Technological Synergies, Heterogenous Firms, and Idiosyncratic Volatility

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Abstract

This paper shows the importance of technological synergies among heterogenous firms for aggregate fluctuations. First, we document five novel empirical facts using micro data that suggest the existence of important technological synergies between trading partners, the presence of positive assortative matching among firms, and their evolution during the business cycle. Next, we embed technological synergies in a general equilibrium model calibrated on firm-level data and show that frictions in forming trading relationships and separation costs explain imperfect sorting between firms in equilibrium. In particular, an increase in the volatility of idiosyncratic productivity shocks significantly decreases aggregate output without resorting to non-convex adjustment costs.

Keywords: Technological synergies, heterogenous firms, idiosyncratic uncertainty.

JEL classification: C63, C68, E32, E37, E44, G12.

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1 Introduction

The premise of our analysis is that technological synergies —cooperation between firms to enhance productivity— are prevalent in modern economies where final output results from complex operations that require strategic partnerships. If one of the technologies, or a partner, fails to fulfill the technical requirements of interlinked manufacturing processes, production decreases, and the partnership may terminate.

An archetypal example of technological synergies is the consortium of Symbian with Nokia for the provision of new operating systems in 2004. At that time, Nokia planned to develop a new user interface for Nokia handsets to start the development of nascent smartphones and formed a trading relationship with Symbian, the leading provider of operating systems for mobile devices in the world. Nokia undertook a series of major hardware updates (by 2008, the Nokia N95 handset had the most advanced camera in the market with 5 megapixels f/2.8 Carl Zeiss Tessar lens), but the old-fashioned `C++` code in the Symbian operating system prevented the development of a functional and fully integrated interface for the new handsets.[1] By 2008, despite Nokia being the leading producer of handsets in the world, sales plummeted, and the company's profitability and stock market value greatly declined. The trading relationship terminated in 2010, Nokia formed a new partnership with Windows Phone and lost half of its market share in mobile technology by the end of the same year. Symbian ceased operations in 2012.

Our example highlights the dynamic nature of technological synergies, which evolve and operate in the economy even without economic distortions such as financial frictions. Despite similar vicissitudes of fortune are ubiquitous in modern economies, the study of technological synergies for aggregate fluctuations has been overlooked by macroeconomists.

With these recurrent business stories in mind, we provide a first attempt to explore the role of technological synergies in the business cycle. We consider two key questions: How do technological synergies influence the sorting between producers with different productivity? How does a heightening in the volatility of idiosyncratic productivity affect aggregate output?

To answer these questions, we use the Compustat fundamental annual data, Compustat Segment data, Factset Supply Chain Relationships data, and the BEA input-output tables to

[1] Alcacer et al. (2014), Doz and Wilson (2017) and Lamberg et al. (2019) provide detailed accounts of the demise of the Nokia-Symbian trading relationship. For a general discussion on the partnership between Symbian and Nokia, see: https://en.wikipedia.org/wiki/Symbian. For details on the technical follies of Symbian, see https://www.silicon.co.uk/mobility/smartphones/symbian-mobile-history-227097.

study the link between technological synergies and idiosyncratic shocks and uncover five novel empirical facts.

Fact 1 is that the economic fundamentals of trading partners, measured as labor productivity, profit-to-sales ratio, profit, and sales growth, are positively correlated. By focusing on the year before establishing the trading relationship, Fact 1 cannot be driven by common shocks to the firms.

Fact 2 is that a firm's output is positively correlated with its partner's productivity, conditional on the firm's productivity. The correlation increases with the level of the firm's productivity, suggesting supermodularity of the production function.

Fact 3 is that the relationship of trading partners with very different economic fundamentals, which we refer to as mismatches, is less durable. Facts 1-3 indicate that positive assortative matching of trading relationships is prominent in the economy and is more stable than mismatching, which can be accounted for parsimoniously by technological synergies between trading partners.

Fact 4 is that a higher absolute value of idiosyncratic productivity shocks to either side of a trading relationship predicts a higher probability of separation in the subsequent years. We show that Fact 4 is not driven by the dominating role of negative or positive shocks. Instead, it is the magnitude of the shocks that lead to separation. Fact 4 can be rationalized by the destabilizing role of idiosyncratic shocks of both signs that make trading partners more different from each other, leading to less efficiency and endogenous separation.

Fact 5 is that higher volatility of idiosyncratic productivity shocks in a sector is correlated with a fall in sectoral output paired with a fall in output in connected sectors. Fact 4 can be explained by the fact that the volatility of idiosyncratic productivity shocks systematically alters the extent of mismatching in the economy. Facts 4 and 5 motivate us to investigate the role of technological synergies and idiosyncratic productivity shocks in a real business cycle model.

Motivated by these empirical facts, we develop a simple and static model with synergies in production input. We assume that manufacturing one unit of output requires a distinct input from firms in each sector whose productivity is either low- or high-type. Synergies in production technology entail a trading relationship with similarly productive firms to produce larger output (our fact 3). High-productivity firms prefer to form a trading relationship with partners of high productivity, a standard assumption in matching theory since the seminal study by Gale and Shapley (1962). Technological synergies indicate that sorting between firms with the same productivity type is the stable (and efficient) equilibrium (our Fact 1). While trading relationships between firms with different types of technology are unstable (our Fact 3) since the partner with high-type of technology optimally terminates the trading relationship with a firm of low-type technology to seek to establish a new trading relationship with a firm of equally high-type technology that yields a larger payoff.

The simple model illustrates that idiosyncratic shocks destabilize trading relationships by transforming positive assortative matching into mismatching (our Fact 4). Technological synergies (our Fact 2) introduce a "bottleneck effect" that generates an asymmetry in output response to negative and positive idiosyncratic shocks. Adverse idiosyncratic shocks that reduce the productivity of one partner (such as the poor coding of Symbian Ltd.) decrease output by impairing the production capacity of the trading relationship. In contrast, positive idiosyncratic shocks (such as Nokia's hardware updates) exert limited benefits to the trading relationship since the partner with low productivity cannot exploit the improved technology.

The asymmetric effect of idiosyncratic shocks implies that the heightening in the volatility of idiosyncratic shocks depresses output on average (our Fact 5). In particular, an increase in the volatility of idiosyncratic productivity in one sector raises the number of trading relationships with a partner of different technology types, for which technological synergies imply sub-optimal production. Misallocation of trading relationships arises due to idiosyncratic shocks and technological synergies. It is intrinsic to business cycle fluctuations and not generated by exogenous distortions in goods or labor markets. Moreover, since technological synergies apply to all interlinked industries, the volatility of idiosyncratic shocks in one industry induces a local contraction in output and a contraction in the connected industries.

We embed the intuition of the simple static model into a quantitative and dynamic framework which allows us to quantify the relevance of the critical mechanisms at play for the importance of technological synergies for economic activity in a more realistic environment that encapsulates the empirical features of the process of trading relationship formation.

The quantitative model has four new features. First, frictions in the matching process across firms prevent the instantaneous and costless formation of trading relationships, which is motivated by the fact that sorting is far from perfect in the data. Second, we assume directed search from both sides of the market to form trading relationships.[2] We show theoretically that, with the above two features, log-supermodularity (which is stronger than supermodularity) of the surplus function is a sufficient condition for the stability of positive assortative matching. Third, the termination of trading relationships with different productivity types is staggered, motivated by the time-consuming separation of trading relationships observed in the data. Fourth, we propose a generalization of the production technology in which the degree of technological synergies is governed by a single parameter, which we estimate using firm-level data.

We use the extended model to quantitatively assess the vital propagation channels for technological synergies. We calibrate the novel parameters in the system that govern the degree of technological complementarity, search frictions, and endogenous separations using Compustat firm-level data. We show that under the benchmark calibration, search frictions and delayed separations generate imperfect sorting of firms and a 21% drop in output in the stationary steady state, which can be decomposed to a 12% decrease in the utilization rate of productive resources and an 11% decrease in the average production efficiency. The size of output losses is comparable to the output gap due to other types of frictions or misallocations.[3]

We find that an increase of 34% in the standard deviation of idiosyncratic productivity shocks, which is of the same magnitude as the increase in uncertainty during the Great Recession, leads to a 1.2% drop in aggregate output. The model shows that the fall in production is explained by a significant increase in the measure mismatch and the persistent rise in the separation rate. The effects of increased idiosyncratic uncertainty are persistent since the termination of ongoing trading relationships with inefficient production is time-consuming, and the process of new trading relationship formation is costly.

Our analysis is related to three realms of research. First, we contribute to the literature on technological synergies by focusing on short-run output fluctuations. Kremer (1993) and Jones (2011) study the implication of technological synergies to economic development and the secular allocation of resources. Technological synergies are also central to the study of strategic mergers and acquisitions of firms (Rhodes-Kropf and Robinson, 2008; Xu, 2017; David, 2021),

[2]Our assumptions of directed search from both sectors are more suitable for our analysis of inter-firm trading relationship than the conventional search models with two-sided heterogeneity, such as in Shimer and Smith (2000) who considered random search, or in Eeckhout and Kircher (2010) who assume that only one side of the matching market conducts a directed search, while the other side posts prices.

[3]For example, the output gap induced by financial constraints, through capital misallocation and inefficiently low capital accumulation, ranges from 35% to 70% (see the literature review by Hopenhayn, 2014).

the magnification effect of technology adoption (Eslava et al., 2015), the slowdown in aggregate productivity (Acemoglu et al., 2023) and patterns of international trade (Demir et al., 2023).[4] Compared to these studies, we investigate the role of technological synergies on short-run output fluctuations and study how they evolve dynamically in response to exogenous disturbances.

Second, we add to the literature on misallocation. The literature attributes misallocation to distortions in physical capital (Banerjee and Duflo, 2005; Hsieh and Klenow, 2009; Restuccia and Rogerson, 2008) or human capital (Alder, 2016; Hsieh and Moretti, 2019; Baley et al., 2022), while we study the misallocation in trading relationships, a new source of dynamic misallocation that yields significant output losses. Our source of misallocation originates from idiosyncratic shocks to technology rather than exogenous distortions such as financial frictions and distortionary taxes and, thus, is endogenous to business cycle fluctuations.

Finally, we link with the literature on idiosyncratic uncertainty (Christiano et al., 2014; Bloom et al., 2018; Arellano et al., 2019) by showing that technological synergies trigger large output losses for the heightening of idiosyncratic volatility without resorting to non-convex adjustment costs.

The rest of the paper is structured as follows. Section 2 documents the five novel facts about the technological synergies embedded in trading relationships. Section 3 develops a simple model to outline the interplay between technological synergies, idiosyncratic shocks, and sorting. Section 4 extends the simple model to a rich general equilibrium framework. Section 5 calibrates the model, and Sections 6-8 explore its quantitative predictions. Section 9 concludes.

2 Empirical evidence

This section documents five facts about the assortative matching of trading relationships. These five facts will motivate our model and offer a benchmark against which to compare our quantitative findings.

Data. We study the formation of trading relationships using two data sets. The first is the Compustat Customer Segment data, which provides information on inter-firm trading for the universe of publicly listed firms in the US. The data have a yearly frequency and cover 1976-2020,

[4]See Fernández-Villaverde et al. (2019, 2021) for alternative sources of complementarities based on the formation of vendor contracts and firm partnerships to study fiscal policy and monopsony power in labor markets.

with approximately 18 thousand firms. Since publicly listed firms must supply the identity of trading partners that account for more than 10% share of yearly sales, we can obtain 72,694 distinct customer-supplier trading relationships. The second dataset is the FactSet Supply Chain Relationships data, which comprises firms' relationship information from public sources such as SEC 10-K annual filings, investor presentations, and press releases since 2003. Using the sample 2003-2021, we obtain 289,239 distinct customer-supplier trading relationships.[5]

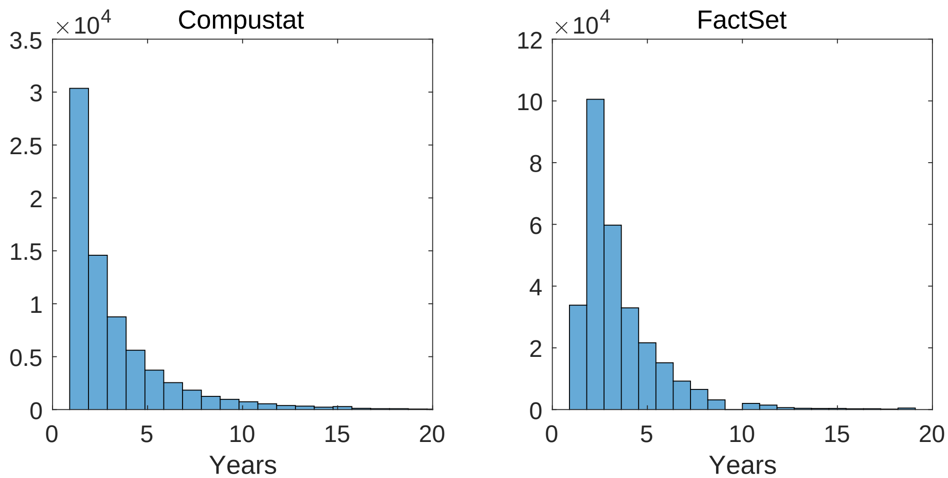


Figure 1: Distribution of duration of trading partnerships (years)

Figure 1 plots the cross-sectional distribution of the yearly duration of the trading partnership (the duration of a relationship that starts in year t_1 and ends in year t_2 is $t_2 - t_1 + 1$) in each dataset. Trading relationships are persistent, with a mean duration of about 4 years in Compustat Segment data and 3.5 years in the Factset data. Since the sample for the two data sets ends in 2020 and 2021, respectively, with many ongoing relationships, 4 and 3.5 years are downward-biased estimates of the true persistence.

Fact 1: Positive assortative matching of trading relationship

Fact 1 is that the economic fundamentals of trading partners are positively correlated. Since the correlation of economic fundamentals between trading partners could be driven by common shocks in addition to the force of positive assortative matching, we control for the effect of common shocks by focusing on the trading partners in a newly formed partnership and assessing the correlation of economic fundamentals in the year before the formation of a trading relationship.

[5]To avoid repetition, we will drop "trading" from "trading relationship" when no ambiguity occurs.

For robustness, Table A.1 in the Appendix reports the panel regression results when the variables are measured during the match.

More concretely, we estimate the regression:

$$decile\left(\pi_{j,k}\right) = \alpha + \beta \times decile\left(\pi_{j,k}^{cus}\right) + \epsilon_{j,k}, \tag{1}$$

for $j \in \{1, 2, \cdots, J\}$ and $k \in \{1, 2, \cdots, J_j\}$. Each observation is a distinct trading relationship in equation (1). The dependent variable $\pi_{j,k}$ is the firm j's economic fundamental in the year before the start of its partnership with its kth customer. The dependent variable $decile\left(\pi_{j,k}\right)$ is the decile of $\pi_{j,k}$ within firm j's three-digit NAICS industry in the year before the start of the partnership, ranging from one (bottom 10%) to ten (top 10%). We will use as fundamentals three different measures: labor productivity (ratio of sales to employment), profit-to-sales ratio (profit is earnings before interest, tax, depreciation, and amortization or EBITDA), and sales growth. The regressor $\pi_{j,k}^{cus}$ is the economic fundamental of firm j's kth customer in the year before the start of the partnership, and $decile\left(\pi_{j,k}^{cus}\right)$ is the decile of $\pi_{j,k}^{cus}$ within the customer firm's three-digit NAICS industry in the year before the start of the partnership.

Table 1: Assortative matching for ranking of economic fundamentals, one year before the match (Compustat Segment)

	(1) Labor productivity	(2) Profit/sales	(3) Sales Growth
$decile\left(\pi_{j,k}^{cus}\right)$	0.320***	0.067***	0.228***
	(0.014)	(0.014)	(0.015)
Constant	4.133***	4.521***	4.562***
	(0.115)	(0.093)	(0.088)
Adjusted R^2	0.07	0.00	0.03
Observations	6,914	7,605	6,854

Note: Sample: 1976-2020. Standard errors are in the parentheses. *** denotes significance level at the 1%.

Tables 1 (for Compustat) and 2 (for Factset) show that the estimate for β is positive and statistically significant for all the alternative measures of economic fundamentals. The results imply that firms with stronger economic fundamentals (compared to other firms in the same industry over the same period) establish trading relationships with customers with stronger economic fundamentals, providing strong evidence of positive assortative matching.

Table 2: Assortative matching for ranking of economic fundamentals, one year before the match (Factset)

	(1) Labor productivity	(2) Profit/sales	(3) Sales Growth
$decile\left(\pi_{j,k}^{cus}\right)$	0.105***	0.052***	0.191***
	(0.014)	(0.008)	(0.008)
Constant	6.783***	5.325***	4.238***
	(0.007)	(0.053)	(0.047)
Adjusted R^2	0.01	0.00	0.03
Observations	17,680	18,151	17,783

Note: Sample: 2003-2021. Standard errors are in the parentheses. *** denotes significance level at the 1%.

Fact 2: Firm's sales comove with partner's labor productivity

Fact 2 is that a firm's output is positively correlated with its partner's productivity, conditional on the firm's labor productivity. This fact implies that firms have the incentive to match with more productive partners, generating positive assortative matching, a key property of our theoretical model. Furthermore, the correlation between a firm's output and its partners' labor productivity increases with its productivity. This finding will motivate us to assume a supermodular production function in the model.

To show these results, we estimate the regression:

$$y_{j,t} = \beta z_{j,t} + \eta \overline{decile}\left(z_{j,t}^{cus}\right) + indu_j + \chi_t + \epsilon_{j,t}, \tag{2}$$

where $y_{j,t}$ and $z_{j,t}$ are firm j's log sales and log labor productivity, respectively, and we remove firm-specific time trends from both variables. The regressor $\overline{decile}\left(z_{j,t}^{cus}\right)$ is the average decile of firm j's partners' labor productivities within partner's three-digit NAICS industries.[6] The variables $indu$ and χ_t are the industry and year fixed effects, respectively. Our main goal is to estimate the contribution of the partner's economic fundamentals to the firm's output rather than estimating the firm's production function, which would require data on inputs that are difficult to measure in our data sets.

Columns (1) and (2) in Table 3 show the estimation results from regression (2) using data

[6]Specifically, $\overline{decile}\left(z_{j,t}^{cus}\right) = \sum_k decile\left(z_{j,k,t}^{cus}\right)/N_{j,t}^{cus}$, where $N_{j,t}^{cus}$ is the number of firm j's partners. We consider the average decile rather than the average labor productivity since the partners can be from different industries, making the level of labor productivity incomparable between each other.

from Compustat Segment and FactSet datasets, respectively. A firm's log sales are increasing in its log labor productivity. The firm's log sales are also positively correlated with the ranking of its partners' labor productivity. Conditional on the firm's labor productivity, increasing a firm's partners' labor productivity decile by one (e.g., from 5th to 6th) would increase the firm's sales by 5.6% and 13.7% for the two data sets, respectively, an economically significant move.

Table 3: Sales comoves with partner's labor productivity

	(1) Compustat Segment	(2) FactSet	(3) Compustat Segment	(4) FactSet
$z_{j,t}$	0.491***	0.477***	0.122***	0.261***
	(0.025)	(0.024)	(0.039)	(0.046)
$\overline{decile}\left(z_{j,t}^{cus}\right)$	0.056***	0.137***	-0.174***	0.038*
	(0.012)	(0.013)	(0.022)	(0.022)
$\overline{decile}\left(z_{j,t}^{cus}\right) \times decile\left(z_{j,t}\right)$			0.028***	0.014***
			(0.002)	(0.002)
Time fixed effect	Yes	Yes	Yes	Yes
Industry fixed effect	Yes	Yes	Yes	Yes
Adjusted R^2	0.20	0.30	0.21	0.31
Observations	14,882	13,664	14,882	13,664

Note: Sample: 1976-2020 for Columns (1) and (3), and 2004-2020 for Columns (2) and (4). The dependent variables are the firm's log sales.

Then, we extend the regression to include the interaction between the partner's labor productivity ranking and the firm's labor productivity ranking as an additional independent variable.

$$y_{j,t} = \beta z_{j,t} + \eta \overline{decile}\left(z_{j,t}^{cus}\right) + \overline{decile}\left(z_{j,t}^{cus}\right) \times decile\left(z_{j,t}\right) + indu_j + \chi_t + \epsilon_{j,t}, \tag{3}$$

Columns (3) and (4) in Table 3 show the estimation results. The estimate for the coefficient of the interaction term is positive, indicating that the correlation between a firm's log sales and its partners' productivity ranking is higher when it has a higher ranking in labor productivity. In other words, the partners' labor productivity is more critical for a more productive firm than a less productive one.

This approach can be motivated by a simple example. Imagine we have firm j and its intermediate goods supplier k. Supplier k produces intermediate goods at a unit cost of $e^{w_t - \zeta_{k,t}}$, where w_t and $\zeta_{k,t}$ are log wage and supplier k's efficiency. We use the term *efficiency* to distinguish this shifter of the cost function from the measured labor productivity in our empirical exercises. Firm i output is determined by $Y_{j,t} = e^{x_t + f\left(\zeta_{j,t},\zeta_{k,t}\right)} L_{j,t}^{\alpha} M_{j,t}^{\gamma},\ \alpha + \gamma < 1$, where x_t is the aggregate TFP. The idiosyncratic TFP $g\left(\zeta_{j,t}, \zeta_{k,t}\right)$ can be potentially determined by both firms' efficiencies, $\zeta_{j,t}$ and $\zeta_{k,t}$. Finally, $L_{j,t}$ and $M_{j,t}$ are firm j's labor and intermediate inputs.

If factor markets are competitive, the firm j's first-order conditions are $\alpha e^{x_t + f\left(\zeta_{j,t},\zeta_{k,t}\right)} L_{j,t}^{\alpha-1} M_{j,t}^{\gamma} = e^{w_t}$ and $\gamma e^{x_t + f\left(\zeta_{j,t},\zeta_{k,t}\right)} L_{j,t}^{\alpha} M_{j,t}^{\gamma-1} = e^{w_t - \zeta_{k,t}}$, which be derived as:

$$\log\left(Y_{j,t}\right) = \underbrace{\log\left(\frac{Y_{j,t}}{L_{j,t}}\right)}_{\text{log labor prod}} + \underbrace{\frac{1}{1-\alpha-\gamma}\left[\gamma\zeta_{k,t} + f\left(\zeta_{j,t}, \zeta_{k,t}\right) + x_t - w_t + \overline{\epsilon}\right]}_{\text{log labor input}}, \tag{4}$$

where $\overline{\epsilon} = \gamma^{\frac{\gamma}{1-\alpha-\gamma}} \alpha^{\frac{1-\gamma}{1-\alpha-\gamma}}$ is a constant, which can be industry-specific. Equation (4) shows that firm j's log output can be decomposed among log labor productivity, the supplier k's efficiency ($\zeta_{k,t}$), a term that captures synergies ($f\left(\zeta_{j,t}, \zeta_{k,t}\right)$), and the aggregate state ($x_t - w_t$), exactly the form of regression (3).

Fact 3: Mismatches are less durable

Fact 3 is that the relationships of trading partners with very different economic fundamentals, which we refer to as mismatches, are less durable. More specifically, the larger the mismatch, the more likely the separation of trading partners.

First, we need a concept of mismatch. We measure the degree of mismatch in a trading relationship as the distance between two partners' deciles in the distribution of economic fundamentals (defined as in Fact 2) in the year preceding the establishment of the trading relationship, measured by $\Delta_{j,k,t} = |decile\left(\pi_{j,k,t}\right) - decile\left(\pi_{j,k,t}^{cus}\right)|$, where the variable $\Delta_{j,k,t}$ measures the degree of mismatch and takes values between zero and nine, with zero indicating no mismatch and a positive value indicating a mismatch.

Second, we estimate the regression:

$$\text{dur}_{j,k,t} = \beta \times \Delta_{j,k,t} + \chi_t + \epsilon_{j,k,t},$$

where $\text{dur}_{j,k,t}$ is the expected annual duration of trading relationship. In particular, the expected duration for a trading relationship that terminates in year $t+p$ had an expected duration of p

years in year t. Also, $\Delta_{j,k,t}$ is the degree of mismatch, and χ_t is the year fixed effect.

Table 4: Partnership duration and the degree of mismatch (Compustat Segment)

	(1) Labor productivity	(2) Profit/sales	(3) Sales Growth
$\Delta_{j,k,t}$	-0.078***	-0.071***	-0.153***
	(0.009)	(0.009)	(0.011)
Time fixed effect	Yes	Yes	Yes
Adjusted R^2	0.10	0.09	0.10
Observations	38,357	29,538	28,151

Note: Sample: 1976-2020. Standard errors are in the parentheses. *** denotes significance level at the 1%.

Table 5: Partnership duration and the degree of mismatch (Factset)

	(1) Labor productivity	(2) Profit/sales	(3) Sales Growth
$\Delta_{j,k,t}$	-0.043***	-0.102***	-0.136***
	(0.008)	(0.007)	(0.007)
Time fixed effect	Yes	Yes	Yes
Adjusted R^2	0.16	0.16	0.16
Observations	75,719	76,009	75,001

Note: Sample: 2003-2021. Standard errors are in the parentheses. *** denotes significance level at the 1%.

Tables 4 and 5 show that the duration of a trading relationship decreases with the degree of mismatch. The regression coefficient is statistically significant and quantitatively large across the alternative measures of economic fundamentals. These findings capture that the instability of the trading relationships increases for partnerships with different economic fundamentals. In contrast, partnerships among firms with similar economic fundamentals are stable and durable. For robustness, Table A.2 in the Appendix shows the cross-sectional results when we focus on the year proceeding the start of matches.

Fact 4: Idiosyncratic shocks lead to separation of trading relationships

Fact 4 is that a higher absolute value of idiosyncratic productivity shocks to either side of a trading relationship predicts a higher probability of separation in the subsequent years.

We proxy idiosyncratic productivity shocks with the change in the labor productivity and profit-to-sales ratio decile. Then, we study the relationship between the absolute value of idiosyncratic shocks to a firm and its trading partner and the subsequent separation of trading relationships by estimating:

$$\text{sep}_{j,k,t} = \beta_1 \times \left|\Delta decile\left(\pi_{j,k,t-1}\right)\right| + \beta_2 \times \left|\Delta decile\left(\pi^{cus}_{j,k,t-1}\right)\right| + \gamma_t + \epsilon_{j,t},$$

where $\text{sep}_{j,k,t}$ is a dummy variable which is equal to 1 if firm j terminates an existing partnership with customer k in year t, and the variables $|\Delta decile\left(\pi_{j,k,t-1}\right)|$ and $|\Delta decile\left(\pi^{cus}_{j,k,t-1}\right)|$ are the absolute value of the change of firm j and customer k's decile of labor productivity or the profit-to-sales ratio. Variable γ_t is a time fixed-effect that controls for the potential comovement between the time trends of separation and the magnitude of idiosyncratic shocks.

Table 6: Absolute value of idiosyncratic shocks and trading relationship separation (Compustat Segment)

	(1)	(2)	(3)	(4)	(5)	(6)
Meas. of fundamental	Labor productivity			Profit/sales		
$\lvert\Delta decile\left(\pi_{j,k,t-1}\right)\rvert$	0.027***			0.011***		
	(0.003)			(0.002)		
$\left\lvert\Delta decile\left(\pi^{cus}_{j,k,t-1}\right)\right\rvert$	0.016***			0.012***		
	(0.006)			(0.003)		
$\lvert\Delta decile\left(\pi_{j,k,t-2}\right)\rvert$		0.013***			0.023***	
		(0.003)			(0.002)	
$\left\lvert\Delta decile\left(\pi^{cus}_{j,k,t-2}\right)\right\rvert$		0.022***			0.014***	
		(0.006)			(0.003)	
$\Delta decile\left(\pi_{j,k,t-2}\right)$			0.004			-0.000
			(0.003)			(0.002)
$\Delta decile\left(\pi^{cus}_{j,k,t-2}\right)$			0.0002			0.002
			(0.006)			(0.003)
Time fixed effect	Yes	Yes	Yes	Yes	Yes	Yes
Adjusted R^2	0.13	0.13	0.13	0.12	0.12	0.12
Observations	23,150	22,000	22,000	25,596	23,997	23,997

Note: Sample: 1976-2020. Standard errors are in the parentheses. *** denotes significance level at the 1% level.

Tables 6 and 7 show the benchmark results in their Column (1). The estimation evinces a significant positive correlation between idiosyncratic shocks to either side of the match and the separation of a trading relationship. Column (2), using the lagged absolute value of the change

Table 7: Absolute value of idiosyncratic shocks and trading relationship separation (Factset)

	(1)	(2)	(3)	(4)	(5)	(6)
Meas. of fundamental	Labor productivity			Profit/sales		
$\lvert \Delta decile\,(\pi_{j,k,t-1}) \rvert$	0.010***			0.003***		
	(0.002)			(0.001)		
$\left\lvert \Delta decile \left(\pi^{cus}_{j,k,t-1}\right) \right\rvert$	0.012***			0.012***		
	(0.003)			(0.001)		
$\lvert \Delta decile\,(\pi_{j,k,t-2}) \rvert$		0.006***			-0.001	
		(0.002)			(0.001)	
$\left\lvert \Delta decile \left(\pi^{cus}_{j,k,t-2}\right) \right\rvert$		0.012***			0.006***	
		(0.003)			(0.001)	
$\Delta decile\,(\pi_{j,k,t-2})$			0.003			0.002
			(0.002)			(0.001)
$\Delta decile \left(\pi^{cus}_{j,k,t-2}\right)$			0.0003			-0.002
			(0.002)			(0.001)
Time fixed effect	Yes	Yes	Yes	Yes	Yes	Yes
Adjusted R^2	0.14	0.14	0.14	0.13	0.13	0.13
Observations	67,187	67,616	67,616	72,141	71,451	71,451

Note: Sample: 2003-2021. Standard errors are in the parentheses. *** denotes significance level at the 1% level.

in profit-to-sales ratio as the independent variable, delivers the same result. Our estimates support the assumption that separations are time-consuming (e.g., due to adjustment costs or long-term contracts) and positively correlated with changes in productivity and profitability in the preceding years.

As a robustness check, we check whether the separation in trading relationships depends on the sign of idiosyncratic shocks (i.e., on whether a firm's ranking or its trading partner rises or falls). Column (3) in the tables shows that if we use the simple change in profit in the regression instead of the absolute change, the coefficient is statistically insignificant, suggesting that the joint combination of positive and negative changes in profits is critical to account for the termination of trading relationships. In particular, our main results in Columns (1) and (2) do not point to large negative shocks dissolving trading relationships but to technological synergies between firms.

Fact 5: Micro uncertainty decreases output in connected industries

Fact 5 is that higher volatility of idiosyncratic productivity shocks in a sector is correlated with a fall in sectoral output paired with a fall in output in connected sectors. We construct measures of the volatility of idiosyncratic productivity using Compustat Fundamentals annual data that provides information on publicly listed firms in the United States. We focus on the post-1998 period since it comprises consistent measures of real output at the 3-digit NAICS industry level.[7]

We proxy the volatility in idiosyncratic productivity in two alternative ways commonly used in the literature as in Bloom et al. (2018). We use the inter-quartile range (IQR) of the profit-to-sales ratio, where sales are measured as net sales and profit as EBITDA. To avoid measurement bias generated by the entry and exit of firms, we trim the data to a fully balanced panel by retaining firms with a continuous record of profits and sales between 1998 and 2019. Moreover, as small industries tend to suffer from severe small sample bias when computing cross-sectional moments, we drop small industries with less than ten firms with a continuous record of profits and sales over our sample periods. Our adjusted panel provides yearly measures of the volatility of idiosyncratic shocks and output growth for 37 industries for 1998-2019.

Next, we construct an index for each industry that measures its connected industries' volatility. We define industry i's connected industries as those industries that account for more than 1% of industry i's trade in intermediate goods as imputed by the BEA input-output tables, which report input-output values of intermediate goods for 66 private industries in 3-digit NAICS. On average, an industry is connected to 20 other industries, and the three most connected industries account for around 45% of the total trade volume in intermediate goods. For each industry i, we derive an index $\sigma_{i,t}^{connect}$ that measures the volatilities in the connected industries by weighting our volatility measures by the value of input-output intermediate goods traded with industry i.

Then, we estimate the panel regression:

$$\Delta y_{i,t} = \beta_1 \times \sigma_{i,t} + \beta_2 \times \sigma_{i,t}^{connect} + \chi_i + \gamma_t + \epsilon_{i,t},$$

where $\Delta y_{i,t}$ is the growth rate of real gross output in each industry i at time t constructed using the BEA dataset, $\sigma_{i,t}$ is the constructed index of the volatility of idiosyncratic shocks

[7] We use GDP by industry data constructed by BEA and available between 1998 and 2018. Historical data for 1947-1997 is not consistent with the data for the post-1997 period.

for industry i, $\sigma_{i,t}^{connect}$ is the constructed measure of the volatility of idiosyncratic shocks for industry i's connected industries, and χ_i and γ_t are industry and time fixed-effect, respectively. The standard errors are clustered by industry.

Table 8: Volatility of idiosyncratic shocks in the connected industries is negatively correlated with output growth

	(1)	(2)
$\sigma_{i,t}$	-0.009**	-0.008***
	(0.004)	(0.003)
$\sigma_{i,t}^{connect}$	-0.080*	-0.062*
	(0.041)	(0.037)
$\Delta y_{i,t}^{connect}$		0.906***
		(0.196)
Time fixed effect	Yes	Yes
Industry fixed effect	Yes	Yes
Adjusted R^2	0.22	0.33
Observations	22×37	22×37

Note: Sample: 1998-2019. Standard errors are in the parentheses. Standard errors are clustered at the industry level. *, **, and *** denote significance level at the 10%, 5%, and 1%, respectively.

Column (1) in Table 8 shows results for our benchmark estimation where the volatility of idiosyncratic shocks is measured by the interquartile range (IQR) of the profit-to-sales ratio. The volatility of idiosyncratic shocks within an industry and in the industry's connected industries has a significant and contractionary effect on sectoral output growth. Important for our analysis, the effect of the volatility of idiosyncratic shocks from other connected industries has a larger negative impact on output growth compared to the volatility of idiosyncratic shocks originating within the same industry. This finding shows that the transmission of changes in the volatility of idiosyncratic shocks across industries is significant and hurts the industry's output.

Unfortunately, as it is common in the literature, we face the measurement of exogenous changes in the volatility of idiosyncratic shocks. Therefore, we cannot identify the causal effect of volatility on connected industries' output.[8] But to partially alleviate the issue induced by the lack of exogenous shock or instrumental variable, in Column (2), we include the output growth in industry i's connected industries, $\Delta y_{i,t}^{connect}$, as a control variable. $\Delta y_{i,t}^{connect}$ is computed as the mean of the gross output growth in industry i's connected industries, weighted by their

[8]See Fernandez-Villaverde and Guerron-Quintana (2020), Fernández-Villaverde et al. (2015), and Mumtaz and Zanetti (2013) for a discussion on the impact of volatility of shocks as a measure of economic uncertainty.

value of intermediate goods input and output traded with industry i. The coefficients of $\sigma_{i,t}$ and $\sigma_{i,t}^{connect}$ are still estimated as negative and statistically significant conditional on $\Delta y_{i,t}^{connect}$. The finding suggests that the negative effect of the volatility of idiosyncratic shocks in connected industries on industrial output is not a byproduct of the fall in output in connected industries that drags down industrial output.

Taking stock

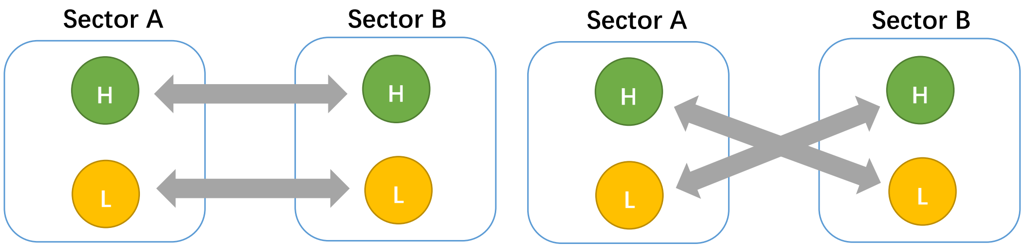
Facts 1-3 above suggest the existence of technological synergies between trading partners and that positive assortative matching of trading relationships is the stable equilibrium of a matching game. Facts 4 and 5 motivate us to investigate the role of technological synergies and idiosyncratic productivity shocks in a business cycle model. We move to do so now in two steps: first, with a simple model that illustrates the main mechanisms at work and, second, with a fully-fledged quantitative model. We will see how the latter replicates all five facts we just documented.

3 A simple model

We present a simple model that illustrates the interplay between inter-firm sorting, technological synergies, and idiosyncratic shocks. The economy is composed of two sectors, A and B. Each sector contains two firms, a firm H with high productivity, z^H, and a firm L with low productivity, z^L, where $z^H > z^L$.

Output $f\left(z^j, z^k\right)$ is produced by a trading relationship formed by two firms, each belonging to a different sector, where z^j is the productivity of the firm in sector A, and z^k is the productivity of firm in sector B. The output from the relationship is divided between a payoff for the firm in sector A, $f_A\left(z^j, z^k\right)$, and a payoff to the firm in sector B, $f_B\left(z^j, z^k\right)$. Aggregate output is $f\left(z^j, z^k\right) + f\left(z^{-j}, z^{-k}\right)$ (where $-i$ and $-k$ denote the other firm in each sector).

We call the relationships formed by firms of the same productivity *positive assortative matchings*, while we call the relationships of different productivity firms as *cross-matchings*. Panels (a) and (b) in Figure 2 illustrate each of these cases. We assume that firms are always matched to focus on the key mechanisms, but we will relax this assumption in Section 4.



(a) Positive assortative matching **(b)** Cross-matching

Figure 2: Alternative matching patterns

3.1 Positive assortative matching

First, we show that a standard assumption of monotonicity in the payoff functions produces positive assortative matching.

Assumption 1. *(Partial monotonicity). The payoff of high-productivity firm strictly increases with the partner's productivity. Specifically,* $f_A(z^H, z^H) > f_A(z^H, z^L)$*, and* $f_B(z^H, z^H) > f_B(z^L, z^H)$*.*

Assumption 1 implies that a high-productivity firm strictly prefers forming a relationship with a high-productivity partner because it generates a larger payoff than matching with a low-productivity partner. Assumption 1 generates positive assortative matching in equilibrium since the H-type firm forms a relationship with a H-type partner and a L-type firm is forced to form a relationship with a L-type partner.

Assumption 1 also generates stable matches in the sense of Gale and Shapley (1962). H-type firms matched with other H-type firms do not want to switch partners. In comparison, cross-matching is unstable since firms of H-type wish to separate from L-type firms and match with an H-type partner. Since cross-matching is unstable, we refer to it as *mismatch.*

Proposition 1 summarizes the effect of monotonicity for the sorting of firms across productivity types (the proof follows directly from Assumption 1).

Proposition 1. *(Positive assortative matching). Under the assumption of partial monotonicity, a trading relationship is stable if and only if it has positive assortative matching.*

3.2 Technological complementarity and its implications

Next, we show that technological complementarities make positive assortative matching the *efficient* equilibrium. But first, let us introduce the concept of supermodularity.

Definition 1. *(Supermodularity). A production function is supermodular and entails technological complementarity if* $f\left(z^H, z^H\right) + f\left(z^L, z^L\right) > f\left(z^H, z^L\right) + f\left(z^L, z^H\right)$.[9]

Definition 1 implies that the output of a relationship is greater with positive assortative matching than in a mismatch. Intuitively, supermodularity implies that firms have a comparative advantage in working with firms of the same productivity type. Supermodularity is embedded in standard production technologies and is widely used in economics (León-Ledesma and Satchi, 2019). For example, the Cobb-Douglas production function, $f\left(z^j, z^k\right) = \left(z^j\right)^\alpha \left(z^k\right)^{1-\alpha}$, is supermodular. Clearly, $\left(z^H\right)^\alpha \left(z^H\right)^{1-\alpha} + \left(z^L\right)^\alpha \left(z^L\right)^{1-\alpha} > \left(z^H\right)^\alpha \left(z^L\right)^{1-\alpha} + \left(z^L\right)^\alpha \left(z^H\right)^{1-\alpha}$ with $z^H > z^L$ and $0 < \alpha < 1$.[10]

Supermodularity delivers key results. For instance, assume that the economy starts from positive assortative matching with aggregate output $y = f\left(z^H, z^H\right) + f\left(z^L, z^L\right)$. Then, suppose that an unexpected idiosyncratic productivity shock hits sector A changing the firm with H-type from z^H to z^L and the firm with L-type from z^L to z^H (but productivity in sector B remains unchanged). If firms cannot re-match, the new aggregate output is $y' = f\left(z^L, z^H\right) + f\left(z^H, z^L\right) < y$. In other words, shocks that change firms' idiosyncratic productivities translate into lower output under supermodularity if firms cannot rearrange their matches.

More pointedly, changes in the variance of the idiosyncratic shock generate movements in aggregate output. To see this, assume that each sector is populated by a continuum of firms of size two (rather than two single firms). Half of the firms are H-type and the other half is L-type. Also, the economy starts from positive assortative matching with a measure one of HH- and LL-type relationship, respectively. The total payoff in sectors A and B are equal to $y_A = f_A\left(z^H, z^H\right) + f_A\left(z^L, z^L\right)$ and $y_B = f_B\left(z^H, z^H\right) + f_B\left(z^L, z^L\right)$.

[9] If the production function is twice differentiable, an equivalent definition of supermodularity is $\frac{\partial^2 f\left(z^j, z^k\right)}{\partial z^j \partial z^k} > 0$.

[10] A production technology can also be submodular, such that output is greater in mismatch than in positive assortative matching: $f\left(z^H, z^H\right) + f\left(z^L, z^L\right) < f\left(z^H, z^L\right) + f\left(z^L, z^H\right)$. An example of submodular production function is $f\left(z^j, z^k\right) = log\left(z^j + z^k\right)$. Moreover, the production function can be neither supermodular nor submodular. For instance, $f\left(z^j, z^k\right) = \left(z^j\right)^\alpha + \left(z^k\right)^\gamma$ implies the same output under positive assortative matching and mismatch.

We let the idiosyncratic shock in sector A follow a Markov-switching process with a transition matrix:

$$\begin{pmatrix} 1-\rho & \rho \\ \rho & 1-\rho \end{pmatrix},$$

where ρ is the probability of changing technology type. We continue to assume that there is no shock in sector B. Assuming a law of large numbers, ρ is also the fraction of firms in sector A that change productivity type and, hence, the share of mismatched relationships. Thus, the expected payoff in the next period for firms in sector A is:

$$y'_A = (1-\rho)\left[f_A\left(z^H, z^H\right) + f_A\left(z^L, z^L\right)\right] + \rho\left[f_A\left(z^L, z^H\right) + f_A\left(z^H, z^L\right)\right], \tag{5}$$

and for firms in sector B:

$$y'_B = (1-\rho)\left[f_B\left(z^H, z^H\right) + f_B\left(z^L, z^L\right)\right] + \rho\left[f_B\left(z^L, z^H\right) + f_B\left(z^H, z^L\right)\right]. \tag{6}$$

We can rewrite equations (5) and (6) as $y'_A = y_A - \rho\Delta y_A$, and $y'_B = y_B - \rho\Delta y_B$, where $\Delta y_i = f_i\left(z^H, z^H\right) + f_i\left(z^L, z^L\right) - f_i\left(z^H, z^L\right) - f_i\left(z^L, z^H\right)$, with $i \in \{A, B\}$, and Δy_i represents the difference of total output between positive assortative matching and mismatch. Thus, $\Delta y_i > 0$ if and only if the payoff function is supermodular. In other words, the total payoff in both sectors is strictly decreasing with ρ, or equivalently, the variance of idiosyncratic productivity shocks in sector A decreases aggregate output if the payoff function is supermodular.

3.3 Takeaways

The simple model establishes four results. First, under the assumptions of monotonicity and supermodularity in technology, positive assortative matching is the stable and efficient equilibrium, corresponding to Facts 1 and 3. Second, mismatching is an unstable and inefficient equilibrium, corresponding to Fact 2. Third, idiosyncratic productivity shocks transform positive assortative matching to mismatching, predicting separation of the relationship, which relates to Fact 4. Fourth, an increase in the variance of idiosyncratic productivity shocks in one sector generates a fall in the output of both sectors, consistent with Fact 5.

While our simple model parsimoniously accounts for our empirical facts, it is unsuitable for quantitative analysis. There are also critical observed patterns that the simple model cannot account for. In particular, sorting is far from perfect in the data, suggesting search frictions exist. Separation of relationship in response to idiosyncratic shocks is staggered rather than instantaneous, implying another form of friction missing in the simple model. Lastly, as shown by Shimer and Smith (2000) and Eeckhout and Kircher (2010), the condition for positive assortative matching becomes more stringent once we have search frictions. The following section will address these issues with a fully-fledged general equilibrium model.

4 A General equilibrium model

In this section, we reforge our simple model by adding households and firms that endogenously create and terminate relationships using directed search.

4.1 Households

There is a representative household of unitary size with a continuum of members and utility function:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\log\left(C_t\right) - N_t\right],$$

where $\mathbb{E}_0$ is the conditional expectation operator at time $t = 0$, C_t is consumption of final goods, N_t is labor input, and $\beta \in (0, 1)$ is the discount factor. For future reference, the household's stochastic discount factor is $\Lambda_{t+1} = \beta C_t / C_{t+1}$.

The household maximizes utility subject to the budget constraint $C_t = W_t N_t + \Pi_t$, where W_t is the wage rate set up in a competitive market, N_t is the total hours, and Π_t is the profit gained by the household from owning the firms.

4.2 Firms and technology

There is a unitary measure of intermediate- and final-goods producers, indexed by $l_I \in [0, 1]$ and $l_F \in [0, 1]$, respectively. An intermediate-goods producer must form a relationship with a final-goods producer to manufacture final goods. Such relationship is indexed by (l_I, l_F). A firm not part of a relationship stays idle. We call it a "single firm."

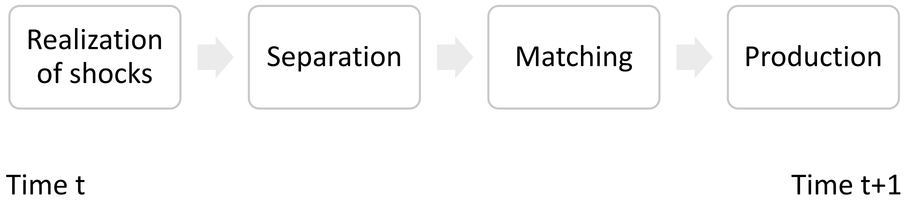


Figure 3: Timeline of firm events

At the beginning of each period t, firms experience aggregate and idiosyncratic productivity shocks and an exogenous separation shock with probability δ. Besides this exogenous shock, firms in a relationship might decide to separate from the current partner and become single. The firms produce if the partnership is not terminated either exogenously or endogenously. Otherwise, single firms search to form new relationships with partners from the opposite sector. At the end of each period t, each relationship sells the produced goods to the households in a competitive market. Figure 3 summarizes the firms' timeline.

The final output in the relationship (l_I, l_F) is $y_t(l_I, l_F) = e^{x_t + f\left(z_{I,t}(l_I), z_{F,t}(l_F)\right)} h_t(l_I, l_F)^{\alpha}$, where $y_t(l_I, l_F)$ is the final-goods output and $h_t(l_I, l_F)$ is labor input. As in Khan and Thomas (2013), we assume decreasing returns to scale (i.e., $0 < \alpha < 1$) to prevent exclusive allocation of labor to the most productive firms. The variables $z_{I,t}(l_I)$ and $z_{F,t}(l_F)$ are the log idiosyncratic productivity (defined below) for the intermediate goods producer and the final goods producers, respectively. The exogenous variable x_t is the log aggregate productivity shock that follows the AR(1) process $x_t = \rho_x x_{t-1} + \sigma_x \epsilon_{x,t}$, where $0 < \rho_x < 1$, and $\epsilon_{I,t} \sim i.i.d.\ \mathcal{N}(0,1)$.

The production function $f(z_{I,t}(l_I), z_{F,t}(l_F))$ determines the efficiency of a relationship in producing final goods. To encompass different degrees of technological complementarity, we consider a generalized technology function:

$$f(z_{I,t}(l_I), z_{F,t}(l_F)) = (1-\gamma)\left[z_{I,t}(l_I) + z_{F,t}(l_F)\right]/2 + \gamma \min\left[z_{I,t}(l_I), z_{F,t}(l_F)\right], \tag{7}$$

where γ encapsulates the degree of technological complementarity.[11] Equation (7) shows that

[11] Another way to encompass different degrees of technological complementarity is to use the CES production function in Jones (2011): $\left[z_{I,t}(l_I)^{\gamma}/2 + z_{F,t}(l_F)^{\gamma}/2\right]^{\frac{1}{\gamma}}$, where a low γ indicates strong technological complementarity. This production function converges to a Leontief technology when $\gamma \to -\infty$. The CES function requires all inputs to be positive, while our generalized production function allows for a negative log productivity $z_{i,t}$.

the log productivity of the relationship is a weighted average of the distinct idiosyncratic productivities in each sector, $z_{I,t}(l_I)$ and $z_{F,t}(l_F)$. The weight assigned to the firm with a lower productivity increases with γ. When $\gamma = 0$, the log productivity of the relationship becomes the unweighted mean of the productivity of the two firms. In this special case, the TFP of the relationship is $e^{x_t + f\left(z_{I,t}(l_I), z_{F,t}(l_F)\right)} = e^{x_t}\left(e^{z_{I,t}}\right)^{1/2}\left(e^{z_{F,t}}\right)^{1/2}$, which is the Cobb-Douglas function of the idiosyncratic productivities of the two firms scaled by the aggregate productivity. When $\gamma > 0$, the log productivity function becomes supermodular by assigning a larger weight to the firm with the lowest productivity in the production process.[12]

To study the interplay between the variance of the idiosyncratic shock and inter-firm sorting, we let the idiosyncratic productivities follow an AR(1) process with time-varying volatility, $z_{i,t}(l_i) = \rho_z z_{i,t-1}(l_i) + \sigma_{z,t}\epsilon_{i,t}(l_i)$, for $i \in \{I, F\}$, where $\epsilon_{i,t} \sim i.i.d.\ N(0,1)$, and $\sigma_{z,t}$ is the standard deviation of the idiosyncratic productivity shocks, which follows a Markov chain. See Fernandez-Villaverde and Guerron-Quintana (2020) for an empirical motivation.

Each relationship (l_I, l_F) chooses the labor input to maximize profits $\pi_t(l_I, l_F) = y_t(l_I, l_F) - h_t(l_I, l_F) W_t$. Profit maximization yields that each relationship produces:

$$y_t(l_I, l_F) = \left\{e^{x_t + f\left(z_{I,t}(l_I), z_{F,t}(l_F)\right)}\right\}^{\frac{1}{1-\alpha}} \left(\frac{W_t}{\alpha}\right)^{-\frac{\alpha}{1-\alpha}}. \tag{8}$$

Since output is identical across relationships with the same idiosyncratic productivities, for parsimony, we re-write equation (8) as:

$$y_t(z_I, z_F) = \left\{e^{x_t + f(z_I, z_F)}\right\}^{\frac{1}{1-\alpha}} \left(\frac{W_t}{\alpha}\right)^{-\frac{\alpha}{1-\alpha}}, \tag{9}$$

and express the profit of a relationship as a function of idiosyncratic productivities:

$$\pi_t(z_I, z_F) = (1-\alpha)\, y_t(z_I, z_F). \tag{10}$$

Equations (9) and (10) show that for $\gamma > 0$, the production function and the profit function

[12] For $\gamma = 1$, the log productivity of the relationship becomes a Leontief production technology, and it is determined by minimum value between $z_{F,t}(l_F)$ and $z_{I,t}(l_I)$, the special case that nests Kremer (1993). When $\gamma < 0$, the log productivity function is submodular.

are log-supermodular, a condition that will play a key role in the next section.[13]

4.3 Directed search and relationship formation

To form a relationship, firms must search for a firm in the opposite sector. We assume directed search: firms in each sector choose optimally the submarket with firms of the productivity type that they want to match with. The matching process is organized in a continuum of submarkets of productivity types, indexed by the idiosyncratic-productivity type of each sector, $(z_I, z_F) \in \mathbb{R}^2$. Specifically, single firms from sector I with a productivity of $\overline{z}_I$ can choose to enter any submarket $(\overline{z}_I, z_F)$, where $z_F \in \mathbb{R}$. Productivity is observable, i.e., firms in sector I with idiosyncratic productivity $\overline{z}_I$ cannot go to an alternative submarket (z_I, z_F) with $z_I \neq \overline{z}_I$. Analogously, single firms in sector F with productivity $\overline{z}_F$ can choose to enter any submarket $(z_I, \overline{z}_F)$ with $z_I \in \mathbb{R}$, but cannot enter a submarket (z_I, z_F) with $z_F \neq \overline{z}_F$.

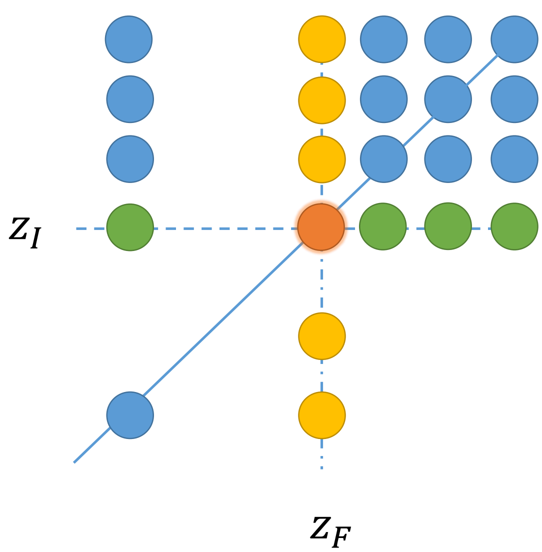


Figure 4: Organization of the submarkets

Figure 4 shows the organization of the submarkets. Each dot represents a submarket. A

[13]Formally, functions (9) and (10) are log-supernormal since:

$$log\left[y_t\left(\overline{z}_{I,t}, \overline{z}_{F,t}\right)\right] + log\left[y_t\left(\underline{z}_{I,t}, \underline{z}_{F,t}\right)\right] > log\left[y_t\left(\underline{z}_{I,t}, \overline{z}_{F,t}\right)\right] + log\left[y_t\left(\overline{z}_{I,t}, \underline{z}_{F,t}\right)\right], \text{ for } \overline{z}_{I,t} > \underline{z}_{I,t}, \overline{z}_{F,t} > \underline{z}_{F,t},$$

which implies that firms have a comparative advantage in working with partners with similar productivities, and positive assortative matching yields higher output and profit than mismatch. As we will show later, log-supermodularity is a sufficient condition for positive assortative matching under our benchmark calibration.

single firm F with productivity z_F can enter any submarket on the dash-dotted vertical line (the yellow and orange dots). Analogously, a single firm I with productivity z_I can enter any submarket on the dashed horizontal line (the green and orange dots). Under sectoral symmetry (i.e., the value functions and the distribution of firms are symmetric between the two sectors), positive assortative matching arises when firms only enter the submarket with firms of the same productivity type in the opposite sectors on the 45-degree line (the orange dot), and the alternative submarkets off the 45-degree line remain empty. We will establish later the sufficient condition for positive assortative matching to be a stable equilibrium, such that no pair of firms prefer to deviate from positive assortative matching and form a relationship with a firm in a submarket off the 45-degree line.

To formalize the process of directed search, we characterize the choice of a single firm I to enter a specific submarket as a function of its idiosyncratic productivity, $z_F^* = \mathsf{z}_{F,t}^*(z_I)$, where z_F^* is the productivity of partner that the single firm I is targeting by entering the submarket (z_I, z_F^*). Analogously, a single firm F's optimal choice of entering submarket is characterized by $z_I^* = \mathsf{z}_{I,t}^*(z_F)$, where z_I^* is the productivity of partner that the single firm F is targeting by entering the submarket (z_I^*, z_F).

Under sectoral symmetry, positive assortative matching is a set of decision rules $\mathsf{z}_{I,t}^*(z)$ and $\mathsf{z}_{F,t}^*(z)$ that satisfies $\mathsf{z}_{I,t}^*(z) = \mathsf{z}_{F,t}^*(z) = z$, for any z. The measure of single firms in sector I with productivity z_I is $\widetilde{n}_I(z_I)$ and the measure of single firms in sector F with productivity z_F is $\widetilde{n}_F(z_F)$. In addition, the measure of single firms from sector I with productivity z_I that choose to enter submarket (z_I, z_F) is $\widetilde{n}_I(z_I, z_F)$. Analogously, $\widetilde{n}_F(z_I, z_F)$ is the measure of single firms from sector F with productivity z_F that choose to enter submarket (z_I, z_F). Since single firms must choose one submarket to enter, the number of single firms in each submarket is equal to:

$$\widetilde{n}_{I,t}(z_I) = \int_{-\infty}^{\infty} \widetilde{n}_{I,t}(z_I, z_F)\, dz_F, \text{and } \widetilde{n}_F(z_F) = \int_{-\infty}^{\infty} \widetilde{n}_F(z_I, z_F)\, dz_I.$$

Under positive assortative matching, a firm enters the submarket with firms in the opposite sector that have the same productivity type, such that:

$$\widetilde{n}_{I,t}(z_I, z_F) = \begin{cases} \widetilde{n}_{I,t}(z_I) & \text{if } z_I = z_F \\ 0 & \text{if } z_I \neq z_F \end{cases} \text{ and } \widetilde{n}_{F,t}(z_I, z_F) = \begin{cases} \widetilde{n}_{F,t}(z_F) & \text{if } z_I = z_F \\ 0 & \text{if } z_I \neq z_F. \end{cases}$$

Our model differs from conventional models of frictional assignment with two-sided heterogeneity (Chade et al., 2017). For example, Shimer and Smith (2000) have symmetric buyers and sellers but assume random search. Eeckhout and Kircher (2010) considers directed search but with asymmetric buyers and sellers: the seller posts a price and the buyer decides in which submarket to shop. Our model aims at representing the process of relationship formation among firms given that their location and productivity are public information (suggesting directed search) and where firms are not inherently different in their position regarding price setting. Nonetheless, our model still delivers positive assortative matching under the mild sufficient conditions we will discuss later, and the equilibrium allocations are similar to those achieved by the studies above.

The formation of relationships in each submarket depends on the measure of single firms from each sector searching in the submarket. A constant-returns-to-scale matching function determines new relationship formation, $M\left(\widetilde{n}_I\left(z_I, z_F\right), \widetilde{n}_F\left(z_I, z_F\right)\right)$, where $\widetilde{n}_I\left(z_I, z_F\right)$ and $\widetilde{n}_F\left(z_I, z_F\right)$ are the measures of single firms in the two sectors defined above.

Conditional that a submarket $\left(z_I, z_F\right)$ has positive measures of visiting firms from both sectors (i.e., $\widetilde{n}_I\left(z_I, z_F\right) > 0$ and $\widetilde{n}_F\left(z_I, z_F\right) > 0$), the matching probability for firms in sector I in the submarket $\left(z_I, z_F\right)$ is:

$$\mu_I\left(z_I, z_F\right) = \frac{M\left(\widetilde{n}_I\left(z_I, z_F\right), \widetilde{n}_F\left(z_I, z_F\right)\right)}{\widetilde{n}_I\left(z_I, z_F\right)} = M\left(1, \theta\left(z_I, z_F\right)\right),$$

and, similarly, the matching probability for firms in sector F in the same submarket $\left(z_I, z_F\right)$ is

$$\mu_F\left(z_I, z_F\right) = \frac{M\left(\widetilde{n}_I\left(z_I, z_F\right), \widetilde{n}_F\left(z_I, z_F\right)\right)}{\widetilde{n}_F\left(z_I, z_F\right)} = M\left(1/\theta\left(z_I, z_F\right), 1\right),$$

where $\theta\left(z_I, z_F\right) = \widetilde{n}_F\left(z_I, z_F\right) / \widetilde{n}_I\left(z_I, z_F\right)$ is the tightness ratio in submarket $\left(z_I, z_F\right)$.

4.4 Firm value functions

Next, we define the firms' Bellman equations. The value $J_{I,t}\left(z_I, z_F\right)$ of the intermediate-goods producer that starts the period t in a relationship is:

$$J_{I,t}\left(z_I, z_F\right) = \left[\delta + \phi s_t\left(z_I, z_F\right)\right] \widetilde{J}_{I,t}\left(z_I\right) + \left[1 - \delta - \phi s_t\left(z_I, z_F\right)\right] \widehat{J}_{I,t}\left(z_I, z_F\right),$$

where $\widetilde{J}_{I,t}(z_I)$ is the value of a single intermediate-goods producer and $\widehat{J}_{I,t}(z_I, z_F) = \pi_{I,t}(z_I, z_F) + \mathbb{E}_t\left[\Lambda_{t+1} J_{I,t+1}\left(z_I^{'}, z_F^{'}\right)\right]$ is the value of continuing the relationship, equal to the flow profit of $\pi_{I,t}(z_I, z_F)$ whose size is established by Nash bargaining (to be described below), plus the expected discounted continuation value $\mathbb{E}_t \Lambda_{t+1} J_{I,t+1}(z_I', z_F')$. The term $s_t(z_I, z_F)$ (derived below) is an indicator of the endogenous termination of a relationship, equal to one if at least one firm prefers to terminate the relationship. The probability of endogenous separation, ϕ, reflects the observed staggered separation process outlined in Section 2.

The value of the final-goods producer $J_{F,t}(z_I, z_F)$ in a relationship at the start of period t is:

$$J_{F,t}(z_I, z_F) = [\delta + \phi s_t(z_I, z_F)] \widetilde{J}_{F,t}(z_F) + [1 - \delta - \phi s_t(z_I, z_F)] \widehat{J}_{I,t}(z_I, z_F),$$

where $\widetilde{J}_{F,t}(z_F)$ is the value of a single intermediate-goods producer and $\widehat{J}_{F,t}(z_I, z_F) = \pi_{F,t}(z_I, z_F) + \mathbb{E}_t\left[\Lambda_{t+1} J_{F,t+1}\left(z_I^{'}, z_F^{'}\right)\right]$ is the value of continuing the relationship.

The value of a single firm in sector I is:

$$\widetilde{J}_{I,t}(z_I) = \mu_{I,t}(z_I, z_F^*) \left\{\pi_{I,t}(z_I, z_F^*) + \mathbb{E}_t\left[\Lambda_{t+1} J_{F,t+1}\left(z_I^{'}, z_F^{*'}\right)\right]\right\},$$

where z_F^* is the productivity of the partner chosen by the single firm in sector I, $\mu_{I,t}(z_I, z_F^*)$ is the probability of forming a relationship in the chosen submarket, $\pi_{I,t}(z_I, z_F^*)$, and $\mathbb{E}_t\left[J_{I,t+1}\left(z_I^{'}, z_F^{*'}\right)\right]$ are the profit and the expected value conditional on relationship formation, respectively.

Similarly, the value of a single firm in sector F is:

$$\widetilde{J}_{F,t}(z_F) = \mu_{F,t}(z_I^*, z_F) \left\{\pi_{F,t}(z_I^*, z_F) + \mathbb{E}_t\left[\Lambda_{t+1} J_{F,t+1}\left(z_I^{*'}, z_F^{'}\right)\right]\right\},$$

where z_I^* is the productivity of partner that the single firm F is targeting.

Finally, we derive the indicator variable for the termination of a relationship. A relationship endogenously terminates if the value of becoming a single firm for any of the partners in the relationship exceeds the value of continuing with the relationship:

$$s_t(z_I, z_F) = \begin{cases} 0 & \text{if } \widehat{J}_{I,t}(z_I, z_F) \geq \widetilde{J}_{I,t}(z_I) \text{ and } \widehat{J}_{F,t}(z_I, z_F) \geq \widetilde{J}_{F,t}(z_F) \\ 1 & \text{if } \widehat{J}_{I,t}(z_I, z_F) < \widetilde{J}_{I,t}(z_I) \text{ or } \widehat{J}_{F,t}(z_I, z_F) < \widetilde{J}_{F,t}(z_F) \end{cases}.$$

4.5 Nash bargaining

The division of profits from the relationship is negotiated after the separation decision and before production. The total surplus of the relationship, $TS_t(z_I, z_F)$, is equal to the sum of the surpluses obtained by each firm in forming a relationship versus remaining a single firm, such that:

$$TS_t(z_I, z_F) = \left[\widehat{J}_{I,t}(z_I, z_F) - \widetilde{J}_{I,t}(z_I)\right] + \left[\widehat{J}_{F,t}(z_I, z_F) - \widetilde{J}_{F,t}(z_F)\right].$$

This surplus is split according to Nash bargaining, and the bargained profits, $\pi_{i,t}$, satisfy $\widehat{J}_{I,t}(z_I, z_F) - \widetilde{J}_{I,t}(z_I) = \tau TS_t$, and $\widehat{J}_{F,t}(z_I, z_F) - \widetilde{J}_{F,t}(z_F) = (1-\tau) TS_t$, where τ is the bargaining share of the intermediate-goods producer. Thus, a firm terminates a relationship if the total surplus becomes negative, and the indicator variable for endogenous termination becomes:

$$s_t(z_I, z_F) = \begin{cases} 0 & \text{if } TS(z_I, z_F) \geq 0 \\ 1 & \text{if } TS(z_I, z_F) < 0 \end{cases}.$$

4.6 Flow motion of firms

The measure of relationships after the realization of shocks and before separation and matching is:

$$m_t(z_I, z_F) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} n_{t-1}(\widehat{z}_I, \widehat{z}_F) \times g_{I,t}(z_I \mid \widehat{z}_I)\, g_{F,t}(z_F \mid \widehat{z}_F)\, d\widehat{z}_I d\widehat{z}_F,$$

where $\widehat{z}_I$ and $\widehat{z}_F$ are the productivities in the period $t-1$, and $n_{t-1}(\widehat{z}_I, \widehat{z}_F)$ is the measure of relationships from the period $t-1$ with productivities $(\widehat{z}_I, \widehat{z}_F)$. The conditional density $g_{j,t}(z_j \mid \widehat{z}_j)$ is the transition probability of idiosyncratic productivity in sectors j, as implied by their AR(1) process. The transition probability functions change over time due to time-varying volatility in the idiosyncratic shocks.

The measure of relationships after separation and matching is:

$$n_t(z_I, z_F) = [1 - \delta - \phi s_t(z_I, z_F)]\, m_t(z_I, z_F) + M_t(z_I, z_F),$$

where $[1 - \delta - \phi s_t(z_I, z_F)]$ is the fraction of relationships that survive separation and $M_t(z_I, z_F)$ is the measure of new relationship formation in the submarket (z_I, z_F).

The measure of single firms in sector I after the realization of shocks and before separation and matching is:

$$\widetilde{m}_{I,t}\left(z_I\right)=\int_{-\infty}^{\infty}\widetilde{n}_{I,t-1}\left(\widehat{z}_I\right)\times g_{I,t}\left(z_I\mid\widehat{z}_I\right)d\widehat{z}_I,$$

where $\widetilde{n}_{I,t-1}\left(\widehat{z}_I\right)$ is the measure of single firms in the previous period $t-1$ with productivity $\widehat{z}_I$.

After separation and matching, the measure of single firms in sector I is:

$$\widetilde{n}_{I,t}\left(z_I\right)=\left[1-\mu_I\left(z_I,z_F^*\left(z_I\right)\right)\right]\widetilde{m}_{I,t}\left(z_I\right)+\int_{-\infty}^{\infty}\left[\delta+\phi s_t\left(z_I,z_F\right)\right]m_t\left(z_I,z_F\right)dz_F, \tag{11}$$

where $\mu_I\left(z_I,z_F^*\left(z_I\right)\right)$ is the probability of forming a relationship in the optimal submarket $\left(z_I,z_F^*\left(z_I\right)\right)$ for the z_I-type single firms. The integrated term on the RHS of equation (11) is the measure of z_I-type single firms newly separated from relationships.

Similarly, the measure of single firms in sector F is:

$$\widetilde{m}_{F,t}\left(z_F\right)=\int_{-\infty}^{\infty}\widetilde{n}_{F,t-1}\left(\widehat{z}_F\right)\times g_{F,t}\left(z_F\mid\widehat{z}_F\right)d\widehat{z}_F$$

and

$$\widetilde{n}_{F,t}\left(z_F\right)=\left[1-\mu_F\left(z_I^*\left(z_F\right),z_F\right)\right]\widetilde{m}_{F,t}\left(z_F\right)+\int_{-\infty}^{\infty}\left[\delta+\phi s_t\left(z_I,z_F\right)\right]m_t\left(z_I,z_F\right)dz_I,$$

where $\widetilde{m}_{I,t}\left(z_I\right)$ and $\widetilde{n}_{I,t}\left(z_I\right)$ are the measure of single firms in sector F before and after separation and matching, respectively.

4.7 Positive assortative matching

Next, we establish sufficient conditions for positive assortative matching to be the stable equilibrium. As discussed in Subsection 4.3, positive assortative matching is the stable equilibrium if no firms prefer to meet in a submarket off the diagonal in Figure 4. We focus our analysis on the case of sectoral symmetry in which the two sectors have the same distribution of single firms, that is, $\widetilde{n}_{I,t}\left(z\right)=\widetilde{n}_{F,t}\left(z\right)$ for any t and z.[14]

[14]The existence of a symmetric equilibrium depends on three conditions: (1) same transition probability functions (i.e., $g_{I,t}\left(z\mid z'\right)=g_{F,t}\left(z\mid z'\right)$); (2) symmetric matching functions (i.e., $M\left(n,n'\right)=M\left(n',n\right)$); (3) symmetric decision rules (i.e., $z_{I,t}^*\left(z\right)=z_{F,t}^*\left(z\right)$, and $s_t\left(z,z'\right)=s_t\left(z',z\right)$). We have already assumed condition (1) and will assume condition (2) in our benchmark calibration. Condition (3) holds under conditions (1) and (2), and the joint surplus is split according to Nash bargaining.

Assortative matching without search frictions. Becker (1973) shows that in markets without search frictions, a supermodular surplus function $TS_t(z_I, z_F)$ is sufficient for positive assortative matching to be the stable equilibrium. To see the intuition, suppose the economy begins from an equilibrium with assortative matching. A pair of firms with idiosyncratic productivity $z_I = z$, $z_F = z'$ $(z \neq z')$ prefer to depart from positive assortative matching and establish a new relationship together if the total surplus in new joint ventures is larger than the total surplus in the ongoing relationship, which occurs if:

$$\tau TS(z, z') > \tau TS(z, z), \text{ and } (1-\tau) TS(z, z') > (1-\tau) TS(z', z') \tag{12}$$

hold simultaneously. Equation (12) implies $2TS(z, z') > TS(z, z) + TS(z', z')$, which cannot be satisfied when TS is supermodular. In other words, supermodularity ensures that no pair of firms prefer to deviate from the equilibrium with positive assortative matching.[15]

Assortative matching with search frictions. The main intuition of Becker (1973) continues to hold in our model. However, search frictions mean that a firm may prefer to enter a submarket with lower productivity firms but with a higher probability of forming a mutually beneficial relationship. Thus, for positive assortative matching to arise as a stable equilibrium, we need more stringent conditions on the supermodularity of the total surplus.

Formally, an intermediate-goods producer with idiosyncratic productivity $z_I = z$ would invite θ measure of final-goods producers with idiosyncratic productivity $z_F = z'$ to meet in the submarket (z, z') submarket (θ is also the tightness ratio for that submarket) if:

$$\tau \mu_I(\theta) TS(z, z') > \tau \mu_I(\theta(z, z)) TS(z, z), \tag{13}$$

where the LHS and the RHS of equation (13) are the expected surplus of the intermediate-goods producer in submarkets (z, z') and (z, z), respectively.

The final-goods producers with productivity z' would accept the invitation by going to the

[15] For simplicity, we assume that the two firms split the total surplus by Nash bargaining. However, the result of Becker (1973) applies to any bargaining rule.

new submarket (z, z') if:

$$(1-\tau)\,\mu_F(\theta)\,TS(z,z') > (1-\tau)\,\mu_F(\theta(z',z'))\,TS(z',z')\,, \tag{14}$$

where the LHS and the RHS of equation (14) are the expected surplus of the final-goods producer in submarkets (z, z') and (z', z'), respectively.

To incentivize the two sides to deviate from positive assortative matching, equations (13) and (14) must hold simultaneously, which implies that:

$$\frac{\mu_I(\theta)\,\mu_F(\theta)}{\mu_I(\theta(x,x))\,\mu_F(\theta(y,y))}TS^2(z,z') > TS(z,z)\,TS(z',z')\,. \tag{15}$$

Under sectoral symmetry, we have that $\theta(z,z) = \theta(z',z') = 1$, and equation (15) becomes:

$$\frac{\mu_I(\theta)\,\mu_F(\theta)}{\mu_I(1)\,\mu_F(1)}TS^2(z,z') > TS(z,z)\,TS(z',z')\,. \tag{16}$$

Equation (16) cannot be satisfied, which implies that equations (13) and (14) cannot hold simultaneously, if $\log[TS(z,z)] + \log[TS(z',z')] > \log(\mu_0) + 2\log[TS(z,z')]$, where we define:

$$\mu_0 = \max_{\theta} \frac{\mu_I(\theta)\,\mu_F(\theta)}{\mu_I(1)\,\mu_F(1)},\ \text{s.t. } \mu_I(\theta) < 1,\ \mu_F(\theta) < 1.$$

Note that $\log(\mu_0)$ can be zero or positive depending on the matching function. Under our benchmark calibration below with $\mu_I(\theta) = \psi\theta^{1/2}$ and $\mu_I(\theta) = \psi\theta^{-1/2}$, we have that $\mu_I(\theta)\,\mu_F(\theta) = \psi^2$ for any θ, and hence $\log(\mu_0) = 0$. In this case, log-supermodularity is a sufficient condition for positive assortative matching. Log-supermodularity is stronger than supermodularity: the former implies the latter, but the opposite does not hold. This is consistent with our previous argument that search frictions make positive assortative matching more difficult to achieve. Interestingly, log-supermodularity is also identified as a sufficient condition for positive assortative matching by Shimer and Smith (2000) and Eeckhout and Kircher (2010), who study alternative models of sorting whose market structures are very different from ours.[16]

[16] In contrast, Lentz (2010) show that supermodularity is sufficient for positive assortative matching when search intensity is endogenous.

4.8 Market-clearing conditions and equilibrium

Given a set of relationships $\Omega_t \subseteq [0,1] \times [0,1]$, aggregate output, $Y_t = \int_{\Omega_t} y_t(l_I, l_F)\, d(l_I, l_F)$, is the sum of final goods output produced by all the relationships in the economy. Notice that $\Omega_t \neq [0,1] \times [0,1]$ due to the presence of single firms that remain idle and forego production. Consumption equals output, $C_t = Y_t$, and the labor market clears when $N_t = \int_{\Omega_t} h_t(l_I, l_F)\, d(l_I, l_F)$. The definition of equilibrium is standard, and we omit it in the interest of space.

5 Calibration

We calibrate the model by matching the steady state of the model to post-WWII U.S. data at a quarterly frequency. Table 9 summarizes the calibration of the model.

Table 9: Calibration

Description	Parameter	Value
Preference and technology		
Discount factor	β	0.987
Labor share	α	0.66
Degree of supermodularity	γ	0.15
Matching, separation, and bargaining		
Matching efficiency	ψ	0.66
Matching elasticity	ι	0.5
Exogenous separation rate	δ	6.17%
Staggerness of endogenous separation	ϕ	0.25
Bargaining share of intermediate goods producers	τ	0.5
Shock process		
Persistence of aggregate productivity (prod.) shock	ρ_x	0.95
Standard deviation (std.) of aggregate prod. shock	σ_x	0.006
Persistence of idiosyncratic prod. shock	ρ_z	0.95
Std of idiosyncratic prod. shock (low uncertainty)	σ_z^L	0.039
Std of idiosyncratic prod. shock (high uncertainty)	σ_z^H	0.052
Transition prob. from low to high uncertainty	$\pi_{L,H}$	0.05
Transition prob. of remaining in high uncertainty	$\pi_{H,H}$	0.92

Conventional parameters. The discount factor, β, equals 0.987 (equivalent to 0.95 at a yearly frequency) to replicate the average annual interest rate of 5% over the sample period. The labor share, α, is set to 0.66 to match the labor share of income.

Following Khan and Thomas (2008), we set the persistence of the AR(1) processes for aggregate and idiosyncratic productivity, x_t and $z_{i,t}\left(l_i\right)$, to 0.95. The aggregate productivity shock standard deviation is 0.006, which implies that the quarterly standard deviation of aggregate productivity is 0.02, consistent with the estimates in Zanetti (2008).

We assume that the process for the time-varying volatility of idiosyncratic productivity shock, $\sigma_{z,t}$, follows a two-state Markov chain. $\sigma_{z,t} \in \left\{\sigma_z^L, \sigma_z^H,\right\}$, where $\Pr\left(\sigma_{z,t+1} = \sigma_z^k \mid \sigma_{z,t} = \sigma_z^j\right) = \pi^{kj}$. Following Bloom et al. (2018), we set σ_z^L, to 0.039. Since the variance of plan-level TFP shocks increased by 76% during the Great Recession of 2008 (an increase of 34% in the standard deviation), we set $\sigma_z^H = 0.052$. Also, after Bloom et al. (2018), we calibrate the transition probability from low to high uncertainty equal to 0.05 and the probability of remaining in high uncertainty equal to 0.92. Conditional on receiving an idiosyncratic shock that makes a relationship mismatched, Section 2 documents it takes one year for firms to separate, which implies $\phi = 0.75$. We let the firms in a relationship split the surplus evenly by setting $\tau = 0.5$.

We assume a standard Cobb-Douglas matching function $M\left(\widetilde{n}_I, \widetilde{n}_F\right) = \psi\left(\widetilde{n}_I\right)^{1-\iota}\left(\widetilde{n}_F\right)^{\iota}$, where ψ is the matching efficiency. We set $\iota = 0.5$ consistent with sectoral symmetry, implying that $\mu_I = \psi\theta^{\iota} = \psi\theta^{0.5}$ and $\mu_F = \psi\theta^{\iota-1} = \psi\theta^{-0.5}$. Hence $\mu_I\mu_F = \psi^2$ for any tightness ratio θ. Under this calibration, log-supermodularity (achieved for $\gamma > 0$) is a sufficient condition for the equilibrium with positive assortative matching.

Model-specific parameters. Three parameters are new in our analysis: the degree of technological complementarity in the production function, γ, the efficiency in the matching function, ψ, and the rate of exogenous separation of relationships, δ. We calibrate these parameters to jointly replicate three moments in the data: the average duration of relationships, the idleness rate (i.e., the fraction of single firms in the model), and the degree of sorting (i.e., the correlation of productivity between partners).

We target the average duration of a relationship to 16 quarters, consistent with the findings from Compustat data documented in Section 2. We target the fraction of single firms equal to the observed 12% average idleness rate in the U.S. non-manufacturing and manufacturing sectors before the Great Recession (Michaillat and Saez, 2015, and Ghassibe and Zanetti, 2022). We target the degree of sorting to 0.6. The correlation of labor productivity between trading partners equals 0.27 in the Compustat Segment data. However, the correlation of productivity is underestimated due to measurement errors. For example, Bils et al. (2021) establish that measurement errors are twice as important as productivity shocks in the measured productivity dispersion, implying that the correlation of productivity is, on average, 2/3 underestimated.[17] Moreover, firms are likely to pick their partners based on only weakly correlated (or uncorrelated) factors with measured productivity. For example, the producer of a mobile device might choose to work with a software company with an operating system with a high potential to build a rich ecosystem for app developers but low current measured productivity. Targeting the correlation of productivity to 0.27 would exaggerate the extent of misallocation and the resulting output loss. Thus, we adjust the original estimate from Compustat data by three to 0.6.

To match these targeted moments, we set $\gamma = 0.15$ such that the production function entails technological complementarity. In particular, using equation (7), the weight of the high-productivity firm in the log-productivity of a relationship is 9%, while the weight of the low-productivity firm is 91%. We set $\psi = 0.66$, implying that forming a relationship takes 1.51 quarters. We calibrate $\delta = 6.17\%$. Given that 0.8% of relationships separate endogenously in each period, the gross separation rate is 6.25%, which implies an average duration of a trading relationship of 16 quarters (see Hamano and Zanetti, 2017, for a discussion on the empirical estimates of the plant separation rate). Since the number of moments equals the number of unknown parameters, we can match our target exactly.

Table 10: The effect of γ, ψ, and δ on selected moments

	γ	ψ	δ
Duration of trading relationship	$\downarrow$	$\downarrow$	$\downarrow$
Degree of sorting	$\uparrow$	$\uparrow$	$\sim$
Idleness rate	$\uparrow$	$\downarrow$	$\uparrow$

Note: The symbols $\downarrow$, $\uparrow$ and $\sim$ indicate a decrease, increase, and unchanged effect of the degree of technological complementarity (γ), the efficiency in the matching frictions (ψ), and the rate of exogenous separation of relationships (δ) on the reported moments.

To illustrate how each parameter is identified, Table 10 and Figure 5 display the comparative statics on the effect of each parameter on the endogenous variables. In each panel of Figure 5, we fix two parameters and let the other parameter move around its calibrated value. The x-axis is the value of the moving parameter. The y-axis is the ratio of the moment implied by the model with the parameter value on the x-axis to the targeted value of the moment, which is

[17] Suppose $\widetilde{z}_{i,t} = z_{i,t} + \widetilde{\epsilon}_{i,t}$, where $\widetilde{z}_{i,t}$ is measured productivity, $\widetilde{\epsilon}_{i,t}$ is measurement error. By assuming that $\sigma\left(\widetilde{\epsilon}_{i,t}\right) \geq \sqrt{2}\sigma\left(z_{i,t}\right)$ and $corr\left(z_{i,t}, \widetilde{\epsilon}_{i,t}\right) = 0$, it entails that $corr\left(\widetilde{z}_{I,t}, \widetilde{z}_{F,t}\right) \geq 3corr\left(z_{I,t}, z_{F,t}\right)$.

equal to one when moment matching is successful (thus, the crossing of the three curves in each panel corresponds to our benchmark calibration).

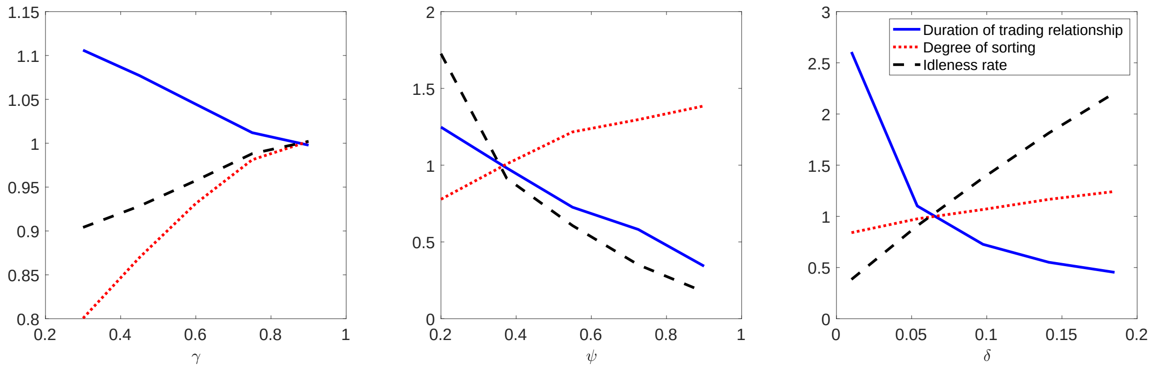


Figure 5: The effect of degree of technological complementarity (γ), the efficiency in the matching frictions (ψ), and the rate of exogenous separation of relationships (δ) on selected moments

We start with γ (the upper row of Table 10 and the left panel of Figure 5). A higher γ increases the degree of sorting. It also induces more endogenous separations, increasing the idleness rate and decreasing the relationships' average duration. Next, we discuss the role of ψ (the middle row of Table 10 and the middle panel of Figure 5). A higher ψ makes the reallocation of a relationship easier, decreasing the relationship duration and increasing the degree of sorting. It also improves the matching speed, leading to a lower idleness rate. Lastly, the bottom row of Table 10 and the right panel of Figure 5 show that a higher δ increases the idleness rate and decreases the average duration of a relationship, yet its effect on the degree of sorting is smaller than its effect on the other moments.

6 Quantitative analysis I: Steady state

This section studies the steady state of the model by fixing the aggregate productivity (x) at the normalized value of one. However, we still have idiosyncratic productivity shocks and compute the stationary distribution for relationships, single firms, and aggregate variables by simulating the model for 100,000 periods (we checked that those were more than enough for convergence).

We also consider three alternative calibrations that abstract from search frictions and staggered separation. In alternative calibration A, we assume that the parameter for matching efficiency ψ is 1 and the rate of staggered separation ϕ equals $1 - \delta$ (since the exogenous separation rate is δ, the gross probability of separation is $\delta + \phi$ for firms who want to separate). This frictionless calibration entails perfect sorting and no single firms. In alternative calibration B, we set $\phi = 1 - \delta$ (search frictions only), and, in alternative calibration C, we set $\psi = 1$ (staggered separation only). By comparing our benchmark calibration with the alternative calibrations, we can measure the role of search frictions and staggered separation for (i) the separation policy of relationships, (ii) the stationary distribution of relationships, and (iii) the level of aggregate output.

6.1 The separation policy

We first investigate the separation of relationships in the steady state by plotting, in grey, the values of the productivity of firms in sectors F (x-axes) and I (y-axes) where an existing relationship continues. For other values, the relationship is dissolved.

The top panel in Figure 6 shows the separation policy for our benchmark calibration. The grey area is wide: we have imperfect sorting because firms endogenously prefer to remain in a trading relationship with a partner of a different productivity type. Search frictions reduce the likelihood of forming a trading relationship upon separation and thereby lower the expected profits of re-matching. Although we do not have non-convex adjustment costs, the optimizing behavior of firms leads to endogenous separation of relationships that is reminiscent of the *Ss* policy rules outlined in Scarf (1963) and exploited in general equilibrium by Bloom (2009).

In comparison, in alternative calibration A (top-right panel), firms only stay in the relationship if they achieve perfect sorting: the region where the relationship continues is the 45-degree line. In alternative calibration B (bottom-left panel), the grey-shaded area remains sizeable. That is, search frictions explain the bulk of imperfect sorting across firms in a trading relationship. In alternative calibration C (bottom-right panel), we are back to a continuation region of just the 45-degree line: while staggered separation will hinder the realization of separation by construction, it does not discourage firms' separation decision without search friction.

6.2 Stationary distribution of relationships

Figure 7 plots the stationary distribution of relationships across different productivity levels for firms in sectors I and F implied by the separation policies above. The top-left panel

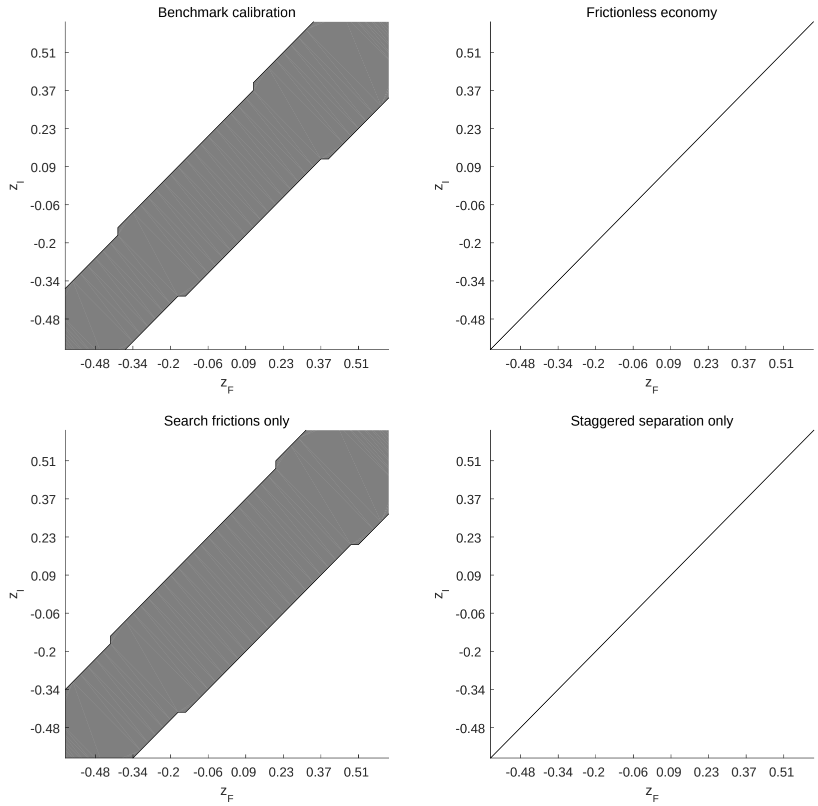


Figure 6: Separation policy: Benchmark and alternative calibrations

shows the distribution of relationships in the benchmark calibration. Despite perfect sorting being predominant in the steady state, as displayed by the larger density in the distribution of relationships along the 45-degree line, the economy entails a sizable fraction of mismatches, exhibited by the positive density off the 45-degree line. Recall that our model replicates the calibrated target of a degree of sorting of 0.6.

The top-right panel shows that the alternative calibration A begets perfect sorting: the distribution of relationships retains a positive mass on the 45-degree line of productivity and zero mass elsewhere. In this case, the degree of sorting equals 1. The bottom-left panel shows alternative calibration B, with a degree of sorting of 0.65, and the bottom-right panel shows alternative calibration C, with a degree of sorting of 0.85. Thus, we learn that the bulk of the mismatch in the steady state comes from search frictions, not staggered separations.

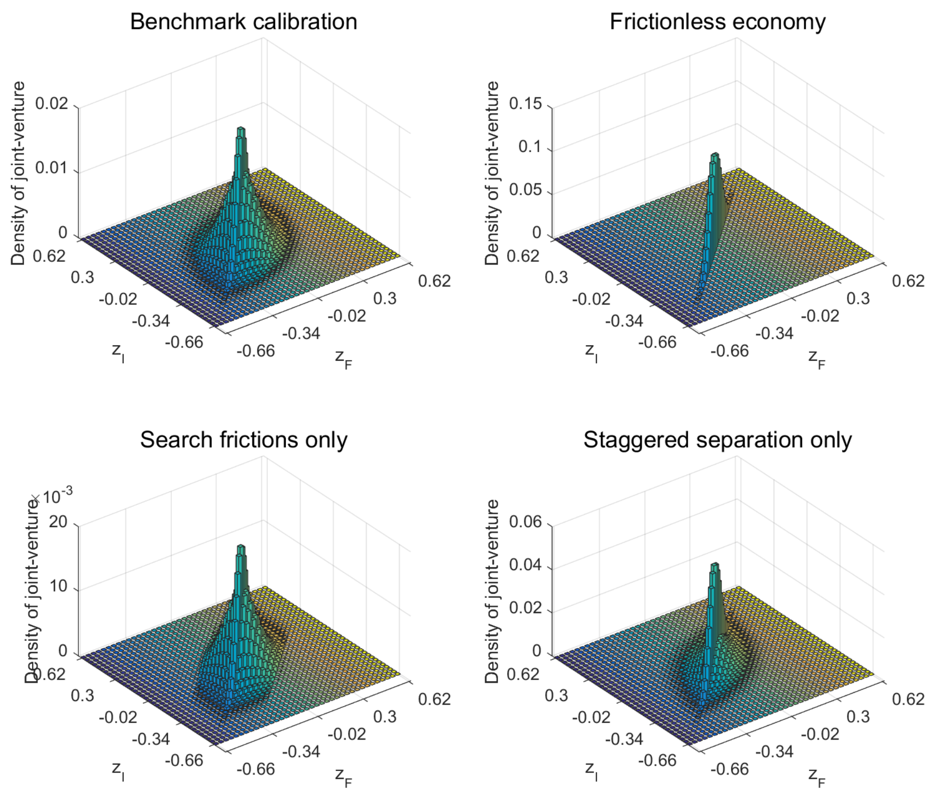


Figure 7: Distribution of relationships

6.3 Comparing model prediction to the firm level data

Next, we re-conduct the empirical analysis in Section 2 with the data simulated by our model and show that our model generates the same regressions as the data. We simulate 18,500 firms for 160 quarters.[18] Then, we convert the remaining quarterly data to yearly series (the time-frequency of Compustat Customer Segment data) with time averaging. Appendix D explains the details of the simulation.

Distribution of trading relationship's duration Figure 8 plots the histogram of match duration (in years) for the model's simulation and the Compustat data (the same as Figure 1), respectively. The figure demonstrates that the model generates a cross-sectional distribution of match duration close to the data.

[18]The simulated model has 37 productivity grids. Each grid accommodates 500 firms in the starting period.

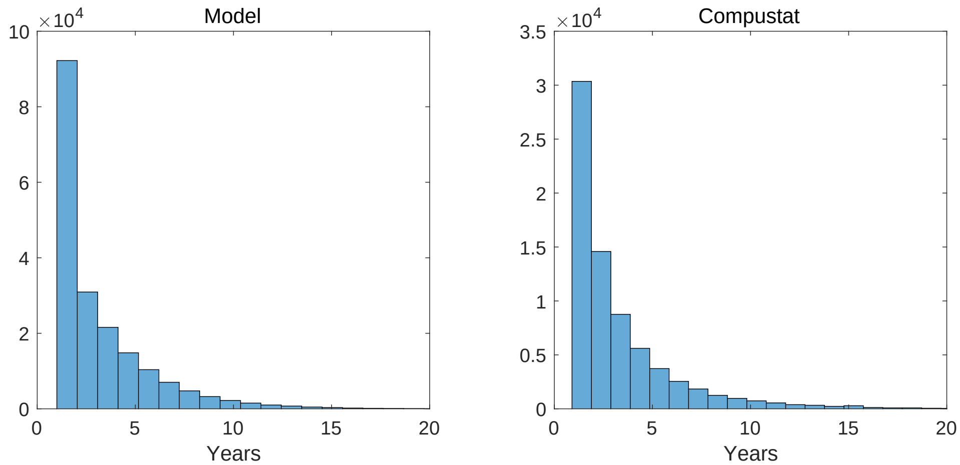


Figure 8: Distribution of trading relationship's duration

Positive assortative matching of relationships Does our model match the positive assortative matching in the data documented in Section 2? Table 11 reports the results of estimation for equation (1) with our simulated data and the actual data, respectively.

Table 11: Assortative matching for ranking of economic fundamental

	(1)	(2)	(3)	(4)
	Model		Data	
		Labor productivity	Profit/sales	Sales Growth
decile $(z_{I,t})$	0.742***	0.320***	0.067***	0.228***
	(0.012)	(0.014)	(0.014)	(0.015)
Constant	1.423***	4.133***	4.521***	4.562***
	(0.054)	(0.115)	(0.093)	(0.088)
Adjusted R^2	0.54	0.07	0.00	0.03
Observations	124,390	6,914	7,605	6,854

Note: Standard errors are in the parentheses. ** and *** denote significance level at the 5% and 1%, respectively.

In Column (1), the dependent variable, *decile* $(z_{F,t})$, is firm F's decile of productivity in the year before the start of the partnership simulated from the model. The independent variable, *decile* $(z_{I,t})$, is firm I's decile of productivity in the year before the start of the partnership. Columns (2)-(5) shows the results estimated with actual data, which is the same as Table 1. Both the model and the data indicate positive assortative matching in partnership formation. However, our model entails a stronger degree of sorting than the data as the coefficient is estimated higher in Column (1) than in Column (2). The high degree of sorting in the year before match formation is driven by construction. In particular, our directed search model predicts a perfect sorting in the quarter of partnership formation. Given that the volatility of idiosyncratic shocks is calibrated to a low value, the degree of sorting must be high the year before the start of the partnership. As we argued earlier, the relatively low degree of sorting measured in the data is likely explained by unobserved firm characteristics and measurement errors. Consequently, we consider the model prediction empirically plausible.

Mismatches are less durable Next, we examine whether our model implies that mismatches are less durable, an empirical fact documented in Section 2. Table 12 shows the estimation result for equation (2) with our simulated data and the actual data, respectively.

Table 12: Partnership duration and the degree of mismatch

	(1)	(2)	(3)	(4)
	Model		Data	
		Labor productivity	Profit/sales	Sales Growth
Δ_t	-0.126***	-0.078***	-0.071***	-0.153***
	(0.010)	(0.009)	(0.009)	(0.011)
Time fixed effect	Yes	Yes	Yes	Yes
Adjusted R^2	0.00	0.10	0.09	0.10
Observations	384,540	38,357	29,538	28,151

Note: Standard errors are in the parentheses. ** and *** denote significance level at the 5% and 1%, respectively.

Column (1) shows the model prediction. The dependent variable is the expected duration of the match. The independent variable is the distance between two partners' deciles in the distribution of productivity, measured by the metric, $\Delta_{I,F,t} = |decile\,(z_{I,t}) - decile\,(z_{F,t})\,|$. The $\Delta_{I,F,t}$ coefficient is estimated as negative and statistically significant, indicating that mismatches are less stable. Columns (2)-(6) report the empirical result estimated with actual data, which is the same as Table 4. The empirical results are close to the prediction of our model. The consistency of our model to the data is impressive, given that our calibration does not use any information from this regression.

Idiosyncratic shocks lead to separation of relationships Lastly, we examine if our model implies that idiosyncratic shocks predict the separation of relationships, an important observation documented in Section 2. Table 13 shows the estimation results for equation (2) with our simulated data and the actual data, respectively.

Table 13: Changes in productivity and match separation

	(1)	(2)	(3)
	Model	Data	
		Labor productivity	Profit/sales
$\lvert\Delta decile\,(z_{I,,t-1})\rvert$	0.008**	0.027***	0.011***
	(0.003)	(0.003)	(0.002)
$\lvert\Delta decile\,(z_{F,,t-1})\rvert$	0.010***	0.016***	0.012***
	(0.003)	(0.006)	(0.003)
Time fixed effect	Yes	Yes	Yes
Adjusted R^2	0.01	0.13	0.12
Observations	247,040	23,150	25,596

Note: Standard errors are in the parentheses. * and ** denote significance level at the 10% and 5%, respectively.

In Column (1), the dependent variable is a dummy variable, $sep_{I,F,t}$, which equals 1 if firm I terminates an existing partnership with firm F in year t,. The independent variables are the absolute values of the change of firm F and firm I's decile of productivity.[19] The result shows a significant and positive correlation between idiosyncratic shocks to either side of the match and the separation of the match. Columns (2) and (3) show the estimation result using actual data, which is the same as Columns (1) and (4) in Table 6. Once again, our model prediction is consistent with data even when our calibration does not depend on the estimated regression coefficient.

6.4 Aggregate output

Finally, we examine in this section the aggregate implication of the model. The aggregate output can be decomposed as the product of the measure of trading relationship and the output per trading relationship:

$$Y_t = \underbrace{\int_{\Omega_t} d\,(l_I, l_F)}_{\text{Measure of trading relationship}} \times \underbrace{\frac{\int_{\Omega_t} y_t\,(l_I, l_F)\,d\,(l_I, l_F)}{\int_{\Omega_t} d\,(l_I, l_F)}}_{\text{Output per trading relationship}}, \tag{17}$$

where Ω_t is the set of relationships. If all firms are matched in relationships, the rate of idleness is zero, and the measure of join-ventures is unitary. In contrast, if some firms fail to form a

[19] In particular, $\lvert\Delta decile\,(z_{I,t-1})\rvert = \lvert decile\,(z_{I,t-1}) - decile\,(z_{I,t-2})\rvert$ and $\lvert\Delta decile\,(z_{F,t-1})\rvert = \lvert decile\,(z_{F,t-1}) - decile\,(z_{F,t-2})\rvert$.

trading relationship, the rate of idleness is positive, and the measure of relationships is equal to one minus the rate of idleness. We can rewrite equation (17) with the more intuitive notation:

$$Y_t = (1 - \text{idleness}_t) \times \overline{y}_t, \tag{18}$$

where idleness_t is the idleness rate, and $\overline{y}_t$ is the output per trading relationship defined by the second term on the RHS of equation (17).

Frictions generate output gap by introducing a positive rate of idleness or reducing the output per trading relationship:

$$Y^n - Y_t \approx (\overline{y}^n - \overline{y}_t) + \overline{y}^n \times \text{idleness}_t, \tag{19}$$

where Y^n and $\overline{y}^n$ are the natural total output and natural output per trading relationship achieved in the steady state in the frictionless economy.[20]

Table 14: The effect of frictions on the the aggregate output

	(1) Benchmark calibration	(2) Frictionless economy	(3) Search frictions only	(4) Staggered separation only
Output	0.81	1.03	0.81	0.97
Output per trading relationship	0.92	1.03	0.93	0.97
Idleness rate	0.12	0	0.11	0

To study the contribution of the different components of output, Table 14 shows the idleness rate and the output per trading relationship for alternative calibration of the model. Column (1) shows the stationary steady state for output, output per trading relationship, and the idleness rate in the benchmark calibration of the model. Column (2) shows the results for the frictionless economy that abstracts from search frictions and staggered separation. Thus, the first and second rows correspond to Y^n and $\overline{y}^n$ in equation (19), respectively. The entries reveal that output is 21% higher in the frictionless economy. This output loss is the joint effect of search frictions and staggered separation, which generate a reduction of 11% in the output per-trading relationship, and an increase in the idleness rate of 12%.

[20]To derive equation (19), we take the total derivative of equation (18) at Y^n, $\overline{y}^n$, and idleness^n (the natural idleness rate):

$$Y_t - Y^n \approx (1 - \text{idleness}^n) \times (\overline{y}_t - \overline{y}^n) - \overline{y}^n \left(\text{idleness}_t - \text{idleness}^n\right),$$

and impose $\text{idleness}^n = 0$ (i.e., the idleness rate is equal to zero in the frictionless economy).

To disentangle the role of staggered separation and search frictions in determining output losses, we simulate the economy abstracting from each of the two frictions in turns. Column (3) shows results for the case of search frictions with instantaneous separation of relationships, while column (4) shows results without search frictions and delayed separation of relationships. Column (3) shows that search frictions alone explain most of the output loss in the benchmark case. In comparison, staggered separation plays a limited role in explaining output losses.

7 Quantitative analysis II: Aggregate TFP shocks

We move now to study the effect of aggregate TFP shocks. Figure 11 shows the impulse-response functions (IRFs) to a negative 10% TFP shock for aggregate output (left panel), the correlation of productivity within trading relationship, which measures the degree of sorting (middle panel), and the separation rate of trading relationship (right panel).

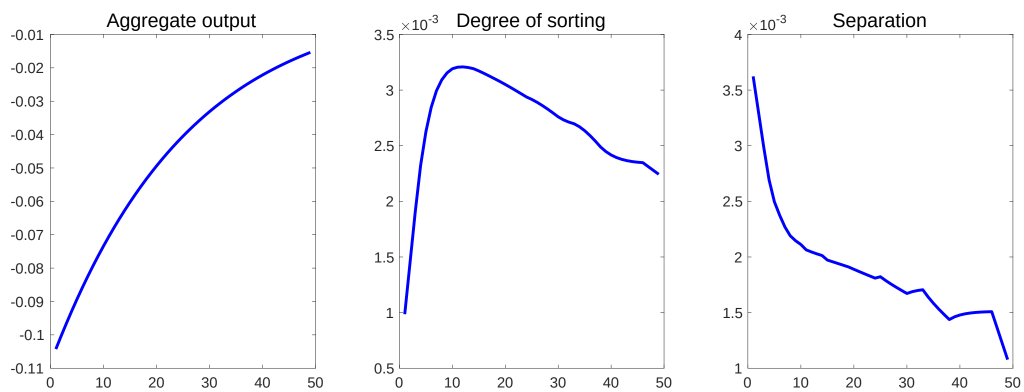


Figure 9: IRFs to a negative 10% TFP shock

The left panel shows that the aggregate output decreases in response to the decline in TFP. The middle and right panels show a slight improvement in the degree of sorting and a mild increase in separation increases, indicating a small cleansing effect of the decreasing TFP. The cleansing effect is as follows: because single firms cannot produce, unsuccessful matching after separation entails a loss of a cash flow stream. These cash flows are lower when TFP is low. Hence, a lower TFP implies a lower opportunity cost of separation. As a result, firms are more willing to separate and search for more efficient matches.

Figure 10 illustrates the cleansing effect by displaying the policy rules for separation in high TFP (10% above SS, dark-shadowed area) and low TFP (10% below SS, light-shadowed area) states, respectively. The figure shows that more mismatches dissolve in the low TFP state than in the high TFP state. This will lead to a more efficient allocation of matches. However, the cleansing effect is negligible since the two shadowed areas almost overlap. In our model, matches with different degrees of sorting are affected by aggregate TFP shock almost uniformly, making the value gap between different matches relatively inelastic to aggregate TFP. Hence the incentive of improving the degree of sorting responds mildly to aggregate TFP. The cleansing effect can be much stronger once we introduce mechanisms that make mismatches more sensitive to aggregate TFP shock.

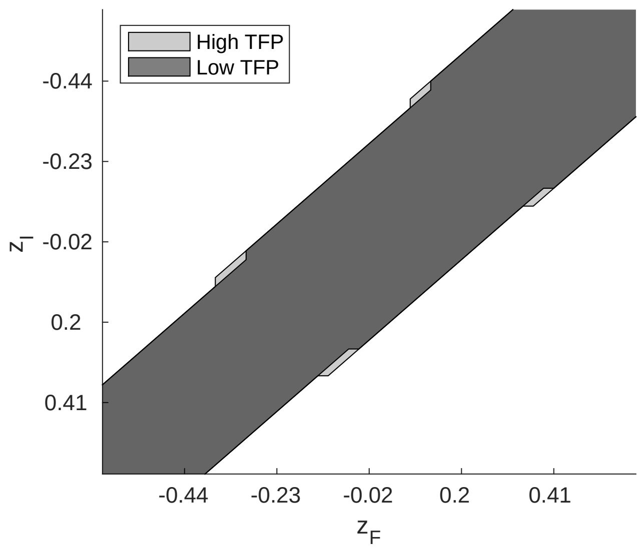


Figure 10: TFP and separation decision

8 Quantitative analysis III: Uncertainty shocks

Our next step is studying the effect of uncertainty shocks that result from an increase in the variance of idiosyncratic productivity shocks. We follow the approach in Bloom et al. (2018) and simulate 400 economies independently for 200 periods. We let each economy have low uncertainty in the first 100 periods (to settle the distribution toward an area where low uncertainty has been prevalent for some time), increase uncertainty to a higher level from period 101 onwards, and let the system evolve according to the Markov-transition process described in Section 3 from period 101 onwards. Then we take the mean of the time series across the simulated economies. Since the stochastic discount factor and the wage rate are functions of the distribution of firms that fluctuates over time due to uncertainty shocks, we implement a dimensionality reduction algorithm inspired by Krusell and Smith (1998). See Appendix E for details.

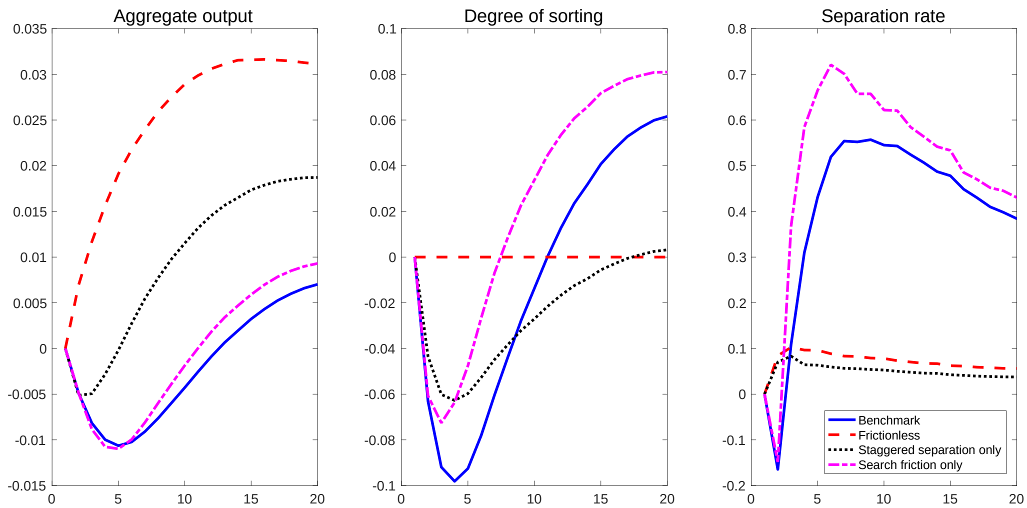


Figure 11: IRFs to an uncertainty shock

Figure 11 shows the IRFs to the uncertainty shock from period 100 to period 150 or aggregate output (left panel), the correlation of productivity within trading relationships (middle panel), and the separation rate of trading relationship separation (right panel). Responses are represented in percentage deviations. The blue solid line and the red dashed line show the responses for the benchmark and frictionless models, and the black-dotted and magenta-dotted lines show the responses for the staggered separation only and search friction only, respectively.

We start by focusing on the benchmark model. The increase in uncertainty reduces aggregate output by 1.2% in the four periods after the shock, before the economy starts recovering. The initial drop in output is driven by the increase in the measure of relationships with mismatched productivity types that generate inefficient production.

In the frictionless economy, output increases in response to the rise in uncertainty. Without frictions, an increase in uncertainty generates a raise in the mass of firms that manufacture output with high idiosyncratic productivity, and firms in relationships with mismatched types of idiosyncratic productivity terminate the ongoing partnerships and form new relationships with equally productive partners. In other words, the fall in output in the benchmark calibration is primarily determined by search frictions since the output drop remains large in the absence of delayed separation of joint ventures, as shown by the magenta dash-dotted line in the left panel of the figure. The economy with delayed separation but without search frictions, represented by the black dotted line, shows a mild initial drop in output, followed by a persistently higher level of output as in the frictionless economy.

The middle panel in Figure 11 studies the degree of sorting, represented as before by the correlation of productivity between partners within relationships. The blue line shows that uncertainty sharply and persistently reduces the degree of sorting. The red dashed line shows that the degree of sorting remains unchanged in the frictionless economy as firms instantaneously establish relationships between equally productive firms. The comparison between the economy with search frictions only (magenta dash-dotted line) and staggered separation only (black dotted line) illustrates that the two frictions evenly contribute to the overall drop in the degree of sorting.

The right panel in Figure 11 plots the rate of trading relationship separation. The uncertainty shock decreases separation in the first period after the shock, followed by a sharp and persistent increase in separation. In the frictionless economy, separation mildly increases, driven by the technological complementarity that indices firms with different productivity types to separate. With search frictions only, separations rise substantially, even more than in the benchmark case. Compared to the benchmark case, firms in mismatched relationships can separate instantaneously. Finally, staggered separations only have less effect on the rate of separation than the frictionless case, as separation is time-consuming.

An important question from our benchmark results is: What drives the decline in the measure of separation in the first period after the shock? To answer the question, we look at the effect of uncertainty on the decision rule for firms' separation. A higher uncertainty increases the probability that a positive assortative matching becomes mismatched (hence reduces the value of positive assortative matching). Meanwhile, a higher uncertainty also increases the probability that a mismatch becomes positive assortative matching (hence increasing the value of mismatch). Therefore, a higher uncertainty narrows the gap of value between mismatched relationships and positive assortative matching, making mismatched trading relationships less willing to pursue a re-match.

Figure 12 shows the policy rules for separation in low uncertainty states (dark-shadowed area) and high uncertainty states (light-shadowed area), respectively. With high uncertainty, many firms remain in mismatched relationships across all productivity levels. In other words, uncertainty discourages endogenous separation, which heightens the degree of mismatch across

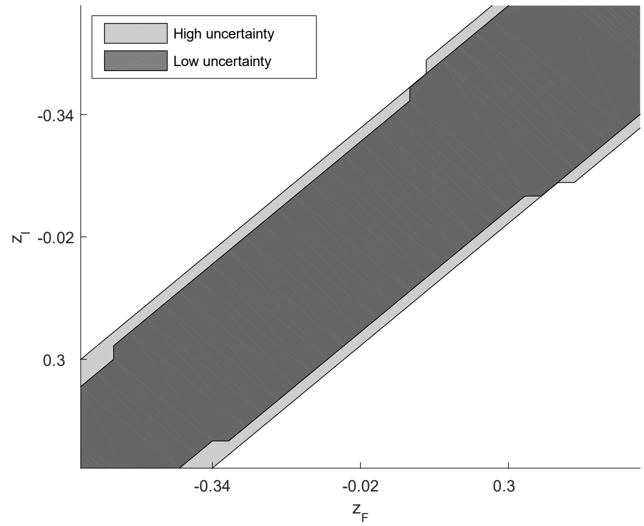


Figure 12: Uncertainty and separation decision

relationships and raises the loss in output, decreasing the degree of sorting (this result resembles the increase in inaction regions after an uncertainty shock in Bloom, 2009). In the second period after the shock, the mass of mismatch relationships increases, uncertainty diminishes, and the rate of trading relationship separation increases. As trading relationship formation is time-consuming, these new separations amplify the drop in output and halt the speed of the output recovery.

9 Conclusion

In this paper, we have documented five empirical facts about the creation of trading relationships among firms. These facts suggest the existence of technological synergies between trading partners that lead to positive assortative matching among firms and their potential impact on aggregate fluctuations.

Then, we have built a general equilibrium model with heterogeneous firms calibrated on new firm-level data and shown that frictions in forming trading relationships and separation costs explain imperfect sorting between firms by matching the model's predictions with the data. Among the most interesting quantitative implications of the model, we have illustrated how an increase in the volatility of idiosyncratic productivity shocks significantly decreases aggregate output without resorting to non-convex adjustment costs.

Our investigation opens many doors for future research, including extending the model to multiple-firm production networks, exploring the consequences of relationship-specific capital, and the effects of IT and automation on technological synergies. We hope to follow some of these ideas shortly.

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Appendix

This appendix provides extra robustness exercises related to our empirical findings and additional details about the computation of the paper.

A Alternative specifications

In the main text, we studied the impact of economic fundamentals on assortative matching using data from the year before the trading relationship was formed to control for the effect of common shocks. Here, Table A.1 shows that our results remain the same if we use data for the year when the relationship is formed.

Table A.1: Assortative matching for ranking of economic fundamentals, during match

	(1) Profit-to-sales ratio	(2) Profit	(3) Sales	(4) Sales Growth
decile $\left(\pi_{j,k,t}^{cus}\right)$	0.037***	0.267***	0.452***	0.215***
	(0.0107)	(0.010)	(0.010)	(0.008)
Time fixed effect	Yes	Yes	Yes	Yes
Adjusted R^2	0.02	0.07	0.17	0.08
Observations	29,982	30,172	32,096	28,597

Note: Sample: 1976-2020. Standard errors are in the parentheses. ** and *** denote significance level at the 5% and 1%, respectively.

Tables 4 and 5 in the main text proved that the duration of a trading relationship decreases with the degree of mismatch. Here, Table A.2 demonstrates the robustness of our results when we focus on the year proceeding the start of matches.

Table A.2: Partnership duration and the degree of mismatch, one year before the match

	(1)	(2)	(3)	(4)
	Profit-to-sales ratio	Profit	Sales	Sales Growth
$\Delta_{j,k}$	-0.062***	-0.202***	-0.223***	-0.136***
	(0.019)	(0.014)	(0.017)	(0.022)
Constant	3.927***	4.545***	4.536***	4.213***
	(0.075)	(0.072)	(0.073)	(0.08)
Adjusted R^2	0.00	0.03	0.02	0.01
Observations	7,750	7,901	8,349	6,969

Note: Sample: 1976-2020. Standard errors are in the parentheses. ** and *** denote significance level at the 5% and 1%, respectively.

B The gain from the establishment of trading relationship

In this appendix, we show that forming trading relationships increases the yearly growth rates of market value and sales, obtained from CRSP and Compustat Fundamentals Annual data, respectively. Therefore, firms have a strong incentive to establish trading relationships.

We measure partnership creation with a dummy variable equal to one if a firm establishes a trading relationship with a new firm in a given year. Thus, the variable $\text{cre}_{j,t}$ describes new joint-venture formalization for firm j in year t. We keep only the firms with continuous records of major customers between 1999 and 2014.

Table B.3: Partnership creation, sales and market value

	(1)	(2)	(3)	(4)
	Market Return	Sales Growth	Market Return	Sales Growth
$\text{cre}_{j,t}$	0.144**	0.026**	0.119*	0.008
	(0.065)	(0.012)	(0.067)	(0.012)
Firm fixed effect	Yes	Yes	Yes	Yes
Time fixed effect	No	No	Yes	Yes
Adjusted R^2	0.00	0.00	0.06	0.08
Observations	2,456	2,219	2,456	2,219

Note: Sample: 1999-2014. Standard errors are in the parentheses. * and ** denote significance level at the 10% and 5%, respectively.

Columns (1) and (2) in Table B.3 show that newly formed trading relationships are associated with a 14.4% increase in the growth rate of a firm's market value and a 2.7% increase in sales growth. Columns (3) and (4) run the same regressions controlling for year fixed effects. The relationship remains significant for market returns but not for sales growth. The analysis supports a positive relationship between joint-venture formation and firm profitability.

C Robustness checks for the effect of sectoral uncertainty shock

Section 2 established a negative and significant correlation between output growth and volatility of idiosyncratic shocks in connected industries. However, it did not identify the direction of causality on whether it is the volatility of idiosyncratic shocks in the connected industry that causes a fall in output growth or the fall in output growth in the connected industries that generate a downturn in output growth, as in Van Nieuwerburgh and Veldkamp (2006) or Bachmann and Moscarini (2011).

A direct assessment of the causality direction would require an instrumental variable that exogenously shifts volatility in connected industries without affecting output growth in the same industries. Unfortunately, such an instrument is not easily constructed since it is difficult to recover a proxy for the primitive exogenous shock to volatility at the industry level. Therefore, we provide support to the direction of causation from the volatility of idiosyncratic shocks in related industries to output growth in a given industry by showing that the data reject two critical implications from the reverse direction of causation.

A first alternative explanation for our result in the main text is that a fall in output in an industry is a byproduct of the fall in output in connected industries rather than a consequence of the increase in volatility of idiosyncratic shocks in connected industries. We can test the validity of this alternative explanation by including a measure of changes in real activity in connected industries ($\Delta y_{i,t}^{connect}$) as an independent variable in the regression (2). Suppose real activity in connected industries is critical to explain the fall in output in a given industry. In that case, the coefficient for the volatility in connected industries ($\sigma_{i,t}^{connect}$) becomes statistically insignificant once we enrich the estimation equation with a measure of real activity in connected industries.

Table C.4: Volatility of idiosyncratic shocks is not synchronized between connected industries

	(1)	(2)	(3)	(4)
Measure of volatility	IQR	IQR	corr. of rank	corr. of rank
$\sigma_{i,t}^{connect}$	0.28	−0.12	0.13	−0.06
	(0.17)	(0.23)	(0.09)	(0.11)
Time fixed effect	No	Yes	No	Yes
Industry fixed effect	Yes	Yes	Yes	Yes
R^2	0.00	0.03	0.02	0.03
Observations	16×42	16×42	16×42	16×42

Note: Sample: 1998 -2013. The dependent variable is the measure of volatility in industry i. $\sigma_{i,t}^{connect}$ and $\Delta y_{i,t}^{connect}$ are the mean of volatility measures and mean of gross output growth in the industry i's connected industries, weighted by their value of intermediate goods input and output traded with industry i obtained from 2007 NIPA input-output table. We keep only the firms with continuous records of major customers between 1998 and 2013. Standard errors are in the parentheses. Standard errors are clustered at the industry level.

Column (3) in Table 8 shows that the coefficient for the volatility of idiosyncratic shocks in connected industries remains negative and statistically significant, despite including the measure of real activity in connected industries in the estimation equation. The negative correlation between real activity and volatility within the industry remains equally significant, and it also shows the positive and significant correlation between output growth in an industry with output growth in related industries. Column (4) shows that results hold for the alternative measure of the volatility of idiosyncratic shocks based on the correlation of rankings. These results allow us to rule out the possibility that the fall in output in the industry correlated with volatility in the connected industries is driven by the fall in real activity in connected industries.

A second alternative explanation is that the fall in output in connected industries increases the volatility of idiosyncratic shocks in the linked industry, generating a fall in real activity in that industry, which our estimation equation interprets as a negative correlation between volatility of idiosyncratic shocks in related industries and output growth of the industry. We test this alternative explanation by studying whether volatility measures in the industry and the connected industries are significantly correlated. We estimate the panel regression:

$$\sigma_{i,t} \quad = \quad \beta \sigma_{i,t}^{connect} + \chi_i + \gamma_t + \epsilon_{i,t}. \tag{20}$$

Table C.4 shows results of the estimation of equation (20) for the volatility of idiosyncratic shocks measured as the interquartile range (IQR) of profit-to-sales ratio (columns (1) and (2)) and the autocorrelation of firms' profit-to-sale ratio ranking between consecutive years (columns (3) and (4)), controlling for industry and time fixed-effect. Entries consistently show that correlation coefficients are insignificant across specifications, thus suggesting that volatility is not synchronized between connected industries and thus ruling out the possibility that the negative effect of volatility in connected industries on output growth is a byproduct of the joint increase in the volatility of idiosyncratic shocks across all industries.

Overall, the analysis shows that the negative effect of the volatility of idiosyncratic shocks in linked sectors on sectoral output is neither a byproduct of the fall in output in connected industries that drags down sectoral output nor a consequence of the rise in sectoral volatility as a result of the increase of the volatility in linked sectors.

D Simulation with finite number of firms

Here, we explain how we simulate the model with idiosyncratic shocks to the firms. A key set of variables of the model are the allocation of single firms across submarkets. In particular, one needs to solve the matrix of $\widetilde{n}_{I,t}(z_I, z_F)$ such that no firm wants to deviate from the allocation given the matrix of tightness ratios, $\theta_t(z_{I,}z_F)$, which are implied by $\widetilde{n}_{I,t}(z_I, z_F)$.

Given the large number of unknowns, this is a difficult numerical problem. Fortunately, we can show that when the distribution of firms is symmetric between the two sectors and when log-supermodularity holds, the model entails positive assortative matching with the following:

$$\widetilde{n}_{I,t}(z_I, z_F) = \begin{cases} \widetilde{n}_{I,t}(z_I) & \text{if } z_I = z_F \\ 0 & \text{if } z_I \neq z_F \end{cases} \text{ and } \widetilde{n}_{F,t}(z_I, z_F) = \begin{cases} \widetilde{n}_{F,t}(z_F) & \text{if } z_I = z_F \\ 0 & \text{if } z_I \neq z_F \end{cases},$$

which further implies that: $\theta_t(z_{I,}z_F) = 1$ and $\mu(z_I, z_F) = \psi$ if $z_I = z_F$, where μ is the matching probability.

These theoretical results simplify our numerical analysis because the transition rule of $\widetilde{n}_{I,t}(z_I, z_F)$ and the equilibrium level of $\theta_t(z_{I,}z_F)$ and $\mu(z_{I,}z_F)$ are all analytically given. However, when we simulate the model with a finite number of firms in both sectors (as one is forced to do in practice), it is impossible to achieve sectoral symmetry given that the idiosyncratic shocks are stochastic and independent, which prevents us from using the above theoretical results. We simulate the model by imposing sectoral symmetry with the following procedure to solve the issue.

In period 0, we randomly draw N firms I. We assume each is matched to a firm F with the same productivity.

In period t:

- Step 1: Each firm I is hit by an idiosyncratic shock.
- Step 2: Each F that is matched to a firm I is hit by an idiosyncratic shock.
- Step 3: Trade relationships decide whether to separate. If they continue to match, they jump to Step 6. If they separate, they proceed to Step 4.
- Step 4: Every single firm I is matched to a firm F with the same productivity with probability ψ. If they fail to match, they jump to period t+1. If they form a new match, they proceed to Step 5. Notice that here we are imposing sectoral symmetry and positive assortative matching to the single firms.
- Step 5: For every single firm I that forms a new match in period t, we simulate the history of productivities for its new partner firm F.
- Step 6: Trading relationships produce according to the production function.

E Solution with uncertainty shocks

When the volatility of idiosyncratic shocks is stochastic, the distribution of firms, Ω_t, becomes a time-varying state variable in a trading relationship's value and policy functions. In particular, a trading relationship state space consists of $(z_{I,t}, z_{F,t}, \sigma_t, \Omega_t)$, which is infinitely dimensional. The intuition is that the stochastic discount factor Λ_{t+1} and the wage W_t, which are used to discount the future utility and to determine the labor demand, depend on aggregate consumption. And since aggregate consumption depends on the distribution of firms, firms need to keep track of the transition of Ω_t to make decisions.

We simplify the model solution with a set of forecasting rules:

$$\Lambda_t = \alpha_{1,\Lambda} + \alpha_{1,\Lambda}\Lambda_{t-1} + \alpha_{1,\Lambda}\sigma_{t-1} + \alpha_{1,\Lambda}\sigma_t$$

$$\Lambda_{t+1}(\sigma_{t+1}) = \alpha_{1,\Lambda} + \rho_{2,\Lambda}\Lambda_{t-1} + \beta_{2,\sigma}\sigma_{t-1} + \gamma_{2,\sigma}\sigma_t + \phi_\sigma\sigma_{t+1}$$

$$W_t = \alpha_W + \rho_W\Lambda_{t-1} + \beta_W\sigma_{t-1} + \gamma_W\sigma_t$$

where $A = (\alpha_{1,\Lambda}, \alpha_{1,\Lambda}, \alpha_{1,\Lambda}, \alpha_{1,\Lambda}, \alpha_{1,\Lambda}, \rho_{2,\Lambda}, \beta_{2,\sigma}, \gamma_{2,\sigma}, \phi_\sigma, \alpha_W, \rho_W, \beta_W, \gamma_W)$ is the vector of coefficients to be determined. The second forecasting rule for $\Lambda_{t+1}(\sigma_{t+1})$ is contingent on the realization of σ_{t+1}. Intuitively, firms do not need to know the transition process of Ω_t to make decisions if the forecast rule is accurate, which reduces the dimension of the space to a finite number. In particular, the new state space of the trading relationship is $(z_{I,t}, z_{F,t}, \sigma_t, \sigma_{t-1}, \Lambda_{t-1})$.

To do so, we proceed as follows:

- Step 1: We initialize the forecasting rule with some initial guess:

$$A^{(0)} = \left(\alpha_{1,\Lambda}^{(0)}, \alpha_{1,\Lambda}^{(0)}, \alpha_{1,\Lambda}^{(0)}, \alpha_{1,\Lambda}^{(0)}, \alpha_{1,\Lambda}^{(0)}, \rho_{2,\Lambda}^{(0)}, \beta_{2,\sigma}^{(0)}, \gamma_{2,\sigma}^{(0)}, \phi_\sigma^{(0)}, \alpha_W^{(0)}, \rho_W^{(0)}, \beta_W^{(0)}, \gamma_W^{(0)}\right)$$

- Step 2: We solve for the value functions, J_F, J_I, $\widehat{J}_F$, $\widehat{J}_I$, $\widetilde{J}_F$, $\widetilde{J}_I$, and the policy functions, s_F and s_I.

- Step 3: We simulate the model for 10,000 periods (disregarding the first 2,000 as a burn-in) with random draws of $(z_{I,t}, z_{F,t}, \sigma_t)$. Then, we compute the series of Λ_t and W_t.

- Step 4: Based on the simulated data, we update the coefficient of the forecast rule $A^{(q)}$ with $A^{(q+1)}$ using ordinary least squares. If $A^{(q)}$ and $A^{(q+1)}$ are sufficiently close to each other, we stop the iteration. Otherwise, we return to Step 2.

The converged forecasting rules explain the fluctuations of Λ_t, $\Lambda_{t+1}(\sigma_{t+1})$, and W_t well, with R^2 of 0.92, 0.95, and 0.87, respectively.