Age and Infertility Revisited

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Results are preliminary and subject to change

Abstract

The consensus view in popular media, official guidance from medical societies, and a body of literature based on small-sample historical populations, is that monthly conception probabilities among noncontracepting married women—"fecundability" in the language of formal demography—is mostly unchanging in the twenties, begins to decline in the early thirties, and then rapidly declines around age 35. The focal acceleration at age 35 has well-documented impacts on individual decision-making, healthcare access, and insurance coverage determinations, including for pregnant women and women seeking fertility treatments. In this paper, we show that the consensus view is not, in fact, supported by the evidence that is typically taken to establish it. The conclusions that have propagated in this area are due to a misinterpretation in the way age-specific fecundability rates are typically estimated, creating a non-linear bias that introduces an artificial concavity in the age-fecundability relationship in the most widely-cited sources. We show, with a theoretical model, that even if the true relationship between fecundability and age were a constant, linear decline, the tools that have been used to estimate the age-fecundability profile would yield a concave, accelerating shape. We then apply this insight to new empirical analysis: We use a dataset of 2.8 million women, assembled from nationally representative samples of 62 countries, to generate new non-parametric estimates of fecundability decline with age. These show that there is no trend-break or acceleration in the mid-thirties, and further that fecundability decline is more rapid through the twenties than the consensus view.

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1 Introduction

How does fecundability—the monthly probability of conceiving a viable pregnancy, among those "at risk" of a new pregnancy—decline with age?[1] It is well-understood that female[2] fecundability declines from a high at young childbearing ages to zero at menopause. The medically, practically, and scientifically relevant question is *what shape* this decline in age takes. Current scientific understanding of the question is astonishingly incomplete.

The modern consensus on female fecundability decline relies primarily on a single 1986 study in *Science*—Menken et al.'s "Age and Infertility." Menken et al. estimate fecundity by studying fertility in several "natural fertility" populations living in times prior to modern birth control and in societies that plausibly made relatively little attempt to limit fertility, such as the "Geneva bourgeoisie, husbands born 1600-49" or "Tunis, marriages of Europeans 1840-59." In its most recent (2022) guidance, the American College of Obstetricians and Gynecologists (ACOG) cites this 1986 paper as its key evidence on the age pattern of fecundability decline, even pasting Figure 1 from Menken et al. (reprinted here as our Figure A1) as the sole visual aid in its committee opinion. ACOG summarizes the consensus view this way: "By age 30, fertility (the ability to get pregnant) starts to decline. This decline becomes more rapid once you reach your mid-30s."[3] The American Society for Reproductive Medicine, interpreting the same data, reports: "A woman's best reproductive years are in her 20s. Fertility gradually declines in the 30s, particularly after age 35." Modern studies of female fecundability published after the landmark 1986 paper, primarily relying on small samples of fertility clinic patients, have been neither designed nor powered to confirm or refute this consensus view of the age profile of fecundability decline, and so this single source has remained the core empirical evidence base that supports the consensus view.[4] In this paper, we show that this consensus view—enshrined in the official guidance of medical societies and widely reflected in popular media—is not supported by this evidence.

The focal age-35 threshold has proven to have important implications for individual decision-making and healthcare practice over the past several decades. ACOG credits the 1986 paper as one of the two factors that led to the definition of *advanced maternal age*—previously known as *geriatric pregnancy*—as greater than age 35 at due date (American College of Obstetricians and Gynecologists, 2022).[5] Today, mothers with an expected date of delivery just a few days after their 35th birthday face a sharp discontinuity in rates of fetal medicine specialist visits, detailed

[1]Except where otherwise noted, throughout we use *fecundability* as an abbreviation for *effective fecundability*—the monthly probability of conceiving a viable pregnancy in the at-risk population that ultimately results in a live birth. In other contexts, *fecundability* is used to denote the monthly probability of conceiving any pregnancy in the at-risk population, without conditioning on the pregnancy outcome.

[2]In this paper, we also contribute a novel, high-resolution characterization of the age-fecundability profile for men, but focus here in this motivation on women, because their fecundability is the at center of the prior literature.

[3]https://www.acog.org/womens-health/faqs/having-a-baby-after-age-35-how-aging-affects-fertility-and-pregnancy

[4]Other potential sources of data on the age pattern of fecundability decline include donor semen studies in which women of various ages are artificially inseminated in clinical settings. But these studies are under-powered to estimate the shape of fecundability decline with age—for example, just 2,200 women in one of the largest of the donor semen studies (Fédération et al., 1982).

[5]See https://www.acog.org/clinical/clinical-guidance/obstetric-care-consensus/articles/2022/08/pregnancy-at-age-35-years-or-older. The other factor is the crossover point for amniocentesis miscarriage risk and the occurrence rate of detectable genetic abnormalities. Per ACOG: "This age cutoff was selected based on *evidence of declining fertility* and concern surrounding increasing risks for genetic abnormalities identified in the offspring of pregnant women older than age 35 years." (our emphasis added)

ultrasounds, and antepartum surveillance, due to medical guidelines adopted by practitioners and insurance coverage determinations built around the advanced maternal age cutoff (Geiger et al., 2021). Because of this discontinuity in the level and intensity of care, the age cutoff has been shown to affect the probability of perinatal death (Geiger et al., 2021). For women not yet pregnant, fertility treatments also differ across this threshold, with women below 35 typically told to try for one year prior to starting treatments, and women 35 and over told to try for six months (American Society for Reproductive Medicine, 2012). And individual decision-making around career and family planning has been influenced by the consensus view that fecundability loss through the twenties is minor but that this loss rapidly accelerates by the mid- to late-thirties. This has been documented in anthropological studies of American women (e.g., Martin, 2021) and is widely evident in popular culture (e.g., appearing in headlines at WebMD, parents.com, reproductivefacts.org, etc.).

A first contribution of our study is to show that the consensus view is largely built around a statistical artifact that is not present in the underlying data-generating process (i.e., that is not present in the actual monthly conception probabilities that lead to the observed data). The bias arises from relying on *fertility* to proxy for the quantity of interest, *fecundability*. As we detail below, fertility rates do not measure fecundability. Fecundability—or more precisely, *effective fecundability*, though we use the shorter *fecundability* throughout—is, roughly, the occurrence-exposure rate of new pregnancies leading to births among women at-risk of pregnancy in some age range and population. Fertility is the occurrence-exposure rate of births among all women in some age range and population, without conditioning on being at-risk of pregnancy. Fertility offers poor alignment to the measurement of fecundability: Intuitively, when a woman is considering her chances of becoming pregnant at some age, she is considering a probability that conditions on, among other restrictions, not already being pregnant at that age. The denominator of an age-specific fertility rate *includes months when women are already pregnant* (and months before postpartum resumption of ovulation and menses), when it would be impossible to begin a new pregnancy for any woman, regardless of her fecundability. This conceptual point is well understood in formal demography (e.g., Bongaarts and Potter, 1983), but has not been applied to the practical problem of fecundability estimation from population-level data before now.

Our key conclusion is that even if the true relationship between fecundability and age were a constant, linear decline, the tools that have been used to estimate the age-fecundability profile would yield a concave, accelerating shape. We demonstrate this in a new analytical model that quantifies the downward bias introduced when using fertility to measure fecundability. This bias is greater when fecundability is higher, such as at younger ages, because more months spent pregnant or postpartum will be inappropriately included in the denominator. These results produce a closed-form expression for the bias that can be used to recover the underlying fecundability rate from fertility rate summary statistics like those reported in Menken et al. Applying our formula unbends the age-profile of fertility (the apparent acceleration in decline around the thirties) in Menken et al.'s reported results, showing that fertility rates yield dropoffs around age-35 *even when the age-profile of fecundability is linear.*

A second contribution of our study is to generate a precise estimate of the age fecundability profile in a modern, diverse sample. Setting aside any bias introduced when proxying for fecundability with fertility, the populations examined in Menken et al. were chosen with the goal of being settings where fertility-regulation behavior would be minimal; the result was samples that are small and unrepresentative, leaving several open questions. For one, the white, European-descended natural fertility populations studied in Menken et al. were living hundreds of years ago in most cases and were facing poorer disease environments, worse nutrition, and worse prenatal medical care than even people in low income countries today. For example, the data on Normandy marriages (1674–1742) in that study come from a time when French life expectancy was about 25 years. For context, life expectancy in the Central African Republic in 2020, which had the lowest life expectancy globally that year, was 54. So describing today's diverse population remains a gap in our scientific understanding. Further, these small-sample statistics were provided with no confidence intervals or other indications of statistical significance or uncertainty. This numerical imprecision is incommensurate with the weight these findings have carried in healthcare policy, healthcare practice, and individual decision-making.

In our empirical results, we both avoid the analytic problem of using fertility as a proxy for fecundability and overcome the data limitation that has hampered precise and representative estimation in prior studies. For estimation, we combine nationally representative survey data from Demographic and Health Surveys (DHS) from 62 low and middle income countries that span Africa, Eastern Europe, Latin America, and Asia, resulting in an ethnically and geographically diverse sample of over 2.8 million women. Because the combined sample of DHS surveys is large, it enables us to characterize fertility and fecundability at fine resolution, down to age in months (e.g., at 37 years and one month of age), significantly clarifying the age pattern of fecundability decline. The improvement in sample size—several million women here, and 230 million woman-months, compared to just a few hundred or thousand women in prior studies—is important because the question of interest concerns the second-derivative of the age-fecundability profile: fecundability declines in age, but does the decline accelerate or decelerate in age? Our sample enables us to generate precise, non-parametric estimates of the shape of the age-fecundability curve. In contrast, the existing literature's reliance on five-year bins for calculating group means (e.g., fecundability at 30–34) implies that the age-profile of fecundability decline (the slope between two adjacent points) is calculated from a 10-year age band (e.g., 35–39 minus 30–34). This has made it difficult, before now, to statistically detect whether the fecundability decline intensifies at, say, age 30 versus 35.

In order to separate fecundability from fertility, we exploit in-depth survey modules on family planning and fertility preferences. We identify the at-risk population that forms the denominator of the monthly fecundability rate by restricting to women who are non-contracepting, married, and not inhibited due to an existing pregnancy or postpartum breastfeeding and amenorrhea. Our restrictions based on contraceptive use at the individual level are analogous to Menken et al.'s restriction at the population level to examining groups prior to the introduction of modern contraception.[6] In some specifications, we additionally restrict on the basis of sexual activity and self reports of desiring more children. Our measure captures how fecundability declines with age given actual behavior

[6]Our fecundability proxy (like Menken et al.'s fecundity proxy and like the later medical literature reviewed in Section 5) does not capture any biological maximum fecundability—nor is that concept well-defined, as we describe in Section 3.1.

among women at risk (in the classic demographic science sense) of becoming pregnant.

Our procedure generates several new empirical findings that complement our main theoretical demonstration and that advance the science of human fecundability. First, we show that, in our large and representative sample, fecundability decline evolves smoothly through the thirties. There is no sudden drop-off in the probability of at-risk women conceiving a viable pregnancy at any age. Simple non-parametric plots by age show that the age profile of fecundability is only gradually changing over all childbearing years, with no steep acceleration in decline around age 30, 35, or 40. This is a major revision to the consensus view. Second, our estimates show that fecundability decline in women occurs sooner than the consensus guidance. It is well underway by the mid-twenties. This carries the implication that delay in the mid twenties or early thirties is more costly than under the consensus view in terms of reducing the probability of achieving a pregnancy after a month or year of trying. Third, our results allow us to improve on the prior literature by separating out the contribution of male infecundability—which is typically impossible to address using the type of demographic data collected for the construction of fertility tables.[7] We show that male fecundability declines monotonically with the male's age, but that this operates independently of the female age-fecundability profile, shifting it up or down, but not altering its shape (i.e., slope and concavity). Finally, we show that our findings robustly characterize the shape of the relationship between age and fecundability across ethnically, culturally, and economically diverse populations. The near-linear decline in fecundability from about 25 to 40 that is a core contribution of this paper is replicated across the global regions in our DHS dataset, as well as in a smaller representative US sample from an alternative data source.

We conclude in Section 5 by reconciling our findings with the consensus view. There we show, in our DHS data for which it is possible to calculate both fecundability and a fertility rate, that the formula we derive for the relationship between fertility and fecundability can successfully recover fecundability using only information on fertility and the length of temporary, postpartum infecundability. This validates our the formula we apply to recover fecundability probabilities from Menken et al.'s reported fertility rates. Thus we show, theoretically and empirically, that the structure of this relationship can account for the characteristic concave age-profile found in Menken et al.

The core contribution of this paper is to complement and improve on the existing, incomplete evidence on female fecundability decline with age, which has included the evidence on fecundity from historical populations existing prior to modern contraception (Menken et al., 1986); prospective studies of women or couples attempting pregnancy, for example through artificial insemination (van Noord-Zaadstra et al., 1991; Fédération et al., 1982; Rothman et al., 2013); and survey-based approaches that ask women directly about their difficulties conceiving (Mosher, 1985, 1988). Our methodological improvements emphasize the practical importance of distinguishing fertiliy and fecundability in future research. And our substantive findings on fecundability rates by age have the potential not only to improve the information currently provided to women, but also to significantly reshape healthcare payment policies of private insurers and clinical decision-making by physicians.

[7] In addition, our sample size allows us to create much more precise estimates of male fecundability by age than the medical literature.

2 Revisiting Fertility as a Proxy for Fecundability

To understand the consequences of Menken et al.'s use of fertility to learn about fecundity, which has been widely interpreted as a proxy for fecundability, we begin by reviewing the difference, at a conceptual level, between fecundability and fertility. We then link these concepts to the practical, empirical approaches to constructing the corresponding demographic rates. The difference between fecundability and fertility and the use of measured fertility rates to proxy for fecundability turns out to be critical in the decades-long understanding that has propagated throughout the literature on the shape of female age-related fecundability decline, with implications for infertility treatment, prenatal care, healthcare spending, individual decision-making, and infant survival.

2.1 Concepts

Fertility and fecundability aim to describe different things. Fertility describes the number of births per woman in some population over some time interval (typically normalized to or measured over one year). Informally, fertility asks how fast a population will grow, holding constant mortality rates. Fecundability describes the chances of conceiving a new pregnancy—sometimes, as effective fecundability, the chances of conceiving a new viable pregnancy—over some time interval (typically normalized to or measured over one month). Informally, fecundability asks what are the chances each cycle of becoming pregnant, among women who are at risk of a new pregnancy.

In some contexts not concerned with measurement, fecundability is meant to signify a biological capacity for conception, divorced from any behavioral or environmental influences that determine whether a conception occurs. Such a capacity is inherently imprecisely defined and unobservable.[8] Here we follow Sheps et al. (1973) in considering a fecundability as a context-dependent probability.

Fecundability is conceptually coherent only for women who are at risk of a new pregnancy. Bongaarts and Potter (1983) provides a useful summary, dividing a woman's months "while married and fecund" into three categories.[9] The first category, "waiting time to conception, also called the fecundable or ovulatory interval," (p. 5) is the time of interest for understanding the age profile of fecundability—it is the exposure period when a woman is "at risk," in the sense that she might or might not become pregnant. The second category, months during pregnancy, is not informative to include in a denominator or exposure for fecundability. Nor is the third category, "the postpartum infecundable interval," in which a woman may be temporarily infecund due to lactation-induced amenorrhea.[10]

[8] It is difficult to see how such a biological concept of fecundability could ever be operationalized as a social science variable or concept, absent behavioral and environmental influences, because humans choose their behaviors and live in environments. We note that no social scientific study of which we are aware—not Menken, et al., not ours—credibly claims to measure such context-free biological fecundability in humans.

[9] See Bongaarts and Potter (1983) (pages 4-5).

[10] Bongaarts and Potter (1983) summarize "Immediately after birth, a woman experiences an infecundable period during which the normal pattern of ovulation and menstruation is absent. The duration of this birth interval segment is primarily a function of breastfeeding behavior. (In a few societies, prolonged postpartum abstinence is practiced and the postpartum infecundable interval then exceeds the anovulatory interval to the extent that abstinence lasts beyond the resumption of ovulation.)" (p. 5). Relatedly, Sheps et al. (1973) explain that one of the four functions important as determinants of conception and birth is: "the time it takes a susceptible woman... to conceive. This is thought of as being a function of her fecundability, or the probability of conception per unit of time."

No woman in her fifth month of pregnancy is wondering about her chances of conception in that month. She knows it to be zero—a biological impossibility. Fecundability, as a probability measure *calculated among women at risk of pregnancy*, is not defined for a woman in her fifth month of pregnancy. Nor is it defined for women who are temporarily unable to conceive following a recent birth before ovulation resumes (postpartum amenorrhea). We follow Bongaarts and Potter (1983) in constructing our measure of fecundability, next.

2.2 Measurement

Measures of both fecundability and fertility are occurrence-exposure rates in formal demography: The numerator counts the occurrences of some event over some period—here, a pregnancy resulting in a live birth—and the denominator tallies the number of person-years of exposure to the risk of that event within some population of interest over the same period.

As occurrence-exposure rates, both fertility and fecundability are calculated as a group property rather than an individual characteristic, for instance describing the count of births (fertility) or the probability (as frequency) of conceptions (fecundability) for some sub-population defined by place, time, and/or cultural group—and almost always defined by female age.

A fertility rate, F_{gt} for some demographic group g (such as married 20-24 year-olds), over some sample period t, is calculated as:

$$F_{gt} = \sum \frac{\text{Births}_{gt}}{\text{Woman-Months}_{gt}}, \tag{1}$$

where we have normalized by woman-months rather than the standard woman-years in order to more easily compare with fecundability. A fecundability probability, f_{gt}, for the same group of married 20-24 year-old women, is calculated as:

$$f_{gt} = \sum \frac{\text{Conceptions}_{gt}}{\text{At-Risk Woman-Months}_{gt}}. \tag{2}$$

An age-specific fertility rate includes all woman-months lived by the group g during period t in its denominator. For example, consider a woman who, over a two-year sample period, experienced a 9-month pregnancy followed by 10 months of lactation-induced amenorrhea. Her contribution to the fertility denominator would be 24 woman-months. The fecundability denominator includes less. The same woman would contribute only 5 at-risk woman-months to the fecundability denominator. That is because woman-months during which a pregnancy is ongoing or while ovulation has not resumed cannot in principle provide any information on fecundability. For the purposes of the simplified model that follows, we focus on effective fecundability (which is more readily observable); this replaces the summand in the numerator of Eq. (2) with conceptions leading to live births.

The logical step that Menken et al. use to justify moving from observing fertility to learning about fecundity is that in the populations studied, women had neither the technology nor the goal of restricting their fertility, so that fertility would be maximal. From Menken et al.: "One indicator of decline in fecundity is the way birth rates among married women change with age in populations in which little or no family limitation is practiced, such as those identified by Henry." However, using these estimates to learn about fecundability neglects this difference in denominators: Fertility's all inclusive population includes women-months who fecundability's narrower, at-risk population, omits, such as pregnant woman-months. In this sense the denominator of an age specific fertility rate includes too much as a measure or proxy of fecundability; it includes months in which there is no actual exposure to the risk of pregnancy.[11]

Of course, texts in formal demography recognize this fact of denominator inclusion (when calculating rates as means) or sample construction (when calculating rates as regression coefficients). But these insights have never been applied to understand and interpret Menken et al.'s data—or to recover *fecundability* probabilities from data on *fertility* rates, which we do below.

One implication of fertility's too-inclusive denominator as a proxy for fecundability is that the levels of fecundability it implies are implausibly low. For example, the single most fertile subpopulation of any in the Menken et al. data is the group of 20-24 year old Hutterites of 1921–30, with a fertility rate of 545 births per 1,000 woman-years. Attempting to interpret that *fertility rate* as *fecundability* would yield a monthly probability of conception of only 4.5% ($= 545/(12 \times 1,000)$). This is only a fraction of external estimates of monthly conception probabilities in the literature. This implausibility about quantitative levels might, in itself, be easily ignored: The goal of Menken et al. was to document the age-profile of fecundity—the shape—rather than to quantify the monthly probabilities. Still, this implausibility is informative about fertiltiy as a proxy for fecundability. Moreover, using the age-profile of fertility as a proxy for the age-profile of fecundability is even more problematic. We show below that the age-shape of fertility is guaranteed to not match the age-shape of fecundability.

As we show next, fertility rates will always be non-linearly biased if misinterpreted as proxy measures of fecundability, *even if fertility is maximally desired and uncontrolled.* Such non-linearity is critical, because in practice it will be correlated with age. In the next subsection, we derive a closed-form analytic characterization of the bias that is introduced by the substitution of a fertility rate for a fecundability rate. It is non-linear and scales with the square of the true underlying fecundability, which will be higher at younger ages.

2.3 Bias: A Wedge Between Fertility and Fecundability

A key contribution of our paper is to illustrate that, although age-specific fertility rates are well-defined demographic quantities, these rates offer poor alignment to questions of fecundability. A simple analytical model can describe this bias—the difference between fecundability and fertility for the same population—and can be used to recover the quantity of interest from summary statistics like those reported in Menken et al. (1986).

To build intuitions before deriving the model, consider a highly stylized example: Begin with a population of women who are maximally fecundable. Call it Population A. Population A women

[11] It is perhaps even clearer to see the reverse: why measures of fecundability would not be good proxies for fertility: Women will not actually get pregnant at their fecundability rate each month. One reason, which Menken et al.'s use of high-fertility populations is intended to address, is that many women choose to limit their fertility. But another reason is the same as we have been emphasizing: A pregnant woman will not become pregnant, no matter how high her fecundability would be before or after her pregnancy.

would succeed in becoming pregnant on every cycle during which they tried, as long as they were not currently pregnant or experiencing postpartum amenorrhea. Assume women in this population try (and succeed) at a new pregnancy in the first month their ovulation resumes following a birth. Solely for mathematical simplicity, assume these women have 24 months of infecundability following each conception: 9 months of pregnancy followed by breastfeeding-induced infecundability lasting for 15 months. Thus, every 24 month period includes one birth per woman. The standard calculation of an age-specific fertility rate would tabulate that 1 out of 2 women each year would give birth, and so would calculate the annual fertility rate to be 50% in this population. In contrast, the fecundability rate is 100% by construction: At every month at-risk of a pregnancy, a Population A woman immediately becomes pregnant.

Now consider Population B, which differs in the following way: fecundability, as monthly rate, is only 25%, so that in expectation, it will take 4 months (4 times as long as in Population A) to become pregnant once a woman in this group begins trying. In Population B, every 27-month period includes one birth per woman in expectation, so the annual fertility rate would be 44% (1 birth per 2.25 years). Comparing the populations, the underlying fecundability would differ by a factor of four and by 75 percentage points, but the measured fertility difference would be an order of magnitude smaller: 6 percentage points. That is because the fertility calculation is including in its denominator many non-exposed months. The difference between fertility and fecundability is highest when birth rates are highest—like in our extreme example here (because there are more non-exposed months due to current pregnancies). And this connects to our question because birth rates are higher at younger ages than at older ages.

To derive a general expression for this bias—the wedge between measured fertility and the object of interest, fecundability—let f be fecundability, the monthly probability that a viable pregnancy begins that will ultimately resolve in a live birth,[12] among women not temporarily unable to become pregnant due to a pregnancy in progress or a recent birth.[13] This follows the treatment in Bongaarts and Potter (1983). For simplicity and tractability, we ignore terminations in this treatment, though see the Appendix for a version that includes a term for the probability of termination.

Write i ("interval") for the number of months spent in pregnancy and postpartum infecundability. This is the number of months following a conception that pass before a women returns to being exposed to the possibility of a new conception (that is, before she returns to "waiting time to conception"). For example, if a pregnancy results in nine months of gestation and half a year of postpartum infecundability, then $i = 9 + 6 = 15$. We are assuming for simplicity in generating proofs and intuitions that i comprises a fixed time interval, rather than a random variable. (And we show below that this deterministic model fits the aggregate data almost perfectly.) Finally, let $e(f, i)$ ("exposure") describe the probability than any randomly-selected month will be one in which

[12] One could alternatively define fecundability as the probability that a pregnancy begins without the further restriction that the pregnancy is viable and carried to a live birth, though this will offer poor alignment to data sources on fertility and natality, which are known to suffer from under-reporting of abortions and miscarriages, even when information on these is solicited.

[13] For the purposes of the analytic model, we ignore infecundability in the several months following a pregnancy termination, though we incorporate this in the empirical analysis below. Ignoring terminations turns out to have a negligible impact empirically (see Figure A7). Ignoring this in the stylized model allows us to generate a simple closed-form expression for the bias in terms of the primitives that are quantitatively most important.

a woman with fecundability f and interval i will be at risk of a new conception (because she is not currently pregnant or in the post-partum temporary infecundable period). The probability of exposure, e, is an intermediate variable that will ultimately be an equilibrium outcome that is a function of the primitives i and and f.

Denote fertility, here the unconditional *monthly* probability of conception, with F. This differs from the definition above in Section 2.2 (Eq. 1) in that we build the model around the timing of the conception event that eventually leads to the birth, rather than the timing of the birth itself. F is weakly smaller than f because its denominator is a weak superset of woman-months, including for example woman-months in which a new pregnancy is biologically impossible due to an ongoing pregnancy. The denominators are equal only when $F = f = 0$.

Given these definitions, the bias between fecundability (f) and fertility ($F = fe$) can be expressed as $f - fe$, where we have used the fact that the unconditional probability of pregnancy leading to birth (fertility) is just the conditional probability of pregnancy among women at risk of a pregnancy (fecundability) times the probability of being at risk of pregnancy (exposure, e). In Appendix B, we prove that exposure, e, is completely pinned down by fecundability and the interval in the following way:

$$e(f,i) = \frac{\frac{1}{f}}{i + \frac{1}{f}}. \tag{3}$$

Intuitively, $\frac{1}{f}$ (one over the monthly conception probability) is the number of months that will pass in expectation before a women exposed to the risk of pregnancy will become pregnant. So Equation 3 reveals that e, the probability that any particular month is exposed to the risk of pregnancy, is the ratio of the expected number of months spent "waiting" for a new pregnancy (in the terminology of Bongaarts and Potter, 1983) to the total months that would occur in expectation between conceptions (equal to these waiting months, plus the months spent pregnant and in postpartum infecundability, which is i).[14]

This leads to the following expression for bias, as a function of the primitives fecundability and interval:

$$\text{Bias} = f - F = \frac{f^2 i}{fi + 1}. \tag{4}$$

This bias term is increasing in both interval and fecundability, and in particular is a nonlinear increasing function of f. The bias will be large at young ages, when f is large. It will converge to 0 at old ages, when f is small. We can recover f, given F and an assumption about the interval, i, by solving Equation 4 for f:

$$f = \frac{F}{1 - Fi}. \tag{5}$$

[14]Evaluating the denominator: Each woman goes through a sequence in which she becomes pregnant and is thus non-exposed for x months, and then is exposed for a period of time that is equal in expectation to $\frac{1}{f}$ months (because she has a probability of f of becoming pregnant in each month); then the sequence begins again. Over the same period, the numerator contains the expected number of months in which a woman is exposed to the possibility of pregnancy (prior to achieving it), which is $\frac{1}{f}$. So the fraction, on average, of each cycle that a woman is exposed to pregnancy risk is $\frac{\frac{1}{f}}{x+\frac{1}{f}}$.

To illustrate the type of distortion this bias can cause if fertility is misinterpreted as fecundability, in Figure 1 we trace out Equation 5, plotting the hypothetical observed fertility rate (vertical axis) against the corresponding fecundability rate (horizontal axis). The range of fecundability rates included along the horizontal axis includes the range we observe in the DHS data in Figure 3. The alternative iso-interval lines show the extent to which the bias scales as the postpartum infecund period goes from 6 months ($i = 15$) to 12 months ($i = 21$). And the 45-degree line plots, for reference, what an unbiased measure of fecundability would return.

It is clear from Figure 1 that using fertility rates as a measure of fecundability introduces a nonlinear bias that is a function of the level of the estimand of interest. Even if the true age-fecundability profile were a linear decline with age, fertility would exhibit a concave, accelerating relationship with age.

2.4 Recovering fecundability from Menken et al.'s fertility rates

The bias expression allows us, with minimal assumptions, to recover fecundability rates from reported age-specific fertility rates, even when the micro-data generating those rates is unavailable. So it is possible to use the formula in (5) to recover fecundability from Menken et al.'s estimates of fertility.

Menken et al. (1986), drawing on data from Henry (1961)—which itself is an aggregation of other reports (e.g., Mashayekhi et al., 1953; Eaton and Mayer, 1953)—report age-specific marital *fertility rates*.[15] The numerator of these rates is births within some age group, typically a five-year bin such 30-34 years of age at last birthday. The denominator is all women-years lived by married women within that age range over that period. Menken et al. report these marital fertility rates in 5-year age groups for each of ten populations.

Figure 2 replots Menken et al.'s data, by consulting the sources for Menken et al.'s Figure 1.[16] To reduce visual clutter, we collapse the age-specific fertility rates from the ten natural fertility populations to a single set of rates by taking the simple mean within each age range. (For completeness, Figure A10 replicates Figure 2 separately for each of these populations.) To create a comparable scale with Panel (a), we divide the "rates per 1,000 women" reported in Henry (1961) by 12,000 to get fertility rates by woman-month. Fertility rates in the figure show the familiar shape of accelerating decline, particularly in the mid-thirties—though it is worth noting that contrary to the way this figure has been interpreted, the acceleration in decline (concavity) is not isolated to, or especially sharp at, 35 in particular.

The *Implied fecundability* plot in the figure takes this reported fertility and de-biases it according to the formula in Equation 5. The free parameter is i. Panel (a) assumes $i = 21$, which corresponds to a 12-month infecundable period postpartum. Below, when we draw on modern, global data on self-reported mean for postpartum amenorrhea and breastfeeding calculated in the DHS sample, we estimate i to be 11.8 months. Panel (b) makes a more conservative assumption about the postpartum infecundability period. It sets $i = 15$, which corresponds to a 6-month infecundable period postpartum, which could happen, for example, if the average woman in Menken et al.'s historical

[15] Menken et al. (1986) take as the source data for their exercise entries in Table 1 of (Henry, 1961).

[16] The original version of Figure 1 in Menken et al. is copied as our Figure A1 for reference.

samples was exclusively breastfeeding for only 4 months and if her ovulation resumed two months after this four-month period ended.

Removing the bias inherent in fertility as a proxy for fecundability changes the shape of the age-fecundability profile. The extreme concavity of the original plot is gone, both in the top panel ($i = 21$), which is most consistent with the available evidence on the length of post-partum temporary infecundability, and in the bottom panel ($i = 15$), which its conservative assumption on i.

The bias correction shows that the consensus view on the age-profile of female fecundability decline is based on misunderstanding fertility to usefully proxy for fecundability. The structure of this bias can account for the concave profile found in Menken et al. (1986), even if the underlying data generating process of fecundability is a near-linear, non-accelerating decline in fecundability throughout the late twenties, thirties, and forties—as in Panel (a) of Figure 2. The widely-cited concave shape with virtually no decline in fecundability through the 20s and a steep dropoff at 35 or 37 is a statistical artifact: Even with conservative assumptions in Panel (b), the concavity is reduced and it is clear that there is significant decline in fecundability through the 20s.

In summary, the characteristic shape of the widely-cited result in Menken et al. does not offer evidence that fecundability is flat in age for younger women and then accelerates in age—even if fertility indeed shows such a concave relationship with age. We have shown that such an accelerating relationship between fertility and age is not merely *consistent* with a linear relationship between fecundability and age: An accelerating relationship between fertility and age *is precisely the shape* that a linear relationship between fecundability and age would generate. And when we use our simple model to invert the mapping from fecundability to fertility, we find that Menken et al.'s data suggest a linear relationship between fecundability and age, if postpartum temporary infecundity lasted about 12 months. Of course, the actual age-profile of fecundability that prevails in today's modern and globally diverse population is a further empirical question. Answering this question would require a large dataset to have the statistical power to estimate a second derivative in age with informative precision.[17] The rest of our paper explores the extent to which large, contemporary demographic survey datasets can inform this further empirical question.

3 New Data and Methods

3.1 Data: Demographic and Health Surveys

To construct new estimates of age-specific fecundability (and fertility), we assemble a global dataset of 230 million women-month observations. This dataset combines Demographic and Health Survey (DHS) data from 62 countries. The DHS are nationally and regionally representative household surveys, with each country-wave interviewing thousands or tens of thousands of respondents. The focus of these surveys is to collect information on female respondents of reproductive age, between 15 and 49 years old in most cases. For the reproductive module, each female respondent is asked to report her full reproductive history, so that dates of birth for each child are reported. In addition, many waves of the DHS collect a "contraceptive calendar," in which information is collected on

[17] Menken et al. do not report any statistical inference on any estimates of this second derivative of interest.

whether the respondent was pregnant, gave birth, experienced a terminated pregnancy, or was using contraception (and if so, what type) for each month of a five-year look-back period.[18,19] A typical contraceptive calendar may involve starting and discontinuing contraception, as well as changing methods. Appendix Figure A2 shows the survey instrument for the contraceptive calendar. These surveys have been conducted in dozens of low and middle income countries for decades, using consistent questionnaires to produce comparable data across contexts.

To construct our estimation sample, we begin with all available country-waves of the DHS in which this contraceptive calendar is included. This set includes 62 countries and 144 survey waves across Africa, East Asia/Pacific, Eastern Europe, Central Asia, Latin America, and South Asia. A list of included countries and survey waves (dates) is provided in Table A1.

Fecundability Sample. We define two estimation samples. The first is the *fecundability sample*, which constructs a denominator in line with the Sheps et al. (1973) definition. This restricts to women-months of married women for which the woman reports (for that month): not contracepting,[20] not currently pregnant, not in a period of postpartum amenorrhea, and not recovering from a termination. This sample conditions on marriage during the month of observation for comparability to prior studies. In our primary specification, we assume a 12-month period of postpartum infecundability for all women. In an alternative specification that yields nearly identical results, we instead exclude from the sample (the implicit denominator) all months when the respondent woman specifically self-reported postpartum amenorrhea or breastfeeding. This alternative definition of the postpartum infecundability period varies at the person-level. To examine sensitivity to assumptions about (and misreporting of) the timing of postpartum amenorrhea, we also generate alternative estimation samples in which some fixed number of postpartum months $\in \{6, 9, 12, 15\}$ are excluded. of beginning a new pregnancy.

We also exclude from the denominator the month in which a pregnancy termination occurred and the two months immediately following. We do so because an stillbirth, induced abortion, or miscarriage may result in several months of abnormal menses, though this latter adjustment makes little difference in the empirical results.

Of the 230 million women months in our sample, we categorize 40.3 million months as being "at risk" of beginning a new pregnancy. The implicit denominator we construct through this sample definition will, of course, include women-month observations in which the respondent women are not actively trying to become pregnant despite being married and having discontinued contraception. Our fecundability measure (like Menken et al.'s fecundity measure and like donor semen studies) does not capture a biological maximum fecundability—nor is that concept well-defined. We cannot, for example, restrict attention to only women who were optimally timing sexual intercourse to their most fertile day (or hour). Nor can we restrict to only women measuring changes in their

[18] Terminations encompass all pregnancies that do not end in a live birth. In most cases, abortions, miscarriages, and stillbirths are not separable in the DHS.

[19] The contraceptive calendar typically contains information for the months leading up to the survey in the calendar year of survey, plus the five prior calendar years.

[20] Contraception takes a value of one for any form contraception, including IUD, injectibles, implants, the pill, condoms, female condoms, emergency contraception, rhythm method, and other traditional methods. We measure contraceptive use in the month prior to the month of observation, so that cases of contraceptive failure do not enter our sample.

body temperature, eating an optimal diet, partnering with a maximally fertile male partner (or, for example, using sorted sperm), or experiencing the most conducive environment in terms of ambient temperature and other exposures. But our proxy measures how fecundability declines with age given actual behavior among women at risk of pregnancy. We discuss this issue in more depth in Section 4.4. There we probe our results with additional data on intercourse frequency and other variables available in the DHS—for example, further restricting attention to women who report recent sexual activity and a desire for more children.

Fertility Sample. The second estimation sample, the *fertility sample*, is meant to parallel, in our individual, person-month-level dataset, the restriction implicit in Menken et al.'s examination of historical populations existing prior to the availability of modern contraception. Measuring age-specific fertility has less strict data requirements than measuring age-specific fecundability: it does not require detailed information on contraception use, postpartum lactation-induced amenorrhea and other individual-by-month specific data. To create the fertility sample, we keep the restriction of non-contraception and marriage from the fecundability sample, but drop all other restrictions. So pregnant woman-months, in particular, enter the denominator of the fertility sample.

For a series of robustness checks, we create the fecundability and fertility samples in a separate US dataset, the National Survey of Family Growth (NSFG). The NSFG is a survey of US respondents with a similar contraceptive calendar methodology to the DHS. The NSFG, like the DHS, gathers information on pregnancy, births, and contraception via a monthly look-back calendar that extends to the 4 years prior to the time of interview. Also like the DHS, the NSFG is nationally representative. We use the 2006–2010 and 2011–2013 survey waves, which are the two most recent waves for which the respondent's month of birth is available.[21] The NSFG is the largest US dataset containing information sufficient to construct our measure of fecundability, but its sample size is too small for precise non-parametric estimation at the age-in-months level, so we collapse certain results to quarter-year-of-age bins. We discuss additional details of the NSFG in Appendix A.

3.2 Estimation

For each age-in-months (for example, 33 years and 2 months old), we compute the fraction of women who become pregnant in that month with a pregnancy that results in a live birth. Using the *fecundability sample* described above (married women who are exposed in that month to the possibility of becoming pregnant), we estimate flexible regressions describing the relationship of fecundability and age:

$$B_{it} = \sum_a \beta^a \cdot I^a_{it} + \epsilon_{it}. \tag{6}$$

Observations in the regression are woman-months. Subscripts i and t index women and calendar month × years, respectively. I^a is an indicator for age in months equal to a. Typically we estimate effects for ages 25 to 49 ($a \in (300, 599)$). The outcome variable, B_{it}, is an indicator for the respondent

[21]For the waves since 2011–2013, it is possible to access a restricted version with birth month data through a Census RDC.

experiencing the first month of a new pregnancy that eventually ended in a live birth.[22]

Our independent variable of interest is a woman's age in months. This is the age resolution that would be ideal in principle for a study of fertility or fecundability (because ovulation is on a roughly monthly cycle), which is why the DHS was designed to collect data this way.

There is complete correspondence between the regression described in Equation (6) and the rate calculation described in Equation (2). Inclusion in the denominator of the rate is synonymous with inclusion in the estimation sample in the regression. The numerator in the rate is simply the summed the binary dependent variable from the regression. The regression in Equation (6) is merely a convenient way to estimate the large set of means and to normalize the level (by choosing the omitted age group) in order to more easily visually compare slopes in different sub-populations. Because our dataset is large, we can primarily rely on this saturated non-parametric approach to estimation, rather than forcing estimands to fit a parameterized curve.

We additionally fit flexible local linear regressions to the same outcomes, plotting the smoothed regressions overlaid with the raw means. These take the form of a first-degree polynomial:

$$B_{it} = \alpha + \gamma\prime(age_{it} - age^0) + \mu_{it}, \tag{7}$$

defined over subsamples $\|age_{it} - age^0\| \leq h$ for bandwidth h, weighting with an Epanechnikov kernel.[23]

Although our estimates of fecundability rates by age necessarily reveal levels (monthly conception probabilities), our research aim is to evaluate changes in the *slope* of the age-fecundability relationship. Not levels, or even slopes, but these *changes in slopes*, have been basis for the consensus view in population science and for medical society committee opinions and healthcare policy decisions. It would be surprising if the regressions described in Equations 6 and 7 found identical coefficients in each of the 62 countries and decades-wide span of our dataset. For example, there is no reason to expect the frequency of sexual intercourse to be identical across contexts. Further within- and across-country differences in female nutrition (such as revealed by BMI and underweight status), are well documented, including in the same DHS data we use here. This poses no necessary problem for our study. The assumptions needed to identify slope changes can be satisfied even in the presence of such level differences. The conclusions drawn from Menken et al. about a slope change in the mid-30s correctly ignored the large differences in levels of fertility across the populations they examined.[24] We do the same, though in Section 4.4, we examine the sensitivity of our conclusions to missing information about the frequency of intercourse in our main estimates by examining subsamples in which intercourse is reported.

[22] We eliminate the last 9 months of each woman's look-back period before the time of interview. This is because some women are pregnant at the time of interview, and we do not observe whether the pregnancy ended in live birth.

[23] Unless otherwise noted, the local linear regressions use a bandwidth h of 15 months.

[24] For example, 406 births per 1,000 woman aged 35 to 39 among the Hutterites (1921-30), compared to 287 births per 1,000 woman aged 35 to 39 among the Geneva bourgeoisie (husbands born 1600-49) in their data.

4 Preliminary Empirical results

4.1 Main empirical result: fecundability declines in age without acceleration

Figure 3 plots the relationship between age and fecundability, our main empirical result. It uses our fecundability sample. Each dot is the sample mean of an indicator for becoming pregnant in that month of age (e.g., 33 years and 7 months) with a pregnancy that ends in a live birth. The means represent the finest age-resolution available in our monthly data, and correspond to the narrowest conceptually relevant interval for considering pregnancy risk. These means are directly interpretable as the monthly probability of beginning a successful pregnancy. The line within each panel is a local linear regression, corresponding to Equation 7.

For our main specification, we focus on the sample of women ages 25-49, or a equal to 300 through 599 months. We make this restriction because our methodology for identifying the age-fecundability slope, which relies on the contraceptive calendar, will be most accurate when there is a stable relationship between contraception and pregnancies across ages, and when most pregnancies ending in a live birth are arising among noncontracepting women seeking to become pregnant—rather than, for example, unintentional pregnancies arising from inconsistent contraception use. We show in Figure A3 that this holds between ages 25 and 40. In contrast, between ages 15 and 20, especially, a large and changing share of pregnancies accrue to non-married women or women who report using contraception.[25] Nonetheless, Figure A4 shows that extending the window further back to age 20 has exactly the same implication as each panel in Figure 3: There is a near-linear decline throughout the twenties and thirties.

The four panels in Figure 3 use variations on sample definitions for isolating women exposed to becoming pregnant. Before detailing each in turn, we note what these panels have in common: their shape. Across all panels, Figure 3 shows a near-linear decline in fecundability: At each month of age throughout the late twenties, thirties, and forties, the probability of conceiving among noncontracepting women declines roughly linearly. This is in contrast with the concavity that is the core result of Menken et al., and that is reflected in the academic and medical consensus.

A second important difference between the patterns in Figure 3 and the consensus view is that we show the decline to be steep in the twenties. This relatively fast decline through the twenties is a significant departure from the scientific claims explicitly adopted by American College of Obstetricians and Gynecologists and the American Society for Reproductive Medicine, which sometimes deny *any* decline in fecundability prior to the early thirties.[26,27] Women guided by such information would incorrectly perceive little cost to delaying their family planning between age 20 and 30. The

[25] For these types of pregnancies, the DHS data are not ideal because the structure of the data makes it impossible to distinguish whether a pregnancy at, say, age 18 was due to a conception in spite of using contraception, or a choice to not use contraception in a particular instance of intercourse.

[26] See, e.g., the 2014 Committee Opinion (reaffirmed 2020) by the American College of Obstetricians and Gynecologists Committee on Gynecologic Practice and The Practice Committee of the American Society for Reproductive Medicine: "The fecundity of women decreases gradually but significantly beginning approximately at age 32 years and decreases more rapidly after age 37 years."

[27] With the caveats noted above, Figure A4 extends further back to age 15 and shows that the steep decline apparent in Figure 3 is present throughout the early twenties as well.

estimated loss in the absolute value of monthly conception probabilities is high between 20 and 30 in these data.

At the oldest ages within the fecund span, Figure 3 shows that the decline *decelerates* as the level of fecundability approaches zero. By the late forties, the slope becomes visibly more shallow. This is the only logically coherent possibility if there is a distribution of underlying fecundability with some long-fecund types, but it is worth noting that Menken et al. (reproduced in Figure A1) do not find this. In their figure, the entire graph is concave: the slope turns decisively to a larger negative value over the 30s (though where in the 30s is hard to pinpoint because of the 5-year age bins and because of inconsistencies across the populations they study), and never inflects again.

These results hold across each of the panels in Figure 3. The panels are differentiated according to the fine details of how fecundabiltity is operationalized in our data. All panels include only women-months in which a woman is married, is not pregnant, and is not using contraception, according to the survey's detailed contraceptive calendar. The panels differ on how exactly they exclude women in months when they are not exposed to becoming pregnant because they are postpartum: Panel (a) is our preferred specification. It excludes women for 12 months after giving birth and for 3 months after a pregnancy termination. Panel (b) retains Panel (a)'s exclusion of 3 months after a termination but does not automatically exclude all women within 12 months of giving birth. Instead, Panel (b) excludes months when the woman specifically reported postpartum amenorrhea or breastfeeding. Thus, in Panel (b), the postpartum infecundability period varies at the person-level and can in principle be as little as zero months. In practice, the interior 90 percentiles of this distribution span from 1 month to 24 months. The median and mean are 10 and 11.8 months after giving birth, respectively.[28] The remaining panels alter the assumptions on postpartum infecundability, while retaining the post-termination exclusion of 3 months: Panel (c) excludes women for 15 months after giving birth. Panel (d) excludes women only for the first 6 months after giving birth. Additional variations in Appendix Figure A7 show that the exclusion of the post-termination period, while conceptually consistent with the exclusion of the postpartum period, has no practical effects on the estimates.

In summary, none of the plausible alternative strategies used to isolate women exposed to becoming pregnant change the characteristic shape (a near-linear decline through the late twenties and thirties), so our conclusion does not hinge on the particular choices made to operationalize postpartum infecundability. To be clear: The different panels of Figure 3 do estimate slightly different *levels* of fecundability. This is unsurprising because the different panels include different sets of woman-months. As discussed previously, we would expect this quantity to depend upon, for example, the nutrition, health, and behavior of different populations in different geographies and times. Our claim is that the *shape* of the relationship between fecundability and age, which has long been the object of interest in the medical and demographic literature, is robust to alternative empirical strategies, which is what the panels of Figure 3 show.

Figure 3 shows there is no visually obvious trend break or inflection point in age-fecundability profile between 25 and 40. To formalize the this finding, in Figure A6 we test for any discontinuity

[28]For some women, amenorrhea and breastfeeding information is missing after a birth. We remove these women from the sample for 12 months after birth.

in the age-fecundability relationship using two alternative methods. First, in Panel (a) of Figure A6, we estimate a series of linear regressions of birth rates on the respondent's age in months, within a bandwidth of 12 months on either side of the month of interest. We then plot the estimated coefficient on the respondent's age in months, which gives an estimate of the localized rate of decline in the monthly probability of beginning a successful pregnancy. In other words, we calculate and display estimates of the slope of the line from Figure 3 at each age-in-months. As this graph makes clear, the rate of decline in fecundability is constant or decreasing in absolute value from 25 to 35, and then clearly becomes *smaller* in the late thirties and early forties. Thus the decline is slower after the mid-thirties in these data, rather than faster as suggested by Menken et al., insofar as it differs in age at all. Second, in Panel (b), we directly investigate whether each age in months marks a discontinuity in the slope of the age-fecundability relationship. To do so, we run a series of regressions of monthly birth rates on the interaction between age in months and an indicator for age in months being greater than the threshold age of interest. Specifically, for every age in months between 300 (25 years) and 600 (50 years), we investigate whether the slope in the two years of age after that month is different from the slope in the two years of age before that month. The results show no evidence of a major discontinuity anywhere in the range of 25-44.

4.2 Male Infecundability

Another question of interest to families planning their reproductive lives is the impact of the male's age on the probability of a successful pregnancy. Prior work by Mineau and Trussell (1982) finds that a couple's fecundability is dictated much more by the woman's age than the man's age, and Menken et al., follow the "demographic convention of attributing their joint reproductive status to the woman."[29]

This convention has been somewhat dictated by data availability in the fertility tables collected for natural fertility populations from which this literature has drawn. Our sample, data, and statistical power allow us to further interrogate the role of male age. The DHS contains the age in years at interview for each respondent's husband or partner, allowing us to examine whether the patterns of fecundability decline by female age is significantly influenced by her partner's age. In Figure 4a, we estimate the female age-fecundability profile separately for five-year male age groups.[30] The separate plots by male age show that a couple's fecundability declines monotonically with the male's age. For example, the monthly probability of a conception ending in live birth for a woman aged 27 falls by about 20% when the husband is age 45-49, rather than 25-29. Nonetheless, the shape (i.e., slope and near-linearity) of the decline in fecundability by female age is remarkably similar, regardless of the age of the husband. Most importantly for the question of this study: There is no combination of male and female ages at which a couple should expect a sharp drop off in the

[29] A few papers in the medical literature, which we discuss further in Section 5, also compare male and female fecundability decline with age, usually attributing declines in fecundability to a greater extent to women. Dunson et al. (2004) use a sample of 782 couples recruited from fertility clinics in Europe, finding that male age is not a significant predictor of a couple's infertility until the late thirties. Rothman et al. (2013) find that male fecundability falls between age 35-40, but to a lesser extent than female fecundability.

[30] For each five-year male age group, we estimate the female age-fecundability relationship for women above the 5th percentile and below the 95th percentile of women with a husband in that age range.

probability of a successful pregnancy.

Given the apparent independence of the male and female age effects in panel (a), panel (b) estimates the age-profile of fecundability in males and females, simultaneously controlling for each, with non-interacted sex-specific coefficients at each age.[31] It shows that the age-profile of fecundability decline is steeper for females than for males, but in that specification as well, the decline for both sexes remains linear.

4.3 Similarity Across Heterogeneous Populations

Historical studies of natural fertility populations like Henry (1961) and Menken et al. (1986) have almost entirely focused on population of European descent, without any claim to be representative of even some particular European population. To give an example, one of the data sets in the landmark Menken et al. (1986) study originates from the genealogies of a mere *19 families* of the Geneva nobility.[32] The same white-European centering is true for studies of fecundability in modern populations using data from fertility clinics, primarily in Europe (e.g., van Noord-Zaadstra et al., 1991; Fédération et al., 1982). In contrast, our DHS sample is an ethnically diverse aggregation of nationally representative samples of countries spanning the globe.

In panel (a) of Figure 5, we estimate the age-fecundability relationship separately for different global geographies. We break the sample into three world regions: Africa, Americas, and Asia.[33] For this figure, we display the monthly probability of pregnancy ending in birth compared to the age 35 level for each respective group in order to focus on comparing the slopes (by removing the vertical shifts influenced by differences in levels of fecundability across subpopulations).

The trajectory of fecundability decline is not precisely the same across regions: Asia seems to have a steeper decline in the twenties and early thirties than the other regions, for example. These differences could be attributed to differences in population health as well as differences in behaviors such as frequency of sexual activity. However, none of the regions display a concave, accelerating relationship; across all regions, fecundability declines steadily and smoothly until it levels off at 0 sometime in the forties.

Panel (b) of Figure 5 estimates this age profile separately for four categories of women's educational attainment at the time of interview: no formal education, primary education only, secondary education, and higher than secondary education. The conclusion is similar: more educated women seem to experience an earlier drop-off in fecundability, but none of the lines display a concave shape.

To complement the results from the middle and low income countries included in the DHS, we draw on the US National Survey of Family Growth (NSFG) to construct the age-fecundability graph for the US. We define our fecundability sample analogously to the construction in the DHS, as described in Section 3: including only woman-months of the look-back period in which the respondent was married, noncontracepting, not already pregnant, not within 12 months of giving birth, and not within 3 months of a termination. Figure 6 presents the results for the US sample.

[31] The model estimated is: $B_{it} = \sum_a [\beta^{a,male} \cdot I_{it}^{a,male}] + \sum_a [\beta^{a,female} \cdot I_{it}^{a,female}] + \epsilon_{it}$.

[32] This is the sample labeled "1600-49; Geneva bourgeoisie, husbands born before 1600," in Menken et al.'s Figure 1.

[33] A list of countries can be found in Table A1. Our sample contains a few Eastern European countries, which we include as part of the Asia sample.

Local polynomial regressions following Equation 7 are estimated at the month-of-age level as in the DHS data, but when plotting mean fecundability by age, we define age in 3 month bins (e.g., a range like [33.0, 33.25)), to accommodate the smaller US sample. Although the US results are less precise, the age-fecundability profile in the US is completely consistent with the global results from poor and middle income countries. The figure shows that (remarkably) the level of fecundability is similar in the US to the global average—despite that TFR in the US over the sample period was below 2 and that African TFR in the sample period was nearly 5. But most importantly, the characteristic non-accelerating decline is replicated in the US context.

4.4 Further Restricting on Intercourse and Desired Family Size

An unavoidable limitation of any study measuring fecundability outside of a controlled lab environment is that the frequency, quality, and timing of intercourse within a monthly cycle is both highly variable across individuals and generally unobservable. Prospective studies of women attempting conception through artificial insemination, such as van Noord-Zaadstra et al. (1991) and Fédération et al. (1982), could escape this limitation in principle. But in practice—leaving aside any issues of non-representative selection into fertility clinic treatments—no such study has yet produced a large enough sample size to solve the practical problem of generating a sufficiently powered sample to estimate the second derivative of the age-fecundability profile in females.

Our fecundability sample is weakly too inclusive to identify any biological maximum: it certainly includes women-months in which there was no sexual activity among the married, noncontracepting female respondents. (This is, of course, also true for the existing estimates—which did not observe individual behavior.) Does this imply that the shape of fecundability decline we estimate—the object of interest here—is uninformative relative to what one might recover from ideal but infeasible laboratory conditions?

To answer this, we exploit additional information about sexual intercourse contained in both the DHS and NSFG surveys, examining whether conditioning on observable sexual activity alters our results. If conditioning on observable intercourse modified the concavity or convexity of the age-fecundability profile recovered in Figure 3, rather than shifting or rotating the near-linear decline, it would suggest that unobservable intercourse intensity might be importantly affecting our conclusions. In the NSFG, each female respondent reports whether or not she had noncontracepting sexual intercourse during each of the look-back months. In the DHS, each female respondent reports whether she had recent intercourse (in the month prior to the DHS interview). Therefore, for the NSFG we can condition on sex without otherwise altering our fecundability sample restriction. For the DHS, we limit the look-back period to recent months prior to interview for closest timing alignment to the recent sex question, and we change the dependent variable to an indicator for whether a conception occurred in the month in question (rather than a conception leading to a live birth, which wouldn't yet be resolved at the time of interview).

Figures A7 through A9 display the results. Figure A7b reproduces our main result for the subsample of women in the DHS who reported recent intercourse at the time of interview. Figure A7c takes the affirmatively sexually active DHS sample (Figure A7b) and additionally restricts to women who report a desire for more children—as revealed by reporting an ideal family size larger than their current family size. Figure A8 makes the same restriction on recent sex as Figure A7b, but limits the look-back period to three, six, or nine months, instead of five years, to better align the timing of the sex question to the at-risk months in question. Figure A9 reproduces our main result for the subsample of women-months in the NSFG for which respondents report intercourse in that month in particular. These specification restrictions reduce power, because they preserve a smaller sample, but none of these alternative approaches finds any evidence of an accelerating decline in fecundability in a woman's 30s. In all cases, our core finding of a near-linear decline in the age profile of female fecundability from 25-40 holds.

5 Reconciling with prior studies

5.1 Estimating fertility, rather than fecundability, with the DHS reproduces a concave, accelerating shape

Section 2 offered a theoretical answer for the question that the DHS data suggest: Why do our DHS results show a robustly non-concave age-fecundability profile, while Menken et al. (1986) report a concave relationship: flat at younger ages, and then steeper after the mid-30s? The theoretical answer was because the *numerators* of fecundability (which we estimate) and fertility (which they and others use as a proxy for fecundity) are essentially the same—a count of births—the different shapes must be because of differences in the *denominators*.

It is straightforward to test whether this fact can indeed account for this difference. Figure 7 addresses this by combining our sample with Menken et al.'s method. We begin by applying Menken et al.'s less restrictive (fertility) denominator construction to our DHS data (meaning, leaving all noncontracepting married women in the age interval). This yields a plot of age-specific fertility rates (at age-in-months intervals). The figure also replicates the main result from Figure 3(a) for reference.

To verify our solution for the analytical expression of bias used to unbias Menken et al.'s result in Figure 2, we apply Equation 5 to each point on the fertility rate line and plot the resulting implied fecundability rate, choosing an interval i that maximizes fit. This is the dotted line in Panel (a), which nearly perfectly overlaps with the true measured fecundability rate. This nonlinear bias correction, which can recover F from f and vice versa, was not calibrated in any way other than the selection of the interval i.

The figure reveals three important facts: First, it shows the hypothesized bias can be quantitatively large for levels of fecundability encountered in practice. Second, we can replicate in our data a fertility plot that shows a concave relationship over the thirties that is not present in the underlying fecundability data, by following Menken et al.'s method of proxying fecundity probabilities with fertility rates. This indicates a clear solution to the question of why our fecundability estimates show an approximately linear decline, while the consensus view—based on using fertility as a proxy for fecundity—agrees on a concave relationship. Third, the result in the figure validates the use of a bias correction term as an empirically-implementable correction that yields a good fit. To emphasize:

Both the fertility and fecundability plots in the figure were calculated from the microdata, while the *Implied fecundability* plot begins from the estimated fertility rate at each age and removes the bias from each point along the curve according to Equation 5.

5.2 What the assisted reproduction studies (don't) tell us

We have thus far focused our attention on Menken et al. (1986), which was pivotal in shaping both the positions of medical associations and popular popular perception of the decline in women's probability of a successful pregnancy with age. However, there are several other studies in the medical literature that are also influential in this area, some of which are more recent. These studies tend to rely on small samples of women who interact with the medical system because they are trying to get pregnant and experiencing infertility issues.

For example, van Noord-Zaadstra et al. (1991) study a sample of fewer than 2,000 women in a donor insemination program in the Netherlands, measuring how long it takes women to become pregnant according to their age. Similarly Fédération et al. (1982) study just over 2,000 women in an artificial insemination with donor semen (AID) program in France. An advantage of using AID programs is that they provide a convenient way of gathering a sample of women attempting conception who are seeking help due to their partners' infertility, rather than their own. Hull et al. (1996) focuses on the probability of successful implantation of embryos after in vitro fertilization. In more recent work, Rothman et al. (2013) gather a more general sample of women trying to get pregnant in Denmark in a prospective cohort study.

These papers differ from the Menken et al. strategy of using natural fertility populations in that they successfully restrict their sample to women that are explicitly trying to get pregnant and are not currently unexposed due to an existing pregnancy or a recent birth. However, none of these have been designed nor powered to confirm or refute the consensus view of a concave age-profile of fecundability decline. To the extent these studies attempt to draw conclusions despite the lack of power, they reach conflicting conclusions, which were often embedded or assumed in the parametric models they attempted to fit. For example, Rothman et al. (2013) concludes that fecundability *peaks* at about age 30 after increasing in the twenties, with stronger declines after 35. In contrast, van Noord-Zaadstra et al. (1991) find a large drop in fecundability after a critical age of 31, fitting a model in which fecundability, by model design, has zero increase or decline until the critical age and then follows a parametric decay. More broadly, these studies tend to focus on finding the "critical age" after which fecundability falls off precipitously, building into their analysis *the assumption that there is one*, rather than considering the possibility of a smooth decline, which our non-parametric analysis permits. In this way, they build-in the consensus view prior to data analysis; they therefore do not provide independent evidence for that view. They are forced to rely on *some* kind of parametric assumption, as all of these studies use small sample sizes that do not allow the authors' to statistically rule out alternative shapes of the age-related decline in fecundability.

6 Conclusion

In this study, we make three important contributions to advance scientific understanding of age-related fecundability decline. First, we show theoretically that, even if the age-profile of fecundability were linear, the age-profile of fertility would show a familiar, concave, accelerating shape. Our model allows us to transform fertility rates into fecundability rates and vice-versa (with the assumption of an interval parameter). We use Menken, et al.'s influential fertility data to show that, once these fertility data are converted to fecundability, the decline in fecundability is nearly linear in age—even in these historical populations and data that have to now been used to support the idea of infecundability acceleration in the mid-thirties.

Second, in our empirical results, we estimate fecundability by age in a sample comprised of millions of modern women across countries representing 52 percent of the global population, a substantial advance over previous studies focused on small historical populations or small samples of women in fertility clinics in Europe. We show that women experience a steady, near-linear decline in fecundability through the late twenties and thirties, with no "critical age" after which the probability of a successful pregnancy plummets. This linear shape holds across sub-populations broken down by world region and educational attainment. We find evidence of a similar shape in a smaller sample of data from the United States.

Third, and substantively, our results indicate that there is no one critical age, before permanent infecundity, that individuals or couples planning their reproductive careers should avoid reaching before completing their family. Instead, fecundability decline begins well before the median age at first marriage in many populations. Our results show that by age 25, the annual decline in fecundability is approximately constant. Contrary to the advice of the The American College of Obstetricians and Gynecologists (ACOG), the American Society for Reproductive Medicine (ASMR), and others, there is no age range that we study in which it is "safe" to wait to have children in the sense of relatively little (or even lesser) loss of fecundability per month or year of age. In addition, we find that the age of the male partner is also an important determinant of a couple's probability of a successful conception. Achieving planned fertility depends on both partners' ages. Our findings give clearer guidance to aid in that planning.

Beyond the potential to improve information currently provided to women and their partners, our results also suggest a benefit of significantly reshaping healthcare payment policies of private insurers and therefore clinical decision-making by physicians. Medical practices that treat women substantially differently after the age of 35 than before the age of 35 on the basis of fecundability decline could be revised. Obstetricians, guided and constrained by health insurance rules that determine what tests and procedures are eligible for reimbursement depending on whether a woman is over or under 35, currently plan and administer care with sharp discrimination at the age-35 threshold. Although women experiencing a "geriatric" pregnancy or advanced maternal age may still be at elevated risk of genetic abnormalities and other complications (this is beyond the scope of this study), our findings suggest that no sharp cutoff of age 35 should be used when evaluating the chance of a conceiving a successful pregnancy.

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van Noord-Zaadstra, Boukje M, Caspar W Looman, Hans Alsbach, J Dik Habbema, Egbert R te Velde, and Jan Karbaat (1991) "Delaying childbearing: effect of age on fecundity and outcome of pregnancy.," *British Medical Journal*, 302 (6789), 1361–1365.

Rothman, Kenneth J, Lauren A Wise, Henrik T Sørensen, Anders H Riis, Ellen M Mikkelsen, and Elizabeth E Hatch (2013) "Volitional determinants and age-related decline in fecundability: a general population prospective cohort study in Denmark," *Fertility and sterility*, 99 (7), 1958–1964.

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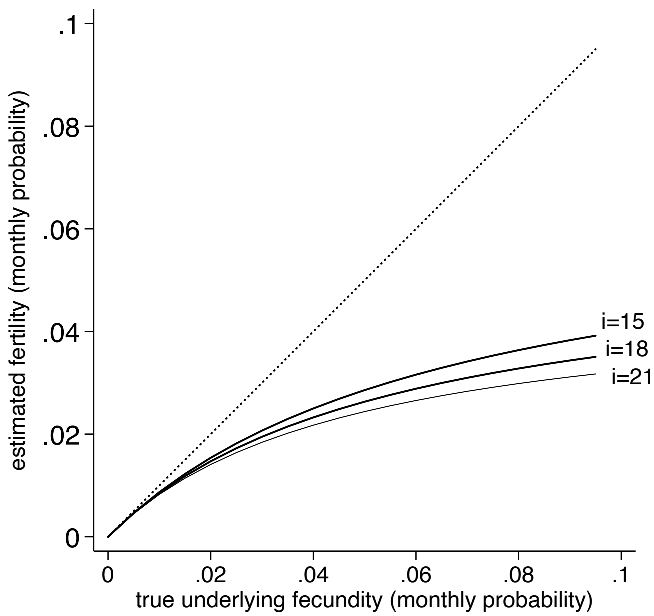
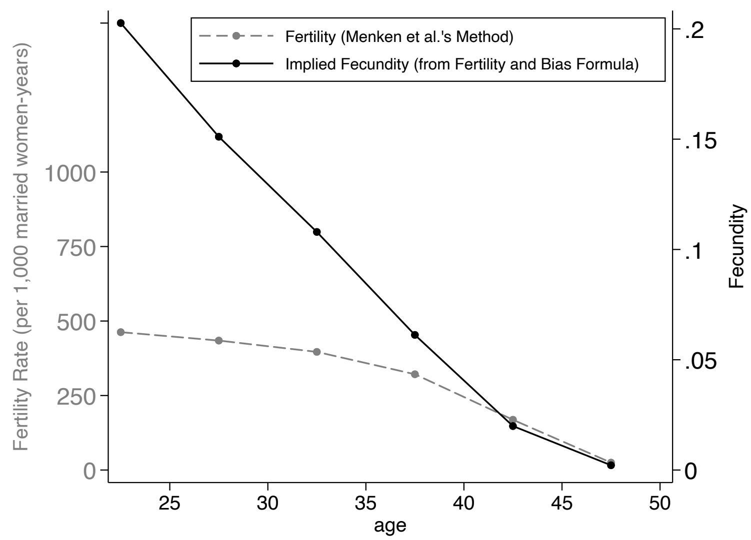
Figure 1: Fertility is biased proxy measure of fecundability that introduces concavity in the age-fecundability profile

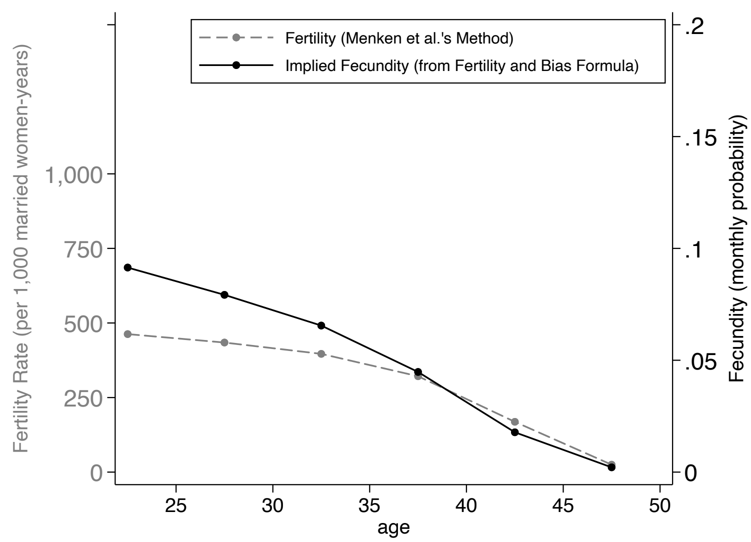


Note: Figure plots theoretical relationship between fecundability (f) and fertility (F) as a function of interval (i). The monthly fertility probabilities, given f and i are calculated according to Equation 5. An interval of $i = 15$ would correspond to a 9-month pregnancy plus a period of 6 months postpartum during which breastfeeding or any other cause would render a new mother temporarily infecundable.

Figure 2: Even if the relationship between fecundability and age is linear, the relationship between fertility and age would be concave and accelerating, Part I: Menken, et al. data.



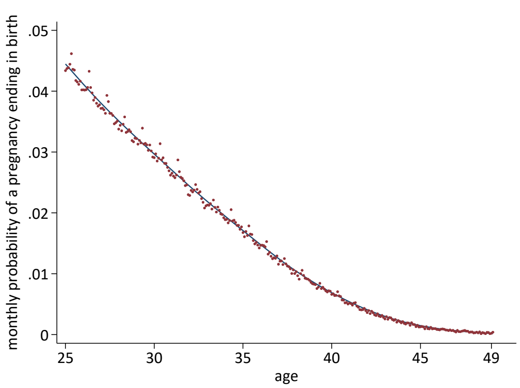
(a) Menken's data, corrected with 12-month postpartum infecundability



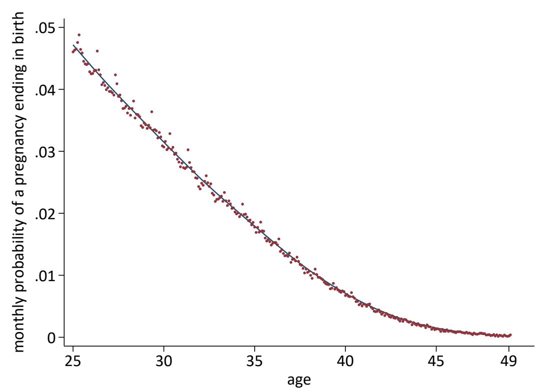
(b) Menken's data, corrected with 6-month postpartum infecundability

Note: Figure presents estimates using Menken et al.'s data (Figure 7 presents similar results using our DHS data.). The fertility plot averages together the age-specific fertility rates for the 10 subpopulations analyzed by Menken et al. and divides by 12,000 get monthly fertility rates (rather than annual births per 1,000 women). The Implied Fecundability plot in Panel (a) takes the measured fertility rate and de-biases it according to the formula in Equation 5 for $i = 21$, which assumes a 12-month infecundable period postpartum, identical to our main result in Panel (a) of Figure 3. The Implied Fecundability plot in Panel (b) does the identical exercise for $i = 15$ (e.g., a nine-month pregnancy plus a six month period of postpartum infecundability).

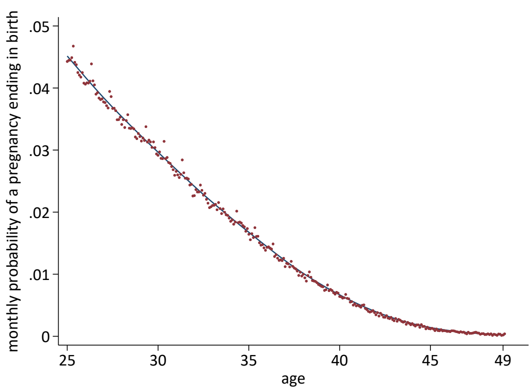
Figure 3: Main empirical result: fecundability declines in age without acceleration in a woman's mid-30s in nationally representative samples of 62 countries.



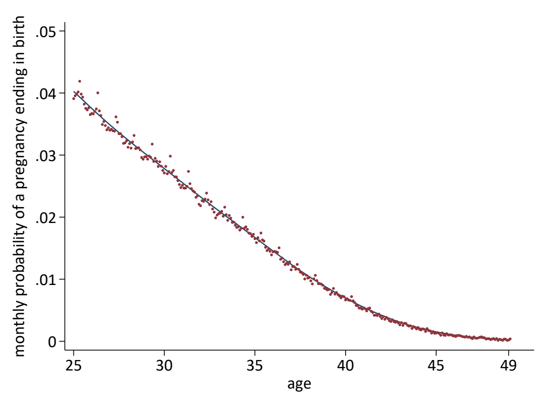
(a) Excluding 12 months postpartum



(b) Using self-reports of amenorrhea



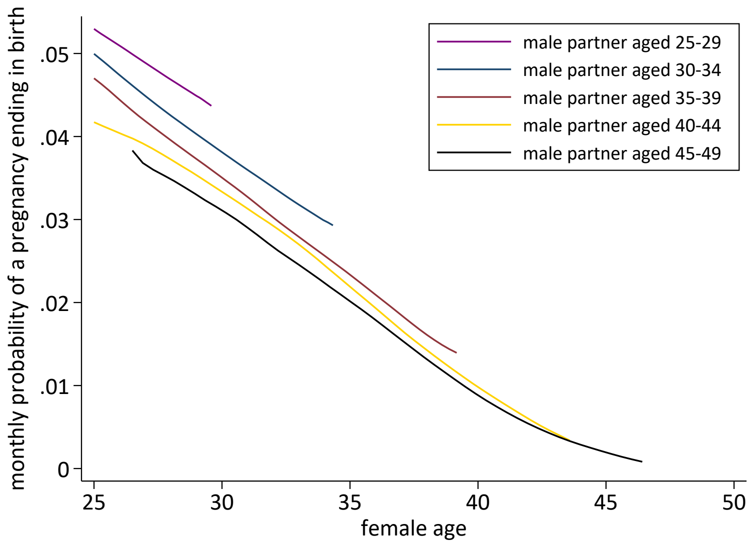
(c) Excluding 15 months postpartum



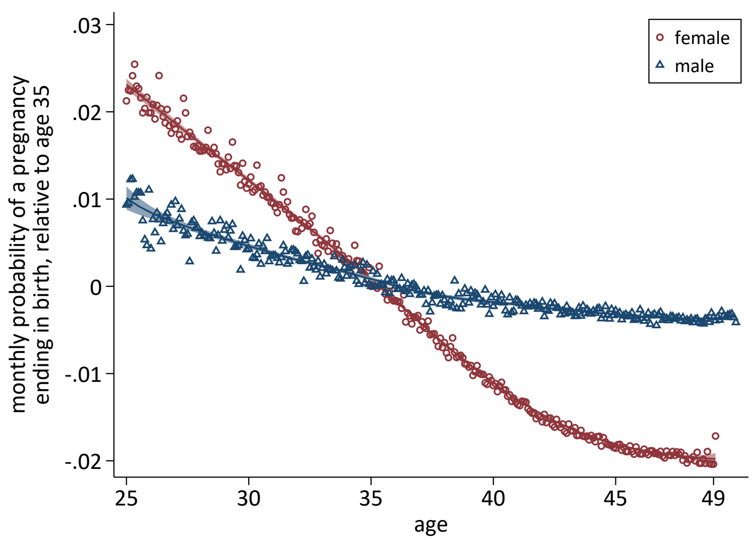
(d) Excluding 6 months postpartum

Note: Each panel presents estimates of fecundability from DHS data. Means for each age-in-month group (e.g., 33 years and 2 months old) are plotted as dots, according to Equation 6. Local polynomials (bandwidth 15, order 1) estimated according to Equation 7 are overlaid. fecundability is measured as the monthly probability of beginning a pregnancy that ultimately ends in live birth, among women who are married, noncontracepting, not pregnant, non-postpartum, and non-post-termination. The panels differ in how each operationalizes the postpartum infecundable period. Panel (a) excludes 12 months postpartum for all women. Panel (b) uses the individual-specific survey responses to exclude months after a birth in which a respondent woman self-reports that she was breastfeeding or experiencing postpartum amenorrhea. Panel (c) excludes 15 months postpartum for all women. Panel (d) excludes 6 months postpartum for all women. All panels exclude the first 3 months following a termination (an abortion or miscarriage).

Figure 4: Fecundability declines in male age, at a slower rate



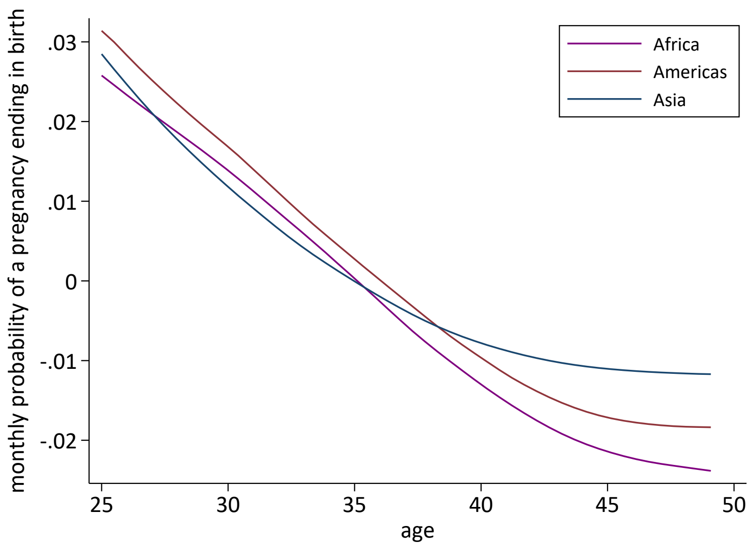
(a) Female fecundability by male partner age



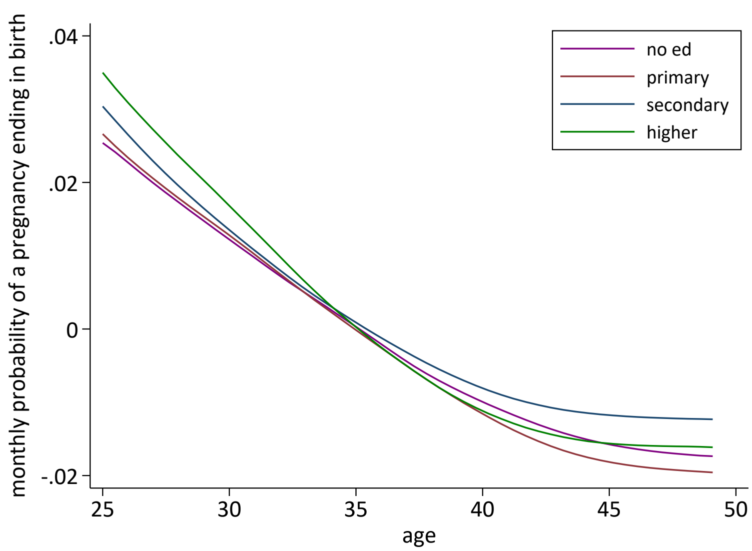
(b) Male and female fecundability

Note: Panel (a) plots estimates of fecundability from DHS data, generated separately by male partner age. Definitions and specifications match panel (a) of Figure 3. The fecundability-age relationship is shown for the middle 90% of female ages partnered with men in the relevant age bin. Panel (b) plots estimates of male and female fecundability, estimated from a single regression where the independent variables are indicators from female age in months and male age in months. Coefficients are interpretable as the effect of male or female age a on the probability of a successful pregnancy, relative to having both partners' ages be 35 years and 0 months.

Figure 5: Heterogeneity: Similarly-shaped relationships between fecundability and age are found within various sub-populations.



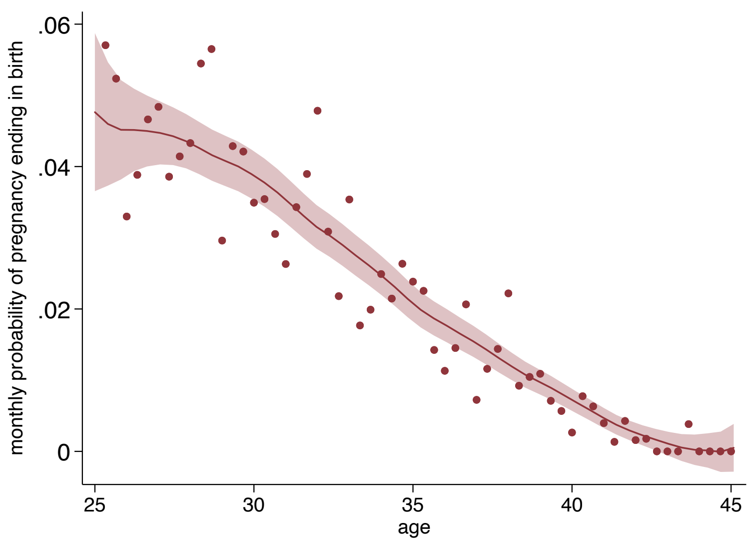
(a) Heterogeneity by world region



(b) Heterogeneity by educational attainment

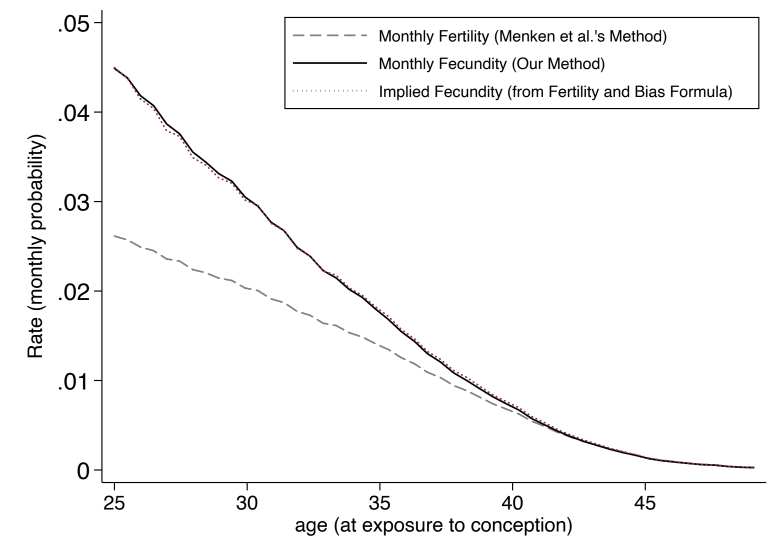
Note: Figure plots estimates of fecundability from DHS data, generated separately for the indicated subpopulations. Definitions and specifications match panel (a) of Figure 3.

Figure 6: Robustness: A non-accelerating relationship between fecundability and age is also seen in the US, although this smaller survey permits less precise estimates.



Note: Figure plots estimates of fecundability from NSFG data: United States, 2006–2010 and 2011–2013 survey waves. Definitions and specifications match panel (a) of Figure 3.

Figure 7: Even if the relationship between fecundability and age is linear, the relationship between fertility and age would be concave and accelerating, Part II: DHS data.



(a) Our Data, Both Methods

Note: Figure plots fecundability and fertility in the DHS samples. The fecundability plot (solid line) reproduces the local polynomial regression from our main result: panel (a) of Figure 3. See Figure 3 notes. The fertility plot (dashed line) uses the *fertility sample* to calculate monthly rates, which does not exclude woman-months in which respondents were pregnant, postpartum, and post-termination. The implied fecundability plot (dotted line) takes the measured fertility rate (dashed line) and de-biases it according to the formula in Equation 5. The debiasing operates on the plotted points, not the micro data.

Online Appendix for:
Infertility and Age Revisited

A National Survey of Family Growth (NSFG)

The National Survey of Family Growth (NSFG) is a US household survey that has collected information on nationally representative samples of women aged 15-44 since 1982 (extended to 15-49 in 2015). The NSFG is administered through in-person interviews by the Center for Disease Control and Prevention's National Center for Health Statistics. The focus of the survey is on reproductive health and behavior: detailed monthly information on marital history, contraceptive use, pregnancies and births, and sexual activity is collected.

10 waves total of the NSFG have been conducted to date, with one underway (2022-2029). Each wave interviews an independent sample of roughly 7,000-20,000 women. Our analysis focuses on the 2006-2010 and 2011-2013 survey waves, which align with our DHS sample period and took place before the respondent's month of birth became a restricted variable in the NSFG in the 2015-2017 wave. These two waves contain data on 33,098 women total between the ages of 15-44, just over 1% of the size of the DHS sample used in our primary analysis. The NSFG collects richer data than the DHS along several dimensions: it records a monthly history of sexual activity (indicators for whether the respondent was sexually active or not for each month of the interview year and the three years prior) as opposed to only at the time of interview. In additional, terminated pregnancies due to miscarriage, abortion, and stillbirth are separated out. Like the DHS, the NSFG collects a monthly history of contraceptive use, and dates of the beginning and end of each pregnancy. We use these data to construct a sample of women-months "at risk" for pregnancy that aligns with our definition in the DHS: we include women who are married, noncontracepting, not already pregnant, and not within 12 months of giving birth or within 3 months of a termination. Also similarly to our DHS sample, we eliminate the last 9 months of the look-back period before the month of interview, to remove pregnancies for which we cannot observe the outcome.

B Bias in Fertility as Fecundability Proxy

Here we derive an expression for the bias that arises when using fertility (F) as a proxy or measure of fecundability (f). For simplicity and ease of comparison to monthly fecundability rates, we will consider age-specific fertility rates expressed per month of age, such as: births per women aged 27 years and 3 months. (In small sample historical studies, age-specific fertility rates are usually calculated in one-year or five-year bins, rather than one-month bins.) Recalling definitions form the main text in Section 5:

- Interval (i) - the count of months that a woman is not exposed to the possibility ("risk") of becoming pregnant once she becomes pregnant. For example $i = 15$ would describe a 9-month pregnancy followed by 6 months of breastfeeding-induced amenorrhea. For simplicity in generating proofs, we assume that i comprises a fixed time interval, rather than a random variable.
- Exposure (e) - the probability than any randomly-selected month will be one in which a woman with fecundability f and interval i will be at risk of a new conception. It will ultimately be an equilibrium outcome that is a function of the primitives i and and f.

Given these definitions, the bias between fecundability (f) and fertility ($F = fe$) can be expressed as $f - fe$, where we have used the fact that the unconditional probability of pregnancy leading to birth (fertility) is just the conditional probability of pregnancy among women at risk of a pregnancy (fecundability) times the probability of being at risk of pregnancy (exposure, e).

To solve for this bias term as a function only of the primitives (f, i), we assume a stationary process and search for a fixed point in a cycle of: conception–birth and postpartum–conception. Consider some index month x that occurred $i+1$ months ago. This is the shortest backward-looking window for which a pregnancy that started then would not inhibit exposure to the possibility of a new pregnancy now. This yields the fixed point equation:

$$\text{probability of exposure } x \text{ months ago} = e = \text{probability of exposure now.}$$

So what is the probability of exposure now? It is the probability of being exposed now conditional on being exposed x months ago plus the probability of being exposed now conditional on not being exposed x months ago.

If woman was exposed x months ago (probability e): To remain exposed, she would have to not become pregnant in any of the next i months, which has a probability $(1-f)^i$, because we have assumed months are independent, conditional on the state variable (an indicator for currently within the interval, i, following a conception).

If woman was not exposed x months ago (probability $1-e$): If the woman was not exposed x months ago, she could have been at any of i equally-likely points in non exposure.

- There is a $\frac{1}{i}$ chance that there will be no exposed months left, so she would then be exposed for sure: $\frac{1}{i} \times 1$.
- There is a $\frac{1}{i}$ chance that there will be 1 exposed month left, so she would then remain exposed with chance $1-f$: $\frac{1}{i} \times (1-f)$.
- There is a $\frac{1}{i}$ chance that there will be 2 exposed months left, so she would then remain exposed with chance $(1-f)^2$: $\frac{1}{i} \times (1-f)^2$.
- And so on until $i-1$.

So the overall probability of being exposed conditional on not being exposed x months ago is:

$$\frac{1}{i}\sum_{j=0}^{i-1}(1-f)^j.$$

Now we have an expression for the probability of being exposed now:

$$e \times \overbrace{(1-f)^i}^{\text{if exposed } x \text{ months ago}} + (1-e) \times \overbrace{\frac{1}{i}\sum_{j=0}^{i-1}(1-f)^j}^{\text{if not exposed } x \text{ months ago}}$$

This lets us close our fixed-point equation:

$$e = e \times (1-f)^i + (1-e) \times \frac{1}{i}\sum_{j=0}^{i-1}(1-f)^j.$$

This works out to, for e as a function of f and i:

$$e = \frac{i^{-1}\sum_{j=0}^{i-1}(1-f)^j}{1-(1-f)^i + i^{-1}\sum_{j=0}^{i-1}(1-f)^j}.$$

Simplifying the finite geometric sequences and canceling terms yields:

$$e = \frac{\frac{1}{f}}{i + \frac{1}{f}}.$$

Evaluating the denominator: Each woman goes through a sequence in which she becomes pregnant and is thus non-exposed for x months, and then is exposed for a period of time that is equal in expectation to 1/f months (because she has a probability of f of becoming pregnant in each month); then the sequence begins again. Over the same period, the numerator contains the expected number of months in which a woman is exposed to the possibility of pregnancy (prior to achieving it), which is 1/f. So the fraction, on average, of each cycle that a woman is exposed to pregnancy risk is $(1/f)/(i + 1/f)$.

Appendix Figure A1: Birth rates per 1,000 wives by age of wife, from Menken et al.

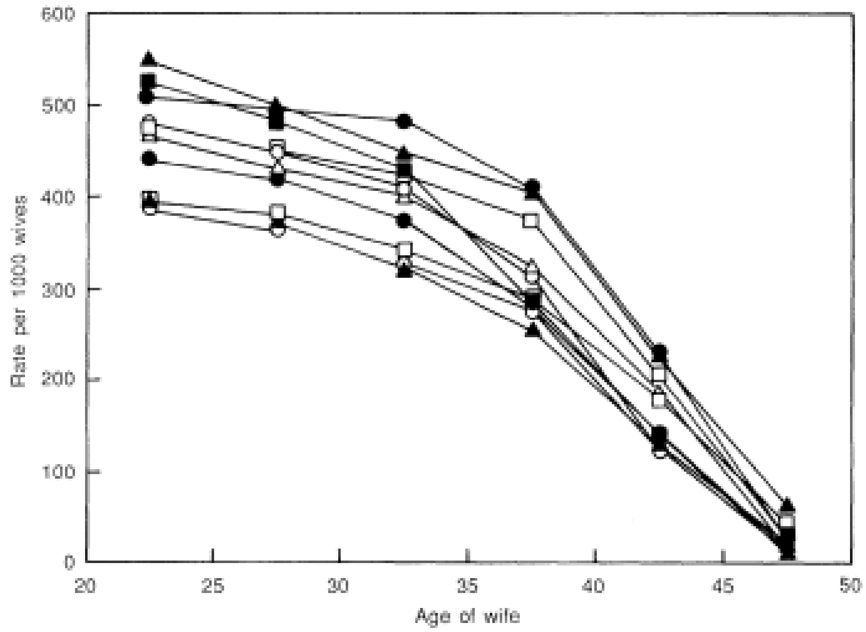


Fig. 1. Marital fertility rates by 5-year age groups (*5*). The ten populations (in descending order at age 20 to 24) are Hutterites, marriages 1921–30 (▲); Geneva bourgeoisie, husbands born 1600–49 (■); Canada, marriages 1700–30 (●); Normandy, marriages 1760–90 (○); Hutterites, marriages before 1921 (□); Tunis, marriages of Europeans 1840–59 (△); Normandy, marriages 1674–1742 (●); Norway, marriages 1874–76 (□); Iran, village marriages, 1940–50 (▲); Geneva bourgeoisie, husbands born before 1600 (○).

Note: Figure 1, as presented in Menken et al. (1986).

Appendix Figure A2: DHS Contraceptive Calendar: Survey Instrument and Completed Example

INSTRUCTIONS:
ONLY ONE CODE SHOULD APPEAR IN ANY BOX.
COLUMN 1 REQUIRES A CODE IN EVERY MONTH.

CODES FOR EACH COLUMN:

COLUMN 1: BIRTHS, PREGNANCIES, CONTRACEPTIVE USE (2)

B BIRTHS
P PREGNANCIES
T TERMINATIONS

0 NO METHOD

1 FEMALE STERILIZATION
2 MALE STERILIZATION
3 IUD
4 INJECTABLES
5 IMPLANTS
6 PILL
7 CONDOM
8 FEMALE CONDOM
9 EMERGENCY CONTRACEPTION
J STANDARD DAYS METHOD
K LACTATIONAL AMENORRHEA METHOD
L RHYTHM METHOD

M WITHDRAWAL
X OTHER MODERN METHOD
Y OTHER TRADITIONAL METHOD

COLUMN 2: DISCONTINUATION OF CONTRACEPTIVE USE

0 INFREQUENT SEX/HUSBAND AWAY
1 BECAME PREGNANT WHILE USING
2 WANTED TO BECOME PREGNANT
3 HUSBAND/PARTNER DISAPPROVED
4 WANTED MORE EFFECTIVE METHOD
5 SIDE EFFECTS/HEALTH CONCERNS

6 LACK OF ACCESS/TOO FAR
7 COSTS TOO MUCH
8 INCONVENIENT TO USE
F UP TO GOD/FATALISTIC
A DIFFICULT TO GET PREGNANT/MENOPAUSAL
D MARITAL DISSOLUTION/SEPARATION
X OTHER ______ (SPECIFY)

Z DON'T KNOW

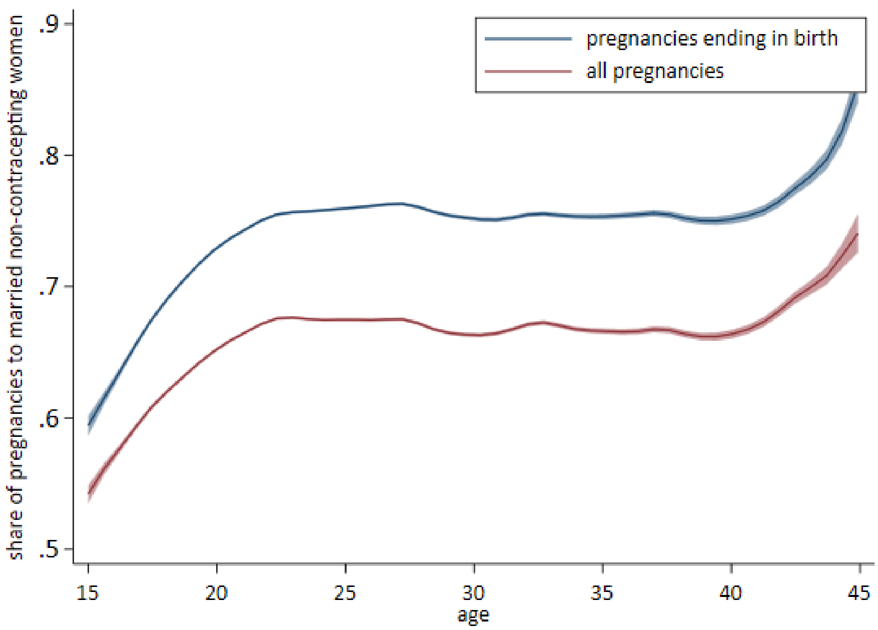
(1) Year of fieldwork is assumed to be 2015. For fieldwork beginning in 2016, all references to calendar years should be increased by one; for example, 2009 should be changed to 2010, 2010 should be changed to 2011, 2011 should be changed to 2012, and similarly for all years throughout the questionnaire.

(2) Response categories may be added for other methods, including fertility awareness methods.

Sara

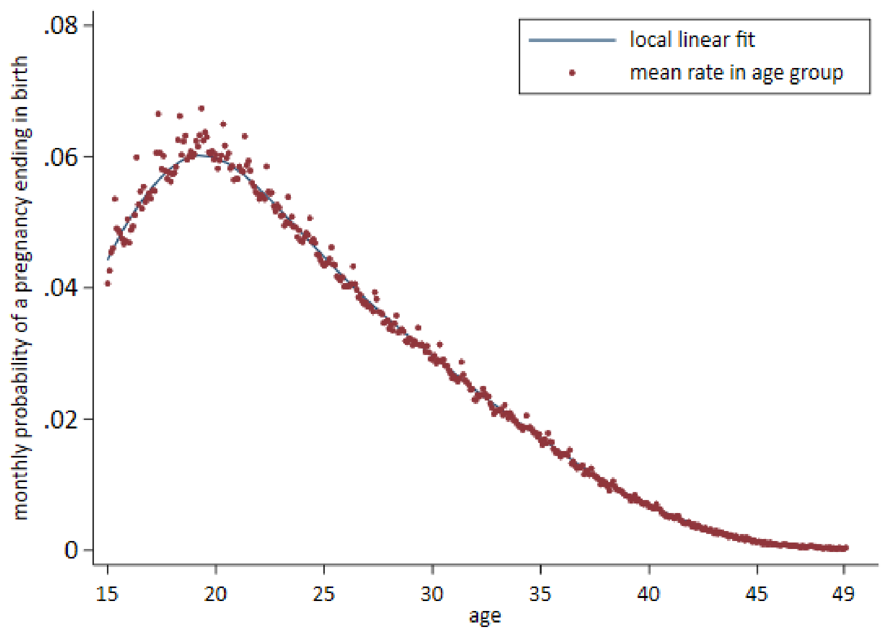
				COL. 1	COL. 2
2015 (1)	12	DEC	01		
	11	NOV	02		
	10	OCT	03		
	09	SEP	04	6	
	08	AUG	05	6	
	07	JUL	06	6	
	06	JUN	07	6	
	05	MAY	08	6	
	04	APR	09	6	
	03	MAR	10	6	
	02	FEB	11	6	
	01	JAN	12	0	
2014	12	DEC	13	0	
	11	NOV	14	0	
	10	OCT	15	B	
	09	SEP	16	P	
	08	AUG	17	P	
	07	JUL	18	P	
	06	JUN	19	P	
	05	MAY	20	P	
	04	APR	21	P	
	03	MAR	22	P	
	02	FEB	23	P	
	01	JAN	24	L	1
2013	12	DEC	25	L	
	11	NOV	26	L	
	10	OCT	27	L	
	09	SEP	28	L	
	08	AUG	29	0	
	07	JUL	30	0	
	06	JUN	31	0	
	05	MAY	32	0	
	04	APR	33	0	
	03	MAR	34	0	
	02	FEB	35	0	
	01	JAN	36	0	
2012	12	DEC	37	0	
	11	NOV	38	6	5
	10	OCT	39	6	
	09	SEP	40	6	
	08	AUG	41	6	
	07	JUL	42	6	
	06	JUN	43	6	
	05	MAY	44	6	
	04	APR	45	6	
	03	MAR	46	0	
	02	FEB	47	0	
	01	JAN	48	0	
2011	12	DEC	49	0	
	11	NOV	50	0	
	10	OCT	51	0	
	09	SEP	52	0	
	08	AUG	53	0	
	07	JUL	54	4	5
	06	JUN	55	4	
	05	MAY	56	4	
	04	APR	57	4	
	03	MAR	58	4	
	02	FEB	59	4	
	01	JAN	60	0	
2010	12	DEC	61	0	
	11	NOV	62	T	
	10	OCT	63	P	
	09	SEP	64	P	
	08	AUG	65	0	
	07	JUL	66	0	
	06	JUN	67	0	
	05	MAY	68	0	
	04	APR	69	0	
	03	MAR	70	0	
	02	FEB	71	0	
	01	JAN	72	0	

Appendix Figure A3: The fraction of pregnancies accruing to married, noncontracepting women is stable from 25-40



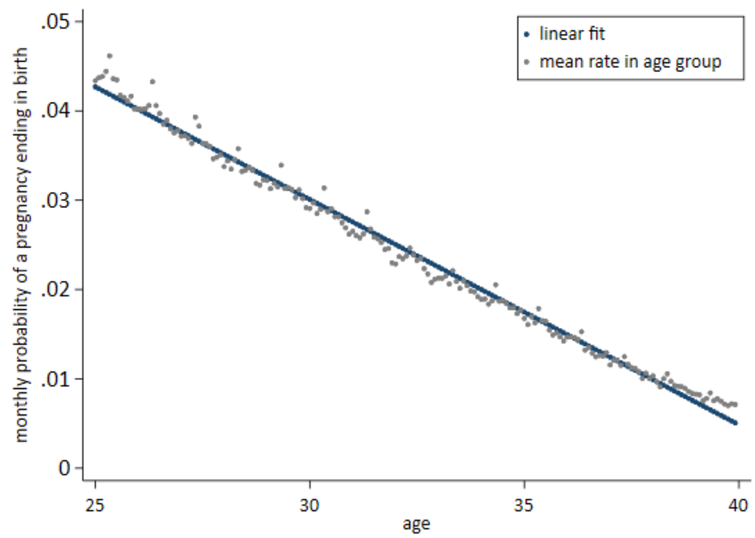
Note: This figure presents the results of local linear regressions where the outcome variable is an indicator equaling 1 if the pregnancy occurs to a woman who is married and noncontracepting, and the independent variable of interest is continuous age in months. The blue line shows the results of this regression run on the sample of pregnancies ending in live birth, and the red line shows the results run on the sample of all pregnancies, including those that end in termination.

Appendix Figure A4: Monthly probability of pregnancy ending in birth, ages 15-49, among women who are noncontracepting, married, not pregnant, and not within 12 months postpartum or recovering from a termination

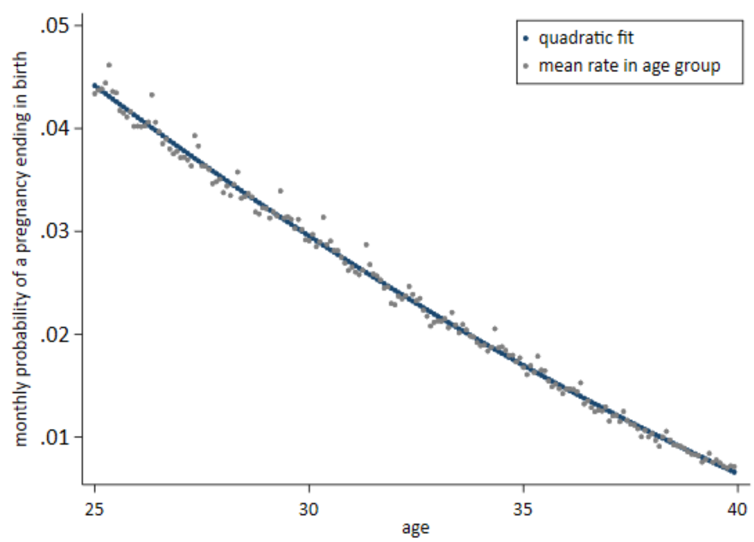


Note: This figure presents estimates of fecundability from DHS data for ages 15-49. Means for each age-in-month group (e.g., 33 years and 2 months old) are plotted as dots, according to Equation 6. Local polynomials (bandwidth 15, order 1) estimated according to Equation 7 are overlaid. fecundability is measured as the monthly probability of beginning a pregnancy that ultimately ends in live birth, among women who are married, noncontracepting, not pregnant, non-postpartum, and non-post-termination.

Appendix Figure A5: Convex quadratic function of age in months best fit for fecundability-age relationship



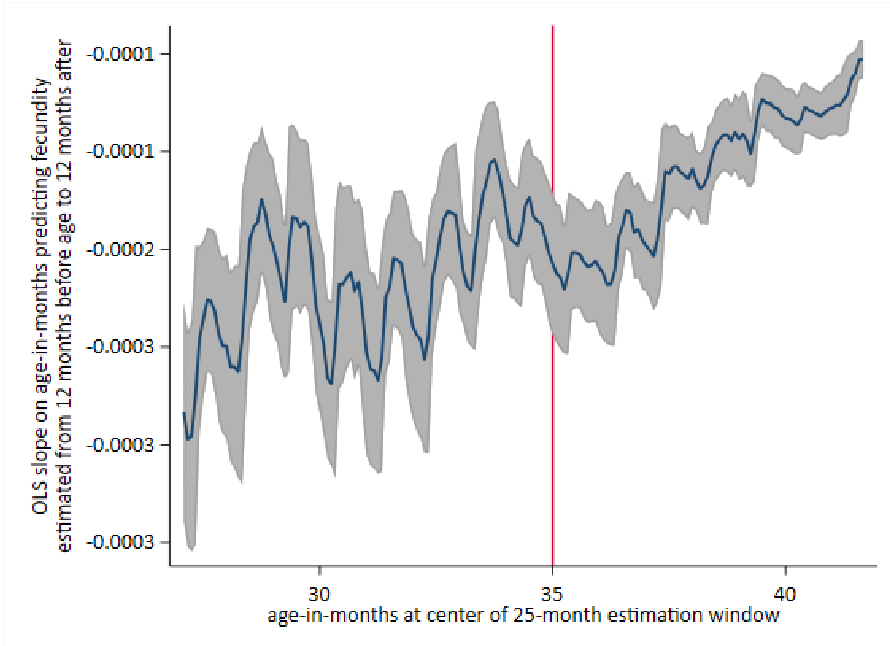
(a) Linear fit



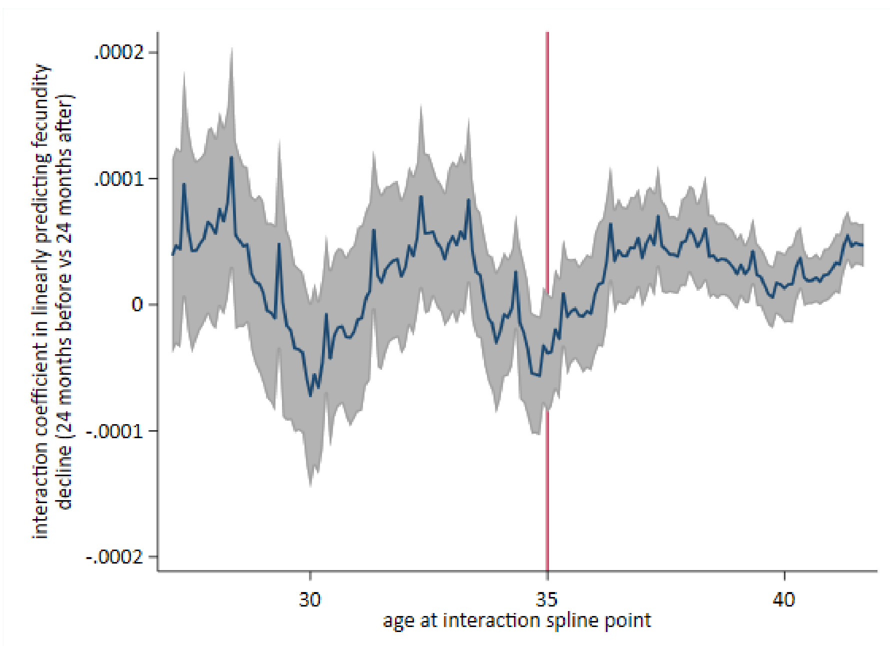
(b) Quadratic fit

Note: Figures plot estimates of fecundability from DHS data, in gray. Definitions and specifications match panel (a) of Figure 3. Panel (a) plots predicted values from a linear regression with continuous age in months as the independent variable, in blue. Panel (b) plots predicted values from a quadratic regression, where the independent variables are continuous age in months, and age in months squared.

Appendix Figure A6: No evidence of sharp change in slope of age-fecundability relationship at 35 or any other age

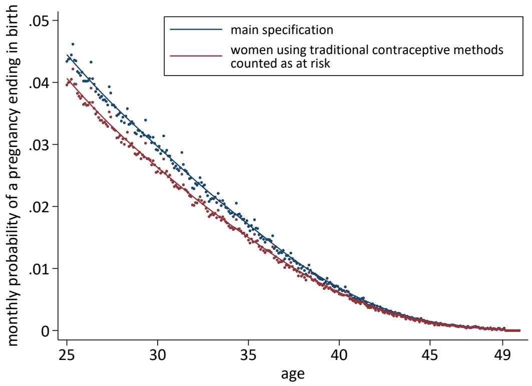


(a) Slopes from a series of local linear regressions of birth rates on women's ages, entered continuously in months. These take the form: $Y_{it} = \alpha + \gamma\prime(age_{it} - age^0) + \mu_{it}$ defined over subsamples $\|age_{it} - age^0\| \leq h$, where age^0 is the index along the horizontal axis in the figure. Here, h is set to 12 months.

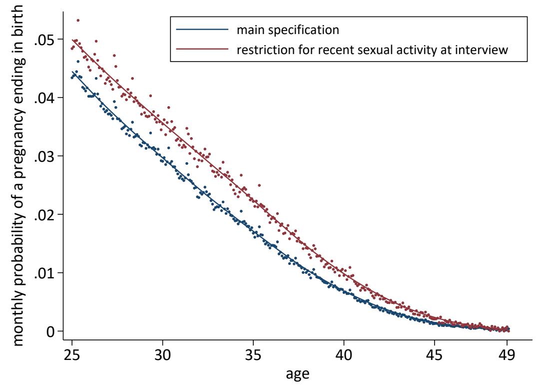


(b) Coefficients from a series of regressions investigating whether the slope in the 24 months after age in months a is different from the slope in the 24 months before. The regressions take the same form as in panel a), except that continuous age in months is interacted with an indicator for age $> age^0$.

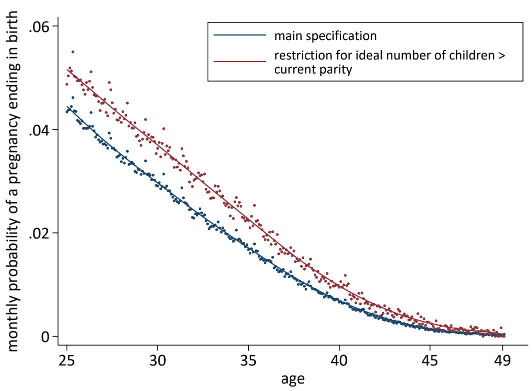
Appendix Figure A7: Robustness: Shape of age-fecundability profile is robust to different specifications of the at risk sample.



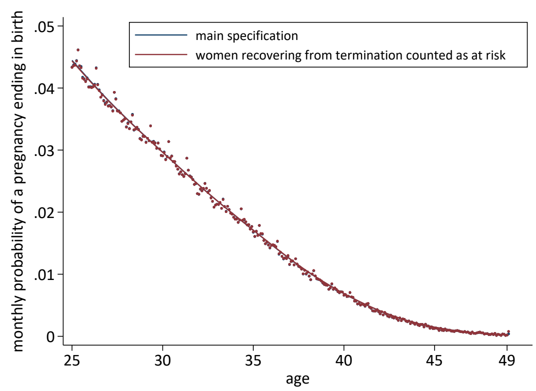
(a) Women using traditional contraception counted as at risk of pregnancy



(b) Sample restriction for recent sexual activity at interview



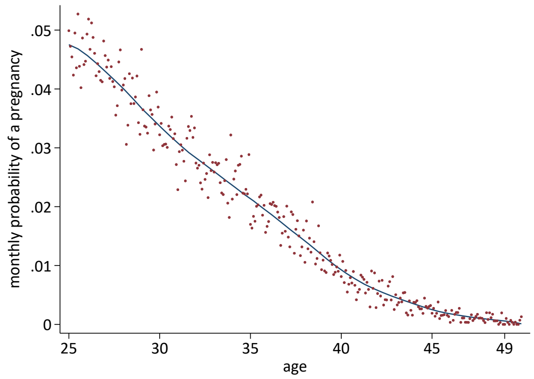
(c) Sample restriction for ideal parity greater than current parity



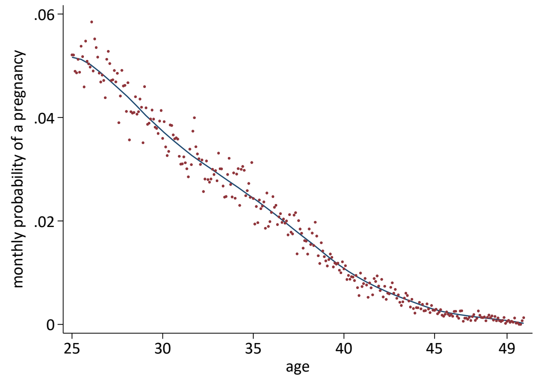
(d) Women recovering from termination counted as at risk of pregnancy

Note: Each panel presents a robustness check on panel (a) of Figure 3. Means for each age-in-month group (e.g., 33 years and 2 months old) are plotted as dots, according to Equation 6. Local polynomials (bandwidth 15, order 1) estimated according to Equation 7 are overlaid. In panel (a), fecundability is measured as the monthly probability of beginning a pregnancy that ultimately ends in live birth, among women who are married, noncontracepting OR using only traditional contraception, not pregnant, non-postpartum, and non-post-termination. Panel (b) displays the monthly probability of beginning a pregnancy that ends in live birth, among women who are married, noncontracepting, not pregnant, non-postpartum, non-post-termination, and who reported being sexually active in the last month at the time of the interview. Panel (c) displays this measure among women who are married, noncontracepting, not pregnant, non-postpartum, non-post-termination, sexually active in the last month at the time of interview, and whose parity was below her self-reported ideal number of children at the time of observation. Panel (d) displays this measure among women who are married, noncontracepting, not pregnant, and non-postpartum. All four panels overlay the results from the altered specification on the results from the main specification, as displayed in Figure 3.

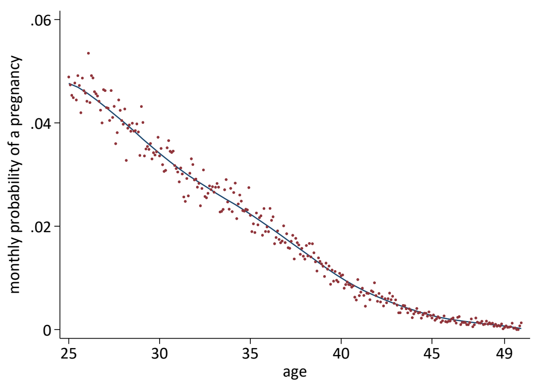
Appendix Figure A8: Robustness: Shape of age-fecundability profile in the DHS is robust to additional restriction on recent intercourse and a shorter look-back window



(a) 3 month lookback



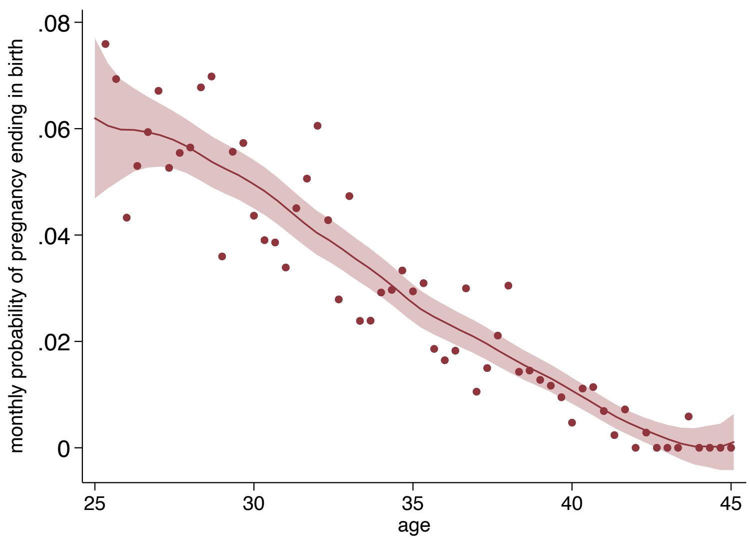
(b) 6 month lookback



(c) 9 month lookback

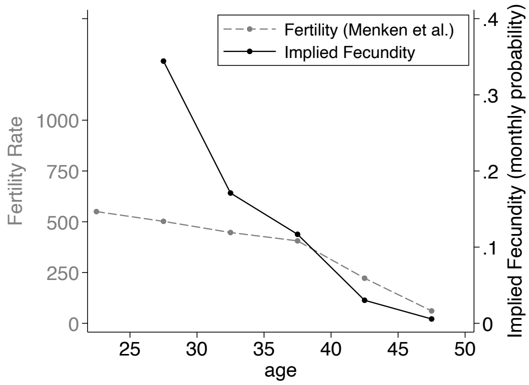
Note: Figure replicates the estimates of Figures 3 and A7 in a narrower sample defined by a smaller look-back period. Each panel makes the same restriction on recent sex as in Figure A7 Panel (b), but limits the look-back period to 3, 6, or 9 months, as indicated, rather than the standard five-year look-back in our main *fecundability sample*. Because of the shortened look-back, we change the outcome variable from an indicator for a pregnancy that resulted in a live birth beginning in that month to an indicator for a pregnancy beginning in that month, as pregnancies will still be ongoing given the short window. Definitions and specifications otherwise match panel (a) of Figure 3.

Appendix Figure A9: Robustness: Shape of age-fecundability profile in the NSFG is robust to additional restriction on woman-months with self-reported intercourse.

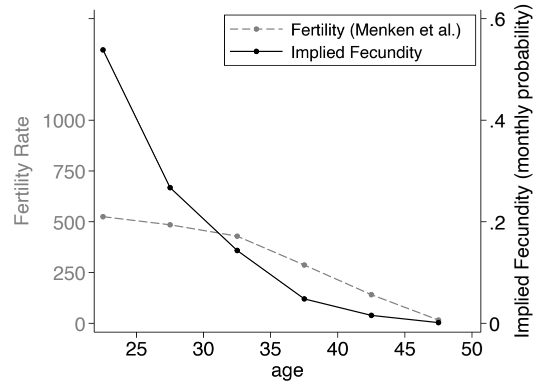


Note: Figure plots estimates of fecundability from NSFG data: United States, 2006–2010 and 2011–2013 survey waves. Replicates Figure 6 with the additional restriction to woman-months with self-reported intercourse. Definitions and specifications match panel (a) of Figure 3.

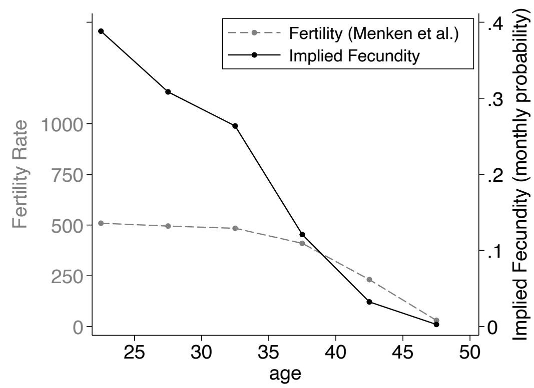
Appendix Figure A10: Breakout of Figure 7; each Menken et al. population separately



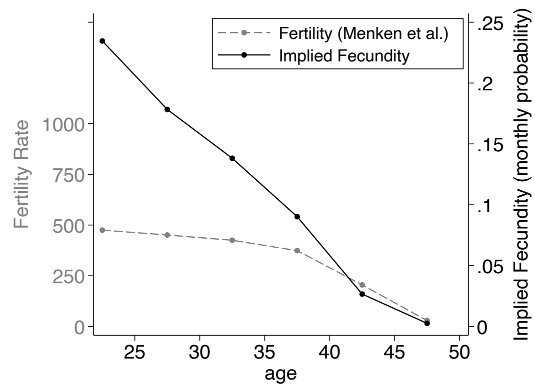
(a) Hutterites, 1921-30



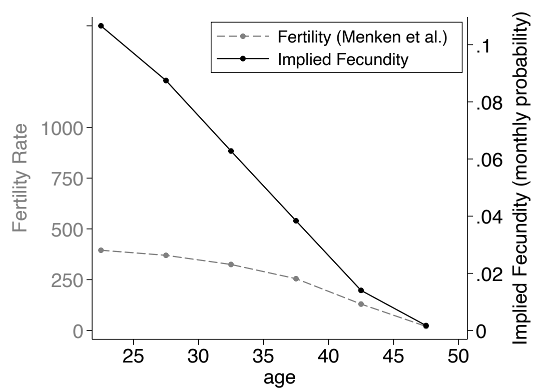
(b) Geneva bourgeoisie, 1600-49



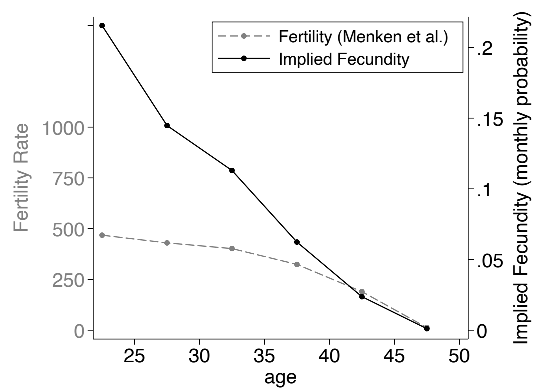
(c) Canada marriages, 1700-30



(d) Hutterites, before 1921



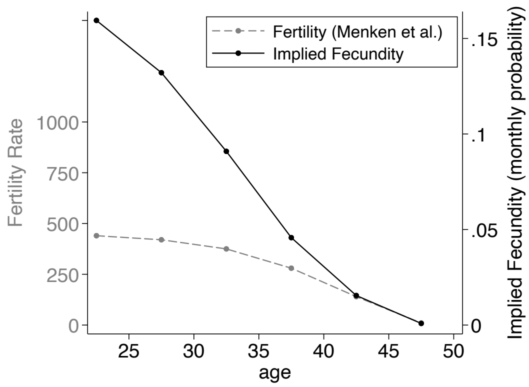
(e) Iran village marriages, 1940-50



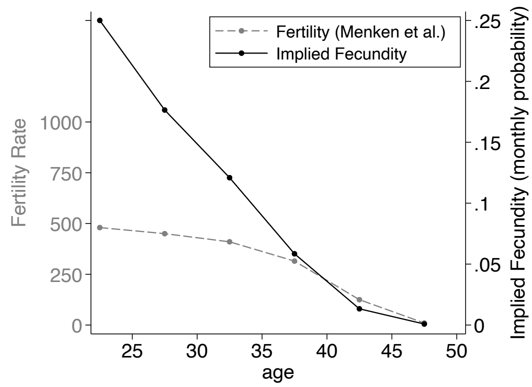
(f) Tunis, marriages of Europeans 1840-59

Note: Figure reproduces fertility rates as displayed in Figure 1 of Menken et al. (1986), separating populations by panel, as indicated in panel captions. (Compare to Figure A1, which pastes the original Menken et al. (1986) figure.) The *Implied fecundability* plot in each panel takes the measured fertility and de-biases it according to the formula in Equation 5 for $i = 21$, which assumes a 12-month infecundable period postpartum, identical to our main result in panel (a) of Figure 3. See Figure 7 for additional detail.

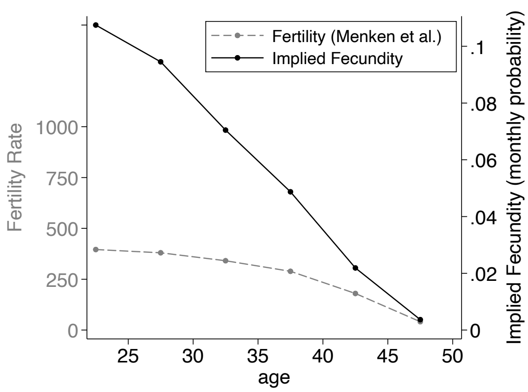
Appendix Figure 10 (Cont.): Breakout of Figure 7; each Menken et al. population separately



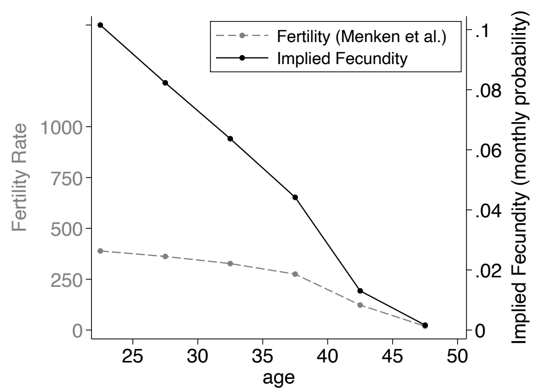
(a) Normandy, marriages 1674-72



(b) Normandy, marriages 1760-90



(c) Norway, marriages 1874-76



(d) Geneva bourgeoisie, before 1600

Note: Figure reproduces fertility rates as displayed in Figure 1 of Menken et al. (1986), separating populations by panel, as indicated in panel captions. (Compare to Figure A1, which pastes the original Menken et al. (1986) figure.) The *Implied fecundability* plot in each panel takes the measured fertility and de-biases it according to the formula in Equation 5 for $i = 21$, which assumes a 12-month infecundable period postpartum, identical to our main result in panel (a) of Figure 3. See Figure 7 for additional detail.

Appendix Table A1: DHS Survey Waves Included in Sample

Country Name	Dates of Fieldwork	Number of Respondents	Number of Respondent-Months	Respondent-Months At Risk
Afghanistan	3/1994-11/1994	29,461	2,386,341	656,475
Albania	10/2008-4/2009	7,584	614,304	72,494
Albania	9/2017-2/2018	15,000	1,215,000	222,046
Angola	10/2015-4/2016	14,379	1,164,699	253,642
Armenia	10/2000-12/2000	6,430	520,830	80,637
Armenia	9/2005-11/2005	6,566	531,846	95,183
Armenia	10/2010-12/2010	5,922	479,682	84,740
Armenia	12/2015-3/2016	6,116	495,396	92,179
Azerbaijan	7/2006-11/2006	8,444	683,964	125,276
Bangladesh	11/1993-3/1994	9,640	780,840	198,975
Bangladesh	11/1996-3/1997	9,127	739,287	175,640
Bangladesh	11/1999-4/2000	10,544	854,064	187,986
Bangladesh	1/2004-5/2004	11,440	926,640	186,316
Bangladesh	3/2007-8/2007	10,996	890,676	179,992
Bangladesh	6/2014-11/2014	17,863	1,446,903	280,477
Bangladesh	7/2011-12/2011	17,842	1,445,202	277,194
Benin	12/2011-4/2012	16,599	781,006	227,465
Benin	11/2017-2/2018	15,928	645,085	164,410
Bolivia	11/1993-6/1994	8,603	348,421	44,624
Bolivia	2/2008-6/2008	16,939	686,030	69,361
Brazil	9/1991-2/1992	6,223	252,031	28,648
Brazil	2/1996-7/1996	12,612	510,786	37,703
Burkina Faso	5/2010-12/2010	17,087	1,384,047	371,067
Burundi	8/2010-1/2011	9,389	380,255	69,245
Burundi	10/2016-3/2017	17,269	699,395	130,558
Cambodia	7/2010-1/2011	18,754	94,941	18,717
Cambodia	6/2014-12/2014	17,578	88,990	15,666
Colombia	5/1990-8/1990	8,644	350,081	31,423
Colombia	3/1995-7/1995	11,140	451,170	37,178
Colombia	6/2004-7/2005	41,344	1,674,432	148,598
Colombia	2/2000-6/2000	11,585	469,192	33,661
Colombia	11/2009-12/2010	53,521	1,762,910	147,971
Colombia	1/2015-3/2016	38,718	784,039	60,750
Comoros	8/2012-12/2012	5,329	26,979	6,458
Dominican Republic	7/1991-11/1991	7,316	148,149	20,976
Dominican Republic	9/1996-12/1996	8,422	170,546	22,996
Dominican Republic	8/1999-10/1999	1,286	26,042	3,167
Dominican Republic	7/2002-12/2002	23,384	473,525	57,605
Egypt	11/1992-2/1993	9,864	199,746	45,447
Egypt	11/1995-2/1996	14,779	299,275	68,902
Egypt	2/2000-5/2000	15,573	315,354	56,189
Egypt	4/2005-7/2005	19,474	394,348	64,520
Egypt	3/2008-6/2008	16,527	334,671	52,063

Kyrgyzstan	8/2012-12/2012	8,208	41,552	8,413
Lesotho	10/2009-1/2010	7,624	38,596	7,352
Lesotho	9/2014-12/2014	6,621	33,519	4,550
Liberia	3/2013-7/2013	9,239	46,772	11,913
Liberia	10/2019-2/2020	8,065	40,830	9,577
Madagascar	11/2008-7/2009	17,375	87,962	20,897
Malawi	1/2004-2/2005	11,698	59,221	13,674
Malawi	6/2010-10/2010	23,020	116,539	21,122
Malawi	10/2015-2/2016	24,562	124,345	22,915
Maldives	1/2009-10/2009	7,131	36,100	11,862
Maldives	3/2016-12/2017	7,699	38,977	13,103
Mali	11/2012-2/2013	10,424	52,771	16,744
Moldova	6/2005-8/2005	7,440	37,664	5,033
Morocco	1/1992-5/1992	9,256	46,858	7,324
Morocco	10/2003-2/2004	16,798	85,041	9,289
Mozambique	5/2011-12/2011	13,745	69,584	18,670
Myanmar	12/2015-7/2016	12,885	65,230	13,058
Namibia	11/2006-3/2007	9,804	49,632	6,087
Namibia	5/2013-9/2013	10,018	50,717	4,158
Nepal	1/2006-7/2006	10,793	54,639	11,027
Nepal	1/2011-5/2011	12,674	64,162	11,875
Nepal	6/2016-1/2017	12,862	65,114	11,547
Nicaragua	12/1997-5/1998	13,634	69,022	9,558
Nigeria	6/2008-11/2008	33,385	169,012	40,997
Nigeria	2/2013-7/2013	38,948	197,174	43,586
Pakistan	10/2012-4/2013	13,558	68,637	18,151
Pakistan	11/2017-4/2018	15,068	76,281	22,508
Papua New Guinea	10/2016-12/2018	15,198	76,941	16,372
Paraguay	5/1990-8/1990	5,827	29,500	3,903
Peru	10/1991-3/1992	15,882	80,403	8,085
Peru	8/1996-12/1996	28,951	146,565	15,542
Peru	7/2000-11/2000	27,843	140,954	14,322
Phillipines	4/1993-7/1993	15,029	76,084	10,976
Phillipines	3/1998-5/1998	13,983	70,788	9,516
Phillipines	6/2003-9/2003	13,633	69,018	10,867
Rwanda	9/2010-4/2011	13,671	69,210	11,193
Rwanda	11/2014-4/2015	13,497	68,328	8,764
Senegal	10/2010-5/2011	15,688	79,420	21,076
Sierra Leone	4/2008-9/2008	7,374	37,331	11,088
Sierra Leone	6/2013-11/2013	16,658	84,331	21,303
Sierra Leone	5/2019-9/2019	15,574	78,844	19,688
South Africa	6/2016-11/2016	8,514	43,103	5,116
Swaziland	7/2006-3/2007	4,987	25,247	5,545
Tajikistan	7/2012-9/2012	9,656	48,884	10,451
Tajikistan	8/2017-11/2017	10,718	54,259	13,247
Tanzania	10/2004-2/2005	10,329	52,291	11,365
Tanzania	12/2009-5/2010	10,139	51,328	8,970

Note: Table provides description of each DHS survey wave in sample.