Investor Betas *

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First draft: September 2020; This draft: April 5, 2023

We provide novel evidence that asset prices reflect covariance risk when betas are measured against institutional investor portfolios. Combining the insights of Markowitz (1952) with recent demand based frameworks, we propose a simple model of holding constrained investors that generates the following prediction: an assets expected return is linear in its beta with respect to each investor's portfolio return, averaged across institutions that hold the asset. Empirically, a unit increase in investor beta commands 5-8% greater annual expected returns in equity markets with a cross-sectional R^2 of approximately 80-90% in test portfolios. We observe a similar investor beta–returns relationship in the bond market where the return increase per unit of investor beta risk is over 100bps. Betas measured from the perspective of otherwise similar institutions that choose not to hold a particular asset are uncorrelated with returns.

KEYWORDS: CAPM, risk, return, idiosyncratic, portfolio, active, investor.

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*We thank Asaf Bernstein, Andrea Buffa, Martjin Boons, Shaun Davies, Francisco Gomes, Valentin Haddad, Ralph Koijen, Tim McQuade, Tyler Muir, Giovanna Nicodano, Jinfei Sheng, Chen Wang, Brian Waters, Jaime Zender and seminar participants at ASU, Baruch, Boulder, Collegio Carlo Alberto, Maryland, Michigan, NTU, Tulane, European Finance Association, and Western Finance Association for helpful comments.

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1 Introduction

Prompted by the failure of CAPM market betas to explain asset returns, numerous strands of finance literature have looked elsewhere for a risk-return trade-off. One approach, typified by Fama and French (1993), relies on ICAPM (Merton, 1973) or APT (Ross, 1976) logic to rationalize empirical factors which do generate robust risk-return relationships. This work drifts away from the canonical Markowitz (1952) insight that investors should demand higher returns for assets with high *portfolio* level covariance. More recently, He and Krishnamurthy (2012, 2013), Haddad and Muir (2021), Adrian, Etula, and Muir (2014), He, Kelly, and Manela (2017), and others emphasize that institutions, not representative households, are the key players in asset markets. Starting with Koijen and Yogo (2019), a large literature estimates reduced-form demand systems which are flexible functions of characteristics of both assets and investors.[1] These papers explicitly account for the large observed heterogeneity in portfolio holdings which is typically ignored in the asset pricing literature. In this paper we connect these two strand of research back to the seminal idea in Markowitz (1952) by demonstrating that an asset's covariance with one's own portfolio is a first-order determinant of perceived risk, and hence, expected return.

To motivate our empirical analysis, we develop an asset pricing model featuring partial segmentation as in Merton (1987), that shares the insight from Koijen and Yogo (2019) that institutional preferences can aggregate to impact prices. Our key innovation is to allow for returns that are "idiosyncratic" relative to a factor model but still have cross-correlations within a portfolio. In our setting, investors demand a higher return for assets that have a high covariance with their portfolio. Segmentation causes these fund level preferences to aggregate and drive a wedge between the risk experienced by the investors who hold an asset and the "factor" risk of the asset. The model predicts that an asset's expected return is linear in its beta with respect to *each* institutional investor's portfolio return, averaged

1. See also Koijen and Yogo (2020); Koijen, Richmond, and Yogo (2020); Jiang, Richmond, and Zhang (2020); Koijen, Koulischer, Nguyen, and Yogo (2021); Haddad, Huebner, and Loualiche (2021); Bretscher, Schmid, Sen, and Sharma (2022).

across all investors who own the asset.

We empirically estimate this relationship separately for equities and corporate bonds using quarterly holdings data to construct a time-series of portfolio returns for each institutional investor. For each asset $\times$ fund pair we estimate a time-series beta in a first stage regression. We then calculate the share-weighted average of the beta across funds, yielding our "investor beta" measure for each asset. Finally, we sort assets into ten decile portfolios according to investor beta and compute average returns by group. Panel A plots average returns vs investor betas for equities and Panel B does the same for corporate bonds. For both asset classes, the cross-sectional fit is nearly perfect. The resulting "investor security market line" for equities has an annualized slope of 6.0%. This is in contrast to the well known pattern of a nearly zero slope when plotting average returns against equity market betas.[2] For bonds, the slope is 1.1%. In panel regressions, after controlling for the market, we find slopes of 5.5% and 2.0% respectively, which, after accounting for differences in the underlying portfolio risk, correspond to implied risk aversion of 6.1 for equity investors and 3.8 for bond investors.

[Insert Figure 1 Near Here]

We perform several robustness tests to ensure our results are not mechanical. First, we exclude asset i when computing the portfolio returns and obtain a nearly identical result. Second, we run 24 month rolling betas excluding the month when we measure returns for the second stage from beta estimation in the first stage and find nearly identical risk premia. Finally, for equities, utilize daily data to estimate higher frequency betas.[3] Each quarter, we first sort stocks into portfolios based on β estimated using odd days, then regress returns on even day return data (and vice versa). The resulting premium for investor portfolio risk is slightly larger than the monthly estimates, further alleviating concerns of endogeneity or look-ahead bias.

2. See, for example, Black, Jensen, and Scholes (1972), Fama and MacBeth (1973), Baker, Bradley, and Wurgler (2011) and Frazzini and Pedersen (2014) among many others.

3. Infrequent trading of most corporate bonds precludes the measurement of daily returns.

If segmentation is not an important feature of markets, that is, if all investors are marginal in all assets, a similar beta/return relationship should exist when beta is measured with respect to funds who *do not* hold the asset. This theoretical result motivates our placebo test of segmentation. For each fund-stock pair, we find a "similar" institution that has approximately the same market beta, total assets, and number of positions but never holds the stock in question. We run the exact same process above but we measure betas between each asset and the placebo fund's returns. Aggregated across placebo funds, there is zero relation between placebo investor betas and stock returns. This finding is suggestive of partially segmented markets, even within an asset class.

Corporate bonds are different from equity shares in many ways. One interesting feature of bonds is that a firms often have multiple bonds outstanding. That is, adding firm fixed effects to a panel regression allows us to test the investor beta risk premium relationship within a given firm. Strikingly, we find that even when comparing bonds within a firm it holds true that bonds associated with higher investor betas exhibit higher excess returns. Only interacting firm fixed effects with time *and* bond maturity bucket fixed effects renders investor beta insignificant. This should not come as a surprise as a significant beta in this last regression specification would suggest the presence of (close to) arbitrage opportunities and a violation of the law of one price.[4]

Since heterogeneous portfolio holdings distort investor betas away from market betas, we investigate the characteristics associated with high investor beta firms and how they differ from those with high market betas. We find a number of stark differences. First, measures relating firm performance to price (e.g. dividend yield, price/sales, return on equity etc.) have opposite correlations between investor and market betas, where the performance to price relation is positively associated with investor betas and negatively associated with market betas. Higher profitability, as a characteristic, is associated with higher returns.

4. When a firm has multiple bond issues outstanding that have similar maturity dates and disagree in their prices, investors can realize close to risk-free arbitrage opportunities. Hence, in the absence of arbitrage opportunities, controlling for time × firm × maturity fixed effects should render the coefficient on the investor beta insignificant.

However, perhaps that return relation is partially attributable to the increased risk these firms impose on the funds that hold them. Second, we find that high investor beta firms have lower R&D expense and higher labor costs, while the opposite is true of high market beta firms. In these cases there is no well documented risk–return relation associated with these factors.

In light of the charactistic loadings, we next investigate whether our results in both equity and bond markets represent a novel form of risk which appears unrelated to previously documented risk factors in three ways. First, we include security or portfolio level beta estimates from the set of factor models we consider in the second stage price of risk regressions. Again, we find that including these betas does not change our results. Second, We re-run our first stage investor risk regressions excluding the market, and including multiple asset class appropriate risk models. We then aggregate these investor betas as before. That is, the resulting investor betas are orthogonalized to the relevant risk model. We find that investor betas are priced no matter the risk model we choose, though the exact slope coefficient can change for some of the models. The few instances of lower coefficients indicate that other successful risk factor models may be implicitly capturing some element of aggregate investor portfolio risk. Lastly, we can replicate all of our analysis using DGTW excess returns and find similar results.

1.1 Related Literature

This paper contributes to a large literature which studies the performance of institutional investors. Van Nieuwerburgh and Veldkamp (2010) show that because information acquisition features increasing returns to specialization, active managers may hold quite dissimilar and underdiversified portfolios. Wermers (2000), Chen, Jegadeesh, and Wermers (2000), Cohen, Gompers, and Vuolteenaho (2002), and Massa, Reuter, and Zitzewitz (2010) show that fund managers do appear to have some stock picking ability. Kacperczyk, Sialm, and Zheng (2005) show that funds which follow a relatively narrow investment mandate tend to outperform funds which follow a broader strategy. Our results contrast with the findings in Fama and French (2010) because we incorporate essential investor heterogeneity whereas that paper does not.

Several other works demonstrate that fund holdings have valuable information content. Wermers, Yao, and Zhao (2012) use past performance to rank funds and show that differences in holdings between skilled and unskilled managers predicts returns. Shumway, Szefler, and Yuan (2011) use a mean-variance framework similar to ours to transform holdings into beliefs to first rank managers by skill and then use the difference in beliefs among skilled and unskilled managers to identify good investments. Using a closely related theoretical setup, Antón, Cohen, and Polk (2020) identify each manager's "best idea" from portfolio holdings and show that these stocks do outperform. Jiang, Verbeek, and Wang (2014) show that stocks which mutual funds, in aggregate, overweight have relatively high future returns. Our results agree with this prior work that holdings contain useful information which aggregates across managers, indicating that managers have some stock-picking ability. However, we additionally show that managers are aware of a risk–return trade off which also meaningfully aggregates.[5] That is, we do not merely posit a risk–return trade off to extract information from holdings. We show this trade off is quantitatively present in the data.

Our finding that expected returns are positively linearly related to covariance with the investor's chosen portfolio may seem at odds with the literature documenting a negative correlation between idiosyncratic variance and average returns.[6] However, the measures are economically different from each other. High investor betas occur when a stock positively covaries with the rest of an investor's portfolio (averaged across holders). This covariance is determined both by the overall composition of the portfolio as well as the weight of the

5. Some results in Shumway, Szefler, and Yuan (2011) also "suggest that fund managers do care about risk when making portfolio decisions."

6. Falkenstein (1994), Ang, Hodrick, Xing, and Zhang (2006), and Ang, Hodrick, Xing, and Zhang (2009) find a negative relation. Fu (2009) finds a positive relation using GARCH instead of realized variances. Bali and Cakici (2008) examine the robustness of these results and find essentially zero relation after excluding small or low priced stocks. We use these same exclusion criteria and find a robustly positive relation between idiosyncratic covariance and returns.

stock in the portfolio. High idiosyncratic risk, in contrast, simply measures residual variance, independent of portfolio composition. Thus there is no inherent relation between investor betas and idiosyncratic risk.

The remainder of the paper is organized as follows. In Section 2 we present the theoretical framework which guides our empirical analysis. Section 3 lays out the methodology and data summary. Section 4 presents our the findings and in Section 8 we conclude.

2 Theoretical Framework

In this section we present a simple partial-equilibrium framework to guide the empirical analysis in Section 4. This analysis yields an empirically useful characterization of expected returns.

As in Merton (1987, henceforth "Merton"), each institutional investor may only invest in a subset of securities. Merton motivates this constraint with fixed holding costs which may vary by asset $i \times$ investor j. It is further motivated by He and Xiong (2013), who show that narrow mandates can arise as optimal contracts for delegated asset management. Our key departure from Merton is that we allow for cross-correlation of "idiosyncratic" asset returns, returns residualized with respect to some factor model such as the CAPM. This is a salient feature of the data. Our model predicts that an asset's alpha (with respect to the factor model) is linear in its beta with respect to investors' portfolio returns (orthogonalized with respect to the same factor model).

In what follows, generally, uppercase letters are matrices, bold lowercase are vectors, and all others are scalars. Rather than solving for an equilibrium by clearing all markets, we derive equilibrium relationships by examining investors' first-order condition for portfolio optimality.

2.1 Model Environment

We assume a two period economy with discrete time $t = 0, 1$ and satisfying no arbitrage. There are N tradable assets in the economy, indexed by $s = 1, ..., N$. The equilibrium net risk-free rate is r_f. We assume the assets feature limited liability. Combined with no arbitrage, this implies strictly positive prices and hence rates of return exist. Let $\tilde{\boldsymbol{r}}$ be the vector of excess returns on the N assets with covariance matrix $\widetilde{\Sigma}$.

Define $k \ll N$ (arbitrary) factors as $\boldsymbol{f} = A\tilde{\boldsymbol{r}}$ with $\text{rank}(A) = k$.[7] In Merton, f is the market portfolio, but here it may be anything (including the empty set). Let $\Omega = \text{cov}\,(f) = A\widetilde{\Sigma}A'$. The betas of the assets with respect to the factors are given by

$$\beta = \left(A\widetilde{\Sigma}A'\right)^{-1}\left(A\widetilde{\Sigma}\right).$$

Define factor neutral excess returns as

$$\boldsymbol{r} = \tilde{\boldsymbol{r}} - \beta'\boldsymbol{f}$$

with covariance matrix Σ.[8] For any portfolio weights $\boldsymbol{\theta}$, the factor-neutral realized return is

$$r_\theta = \boldsymbol{\theta}'\boldsymbol{r} = \boldsymbol{\theta}'\tilde{\boldsymbol{r}} - [\boldsymbol{\theta}'\beta']\,\boldsymbol{f}.$$

Hence, any portfolio can be written as

$$\tilde{r}_\theta = \boldsymbol{b}'\boldsymbol{f} + \boldsymbol{\theta}'\boldsymbol{r},$$

with $\boldsymbol{b}' = \boldsymbol{\theta}'\beta'$ and $\text{cov}\,(\boldsymbol{r}, \boldsymbol{f}) = 0$.

7. Without loss of generality, the factors are tradable since in what follows any factor can be replaced by its mimicking portfolio.

8. In general, Σ has rank $N - k$.

2.2 Investors and "Equilibrium"

There are J masses of institutional investors ("funds") having initial wealth W_j and mean-variance utility over time 1 wealth. That is, they maximize

$$\mathcal{U}(\theta) = \mathbb{E}_j(\tilde{r}_\theta W_j) - \frac{\gamma}{2W_j}\mathbb{V}(\tilde{r}_\theta W_j), \tag{1}$$

where subscripts indicate subjective beliefs. We assume investors agree on covariances. Their beliefs about means are dogmatic; investors agree to disagree. There may be other traders with arbitrary preferences, beliefs, and endowments.

Like in Merton (1987), each investor of type j may only hold a subset $\mathbb{S}_j \subseteq \{1 \dots N\}$ of the universe of assets, as well as the risk-free asset and factor ETFs. Let S_j be a diagonal matrix with entries equal to 1 if the asset is in $\mathbb{S}_j$ and 0 otherwise; S_j is called a "selector" matrix. Maximizing utility given in Eq. (1), fund j's mean-variance optimal portfolio is given by

$$\boldsymbol{b_j} = \frac{1}{\gamma}(\Omega)^+ \boldsymbol{\mu_j} \tag{2}$$

$$\boldsymbol{\theta_j} = \frac{1}{\gamma}(S_j \Sigma S_j)^+ \boldsymbol{\alpha_j}, \tag{3}$$

where $(\cdot)^+$ indicates the Moore-Penrose pseudoinverse, and $\boldsymbol{\mu_j}$ and $\boldsymbol{\alpha_j} = \mathbb{E}_j(\boldsymbol{r})$ are fund j's subjective beliefs about factor risk premia and alphas, respectively.[9] This is the standard mean-variance solution with the constraint that if asset $i \notin \mathbb{S}_j$ then $\theta_{j,i} = 0$. Let $r_{\theta,j} = \boldsymbol{\theta_j}'\boldsymbol{r}$ be the factor-neutralized return on fund j's portfolio. Note that if the covariance matrix, Σ, is diagonal, Eq. (3) matches equation (9.b) of Merton.

Given the optimal portfolio, we can compute the covariance of any stock $r_i \in \mathbb{S}_j$ with $r_{\theta,j}$ to obtain the investor's subjective risk-return trade-off for factor-neutral "idiosyncratic"

9. We use the convention that if asset i is not in $\mathbb{S}_j$ (not allowed to be held by investor j) then $\alpha_{j,i} = \alpha_i$, the objective value.

betas

$$\alpha_{j,i} = \gamma \, \hat{\mathrm{cov}} \left(r_i, \, r_{\theta,j} \right). \tag{4}$$

Each investor adjusts her portfolio weights so that her Euler equation holds for all assets which she can trade.

2.3 Aggregation

For the model to have any empirical content, we require some restriction on beliefs, $\alpha_{i,j}$. Unlike much of the literature which assumes strict rationality, we assume a weaker condition, "group rationality." We make no assumption about subjective factor premia, μ_j.

Assumption 1 (Group Rationality). A weighted-average belief across investors who hold an asset is correct, where the weights are observable to the econometrician. Let $\kappa_i \subseteq \mathbb{K}$ be the set of investors who may hold asset i. Then group rationality requires

$$\alpha_i = \sum_{\kappa_i} \omega_{j,i} \, \theta_{j,i} \, \alpha_{j,i}, \tag{5}$$

Two particular choices of weights, shares or wealth, are motivated in Appendix B. There we show that in an unconstrained model, belief errors "wash out" (do not affect equilibrium prices) using wealth weights. With Merton-style constraints, this obtains using share weights.

Combining Eq. (4) with Assumption 1 we obtain the objective pricing relation

$$\alpha_i = \gamma \sum_{\kappa_i} \omega_{j,i} \, \mathrm{cov} \left(r_i, \, r_{\theta,j} \right) = \gamma \, \mathrm{cov} \left(r_i, \, \sum_{\kappa_i} \omega_{j,i} \, r_{\theta,j} \right). \tag{6}$$

2.3.1 β Representation

Since most empirical work uses betas instead of covariances, we now derive a beta version of eq. (6). Start with eq. (4), multiply by $1 = \mathbb{V}(z_j) / \mathbb{V}(z_j)$, and rearrange to obtain a β

representation for fund $j's$ subjective beliefs,

$$\alpha_{j,i} = \beta_{j,i} \left[\gamma \, \mathbb{V} \left(r_{\theta,j} \right) \right], \tag{7}$$

where $\beta_{j,i}$ is the regression beta of r_i on $r_{\theta,j}$. If $\mathbb{V}\left(r_{\theta,j}\right)$ is approximately equal across funds, then we obtain the objective pricing relation

$$\alpha_i = \left[\gamma \, \mathbb{V} \left(r_\theta \right) \right] \sum_{\kappa_i} \omega_{j,i} \, \beta_{j,i} = \left[\gamma \, \mathbb{V} \left(r_\theta \right) \right] \beta_i^{Investor} \tag{8}$$

That is, a securities α is proportional to the investor beta, $\beta_i^{Investor}$, where $\beta_i^{Investor} = \sum_{\kappa_i} \omega_{j,i} \, \beta_{j,i}$. In our empirical analysis we primarily focus on this expected return investor beta relationship.

2.3.2 A General Equilibrium Example

In Appendix B we present a general equilibrium framework consistent with the above partial equilibrium model. There we also solve a stark four asset, four investor example to highlight the mechanism. The economy offers near arbitrage (very high Sharpe ratio) trading opportunities. Without Merton-style constraints, investors trade aggressively against these opportunities. In equilbrium, mispricing almost disappears. With constraints, no investor is able to fully trade against the arbitrages, leading to large equilibrium mispricing.

Figure 2 plots the stark 4 asset case with the following conditions. A) while investors do not face strict segmentation, no investor can simultaneously hold assets 1 and 3 or 2 and 4. B) Assets 1 and 3 are negatively correlated with eachother as are assets 2 and 4. C) Assets 1 and 3 are in high supply while 2 and 4 are in low supply, with the market positively correlated with assets 1 and 4.

[Insert Figure 2 Near Here]

When measured against a CAPM model, asset 3 appears to offer extremely high returns for its level of risk while asset 4 offers low returns as depicted in Panel A. However, after accounting for the holding constraints by measuring betas with respect to the investors that hold each asset, investor betas correctly line up with the returns on each asset. This simple example delivers the main insight of the model: segmentation can drive a wedge between investor perceived covariance risk and market level covariance risk. As long as investor covariance is correlated across investors (e.g. does not wash out when aggregated) then investor level betas will correctly price assets and the security market line predicted by the CAPM model will be too flat.

3 Methodology

In this section we first discuss how we empirically measure investor betas and how it maps into the theoretical framework discussed in Section 2. Secondly, we provide a description of the data and sample summary stats.

3.1 Investor Beta Measurement

The central assumption in our model that investors largely care about an asset's covariance with their chosen portfolio. More precisely, given the importance of relative pay performance contracts and the likelihood that the market portfolio is easily investable, our model suggests that investors care about the covariance which obtains after first orthogonalizing the asset and portfolio with respect to the market. Thus, we estimate multi-factor betas for each manager using the two factor specification in Equation (9)

$$r_{it} = a_{j,i} + \beta_{j,i} r_{j,t} + \beta_{j,i}^{mkt} r_t^{mkt} + e_{j,i,t}, \tag{9}$$

for fund j, asset i, and time t where r_t^{mkt} is the return on the market and $\beta_{j,i}^{mkt}$ the fund and asset specific market beta. By the Frisch-Waugh-Lovell theorem, $\beta_{j,i}$ exactly maps to equation (7) in the model.

In our base specification, we calculate the monthly portfolio return $r_{j,t}$ for each investor by measuring the set of portfolio holdings at the quarterly date q, the most recent quarter relative to time t. That is $q < t$ and $q + 3 \geq t$. We then obtain the investment weights for each asset i at time t as follows:

$$w_{j,i,t} = \frac{X_{j,i,q} \times P_{i,t-1}}{\sum_{k=1}^{N} X_{j,k,q} \times P_{k,t-1}}, \tag{10}$$

where $X_{j,i,q}$ is the number of shares that investor j owns of stock i at the end of quarter q and $P_{i,t-1}$ is the price of asset i at the end of the previous month. Monthly portfolio returns are defined as the weighted average return of all assets held by an investor j.

$$r_{j,t} = \sum_{k=1}^{N} w_{j,k,t} \times r_{k,t} \tag{11}$$

Thus the regression equation (9) can be rewritten to reflect the portfolio return calculation as follows:

$$r_{i,t} = a_{j,i} + \beta_{j,i} \sum_{k=1}^{N} w_{j,k,t} \times r_{k,t} + \beta_{j,i}^{mkt} r_t^{mkt} + e_{j,i,t} \tag{12}$$

Equation (12) is akin to the first stage of a two factor model, estimated at the fund by asset level. While the covariance of each asset with the market includes the asset's individual variance term, they are, in practice extremely small and unlikely to play much of a role in the calculation of market betas. For the beta on the portfolio return, $\beta_{j,i}$, this may not be the case. The variance term of an over-weighted asset in a small portfolio of assets may contribute in a meaningful way to the overall portfolio variance. This is not a shortcoming of the framework, since the first-order condition of a constrained manager necessarily embeds the contribution of each asset held to the covariance of a newly considered asset with the entire portfolio. However, in order to address concerns that our results are driven by a mechanical relation between variance and returns, we consider two alternative ways of calculating the portfolio return betas, $\beta_{j,i}$.

First, we exclude the reference asset from the fund's portfolio as follows:

$$r_{i,t} = a_{j,i} + \beta_{j,i} \sum_{k \neq i}^{N} w_{j,k,t} \times r_{k,t} + \beta_{j,i}^{mkt} r_t^{mkt} + e_{j,i,t} \tag{13}$$

The portfolio return no longer includes asset i, so we explicitly break any link between an investors' betas and asset returns. Second, we calculate betas excluding the month we use to measure asset returns. Thus in our second stage when we compare realized returns to our beta estimates, it is not possible that the betas are mechanically related to those returns since they are measured using a non-overlapping return sample.

In order to move from fund–CUSIP betas to our asset level investor beta, we aggregate our betas in a way consistent with equation (8) of our model presented in Section 2. In particular the Investor Beta for asset i at time t is the weighted sum of each investor beta for that asset in that period. We aggregate according to equation (14),

$$\beta_i^{Investor} = \sum_{\kappa_{i,t}} \frac{X_{j,i,q}}{\sum_{\kappa_{i,t}} X_{j,i,q}} \beta_{j,i} = \omega_{j,i,t} \beta_{j,i} \tag{14}$$

where $\kappa_{i,t}$ indexes the set of funds holding asset i at time t and $X_{j,i,q}$ is the number of shares of asset i held by investor j at the beginning of quarter q. Since we focus on managers we consider constrained in holding a particular portfolio, we weight the betas relative to all other constrained managers who hold that asset at that time.[10]

3.2 Data and Sample Summary Stats

3.2.1 Data

We empirically examine our model implications both in the context of equities and corporate bonds. That is, for both asset classes, we combine holdings data with pricing infor-

10. In the baseline analysis we discard first stage betas that are either lower than the 10th or larger than the 90th percentile. Importantly, this does not constitute dropping assets from the final analysis since all assets are held by multiple funds. Our main results are robust to including these noisy betas in the aggregation. Doing so flattens the investor beta–return relationship due to the two extreme portfolios.

mation and asset specific static characteristics such as, for example, the name of the issuing company. For equities, we rely on the 13F filings to identify investors' quarterly portfolio holdings. We then merge the holdings data with either the CRSP daily or monthly return files which allows us to construct equity portfolio returns as outlined in (11). Moreover, we narrow the sample by additionally merging our data set with the characteristic portfolios formed in Fama and French (1992). Approximately 70% of the initial CUSIPs also appear in FF25 portfolios. The CUSIPs that do not appear in the FF25 portfolios are extremely small firms with insufficient accounting data. In fact, we are able to calculate investor betas for over 99% of the assets that appear in FF25 portfolios. We further condition each stock to have greater than $2 share price and drop microcaps (the bottom quintile of firms in any period).[11] Finally, we only include equity investors with portfolios consisting of at least ten shares at a given point in time.

Similarly, we calculate corporate bond portfolio returns by merging quarterly bond holdings data from Thomson Reuters eMAXX, monthly return data from the WRDS Bond Returns, and static bond characteristics from the Fixed Income Securities Database (FISD).

The monthly WRDS Bond Returns database is based on the corporate bond transactions reported in the TRACE (Trade Reporting and Compliance Engine) database. As some bonds may not trade frequently and thus may not be present in the WRDS database, we check the quality of the coverage with respect to the overall U.S. corporate bond universe. To construct the U.S. corporate bond universe, we follow an approach similar to Asquith, Au, Covert, and Pathak (2013) and identify corporate bonds in FISD that are denominated in U.S. dollars, are issued by firms domiciled in U.S., and are publicly traded. Our definition of U.S. publicly traded corporate bond universe yields the total outstanding (by par value) of 6.5 trillion US dollars in 2019.[12] During our sample period, on average, we observe bond holdings that collectively account for 45% to 50% of the total par amount outstanding in

11. Note, this eliminates about 1/2 of the firms in the bottom FF25 size quintile since those cutoffs are based on NYSE breakpoints

12. According to SIFMA total U.S. corporate bond outstanding is $9 trillion as of 2019.

the corporate bond market (for a detailed year by year statistics see Appendix Table A.1).[13]

In a final step, we merge each of our two data sets with Compustat to additionally obtain issuer specific information. Further, we use the CRSP value-weighted index (bond market index of Bai, Bali, and Quan (2021)) as a proxy for the equity (bond) market factor.

3.2.2 Sample Summary Stats

Panel A of Table 1 reports some of the characteristics of our final equity sample. The sample period starts with the 13F coverage in 1980. That is, we calculate betas using data for 9,541 distinct 13F managers and we end up with 1.45 million share x month observations. Equity investors in our sample hold on average 173 positions, though this is right skewed with the median fund averaging 73 assets per reporting period. Portfolio returns are positively correlated with the market. The average R^2 in a univariate regression of investor portfolio returns against the market portfolio is 0.73. The average market capitalization of our sample firms equals \$2.9bn with the median firm being much smaller at \$306mm.

[Insert Table 1 Near Here]

Panel B of Table 1 reports analogous statistics for the corporate bond sample. Even though the bond sample only starts in 2006, we observe 9,520 different investors and about 740 thousands bond x month observations. The average bond investor holds 269 bonds while the median investors has 113 positions. The average bond in our sample has a face of about 600mm and 7.5 years time to maturity.

Before calculating investor betas, we analyze the extent to which we observe partial segmentation in equity markets across equity and corporate bond investors. Rather than looking at the overlap of holdings directly, we can answer this question by examining the covariance matrix of their returns orthogonalized to the market. We orthogonalize the returns with respect to the market to be consistent with our baseline specification of investor betas

13. for which we observe holdings for more than 25% of the total amount outstanding. However, all results remain unchanged if we instead include all the bonds in our sample.

from equation (9). Figure 3 shows a scree plot of the cumulative percentage of variance explained from successively increasing the number of principal components used to explain the return covariance matrix.[14] Even 100 PCs explain less than 50% of the total variance, implying that there is substantial heterogeneity across portfolios. This is important, since if they were all perfectly correlated, an asset's covariance with investors' portfolio returns could not vary across investors.

[Insert Figures 3 And 4 Near Here]

A high correlation between fund returns and market returns implies that betas measured in a single factor framework likely conflate the two sources of risk. Figure 4 Panel (A) illustrates that single factor beta estimates for investor betas and market betas are highly correlated. The cross-sectional correlation is 56%. However, as shown in Panel (B), the multi-factor investor beta and single-factor market beta have nearly zero correlation.

To sum up, both equity and bond investors' portfolio holdings are very segmented. As a consequence, there is a lot of cross-sectional variation in investor betas. In fact, Panels A.3 and B.3 of Table 1 show that there is as much variation in investor betas as in market betas both for equities as well as for corporate bonds. Moreover, as expected, the single factor market and investor betas are each close to one with 1.10 and 1.05 for equities and 0.90 and 0.93 for bonds, respectively.

Further, we examine whether investor betas capture characteristics that are known to be associated with returns from previous studies. To that end, Table 2 reports time-series averages of selected WRDS financial ratios for portfolios sorted on investor and market betas, respectively.[15]

[Insert Table 2 Near Here]

14. We include funds with at least 180 months of data. Missing correlations are replaced with zero, approximately the average non-missing value.

15. Table A.2 in the appendix contains the full comparison of all variables.

Some stark differences among equity investors in Panel A of Table 2 deserve mention. Whereas market beta is negatively correlated with a firm's dividend yield, investor beta is not. A similar pattern holds for the dividend payout ratio, the after-tax return on common equity, and labor expense to sales ratios. Opposite signed patterns across investor or market beta quintiles hold for measures of sales and R&D expenses. To summarize, investor betas are positively correlated with various price and fundamental measures of value which have been previously shown to be related to anomalous average returns which suggests that equity investors are decidedly choosy.

Panel B of Table 2 reports that also betas of corporate bond investors are differently correlated with firm fundamentals compared to bond market beta. For example, as for equity investors, bond investor betas are positively with dividend payout ratio whereas market betas are uncorrelated. Similarly the correlation of investor and market betas with total debt to asset ratio is positive and negative, respectively. Further, the labor expense to sales ratio and the after-tax return on equity exhibit a similar pattern with opposite signs.

4 Results

The main finding of this paper is reported in Figure 1. For each month, we generate 10 portfolios sorted by multi-factor investor betas as measured according to equations (12) and (14). We plot the investor beta of each portfolio (x-axis) against the annualized historical excess returns in excess of the risk-free rate (y-axis). The figure displays a clear, approximately linear, risk reward relation when risk is measured using investor portfolio betas. The slope of this "SML" implies additional returns of 6.0% per year per unit increase in a portfolio's investor beta for equities and 1.1% per year for bonds.

Controlling for the market return in the first stage regression eliminates any correlation between investor betas and market betas, as illustrated previously in Figure 4. However, since investor betas and market betas are correlated when estimating using a single factor model, and previous research has shown the CAPM SML is too flat, we expect that there will be some attenuation of the price of risk estimate using a single factor investor beta in equities. Figure 5 plots the same relation but using a univariate model to estimate investor betas (not controlling for the market return in the first stage estimation of these betas). While higher univariate investor betas appear to yield higher returns, the compensation per unit of risk in equity markets is substantially lower: 4.41% additional annual return per unit beta increase (see Panel A). For corporate bonds the opposite is true. That is, as the corporate bond market factor is priced in the cross-section of bond returns, excluding the bond market from the first stage results in a higher return compensation per unit of risk. As reported in Panel B of Figure 5, the additional return from a unit increase in the estimated single factor investor beta is 3.30% per year.

[Insert Figure 5 Near Here]

In a next step, we augment our analysis by testing the relationship between investor beta and excess returns in the context of panel regressions. For equities, we start with 25 size and book to market portfolios as calculated according to Fama and French. Within each portfolio we subsequently sort on investor beta.[16] We then run a panel regression of monthly portfolio excess returns against either only investor betas or additionally including market betas. In columns (1) and (2) of Table 3, betas for each fund×CUSIP pair are estimated over the full sample for which the fund holds that particular security. The estimated premium on investor beta is 6% per year, and does not substantially change when including market beta. Consistent with prior literature, we find essentially zero premium associated with market beta.

[Insert Tables 3 And 4 Near Here]

The panel regression results deliver two important insights beyond the baseline analysis in Figures 1 and 5. First, we are able to two-way cluster on both portfolio and time dimensions

16. Performing conditional sorts assures that all portfolios are populated, though our results are unchanged if we sort unconditionally on investor beta.

which eliminates the possibility that correlation in the errors within portfolio across time or across portfolios for each time period, both common concerns in asset pricing, artificially increase the statistical significance of our results. Second, presorting on the FF25 portfolios helps to eliminate concerns that our main sorts are simply capturing previously documented risk factors. We discuss the latter further in Section 7.

4.1 Robustness

If returns are not normally distributed then investor betas may be mechanically related to realized asset returns over the measurement window. Therefore, we calculate two alternative robust investor betas to rule out any mechanical relationship between returns and investor betas. First, we modify our first stage estimation according to equation (13) which excludes the asset of interest from the investor's portfolio when calculating betas. That is, we remove the returns of asset i from the portfolio of investor j when estimating the beta of asset i's return, $r_{i,t}$, with respect to investor j's portfolio return, $r_{j,t}$. Doing so rules out the possibility that high realized returns on asset i affect $\beta_{j,i}$.

The second approach we take to eliminate concerns of a mechanical investor beta–return relation is to exclude the month in which we measure the return in the second stage from the beta estimation. That is, we calculate $\beta_{j,i,t}$ according to equation (12) based on rolling windows 12 months prior and 12 months after the return date, excluding the return date t. Both methods rule out the possibility that our high beta portfolios are mechanically associated with high, contemporaneous returns.

For equities, columns (3) to (6) of Table 3 report the results for these alternative betas Importantly, we observe similar price of risk estimates for the investor beta factor across both robustness methodologies. That is, a unit increase in investor beta risk is associated with 3-4% higher annual return. The coefficients are all highly statistically significant. The market price of risk estimates are slightly noisier in these regressions, but lie always within the 95% confidence bounds of the baseline estimate in column (2).

Finally, we replicate the monthly analysis for equities using daily return data to estimate betas. The higher frequency permits us to allow for more time-variation in betas which may sharpen our estimates. We calculate betas at the asset $\times$ fund $\times$ quarter level using daily returns. In order to exclude any possibility of a mechanical relation between returns and betas, we estimate two separate betas in the spirit of Jegadeesh, Noh, Pukthuanthong, Roll, and Wang (2019), one during odd days in the quarter and one during even days. We then project even day returns on odd day betas and vice versa.

[Insert Figure 6 Near Here]

In Figure 6 we plot annualized excess returns (y-axis) vs betas measured over the opposite days (x-axis). The figure shows a tight fit with a slope of approximately 8% per year, and an R^2 of approximately 90%. The slope is larger than what we observe in our monthly analysis, likely due to more precise beta measurement at the daily frequency.

Columns (7) and (8) of Table 3 present the analogous panel regression results for the daily analysis. Again, we observe higher coefficients. This is as expected since monthly betas contain measurement error which leads to attenuation bias in the second stage estimation of the price of risk. As with monthly estimation, the estimated price of risk for market beta is nearly zero.

In summary, our results do not appear to be driven by a mechanical relation between returns and investor betas. Breaking any possible link between the measurement of beta and the dependent variable in the regression (either through excluding the stock from the portfolio estimate or excluding the return date t from the rolling beta measurement window) yields risk estimates that are stable across methods, though slightly higher when we focus at higher frequency beta estimation.

4.2 Corporate Bonds

As for equities, we augment the baseline figures by running panel regressions of monthly corporate bond excess returns on either investor betas alone and including market betas. In contrast to our analysis for equities, we do not form portfolios but run the regressions at the security level and saturate them with either time or time×rating fixed effects.[17] Table 4 reports the estimation results. Columns (1) and (2) contain the estimated coefficients of betas for each fund×CUSIP pair that are estimated over the entire period an investor holds that particular bond. The estimated premium on investor beta is 1.2% per year, about four times smaller compared to equities. However, similar to equities, the coefficient does not substantially change when additionally controlling for the market beta in column (2).

Column (2) allows us to compare the magnitude of Investor Betas with that of Bond Market betas. We find that a unit increase in investor betas commands about double the return premium of a unit increase in market betas in the corporate bond market. To put this magnitude in context, this is approximately the size of moving from a AAA bond to a BBB bond in terms of expected return differential.

We also run a corresponding analysis to columns (3) to (6) of Table 3 in the bond universe. There, we eliminate concerns of a mechanical relation between returns and betas by estimating investor betas in two different ways: i) excluding the specific bond from the investment portfolio and ii) excluding the return measurement period from the beta estimation. As with equities, our results remain stable over these specification. In fact, here the correlation across measures is higher, and the time series stability of monthly beta measurements allows for a larger variety of rolling windows estimates. Due to the infrequent nature of bond trading, a daily analysis is not possible with corporate bonds.

4.3 Alternative Beta Measurement

In principal, share weighted investor betas should be approximately proportional to measuring betas for each stock using a two factor model: the market, and the share weighted

17. Forming portfolios at the bond level would inhibit some of the bond-specific analysis we run later in the paper. Our results are robust to pre-forming rating and maturity based portfolios.

portfolio returns of the set of investors that hold an asset as defined in equation (15).

$$r_{i,t}^{portfolios} = \frac{\sum_{j=1}^{J} r_{j,t} * X_{j,i,q}}{\sum_{j=1}^{J} X_{j,i,q}}. \tag{15}$$

Given the "portfolio" return, $r_{it}^{portfolios}$ (which differs across assets i), we estimate betas at the stock level using the traditional first stage time series regression:

$$r_{i,t} = a_{j,i} + \hat{\beta}_i^{Investor} r_{i,t}^{portfolios} + \beta_i^{mkt} r_t^{mkt} + e_{i,t}. \tag{16}$$

There are two issues with this approach, one theoretical and one empirical. First, $\hat{\beta}_i^{Investor}$ will not be equal to the share-weighted average investor beta. Mathematically, the two will have the same numerator, but $\hat{\beta}_i^{Investor}$ has a much smaller denominator due to diversification across funds. Hence, the price of risk estimate no longer has the interpretation of reward per unit of idiosyncratic risk from investors' perspective. Second, this same diversification leads to a collinearity problem in estimating first stage betas. Calculated at the asset level, $r_{i,t}^{portfolios}$ is 95% correlated with the market. In our baseline analysis we exclude asset/investor pairs that have outlier betas before aggregating. This exclusion does not meaningfully reduce our sample—all stocks are held by at least a few investors with reasonable beta estimates. Nevertheless, trimming outlier betas excludes some stocks entirely from our sample and as such we view this method as inferior.

[Insert Figure 7 Near Here]

However, we find that investor betas calculated using this alternative methodology still convey meaningful information about both stock and bond returns as can be seen from Figure 7. That is, we find a similar, robust, upward slope with a high R^2 at the portfolio level both for equities as well as for corporate bonds. Thus, our results are qualitatively robust to aggregating portfolio returns for each stock and then calculating the price of risk estimates by means of a traditional two factor time series regression.

5 Placebo Investor Betas

The model draws a clear connection between the perceived risk of investors that choose to hold an asset and the expected returns of that asset. Conversely, investors that choose *not* to hold an asset should have no impact on the expected returns of that asset.

We test this idea by matching a placebo fund that does not hold a specific asset as follows. For each investor A that holds asset X we find a set of investors that are similar to investor A but do not hold asset X. We identify similar investors according to size, market risk, and number of portfolio positions. That is, the set of investors similar to investor A consists of all investors that do *not* hold asset X and belong to the same decile in terms of market beta (equity market for equity investors, bond market for bond investors), net asset value, and number of portfolio positions, where the deciles are formed based on time-series averages of characteristics for each fund. Within this set of possible placebo investors for the investor asset combination A, X, we select the investor B that has the fewest common portfolio positions compared to investor A. This process yields a placebo investor B for each investor that holds asset X. For example, if 100 13f filers indicate they own Netflix, we determine 100 placebo investors that do not hold Netflix, though those 100 investors may contain duplicates.

[Insert Figure 8 Near Here]

Across these placebo investors, we estimate placebo investor betas using the exact same methodology detail in section 3.1. Figure 8 illustrates a flat relationship between placebo investor betas and returns. That is, low placebo investor beta assets have approximately the same returns as high placebo investor beta assets.

This placebo test compliments the main insight of the paper. Our primary analysis demonstrates that assets that add to (reduce) the risk of investors that hold them tend to have high (low) returns. Conversely, an asset might present a perfect hedge for a set of portfolios, but if those managers cannot purchase that asset, this low covariance does not translate into low returns.

6 Bond Market Specific Results

In the context of corporate bonds, we additionally test whether this very apparent linear relationship between excess returns and investor betas is driven by one investor group or whether the relationship holds across all investor groups. That is, we repeat our analysis from Panel (B) in Figure 1 for the groups of life and P&C insurers, mutual funds, variable annuities, and foreign investors. Figure 9 reports the results. Remarkably, the risk–return relationship also holds up within investor groups. The additional return for a unit increase in investor beta ranges from 0.16% for variable annuities to 0.73% for life insurers. Moreover, the beta–return relationship seems most apparent for life insurers and mutual funds and is least obvious for variable annuities.

[Insert Figure 9 And Table 5 Near Here]

In Table 5, we run similar panel regressions as in Table 4, however, our focus is on companies that have multiple bond issues outstanding.[18] Controlling for time × firm fixed effects allows us to test whether the investor beta return relationship holds for corporate bonds issued by the same company. As in Table 4 we repeat the analysis for investor betas computed in three alternative ways. The coefficient on the investor beta is highly statistically significant throughout all regression specification which include time × firm fixed effects, even when additionally controlling for market betas.

When a firm has multiple bond issues outstanding that have similar maturity dates and disagree in their prices, investors can realize close to risk-free arbitrage opportunities. Hence, in the absence of arbitrage opportunities, controlling for time × firm × maturity fixed effects

18. In our sample, many firms have multiple bonds outstanding. In fact, only 30% of all firm×month observations are due to firms with only one bond. Among the sample firms with multiple bonds, having two or three bonds at the same time is the most likely scenario as reported in Appendix Table A.4. Moreover, Appendix Table A.5 shows that the dispersion in investor betas within firm is similar for firms with more than ten bonds outstanding is on average slightly higher compared to firms with fewer than 10 bonds outstanding.

should render the coefficient on the investor beta insignificant. The results in columns (3), (6), and (9) confirm this intuition.

7 Relation to Existing Risk Factors

Next we examine the relation between investor betas and a set of risk factors that have been previously documented in the literature. In Section 3.2.2 we document that portfolio betas are correlated with a number of firm level characteristics, namely those associated with "value" strategies. Thus our objective is to analyze whether the priced component of investor betas is subsumed by other factors that demand a risk premium.

First, we follow the methodology in He, Kelly, and Manela (2017) to test whether the explanatory power of idiosyncratic investor betas is subsumed by other commonly studied factors. The alternative factor models we consider are Fama-French 3 factor, Carhart 4 factor, Fama-French 5 factor, He, Kelly, and Manela intermediary factor, and Hue, Xue, and Zhang q-factor.[19]

We run the same panel regressions as Table 3, but include the estimated betas for all factor models we consider according to corresponding methodology. We display the results in Table 7. Panel A uses the monthly full sample betas while Panel B uses daily odd even betas, projected onto the opposite return window. The estimated price of investor beta risk is stable across models and very similar to the estimates for both the monthly samples and daily instrumented approach. Incorporating betas from alternative risk models does not meaningfully impact our results. Moreover, while we supress the second stage estimates for alternative factor models, the associated price of risk appears in line with what those papers document. Thus, the stability of our coefficient is not a byproduct of noisly measured betas from other factor models.

Next, we re-estimate our investor betas using a different set of factor models in the first stage, Equation (9). Our primary analysis orthogonalizes fund by asset returns to the

19. See and Fama and French (1993), Carhart (1997), Fama and French (2016), Fama and French (2016),and Hou, Xue, and Zhang (2015) respectively.

appropriate market benchmark when estimating investor betas. In Table 8 we rerun this beta estimation first by only including the investor portfolio as a single risk factor in the first stage, and second by replacing the CAPM model with a number of models which have been shown to drive returns in their respective asset markets.

Table 8 displays the results of this analysis. For each model listed, we report the slope coefficient, standard error, and adjusted R^2s. Aside from two notable exceptions, we find largely stable coefficients across the set of models used. First, in equity markets, including the market as a first stage control increases the slope of the coefficient by about 30% relative to the single factor investor beta. Single factor investor betas are highly correlated with the market. At the same time, previous research has established that the market factor has a "too flat" slope in second stage regressions. Thus, orthogonalizing to the market increases the return for risk measured using investor betas.

Second, the opposite effect appears to be true in the bond market. There, as shown by Bai, Bali, and Quan (2021), the bond market does appear to be a priced factor. We show that including the bond market diminishes the second stage slope coefficient considerably. However, the price of investor beta risk in the bond market is still about three times as large as the price of bond market risk.

8 Conclusion

Essentially all equilibrium asset pricing models imply a basic risk-return relation; expected returns should be increasing in beta with respect to the return on investors' wealth. Empirically, using the aggregate market as a proxy for wealth, the estimated relation between beta and returns is flat. However, many investors' holdings substantially deviate from the market portfolio. We propose a simple model which predicts a linear relationship between expected returns and the idiosyncratic beta of a stock with respect to active investors' portfolio returns. Empirically, we find that a unit increase in investor betas is associated with a 5-10% increase in annual expected returns. The estimated risk premium is robust across a variety of empirical specifications. In sum, institutional investors appear to be compensated for holding stocks that have a high covariance with the idiosyncratic component of their portfolio.

Our findings compliment a growing literature that argues that the market is not the correct proxy for wealth when investors systematically deviate from holding the market portfolios. The approach of these papers has, heretofore, been to model the behavior of the constrained investors and determine an appropriate factor that captures their wealth process. This approach has been highly successful across a number of dimensions, most notably the intermediary asset pricing factors. However, the aggregate portfolio of "active" managers is not a successful factor in the cross section. Measuring betas at the portfolio$\times$asset level, a method that implicitly accounts for specific constraints investors face, yields a clear risk-reward trade off.

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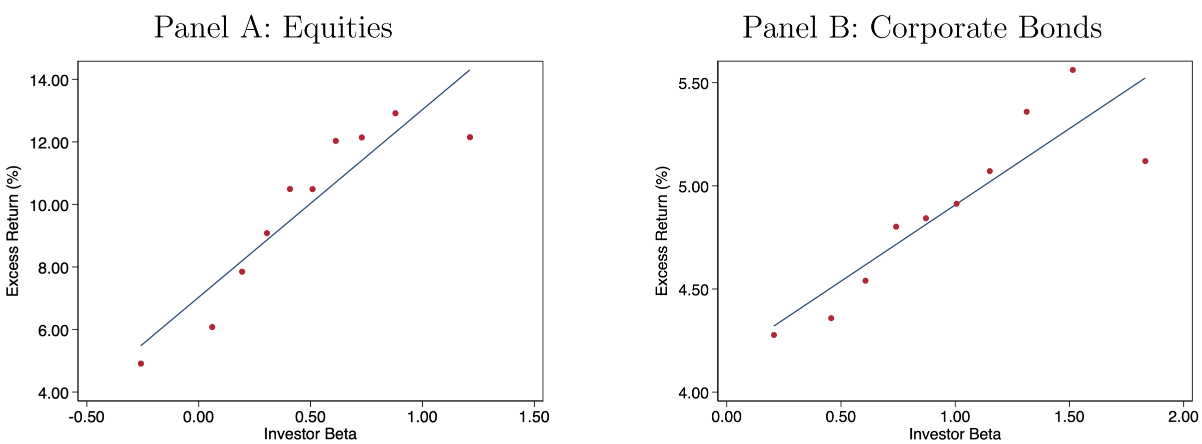
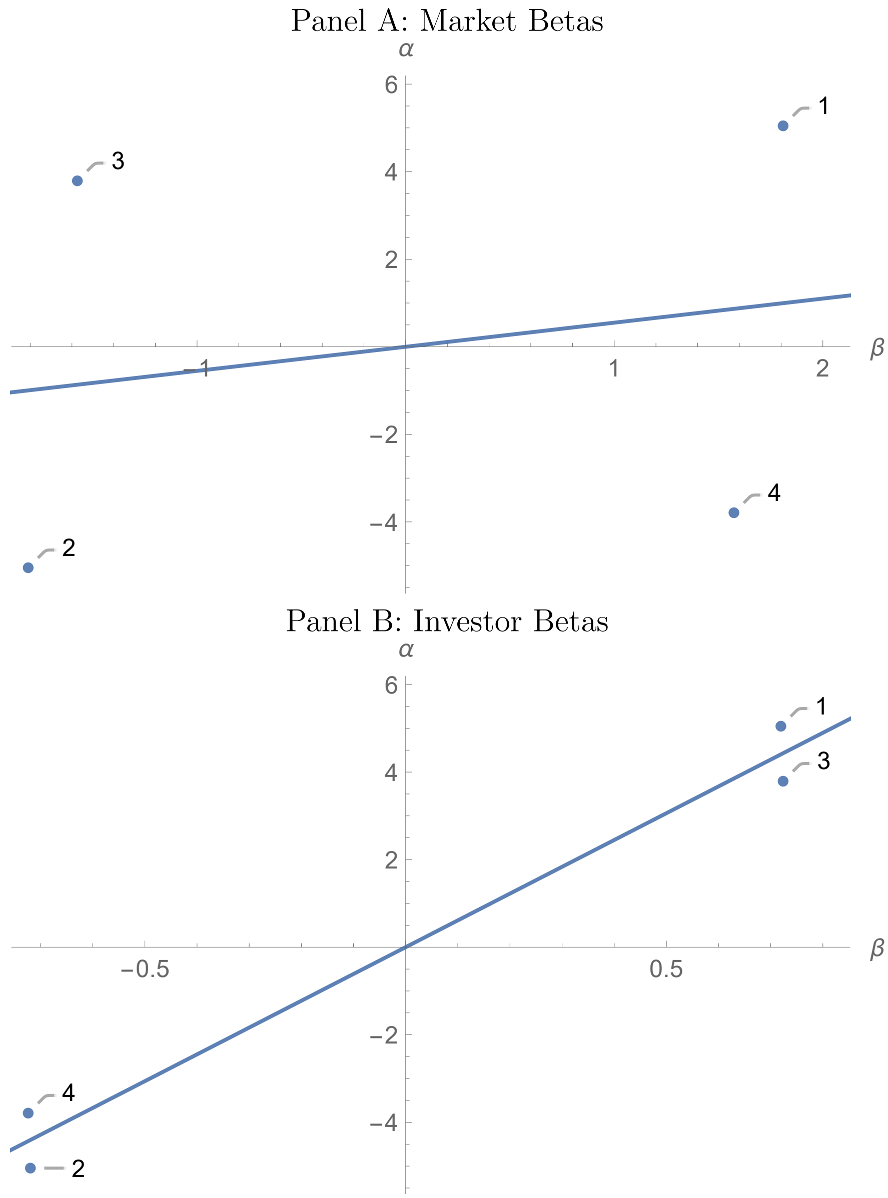
Fig. 1. Multi-Factor Investor Betas and Returns



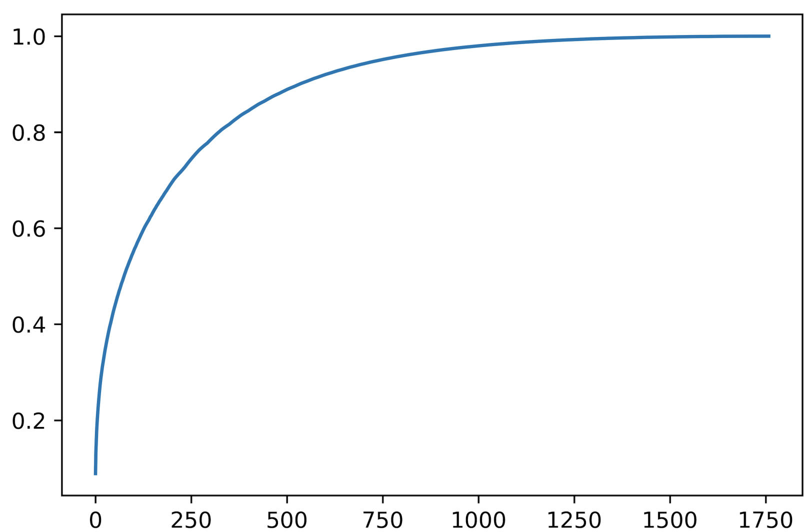
Note: This figure displays the beta–return relationship for 10 portfolios of stocks in Panel (A) and corporate bonds in Panel (B) sorted on multi-factor investor betas. Annualized excess returns are along the y-axis and the estimated beta of each portfolio are along the x-axis. Betas are calculated over the full sample for each investor–stock pair (investor–bond pair, respectively) using a multi-factor first stage with investor portfolio returns and equity market (bond market) returns as the factors as in Section 3.1. Betas are aggregated using share (market value) weights to the stock x month (bond x month) observation level.

Fig. 2. α vs Average Investor β



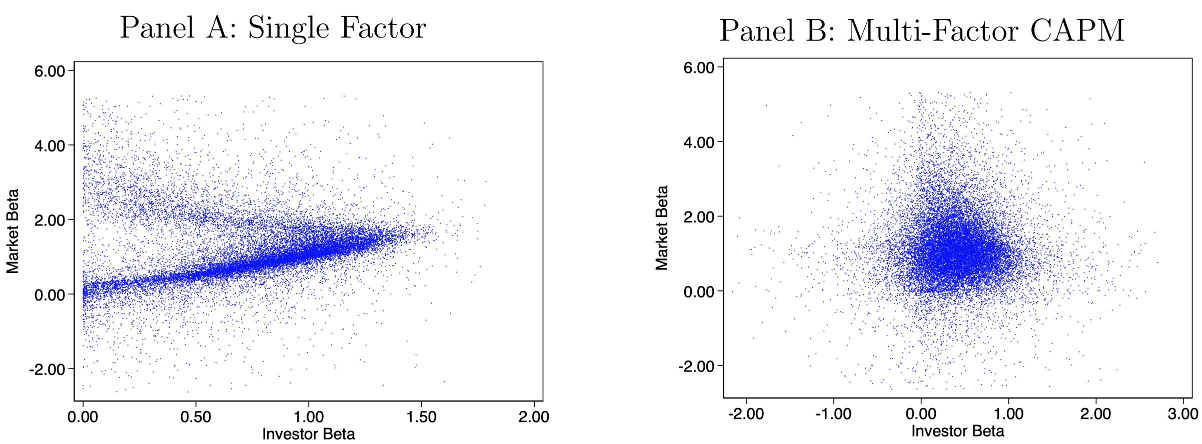
Note: This figure displays the equilibrium α on the vertical axis against average market beta (panel A) and investor beta (panel B) on the horizontal axis. α and β in both figures are both computed under the model where investors face holding constraints.

Fig. 3. Scree Plot of Residual Covariance



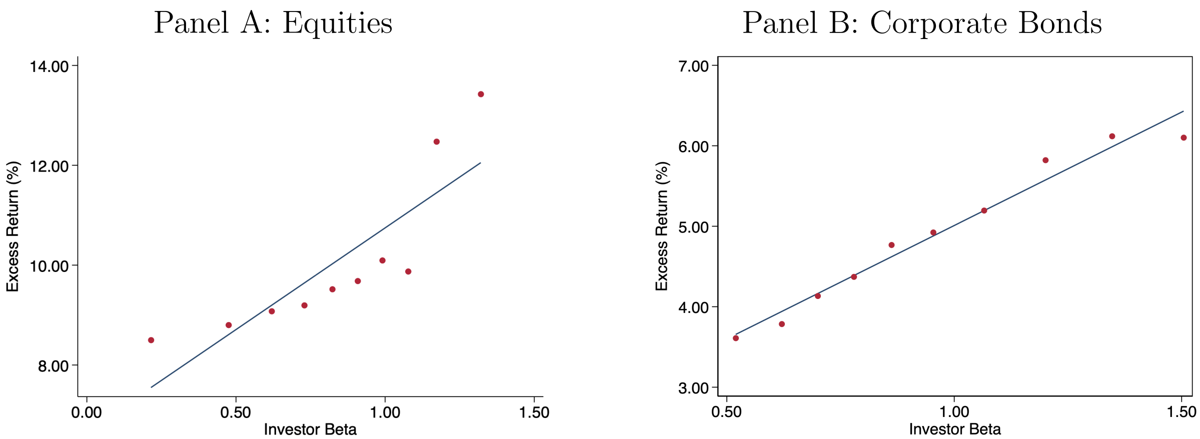
Note: This figure displays a scree plot of the cumulative percent of variance explained by successively more principal components of covariance matrix of market-neutral investor returns. We include investors with at least 180 months of return data. Missing correlations are replaced with zero, approximately the average non-missing value.

Fig. 4. Correlation: Investor Beta and Market Beta



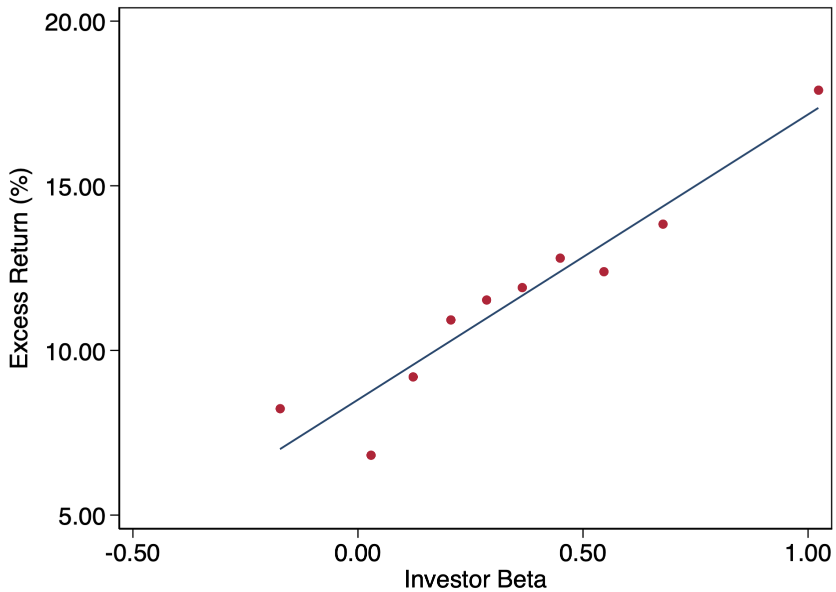
Note: This figure plots the average Investor Beta (x-axis) against the Market Beta (y-axis) for each firm in our sample. Panel (A) plots the average single factor betas while Panel (B) plots the multi-factor investor beta estimated using the methodology detailed in Section 3.1. Investor Betas are calculated by dropping any fund×CUSIP betas that exceed the first or 99th percentile (approximately 0 to 3) for the single factor investor beta. To preserve scale, outliers are trimmed at the 0.1% level.

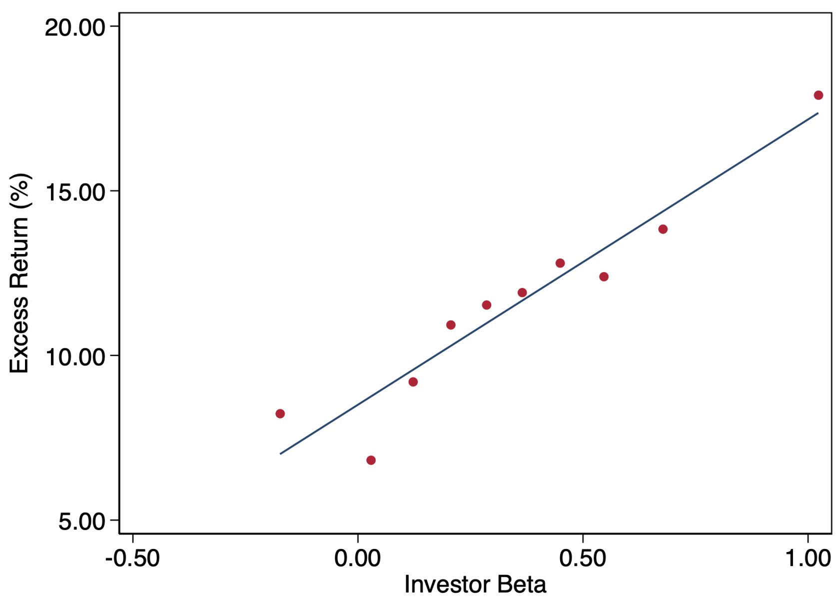
Fig. 5. Single Factor Investor Betas and Returns



Note: This figure displays the beta–return relationship for 10 portfolios of stocks in Panel (A) and corporate bonds in Panel (B) sorted on single factor investor betas. Annualized excess returns are along the y-axis and the estimated beta of each portfolio are along the x-axis. Betas are calculated over the full sample for each investor–stock pair (investor–bond pair, respectively) using the investor's portfolio return as a single factor at the stock (bond) by investor level. Betas are aggregated using share (market value) weights to the stock x month (bond x month) observation level.

Fig. 6. Daily Instrumented Portfolios

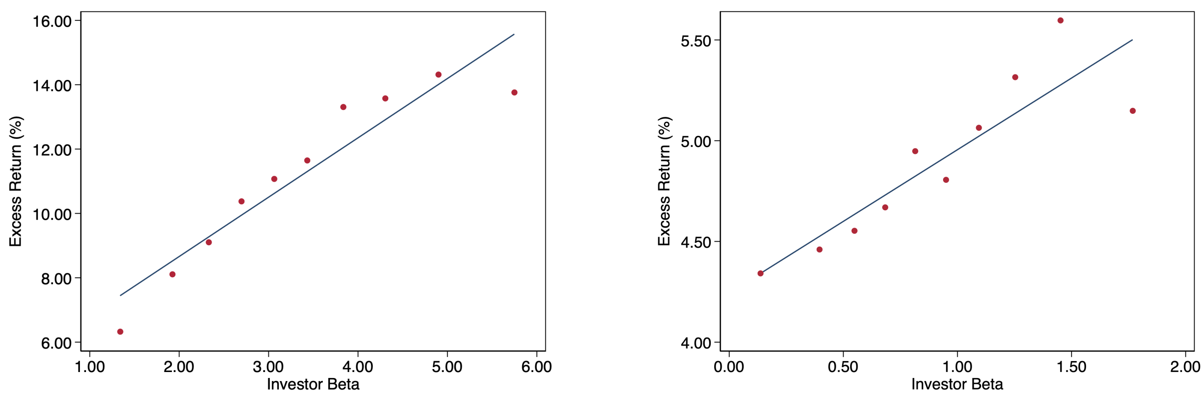




Note: This figure displays the investor beta–return relationship for ten portfolios of stocks sorted on multi-factor investor betas. Two betas are calculated for each quarter: one using odd days and one using even days. Ten portfolios are formed based on odd day betas, but we retain the corresponding even day return and even day "post-ranking" investor beta for those portfolios. Another ten are formed using the even day betas, but we retain corresponding the odd day returns and betas. Finally, we average the returns and post-ranking betas by decile. Excess returns are along the y-axis and the post-ranking betas are along the x-axis.

Fig. 7. Alternative Method: Investor Betas Calculated Using Average Returns of Holders

Panel A: Equities Panel B: Corporate Bonds



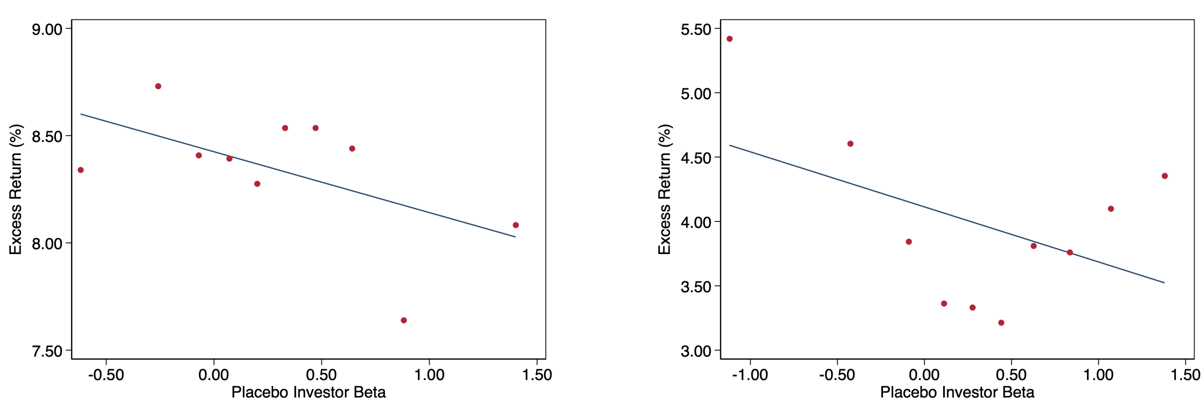
Note: This figure displays the investor beta–return relationship for ten portfolios of stocks sorted on investor betas estimated in the following first stage regression:

$$r_{it} = a_{sf} + \beta_i^{inv} r_{it}^{portfolios} + \beta_i^{mkt} r_{mt} + e_{it}$$

where r_{it} are the returns for stock i at time t, $r_{it}^{portfolios}$ is the share weighted average returns in time t for all 13f filing funds that hold stock i, and r_{mt} is the returns on the market at time t. Excess returns are along the y-axis and investor betas are along the x-axis. Due to high covariance between average holding portfolio returns and the market, this regression produces extreme outlier beta estimates, thus we truncate investor betas between -3 and 3 for this picture.

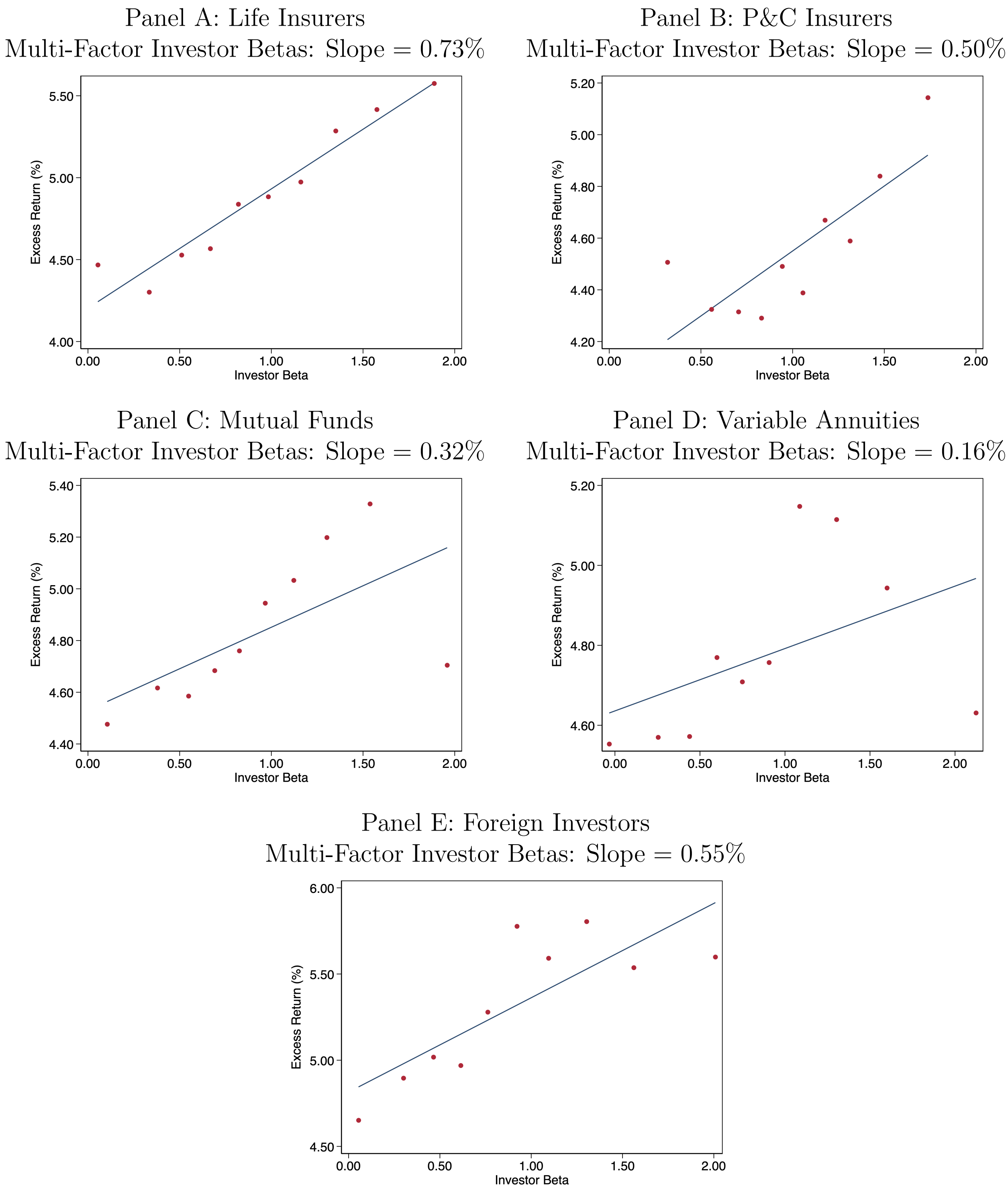
Fig. 8. Placebo Investor Betas

Panel A: Equities Panel B: Corporate Bonds



Note: This figure displays the placebo investor beta–return relationship for 10 portfolios of stocks (Panel A) and bonds (Panel B) sorted multi-factor placebo investor betas. Excess returns are along the y-axis and the estimated beta of each portfolio are along the x-axis. Placebo funds are matched to each fund x stock pair using market beta, average number of holdings and assets under management. Placebo investor betas are calculated over the full sample for each placebo fund–stock pair using a multi-factor first stage with investor fund returns and market returns as the factors as in Section 3.1. Betas are aggregated using share weights to the stock x month observation level.

Fig. 9. Investor Group Specific Corporate Bond Investor Betas and Returns



Note: This figure displays the beta–return relationship for 10 portfolios of corporate bonds sorted on multi-factor investor betas. That is, betas are calculated over the full sample for each investor–bond pair using a multi-factor first stage with investor bond portfolio returns and bond market returns as the factors. Annualized excess returns are along the y-axis and the estimated beta of each portfolio are along the x-axis. Panels (A) to (E) plot the results for five investor types including life insurers, property and casualty insurers, mutual funds, variable annuities, and foreign investors. Betas are aggregated for each investor type subsample using market value weights to the bond x month observation level.

Table 1. Summary Statistics

	Mean	25th Pct	Median	75th Pct	StDev	N
A: Equities						
A.1. Fund Characteristics						
Fund R-Squared with Market	0.71	0.58	0.80	0.91	0.25	8,720
Number of Positions	171.05	26.33	71.71	165.46	327.88	8,720
A.2 Stock Characteristics						
Firm Size ($mm)	3022.52	143.32	402.87	1429.85	14011.29	1,530,430
Excess Return (% Ann)	9.16	-66.20	4.81	77.02	170.47	1,535,646
A.3 Betas						
Single Factor Market Beta	1.11	0.70	1.04	1.42	0.88	1,535,469
Single Factor Investor Beta	0.81	0.57	0.85	1.07	0.35	1,508,040
Multi-factor Investor Beta	0.47	0.21	0.47	0.73	0.41	1,535,646
B: Corporate Bonds						
B.1. Fund Characteristics						
Fund R-Squared with Market	0.65	0.50	0.71	0.84	0.23	9,520
Number of Positions	269	36	113	299	468	257,332
B.2 Bond Characteristics						
Face Value (mm)	613.72	300.00	475.00	750.00	553.13	797,683
Average Return (% Ann)	4.93	-6.04	3.41	15.85	32.06	797,683
Time to Maturity (Years)	9.72	3.98	6.77	11.62	8.19	797,683
B.3 Betas						
Single Factor Market Beta	0.91	0.52	0.83	1.19	0.53	795,713
Single Factor Investor Beta	0.97	0.74	0.94	1.20	0.29	797,683
Multi-factor Investor Beta	0.98	0.63	0.95	1.32	0.47	796,447

Note: Panel A reports summary statistics for equities and describes the sample of firms that i) have investor betas calculated as in Section 3.1; ii) match with the constituents of FF25 portfolios; iii) are not in the bottom quintile of market cap one year prior to their FF25 portfolio formation date. For stock and beta statistics, we report the number of firm×month observations. Panel B reports corresponding results for corporate bonds. That is, fund characteristics statistics are reported at the fund and at the fund×quarter level, respectively. Moreover, bond and beta statistics are reported at the bond×month level.

Table 2. Selected Characteristics by Beta Quintile

	β_{inv}Q1	β_{inv}Q5	Q5-Q1	β_{mkt}Q1	β_{mkt}Q5	Q5-Q1
A. Equities						
Market Beta	1.2	1.0	-0.2	0.4	1.9	1.4
Investor Beta	0.1	0.8	0.7	0.4	0.3	-0.1
Dividend Yield %	3.0	2.8	-0.2	3.8	2.2	-1.6
Book/Market	0.7	0.7	0.0	0.8	0.6	-0.2
Price/Sales	2.4	1.7	-0.7	1.8	2.8	1.0
Dividend Payout Ratio %	19.1	25.0	5.9	29.1	10.7	-18.4
A.T. ret on Common Equity %	-1.6	6.6	8.2	8.0	-5.4	-13.4
Total Debt/Total Assets %	21.7	21.0	-0.6	22.7	20.9	-1.7
R&D Expense/Sales %	6.5	2.8	-3.6	1.3	10.0	8.7
Labor Expense/Sales %	4.8	5.9	1.1	8.3	2.5	-5.8
B. Corporate Bonds						
Market Beta	0.8	1.0	0.2	0.2	2.0	1.8
Investor Beta	0.4	1.3	0.8	0.8	0.9	0.1
Dividend Yield %	2.1	2.6	0.5	2.4	2.4	0.0
Book/Market	0.6	0.8	0.2	0.7	0.7	0.0
Price/Sales	1.7	1.7	0.0	1.7	1.7	0.0
Dividend Payout Ratio %	47.8	80.1	32.3	58.9	58.3	-0.6
A.T. ret on Common Equity %	7.9	5.2	-2.7	14.5	15.5	0.9
Total Debt/Total Assets %	31.4	34.2	2.8	34.7	29.9	-4.9
R& D Expense/Sales %	1.5	1.3	-0.3	1.5	1.2	-0.2
Labor Expense/Sales %	5.4	2.8	-2.6	2.8	6.2	3.4

Note: Panel A reports time-series averages of various firm characteristics by investor and market beta quintile for equities. All values are averaged each quarter by quintile and then a time-series average is taken over the period. Panel B displays corresponding values for corporate bonds. For firms which have mutliple bonds outstanding contemporaneously, we consider the average investor and market beta, respectively. All firm characteristics are averaged each quarter by quintiles and then a time-series average is taken over the sample period between 2006 and 2020.

Table 3. Panel Regressions

	Main		Excluded		Rolling		Daily	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Investor Beta	6.042***	5.258***	4.460**	3.960**	3.517***	3.457***	7.349***	8.657***
	(3.03)	(3.29)	(2.04)	(2.30)	(3.29)	(3.47)	(3.54)	(4.65)
Market Beta		-4.154		-2.601		-2.812		-3.338
		(-0.71)		(-0.39)		(-0.43)		(-0.91)
Fixed Effects	T	T	T	T	T	T	T	T
Adjusted R^2	0.704	0.704	0.652	0.653	0.663	0.664	0.645	0.645
Observations	54,969	54,968	54,960	54,959	54,831	54,830	54,967	54,967

Note: This table estimates the OLS panel regression using 125 portfolios. We first sort stocks each month into 25 portfolios formed on size and value then within each such portfolio sort into quintiles based on investor beta. The dependent variable is excess return and the independent variables are investor and market betas. All columns include a month fixed effect. Columns (1-2) correspond to our baseline estimation of investor betas. Columns (3-4) of the table calculate investor betas by excluding the reference firm from the fund portfolio returns Columns (5-6) use rolling four year window betas but exclude the reference month from each beta calculation. Lastly, Columns (7-8) measures separate daily betas within each holding quarter for odd and even days. We match odd return days to even day betas and vise versa to estimate the second stage price of risk. Reported t-statistics in parentheses are heteroskedasticity-robust and double clustered by month and portfolio. ***p<0.01, **p<0.05, *p<0.1.

Table 4. Panel Regressions - Corporate Bonds

	Main		Excluded		Rolling	
	(1)	(2)	(3)	(4)	(5)	(6)
Investor Beta	1.338**	1.908**	1.270*	1.848**	1.128***	1.380***
	(0.656)	(0.744)	(0.648)	(0.739)	(0.348)	(0.380)
Market Beta		0.672***		0.671***		0.513***
		(0.192)		(0.191)		(0.169)
Fixed Effects	T x R	T x R	T x R	T x R	T x R	T x R
Adjusted R^2)	0.33	0.33	0.33	0.33	0.33	0.33
Observations	791,248	791,248	791,214	791,214	588,708	588,708

Note: This table estimates the OLS panel regression using monthly observations. The dependent variable is monthly bond excess returns and the independent variables are investor and market betas. Columns (1-2) correspond to our baseline estimation of investor betas. Columns (3-4) of the table calculate investor betas by excluding the reference firm from the fund portfolio returns Columns (5-6) use rolling five year window betas but exclude the reference month from each beta calculation. Investor betas are aggregated using market value weights to the bond×month observation level. The two panel regressions specifications estimated for each investor beta include time x bond rating groups (from Standard & Poor's), T x R, fixed effects. Reported standard t-statistics in parentheses are heteroskedasticity-robust and clustered at the time by letter rating group level. ***p<0.01, **p<0.05, *p<0.1.

Table 5. Panel Regressions with Firm Fixed Effects - Corporate Bonds

	Main			Excluded			Rolling		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Investor Beta	1.828***	2.272***	0.305	1.745**	2.208***	0.293	1.240***	1.379***	0.100
	(0.691)	(0.796)	(0.564)	(0.680)	(0.788)	(0.539)	(0.407)	(0.439)	(0.238)
Market Beta		0.599***	0.037		0.594***	0.035		0.410***	0.016
		(0.185)	(0.097)		(0.186)	(0.099)		(0.154)	(0.108)
Fixed Effects	T x ID	T x ID	T x ID x M	T x ID	T x ID	T x ID x M	T x ID	T x ID	T x ID x M
Firm Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Adjusted R^2	0.51	0.51	0.62	0.51	0.51	0.62	0.51	0.51	0.64
Observations	638,021	638,021	452,907	638,066	638,066	452,864	472,413	472,413	328,282

Note: This table estimates the OLS panel regression using monthly observations. The dependent variable is monthly bond excess returns and the independent variables are investor and market betas. Columns (1-3) correspond to our baseline estimation of investor betas. Columns (4-6) of the table calculate investor betas by excluding the reference firm from the fund portfolio returns Columns (7-9) use rolling five year window betas but exclude the reference month from each beta calculation. Investor betas are aggregated using market value weights to the bond×month observation level. The three panel regressions specifications estimated for each investor beta include either time x firm, T x ID, or time x firm x maturity bucket, T x ID x M, fixed effects. That is, every month, all available bonds are sorted in 6 equally populated quantiles according to the bonds' remaining time to maturity. Reported t-statistics in parentheses are heteroskedasticity-robust and clustered at the time and letter rating group level. ***p<0.01, **p<0.05, *p<0.1.

Table 6. Correlation with Commonly Priced Factors

		FF-3			Carhart				FF-5					HKM		HXZ-q			
	I-Beta	Rm-Rf	SMB	HML	Rm-Rf	SMB	HML	UMD	Rm-Rf	SMB	HML	RMW	CMA	Rm-Rf	HKM	Rm-Rf	Size	Invest.	Prof.
Investor Beta	1.00																		
Rm-Rf	0.25	1.00																	
SMB	0.03	0.21	1.00																
HML	-0.05	0.21	0.10	1.00															
Rm-Rf	0.21	0.89	0.20	0.17	1.00														
SMB	0.02	0.17	0.97	0.08	0.23	1.00													
HML	-0.08	0.15	0.10	0.86	0.18	0.12	1.00												
UMD	0.04	0.00	-0.04	-0.03	-0.05	-0.06	0.04	1.00											
Rm-Rf	0.21	0.90	0.20	0.22	0.85	0.18	0.17	0.01	1.00										
SMB	0.01	0.17	0.95	0.11	0.19	0.94	0.11	-0.04	0.25	1.00									
HML	-0.02	0.20	0.09	0.79	0.17	0.07	0.72	-0.03	0.23	0.12	1.00								
RMW	-0.05	-0.06	-0.03	0.08	-0.00	-0.02	0.04	0.08	0.13	0.16	0.25	1.00							
CMA	-0.04	-0.10	0.01	0.08	-0.05	0.02	0.05	0.02	0.03	0.03	-0.34	0.08	1.00						
Rm-Rf	0.38	0.51	0.29	-0.23	0.49	0.27	-0.21	0.04	0.48	0.28	-0.14	-0.05	-0.09	1.00					
HKM	-0.03	0.04	-0.08	0.24	-0.00	-0.09	0.19	-0.16	0.03	-0.08	0.21	-0.04	-0.05	-0.50	1.00				
Rm-Rf	0.23	0.81	0.06	-0.04	0.77	0.05	-0.05	0.05	0.80	0.05	-0.07	-0.05	0.03	0.53	0.04	1.00			
Size	0.03	0.15	0.93	-0.00	0.16	0.92	0.03	-0.01	0.16	0.92	-0.01	-0.02	0.02	0.32	-0.09	0.11	1.00		
Invest.	-0.05	-0.00	0.02	0.44	0.03	0.03	0.39	0.04	0.09	0.04	0.14	0.07	0.61	-0.16	0.11	0.08	0.02	1.00	
Prof.	0.01	-0.10	-0.24	-0.26	-0.06	-0.21	-0.22	0.34	-0.04	-0.17	-0.17	0.30	-0.04	-0.03	-0.15	0.02	-0.10	-0.07	1.00

Note: This table estimates the correlation matrix between the Investor Beta and other commonly priced factors. We consider factors from Fama-French 3 factor, Carhart, Fama-French 5 Factor, He, Kelly, and Manela Intermediary. Investor betas are calculated using a 90 day rolling window sample for each fund–stock pair using a two-factor first stage with investor fund returns and market returns as the factors. Investor betas are aggregated using share weights to the stock x month observation level. Betas from other factor models are estimated over the same windows. In all cases, we instrument for odd day betas using even day betas and vise versa. To form portfolios, we first sort stocks each month into 25 portfolios formed on size and value then within each such portfolio sort into quintiles based on investor betas, for a total of 125 portfolios.

Table 7. Comparison with Commonly Priced Factors

A: Monthly Full Sample

	(1)	(2)	(3)	(4)	(5)
Investor Beta	4.876***	4.206**	6.479***	5.563***	5.265***
	(2.78)	(2.59)	(3.83)	(3.10)	(3.37)
Model	FF3	Carhart	FF5	HKM	HXZ-q
R^2	0.707	0.707	0.707	0.706	0.707
Observations	54,969	54,969	54,969	54,969	54,969

B: Daily Odd-Even

	(1)	(2)	(3)	(4)	(5)
Investor Beta	7.599***	6.925***	8.199***	10.951***	7.620***
	(4.46)	(3.55)	(4.57)	(3.17)	(4.19)
Model	FF3	Carhart	FF5	HKM	HXZ-q
R^2	0.648	0.649	0.648	0.660	0.648
Observations	54,967	54,967	54,967	25,500	54,967

Note: This table estimates the OLS panel regression using market and investor beta sorted portfolios. The dependent variable is excess returns and the independent variables are investor betas and the betas measured from a variety of other factor models (Fama-French 3 factor, Carhart, Fama-French 5 Factor, He, Kelly, and Manela Intermediary, and Hue, Xue, and Zhang q factor). Investor betas are calculated using a 90 day rolling window sample for each fund–stock pair using a two-factor first stage with investor fund returns and market returns as the factors. Investor betas are aggregated using share weights to the stock x month observation level. Betas from other factor models are estimated over the same windows at the stock level. In all cases, we instrument for odd day betas using even day betas and vise versa. To form test portfolios, we first sort stocks each month into 25 portfolios formed on size and value then within each such portfolio sort into quintiles based on investor betas, for a total of 125 portfolios. Reported t-statistics in parentheses are heteroskedasticity-robust and double clustered by return date and portfolio. ***p<0.01, **p<0.05, *p<0.1.

Table 8. Investor Betas and Risk Models

A. Equities

	Slope	SE	Adj. R^2
Single Factor	4.755	0.789	0.797
CAPM	5.997	0.912	0.824
FF3	4.276	0.793	0.757
Carhart	4.573	0.738	0.806
FF5	3.748	0.598	0.809
HKM	4.48	0.836	0.755
HXZQ	3.767	0.835	0.682

B. Corporate Bonds

	Slope	SE	Adj. R^2
Single Factor	3.300	0.072	0.996
(Bond)-CAPM	1.082	0.081	0.951
FF3	3.140	0.166	0.975
FF5	2.985	0.141	0.980
Carhart	2.631	0.091	0.989
HKM	3.679	0.191	0.976
PS	3.006	0.186	0.967
BBW	1.060	0.070	0.962

Note: This Table presents slopes, standard errors, and adjusted R^2s from regressing ten portfolios sorted on investor betas on portfolio excess returns when controlling for various well-known risk models in the first stage (i.e., calculation of investor beta). First stage betas are share-weighted as outlined in equation (14).

Internet Appendix For "Investor Betas"

A Additional Tables

Table A.1. Summary of Corporate Bond Holdings

Year	Number of Investors	% of Market Held	AUM (USD Million)		Number of Bonds	
			Median	90th Percentile	Median	90th Percentile
2006	1,281	49.0	53.7	628.5	48	162
2007	1,360	44.9	54.7	622.9	51	168
2008	1,570	44.7	54.7	617.6	53	182
2009	1,972	46.4	58.6	639.3	57	212
2010	2,036	49.6	63.5	725.5	58	216
2011	2,172	48.4	64.8	756.7	60	229
2012	2,444	49.2	67.8	769.8	64	236
2013	2,486	47.9	70.8	831.3	68	252
2014	2,622	46.6	70.4	852.6	67	258
2015	2,676	45.5	70.1	872.4	69	278
2016	3,260	45.2	66.7	792.0	68	282
2017	3,666	48.0	68.8	847.8	74	305
2018	3,297	44.6	71.6	878.9	79	331
2019	3,960	45.4	68.2	805.7	78	328
2020	3,478	44.1	76.1	983.4	86	377

Note: This table reports the summary statistics of the quarterly corporate bond holdings in our sample. Each cell is the time-series mean of the quarterly summary statistic within the given year. The sample period includes 55 quarters from 2006:Q1 to 2020:Q3.

Table A.2. Part I: WRDS Characteristics by Beta Quintile

	β_{inv}Q1	β_{inv}Q5	Q5-Q1	β_{mkt}Q1	β_{mkt}Q5	Q5-Q1
Market Beta	1.2	1.0	-0.2	0.4	1.9	1.4
Investor Beta	0.1	0.8	0.7	0.4	0.3	-0.1
Market Cap.	2.3	2.4	0.1	3.3	1.6	-1.7
Cyclically Adjusted P/E	12.6	17.8	5.2	17.7	12.2	-5.5
Book/Market	0.7	0.7	0.0	0.8	0.6	-0.2
Enterprise Value Multiple	7.4	8.9	1.5	9.3	7.2	-2.1
P/Oper. Earn. (Basic)	9.0	13.1	4.1	14.1	6.8	-7.3
P/Oper. Earn (Diluted)	9.9	14.3	4.4	16.0	7.5	-8.5
P/E (Diluted, Excl. EI)	9.0	12.9	3.9	13.5	7.5	-6.0
P/E (Diluted, Incl. EI)	8.7	12.6	3.8	13.2	7.3	-6.0
Price/Sales	2.4	1.7	-0.7	1.8	2.8	1.0
Price/Cash flow	5.8	8.4	2.6	7.7	6.2	-1.5
Dividend Payout Ratio %	19.1	25.0	5.9	29.1	10.7	-18.4
Net Profit Margin %	-8.2	1.7	9.9	5.1	-14.0	-19.1
Op. Margin Before Depr %	7.2	14.0	6.8	20.0	1.3	-18.7
Op. Margin After Depr %	0.1	8.9	8.8	14.3	-6.5	-20.8
Gross Profit Margin %	36.0	36.1	0.0	39.0	34.8	-4.2
P.T. Profit Margin %	-6.3	4.7	11.0	8.9	-12.8	-21.7
Cash Flow Margin	-0.0	0.1	0.1	0.1	-0.1	-0.2
Return on Equity %	6.4	10.4	4.0	9.3	5.8	-3.5
Return on Equity	-1.6	6.2	7.8	7.3	-5.4	-12.6
Return on Capital Employed %	5.0	11.0	6.0	11.2	2.5	-8.7
Effective Tax Rate %	31.0	32.4	1.3	32.3	29.8	-2.5
A.T. ret on Common Equity %	-1.6	6.6	8.2	8.0	-5.4	-13.4
A.T. ret on Invested Capital %	1.9	6.2	4.3	5.8	0.6	-5.2
A.T. ret on Total Equity %	-1.3	6.7	8.0	8.1	-5.0	-13.0
P.T. ret on Net Op. Assets %	5.4	15.1	9.8	15.9	2.7	-13.3
P.T. ret on Total Earning Assets %	2.8	9.9	7.1	10.5	1.0	-9.5
Gross Profit/Total Assets %	25.8	28.6	2.8	21.1	30.1	9.0
Common Equity/Invested Capital %	70.0	70.6	0.6	65.8	72.8	7.0
LT Debt/Invested Capital %	28.2	28.1	-0.2	32.4	25.6	-6.9
Capitalization Ratio %	41.1	40.1	-1.0	47.1	35.2	-11.9
Capitalization Ratio	28.3	28.1	-0.2	32.5	25.7	-6.8
Interest/LT Debt %	15.0	13.2	-1.8	11.9	16.3	4.4
Interest/Total Debt %	9.0	8.3	-0.7	7.9	9.3	1.4

Note: This table displays time-series averages of various firm characteristics by investor and market beta quintile. All values are first averaged each quarter by quintile and then a time-series average is taken over the period 1980-2018.

Table A.2. Part II: WRDS Characteristics by Beta Quintile

	β_{inv}Q1	β_{inv}Q5	Q5-Q1	β_{mkt}Q1	β_{mkt}Q5	Q5-Q1
Cash/Total Liabilities %	64.2	40.7	-23.5	28.6	88.2	59.5
Inventory/Current Assets %	20.2	26.2	6.1	23.4	19.9	-3.5
Receivables/Current Assets %	33.7	37.7	4.0	38.5	31.4	-7.1
Total Debt/Total Assets %	21.7	21.0	-0.6	22.7	20.9	-1.7
Total Debt/EBITDA	2.4	2.5	0.1	3.0	2.0	-1.1
LT Debt/Total Liab. %	29.4	27.7	-1.7	27.8	28.5	0.7
Current Debt/Total Liab. %	54.2	51.6	-2.6	43.3	58.6	15.4
Long-term Debt/Total Liabilities	29.0	28.5	-0.5	29.0	29.3	0.3
Profit Before Depr./Current Liab. %	31.1	61.0	29.9	74.1	21.0	-53.1
Oper. CF/Current Liab. %	17.6	40.2	22.6	49.8	8.6	-41.2
Cash Flow/Total Liab. %	6.6	14.9	8.3	12.6	4.9	-7.8
Free Cash Flow/Oper. CF %	4.6	18.5	13.9	16.1	-2.5	-18.6
Total Liabilities/Total Tangible Assets	14.7	13.9	-0.8	20.5	8.7	-11.8
LT Debt/Book Equity %	60.1	55.3	-4.7	62.9	55.9	-7.0
Total Debt/Total Assets %	55.0	56.0	1.0	63.3	48.2	-15.1
Total Debt/Total Capital %	45.3	45.1	-0.2	53.9	38.0	-15.9
Total Debt/Equity	2.9	2.7	-0.1	3.7	2.0	-1.7
After-tax Interest Coverage	2.7	8.8	6.1	8.5	0.5	-8.0
Interest Coverage Ratio	7.6	14.1	6.4	12.5	6.3	-6.3
Cash Ratio	1.4	0.9	-0.5	0.7	1.6	0.9
Quick Ratio (Acid Test)	2.3	1.8	-0.5	1.6	2.6	1.0
Current Ratio	2.8	2.5	-0.3	2.1	3.1	1.1
Cash Conversion Cycle (Days)	100.4	100.8	0.3	82.6	105.9	23.4
Inventory Turnover	17.4	14.3	-3.1	19.6	13.5	-6.1
Asset Turnover	0.8	0.9	0.1	0.7	0.9	0.2
Receivables Turnover	7.2	7.2	0.0	7.1	7.4	0.2
Payables Turnover	10.2	10.5	0.4	9.1	10.7	1.6
Sales/Invested Capital	1.4	1.5	0.2	1.2	1.4	0.2
Sales/Stockholders Equity	2.2	2.3	0.1	2.0	2.3	0.3
Sales/Working Capital	7.2	8.2	1.0	11.5	5.6	-5.9
R&D Expense/Sales %	6.5	2.8	-3.6	1.3	10.0	8.7
Advertising Expense/Sales %	0.8	0.7	-0.1	0.7	0.8	0.1
Labor Expense/Sales %	4.8	5.9	1.1	8.3	2.5	-5.8
Accruals/Assets %	4.9	4.0	-0.9	3.4	5.5	2.1
Price	19.4	26.1	6.7	26.7	18.1	-8.6
Price/Book	2.5	2.2	-0.3	1.9	2.9	0.9
Trailing PEG ratio	1.0	1.3	0.3	1.6	0.7	-0.9
Dividend Yield %	3.0	2.8	-0.2	3.8	2.2	-1.6
Forward PEG ratio	0.1	0.4	0.2	0.6	-0.0	-0.6

Table A.3. Part I: WRDS Characteristics by Beta Quintile - Corporate Bonds

	β_{inv}Q1	β_{inv}Q5	Q5-Q1	β_{mkt}Q1	β_{mkt}Q5	Q5-Q1
Market Beta	0.8	1.0	0.2	0.2	2.0	1.8
Investor Beta	0.4	1.3	0.8	0.8	0.9	0.1
Book/Market	0.6	0.8	0.2	0.7	0.7	0.0
Enterprise Value Multiple	11.7	14.9	3.2	14.4	12.7	-1.7
P/E (Diluted, Excl. EI)	15.9	15.1	-0.8	13.2	18.8	5.6
P/E (Diluted, Incl. EI)	15.8	13.6	-2.2	12.3	18.2	5.9
Price/Sales	1.7	1.7	0.0	1.7	1.7	0.0
Dividend Payout Ratio %	47.8	80.1	32.3	58.9	58.3	-0.6
Net Profit Margin %	7.7	5.1	-2.7	6.1	8.5	2.4
Op. Margin Before Depr %	21.6	24.8	3.2	23.1	23.6	0.5
Op. Margin After Depr %	16.4	15.1	-1.3	14.9	17.9	3.0
Gross Profit Margin %	39.2	38.0	-1.3	40.0	38.7	-1.2
P.T. Profit Margin %	11.3	8.0	-3.3	8.9	12.4	3.4
Cash Flow Margin	0.1	0.2	0.0	0.2	0.2	0.0
Return on Equity %	15.5	9.6	-5.8	12.4	15.0	2.6
Return on Capital Employed %	16.9	13.4	-3.4	14.1	16.0	1.9
Effective Tax Rate %	26.2	26.6	0.4	21.0	25.1	4.1
A.T. ret on Common Equity %	7.9	5.2	-2.7	14.5	15.5	0.9
A.T. ret on Invested Capital %	12.1	8.9	-3.2	10.1	11.7	1.6
A.T. ret on Total Equity %	10.1	0.0	-10.1	13.9	14.4	0.4
P.T. ret on Net Op. Assets %	39.4	3.5	-35.9	25.3	28.0	2.8
P.T. ret on Total Earning Assets %	20.2	14.6	-5.7	17.0	20.0	3.0
Gross Profit/Total Assets %	27.2	23.3	-3.9	28.0	22.4	-5.6
Common Equity/Invested Capital %	48.9	45.2	-3.8	45.5	47.9	2.4
LT Debt/Invested Capital %	49.4	52.7	3.3	52.7	50.5	-2.2
Capitalization Ratio %	49.7	49.9	0.2	49.8	50.6	0.8
Interest/LT Debt %	6.8	8.9	2.1	7.8	8.5	0.7
Interest/Total Debt %	5.9	6.0	0.1	6.1	5.6	-0.6

Note: This table displays time-series averages of various firm characteristics by investor and market beta quintile. For firms which have multiple bonds outstanding contemporaneously, we consider the average investor and market beta, respectively. All firm characteristics are averaged each quarter by quintiles and then a time-series average is taken over the full sample period between 2006 and 2020.

Table A.3. Part II: WRDS Characteristics by Beta Quintile - Corporate Bonds

	β_{inv}Q1	β_{inv}Q5	Q5-Q1	β_{mkt}Q1	β_{mkt}Q5	Q5-Q1
Cash/Total Liabilities %	16.2	14.1	-2.1	15.1	14.8	-0.3
Inventory/Current Assets %	24.6	23.2	-1.4	23.4	23.6	0.2
Receivables/Current Assets %	37.6	35.9	-1.7	37.8	36.9	-0.8
Total Debt/Total Assets %	31.4	34.2	2.8	34.7	29.9	-4.9
Total Debt/EBITDA	3.5	5.4	2.0	4.6	4.6	-0.1
LT Debt/Total Liab. %	40.9	44.5	3.6	46.6	36.8	-9.7
Current Debt/Total Liab. %	34.3	29.1	-5.2	31.3	33.8	2.5
Profit Before Depr./Current Liab. %	87.4	98.0	10.6	93.3	87.5	-5.8
Oper. CF/Current Liab. %	62.2	73.6	11.4	69.9	63.7	-6.2
Cash Flow/Total Liab. %	15.5	14.8	-0.7	16.6	13.1	-3.4
Free Cash Flow/Oper. CF %	37.8	-28.3	-66.2	11.2	19.4	8.3
Total Liabilities/Total Tangible Assets	26.7	27.6	0.8	21.8	44.8	23.0
LT Debt/Book Equity %	120.7	160.9	40.2	164.3	116.9	-47.3
Total Debt/Total Capital %	59.5	59.3	-0.2	57.8	61.9	4.1
Total Debt/Equity	2.7	8.8	6.1	2.7	3.9	1.2
After-tax Interest Coverage	7.8	25.9	18.1	7.6	24.6	17.0
Interest Coverage Ratio	11.2	41.1	29.9	11.4	38.8	27.3
Cash Ratio	0.5	0.5	0.0	0.5	0.5	0.0
Quick Ratio (Acid Test)	1.3	1.2	-0.1	1.3	1.3	-0.1
Current Ratio	1.8	1.6	-0.2	1.7	1.7	-0.1
Cash Conversion Cycle (Days)	106.0	222.4	116.4	124.4	185.6	61.2
Inventory Turnover	31.9	91.1	59.2	17.7	107.7	90.0
Asset Turnover	0.9	0.7	-0.2	0.9	0.7	-0.1
Receivables Turnover	19.6	17.3	-2.3	17.1	20.4	3.3
Payables Turnover	12.6	9.8	-2.9	12.0	10.0	-2.0
Sales/Invested Capital	1.6	1.4	-0.2	1.6	1.4	-0.2
Sales/Stockholders Equity	4.1	8.1	4.0	5.8	3.4	-2.4
Sales/Working Capital	32.9	60.1	27.1	45.4	21.9	-23.5
R& D Expense/Sales %	1.5	1.3	-0.3	1.5	1.2	-0.2
Advertising Expense/Sales %	1.1	1.0	-0.1	1.2	1.1	-0.1
Labor Expense/Sales %	5.4	2.8	-2.6	2.8	6.2	3.4
Accruals/Assets %	-4.1	-5.6	-1.5	-5.4	-3.7	1.7
Price/Book	3.0	2.7	-0.2	3.0	2.8	-0.2
Trailing PEG ratio	2.1	2.5	0.4	2.1	2.2	0.1
Dividend Yield %	2.1	2.6	0.5	2.4	2.4	0.0
Forward PEG ratio	0.4	0.1	-0.3	0.2	0.6	0.3

Table A.4. Number of Bonds per Company

Number of Bonds	Firm Months	Relative Frequency (in %)
1	45,715	31.05
2	24,709	16.78
3	15,908	10.81
4	11,836	8.04
5	8,678	5.89
6 to 10	18,869	12.81
More than 10	21,502	14.62
Total	147,217	100.00

Table A.5. Dispersion in Bond Investor Betas within Company

A.Within Firm Standard Deviation in Investor Betas						
	mean	p25	median	p75	sd	N
2 to 5 Bonds	0.21	0.11	0.19	0.29	0.13	57,478
5 to 10 Bonds	0.26	0.20	0.26	0.32	0.08	22,841
more than 10 Bonds	0.28	0.24	0.29	0.32	0.06	17,505
B. .Within Firm Standard Deviation in Investor Betas (excluded)						
	mean	p25	median	p75	sd	N
2 to 5 Bonds	0.21	0.11	0.19	0.30	0.13	57,323
5 to 10 Bonds	0.26	0.20	0.27	0.32	0.08	22,839
more than 10 Bonds	0.28	0.25	0.29	0.32	0.06	17,505

B General Equilibrium

We model a multi-asset market similar to that in Merton (1987), with two key generalizations. First, we allow risk, even after it has been residualized against a common factor, to be correlated across stocks rather than uncorrelated. Second, we do not assume strict rationality of all investors. Rather, traders in our economy feature, “group rationality”, a weaker condition. In what follows, uppercase letters are matrices, bold lowercase are vectors, and all else are scalars.

B.1 Model Environment

We assume a two period economy with discrete time $t = 0, 1$. There are N stocks in the economy, indexed by $s = 1, ..., N$, each in unit supply. A risk-free bond is available in perfectly elastic supply with a gross interest rate of r_f. Stock s earns time-1 dividends of d_s per share. We collect the individual-stock dividends in the column vector $\boldsymbol{d}$ and assume that $\boldsymbol{d} \sim \mathcal{N}(\boldsymbol{\mu_d}, \Gamma)$.

Let $R_m \equiv q'D - P_m R_f$ denote the dollar excess return on the market. Similarly, let $\tilde{R} \equiv D - P\, R_f$ denote the vector of dollar excess returns on the N stocks and let $r \equiv \tilde{R} - \beta_m R_m$ be the vector of market-neutral returns.[20]

Since prices are observable, the covariance matrix of returns equals the covariance matrix of dividends, Γ, with eigenvalue decomposition $\Gamma = Q \Lambda Q'$. We assume that the first eigenvector (associated with largest eigenvalue) is proportional to market weights, ι.

Since the market portfolio is the first PC of Γ, we have that

$$\Sigma \equiv \mathrm{cov}\,(r) = Q \Lambda_{-1} Q', \tag{C.1}$$

where Λ_{-1} equals Λ with the (1,1) element set to zero. The variance of returns on the market

20. β_m is the vector of market betas.

portfolio is

$$\sigma_m^2 = \text{Var}(R_m) = \iota' q_1 q_1' \iota \lambda_1 = N\lambda_1, \tag{C.2}$$

where λ_1 is the first eigenvalue of Λ.

B.2 Investors

There are K masses of investors ("funds") with exponential utility and mass $\mathcal{W}_k$, which we refer to as wealth. Let $\mathcal{W} = \sum \mathcal{M}_k$ be the total wealth of investors. Their problem is

$$\max E_k \left[-\exp\left(-\rho W_{k1}\right)\right] \tag{C.3}$$

$$\text{s.t. } W_{k1} = \left(W_{k0} - C_{k0}\right) R_F + yR_m + w_k \left(R_1\right), \tag{C.4}$$

where E_k represents an expectation taken under investor ks beliefs. Each investor k may only hold a subset $\mathbb{S}_k \subseteq \{1 \ldots N\}$ of the universe of assets, as well as the risk-free asset and market ETF.[21] This constraint is equivalent to the "knowledge" limitation in Merton (1987). It is further motivated by He and Xiong (2013), who show that narrow mandates can arise as optimal contracts for delegated asset management. We assume investors know objective covariances, Σ, and then aggregate, but may disagree about the market risk premium, μ_m, and CAPM deviations, α. Let $\nu_k = \alpha_k - \alpha$ be the vector of cross-sectional belief distortions of fund k. We use the convention that if asset s is not in $\mathbb{S}_k$ (not allowed to be held by investor k) then $\nu_{ks} = 0$. Note that since we deal with dollar returns, disagreement about expected cash flows and returns are equivalent.

As is well known, an investor's problem can be separated into two parts; choosing an optimal quantity of the market ETF and independently, choosing how much of each market-neutralized stock to hold. Let S_k be a diagonal matrix with entries equal to 1 if the asset is in $\mathbb{S}_k$ and 0 otherwise. S_k is called a "selector" matrix. Fund k's optimal portfolio is given

21. Equivalently, investors face fixed holding costs for individual stocks, which may vary by asset $i\times$ investor k. Then $\mathbb{S}_k$ can be interpreted as the set of assets the investor chooses to hold.

by

$$y_k = \frac{\mu_{m,k}}{\rho\sigma_m^2} \tag{C.5}$$

$$\theta_k = \frac{1}{\rho}\left(S_k \Sigma S_k\right)^+ \alpha_k, \tag{C.6}$$

where $(\cdot)^+$ indicates the Moore-Penrose pseudo-inverse.

In addition, there may be traders with inelastic demand vector $\tilde{\delta}$. These traders can be viewed as sentiment investors, or investors with un-modeled hedging demands. Linearly projecting δ on to the market portfolio weights, we have $\tilde{\delta} = \delta_m + \delta$ where $\delta_m = \left(q'\tilde{\delta}\right) q$.

B.3 Equilibrium

First, for the aggregate index, market clearing gives

$$1 = \frac{1}{\rho\sigma_m^2}\sum \mathcal{W}_k \mu_{m,k} + \delta_m. \tag{C.7}$$

Since we make no assumption regarding $\mu_{m,k}$, the model is silent about the objective market risk premium, μ_m.

From this point, we can "solve" the model in two ways. First, we briefly show the explicit solution for α, which turns out to be somewhat ugly. Next, we exploit the first order condition in eq. (C.6) to obtain each investor's subjective risk-return relationship and then aggregate. This analysis yields an empirically useful characterization of the equilibrium without providing a direct construction for expected returns.

B.3.1 Explicit Solution

Start with eq. (C.6) and aggregate across funds. Market clearing implies

$$\alpha = -\Omega^{-1}\delta - \sum_1^K \mathcal{W}_k \left[S_k \Sigma S_k\right]^+ \nu_k \tag{C.8}$$

$$\Omega = \sum_1^K \mathcal{W}_k \left[S_k \Sigma S_k \right]^+ . \tag{C.9}$$

If investors are unconstrained (can hold all assets), the solution simplifies to

$$\alpha = -\Sigma\delta - \sum_1^K \mathcal{W}_k \nu_k. \tag{C.10}$$

Eq. (C.8) shows that in the constrained model, investors' belief errors, ν_k, affect equilibrium prices unless the share-weighted average error is zero. Eq. (C.10) shows that in the unconstrained model, investors' belief errors, ν_k, affect equilibrium prices unless the wealth-weighted average error is zero. In our empirical analysis, we use both weighting schemes and obtain similar findings.

B.3.2 Risk-Return

Let $z_k = \theta_k' r$ be investor k's portfolio return orthogonalized with respect to the market. Start with the optimal portfolio weights eq. (C.6), compute the covariance of any stock $r_s \in \mathbb{S}_k$ with z_k to obtain the investor's subjective risk-return trade off for market-neutral betas

$$\alpha_{s,k} = \rho \operatorname{cov}\left(r_s,\, z_k\right), \tag{C.11}$$

which resembles a subjective "idiosyncratic CAPM." Note that the wealth of each investor type, $\mathcal{W}_k$, does not explicitly show up in this expression. Of course the distribution of wealth affects the equilibrium, but the investor optimization adjusts her portfolio weights so that her Euler equation holds for all assets she can trade.

Combined with group rationality we obtain the objective pricing relation

$$\alpha_s = \rho \sum_{\kappa_s} \omega_k \operatorname{cov}\left(r_s,\, z_k\right) = \rho \operatorname{cov}\left(r_s,\, \sum_{\kappa_s} \omega_k z_k\right). \tag{C.12}$$

Notice that the above representation makes no assumptions about ν_k or δ, and thus holds no matter what values these distortions take.

B.4 An Example

We now present a simple example of the model to highlight the main mechanism. The model environment conforms to the above general framework. The specific configuration of covariance, constraints, and noise trading are stark in order to clearly demonstrate the possible consequences of partial segmentation. There are four assets, (1, 2, 3, 4) and four unit masses of investors, $(A,\ B,\ C,\ D)$, each of whom can invest in only two of the four assets and have unit risk aversion. A can invest in assets 1 and 2, B in 2 and 3, and C in 3 and 4 and D in 1 and 4. In addition, each investor can invest in the market ETF.

Recall Γ is the covariance matrix of returns, with eigen decomposition $\Gamma = Q\Lambda Q'$. We assume Q and Λ are

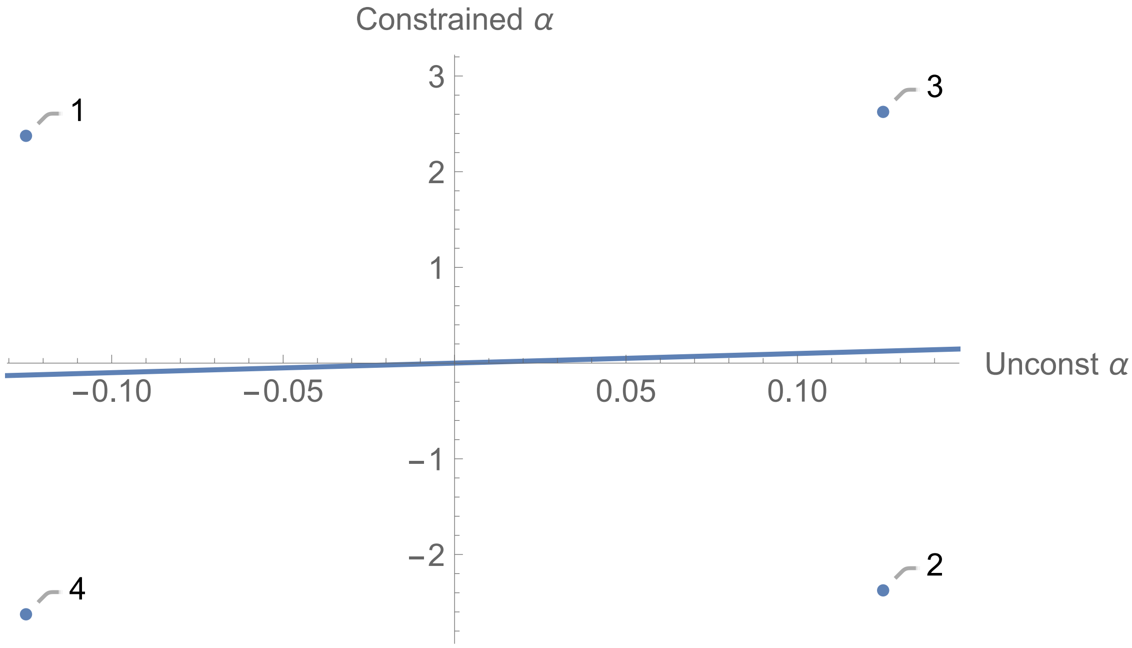
$$Q = \begin{bmatrix} \frac{1}{2} & \frac{1}{\sqrt{2}} & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{\sqrt{2}} & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & -\frac{1}{\sqrt{2}} & -\frac{1}{2} \end{bmatrix}, \quad \Lambda = \begin{bmatrix} \Xi & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \xi \end{bmatrix}, \tag{C.13}$$

where $\Xi \gg 1$ and $\xi \ll 1$. The residual correlation matrix (after orthogonalizing with respect to the market) is

$$\mathrm{corr}\,(r) = \begin{bmatrix} 1 & -\frac{1}{2\xi} & -1+\frac{1}{2\xi} & -\frac{1}{2\xi} \\ -\frac{1}{2\xi} & 1 & -\frac{1}{2\xi} & -1+\frac{1}{2\xi} \\ -1+\frac{1}{2\xi} & -\frac{1}{2\xi} & 1 & -\frac{1}{2\xi} \\ -\frac{1}{2\xi} & -1+\frac{1}{2\xi} & -\frac{1}{2\xi} & 1 \end{bmatrix} \approx \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}, \tag{C.14}$$

Finally, we assume $\delta' \propto [1-\varepsilon\ 1+\varepsilon\ 1-\varepsilon\ 1+\varepsilon]$ where $\varepsilon \approx 0$. Immediately one can see that as $\xi \to 0$ the model admits strict arbitrage. However, no investor is able to exploit this

Fig. B.1. Unconstrained and Constrained Equilibria



Note: This figure displays equilibrium CAPM αs from the unconstrained model on horizontal axis and αs from the constrained model on the vertical axis.

opportunity.

Using the constrained and unconstrained solutions given in B.3.1, we obtain equilibrium CAPM-αs under both settings. The figure shows that constraints can change the relative ordering of αs, can cause some αs to change sign, and other αs to increase in magnitude. In short, measuring the average investor β rather than β with respect to the aggregate investor portfolio is key. Looking at the scale of the axes, the cross-sectional dispersion in α is almost zero in the unconstrained setting. In contrast, the constrained setting generates two orders of magnitude greater dispersion. This obtains because no investor can take advantage of the approximate arbitrages available due to the Merton-style holding restrictions.