SHADOW-RATE VARs

Andrea Carriero[1] Todd E. Clark[2]
Massimiliano Marcellino[3] Elmar Mertens[4]

[1]Queen Mary University of London, [2]Federal Reserve Bank of Cleveland,
[3]Bocconi University, IGIER, and CEPR, [4]Deutsche Bundesbank

NBER Summer Institute

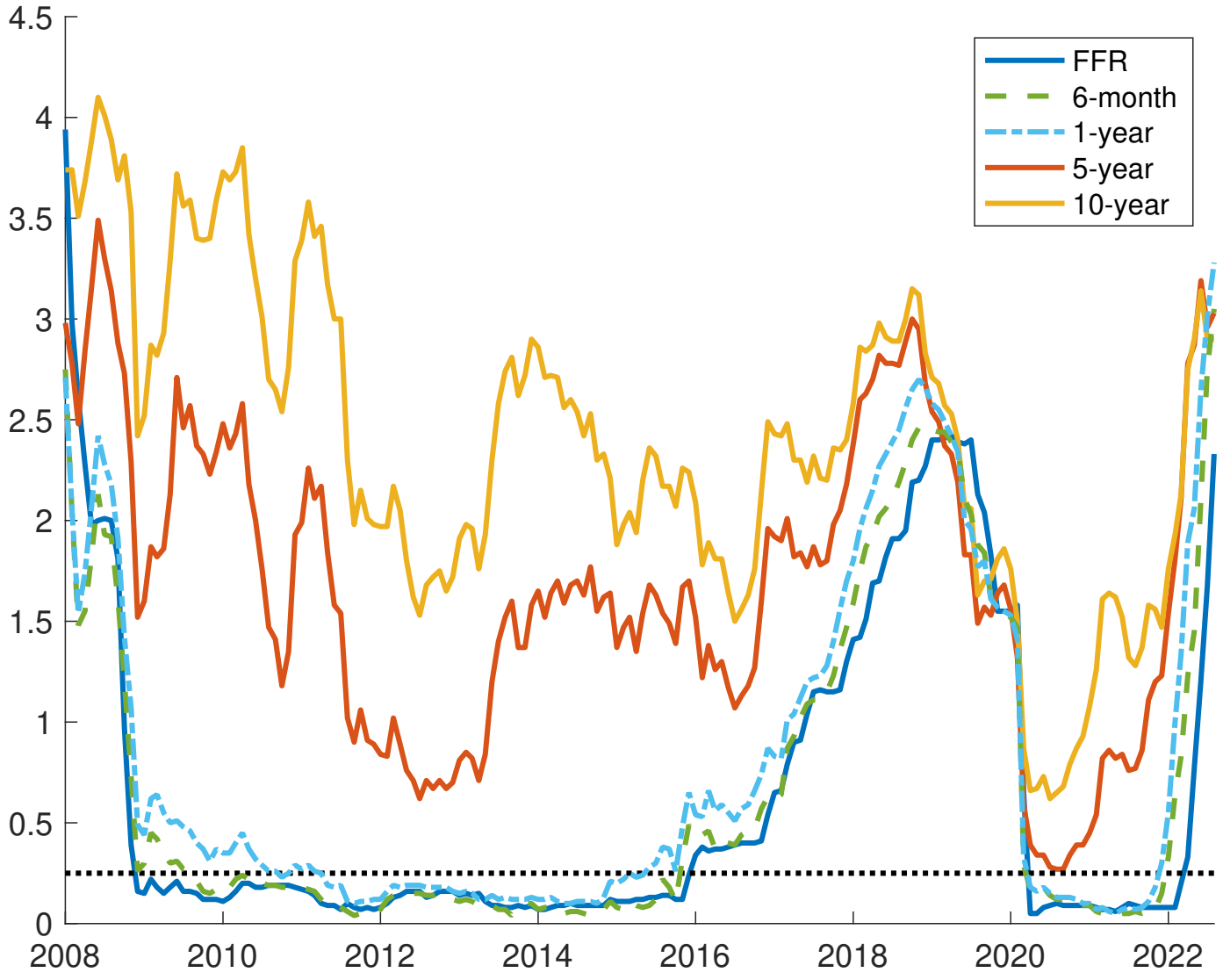
Workshop on Methods and Applications
for Dynamic Equilibrium Models

July 2023

The results presented here do not necessarily represent the views of the Federal Reserve Bank of Cleveland, the Federal Reserve System, the Deutsche Bundesbank, the Eurosystem, or their respective staffs.

NOMINAL INTEREST RATES SINCE 2008

Binding ELB constraint has become a recurring event



VARs AND ELB DATA

How to use VARs
w/nominal-interest rates at the ELB?

Bayesian VARs are a great time series tool

But, VARs are ill-equipped to handle bounded data

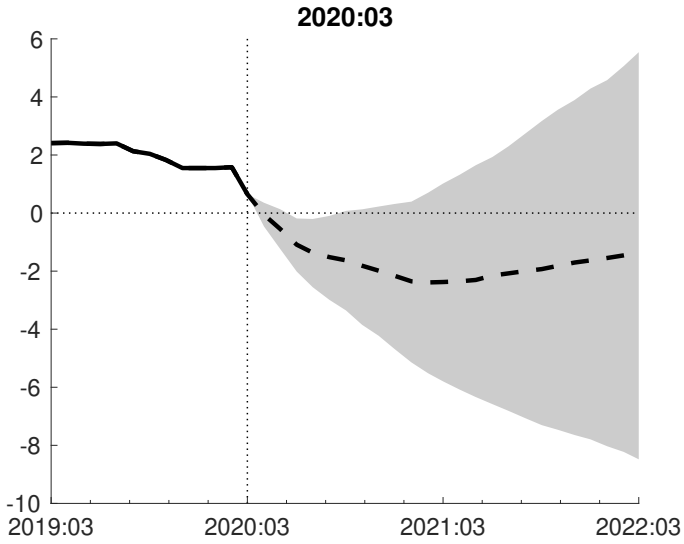
Problem: the standard VAR is linear and unbounded

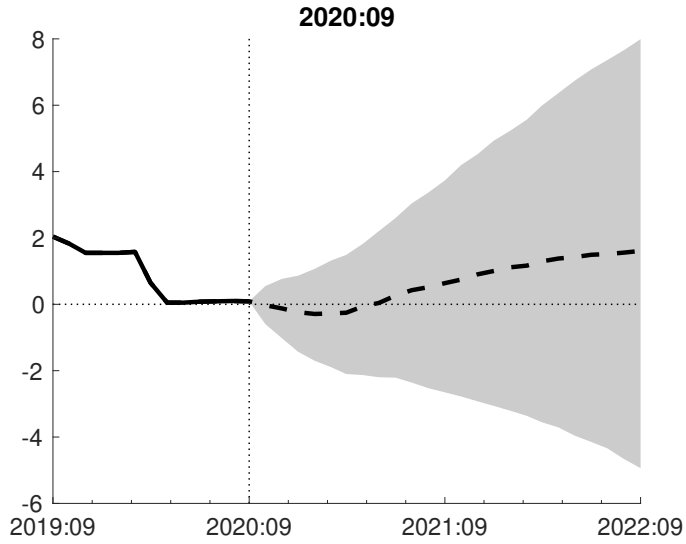
$$y_t = \Pi_0 + \Pi(L) y_{t-1} + v_t\,, \quad v_t \sim N(0, \Sigma_t)$$

VAR FORECASTS FOR INTEREST RATES IN 2020

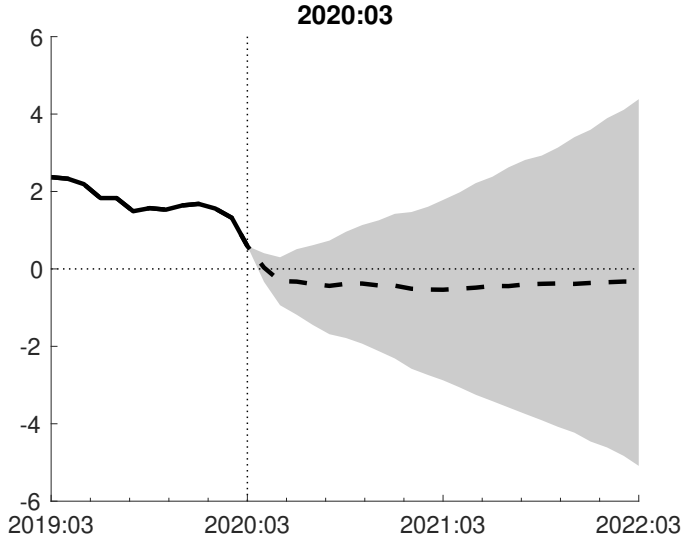
Linear forecasts from BVAR-SV with 20 macro and financial variables

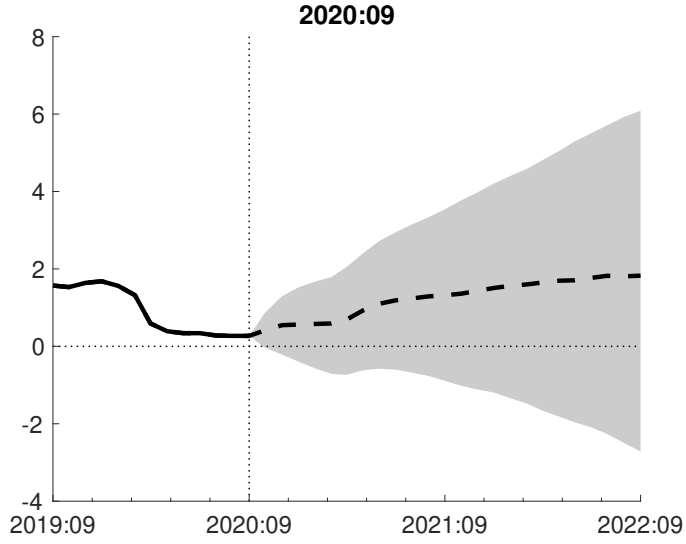
Federal funds rate





5-year Treasury yield





Medians and 68% bands

WAYS TO ACCOMODATE ELB

- **Use longer-term yields** and no short-term policy rates (Swanson & Williams, 2014; Debortoli et al., 2019)

  Applicable when 5y rate at ELB (US in 2020, or Japan)?

WAYS TO ACCOMODATE ELB

- Use longer-term yields and no short-term policy rates (Swanson & Williams, 2014; Debortoli et al., 2019)

  Applicable when 5y rate at ELB (US in 2020, or Japan)?

- **No-arbitrage shadow-rate models** as in Black (1995)

  Mostly limited to affine, homoskedastic, Gaussian models

WAYS TO ACCOMODATE ELB

- Use longer-term yields and no short-term policy rates (Swanson & Williams, 2014; Debortoli et al., 2019)

  Applicable when 5y rate at ELB (US in 2020, or Japan)?

- No-arbitrage shadow-rate models as in Black (1995)

  Mostly limited to affine, homoskedastic, Gaussian models

- **Plug-in VAR**: External shadow rate estimates as data

  Generated regressor and inconsistent estimates (Mavroeidis, 2021; Krippner, 2020)

WAYS TO ACCOMODATE ELB

- Use longer-term yields and no short-term policy rates (Swanson & Williams, 2014; Debortoli et al., 2019)

  Applicable when 5y rate at ELB (US in 2020, or Japan)?

- No-arbitrage shadow-rate models as in Black (1995)

  Mostly limited to affine, homoskedastic, Gaussian models

- Plug-in VAR: External shadow rate estimates as data

  Generated regressor and inconsistent estimates (Mavroeidis, 2021; Krippner, 2020)

Our approach: censored data in a VAR

"Shadow rate" as latent state variable,
informed by bond yields, financial and macro variables
in a VAR model

RELATED LITERATURE

No-arbitrage term structure models w/shadow rates

- Black (1995), Krippner (2013/15/20), Wu & Xia (2016/20)
- Bauer & Rudebusch (2014/17), Kim & Singleton (2011), Priebsch (2017), Christensen & Rudebusch (2015, 2016)

Macro models w/negative-rate substitutes

- Sims & Wu (2021), Wu & Zhang (2019)
- Gust et al (2017), Wolf (2021)
- Kulish, Morley, Robinson (2017), Jones, Kulish, Morley (2022)

Time series models w/bounded or censored rates

- Mavroeidis (2021), Ikeda et al (2022), Aruoba, et al (2022)
- Johannsen & Mertens (2021)
- Goncalves et al (2021)
- Guerron-Quintana, Khazanov & Zhong (2023)
- Chib (1992), Chib & Greenberg (1998)
- Gonzalez-Astudillo & Laforte (2020)
- Iwata & Wu (2006), Nakajima (2011), Koop & Potter (2011), Chan & Strachan (2014), Bäurle, et al. (2016)

AGENDA

1. **Shadow-rate VAR concept**
2. **Estimation method**
3. **Shadow rate estimates and macro predictions**
4. **SVAR responses to financial conditions shock**
5. **Conclusions and additional results**

AGENDA

1. **Shadow-rate VAR concept**
2. Estimation method
3. Shadow rate estimates and macro predictions
4. SVAR responses to financial conditions shock
5. Conclusions and additional results

THE SHADOW RATE OF INTEREST

Shadow rate s_t

Nominal interest rate that would prevail in the absence of lower bound constraint

Observed Rate i_t

$$i_t = \max(s_t, ELB)$$

THE SHADOW RATE OF INTEREST

BLACK (1995)

Shadow rate s_t

Nominal interest rate that would prevail in the absence of lower bound constraint

Observed Rate i_t

$$i_t = \max(s_t, ELB)$$

Our paper

- VAR model for joint dynamics of s_t and other variables
- Estimation treats i_t as censored variable
- Shadow rates identified from historical comovements between interest rates, macro and financial variables
- We study the role of s_t and i_t as predictors in VAR

SIMPLE AND HYBRID SHADOW-RATE VARs

All expressed in companion form omitting intercepts

Setup

- Partition the variable vector:

$$\boldsymbol{y}_t = \begin{bmatrix} \boldsymbol{x}_t \\ \boldsymbol{i}_t \end{bmatrix}$$

SIMPLE AND HYBRID SHADOW-RATE VARs

All expressed in companion form omitting intercepts

Setup

- Partition the variable vector:

$$y_t = \begin{bmatrix} x_t \\ i_t \end{bmatrix}$$

- Define a corresponding shadow-rate VAR vector

$$z_t = \begin{bmatrix} x_t \\ s_t \end{bmatrix}, \qquad i_t = \max(s_t, ELB)$$

Simple shadow-rate VAR

$$z_t = \Pi z_{t-1} + v_t$$

SIMPLE AND HYBRID SHADOW-RATE VARs

All expressed in companion form omitting intercepts

Setup

- Partition the variable vector:

$$y_t = \begin{bmatrix} x_t \\ i_t \end{bmatrix}$$

- Define a corresponding shadow-rate VAR vector

$$z_t = \begin{bmatrix} x_t \\ s_t \end{bmatrix}, \qquad i_t = \max(s_t, ELB)$$

Simple shadow-rate VAR

$$z_t = \Pi\, z_{t-1} + v_t$$

Hybrid shadow-rate VAR

$$x_t = \Pi_{xx}\, x_{t-1} + \Pi_{xi}\, i_{t-1} + v_t^x$$
$$s_t = \Pi_{sx}\, x_{t-1} + \Pi_{ss}\, s_{t-1} + v_t^s$$

INTERPRETATION OF SHADOW RATE

In our VAR models, s_t is ...

- Vector with as many elements as number of interest rate measures, $\boldsymbol{i_t}$, included in VAR
- Latent state for forecasting future $\boldsymbol{x_t}$, $\boldsymbol{s_t}$, and $\boldsymbol{i_t}$
- Informed by VAR estimates of comovements between interest rates, macro and financial variables
- Above ELB: identical to actual interest rates $\boldsymbol{i_t}$
- Below ELB: reflects historical comovements with other macro and yield variables, $\boldsymbol{x_t}$ and $\boldsymbol{i_t}$

RELATIONSHIP TO SVAR OF MAVROEIDIS (2021)

Mavroeidis' "censored and kinked" SVAR

$$\begin{bmatrix} A_{11} & A_{12}^* & A_{12} \\ A_{21} & A_{22}^* & A_{22} \end{bmatrix} \begin{bmatrix} x_t \\ s_t \\ i_t \end{bmatrix} = BX_t + B^* X_t^* + \varepsilon_t$$

time-invariant SVAR parameters
but time-varying reduced-form parameters

with lags of x_t and i_t in X_t and lags of s_t in X_t^*

RELATIONSHIP TO SVAR OF MAVROEIDIS (2021)

Mavroeidis' "censored and kinked" SVAR

$$\begin{bmatrix} A_{11} & A_{12}^* & \mathbf{0} \\ A_{21} & A_{22}^* & \mathbf{0} \end{bmatrix} \begin{bmatrix} x_t \\ s_t \\ i_t \end{bmatrix} = BX_t + B^*X_t^* + \varepsilon_t$$

time-invariant SVAR parameters
but time-varying reduced-form parameters

with lags of x_t and i_t in X_t and lags of s_t in X_t^*

Our shadow-rate VARs add restrictions:

zero restrictions on shock impacts $A_{\cdot 2}$
yield a time-invariant reduced-form VAR

- Simple shadow-rate VAR: "censored" SVAR
- Hybrid shadow-rate VAR: "censored and kinked" SVAR (w/added zero restrictions)

AGENDA

1. Shadow-rate VAR concept
2. **Estimation method**
3. Shadow rate estimates and macro predictions
4. SVAR responses to financial conditions shock
5. Conclusions and additional results

VAR parameters

$$z_t = \Pi_0 + \Pi(L) z_{t-1} + \Pi_i(L) i_{t-1} + v_t$$
$$v_t \sim N(0, \Sigma_t)$$

- transition coefficients Π
- stochastic volatility in shock vector $\Sigma_t = A^{-1} \Lambda_t A^{-1\prime}$

Given values for s_t, we know how to draw Π and Σ_t

Additional step: draw shadow rates consistent with ELB

VAR parameters

$$z_t = \Pi_0 + \Pi(L) z_{t-1} + \Pi_i(L) i_{t-1} + v_t$$
$$v_t \sim N(0, \Sigma_t)$$

- transition coefficients Π
- stochastic volatility in shock vector $\Sigma_t = A^{-1} \Lambda_t A^{-1\prime}$

Given values for s_t, we know how to draw Π and Σ_t

Additional step: draw shadow rates consistent with ELB

Next slide:

Sample path for s_t from truncated "missing data" distribution

Throughout, we treat ELB as known value of 25bp

SHADOW-RATE SAMPLING

Data augmentation step (Chib, 1992) within MCMC sampler for VAR

Shadow-rate setup in static form

- $\boldsymbol{Y}$, vector of all $\boldsymbol{y_t} = \begin{bmatrix} \boldsymbol{x_t} \\ \boldsymbol{i_t} \end{bmatrix}$
- $\bar{\boldsymbol{Y}}$, all of $\boldsymbol{Y}$ except for $\boldsymbol{i_t}$ when ELB binds
- $\boldsymbol{S}$, all shadow rates $\boldsymbol{s_t}$ when ELB binds

SHADOW-RATE SAMPLING

Data augmentation step (Chib, 1992) within MCMC sampler for VAR

Shadow-rate setup in static form

- $\boldsymbol{Y}$, vector of all $\boldsymbol{y_t} = \begin{bmatrix} \boldsymbol{x_t} \\ \boldsymbol{i_t} \end{bmatrix}$
- $\bar{\boldsymbol{Y}}$, all of $\boldsymbol{Y}$ except for $\boldsymbol{i_t}$ when ELB binds
- $\boldsymbol{S}$, all shadow rates $\boldsymbol{s_t}$ when ELB binds

Missing-value problem (given Π, Σ_t, and $\forall\, t$)

$$\boldsymbol{S}|\bar{\boldsymbol{Y}} \sim \boldsymbol{N}(\mu, \Omega)$$

can be obtained from standard Kalman smoothing

SHADOW-RATE SAMPLING

Data augmentation step (Chib, 1992) within MCMC sampler for VAR

Shadow-rate setup in static form

- $\boldsymbol{Y}$, vector of all $\boldsymbol{y_t} = \begin{bmatrix} \boldsymbol{x_t} \\ \boldsymbol{i_t} \end{bmatrix}$
- $\bar{\boldsymbol{Y}}$, all of $\boldsymbol{Y}$ except for $\boldsymbol{i_t}$ when ELB binds
- $\boldsymbol{S}$, all shadow rates $\boldsymbol{s_t}$ when ELB binds

Missing-value problem (given Π, Σ_t, and $\forall\, t$)

$$\boldsymbol{S}|\bar{\boldsymbol{Y}} \sim \boldsymbol{N}(\mu, \Omega)$$

can be obtained from standard Kalman smoothing

Shadow-rate sampling problem

$$\boldsymbol{S}|\boldsymbol{Y} \sim \mathbf{truncN}\left(\mu, \Omega, \boldsymbol{S} \leq \boldsymbol{ELB}\right)$$

SHADOW-RATE SAMPLING

Data augmentation step (Chib, 1992) within MCMC sampler for VAR

Shadow-rate setup in static form

- $\boldsymbol{Y}$, vector of all $\boldsymbol{y_t} = \begin{bmatrix} \boldsymbol{x_t} \\ \boldsymbol{i_t} \end{bmatrix}$
- $\bar{\boldsymbol{Y}}$, all of $\boldsymbol{Y}$ except for $\boldsymbol{i_t}$ when ELB binds
- $\boldsymbol{S}$, all shadow rates $\boldsymbol{s_t}$ when ELB binds

Missing-value problem (given Π, Σ_t, and $\forall\, t$)

$$\boldsymbol{S}|\bar{\boldsymbol{Y}} \sim \boldsymbol{N}(\mu, \Omega)$$

can be obtained from standard Kalman smoothing

Shadow-rate sampling problem

$$\boldsymbol{S}|\boldsymbol{Y} \sim \mathbf{truncN}\left(\mu, \Omega, \boldsymbol{S} \leq \boldsymbol{ELB}\right)$$

- Johannsen & Mertens (2021): Rejection sampling
- Here: Direct sampling from truncated multivariate normal

AGENDA

1. Shadow-rate VAR concept
2. Estimation method
3. **Shadow rate estimates and macro predictions**
4. SVAR responses to financial conditions shock
5. Conclusions and additional results

DATA AND ESTIMATION SETUP

Data

- 20 variables, including FFR and 5 other yields (6m-10y maturities)
- All data from same FRED-MD vintage
- Monthly observations from 1959:03 – 2022:08

Full-sample (smoothed) estimates

reflect data through 2022:08

Quasi-real time estimates

- MCMC over growing estimation windows
- Evaluation window 2009:01 – 2017:12 (similar results through 2022:08)
- Forecasts up to two years out ($\boldsymbol{h = 24}$)

20-VARIABLE DATA SET

Monthly obs from 1959:03 to 2022:08; FRED-MD vintage 2022:09

Variable	FRED-MD code	transformation
Real Income	RPI	$\Delta \log(x_t) \cdot 1200$
Real Consumption	DPCERA3M086SBEA	$\Delta \log(x_t) \cdot 1200$
IP	INDPRO	$\Delta \log(x_t) \cdot 1200$
Capacity Utilization	CUMFNS	
Unemployment	UNRATE	
Nonfarm Payrolls	PAYEMS	$\Delta \log(x_t) \cdot 1200$
Hours	CES0600000007	
Hourly Earnings	CES0600000008	$\Delta \log(x_t) \cdot 1200$
PPI (Fin. Goods)	WPSFD49207	$\Delta \log(x_t) \cdot 1200$
PPI (Metals)	PPICMM	$\Delta \log(x_t) \cdot 1200$
PCE Prices	PCEPI	$\Delta \log(x_t) \cdot 1200$
Housing Starts	HOUST	$\log(x_t)$
S&P 500	SP500	$\Delta \log(x_t) \cdot 1200$
USD / GBP FX Rate	EXUSUKx	$\Delta \log(x_t) \cdot 1200$
Federal Funds Rate	FEDFUNDS	
6m Tbill	TB6MS	
1-Year Yield	GS1	
5-Year Yield	GS5	
10-Year Yield	GS10	
Corporate Bond Yield	BAA	

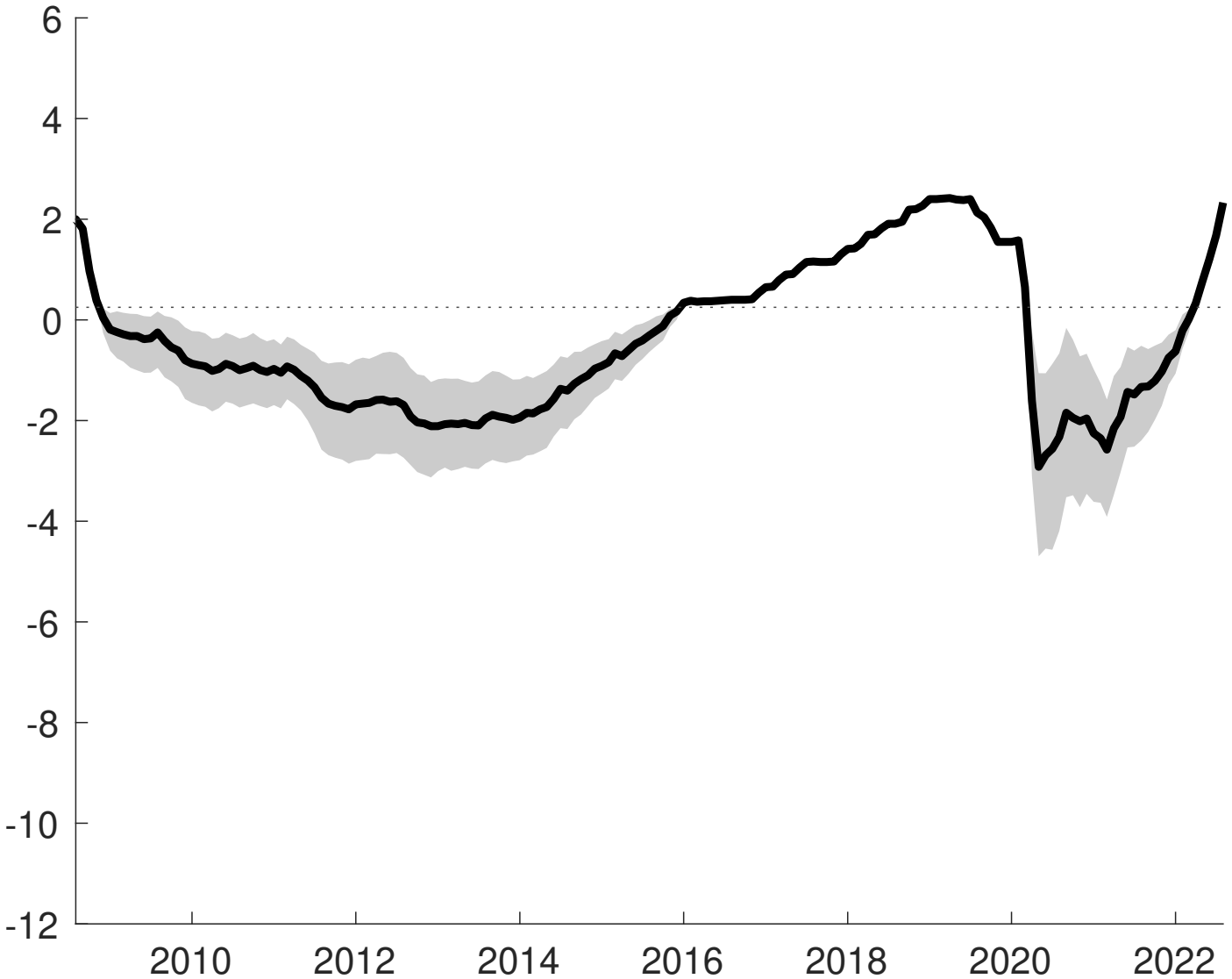
AGENDA

1. Shadow-rate VAR concept
2. Estimation method
3. **Shadow rate estimates and macro predictions**
   - Shadow rate estimates
   - Interest rate predictions
   - Expected paths of short rates
   - Forecast comparison for macro and financial variables
4. SVAR responses to financial conditions shock
5. Conclusions and additional results

SHADOW RATE ESTIMATES

FFR

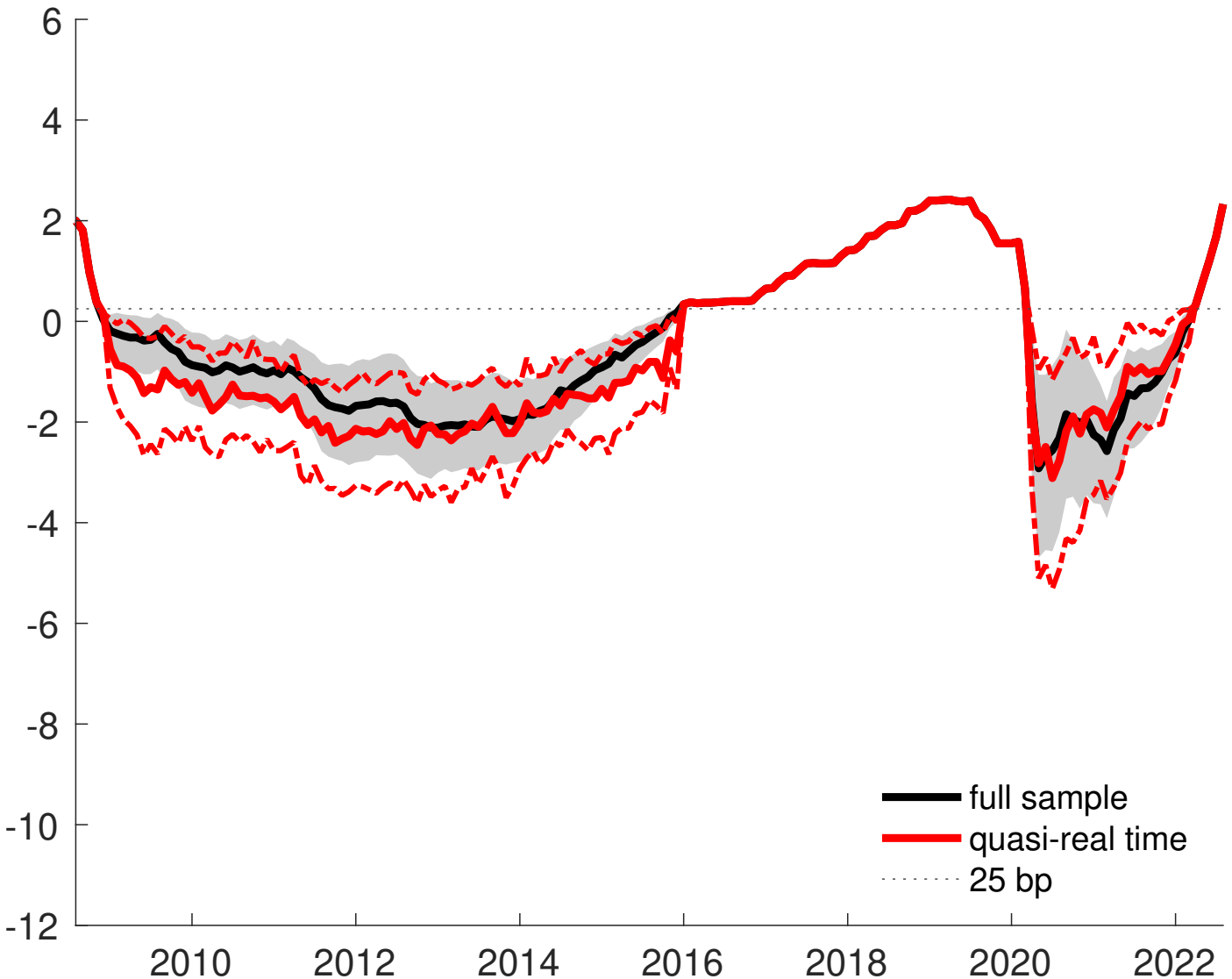
simple shadow-rate VAR (w/yields), full-sample median and 90% bands



Reflects historical comovements of FFR w/other variables

SHADOW RATE ESTIMATES

simple shadow-rate VAR, median and 90% bands

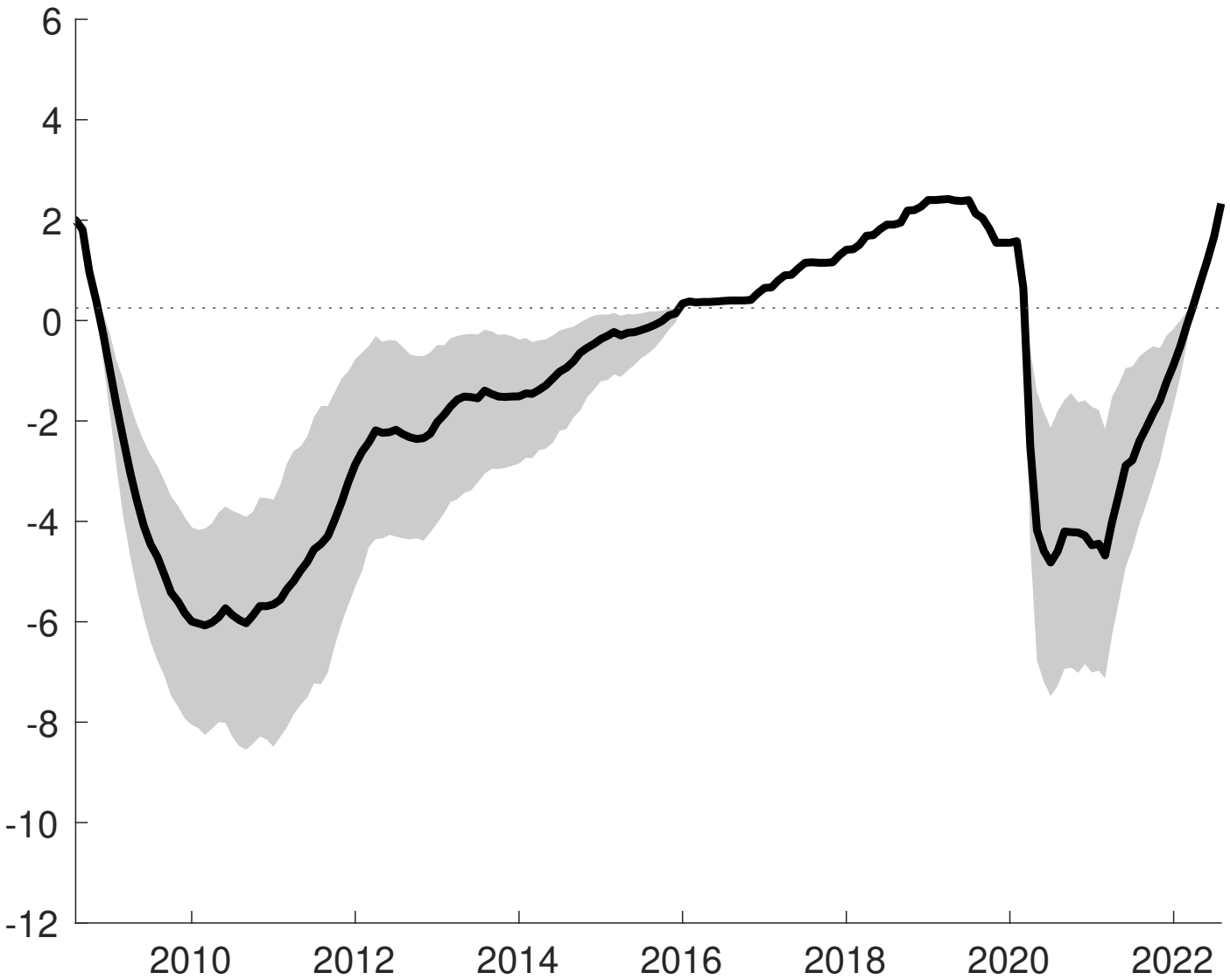


decent quasi-real time properties

SHADOW RATE ESTIMATED W/O YIELDS

FFR

simple shadow-rate VAR, full-sample median and 90% bands

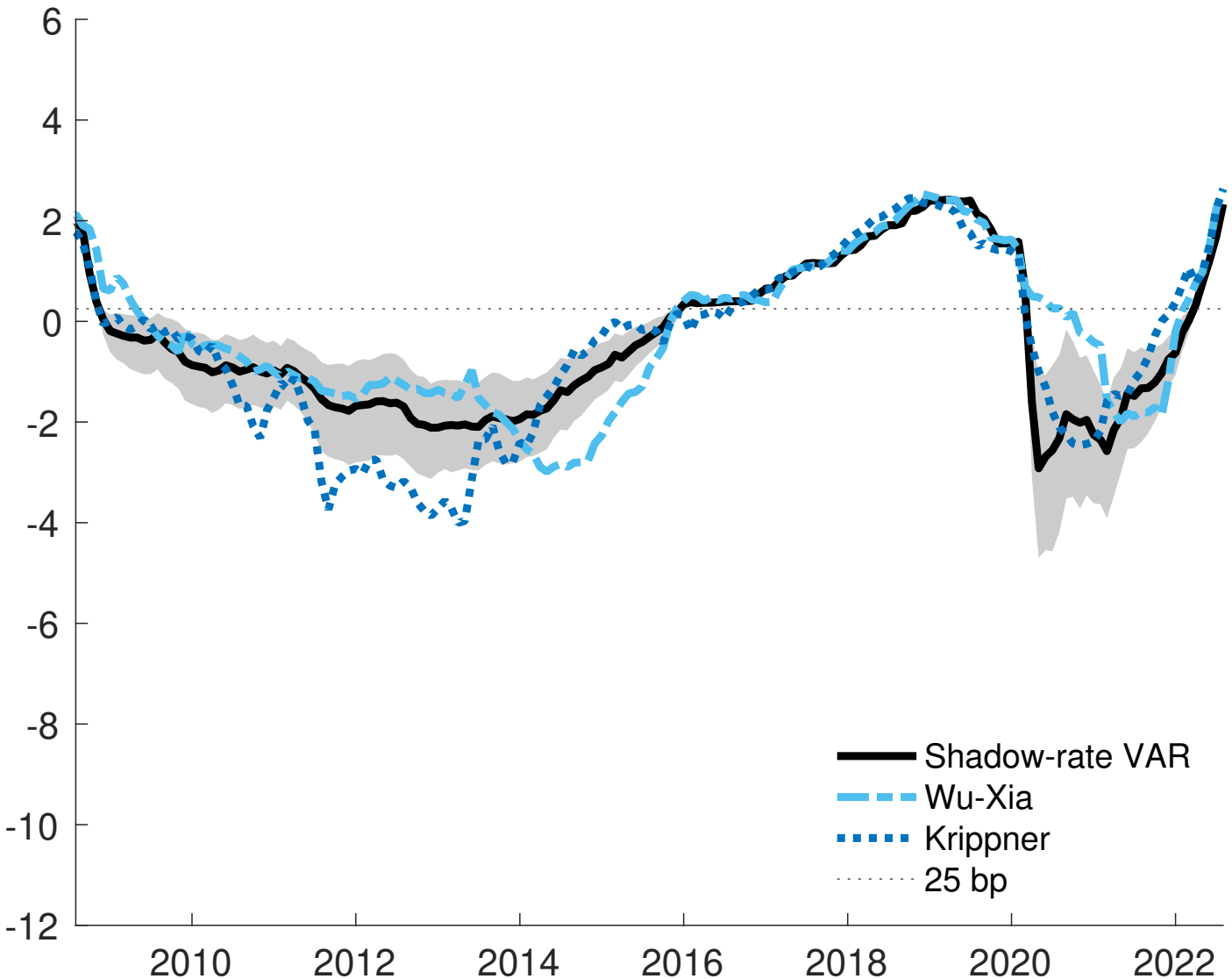


Deeper path when not informed by yields

SHADOW RATE ESTIMATES

FFR

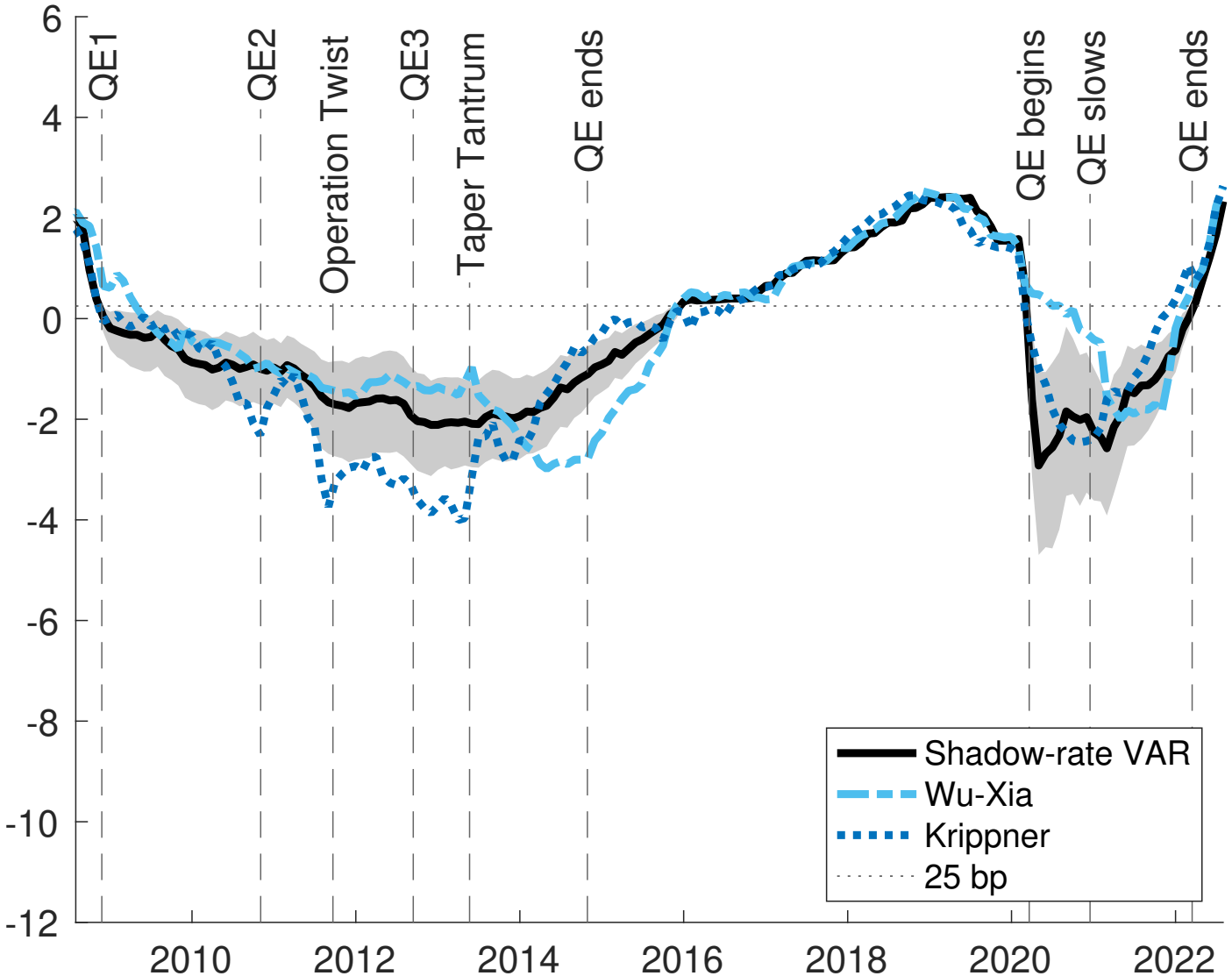
simple shadow-rate VAR (w/yields), full-sample median and 90% bands



Between estimates of Krippner and Wu-Xia

SHADOW RATE ESTIMATES

simple shadow-rate VAR (w/yields), full-sample median and 90% bands



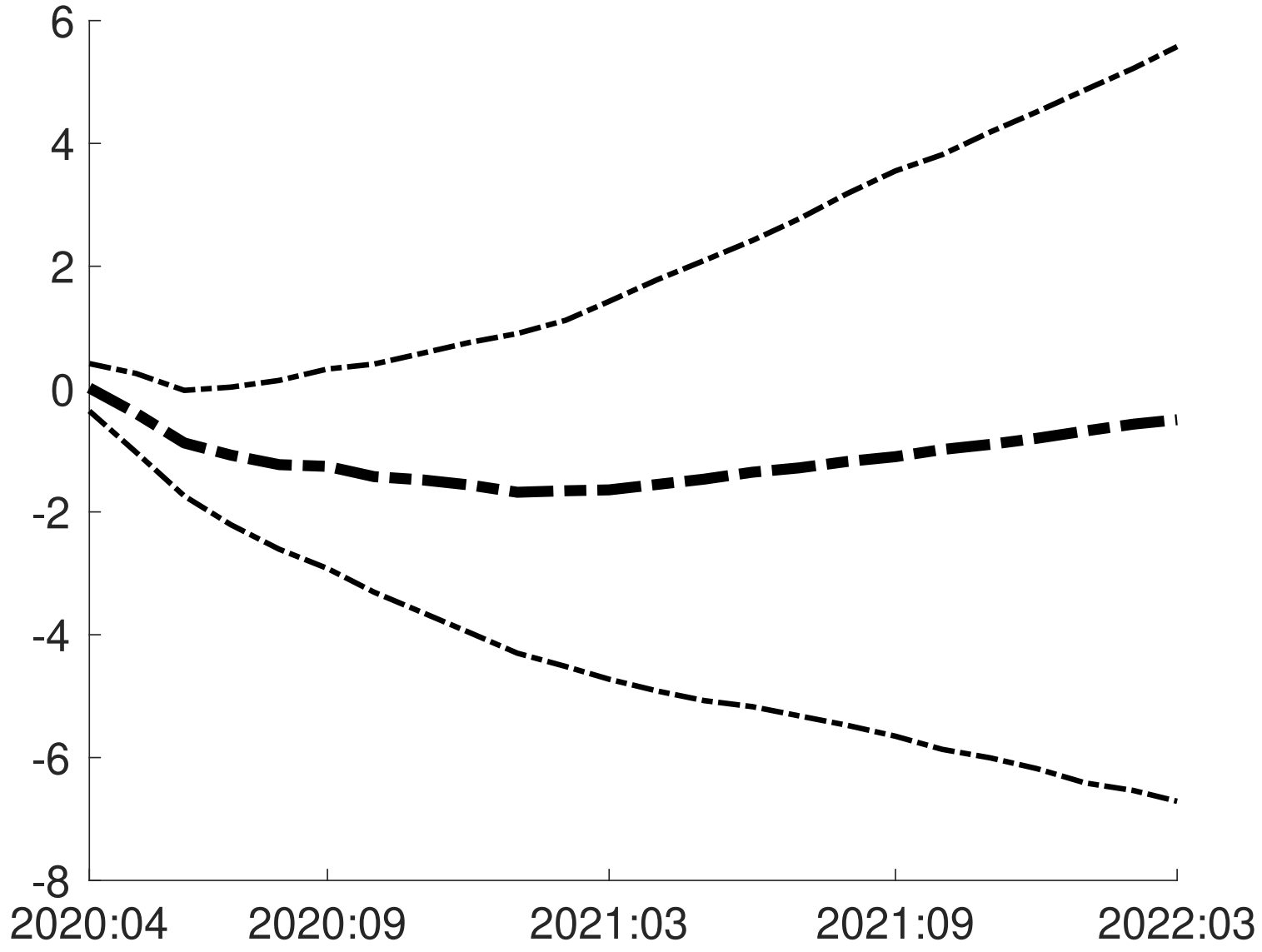
Some reactions to major balance sheet events

AGENDA

1. Shadow-rate VAR concept
2. Estimation method
3. **Shadow rate estimates and macro predictions**
   - Shadow rate estimates
   - Interest rate predictions
   - Expected paths of short rates
   - Forecast comparison for macro and financial variables
4. SVAR responses to financial conditions shock
5. Conclusions and additional results

STANDARD VAR: FUNDS RATE PREDICTIONS

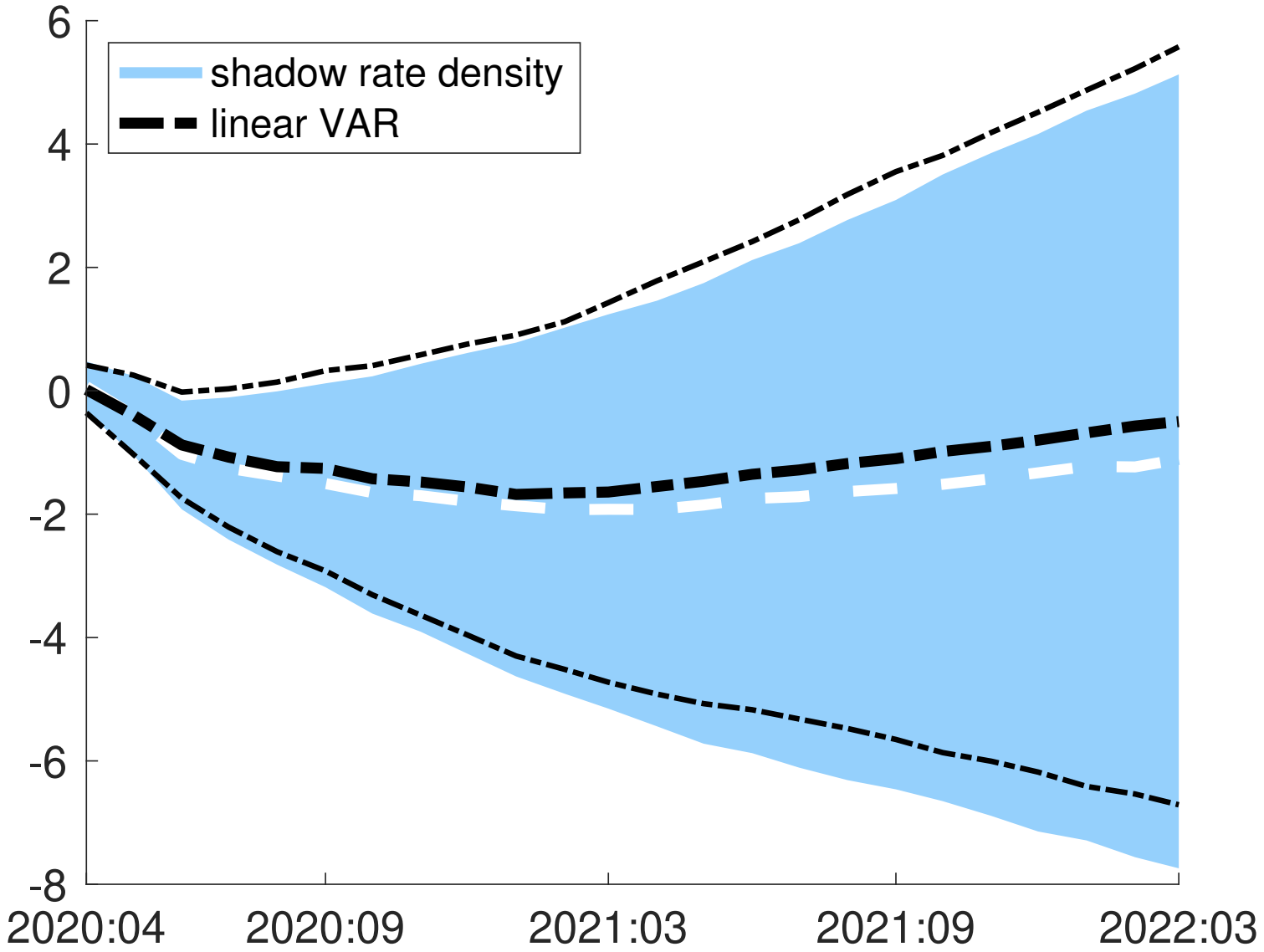
2020:03



Median and 68% bands of predictive densities.

STANDARD VS SHADOW-RATE PREDICTIONS

2020:03

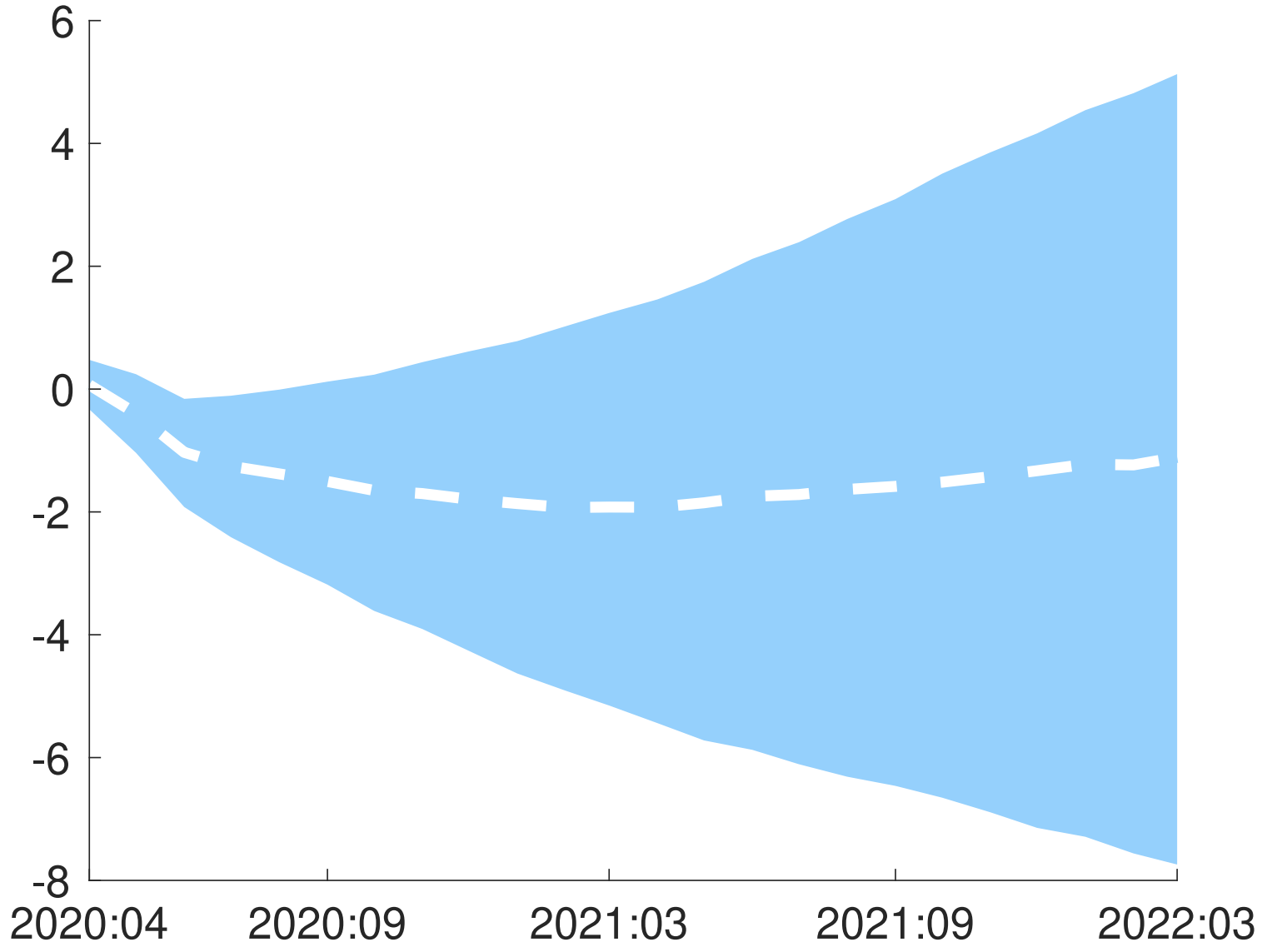


Median and 68% bands of predictive densities.

SHADOW-RATE VAR PREDICTIONS

2020:03

Shadow-rate density (light blue)

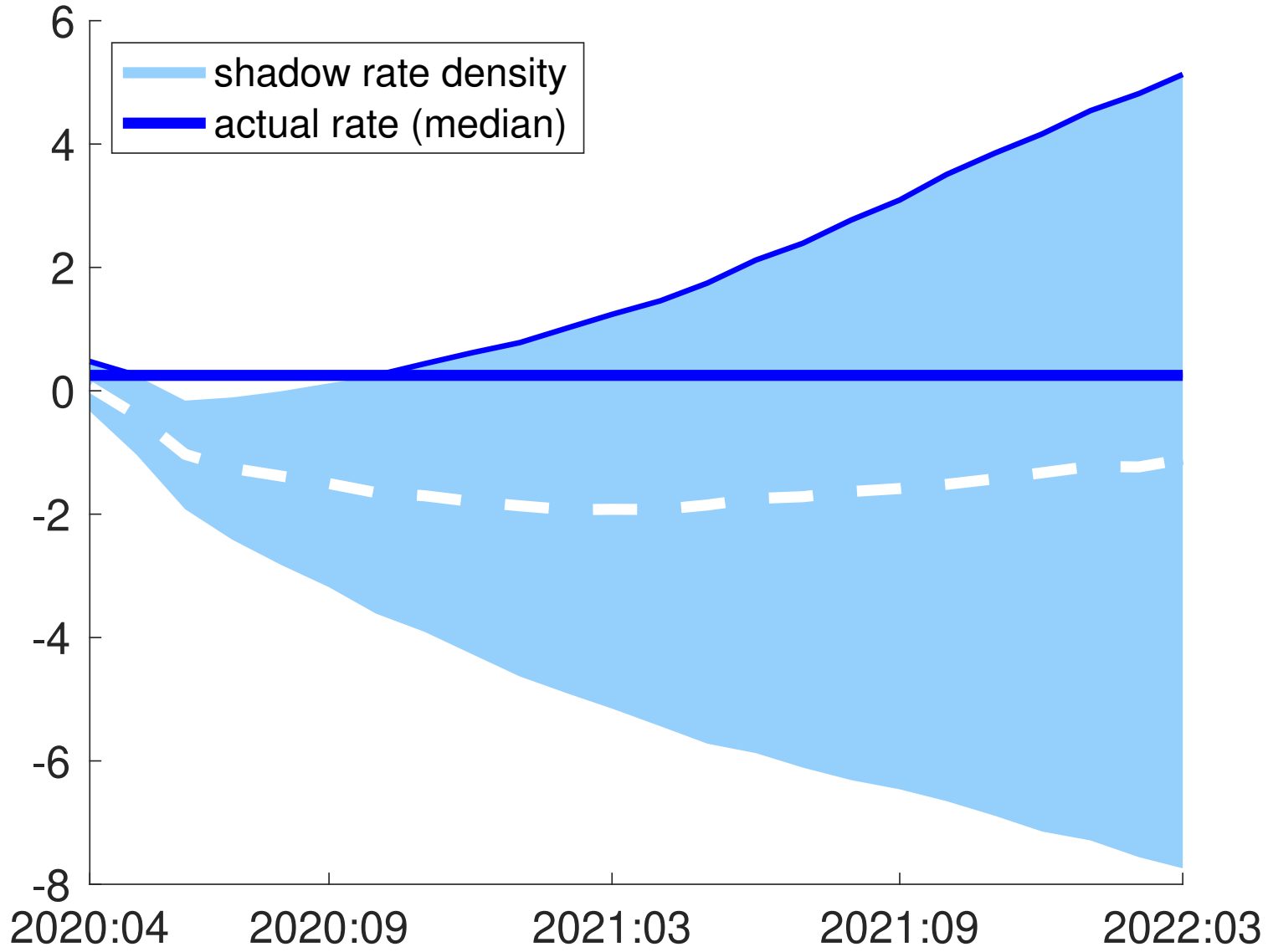


Medians and 68% bands of predictive densities.

SHADOW-RATE VAR PREDICTIONS

2020:03

Shadow-rate (light blue) and actual-rate (dark blue) densities

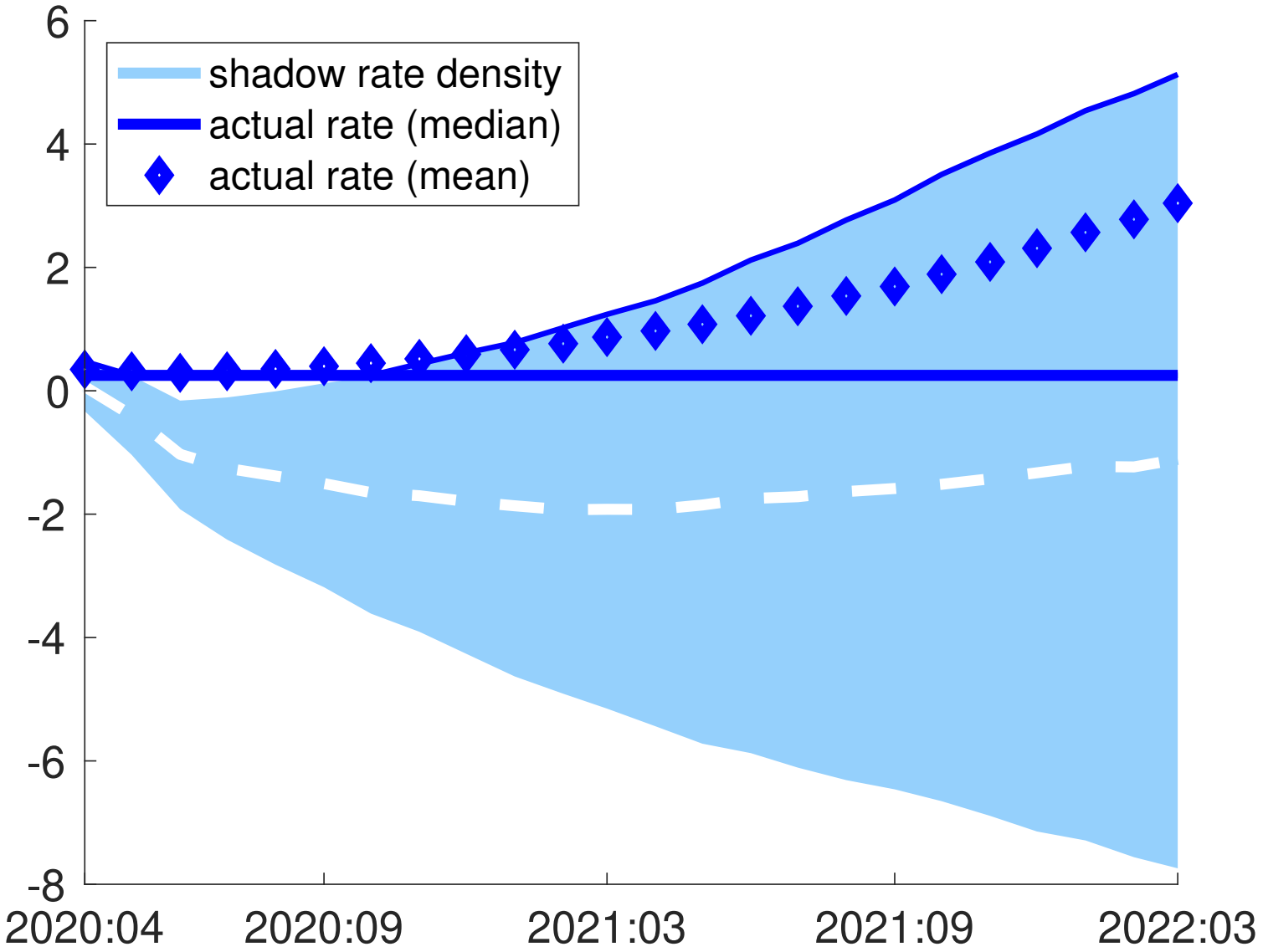


Medians and 68% bands of predictive densities.

SHADOW-RATE VAR PREDICTIONS

2020:03

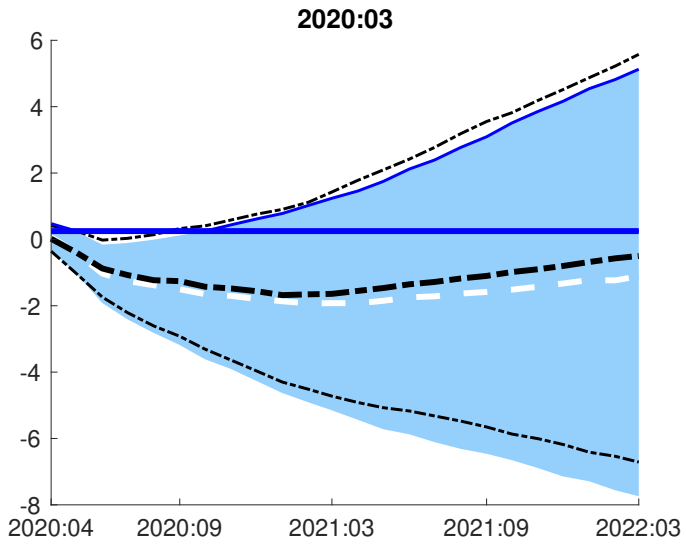
Shadow-rate (light blue) and actual-rate (dark blue) densities



Medians and 68% bands of predictive densities.

FFR PREDICTIONS SINCE COVID-19

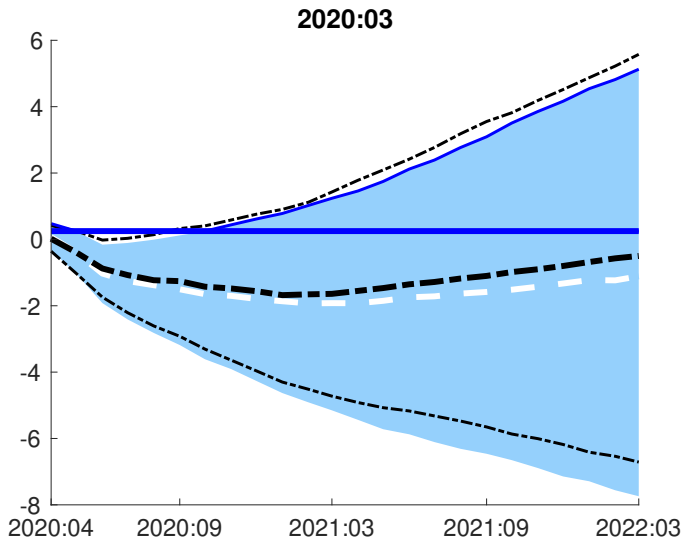
Shadow rate (light blue), actual rate (dark blue), linear VAR (black)

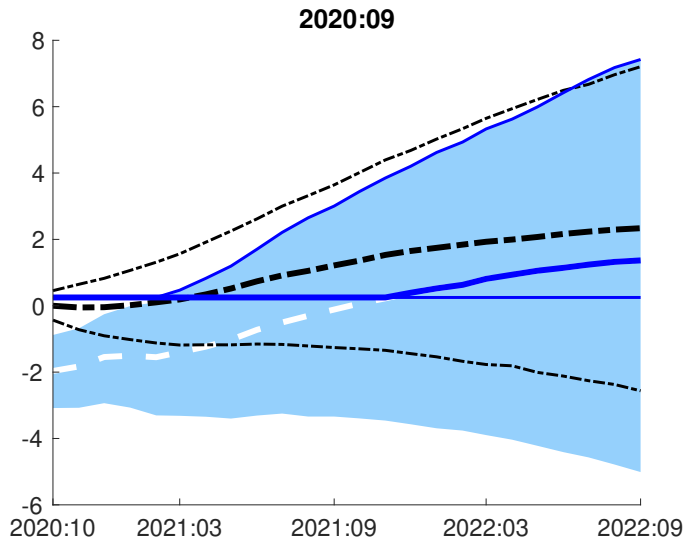


Medians and 68% bands of predictive densities.

FFR PREDICTIONS SINCE COVID-19

Shadow rate (light blue), actual rate (dark blue), linear VAR (black)

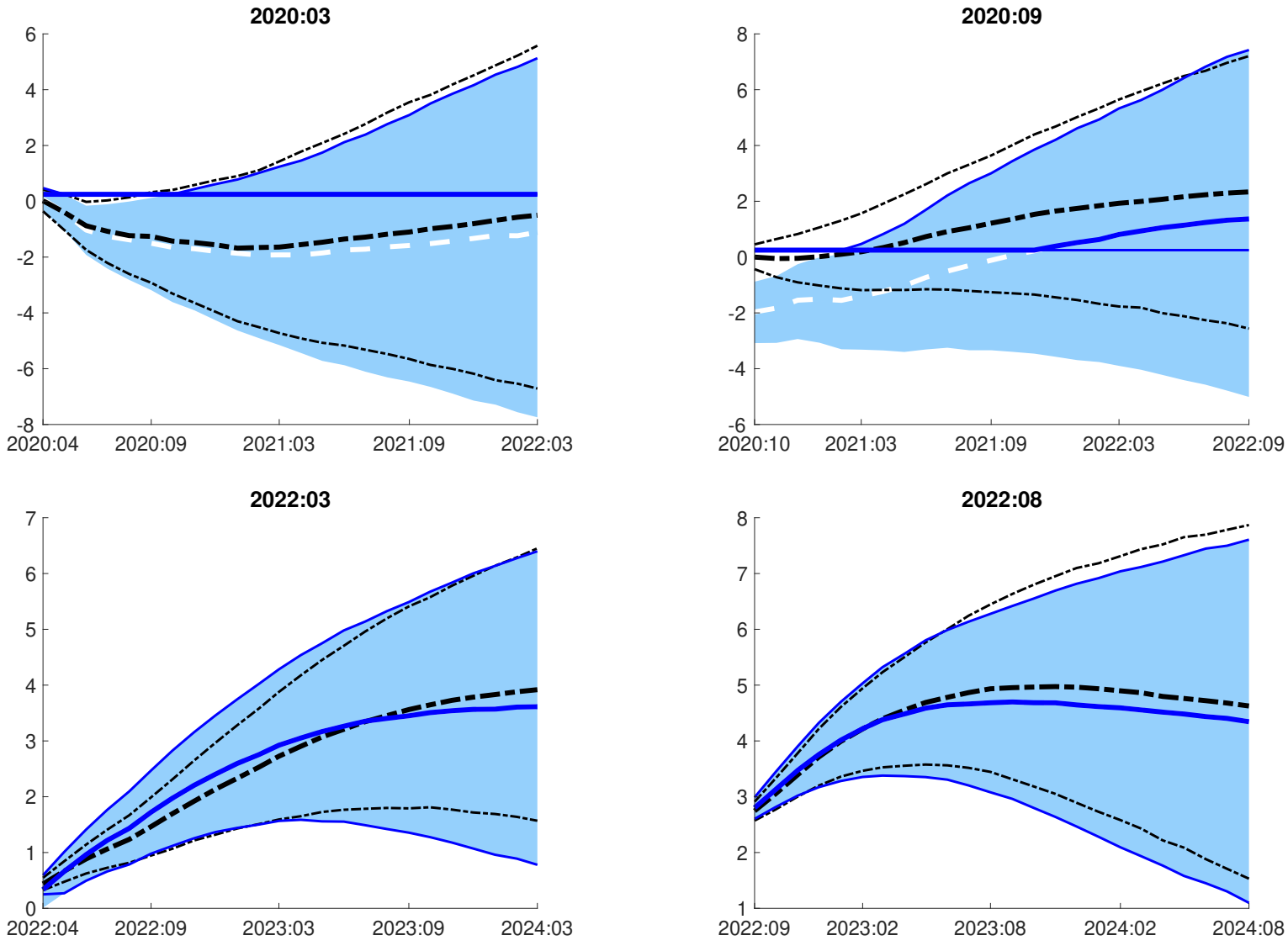




Medians and 68% bands of predictive densities.

FFR PREDICTIONS SINCE COVID-19

Shadow rate (light blue), actual rate (dark blue), linear VAR (black)



Medians and 68% bands of predictive densities.

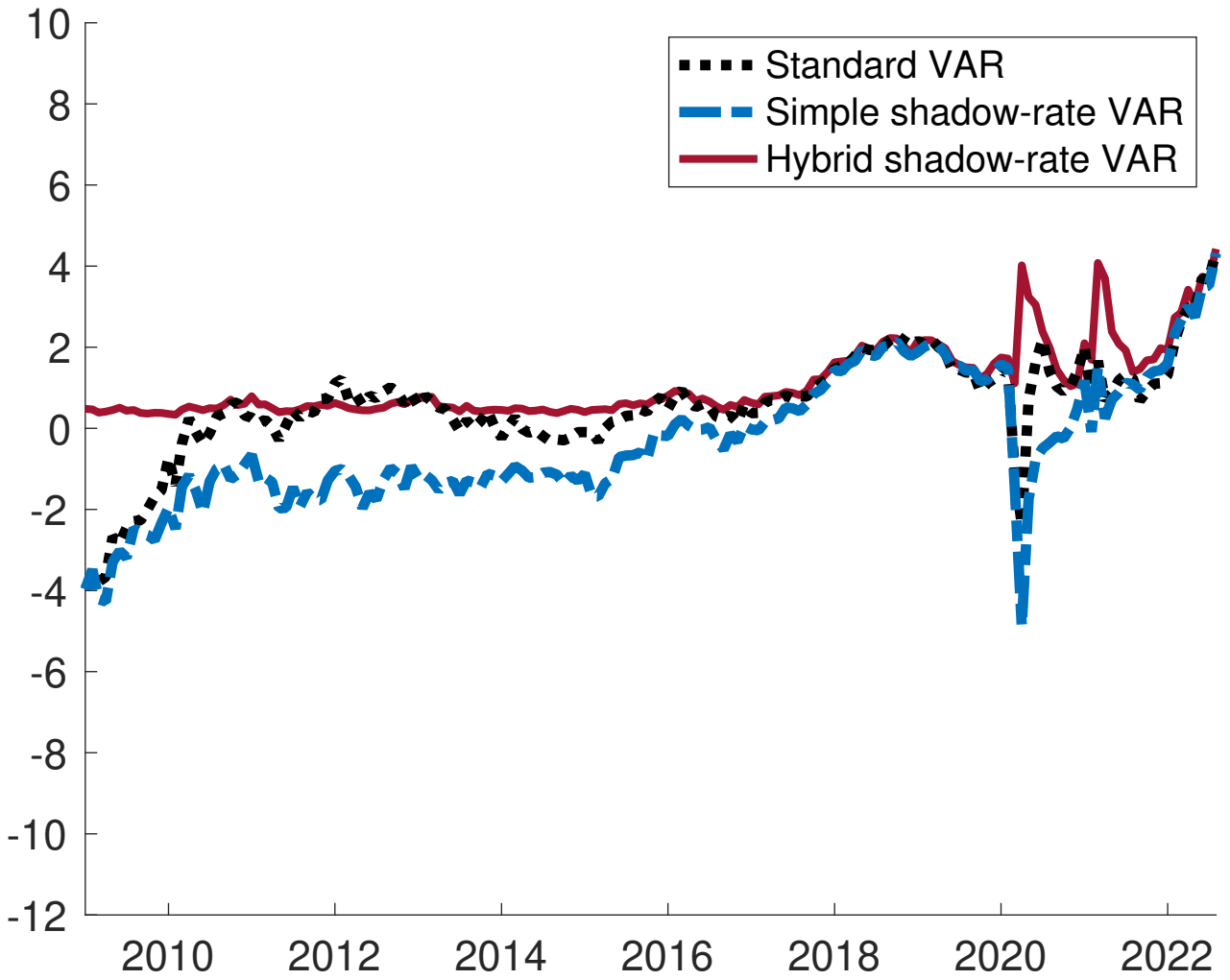
AGENDA

1. Shadow-rate VAR concept
2. Estimation method
3. **Shadow rate estimates and macro predictions**
   - Shadow rate estimates
   - Interest rate predictions
   - Expected paths of short rates
   - Forecast comparison for macro and financial variables
4. SVAR responses to financial conditions shock
5. Conclusions and additional results

SHORT-RATE EXPECTATIONS

FEDFUNDS

Average two-year-ahead (shadow-)interest rates at different forecast origins



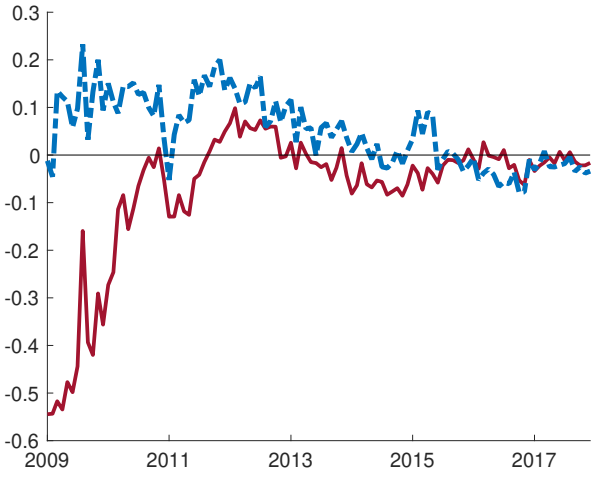
Stronger stimulus predicted by simple shadow-rate VAR, and linear VAR 2008-11

DIFFERENCES IN FORECASTS BETWEEN MODELS

Simple (blue) and hybrid (red) shadow-rate VARs relative to linear, $h = 24$

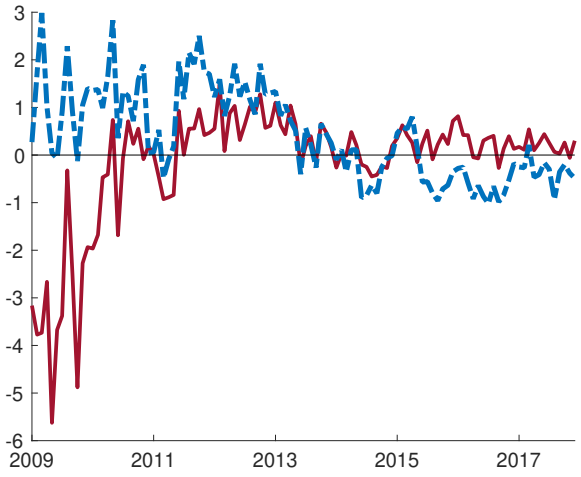
Housing starts

predictive means



Capacity utilization

predictive means



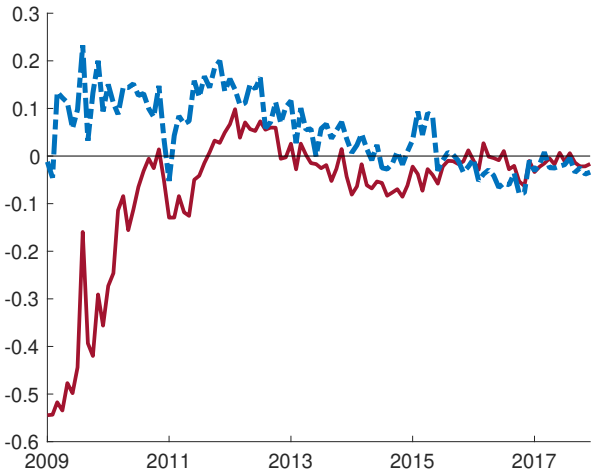
Hybrid VAR (red) predicts lower stimulus and lower activity after GFC

DIFFERENCES IN FORECASTS BETWEEN MODELS

Simple (blue) and hybrid (red) shadow-rate VARs relative to linear, $h = 24$

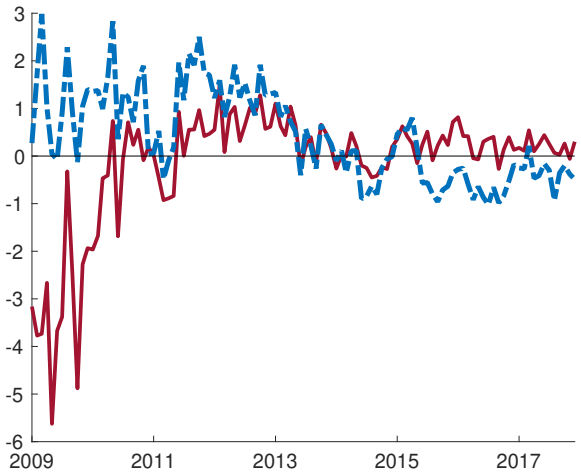
Housing starts

predictive means

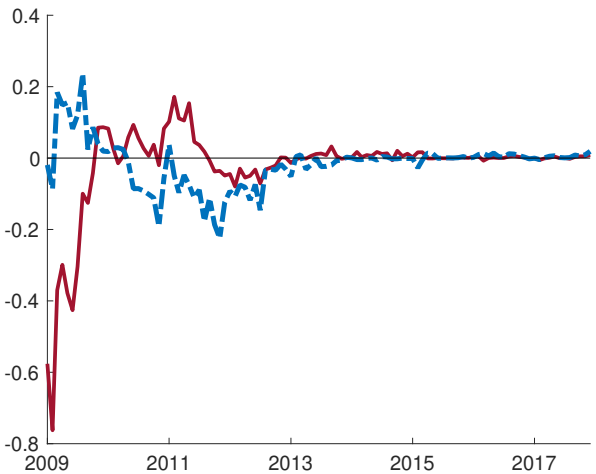


Capacity utilization

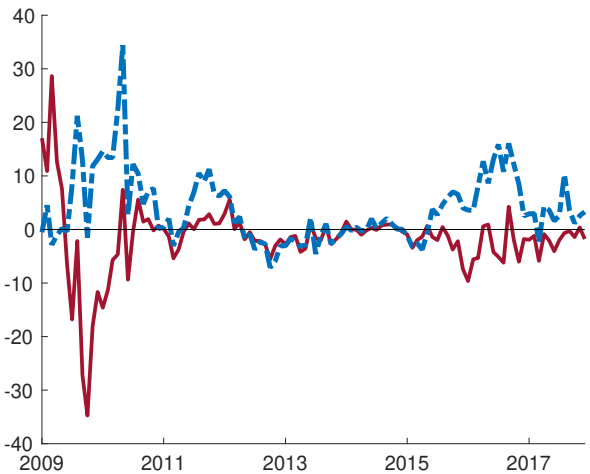
predictive means



squared errors



squared errors



Hybrid VAR (red) predicts lower stimulus and lower activity after GFC, which leads to lower squared errors

AGENDA

1. Shadow-rate VAR concept
2. Estimation method
3. **Shadow rate estimates and macro predictions**
   - Shadow rate estimates
   - Interest rate predictions
   - Expected paths of short rates
   - Forecast comparison for macro and financial variables
4. SVAR responses to financial conditions shock
5. Conclusions and additional results

HYBRID VS STANDARD VAR

Values below one indicate improvement of hybrid over standard VAR

	RMSE			MAE			CRPS		
Var. / Hor.	3	12	24	3	12	24	3	12	24
Income									
Consumption									
IP									
Cap. Util.									
Unemp.									
Nfm Pyrlls									
Hours									
H. Earnings									
PPI (Fin.)									
PPI (Metals)									
PCE Prices									
Hsng Strts									
S&P 500									
USD / GBP									
FFR									
6m Tbill									
1y Trsy									
5y Trsy									
10y Trsy									
BAA Yld									

Note: Stars denote DMW significance. Eval from 2009:01 through 2017:12.

HYBRID VS STANDARD VAR

Values below one indicate improvement of hybrid over standard VAR

	RMSE			MAE			CRPS		
Var. / Hor.	3	12	24	3	12	24	3	12	24
Income									
Consumption									
IP									
Cap. Util.									
Unemp.									
Nfm Pyrlls									
Hours									
H. Earnings									
PPI (Fin.)									
PPI (Metals)									
PCE Prices									
Hsng Strts									
S&P 500									
USD / GBP									
FFR	0.21^{**}	0.29	0.51	0.13^{***}	0.28^{*}	0.38^{**}	0.15^{***}	0.25^{**}	0.39^{***}
6m Tbill	0.36^{*}	0.51	0.70	0.24^{***}	0.35^{**}	0.47^{***}	0.26^{***}	0.40^{**}	0.50^{***}
1y Trsy	0.63^{*}	0.66	0.78	0.51^{***}	0.44^{**}	0.55^{***}	0.52^{***}	0.52^{***}	0.56^{***}
5y Trsy	0.94	0.80^{***}	0.75^{*}	0.95	0.89	0.87^{*}	0.97	0.87^{**}	0.82^{***}
10y Trsy	0.96	0.86^{**}	0.77^{***}	0.96	0.87^{*}	0.93	0.97	0.91	0.93
BAA Yld	0.97	0.99	0.95	0.98	1.00	1.04	0.99	1.01	1.06

Note: Stars denote DMW significance. Eval from 2009:01 through 2017:12.

HYBRID VS STANDARD VAR

Values below one indicate improvement of hybrid over standard VAR

	RMSE			MAE			CRPS		
Var. / Hor.	3	12	24	3	12	24	3	12	24
Income	1.00	1.01	0.70	0.99	1.01	0.96	1.00	1.01*	0.99
Consumption	1.00	0.99	0.89*	0.99	1.02	0.90*	1.00	1.00	0.97*
IP	1.01	1.01	0.96	1.02*	1.01	0.97	1.01	1.01	1.00
Cap. Util.	1.01	1.01	0.94**	1.00	1.00	0.94*	1.00	1.01	0.96*
Unemp.	1.00	1.01	0.92***	1.00	1.00	0.92*	1.00	1.00	0.95***
Nfm Pyrlls	0.99	1.06	0.99	0.99	1.01	0.92	0.99	1.02	0.98
Hours	1.02**	1.03	1.03	1.01	1.03	1.04	1.01***	1.02	1.02
H. Earnings	1.01	1.01	1.03***	1.00	1.01	1.02*	1.01*	1.01	1.02***
PPI (Fin.)	1.00	0.97	1.00	1.00	0.98	1.01	1.00	0.98	1.00
PPI (Metals)	0.99	0.99*	1.00	1.00	0.99	0.99	1.00	0.99	1.00
PCE Prices	1.00	0.97	1.06*	1.00	0.98	1.08***	1.00	0.99	1.05***
Hsng Strts	1.00	0.90	0.90	1.01	0.91	0.90	1.01	0.94	0.93
S&P 500	1.00	1.02**	1.09**	0.99**	1.01	1.02*	1.00	1.01*	1.01***
USD / GBP	1.00	1.00	1.01	1.00	1.01	1.05	1.00	1.00	1.01
FFR	0.21**	0.29	0.51	0.13***	0.28*	0.38**	0.15***	0.25**	0.39***
6m Tbill	0.36*	0.51	0.70	0.24***	0.35**	0.47***	0.26***	0.40**	0.50***
1y Trsy	0.63*	0.66	0.78	0.51***	0.44**	0.55***	0.52***	0.52***	0.56***
5y Trsy	0.94	0.80***	0.75*	0.95	0.89	0.87*	0.97	0.87**	0.82***
10y Trsy	0.96	0.86**	0.77***	0.96	0.87*	0.93	0.97	0.91	0.93
BAA Yld	0.97	0.99	0.95	0.98	1.00	1.04	0.99	1.01	1.06

Note: Stars denote DMW significance. Eval from 2009:01 through 2017:12.

SUMARY OF FORECAST RESULTS

Compared to the standard VAR, the hybrid . . .

- . . . delivers **superior interest-rate forecasts**
- . . . with **some gains for macro** variables

Also: Hybrid VAR . . .

- . . . **does better than simple shadow-rate VAR** (which generates somewhat poorer macro forecasts than the standard VAR)
- . . . **beats plug-in VAR** (with Wu-Xia or Krippner rates)
- . . . **censored predictions from linear VAR** expect ELB departure too early

AGENDA

1. Shadow-rate VAR concept
2. Estimation method
3. Shadow rate estimates and macro predictions
4. **SVAR responses to financial conditions shock**
5. Conclusions and additional results

Responses to Gilchrist-Zakrajsek EBP shock

- EBP added at top of VAR
- At MCMC draw m:

$$\Psi_{t,h}^{m} \equiv \boldsymbol{E}_{t}^{m}\left(\boldsymbol{y}_{t+h} \mid \tilde{\varepsilon}_{t+1}^{EBP} = \varepsilon_{t+1}^{EBP} + \sigma\right) - \boldsymbol{E}_{t}^{m}\left(\boldsymbol{y}_{t+h}\right)$$

  while simulating all other shocks from baseline density
- Full-sample parameters, SV simulated at jump off

Responses to Gilchrist-Zakrajsek EBP shock

- EBP added at top of VAR
- At MCMC draw m:

$$\Psi_{t,h}^{m} \equiv \boldsymbol{E}_{t}^{m}\left(\boldsymbol{y}_{t+h} \mid \tilde{\varepsilon}_{t+1}^{EBP} = \varepsilon_{t+1}^{EBP} + \sigma\right) - \boldsymbol{E}_{t}^{m}\left(\boldsymbol{y}_{t+h}\right)$$

  while simulating all other shocks from baseline density
- Full-sample parameters, SV simulated at jump off
- Mean IRF integrates over m: $\hat{\Psi}_{t,h} = \sum_{m} \Psi_{t,h}^{m}$
- Uncertainty bands reflect distribution of $\Psi_{t,h}^{m}$ across m

SVAR SETUP

IRF of Goncalves et al (2021/23)

Responses to Gilchrist-Zakrajsek EBP shock

- EBP added at top of VAR
- At MCMC draw m:

$$\Psi_{t,h}^{m} \equiv \boldsymbol{E}_{t}^{m}\left(\boldsymbol{y}_{t+h} \mid \tilde{\varepsilon}_{t+1}^{EBP} = \varepsilon_{t+1}^{EBP} + \sigma\right) - \boldsymbol{E}_{t}^{m}\left(\boldsymbol{y}_{t+h}\right)$$

  while simulating all other shocks from baseline density
- Full-sample parameters, SV simulated at jump off
- Mean IRF integrates over m: $\hat{\Psi}_{t,h} = \sum_{m} \Psi_{t,h}^{m}$
- Uncertainty bands reflect distribution of $\Psi_{t,h}^{m}$ across m

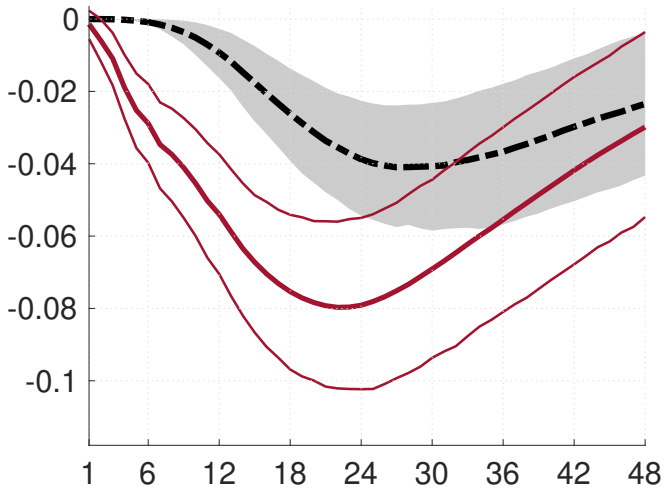
Hybrid VAR generates differences at/away from ELB

- Directly on interest rates
- Indirectly on all variables via lagged i_t

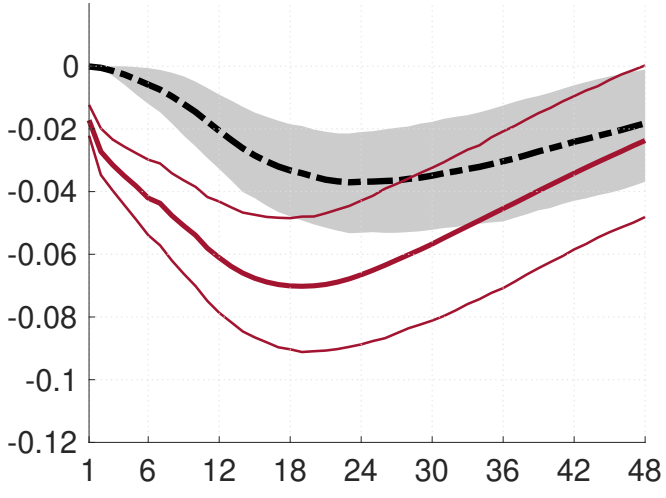
IRF: 2006 vs 2012

Away from ELB (red, 2006 Dec) vs at ELB (black, 2012 Dec)

Fed Funds Rate



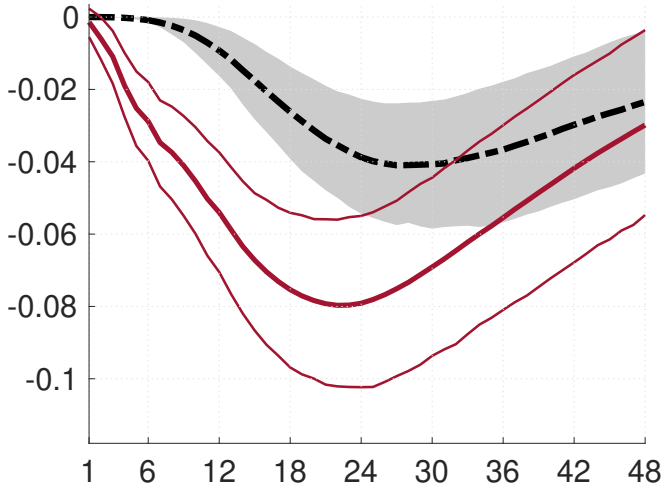
1-year yield



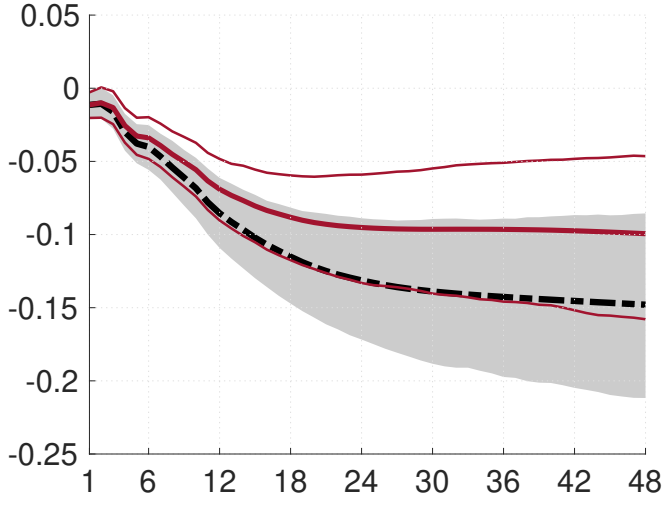
IRF: 2006 vs 2012

Away from ELB (red, 2006 Dec) vs at ELB (black, 2012 Dec)

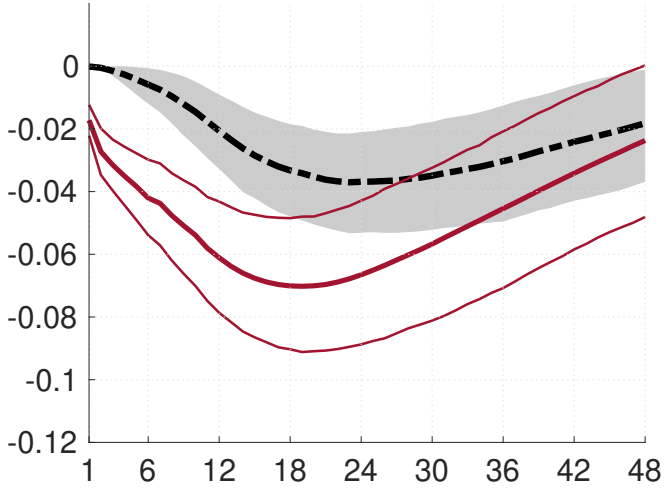
Fed Funds Rate



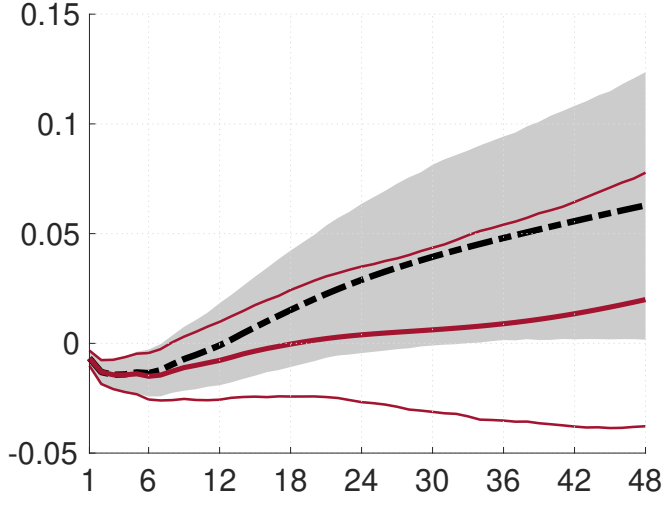
Consumption



1-year yield



PCE prices



SVAR SUMMARY

In response to adverse shock to financial conditions . . .

- Similar to Gilchrist et al (2019), Arouba et al (2021)
- **Pre 2008:** (G)IRF of hybrid **similar to linear** VAR
- **Persistence** of shadow rates **prolongs stay at ELB**
- **Negative nominal-rate responses** obey the ELB, and **reflect path running below baseline** forecast
- **Macro responses affected by prolonged expectation** of actual rates at ELB
  - Differ between origins at or away from ELB
  - For given origin t: (almost) linear and symmetric

AGENDA

1. Shadow-rate VAR concept
2. Estimation method
3. Shadow rate estimates and macro predictions
4. SVAR responses to financial conditions shock
5. **Conclusions and additional results**

ADDITIONAL RESULTS

Time-invariant VAR parameters

- Linear VAR exhibits significant drift in VAR parameters (in particular for interest rate equations)
- Shadow-rate VAR does not (except for COVID-19)
- Poor performance when shock impact matrix $\boldsymbol{A}$ allowed to switch at ELB

Estimates w/o yields (other than FFR)

- on balance, poorer macro forecasts
- deeper, and more uncertain shadow rate path
- IRF qualitatively similar

Alternative ELB values

- 12.5bp: Similar (albeit limited) differences as w/25bp
- 50bp: problematic for hybrid (b/o assumed 50bp path)

CONCLUSIONS

Our solution to handle ELB: Shadow-rate VARs

- **Internally consistent** inference
- **Scalable** to multiple interest-rate maturities at ELB
- **Hybrid VAR** conditions macro variables on actual rates

Structural Analysis at/away from ELB:

- strong differences in nominal rate responses
- some differences in macro responses
- consistent w/some effectiveness of unconventional policies

Forecasts for interest rates and macro variables

- **Interest-rate forecasts superior** to standard VAR
- **Hybrid** typically better than simple shadow-rate VAR
- . . . and with **some gains in macro** over linear VAR

APPENDIX

- **Additional shadow rate estimates**
  - Shadow rate estimates for other short rates
  - Shadow rate estimates from hybrid VAR
  - Shadow rate estimates w/o yields
- **Additional forecast comparisons**
- **Shadow-rate vs. missing-data sampling**
- **Stability of VAR coefficients over time**

AGENDA

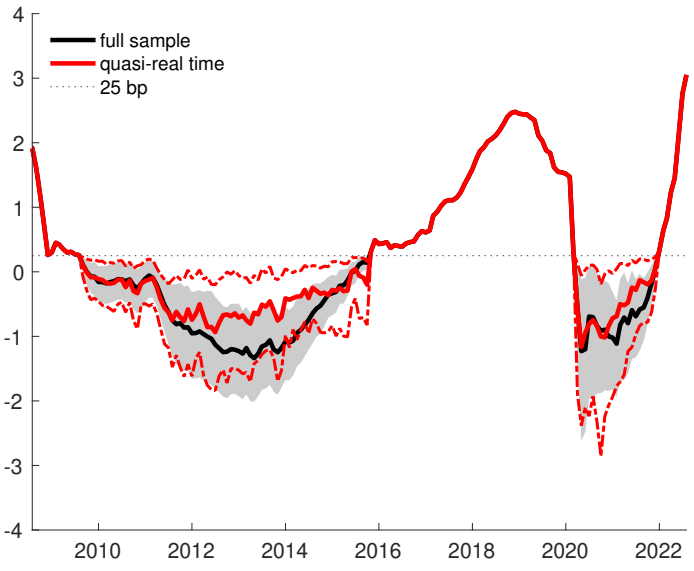
- **Additional shadow rate estimates**
  - Shadow rate estimates for other short rates
  - Shadow rate estimates from hybrid VAR
  - Shadow rate estimates w/o yields
- **Additional forecast comparisons**
- **Shadow-rate vs. missing-data sampling**
- **Stability of VAR coefficients over time**

SHADOW RATE ESTIMATES FOR 6M and 1Y RATES

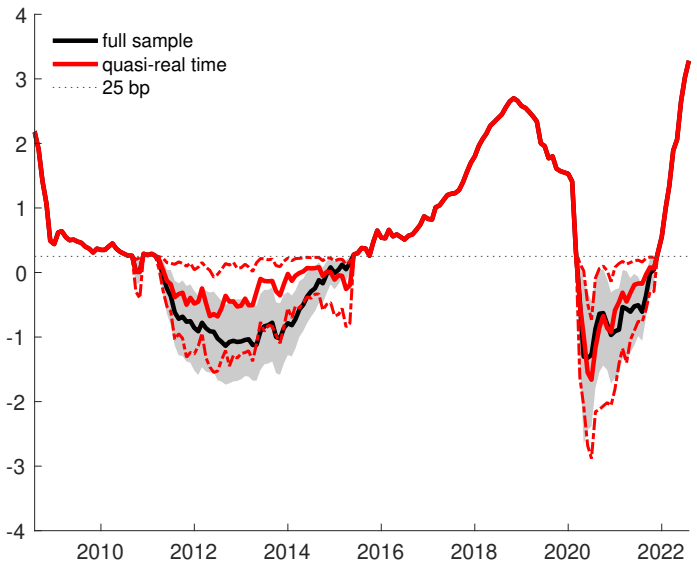
medians and 90% bands, simple shadow-rate VAR

in addition to FFR,
6M Tbill and 1y yield have data at ELB

6m Tbill



1y yield



(similar results with hybrid shadow-rate VAR)

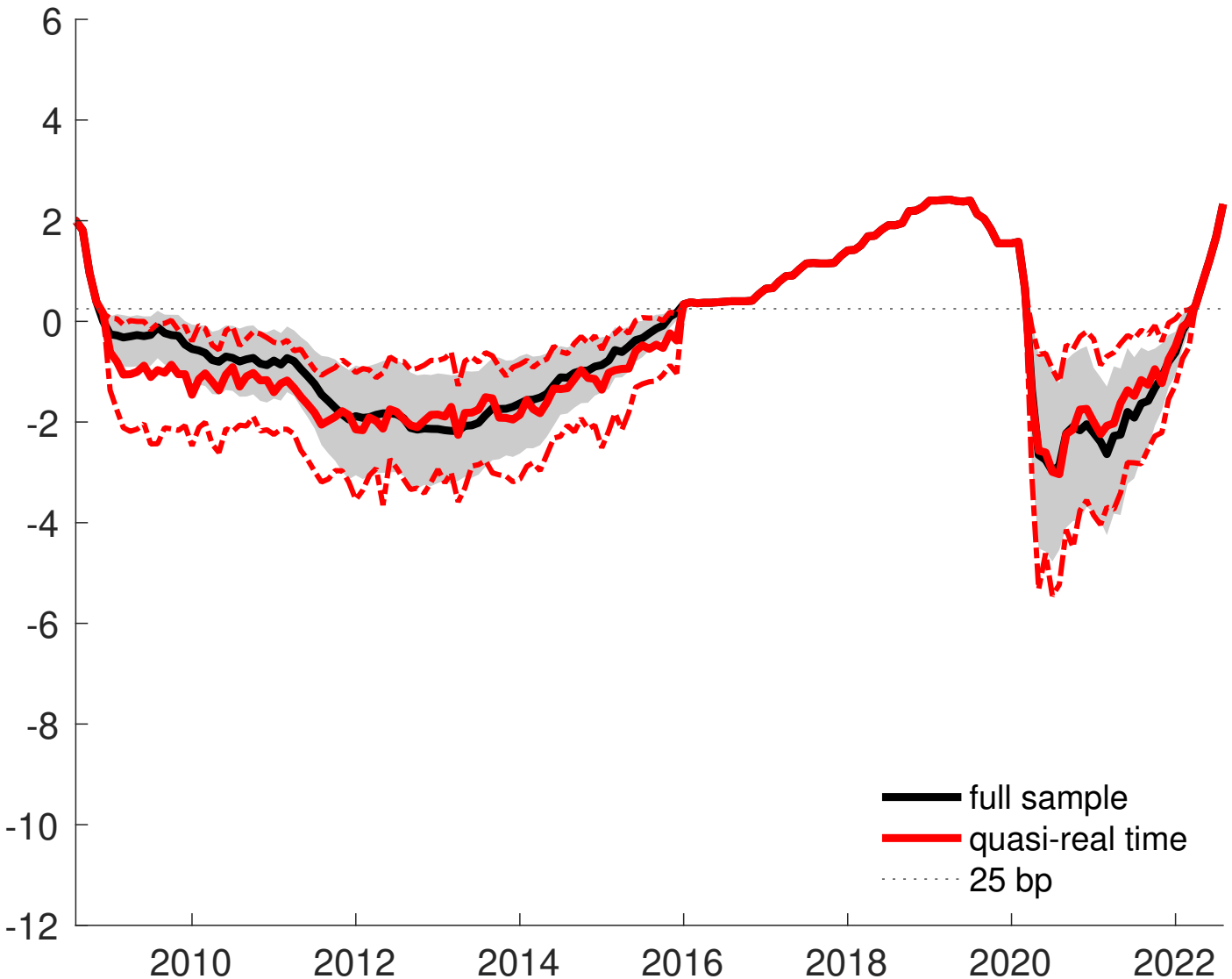
AGENDA

- **Additional shadow rate estimates**
  - Shadow rate estimates for other short rates
  - Shadow rate estimates from hybrid VAR
  - Shadow rate estimates w/o yields
- **Additional forecast comparisons**
- **Shadow-rate vs. missing-data sampling**
- **Stability of VAR coefficients over time**

SHADOW RATE ESTIMATES

FFR

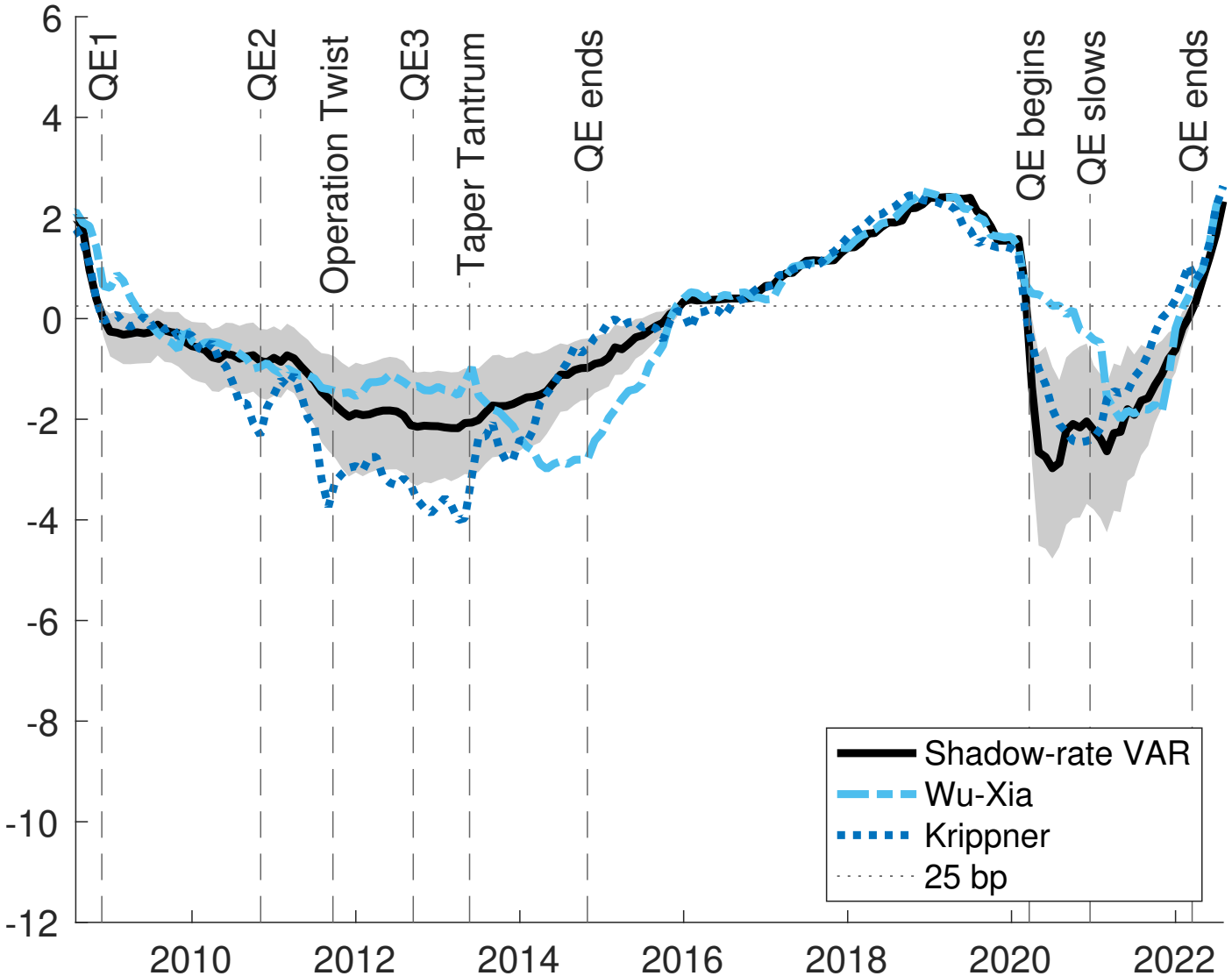
hybrid shadow-rate VAR, full-sample median and 90% bands



hybrid VAR: somewhat more uncertainty in quasi-real time

SHADOW RATE ESTIMATES

hybrid shadow-rate VAR, full-sample median and 90% bands

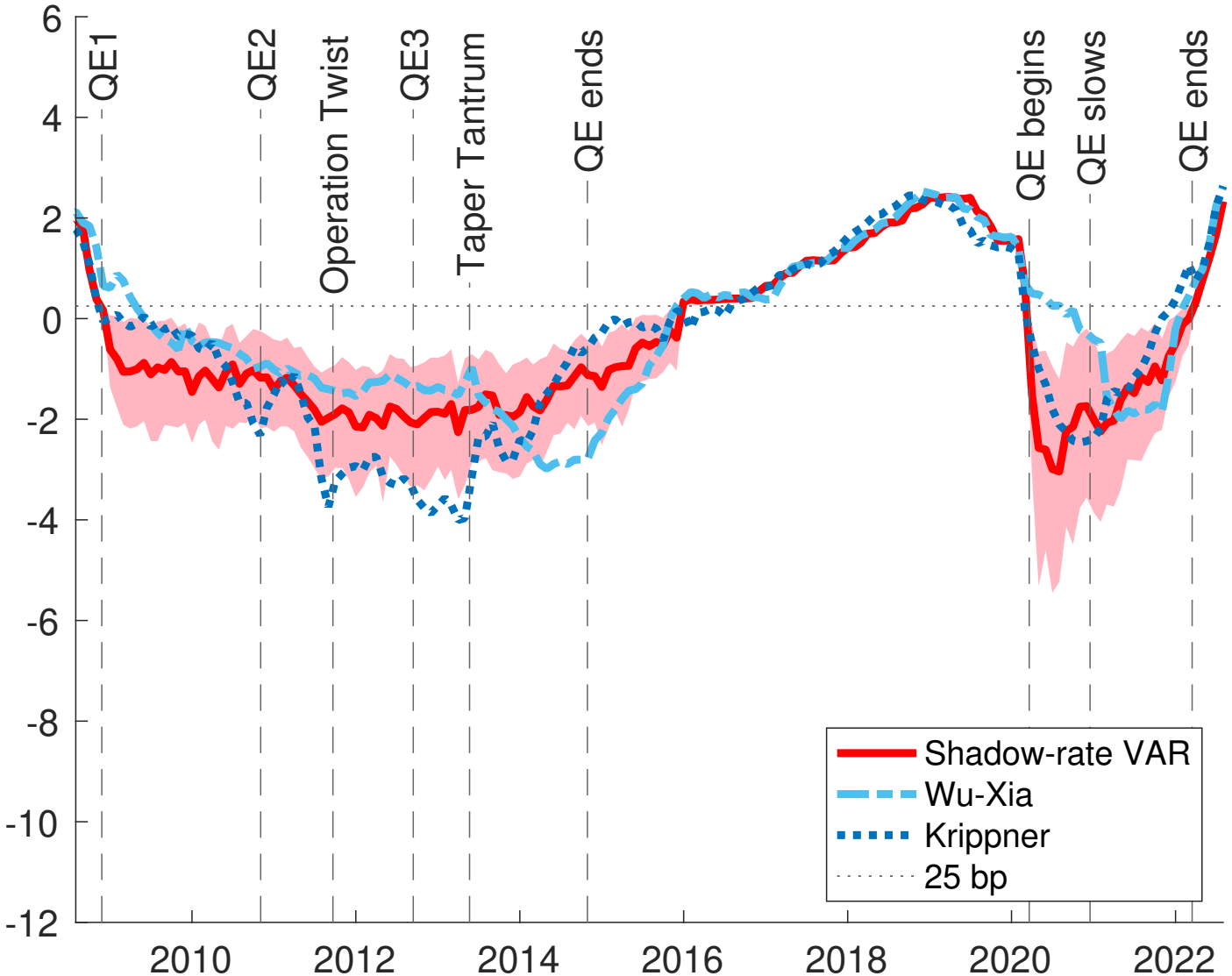


Some reactions to major balance sheet events

SHADOW RATE ESTIMATES

FFR

hybrid shadow-rate VAR in quasi-real time, median and 90% bands



Some reactions to major balance sheet events

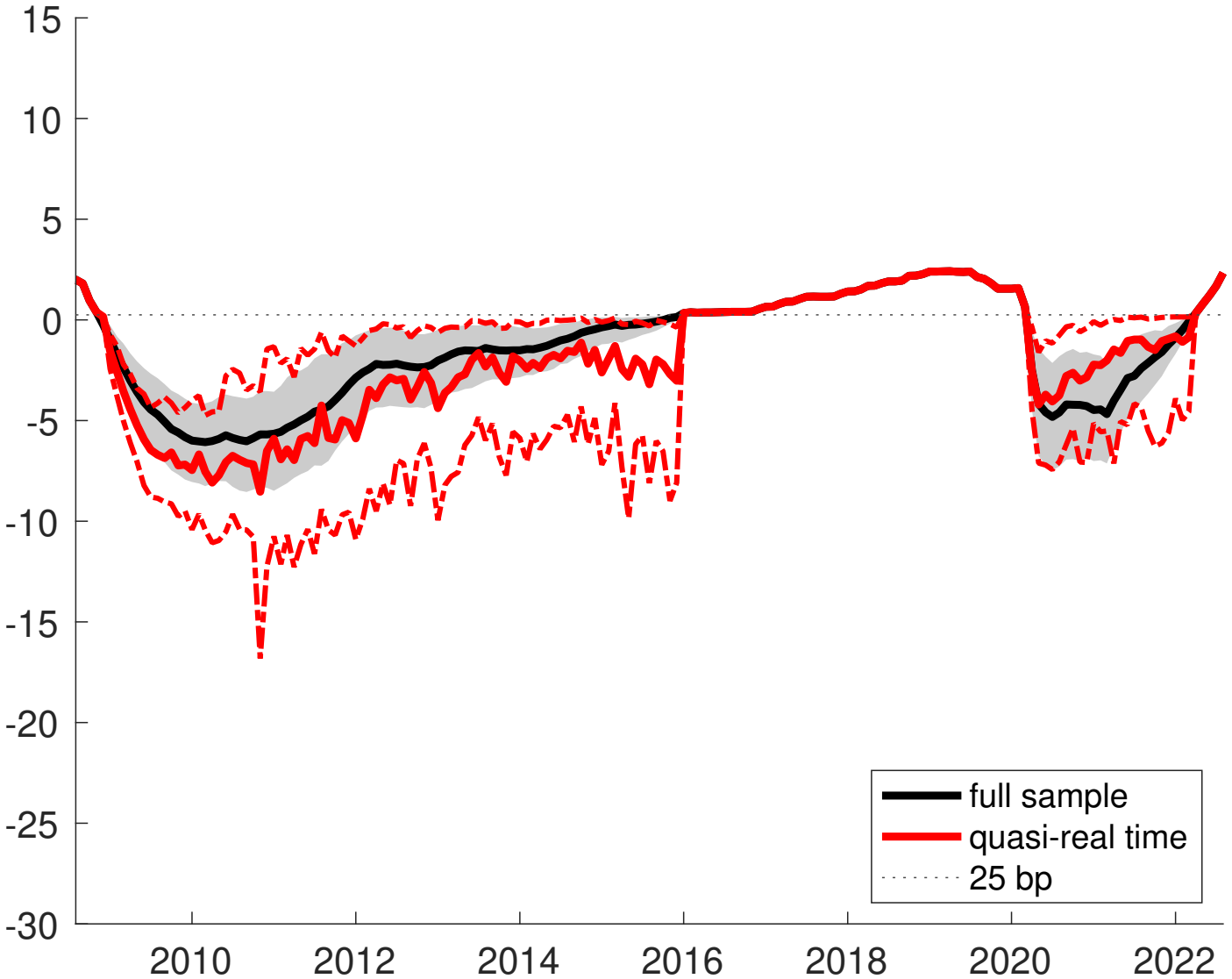
AGENDA

- **Additional shadow rate estimates**
  - Shadow rate estimates for other short rates
  - Shadow rate estimates from hybrid VAR
  - Shadow rate estimates w/o yields
- **Additional forecast comparisons**
- **Shadow-rate vs. missing-data sampling**
- **Stability of VAR coefficients over time**

SHADOW RATE ESTIMATED W/O YIELDS

FFR

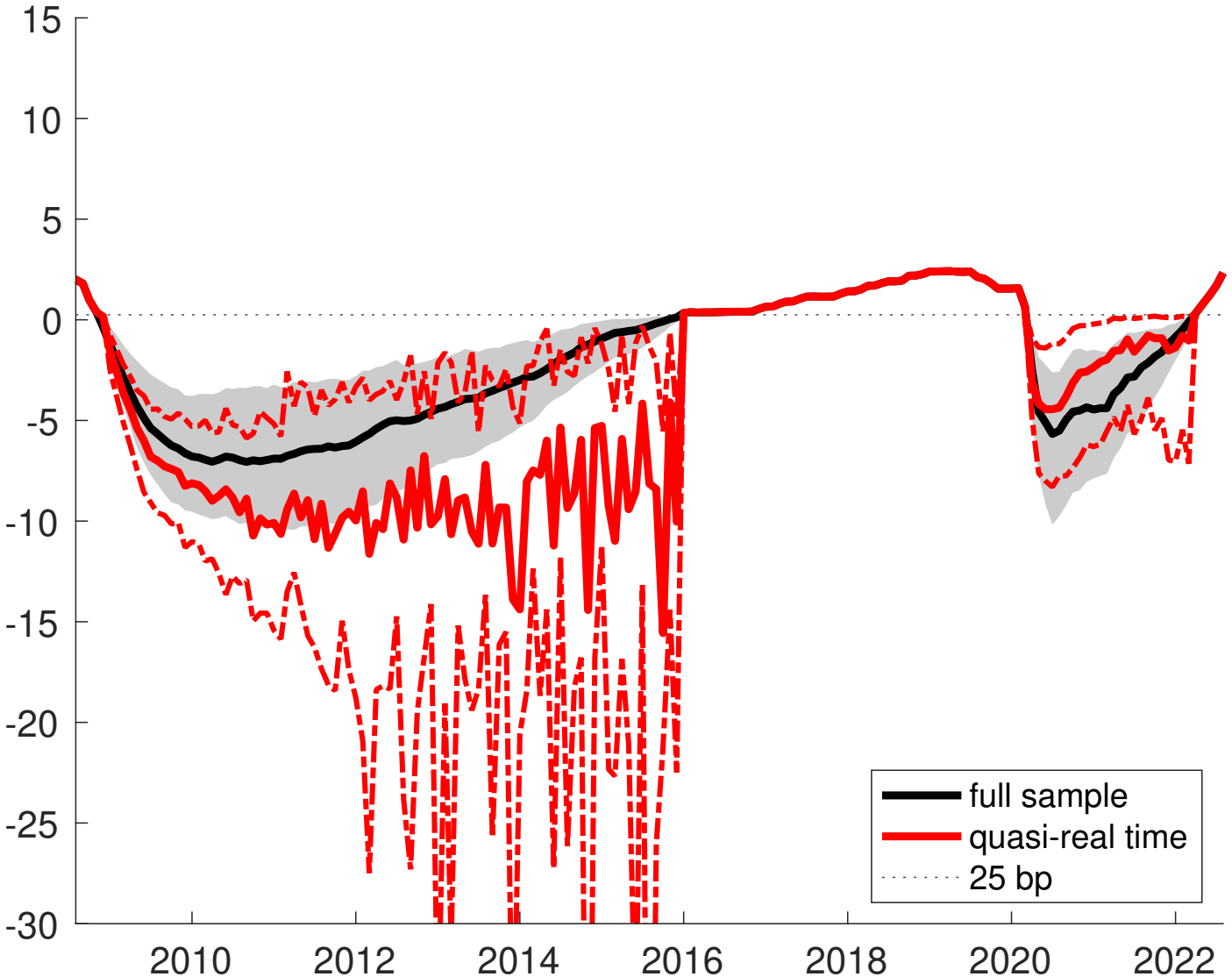
simple shadow-rate VAR, median and 90% bands



more quasi-real-time uncertainty w/o yields

SHADOW RATE ESTIMATED W/O YIELDS

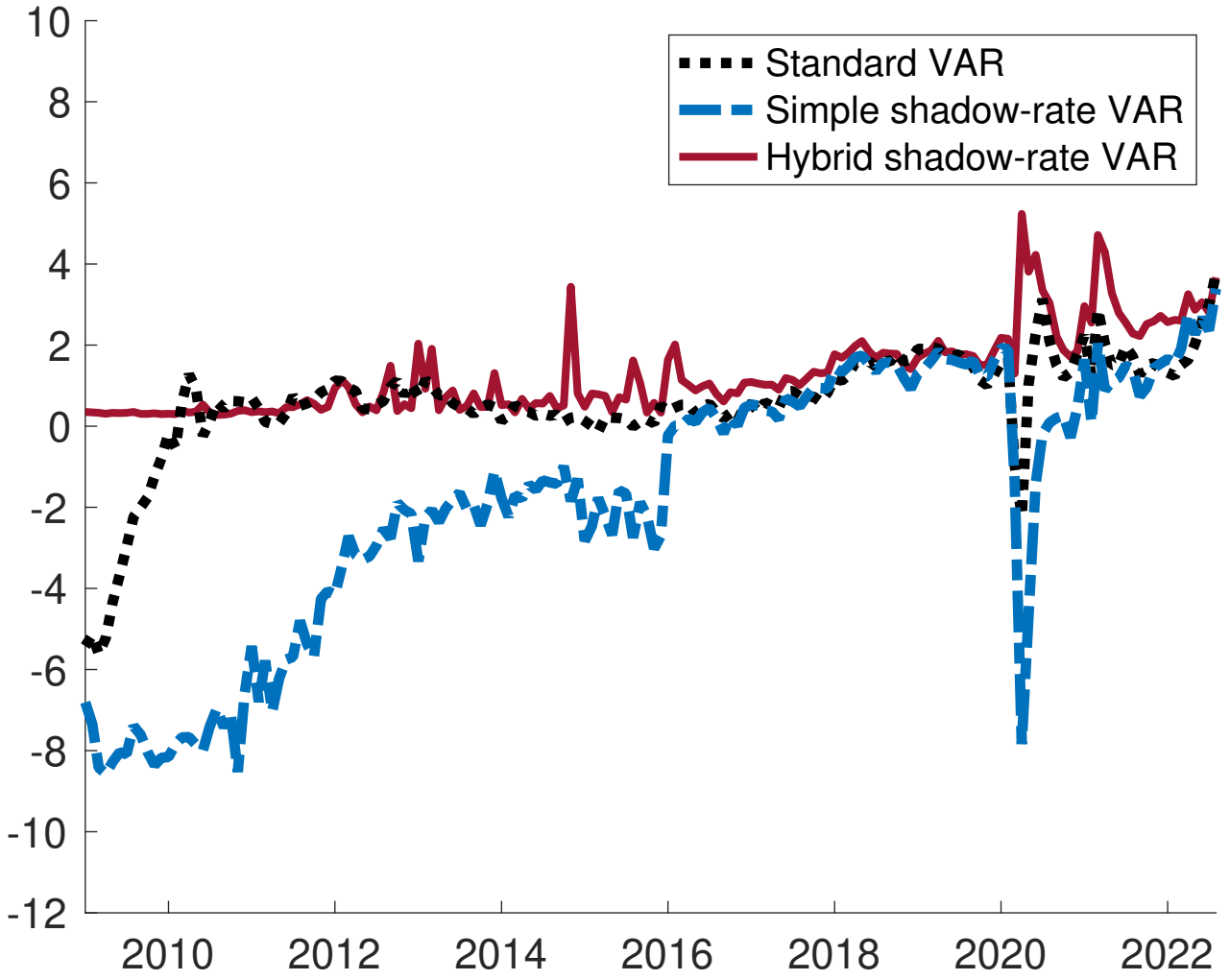
hybrid shadow-rate VAR, median and 90% bands



much more quasi-real-time uncertainty for hybrid w/o yields

SHORT-RATE EXPECTATIONS

Average two-year-ahead (shadow-)interest rates at different forecast origins



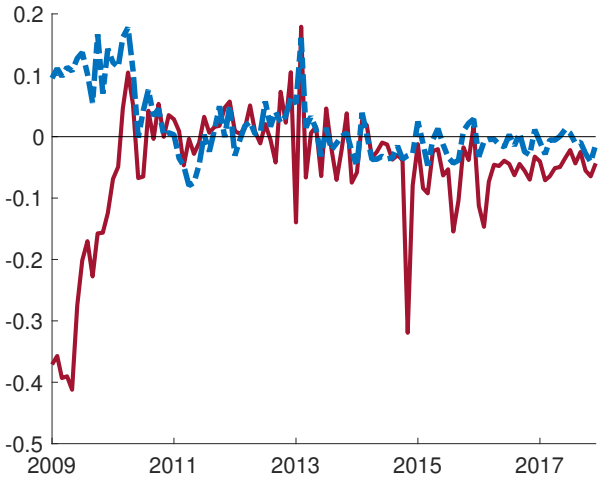
Absent yield data, even stronger stimulus predicted by simple shadow-rate VAR (but not by hybrid VAR)

DIFFERENCES IN FORECASTS

Simple (blue) and hybrid (red) shadow-rate VARs relative to linear, $h = 24$

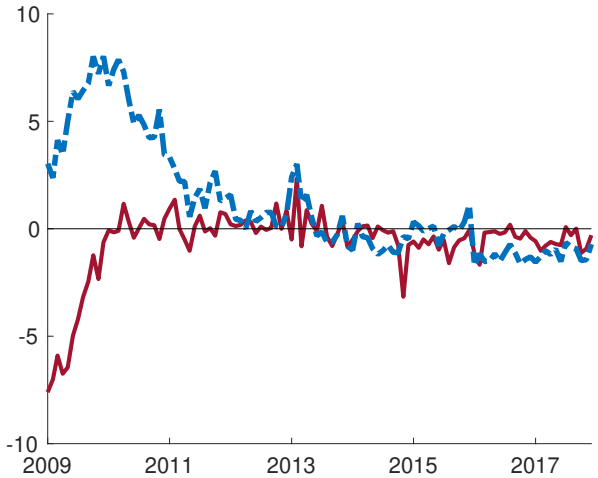
Housing starts

predictive means

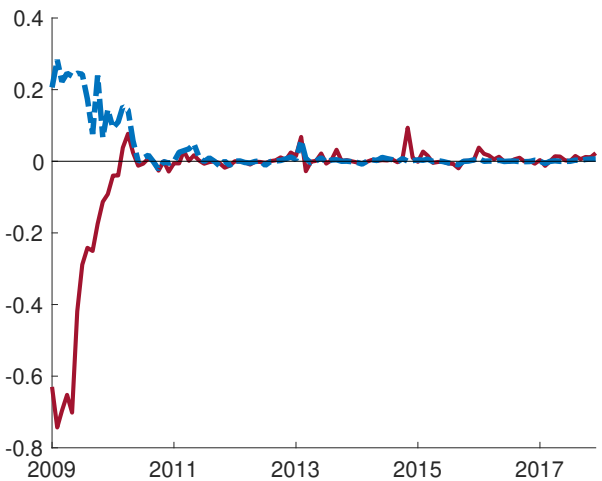


Capacity utilization

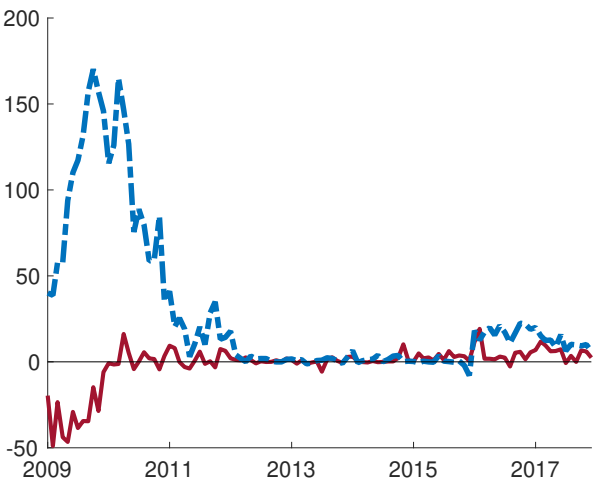
predictive means



squared errors



squared errors



Even starker differences between simple and hybrid VAR when yields (other than FFR) are excluded from model

APPENDIX

- Additional shadow rate estimates
- **Additional forecast comparisons**
- Shadow-rate vs. missing-data sampling
- Stability of VAR coefficients over time

SIMPLE SHADOW-RATE VS LINEAR VAR

Values below one indicate improvement over standard VAR

	RMSE			MAE			CRPS		
Var. / Hor.	3	12	24	3	12	24	3	12	24
Income	1.00	1.00	0.63	0.98*	0.99	0.99	0.99**	1.01	1.00
Consumption	1.00	1.02	0.96	1.00	1.02	0.99	1.00	1.02	1.00
IP	1.03***	1.06***	0.98	1.04***	1.06***	1.03**	1.03***	1.05***	1.02***
Cap. Util.	1.04**	1.11**	1.13**	1.05**	1.16***	1.12*	1.03***	1.11***	1.10***
Unemp.	1.00	1.02	1.03	1.00	0.97	0.98	1.00	1.02	1.01
Nfm Pyrlls	0.98	1.04	1.05	0.99	1.11*	1.03	1.00	1.06	1.03
Hours	1.02*	1.01	1.04	1.02	1.03	1.06	1.02**	1.02	1.02
H. Earnings	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	1.00
PPI (Fin.)	1.00	1.00	0.98**	1.00	1.00	0.99	1.00	1.00	1.00
PPI (Metals)	0.99*	1.00	1.01	0.99	1.00	1.00	0.99	1.00	1.00
PCE Prices	1.00	1.00	0.99	1.00	1.00	1.02**	1.00	1.00	1.01**
Hsng Strts	1.01	0.97	0.92	1.00	0.99	0.93	1.01	0.97	0.93
S&P 500	0.99	1.01	1.08	0.97	1.00	1.01**	0.99	1.01	1.01***
USD / GBP	0.99**	0.99	0.98**	0.98**	0.98*	0.98**	0.98***	0.99*	0.99
FFR	0.22**	0.30	0.53	0.13***	0.29*	0.39**	0.15***	0.26**	0.43**
6m Tbill	0.37*	0.54	0.76	0.24***	0.39*	0.55**	0.26***	0.44**	0.58***
1y Trsy	0.64	0.70	0.85	0.51***	0.52**	0.65**	0.53***	0.58**	0.64***
5y Trsy	0.94	0.84***	0.83	0.94	0.89**	0.89***	0.97	0.89***	0.85***
10y Trsy	0.95	0.88***	0.80***	0.95	0.89**	0.89*	0.97	0.93	0.93*
BAA Yld	0.97	1.02	0.93	0.99	1.03	1.01	0.99	1.03	1.04

Note: Stars denote DMW significance. Eval from 2009:01 through 2017:12.

HYBRID VS SIMPLE SHADOW-RATE VAR

Values below one indicate improvement over simple shadow-rate VAR

	RMSE			MAE			CRPS		
Var. / Hor.	3	12	24	3	12	24	3	12	24
Income	0.99	1.01	1.11	1.01	1.02	0.97	1.01*	1.01	0.99*
Consumption	1.00	0.98	0.93	0.99	1.00	0.90	1.00	0.98	0.97*
IP	0.98**	0.96	0.98	0.98	0.95*	0.94	0.98**	0.96**	0.98
Cap. Util.	0.97	0.91	0.83**	0.95**	0.86**	0.84**	0.97**	0.91**	0.88***
Unemp.	1.00	0.98	0.89	1.00	1.03	0.94	1.00	0.98	0.94
Nfm Pyrlls	1.01	1.01	0.94	1.00	0.91	0.90*	0.99	0.96	0.95**
Hours	1.00	1.02	0.99	0.99	1.01	0.98	1.00	1.00	0.99
H. Earnings	1.01	1.01**	1.04**	1.00	1.01	1.02**	1.00	1.01	1.02**
PPI (Fin.)	0.99	0.97	1.02	1.00	0.98	1.02	1.00	0.99	1.00
PPI (Metals)	1.00	0.99***	1.00	1.00	0.99*	0.99	1.00	0.99***	1.00
PCE Prices	1.00	0.97	1.07**	1.00	0.98	1.06***	1.00	0.99	1.04***
Hsng Strts	0.99	0.92	0.97	1.00	0.92	0.96	0.99	0.96	1.00
S&P 500	1.01	1.01	1.01	1.02	1.01	1.00	1.00	1.01	1.00
USD / GBP	1.01	1.01	1.03	1.02**	1.03	1.07	1.02*	1.01	1.02
FFR	0.97	0.98	0.96	1.00	0.95*	0.99	0.96	0.94	0.92*
6m Tbill	0.98	0.94***	0.91***	0.97	0.91**	0.85***	0.97	0.91***	0.87***
1y Trsy	0.99	0.94***	0.92***	1.00	0.86***	0.85***	0.99	0.91***	0.88***
5y Trsy	1.00	0.96	0.91***	1.01	1.00	0.98	1.00	0.97	0.96
10y Trsy	1.00	0.98	0.96*	1.01	0.98	1.05	1.00	0.99	1.00
BAA Yld	1.00	0.97	1.02	0.99	0.98	1.03	0.99	0.98	1.02

Note: Stars denote DMW significance. Eval from 2009:01 through 2017:12.

HYBRID VS PLUG-IN VAR W/WU-XIA RATE

Values below one indicate improvement over plug-in VAR

	RMSE			MAE			CRPS		
Var. / Hor.	3	12	24	3	12	24	3	12	24
Income	0.99	1.00	0.86	0.98	0.96	0.89**	0.99	0.98	0.96**
Consumption	0.99	0.95**	0.82**	0.98	0.97	0.82***	0.99	0.96***	0.92***
IP	1.03*	1.01	0.78	1.03*	1.01	0.95	1.02*	1.01	0.98
Cap. Util.	1.00	0.98	0.82**	0.99	0.95	0.81**	0.99	0.97	0.86***
Unemp.	0.99	0.99	0.82***	1.01	1.01	0.81***	0.99	0.98	0.86***
Nfm Pyrlls	1.00	1.02	0.82**	1.01	0.97	0.82**	1.00	0.99	0.90***
Hours	0.99	0.99	0.92*	0.97	0.99	0.91	0.98	0.98	0.93*
H. Earnings	0.99	0.98	1.03	0.98	0.99	1.02	0.99	0.98	1.00
PPI (Fin.)	1.00	0.94**	0.90*	1.00	0.96**	0.94	1.00	0.96***	0.94
PPI (Metals)	0.98	0.96***	0.93	0.98	0.95***	0.95	0.97**	0.96***	0.98
PCE Prices	0.98	0.94**	1.01	0.98	0.96	0.99	0.97	0.95*	0.99
Hsng Strts	0.99	0.88	0.93	1.00	0.85	0.83	0.99	0.91	0.92
S&P 500	0.98**	1.01	1.11	0.98	1.00	1.00	0.99	1.01	1.01**
USD / GBP	1.00	1.00	1.00	1.01	1.03	1.06	1.00	1.00	1.01
Policy Rate	0.57*	0.96	1.13*	0.55*	1.06	1.10	0.61**	1.03	1.10
6m Tbill	0.86	0.98	0.99	0.93	0.93	0.92	0.84	0.97	1.00
1y Trsy	1.00	1.07	1.04	1.06	0.98	0.99	0.97	1.05	1.02
5y Trsy	0.94	1.05	1.08	0.88	0.97	1.08	0.90	0.94	1.02
10y Trsy	0.96	0.96	0.99	0.91	0.90	1.00	0.93	0.90	0.97
BAA Yld	0.90*	0.75*	0.78*	0.92*	0.80**	0.78*	0.91**	0.81**	0.84***

Note: Stars denote DMW significance. Eval from 2009:01 through 2017:12.

HYBRID VS PLUG-IN VAR W/KRIPPNER RATE

Values below one indicate improvement over plug-in VAR

	RMSE			MAE			CRPS		
Var. / Hor.	3	12	24	3	12	24	3	12	24
Income	0.99*	1.00	0.90	0.97	0.98	0.92**	0.98	0.99	0.95***
Consumption	0.99	0.96	0.88*	0.99	0.97	0.85*	0.99	0.97**	0.93***
IP	1.00	0.98	0.95	1.02	0.98	0.93	1.00	0.97*	0.95
Cap. Util.	1.02	0.98	0.81**	1.01	0.98	0.81*	0.99	0.97	0.84***
Unemp.	0.96	1.02	0.92	0.98	1.03	0.97	0.97	1.00	0.93
Nfm Pyrlls	1.01	1.05	0.87	1.01	0.99	0.84	1.00	0.99	0.89**
Hours	0.98	1.03	0.94	0.97	1.03	0.94	0.97	1.00	0.92
H. Earnings	1.01	1.01	1.05***	0.99	1.01	1.05***	1.00	1.00	1.01
PPI (Fin.)	0.99	0.96	0.96	1.00	0.98	0.97	0.99	0.97*	0.96
PPI (Metals)	0.97	0.94**	0.94	0.98	0.92**	0.92*	0.96*	0.95***	0.95*
PCE Prices	0.98	0.95	1.04	0.99	0.96	1.03	0.99	0.96*	0.99
Hsng Strts	1.00	0.98	1.04	1.00	1.00	1.02	1.00	0.99	1.01
S&P 500	0.98	1.00	1.07	1.01	1.00	0.98	0.98	1.00	0.99
USD / GBP	1.01	1.01	1.02	1.02	1.04	1.06	1.01	1.01	1.01
Policy Rate	0.89	0.89*	0.98	1.05	0.99	1.03	1.01	0.96	1.07
6m Tbill	0.56***	0.77*	0.78*	1.13	0.94	0.90	0.61***	0.84	0.88
1y Trsy	0.69***	0.88	0.85	1.16	1.02	1.02	0.78**	0.97	0.95
5y Trsy	1.01	1.09	1.02	0.95	1.00	0.87	0.94	0.96	0.95
10y Trsy	0.98	1.01	1.03	0.97	0.99	0.89	0.96	0.93	0.92
BAA Yld	0.95	0.89	0.90**	0.97	0.95	0.85**	0.96	0.90**	0.86***

Note: Stars denote DMW significance. Eval from 2009:01 through 2017:12.

HYBRID VS CENSORING OF LINEAR VAR

Values below one indicate improvement over censoring of linear VAR

	RMSE			MAE			CRPS		
Var. / Hor.	3	12	24	3	12	24	3	12	24
Income	1.00	1.01	0.70	0.99	1.00	0.98	1.00	1.01**	1.00
Consumption	0.99	0.99	0.96	0.99	1.00	0.99	0.99	1.00	0.99
IP	1.01	1.02	1.01	1.01	1.02	1.05**	1.00	1.02*	1.03**
Cap. Util.	1.01	0.98	0.92*	1.01	0.97	0.92	1.01	0.99	0.96
Unemp.	1.00	0.98	0.84***	1.00	0.97**	0.83***	1.00	0.98	0.89***
Nfm Pyrlls	1.00	0.94**	0.82***	1.00	0.94**	0.80***	1.00	0.96**	0.91***
Hours	1.02**	1.00	0.98	1.01	1.01	0.97	1.01***	1.00	0.99
H. Earnings	1.00	1.01	1.02	1.00	1.00	1.02	1.00	1.00	1.01
PPI (Fin.)	1.00	0.99	0.99	1.00	1.00	1.00	1.00	0.99	1.00
PPI (Metals)	0.99*	0.99*	1.00	1.00	1.00	1.00	1.00	1.00	1.00
PCE Prices	1.01**	1.01	1.05*	1.01	1.03***	1.07***	1.01*	1.02***	1.05***
Hsng Strts	1.00	0.92*	0.88**	1.00	0.93*	0.85***	1.00	0.94*	0.89**
S&P 500	0.99	1.01*	1.09*	0.98***	1.00	1.01	1.00	1.01	1.01***
USD / GBP	0.99**	1.00	1.00	0.99	1.00	1.01	0.99	1.00	1.01*
FFR	0.60***	0.60**	0.67**	0.38***	0.49*	0.64	0.42***	0.48**	0.67
6m Tbill	0.76***	0.79**	0.81*	0.53***	0.55**	0.68	0.57***	0.64**	0.78
1y Trsy	0.79***	0.82**	0.83*	0.60***	0.58**	0.72	0.68***	0.69**	0.80
5y Trsy	0.94	0.80**	0.68	0.95	0.89	0.91	0.97	0.89	0.90
10y Trsy	0.96	0.83*	0.66	0.96	0.84	0.88	0.97	0.90	0.95
BAA Yld	0.96	0.90	0.90	0.97	0.93	1.13	0.98	0.97	1.10

Note: Stars denote DMW significance. Eval from 2009:01 through 2017:12.

APPENDIX

- Additional shadow rate estimates
- Additional forecast comparisons
- **Shadow-rate vs. missing-data sampling**
- Stability of VAR coefficients over time

DETAILS ABOUT SHADOW-RATE SAMPLING

Problem: $S|Y \sim \text{truncN}(\mu, \Omega, S \leq ELB)$

- μ and Ω implied by VAR and $\bar{Y}$
- With T^* obs at the ELB, S is large ($T^* \cdot N_s$)

DETAILS ABOUT SHADOW-RATE SAMPLING

Problem: $S|Y \sim \text{truncN}(\mu, \Omega, S \leq ELB)$

- μ and Ω implied by VAR and $\bar{Y}$
- With T^* obs at the ELB, S is large ($T^* \cdot N_s$)
- $S \leq ELB$ holds elements-wise for entire trajectory

DETAILS ABOUT SHADOW-RATE SAMPLING

Problem: $S|Y \sim \text{truncN}(\mu, \Omega, S \leq ELB)$

- μ and Ω implied by VAR and $\bar{Y}$
- With T^* obs at the ELB, S is large ($T^* \cdot N_s$)
- $S \leq ELB$ holds elements-wise for entire trajectory

Implementation

DETAILS ABOUT SHADOW-RATE SAMPLING

Problem: $S|Y \sim \text{truncN}(\mu, \Omega, S \leq ELB)$

- μ and Ω implied by VAR and $\bar{Y}$
- With T^* obs at the ELB, S is large ($T^* \cdot N_s$)
- $S \leq ELB$ holds elements-wise for entire trajectory

Implementation

- We do Gibbs sampling from multivariate truncN

$$s_t | s_{1:t-1}, s_{t+1:T}, \bar{Y} \qquad \text{(Geweke, 1991)}$$

DETAILS ABOUT SHADOW-RATE SAMPLING

Problem: $S|Y \sim \text{truncN}(\mu, \Omega, S \leq ELB)$

- μ and Ω implied by VAR and $\bar{Y}$
- With T^* obs at the ELB, S is large ($T^* \cdot N_s$)
- $S \leq ELB$ holds elements-wise for entire trajectory

Implementation

- We do Gibbs sampling from multivariate truncN

$$s_t | s_{1:t-1}, s_{t+1:T}, \bar{Y} \qquad \text{(Geweke, 1991)}$$

- **. . . while exploiting Markov structure of VAR**

INFORMATION CONTENT OF BINDING ELB FOR VAR

Let's consider the following thought experiments

Let's record . . .

1. Shadow-rate draws $\boldsymbol{S}|\boldsymbol{Y}$, from shadow-rate VAR

Purpose:

1. This is our baseline

INFORMATION CONTENT OF BINDING ELB FOR VAR

Let's consider the following thought experiments

Let's record . . .

1. Shadow-rate draws $\boldsymbol{S}|\boldsymbol{Y}$, from shadow-rate VAR
2. Missing-data draws, $\boldsymbol{S}|\bar{\boldsymbol{Y}}$, from shadow-rate VAR

Purpose:

1. This is our baseline
2. Tells us to what extent ELB is binding for the sampler *at the parameters estimated with shadow-rate VAR*

INFORMATION CONTENT OF BINDING ELB FOR VAR

Let's consider the following thought experiments

Let's record . . .

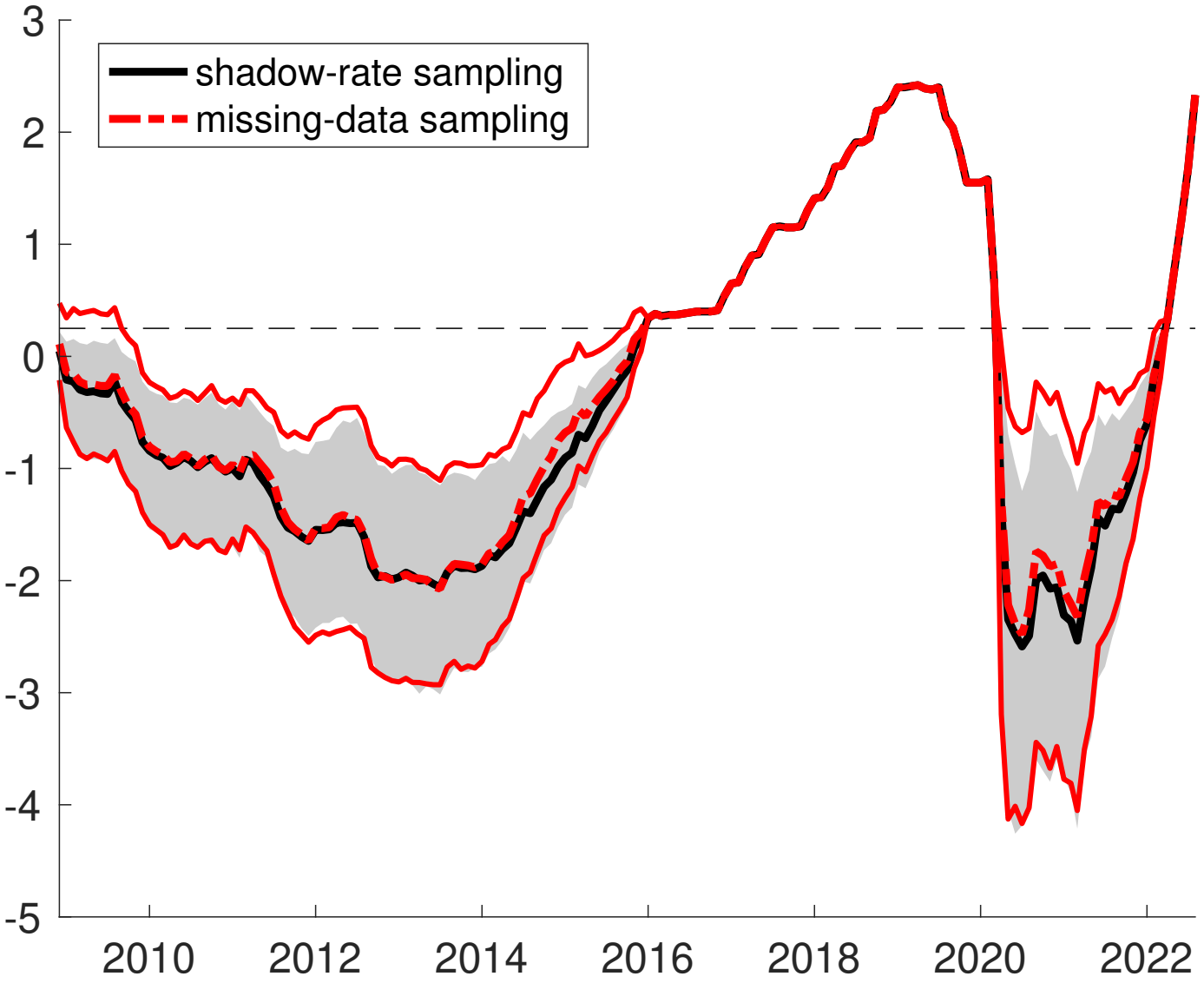
1. Shadow-rate draws $\boldsymbol{S}|\boldsymbol{Y}$, from shadow-rate VAR
2. Missing-data draws, $\boldsymbol{S}|\bar{\boldsymbol{Y}}$, from shadow-rate VAR
3. Missing-data draws, $\boldsymbol{S}|\bar{\boldsymbol{Y}}$, from a missing-data VAR estimated on $\bar{\boldsymbol{Y}}$ rather than $\boldsymbol{Y}$
   (e.g. Del Negro, et al., BPEA, 2017)

Purpose:

1. This is our baseline
2. Tells us to what extent ELB is binding for the sampler *at the parameters estimated with shadow-rate VAR*
3. Shows us if ELB sampling shifted VAR parameters

EFFECT OF CONDITIONING ON ELB

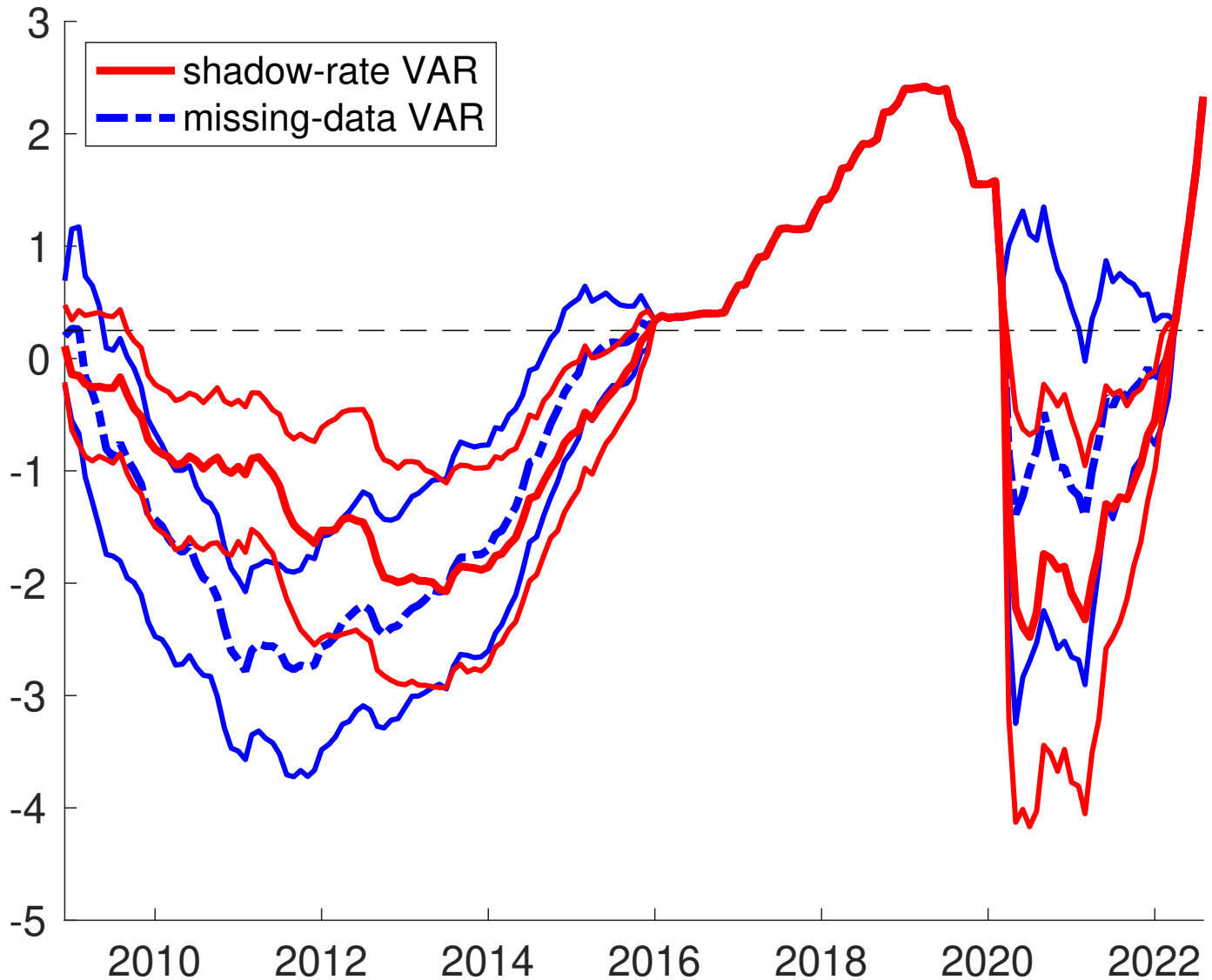
Cases 1&2: $S|Y$ (black) vs $S|\bar{Y}$ (red) with $\Pi, \Sigma_t|Y$, median/90% bnds.



Truncation appears negligible with shadow-rate parameters and SV

EFFECT OF CONDITIONING ON ELB

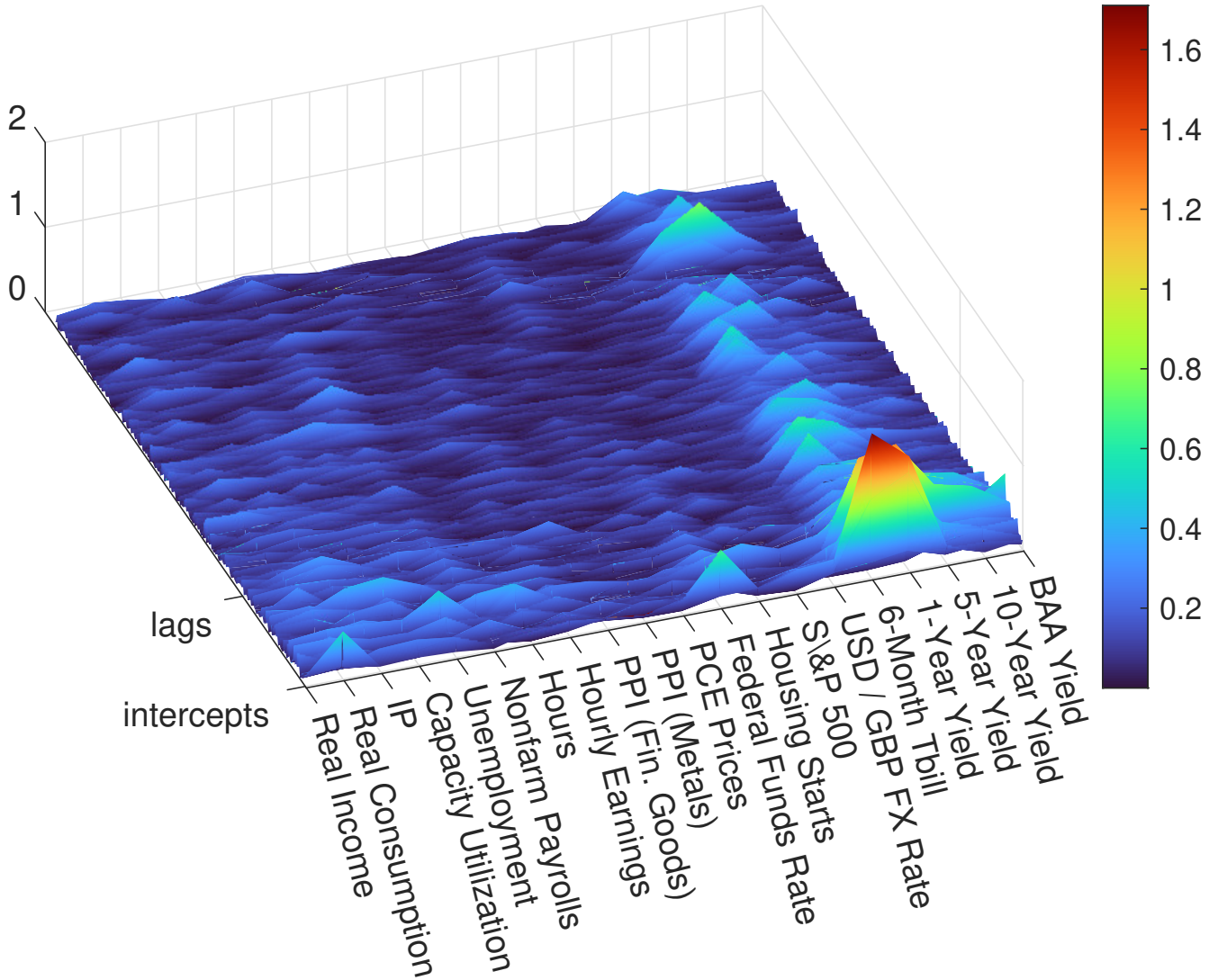
Cases 2&3: $S|\bar{Y}$ from $\Pi, \Sigma_t|\bar{Y}$ (blue) or $\Pi, \Sigma_t|Y$ (red), median/90% bnds.



$S|\bar{Y} > ELB$ more often w/missing-data SV & parameters

EFFECTS ON Π FROM CONDITIONING ON ELB

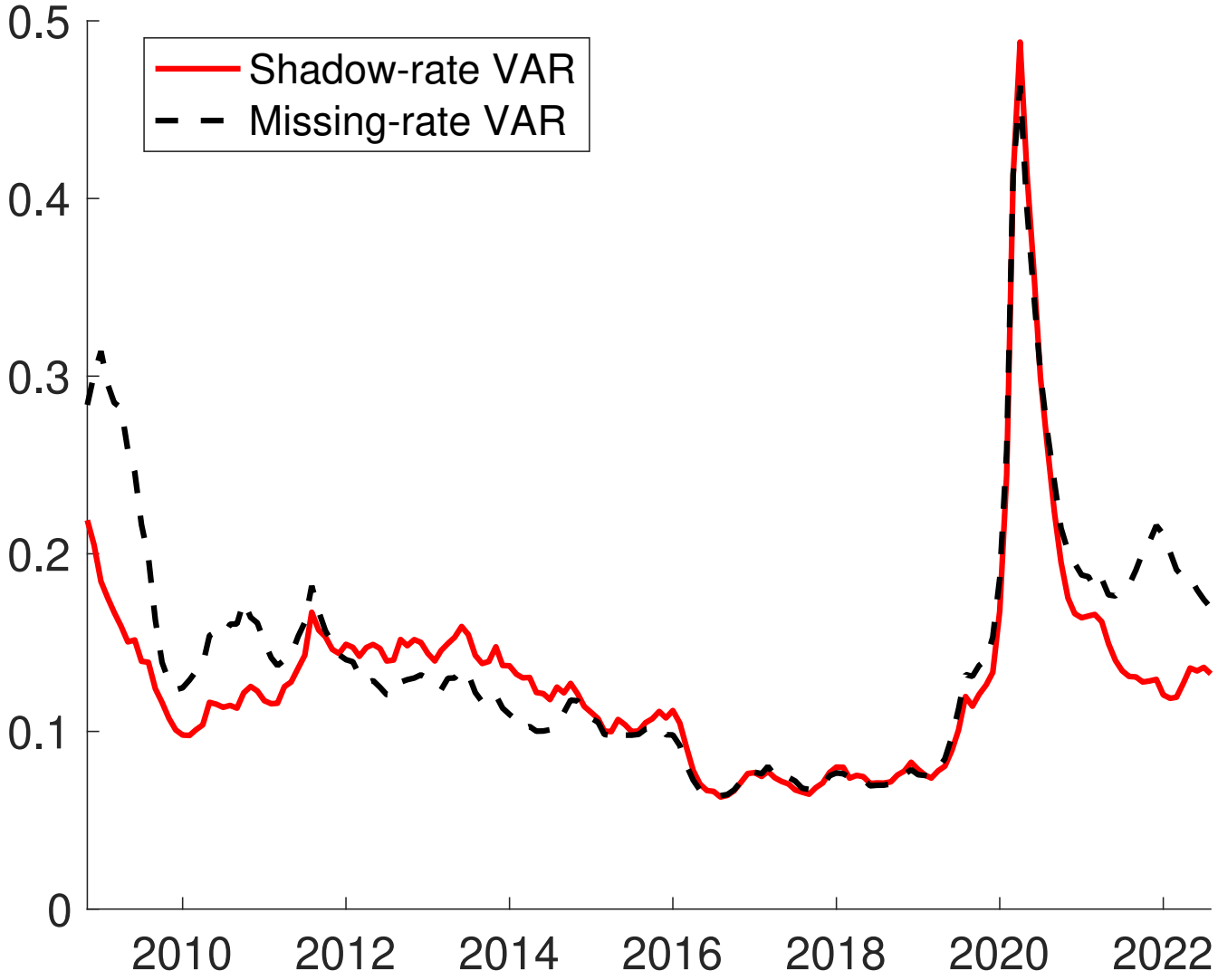
$| E(\Pi^{shadow}) - E(\Pi^{missing}) | / \mathrm{Vol}(\Pi^{missing})$



Some changes in yield equations

EFFECTS ON SV FROM CONDITIONING ON ELB

SV of Choleski residual in shadow-rate equation of FFR

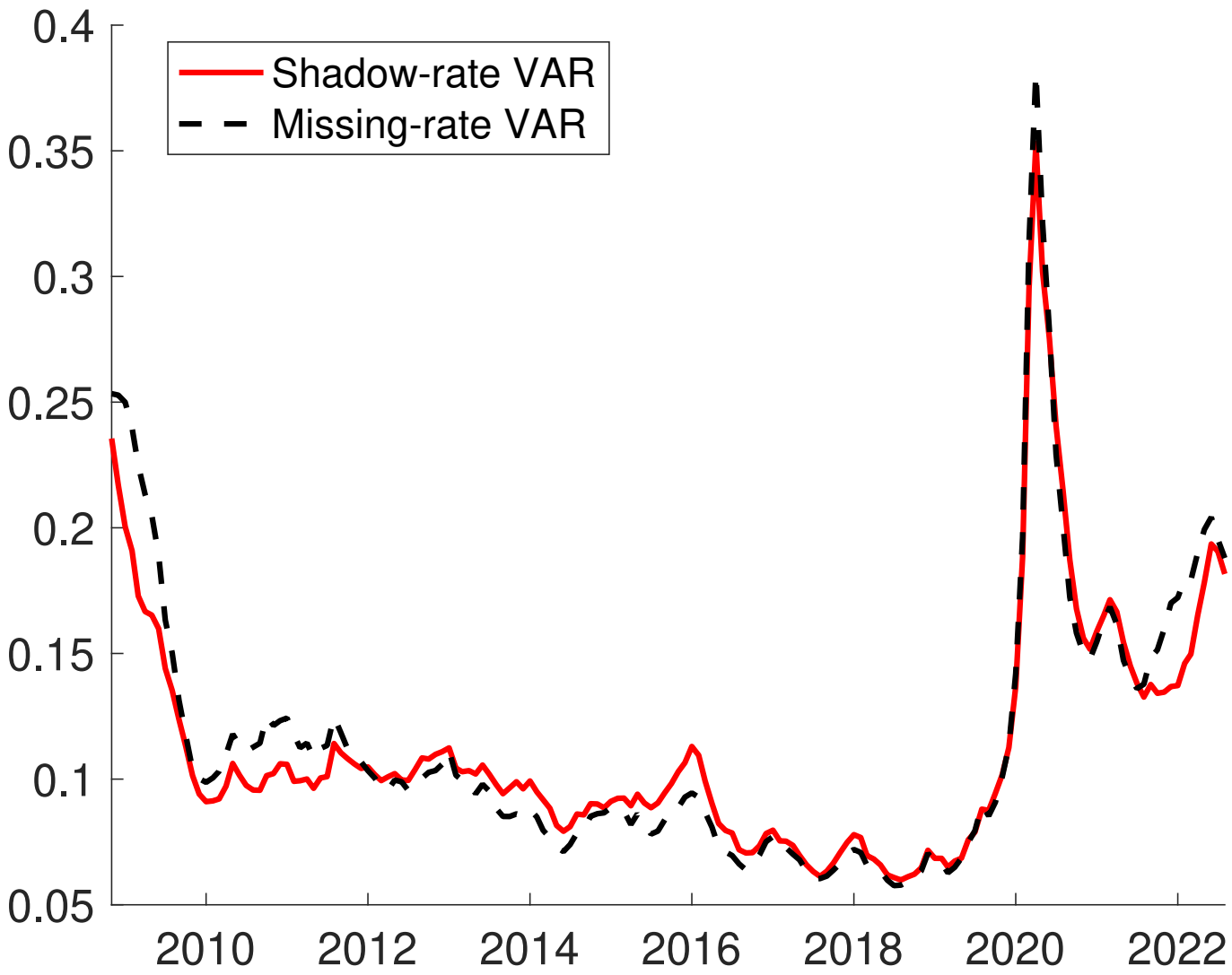


Different shadow-rate SV in 2009/2010

(with other SV paths essentially unchanged)

EFFECTS ON SV FROM CONDITIONING ON ELB

SV of Choleski residual in shadow-rate equation of 6M Tbill

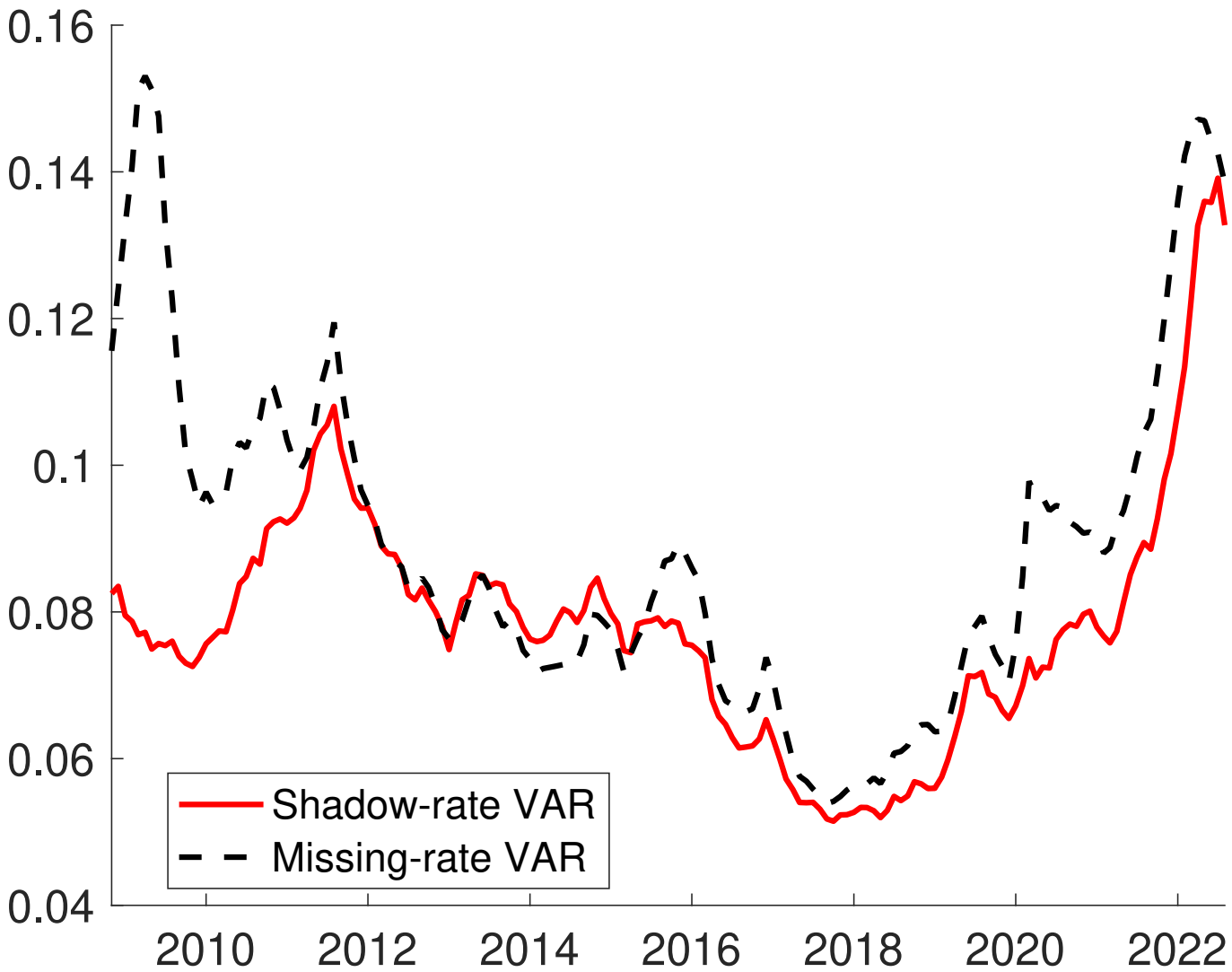


Different shadow-rate SV in 2009/2010

(with other SV paths essentially unchanged)

EFFECTS ON SV FROM CONDITIONING ON ELB

SV of Choleski residual in shadow-rate equation of 1Y Tbond



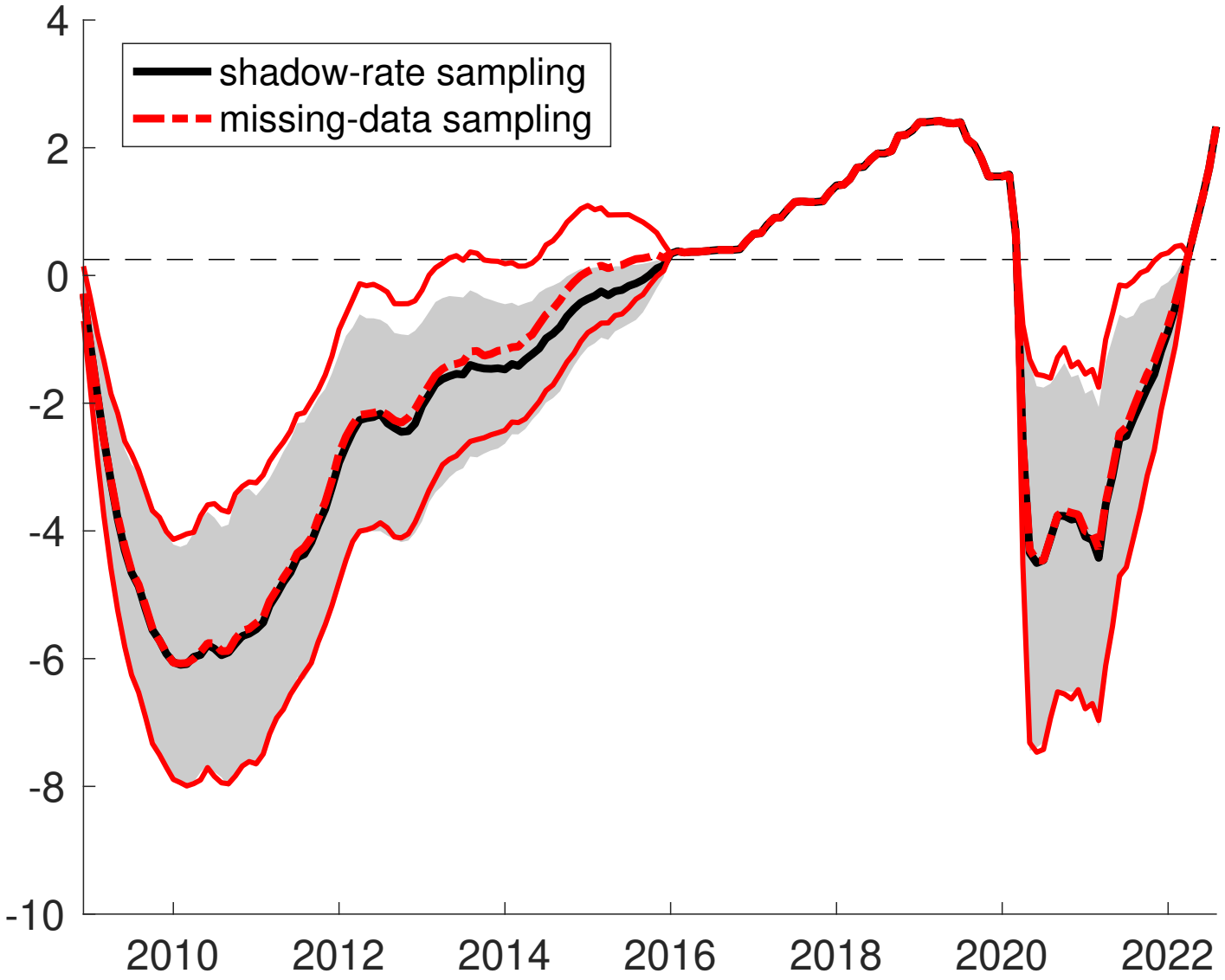
Different shadow-rate SV in 2009/2010

(with other SV paths essentially unchanged)

EFFECT OF CONDITIONING ON ELB

EX YIELDS

Cases 1&2: $S|Y$ (black) vs $S|\bar{Y}$ (red) with $\Pi, \Sigma_t|Y$, median/90% bnds.

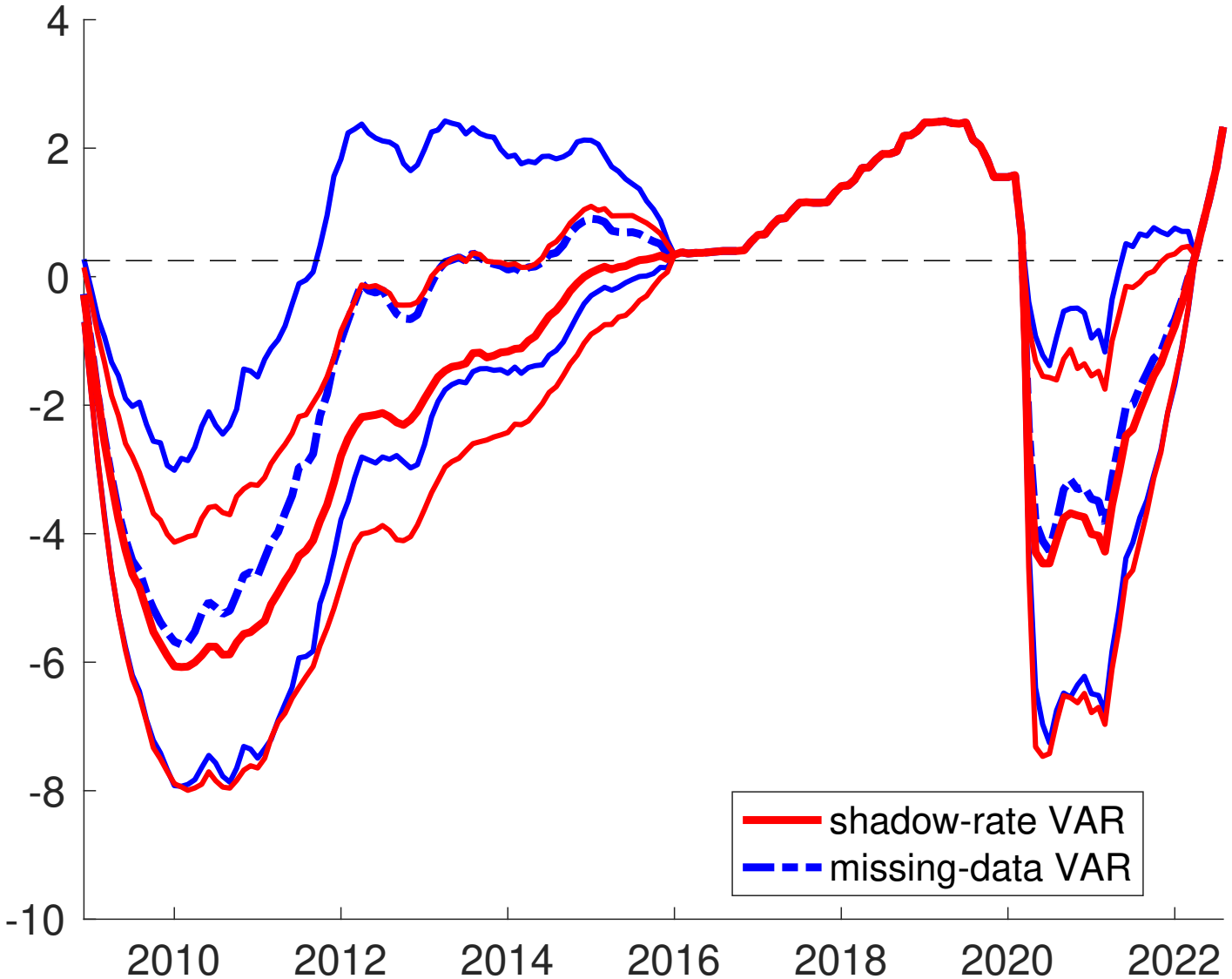


Truncation appears negligible with shadow-rate parameters and SV

EFFECT OF CONDITIONING ON ELB

EX YIELDS

Cases 2&3: $S|\bar{Y}$ from $\Pi, \Sigma_t|\bar{Y}$ (blue) or $\Pi, \Sigma_t|Y$ (red), median/90% bnds.

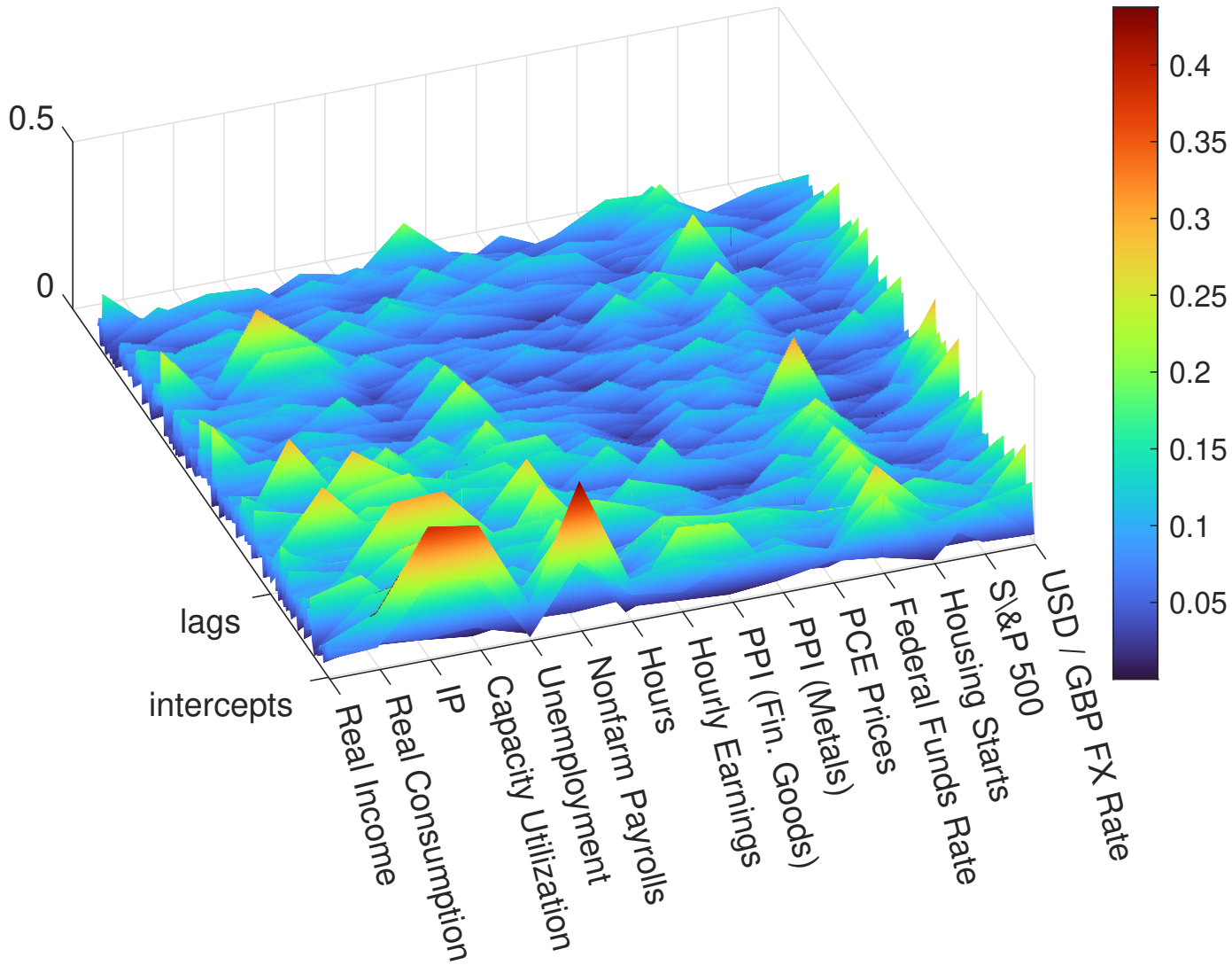


$S|\bar{Y} > ELB$ more often w/missing-data SV & parameters

EFFECTS ON Π FROM CONDITIONING ON ELB

EX YIELDS

$$| E(\Pi^{shadow}) - E(\Pi^{missing}) | / \text{Vol}(\Pi^{missing})$$



Almost no relevant coefficient changes

APPENDIX

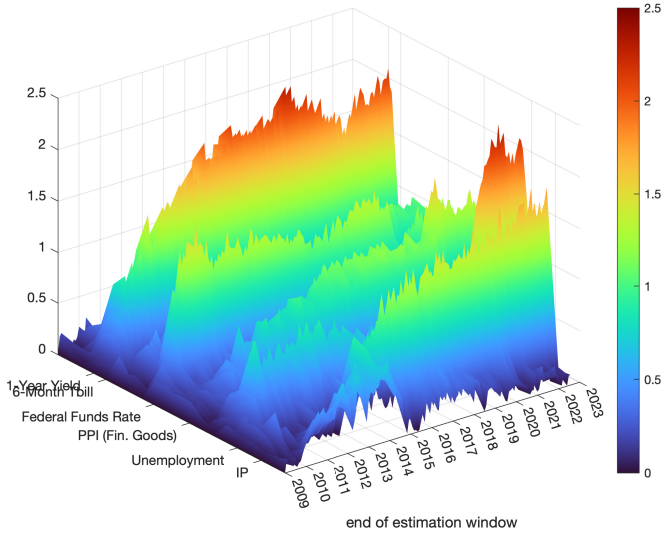
- Additional shadow rate estimates
- Additional forecast comparisons
- Shadow-rate vs. missing-data sampling
- **Stability of VAR coefficients over time**

CHANGES IN RECURSIVE COEFFICIENT ESTIMATES

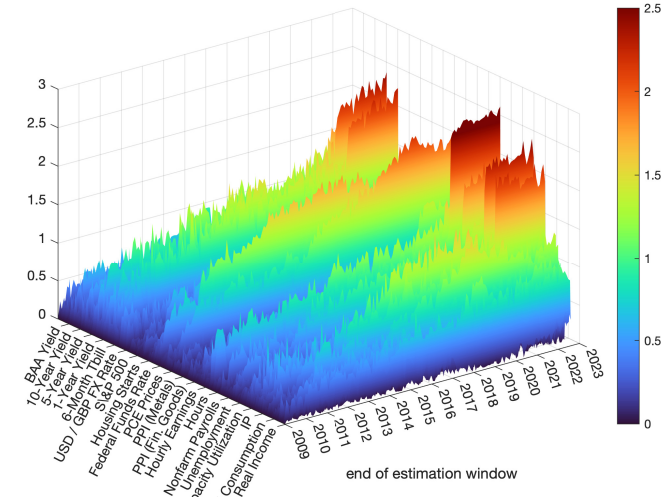
Abs. change relative to 2009:01 estimate of linear VAR (standardized)

Standard VAR

intercepts

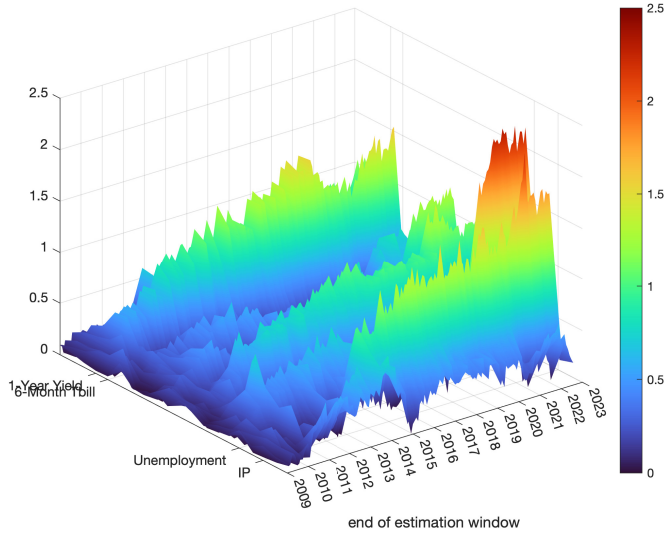


1st lag

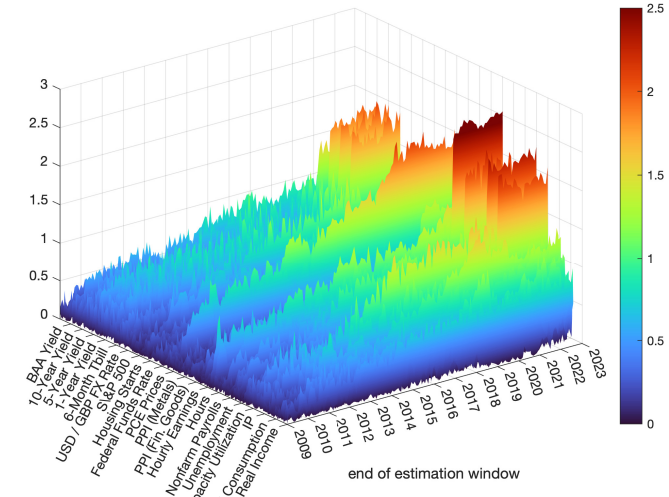


Shadow-rate VAR

intercepts



1st lag



CONCLUSIONS

Our solution to handle ELB: Shadow-rate VARs

- **Internally consistent** inference
- **Scalable** to multiple interest-rate maturities at ELB
- **Hybrid VAR** conditions macro variables on actual rates

Structural Analysis at/away from ELB:

- strong differences in nominal rate responses
- some differences in macro responses
- consistent w/some effectiveness of unconventional policies

Forecasts for interest rates and macro variables

- **Interest-rate forecasts superior** to standard VAR
- **Hybrid** typically better than simple shadow-rate VAR
- . . . and with **some gains in macro** over linear VAR