General Equilibrium Analysis of Multinational Financial and Trade Linkages

Chenyue Hu

UC Santa Cruz

Motivation

Existing Literature

- Few multi-country structural models with both trade and finance
  - Trade literature: takes asset positions as exogenous
  - Macro literature: studies a small number of countries
- General equilibrium effects hence not fully captured

This Paper

- Combines macro/trade methodological breakthroughs
- Solves for countries' multilateral linkages in both channels
- Delivers policy implications with comparative static analysis

Why Need An Integrated Framework with Trade and Finance?

Questions not fully answered by existing literature

Trade Influences Finance

- How do cross-country input-output linkages draw the global capital allocation map?
- How might a trade war affect countries' optimal portfolio choice?

Finance Influences Trade

- How would international trade respond to regional financial integration?
- How would the tightening of global financial markets influence the direction and volume of trade?

Relation to Literature

Portfolio choice in DSGE framework

- Literature: Devereux and Sutherland (DS), Tille and Van Wincoop
- Idea: 2nd-order approx of portfolio eqn + 1st-order approx of others
  $\Rightarrow$ steady-state (s.s.) portfolio
- Shortcoming: computationally hard to implement across many countries

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Exact hat algebra from trade literature

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- Idea: characterize variables' changes instead of levels $\hat{X} = \frac{X^{ctf}}{X^{org}}$
- Limitation: finance is exogenously taken from data or endogenously derived under extreme cases (autarky or complete markets)

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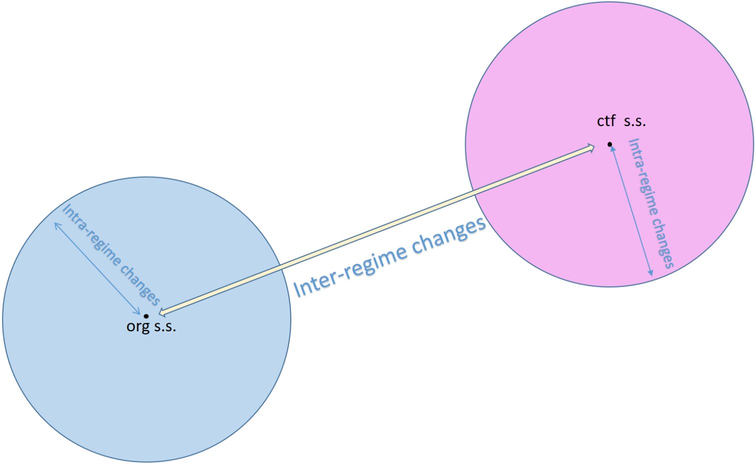
Contribution of this paper

- To trade literature: adds endogenous financial allocations under intertemporal utility maximization + cross-country frictions
- To macro literature: solves financial and trade linkages in a multi-country framework with a new approach

Main Idea of the New Approach



- Policies both shift locations of s.s. and affect how variables behave around s.s.
- Second-moment variables around s.s. matter for portfolio choice
- Hat algebra is used for both inter- and intra-regime analyses

Presentation Outline

Introduction

Model

Appendix

Goods Market

- A single tradable sector with intermediate goods

$$Q_{i,t} = \int_0^1 [q_{iu,t}(u)^{\frac{\epsilon-1}{\epsilon}} du]^{\frac{\epsilon}{\epsilon-1}}$$

- Productivity drawn from Fréchet distribution

$$F_{i,t}(z) = exp(-T_{i,t} z^{-\theta})$$

- Country-level productivity $T_{i,t}$ follows an AR(1) subject to shocks with cross-country covariance Σ_T around mean $\bar{T}_i$

$$T_{i,t} = \rho T_{i,t-1} + (1-\rho)\bar{T}_i + \epsilon_{i,t}$$

- Fixed labor and capital endowments with wage $w_{i,t}$ and rent $r_{i,t}$, final goods used as input with price $P_{i,t}$
- Bilateral trade shares given iceberg trade cost τ_{ij}

$$\pi_{ij,t} = \frac{T_{i,t}[\tau_{ij}(r_{i,t}^{\mu} w_{i,t}^{1-\mu})^{\eta} P_{i,t}^{1-\eta}]^{-\theta}}{\Phi_{j,t}}, \ \Phi_{j,t} = \sum_{k=1}^{I} T_{k,t}[\tau_{kj}(r_{k,t}^{\mu} w_{k,t}^{1-\mu})^{\eta} P_{k,t}^{1-\eta}]^{-\theta}$$

- Goods market clearing:
  $Y_{i,t} = \sum_{j=1}^{I} \pi_{ij,t} X_{j,t},$ where $X_{j,t} = (1-\eta) Y_{j,t} + P_{j,t} C_{j,t}$

Financial Market

- Wealth constraint

$$\mathcal{W}_{i,t+1} = \mathcal{W}_{i,t} R_{I,t+1} + \sum_{j=1}^{I-1} \alpha_{ij,t}(R_{j,t+1} - R_{I,t+1}) + \eta(1-\mu) Y_{i,t+1} - X_{i,t+1}$$

- Asset positions $\mathcal{W}_{i,t} = \sum_{j=1}^{I} \alpha_{ij,t}$
- Countries issue equities as claims to capital income with dividends ($d_{i,t}$), prices ($q_{i,t}$), and returns ($R_{i,t}$)

$$d_{i,t} = \mu\eta Y_{i,t}, \qquad R_{i,t+1} = \frac{d_{i,t+1} + q_{i,t+1}}{q_{i,t}}$$

- Households' objective: $\max \sum_{t=0}^{\infty} \beta^t \frac{C_{i,t}^{1-\gamma}}{1-\gamma}$
- Euler equation

$$\frac{C_{i,t}^{-\gamma}}{P_{i,t}} = E_t[\frac{C_{i,t+1}^{-\gamma}}{P_{i,t+1}} R_{i,t+1}] = \beta E_t[\frac{C_{i,t+1}^{-\gamma}}{P_{i,t+1}} e^{-f_{ij}} R_{j,t+1}], \forall i,j \in [1, I].$$

  - Inter-temporal investment influenced by $\beta, \gamma, P_{i,t+1}, R_{i,t+1}$
  - Intra-temporal investment influenced by Σ_T, f_{ij}

Portfolio Choice

Solution method developed by DS, similar to Tille and Van Wincoop (2010)

- Log-deviation of any variable A from s.s.

$$\widetilde{A}_t = \ln(\frac{A_t - \bar{A}}{\bar{A}})$$

- Log-deviation of any cross-country ratio from s.s.

$$\widetilde{B}_{i/j,t} = \widetilde{B}_{i,t} - \widetilde{B}_{j,t} \text{ for } B_{i/j,t} = \frac{B_{i,t}}{B_{j,t}}$$

- Vector of excess returns relative to the numeraire asset I

$$\widetilde{R}'_{x,t+1} = [\widetilde{R}_{1,t+1} - \widetilde{R}_{I,t+1}, \widetilde{R}_{2,t+1} - \widetilde{R}_{I,t+1}, ..., \widetilde{R}_{I\text{-}1,t+1} - \widetilde{R}_{I,t+1}],$$

- Second-order Taylor expansion of Euler equations

$$E_t[\frac{U'(C_{i,t+1})}{P_{i,t+1}} e^{-f_{iI}} R_{I,t+1}] = E_t[\frac{U'(C_{i,t+1})}{P_{i,t+1}} e^{-f_{i1}} R_{1,t+1}]$$
$$... = E_t[\frac{U'(C_{i,t+1})}{P_{i,t+1}} e^{-f_{iI\text{-}1}} R_{I\text{-}1,t+1}].$$

Standard DSGE Approach

- Portfolio determination equation

$$E_t[(\gamma \widetilde{C}_{i/I,t+1} + \widetilde{P}_{i/I,t+1})\widetilde{R}'_{x,t+1}] = \frac{1}{2} F_{iI}$$

- Financial frictions

$$F_{iI} = [f_{iI} - f_{II}] \times \text{ones}(I-1,1) - [f_{i1} - f_{I1}, f_{i2} - f_{I2}, ..., f_{iI\text{-}1} - f_{II\text{-}1}].$$

- Take the derivative of two LHS terms wrt productivity shocks and derive portfolios that support consumption allocations

$$E_t(\underbrace{G_{t+1}}_{1\times I} \times \underbrace{\Sigma_T}_{I\times I} \times \underbrace{H'_{t+1}}_{I\times(I\text{-}1)}) = \frac{1}{2}\underbrace{F_{iI}}_{1\times(I\text{-}1)} + \mathcal{O}(\epsilon^3).$$

Blanchard-Kahn Method

- Solve s.s. values of all the variables
- Log-linearize all the equations
- Evaluate how variables behave around s.s. to the second order

New Approach — Apply Hat Algebra around s.s.

Implementation

- Treat s.s. as initial level around which simulate productivity shocks ($\widehat{T}$)
- Compute hat variables in response to shocks $\widehat{B}_i = \frac{B_{i,t}}{\bar{B}_i}$
- Convert hat variables to tilde variables

$$\widetilde{B}_{i/j,t} = \widetilde{B}_{i,t} - \widetilde{B}_{j,t} = \ln\left(\frac{\widehat{B}_{i,t}}{\widehat{B}_{j,t}}\right) = \ln(\widehat{B}_{i/j,t}).$$

- Calculate second moments in portfolio equations

Tractability

- No need for loglinearization to predict variables' reactions to shocks
- No need to calculate variables' s.s. values
- Need few sufficient statics which already embed trade/finance frictions

▸ Calibration 1 ▸ Calibration 2 (Productivity)

New Approach — Apply Hat Algebra around s.s.

Goal: Compute second-moment variables under 'simulated' productivity shocks

- Vectors to solve for: wage and price

$$\widehat{w'} = [\widehat{w}_1, \widehat{w}_2, ..., \widehat{w}_I], \qquad \widehat{P'} = [\widehat{P}_1, \widehat{P}_2, ..., \widehat{P}_I]$$

- Characterizing equations

$$\widehat{P}_i^{-\theta} = \sum_{j=1}^{I} \bar{\pi}_{ji} \widehat{T}_j (\widehat{w}_j^{\eta} \widehat{P}_j^{1-\eta})^{-\theta},$$

$$\widehat{w}_i \bar{Y}_i = \sum_{j=1}^{I} \frac{\bar{\pi}_{ij} \widehat{T}_i (\widehat{w}_i^{\eta} \widehat{P}_i^{1-\eta})^{-\theta}}{\sum_{k=1}^{I} \bar{\pi}_{kj} \widehat{T}_k (\widehat{w}_k^{\eta} \widehat{P}_k^{1-\eta})^{-\theta}} \widehat{X}_j \bar{X}_j.$$

- Sufficient statistics $\bar{Y}, \bar{\pi}, \bar{D}, \bar{\alpha}$ directly observable from data
- Express variables as functions of $\widehat{w}_i, \widehat{P}_i$, convert 'hat' to 'tilde' by taking log, and compute products

$$\widetilde{Y}\widetilde{R}', \qquad \widetilde{P}\widetilde{R}', \qquad \widetilde{R}\widetilde{R}'$$

- Next step: Add wealth as a state variable and find its covariance with $\widehat{w}_i, \widehat{P}_i$ through linearization

Mechanisms for Portfolio Choice

- Asset home bias literature (surveyed by Coeurdacier and Rey 2013)

$$\bar{\alpha}_{ii} = \underbrace{\frac{1}{2}}_{\substack{\text{Risk sharing}\\\text{(Diversification)}}} - \underbrace{\frac{1}{2}\frac{1-\mu}{\mu}\frac{cov(\widetilde{w}_{i/j}, \widetilde{R}_{i/j})}{var(\widetilde{R}_{i/j})}}_{\text{Hedging labor income risk}} + \underbrace{\frac{1}{2}\frac{1-1/\gamma}{\mu}\frac{cov(\widetilde{P}_{i/j}, \widetilde{R}_{i/j})}{var(\widetilde{R}_{i/j})}}_{\text{Hedging RER risk}}.$$

- Portfolio should be diversified for risk sharing (Lucas 1982)
- Portfolio should tilt toward assets whose income
  - decreases with domestic labor income
  - increases with real exchange rate (RER)

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Extensions made in this paper

- complete ⇒ incomplete markets: 2nd-moments including asset covariances matter
- 2 ⇒ 43 countries: individual foreign asset returns less correlated with domestic fundamentals
- s.s. no longer fixed and exogenous: financial/real variables jointly respond to trade/financial frictions

Disentangling Mechanisms for Portfolio Choice

- General portfolio equation

$$E_t[\gamma(1-\beta)\widetilde{Y}\widetilde{R}' + (1-\gamma+\beta\gamma)\widetilde{P}\widetilde{R}' + \gamma(1-\beta)\breve{\alpha}\widetilde{R}\widetilde{R}'] = \frac{1}{2}F$$

- No risk sharing — impose the Backus-Smith condition

$$E_t[(\gamma\widetilde{C} + \widetilde{P})] = E_t[(\gamma(1-\beta)\widetilde{Y} + (1-\gamma+\beta\gamma)\widetilde{P} + \gamma(1-\beta)\breve{\alpha}\widetilde{R})] = 0.$$

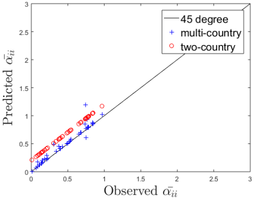
- No labor risk hedging — exclude labor income from wealth constraint

$$E_t[\gamma(1-\beta)(1-\eta+\eta\mu)\widetilde{Y}\widetilde{R}' + (1-\gamma+\beta\gamma)\widetilde{P}\widetilde{R}' + \gamma(1-\beta)\breve{\alpha}\widetilde{R}\widetilde{R}'] = \frac{1}{2}F.$$

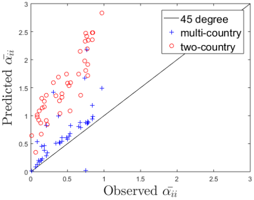
- No RER risk hedging — assume log utility ($\gamma = 1$)

$$E_t[(1-\beta)\widetilde{Y}\widetilde{R}' + \beta\widetilde{P}\widetilde{R}' + (1-\beta)\breve{\alpha}\widetilde{R}\widetilde{R}'] = \frac{1}{2}F.$$

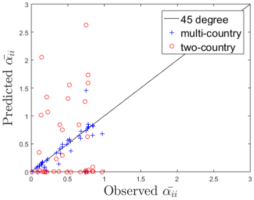
Domestic Asset Holdings ($\bar{\alpha}_{ii}$) in 2- vs 43-countries



(a) No labor risk hedging



(b) No RER risk hedging



(c) No risk sharing

Finding: Risk-hedging becomes less important in explaining asset positions given countries' multilateral linkages.

Bilateral Trade and Financial Linkages

- High correlation of bilateral linkages across two channels

$$Corr(\alpha_{ij}, \pi_{ij}) = 0.835, \quad \text{where } \pi_{ij} = \frac{\bar{\pi}_{ij} + \bar{\pi}_{ji}}{2}, \quad \alpha_{ij} = \frac{\bar{\alpha}_{ij} + \bar{\alpha}_{ji}}{2}, \quad \forall i, j \in [1, I]$$

- Two potential explanations (assuming given trade structure)
  - Trade influences second moments and therefore risk-sharing/-hedging patterns
  - Bi-trade and bi-finance face common barriers such as policy/information/cultural frictions
- Counterfactual exercises based on the structural model

$Corr(\alpha_{ij}, \pi_{ij})$ data 0.835	No risk sharing	No bi-friction differential
	0.587	0.524
	No labor hedging	No RER hedging
	0.835	0.696

Empirical Evidence from Bilateral Asset Positions

Dep. Var: log(Bilateral Holdings)	(1)	(2)	(3)	(4)
$\log(GDP_o)$	1.245 ***	1.108 ***	1.085 ***	1.067 ***
	(0.034)	(0.061)	(0.062)	(0.061)
$\log(GDP_d)$	1.442 ***	-0.012	0.042	0.048
	(0.032)	(0.093)	(0.094)	(0.090)
log(dist)	-0.709 ***	-1.167 ***	-1.202 ***	-1.099 ***
	(0.037)	(0.021)	(0.022)	(0.021)
Chinn-Ito			0.674 **	1.412 ***
			(0.298)	(0.298)
cov(T)				2.498 ***
				(0.224)
Chinn $\times$ cov(T)				-2.093 ***
				(0.249)
Fixed Effects	N	Y	Y	Y
Gravity Var	N	Y	Y	Y
Observations	22,448	22,448	20,807	20,807
R^2	0.123	0.957	0.959	0.960

Robust standard errors in parentheses. Asset positions are from Factset/Lionshare. Fixed Effects include origin-, destination-, and time-FE. Gravity variables from CEPII. Chinn-Ito measures capital openness. cov(T) is estimated productivity covariance.

Joint Determination of Trade and Finance

- Two policy regimes: $s \in \{org, ctf\}$
- Tariff changes: $\hat{\tau} = \frac{\tau_{ij}^{ctf}}{\tau_{ij}^{org}} = 1.5, \forall i \neq j \in [1, I]$
- Two types of changes

$$\underbrace{\ln(Y_{i/j,t+1}^{ctf}) - \ln(Y_{i/j,t+1}^{org})}_{\text{Total changes}} = \underbrace{[\ln(\bar{Y}_{i/j}^{ctf}) - \ln(\bar{Y}_{i/j}^{org})]}_{\text{Inter-regime changes}}$$

$$+ \underbrace{[\ln(Y_{i/j,t+1}^{ctf}) - \ln(\bar{Y}_{i/j}^{ctf})] - [\ln(Y_{i/j,t+1}^{org}) - \ln(\bar{Y}_{i/j}^{org})]}_{\text{Intra-regime changes}}.$$

Inter-regime changes

- Employ hat algebra similar to DEK: $\widehat{\bar{A}} = \frac{\bar{A}^{ctf}}{\bar{A}^{org}}$
- Variables to solve for: wage and price

$$\widehat{\bar{w}'} = [\widehat{\bar{w}}_1, \widehat{\bar{w}}_2, ..., \widehat{\bar{w}}_I], \qquad \widehat{\bar{P}'} = [\widehat{\bar{P}}_1, \widehat{\bar{P}}_2, ..., \widehat{\bar{P}}_I]$$

- Characterizing equations

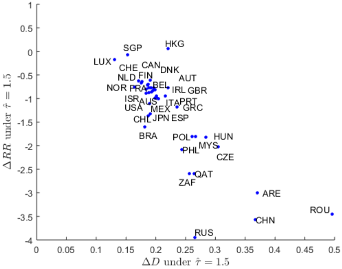
$$\widehat{\bar{P}}_i^{-\theta} = \sum_{j=1}^{I} \bar{\pi}_{ji}^{org} \widehat{\tau}_{ji}^{-\theta} (\widehat{\bar{w}}_j^{\eta} \widehat{\bar{P}}_j^{1-\eta})^{-\theta},$$

$$\widehat{\bar{w}}_i \bar{Y}_i^{org} = \sum_{j=1}^{I} \frac{\bar{\pi}_{ij}^{org} \widehat{\tau}_{ij}^{-\theta} (\widehat{\bar{w}}_i^{\eta} \widehat{\bar{P}}_i^{1-\eta})^{-\theta}}{\sum_{k=1}^{I} \bar{\pi}_{kj}^{org} \widehat{\tau}_{kj}^{-\theta} (\widehat{\bar{w}}_k^{\eta} \widehat{\bar{P}}_k^{1-\eta})^{-\theta}} \widehat{\bar{w}}_j \bar{Y}_j^{org} [1 - \bar{D}_j^{ctf} (1 - \frac{1}{\beta})]$$

- $\bar{D}^{ctf}$ (wealth/GDP) solved under intra-regime changes

Comparative Statics under Higher Tariffs

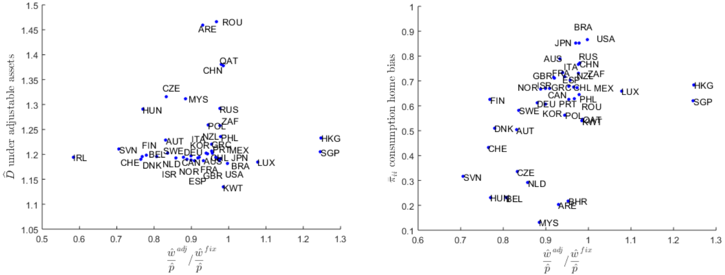
Figure: Country *i*'s Median Second Moments and Asset Positions



(a) $\Delta \bar{D}_i$ and $\Delta \tilde{R}_i \tilde{R}'$

Finding:
$\tilde{R}_i \tilde{R}' \Downarrow \Rightarrow$ Better risk sharing by foreign assets $\Rightarrow \bar{D}_i \Uparrow$

Real wages under asset positions adjustable to tariffs



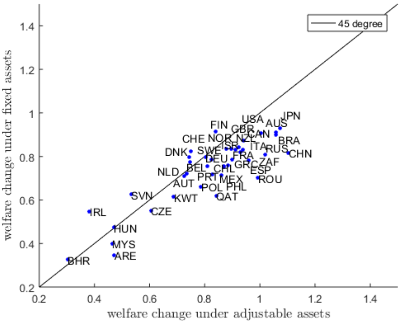
(a) $\frac{\widehat{\bar{w}}}{\widehat{\bar{p}}}$ and $\widehat{\bar{D}}_i$ **(b)** $\frac{\widehat{\bar{w}}}{\widehat{\bar{p}}}$ and $\bar{\pi}_{ii}$

Finding:

Real wages tend to rise more under higher $\widehat{\bar{D}}_i$, especially for countries with stronger expenditure home bias $\bar{\pi}_{ii}$.

Welfare under fixed and adjustable asset positions

$$\widehat{\mathbb{W}}_i = \frac{\widehat{\bar{w}}_i}{\widehat{\bar{P}}_i} \widehat{\bar{D}}_i$$



Finding: Treating finance as fixed may overestimate welfare loss.

Conclusion

This paper

- Combines portfolio choice solution method and hat algebra technique to solve a general equilibrium model with trade and finance
- Delivers policy implications with counterfactual analyses with tariffs and financial frictions

Future work

- Expands into a full-fledged macro model

  Solve with hybrid linearization + hat algebra
- Traces dynamic portfolio choice

  3rd-order approx of Euler eqn + 2nd-order approx of other eqns $\Rightarrow$ 1st-order dynamic portfolios

Presentation Outline

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Model

Appendix

Country List

Name	Code	Name	Code	Name	Code	Name	Code
Australia	AUS	France	FRA	Luxembourg	LUX	Russia	RUS
Austria	AUT	Germany	DEU	Malaysia	MYS	Singapore	SGP
Bahrain	BHR	Greece	GRC	Mexico	MEX	Slovenia	SVN
Belgium	BEL	Hong Kong	HKG	Netherlands	NLD	Spain	ESP
Brazil	BRA	Hungary	HUN	New Zealand	NZL	Sweden	SWE
Canada	CAN	Ireland	IRL	Norway	NOR	Switzerland	CHE
Chile	CHL	Israel	ISR	Philippines	PHL	U.A.E.	ARE
China	CHN	Italy	ITA	Poland	POL	United Kingdom	GBR
Czech	CZE	Japan	JPN	Portugal	PRT	United States	USA
Denmark	DNK	Korea	KOR	Qatar	QAT	South Africa	ZAF
Finland	FIN	Kuwait	KWT	Romania	ROU		

Calibration

- 43 countries plus the rest of the world (ROW)
- GDP, NFA from Penn World Table (PWT)
- bilateral trade shares from DOTS, bilateral portfolio weights from Factset/Lionshare
- annual discount factor $\beta = .9$, coefficient of relative risk aversion $\gamma = 2$
- share of intermediate input in production $\eta = .312$ following DEK, share of labor input $1 - \mu$ as country-specific labor income share from PWT
- trade elasticity $\theta = 4$ following Simonovska and Waugh (2014)

Back

Estimating Dynamic Productivity

Method developed by Levchenko and Zhang (2014)

- Theoretical prediction based on Eaton-Kortum model

$$\frac{\pi_{ij,t}}{\pi_{jj,t}} = \frac{T_{i,t}}{T_{j,t}}\left(\frac{\tau_{ij,t}c_{i,t}}{c_{j,t}}\right)^{-\theta}.$$

- Regression equation

$$\ln\left(\frac{\pi_{ij,t}}{\pi_{jj,t}}\right) = \ln(T_{i,t}\hat{c}_{i,t}^{-\theta}) - \ln(T_{j,t}\hat{c}_{j,t}^{-\theta}) - \theta\hat{\tau}_{ij,t} + \gamma_{ij,t},$$

- Exponentiating importer FE

$$\widehat{Tc}_{j,t} = T_{j,t}\hat{c}_{j,t}^{-\theta}.$$

- Estimating production cost using PWT data

$$\hat{c}_{i,t} = (r_{i,t}^{\mu} w_{i,t}^{1-\mu})^{\eta} P_{i,t}^{1-\eta}.$$

- Recovering Ricardian productivity

$$T_{j,t} = T_{US,t}\frac{\widehat{Tc}_{j,t}}{\widehat{Tc}_{US,t}}\left(\frac{\hat{c}_{j,t}}{\hat{c}_{US,t}}\right)^{\theta}$$

- Simulating productivity shocks with $\bar{T}$ and Σ_T or with Bootstrap

Back

Intra-regime changes

- Use hat algebra again in each regime

$$\widehat{w}^s = \frac{w_t^s}{\bar{w}^s}, \qquad \widehat{P}^s = \frac{P_t^s}{\bar{P}^s}, \qquad s \in \{org, ctf\}$$

- Characterizing equations

$$\widehat{P}_i^{-\theta,s} = \sum_{j=1}^{I} \bar{\pi}_{ji}^s \widehat{T}_j (\widehat{w}_j^{\eta,s} \widehat{P}_j^{1-\eta,s})^{-\theta},$$

$$\widehat{w}_i^s \bar{Y}_i^s = \sum_{j=1}^{I} \frac{\bar{\pi}_{ij}^s \widehat{T}_i (\widehat{w}_i^{\eta,s} \widehat{P}_i^{1-\eta,s})^{-\theta}}{\sum_{k=1}^{I} \bar{\pi}_{kj}^s \widehat{T}_k (\widehat{w}_k^{\eta,s} \widehat{P}_k^{1-\eta,s})^{-\theta}} \widehat{X}_j^s \bar{X}_j^s$$

- $\bar{Y}^{org}, \bar{\pi}^{org}$ calibrated to data, $\bar{Y}^{ctf}, \bar{\pi}^{ctf}$ predicted by inter-regime changes
- Express variables as functions of $\widehat{w}_i^s$, $\widehat{P}_i^s$, convert 'hat' to 'tilde' variables by taking log, and compute products

$$\widetilde{Y}\widetilde{R}'^s, \qquad \widetilde{P}\widetilde{R}'^s, \qquad \widetilde{R}\widetilde{R}'^s$$

Back

Combined intra- and inter-regime changes

- Evaluate the portfolio equation within each regime

$$E_t[(\gamma(1-\beta)\widetilde{R}\widetilde{R}'^{s}(1+\breve{\alpha}^s)+(1-\gamma+\beta\gamma)\widetilde{P}\widetilde{R}'^{s}] = \frac{1}{2}F^s,\ s \in \{org, ctf\}$$

- Take cross-regime difference to find portfolio changes

$$E_t[(\gamma(1-\beta)\Delta\widetilde{R}\widetilde{R}'(1+\Delta\breve{\alpha}) + (1-\gamma+\beta\gamma)\Delta\widetilde{P}\widetilde{R}'] = \frac{1}{2}\widetilde{F}F^{org}.$$

- Use solved $\bar{D}^{ctf}$ to update inter-regime changes

$$\widehat{\bar{w}}_i \bar{Y}_i^{org} = \sum_{j=1}^{I} \frac{\bar{\pi}_{ij}^{org}\widehat{\tau}_{ij}^{-\theta}(\widehat{\bar{w}}_i^{\eta}\widehat{\bar{P}}_i^{1-\eta})^{-\theta}}{\sum_{k=1}^{I}\bar{\pi}_{kj}^{org}\widehat{\tau}_{kj}^{-\theta}(\widehat{\bar{w}}_k^{\eta}\widehat{\bar{P}}_k^{1-\eta})^{-\theta}} \widehat{\bar{w}}_j \bar{Y}_j^{org}\bar{D}_j^{ctf}$$

- Keep iterating until a joint fixed-point problem with $(\widehat{\bar{w}}, \widehat{\bar{P}}, \Delta\bar{D})$ is solved. ▸ Existence ▸ Algorithm

Existence and uniqueness of solution

Excess demand

$$\mathbb{Z}_i(\widehat{\bar{w}}_i) = \frac{1}{\widehat{\bar{w}}}[\widehat{\bar{w}}_i \bar{Y}_i^{org} - \sum_{j=1}^{I} \frac{\bar{\pi}_{ij}^{org} \widehat{\tau}_{ij}^{-\theta} (\widehat{\bar{w}}_i^{\eta} \widehat{\bar{P}}_i^{1-\eta})^{-\theta}}{\sum_{k=1}^{I} \bar{\pi}_{kj}^{org} \widehat{\tau}_{kj}^{-\theta} (\widehat{\bar{w}}_k^{\eta} \widehat{\bar{P}}_k^{1-\eta})^{-\theta}} \widehat{\bar{w}}_j \bar{Y}_j^{org} \bar{D}_j^{ctf}].$$

Properties of $\mathbb{Z}_i(w)$ by Alvarez and Lucas (2007)

- continuous
- homogenous of degree zero
- has the gross substitute property $\frac{\partial \mathbb{Z}_i(w)}{\partial w_j} > 0$
- satisfies Walras's Law ($\sum_i w_i \mathbb{Z}_i(w) = 0$)
- faces a lower but not upper bound
  $\mathbb{Z}_i(w) > -\max_j L_j, \max_i \mathbb{Z}_i(w \to w^{org}) \to \infty$

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Algorithm

- Step 1. Calibrate the original steady state (s.s.) of the economy
  GDP, NFA, bilateral trade and portfolio weights
- Step 2. Form initial guesses about inter-regime changes

$$\hat{\bar{w}}^0 = \hat{\bar{P}}^0 = ones(I, 1), \Delta\bar{D} = zeros(I, 1)$$

- Step 3. Given predicted s.s., characterize intra-regime changes to get

$$\widetilde{Y}\widetilde{R}'^s, \widetilde{P}\widetilde{R}'^s, \widetilde{R}\widetilde{R}'^s, \; s \in \{org, ctf\}$$

- Step 4. Derive the change in portfolios across regimes

$$E_t[(\gamma(1-\beta)\Delta\widetilde{R}\widetilde{R}'(1+\Delta\check{\alpha}) + (1-\gamma+\beta\gamma)\Delta\widetilde{P}\widetilde{R}'] = \frac{1}{2}\widetilde{F}F^{org}$$

- Step 5. Use the solved portfolio to update inter-regime changes

$$\hat{\bar{w}}^1 = M(\hat{\bar{w}}^0) = \hat{\bar{w}}^0(1 + \nu\frac{\mathbb{Z}_i(\hat{\bar{w}}^0)}{\bar{Y}_i^{org}})$$

- Step 6. Repeat steps 3-5 until convergence
  where the joint fixed-point problem $(\hat{\bar{w}}, \hat{\bar{P}}, \Delta\bar{D})$ is solved

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