The Dean and The Chair*

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Abstract

A principal needs the expertise of a biased agent in order to assess applicants who are available for hire. The principal can commit to a mechanism in order to make use of the agent's expertise, while mitigating the consequences of the agent's bias. The optimal mechanism is such that the agent is rewarded or punished in real time for his hiring decisions. If the agent hires an applicant, the mechanism rewards him with an increase in value. If the agent does not hire an applicant, the mechanism punishes him with a decrease in value. The system of punishments and rewards moves the agent's reservation quality towards the one preferred by the principal and, hence, reduces the impact of the agent's bias on the hiring. The punishments and rewards are ultimately delivered by the mechanism as changes in probability with which the hiring decision is taken away from the agent. Once the hiring decision is taken away from the agent, the principal permanently takes over and hires every applicant.

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1 Introduction

A principal needs the expertise of a biased agent in order to assess applicants who are available to fill the principal's vacancies. The principal could control the hiring process directly, but in doing so he would have to forego the expertise of the agent. The principal could fully delegate the hiring process to the agent, but in doing so he would have to suffer the consequences of the agent's bias. The principal could assess the hiring record of the agent every year and, depending on the fraction of applicants hired by the agent, he could take over the hiring process or leave it in the hands of the agent. In this paper, we show that none of these mechanisms—direct control of hiring by the principal, unconditional delegation of hiring to the agent, or hiring quotas—are optimal.

The optimal mechanism, instead, is such that the agent is rewarded or punished in real time for his hiring decisions. If the agent hires an applicant, the mechanism rewards him with an increase in value. If the hire does not hire an applicant, the mechanism punishes him with a decrease in value. The system of punishments and rewards moves the agent's reservation quality towards the one preferred by the principal and, hence, reduces the impact of the agent's bias on the hiring outcomes. The punishments and rewards are ultimately delivered by the mechanism as changes in probability with which the hiring decision is taken away from the agent. Once the hiring decision is taken away from the agent, the principal permanently takes over and hires every applicant.

A fitting example of the type of problems studied in this paper is the relationship between the Dean at a University and the Chair of one of the departments that fall under the purview of the Dean. The Dean and the Chair both care about the quality of new department hires. The Chair naturally has superior information about the quality of different candidates. However, the Chair is biased against candidates belonging to some demographic group or, equivalently, the Dean places some extra value on candidates belonging to some demographic group. Occasionally, a candidate from such a demographic group is available for hire. The paper characterizes the features of the mechanism that the Dean should implement in order to minimize the consequences of the Chair's bias.

In Section 2, we consider a basic version of the problem. In every period, there is a vacancy and an applicant that may be hired to fill the vacancy. If the applicant is hired, the principal's payoff is given by the quality x of the applicant and the agent's payoff is the given by the quality x of the applicant net of a constant $\eta > 0$, which is meant to capture the extent of the agent's bias. If the applicant is not hired, the principal's and the agent's payoffs are equal to zero. The quality of the applicant is privately observed by the agent and it is drawn from some commonly known distribution. No monetary transfers between the principal and the agent are feasible.

In the first period, the principal commits to a direct-revelation mechanism. In each period, the mechanism elicits from the agent a report about the quality of the applicant.

Depending on the realized history of play, the mechanism may make use of the agent's report to decide whether the applicant is hired, or it may ignore the agent's report. In the first case, the mechanism produces the same outcomes as if the agent was choosing weather or not to hire the applicant taking as given the consequences of his decision on the value that he will receive from the mechanism going forward. For this reason, we refer to the first case as "delegation". In the second case, the mechanism produces the same outcomes as if the principal was directly choosing whether to hire the applicant. For this reason, we refer to the second case as "control". Since the upper envelope of the principal's values in delegation and control is unlikely to be a concave function of the agent's value, we allow the mechanism to specify, at the beginning of each period, a public lottery between delegation and control. In Section 2, we restrict attention to mechanisms with the property that: (i) once the mechanism enters control, it remains in control forever; (ii) during control, the principal hires the applicant. In Section 3, we prove that these restrictions are innocuous.

We formulate the optimal mechanism design recursively, using the agent's promised value as an auxiliary state variable. We first characterize the optimal lottery between delegation and control as a function of the agent's promised value V. We show that, if V is lower than some threshold V_C, the optimal lottery is non-degenerate, in the sense that it assigns positive probability to both delegation and control. If the outcome of the lottery is control, the principal hires every future candidate and the agent's value falls to some V_P. If the outcome of the lottery is delegation, the agent keeps making the hiring decisions and his value moves up to the threshold V_C. If the agent's promised value is higher than the threshold V_C, the optimal lottery is degenerate. In particular, the agent keeps making the hiring decisions and his value remains equal to the promised value V.

We then characterize the optimal incentives in delegation as a function of the agent's promised value $\hat{V}$. We show that, if the agent hires the applicant in the current period, he gets rewarded with a continuation value V_1 that is strictly greater than his promised value $\hat{V}$. If the agent does not hire the applicant, he gets punished with a continuation value V_0 that is strictly smaller than his promised value $\hat{V}$. The gap between the agent's continuation value conditional of hiring and not hiring moves the agent's reservation quality R away from the agent's preferred reservation quality η and towards the principal's preferred reservation quality 0. If the agent keeps on hiring candidates, his value converges towards (but never attains) V_F, which is the value that the agent could obtain if he was given unfettered discretion over hiring. In this limit, the mechanism gives no incentives to the agent and his reservation quality R converges to η. If the agent keeps not hiring candidates, his value eventually falls below the threshold V_C and he is faced with the threat of control. After any history of play, the agent faces a strictly positive probability of having to permanently hand over hiring to the principal because the probability of not hiring an applicant is always strictly positive. In contrast, after any history of play, the agent has no chance of having permanent discretion over hiring because, even though the probability of hiring an applicant is always strictly positive, the mechanism never rewards the agent with the value V_F.

In Section 4, we show that the characterization of the optimal mechanism can be directly applied to several generalizations of the baseline environment. First, we consider a version of the model in which the agent is positively, rather than negatively, biased against the applicants. The optimal mechanism in this version of the model has the same features as in the baseline, except that the agent is rewarded with a higher value for not hiring the applicant rather than for hiring the applicant. Second, we consider a version of the model in which there are $n > 1$ applicants for each vacancy. We show that the optimal mechanism design problem for this version of the model is isomorphic to the optimal mechanism design problem in the baseline, except that the quality distribution is replaced with the distribution of the maximum of n draws from the quality distribution. Third, we consider a version of the model in which, for each vacancy, there is both a contentious applicant—an applicant from a demographic group against which the agent is biased—and an uncontentious applicant—an applicant from a demographic group towards which the agent is unbiased. We show that the optimal mechanism design problem for this version of the model is isomorphic to the optimal mechanism design problem in the baseline, except that the quality distribution is replaced with the distribution of the difference between the quality of the contentious applicant and the quality of the uncontentious applicant. Lastly, we consider a version of the model in which the principal does not know whether an applicant is available or not. We show that the optimal mechanism design problem for this version of the model is nearly identical to the optimal mechanism design problem in the baseline, except that not hiring an applicant and not having an applicant are treated as identical events by the mechanism.

The paper relates to the literature on delegation, broadly defined as mechanism design without transfers (Holmstrom 1977, Amador and Bagwell 2013). Important examples of delegation include setting consumption rules for an individual with time-inconsistent preferences (Angeletos, Werning and Amador 2006), setting rules of conduct for a time-inconsistent monetary authority (Athey, Atkeson and Kehoe 2005), setting fiscal rules for a time-inconsistent government (Halac and Yared 2018), setting hiring rules for a biased employer (Frankel 2021). A common finding in this literature is that, as long as shocks are independently drawn over time, the optimal mechanism is static, in the sense that it makes the same prescriptions independently of the realized history of play. In contrast to this literature, we find that the optimal mechanism is dynamic. Intuitively this is so because, in our model the principal and the agent disagree not only today, but also in the future.

Other papers on delegation find, like us, that the optimal mechanism is history-dependent. Guo and Horner (2020) study the problem of a principal that needs to rely on a biased agent to assess the quality of investment opportunities. They restrict attention to an environment in which the investment can be of one of two qualities, while we study an environment in which the investment can take on a continuum of qualities. The difference is not a pure technicality, as it affects both the approach to characterizing the optimal mechanism as well as the properties of the optimal mechanism. For example, with a continuum of qualities, the mechanism delivers higher and lower value to the agent by also inducing him to change his reservation quality. With two qualities, this margin is inoperative. Moreover, with a continuum of qualities, it is immediate to extend the analysis of the optimal mechanism to an environment with multiple investment opportunities, some of which may be contentious and some of which are not.

Lipnowski and Ramos (2020) consider the same model as Guo and Horner (2020), but under the assumption that the principal lacks commitment and, hence, they focus on public perfect equilibria rather than optimal mechanisms. Jackson and Sonnenschein (2007) study dynamic delegation in the limit as the principal and the agent become infinitely patient. They find that, in the limit, the optimal mechanism can support the first-best payoff of the principal and it can do so by leveraging the fact that in the long-run the principal knows the realization of the quality of investment projects faced by the agent. While the findings of Jackson and Sonneschein (2007) apply to our environment, we focus on the case of impatient principal and agent.

2 Baseline Model

In this section, we describe the mechanism design problem of a principal who needs to rely on the expertise of a biased agent in order to evaluate the quality of applicants that are available for hire. Conditional on the realized history of play, a mechanism specifies the probabilities of a lottery between delegation–where the agent's report about the quality of the applicant are used to decide whether to hire the applicant or not–and control–where the agent's report in which the agent's report about the quality of the applicant is ignored. Conditional on the realized history of play and on the delegation outcome of the lottery, a mechanism specifies which agent's reports about the quality of the applicant are such that the applicant is hired and which reports are such that the agent is not hired. We restrict attention to mechanisms such that, when the agent's reports are ignored, they are ignored permanently and the mechanism hires all the applicants. In section 2.2, we formulate the optimal mechanism design problem recursively, using the agent's promised value as an auxiliary state variable. In Section 2.3, we characterize the properties of the optimal lottery between delegation and control. In Section 2.4, we characterize the properties of the optimal delegation. In Section 3, we show that the restriction to the space of mechanisms is without loss in generality.

2.1 Environment

In every period $t = 0, 1, 2, ...$, there is a vacancy that needs to be filled by the principal. In every period, there is an applicant that is available to fill the vacancy. The quality of the applicant x is drawn from a distribution $F(x)$ with mean 0 and support $X = [\underline{x}, \overline{x}]$. Moreover, the distribution F is assumed to be continuously differentiable. If the applicant is hired, the flow payoff to the principal is $(1-\beta)x$, where $\beta \in (0,1)$ is the rate at which the principal discounts future payoffs. If the applicant is not hired, the flow payoff to the principal is 0. The quality of the applicant is not observed by the principal, but it is observed by the agent. The agent, however, does not have the same preferences as the principal. In particular, if the applicant is hired, the flow payoff to the agent is $(1-\beta)(x-\eta)$, where β is the rate at which the agent discounts future payoffs and $\eta \in (0, \overline{x})$ is a measure of the agent's negative bias against the applicants. If the applicant is not hired, the flow payoff to the agent is 0. No monetary transfers between the principal and the agent are possible.

One natural interpretation of the environment has the principal being a dean and the agent being the chair of one of the departments that fall under the perview of the dean. A candidate from some particular demographic group can be hired to the department. The dean and the chair both care about the quality of the candidate. The chair, however, is the only one with the expertise to evaluate the candidate, and he is biased against the candidate because of his demographic characteristics.

If the principal and the agent had the same preferences, the principal could permanently delegate hiring to the agent. Then, the agent would hire an applicant if and only if his quality x was greater than 0 and, in doing so, he would maximize the present value of payoffs to himself and the principal. However, since the agent is biased against the applicants, delegating hiring to the agent does not work quite as well. Indeed, the agent would only hire an applicant if his quality x exceeds η. In doing so, the agent would maximize the present value of his own payoffs, but not the present value of the principal's payoffs. If, alternatively, the principal took control of hiring, the present value of his payoffs would only be 0. The principal however can do something more sophisticated than either permanently delegate hiring to the agent or permanently control the hiring process. The principal can use the threat of taking over hiring to reward and punsih the agent for his hiring decisions and, in doing so, he can incentivize the agent to hire some applicants with quality lower than η.

Formally, the principal can commit to a direct-revelation mechanism in period $t = 0$. The principal chooses the mechanism so as to maximize the expectation of the present discounted value of his payoffs. In every period $t = 0, 1, 2, ...$, the mechanism solicits a report $\hat{x} \in X$ from the applicant about the quality $x \in X$ of the candidate and decides whether or not to hire the applicant $a \in \{0, 1\}$. The mechanism may use the report from the agent, in the sense that it does or does not hire the applicant depending on the report, or it may ignore the report from the agent, in the sense that it may or may not hire the applicant irrespective of the report. In the first case, we say that the mechanism is delegating the hiring decision to the agent. In the second case, we say that the hiring decision is controlled by the principal.

Consider the case in which hiring is delegated to the agent. Let X_0 denote the set of reports such that the mechanism does not hire the applicant, i.e. $a(\hat{x}) = 0$ for all $\hat{x} \in X_0$. Similarly, let X_1 denote the set of reports such that the mechanism does not hire the applicant, i.e. $a(\hat{x}) = 1$ for all $\hat{x} \in X_1$. If the agent reports $\hat{x}$, his flow payoff is $a(\hat{x})(1-\beta)(x-\eta)$. If the agent reports $\hat{x}$, let $\beta V(\hat{x})$ denote the agent's discounted continuation value from the mechanism. Since the mechanism must induce the agent to report the applicant's quality truthfully, it follows immediately that $V(\hat{x}) = V_0$ for all $\hat{x} \in X_0$ and $V(\hat{x}) = V_1$ for all $\hat{x} \in X_1$. That is, the mechanism must give the agent the same continuation value V_0 for all reports that lead the mechanism to pass on the applicant, and the same continuation value V_1 for all reports that lead the mechanism to hire the applicant. Moreover, since the mechanism must induce the agent to report the applicant's quality truthfully, it has to be the case that $X_0 = [\underline{x}, R)$.and $X_1 = (R, \overline{x}]$, where R is the quality of the candidate that makes the agent indifferent between $(1-\beta)(x-\eta)+\beta V_1$ and βV_0. Overall, when the mechanism uses the agent's report, it is as if the agent was directly choosing whether or not to hire the applicant, taking into account that his continuation value is V_0 if he does not hire the applicant and V_1 if he hires the applicant.

Consider the case in which hiring is controlled by the principal. In this case, the mechanism either does not hire the applicant irrespective of the agent's report, i.e. $a(\hat{x}) = 0$ for all $\hat{x} \in X$, ot it hires the applicant irrespective of the agent's report, i.e. $a(\hat{x}) = 0$ for all $\hat{x} \in X$. In either case, since the mechanism must induce the agent to report truthfully the quality of the applicant, the agent's continuation value is independent of the agent's report. Overall, when the mechanism ignores the agent's report, it is as if the principal was controlling the hiring process.

The agent's and principal's payoff combinations that can be achieved under delegation and under control are likely to be different and, in turn, the upper envelope of the agent's and principal's payoffs under delegation and under control need not be concave. Therefore, it is natural to let the mechanism specify a public lottery between delegation and control at the beginning of every period. Specifically, at the beginning of every period, the mechanism specifies some probability p with which hiring is controlled by the principal and some probability $1-p$ with which hiring is delegated to the agent, where $p \in [0, 1]$.

In the remainder of this section, we are going to restrict attention to mechanisms such that: (i) if the mechanism hands the control of the hiring process to the principal, it does so forever; (ii) if the mechanism assigns control of the hiring process to the principal, the principal hires the applicant. In Section 3, we show that these restrictions are without loss in generality. Even though starting with a restriction on the space of mechanisms and then shwoing that it is without loss in generality may appear backwards to some of our readers, it does afford for a simpler analysis of the optimal mechanism.

2.2 Recursive Formulation and Preliminaries

The principal's mechanism design problem can be formulated recursively, using the agent's promised value as an auxiliary state variable. In the first-stage of the recursive problem, the principal chooses a lottery between delegation and control, subject to delivering the promised value to the agent. In the second-stage of the recursive problem, which is the stage associated with the delegation branch of the lottery, the principal chooses the agent's continuation value conditional on the agent not hiring the applicant and the agent's continuation value conditional on the agent hiring the applicant, subject to delivering the promised value to the agent.

Formally, the first-stage problem is

$$J(V) = \max_{p \in [0,1], \hat{V} \in \hat{\mathcal{V}}} pJ_P + (1-p)\hat{J}(\hat{V}) \tag{2.1}$$

subject to the promise-keeping constraint

$$pV_P + (1-p)\hat{V} = V. \tag{2.2}$$

The first-stage problem is easy to understand. The mechanism chooses the probability p with which the hiring decision is permanently made by the principal, the probability $1-p$ with which the hiring decision is delegated to the agent and the value $\hat{V}$ of delegation to the agent. The mechanism maximizes the value to the principal, subject to delivering the value V to the agent. If hiring is controlled by the principal, the value of the mechanism to the principal is J_P and the avlue to the agent is V_P. If hiring is delegated to the agent, the value to the principal is $\hat{J}(\hat{V})$ and the value to the agent is $\hat{V}$. The values J_P and V_P are, respectively, given by

$$J_P \equiv \int_{\underline{x}}^{\overline{x}} x dF(x) = 0, \; V_P \equiv \int_{\underline{x}}^{\overline{x}} (x-\eta)\, dF(x) = -\eta. \tag{2.3}$$

The second-stage problem is

$$\hat{J}(\hat{V}) = \max_{V_0, V_1 \in \mathcal{V}} (1-\beta) \int_R x dF(x) + \beta \left[F(R)J(V_0) + (1-F(R))J(V_1)\right] \tag{2.4}$$

subject to the promise-keeping constraint

$$\hat{V} = (1-\beta) \int_R (x-\eta) dF(x) + \beta \left[F(R)V_0 + (1-F(R)))V_1\right], \tag{2.5}$$

and the incentive-compatibility constraint

$$R = \eta - \frac{\beta}{1-\beta}\left(V_1 - V_0\right). \tag{2.6}$$

The second-stage problem is also easy to understand. The mechanism chooses the agent's continuation value V_0 if the agent does not hire the applicant and the agent's continuation value V_1 if the agent does hire the applicant. Given the continuation values V_0 and V_1, the agent finds it optimal to hire the applicant if and only if the applicant's quality x exceeds the reservation threshold R in (2.6). The mechanism maximizes the value to the principal, subject to delivering the value V to the agent. The value to the principal in the current period is $(1-\beta)\int_R x dF(x)$. The continuation value to the principal is $J(V_0)$ if the agent does not hire the applicant, an event that occurs with probability $F(R)$, and $J(V_1)$ if the agent hires the applicant, an event that occurs with probability $1-F(R)$. The value to the agent in the current period is $(1-\beta)\int_R (x-\eta)dF(x)$. The continuation value to the agent is V_0 if he does not hire the applicant and V_1 if he does.

The formulation of the first and second-stage problems (2.1) and (2.4) is not complete yet, as it does not specify the feasible sets $\hat{\mathcal{V}}$ and $\mathcal{V}$ for the agent's continuation values. The agent's continuation value $\hat{V}$ in (2.1) must be deliverable by the mechanism in the second-stage problem, in the sense that there exists a mechanims that delivers $\hat{V}$ to the agent in the second stage. The agent's continuation values V_0 and V_1 in (2.4) must be deliverable in the first-stage problem, in the sense that there exists a mechanism that delivers V_0 and V_1 to the agent in the first stage. The set $\mathcal{V}$ denotes the values that can be delivered in the first-stage to the agent. The set $\hat{\mathcal{V}}$ denotes the values that can be delivered in the second-stage to the agent. The following lemma characterizes $\mathcal{V}$ and $\hat{\mathcal{V}}$.

Lemma 1. *The sets $\mathcal{V}$ and $\hat{\mathcal{V}}$ are, respectively, given by*

$$\mathcal{V} = [V_P, V_F], \quad \hat{\mathcal{V}} = [V_\ell, V_F], \tag{2.7}$$

where V_ℓ and V_F are defined as

$$V_\ell \equiv (1-\beta)V_F + \beta V_P, \tag{2.8}$$

$$V_F \equiv \int \max\{x-\eta, 0\}dF(x) = \int_\eta (x-\eta)dF(x). \tag{2.9}$$

Proof. Consider the first-stage problem (2.1). Irrespective of what the set $\hat{\mathcal{V}}$ is, the mechanism $p=1$ delivers the value V_P to the agent. Hence, $V_P \in \mathcal{V}$. Now, consider the second-stage problem (2.2). The mechanism $(V_0, V_1) = (V_P, V_P)$ is feasible, as $V_P \in \mathcal{V}$, and delivers to the agent the value

$$\begin{aligned} V^1 &= (1-\beta)\int_\eta (x-\eta)dF(x) + \beta V_P \\ &= (1-\beta)V_F + \beta V_P. \end{aligned} \tag{2.10}$$

Hence, $V^1 \in \hat{\mathcal{V}}$. Notice that $V^1 \in (V_P, V_F)$ since $V_F > 0$ and $V_P = -\eta$.

Return to the first-stage problem (2.1). Since $V^1 \in \hat{\mathcal{V}}$ and $V_1 > V_P$, the mechanism can deliver to the agent any value $V \in [V_P, V^1]$. Indeed, the mechanism $(p, \hat{V}) = ((V_1 - V)/(V_1 - V_P), V^1)$ is feasible for all $V \in [V_P, V^1]$ and delivers to the agent the value V. Therefore, $[V_P, V^1] \subseteq \hat{\mathcal{V}}$. Now, consider the second-stage problem (2.2). Since $[V_P, V^1] \subseteq \hat{\mathcal{V}}$, the mechanism can deliver to the agent any value $\hat{V} \in [V^1, V^2]$, where

$$V^2 = (1-\beta)V_F + \beta V^1. \tag{2.11}$$

Indeed, the mechanism $(V_0, V_1) = (V, V)$ with $V = (\hat{V} - (1-\beta)V_F)/\beta$ is feasible for all $\hat{V} \in [V^1, V^2]$ and delivers to the agent the value $\hat{V}$. Therefore, $[V^1, V^2] \subseteq \hat{\mathcal{V}}$. Notice that $V^2 \in (V^1, V_F)$, since $V^1 < V_F$.

Iterating the argument n times, one obtains that $[V_P, V^n] \subseteq \mathcal{V}$ and $[V^1, V^{n+1}] \subseteq \hat{\mathcal{V}}$, where

$$V^k = (1-\beta)V_F + \beta V^{k-1} \text{ for } k = 1, 2, ...n+1. \tag{2.12}$$

The sequence $\{V^k\}_{k=1}^n$ is strictly increasing and converges to V_F for $n \to \infty$. Therefore, $[V_P, V_F) \subseteq \mathcal{V}$ and $V \in [V^1, V_F) \subseteq \hat{\mathcal{V}}$.

The flow payoff v to the agent is such that

$$v \geq \min\left\{\min_R (1-\beta)\int_R (x-\eta)dF(x), (1-\beta)\int_{\underline{x}}^{\overline{x}} (x-\eta)dF(x)\right\}. \tag{2.13}$$

The first term on the right-hand side of (2.13) denotes the lowest flow payoff that the agent may attain when the agent is in charge of hiring. The second term on the right-hand side denotes the flow payoff that the agent attains when the principal is in charge of hiring. Since the value of the right-hand side of (2.13) is $-(1-\beta)\eta$, it follows that no mechanism can deliver a value $V < V_P = -\eta$ to the agent. Hence, $V \notin \mathcal{V}$, $\hat{\mathcal{V}}$ for any $V < V_P$.

The flow payoff v to the agent is such that

$$v \leq \max\left\{\max_R (1-\beta)\int_R (x-\eta)dF(x), (1-\beta)\int_{\underline{x}}^{\overline{x}} (x-\eta)dF(x)\right\}. \tag{2.14}$$

The first term on the right-hand side of (2.14) denotes the highest flow payoff that the agent may attain when the agent is in charge of hiring. The second term on the right-hand side denotes the flow payoff that the agent attains when the principal is in charge of hiring. Since the value of the right-hand side of (2.14) is $(1-\beta)V_F$, it follows that no mechanism can deliver a value $V > V_F$ to the agent. Hence, $V \notin \mathcal{V}$, $\hat{\mathcal{V}}$ for any $V > V_F$.

Also, a mechanism can deliver the value V_F to the agent by letting him choose whom to hire in every period and giving him continuation values that are independent from his hiring decision. Hence, $V_F \in \mathcal{V}, \hat{\mathcal{V}}$.

Lastly, notice that $\hat{V} \in \hat{\mathcal{V}}$ for any $\hat{V} < V^1 = V_\ell$. To see why this is the case notice that, in the second stage, the agent's value is such that

$$\begin{aligned} \hat{V} &= \max_R (1-\beta) \int_R (x-\eta) dF(x) + \beta F(R) V_0 + \beta(1-F(R)) V_1 \\ &\geq \max_R (1-\beta) \int_R \{x-\eta\} dF(x) + \beta F(R) V_P + \beta(1-F(R)) V_P \\ &= (1-\beta) V_F + \beta V_P = V_\ell. \end{aligned} \tag{2.15}$$

■

2.3 Optimal Lottery between Delegation and Control

We now want to characterize the solution of the first-stage problem. To this aim, it is useful to study the solution of the first stage problem at $V \in \{V_P, V_F\}$ and the solution of the second-stage problem at $\hat{V} \in \{V_\ell, V_F\}$.

Consider the first-stage problem. The only feasible mechanism for $V = V_P$ is such that $(p, \hat{V}) = (1, \hat{V})$ for any $\hat{V}$. Hence, $J(V_P) = J_P$. Similarly, the only feasible mechanism for $V = V_F$ is $(p, \hat{V}) = (0, V_F)$. Hence, $J(V_F) = \hat{J}(V_F)$. Consider the second-stage problem. It is immediate to verify that the only feasible mechanism for $\hat{V} = V_F$ is $(V_0, V_1) = (V_F, V_F)$. Since $V_0 = V_1$ implies $R = \eta$, $\hat{J}(V_F)$ is such that

$$\begin{aligned} \hat{J}(V_F) &= (1-\beta) \int_\eta x dF(x) + \beta J(V_F) \\ &= (1-\beta) \int_\eta x dF(x) + \beta \hat{J}(V_F), \end{aligned} \tag{2.16}$$

where the second line follows from the fact that $J(V_F) = \hat{J}(V_F)$. Solving the above equation for $\hat{J}(V_F)$ yields

$$J_F \equiv \hat{J}(V_F) = \int_\eta x dF(x). \tag{2.17}$$

Similarly, it is immediate to verify that the only feasible mechanism for $\hat{V} = V_\ell$ is $(V_0, V_1) = (V_P, V_P)$. Since $V_0 = V_1$ implies $R = \eta$, $\hat{J}(V_\ell)$ is such that

$$\begin{aligned} J_\ell \equiv \hat{J}(V_\ell) &= (1-\beta) \int_\eta x dF(x) + \beta J(V_P) \\ &= (1-\beta) J_F + \beta J_P, \end{aligned} \tag{2.18}$$

where the second line follows from the definition of J_F and from the fact that $J(V_P) = J_P$.

The previous analysis has identified three key points. The "punishment" point $P = (V_P, J_P)$, which is equal to $(V_P, J(V_P))$, denotes the value to the agent and to the principal when the principal permanently takes over the hiring decision. We refer to this point as the punishment point. The "freedom" point $F = (V_F, J_F)$, which is equal to $(V_F, J(V_F))$ and $(V_F, \hat{J}(V_F))$, denotes the value to the agent and to the principal when the agent always decides whom to hire without any interference from the principal. Indeed, the implementation of V_F is such that, in the current period, the agent hires an applicant if and only if the applicant's quality $x \geq \eta$, exactly as he would do without a principal. The implementation of V_F is also such that, in the next period, the agent's continuation value is V_F whether he does or does not hire today's applicant. Hence, in the next period (and in any period after that), the agent hires an applicant if and only if the applicant's quality $x \geq \eta$. The point $L = (V_\ell, J_\ell)$, which is equal to $(V_\ell, \hat{J}(V_\ell))$, denotes the value to the agent and to the principal when the agent freely chooses whom to hire in the current period and the principal takes over hiring from tomorrow on. It is easy to see that the points P, L and F lie on an upward sloping line in the (V, J) space. Indeed, P, L and F are all points on the $(V, H(V))$ line, where

$$H(V) = J_P + \frac{J_F - J_P}{V_F - V_P}(V - V_P). \tag{2.19}$$

Let $S = (V^*, J^*)$ denote the point where $\hat{J}(\hat{V})$ is maximized. That is, let V^* and J^* denote, respectively, the argmax and the max of the function $\hat{J}(\hat{V})$ with respect to $\hat{V} \in \hat{\mathcal{V}}$. Since $J(V)$ is a convex combination between J_P and some $\hat{J}(\hat{V})$ and $J_P < \hat{J}(V_F)$, V^* and J^* are also, respectively, the argmax and the max of the function $J(V)$ with respect to $V \in \mathcal{V}$. Therefore, when the principal chooses a mechanism in period $t = 0$, it will pick a mechanism that delivers the point $S = (V^*, J^*)$. For some parameter values, $J^* = J_F$ and, in turn, $V^* = V_F$. In this case, the mechanism chosen by the principal is such that the hiring decisions are permanently delegated to the agent. For other parameter values, $J^* > J_F$ and, in turn, $V^* \in (V_\ell, V_F)$. In this case, the mechanism chosen by the principal will not involve permanent, unfettered delegation. In the following lemma, we identify a condition on the parameters of the model such that $J^* > J_F$. In the remainder of the paper, we assume that the condition is satisfied.[1]

Lemma 2. *The optimal mechanism $S = (V^*, J^*)$ is such that $J^* > J_F$ as long as η and F are such that*

$$\frac{\eta F'(\eta)}{F(\eta)} > \frac{\int_\eta x dF(x)}{\int_\eta x dF(x) + \eta F(\eta)}. \tag{2.20}$$

Proof. Consider the following feasible mechanism. In the current period, the agent chooses whether or not to hire the applicant. If the agent does hire the applicant in the current period, the agent continuation value V_1 is set to V_F. If the agent does hire the applicant

[1] If, for instance, the quality distribution F is uniform over the interval $[-\delta, \delta]$, the sufficient condition for $J^* > J_F$ is $\eta > \delta/2$.

in the current period, the agent continuation value V_0 is set to $\epsilon V_P + (1-\epsilon)V_F$. The continuation value V_0 is delivered as a lottery that assigns probability ϵ to V_P and probability $1-\epsilon$ to V_F. The mechanism need not be optimal, but it is feasible.

The value $\Gamma(\epsilon)$ of the mechanism to the principal is

$$\Gamma(\epsilon) = (1-\beta)\int_R xF'(x)dx + \beta F(R)\left[\epsilon J_P + (1-\epsilon)J_F\right] + \beta(1-F(R))J_F, \tag{2.21}$$

where the reservation quality R is

$$R = \eta - \frac{\beta}{1-\beta}\left[V_F - \epsilon V_P - (1-\epsilon)V_F\right]. \tag{2.22}$$

The derivative of Γ with respect to ϵ is

$$\Gamma'(\epsilon) = -\beta F(R)\left(J_F - J_P\right) - \frac{dR}{d\epsilon}\left\{(1-\beta)RF'(R) + \beta F'(R)\epsilon\left[J_F - J_P\right]\right\}, \tag{2.23}$$

where

$$\frac{dR}{d\epsilon} = -\frac{\beta}{1-\beta}(V_F - V_P). \tag{2.24}$$

When evaluated at $\epsilon = 0$, the derivative of Γ with respect to ϵ becomes

$$\Gamma'(0) = -\beta F(\eta)\left(J_F - J_P\right) + \beta\eta F'(\eta)(V_F - V_P). \tag{2.25}$$

The above expression is strictly positive as long as

$$\frac{\eta F'(\eta)}{F(\eta)} > \frac{J_F - J_P}{V_F - V_P} = \frac{\int_\eta xdF(x)}{\int_\eta xdF(x) + \eta F(\eta)}. \tag{2.26}$$

The value $\Gamma(\epsilon)$ of the mechanism to the principal cannot exceed J^*, as the mechanism is a feasible solution to the second-stage problem. The value $\Gamma(\epsilon)$ of the mechanism to the principal is equal to J_F for $\epsilon = 0$. Hence, if condition (2.26) is satisfied, there exists an $\epsilon^* > 0$ such that $\Gamma(\epsilon^*) > J_F$. Since $J^* \geq \Gamma(\epsilon^*)$, it follows that $J^* > J_F$. ■

In order to simplify the characterization of the optimal mechanism, we are going to conjecture that the second-stage value function $\hat{J}$ is strictly concave. In most of our numerical simulations, we find that $\hat{J}$ is indeed strictly concave. Having conjectured that $\hat{J}$ is strictly concave, we can establish some useful properties relating $\hat{J}(\hat{V})$ to the line $H(\hat{V})$ that connects the points P, L and F. Since $\hat{J}(V_\ell) = H(V_\ell)$ and $\hat{J}(V^*) > J_F > H(V^*)$, the

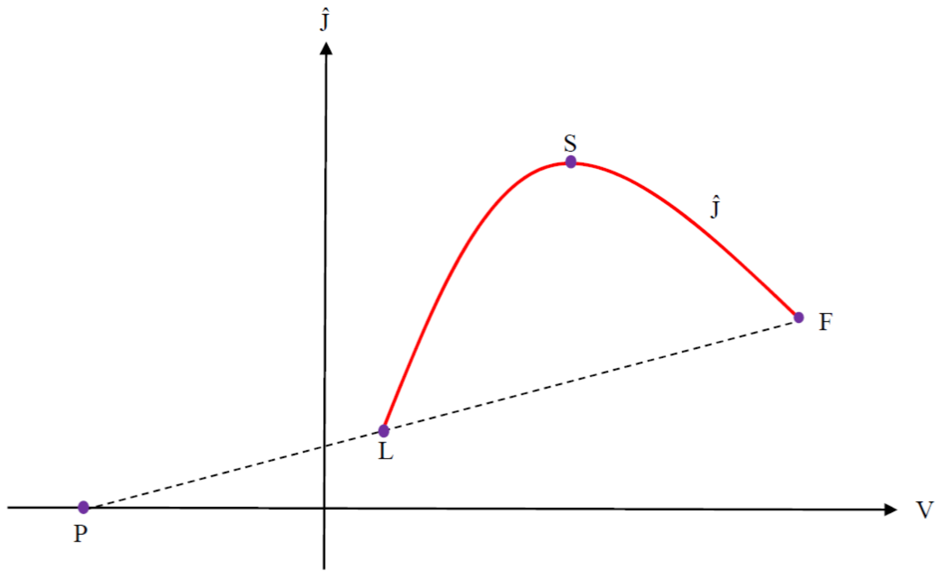

Figure 1: Points P, L, F and S, and locus $(\hat{V}, \hat{J}(\hat{V}))$.

concavity of $\hat{J}$ implies that $\hat{J}'(V_\ell) > H' = (J_F - J_P)/(V_F - V_P)$. Since $\hat{J}(V_F) = H(V_F)$, the concavity of $\hat{J}$ implies that $\hat{J}'(V_F) < H'$. Since $J(V_\ell) = H(V_\ell)$, $\hat{J}(V_F) = H(V_F)$ and $\hat{J}(V^*) > H(V^*)$, the concavity implies that $\hat{J}(\hat{V}) > H(\hat{V})$ for all $\hat{V} \in (V_\ell, V_F)$.

We are now ready to characterize the solution of the first-stage problem (2.1). Using the promise-keeping constraint (2.2) to substitute the probability p, we can rewrite (2.1) as

$$\begin{aligned} J(V) = \quad & \max_{\hat{V} \in [V_\ell, V_F]} \frac{\hat{V} - V}{\hat{V} - V_P} J_P + \frac{V - V_P}{\hat{V} - V_P} \hat{J}(\hat{V}), \text{ s.t.} \\ & \text{s.t. } \hat{V} \geq V. \end{aligned} \tag{2.27}$$

The necessary condition for the optimality of $\hat{V}$ is

$$\frac{V - V_P}{\hat{V} - V_P} \left[\hat{J}'(\hat{V}) - \frac{\hat{J}(\hat{V}) - J_P}{\hat{V} - V_P} \right] \begin{cases} = 0 & \text{if } \hat{V} \in (\max\{V, V_\ell\}, V_F), \\ \leq 0 & \text{if } \hat{V} = \max\{V, V_\ell\}, \\ \geq 0 & \text{if } \hat{V} = V_F. \end{cases} \tag{2.28}$$

Let us examine the term in square brackets on the left-hand side of (2.27). The function $\hat{J}'(\hat{V})$ is the derivative of the second-stage problem value function. Since $\hat{J}$ is strictly concave, $\hat{J}'(\hat{V})$ is strictly decreasing and such that $J'(V_\ell) > 0$, $\hat{J}'(V^*) = 0$, $J'(V_F) < 0$. The function $(\hat{J}(\hat{V}) - J_P)/(\hat{V} - V_P)$ is the slope of the line connecting the points (V_P, J_P) and $(\hat{V}, J(\hat{V}))$. Since $\hat{J}(V_\ell) = H(V_\ell)$ and $\hat{J}(V_F) = H(V_F)$, $(\hat{J}(\hat{V}) - J_P)/(\hat{V} - V_P)$ is equal to $H' = (J_F - J_P)/(V_F - V_P)$ at V_ℓ and V_F. Since $\hat{J}(\hat{V}) > H(\hat{V})$ for all $\hat{V} \in (V_\ell, V_F)$, $(\hat{J}(\hat{V}) - J_P)/(\hat{V} - V_P) > H'$ for all $\hat{V} \in (V_\ell, V_F)$. The strict concavity of $\hat{J}$ implies that

$H' < J'(\hat{V}_\ell)$ and $H' > J'(V_F)$. Since $(\hat{J}(\hat{V})-J_P)/(\hat{V}-V_P)$ is strictly smaller than $J'(\hat{V})$ at $\hat{V} = V_\ell$ and strictly greater than $J'(\hat{V})$ at $\hat{V} = V_F$, there exists at least one $V_C \in (V_\ell, V_F)$ such that $(\hat{J}(V_C) - J_P)/(V_C - V_P) = J'(V_C)$. Since $(\hat{J}(\hat{V}) - J_P)/(\hat{V} - V_P) \geq H'$ and $H' > 0$, V_C is strictly smaller than V^*. Since the derivative of $(\hat{J}(\hat{V}) - J_P)/(\hat{V} - V_P)$ with respect to $\hat{V}$ is zero at any V_C, V_C is unique. Moreover, for all $\hat{V} \in [V_\ell, V_c)$, $\hat{J}'(\hat{V})$ is strictly greater than $(\hat{J}(\hat{V}) - J_P)/(\hat{V} - V_P)$. For all $\hat{V} \in (V_C, V_F]$, $\hat{J}'(\hat{V})$ is strictly smaller than $(\hat{J}(\hat{V}) - J_P)/(\hat{V} - V_P)$.

The above discussion implies that, for any $V \in [V_P, V_C)$, the optimality condition (2.28) and the promise-keeping constraint (2.2) are satisfied if and only if

$$\hat{V} = V_C, \quad p = \frac{V_C - V}{V_C - V_P} > 0. \tag{2.29}$$

Plugging the optimal choice for $\hat{V}$ in (2.27) yields

$$J(V) = J_P + \frac{V - V_P}{V_C - V_P}(J_C - J_P). \tag{2.30}$$

where $J_C \equiv \hat{J}(V_C)$. Differentiating (2.30) with respect to V yields

$$J'(V) = \frac{J_C - J_P}{V_C - V_P} > \hat{J}'(V), \tag{2.31}$$

where the last inequality follows from the fact that $(\hat{J}(V_C) - J_P)/(V_C - V_P) = \hat{J}'(V_C)$ and $\hat{J}'(V) > \hat{J}'(V_C)$.

For any $V \in [V_C, V_F]$, the optimality condition (2.28) and the promise-keeping constraint (2.2) are satisfied if and only if

$$\hat{V} = V, \quad p = 0. \tag{2.32}$$

Plugging the optimal choice for $\hat{V}$ in (2.27) yields

$$J(V) = \hat{J}(V). \tag{2.33}$$

Differentiating (2.33) with respect to V yields

$$J'(V) = \hat{J}'(V). \tag{2.34}$$

The proposition below summarizes the characterization of the first-stage problem.

Proposition 3. *(Optimal lottery) The solution to the first-stage problem is such that:*

1. *For $V \in [V_P, V_C)$, the principal takes over hiring with probability p and the agent retains control of hiring with probability $1-p$, where $p = (V_C - V)/(V_C - V_P)$. If the agent retains control of hiring, his continuation value $\hat{V}$ is V_C.*

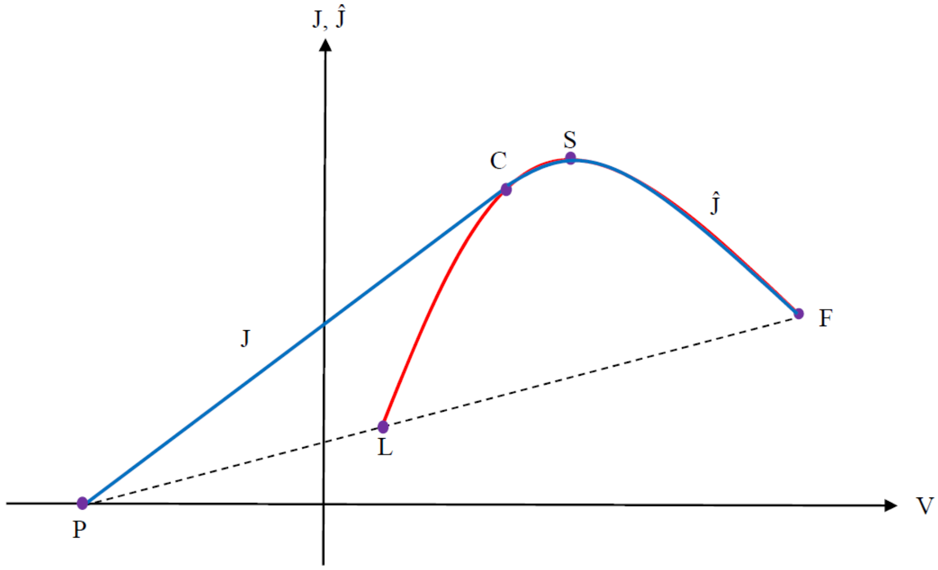

Figure 2: Value functions $J(V)$ and $\hat{J}(\hat{V})$.

2. *For $V \in [V_C, V_F]$, the principal never takes over hiring. The agent's continuation value $\hat{V}$ is V.*
3. *The value to the principal is*

$$J(V) = \begin{cases} J_P + \dfrac{V - V_P}{V_C - V_P}(J_C - J_P), & \text{if } V \in [V_P, V_C) \\ \hat{J}(V) & \text{if } V \in [V_C, V_F] \end{cases} \quad (2.35)$$

4. *The point (V_C, J_C) is such that $J_C = \hat{J}(V_C)$ and $V_C \in (V_\ell, V^*)$ is the unique solution to*

$$\hat{J}'(V_C) = \frac{J_C - J_P}{V_C - V_P}. \quad (2.36)$$

The properties of the optimal lottery between delegation and control are intuitive. The lottery between control and delegation allows the mechanism to achieve any combination of the agent's and the principal's values that lie on a line connecting the control values, given by the point P, and one of the delegation values, given by some point on the locus $(\hat{V}, \hat{J}(\hat{V}))$. The optimal lottery between control and delegation is the highest line connecting P to a point on the locus $(\hat{V}, \hat{J}(\hat{V}))$, and it is obviously such that P is connected to $C = (V_C, J_C)$, where C is the point where $\hat{J}$ tangent to the line connecting P and C. Hence, the optimal lottery is such that, for any $V \in (V_P, V_C)$, the lottery randomizes with between P and C with non-degenerate probabilities. For any $V \in [V_C, V_F]$, the optimal lottery is degenerate and the hiring decision is delegated to the agent with probability 1.

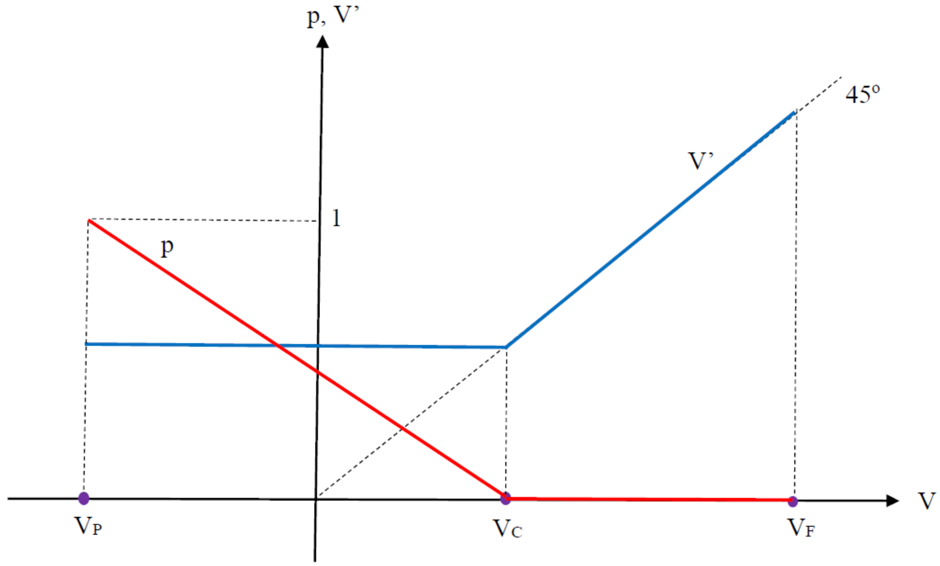

Figure 3: Optimal lottery p and $\hat{V}$ as function of the promised value V.

2.4 Optimal Delegation

We now turn to the analysis of the second-stage problem. In the next proposition, we show that the optimal mechanism induces the agent to follow a reservation quality that is interior to the support of the quality distribution. The proposition implies that the optimal mechanism is such that there is a strictly positive probability that the agent does not hire the applicant, and a strictly positive probability that the agent hires the applicant.

Proposition 4. *(Optimal Reservation Quality) For all $\hat{V} \in [V_C, V_F]$, the solution to the second-stage problem is such that the reservation quality R belongs to $(\underline{x}, \overline{x})$.*

Proof. First, suppose that for some $\hat{V}_a \in [V_C, V_F]$, the solution to the second-stage problem (2.4) specifies continuation values V_0, $V_1 \in [V_P, V_F]$ such that

$$R = \eta - \frac{\beta}{1-\beta}(V_1 - V_0) \leq \underline{x}. \tag{2.37}$$

The value to the agent becomes

$$\begin{aligned} \hat{V}_a &= (1-\beta)\int_{\underline{x}}^{\overline{x}}(x-\eta)dF(x) + \beta V_1 \\ &= (1-\beta)V_P + \beta V_1, \end{aligned} \tag{2.38}$$

The value to the principal becomes

$$\begin{aligned} \hat{J}(\hat{V}_a) &= (1-\beta)\int_{\underline{x}}^{\overline{x}} x dF(x) + \beta J(V_1) \\ &= (1-\beta) J_P + \beta J(V_1) \\ &= (1-\beta) J_P + \beta \hat{J}(V_1) \end{aligned} \tag{2.39}$$

where the second line in (2.39) makes use of the definition of V_P, the second line in (2.39) makes use of the definition of V_P, and the third line in (2.39) makes use of the fact that $\hat{V}_a > V_P$ implies $V_1 > \hat{V}_a > V_C$ and, hence, $J(V_1) = \hat{J}(V_1)$.

Consider now the first-stage problem (2.1) for $V = \hat{V}_a$. The problem is

$$\begin{aligned} J(\hat{V}_a) = \quad & \max_{p \in [0,1], \hat{V} \in \hat{\mathcal{V}}} pJ_P + (1-p)\hat{J}(\hat{V}), \\ & \text{s.t. } \hat{V}_a = pV_P + (1-p)\hat{V}. \end{aligned} \tag{2.40}$$

Since $p = \beta$ and $\hat{V} = V_1$ is feasible. Since $V_1 \neq V_C$, $p = \beta$ and $\hat{V} = V_1$ is not optimal. Therefore, $J(\hat{V}_a) > \hat{J}(\hat{V}_a)$. However, $J(\hat{V}_a) = \hat{J}(\hat{V}_a)$ because $\hat{V}_a \geq V_C$ and $\hat{J}(\hat{V}) = J(\hat{V})$ for all $\hat{V} \geq V_C$. A contradiction.

Next, suppose that for some $\hat{V}_b \in [V_c, V_F]$, the solution to the second-stage problem (2.40) specifies continuation values V_0, $V_1 \in [V_P, V_F]$ such that

$$R = \eta - \frac{\beta}{1-\beta}(V_1 - V_0) \geq \overline{x}. \tag{2.41}$$

In this case, the value to the agent becomes

$$\hat{V}_b = \beta V_0. \tag{2.42}$$

The value to the principal becomes

$$\begin{aligned} \hat{J}(\hat{V}_b) &= 0 + \beta J(V_0) \\ &< (1-\beta)J(0) + \beta J(V_0) \\ &\leq J((1-\beta)0 + \beta V_0) \\ &= J(\hat{V}_b) \end{aligned} \tag{2.43}$$

The second line makes use of the fact that $J(V) \geq H(V) > 0$ for all $V > V_P$. The third line makes use of the fact that $J(V)$ is weakly concave, as $J'(V) = \hat{J}'(V_c)$ for all $V \in [V_\ell, V_C]$ and $J'(V) = \hat{J}'(V)$ for all $V \in [V_C, V_F]$. The fourth line makes use of the fact that $V_0 = \hat{V}_b/\beta$. Comparing the first and the last line, we obtain $\hat{J}(\hat{V}_b) < J(\hat{V}_b)$. However, $J(\hat{V}_b) = \hat{J}(\hat{V}_b)$ because $\hat{V}_b \geq V_C$ and $\hat{J}(\hat{V}) = J(\hat{V})$ for all $\hat{V} \geq V_C$. A contradiction. ■

We now turn to the characterization of the optimal continuation values for the agent in the second-stage problem. For $V_0 \in (V_P, V_F)$, the necessary condition for optimality is

$$\begin{aligned} 0 = \quad & \beta F(R) J'(V_0) + \beta F'(R) \left[J(V_0) - J(V_1) \right] \frac{dR}{dV_0} - (1-\beta) R F'(R) \frac{dR}{dV_0} \\ & + \lambda \left\{ \beta F(R) + \beta F'(R)(V_0 - V_1) \frac{dR}{dV_0} - (1-\beta)(R-\eta) F'(R) \frac{dR}{dV_0} \right\}, \end{aligned} \tag{2.44}$$

where λ denotes the Lagrange multiplier on the promise-keeping constraint (2.5) and dR/dV_0 denotes the derivative of the agent's reservation quality (2.6) with respect to V_0, i.e.

$$\frac{dR}{dV_0} = \frac{\beta}{1-\beta}. \tag{2.45}$$

The first line on the right-hand side of (2.44) is the derivative of the principal's value with respect to V_0. This derivative is given by the change in the principal's continuation value due to the change in V_0 and the change in the principal's flow payoff and continuation value due to change in the agent's reservation quality R. The second line on the right-hand side of (2.44) is the derivative of the agent's value with respect to V_0 multiplied by λ. The derivative is given by the change in the agent's continuation value due to the change in V_0 and the change in the agent's flow payoff and continuation value due to change in the agent's reservation quality R.

Using the fact that the agent's reservation quality R is given by (2.6) and that the derivative dR/dV_0 of the agent's reservation quality with respect to V_0 is given by (2.45), we can rewrite (2.44) as

$$J'(V_0) + \lambda = \frac{F'(R)}{F(R)} \left\{ R + \frac{\beta}{1-\beta} \left[J(V_1) - J(V_0) \right] \right\}. \tag{2.46}$$

Similarly, for $V_1 \in (V_P, V_F)$, the necessary condition for optimality is

$$\begin{aligned} 0 = \quad & \beta (1 - F(R)) J'(V_1) + \beta F'(R) \left[J(V_0) - J(V_1) \right] \frac{dR}{dV_1} - (1-\beta) R F'(R) \frac{dR}{dV_1} \\ & + \lambda \left\{ \beta (1 - F(R)) + \beta F'(R)(V_0 - V_1) \frac{dR}{dV_1} - (1-\beta)(R-\eta) F'(R) \frac{dR}{dV_1} \right\}, \end{aligned} \tag{2.47}$$

where dR/dV_1 denotes the derivative of the agent's reservation quality (2.6) with respect to V_1

$$\frac{dR}{dV_1} = -\frac{\beta}{1-\beta}. \tag{2.48}$$

Using the fact that the agent's reservation quality R is given by (2.6) and that the derivative dR/dV_1 of the agent's reservation quality with respect to V_0 is given by (2.48), we can rewrite (2.47) as

$$J'(V_1) + \lambda = -\frac{F'(R)}{1 - F(R)} \left\{ R + \frac{\beta}{1-\beta} \left[J(V_1) - J(V_0) \right] \right\}. \tag{2.49}$$

Finally, the derivative $\hat{J}'(\hat{V})$ of the principal's value with respect to the agent's promised value $\hat{V}$ is equal to the Lagrange multiplier λ on the promise-keeping constraint (2.5). That is,

$$\hat{J}'(\hat{V}) = -\lambda. \tag{2.50}$$

In the next proposition, we first use the necessary conditions (2.46) and (2.49) to show that the optimal continuation values V_0 and V_1 are such that $V_0 \neq V_1$ for any agent's promised value $\hat{V} \in (V_\ell, V_F)$. Second, under the conjecture that the optimal continuation values V_0 and V_1 are continuous with respect to $\hat{V}$, we show that the optimal continuation values are such that $V_0 < V_1$ for any agent's promised value $\hat{V} \in (V_\ell, V_F)$. Third, we use the necessary conditions (2.46) and (2.49) and the envelope condition (2.50) to show that the optimal continuation values are such that $V_0 < \hat{V} < V_1$ for any $\hat{V} \in (V_C, V_F)$. Fourth, we show that $\hat{J}'(V_F) = -\infty$ and, hence, $V_1 < V_F$ for any $\hat{V} \in (V_\ell, V_F)$. Lastly, we establsih that $V_0 = V_P$ and $V_1 > V_C$ at $\hat{V} = V_C$.

Proposition 5. *(Optimal Incentives in Delegation). The solution to the first-stage problem is such that:*

1. *For any promised value $\hat{V} \in (V_\ell, V_F)$, the optimal continuation values V_0 and V_1 are such that $V_0 \neq V_1$. For $\hat{V} = V_\ell$, both V_0 and V_1 are equal to V_P. For $\hat{V} = V_F$, both V_0 and V_1 are equal to V_F.*
2. *For any $\hat{V} \in (V_\ell, V_F)$, V_0 and V_1 are such that $V_0 < V_1$ and, hence, the agent's reservation quality R is strictly smaller than η.*
3. *For any $\hat{V} \in (V_C, V_F)$, V_0 and V_1 are such that $V_0 < \hat{V}$ and $V_1 > \hat{V}$.*
4. *For any $\hat{V} \in (V_\ell, V_F)$, V_1 is such that $V_1 < V_F$.*
5. *For $\hat{V} = V_C$, V_0 and V_1 are such that $V_0 = V_P$ and $V_1 > V_C$.*

Proof. **Part 1**. Consider some promised value $\hat{V} \in (V_\ell, V_F)$. Suppose that the optimal continuation values V_0 and V_1 are both equal to $V \in (V_P, V_F)$. Then, the necessary conditions (2.46) and (2.49) are

$$\begin{aligned} J'(V_0) + \lambda &= \frac{F'(R)}{F(R)} \left\{ R + \frac{\beta}{1-\beta} \left[J(V_1) - J(V_0) \right] \right\} = \frac{F'(\eta)}{F(\eta)} \eta, \\ J'(V_1) + \lambda &= -\frac{F'(R)}{1-F(R)} \left\{ R + \frac{\beta}{1-\beta} \left[J(V_1) - J(V_0) \right] \right\} = -\frac{F'(\eta)}{1-F(\eta)} \eta, \end{aligned} \tag{2.51}$$

where the second equalities make use of the fact that $V_0 = V_1$ and, hence, $J(V_1) = J(V_0)$ and $R = \eta$. The necessary conditions (2.51) imply that $J'(V_0) > J'(V_1)$. However, $V_0 = V_1$ implies that $J'(V_0) = J'(V_1)$. A contradiction.

Next suppose that the optimal continuation values V_0 and V_1 are both equal to V_P. Then the promise-keeping constraint becomes

$$\begin{aligned}\hat{V} &= (1-\beta)\int_R (x-\eta)dF(x) + \beta V_P \\ &= (1-\beta)\int_\eta (x-\eta)dF(x) + \beta V_P \\ &= (1-\beta)V_F + \beta V_P = V_\ell.\end{aligned} \tag{2.52}$$

where the second line makes use of the fact that $R = \eta$ if $V_0 = V_1$, and the third line makes use of the definitions of V_F and V_ℓ. Similarly, if the optimal continuation values V_0 and V_1 are both equal to V_F, the promise-keeping constraint becomes

$$\begin{aligned}\hat{V} &= (1-\beta)\int_R (x-\eta)dF(x) + \beta V_F \\ &= (1-\beta)\int_\eta (x-\eta)dF(x) + \beta V_F \\ &= (1-\beta)V_F + \beta V_F = V_F.\end{aligned} \tag{2.53}$$

In both cases, we have a contradiction because the agent's promised value $\hat{V} \in (V_\ell, V_F)$.

Part 2. Conjecture that the optimal continuation values V_0 and V_1 are continuous with respect to $\hat{V}$. Since $V_0 \neq V_1$ for all $\hat{V} \in (V_\ell, V_F)$ and V_0 and V_1 are continuous, it follows that either $V_0 > V_1$ for all $\hat{V} \in (V_\ell, V_F)$ or $V_0 < V_1$ for all $\hat{V} \in (V_\ell, V_F)$. On the way to a contradiction, suppose that $V_0 > V_1$ for all $\hat{V} \in (V_\ell, V_F)$. Then, the agent's reservation quality R is greater than η for all $\hat{V} \in (V_\ell, V_F)$. For $\hat{V} = V_\ell$, the only feasible continuation values are $V_0 = V_1 = V_P$ and, hence, the agent's reservation quality R is equal to η. For $\hat{V} = V_F$, the only feasible continuation values are $V_0 = V_1 = V_F$ and, hence, the agent's reservation quality R is equal to η. Overall, as long as the agent retains control over whom to hire, the agent's reservation quality is greater than R and the principal's flow payoff is

$$\begin{aligned}v &\leq \max_{R \geq \eta} \int_R x dF(x) \\ &= \int_\eta x dF(x) = (1-\beta)J_F,\end{aligned} \tag{2.54}$$

where the second line uses the fact that the integral is strictly decreasing in R for all $R > 0$ and the definition of J_F. Once the principal takes control over whom to hire, the principal's flow payoff is

$$v_P = \int_{\underline{x}}^{\overline{x}} x dF(x) = (1-\beta)J_P, \tag{2.55}$$

where the second equality uses the definition of J_P. Since the value of the mechanism to the principal is the discounted sum of flow payoffs, it follows that

$$J(\hat{V}) \leq \frac{(1-\beta)\max\{J_F, J_P\}}{1-\beta} = J_F, \text{ for all } \hat{V} \in [V_\ell, V_F]. \tag{2.56}$$

However, $V^* \in (V_\ell, V_F)$ and $J(V^*) > J_F$. A contradiction.

Part 3. Consider some promised value $\hat{V} \in (V_C, V_F)$. The optimal continuation values V_0 and V_1 are such that $V_0 < V_1$. Suppose that the continuation values V_0 and V_1 are both interior, in the sense that $V_0, V_1 \in (V_P, V_F)$. Using the envelope condition (2.50) to substitute out the Lagrange multiplier λ, we can rewrite the necessary conditions for the optimality (2.46) and (2.49) as

$$\begin{aligned} J'(V_0) - \hat{J}'(\hat{V}) &= \frac{F'(R)}{F(R)} \left\{ R + \frac{\beta}{1-\beta} \left[J(V_1) - J(V_0) \right] \right\}, \\ J'(V_1) - \hat{J}'(\hat{V}) &= -\frac{F'(R)}{1-F(R)} \left\{ R + \frac{\beta}{1-\beta} \left[J(V_1) - J(V_0) \right] \right\}. \end{aligned} \tag{2.57}$$

If $V_0 > V_C$, then subtract the second equation from the first equation in (2.57) to obtain

$$J'(V_0) - J'(V_1) = \left[\frac{F'(R)}{F(R)} + \frac{F'(R)}{1-F(R)} \right] \left\{ R + \frac{\beta}{1-\beta} \left[J(V_1) - J(V_0) \right] \right\}. \tag{2.58}$$

Since $V_0, V_1 > V_C$, $J'(V_0) = \hat{J}'(V_0)$ and $J'(V_1) = \hat{J}'(V_1)$. Since $V_1 > V_0$ and $\hat{J}$ is strictly concave, $\hat{J}'(V_0) > \hat{J}'(V_1)$. From these observations, it follows that $J'(V_0) > J'(V_1)$ and, in turn,

$$R + \frac{\beta}{1-\beta} \left[J(V_1) - J(V_0) \right] > 0. \tag{2.59}$$

The inequality in () implies that $J'(V_0) > \hat{J}'(\hat{V})$. Since $J'(V_0) = \hat{J}'(V_0)$ and $\hat{J}$ is strictly concave, $V_0 < \hat{V}$. Similarly, the inequality in () implies that $J'(V_1) < \hat{J}'(\hat{V})$. Since $J'(V_1) = \hat{J}'(V_1)$ and $\hat{J}$ is strictly concave, $V_1 > \hat{V}$. If $V_0 < V_C$, then $J'(V_0) = \hat{J}'(V_C)$. Since $\hat{V} > V_C$ and $\hat{J}$ is strictly concave, $J'(\hat{V}) < \hat{J}'(V_C)$. From these observations, it follows that

$$R + \frac{\beta}{1-\beta} \left[J(V_1) - J(V_0) \right] > 0. \tag{2.60}$$

The inequality in () implies that $J'(V_1) < \hat{J}'(\hat{V})$. Since $\hat{V} > V_C$, $\hat{J}'(\hat{V}) = J'(\hat{V})$. Since J is weakly concave, $V_1 > \hat{V}$. Therefore, $V_0 < \hat{V}$ and $V_1 > \hat{V}$. Similar arguments allow us to prove that $V_0 < \hat{V}$ and $V_1 > \hat{V}$ also in case V_0 and/or V_1 are not interior.

Part 4. Since V_0 is a continuous function of $\hat{V}$ such that $V_0 = V_F$ for $\hat{V} = V_F$ and $V_0 < \hat{V}$ for all $\hat{V} \in (V_C, V_F)$, there must exist an interval $(\hat{V}_a, V_F)$ such that $V_0 \in (V_C, V_F)$ for all $\hat{V} \in (\hat{V}_a, V_F)$. Hence, for all $\hat{V} \in (\hat{V}_a, V_F)$, the first-order condition for V_0 is given by

$$\frac{\hat{J}'(V_0) - \hat{J}'(\hat{V})}{\hat{J}'(\hat{V})} = \frac{1}{\hat{J}'(\hat{V})} \frac{F'(R)}{F(R)} \left\{ R + \frac{\beta}{1-\beta} \left[J(V_1) - J(V_0) \right] \right\}, \tag{2.61}$$

where the above expression is obtained by dividing (2.46) by $\hat{J}'(\hat{V})$ and using $J'(V_0) = \hat{J}'(V_0)$ and $\lambda = -\hat{J}'(\hat{V})$. On the way to a contradiction, suppose that $\hat{J}'(\hat{V})$ converges to some k for $\hat{V} \to V_F$. If so, the left-hand side of (2.61) converges to 0 for $\hat{V} \to V_F$.

The right-hand side of (2.61) converges to $F'(\eta)\eta/F(\eta)k \neq 0$ since R converges to η for $\hat{V} \to V_F$. Therefore, $\hat{J}'(\hat{V})$ must diverge for $\hat{V} \to V_F$. Since $\hat{J}'(\hat{V}) < 0$ for all $\hat{V} > V^*$, $\hat{J}'(\hat{V}) = -\infty$ for $\hat{V} \to V_F$.

For any $\hat{V} \in (V_C, V_F)$, $V_1 = V_F$ is optimal only if

$$\hat{J}'(V_F) - \hat{J}'(\hat{V}) \geq -\frac{F'(R)}{1-F(R)} \left\{ R + \frac{\beta}{1-\beta} \left[J_F - J(V_0) \right] \right\}. \tag{2.62}$$

The left-hand side of (2.62) is $-\infty$ since $\hat{J}'(V_F) = -\infty$ and $\hat{J}'(\hat{V})$ is finite. The right-hand side of (2.62) is finite, since $R \in (\underline{x}, \overline{x})$, $F'(R)/(1-F(R))$ is finite, and $J_F - J(V_0)$ is bounded between $J_F - J^*$ and $J_F - J_P$. Hence, the condition for the optimality of $V_1 = V_F$ cannot hold for any $\hat{V} \in (V_C, V_F)$.

Part 5. Since V_0 and V_1 are continuous functions of $\hat{V}$ such that $V_0 < \hat{V}$ for all $\hat{V} \in (V_C, V_F)$ and $V_1 > \hat{V}$ for all $\hat{V} \in (V_C, V_F)$, it follows that, for $\hat{V} = V_C$, the optimal continuation value $V_0 \leq V_C$ and the optimal continuation value $V_1 \geq V_C$.

On the way to a contradiction, let us suppose that $V_1 = V_C$. If this is the case, the solution to the second-stage problem for $\hat{V} = V_C$ has a very simple structure. The value of the second-stage problem to the principal is $J_C = \hat{J}(V_C)$. The value of the second-stage problem to the agent is V_C. If, in the current period, the agent hires the applicant, the agent retains control of hiring, the continuation value to the agent is V_1 and the continuation value to the principal is $J(V_1)$. Since $V_1 = V_C$, in the next period, the mechanism delivers V_C to the agent and J_C to the principal. If, in the current period, the agent does not hire the applicant, his continuation value is V_0 and the principal's continuation value is $J(V_0)$. Since $V_0 < V_C$, in the next period, the mechanism delivers a lottery between the the principal taking over hiring with probability p, in which case the agent and principal values are V_P and J_P, and the agent retaining control over hiring with probability $1-p$, in which case the agent and the principal values are V_C and J_C. Therefore, the value of the second-stage problem to the principal can be written as

$$J_C = (1-\beta) \int_R x dF(x) + \beta F(R) p J_P + \beta(1 - F(R)p) J_C. \tag{2.63}$$

The value to the agent can be written as

$$V_C = (1-\beta) \int_R (x-\eta) dF(x) + \beta F(R) p V_P + \beta(1 - F(R)p) V_C. \tag{2.64}$$

The reservation quality can be written as

$$R = \eta - \frac{\beta p}{1-\beta} (V_C - V_P). \tag{2.65}$$

Now consider the following mechanism. As long as the agent has control over whom to hire, the mechanism specifies that, if the agent hires the applicant in the current period, the agent will have control over whom to hire in the next period. If the agent does not hire the applicant in the current period, the principal will permanently take over hiring with some probability p and the agent will retain control over hiring in the next period with probability $1-p$. Let V denote the value of this mechanism to the agent and let $\Gamma(V)$ denote the value of this mechanism to the principal. Conditional on the agent having control over hiring in the current period, the value of the mechanism to the principal is

$$\Gamma(V) = (1-\beta)\int_R x dF(x) + \beta F(R) p J_P + \beta(1 - F(R)p)\Gamma(V). \tag{2.66}$$

The value of the mechanism to the agent is

$$V = (1-\beta)\int_R (x-\eta) dF(x) + \beta F(R) p V_P + \beta(1 - F(R)p)V. \tag{2.67}$$

The reservation quality is

$$R = \eta - \frac{\beta p}{1-\beta}\left(V - V_P\right). \tag{2.68}$$

For $V = V_C$, the mechanism is the same as the solution of the second-stage problem. That is $\Gamma(V_C) = \hat{J}(V_C)$. For any $V \neq V_C$, the mechanism is feasible but need not be the solution of the second-stage problem. That is $\Gamma(V) \leq \hat{J}(V)$. Therefore, the derivative of $\Gamma(V)$ is equal to the derivative of $\hat{J}(V)$ for $V = V_C$. That is, $\Gamma'(V_C) = \hat{J}'(V_C)$. The derivative of $\Gamma(V)$ with respect to V is such that

$$\begin{aligned} & \left(1 - \beta(1 - pF(R))\right)\Gamma'(V) \\ = \quad & -\beta F(R)(\Gamma(V) - J_P)\frac{dp}{dV} - \left[(1-\beta)R + \beta p(\Gamma(V) - J_P)\right]F'(R)\frac{dR}{dV}, \end{aligned} \tag{2.69}$$

where dp/dV is such that

$$1 - \beta(1 - pF(R)) = -\beta F(R)(V - V_P)\frac{dp}{dV}, \tag{2.70}$$

and dR/dV is

$$\frac{dR}{dV} = -\frac{\beta}{1-\beta}\left[p + (V - V_P)\frac{dp}{dV}\right]. \tag{2.71}$$

When we evaluate $\Gamma'(V)$ at $V = V_C$, we obtain

$$\begin{aligned} & (1-\beta(1-pF(R)))\,\Gamma'(V_C) \\ = \quad & -\beta F(R)(J_C - J_P)\frac{dp}{dV} - [(1-\beta)R + \beta p(J_C - J_P)]\, F'(R)\frac{dR}{dV}, \end{aligned} \tag{2.72}$$

where dp/dV is

$$\frac{dp}{dV} = -\frac{1-\beta(1-pF(R))}{\beta F(R)}\frac{1}{V_C - V_P}, \tag{2.73}$$

and dR/dV is

$$\frac{dR}{dV} = -\frac{\beta}{1-\beta}\left[p + (V_C - V_P)\frac{dp}{dV}\right]. \tag{2.74}$$

Combining (2.72), (2.73) and (2.74), we find that the derivative of $\Gamma(V)$ at $V = V_C$ is

$$\Gamma'(V_C) = \frac{J_C - J_P}{V_C - V_P} - \frac{(1-\beta)R + \beta p(J_C - J_P)}{1-\beta(1-pF(R))}\frac{F'(R)}{F(R)}. \tag{2.75}$$

The derivative of $\hat{J}(V)$ at $V = V_C$ is

$$J'(V_C) = \frac{J_C - J_P}{V_C - V_P}. \tag{2.76}$$

Therefore $\Gamma'(V_C) < J'(V_C)$, which contradicts $\Gamma'(V_C) = J'(V_C)$.

We have established that, for $\hat{V} = V_C$, the optimal continuation value V_1 is such that $V_1 > V_C$. The necessary condition for the optimality of V_0 can be written as

$$J'(V_0) - \hat{J}'(V_C) \leq \frac{F'(R)}{F(R)}\left\{R + \frac{\beta}{1-\beta}[J(V_1) - J(V_0)]\right\} \tag{2.77}$$

and $V_0 \geq V_P$, where the two inequalities hold with complementary slackness. The necessary condition for the optimality of V_1 can be written as

$$J'(V_1) - \hat{J}'(V_C) \geq -\frac{F'(R)}{1-F(R)}\left\{R + \frac{\beta}{1-\beta}[J(V_1) - J(V_0)]\right\} \tag{2.78}$$

and $V_1 \leq V_F$, where the two inequalities hold with complementary slackness.

Since $V_1 > V_C$, $J'(V_1) = \hat{J}'(V_1)$. Since $\hat{J}$ is strictly concave, $\hat{J}'(V_1) < \hat{J}'(V_C)$. From these observations, it follows that

$$R + \frac{\beta}{1-\beta}[J(V_1) - J(V_0)] > 0. \tag{2.79}$$

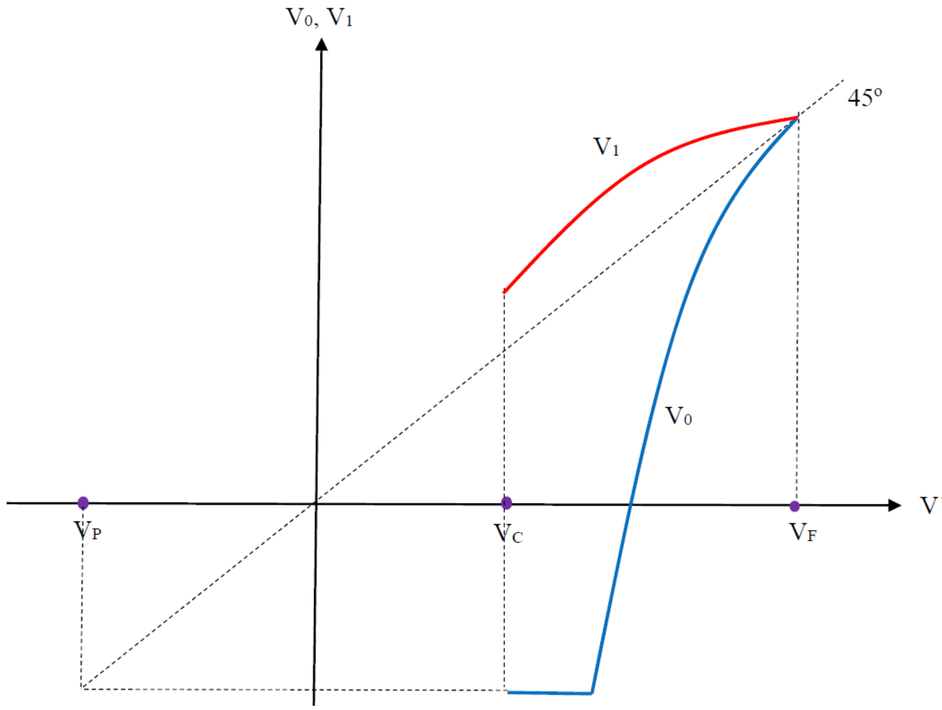

Figure 4: Optimal continuation values V_0 and V_1 as functions of the promised value $\hat{V}$.

Since $V_0 \leq V_C$, $J'(V_0) = \hat{J}'(V_C)$. From this observation and (2.79), it follows that

$$J'(V_0) - \hat{J}'(V_C) = 0 < \frac{F'(R)}{F(R)} \left\{ R + \frac{\beta}{1-\beta} \left[J(V_1) - J(V_0) \right] \right\}. \tag{2.80}$$

Therefore, $V_0 = V_P$. ■

The properties of the optimal mechanism in delegation are intuitive. The principal would like to hire all and only applicants whose quality exceeds 0. The agent would like to hire all and only applicants whose quality exceeds $\eta > 0$. In order to move the agent's preferences towards the principal's preferences, the mechanism gives a higher continuation value to the agent if the hires the applicant than if he does not. Moreover, since the principal's value function is conjectured to be strictly concave in the agent's value, the mechanism is such that the agent's continuation value is strictly greater than the agent's promised value if he hires the applicant, and strictly smaller than the agent's promised value if he does not hire the applicant.

Combining the findings in Propositions 3, 4 and 5, we can characterize the dynamic behavior generated by the optimal mechanism. In period $t = 0$, the mechanism gives the value $V^* \in (V_C, V_F)$ to the agent. If the agent hires the applicant, the agent's value strictly increases. If the agent does not hire the applicant, the agent's value strictly decreases. The probability of both events is strictly positive because the reservation quality $R \in (\underline{x}, \eta]$ for all $V \in [V_C, V_F]$. If the agent fails to hire an applicant enough times, the agent's continuation value does fall below V_C. When this happens, the mechanism runs a lottery between control and delegation that assigns a strictly positive probability p to control. If the outcome of the lottery is control, the principal takes over. In this case, the principal hires all the applicants and the agent's value becomes $V_P = -\eta$. If the outcome of the lottery is delegation, the agent's value is bumped up to V_C. In this case, if the agent does not hire the applicant, the principal takes over hiring with probability 1. If the agent hires the applicant, the mechanism increases the value of the agent above V_C, where the agent will have control over hiring for at least one more period with probability 1.

As long as the agent keeps hiring applicants, the agent's value keeps growing. Depending on the shape of V_1 as a function of V, the agent's value converges to V_F. When the agent is in a neighborhood of V_F, the mechanism is such that the agent hires candidates whose quality is above a threshold R that is approximately equal to η. Moroever, the agent's continuation values are both either approximately equal to V_F. However, since the agent approaches V_F without ever reaching it, his value eventually slides back down and may reach any point with positive probability.

In period $t = 0$, the mechanism gives the value J^* to the principal. Since $J^* > J_P$, the value of the mechanism to the principal exceeds what the principal could obtain by directly controlling the hiring process without the expertise of the agent. Since $J^* > J_F$, the value of the mechanism to the principal exceeds what the principal could obtain by giving the agent complete discretion over hiring and suffering the consequences of the agent's bias. The mechanism delivers a higher value to the principal because it succeeds in making use of the expertise of the agent without giving the agent complete discretion. In particular, the mechanism induces the agent to lower his hiring standards by rising and lowering the threat of giving control to the principle depending on whether the agent hires or does not hire an applicant.

3 Optimal Control

Up to this point, we have restricted attention to mechanisms such that: (i) when the mechanism assigns the hiring decision to the principal, it does so permanently; (ii) when the mechanism assigns the hiring decision to the principal, it prescribes that the principal hires the applicant. These seemingly aribitrary restrictions to the space of mechanisms are made without loss in generality.

First, let us consider the space of mechanisms that allow for the hiring decision to be assigned to either the principal or the agent in any period and after any history. The first-stage problem becomes

$$\Gamma(V) = \max_{p,\tilde{V},\hat{V}} p\tilde{J}(\tilde{V}) + (1-p)\hat{J}(\hat{V}), \text{ s.t.}$$
$$V = p\tilde{V} + (1-p)\hat{V}, \tag{3.1}$$
$$p \in [0,1], \quad \tilde{V} \in \mathcal{V}, \quad \hat{V} \in \hat{\mathcal{V}},$$

where $\hat{J}$ denotes the value of the mechanism if the hiring decision in the current period is assigned to the agent, and G denotes the value of the mechanism to the principal if the hiring decision in the current period is assigned to the principal (who is still instructed to hire the applicant). The value function $\tilde{J}$ is given by (2.4). The value function G is given by

$$\begin{aligned} \tilde{J}(\tilde{V}) &= (1-\beta)\int_{\underline{x}}^{\overline{x}} x dF(x) + \beta\Gamma(V_+) \\ &= (1-\beta)J_P + \beta\Gamma(V_+), \end{aligned} \tag{3.2}$$

where

$$\begin{aligned} \tilde{V} &= (1-\beta)\int_{\underline{x}}^{\overline{x}} (x-\eta) dF(x) + \beta V_+ \\ &= (1-\beta)V_P + \beta V_+. \end{aligned} \tag{3.3}$$

Let V_0 denote some arbitrary value that the mechanism needs to deliver to the agent. Using the definitions of Γ and $\tilde{J}$, we can write $\Gamma(V_0)$ as

$$\Gamma(V_0) = p_0(1-\beta)J_P + (1-p_0)\hat{J}(\hat{V}_0) + p_0\beta\Gamma(V_1), \tag{3.4}$$

where p_0, $\tilde{V}_1$ and $\hat{V}_0$ are optimal given V_0, and V_1 is such that $\tilde{V}_1 = (1-\beta)V_P + \beta V_1$. Notice that the coefficients in front of J_P, $\hat{J}(\hat{V}_0)$ and $\Gamma(V_1)$ are all positive and they sum up to 1.

Using the definitions of Γ and $\tilde{J}$ to replace $\Gamma(V_1)$, we can write $\Gamma(V_0)$ as

$$\begin{aligned} \Gamma(V_0) &= p_0(1-\beta)J_P + \beta p_0 p_1(1-\beta)J_P \\ &+(1-p_0)\hat{J}(\hat{V}_0) + \beta p_0(1-p_1)\hat{J}(\hat{V}_1) + \beta^2 p_0\Gamma(V_2), \end{aligned} \tag{3.5}$$

where p_1, $\tilde{V}_2$ and $\hat{V}_1$ are the optimal choices given V_1, and V_2 is such that $\tilde{V}_2 = (1-\beta)V_P + \beta V_2$. Again, notice that the coefficient in front of J_P, $\hat{J}(\hat{V}_0)$, $\hat{J}(\hat{V}_0)$ and $\Gamma(V_2)$ are all positive and they sum up to 1. This is because, when we replaced $\Gamma(V_1)$ with $p_1\left[(1-\beta)J_P + \beta\Gamma(V_2)\right] + (1-p_1)\hat{J}(\hat{V}_1)$ in (3.4), the coefficients on J_P, $\Gamma(V_2)$ and $\hat{J}(\hat{V}_1)$ are positive and sum up to 1.

Similarly, using the definitions of Γ and $\tilde{J}$ to replace $\Gamma(V_\tau)$ for $\tau = 2, 3, ...T$, we can write

$$\begin{aligned}\Gamma(V_0) &= \left[(1-\beta)\sum\nolimits_{t=0}^{T}\beta^t\left(\prod\nolimits_{i=0}^{t}p_i\right)\right]J_P \\ &+\sum\nolimits_{t=0}^{T}\left[\beta^t(1-p_t)\left(\prod\nolimits_{i=0}^{t-1}p_i\right)\hat{J}(\hat{V}_t)\right]+\left(\beta^{T+1}\prod\nolimits_{i=0}^{T}p_i\right)\Gamma(V_{T+1}).\end{aligned} \tag{3.6}$$

For $T\to\infty$, the expression above becomes

$$\begin{aligned}\Gamma(V_0) &= \left[(1-\beta)\sum\nolimits_{t=0}^{\infty}\beta^t\left(\prod\nolimits_{i=0}^{t}p_i\right)\right]J_P \\ &+\sum\nolimits_{t=0}^{\infty}\left[\beta^t(1-p_t)\left(\prod\nolimits_{i=0}^{t-1}p_i\right)\hat{J}(\hat{V}_t)\right].\end{aligned} \tag{3.7}$$

As before, the coefficients in front of J_P and $\hat{J}(\hat{V}_t)$ are all positive and sum up to 1.

Notice that we can write (3.7) as

$$\begin{aligned}\Gamma(V_0) &= \overline{p}J_P+(1-\overline{p})\sum\nolimits_{t=0}^{\infty}\left[\frac{\beta^t(1-p_t)\left(\prod\nolimits_{i=0}^{t-1}p_i\right)}{1-\overline{p}}\hat{J}(\hat{V}_t)\right] \\ &\le \overline{p}J_P+(1-\overline{p})\hat{J}\left(\overline{V}\right),\end{aligned} \tag{3.8}$$

where $\overline{p}$ and $\overline{V}$ are given by

$$\overline{p}=(1-\beta)\sum_{t=0}^{\infty}\beta^t\left(\prod_{i=0}^{t}p_i\right) \text{ and } \overline{V}=\frac{\beta^t(1-p_t)\left(\prod\nolimits_{i=0}^{t-1}p_i\right)}{1-\tilde{p}}\hat{V}_t, \tag{3.9}$$

and the second line in (3.8) makes use of the concavity of $\hat{J}$.

Now consider the first-stage problem when the space of mechanisms is restricted to those that, once they assign the hiring decision to the principal, they do so forever. This first-stage problem is

$$\begin{aligned}J(V_0) = \quad & \max_{p,\hat{V}} pJ_P+(1-p)\hat{J}(\hat{V}), \text{ s.t.} \\ & V_0 = pV_P+(1-p)\hat{V}, \\ & p\in[0,1], \quad \hat{V}\in\hat{\mathcal{V}}.\end{aligned} \tag{3.10}$$

Since $(p,\hat{V})=(\overline{p},\overline{V})$ is a feasible choice for (3.10), it follows that $J(V_0)\ge\Gamma(V_0)$. Since any mechanism that assigns control of hiring to the principal permanently is feasible in the space of mechanisms that allow hiring to be assigned to the principal or the agent in any period and after any history, it follows that $\Gamma(V_0)\ge J(V_0)$. Combining the two inequalities yields $\Gamma(V_0)=J(V_0)$. In other words, any mechanism that is optimal in the class of mechanisms that flexibly assigns control of hiring is payoff equivalent to some mechanism that assigns control of hiring to the principal permanently.

Next, let us consider the space of mechanisms that can specify that the principal either hires or does not hire applicants when he is in control. We maintain the restriction to mechanisms that turn the control over hiring to the principal permanently. The first-stage problem now reads

$$\begin{aligned}\Gamma(V) = \quad & \max_{p,\hat{V}} pJ_P + qJ_Q + (1-p-q)\hat{J}(\hat{V}), \text{ s.t.} \\ & V = pV_P + qV_Q + (1-p-q)\hat{V}, \\ & p \in [0,1], \quad \hat{V} \in \hat{\mathcal{V}},\end{aligned} \tag{3.11}$$

where J_Q and V_Q are the values to the principal and the agent if the principal has control over hiring and does not hire the applicants, i.e. $(J_Q, V_Q) = (0,0)$, and J_P and V_P are the values to the principal and the agent if the principal has control over hiring and hires the applicants, i.e. $(J_P, V_P) = (0, -\eta)$.

Let p_0, q_0 and $\hat{V}_0$ denote a solution to (3.11) for some V_0. Suppose that $V_Q \geq V_\ell$. Notice that the value to the principal $\Gamma(V_0)$ is such that

$$\begin{aligned}\Gamma(V_0) \quad & = p_0 J_P + q_0 J_Q + (1-p_0-q_0)\hat{J}(\hat{V}_0) \\ & \leq p_0 J_P + (1-p_0)\left[\frac{q_0}{1-p_0}\hat{J}(V_Q) + \frac{1-p_0-q_0}{1-p_0}\hat{J}(\hat{V}_0)\right] \\ & \leq p_0 J_P + (1-p_0)\hat{J}\left(\overline{V}\right),\end{aligned} \tag{3.12}$$

where

$$\overline{V} = \frac{q_0}{1-p_0}V_Q + \frac{1-p_0-q_0}{1-p_0}\hat{V}_0. \tag{3.13}$$

The second line in (3.12) uses the fact that $\hat{J}(V_Q) > J_Q = 0$. The third line in (3.12) makes use of the strict concavity of $\hat{J}$. Similarly, notice that the value to the agent V_0 is such that

$$V_0 = p_0 V_P + (1-p_0)\overline{V}. \tag{3.14}$$

Therefore, $(p, \hat{V}) = (p_0, \hat{V})$ is a feasible choice for the first-stage problem (3.10). Hence, $J(V_0) \geq \Gamma(V_0)$. Since any mechanism that involves the principal hiring applicants when in control is a feasible choice for (3.11), it follows that $\Gamma(V_0) \geq J(V_0)$. Combining the two inequalities yields $\Gamma(V_0) = J(V_0)$. That is, any optimal mechanism in the space of mechanisms that can instruct the principal to either hire or not hire applicants is payoff equivalent to some mechanism in the restricted space of mechanisms.

Now, suppose that $V_Q < V_\ell$ and, hence, $V_0 > V_Q = 0$. Notice that the value to the principal $\Gamma(V_0)$ is such that

$$\begin{aligned}\Gamma(V_0) \quad & = p_0 J_P + q_0 J_Q + (1-p_0-q_0)\hat{J}(\hat{V}_0) \\ & \leq \overline{p} J_P + (1-\overline{p})\,\hat{J}(\hat{V}_0),\end{aligned} \tag{3.15}$$

where $\overline{p}$ is defined as

$$\overline{p} = p_0 + q_0 \frac{\hat{V}_0 - V_Q}{\hat{V}_0 - V_P} \in [p_0, p_0 + q_0]\,. \tag{3.16}$$

The inequality in (3.15) makes use of the fact that $1 - \overline{p} \geq 1 - p_0 - q_0$ and $\hat{J}(\hat{V}_0) > J_P$. Similarly, notice that the value to the agent V_0 is such that

$$\begin{aligned} V_0 &= p_0 V_P + q_0 V_Q + (1 - p_0 - q_0)\hat{V}_0. \\ &= \overline{p} V_P + (1 - \overline{p})\,\hat{V}_0. \end{aligned} \tag{3.17}$$

Therefore, $(p, \hat{V}) = (p_0, \hat{V})$ is a feasible choice for the first-stage problem (3.10). Hence, $J(V_0) \geq \Gamma(V_0)$. Since any mechanism that involves the principal hiring applicants when in control is a feasible choice for (3.11), it follows that $\Gamma(V_0) \geq J(V_0)$. Combining the two inequalities yields $\Gamma(V_0) = J(V_0)$.

Combining the above arguments, it is easy to immediate to verify that, for any optimal mechanism in the class of mechanisms that can assign control of hiring to either the principal or the agent in any period and after any history, and that can specify in every period and after any history whether the principal hires or does not hire applicants, there exists a payoff-equivalent mechanism is the class of restricted mechanism considered in Section 2. We have thus established the following proposition.

Proposition 6. *(Optimal Control). Consider the class of mechanisms that can assign control of hiring to either the principal or the agent in any period and after any history, and that can specify in every period and after any history whether the principal hires or does not hire applicants. Given this class of mechanisms, let* Γ *denote the first-stage value function and let* J *be given by (2.1). Then,* $\Gamma = J$.

4 Extensions

In this section, we show that the analysis of the baseline model can be immediately extended in a number of natural directions. First, we consider a version of the model in which the agent is positively biased in favor of the applicants. We show that the optimal mechanism has the same properties as in the baseline model, except that the agent is rewarded for not hiring the applicant and punished for hiring the applicant. Second, we consider a version of the model in which there are n applicants for each vacancy. We show that the mechanism design problem has exactly the same structure as in the baseline, except that the quality distribution F needs to be replaced with the distribution F^n of the maximum of n independent draws from F. Third, we consider a version of the model in which the vacancy attracts both contentious applicants–applicants against whom the agent is biased–and uncontentious applicants–applicants that the agent evaluates in an unbiased way. We show that the mechanism design problem has exactly the same structure as in the baseline, except that the relevant distribution is the distribution of the gap between the quality of the best contentious applicant and the quality of the best uncontentious applicant. Lastly, we consider a version of the model in which the principal does not know if an applicant is available for hire. We show that the mechanism design problem has the same structure as in the baseline, except that the agent is punished equally for not hiring an applicant and not meeting an applicant.

4.1 Positive Bias

We consider a version of the baseline model in which the agent is positively biased towards the applicants, rather than negatively biased. In particular, the difference between the payoff to the agent and the payoff to the principal if an applicant is hired is some $\phi > 0$.

For this version of the model, the first-stage problem is

$$J_\phi(V_\phi) = \max_{p\in[0,1],\hat{V}\in\hat{\mathcal{V}}_\phi} pJ_P^\phi + (1-p)\hat{J}_\phi(\hat{V}_\phi) \tag{4.1}$$

subject to the promise-keeping constraint

$$qV_P^\phi + (1-q)\hat{V}_\phi = V_\phi. \tag{4.2}$$

Adapting the arguments in Section 3, it is easy to show that the optimal mechanism is such that, when the principal has control over hiring, he retains control forever and he does not hire any of the applicants. Therefore, the "punishment" payoffs to the principal and the agent are

$$J_P^\phi = V_P^\phi = 0. \tag{4.3}$$

The second-stage problem is

$$\hat{J}_\phi(\hat{V}_\phi) = \max_{V_0^\phi,V_1^\phi\in\mathcal{V}_\phi} (1-\beta)\int_{R_\phi} xdF(x) + \beta\left[F(R_\phi)J(V_0^\phi) + (1-F(R_\phi))J(V_1^\phi)\right] \tag{4.4}$$

subject to the promise-keeping constraint

$$\hat{V}_\phi = (1-\beta)\int_{R_\phi}(x+\phi)dF(x) + \beta\left[F(R_\phi)V_0^\phi + (1-F(R_\phi)))V_1^\phi\right], \tag{4.5}$$

and the incentive-compatibility constraint

$$R_\phi = -\phi - \frac{\beta}{1-\beta}\left(V_1^\phi - V_0^\phi\right). \tag{4.6}$$

Following the same argument as in Lemma 1, it is straightforward to show that the set $\mathcal{V}_\phi$ is given by the interval $[V_P^\phi, V_F^\phi]$, and the set $\hat{\mathcal{V}}_\phi$ is given by the interval $[V_\ell^\phi, V_F^\phi]$, where

$$V_\ell^\phi \equiv (1-\beta)V_F^\phi + \beta V_P^\phi, \quad V_F^\phi \equiv \int_{-\phi}(x+\phi)dF(x). \tag{4.7}$$

Following the same argument as in Lemma 2, it is straightforward to show that J_ϕ^*, which is defined as the maximum of $\hat{J}_\phi(\hat{V})$ with respect to $\hat{V}$, is strictly greater than J_F^*, which is defined as $\hat{J}_\phi(V_F)$ and is equal to $\int_{-\phi} x dF(x)$, if the quality distribution F and the bias ϕ are such that

$$\frac{\phi F'(-\phi)}{1 - F(-\phi)} > \frac{\int_{-\phi} x dF(x)}{\int_{-\phi} (x + \phi) dF(x)} > 0. \tag{4.8}$$

Under condition (4.8), we can follow the same arguments as in Section 2 to show that the optimal mechanism has the same qualitative features when the agent is positively biased as when the agent is negatively biased. There is only one difference between the optimal mechanism when the agent is positively biased and the optimal mechanism when the agent is negatively biased. When the agent is negatively biased, the mechanism rewards the agent for hiring an applicant by increasing his value, and it punishes the agent for not hiring an applicant by lowering his value. When the agent is positively biased, the mechanism rewards the agent for not hiring an applicant and punsihes him for hiring an applicant. This difference between the optimal mechanisms is easy to understand. When the agent is negatively biased, the agent wants to hire fewer applicants than the principal. The optimal mechanism reduces the gap between the preferences of the agent and the principal by rewarding the agent for hiring applicants and by punishing him for not hiring applicants. When the agent is positively biased, the agent wants to hire more applicants than the principal. The optimal mechanism reduces the gap between the preferences of the agent and the principal by rewarding the agent for not hiring applicants and by punishing him for not hiring applicants. The following proposition contains a complete characterization of the optimal mechanism when the agent is positively biased.

Proposition 7. *(Optimal Mechanism with Positive Bias)*

1. *For all $V_\phi \in [V_P^\phi, V_C^\phi)$, the optimal lottery between control and delegation is such that the probability p of control is such that $p > 0$ and, conditional on delegation, the agent's value $\hat{V}_\phi$ is V_C^ϕ. For all $V_\phi \in [V_C^\phi, V_F^\phi]$, $p = 0$ and $\hat{V}_\phi = V$. The critical value V_C^ϕ is such that*

$$\hat{J}'_\phi(V_C^\phi) = \frac{\hat{J}_\phi(V_C^\phi) - J_P^\phi}{V_C^\phi - V_P^\phi}. \tag{4.9}$$

2. *For all $\hat{V}_\phi \in (V_C^\phi, V_F^\phi)$, the optimal agent's continuation values V_0^ϕ and V_1^ϕ are such that*

$$V_1^\phi < \hat{V} < V_0^\phi < V_F^\phi. \tag{4.10}$$

For $\hat{V}_\phi = V_C^\phi$, the optimal continuation values are $V_0^\phi > V_C^\phi$ and $V_1^\phi = V_P^\phi$. For $\hat{V}_\phi = V_F^\phi$, the optimal continuation values are $V_0^\phi = V_F^\phi$ and $V_1^\phi = V_F^\phi$. For all $\hat{V}_\phi \in [V_C^\phi, V_F^\phi]$, the optimal continuation values induce the agent to use a reservation *quality R_ϕ such that $R_\phi \in (-\phi, \overline{x})$.*

A tight connection between the optimal mechanism with positive bias and the optimal mechanism with negative bias can be drawn when the quality distribution is symmetric around 0, i.e. $F(x) = 1 - F(-x)$ for all $x \in [\underline{x}, \overline{x}]$, and the biases have the same magnitude, i.e. $\phi = \eta$. The connection is presented in the following proposition.

Proposition 8. *(Positive and Negative Bias with Symmetry). Let the quality distribution of applicants be symmetric around 0, in the sense that $F(x) = 1 - F(-x)$ for all $x \in [\underline{x}, \overline{x}]$, and let $\phi = \eta$. Then:*

1. *The first-stage and second-stage value functions J_ϕ and $\hat{J}_\phi$ are such that*

$$J_\phi(V + \eta) = J(V), \quad \hat{J}_\phi(\hat{V} + \eta) = \hat{J}(\hat{V}).$$

2. *Given the promised value $V_\phi = V + \eta$, the lottery $(p_\phi, \hat{V}_\phi)$ that solves (4.1) is such that $p_\phi = p$ and $\hat{V}_\phi = \hat{V} + \eta$, where $(p_\phi, \hat{V}_\phi)$ solves (2.1) given the promised value V.*
3. *Given the promised value $\hat{V}_\phi = \hat{V} + \eta$, the continuation values V_0^ϕ and V_1^ϕ that solve (4.4) are $V_0^\phi = V_1 + \eta$ and $V_1^\phi = V_0 + \eta$, where V_0 and V_1 are the continuation values that solve (2.4) given the promised value $\hat{V}$.*
4. *Given the promised value $\hat{V}_\phi = \hat{V} + \eta$, the agent's reservation quality R_ϕ that solves (4.4) is $R_\phi = -R$, where R is the agent's reservation quality that solves (2.4) given the promised value $\hat{V}$.*

Proof. For $\hat{V}_\phi = \hat{V} + \eta$, the functional equation (4.4) can be written

$$\begin{aligned} & \hat{J}_\phi(\hat{V} + \eta) \\ = \quad & \max_{V_0^\phi, V_1^\phi} (1-\beta) \int_{R_\phi} x dF(x) + \beta \left[F(R_\phi) J_\phi(V_0^\phi) + (1 - F(R_\phi)) J_\phi(V_1^\phi) \right], \\ \text{s.t.} \quad & \hat{V} + \eta = (1-\beta) \int_{R_\phi} (x + \phi) dF(x) + \beta \left[F(R_\phi) V_0^\phi + (1 - F(R_\phi))) V_1^\phi \right] \\ & R_\phi = -\phi - \frac{\beta}{1-\beta}(V_1^\phi - V_0^\phi), \quad V_0^\phi, V_1^\phi \in [V_P^\phi, V_F^\phi]. \end{aligned} \tag{4.12}$$

Let us define V_0 and V_1 as, respectively, $V_1^\phi - \eta$ and $V_0^\phi - \eta$. Since the choice variables V_0^ϕ and V_1^ϕ must belong to the interval $[V_P^\phi, V_F^\phi]$, the alternative choice variables V_0

and V_1 must belong to the interval

$$\begin{aligned}
\left[V_P^\phi - \eta, V_F^\phi - \eta\right] &= \left[-\eta, \int_{-\phi} (x+\phi) dF(x) - \eta\right] \\
&= \left[-\eta, \int_{-\phi} x dF(x) + F(-\phi)\phi - \eta\right] \\
&= \left[-\eta, \int_{\eta} x dF(x) + (1 - F(\eta))\eta - \eta\right] \\
&= \left[-\eta, \int_{\eta} (x - \eta) dF(x)\right] = [V_P, V_F]
\end{aligned} \tag{4.13}$$

The first line makes use of the definitions of V_P^ϕ and V_F^ϕ. The third line makes use of the fact that $F(-\phi)$ equals $1 - F(\phi)$ and $\phi = \eta$, as well as of the fact that, by symmetry of the quality distribution, $\int_{-\phi} x dF(x)$ equals $\int_{\phi} x dF(x)$. The 0 and, hence, $\int_{-\phi} x dF(x)$ equals $\int_{\phi} x dF(x)$. The fourth line makes use of the definition of V_P and V_F.

Using the above definitions and observations, we can rewrite (4.12) as

$$\begin{aligned}
&\hat{J}_\phi(\hat{V} + \eta) \\
&= \max_{V_0, V_1} (1-\beta) \int_{R_\phi} x dF(x) + \beta \left[F(R_\phi) J_\phi(V_1 + \eta) + (1 - F(R_\phi)) J_\phi(V_0 + \eta)\right], \text{ s.t.} \\
&\quad \hat{V} + \eta = (1-\beta) \int_{R_\phi} (x+\phi) dF(x) + \beta \left[F(R_\phi) V_1 + (1 - F(R_\phi)) V_0 + \eta\right] \\
&\quad R_\phi = -\eta - \frac{\beta}{1-\beta}(V_0 - V_1), \quad V_0, V_1 \in [V_P, V_F].
\end{aligned} \tag{4.14}$$

Defining R as $-R_\phi$ and using the fact that $F(R_\phi) = 1 - F(-R_\phi)$, we can rewrite (4.14) as

$$\begin{aligned}
&\hat{J}_\phi(\hat{V} + \eta) \\
&= \max_{V_0, V_1} (1-\beta) \int_{R_\phi} x dF(x) + \beta \left[(1 - F(R)) J_\phi(V_1 + \eta) + F(R) J_\phi(V_0 + \eta)\right], \text{ s.t.} \\
&\quad \hat{V} + \eta = (1-\beta) \int_{R_\phi} (x+\phi) dF(x) + \beta \left[(1 - F(R)) V_1 + F(R) V_0 + \eta)\right] \\
&\quad R = \eta - \frac{\beta}{1-\beta}(V_1 - V_0), \quad V_0, V_1 \in [V_P, V_F].
\end{aligned} \tag{4.15}$$

Using the symmetry of the quality distribution and the definition of R, we can now rewrite (4.15) as

$$\begin{aligned} &\hat{J}_\phi(\hat{V}+\eta) \\ &= \max_{V_0,V_1}(1-\beta)\int_R x dF(x) + \beta\left[(1-F(R))J_\phi(V_1+\eta) + F(R)J_\phi(V_0+\eta)\right], \text{ s.t.} \\ &\quad \hat{V} = (1-\beta)\int_R (x-\eta)dF(x) + \beta\left[(1-F(R))V_1 + F(R)V_0\right] \\ &\quad R = \eta - \frac{\beta}{1-\beta}(V_1 - V_0), \quad V_0,\, V_1 \in [V_P, V_F]. \end{aligned} \tag{4.16}$$

In the objective function, we used the fact that $\int_R x dF(x)$ equals $\int_{-R} x dF(x)$ and the definition $R_\phi = -R$. In the promise-keeping constraint, we collected η on the right-hand side and used the fact that $\int_R x dF(x)$ equals $\int_{R_\phi} x dF(x)$.

For $\hat{V}+\eta$, the functional equation (4.1) can be written as

$$\begin{aligned} J_\phi(V+\eta) &= \max_{p,\hat{V}_\phi} p J_P^\phi + (1-p)\hat{J}_\phi(\hat{V}_\phi) \\ \text{s.t. } V+\eta &= pV_P^\phi + (1-p)\hat{V}_\phi, \\ \hat{V}_\phi &\in \left[V_\ell^\phi, V_F^\phi\right],\ p\in[0,1]. \end{aligned} \tag{4.17}$$

Let $\hat{V}$ be defined as $\hat{V}_\phi - \eta$. Since $\hat{V}_\phi$ belongs to the interval $[V_\ell^\phi, V_F^\phi]$, $\hat{V}$ belongs to the interval

$$\begin{aligned} \left[V_\ell^\phi - \eta, V_F^\phi - \eta\right] &= \left[(1-\beta)V_P^\phi + \beta V_F^\phi - \eta, V_F^\phi - \eta\right] \\ &= \left[(1-\beta)\left(V_P^\phi - \eta\right) + \beta\left(V_F^\phi - \eta\right), V_F^\phi - \eta\right] \\ &= [(1-\beta)V_P + \beta V_F, V_F] = [V_\ell, V_F]. \end{aligned} \tag{4.18}$$

The third line makes use of the fact that

$$V_P^\phi - \eta = -\eta = V_P. \tag{4.19}$$

The third line also makes use of the fact that

$$\begin{aligned} V_F^\phi - \eta &= \int_{-\phi}(x+\phi)dF(x) - \eta \\ &= \int_{-\phi} x dF(x) + \phi(1-F(-\phi)) - \eta \\ &= \int_\eta (x-\eta)dF(x) = V_F. \end{aligned} \tag{4.20}$$

Using the definition of $\hat{V}$ and the above observations, we can rewrite (4.17) as

$$\begin{aligned} J_\phi(V+\eta) &= \max_{p,\hat{V}} pJ_P^\phi + (1-p)\hat{J}_\phi(\hat{V}+\eta) \\ \text{s.t. } V+\eta &= pV_P^\phi + (1-p)(\hat{V}+\eta), \\ \hat{V} &\in [V_\ell, V_F],\ p \in [0,1]. \end{aligned} \tag{4.21}$$

Rearranging terms in the objective function and using the fact that $J_P^\phi = J_P$, we can rewrite (4.21) as

$$\begin{aligned} J_\phi(V+\eta) &= \max_{p,\hat{V}} pJ_P + (1-p)\hat{J}_\phi(\hat{V}+\eta) \\ \text{s.t. } V &= pV_P + (1-p)\hat{V}, \\ \hat{V} &\in [V_\ell, V_F],\ p \in [0,1]. \end{aligned} \tag{4.22}$$

The value functions $\hat{J}_\phi(\hat{V}+\eta) = \hat{J}(\hat{V})$ and $J_\phi(V+\eta) = J(V)$ solve the functional equations (4.16) and (4.22), since $\hat{J}(\hat{V})$ and $J(V)$ are the solution to (2.4) and (2.1), (4.16) is identical to (2.4), and (4.22) is identical to (2.1). Parts 2, 3 and 4 of the proposition directly follow from the change in choice variables implemented in the derivation of (4.16) and (4.22). ■

4.2 Multiple Applicants

We now consider a version of the baseline model in which there are $n \in \mathbb{N}$ applicants for each vacancy. Every applicant has a quality x that is independently from the distribution $F(x)$. Every applicant has a quality that is observed by the agent but not by the principal. It is useful to denote as x_n the maximum of the quality of the n applicants. It is also useful to denote as $F_n(x_n)$ the distribution of x_n, i.e. $F_n(x_n) = (F(x_n))^n$.

For this version of the model, the first-stage problem is

$$\begin{aligned} J(V) &= \max_{p,\hat{V}} pJ_P + (1-p)\hat{J}(\hat{V}) \\ \text{s.t. } V &= pV_P + (1-p)\hat{V}, \\ p &\in [0,1],\ \hat{V} \in \hat{\mathcal{V}}. \end{aligned} \tag{4.23}$$

The second-stage problem is

$$\begin{aligned}
&\hat{J}(\hat{V}) \\
&= \max_{V_0,V_1}(1-\beta)\int_R x_n dF_n(x_n) + \beta\left[F_n(R)J(V_0) + (1-F_n(R))J(V_1)\right], \text{ s.t.} \\
&\quad \hat{V} = (1-\beta)\int_R (x_n-\eta)dF_n(x) + \beta\left[F_n(R)V_0 + (1-F_n(R)))V_1\right] \\
&\quad R = \eta - \frac{\beta}{1-\beta}(V_1 - V_0), \quad V_0,\, V_1 \in \mathcal{V}.
\end{aligned} \tag{4.24}$$

Following the logic of Section 3, we restrict attention to mechanisms such that, when the principal takes over hiring, he does so forever. Moroever, we restrict attention to mechanisms such that, whenever the principal is in control of hiring, the agent reports the ranking of the applicants and the principal hires the highest ranked one. The agent has an incentive to correctly report the ranking of applicants because he is better off if the principal hires an applicant with higher quality than an applicant with lower quality. In our numerical examples, we verify that this restriction is without loss in generality. Under this restrictions, the "punishment" payoffs to the principal and the agent are

$$J_P = \int_{\underline{x}}^{\overline{x}} x_n dF_n(x_n), \quad V_P = \int_{\underline{x}}^{\overline{x}} (x_n - \eta) dF_n(x_n). \tag{4.25}$$

Following the same argument as in Lemma 1, it is easy to show that the set $\mathcal{V}$ is given by the interval $[V_P, V_F]$, and the set $\hat{\mathcal{V}}$ is given by the interval $[V_\ell, V_F]$, where

$$V_\ell \equiv (1-\beta)V_F + \beta V_P, \quad V_F \equiv \int_\eta (x_n - \eta) dF_n(x_n). \tag{4.26}$$

Following the same argument as in Lemma 2, it is straightforward to show that

$$J^* \equiv \max_{\hat{V}\in\hat{\mathcal{V}}} J(\hat{V}) > \int_\eta x_n dF_n(x_n) \equiv J_F \tag{4.27}$$

if the quality distribution F_n and the bias η are such that

$$\begin{aligned}
\frac{\eta F_n'(\eta)}{1-F_n(\eta)} &> \frac{J_F - J_P}{V_F - V_P} \\
&= \frac{\int_\eta x_n dF_n(x_n) - \int_{\underline{x}}^{\overline{x}} x_n dF_n(x_n)}{\int_\eta (x_n-\eta) dF_n(x_n) - \int_{\underline{x}}^{\overline{x}} (x_n-\eta) dF_n(x_n)}.
\end{aligned} \tag{4.28}$$

The recursive formulation of the optimal mechanism for a model with n applicants is identical to the recursive formulation of the optimal mechanism for the model with a single applicant, except that the quality distribution F_n replaces the quality distribution

F. Therefore, under condition (4.28), it is straightforward to generalize the characterization of the optimal mechanism for the model with a single applicant to a model with n applicants. The charcaterization of the optimal mechanism for a model with n applicants is summarized in the following proposition.

Proposition 9. *(Optimal Mechanism with Multiple Applicants)*

1. *For all $V \in [V_P, V_C)$, the optimal lottery between control and delegation is such that the probability p of control is such that $p > 0$ and, conditional on delegation, the agent's value $\hat{V}$ is V_C. For all $V \in [V_C, V_F]$, $p = 0$ and $\hat{V} = V$. The critical value V_C is such that $\hat{J}'(V_C)$ equals $(\hat{J}(V_C) - J_P)/(V_C - V_P)$.*
2. *For all $\hat{V} \in (V_C, V_F)$, the optimal agent's continuation values V_0 and V_1 are such that $V_0 < \hat{V} < V_1 < V_F$. For $\hat{V} = V_C$, the optimal continuation values are $V_0 = V_P$ and $V_1 > V_C$. For $\hat{V} = V_F$, the optimal continuation values are $V_0 = V_F$ and $V_1 = V_F$. For all $\hat{V} \in [V_C, V_F]$, the optimal continuation values induce the agent to use a reservation quality R such that $R \in (\underline{x}, \eta)$.*

4.3 Contentious and Uncontentious Applicants

As in the previous extension, we consider a version of the model in which in there are multiple applicants for every vacancy. In contrast to the previous extension, applicants come from different groups. In particular, some applicants come from group X and some applicants come from group Y. The quality x of an X-applicant is drawn from some distribution $F_x(x)$, with mean 0 and support $[\underline{x}, \overline{x}]$. The quality y of a Y-applicant is independently drawn from some distribution $F_y(y)$, with mean 0 and support $[\underline{y}, \overline{y}]$. The agent is negatively biased towards X-applicants, with a bias equal to some $\eta > 0$. The agent is unbiased towards Y-applicants. In this sense, X-applicants are contentious and Y-applicants are not contentious. We assume that the group from which an applicant comes from is known to both the principal and the agent, as it may refect readily observable demographic characteristics. We assume that the quality of a particular applicant is known only to the agent.

In order to streamline the analysis, let us assume that there are one X-applicant and one Y-applicant per vacancy. It is useful to denote as $\hat{y}$ the maximum between the quality y of a Y-applicant and 0. It is also useful to denote as z the difference between the quality x of an X-applicant and $\hat{y}$. That is, $\hat{y} = \max\{y, 0\}$ and $z = x - \hat{y}$. The joint distribution of the random variables $\hat{y}$ and z is determined by the quality distribution F_x and F_y for X and Y-applicants. For our purposes, it is useful to describe the joint distribution of $\hat{y}$ and z with the marginal distribution $F_z(z)$ of the random variable z and the conditional distribution $G(\hat{y}|z)$ of the random variable $\hat{y}$.

For this version of the model, the first-stage problem is

$$\begin{aligned} J(V) &= \max_{p,\hat{V}} pJ_P + (1-p)\hat{J}(\hat{V}) \\ \text{s.t. } V &= pV_P + (1-p)\hat{V}, \\ & p \in [0,1],\ \hat{V} \in \hat{\mathcal{V}}, \end{aligned} \tag{4.29}$$

where J_p and V_P are respectively given by

$$J_P = 0, \quad V_P = -\eta. \tag{4.30}$$

As in Section 3, we can show that the optimal mechanism is such that, once the principal takes control of hiring, he does so forever and, whenever the principal controls hiring, he always hire the contentious applicant.

At the second stage, the value of the mechanism to the agent is

$$\hat{V} = \int_z \left[\int_{\hat{y}} \max \left\{ (1-\beta)(\hat{y}+z-\eta) + \beta V_1, (1-\beta)\hat{y} + \beta V_0 \right\} dG_{\hat{y}}(\hat{y}|z) \right] dF_z(z). \tag{4.31}$$

The above expression is easy to understand. Consider a particular realization of the random variables $\hat{y}$ and z. If the agent hires the X-applicant, the agent's flow payoff is $(1-\beta)(\hat{y}+z-\eta)$, where $\hat{y}+z$ is the quality x of the X-applicant, and the agent's continuation value is βV_1. If the agent does not hire the X-applicant, the agent's flow payoff is $(1-\beta)\hat{y}$, where $\hat{y}$ is the maximum between the quality y of the Y-applicant and the flow payoff from not hiring, and the agent's continuation payoff is βV_0. The agent chooses whether to hire or not hire the Y-applicant so as to maximize the sum of its flow and continuation payoffs. Therefore, the agent hires the Y-applicant if and only if $z \geq R$, where

$$R = \eta - \frac{\beta}{1-\beta}(V_1 - V_0). \tag{4.32}$$

Using the definition of the reservation quality R, we can rewrite (4.31) as

$$\begin{aligned} \hat{V} &= \int^R \left[\int_{\hat{y}} \left[(1-\beta)\hat{y} + \beta V_0 \right] dG_{\hat{y}}(\hat{y}|z) \right] dF_z(z) \\ &\quad + \int_R \left[\int_{\hat{y}} \left[(1-\beta)(\hat{y}+z-\eta) + \beta V_1 \right] dG_{\hat{y}}(\hat{y}|z) \right] dF_z(z) \\ &= (1-\beta)\left[\kappa + \int_R (z-\eta)\, dF_z(z) \right] + \beta F_z(R) V_0 + \beta \left(1 - F_z(R)\right) V_1 \end{aligned} \tag{4.33}$$

where κ denotes the unconditional mean of the random variable $\hat{y}$, i.e.,

$$\kappa = \int_z \left[\int_{\hat{y}} \hat{y}\, dG_{\hat{y}}(\hat{y}|z) \right] dF_z(z). \tag{4.34}$$

Similarly, at the second stage, the value of the mechanism to the agent is

$$\begin{aligned}\hat{J} &= \int^{R} \left[\int_{\hat{y}} \left[(1-\beta)\hat{y} + \beta J(V_0) \right] dG_{\hat{y}}(\hat{y}|z) \right] dF_z(z) \\ &+ \int_{R} \left[\int_{\hat{y}} \left[(1-\beta)(\hat{y}+z) + \beta J(V_1) \right] dG_{\hat{y}}(\hat{y}|z) \right] dF_z(z) \\ &= (1-\beta)\left[\kappa + \int_{R} z dF_z(z) \right] + \beta F_z(R) J(V_0) + \beta \left(1 - F_z(R)\right) J(V_1)\end{aligned} \tag{4.35}$$

The above expression is also easy to understand. Consider a particular realization of the random variables $\hat{y}$ and z. If $z \geq R$, the agent hires the X-applicant. The principal's flow payoff is $(1-\beta)(\hat{y}+z)$, where $\hat{y}+z$ is the quality x of the X-applicant, and the principal's continuation value is $\beta J(V_1)$. If $z < R$, the agent does not hire the X-applicant. The agent's flow payoff is $(1-\beta)\hat{y}$, where $\hat{y}$ is the maximum between the quality y of the Y-applicant and the flow payoff from not hiring, and the principal's continuation payoff is $\beta J(V_0)$.

Combining (4.33) and (4.35), we can write the second-stage problem as

$$\begin{aligned}&\hat{J}(\hat{V}) \\ &= \max_{V_0, V_1} (1-\beta)\left[\kappa + \int_{R} z dF_z(z) \right] + \beta \left[F_z(R) J(V_0) + (1 - F_z(R)) J(V_1) \right], \text{ s.t.} \\ &\quad \hat{V} = (1-\beta)\left[\kappa + \int_{R} (z-\eta)\, dF_z(z) \right] + \beta F_z(R) V_0 + \beta \left(1 - F_z(R)\right) V_1 \\ &\quad R = \eta - \frac{\beta}{1-\beta}(V_1 - V_0), \quad V_0,\, V_1 \in \mathcal{V}.\end{aligned} \tag{4.36}$$

Following the same argument as in Lemma 1, it is easy to show that the set $\mathcal{V}$ is given by the interval $[V_P, V_F]$, and the set $\hat{\mathcal{V}}$ is given by the interval $[V_\ell, V_F]$, where

$$V_\ell \equiv (1-\beta)V_F + \beta V_P, \quad V_F \equiv \kappa + \int_{\eta} (z-\eta) dF_z(z). \tag{4.37}$$

Following the same argument as in Lemma 2, it is straightforward to show that

$$J^* \equiv \max_{\hat{V} \in \hat{\mathcal{V}}} J(\hat{V}) > \kappa + \int_{\eta} z dF_z(z) \equiv J_F \tag{4.38}$$

if the marginal distribution F_z and the bias η are such that

$$\begin{aligned}\frac{\eta F_z'(\eta)}{1 - F_z(\eta)} &> \frac{J_F - J_P}{V_F - V_P} \\ &= \frac{\kappa + \int_{\eta} z dF_z(z)}{\kappa + \int_{\eta} (z-\eta) dF_z(z) + \eta}.\end{aligned} \tag{4.39}$$

Under condition (4.39), we can establish the following.

Proposition 10. *(Optimal Mechanism with Contentious and Uncontentious Applicants)*

1. *For all $V \in [V_P, V_C)$, the optimal lottery between control and delegation is such that the probability p of control is such that $p > 0$ and, conditional on delegation, the agent's value $\hat{V}$ is V_C. For all $V \in [V_C, V_F]$, $p = 0$ and $\hat{V} = V$. The critical value V_C is such that $\hat{J}'(V_C)$ equals $(\hat{J}(V_C) - J_P)/(V_C - V_P)$.*
2. *For all $\hat{V} \in (V_C, V_F)$, the optimal agent's continuation values V_0 and V_1 are such that $V_0 < \hat{V} < V_1 < V_F$. For $\hat{V} = V_C$, the optimal continuation values are $V_0 = V_P$ and $V_1 > V_C$. For $\hat{V} = V_F$, the optimal continuation values are $V_0 = V_F$ and $V_1 = V_F$. For all $\hat{V} \in [V_C, V_F]$, the optimal continuation values induce the agent to use a reservation quality R such that $R \in (\underline{z}, \eta)$, with $\underline{z} \equiv \underline{x} - \overline{y}$ and $\overline{z} \equiv \overline{x}$.*

4.4 Unobservable Arrival of Applicants

Lastly, we consider a version of the model in which an applicant may not always be available. We assume that an applicant is available to fill the vacancy with probability ϕ, and no applicant is available to fill the vacancy with probability $1 - \phi$, with $\phi \in (0, 1)$. If both the principal and the agent observe whether an applicant is available, this version of the model simply boils down to the baseline model with flow payoffs premultiplied by the factor ϕ. If the agent privately observes whether an applicant is available, this version of the model is qualitatively different from the baseline model, as the principal cannot tell whether the agent has not hired an applicant because none was available or because the applicant's quality was not high enough.

For this version of the model, the second-stage problem is

$$\begin{aligned}
&\hat{J}(\hat{V}) \\
&= \max_{V_0, V_1} (1-\beta)\phi \int_R x dF(x) + \beta \left[(1 - \phi(1 - F(R)))\, J(V_0) + \phi(1 - F(R)) J(V_1) \right], \text{ s.t.} \\
&\quad \hat{V} = (1-\beta)\phi \int_R (x - \eta) dF(x) + \beta \left[(1 - \phi(1 - F(R)) V_0 + \phi(1 - F(R))) V_1 \right] \\
&\quad R = \eta - \frac{\beta}{1-\beta}(V_1 - V_0), \quad V_0,\, V_1 \in \mathcal{V}.
\end{aligned} \tag{4.40}$$

The problem above is easy to understand. In the current period, an applicant is available with probability ϕ. If the applicant's quality x is higher than R, the agent hires the applicant. In this case, the principal's flow payoff is x, the agent's flow payoff is $x - \eta$, the principal's continuation value is $J(V_1)$, and the agent's continuation value is V_1. If an applicant is not available or if an applicant is available and his quality x is lower than R, the agent does not hire. In this case, the principal's and the agent's flow payoffs are

0, the principal's continuation value is $J(V_0)$, and the agent's continuation value is V_0. Clearly, since the principal does not observe whether an applicant is or is not available, the agent's continuation payoff can only be contingent on whether an applicant is hired or not.

The first-stage problem is

$$\begin{aligned} J(V) = \max_{p,\hat{V}} \; & pJ_P + (1-p)\hat{J}(\hat{V}) \\ \text{s.t. } & V = pV_P + (1-p)\hat{V}, \\ & p \in [0,1],\ \hat{V} \in \hat{\mathcal{V}}, \end{aligned} \tag{4.41}$$

where J_p and V_P are respectively given by

$$J_P = 0, \quad V_P = 0. \tag{4.42}$$

The first-stage problem is the same as in the baseline. The "punishment" payoffs are different than in the baseline, and they are the values from not hiring any applicants. In fact, when the principal has control over hiring, the agent needs to report whether an applicant is available or not. Since the agent is better off not hiring an applicant than hiring an applicant irrespective of his quality, he would always report than no applicant is available.

Following the same argument as in Lemma 1, it is easy to show that the set $\mathcal{V}$ is given by the interval $[V_P, V_F]$, and the set $\hat{\mathcal{V}}$ is given by the interval $[V_\ell, V_F]$, where

$$V_\ell \equiv (1-\beta)V_F + \beta V_P, \quad V_F \equiv \phi \int_\eta (x-\eta)dF(x). \tag{4.43}$$

Following the same argument as in Lemma 2, it is straightforward to show that

$$J^* \equiv \max_{\hat{V} \in \hat{\mathcal{V}}} J(\hat{V}) > \phi \int_\eta x dF(x) \equiv J_F \tag{4.44}$$

if the quality distribution F and the bias η are such that

$$\frac{\phi \eta F'(\eta)}{1 - \phi(1-F(\eta))} > \frac{J_F - J_P}{V_F - V_P}. \tag{4.45}$$

Under condition (4.45), we can establish the following.

Proposition 11. *(Optimal Mechanism with Unobservable Arrival)*

1. *For all $V \in [V_P, V_C)$, the optimal lottery between control and delegation is such that the probability p of control is such that $p > 0$ and, conditional on delegation, the*

agent's value $\hat{V}$ is V_C. For all $V \in [V_C, V_F]$, $p = 0$ and $\hat{V} = V$. The critical value V_C is such that $\hat{J}'(V_C)$ equals $(\hat{J}(V_C) - J_P)/(V_C - V_P)$.

2. *For all $\hat{V} \in (V_C, V_F)$, the optimal agent's continuation values V_0 and V_1 are such that $V_0 < \hat{V} < V_1 < V_F$. For $\hat{V} = V_C$, the optimal continuation values are $V_0 = V_P$ and $V_1 > V_C$. For $\hat{V} = V_F$, the optimal continuation values are $V_0 = V_F$ and $V_1 = V_F$.*

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