^P_{i,k}Z_{i,k}}$. With these two adjustments, for every country we re-scale the emission and tax data from the production side to make their aggregates equal to the observed levels in the entire economy.

Next, we explain our data construction regarding that we do not observe environmentally-related taxes by industry disaggregation in non-European countries. For country i, we observe from OECD-PINE the emission taxes collected as percentage of GDP. From here, we calculate total emission taxes T^E_i in country i. We observe these taxes by industry disaggregation, $T^E_{i,k}$, only for the EU. For non-European countries, we make the assumption that $\tau_{i,k}/\tau_{i,k_0} = \tau_{EU,k}/\tau_{EU,k_0}$ —that is, the relative emission tax rate of industry k to that of a reference industry is the same between the EU and that in other countries. For a non-EU country i, the accounting of taxes and emissions require that: $T_i^E = \sum_k \tau_{i,k} Z_{i,k}$, which then implies $T_{i,k}^E = \frac{\tau_{EU,k} Z_{i,k}}{\sum_k \tau_{EU,k} Z_{i,k}} T_i^E$. Using this proportionality assumption, we construct industry-level emission taxes for non-European countries in a way that is consistent with their total emission taxes which we observe in OECD-PINE data.

Emission Disutility Parameters. Our calibration of countries' perceived CPI-adjusted disutility from emissions is based on two assumptions: (a) unilaterally optimal domestic emission tax equals the currently-applied energy tax in a country; (b) the globally optimal CO_2 tax equals world's disutility from CO_2 emissions,

$$\begin{cases} T_i^E = \sum_k (\tilde{\phi}_{i,k} + \tilde{\phi}_i) Z_{i,k} & \text{(a)} \\ SCC = \sum_i \tilde{\phi}_i & \text{(b)} \end{cases}. \tag{E.1}$$

We let $\tilde{\phi}_i$ be proportional to country i's share of world GDP adjusted for differences in energy tax rates across countries. Specifically, we recover relative values of $\tilde{\phi}_i$ across countries, by making two assumptions: (i) If every individual person cares equally about global warming, the aggregate care of a larger country will be proportionally larger. To reflect the importance of size, it is more plausible to denote the damage from climate change as a percentage of countries' real GDP. This means that, $\frac{\phi_i}{\phi_j} \propto \frac{Y_i/\tilde{P}_i}{Y_j/\tilde{P}_j}$, which is equivalent to $\frac{\tilde{\phi}_i}{\tilde{\phi}_j} \propto \frac{Y_i}{Y_j}$. (ii) We then acknowledge that countries do not care equally about carbon taxes. We take a stand that countries' care about climate change is reflected in their current policy toward the environmental damage of burning fossil fuels. As such, we make the assumption that country i relative to j's care about climate change is proportional to their observed emission taxes per tonne of CO_2, which means: $\frac{\tilde{\phi}_i}{\tilde{\phi}_j} \propto \frac{\left(T_i^E/Z_i\right)}{\left(T_j^E/Z_j\right)}$. Putting these two assumptions together, we can specify $\tilde{\phi}_i$ as

$$\tilde{\phi}_i = \bar{h} y_i (T_i^E / Z_i), \qquad y_i \equiv Y_i / Y_W.$$

Equation (42)-b requires that if countries could act cooperatively, they would target the social cost of carbon with their domestic CO_2 taxes, $\sum_i \tilde{\phi}_i = SCC$. This pins down scalar $\bar{h}$ in the above equation, $\bar{h} = \frac{SCC}{\sum_i y_i (T_i^E/Z_i)}$, and delivers $\tilde{\phi}_i$ as

$$\tilde{\phi}_i = \frac{y_i T_i^E / Z_i}{\sum_i y_i (T_i^E / Z_i)} SCC.$$

Using equation (42)-a, we can then calibrate the local adjustment to carbon taxes, $\tilde{\phi}_{i,k}$, by attributing the difference between observed emission taxes and the calibrated $\tilde{\phi}_i$ to the local component of emission taxes,$\tilde{\phi}_{i,k}$, $T_i^E = \sum_k (\tilde{\phi}_i + \tilde{\phi}_{i,k}) Z_{i,k} \Rightarrow \sum_k \tilde{\phi}_{i,k} Z_{i,k} = T_i^E - \tilde{\phi}_i Z_i$. Notice that, $\tilde{\phi}_{i,k} \equiv \tilde{\phi}_i^0 \bar{\zeta}_{i,k}$ where we observe $\bar{\zeta}_{i,k}$ in the data as the amount of local pollutants generated in country-industry i, k (for every one tonne of CO_2). Putting together, this delivers $\tilde{\phi}_{i,k}$ as

$$\tilde{\phi}_{i,k} = \frac{\zeta_{i,k}(1 - h_i) T_i^E}{\sum_k \zeta_{i,k} Z_{i,k}}, \qquad h_i = \frac{y_i SCC}{\sum_i y_i (T_i / Z_i)}.$$

F Additional Quantitative Exercises

F.1 Globally Optimal Carbon Border Taxes

In this appendix, we quantify the CO_2 reduction and welfare gains associated with cooperative carbon border taxes (see Section 4.2). These consequences are then compared to those of cooperative production and carbon taxes. This comparison illuminates the reasons behind the inefficacy of border taxes at cutting CO_2 emissions.

Following Section 4.2, the change in tariffs when transitioning from the applied rate to the globally optimal carbon border tax rate is described by:

$$\widehat{1+t_{ji,k}} = \frac{\left(1+(\sigma_k-1)\hat{\lambda}_{ii,k}\lambda_{ii,k}\right)\left(1+t_{ji,k}\right)^{-1}}{1+\left[1+\sum_{n\neq i}\left[\tilde{\phi}_n\hat{P}_n\right]\hat{v}_{i,k}\nu_{i,k}\right](\sigma_k-1)\hat{\lambda}_{ii,k}\lambda_{ii,k}}\left(1+\sum_{n\neq j}\left[\tilde{\phi}_n\hat{P}_n\right]\hat{v}_{j,k}\nu_{j,k}\right).$$

Solving the above equation alongside equilibrium conditions 32-37, determines the change to equilibrium variables $\mathcal{R}\equiv\{\mathcal{R}_v,\mathcal{R}_t\}$ as a function of baseline data and parameters $\mathcal{B}\equiv\{\mathcal{B}_v,\mathcal{B}_t,\mathcal{B}_\phi,\mathcal{B}_e\}$. In this process we are assuming away changes in export subsidies and production taxes, i.e., $\widehat{1+x_{ij,k}}=\widehat{1+s_{i,k}}=1$, and setting carbon taxes to their unilaterally optimal level.

The first two columns in Table A.1 report the CO_2 reduction and welfare effects implied by globally optimal (or cooperative) carbon border taxes. The adoption of cooperative carbon border taxes by all countries can cut global CO_2 emissions by 0.3% and raise global welfare by a mere 0.03%. Clearly, cooperative carbon border taxes prevent the substantive welfare loss implied by non-cooperative border taxes. However, they deliver a smaller reduction in CO_2 emissions compared to the non-cooperative border taxes (Table 3). There is a simple intuition behind this contrast. Cooperative carbon border taxes cut CO_2 emissions purely through composition effects. Noncooperative border taxes, on the other hand, cut CO_2 emissions through both composition and scale effects. The latter effect results from a sever shrinkage of global output under the non-cooperative Nash equilibrium.

Table A.1: Cooperative border taxes vs. other cooperative tax instruments

	Cooperative Taxes (CRS)					
	Carbon Border Tax		Production Tax		Carbon Tax	
Country	ΔCO2e	ΔW	ΔCO2e	ΔW	ΔCO2e	ΔW
AUS	0.9%	0.14%	-6.2%	0.19%	-47.3%	0.46%
EU	1.0%	0.19%	0.0%	0.37%	-9.2%	1.62%
BRA	-0.1%	0.00%	-6.9%	-0.01%	-69.1%	-0.31%
CAN	0.1%	0.00%	-2.0%	0.05%	-46.4%	0.23%
CHN	-0.5%	-0.06%	-9.2%	-0.12%	-68.8%	-0.44%
IDN	-0.7%	-0.04%	-6.7%	-0.07%	-66.5%	-0.48%
IND	-1.5%	-0.17%	-15.3%	-0.36%	-75.6%	-0.98%
JPN	0.3%	0.16%	-1.2%	0.32%	-22.5%	1.34%
KOR	-2.3%	0.11%	-8.5%	0.17%	-45.6%	0.26%
MEX	-0.3%	-0.09%	-5.3%	-0.12%	-78.8%	-0.28%
RUS	-0.6%	-0.43%	-10.9%	-0.58%	-80.4%	-0.72%
TUR	0.2%	0.11%	-2.6%	0.23%	-39.5%	0.15%
TWN	-4.9%	0.33%	-16.6%	0.33%	-59.1%	0.18%
USA	-0.1%	0.07%	-7.1%	0.10%	-47.9%	0.28%
RoW	-0.6%	-0.23%	-6.0%	-0.30%	-84.5%	-0.34%
Global	-0.3%	0.03%	-7.0%	0.08%	-60.3%	0.47%

In summary, the first two columns in Table A.1 re-establish the inefficacy of carbon border taxes. The remaining columns clarify why that is the case. As previously argued, carbon taxes lack efficacy for two reasons: (1) most CO_2emissions are produced by industries that are less-tradable, and (2) carbon border taxes are not granular enough to induce firm-level abatement. The second two columns in Table A.1 elucidate Reason (1). If cooperative governments are willing to tax all production–not just traded production—for climate objectives, they can cut CO_2 emissions by 7%. To put it differently, the ability to tax non-traded production yields a more than 20-fold increase in efficacy. Carbon taxes are even more effective because they can directly target firm-level abatement. The last two columns in Table A.1 report the gains from cooperative carbon taxes, which elucidate Reason (2). Cooperative carbon taxes can cut CO_2 emissions by 60%, which is close to 9-times higher than what is achievable via production taxes.

F.2 Impact of Carbon Border Adjustments Adopted Unilaterally by the EU

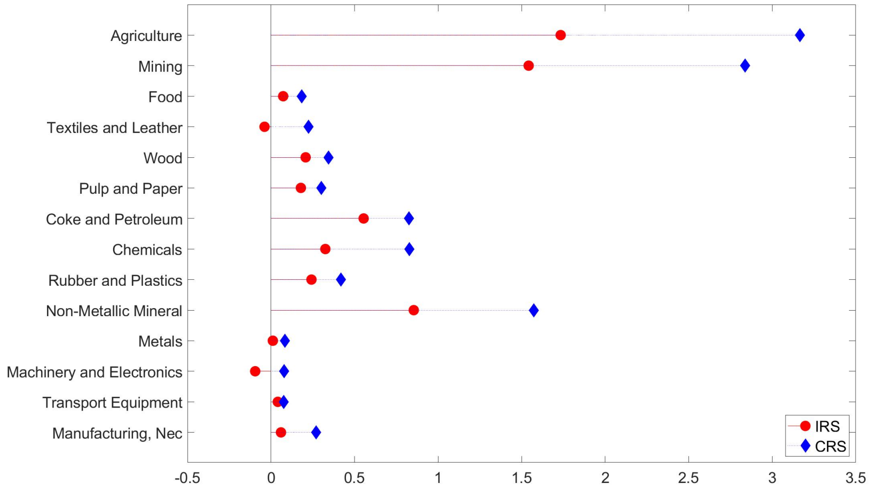
Recall from Section 3.2 that unilaterally optimal border taxes exhibit a *terms-of-trade-driven* component and a *carbon-reducing* component. In this appendix, we attempt to isolate the pure impact of the carbon-reducing component. Specifically, we consider a counterfactual scenario wherein the EU adopts its unilaterally optimal policy while other countries remain passive. We contrast the results from this exercise to a policy scenario in which the EU does not incorporate the carbon border tax component in its policy schedule (i.e., by setting $\delta_{ni,k} = 0$ for $i = EU, \quad n \neq EU$ while keeping $\delta_{EU,EU,k}$ at its original value). We report the difference in the equilibrium outcomes between these two policy scenarios as the effect of the carbon-reducing component of the EU's unilaterally optimal trade policy.

We find that the optimal carbon border adjustments to import tariffs and export subsidies are on average 0.71 and 0.95 percentage points, respectively. These numbers reflect the carbon content of the EU's optimal border taxes. Figure A.1 shows the carbon border adjustments associated with the EU's optimal import tariffs and export subsidies across non-service industries. As explained above, these reported values are calculated as the difference between two different equilibria. Hence, these values differ from a simple calculation of border adjustments at the same baseline values. This also explains why in a few cases, the border adjustment is slightly negative. In any event, carbon border adjustments tend to be larger under the constant-returns to scale assumption.

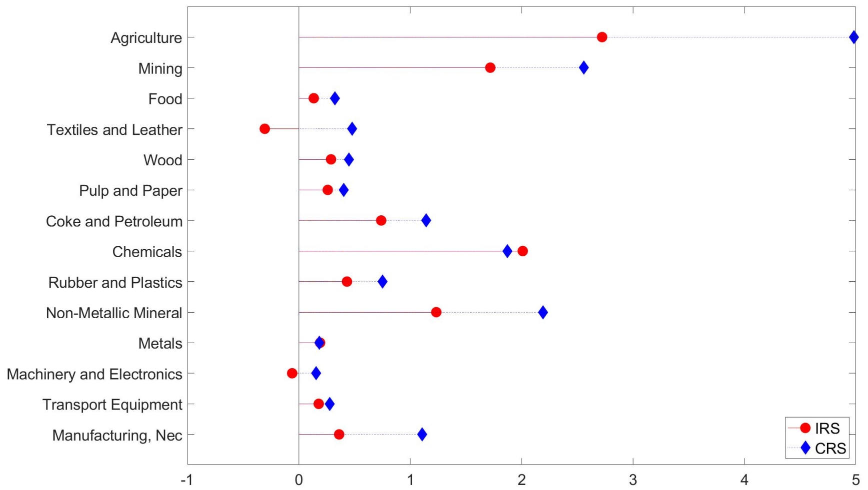
Before concluding this appendix, we should clarify a basic point about unilaterally optimal carbon border taxes. These taxes induce a relocation of firms from foreign countries into the EU, in which case the EU can tax their carbon content more effectively. As a result of these firm delocation effects, CO_2 emissions rise slightly in the EU but decrease elsewhere, prompting a 0.17% reduction in global emissions. This is, however, a modest effect if we take into account that the EU is the largest region in terms of GDP and has the highest aversion to CO_2 emissions in our sample.

Figure A.1: Unilaterally Optimal Border Carbon Taxes of the EU

(a) Carbon border taxes (as a part of tariffs)



(b) Carbon border subsidies (as a part of export subsidies)

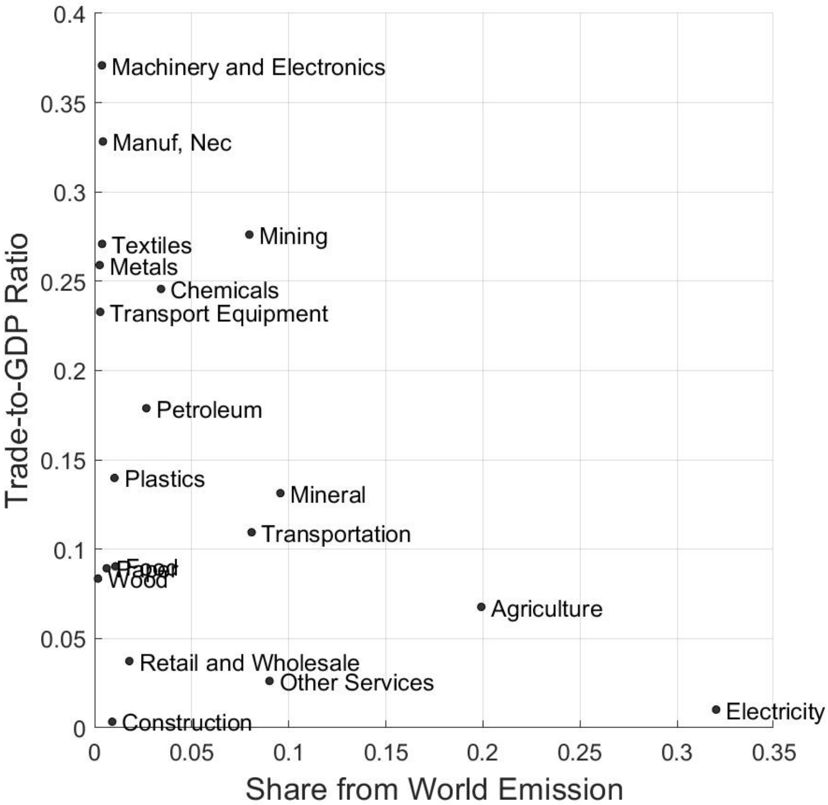


Notes: This figure shows optimal carbon border adjustments in tariffs and export subsidies set by the EU. Each dot is an average across trade partners for an industry reported in percentage points. CRS stands for constant returns to scale, and IRS for increasing returns to scale (as our main specification).

G Additional Figures and Tables

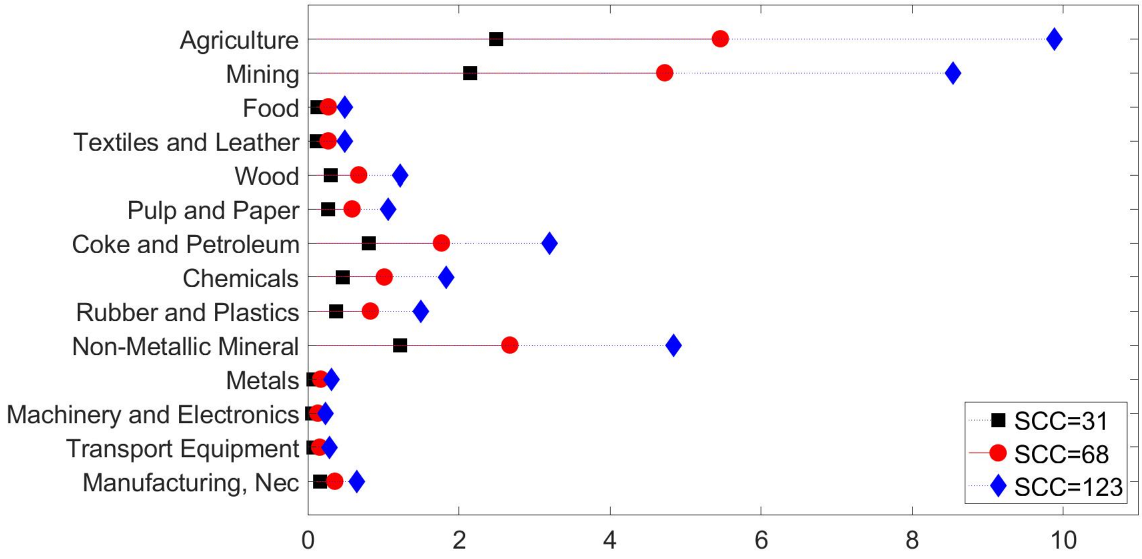
Additional Figures

Figure A.2: Industry-level Share of Global Emissions vs Trade-to-GDP Ratio



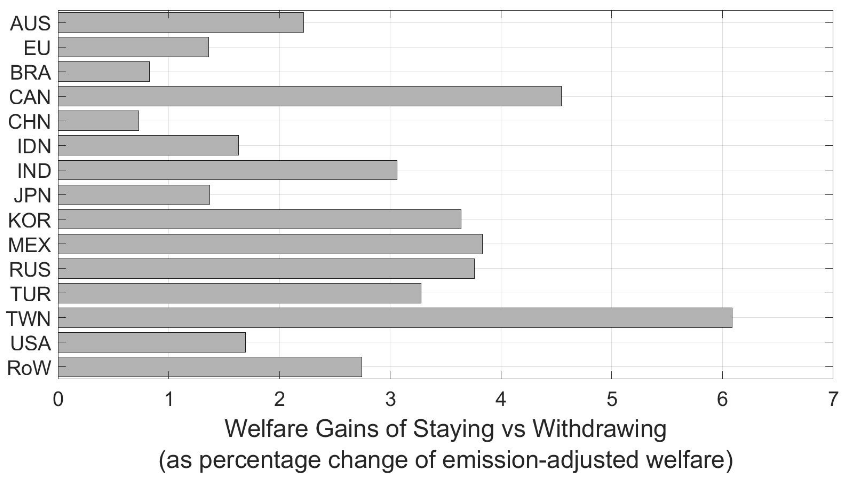
Note: This figure shows the scatter plot of CO2 emission of every industry as a share of total emissions against trade-to-GDP ratios.

Figure A.3: Optimal Carbon Border Taxes for the EU



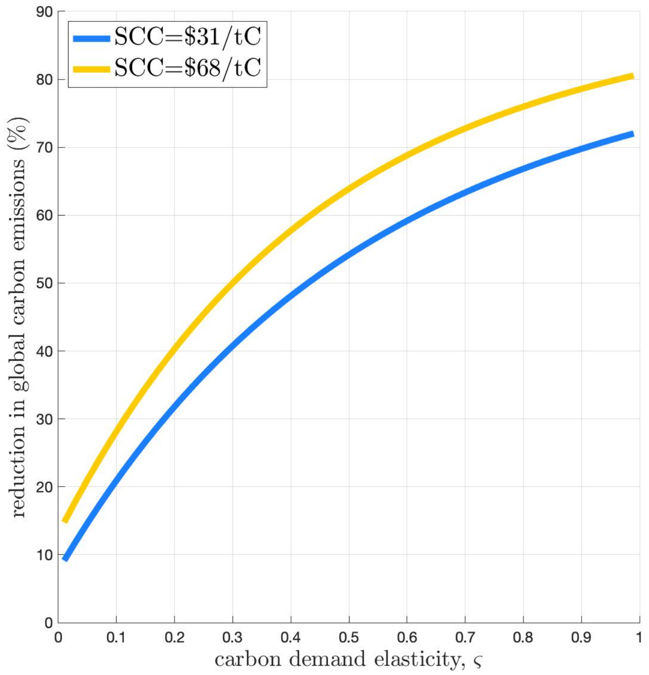
Note: This figure shows for every industry the carbon border taxes adopted optimally and unilaterally by the EU, at our baseline data and parameter estimates, for three values of SCC.

Figure A.4: Welfare Gains of Staying vs Leaving the Club-of-all-nations - Constant Returns to Scale



Note: This figure shows for the case of constant-returns-to-scale (CRS), for every country, the percentage change to welfare of staying in the club-of-all-nations relative to leaving the club-of-all-nations unilaterally.

Figure A.5: Reduction in CO2 Emissions under Globally Optimal Carbon Taxes



Note: This figure shows the absolute value of percentage change to CO2 emissions under the globally first-best carbon tax, for all values of carbon demand elasticity between zero and one, and two values of SCC, 31 $/tC and 68 $/tC.

Additional Tables

Table A.2: Countries and their Select Characteristics

Country	Share of World GDP	Share of World CO2	Carbon Intensity ($\bar{v}_i$)	Emission Tax Rate ($\bar{\tau}_i$)	CO2 Disutility ($\tilde{\phi}_i$)	Normalized $\tilde{\phi}_i$
AUS	1.7%	1.4%	100.00	32.51	0.49	40.43
EU	27.2%	12.1%	53.57	80.41	19.12	100.00
BRA	2.4%	2.4%	121.33	13.43	0.28	16.70
CAN	2.0%	1.7%	102.68	20.83	0.37	25.90
CHN	13.6%	23.1%	204.31	6.93	0.82	8.61
IDN	1.0%	1.8%	218.95	8.43	0.07	10.48
IND	2.2%	6.5%	359.48	5.25	0.10	6.53
JPN	8.4%	2.9%	40.99	69.13	5.08	85.97
KOR	1.9%	1.6%	99.68	26.80	0.44	33.33
MEX	1.2%	1.4%	137.31	3.76	0.04	4.67
RUS	2.0%	5.8%	344.11	3.69	0.07	4.59
TUR	1.0%	0.9%	116.09	48.45	0.41	60.25
TWN	0.7%	0.8%	139.84	13.69	0.09	17.03
USA	21.1%	15.3%	87.32	18.18	3.35	22.61
RoW	13.5%	22.1%	197.23	2.21	0.26	2.75

Note: This table shows for every of the 15 regions (13 countries + the EU + the RoW), their share of world GDP, share of world CO2 emissions, carbon intensity (CO2 emissions per dollar of output) normalized by that of Australia, emission tax rate (dollar per tonne of CO2), calibrated CPI-adjusted disutility parameter from one tonne of CO2 emission ($\tilde{\phi}_i$), and the ratio of $\tilde{\phi}_i$ to country i's GDP normalized to 100 for the EU. All CO2 measures are CO2 equivalent.

Table A.3: Non-cooperative and Cooperative Outcomes - Alternative Baseline Equilibrium

	Increasing Returns to Scale					
	Non-Cooperative			Global Cooperation		
Country	ΔCO_2	ΔV	ΔW	ΔCO_2	ΔV	ΔW
AUS	10.5%	0.5%	0.2%	-36.6%	1.8%	2.4%
EU	18.8%	1.8%	1.3%	7.0%	3.1%	4.5%
BRA	21.8%	1.6%	1.5%	-62.1%	1.6%	2.0%
CAN	7.6%	-1.3%	-1.5%	-34.8%	2.4%	3.0%
CHN	22.8%	1.6%	1.5%	-63.4%	1.3%	1.6%
IDN	23.7%	0.8%	0.7%	-57.8%	1.1%	1.5%
IND	25.0%	-0.6%	-0.7%	-70.4%	1.2%	1.6%
JPN	21.7%	2.4%	2.0%	-9.5%	3.2%	4.5%
KOR	37.8%	1.6%	1.3%	-39.3%	2.7%	3.4%
MEX	19.7%	0.7%	0.7%	-75.0%	3.3%	3.4%
RUS	8.6%	1.1%	1.1%	-76.7%	3.7%	3.9%
TUR	14.8%	1.7%	1.2%	-27.3%	4.4%	5.5%
TWN	67.3%	0.3%	-0.0%	-57.8%	1.2%	1.7%
USA	16.7%	1.2%	1.0%	-40.4%	2.6%	3.1%
RoW	13.3%	-2.5%	-2.6%	-81.5%	-1.2%	-1.1%
Global	18.6%	0.9%	0.6%	-53.4%	2.0%	2.8%

Note: This table shows for every country the change to CO2 emission, real consumption, and welfare from the baseline to non-cooperative and cooperative equilibrium. The baseline is observed data, which means that the effects of optimal production taxes, which restore marginal-cost pricing, are also considered.

Table A.4: Non-cooperative and Cooperative Outcomes - CRS

	Constant Returns to Scale					
	Non-Cooperative			Global Cooperation		
Country	ΔCO_2	ΔV	ΔW	ΔCO_2	ΔV	ΔW
AUS	-5.2%	-1.8%	-1.8%	-47.3%	-0.4%	0.5%
EU	0.2%	-1.0%	-1.0%	-9.2%	-0.0%	1.6%
BRA	-5.5%	-1.1%	-1.1%	-69.1%	-0.7%	-0.3%
CAN	-8.7%	-4.1%	-4.1%	-46.4%	-0.4%	0.2%
CHN	2.1%	-0.9%	-0.9%	-68.8%	-0.8%	-0.4%
IDN	-2.2%	-1.5%	-1.5%	-66.5%	-0.9%	-0.5%
IND	-0.3%	-3.6%	-3.6%	-75.6%	-1.4%	-1.0%
JPN	0.7%	-1.0%	-1.0%	-22.5%	-0.1%	1.3%
KOR	1.9%	-2.9%	-2.9%	-45.6%	-0.5%	0.3%
MEX	-1.2%	-3.5%	-3.5%	-78.8%	-0.4%	-0.3%
RUS	-6.7%	-4.1%	-4.1%	-80.4%	-1.0%	-0.7%
TUR	-4.3%	-3.5%	-3.5%	-39.5%	-1.1%	0.1%
TWN	9.9%	-4.8%	-4.9%	-59.1%	-0.4%	0.2%
USA	-0.1%	-1.5%	-1.5%	-47.9%	-0.2%	0.3%
RoW	-4.7%	-2.6%	-2.6%	-84.5%	-0.5%	-0.3%
Global	-1.3%	-1.6%	-1.6%	-60.3%	-0.3%	0.5%

Note: For the case of constant-returns to scale (CRS), this table shows for every country the change to CO2 emission, real consumption, and welfare from the baseline to non-cooperative and cooperative equilibrium.

Table A.5: Alternative Specifications and Policy Scenarios for Carbon Border Taxes

	$\Delta CO2$	$\Delta CO2$ as % of 1st-best	ΔV
Main specification	-0.62%	1.02%	-1.71%
Main specification with SCC=68 $/tC	-0.71%	1.01%	-1.72%
Main specification with $\varsigma = 1$ (Cobb-Douglas)	-2.07%	2.85%	-1.64%
CRS	-1.29%	2.16%	-1.63%
CRS with SCC=68 $/tC	-1.42%	2.04%	-1.64%
CRS with $\varsigma = 1$ (Cobb-Douglas)	-2.70%	3.74%	-1.64%
No ToT border taxes (base: zero tariffs)	-0.87%	1.42%	-0.01%
No ToT border taxes (base: applied tariffs)	-0.31%	0.51%	0.01%
Cooperative border taxes	-0.34%	0.56%	0.03%

Note: This table shows the effects of optimal border taxes (Proposal 1) on emission and consumption under alternative scenarios/specifications. The columns report the change to CO2 emissions, and that as percentage of the reduction attainable in the globally first-best outcome, as well as the change to real consumption. The 1st column reproduces the results of the main specification. The 2nd column reports results of the main specification under SCC of 68 $/tC. The 3rd column reports results of the main specification under the Cobb-Douglas production function corresponding to $\varsigma = 1$. The 4th-6th columns repeat the same exercises for the case of constant-returns-to-scale (CRS). The 7th and 8th columns report results for the cases in which carbon border taxes exclude terms-of-trade driven components and include only carbon-reducing components, with the baselines of zero tariffs or currently applied tariffs. The last column reports results of optimal cooperative border taxes.

Table A.6: Climate Club Game - Successive Dominant Strategies - Constant Returns to Scale

Core Members	*1st Round*	*2nd Round*	*3rd Round*
EU, USA	CAN, MEX, ROW	AUS, BRA, IND, JPN, KOR, RUS, TUR, TWN	CHN, IDN

Table A.7: Climate Club Game, SCC= 31, $\varsigma = 0.25$

Core Members	*1st Round*	*2nd Round*	*3rd Round*	*Remain Outside of the Club*
EU, USA	CAN, ROW	AUS, JPN, KOR, RUS, TUR, TWN	MEX	BRA, IND, CHN, IDN

Table A.8: Climate Club Game, SCC= 31, $\varsigma = 0.99$

Core Members	*1st Round*	*2nd Round*	*3rd Round*
EU, USA	CAN, ROW	AUS, IND, JPN, KOR, MEX, RUS, TUR, TWN	BRA, CHN, IDN

Table A.9: Climate Club Game, SCC= 68, $\varsigma = 0.63$

Core Members	*1st Round*	*2nd Round*	*3rd-5th Round*	*Remain Outside of the Club*
EU, USA	CAN, ROW	AUS, JPN, TWN	KOR, MEX, RUS, TUR	BRA, IND, CHN, IDN