BANCA D'ITALIA
EUROSISTEMA

Aggregate Uncertainty, HANK, and the ZLB

Alessandro Lin (Bank of Italy) and Marcel Peruffo (USydney)

Workshop on Methods and Applications for Dynamic Equilibrium Models - NBER SI 2023

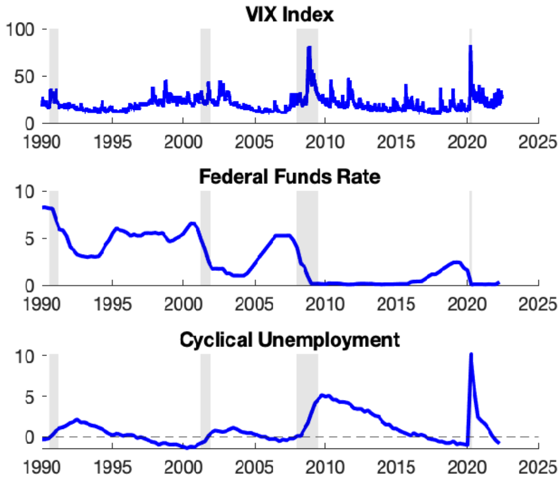
July 14th, 2023

Disclaimer

The views expressed herein are those of the authors and do not necessarily reflect those of the Bank of Italy or its executive board.

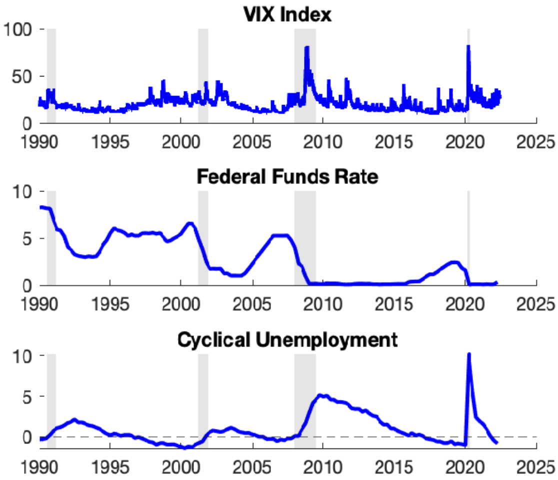
Outline

Figure 1: Mon. Policy, Micro-Macro uncertainty



1. Uncertainty rises in recessions : Bloom et al. (2018), Bloom (2014), Guvenen et al. (2014), Shimer (2005)

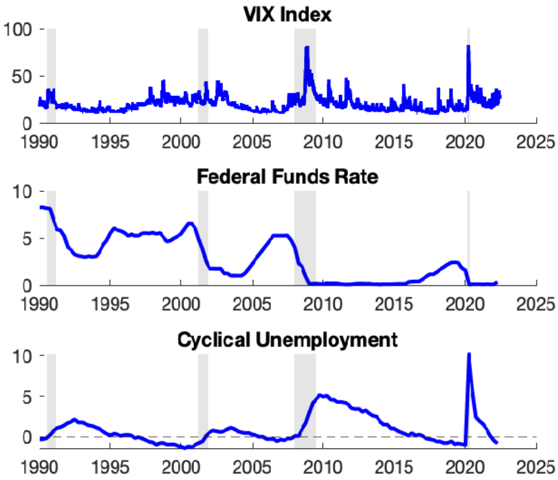
Figure 1: Mon. Policy, Micro-Macro uncertainty



1. Uncertainty rises in recessions : Bloom et al. (2018), Bloom (2014), Guvenen et al. (2014), Shimer (2005)
2. Aggr. uncertainty interacts with ZLB: Basu and Bundick (2016), Basu and Bundick (2017), Caggiano et al. (2017)

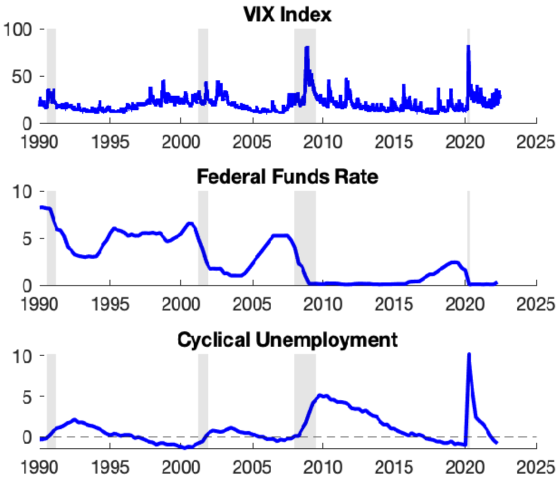
Introduction (1) - Motivation

Figure 1: Mon. Policy, Micro-Macro uncertainty



1. Uncertainty rises in recessions : Bloom et al. (2018), Bloom (2014), Guvenen et al. (2014), Shimer (2005)
2. Aggr. uncertainty interacts with ZLB: Basu and Bundick (2016), Basu and Bundick (2017), Caggiano et al. (2017)
3. What about idiosyncratic risk at the ZLB?

Figure 1: Mon. Policy, Micro-Macro uncertainty



1. Uncertainty rises in recessions : Bloom et al. (2018), Bloom (2014), Guvenen et al. (2014), Shimer (2005)
2. Aggr. uncertainty interacts with ZLB: Basu and Bundick (2016), Basu and Bundick (2017), Caggiano et al. (2017)
3. What about idiosyncratic risk at the ZLB?
   - We are interested in understanding this interaction...
   - via HANK-DSGE-model

Introduction (2) - Literature (Model)

Introduction (2) - Literature (Model)

1. HANK: Kaplan et al. (2018), Achdou et al. (2022), Ahn et al. (2018), Auclert et al. (2021), Auclert (2019), Bayer et al. (2019) ... Auclert et al. (2021)

Introduction (2) - Literature (Model)

1. HANK: Kaplan et al. (2018), Achdou et al. (2022), Ahn et al. (2018), Auclert et al. (2021), Auclert (2019), Bayer et al. (2019) ... Auclert et al. (2021)
2. ZLB: Eggertsson and Woodford (2003), Christiano et al. (2011), Eggertsson et al. (2021)

1. HANK: Kaplan et al. (2018), Achdou et al. (2022), Ahn et al. (2018), Auclert et al. (2021), Auclert (2019), Bayer et al. (2019) ... Auclert et al. (2021)
2. ZLB: Eggertsson and Woodford (2003), Christiano et al. (2011), Eggertsson et al. (2021)
3. HANK + ZLB: Eggertsson and Krugman (2012), Guerrieri and Lorenzoni (2017), McKay et al. (2016), Benigno et al. (2020)...

Introduction (2) - Literature (Model)

1. HANK: Kaplan et al. (2018), Achdou et al. (2022), Ahn et al. (2018), Auclert et al. (2021), Auclert (2019), Bayer et al. (2019) ... Auclert et al. (2021)
2. ZLB: Eggertsson and Woodford (2003), Christiano et al. (2011), Eggertsson et al. (2021)
3. HANK + ZLB: Eggertsson and Krugman (2012), Guerrieri and Lorenzoni (2017), McKay et al. (2016), Benigno et al. (2020)...perfect foresight (PF)

Introduction (2) - Literature (Model)

1. HANK: Kaplan et al. (2018), Achdou et al. (2022), Ahn et al. (2018), Auclert et al. (2021), Auclert (2019), Bayer et al. (2019) ... Auclert et al. (2021)
2. ZLB: Eggertsson and Woodford (2003), Christiano et al. (2011), Eggertsson et al. (2021)
3. HANK + ZLB: Eggertsson and Krugman (2012), Guerrieri and Lorenzoni (2017), McKay et al. (2016), Benigno et al. (2020)...perfect foresight (PF)
4. HANK + ZLB + Aggregate Uncertainty: Fernández-Villaverde et al. (2021), Kase et al. (2022), Schaab (2020)

Introduction (3) - Contribution

- Novel solution strategy for HANK models w/aggregate uncertainty (AU) and ZLB:
- In practice: take standard HANK, add ZLB, add tractable AU, compare to PF
- Solution allows to quantify interactions between AU-ZLB-HA

Introduction (3) - Contribution

- Novel solution strategy for HANK models w/aggregate uncertainty (AU) and ZLB:
- In practice: take standard HANK, add ZLB, add tractable AU, compare to PF
- Solution allows to quantify interactions between AU-ZLB-HA

	No ZLB		ZLB	
	Perf. Fores.	Agg. Unc.	Perf. Fores.	Agg. Unc.
Repr. Agent	A	A	B	C
Het. Agents	A	A	D	E

Introduction (3) - Contribution

- Novel solution strategy for HANK models w/aggregate uncertainty (AU) and ZLB:
- In practice: take standard HANK, add ZLB, add tractable AU, compare to PF
- Solution allows to quantify interactions between AU-ZLB-HA

	No ZLB		ZLB	
	Perf. Fores.	Agg. Unc.	Perf. Fores.	Agg. Unc.
Repr. Agent	A	A	B	C
Het. Agents	A	A	D	E

1. Quantify effect of aggregate uncertainty in HANK at the ZLB (E vs D)

Introduction (3) - Contribution

- Novel solution strategy for HANK models w/aggregate uncertainty (AU) and ZLB:
- In practice: take standard HANK, add ZLB, add tractable AU, compare to PF
- Solution allows to quantify interactions between AU-ZLB-HA

	No ZLB		ZLB	
	Perf. Fores.	Agg. Unc.	Perf. Fores.	Agg. Unc.
Repr. Agent	A	A	B	C
Het. Agents	A	A	D	E

1. Quantify effect of aggregate uncertainty in HANK at the ZLB (E vs D)
2. Quantify effect of aggregate uncertainty in RANK at the ZLB (C vs B)

Introduction (3) - Contribution

- Novel solution strategy for HANK models w/aggregate uncertainty (AU) and ZLB:
- In practice: take standard HANK, add ZLB, add tractable AU, compare to PF
- Solution allows to quantify interactions between AU-ZLB-HA

	No ZLB		ZLB	
	Perf. Fores.	Agg. Unc.	Perf. Fores.	Agg. Unc.
Repr. Agent	A	A	B	C
Het. Agents	A	A	D	E

1. Quantify effect of aggregate uncertainty in HANK at the ZLB (E vs D)
2. Quantify effect of aggregate uncertainty in RANK at the ZLB (C vs B)
3. Decompose the role of HA in the amplification (E-D vs C-B)

Introduction (3.1) - Contribution Extra

- Novel solution strategy...
- but not limited to ZLB, can accomodate more general non-linearities (kinky PC, aggregate borrowing constraints/financial accelerator, downward wage rigidity...)

Simple Model (1) - Description

- Simple model to define ZLB-AU interactions

Simple Model (1) - Description

- Simple model to define ZLB-AU interactions
- Infinitely lived households, standard consumption-savings decisions, CRRA preferences, exogenous discount factor β_t, rigid prices, Taylor rule

Simple Model (1) - Description

- Simple model to define ZLB-AU interactions
- Infinitely lived households, standard consumption-savings decisions, CRRA preferences, exogenous discount factor β_t, rigid prices, Taylor rule
- Idiosyncratic shock
  - c: constrained, no access to financial markets, earn $z_c Y_t$
  - u: unconstrained, access to financial markets, earn $z_u Y_t$

Simple Model (1) - Description

- Simple model to define ZLB-AU interactions
- Infinitely lived households, standard consumption-savings decisions, CRRA preferences, exogenous discount factor β_t, rigid prices, Taylor rule
- Idiosyncratic shock
  - c: constrained, no access to financial markets, earn $z_c Y_t$
  - u: unconstrained, access to financial markets, earn $z_u Y_t$

$$
\begin{aligned}
{Y_t}^{-\sigma} &= \frac{\beta_t R_t}{\beta R_{ss}} \mathbb{E}_t {Y_{t+1}}^{-\sigma}, && (1) \\
R_t &= \max\left\{\underline{R}, R_{ss} {Y_t}^{\phi}\right\} && (2) \\
R_{ss} &= \frac{1}{\beta\left\{p\left(z_u^{-\sigma}\right) + (1-p)\left[\left(\frac{1-\lambda z_u}{1-\lambda}\right)^{-\sigma}\right]\right\}}
\end{aligned}
$$

Simple Model (2) - Solution

$$Y_t = f\left(\mathbb{E}_t Y_{t+1}^{-\sigma}, \beta_t \middle| \beta, \sigma, \phi, \underline{R}, R_{ss}\right)$$
$$= \begin{cases} \left(\frac{\beta_t}{\beta}\mathbb{E}_t {Y_{t+1}}^{-\sigma}\right)^{-\frac{1}{\sigma+\phi}} & \text{if } \beta_t \leq \beta\left(\frac{R_{ss}}{\underline{R}}\right)^{\frac{\sigma+\phi}{\phi}}\left(\mathbb{E}_t Y_{t+1}^{-\sigma}\right)^{-1} \\ \left(\frac{\beta_t}{\beta}\frac{\underline{R}}{R_{ss}}\mathbb{E}_t Y_{t+1}^{-\sigma}\right)^{-\frac{1}{\sigma}} & \text{otherwise} \end{cases}, \tag{3}$$

- Higher future MU (or larger discount factor) leads to larger recession

Simple Model (3) - Shocks

Simple Model (3) - Shocks

1. The economy is at steady state at $t = 0$.

Simple Model (3) - Shocks

1. The economy is at steady state at $t = 0$.
2. Unexpected shock:
   - $\beta_0 = \beta$
   - $\beta_t = \beta$ for any $t > 1$

Simple Model (3) - Shocks

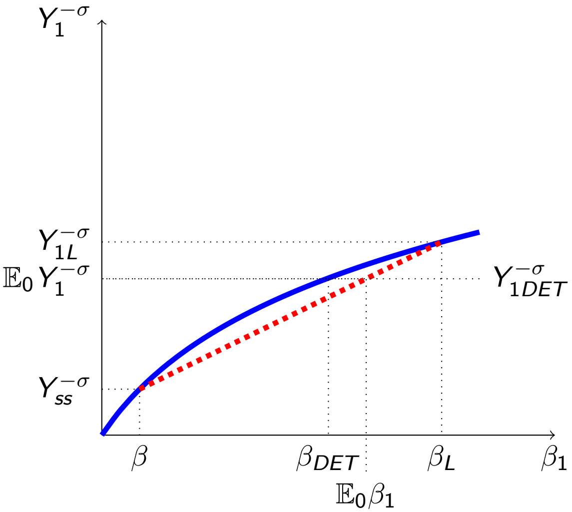
1. The economy is at steady state at $t = 0$.
2. Unexpected shock:
   - $\beta_0 = \beta$
   - $\beta_t = \beta$ for any $t > 1$
   - AU) $\beta_1 = \beta_L > \beta$ with probability μ, $\beta_1 = \beta$ otherwise

Simple Model (3) - Shocks

1. The economy is at steady state at $t = 0$.
2. Unexpected shock:
   - $\beta_0 = \beta$
   - $\beta_t = \beta$ for any $t > 1$
   - AU) $\beta_1 = \beta_L > \beta$ with probability μ, $\beta_1 = \beta$ otherwise
   - PF) $\beta_1 = \beta_{DET}$ such that same effect absent ZLB (i.e. $\underline{R} = -\infty$)

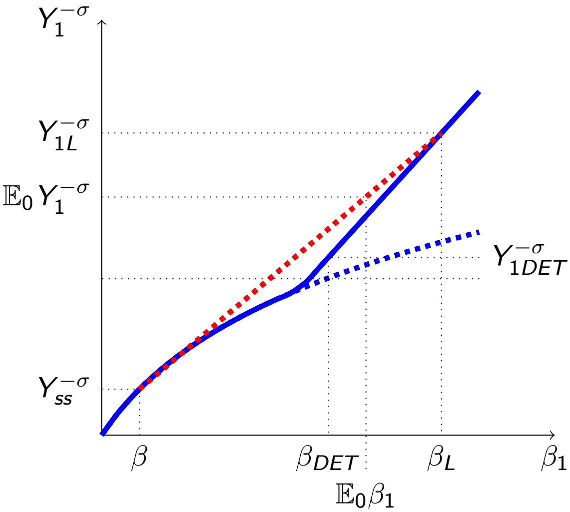
Simple Model (4) - Graphical Intuition

Figure 2: Equilibrium in the Simple Model



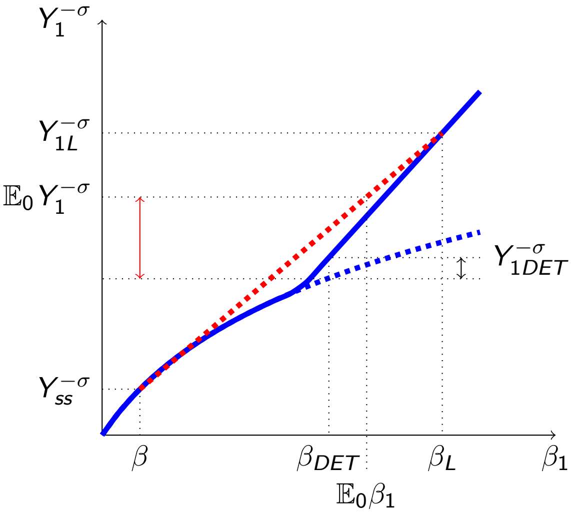
Simple Model (4) - Graphical Intuition

Figure 2: Equilibrium in the Simple Model



Simple Model (4) - Graphical Intuition

Figure 2: Equilibrium in the Simple Model



Simple Model (5) - Results' Intuitions

1. ZLB amplifies effect of shock because interest rate higher than it would have been.

Simple Model (5) - Results' Intuitions

1. ZLB amplifies effect of shock because interest rate higher than it would have been. True even with PF...

Simple Model (5) - Results' Intuitions

1. ZLB amplifies effect of shock because interest rate higher than it would have been. True even with PF...
2. AU shock interacts with ZLB, implies further amplification, because of Jensen's inequality.

Simple Model (5) - Results' Intuitions

1. ZLB amplifies effect of shock because interest rate higher than it would have been. True even with PF...
2. AU shock interacts with ZLB, implies further amplification, because of Jensen's inequality. True even with RA...

Simple Model (5) - Results' Intuitions

1. ZLB amplifies effect of shock because interest rate higher than it would have been. True even with PF...
2. AU shock interacts with ZLB, implies further amplification, because of Jensen's inequality. True even with RA...
3. So what is the role of HA in this amplification?

Simple Model (5) - Results' Intuitions

1. ZLB amplifies effect of shock because interest rate higher than it would have been. True even with PF...
2. AU shock interacts with ZLB, implies further amplification, because of Jensen's inequality. True even with RA...
3. So what is the role of HA in this amplification?
   - in the steady state (closer to the ZLB in the steady state because of precautionary savings - kink more to the left)
   - in the business cycle (lowers R_t towards ZLB because of precautionary savings and MPCs - steeper slope)

HANK Model (1) - Overview

HANK Model (1) - Overview

- Standard one-asset HANK model (McKay et al. (2016), Guerrieri and Lorenzoni (2017)):
  - Demand side (idiosyncratic risk, borrowing constraint)
  - New-Keynesian Phillips Curve
  - Supply of bonds from government
  - Taylor rule + ZLB
  - Preference shock

HANK Model (1) - Overview

- Standard one-asset HANK model (McKay et al. (2016), Guerrieri and Lorenzoni (2017)):
  - Demand side (idiosyncratic risk, borrowing constraint)
  - New-Keynesian Phillips Curve
  - Supply of bonds from government
  - Taylor rule + ZLB
  - Preference shock
- Calibration: standard parameter values + Great Recession

HANK Model (2) - Households

HANK Model (2) - Households

- Household i with assets a_{it-1} and shock z_{it} maximizes:

$$V_t\left(z_{it}, a_{it-1}\right) = \max_{c_{it}, a_{it} \geq \underline{a}} \frac{c_{it}^{1-\sigma}}{1-\sigma} + \beta_t \mathbb{E}_t V_{t+1}\left(z_{it+1}, a_t\right)$$

subject to:

$$c_{it} + \frac{a_{it}}{R_t} = \frac{a_{it-1}}{\Pi_t} + z_{it}\left(Y_t - t_t\right)$$

HANK Model (2) - Households

- Household i with assets a_{it-1} and shock z_{it} maximizes:

$$V_t\left(z_{it}, a_{it-1}\right) = \max_{c_{it}, a_{it} \geq \underline{a}} \frac{c_{it}^{1-\sigma}}{1-\sigma} + \beta_t \mathbb{E}_t V_{t+1}\left(z_{it+1}, a_t\right)$$

subject to:

$$c_{it} + \frac{a_{it}}{R_t} = \frac{a_{it-1}}{\Pi_t} + z_{it}\left(Y_t - t_t\right)$$

- $z_{it} \sim$ a Markov chain following $Q = P(z_{it+1}|z_{it})$ (time invariant)
  - $\implies$ earnings risk is acyclical

- Household i with assets a_{it-1} and shock z_{it} maximizes:

$$V_t\left(z_{it}, a_{it-1}\right) = \max_{c_{it}, a_{it} \geq \underline{a}} \frac{c_{it}^{1-\sigma}}{1-\sigma} + \beta_t \mathbb{E}_t V_{t+1}\left(z_{it+1}, a_t\right)$$

subject to:

$$c_{it} + \frac{a_{it}}{R_t} = \frac{a_{it-1}}{\Pi_t} + z_{it}\left(Y_t - t_t\right)$$

- $z_{it} \sim$ a Markov chain following $Q = P(z_{it+1}|z_{it})$ (time invariant)
  - $\Longrightarrow$ earnings risk is acyclical

Aggregate Asset Demand

$$A_t = \int g_t^a(z, a) dD_t(z, a)$$

HANK Model (3) - Rest of economy

HANK Model (3) - Rest of economy

- New Keynesian Phillips Curve (from Rotemberg):

$$\left(\Pi_t - \overline{\Pi}\right)\Pi_t = \mathbb{E}_t \beta_t \left(\frac{Y_{t+1}}{Y_t}\right)^{1-\sigma} \times \left(\Pi_{t+1} - \overline{\Pi}\right)\Pi_{t+1} + \tilde{\kappa}\left[Y_t^{\omega+\sigma} - 1\right]$$

- Government Budget and Fiscal Policy

$$T_t + \frac{b_t}{R_t} = \frac{b_{t-1}}{\Pi_t} \qquad\qquad b_t = \overline{b}$$

- Market Clearing

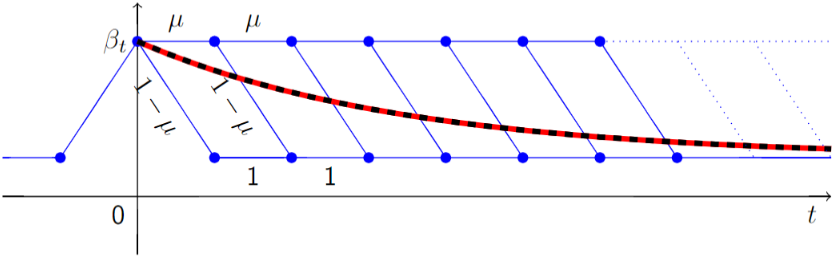
$$b_t = \int g_t^a(a,z)\, dD_t(z,a)$$

- Monetary Policy

$$R_t = \max\left\{1, \overline{R}\left(\frac{\Pi_t}{\overline{\Pi}}\right)^{\phi_\pi}\left(\frac{Y_t}{\bar{Y}}\right)^{\phi_Y}\right\}$$

HANK Model (4) - Shock Structure

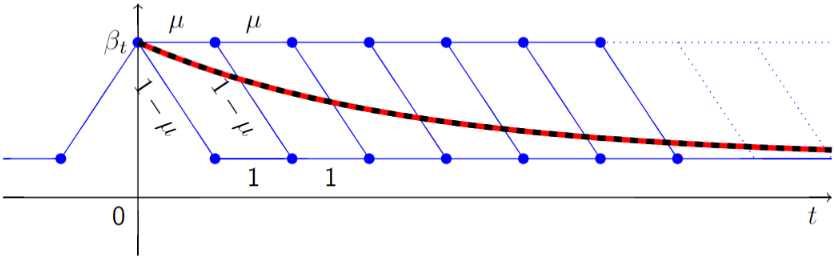
Graphical Representation of Deterministic and Stochastic Shocks



- economy at steady state ($t = -1$)
- time preference shock β materializes: $\beta_0 = \beta_L$
- every period: probability $1 - \mu$ to revert (and a *contingency* realizes)
- Compare to deterministic shock: $\beta_t^{DET} = \mathbb{E}_0 \beta_t$
- no restrictions on values, only on μ (must be the same).

HANK Model (4) - Shock Structure

Graphical Representation of Deterministic and Stochastic Shocks



$$\beta_t = \begin{cases} \beta & \text{w.p.} = 1, \text{ if } \beta_{t-1} = \beta \\ \beta & \text{w.p.} = 1\text{-}\mu, \text{ if } \beta_{t-1} = \beta_L \\ \beta_L & \text{w.p.} = \mu, \text{ if } \beta_{t-1} = \beta_L \end{cases} \qquad \beta_t^{PF} = \mu^t \beta_L + \left(1 - \mu^t\right) \beta \tag{4}$$

HANK Model (5) - Calibration

Table 1: Calibration

Parameter	Value	Source	Note
σ	1.5	Smets and Wouters (2007)	EIS
β	0.9805	Calibrated	Discount Factor
κ	0.01	Eggertsson et al. (2021)	NKPC
Π	$1.02^{0.25}$	Standard	Inflation target
ϕ_π	1.5	Standard	Monetary Policy
ϕ_y	0.125	Standard	Monetary Policy
z		Guerrieri and Lorenzoni (2017)	Idiosyncratic Shocks
Q		Guerrieri and Lorenzoni (2017)	Idiosyncratic Shocks
μ	0.9	Eggertsson et al. (2021)	Switching Probability
β_L	0.993	Calibrated	Shock
T	300	-	Horizon Truncation
τ^{max}	100	-	Largest Contingency

HANK Model (6) - Challenges

HANK Model (6) - Challenges

- Wealth distribution is state variable
  - affects the evolution of the economy
  - infinite-dimensional object $\implies$ curse of dimensionality

HANK Model (6) - Challenges

- Wealth distribution is state variable
  - affects the evolution of the economy
  - infinite-dimensional object $\implies$ curse of dimensionality
- Known solutions:
  - summarize distribution in few moments (Krusell and Smith, 1998)
  - aggregate economy behaves linearly (Reiter, 2009) (certainty equivalence)

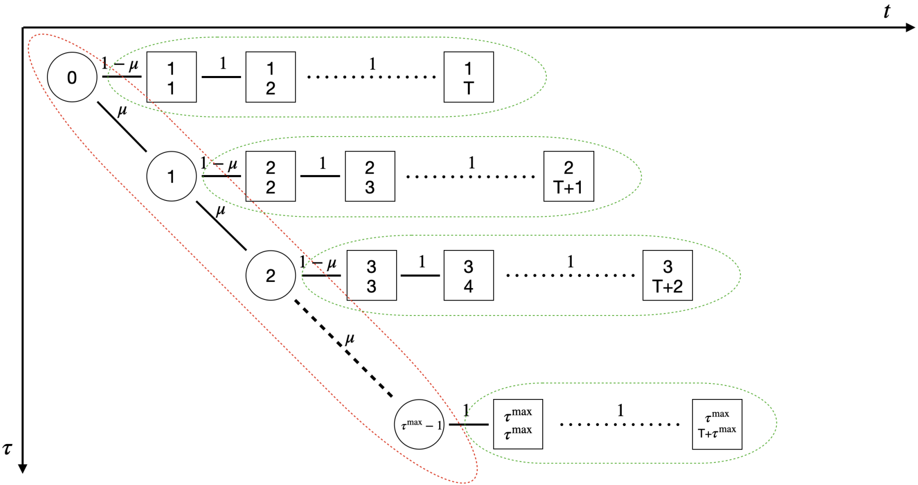
HANK Model (6) - Challenges

- Wealth distribution is state variable
  - affects the evolution of the economy
  - infinite-dimensional object $\implies$ curse of dimensionality
- Known solutions:
  - summarize distribution in few moments (Krusell and Smith, 1998)
  - aggregate economy behaves linearly (Reiter, 2009) (certainty equivalence)
- ZLB introduces aggregate nonlinearity
  - potentially computationally burdensome
  - some solutions: Schaab (2020), Fernández-Villaverde et al. (2021), Kase et al. (2022)

HANK Model (6) - Challenges

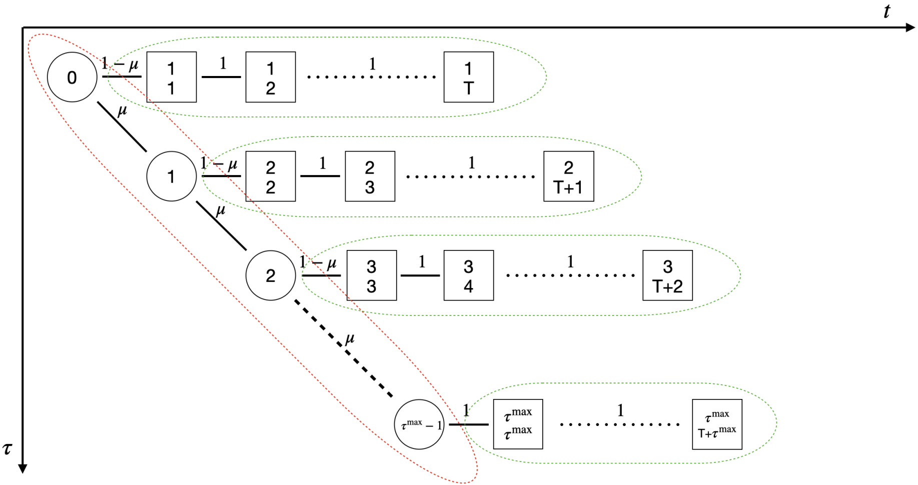
- Wealth distribution is state variable
  - affects the evolution of the economy
  - infinite-dimensional object $\implies$ curse of dimensionality
- Known solutions:
  - summarize distribution in few moments (Krusell and Smith, 1998)
  - aggregate economy behaves linearly (Reiter, 2009) (certainty equivalence)
- ZLB introduces aggregate nonlinearity
  - potentially computationally burdensome
  - some solutions: Schaab (2020), Fernández-Villaverde et al. (2021), Kase et al. (2022)
- We instead solve the model in the space of sequences
  - shock structure $\implies$ finite # of paths the economy can follow

Solution Approach (1) - Economy Overview



Notes: x-axis is time t, y-axis is contingency τ.

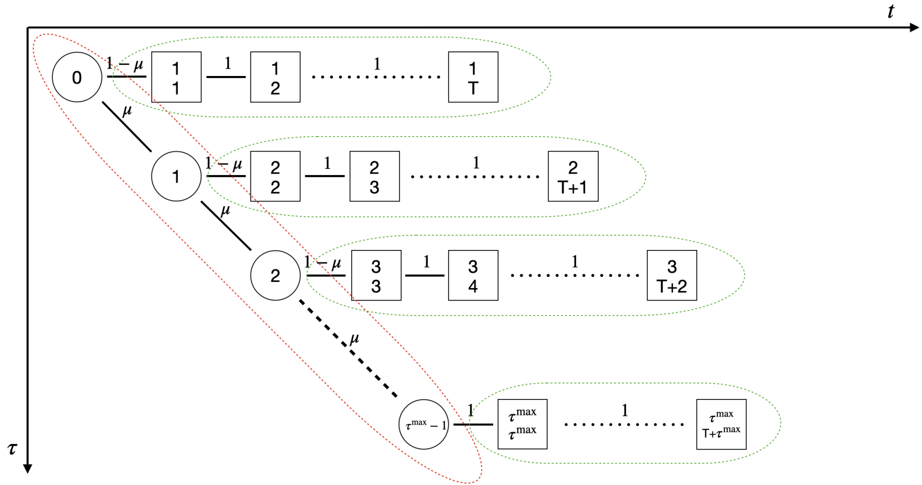
Solution Approach (1) - Economy Overview



Notes: x-axis is time t, y-axis is contingency τ.

Solution: large system of equations $(\tau^{\max} \times (T+1) \times nX)$.

Solution Approach (1) - Economy Overview



Notes: x-axis is time t, y-axis is contingency τ.

Solution: large system of equations $(\tau^{\max} \times (T+1) \times nX)$. We split diagonal/contingencies

Solution Approach (2) - Notation and Terminology

Solution Approach (2) - Notation and Terminology

- Notation and terminology:

Solution Approach (2) - Notation and Terminology

- Notation and terminology:
  - A *contingency* refers to the time τ when then shock switched back

Solution Approach (2) - Notation and Terminology

- Notation and terminology:
  - A *contingency* refers to the time τ when then shock switched back
  - as well to the aggregate equilibrium dynamics following such event.

Solution Approach (2) - Notation and Terminology

- Notation and terminology:
  - A *contingency* refers to the time τ when then shock switched back
  - as well to the aggregate equilibrium dynamics following such event.
  - x_t^τ is the value of economic object x at time t under contingency τ,

Solution Approach (2) - Notation and Terminology

- Notation and terminology:
  - A *contingency* refers to the time τ when then shock switched back
  - as well to the aggregate equilibrium dynamics following such event.
  - x_t^τ is the value of economic object x at time t under contingency τ,
  - x_t is the value of economic object x at time t, if the shock has not yet reverted.

Solution Approach (2) - Notation and Terminology

- Notation and terminology:
  - A *contingency* refers to the time τ when then shock switched back
  - as well to the aggregate equilibrium dynamics following such event.
  - x_t^τ is the value of economic object x at time t under contingency τ,
  - x_t is the value of economic object x at time t, if the shock has not yet reverted.
- Value functions become

$$V_t\left(z_{it}, a_{it-1}\right) = \max_{c_{it}, a_{it} \geq \underline{a}} \frac{c_{it}^{1-\sigma}}{1-\sigma} + \beta_t \mathbb{E}_t\left[\mu V_{t+1}\left(z_{it+1}, a_t\right) + (1-\mu) V_{t+1}^{t+1}\left(z_{it+1}, a_t\right)\right]$$

$$V_t^\tau\left(z_{it}, a_{it-1}\right) = \max_{c_{it}, a_{it} \geq \underline{a}} \frac{c_{it}^{1-\sigma}}{1-\sigma} + \beta_t^\tau \mathbb{E}_t V_{t+1}^\tau\left(z_{it+1}, a_t\right)$$

Solution Approach (2) - Notation and Terminology

- Notation and terminology:
  - A *contingency* refers to the time τ when then shock switched back
  - as well to the aggregate equilibrium dynamics following such event.
  - x_t^τ is the value of economic object x at time t under contingency τ,
  - x_t is the value of economic object x at time t, if the shock has not yet reverted.
- Value functions become

$$V_t\left(z_{it}, a_{it-1}\right) = \max_{c_{it}, a_{it} \geq \underline{a}} \frac{c_{it}^{1-\sigma}}{1-\sigma} + \beta_t \mathbb{E}_t \left[\mu V_{t+1}\left(z_{it+1}, a_t\right) + (1-\mu) V_{t+1}^{t+1}\left(z_{it+1}, a_t\right)\right]$$

$$V_t^\tau\left(z_{it}, a_{it-1}\right) = \max_{c_{it}, a_{it} \geq \underline{a}} \frac{c_{it}^{1-\sigma}}{1-\sigma} + \beta_t^\tau \mathbb{E}_t V_{t+1}^\tau\left(z_{it+1}, a_t\right)$$

- (linearized) NK Phillips curve becomes

$$\hat{\Pi}_t = \beta\left[\mu \hat{\Pi}_{t+1} + (1-\mu)\hat{\Pi}_{t+1}^{t+1}\right] + \kappa \hat{Y}_t$$

$$\hat{\Pi}_t^\tau = \beta \hat{\Pi}_{t+1}^\tau + \kappa \hat{Y}_t^\tau$$

Solution Approach (3) - Equilibrium

Solution Approach (3) - Equilibrium

- Numerically, an equilibrium is represented by systems of equations

$$0 = \mathbf{F}^{PF}(\mathbb{X}^{\tau}, \mathbb{Z}^{\tau}; X_{\tau-1}, D_{\tau}^{\tau}) \tag{5}$$

$$0 = \mathbf{F}^{TS}(\mathbb{X}^{TS}, \mathbb{Z}^{TS}; \mathbb{V}_1^{PF}, \mathbb{X}_1^{PF}) \tag{6}$$

Solution Approach (3) - Equilibrium

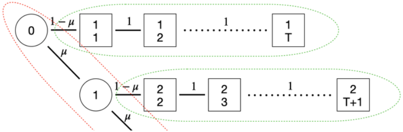
- Numerically, an equilibrium is represented by systems of equations

$$0 = \mathbf{F}^{PF}(\mathbb{X}^{\tau}, \mathbb{Z}^{\tau}; X_{\tau-1}, D_{\tau}^{\tau}) \tag{5}$$

$$0 = \mathbf{F}^{TS}(\mathbb{X}^{TS}, \mathbb{Z}^{TS}; \mathbb{V}_1^{PF}, \mathbb{X}_1^{PF}) \tag{6}$$

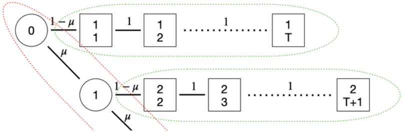
- where $\mathbb{X}^{\tau}$ ($\mathbb{Z}^{\tau}$) contains all the aggregate variables (shocks) in contingency τ
- $\mathbb{X}^{TS}$ ($\mathbb{Z}^{TS}$) contains all the aggr. variables (shocks) on the "uncertain" diagonal
- $\mathbb{V}_1^{PF}, \mathbb{X}_1^{PF}$ contain all the "forward looking" information relevant for the diagonal

Solution Approach (4) - Algorithm Details



1. Guess path of state variable(s) on uncertain path

Solution Approach (4) - Algorithm Details



1. Guess path of state variable(s) on uncertain path
2. Solve the $\tau^{\max}$ PF paths

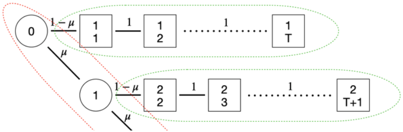
$$0 = \mathbf{F}^{PF}(\mathbb{X}^{\tau}, \mathbb{Z}^{\tau}; X_{\tau-1}, D^{\tau}_{\tau})$$

$$d\mathbb{X}^{\tau} = \mathbf{F}_{X}^{PF\,-1}\left(F_{D}^{PF}\, dD^{\tau}_{\tau} + \mathbf{F}^{PF}_{X_{\tau-1}}\, dX_{\tau-1}\right)$$

$$\mathbf{F}^{PF}_{D}\, dD^{\tau}_{\tau} \approx \underbrace{\mathbf{F}^{PF}\left(\mathbb{X}^{PF}_{ss} | D^{\tau}_{\tau}, X_{ss}\right)}_{y^{a\,\prime}_{ss}(\Lambda^{\prime}_{ss})^{t-\tau}\, dD^{\tau}_{\tau}} - \mathbf{F}^{PF}\left(\mathbb{X}^{PF}_{ss} | D_{ss} X_{ss}\right)$$

- use Sequence Space Jacobian + OccBin
- collect value functions and forw. looking vars at first period of PFs

Solution Approach (4) - Algorithm Details



1. Guess path of state variable(s) on uncertain path
2. Solve the $\tau^{\max}$ PF paths
3. Given the value functions and forw. looking vars, solve the uncertain path*

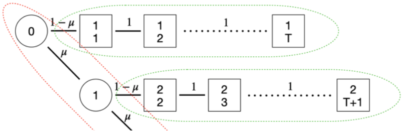
$$0 = \mathbf{F}^{TS}(\mathbb{X}^{TS}, \mathbb{Z}^{TS}; \mathbb{V}_1^{PF}, \mathbb{X}_1^{PF})$$

$$d\mathbb{X}^{TS} = (\mathbf{F}_X^{TS})^{-1}\left(\mathbf{F}_{\mathbb{Z}}^{TS} d\mathbb{Z} + \mathbf{F}_{\mathbb{X}}^{TS} d\mathbb{X}_1^{PF} + \mathbf{F}_{\mathbf{V}}^{TS} d\mathbb{V}_1^{PF}\right)$$

$$\mathbf{F}_{\mathbb{V}}^{TS} d\mathbb{V}_1^{PF} \approx \mathbf{F}^{TS}\left(\mathbb{X}_{ss}, \mathbb{Z}_{ss} | \mathbb{X}_{ss}, \mathbb{V}_1^{PF}\right) - \mathbf{F}^{TS}\left(\mathbb{X}_{ss}^{TS}, \mathbb{Z}_{ss} | \mathbb{X}_{ss}^{TS}, \mathbb{V}_{ss}\right)$$

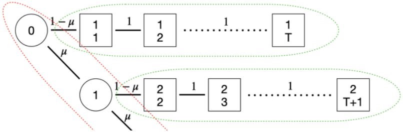
- use Sequence Space Jacobian (modified for AU) + OccBin
- recover new set of state variables

Solution Approach (4) - Algorithm Details



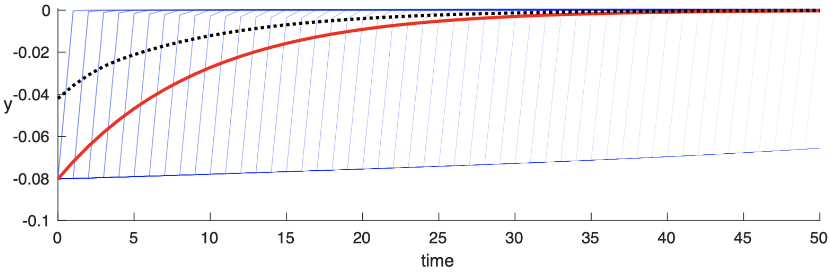
1. Guess path of state variable(s) on uncertain path
2. Solve the $\tau^{\max}$ PF paths
3. Given the value functions and forw. looking vars, solve the uncertain path*
4. Iterate until convergence

Solution Approach (4) - Algorithm Details



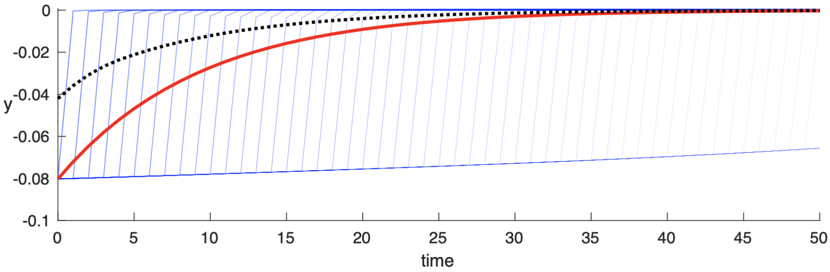
1. Guess path of state variable(s) on uncertain path
2. Solve the $\tau^{\max}$ PF paths
3. Given the value functions and forw. looking vars, solve the uncertain path*
4. Iterate until convergence
5. (optional) Use quasi-Newton method for higher order

Results (0) - Fixing ideas on what we are after



Measuring amplification due to uncertainty:

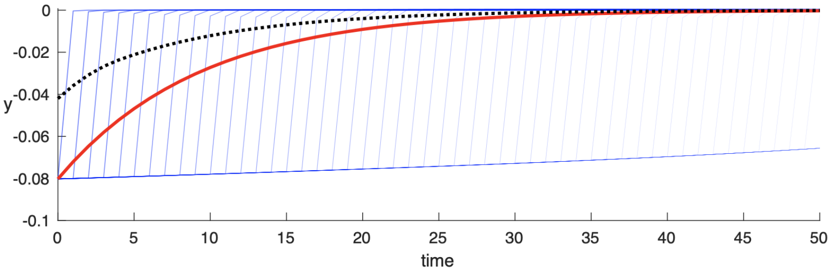
Results (0) - Fixing ideas on what we are after



Measuring amplification due to uncertainty:

- Aggregate Uncertainty vs Deterministic Counterfactual

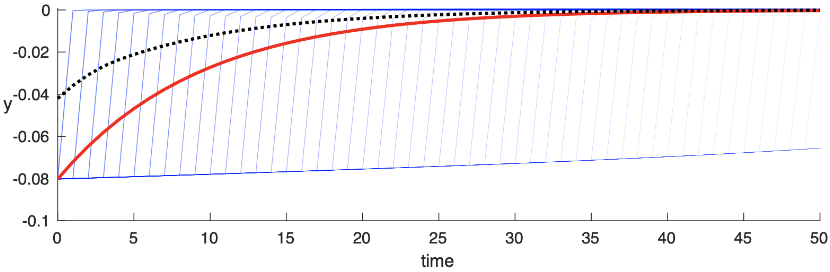
Results (0) - Fixing ideas on what we are after



Measuring amplification due to uncertainty:

- Aggregate Uncertainty vs Deterministic Counterfactual
- Shocks with the same expected values from $t = 0$ perspective

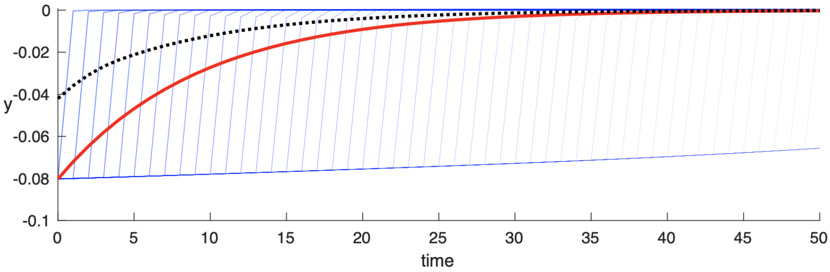
Results (0) - Fixing ideas on what we are after



Measuring amplification due to uncertainty:

- Aggregate Uncertainty vs Deterministic Counterfactual
- Shocks with the same expected values from $t = 0$ perspective
- IRF-AU: $\mathbb{E}_0 Y_t - \overline{Y}$, weighted average of all contingencies. IRF-PF: $Y_t^{PF} - \bar{Y}$

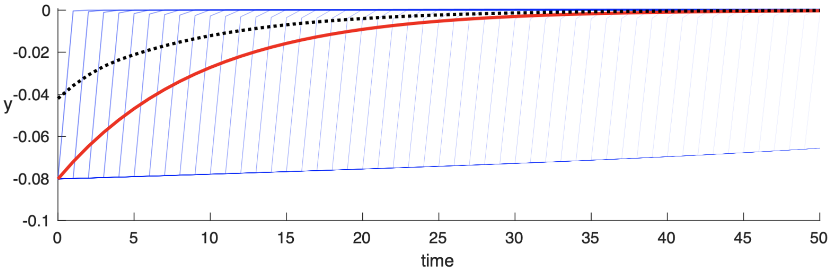
Results (0) - Fixing ideas on what we are after



Measuring amplification due to uncertainty:

- Aggregate Uncertainty vs Deterministic Counterfactual
- Shocks with the same expected values from $t = 0$ perspective
- IRF-AU: $\mathbb{E}_0 Y_t - \overline{Y}$, weighted average of all contingencies. IRF-PF: $Y_t^{PF} - \bar{Y}$
- Amplification if $\left|\mathbb{E}_0 Y_t - Y_t^{PF}\right| >> 0$

Results (0) - Fixing ideas on what we are after

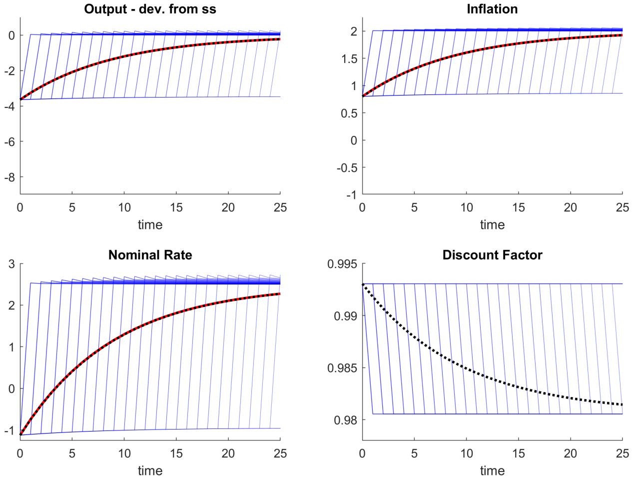


Measuring amplification due to uncertainty:

- Aggregate Uncertainty vs Deterministic Counterfactual
- Shocks with the same expected values from $t = 0$ perspective
- IRF-AU: $\mathbb{E}_0 Y_t - \overline{Y}$, weighted average of all contingencies. IRF-PF: $Y_t^{PF} - \bar{Y}$
- Amplification if $\left|\mathbb{E}_0 Y_t - Y_t^{PF}\right| >> 0$
- Quantification with PDV: $\sum_{t=0}^{\infty} \beta^t \left(\mathbb{E}_0 Y_t - \overline{Y}\right)$ vs $\sum_{t=0}^{\infty} \beta^t \left(Y_t^{PF} - \overline{Y}\right)$

Results (1) - Uncertainty and Amplification

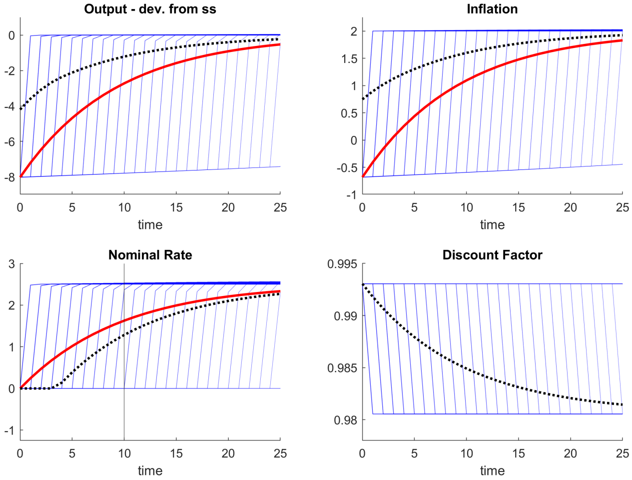
Figure 3: IRF - HANK - No ZLB



- AU vs PF in HANK without the ZLB
  - linear behavior to aggregate shocks
  - $\mathbb{E}_0 Y_t \approx Y_t^{PF}$
  - certainty equivalence

Results (2) - Uncertainty and Amplification

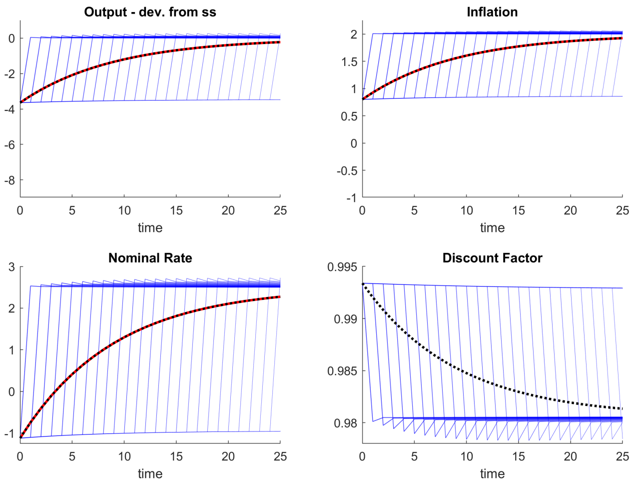
Figure 4: IRF - HANK - with ZLB



- AU vs PF in HANK at the ZLB
- certainty equivalence broken
  - $\mathbb{E}_0 Y_t < Y_t^{PF}$
  - amplification in PDV: **2x**
- Uncertainty: ZLB binds for longer
  - On average, 10 quarters
  - vs. **4** quarters in PF

Results (3) - Uncertainty and Amplification in RANK

Figure 5: IRF - RANK - without ZLB

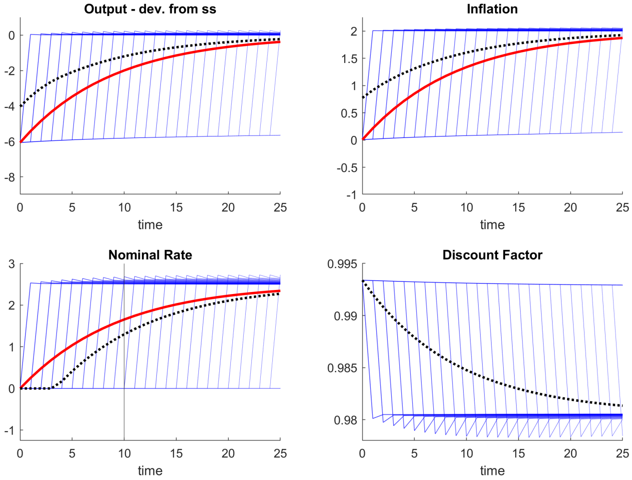


- AU vs PF in RANK at the ZLB
- Calibrate shocks such that:
- RA economy exhibits same response as HANK w/o ZLB (Werning, 2015)

$$X_t(RANK) = X_t(HANK)$$
$$X_t^\tau(RANK) = X_t^\tau(HANK)$$

Results (3) - Uncertainty and Amplification in RANK

Figure 5: IRF - RANK - with ZLB



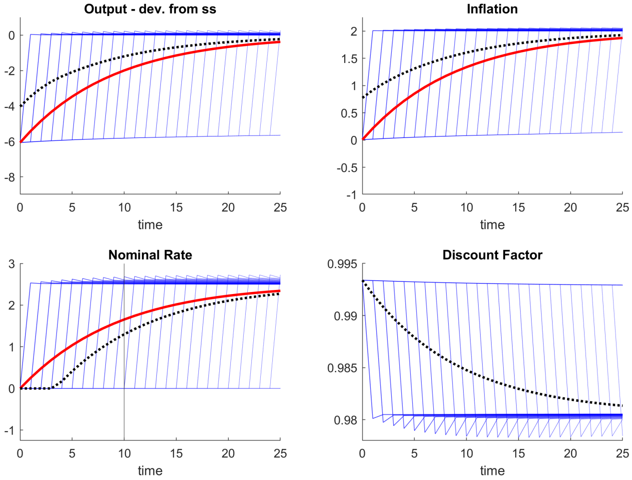
- AU vs PF in RANK at the ZLB
- Calibrate shocks such that:
- RA economy exhibits same response as HANK w/o ZLB (Werning, 2015)

$$X_t(RANK) = X_t(HANK)$$
$$X_t^{\tau}(RANK) = X_t^{\tau}(HANK)$$

- Introduce ZLB

Results (3) - Uncertainty and Amplification in RANK

Figure 5: IRF - RANK - with ZLB

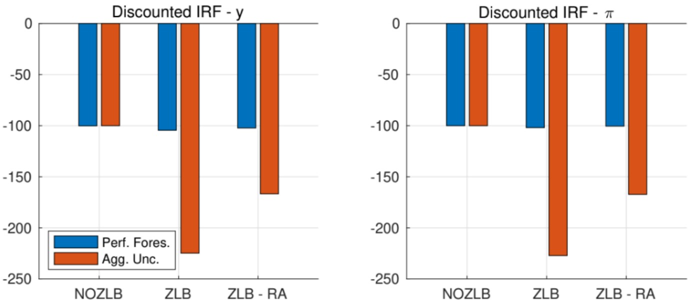


- AU vs PF in RANK at the ZLB
- Calibrate shocks such that:
- RA economy exhibits same response as HANK w/o ZLB (Werning, 2015)

$$X_t(RANK) = X_t(HANK)$$
$$X_t^{\tau}(RANK) = X_t^{\tau}(HANK)$$

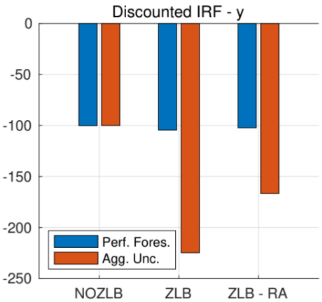
- Introduce ZLB
- Amplification in PDV: **1.6x**

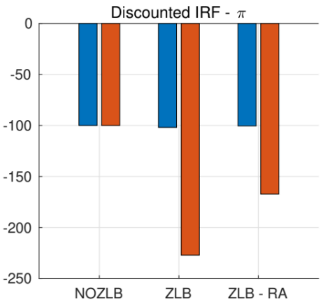
Results (4) - Uncertainty and Amplification - Summary (Y)



	No ZLB		ZLB		
	Perf. Fores.	Agg. Unc.	Perf. Fores.	Agg. Unc.	
Repr. Agent	A	A	B	C	C-B
Het. Agents	A	A	D	E	E-D

Results (4) - Uncertainty and Amplification - Summary (Y)





	No ZLB		ZLB		
	Perf. Fores.	Agg. Unc.	Perf. Fores.	Agg. Unc.	
Repr. Agent	100	100	102.3	166.7	64.4
Het. Agents	100	100	104.5	225	120.5

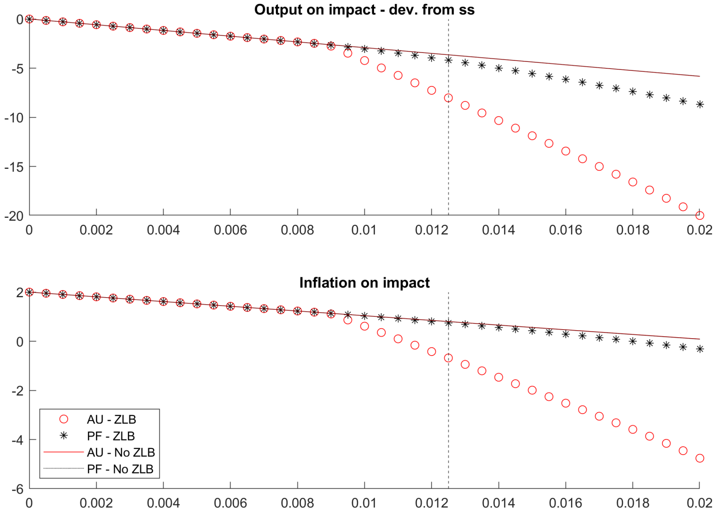
Results (5) - Computing Time

Table 2: Running Times - Seconds

Specification Step	Benchmark Time	Benchmark Max. Err.	MNP Time	MNP Max. Err.
Steady State	0.7	-	6	-
All Jacobians	4	-	179	-
Algorithm 1 - First-Order	20	0.5%	144	0.5%
Algorithm 1 - Exact only on TS	26	0.008%	216	0.002%
Algorithm 1 - Exact Equilibrium	116	0.00000006%	7735	0.0000002%

- Matlab, ASUS laptop, 1.80Ghz processor, 16GB RAM, and 8 cores
- MNP: Mendicino et al. (2021), richer income risk

Figure 6: Effects on Impact as a function of shock size



Results (7) - Decomposition of Consumption Demand

- Define a consumption function:

$$\mathbb{C}^{PF} = \mathcal{C}^{PF}\left(\mathbb{X}^{PF}, \mathbb{Z}^{PF}\right) \tag{7}$$

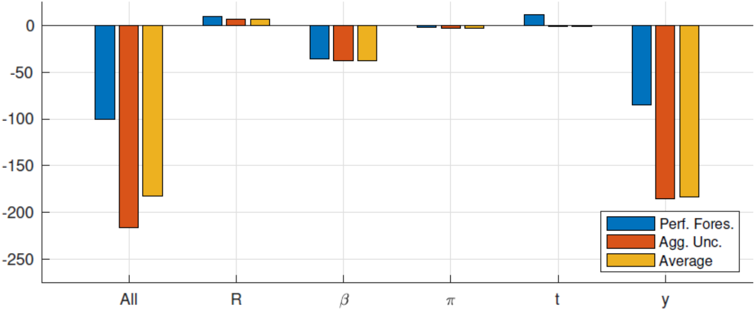
$$\mathbb{C}^{TS} = \mathcal{C}^{AU}\left(\mathbb{X}^{TS}, \mathbb{Z}^{TS}, \{\mathbb{X}^{\tau}\}_{\tau=1}^{\tau^{\max}}, \{\mathbb{Z}^{\tau}\}_{\tau=1}^{\tau^{\max}}\right) \tag{8}$$

$$\mathbb{C}^{Interm} = \mathcal{C}^{PF}\left(IRF\left(\mathbb{X}^{TS}, \mathbb{Z}^{TS}, \{\mathbb{X}^{\tau}\}_{\tau=1}^{\tau^{\max}}, \{\mathbb{Z}^{\tau}\}_{\tau=1}^{\tau^{\max}}\right)\right) \tag{9}$$

- Which aggregate is driving the amplification? Y, Π, R, t, β
- Is uncertainty important per se? Feed **IRF** of AU in a deterministic world (captures indirect effect via "average" aggregates).

Results (8) - Decomposition of Consumption Demand

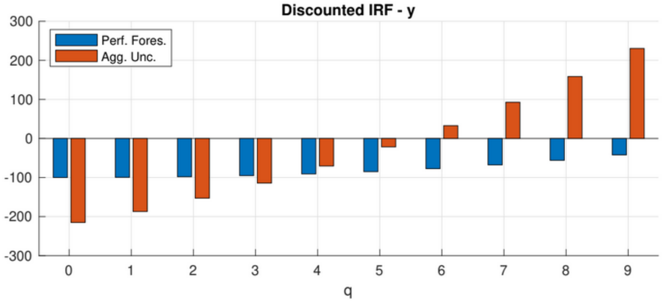
Figure 7: Decomposition of discounted IRF - Consumption



- What price is driving the amplification?
- Feed **average** prices of AU, in a deterministic world
- Aggregate income is largest driver.
- Expected aggregate income does most of it.

Other Applications (1) - Forward Guidance

- One policy application: forward guidance
- Central bank announces to keep rates at ZLB for q quarters on top of what prescribed in main exercise
- FG powerful when there is uncertainty (6 quarters can revert recession)



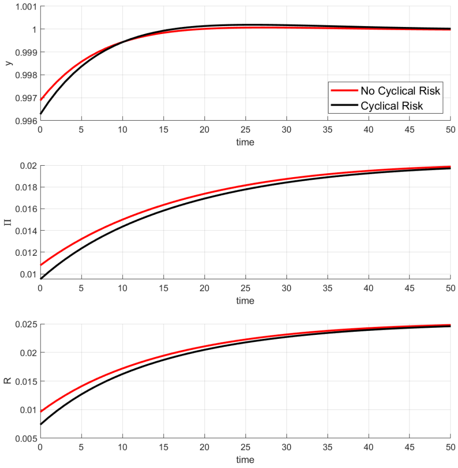
Application (2) - Two Asset HANK

1. Households
   - illiquid physical capital
   - Calvo fairy for portfolio re-balancing

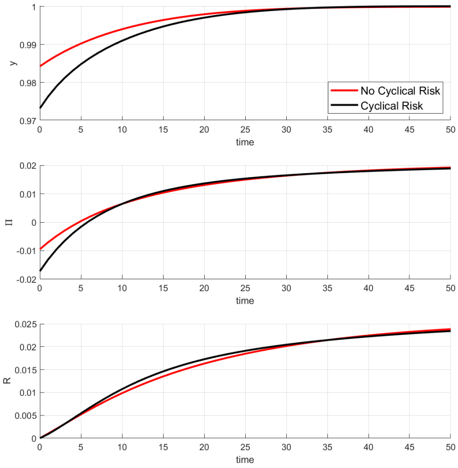
   ⇒ heterogeneous in income, wealth, and portfolio composition
2. Other Blocks
   - intermediate-goods producer - Cobb-Douglas production
   - Labor Union with adjustment costs (Wage Phillips Curve)
   - Capital production subject to adjustment cost

Application (2.1) - Cyclical Income Risk

Deterministic Shock

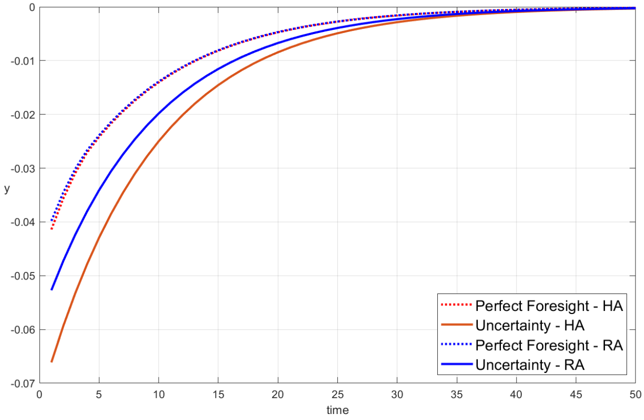


Stochastic Shock



Application (2.2) - Earnings Risk

Earnings risk as in Mendicino et al. (2021):



Conclusions

- We study the interaction b/w aggregate uncertainty and household heterogeneity:
  - new methodology for HANK models with aggregate uncertainty and non-linearities
  - simulations suggest that interaction is strong at the ZLB, even with acyclical risk
  - quantify the interaction in a simple way, during GR (55% amplification)
- Applications:
  - Forward Guidance
  - Two Asset HANK
- Methodology can be used for many applications involving HA, AU, aggregate non-linearities

References I

Achdou, Y., Han, J., Lasry, J.-M., Lions, P.-L., and Moll, B. (2022). Income and wealth distribution in macroeconomics: A continuous-time approach. *Review of Economic Studies*, 89(1):45–86.

Ahn, S., Kaplan, G., Moll, B., Winberry, T., and Wolf, C. (2018). When inequality matters for macro and macro matters for inequality. *NBER macroeconomics annual*, 32(1):1–75.

Auclert, A. (2019). Monetary policy and the redistribution channel. *American Economic Review*, 109(6):2333–2367.

Auclert, A., Bardóczy, B., Rognlie, M., and Straub, L. (2021). Using the sequence-space jacobian to solve and estimate heterogeneous-agent models. *Econometrica*, 89(5):2375–2408.

References II

Basu, S. and Bundick, B. (2016). Endogenous volatility at the zero lower bound: Implications for stabilization policy. Technical report, National Bureau of Economic Research.

Basu, S. and Bundick, B. (2017). Uncertainty shocks in a model of effective demand. *Econometrica*, 85(3):937–958.

Bayer, C., Lütticke, R., Pham-Dao, L., and Tjaden, V. (2019). Precautionary savings, illiquid assets, and the aggregate consequences of shocks to household income risk. *Econometrica*, 87(1):255–290.

Benigno, P., Eggertsson, G. B., and Romei, F. (2020). Dynamic debt deleveraging and optimal monetary policy. *American Economic Journal: Macroeconomics*, 12(2):310–50.

Bloom, N. (2014). Fluctuations in uncertainty. *Journal of Economic Perspectives*, 28(2):153–76.

References III

Bloom, N., Floetotto, M., Jaimovich, N., Saporta-Eksten, I., and Terry, S. J. (2018). Really uncertain business cycles. *Econometrica*, 86(3):1031–1065.

Caggiano, G., Castelnuovo, E., and Pellegrino, G. (2017). Estimating the real effects of uncertainty shocks at the zero lower bound. *European Economic Review*, 100:257–272.

Christiano, L., Eichenbaum, M., and Rebelo, S. (2011). When is the government spending multiplier large? *Journal of Political Economy*, 119(1):78–121.

Eggertsson, G. B., Egiev, S. K., Lin, A., Platzer, J., and Riva, L. (2021). A toolkit for solving models with a lower bound on interest rates of stochastic duration. *Review of Economic Dynamics*.

Eggertsson, G. B. and Krugman, P. (2012). Debt, Deleveraging, and the Liquidity Trap: A Fisher-Minsky-Koo Approach*. *Quarterly Journal of Economics*, 127(3):1469–1513.

Eggertsson, G. B. and Woodford, M. (2003). Zero bound on interest rates and optimal monetary policy. *Brookings papers on economic activity*, 2003(1):139–233.

Fernández-Villaverde, J., Marbet, J., Nuño, G., and Rachedi, O. (2021). Inequality and the zero lower bound. Technical report.

Guerrieri, V. and Lorenzoni, G. (2017). Credit Crises, Precautionary Savings, and the Liquidity Trap. *Quarterly Journal of Economics*, 132(3):1427–1467.

Guvenen, F., Ozkan, S., and Song, J. (2014). The nature of countercyclical income risk. *Journal of Political Economy*, 122(3):621–660.

Kaplan, G., Moll, B., and Violante, G. L. (2018). Monetary policy according to hank. *American Economic Review*, 108(3):697–743.

Kase, H., Melosi, L., and Rottner, M. (2022). Estimating nonlinear heterogeneous agents models with neural networks.

Krusell, P. and Smith, Jr, A. A. (1998). Income and wealth heterogeneity in the macroeconomy. *Journal of political Economy*, 106(5):867–896.

References V

McKay, A., Nakamura, E., and Steinsson, J. (2016). The power of forward guidance revisited. *American Economic Review*, 106(10):3133–58.

Mendicino, C., Nord, L., and Peruffo, M. (2021). Distributive effects of banking sector losses.

Reiter, M. (2009). Solving heterogeneous-agent models by projection and perturbation. *Journal of Economic Dynamics and Control*, 33(3):649–665.

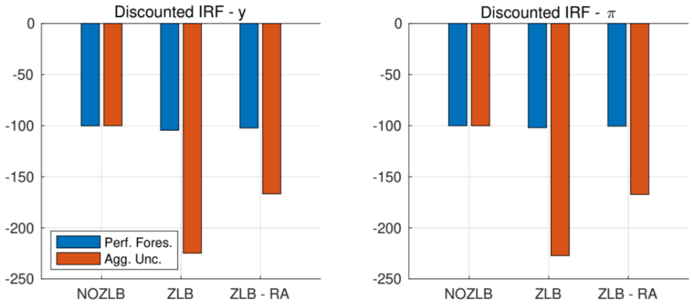
Schaab, A. (2020). Micro and macro uncertainty. Technical report, Working Paper.

Shimer, R. (2005). The cyclical behavior of equilibrium unemployment and vacancies. *American Economic Review*, 95(1):25–49.

Smets, F. and Wouters, R. (2007). Shocks and frictions in us business cycles: A bayesian dsge approach. *American Economic Review*, 97(3):586–606.

Werning, I. (2015). Incomplete markets and aggregate demand. Technical report, National Bureau of Economic Research.

Results (5) - Uncertainty and Amplification - Summary (Π)



	No ZLB		ZLB (y/Pi)	
	Perf. Fores.	Agg. Unc.	Perf. Fores.	Agg. Unc.
Repr. Agent	100	100	100.45	167.3
Het. Agents	100	100	101.9	227

Solution Approach (3) - Heterogeneous Agents Block

- Similar to Sequence-Space Jacobian. Heterogeneous block is represented by:

$$\mathbf{v}_t = v^{TS}\left(\mathbf{v}_{t+1}, \mathbf{v}_{t+1}^{t+1}, X_t\right) \tag{10}$$

$$D_{t+1}^{t+1} = D_{t+1} = \Lambda^{TS}\left(\mathbf{v}_{t+1}, \mathbf{v}_{t+1}^{t+1}, X_t\right)' D_t \tag{11}$$

$$\mathcal{Y}_t = y^{TS}\left(\mathbf{v}_{t+1}, \mathbf{v}_{t+1}^{t+1}, X_t\right)' D_t \tag{12}$$

$$\mathbf{v}_t^{\tau} = v\left(\mathbf{v}_{t+1}^{\tau}, X_t^{\tau}\right) \tag{13}$$

$$D_{t+1}^{\tau} = \Lambda\left(\mathbf{v}_{t+1}^{\tau}, X_t^{\tau}\right)' D_t^{\tau} \tag{14}$$

$$\mathcal{Y}_t^{\tau} = y\left(\mathbf{v}_{t+1}^{\tau}, X_t^{\tau}\right)' D_t^{\tau} \tag{15}$$

- A competitive equilibrium is
  - Aggregate variables
    - a sequence $\{Y_t, \Pi_t, R_t, b_t, t_t\}_{t=0}^{\tau^{\max}-1} = \{X_t\}_{t=0}^{\tau^{\max}-1} = \mathbb{X}^{TS}$
    - $\tau^{\max}$ sequences $\left\{\{Y_t^\tau, \Pi_t^\tau, R_t^\tau, b_t^\tau, t_t^\tau\}_{t=\tau}^{\tau+T}\right\}_{\tau=1}^{\tau^{\max}} = \left\{\{X_t^\tau\}_{t=\tau}^{\tau+T}\right\}_{\tau=1}^{\tau^{\max}} = \{\mathbb{X}^\tau\}_{\tau=1}^{\tau^{\max}}$,
  - Individual agents objects (wealth distribution, value function)
    - a sequence $\{D_{t+1}, V_t\}_{t=0}^{\tau^{\max}-1}$
    - $\tau^{\max}$ sequences $\left\{\{D_t^\tau, V_t^\tau\}_{t=\tau}^{\tau+T}\right\}_{\tau=1}^{\tau^{\max}}$
  - s.t. given exogenous processes $\{\beta_t\}_{t=0}^{\tau^{\max}} = \mathbb{Z}^{TS}$ and $\left\{\{\beta_t^\tau\}_{t=0}^{\tau^{\max}}\right\}_{\tau=1}^{\tau^{\max}} = \{\mathbb{Z}^\tau\}_{\tau=1}^{\tau^{\max}}$, aggregate equations hold, agents solve their maximization problem, and $D_t = D_t^t$

Solution Approach (6) - Occasionally Binding

- Occasionally binding constraints sub-algorithm
  1. Consider $d\mathbb{X}^{TS} = (\mathbf{F}_X^{TS})^{-1}\left(\mathbf{F}_{\mathbb{Z}}^{TS} d\mathbb{Z} + \mathbf{F}_{\mathbb{X}}^{TS} d\mathbb{X}_1^{PF} + \mathbf{F}_{\mathbf{V}}^{TS} d\mathbb{V}_1^{PF}\right)$
  2. Guess periods in which the constraint binds, place them in an binary vector I_{ZLB}
  3. Adjust the main matrix so that

$$
\begin{aligned}
d\mathbb{X}^{TS} &= [(1 - I_{ZLB}) \times \mathbf{F}_X^{TS} + I_{ZLB} \times \tilde{\mathbf{F}}_X^{TS}]^{-1} \\
&\quad [(1 - I_{ZLB}) \times \left(\mathbf{F}_{\mathbb{Z}}^{TS} d\mathbb{Z} + \mathbf{F}_{\mathbb{X}}^{TS} d\mathbb{X}_1^{PF} + \mathbf{F}_{\mathbf{V}}^{TS} d\mathbb{V}_1^{PF}\right) + I_{ZLB} \times (\underline{R} - \overline{R})] \qquad (16)
\end{aligned}
$$

  where $\tilde{\mathbf{F}}_X^{TS}$ substitutes the Taylor rule with $R_t = \underline{R}$
- Generate shadow rates by simply multiplying $d\mathbb{X}^{TS}$ and $\mathbf{F}_X^{TS}$, is the guess is correct, stop. Otherwise go to 2 and update I_{ZLB}.

Solution Approach (7) - Others

Solution Approach (7) - Others

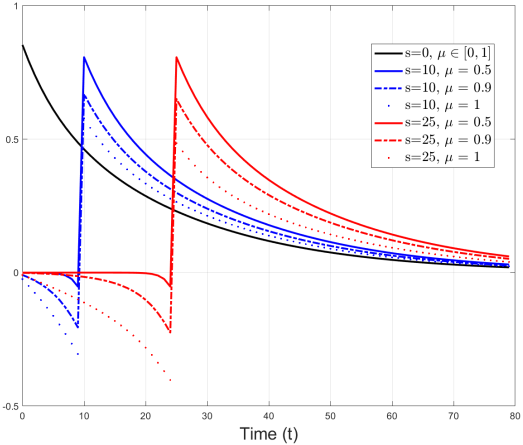
- We exploit:
  - fake news algorithm
  - expectations vector

Solution Approach (7) - Others

- We exploit:
  - fake news algorithm
  - expectations vector
- We do not exploit:
  - DAG-part

Solution Approach (7) - Others

- We exploit:
  - fake news algorithm
  - expectations vector
- We do not exploit:
  - DAG-part
- New Jacobian $\frac{\partial A_t}{\partial Y_s}$:
  - different μ



Results (6) - Shock Size

Figure 8: Effects on Discounted IRF as a function of shock size