Optimal Monetary Policy with $r^* < 0$

Roberto Billi Jordi Galí Anton Nakov

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Motivation

- Widespread consensus on a substantial decline in the *average* natural rate of interest $r^* \equiv \mathbb{E}\{r_t^n\}$. Recent estimates: $r^* < 0$.
- Driving forces: productivity, demography, inequality, relative price of investment goods, uncertainty. Likely to be persistent

Motivation

- Widespread consensus on a substantial decline in the *average* natural rate of interest $r^* \equiv \mathbb{E}\{r_t^n\}$. Recent estimates: $r^* < 0$.
- Driving forces: productivity, demography, inequality, relative price of investment goods, uncertainty. Likely to be persistent
- With a ZLB constraint: increased incidence of binding ZLB episodes, greater macro instability
- Existing literature: optimal monetary policy under a ZLB constraint and $r^* > 0$. Normal times: $r_t^n > 0 \Rightarrow i_t > 0$, successful stabilization of inflation and the output gap. Occasional episodes with $r_t^n < 0 \Rightarrow i_t = 0$, macro instability. Key role for forward guidance.
- *This paper*: optimal monetary policy under a ZLB constraint with $r^* < 0$. "New normal": $r_t^n < 0$. Occasional episodes with $r_t^n > 0$. Summers' "secular stagnation" speech.

What does the optimal monetary policy look like in that environment?
What are its implications for macro outcomes?

Outline

- The optimal monetary policy problem
- Equilibrium under the optimal policy: The Deterministic Case
- Equilibrium under the optimal policy: The Stochastic Case
- Implementation

  byproduct: sufficient condition for determinacy in models with endogenous regime switches
- Concluding remarks

Related Literature

- Evidence on r^* and its driving forces: Holston et al. (2017), Del Negro et al. (2019), Eggertsson et al. (2019), Rachel and Summers (2019), Brand and Mazelis (2019), Davis et al. (2023)
- Optimal monetary policy under the ZLB: Eggertsson and Woodford (2003), Jung, Teranishi and Watanabe (2005), Adam and Billi (2006), Nakov (2008),
- Optimal choice of an inflation target, conditional on a given interest rate rule: Coibion et al. (2012), Bernanke et al. (2019), and Andrade et al. (2020, 2021).
- Equilibrium determinacy in regime-switching models: Davig and Leeper (2007), Farmer et al. (2009) and Barthélemy and Marx (2017, 2019).

The Optimal Monetary Policy Problem

$$\min \frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\pi_t^2 + \vartheta y_t^2\right)$$

subject to

$$\pi_t = \beta \mathbb{E}_t\{\pi_{t+1}\} + \kappa y_t \quad (1)$$

$$y_t = \mathbb{E}_t\{y_{t+1}\} - \frac{1}{\sigma}(i_t - \mathbb{E}_t\{\pi_{t+1}\} - r_t^n) \quad (2)$$

$$i_t \geq 0 \quad (3)$$

$$r_t^n = r^* + z_t \quad (4)$$

all for $t = 0, 1, 2, ..$where $z_t \sim AR(1)$ and

$$r^* < 0$$

A Brief Detour: A Microfounded NK Model with r*<0

- Based on the NK-OLG model in Galí (2021)
- Consumers: constant "life" and "activity" survival rates (γ, υ). Objective function for consumer born in period s:

$$\mathbb{E}_s \sum_{t=s}^{\infty} (\beta\gamma)^{t-s} Z_t \log C_{t|s}$$

where $\beta \equiv \exp\{-\rho\}$ and $z_t \equiv \log Z_t \sim AR(1)$

- Firms: attached to founder, hence survival rate $\gamma\upsilon$. Calvo pricing.
- Steady state:

$$r^* = \rho + \log \upsilon$$

- Condition for $r^* < 0$

$$\upsilon < \beta$$

- Linearized equilibrium conditions:

$$\pi_t = \beta\gamma \mathbb{E}_t\{\pi_{t+1}\} + \kappa y_t$$

$$y_t = \mathbb{E}_t\{y_{t+1}\} - (i_t - \mathbb{E}_t\{\pi_{t+1}\} - r_t^n)$$

with $r_t^n = r^* + (1 - \rho_z) z_t$

The Optimal Monetary Policy Problem

$$\min \frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\pi_t^2 + \vartheta y_t^2\right)$$

subject to

$$\pi_t = \beta \mathbb{E}_t\{\pi_{t+1}\} + \kappa y_t \quad (5)$$

$$y_t = \mathbb{E}_t\{y_{t+1}\} - \frac{1}{\sigma}(i_t - \mathbb{E}_t\{\pi_{t+1}\} - r_t^n) \quad (6)$$

$$i_t \geq 0 \quad (7)$$

$$r_t^n = r^* + z_t \quad (8)$$

all for $t = 0, 1, 2, ..$where $z_t \sim AR(1)$ and

$$r^* < 0$$

The Optimal Monetary Policy Problem

- Optimality conditions:

$$\pi_t = \xi_{1,t} - \xi_{1,t-1} + \beta^{-1}\xi_{2,t-1} \tag{9}$$

$$\vartheta y_t = -\kappa\xi_{1,t} - \sigma\xi_{2,t} + \sigma\beta^{-1}\xi_{2,t-1} \tag{10}$$

$$\xi_{2,t} \geq 0 \tag{11}$$

$$\xi_{2,t} \; [r_t^n + \mathbb{E}_t\{\pi_{t+1}\} + \sigma(\mathbb{E}_t\{y_{t+1}\} - y_t)] = 0 \tag{12}$$

with initial conditions $\xi_{1,-1} = \xi_{2,-1} = 0$.

Optimal Policy: The Deterministic Case

- Initial steady state:

$$i_t = r_t^n = r^* > 0$$

$$y_t = \pi_t = 0$$

- MIT shock at time 0:

$$r_t^n = r^* < 0$$

  for $t = 0, 1, 2, ...$

- New steady state

$$\pi = \beta^{-1}\xi_2 \geq 0$$

$$\vartheta y = -\kappa\xi_1 + \sigma(\beta^{-1} - 1)\xi_2$$

$$\xi_2 \geq 0 \quad ; \quad r^* + \pi \geq 0$$

$$\xi_2(r^* + \pi) = 0$$

$$\Rightarrow \quad \pi \geq -r^* > 0$$

$$\Rightarrow \quad \xi_2 > 0 \quad \Rightarrow \quad i = 0 \quad \Rightarrow \quad \pi = -r^*$$

Optimal Policy: The Deterministic Case

- Transitional dynamics

$$\widehat{\pi}_t = \beta \widehat{\pi}_{t+1} + \kappa \widehat{y}_t$$

$$\widehat{\pi}_t = \widehat{\xi}_{1,t} - \widehat{\xi}_{1,t-1} + \beta^{-1} \widehat{\xi}_{2,t-1}$$

$$\vartheta \widehat{y}_t = -\kappa \widehat{\xi}_{1,t} - \sigma \widehat{\xi}_{2,t} + \sigma \beta^{-1} \widehat{\xi}_{2,t-1}$$

$$\widehat{\xi}_{2,t} + \xi_2 \geq 0$$

$$\widehat{\pi}_{t+1} + \sigma(\widehat{y}_{t+1} - \widehat{y}_t) \geq 0$$

$$(\widehat{\xi}_{2,t} + \xi_2) \; [\widehat{\pi}_{t+1} + \sigma(\widehat{y}_{t+1} - \widehat{y}_t)] = 0$$

for $t = 0, 1, 2, ...$with initial conditions $\widehat{\xi}_{1,-1} = -\xi_1$ and $\widehat{\xi}_{2,-1} = -\xi_2$ and such that $\lim_{t\to\infty} \widehat{x}_t = 0$ for $\widehat{x}_t \in \{\widehat{\pi}_t, \widehat{y}_t, \widehat{\xi}_{1,t}, \widehat{\xi}_{2,t}\}$

- Simulations for a calibrated economy

$\sigma = 1$, $\beta = 0.99$, $\kappa = 0.1717$, $\vartheta = 0.0191$ (Galí (2015))
$r^* = -0.0025$

Optimal Policy: Transitional Dynamics

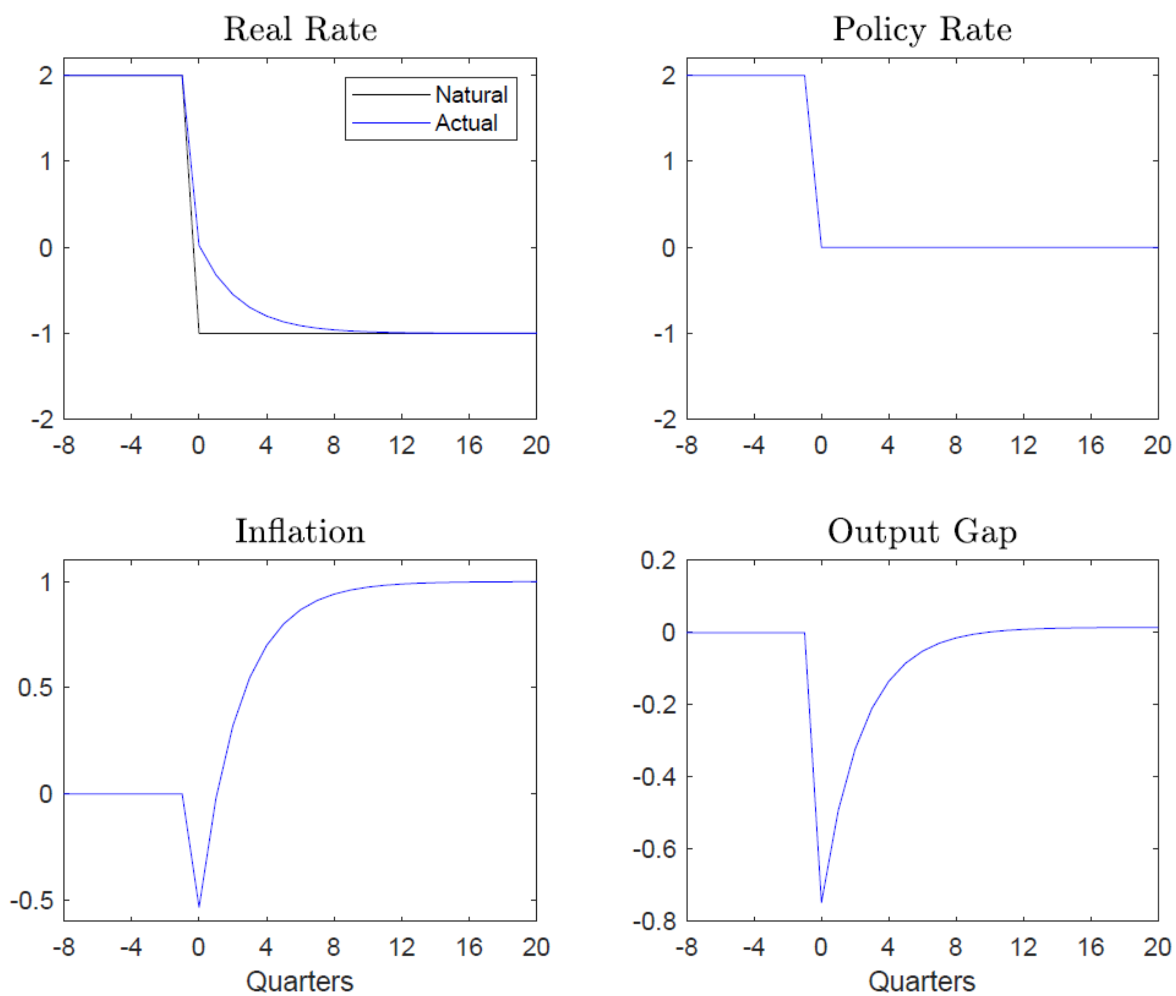


Figure 1: Transitional dynamics under the optimal monetary policy. Inflation and interest rates in annualized terms.

Optimal Policy: The Stochastic Case

- Stochastic equilibrium

$$\widehat{\pi}_t = \beta \mathbb{E}_t\{\widehat{\pi}_{t+1}\} + \kappa \widehat{y}_t$$

$$\widehat{\pi}_t = \widehat{\xi}_{1,t} - \widehat{\xi}_{1,t-1} + \beta^{-1}\widehat{\xi}_{2,t-1}$$

$$\vartheta \widehat{y}_t = -\kappa \widehat{\xi}_{1,t} - \widehat{\xi}_{2,t} + \beta^{-1}\widehat{\xi}_{2,t-1}$$

$$\widehat{\xi}_{2,t} + \xi_2 \geq 0$$

$$\sigma(\mathbb{E}_t\{\widehat{y}_{t+1}\} - \widehat{y}_t) + \mathbb{E}_t\{\widehat{\pi}_{t+1}\} + z_t \geq 0$$

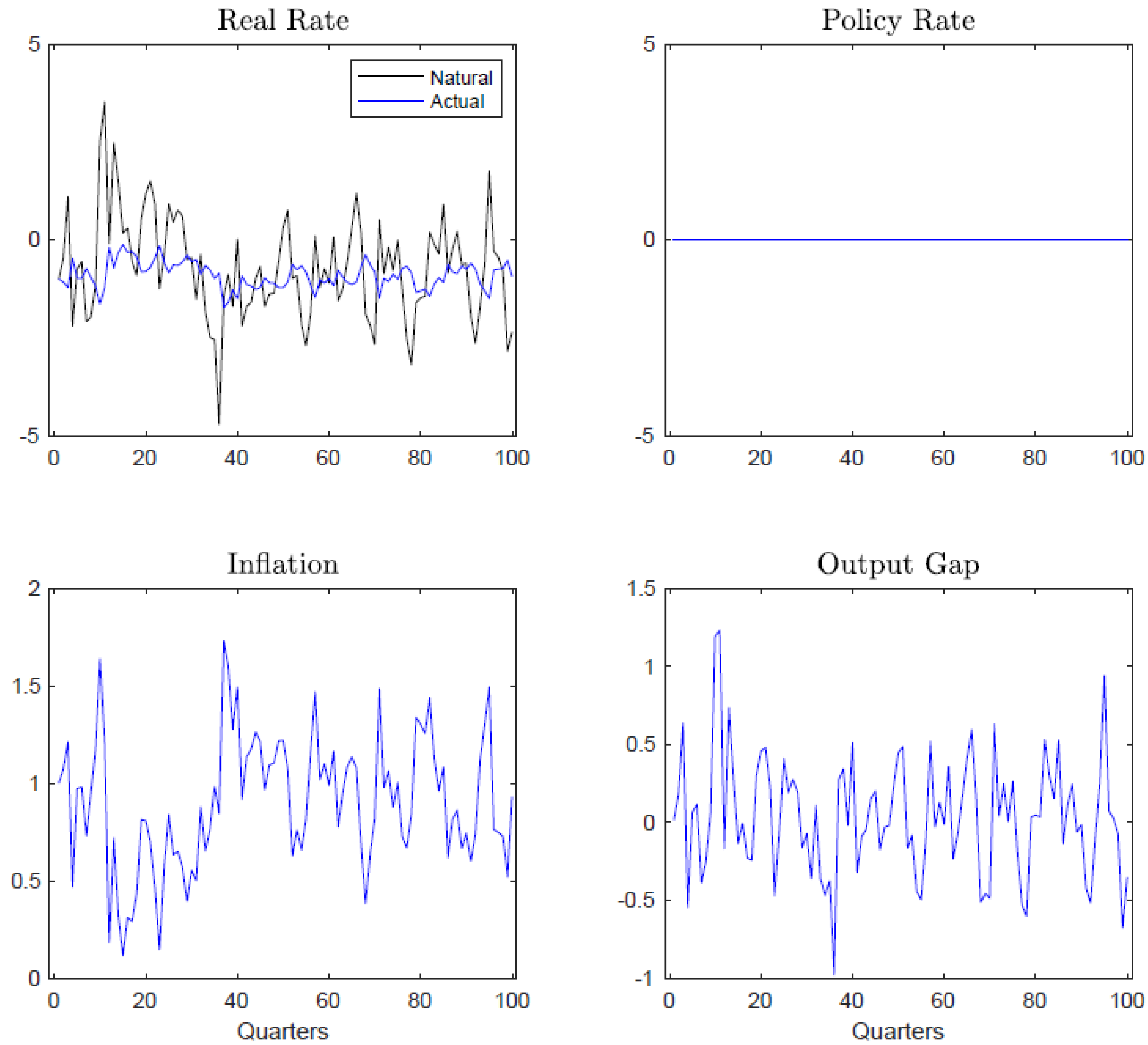
$$[\widehat{\xi}_{2,t} + \xi_2][\sigma(\mathbb{E}_t\{\widehat{y}_{t+1}\} - \widehat{y}_t) + \mathbb{E}_t\{\widehat{\pi}_{t+1}\} + z_t] = 0$$

for $t = 0, 1, 2, ...$with initial conditions given by $\widehat{\xi}_{1,-1} = 0$ and $\widehat{\xi}_{2,-1} = 0$.

- Simulations: $(\rho_z, \sigma_z) = (0.5, 0.0025)$

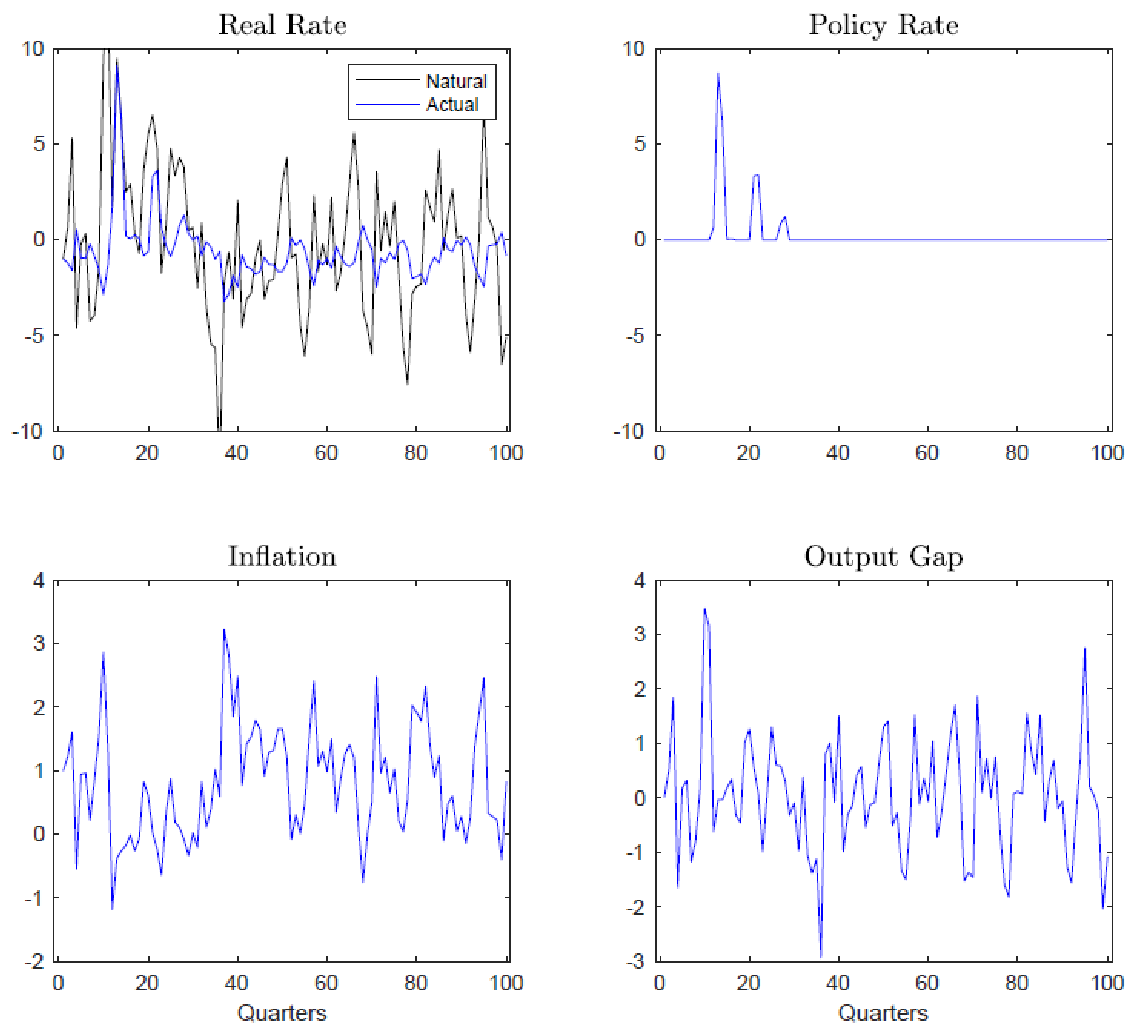
Optimal Policy: Fluctuations around the Steady State

Baseline Calibration

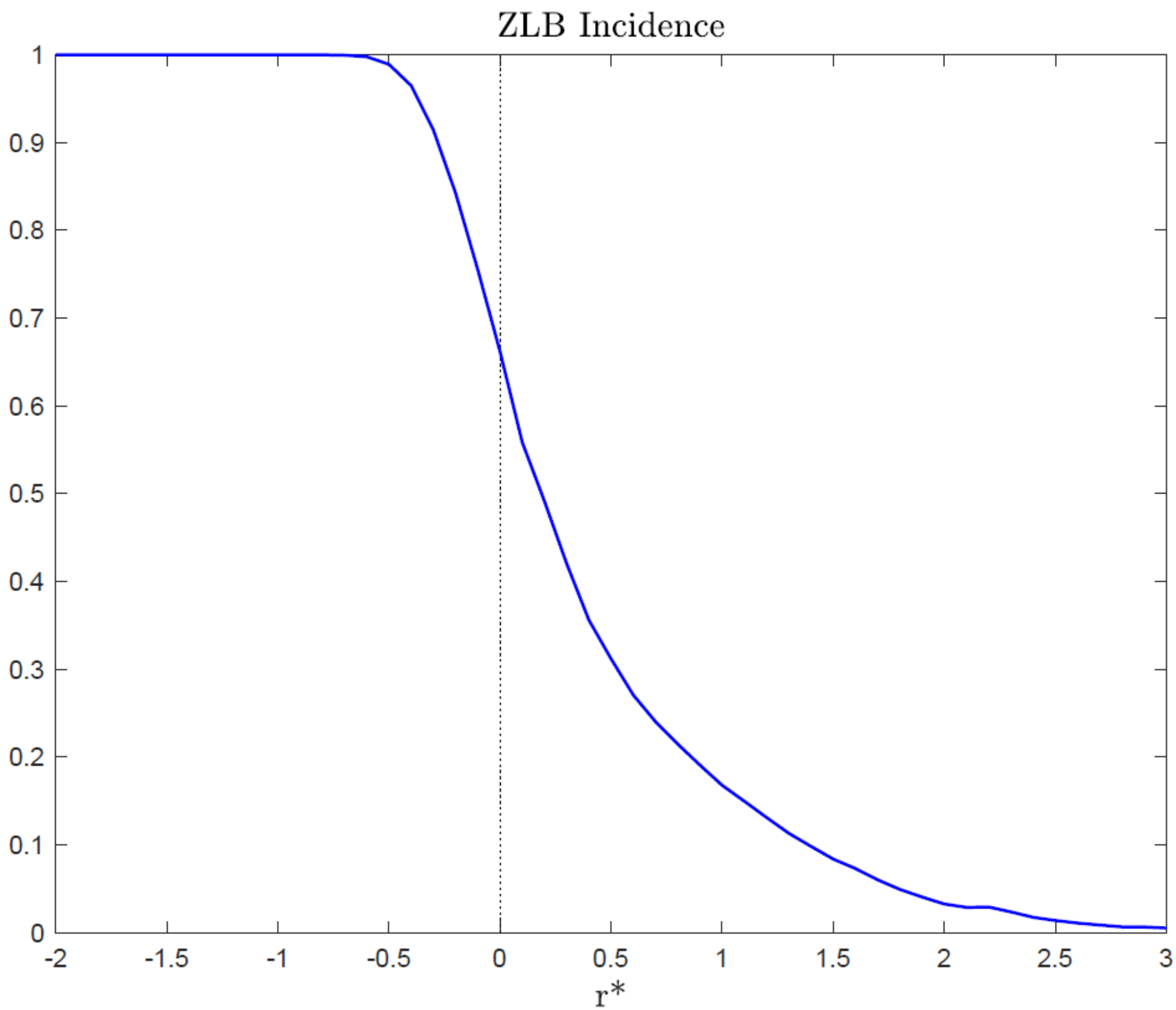


Optimal Policy: Fluctuations around the Steady State

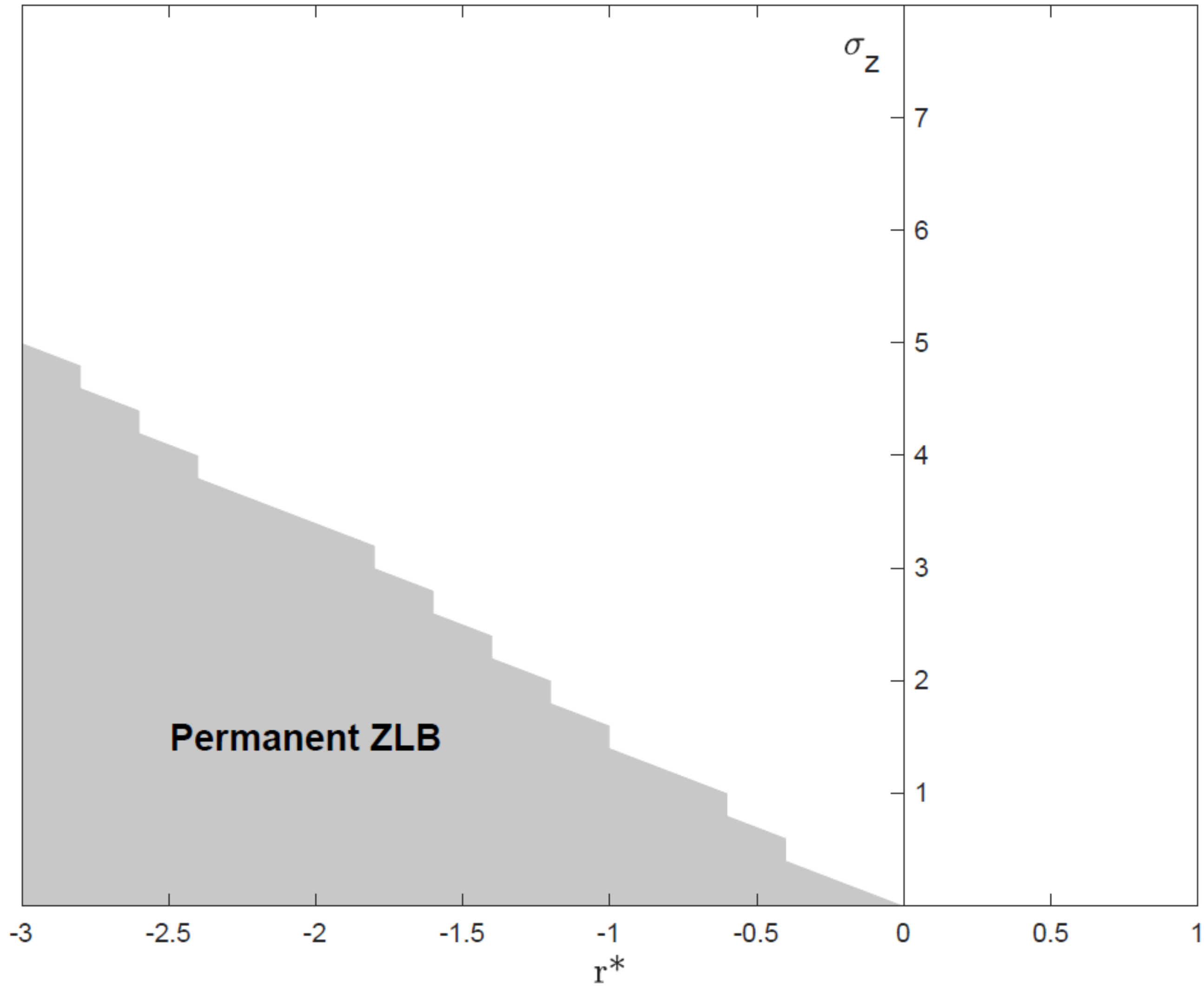
High Volatility Calibration



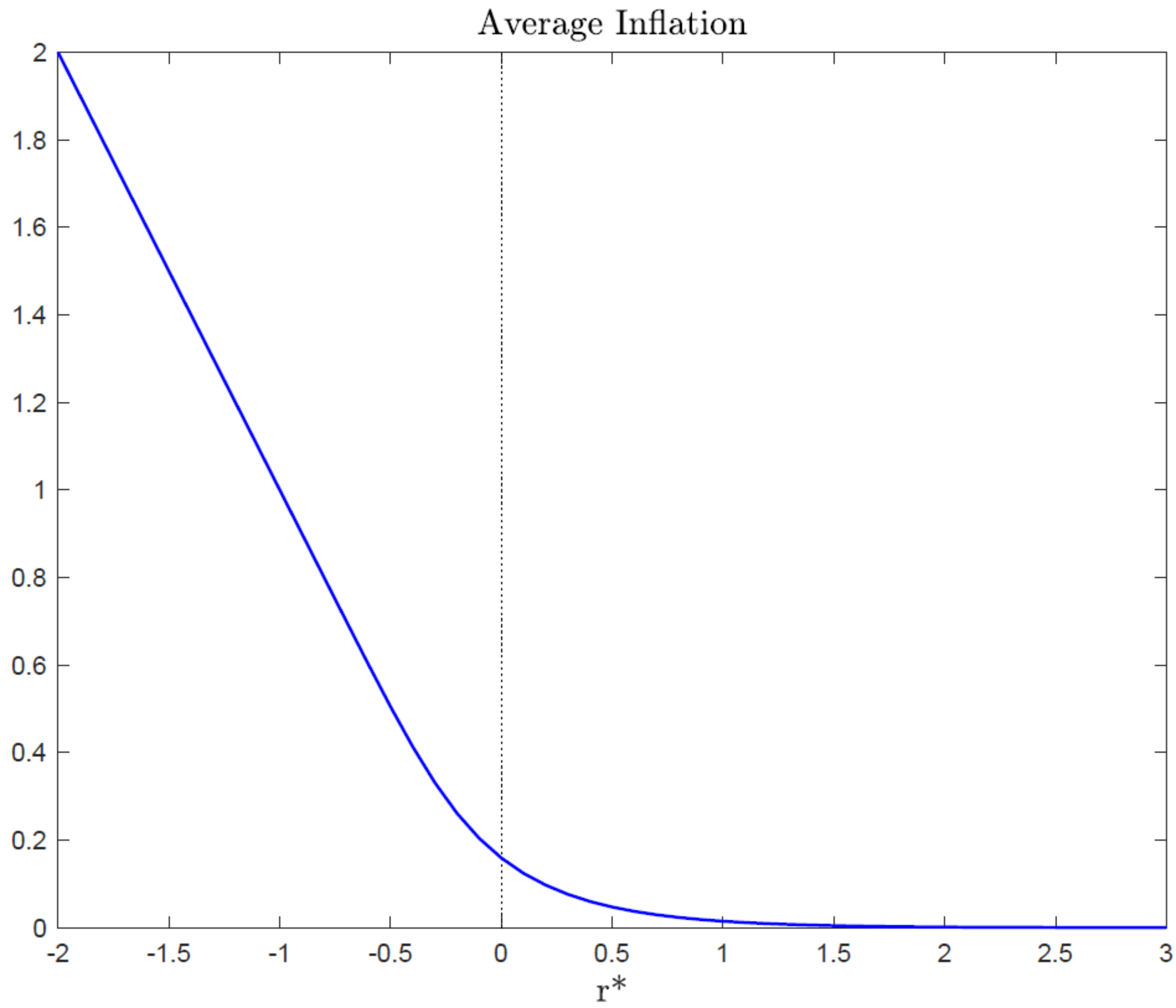
r* and ZLB Incidence under the Optimal Policy



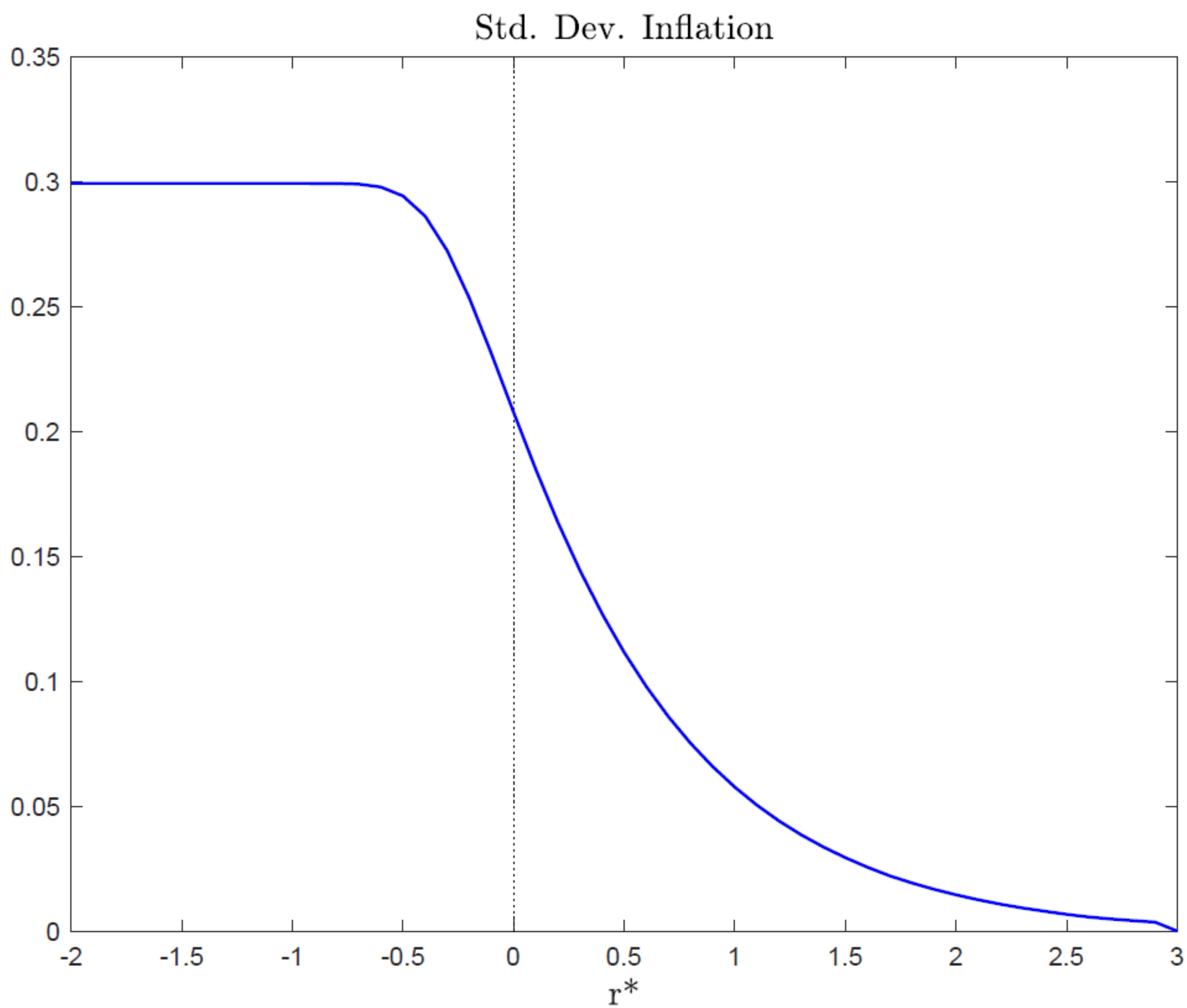
Conditions for a Permanent ZLB



r* and Average Inflation under the Optimal Policy



r* and Inflation Volatility under the Optimal Policy



Optimal Policy: The Stochastic Case

- Stochastic equilibrium

$$\widehat{\pi}_t = \beta \mathbb{E}_t\{\widehat{\pi}_{t+1}\} + \kappa \widehat{y}_t$$

$$\widehat{\pi}_t = \widehat{\xi}_{1,t} - \widehat{\xi}_{1,t-1} + \beta^{-1}\widehat{\xi}_{2,t-1}$$

$$\vartheta \widehat{y}_t = -\kappa \widehat{\xi}_{1,t} - \widehat{\xi}_{2,t} + \beta^{-1}\widehat{\xi}_{2,t-1}$$

$$\widehat{\xi}_{2,t} + \xi_2 \geq 0$$

$$\sigma(\mathbb{E}_t\{\widehat{y}_{t+1}\} - \widehat{y}_t) + \mathbb{E}_t\{\widehat{\pi}_{t+1}\} + z_t \geq 0$$

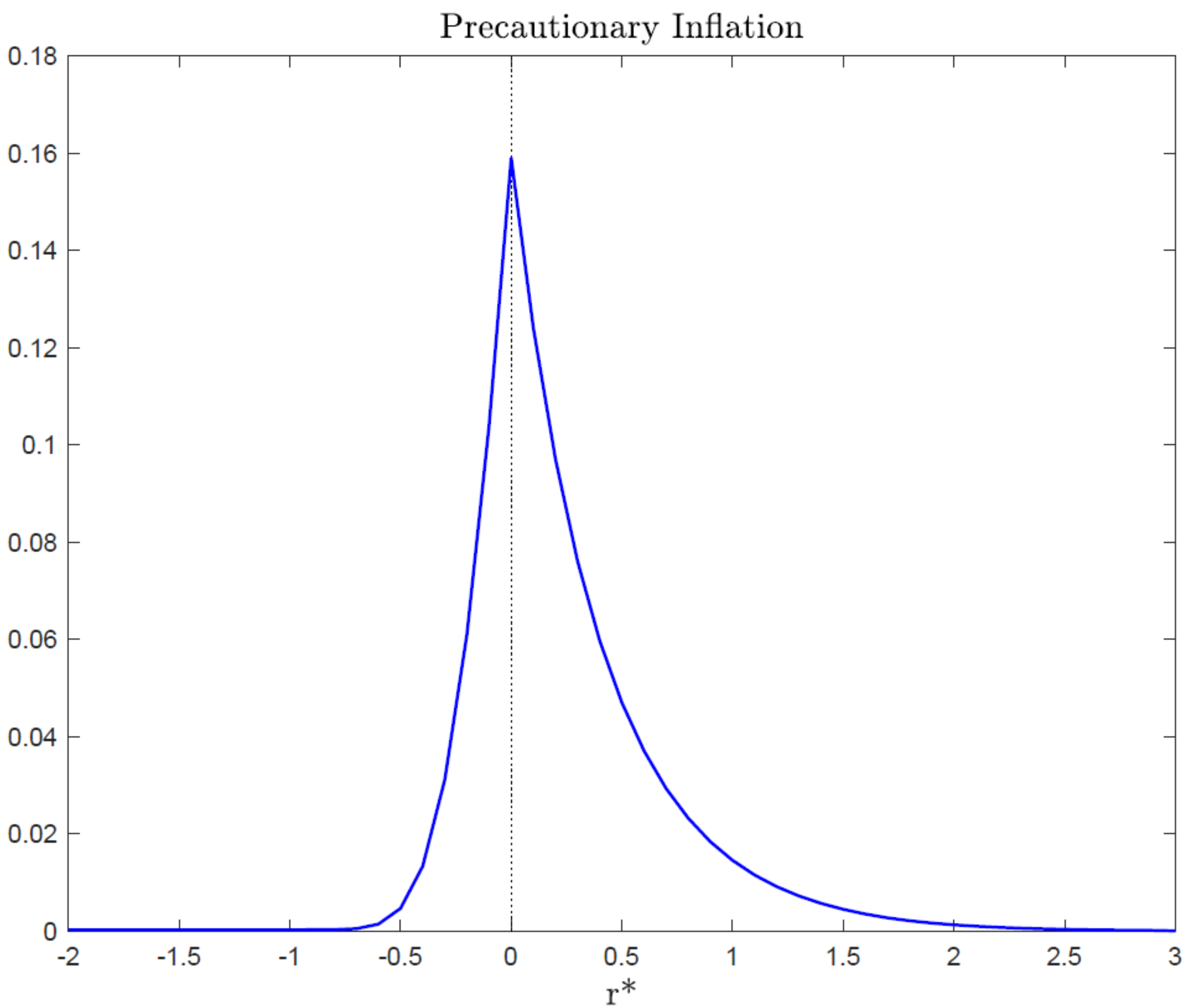
$$[\widehat{\xi}_{2,t} + \xi_2][\sigma(\mathbb{E}_t\{\widehat{y}_{t+1}\} - \widehat{y}_t) + \mathbb{E}_t\{\widehat{\pi}_{t+1}\} + z_t] = 0$$

for $t = 0, 1, 2, ...$with initial conditions given by $\widehat{\xi}_{1,-1} = 0$ and $\widehat{\xi}_{2,-1} = 0$.

- Simulations: $(\rho_z, \sigma_z) = (0.5, 0.0025)$
- Precautionary inflation

$$\overline{\pi}^p(r^*) = \overline{\pi}(r^*) - \max\{0, -r^*\}$$

r* and Precautionary Inflation



Optimal Policy: Implementation

- Equilibrium outcomes under optimal policy: (i_t^*, y_t^*, π_t^*)
- Non-Policy block, in deviations from optimal path:

$$\widetilde{\pi}_t = \beta \mathbb{E}_t\{\widetilde{\pi}_{t+1}\} + \kappa \widetilde{y}_t$$

$$\widetilde{y}_t = \mathbb{E}_t\{\widetilde{y}_{t+1}\} - \frac{1}{\sigma}(\widetilde{i}_t - \mathbb{E}_t\{\widetilde{\pi}_{t+1}\})$$

$$\widetilde{i}_t \geq -i_t^*$$

where $\widetilde{x}_t \equiv x_t - x_t^*$ for $x \in \{\pi, y, i\}$

- *Wanted*: policy rule that guarantees $(\widetilde{\pi}_t, \widetilde{y}_t, \widetilde{i}_t)$ is the only equilibrium.
- Candidate rule:

$$i_t = i_t^* \Leftrightarrow \widetilde{i}_t = 0$$

for all t. Combined with non-policy block $\Rightarrow$ multiplicity of solutions in addition to (i_t^*, y_t^*, π_t^*)

Optimal Policy: Implementation

- Proposed rule:

$$i_t = i_t^* + \phi_\pi^{(q)} \widetilde{\pi}_t + \phi_y^{(q)} \widetilde{y}_t$$

where $q \in \{1, 2, 3, 4\}$, satisfying:

$$\begin{aligned}
q = 1 &: \widetilde{\pi}_t \geq 0, \widetilde{y}_t \geq 0 \Rightarrow \phi_\pi^{(1)} \geq 0, \phi_y^{(1)} \geq 0 \\
q = 2 &: \widetilde{\pi}_t < 0, \widetilde{y}_t < 0 \Rightarrow \phi_\pi^{(2)} \leq 0, \phi_y^{(2)} \leq 0 \\
q = 3 &: \widetilde{\pi}_t \geq 0, \widetilde{y}_t < 0 \Rightarrow \phi_\pi^{(3)} \geq 0, \phi_y^{(3)} \leq 0 \\
q = 4 &: \widetilde{\pi}_t < 0, \widetilde{y}_t \geq 0 \Rightarrow \phi_\pi^{(4)} \leq 0, \phi_y^{(4)} \geq 0
\end{aligned}$$

Optimal Policy: Implementation

- Regime switching representation

$$\begin{bmatrix} \widetilde{y}_t \\ \widetilde{\pi}_t \end{bmatrix} = \mathbf{A}_t \begin{bmatrix} \mathbb{E}_t\{\widetilde{y}_{t+1}\} \\ \mathbb{E}_t\{\widetilde{\pi}_{t+1}\} \end{bmatrix}$$

where $\mathbf{A}_t = \mathbf{A}^{(q)}$ if the economy is in "regime" q in period t, with

$$\mathbf{A}^{(q)} \equiv \frac{1}{\sigma + \phi_y^{(q)} + \kappa\phi_\pi^{(q)}} \begin{bmatrix} \sigma & 1 - \beta\phi_\pi^{(q)} \\ \sigma\kappa & \kappa + \beta(\sigma + \phi_y^{(q)}) \end{bmatrix}$$

for $q \in \{1, 2, 3, 4\}$

Equilibrium Determinacy in (Endogenous) Regime Switching Models

- A benchmark regime switching model

$$\mathbf{x}_t = \mathbf{A}_t \mathbb{E}_t\{\mathbf{x}_{t+1}\} \tag{13}$$

where $\mathbf{x}_t$ is an $(n \times 1)$ vector of non-predetermined variables and A_t is an $(n \times n)$ non-singular matrix. Assume $\mathbf{A}_t \in \mathcal{A} \equiv \{\mathbf{A}^{(1)}, \mathbf{A}^{(2)}, ..., \mathbf{A}^{(Q)}\}$.

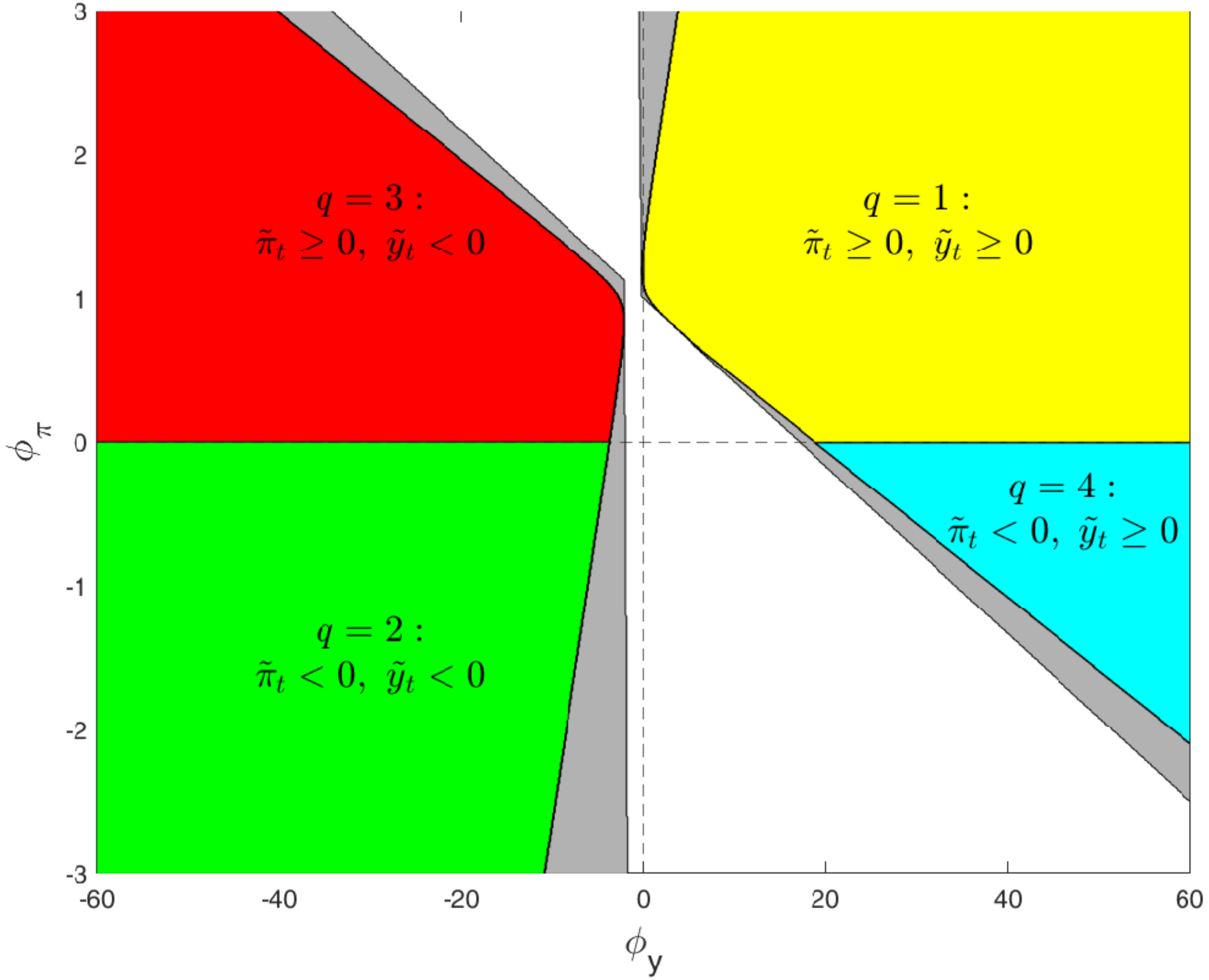
- Goal: establish *sufficient* conditions on $\mathcal{A}$ that guarantee that $\mathbf{x}_t = 0$ all t is the only bounded solution to (13), i.e. $\lim_{T \to +\infty} \mathbb{E}_t\{\|\mathbf{x}_{t+T}\|\} > M\|\mathbf{x}_t\|$ for any $M > 0$ and $\mathbf{x}_t \neq 0$, and where $\|\cdot\|$ is the usual L^2 norm.
- Define $\left\|\mathbf{A}^{(q)}\right\| \equiv \max_{\mathbf{x}} \left\|\mathbf{A}^{(q)}\mathbf{x}\right\|$ subject to $\|\mathbf{x}\| = 1$. In addition, $\alpha \equiv \max\{\left\|\mathbf{A}^{(1)}\right\|, \left\|\mathbf{A}^{(2)}\right\|, ..., \left\|\mathbf{A}^{(Q)}\right\|\} > 0$.

Theorem *[sufficient condition for determinacy]*: If $\alpha < 1$, then $\mathbf{x}_t = 0$ for all t is the only bounded solution to (13)

Optimal Policy: Implementation

- Determinacy region

Implementation: Determinacy Region



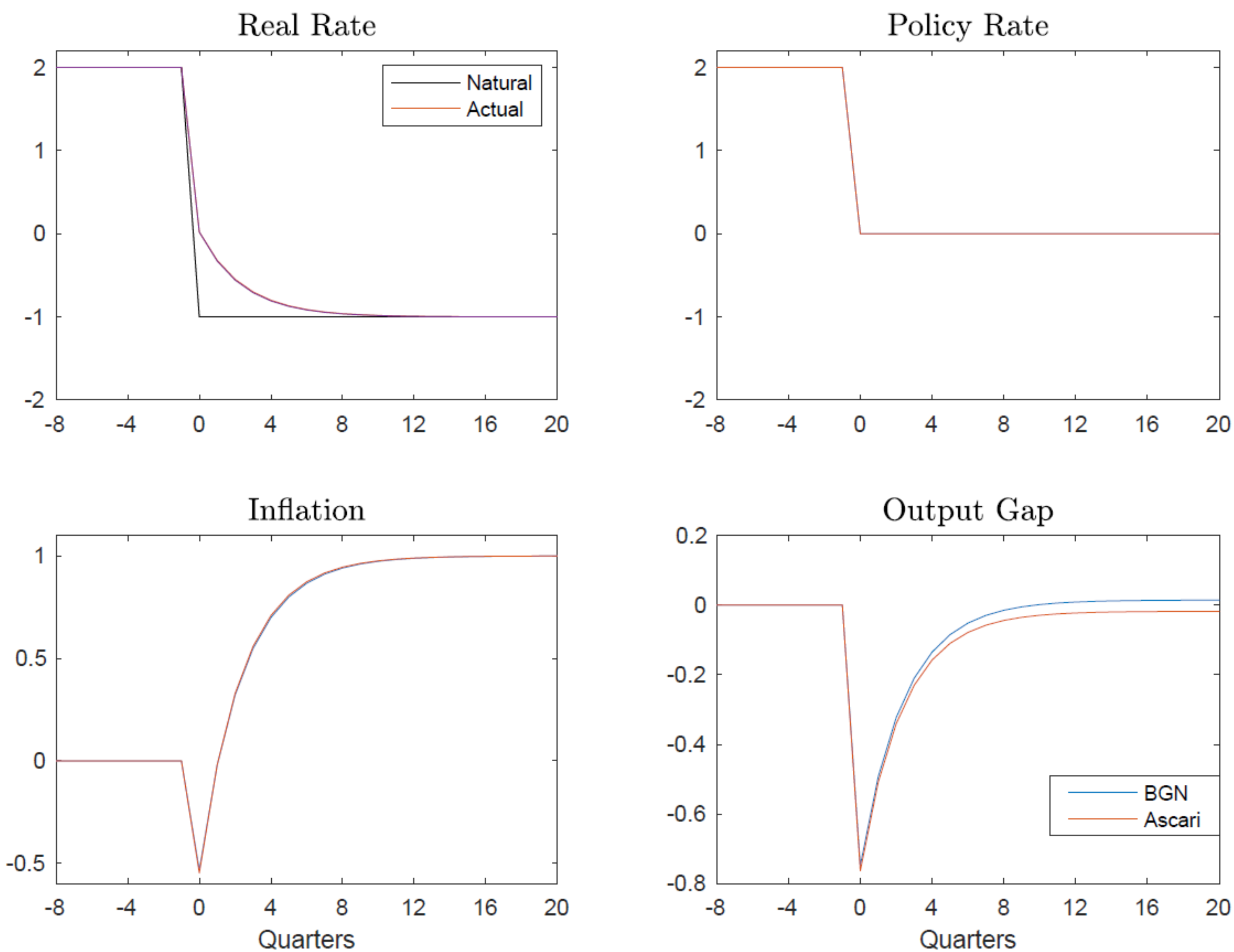
Optimal Policy: Implementation

- Determinacy region
- Discussion: time inconsistency, credibility

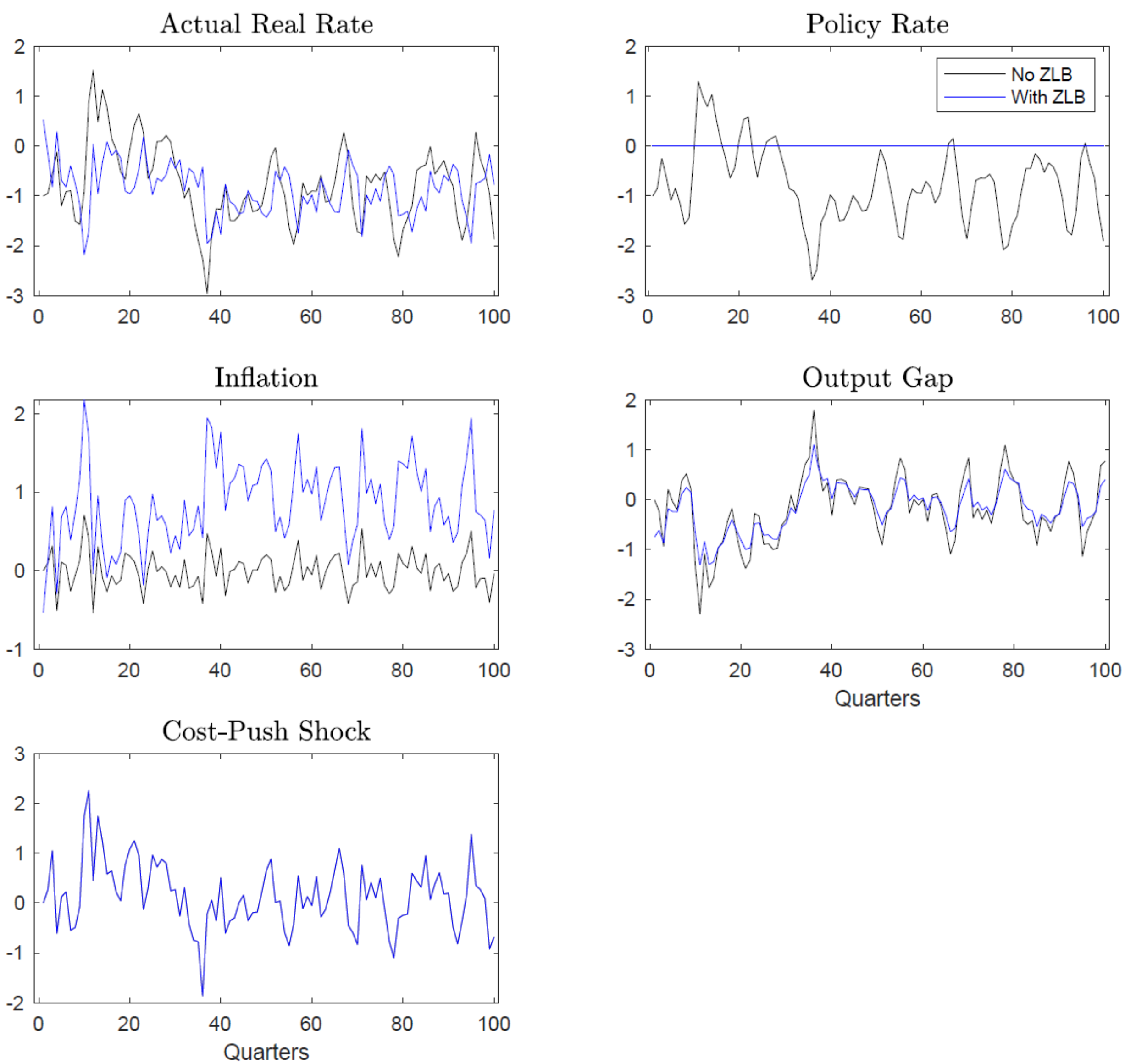
Concluding remarks

- Optimal monetary policy with a ZLB constraint and $r^* < 0$.
- The optimal policy aims to approach *gradually* a steady state with positive average inflation and a binding ZLB.
- Around that steady state, inflation and the output gap display (second-best) fluctuations in response to shocks. Those fluctuations coexist with a nominal rate that remains at its ZLB most (or all) of the time.
- The central bank can implement the optimal policy as a (locally) unique equilibrium by means of an appropriate state-contingent rule.
- In order to establish that result, we derive a sufficient condition for local determinacy in a general model with endogenous regime switches, a finding that may be of interest beyond the problem studied in the present paper.

Correction for Positive Trend Inflation



Optimal Policy with Cost-Push Shocks: Baseline Calibration



Optimal Policy with Cost-Push Shocks: High Volatility Calibration