General Equilibrium Analysis of
Multinational Financial and Trade Linkages

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Abstract

This paper develops a framework where both trade and financial flows are characterized in a multi-country setting. The structural model captures the potential interaction of finance and trade in general equilibrium: financial allocations reflect agents' risk-sharing incentives influenced by the global trade pattern, and countries' asset positions shift demand in the world commodity market. The solution of the model is derived using a novel approach that combines the linearization method for portfolio choice analysis in a DSGE framework and the hat algebra technique from the trade literature. Comparative static analyses employing the approach elucidate the impacts of tariffs and financial frictions as obstacles to globalization.

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1 Introduction

Cross-country commodity and capital flows serve as two paramount engines of globalization. Nonetheless, few models have been proposed to characterize both trade and financial flows in a multi-country structural framework. This paper develops a new approach to build and solve a general equilibrium model with both trade and financial channels. It has the potential to answer many unexplored questions about how the two channels influence each other, which provide economic insights and policy implications. Examples of such questions include how input-output linkages draw the map of global capital allocation, how a trade war reshapes the pattern of financial flows across economies, and how regional financial integration influences the direction and volume of global trade flows.

The new approach not only combines the recent breakthroughs from both international macro and trade literatures, but also mitigates the methodological challenges faced by each strand and yields different predictions from existing works. Compared to the trade literature that typically takes countries' asset positions as exogenous, this paper solves global financial allocations under agents' intertemporal utility maximization decisions and bilateral financial frictions. Compared to the international macro literature that usually studies a small number of countries, this paper examines the multilateral linkages across many economies of uneven sizes. Specifically, I embed portfolio choice analysis in a quantitative macro-trade model, and examine 43 calibrated economies linked through trade and financial exchanges. The endogenous portfolio captures agents' risk-sharing motives shaped by the global trade pattern. Furthermore, countries' asset positions shift the world demand system in the commodity market. Therefore, this approach permits a higher degree of interplay between the two channels of globalization, and facilitates a new and comprehensive understanding in the patterns and determinants of cross-country economic linkages.

To illustrate the main idea of the approach, I develop a model that builds on the Eaton and Kortum (2002) trade framework with immobile human and physical capital endowments and a single tradable sector subject to iceberg trade costs. Besides intermediate goods, financial assets which are claims to countries' capital income are traded across countries. Financial frictions that vary across country pairs add costs to households' repatriation of foreign returns. Nevertheless, households hold different assets to reduce the impact of country-specific productivity shocks on their consumption through international risk sharing. To derive asset positions in this open economy macro model, I follow Devereux and Sutherland (2011)'s method which combines a second-order approximation of agents' Euler equation with a first-order approximation of other model equations to determine a steady-state portfolio. The method is flexible enough to be applied to a wide range of DSGE frameworks, but it is computationally challenging to implement when the number of variables rises substantially with the number of countries.[1]

I employ the 'exact hat algebra' technique developed by Dekle et al. (2007) from the trade literature to overcome this computational challenge. This tractable technique enables comparative static analyses in a rich environment with numerous economies of uneven sizes connected through multilateral economic linkages. In response to changes in cross-country trade or financial frictions, the total changes of any variable may include 1) the change of its steady-state value under different policy regimes (inter-regime changes), and 2) the deviation of the variable from its steady state under shocks within a specific policy regime (intra-regime changes). I use the hat algebra method to characterize both types of changes: one globally to measure the distance between steady states across original and counterfactual regimes, and the other locally within each regime around its steady state. Countries' asset positions, determined by intra-regime changes of second-moment variables in response to shocks, will simultaneously influence inter-regime changes by shifting countries' expenditure. The counterfactual outcome is obtained as the solution to a joint fixed-point problem of changes to wage on the real side and to portfolio on the financial side of the economy. By characterizing both the size and composition of countries' financial allocations in general equilibrium, this approach makes an important contribution to the trade literature, which typically treats financial flows either as exogenously determined with values taken from the data or endogenously derived under extreme circumstances such as financial autarky or complete markets.[2]

The intra-regime analysis for portfolio choice is able to explain observed cross-country financial flows when the model is calibrated to the data. Therefore, I use this rich structural framework to evaluate the factors shaping countries' asset positions proposed in the international macro literature on the topic of asset home bias (see a comprehensive

[1]This is because the local perturbation method typically requires the computation of steady-state values of all the variables and the loglinearization of all the equations in the model. Moreover, its numerical results may not be accurate since the coefficient matrices in a large-scale multi-country setting are badly scaled with countries' uneven sizes and sparse bilateral ties. This is a common challenge for international macro including asset home bias literature, which normally have to limit their inquiry scope to a small open economy or two symmetric economies. Such models cannot capture all the cross-country bilateral linkages and multilateral resistances in the spirit of Anderson and Van Wincoop (2003).

[2]For example, Dekle et al. (2007) examines counterfactual trade patterns under financial autarky. Eaton et al. (2016) study the puzzles in international macroeconomics under complete markets. It is challenging to analyze general cases between these two extreme financial arrangements due to the difficulty of solving the portfolio choice problem in a multi-country general equilibrium model.

survey by Coeurdacier and Rey (2013)). Through numerical exercises that compare predictions from complete markets with 2 economies and from incomplete markets with 43 economies, I highlight the contribution of this paper to the home bias literature by quantifying the influences of different mechanisms including risk sharing, risk hedging, and financial frictions on countries' portfolio choice in a general setting. Furthermore, the model predicts bilateral asset positions besides countries' holdings of domestic assets, which is a substantial extension of the home bias literature. I conduct counterfactual analyses to explain the strong correlation of bilateral trade and financial linkages by exploiting the variation across country pairs. The results suggest that both risk-sharing motives and common barriers contribute to the deep connection between cross-country financial and trade linkages.

Another source of the connection originates from the joint dynamics of the two channels when responding to global economic conditions. To capture their interactions, I perform both inter- and intra-regime analyses to examine how financial and real variables react to higher tariffs. The numerical results suggest that, most countries witness greater declines to real wages yet increases to financial holdings when asset positions are adjustable. This happens because higher tariffs impair international output synchronization, which reduces cross-country asset covariances in the financial market and hence induces risk-averse agents to hold more assets. The increased holdings allow agents to raise expenditure, therefore their welfare loss would be overestimated if endogenous financial adjustments were not considered. This comparison between fixed and adjustable asset positions underscores the importance of incorporating a financial channel in trade models. Meanwhile, the feedback of financial allocations on real variables is missing from the asset home bias literature. Therefore, the general equilibrium effects predicted by this approach differ from those of both strands of existing literatures.

I conduct two counterfactual exercises as applications of the new approach. The first examines a universal rise of cross-country financial frictions to replicate the tightening of international markets during a global financial crisis. The model predicts that most countries reduce foreign holdings under greater barriers to global investment. Nevertheless, they turn out to adjust portfolios less to financial frictions when assets' covariance structure offers risk-sharing benefits. The second application calibrates the model to the tariff increase during the 2018 China-US trade war. The results suggest that the majority of countries suffer a decline in real wages due to the rise in commodity prices, yet they raise asset holdings which partially offset the welfare shortfall. Moreover, I examine a counterfactual scenario where China sells off its holding of US assets and reconstructs the portfolio under the asset covariance structure. The welfare of China would decrease by another 1.3 percent if this financial retaliation occurred based on the numeral results. Therefore, the decoupling of the two major economies in trade and financial channels is very likely to bring more costs than benefits to themselves and to the world economy.

This paper contributes to both international macro and trade literatures. The portfolio choice analysis employs the method developed by Devereux and Sutherland (2011), with similar insights by Judd and Guu (2001) and Tille and Van Wincoop (2010). They use a second-order approximation to overcome the certainty equivalence in a first-order approximation. The method offers a powerful tool in international macro to solve perplexing puzzles like asset home bias (for example, Coeurdacier and Rey (2013) and Coeurdacier and Gourinchas (2016)). Compared to the portfolio techniques driven by investors' specific preference for assets, such as the asset demand system employed by Liu et al. (2022) and the rational inattention logit demand system adopted by Pellegrino et al. (2021), this method does not require separate utility assumptions for agents' intratemporal financial allocation, which is determined by endogenous second-moment variables instead. I extend the method by combining it with hat algebra, in order to perform portfolio analysis across many economies. Furthermore, inter-regime analyses using this approach predict the influences of financial allocation on the level of real variables and hence characterize two-way interactions of trade and financial channels, while other applications of the Devereux and Sutherland (2011) method focus more on portfolio choice taking the world trade structure as given (Heathcote and Perri (2013), Steinberg (2018), Chau (2020), and Hu (2020)).

This paper contributes to the trade literature by modeling cross-country financial flows endogenously. Most trade models are static and ignore intertemporal decisions, with recent exceptions Alvarez (2017) and Kleinman et al. (2021) who introduce forward-looking physical capital accumulation but still assume exogenous financial allocation. My approach instead generates predictions for how a country's asset position is formed by households' intertemporal decisions. Plus, I derive counterfactual financial linkages with the hat algebra technique (Dekle et al. (2007)), which has been commonly used in quantitative spatial models (see Redding and Rossi-Hansberg (2017) for a survey). I add financial flows, which exhibit similar geographic patterns to trade and migration flows according to Portes and Rey (2005), to complete the spatial analysis.

Lastly, this paper contributes to the literature that examines the interaction of trade and finance, including Antras and Caballero (2009) and Jin (2012). I complement these works by developing computational techniques to characterize financial and trade linkages under bilateral frictions in a multi-country structural model. Such papers are particularly meaningful for understanding the general equilibrium effects of globalization.

The paper proceeds as follows: Section 2 develops a multi-country model with trade and finance and describes the method to solve it. Section 3 starts with intra-regime analysis to explain countries' observed asset positions, and then adds inter-regime analysis under higher tariffs to study the joint changes of trade and finance. Section 4 conducts two counterfactuals with either financial or trade frictions as applications of the model to elucidate the impacts of these barriers to globalization. Section 5 concludes.

2 Theory

2.1 Model Setup

The world comprises I countries indexed $i = 1, ..., I$. Each country i produces a final composite good using a continuum of intermediate goods $u \in [0, 1]$ traded across countries

$$Q_{i,t} = \int_0^1 [q_{iu,t}(u)^{\frac{\epsilon-1}{\epsilon}} du]^{\frac{\epsilon}{\epsilon-1}}, \tag{1}$$

where ϵ is the elasticity of substitution in the CES aggregator. The composite good can be used either for consumption $C_{i,t}$ or for the production of intermediate goods $Y_{i,t}$. Country i's technology of producing u, denoted as $Z_{i,t}(u)$, is drawn from a Fréchet distribution as in the Eaton and Kortum (2002) model

$$\Pr(S_i \leq z) = \exp(-T_{i,t} z^{-\theta}). \tag{2}$$

To characterize the risks of the economy for portfolio analysis, I assume $T_{i,t}$ follows an AR(1) process and is subject to serially independent shocks $\epsilon_{i,t}$ drawn from a joint normal distribution with a cross-country covariance matrix Σ_T that contains $cov(\epsilon_{i,t}, \epsilon_{j,t}) = \sigma_{ij}, \ \forall i, j \in [1, I]$ around its mean value over time $\bar{T}_i$:[3]

$$T_{i,t} = \rho T_{i,t-1} + (1 - \rho)\bar{T}_i + \epsilon_{i,t}. \tag{3}$$

[3]This assumption about productivity shocks is in the same spirit as the international real business cycle literature, such as canonical works by Mendoza (1991) and Backus et al. (1992). Besides productivity shocks, this model can be adapted to accommodate other risks which drive countries' output fluctuations that induce agents to construct portfolios for international risk sharing.

Production of intermediate goods combines country-specific labor and capital endowments denoted as L_i and K_i which are assumed to be fixed in supply.[4] Moreover, it uses country i's composite good as an input for production. Let $w_{i,t}$, $r_{i,t}$, and $P_{i,t}$ be the prices of these inputs, and let τ_{ij} be the iceberg trade cost for exports from country i to j, country i's cost of serving a specific good u to country j at time t is hence given by

$$p_{ij,t}(u) = \frac{\tau_{ij}[(r_{i,t}^{\mu} w_{i,t}^{1-\mu})^{\eta} P_{i,t}^{1-\eta}]}{Z_{i,t}(u)}, \tag{4}$$

where μ is the share of capital and $1-\eta$ is the share of the composite good in production. The share of i's goods in j's expenditure is

$$\pi_{ij,t} = \frac{T_{i,t}[\tau_{ij}(r_{i,t}^{\mu} w_{i,t}^{1-\mu})^{\eta} P_{i,t}^{1-\eta}]^{-\theta}}{\Phi_{j,t}}, \qquad where \qquad \Phi_{j,t} = \sum_{k=1}^{I} T_{k,t}[\tau_{kj}(r_{k,t}^{\mu} w_{k,t}^{1-\mu})^{\eta} P_{k,t}^{1-\eta}]^{-\theta}, \tag{5}$$

while $\Phi_{j,t}$ is linked to the price level in country j through

$$P_{j,t} = \Gamma \Phi_{j,t}^{-\frac{1}{\theta}}, \tag{6}$$

in which Γ represents a Gamma function: $\Gamma(\frac{1-\epsilon}{\theta}+1)^{\frac{1}{1-\epsilon}}$.

The goods market clearing condition of country i follows

$$Y_{i,t} = \sum_{j=1}^{I} \pi_{ij,t} X_{j,t}, \qquad \text{where} \qquad X_{j,t} = (1-\eta)Y_{j,t} + P_{j,t}C_{j,t}. \tag{7}$$

$X_{j,t}$ is country j's expenditure and $Y_{i,t}$ is i's output. The difference between the expenditure and output of a country adds to its wealth accumulation

$$\mathcal{W}_{i,t+1} = \mathcal{W}_{i,t} R_{I,t+1} + \sum_{j=1}^{I-1} \alpha_{ij,t}(R_{j,t+1} - R_{I,t+1}) + \eta(1-\mu)Y_{i,t+1} - X_{i,t+1}. \tag{8}$$

$\mathcal{W}_{i,t+1}$ is country i's total claims of different country's assets, for which $\alpha_{ij,t}$ which denotes

[4] To deliver the main idea of the method, I assume physical capital is a fixed endowment for tractability and focus on financial capital as means of consumption risk sharing. Deriving physical capital accumulation in such a DSGE framework under incomplete markets typically employs local perturbation or global value (or policy) function iteration, both of which face the curse of dimensionality in a large-scale multi-country environment. Future extensions of the model may consider deep-learning algorithm proposed by Fernandez-Villaverde et al. (2020) to overcome this computational challenge.

i's holding of j's assets

$$\mathcal{W}_{i,t} = \sum_{j=1}^{I} \alpha_{ij,t}. \tag{9}$$

On the financial side, I follow the international macro literature including Coeurdacier and Rey (2013) to assume countries issue one-period equities whose dividends are claims to their capital income

$$d_{i,t} = \mu\eta Y_{i,t}, \tag{10}$$

which together with equity prices $q_{i,t}$ define financial returns

$$R_{i,t+1} = \frac{d_{i,t+1} + q_{i,t+1}}{q_{i,t}}. \tag{11}$$

Households hold assets to maximize expected lifetime utility

$$\max \sum_{t=0}^{\infty} \beta^t \frac{C_{i,t}^{1-\gamma}}{1-\gamma}, \tag{12}$$

where β is the discount factor and γ is the coefficient of relative risk aversion in the CRRA utility function, both of which appear in the intertemporal Euler equation

$$\frac{C_{i,t}^{-\gamma}}{P_{i,t}} = \beta E_t[\frac{C_{i,t+1}^{-\gamma}}{P_{i,t+1}} e^{-f_{ij}} R_{j,t+1}], \qquad \forall i, j \in [1, I]. \tag{13}$$

Markets are incomplete due to the existence of barriers to global financial investment. In particular, financial frictions potentially vary across country pairs, which justifies the gravity model of capital flows documented by Portes and Rey (2005). I follow Heathcote and Perri (2004) and Aviat and Coeurdacier (2007) by introducing financial frictions as iceberg transaction costs $f_{ij} \geq 0$, so that households in country i expect to collect $e^{-f_{ij}} R_{j,t+1}$ when repatriating asset returns from country j.[5] Besides, these frictions are second-order in magnitude (proportional to the variance of shocks in the model) so that I can employ the solution method for portfolio choice in an open economy DSGE

[5]This modeling assumption about financial frictions, which is analogous to trade costs in the commodity market, makes the portfolio choice problem tractable. However, I do not take a strong stand on either the underlying structure or the theoretical foundation of these barriers to international financial investment. Specifically, the bilateral friction f_{ij} can reflect a mix of worldwide factors including global financial liquidity, country-specific factors including capital account openness, and pair-specific factors including geographic distance and bilateral financial agreements. It can take alternative forms such as informational frictions, as Okawa and Van Wincoop (2012) find that these types of frictions yield comparable implications for the gravity model of financial flows.

framework developed by Devereux and Sutherland (2011). Acknowledging that assets are distinguishable by their risk characteristics, these authors develop a method that combines a second-order approximation of the portfolio choice equation derived from the Euler equation with a first-order approximation of other equations of the model in order to determine a zero-order (i.e. steady-state) portfolio.

Country i's portfolio choice equation is characterized by

$$E_t[\frac{C_{i,t+1}^{-\gamma}}{P_{i,t+1}} R_{i,t+1}] = E_t[\frac{C_{i,t+1}^{-\gamma}}{P_{i,t+1}} e^{-f_{ij}} R_{j,t+1}], \qquad \forall i, j \in [1, I], \tag{14}$$

It is worth noting that portfolio choice derived from the Euler equations (13 and 14) captures both inter-temporal and intra-temporal investment decisions of households to maximize their expected lifetime utility (12). Inter-temporally, households decide between financial investment and current consumption, given their patience (β) and elasticity of intertemporal substitution ($\frac{1}{\gamma}$), upon expected asset returns $R_{j,t+1}$ and inflation ($P_{i,t+1}$). Intra-temporally, the covariance matrix of different countries' productivity shocks (Σ_T) and the matrix of bilateral financial frictions will be reflected in the second-order Taylor expansion of the Euler equation 14, evaluating which determines portfolio choice. Therefore, households will naturally prefer assets from countries whose shocks are less correlated with their home country's under risk-sharing motives, and whose assets are subject to lower transaction costs to maximize financial payoff.

To summarize the description of the model setup, the general equilibrium of the model consists of a set of prices and quantities such that 1) households choose consumption and construct portfolio to maximize expected lifetime utility, 2) firms set output and price to maximize profit, and 3) factor, commodity, and asset markets clear. The economy is deemed in a steady state when the endogenous variables that satisfy all the equilibrium conditions are constant over time.

2.2 Solution Techniques

Solving for the equilibrium portfolio choice in a DSGE framework with the perturbation method developed by Devereux and Sutherland (2011) involves log-linearizing the model around the steady state of the economy. Let $\widetilde{A}_t$ represent the log-deviation of any variable A from its steady state $\bar{A}$ at t

$$\widetilde{A}_t = \ln(\frac{A_t - \bar{A}}{\bar{A}}), \tag{15}$$

then the cross-country ratio of any country-specific variable $B_{i,t}$ defined as

$$B_{i/j,t} = \frac{B_{i,t}}{B_{j,t}} \tag{16}$$

has its deviation from the steady state expressed as

$$\widetilde{B}_{i/j,t} = \widetilde{B}_{i,t} - \widetilde{B}_{j,t}. \tag{17}$$

I assume I is a numeraire country when solving the world matrix of steady-state portfolio weights[6]

$$\begin{bmatrix} \bar{\alpha}_{11} & \bar{\alpha}_{12} & \cdots & \bar{\alpha}_{1I\text{-}1} \\ \bar{\alpha}_{21} & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ \vdots & & & \bar{\alpha}_{I\text{-}2I\text{-}1} \\ \bar{\alpha}_{I\text{-}11} & \cdots & \bar{\alpha}_{I\text{-}1I\text{-}2} & \bar{\alpha}_{I\text{-}1I\text{-}1} \end{bmatrix} \tag{18}$$

whose elements in the i^{th} row are decided by country i's Euler equation (14) which satisfies

$$E_t[\frac{U'(C_{i,t+1})}{P_{i,t+1}} e^{-f_{iI}} R_{I,t+1}] = E_t[\frac{U'(C_{i,t+1})}{P_{i,t+1}} e^{-f_{i1}} R_{1,t+1}] = ... = E_t[\frac{U'(C_{i,t+1})}{P_{i,t+1}} e^{-f_{iI\text{-}1}} R_{I\text{-}1,t+1}]. \tag{19}$$

Portfolio is derived from the second-order Taylor expansion of equation 19 while taking the difference between the numeraire asset I and all the other assets:

$$E_t[\widetilde{R}_{x,t+1} + \frac{1}{2}\widetilde{R}^2_{x,t+1} - \widetilde{R}_{x,t+1}(\gamma\widetilde{C}_{i,t+1} + \widetilde{P}_{i,t+1})] = -\frac{1}{2}F_i + \mathcal{O}(\epsilon^3), \tag{20}$$

where $R_{x,t+1}$ denotes a vector of excess returns relative to the numeraire asset

$$\widetilde{R}'_{x,t+1} = [\widetilde{R}_{1,t+1} - \widetilde{R}_{I,t+1}, \widetilde{R}_{2,t+1} - \widetilde{R}_{I,t+1}, ..., \widetilde{R}_{I\text{-}1,t+1} - \widetilde{R}_{I,t+1}], \tag{21}$$

$R^2_{x,t+1}$ denotes the vector of excess squared returns

$$\widetilde{R}^{2'}_{x,t+1} = [\widetilde{R}^2_{1,t+1} - \widetilde{R}^2_{I,t+1}, \widetilde{R}^2_{2,t+1} - \widetilde{R}^2_{I,t+1}, ..., \widetilde{R}^2_{I-1,t+1} - \widetilde{R}^2_{I,t+1}], \tag{22}$$

[6] The portfolio matrix's dimension is $(I\text{-}1) \times (I\text{-}1)$ instead of $I \times I$. For the remaining assets positions, country i's holding of the numeraire asset is decided by the difference between its aggregate wealth position $\bar{D}_i$ and its holding of other assets. Meanwhile, country I's holding of any asset j is decided by j's market clearing condition such that the supply of the asset equals its demand given by $\sum_{i=1}^{I} \bar{\alpha}_{ij} Y_i$.

and F_i denotes i's vector of financial frictions defined as

$$F_i' = [f_{iI} - f_{i1}, f_{iI} - f_{i2}, ..., f_{iI} - f_{iI\text{-}1}], \tag{23}$$

whose k^{th} element represents the additional financial friction country i's households incur when holding I's relative to k's asset. $\mathcal{O}(\epsilon^3)$ captures all terms of order higher than two.

The difference between any country i's and the numeraire country I's expanded Euler equations (20) follows

$$E_t[(\gamma\widetilde{C}_{i/I,t+1} + \widetilde{P}_{i/I,t+1})\widetilde{R}'_{x,t+1}] = \frac{1}{2}F_{iI} + \mathcal{O}(\epsilon^3), \qquad \forall i \in [1, I\text{-}1], \tag{24}$$

where F_{iI} stands for the excess financial frictions faced by country i relative to by I

$$F_{iI} = F_i' - F_I'. \tag{25}$$

Equation 24 is country i's relative to I's portfolio determination equation: the variables on its left covary with country i's relative asset positions which are also influenced by financial frictions F_{iI} on the right. When evaluating the equation to pin down the equilibrium portfolio, we consider the responses of $\gamma\widetilde{C}_{i/I,t+1}+\widetilde{P}_{i/I,t+1}$ and $\widetilde{R}_{x,t+1}$ to all the productivity shocks. A standard DSGE approach would take the first-order derivative of the two variables with respect to productivity shocks, denoted as G_{t+1} and H_{t+1} respectively in

$$E_t(\underbrace{G_{t+1}}_{1\times I} \times \underbrace{\Sigma_T}_{I\times I} \times \underbrace{H'_{t+1}}_{I\times(I\text{-}1)}) = \frac{1}{2}\underbrace{F_{iI}}_{1\times(I\text{-}1)} + \mathcal{O}(\epsilon^3). \tag{26}$$

Nonetheless, solving the portfolio choice problem with a standard DSGE approach is challenging especially in a multi-country framework, because it is a daunting task to compute all the steady-state values and loglinearize all the equations when their numbers grow exponentially with the number of countries. Moreover, the numerical results solved by the log-linearization method can be inaccurate given the large coefficient matrices that cover the world economy are badly scaled with countries' uneven sizes and with their sparse bilateral linkages.

I find these challenges can be mitigated by the hat algebra technique developed by the trade literature, if the technique is applied locally around a steady state of the economy. First, the technique employs an efficient computation procedure, whose solution captures the comovement of all the countries' variables, replaces the need for loglinearing equations in the model to predict variables' responses to shocks. Second, the technique requires few observable moments which will be sufficient statistics to predict all the variables' deviations from the steady state, so that it is no longer necessary to solve for their steady-state values. Plus, taking advantage of sufficient statistics, this approach already embeds the impacts of trade and financial frictions across all the countries when solving for portfolios. These frictions would be difficult to calibrate for the standard log-linearization method to work. For these reasons, the computation for portfolio analysis in a multi-country setting becomes significantly more tractable and efficient. Therefore, instead of taking a first-order approximation to evaluate equation 26, I employ hat algebra by simulating productivity shocks to directly compute variables' responses around the steady state of the economy.

To solve for portfolios using this new approach, I rewrite equation 24 as[7]

$$E_t[(\gamma(1-\beta)\widetilde{Y}_{i/I,t+1} + (1-\gamma+\beta\gamma)\widetilde{P}_{i/I,t+1} + \gamma(1-\beta)(\breve{\boldsymbol{\alpha}}_i - \breve{\boldsymbol{\alpha}}_I)\widetilde{R}_{x,t+1})\widetilde{R}'_{x,t+1}] = \frac{1}{2}F_{iI}, \quad (27)$$

where portfolio weights are scaled by countries' value-added output and discount factor:

$$\breve{\alpha}_{ik} = \frac{\bar{\alpha}_{ik}}{\beta\eta\bar{Y}_i}, \quad (28)$$

which constitute the country's vector of bilateral asset holdings

$$\breve{\boldsymbol{\alpha}}_i = [\breve{\alpha}_{i1}, \breve{\alpha}_{i2}, ..., \breve{\alpha}_{iI\text{-}1}]. \quad (29)$$

Equation 27, if stacked vertically with each row representing a country, constructs a system of equations for the world bilateral portfolio matrix (defined in 18) to be solved. When the real side of the economy is calibrated to the data, using hat algebra to evaluate equation 27 is able to explain the drivers for observed cross-country financial flows, assuming real variables are taken as given. However, what is even more interesting is how financial and real variables jointly respond to evolving global economic conditions. I discuss the computation strategy for such a general equilibrium analysis in detail.

Policy experiments under counterfactual trade and financial frictions are conducted to examine the impacts of these barriers to globalization. Given the counterfactual frictions, equation 27 is evaluated in original and counterfactual cases, where variables are

[7] See Appendix B for the detailed derivation. This step uses countries' wealth constraint (equation 8) to substitute out consumption as a function of other variables solvable by hat algebra.

superscripted $s = \{org, ctf\}$ respectively:

$$E_t[(\gamma(1-\beta)\widetilde{Y}^{org}_{i/I,t+1}+(1-\gamma+\beta\gamma)\widetilde{P}^{org}_{i/I,t+1}+\gamma(1-\beta)(\breve{\alpha}^{org}_i-\breve{\alpha}^{org}_I)\widetilde{R}^{org}_{x,t+1})\widetilde{R}'^{org}_{x,t+1}] = \frac{1}{2}F^{org}_{iI}, \tag{30}$$

$$E_t[(\gamma(1-\beta)\widetilde{Y}^{ctf}_{i/I,t+1}+(1-\gamma+\beta\gamma)\widetilde{P}^{ctf}_{i/I,t+1}+\gamma(1-\beta)(\breve{\alpha}^{ctf}_i-\breve{\alpha}^{ctf}_I)\widetilde{R}^{ctf}_{x,t+1})\widetilde{R}'^{ctf}_{x,t+1}] = \frac{1}{2}F^{ctf}_{iI}. \tag{31}$$

The difference between equations 30 and 31 will decide changes in country i's relative to the numeraire country I's bilateral asset holdings

$$(\breve{\alpha}^{ctf}_i - \breve{\alpha}^{org}_i) - (\breve{\alpha}^{ctf}_I - \breve{\alpha}^{org}_I) \tag{32}$$

in response to changes in the products of variables from the equations including

$$\widetilde{Y}^{s}_{i/I,t+1}\widetilde{R}'^{s}_{x,t+1},\ \widetilde{P}^{s}_{i/I,t+1}\widetilde{R}'^{s}_{x,t+1},\ \widetilde{R}^{s}_{x,t+1}\widetilde{R}'^{s}_{x,t+1}, \qquad s \in \{org, ctf\} \tag{33}$$

as well as in financial frictions $F^{ctf}_{iI} - F^{org}_{iI}$ across original and counterfactual scenarios. The resulting bilateral asset holdings will add up to countries' equilibrium aggregate position (the steady state value of $\mathcal{W}_{i,t}$ in equation 8) as shares of output denoted as

$$\bar{D}_i = \beta\eta\sum_{k=1}^{I}\breve{\alpha}_{ik}. \tag{34}$$

Counterfactual trade or financial frictions affect both the first moments (levels) and second moments (covariances) of variables. Using the relative output of country i to j at time $t+1$ as an example, I decompose its total dynamics under counterfactual versus original frictions into two parts

$$\underbrace{\ln(Y^{ctf}_{i/j,t+1}) - \ln(Y^{org}_{i/j,t+1})}_{\text{Total changes}} = \underbrace{[\ln(\bar{Y}^{ctf}_{i/j}) - \ln(\bar{Y}^{org}_{i/j})]}_{\text{Inter-regime changes}} + \underbrace{[\ln(Y^{ctf}_{i/j,t+1}) - \ln(\bar{Y}^{ctf}_{i/j})] - [\ln(Y^{org}_{i/j,t+1}) - \ln(\bar{Y}^{org}_{i/j})]}_{\text{Intra-regime changes}}. \tag{35}$$

From this decomposition, the total changes of a variable reflect 1) the change of its steady-state value under different policy regimes (inter-regime changes), and 2) the deviation of the variable from its steady state under shocks within a specific policy regime (intra-regime changes). Inter-regime changes are derived in a similar way as in trade models such as Dekle et al. (2007), while examining intra-regime changes of how second-moment variables behave around a steady state is necessary to solve the equilibrium asset position. Now I describe how inter- and intra-regime changes are both characterized by hat algebra.

To start with the inter-regime analysis, let the ratio of any variable A's counterfactual to original steady-state value under policy changes be denoted as

$$\widehat{\bar{A}} = \frac{\bar{A}^{ctf}}{\bar{A}^{org}}. \tag{36}$$

It follows that the vectors of all the countries' wage and price

$$\widehat{\bar{w}}' = [\widehat{\bar{w}}_1, \widehat{\bar{w}}_2, ..., \widehat{\bar{w}}_I], \qquad \widehat{\bar{P}}' = [\widehat{\bar{P}}_1, \widehat{\bar{P}}_2, ..., \widehat{\bar{P}}_I] \tag{37}$$

are obtained by an iterative computation procedure to solve a fixed-point problem for a pair of vectors $(\widehat{\bar{w}}, \widehat{\bar{P}})$ based on all the countries' price determination and goods market clearing conditions (see Appendix B for the derivation of equations):

$$\widehat{\bar{P}}_i^{-\theta} = \sum_{j=1}^{I} \bar{\pi}_{ji}^{org} \widehat{\tau}_{ji}^{-\theta} (\widehat{\bar{w}}_j^{\eta} \widehat{\bar{P}}_j^{1-\eta})^{-\theta}, \tag{38}$$

$$\widehat{\bar{w}}_i \bar{Y}_i^{org} = \sum_{j=1}^{I} \frac{\bar{\pi}_{ij}^{org} \widehat{\tau}_{ij}^{-\theta} (\widehat{\bar{w}}_i^{\eta} \widehat{\bar{P}}_i^{1-\eta})^{-\theta}}{\sum_{k=1}^{I} \bar{\pi}_{kj}^{org} \widehat{\tau}_{kj}^{-\theta} (\widehat{\bar{w}}_k^{\eta} \widehat{\bar{P}}_k^{1-\eta})^{-\theta}} \widehat{\bar{w}}_j \bar{Y}_j^{org} (1 - \bar{D}_j^{ctf} (1 - \frac{1}{\beta})). \tag{39}$$

After solving for $\widehat{\bar{w}}, \widehat{\bar{P}}$ using the procedure, changes to other macroeconomic variables including output $\widehat{\bar{Y}}$ can be derived through their relations with $\widehat{\bar{w}}, \widehat{\bar{P}}$ based on the equilibrium equations in the model. The hat algebra method is easy to implement and it only requires calibrating countries' initial output $(\bar{Y}_i^{org})$ and bilateral trade shares $(\bar{\pi}_{ij}^{org})$ in the original steady state, which can be regarded as their long-term average values directly observable in the data.

This paper departs from the trade literature in the characterization of $\bar{D}_j^{ctf}$, which is determined by equations 30 and 31. To solve for portfolio choice, I apply the hat algebra method around original and counterfactual steady states respectively to determine 'intra-regime changes' in 35. If the ratio of any variable A to its steady-state value in either regime is denoted as

$$\widehat{A}_t^s = \frac{A_t^s}{\bar{A}^s}, \qquad where \qquad s \in \{org, ctf\}, \tag{40}$$

For any country-specific variable $B_{i,t}^s$, its cross-country ratio $B_{i/j,t}^s$ defined in equation 16 has its percentage deviation from the steady state approximated as

$$\widetilde{B}^s_{i/j,t} = \widetilde{B}^s_{i,t} - \widetilde{B}^s_{j,t} = \ln(\frac{\widehat{B}^s_{i,t}}{\widehat{B}^s_{j,t}}) = \ln(\widehat{B}^s_{i/j,t}). \tag{41}$$

Therefore, the 'tilde' variables that appear in the portfolio determination equations 30-31 can be converted from the corresponding 'hat' variables.[8]

These hat variables can be characterized as their responses to the vector of countries' productivity shocks around steady-state values

$$\widehat{T}' = [\widehat{T}_1, \widehat{T}_2, ..., \widehat{T}_I] = [\frac{T_1}{\bar{T}_1}, \frac{T_2}{\bar{T}_2}, ..., \frac{T_I}{\bar{T}_I}], \qquad \forall i \in [1, I]. \tag{42}$$

Elements in this vector of productivity shocks $\widehat{T}_i$ will drive the dynamics of variables around the steady state in each regime $s \in \{org, ctf\}$ including wage and price:

$$\widehat{P}_i^{-\theta,s} = \sum_{j=1}^{I} \bar{\pi}^s_{ji} \widehat{T}_j (\widehat{w}_j^{\eta,s} \widehat{P}_j^{1-\eta,s})^{-\theta}, \tag{43}$$

$$\widehat{w}_i^s \bar{Y}_i^s = \sum_{j=1}^{I} \frac{\bar{\pi}^s_{ij} \widehat{T}_i (\widehat{w}_i^{\eta,s} \widehat{P}_i^{1-\eta,s})^{-\theta}}{\sum_{k=1}^{I} \bar{\pi}^s_{kj} \widehat{T}_k (\widehat{w}_k^{\eta,s} \widehat{P}_k^{1-\eta,s})^{-\theta}} \widehat{X}_j^s \bar{X}_j^s. \tag{44}$$

Here the initial levels of output and trade shares using which hat algebra is performed are set as their steady-state values in each regime $\bar{Y}_i^s, \bar{\pi}^s_{ij}, s \in \{org, ctf\}$.[9]

It is noteworthy that equations 43 and 44 for intra-regime analysis are counterparts to equations 38 and 39 for inter-regime analysis. Within either regime, tariffs and equilibrium portfolios are constant and therefore do not influence how variables behave around the steady state. Instead, the main driver for the dynamics of variables are simulated productivity shocks $\widehat{T}$, under which $\widehat{w}^s, \widehat{P}^s$ are solved with the same iterative procedure as for inter-regime analysis. I then express the changes of the second-moment variables in portfolio determination equations (listed in 33) as functions of $\widehat{w}^s, \widehat{P}^s$ to derive asset changes across original and counterfactual regimes ($\Delta\bar{D}$) from equations 30 and 31. Asset positions in the financial channel will affect countries' expenditure in the trade channel. Hence, I update inter-regime dynamics $\widehat{\widehat{w}}, \widehat{\widehat{P}}$ with $\Delta\bar{D}$ using equations 38 and 39. Gen-

[8]Since the variables in these portfolio equations are all expected to be realized at $t+1$, I omit the time subscript for brevity in the following intra-regime analysis.

[9]As mentioned above, the original steady state is calibrated to the data. The counterfactual steady state $\bar{Y}_i^{ctf}, \bar{\pi}_{ij}^{ctf}$ will be predicted under inter-regime changes $\widehat{\widehat{w}}, \widehat{\widehat{P}}$ imposed on the original steady state.

eral equilibrium of this framework, where trade and financial channels interact with each other, can be characterized by the solution to a joint fixed-point problem of $(\widehat{\bar{w}}, \widehat{\bar{P}}, \Delta\bar{D})$.

Appendix B provides more technical details of the model and its solution: Section B.1 derives model equations used to perform hat algebra and portfolio analysis. Section B.2 discusses the existence and uniqueness of the model solution. Section B.3 outlines the algorithm which builds a recursive structure that combines inter- and intra-regime analyses to solve the fixed-point problem.

2.3 Calibration

I calibrate the model to a world economy that consists of 43 countries (listed in table A.1) plus the rest of the world (ROW). On the real side of the economy, I only need countries' GDP data from the Penn World Table (PWT) and bilateral trade shares from the Direction of Trade Statistics when using the hat algebra technique to predict counterfactuals. Analogously on the financial side, I obtain countries' net foreign asset positions (NFA) from the World Bank and bilateral portfolio weights from Factset/Lionshare (see Appendix C for details). The time-averaged values over the sample period 2001-2021 are used as the steady-state values of these variables in the original regime.

The risk characteristics of the economy are reflected as productivity fluctuations. Therefore, I estimate countries' dynamic productivity consistent with the Eaton and Kortum (2002) model following Levchenko and Zhang (2014)'s approach and compute its corresponding mean value and covariance matrix (see Appendix C). Productivity shocks for intra-regime analyses can either be simulated with a joint normal distribution featuring the estimates or directly with the bootstrap method.[10] Other parametric assumptions include annual discount factor $\beta = .9$, coefficient of relative risk aversion $\gamma = 2$, share of intermediate input in production $\eta = .312$ following Dekle et al. (2007), share of labor input $1 - \mu$ as country-specific labor income share from the PWT, and trade elasticity $\theta = 4$ based on Simonovska and Waugh (2014).

Several quantitative exercises will compare model predictions in settings with 2 and with 43 countries. For the former, I collapse the multi-country to a two-country model where each country is treated as the domestic economy and the sum of all the other countries from this country's perspective as the foreign economy. Following this rule,

[10] Bootstrap is more feasible when the estimated covariance matrix is not positive semi-definite, due to the collinearity problem given the large-scale data, to simulate random samples from the specified multivariate normal distribution. In the baseline exercises to be conducted in section 3, I draw 1000 productivity vectors with bootstrap when simulating shocks.

I calculate the two-by-two matrices of financial and trade shares, and re-estimate the productivity of domestic and foreign economies based on country sizes and trade flows.

3 Model Predictions

This section consists of two sets of quantitative analyse based on the model. The first performs intra-regime analysis of the calibrated original regime to evaluate the determinants of portfolio choice observed in the data. The second combines intra- and inter-regime analyses under counterfactual tariffs to elucidate the comovement of financial and trade channels in general equilibrium.

3.1 Mechanisms for Portfolio Choice

To examine how trade influences finance in this multi-country framework, I compare the model predictions with those from the international macro literature on the topic of asset home bias including Coeurdacier (2009) and Heathcote and Perri (2013). I start with their assumption that there are two symmetric countries in a world with complete risk sharing.[11] Under the assumption, Coeurdacier and Rey (2013) derive the general expression for the share of domestic assets in a country's portfolio

$$\bar{\alpha}_{ii} = \underbrace{\frac{1}{2}}_{\substack{\text{Risk sharing} \\ \text{(Diversification)}}} - \underbrace{\frac{1}{2}\frac{1-\mu}{\mu}\frac{cov(\widetilde{w}_{i/j}, \widetilde{R}_{i/j})}{var(\widetilde{R}_{i/j})}}_{\text{Hedging labor income risk}} + \underbrace{\frac{1}{2}\frac{1-1/\gamma}{\mu}\frac{cov(\widetilde{P}_{i/j}, \widetilde{R}_{i/j})}{var(\widetilde{R}_{i/j})}}_{\text{Hedging RER risk}}. \tag{45}$$

The first term reflects households' incentive to diversify portfolios across different countries' assets in order to reduce the impact of country-specific shocks on consumption (as in Lucas (1982)). The second term captures the hedging against labor income risk, which induces households to hold assets whose return decreases with domestic labor income to avoid the simultaneous shortfall in financial and wage income (Baxter and Jermann (1997)). The third term represents households' hedging of real exchange rate (RER) risk which motivates them to hold assets whose return increases with the domestic price level

[11] Risk sharing can be derived under various assumptions. For example, Coeurdacier and Rey (2013) follow the Backus and Smith (1993) condition implied by complete markets to solve for portfolio choice. Heathcote and Perri (2013) assume Cobb-Douglas preference for domestic and foreign goods in the expenditure bundle, which automatically changes terms-of-trade in response to shocks to make risk sharing achievable by time-invariant asset positions. Coeurdacier and Gourinchas (2016) show markets are locally complete to support risk sharing, as long as the set of asset returns spans the space of shocks.

in order to sustain purchasing power (Kollmann (2006)).

Heathcote and Perri (2013) and Coeurdacier (2009) focus on the two hedging components in 45 respectively when studying the effects of international trade on asset home bias. Specifically, Heathcote and Perri (2013) assume log-utility ($\gamma = 1$) in their baseline case where the hedging of RER risk is turned off. For the hedging of labor income risk, they argue that lower trade openness will alleviate the terms-of-trade (TOT) depreciation that decreases labor income in response to a positive productivity shock at home, while the shock simultaneously reduces domestic dividends given higher capital investment.[12] Therefore, domestic labor and financial income become more negatively correlated to generate stronger asset home bias. In contrast, Coeurdacier (2009) considers capital as the only production factor ($\mu = 1$) to shut down the labor income risk. For the hedging against RER risk, he finds that higher trade costs weaken asset home bias because domestic financial income increases less closely with RER: A positive domestic productivity shock raises domestic dividends as claims to capital income but triggers RER depreciations when domestic and foreign goods are substitutes.[13] The contradiction of these two papers' predictions is resolved when the elasticity of substitution between domestic and foreign goods is above 4 (which is the parametric assumption adopted by Eaton and Kortum (2002)) — both labor income risk and RER risk will tilt portfolios towards foreign assets in the presence of trade costs.[14]

This paper departs from these home bias papers along three dimensions. First, I acknowledge the existence of financial frictions and solve for portfolio choice in incomplete markets. International risk sharing is hence not complete and the first term in equation 45 is no longer a constant that reflects a country's relative size in the world economy. Instead, households need to consider second-moment variables in the financial market including

[12] Dividends are defined as the difference between capital income (μY) and physical capital investment expenditure, while labor income is proportional to nominal output ($(1-\mu)Y$) in their framework. Moreover, they assume domestic goods account for a greater share in the Cobb-Douglas bundle. Under these assumptions, domestics dividends will negatively correlate with labor income to provide a hedge.

[13] The reasoning is that, when the elasticity of substitution between domestic and foreign goods is (roughly) above unity, in response to a positive domestic productivity shock, the output rise dominates the TOT drop to raise domestic financial income. When the consumption bundle is skewed more towards domestic goods due to higher trade costs, the TOT drop translates to a greater RER depreciation, which strengthens the negative comovement between RER and domestic return that induces households to hold foreign assets for the hedging against RER risk.

[14] See the sensitivity analysis conducted by Heathcote and Perri (2013) which shows their model with physical capital also predicts asset foreign bias when the elasticity is sufficiently large. TOT adjustment is so small in this case that a positive domestic productivity shock will increase both financial and labor income at home. The positive comovement between the two types of income will hence tilt portfolios towards foreign assets. Skewness of expenditure towards domestic goods in their model is generated by households' preference, which can also be replicated by the assumption of trade costs.

asset covariances and market frictions when constructing the optimal portfolio for the purpose of risk sharing. Second, in this framework with I economies, there are $I-1$ foreign countries instead of one. The correlations between individual foreign countries' asset returns and domestic macro fundamentals in this setting are lower while the diversification benefits offered by a large group of countries with non-perfectly correlated shocks are higher. For these two reasons, risk sharing becomes relatively more important than risk hedging in determining financial flows in this multi-country framework with incomplete markets than in a two-country framework with complete markets. Third, this paper examines inter-regime changes where the steady state of the economy shifts under the joint forces of real and financial variables in response to policy changes. In this process, portfolio allocation reflects agents' risk-sharing and risk-hedging motives shaped by the global trade pattern, while countries' asset positions shift the world demand system in the commodity market.[15,16] This general equilibrium analysis differs from the home bias literature which typically studies portfolio taking the real side of the economy as given around a fixed steady state.

I conduct two sets of numerical exercises to highlight these new mechanisms separately. The first focuses on the original regime, where the model is calibrated to the data, to explain observed cross-country financial flows. In particular, I will disentangle risk-sharing and risk-hedging mechanisms in a two-country and in a 43-country setting respectively, to underscore the contribution of this paper to the home bias literature by quantifying the influences of these mechanisms separately in a more general setting. The second adds inter-regime changes under counterfactual tariffs to examine the two-way interactions of trade and financial channels.

To start the analysis of financial flows observed in the data, I write the portfolio determination equation (30) in the original regime $s = \{org\}$ for simplicity as

$$E_t[\gamma(1-\beta)\widetilde{Y}\widetilde{R}' + (1-\gamma+\beta\gamma)\widetilde{P}\widetilde{R}' + \gamma(1-\beta)\breve{\alpha}\widetilde{R}\widetilde{R}'] = \frac{1}{2}F \tag{46}$$

[15]Global trade linkages affect portfolio choice by influencing second-moment variables, such as cross-country output covariances, and comovments of macro variables (labor income and RER) with financial income. These second-moment variables will enter the portfolio determination equations to influence countries' financial allocations. Meanwhile, financial allocations influence international trade because a country's asset position decides the size of its expenditure in the commodity market.

[16]It is noteworthy that portfolio choice does not influence the steady-state value of real variables within a regime as a condition to use the Devereux and Sutherland (2011) method. The level of variables only shifts across regimes driven by the joint changes of wage, price, and portfolio choice (derived in original and counterfactual regimes separately) until the fixed point problem of $(\widehat{\bar{w}}, \widehat{\bar{P}}, \Delta\bar{D})$ is solved.

Table 1: Median Asset Positions in Different Scenarios

	Data	Setting	No risk sharing	No labor risk hedging	No RER risk hedging
$\bar{\alpha}_{ii}$	0.427	multi-country	0.373	0.490	0.612
		two-country	0.088	0.630	1.395
$\bar{D}_i$	0.971	multi-country	0.965	0.992	1.048
		two-country	0.923	1.028	1.289

This table presents the median size of portfolios $\bar{D}_i$ and the median domestic asset holding $\bar{\alpha}_{ii}$ both as shares of GDP across countries in the sample in different scenarios. Results are reported for: 1) a multi-country case where there are 43 countries with bilateral trade and financial linkages, and 2) a two-country case where each of the countries in the sample treats itself as the domestic economy and all the other countries in the world as the aggregate foreign economy. See calibration strategies of both settings in section 2.3, cross-country plots in figure 1, and complete results by country in table A.2.

where F and $\breve{\alpha}$ stand for the vectors of all the countries' financial frictions and asset holdings (defined by 25 and 29) relative to the numeraire country I:

$$F^{'} = [F_{1I}, F_{2I}, ..., F_{I-1I}], \qquad \breve{\alpha}^{'} = [\breve{\alpha}_1, \breve{\alpha}_2, ..., \breve{\alpha}_{I-1}] - \breve{\alpha}_I.$$

$\widetilde{Y}\widetilde{R}^{'}$, $\widetilde{P}\widetilde{R}^{'}$, and $\widetilde{R}\widetilde{R}^{'}$ represent the second-moment variables listed in 33 for brevity.

The following numerical exercises disentangle portfolios shaped by risk-sharing and by risk-hedging motives. First, to isolate the risk-sharing channel, I derive portfolios in complete markets under the Backus and Smith (1993) condition

$$E_t[(\gamma\widetilde{C} + \widetilde{P})] = E_t[(\gamma(1-\beta)\widetilde{Y} + (1-\gamma+\beta\gamma)\widetilde{P} + \gamma(1-\beta)\breve{\alpha}\widetilde{R})] = 0. \tag{47}$$

Second, to shut down the hedging of RER risk, I assume log-utility ($\gamma = 1$) under which assumption the portfolio equation becomes

$$E_t[(1-\beta)\widetilde{Y}\widetilde{R}^{'} + \beta\widetilde{P}\widetilde{R}^{'} + (1-\beta)\breve{\alpha}\widetilde{R}\widetilde{R}^{'}] = \frac{1}{2}F. \tag{48}$$

Lastly, labor income is excluded from households' wealth constraint (8) to turn off the hedging against labor income risk. The portfolio equation in this situation is

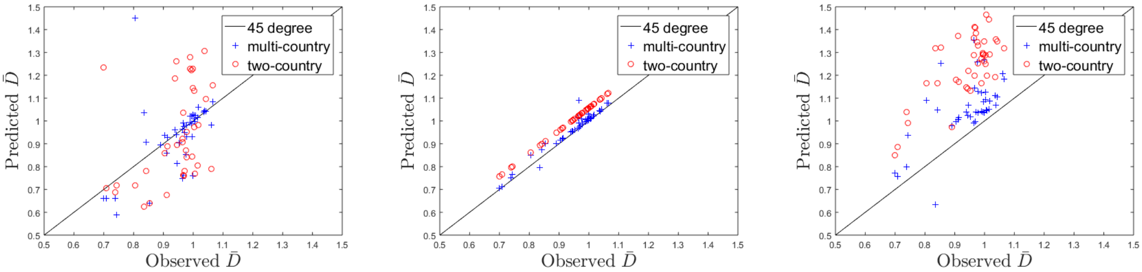
$$E_t[\gamma(1-\beta)(1-\eta+\eta\mu)\widetilde{Y}\widetilde{R}^{'} + (1-\gamma+\beta\gamma)\widetilde{P}\widetilde{R}^{'} + \gamma(1-\beta)\breve{\alpha}\widetilde{R}\widetilde{R}^{'}] = \frac{1}{2}F. \tag{49}$$

The shifts of asset positions characterized by equations 47-49 from those in the data reflect the potential impacts of these risk-sharing and -hedging factors on financial flows.

Table 1 and figure 1 report the numerical results for the size and composition of portfolios in different scenarios. When portfolios are derived under the assumption of com-

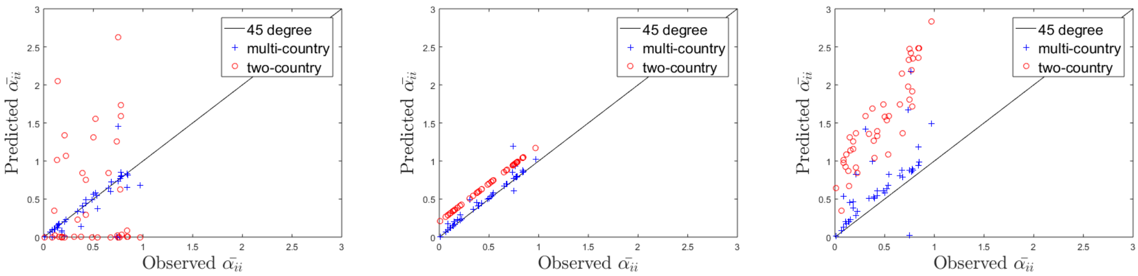
Figure 1: Disentangling Risk-sharing and Risk-hedging Channels

(I) Aggregate Asset Positions ($\bar{D}_i$)



(a) No risk sharing (b) No labor risk hedging (c) No RER risk hedging

(II) Domestic Asset Holdings ($\bar{\alpha}_{ii}$)



(d) No risk sharing (e) No labor risk hedging (f) No RER risk hedging

This figure examines countries' portfolio choice in three scenarios: 1) in complete markets with no additional need for risk sharing, 2) where there is no labor income risk hedging, and 3) where there is no RER risk hedging. Panel (I) reports the size of a country's portfolio ($\bar{D}_i$) and (II) reports its holding of domestic assets ($\bar{\alpha}_{ii}$) both as shares of GDP. Portfolios in the data are on the horizontal axis and model-predicted portfolios under counterfactual assumptions are on the vertical axis. Red circles and blue stars represent predictions from settings with 2 and 43 countries respectively. See calibration strategies of both settings in section 2.3, cross-country median values in table 1, and complete results by country in table A.2.

plete markets (equation 47), the median holding of domestic assets drops from 0.427 to 0.373 in a multi-country setting and plummets to 0.088 in a two-country setting,[17] which contributes to the shrinkage of countries' aggregate asset positions to a median level of 0.965 and 0.923 as shares of GDP in the two cases respectively. The fact that counterfactual asset positions change little in a 43-country case suggests that the risk-sharing (diversification) benefits from holding assets of many economies with their non-perfectly correlated shocks almost replicate the complete market allocation. Such diversification benefits cannot be fully captured in a two-country setting, which explains the drastic portfolio reallocations from incomplete to complete markets.

In terms of risk hedging, when portfolio choice abstracts from labor income and RER risk considerations (equations 48 and 49 respectively), domestic asset holdings increase for almost all the countries. This result suggests that both labor income and RER risks tilt portfolios towards foreign assets, consistent with predictions from the home bias literature including Baxter and Jermann (1997), Coeurdacier (2009), and Heathcote and Perri (2013) under comparable parametric assumptions. Between the two risks, the magnitude of portfolio changes under the labor income risk is nearly identical across countries, which stems from the assumption of Cobb-Douglas production with labor and capital endowments.[18] On the other hand, the RER risk generates more disparate patterns across countries, shaped by the sophisticated trade structure characterized in the Eaton and Kortum (2002) model. When comparing two- and multi-country predictions, we observe that the deviations of counterfactual from actual portfolios overall are substantially larger in a two-country than in a multi-country setting. This result implies that risk-hedging mechanisms proposed in the home bias literature may play a less significant role in shaping portfolios given countries' multilateral trade and financial linkages in the real world.

Another major contribution of this multi-country model to the international macro literature lies in its ability to examine drivers for bilateral financial flows. Beyond countries' domestic holdings, the model also predicts bilateral asset positions across all the country pairs under the influences of second-moment variables shaped by the global trade

[17]The holding of domestic assets in complete markets with two countries is heavily influenced by the distribution of country sizes. In a two-country symmetric case, the domestic share is $\frac{1}{2}$ in equation 45. Here country sizes are calibrated to the data where the home country is small as shares of world GDP, which contributes to the low shares of domestic asset holdings for most countries.

[18]Future extensions of the model can consider methods to reduce this strong comovement between labor and financial income, such as introducing endogenous labor supply as in Jermann (2002) and Matsumoto (2007) or endogenous capital accumulation as in Heathcote and Perri (2013).

pattern. To simplify the analysis, I rewrite the portfolio determination equation (46) as

$$E_t[\gamma(1-\beta)\widetilde{R}\widetilde{R}'(1+\check{\alpha})+(1-\gamma+\beta\gamma)\widetilde{P}\widetilde{R}'] = \frac{1}{2}F \tag{50}$$

under the model assumption that financial income flows are proportional to output such that $\widetilde{Y}\widetilde{R}' = \widetilde{Y}\widetilde{Y}' = \widetilde{R}\widetilde{R}'$.

In equation 50, $\widetilde{R}\widetilde{R}'$, $\widetilde{P}\widetilde{R}'$, F, and portfolio $\check{\alpha}$ are all $(I\text{-}1) \times (I\text{-}1)$ matrices. In particular, the matrix $\widetilde{R}\widetilde{R}'$ reflects cross-country asset covariances important for risk sharing in incomplete markets, while $\widetilde{P}\widetilde{R}'$ reflects the covariance of financial income with price fluctuations relevant for RER risk hedging.[19] The elements in the i^{th} row j^{th} column of $\widetilde{R}\widetilde{R}'$ and $\widetilde{P}\widetilde{R}'$ capture the comovement of country i's financial income and price level respectively with country j's financial income under the shocks in the economy. These elements will determine i's holding of j's assets affected also by the corresponding bilateral financial friction in the i^{th} row j^{th} column of the friction matrix F.

I use this structural model to explain the strong correlation between bilateral trade and financial ties in the data. When calculating trade and financial linkages measured as the mean value of bidirectional shares averaged over time for a pair of country:

$$\pi_{ij} = \frac{\bar{\pi}_{ij}+\bar{\pi}_{ji}}{2}, \qquad \alpha_{ij} = \frac{\bar{\alpha}_{ij}+\bar{\alpha}_{ji}}{2}, \qquad \forall i,j \in [1, I], \tag{51}$$

I find the correlation between trade (π_{ij}) and financial (α_{ij}) linkages to be as high as 0.835 in the sample of country pairs, which verifies the strong connection between the two channels of globalization. There are two potential explanations for this empirical regularity based on the model.[20] First, trade linkages influence cross-country covariances of financial and macro variables, including $\widetilde{R}\widetilde{R}'$ and $\widetilde{P}\widetilde{R}'$, to the extent that bilateral trade flows predict financial flows. Second, cross-country financial and trade frictions are shaped by common factors such as geographic distance, cultural similarity, information accessibility, and regional economic integration, all of which are embedded in the friction matrix F.[21] I use the model to distinguish between these two explanations.

[19] The term $\widetilde{R}\widetilde{R}'$ also matters for labor income risk hedging under the comovement of financial and labor income in the model which implies $\widetilde{R} = \widetilde{w}$. I disentangle the two channels in table 2 and find risk sharing to be much more important in explaining the variation in bilateral financial flows driven by $\widetilde{R}\widetilde{R}'$.

[20] This analysis exploits the cross-sectional variation across country pairs and takes the trade structure as given to investigate its implications for asset positions. In the following sections I will examine the joint determination of trade and financial flows under policy changes by adding the inter-regime analysis.

[21] As mentioned in footnote 5, cross-country barriers for financial flows in this model may take various forms including the factors mentioned here. These factors, besides through their potential inferences for $\widetilde{R}\widetilde{R}'$ and $\widetilde{P}\widetilde{R}'$, shape countries' portfolio choice with their impacts on the elements of F.

Table 2 reports the correlation coefficient between bilateral trade and financial linkages in different scenarios. By comparing the portfolio changes across counterfactual scenarios, risk sharing and bilateral frictions play a more significant role than risk hedging in contributing to the connection between trade and financial linkages. Under the assumption of complete markets, the correlation declines significantly from 0.835 to 0.587. This result suggests that cross-country financial flows are strongly directed by countries' need for risk sharing whose pattern is shaped by the global trade structure,[22] which explains the sharp drop in the comovement between trade and financial linkages in complete markets where portfolio choice no longer serves of the purpose of risk sharing. Moreover, in the case where all the elements in F are set as zero to turn off the variation in frictions, the correlation plummets to 0.524, which suggests substantial impacts of common barriers facing both channels on their deep connection.

Table 2: Bilateral Financial and Trade Linkages

	Data	No risk sharing (1)	No labor risk hedging (2)	No RER hedging (3)	No friction difference (4)
$Corr(\alpha_{ij}, \pi_{ij})$	0.835	0.587	0.835	0.696	0.524

This table presents the correlation coefficient between bilateral trade and financial linkages (defined in 51) across country pairs in the sample in three scenarios: 1) in complete markets with no need for risk sharing, 2) where there is no labor income risk hedging, 3) where there is no RER risk hedging, and 4) where the elements of the bilateral friction matrix F are set as zero.

To empirically validate the importance of risk sharing and bilateral frictions for portfolio choice, I test whether their measures covary with observed bilateral asset positions. To proxy risk sharing, I consider cross-country covariance of productivity (see Appendix C for estimation) which reflects risks in the model. For bilateral frictions, I consider geographic distance, the mean value of a country-pair's Chinn-Ito index values to measure their capital account openness, and other gravity variables including dummies for contiguity, for regional trade agreements, and for common official language, religion, and legal origin. All of these gravity variables are sourced from the CEPII dataset. Moreover, I consider a country pair's bilateral RER as a control variable and get its measure based on the ratio of their CPI-based real effective exchange rates (REER) constructed by IMF divided by the pair's ratio of nominal exchange rates.[23]

[22]Trade influences risk sharing through multiple channels. In particular, trade structure matters for the degree of TOT adjustments in response to countries' idiosyncratic productivity shocks, a mechanism examined in the international macro literature such as Corsetti et al. (2008). This paper contributes to this literature by embedding the global trade structure in a multi-country DSGE framework, whose

Table 3: Sources of Variation for Bilateral Asset Positions

Dep. Var: log(Bilateral Holdings)	(1)	(2)	(3)	(4)	(5)
log(GDP$_o$)	1.245 ***	1.108 ***	1.085 ***	1.067 ***	1.151 ***
	(0.034)	(0.061)	(0.062)	(0.061)	(0.067)
log(GDP$_d$)	1.442 ***	-0.012	0.042	0.048	0.121
	(0.032)	(0.093)	(0.094)	(0.090)	(0.104)
log(dist)	-0.709 ***	-1.167 ***	-1.202 ***	-1.099 ***	-1.200 ***
	(0.037)	(0.021)	(0.022)	(0.021)	(0.019)
Chinn-Ito			0.674 **	1.412 ***	2.343 **
			(0.298)	(0.298)	(0.942)
cov(T)				2.498 ***	3.092 ***
				(0.224)	(0.244)
Chinn × cov(T)				-2.093 ***	-2.634 ***
				(0.249)	(0.270)
RER					0.320
					(0.711)
Chinn × RER					-0.678
					(0.879)
Fixed Effects	N	Y	Y	Y	Y
Gravity Var	N	Y	Y	Y	Y
Observations	22,448	22,448	20,807	20,807	17,105
R^2	0.123	0.957	0.959	0.960	0.963

Robust standard errors in parentheses.***significant at 1% and ** significant at 5%. The dependent variable is bilateral asset holdings in logs from Factset/Lionshare. GDP$_o$ and GDP$_d$ are the GDPs of the investment origin (holder) and destination (asset) country. dist is the population-weighted distance between countries from CEPII. Chinn-Ito is a capital account openness measure here averaged over the country pair. cov(T) is estimated bilateral covariance of productivity (see Appendix C for estimation). Fixed Effects include origin-, destination-, and time-FE. Gravity variables include CEPII's dummy variables for contiguity, regional trade agreements, common official language, religion, and legal origin. RER is the ratio of origin to destination country's real exchange rate, computed as their IMF's REER divided by nominal exchange rate ratios.

Table 3 reports the empirical results for countries' bilateral asset positions. Column (1) shows that bilateral holdings increase in the GDPs of the holders' (origin) and assets' (destination) countries and decrease in geographic distance, consistent with the gravity model of international finance documented by Portes and Rey (2005). Column (2), by adding origin-, destination-, time-fixed effects and gravity variables, accounts for most of the variability of asset positions in the data implied by the high R^2 value. Moreover, column (3) further improves the prediction and shows that capital account openness facilitates financial investment. Controlling for this institutional feature, stronger productivity

predictions for countries' TOT and RER variability differ from those in a standard two-country setup.

[23]Adjusting variables for nominal exchange rate is a common empirical strategy from the home bias literature (for example Coeurdacier and Gourinchas (2016)). The purpose is to control for nominal exchange rate fluctuations in the data which cannot be explained by economic fundamentals from an open economy macro model (also known as the exchange rate disconnect puzzle).

comovement reduces bilateral asset positions, as suggested by the negative coefficient for the interaction term of productivity covariance with the Chinn-Ito index in column (4). This finding that portfolio choice is influenced by risk sharing echoes Coeurdacier and Guibaud (2011) and Bergin and Pyun (2016) who empirically and theoretically establish the importance of cross-country covariance structure for financial flows. Besides, column (5) considers RER but does not find it to be an important determinant of bilateral holdings. This empirical finding is consistent with the earlier argument that RER risk hedging may not be as essential in this multi-country setting where prices are jointly determined by all the economies, which reduces the correlation between a country's price level with an individual foreign country's asset returns to influence bilateral asset positions.

So far I have performed intra-regime analysis to explain countries' asset positions observed in the data. Next I add inter-regime analysis to examine the joint determination of trade and finance under policy changes.

3.2 Interaction of Trade and Finance under Tariffs

This section conducts comparative static analyses with higher trade costs to predict the general equilibrium changes of trade and finance. In this policy experiment I assume that bilateral tariffs among all the country pairs uniformly increase by a half

$$\hat{\tau} = \frac{\tau_{ij}^{ctf}}{\tau_{ij}^{org}} = 1.5, \qquad \forall i \neq j \in [1, I], \tag{52}$$

and examine inter-regime changes of variables denoted with a Δ. For any variable A,

$$\Delta A = A^{ctf} - A^{org}. \tag{53}$$

To derive countries' asset positions under the tariff changes, I examine the portfolio determination equation (50) in both original and counterfactual regimes

$$E_t[\gamma(1-\beta)\widetilde{R}\widetilde{R}'^s(1+\breve{\alpha}^s) + (1-\gamma+\beta\gamma)\widetilde{P}\widetilde{R}'^s] = \frac{1}{2}F^s, \ s \in \{org, ctf\} \tag{54}$$

and take their difference to find the inter-regime portfolio changes

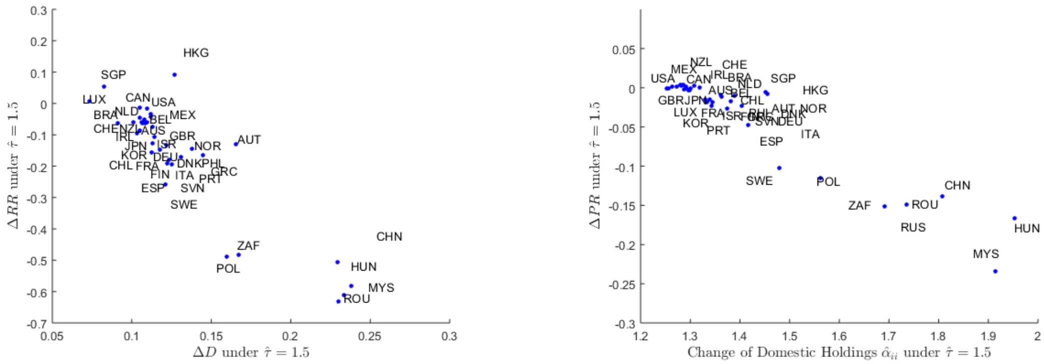
$$E_t[\gamma(1-\beta)\Delta(\widetilde{R}\widetilde{R}'(1+\breve{\alpha})) + (1-\gamma+\beta\gamma)\Delta\widetilde{P}\widetilde{R}'] = \frac{1}{2}\Delta F. \tag{55}$$

Figure 2 presents the numerical results for countries' cross-regime changes of asset positions and second-moment variables (see table A.3 for complete results). Figure 2a documents a strong negative correlation between the change to any country i's aggregate asset position ($\Delta \bar{D}_i$) and the change to its median asset covariance with others ($\Delta \tilde{R}_i \tilde{R}'$). This finding echoes the earlier risk-sharing mechanism: Lower asset covariance yields greater risk-sharing benefits, which creates incentives for risk-averse households to raise asset positions. Therefore, exporting countries such as China and Malaysia are predicted to increase holdings significantly since they experience large reductions in asset covariance attributable to reductions in their output synchronization with other countries under higher tariffs. In contrast, asset covariances of Hong Kong, Singapore, and Luxembourg are less affected partly due to the relatively symmetric impacts of tariffs on exports and imports of these port countries. Hence, their portfolios increase less due to fewer increased diversification benefits offered by international assets.

Figure 2b shows the relationship between the change to a country's domestic asset holding ($\hat{\alpha}_{ii}$) and the change to the median covariance of its price level with other countries' financial income ($\Delta \tilde{P}_i \tilde{R}'$). The covariance change $\Delta \tilde{P}_i \tilde{R}'$ is very small for the majority of the countries. This is not only driven by the assumption of uniform tariff changes but also by the fact that price is jointly determined by all the economies which makes it less volatile in this multi-country setting. Moreover, the figure shows that there is a negative correlation between $\hat{\alpha}_{ii}$ and $\Delta \tilde{P}_i \tilde{R}'$, which supports the risk-hedging mechanism discussed earlier: When returns to international assets positively comove with a country's price level, households have an incentive to substitute foreign for domestic assets to hedge against the RER risk. Therefore, countries such as Russia, Hungary, and China are predicted to reduce their domestic asset holdings by a greater magnitude.

Figure 3 plots real wages, defined as countries' wage-to-price ratios, under the tariff increase. In particular, I compare changes to real wages in the case where countries' asset positions are adjustable to tariffs and in the case where they are fixed ($\widehat{\bar{D}} = 1$ or equivalently $\Delta \bar{D} = 0$). The wedge of the two real wages (denoted as $\frac{\widehat{\bar{w}}}{\widehat{\bar{p}}}^{adj} / \frac{\widehat{\bar{w}}}{\widehat{\bar{p}}}^{fix}$) quantifies the feedback effects of financial allocations on real variables. Figure 3a documents a weakly positive correlation between the wedge and countries' adjustable asset positions under tariffs. This result can be understood from the fact that higher asset positions allow countries to increase expenditure, which in turn raises their wages through greater demand for labor, if much of the expenditure is spent on domestic goods. Therefore, the prediction of asset positions for real wages in figure 3a is not monotonic, because the composition of a country's expenditure also influences its wage-to-price ratio. Figure 3b plots the wedge in real wages and the share of domestic expenditure in the data, which

Figure 2: Changes of Second Moments and Asset Positions under Higher Tariffs



(a) $\Delta\bar{D}_i$ and $\Delta\tilde{R}_i\tilde{R}'$ (b) $\hat{\alpha}_{ii}$ and $\Delta\tilde{P}_i\tilde{R}'$

This figure plots the changes in the size ($\Delta\bar{D}_i$) and domestic share of country i' asset position ($\hat{\alpha}_{ii} = \alpha_{ii}^{ctf}/\alpha_{ii}^{org}$) against the median covariance change of other countries' financial income with country i's price level ($\Delta\tilde{P}_i\tilde{R}'$) and financial income ($\Delta\tilde{R}_i\tilde{R}'$) between original and counterfactual regimes when the bilateral tariffs increase by $\hat{\tau} = 1.5$. See table A.3 for complete results.

exhibits a positive comovement between the two variables. Countries including Japan and the US show stronger expenditure home bias with a higher $\bar{\pi}_{ii}$, which contributes to their higher rise in real wages when asset positions are adjustable. This occurs because the increased financial holdings mostly boost their demand for domestic goods and therefore raise local wage compensation.

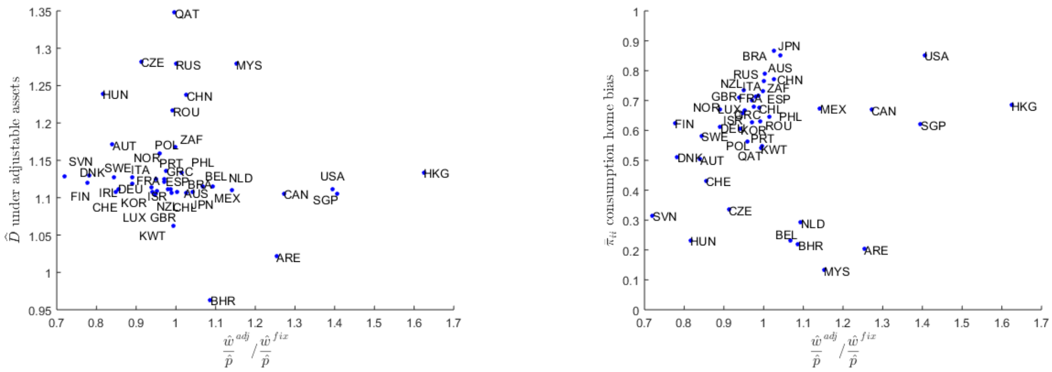
Next I combine trade and financial channels to conduct welfare analysis. The trade literature including Dekle et al. (2007) measures welfare, with the cost and size of expenditure considered, as real wages multiplied by asset positions:

$$\widehat{\mathbb{W}}_i = \frac{\widehat{\bar{w}}_i}{\widehat{\bar{\bar{P}}}_i}\widehat{\bar{D}}_i. \tag{56}$$

The two components are real and financial aspects of welfare respectively. Using this decomposition, I compare welfare under fixed and under adjustable positions, to capture the departure of this new approach from the existing literature.

Figure 4a compares counterfactual real wages under fixed and adjustable asset positions. Most countries' real wages are lower in either case ($\frac{\widehat{\bar{w}}}{\widehat{\bar{\bar{P}}}} < 1$) given a tariff change $\hat{\tau} = 1.5$ relative to in the original situation without tariffs. This happens as higher tariffs prohibit cross-country commodity flows and raise the price paid by households more than their wage. Between the two cases, real wages decrease by a greater magnitude under adjustable than under fixed asset positions as most countries lie above the 45 degree line

Figure 3: Changes of Real Wages under Higher Tariffs



(a) $\frac{\widehat{w}}{\widehat{\bar{P}}}^{adj} / \frac{\widehat{w}}{\widehat{\bar{P}}}^{fix}$ and $\widehat{\bar{D}}_i$

(b) $\frac{\widehat{w}}{\widehat{\bar{P}}}^{adj} / \frac{\widehat{w}}{\widehat{\bar{P}}}^{fix}$ and $\bar{\pi}_{ii}$

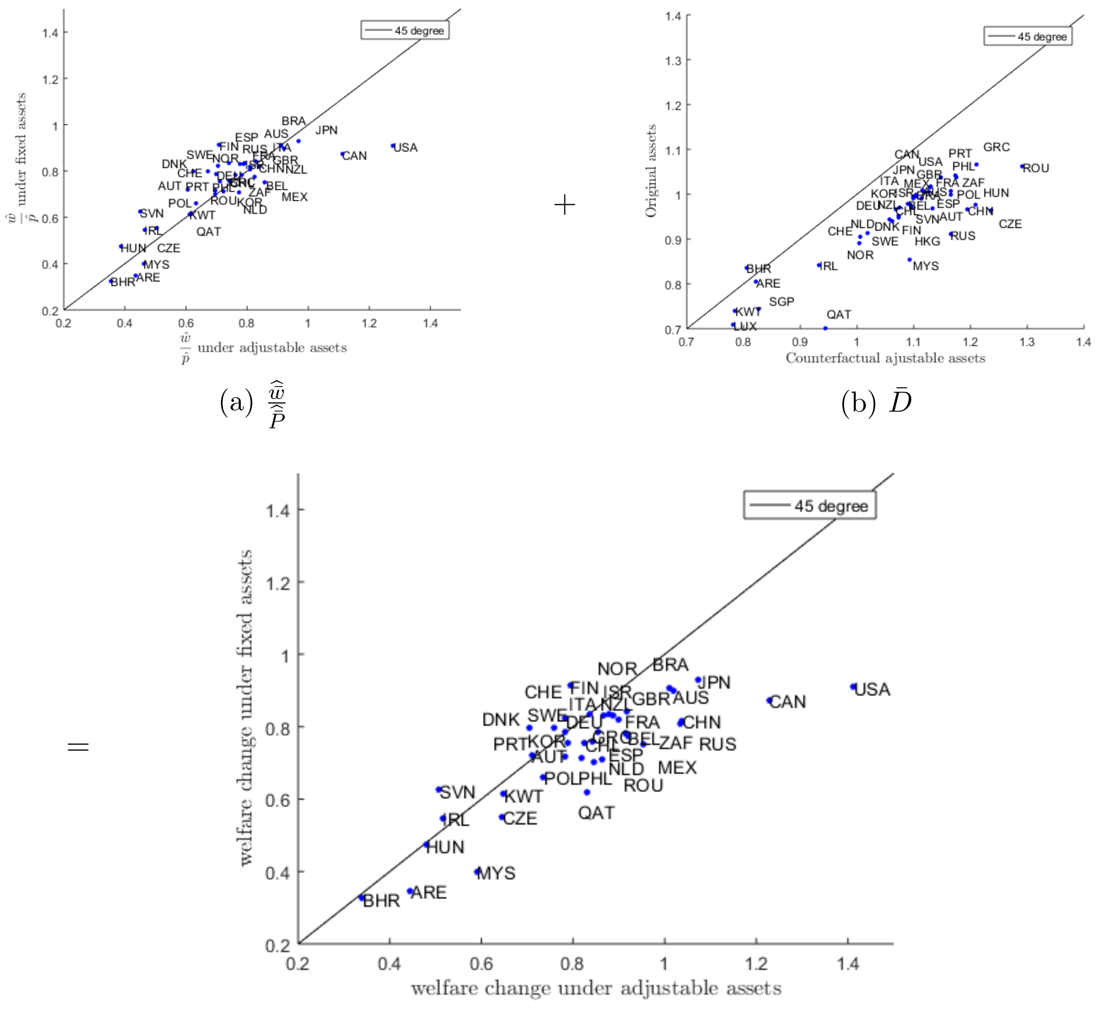
This figure plots the changes to real wage $\frac{\widehat{w}}{\widehat{\bar{P}}}$ when bilateral tariffs increase by $\hat{\tau} = 1.5$. The variable on the x axis $\frac{\widehat{w}}{\widehat{\bar{P}}}^{adj} / \frac{\widehat{w}}{\widehat{\bar{P}}}^{fix}$ is the ratio of real wage changes under adjustable (superscripted adj) relative to fixed (fix) asset positions. The variable on the y axis is changes to asset positions ($\widehat{\bar{D}}_i$) if adjustable to tariffs in figure 3a and countries' share of domestic goods in expenditure ($\bar{\pi}_{ii}$) in figure 3b.

in the diagram. This result suggests that households witness greater deteriorations to the purchasing power of their labor income when they can adjust their financial holdings in response to tariffs. Nonetheless, the financial holdings partially offset the impact of labor income loss on households' welfare, as figure 4b shows that most countries increase their holdings under optimal intertemporal decisions.[24] This happens since assets provide more diversification benefits (reflected as $\Delta\widetilde{R}\widetilde{R}' < 0$ in figure 2a) when the trade channel, which facilitates international output synchronization that generates a higher $\widetilde{R}\widetilde{R}'$, faces greater barriers. This creates incentives for risk-averse households to increase asset positions. Another angle to interpret this result is that, TOT adjustments in the trade channel, which would help reduce the impacts of idiosyncratic output shocks on consumption, are restricted by higher tariffs. Therefore, households switch from trade to financial channels for international risk sharing.

Figure 4c illustrates the welfare changes which combine real wages in figure 4a and asset positions in figure 4b. If the original welfare without tariff increase is normalized

[24]It is worth noting that figures 4a and 4b should be examined simultaneously because they are both derived from the model solution to a joint fixed-point problem of $(\widehat{w}, \widehat{\bar{P}}, \Delta\bar{D})$ under the normalization condition that treats the world output as a numeraire $\sum_{i=1}^{I} \widehat{w}\bar{Y}_i^{org} = 1$ and the world resource constraint $\sum_{i=1}^{I} \widehat{w}\bar{Y}_i^{org} = \sum_{i=1}^{I} \widehat{w}\bar{Y}_i^{org}\bar{D}_i^{ctf}$. Under these two conditions, countries' increased borrowing is sustainable since their wages decline concurrently under tariffs. Countries' new asset positions reflect households' optimal lifetime utility maximization decisions given the tariff changes.

Figure 4: Counterfactual Welfare under Fixed and Adjustable Asset Positions



(a) $\frac{\widehat{w}}{\widehat{\bar{P}}}$ (b) $\bar{D}$ (c) $\widehat{\mathbb{W}}$

This figure plots the counterfactual changes to real wage $\frac{\widehat{w}}{\widehat{\bar{P}}}$ in (a), counterfactual equilibrium asset positions as shares of output $\bar{D}$ in (b), and counterfactual welfare changes $\widehat{\mathbb{W}}$ in (c) when bilateral tariffs in the world economy uniformly increase by $\hat{\tau} = 1.5$. Variables on the horizontal axis represent the case where asset positions are adjustable in response to tariffs and the variables on the vertical axis represent the case where asset positions are fixed. See table A.3 for complete results.

to 1, the median counterfactual welfare across countries is 0.743 under fixed and 0.785 under adjustable asset positions (see table A.3 for complete results). For expositional purposes, the figures miss extreme outliers especially port countries which experience enormous welfare changes, a common prediction from structural trade models (see, for example, Alvarez and Lucas (2007)). Aside from those outliers, the US and Canada are among the economies with greatest welfare increases, largely due to their expenditure home bias mentioned earlier for figure 3b, which explains their higher real wages under adjustable portfolios. Furthermore, figure 4c suggests that most countries have higher welfare when financial holdings can be adjusted. The welfare loss of these countries would be overestimated under the assumption that their asset positions were fixed given the tariff changes.

The discrepancy between the two cases occurs because a standard trade model excludes finance, which is a major means for risk-averse agents to achieve consumption smoothing for utility maximization. When the world economy faces higher tariffs which hinder output synchronization, agents endogenously increase asset holdings under intertemporal decisions to take advantage of the risk-sharing benefits given lower cross-country covariances in the financial market. Ignoring this margin of adjustment may miscalculate welfare. Hence, this comparison between fixed and flexible asset positions underscores the importance of incorporating a financial channel in a trade model. Meanwhile, this feedback of countries' asset positions on real variables is missing in the home bias literature where portfolio is typically solved taking the the real side of the economy including trade as fixed. Therefore, this paper contributes to both international macro and trade literatures by capturing the two-way interactions of trade and finance.

4 Applications

This section examines two counterfactuals as applications of the new approach which determines both financial and real variables. The first analyzes a scenario where cross-country financial frictions universally increase as in the 2008 global financial crisis. The second studies the implications of tariffs during the 2018 China-US trade war.

4.1 Tightening of Global Financial Markets

To examine how the world economy reacts to financial frictions, I impose a uniform increase of bilateral frictions among all the country pairs by

$$\widetilde{F} = \frac{F^{ctf} - F^{org}}{F^{org}} \tag{57}$$

to replicate a scenario where countries face higher costs in foreign investment as in a global financial crisis. The changes in bilateral frictions are in relative terms to the frictions faced by the numeriare country I which is assumed to be ROW, as the element in the i^{th} row j^{th} column of F^s in regime $s \in \{org, ctf\}$

$$F^s(i,j) = (f_{iI} - f_{ij}) - (f_{II} - f_{Ij}). \tag{58}$$

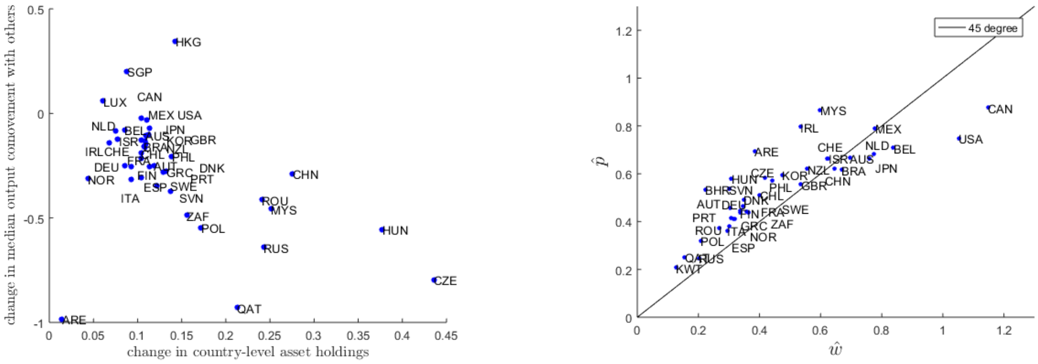
Everything else equal, an increase in bilateral friction f_{ij}, which stands for country i's friction when holding j's asset, will generate a negative $\widetilde{F}(i,j)$. The counterfactual exercise assumes $\widetilde{F}(i,j)$ = -0.2 for $\forall i \neq j \in [1, I]$, which represents a universal 20% relative increase of bilateral financial frictions. Incorporating this increase in the cross-regime difference of the portfolio equations yields

$$E_t[\gamma(1-\beta)\Delta(\widetilde{R}\widetilde{R}'(1+\breve{\alpha})) + (1-\gamma+\beta\gamma)\Delta\widetilde{P}\widetilde{R}'] = \frac{1}{2}\widetilde{F}F^{org}. \tag{59}$$

From this equation, an increase in financial frictions will generate a decrease in asset holdings $\breve{\alpha}$. Nonetheless, the magnitude of the decrease also depends on $\widetilde{R}\widetilde{R}'$ whose values simultaneously shift across policy regimes under the influence of the frictions. Specifically, asset holdings will drop less if $\Delta\widetilde{R}\widetilde{R}' < 0$. The earlier intuition applies here: $\widetilde{R}\widetilde{R}'$ as the asset covariance matrix captures the comovement across countries' financial income and reflects the risk-sharing benefits offered by international assets. Therefore, by constructing diversified portfolios of different countries' assets, households have more stable financial income less subject to country-specific risks for consumption smoothing, which creates greater incentives to hold the assets despite higher financial frictions.

Figure 5a verifies this risk-sharing mechanism: Countries with a higher median asset covariance increase with others are expected to curtail foreign holdings to a greater extent. Many countries switch to domestic assets, which contributes the increase in their aggregate asset positions. Figure 5a is similar to figure 2a under tariffs, but in this scenario the friction originates from the financial channel. Financial frictions affect households' port-

Figure 5: Changes of Financial and Real Variables under Financial Frictions



(a) $\Delta \bar{D}_i$ and $\Delta \tilde{R}_i \tilde{R}'$ (b) $\widehat{\overline{w}}$ and $\widehat{\overline{P}}$

Figure (a) shows the changes in country i's asset position $\Delta \bar{D}_i$ and in i's median asset covariance with other countries $\Delta \tilde{R}_i \tilde{R}'$, and (b) plots the changes in countries' wage $\widehat{\overline{w}}$ and price $\widehat{\overline{P}}$, both under universally higher financial frictions $\widetilde{F} = $ -0.2.

folio choice on impact, which then transmit to their expenditure in the trade channel to cause inter-regime changes of $\Delta \widetilde{R} \widetilde{R}'$ until equation 59 is restored in general equilibrium.

On the real side of the general equilibrium, countries' wage ($\widehat{\overline{w}}$) and price ($\widehat{\overline{P}}$) changes are shown in figure 5b. Affected by the financial frictions, most countries experience lower wage which declines more than price compared to the original regime ($\widehat{\overline{w}} < \widehat{\overline{P}} < 1$). Therefore, their real wages deteriorate when international financial allocations face higher barriers which impair economic efficiency.

4.2 China-US Trade War

In 2018, the Trump administration started to raise tariffs and other trade barriers on China, which took retaliatory actions in response. Besides reciprocal tariff increases, selling off US assets was considered a possible means for China to strike back.[25] I conduct analyses with both trade and financial measures based on the structural model to quantify the welfare implications of the trade war.

In the trade channel, I calibrate bilateral tariff changes as $\widehat{\tau}_{US,CN} = 1.189$, $\widehat{\tau}_{CN,US} = 1.186$ based on the estimates from Li et al. (2020).[26] Table A.3 panel (III) reports

[25]Media coverage of this possibility is found in New York Times, Politico, among others. These media agree that although the possibility is not high, the consequences can be catastrophic for both countries once China weaponizes its portfolio ownership including over 1 trillion dollars US government debt.

[26]They merge industry-level tariff changes from government announcements with sectoral trade data from the UN Comtrade, to calculate the trade-weighted cumulative tariff increases by March 2020.

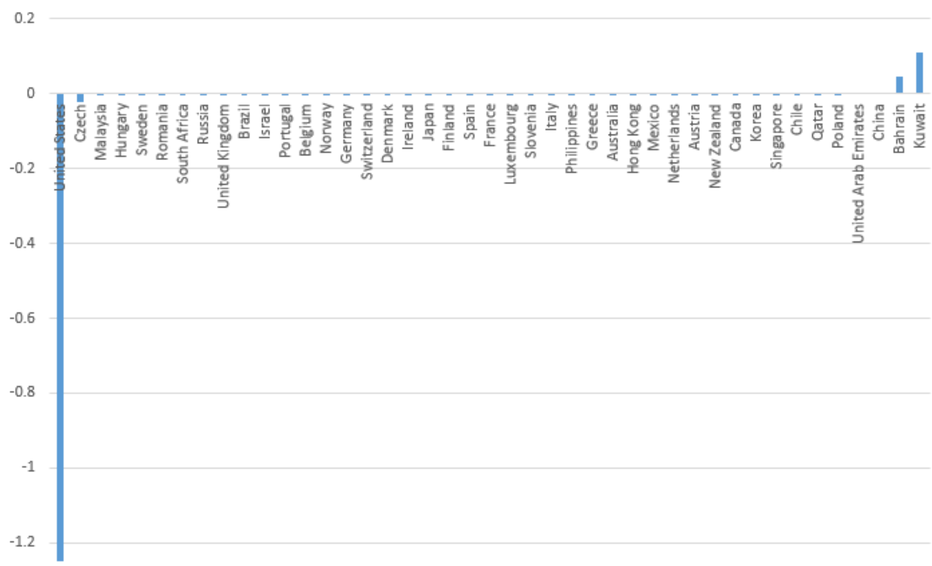
the model-predicted real and financial variables under the tariffs, which share many similarities with the earlier findings in section 3.2 under uniform tariff changes. In this scenario, the tariff shocks only hit the two major economies, however all the 43 countries are affected due to the interconnected global trade and financial structures. Specifically, the model predicts that most countries suffer loss in real wage as $\frac{\widehat{\tilde{w}}}{\widehat{\tilde{P}}} < 1$. The magnitude of this loss tends to be larger than the prediction from existing literature for two major reasons.[27] First, this model assumes a single tradable sector, while many trade models consider both nontradable and tradable sectors with global input-output linkages. Cross-sector substitution reduces the impacts of industry-specific tariffs to leave wages less affected. Second and more importantly, this paper considers countries' portfolio changes while most trade literature ignores this margin of adjustment. Table A.3 reports that most countries increase their asset holdings which can be explained by the same reason as for figure 4b: Cross-country asset covariances decrease when tariffs increase to impair output synchronization. Hence, countries have stronger incentives to hold each others' assets for international risk sharing. This financial allocation requires that wages be adjusted to a lower level to satisfy the world resource constraint, which magnifies the adjustment of real variables compared to in a standard trade model. When combining the changes in real wages and in asset positions, I find a median welfare change of $\widehat{\mathbb{W}} = 0.912$ across countries in the sample. Nevertheless, the model does not suggest welfare loss for China and the US, which are predicted to experience increased asset positions that also generate higher real wages instead.[28] However, these predictions do not consider the implications of trade policy uncertainty which is an important feature of the event (Handley and Limão (2022)). A future extension is to add uncertainties, which can be embedded in the friction matrix F containing second moments including volatilities, to examine their impacts on the financial aspects of welfare.

I proceed to discuss the possibility that China gives up its holding of US assets as means of financial retaliation. To conduct this counterfactual analysis, I set $\alpha_{CN,US} = 0$ and solve for China's portfolio choice among the remaining countries' assets under their

[27] For example, Caliendo and Parro (2021) estimate with a multi-sector multi-region quantitative spatial model that the U.S. aggregate real wages decline by 0.16 percent due to the trade war. Besides, Li et al. (2020) build a computational general equilibrium (CGE) model to predict that China's welfare falls by 1.7%, and that other countries have disparate but smaller changes to real wage.

[28] Both countries exhibit strong expenditure home bias as shown in figure 3. Therefore, when their financial holdings increase under lower asset covariances, demand for domestic expenditure increases to raise real wages. For comparison, I conduct another experiment under calibrated tariffs and fixed asset positions, and find the real wages of China and the US to be 0.82 and 0.96 times of their original values without the trade war.

Figure 6: Changes in China's Asset Holdings by Destination



This figure plots the difference in the changes to China's holdings of assets between the case with financial retaliation (denoted as ctf_1 below) where $\alpha_{CN,US} = 0$ and the case without retaliation (ctf_2), both relative to the situation where there is no trade war (org). The values presented in the figure are calculated as $\Delta\alpha_{CN,i} = \frac{\alpha^{ctf_1}_{CN,i}}{\alpha^{org}_{CN,i}} - \frac{\alpha^{ctf_2}_{CN,i}}{\alpha^{org}_{CN,i}}, \forall i \in [1, I]$

covariance structure. Figure 6 plots the difference in the changes of China's asset holdings between the cases with and without financial retaliation. If China had to reconstruct its portfolio, the model predicts that it would replace US assets with either domestic assets or the assets of oil exporters including UAE, Bahrain, and Kuwait, while keeping most of the other asset positions slightly lower. Compared to the case with no financial retaliation, China's aggregate asset holding $\bar{D}^{ctf}$ would drop more by 0.16% as shares of GDP and its wage-to-price ratio would drop by 1.14%. Therefore, based on the welfare definition from equation 56, China's welfare loss during the trade war would be exacerbated by 1.3% if the country retaliated in the financial channel. Nevertheless, this estimate does not consider complications beyond the scope of this model, including the impacts of the asset sale on Chinese exchange rate policy or on US monetary policy. Understanding such policy-relevant questions requires incorporating additional modeling ingredients and techniques from the open economy macro literature, which will be interesting topics to explore for future research.

5 Conclusion

This paper develops a multi-country model where financial and trade channels influence each other. The general equilibrium effects captured by the model provide new insights about the patterns and determinants of cross-country economic linkages. The solution of the model is derived with a novel approach that combines the portfolio choice solution method and the hat algebra technique. This approach can readily be applied to a wide range of topics in both international macro and trade literatures, meanwhile it has many potentials for extensions. I hereby discuss two directions for future work.

First, this paper focuses on comparative statics across steady states under specific policy regimes, without tracing the dynamic path of the world economy between steady states. Although the portfolio choice problem considers agents' intertemporal investment decisions, the derived equilibrium portfolio is static in nature. If future research questions involve time-series patterns of economic activities, solving dynamic portfolios requires extending the current method to higher-order approximations of the model. Devereux and Sutherland (2010) show that the first-order dynamics of portfolios is obtained by combining a third-order approximation of the portfolio determination equation with a second-order approximation to the rest of the model. On the real side of the economy, either 'dynamic exact hat algebra' techniques from the quantitative trade literature (Caliendo et al. (2019) and Kleinman et al. (2021) among others) or perturbation methods from the extensive DSGE literature (see Fernández-Villaverde et al. (2016) for a comprehensive survey) can be applied, depending on the specific context of interest. Such dynamic analyses characterize the world economy's pattern and speed of convergence towards a steady state, which is important in quantifying both persistent and transitory economic outcomes when general equilibrium is being restored.

Second, this paper uses the local linearization method to derive portfolio choice around a deterministic steady state. The method offers a powerful yet tractable toolkit widely applicable to DSGE models. Furthermore, the derived solution is extremely close to the exact around the point of approximation. However, it is less accurate when there are large deviations from the steady state or when the problem exhibits strong non-linearity (see a detailed discussion in Coeurdacier and Rey (2013)). Therefore, if global solution methods (such as policy or value function iterations) for the portfolio choice problem with comparable applicability and tractability become available in the future, financial investment can be endogenously determined in more general economic environments. Important questions including those related to sovereign defaults and disaster risks can be answered with the development of such solution techniques.

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Appendices

A Tables

Table A.1: List of Sample Countries

Name	Code	Name	Code	Name	Code	Name	Code
Australia	AUS	France	FRA	Luxembourg	LUX	Russia	RUS
Austria	AUT	Germany	DEU	Malaysia	MYS	Singapore	SGP
Bahrain	BHR	Greece	GRC	Mexico	MEX	Slovenia	SVN
Belgium	BEL	Hong Kong	HKG	Netherlands	NLD	Spain	ESP
Brazil	BRA	Hungary	HUN	New Zealand	NZL	Sweden	SWE
Canada	CAN	Ireland	IRL	Norway	NOR	Switzerland	CHE
Chile	CHL	Israel	ISR	Philippines	PHL	U.A.E.	ARE
China	CHN	Italy	ITA	Poland	POL	United Kingdom	GBR
Czech	CZE	Japan	JPN	Portugal	PRT	United States	USA
Denmark	DNK	Korea	KOR	Qatar	QAT	South Africa	ZAF
Finland	FIN	Kuwait	KWT	Romania	ROU		

B Details of the Model

B.1 Model Equations

This section derives the equations for hat algebra and portfolio analysis based on the model. Unless otherwise noted, a variable marked with a hat denotes the ratio of its realized value (marked with a prime) relative to the original steady state (marked with a bar):

$$\widehat{A} = \frac{A'}{\bar{A}}. \tag{B.1}$$

I do not distinguish between inter-regime ($\widehat{\widehat{A}}$) or intra-regime ($\widehat{A}^s, s \in \{org, ctf\}$) changes because most equations in this section can be used to characterize either case with minor modifications. Moreover, I omit time subscripts for brevity because these equations that contain hat variables characterize intra-temporal allocations.

On the production side, factor prices are determined by firms' profit maximization condition which holds both in the steady state and for variables' realized values:

$$\frac{\bar{w}_i \bar{L}_i}{\bar{r}_i \bar{K}_i} = \frac{1-\mu}{\mu}, \qquad \frac{w_i' L_i'}{r_i' K_i'} = \frac{\widehat{w}_i \bar{w}_i \widehat{L}_i \bar{L}_i}{\widehat{r}_i \bar{r}_i \widehat{K}_i \bar{K}_i} = \frac{1-\mu}{\mu} \tag{B.2}$$

Therefore, in this economy with labor and capital as endowments $\widehat{L}_i = \widehat{K}_i = 1$, the change in capital rental fee equals that in wage:

$$\widehat{r}_i = \widehat{w}_i. \tag{B.3}$$

These factor prices are reflected in a country's total income

$$Y_i' = w_i' \bar{L}_i + r_i' \bar{K}_i, \tag{B.4}$$

Table A.2: Disentangling Risk-sharing and Risk-hedging Channels

Country	Data		No Risk Sharing				No Labor Risk Hedging				No RER Risk Hedging			
			multi-co		two-co		multi-co		two-co		multi-co		two-co	
	$\bar{\alpha}_{ii}$	$\bar{D}_i$	$\bar{\alpha}_{ii}$	$\bar{D}_i$	$\bar{\alpha}_{ii}$	$\bar{D}_i$	$\bar{\alpha}_{ii}$	$\bar{D}_i$	$\bar{\alpha}_{ii}$	$\bar{D}_i$	$\bar{\alpha}_{ii}$	$\bar{D}_i$	$\bar{\alpha}_{ii}$	$\bar{D}_i$
AUS	0.68	1.01	0.73	1.02	4.41	2.05	0.69	1.01	0.88	1.07	0.78	1.04	1.36	1.20
AUT	0.11	0.97	0.11	0.99	0.35	1.04	0.12	0.99	0.31	1.02	0.20	1.15	0.93	1.20
BHR	0.75	0.84	1.45	1.04	0.00	0.62	0.61	0.80	0.95	0.89	0.02	0.63	2.47	1.32
BEL	0.12	0.98	0.10	0.93	0.02	0.95	0.12	0.99	0.32	1.03	0.17	1.13	1.09	1.25
BRA	0.78	1.00	0.85	1.03	1.59	1.23	0.79	1.00	0.98	1.06	0.89	1.03	1.72	1.26
CAN	0.50	0.99	0.56	1.02	1.31	1.22	0.51	1.00	0.70	1.05	0.58	1.04	1.58	1.30
CHL	0.77	0.96	0.77	0.97	0.62	0.92	0.78	0.97	0.98	1.02	0.86	0.99	2.19	1.36
CHN	0.74	0.97	0.00	0.76	0.02	0.76	1.20	1.09	0.95	1.02	4.39	1.98	2.33	1.41
CZE	0.21	0.96	0.01	0.75	0.00	0.91	0.30	1.02	0.41	1.02	0.83	1.35	1.36	1.29
DNK	0.14	0.94	0.15	0.96	1.02	1.19	0.14	0.95	0.34	1.00	0.20	1.04	0.87	1.15
FIN	0.49	0.98	0.49	0.98	0.00	0.84	0.50	0.99	0.69	1.03	0.61	1.04	1.74	1.33
FRA	0.39	1.00	0.41	1.02	0.84	1.13	0.41	1.02	0.59	1.06	0.53	1.09	1.26	1.25
DEU	0.21	0.94	0.20	0.94	1.34	1.26	0.22	0.95	0.41	1.00	0.28	1.02	0.91	1.14
GRC	0.43	1.07	0.44	1.08	0.75	1.16	0.45	1.08	0.63	1.12	0.61	1.18	1.33	1.32
HKG	0.18	0.95	0.07	0.82	0.00	0.90	0.22	0.96	0.39	1.00	0.38	1.07	1.15	1.22
HUN	0.38	0.98	0.14	0.85	0.00	0.87	0.46	1.01	0.58	1.03	0.99	1.26	1.69	1.34
IRL	0.15	0.84	0.17	0.91	-0.06	0.78	0.20	0.87	0.35	0.90	0.46	1.05	1.31	1.17
ISR	0.75	0.99	0.73	0.99	2.62	1.52	0.77	1.00	0.95	1.05	0.88	1.04	1.81	1.29
ITA	0.22	0.99	0.23	0.99	1.07	1.23	0.24	1.01	0.43	1.05	0.34	1.12	0.85	1.17
JPN	0.54	1.00	0.37	0.93	0.00	0.84	0.58	1.01	0.75	1.05	0.82	1.08	1.59	1.29
KOR	0.79	0.97	0.81	0.98	0.03	0.76	0.80	0.97	0.99	1.03	0.88	1.00	2.35	1.41
KWT	0.18	0.74	0.09	0.66	0.00	0.69	0.23	0.75	0.39	0.80	0.47	0.80	1.25	1.04
LUX	0.01	0.71	0.01	0.66	0.00	0.71	0.01	0.71	0.21	0.77	0.01	0.76	0.64	0.89
MYS	0.76	0.85	0.00	0.64	0.00	0.64	0.93	0.90	0.96	0.91	2.17	1.25	2.42	1.32
MEX	0.84	1.02	0.83	1.01	0.09	0.81	0.85	1.02	1.04	1.07	0.94	1.05	2.36	1.44
NLD	0.09	0.91	0.09	0.93	0.00	0.89	0.09	0.93	0.29	0.97	0.13	1.01	1.01	1.17
NZL	0.52	0.99	0.58	1.02	1.55	1.28	0.54	1.00	0.73	1.05	0.63	1.04	1.53	1.27
NOR	0.06	0.89	0.07	0.89	3.08	1.74	0.07	0.90	0.27	0.95	0.09	0.98	0.35	0.97
PHL	0.54	1.04	0.54	1.04	6.03	2.58	0.56	1.04	0.74	1.09	0.68	1.11	1.09	1.19
POL	0.85	1.01	0.82	1.00	0.00	0.77	0.86	1.01	1.05	1.06	0.99	1.05	2.48	1.47
PRT	0.65	1.04	0.64	1.04	0.84	1.10	0.67	1.05	0.85	1.10	0.81	1.11	1.74	1.35
QAT	0.14	0.70	0.12	0.66	2.05	1.24	0.15	0.71	0.34	0.76	0.20	0.77	0.67	0.85
ROU	0.97	1.06	0.68	0.98	0.00	0.79	1.02	1.08	1.17	1.12	1.49	1.21	2.83	1.58
RUS	0.84	0.91	0.65	0.86	0.00	0.68	0.88	0.92	1.04	0.97	1.18	1.01	2.48	1.37
SGP	0.09	0.74	0.00	0.59	0.00	0.72	0.17	0.77	0.29	0.80	0.54	0.94	0.97	0.99
SVN	0.67	0.97	0.60	0.95	0.00	0.78	0.70	0.98	0.88	1.03	0.88	1.04	2.15	1.38
ESP	0.34	1.00	0.34	1.01	0.23	0.97	0.36	1.02	0.54	1.06	0.51	1.12	1.35	1.29
SWE	0.40	0.95	0.32	0.90	3.62	1.86	0.41	0.96	0.60	1.01	0.51	1.02	1.04	1.13
CHE	0.16	0.90	0.18	0.94	0.00	0.86	0.17	0.92	0.36	0.96	0.23	1.00	1.14	1.18
ARE	0.30	0.80	3.08	1.45	0.00	0.72	0.49	0.85	0.51	0.86	1.42	1.09	1.59	1.16
GBR	0.42	1.02	0.49	1.06	0.30	0.98	0.44	1.03	0.63	1.07	0.55	1.09	1.39	1.29
USA	0.78	1.04	0.80	1.05	1.74	1.31	0.79	1.04	0.98	1.09	0.88	1.07	1.91	1.36
ZAF	0.74	1.00	0.00	0.76	1.26	1.14	0.85	1.03	0.94	1.06	1.67	1.27	1.98	1.35

This table presents the model predictions for the size and domestic holding of countries' portfolios in three scenarios: 1) in complete markets with no need for risk sharing, 2) where there is no labor income risk, and 3) where there is no RER risk. Results are reported for: 1) a multi-country case (labeled "multi-co") where there are 43 countries with bilateral trade and financial linkages, and 2) a two-country case (labeled "two-co") with each of the countries in the sample treating itself as the domestic economy and all the other countries in the world as the aggregate foreign economy. The median values and plots across countries are presented in table 1 and figure 1.

Table A.3: Comparative Statics under Higher Tariffs

Country	Second Moments and Portfolios (I)				Fixed Asset Positions (II)			Adjustable Positions (II)			Trade War (III)		
	$\Delta\bar{D}_i$	$\Delta\tilde{R}_i\tilde{R}'$	$\hat{\alpha}_{ii}$	$\Delta\tilde{P}_i\tilde{R}'$	$\frac{\widehat{w}}{\widehat{P}}$	$\bar{D}$	$\widehat{\mathbb{W}}$	$\frac{\widehat{w}}{\widehat{P}}$	$\bar{D}$	$\widehat{\mathbb{W}}$	$\frac{\widehat{w}}{\widehat{P}}$	$\bar{D}$	$\widehat{\mathbb{W}}$
AUS	0.11	-0.05	1.28	0.00	0.91	1.01	0.91	0.91	1.12	1.01	1.02	1.11	1.12
AUT	0.17	-0.13	1.36	-0.01	0.72	0.97	0.72	0.61	1.13	0.71	0.70	1.17	0.84
BHR	-0.03	-1.80	0.70	-0.64	0.33	0.84	0.33	0.35	0.81	0.34	0.41	0.80	0.39
BEL	0.11	-0.04	1.32	0.00	0.77	0.98	0.77	0.83	1.09	0.92	1.22	1.09	1.36
BRA	0.11	-0.06	1.28	0.00	0.90	1.00	0.90	0.92	1.10	1.02	0.99	1.10	1.09
CAN	0.10	-0.01	1.29	0.00	0.87	1.00	0.87	1.11	1.10	1.23	1.39	1.10	1.53
CHL	0.10	-0.10	1.30	0.00	0.75	0.97	0.75	0.74	1.07	0.82	0.78	1.06	0.86
CHN	0.23	-0.51	1.81	-0.14	0.82	0.97	0.82	0.84	1.20	1.04	1.03	1.24	1.33
CZE	0.27	-0.84	2.62	-0.29	0.55	0.96	0.55	0.50	1.24	0.65	0.74	1.44	1.10
DNK	0.12	-0.13	1.36	-0.01	0.80	0.94	0.80	0.62	1.06	0.70	0.73	1.07	0.83
FIN	0.12	-0.15	1.33	-0.02	0.91	0.98	0.91	0.71	1.09	0.79	0.77	1.09	0.86
FRA	0.11	-0.11	1.30	0.00	0.83	1.00	0.83	0.78	1.12	0.87	0.83	1.12	0.92
DEU	0.11	-0.16	1.38	-0.02	0.79	0.94	0.79	0.70	1.06	0.78	0.77	1.06	0.86
GRC	0.15	-0.17	1.34	-0.02	0.76	1.07	0.76	0.74	1.21	0.84	0.77	1.22	0.88
HKG	0.13	0.09	1.46	-0.01	5.34	0.95	5.34	8.68	1.07	9.84	16.04	1.08	18.38
HUN	0.23	-0.61	1.73	-0.15	0.47	0.98	0.47	0.39	1.21	0.48	0.54	1.37	0.75
IRL	0.09	-0.06	1.28	0.00	0.55	0.84	0.55	0.46	0.93	0.52	0.70	0.93	0.77
ISR	0.11	-0.06	1.30	0.00	0.83	1.00	0.83	0.79	1.10	0.88	0.95	1.10	1.05
ITA	0.12	-0.18	1.40	-0.02	0.83	0.99	0.83	0.79	1.11	0.89	0.82	1.12	0.93
JPN	0.11	-0.06	1.30	0.00	0.93	1.00	0.93	0.97	1.10	1.07	1.08	1.10	1.19
KOR	0.10	-0.09	1.29	0.00	0.76	0.97	0.76	0.71	1.07	0.79	0.79	1.07	0.88
KWT	0.05	-2.78	0.58	-1.17	0.61	0.74	0.61	0.61	0.79	0.65	0.61	0.78	0.64
LUX	0.07	0.01	1.26	0.00	2.13	0.71	2.13	2.02	0.78	2.23	3.08	0.77	3.37
MYS	0.24	-0.58	1.95	-0.17	0.40	0.85	0.40	0.46	1.09	0.59	0.70	1.11	0.90
MEX	0.11	-0.03	1.28	0.00	0.75	1.02	0.75	0.86	1.13	0.95	1.02	1.12	1.13
NLD	0.10	-0.05	1.31	0.00	0.71	0.91	0.71	0.77	1.02	0.86	1.16	1.02	1.29
NZL	0.11	-0.06	1.28	0.00	0.82	0.99	0.82	0.81	1.10	0.90	0.89	1.10	0.99
NOR	0.11	-0.13	1.39	-0.01	0.83	0.89	0.83	0.74	1.00	0.84	0.80	1.01	0.91
PHL	0.14	-0.14	1.34	-0.02	0.71	1.04	0.71	0.72	1.17	0.82	0.78	1.18	0.89
POL	0.16	-0.49	1.48	-0.10	0.66	1.01	0.66	0.63	1.17	0.74	0.65	1.18	0.76
PRT	0.13	-0.17	1.33	-0.02	0.72	1.04	0.72	0.70	1.17	0.78	0.73	1.17	0.82
QAT	0.24	-1.13	2.46	-0.30	0.62	0.70	0.62	0.62	0.94	0.83	0.62	0.96	0.85
ROU	0.23	-0.63	1.69	-0.15	0.70	1.06	0.70	0.69	1.29	0.85	0.72	1.31	0.89
RUS	0.25	-0.93	1.91	-0.23	0.81	0.91	0.81	0.81	1.17	1.03	0.81	1.16	1.04
SGP	0.08	0.05	1.45	-0.01	2.77	0.74	2.77	3.87	0.83	4.30	5.92	0.82	6.55
SVN	0.13	-0.20	1.34	-0.02	0.62	0.97	0.62	0.45	1.10	0.51	0.56	1.10	0.64
ESP	0.12	-0.19	1.37	-0.03	0.78	1.00	0.78	0.76	1.13	0.85	0.79	1.13	0.89
SWE	0.12	-0.26	1.42	-0.05	0.80	0.95	0.80	0.67	1.07	0.76	0.75	1.08	0.84
CHE	0.10	-0.06	1.27	0.00	0.82	0.91	0.82	0.71	1.01	0.78	0.95	1.01	1.06
ARE	0.02	-1.00	0.01	-0.31	0.35	0.81	0.35	0.44	0.82	0.45	0.56	0.83	0.57
GBR	0.11	-0.07	1.25	0.00	0.84	1.02	0.84	0.83	1.13	0.92	0.96	1.13	1.07
USA	0.11	-0.02	1.26	0.00	0.91	1.04	0.91	1.28	1.15	1.41	1.50	1.14	1.65
ZAF	0.17	-0.48	1.56	-0.12	0.78	1.00	0.78	0.78	1.17	0.91	0.80	1.16	0.93

Panels (I) and (II) report the model predictions from section 3.2 for counterfactual financial and real variables under universal higher tariffs $\hat{\tau} = 1.5$. Panel (I) presents the median changes of second-moment variables and asset positions corresponding to figure 2. Panel (II) presents the comparison of scenarios with fixed versus adjustable asset positions corresponding to figure 4. Panel (III) lists the changes of variables under calibrated tariffs to the trade war in section 4.2. For welfare analyses in panels (II) and (III), $\frac{\widehat{w}}{\widehat{P}}$ is the change to wage-to-price ratio, $\bar{D}$ is the counterfactual equilibrium asset position as shares of output, and $\widehat{\mathbb{W}}$ is the counterfactual welfare change relative to the original regime.

which, with the definition of hat variables B.1 and B.3, is re-written as

$$Y_i' = \widehat{w}_i \bar{w}_i \bar{L}_i + \widehat{r}_i \bar{r}_i \bar{K}_i = \widehat{w}_i (1-\mu)\bar{Y}_i + \widehat{w}_i \mu \bar{Y}_i. \tag{B.5}$$

Hence the change in income also equals that in wage

$$\widehat{Y}_i = \frac{Y_i'}{\bar{Y}_i} = \widehat{w}_i. \tag{B.6}$$

Now I characterize the changes to wage ($\widehat{w}_i$) and price ($\widehat{P}_i$) which, with minor modifications, can be applied to the analysis both within (equations 38 and 39) and across (equations 43 and 44) regimes. With equations 5 and 6, price in country i is given by

$$P_i'^{-\theta} = \Gamma^{-\theta} \sum_{j=1}^{I} T_j' \tau_{ji}'^{-\theta} (w_j'^{\eta\mu} r_j'^{\eta(1-\mu)} P_j'^{1-\eta})^{-\theta}, \tag{B.7}$$

Dividing this by the steady-state price while imposing B.3 yields the change of price

$$\widehat{P}_i^{-\theta} = \sum_{j=1}^{I} \bar{\pi}_{ji} \widehat{T}_j \widehat{\tau}_{ji}^{-\theta} (\widehat{w}_j^{\eta} \widehat{P}_j^{1-\eta})^{-\theta}. \tag{B.8}$$

Meanwhile, wage $\widehat{w}_j$ in the equation is derived from the goods market clearing condition. Plugging equation 5 in 7 gives

$$Y_i' = \sum_{j=1}^{I} \frac{T_i' [\tau_{ij}' (r_i'^{\mu} w_i'^{1-\mu})^{\eta} P_i'^{1-\eta}]^{-\theta}}{P_j'^{-\theta} / \Gamma^{-\theta}} X_j'. \tag{B.9}$$

In the steady state, a country's output, expenditure, asset positions are linked through

$$\bar{Y}_i \bar{D}_i (1-\bar{R}) = \bar{Y}_i \bar{D}_i (1 - \frac{1}{\beta}) = \bar{Y}_i - \bar{X}_i. \tag{B.10}$$

Combining equations B.3, B.6, B.8, and B.10 yields inter-regime changes of output

$$\widehat{w}_i \bar{Y}_i = \sum_{j=1}^{I} \frac{\bar{\pi}_{ij} \widehat{T}_i \widehat{\tau}_{ij}^{-\theta} (\widehat{w}_i^{\eta} \widehat{P}_i^{1-\eta})^{-\theta}}{\sum_{k=1}^{I} \bar{\pi}_{kj} \widehat{T}_k \widehat{\tau}_{kj}^{-\theta} (\widehat{w}_k^{\eta} \widehat{P}_k^{1-\eta})^{-\theta}} \widehat{w}_j \bar{Y}_j (1 - \bar{D}_j^{ctf} (1 - \frac{1}{\beta})). \tag{B.11}$$

Within a regime, we compute expenditure $X_{j,t+1}$ as the sum of intermediate input $(1-\eta) Y_{j,t+1}$ and consumption $P_{j,t+1} C_{j,t+1}$, where consumption will be derived from an intertemporal utility maximization with portfolio choice problem. The following derivation is modified from the two-country analysis by Devereux and Sutherland (2011). The first-order dynamics of the wealth constraints (equation 8) around a steady state in this model is

$$\widetilde{\mathcal{W}}_{i,t+1} = \frac{1}{\beta}\widetilde{\mathcal{W}}_{i,t} + \sum_{k=1}^{I-1}\frac{\bar{\alpha}_{ik}}{\beta\bar{Y}_i}(\widetilde{R}_{k,t+1} - \widetilde{R}_{I,t+1}) + \eta\widetilde{Y}_{i,t+1} - \eta\widetilde{P}_{i,t+1} - \eta\widetilde{C}_{i,t+1} + \mathcal{O}(\epsilon^2), \quad \text{(B.12)}$$

$$\widetilde{\mathcal{W}}_{I,t+1} = \frac{1}{\beta}\widetilde{\mathcal{W}}_{I,t} + \sum_{k=1}^{I-1}\frac{\bar{\alpha}_{Ik}}{\beta\bar{Y}_I}(\widetilde{R}_{k,t+1} - \widetilde{R}_{I,t+1}) + \eta\widetilde{Y}_{I,t+1} - \eta\widetilde{P}_{I,t+1} - \eta\widetilde{C}_{I,t+1} + \mathcal{O}(\epsilon^2), \quad \text{(B.13)}$$

where wealth is normalized by a country's output $\widetilde{\mathcal{W}}_{i,t} = (\mathcal{W}_{i,t} - \bar{\mathcal{W}}_i)/\bar{Y}_i$.

Taking the cross-country difference between equations B.12 and B.13, iterating forward over the infinite time horizon to sum up the discounted consumption flows, and imposing a transversality condition to drop $\widetilde{\mathcal{W}}_{i/I,t+\infty}$ yields

$$\begin{aligned}\sum_{s=0}^{\infty}\beta^s(\eta\widetilde{C}_{i/I,t+1+s}) = \frac{1}{\beta}\widetilde{\mathcal{W}}_{i/I,t} + \sum_{s=0}^{\infty}\beta^s[\eta\widetilde{Y}_{i/I,t+1+s} - \eta\widetilde{P}_{i/I,t+1+s} \\ + \sum_{k=1}^{I-1}(\frac{\bar{\alpha}_{ik}}{\beta\bar{Y}_i} - \frac{\bar{\alpha}_{Ik}}{\beta\bar{Y}_I})(\widetilde{R}_{k,t+1+s} - \widetilde{R}_{I,t+1+s})] + \mathcal{O}(\epsilon^2).\end{aligned} \quad \text{(B.14)}$$

If country-level productivity follows an AR(1) process as in a standard DSGE model

$$T_{i,t} = \rho T_{i,t-1} + (1-\rho)\bar{T}_i + \epsilon_{i,t}, \quad \text{(B.15)}$$

under households' consumption smoothing characterized by the Euler equation (13), the expected consumption differential between the two countries at $t+1$ follows

$$\widetilde{C}_{i/I,t+1} = \frac{1-\beta}{\beta\eta}\widetilde{\mathcal{W}}_{i/I,t} + \frac{1-\beta}{1-\rho\beta}\widetilde{Y}_{i/I,t+1} - \frac{1-\beta}{1-\rho\beta}\widetilde{P}_{i/I,t+1} + \frac{1-\beta}{1-\rho\beta}(\breve{\boldsymbol{\alpha}}_i - \breve{\boldsymbol{\alpha}}_I)\widetilde{R}_{x,t+1} + \mathcal{O}(\epsilon^2), \quad \text{(B.16)}$$

where $\breve{\alpha}$ denotes a country's vector of asset holdings scaled by its value-added output and discount factor

$$\breve{\boldsymbol{\alpha}}_i = [\breve{\alpha}_{i1}, \breve{\alpha}_{i2}, ..., \breve{\alpha}_{iI\text{-}1}], \qquad \text{where} \qquad \breve{\alpha}_{ik} = \frac{\bar{\alpha}_{ik}}{\beta\eta\bar{Y}_i}. \quad \text{(B.17)}$$

We can then derive expenditure (X'_j in B.9) for intra-regime analysis

$$\widetilde{X}_{j,t+1} = \frac{1-\eta}{1-\bar{D}_j(1-\frac{1}{\beta})}\widetilde{Y}_{j,t+1} + \frac{\eta-\bar{D}_j(1-\frac{1}{\beta})}{1-\bar{D}_j(1-\frac{1}{\beta})}(\widetilde{P}_{j,t+1} + \widetilde{C}_{j,t+1}) \quad \text{(B.18)}$$

If there is no productivity persistence ($\rho = 0$), plugging the consumption differential from B.16 in the second-order approximation of 24 yields the portfolio determination equation:

$$E_t[(\gamma(1-\beta)\widetilde{Y}_{i/I,t+1} + (1-\gamma+\beta\gamma)\widetilde{P}_{i/I,t+1} + \gamma(1-\beta)(\breve{\boldsymbol{\alpha}}_i - \breve{\boldsymbol{\alpha}}_I)\widetilde{R}_{x,t+1})\widetilde{R}'_{x,t+1}] = \frac{1}{2}F_{iI}. \quad \text{(B.19)}$$

In this equation, asset returns $R_{i,t}$ are determined by dividends $d_{i,t}$ and prices $q_{i,t}$:

$$E_t(R_{i,t+1}) = E_t(\frac{d_{i,t+1} + q_{i,t+1}}{q_{i,t}}). \tag{B.20}$$

Log-linearizing the equation yields the changes to returns around a steady state

$$E_t(\widetilde{R}_{i,t+1}) = (1-\beta)E_t(\widetilde{d}_{i,t+1}) + \beta E_t(\widetilde{q}_{i,t+1}) - \widetilde{q}_{i,t}. \tag{B.21}$$

To the first order approximation, excess return of any asset i relative to I should be 0:

$$E_t(\widetilde{R}_{i/I,t+1}) = (1-\beta)E_t(\widetilde{d}_{i/I,t+1}) + \beta E_t(\widetilde{q}_{i/I,t+1}) - \widetilde{q}_{i/I,t} = 0. \tag{B.22}$$

Relative asset price in the following period is then given by

$$E_t(\widetilde{q}_{i/I,t+1}) = \frac{1}{\beta}\widetilde{q}_{i/I,t} - \frac{1-\beta}{\beta}E_t(\widetilde{d}_{i/I,t+1}). \tag{B.23}$$

Iterating forward to the next period

$$\begin{aligned} E_t(\widetilde{q}_{i/I,t+2}) &= \frac{1}{\beta}E_t(\widetilde{q}_{i/I,t+1}) - \frac{1-\beta}{\beta}\widetilde{E}_t(d_{i/I,t+2}) \\ &= \frac{1}{\beta}(\frac{1}{\beta}\widetilde{q}_{i/I,t} - \frac{1-\beta}{\beta}E_t(\widetilde{d}_{i/I,t+1})) - \frac{1-\beta}{\beta}E_t(\widetilde{d}_{i/I,t+2}). \end{aligned} \tag{B.24}$$

More generally, the asset price in any period s is

$$E_t(\widetilde{q}_{i/I,t+s}) = \frac{1}{\beta^s}\widetilde{q}_{i/I,t} - \frac{1-\beta}{\beta}\sum_{u=1}^{s}(\frac{1}{\beta})^{s-u}E_t(\widetilde{d}_{i/I,t+u}). \tag{B.25}$$

Hence the current price is the expected present value of dividends when s goes to infinity

$$\widetilde{q}_{i/I,t} = \beta^s E_t(\widetilde{q}_{i/I,t+s}) + \frac{1-\beta}{\beta}\sum_{u=1}^{s}\beta^u E_t(\widetilde{d}_{i/I,t+u}). \tag{B.26}$$

When dividends follow an AR(1) process with persistence ρ

$$E_t(\widetilde{d}_{i/I,t+1}) = \rho\, \widetilde{d}_{i/I,t}, \tag{B.27}$$

the present value of dividends is proportional to the current dividend

$$\widetilde{q}_{i/I,t} = \frac{1-\beta}{\beta}\sum_{u=1}^{s}\beta^u E_t(\widetilde{d}_{i/I,t+u}) = \frac{\rho(1-\beta)}{\beta(1-\beta\rho)}\widetilde{d}_{i/I,t}. \tag{B.28}$$

The expected change of price is

$$E_t(\widetilde{q}_{i/I,t+1}) - \widetilde{q}_{i/I,t} = \frac{\rho(1-\beta)(\rho-1)}{\beta(1-\beta\rho)}\widetilde{d}_{i/I,t}. \tag{B.29}$$

In the case with no persistence ($\rho = 0$), capital gains will be zero so that excess asset returns are determined by relative dividends, which are proportional to relative output.

B.2 Existence and Uniqueness of Solution

Dekle et al. (2007) (DEK hereforth) follow the theorems of Alvarez and Lucas (2007) to establish the existence and uniqueness of the model solution to a fixed-point problem of $(\widehat{\bar{w}}, \widehat{\bar{P}})$ given counterfactual asset positions.[29] Many properties of their numerical solutions are maintained under the assumptions specified in this model, for example there exists a unique price vector $\widehat{P}^s$ within a regime given the corresponding wage vector $\widehat{w}^s$. After solving the portfolio choice problem by evaluating the second-moment variables as functions of $\widehat{w}^s$ in each regime $s \in \{org, ctf\}$, the resulting portfolio $\bar{D}^{ctf}$ is then used to characterize an excess demand system across regimes

$$\mathbb{Z}_i(\widehat{\bar{w}}_i) = \frac{1}{\widehat{\bar{w}}}[\widehat{\bar{w}}_i \bar{Y}_i^{org} - \sum_{j=1}^{I} \frac{\bar{\pi}_{ij}^{org} \widehat{\tau}_{ij}^{-\theta} (\widehat{\bar{w}}_i^{\eta} \widehat{\bar{P}}_i^{1-\eta})^{-\theta}}{\sum_{k=1}^{I} \bar{\pi}_{kj}^{org} \widehat{\tau}_{kj}^{-\theta} (\widehat{\bar{w}}_k^{\eta} \widehat{\bar{P}}_k^{1-\eta})^{-\theta}} \widehat{\bar{w}}_j \bar{Y}_j^{org} \bar{D}^{ctf}]. \tag{B.30}$$

$\bar{D}^{ctf}$ appears as a finite multiplier and hence does not change most properties described in footnote 29 of $\widehat{\bar{w}}$ computed as the solution to equation B.30.[30] For example, under the world resource constraint

$$\sum_{i=1}^{I} \widehat{\bar{w}} \bar{Y}_i^{org} = \sum_{i=1}^{I} \widehat{\bar{w}} \bar{Y}_i^{org} \bar{D}^{ctf}, \tag{B.31}$$

Walras's Law is satisfied:

$$\sum_{i=1}^{I} \widehat{\bar{w}}_i \mathbb{Z}_i(\widehat{\bar{w}}) = \sum_{i=1}^{I} (\widehat{\bar{w}}_i \bar{Y}_i^{org} - \sum_{j=1}^{I} \bar{\pi}_{ij}^{ctf} \widehat{\bar{w}}_j \bar{Y}_j^{org} \bar{D}^{ctf}) = 0, \tag{B.32}$$

which is necessary to establish the existence of the solution to inter-regime changes. The solution will characterize the counterfactual outcome $(\widehat{\bar{w}}, \widehat{\bar{P}}, \Delta\bar{D})$ which encompasses both

[29] Alvarez and Lucas (2007) show that under the assumptions that $\eta < 1$, $1 + \theta(1-\epsilon) > 0$, $\tau_{ij} \geq 1$, a unique solution to counterfactual w exists to ensure zero excess demand (denoted as $\mathbb{Z}_i(w)$ in equation B.30 for the level of wage) in the commodity market. They prove these theorems by showing that $\mathbb{Z}_i(w)$ is continuous, homogenous of degree zero, has the gross substitute property $\frac{\partial \mathbb{Z}_i(w)}{\partial w_j} > 0$, satisfies Walras's Law ($\sum_i w_i \mathbb{Z}_i(w) = 0$), faces a lower but not upper bound $\mathbb{Z}_i(w) > -\max_j L_j, \max_i \mathbb{Z}_i(w \to w^{org}) \to \infty$.

[30] Certain properties such as continuity do change if $\bar{D}^{ctf}$ appears as a multiplicative instead of an additive term as in DEK. In most computational exercises, I find the iteration process to monotonically converge to an accurate result just like DEK, especially under tariff changes only. But under very large financial friction changes, it takes more effort to converge to a specific solution for $\widehat{\bar{w}}$. Despite this, the convergence is good and close to 1e-12 which is the default tolerance level used in the code.

real and financial sides of the economy.

B.3 Algorithm

Step 1. Collect data to calibrate the original steady state of the economy

Obtain the timed-averaged country-level GDP and NFA, and bilateral trade shares and bilateral portfolio weights to calibrate the steady state of the original regime.

Step 2. Form initial guesses about inter-regime changes[31]

Start with the guess that original and counterfactual regimes have the same steady-state values for wage, price, and asset positions

$$\hat{\bar{w}}^0 = \hat{\bar{P}}^0 = ones(I,1), \ \Delta\bar{D}^0 = zeros(I,1) \tag{B.33}$$

Step 3. Characterize intra-regime changes[32]

Simulate calibrated productivity shocks and solve for the responses of $\widehat{w}^s, \widehat{P}^s$ with 43 and 44 around original and counterfactual steady states respectively ($s \in \{org, ctf\}$). Calculate other variables including $\widehat{Y}^s, \widehat{R}^s$ as functions of the solved $\widehat{w}^s, \widehat{P}^s$ and these variables' products (listed in 33) in response to the simulated productivity shocks and take the mean values across simulations to get the second-moment variables.

Step 4. Solve the portfolio choice problem

Use the second-moment variables from step 3 in the original and counterfactual portfolio determination equations (30 and 31) to yield

$$E_t[(\gamma(1-\beta)\widetilde{Y}\widetilde{R}'^s + (1-\gamma+\beta\gamma)\widetilde{P}\widetilde{R}'^s + \gamma(1-\beta)\breve{\alpha}^s\widetilde{R}\widetilde{R}'^s] = \frac{1}{2}F^s, \ s \in \{org, ctf\}. \tag{B.34}$$

where F^s and $\breve{\alpha}^s$ stand for the vectors of all the countries' relative financial frictions and asset holdings (defined by 25 and 29):

$$F^{s'} = [F^s_{1I}, F^s_{2I}, ..., F^s_{I-1I}], \qquad \breve{\alpha}^{s'} = [\breve{\alpha}^s_1, \breve{\alpha}^s_2, ..., \breve{\alpha}^s_{I-1}] - \breve{\alpha}^s_I. \tag{B.35}$$

[31] The values used for the initial guesses do not matter significantly due to the existence and uniqueness of the model solution discussed earlier. Therefore, the iterative computation procedure described in the algorithm will update ($\hat{\bar{w}}, \hat{\bar{P}}, \Delta\bar{D}$) to reach the unique solution from alternative initial guesses.

[32] The purpose of this step is to calculate second-moment variables that appear in portfolio determination equations (30 and 31). To do the calculation, we need the percentage deviation of variables from a steady state under the shocks of the economy. For this intra-regime analysis solved by hat algebra, the initial levels of output and trade shares are set as their steady-state values $\bar{Y}^s, \bar{\pi}^s_{ij}$. The original steady state is from step 1, and the counterfactual steady state is predicted by inter-regime changes ($\hat{\bar{w}}$ and the corresponding $\hat{\bar{P}}$ that satisfies 38) imposed on the original steady state.

$\widetilde{Y}\widetilde{R}'^s$, $\widetilde{P}\widetilde{R}'^s$, and $\widetilde{R}\widetilde{R}'^s$ represent the second-moment variables listed in 33 for brevity. Take the difference of equation B.34 between the two regimes to determine the shift of bilateral asset holdings driven by second-moment variables including potential changes to financial frictions. Numeraire country I's holding will be derived, after other countries' holdings relative to it are solved, to satisfy the assets' market clearing condition that asset supply (whose increase is proportional to a country's output increase) equals all the countries' total holdings of the asset. Add up a country's bilateral holdings to yield its aggregate asset positions, which will update $\Delta\bar{D}^0$ to $\Delta\bar{D}^1$.

Step 5. Update Inter-regime changes given the solved portfolio

Update inter-regime changes of $\widehat{\bar{w}}$ using the portfolio $\Delta\bar{D}$ from step 4. This involves employing an iterative computation procedure similar to that described by Alvarez and Lucas (2007) and DEK with function M, which denotes the mapping of $\widehat{\bar{w}}^0$ to $\widehat{\bar{w}}^1$ with a constant $\nu \in (0,1)$

$$\widehat{\bar{w}}^1 = M(\widehat{\bar{w}}^0) = \widehat{\bar{w}}^0(1 + \nu\frac{\mathbb{Z}_i(\widehat{\bar{w}}^0)}{\bar{Y}_i^{org}}), \tag{B.36}$$

and excess demand $\mathbb{Z}_i(\widehat{\bar{w}})$

$$\mathbb{Z}_i(\widehat{\bar{w}}_i) = \frac{1}{\widehat{\bar{w}}}[\widehat{\bar{w}}_i\bar{Y}_i^{org} - \sum_{j=1}^{I}\frac{\bar{\pi}_{ij}^{org}\widehat{\tau}_{ij}^{-\theta}(\widehat{\bar{w}}_i^{\eta}\widehat{\bar{P}}_i^{1-\eta})^{-\theta}}{\sum_{k=1}^{I}\bar{\pi}_{kj}^{org}\widehat{\tau}_{kj}^{-\theta}(\widehat{\bar{w}}_k^{\eta}\widehat{\bar{P}}_k^{1-\eta})^{-\theta}}\widehat{\bar{w}}_j\bar{Y}_j^{org}\bar{D}^{ctf}]. \tag{B.37}$$

Note that the mapping is bounded by one under Walras's Law (equation B.32)

$$\sum_{i=1}^{I}M(\widehat{\bar{w}}_i^0)\bar{Y}_i^{org} = \sum_{i=1}^{I}\widehat{\bar{w}}_i^0(1+\nu\frac{\mathbb{Z}_i(\widehat{\bar{w}}_i^0)}{\bar{Y}_i^{org}})\bar{Y}_i^{org} = \sum_{i=1}^{I}\widehat{\bar{w}}^0\bar{Y}_i^{org} + \nu\sum_{i=1}^{I}\widehat{\bar{w}}_i^0\mathbb{Z}_i(\widehat{\bar{w}}^0) = 1, \tag{B.38}$$

and the normalization condition that treats the world output as a numeraire

$$\sum_{i=1}^{I}\widehat{\bar{w}}\bar{Y}_i^{org} = 1. \tag{B.39}$$

Step 6. Repeat steps 3-5 until convergence

Use the updated $\widehat{\bar{w}}^1$ from step 5 and $\Delta\bar{D}^1$ from step 4 as new guesses, and repeat the procedures from step 3 to 5 for both inter-regime and intra-regime analyses to reach new updated $\widehat{\bar{w}}^2, \widehat{\bar{P}}^2$ and $\Delta\bar{D}^2$. This continues until the difference between the k^{th} and the $k{+}1^{th}$ iteration $\mid \widehat{\bar{w}}^{k+1} - \widehat{\bar{w}}^k \mid, \mid \widehat{\bar{P}}^{k+1} - \widehat{\bar{P}}^k \mid, \mid \Delta\bar{D}^{k+1} - \Delta\bar{D}^k \mid$ is sufficiently small, which solves the joint fixed-point problem of $(\widehat{\bar{w}}, \widehat{\bar{P}}, \Delta\bar{D})$ necessary to characterize counterfactual outcomes under alternative tariffs or financial frictions.

C Data and Calibration

This section describes the data source and calibration strategy for both the real and financial sides of the economy. The sample of economies includes 43 countries (listed in table A.1) and the rest of the world (ROW). The time-averaged values of variables over the sample period from 2001-2021 will be used as their original steady-state values, including countries' output obtained from the Penn World Table (PWT) and net foreign asset positions from the World Bank. The values of the ROW's variables are the difference between the world aggregate values and those of the countries in the sample.

C.1 Bilateral Trade and Financial Shares

Cross-country trade data are obtained from the Direction of Trade Statistics (DOTS) compiled by the IMF. I use the bilateral import (CIF) data to calculate a country's spending on goods sourced from other countries. A country's spending on its own goods is computed as the difference between its gross expenditure and total imports, both available from the World Development Indicators (WDI) compiled by the World Bank.

Financial data are sourced from Factset/Lionshare, a dataset that provides information on institutional investors' asset holdings. It has comprehensive coverage of institutional holdings across countries. I describe its details in Hu (2022) and its consistency in terms of portfolio composition with macro-level datasets such as IMF's International Financial Statistics. Factset/Lionshare compiles financial investment by investors' origin and their investment destination including for domestic assets, using which I calculate bilateral portfolio weights directly. Ideally, bilateral holdings should include all forms of capital, such as equity, debt, derivatives, and FDI. However, such comprehensive cross-country financial datasets are scarce. Another popular data source for the purpose is the Coordinated Portfolio Investment Survey (CPIS) and Coordinated Direct Investment Survey (CDIS). I look into these data and find their coverage to be much smaller than Factset/Lionshare's especially for non-OECD countries. Meanwhile, their methodology documentation states that data construction involves much imputation based on the information provided by reporting countries (normally asset holders). This may have caused data anomaly such as negative assets or liabilities which is difficult to interpret and treat properly, as excluding all the extreme values makes the matrix of bilateral portfolio weights even more sparse. For these reasons, I use Factset/Lionshare as the data source for the calibration of portfolio weights, knowing it is not perfect either.

As the analysis in this paper covers two channels, the sample of countries includes those with trade and finance data both available (see table A.1).

C.2 Productivity

The estimation of productivity consistent with the Eaton and Kortum (2002) model is modified from the approach developed by Levchenko and Zhang (2014), who infer Ricardian productivity from bilateral trade data.

Let country i's production cost be denoted as

$$c_{i,t} = (r_{i,t}^{\mu} w_{i,t}^{1-\mu})^{\eta} P_{i,t}^{1-\eta}. \tag{C.1}$$

It follows from equation 5 that trade shares for any destination country j should satisfy

$$\frac{\pi_{ij,t}}{\pi_{jj,t}} = \frac{T_{i,t}}{T_{j,t}} (\frac{\tau_{ij,t} c_{i,t}}{c_{j,t}})^{-\theta}. \tag{C.2}$$

As the left hand side is directly observable from the trade data, we can recover relative productivity $\frac{T_{i,t}}{T_{j,t}}$ after estimating bilateral trade friction $\tau_{ij,t}$ and relative input cost $\frac{c_{i,t}}{c_{j,t}}$.

I follow the trade literature by estimating bilateral trade costs $\hat{\tau}_{ij,t}$ from a combination of gravity variables including geographic distance divided into intervals set by Eaton and Kortum (2002), dummies for contiguity, common language, common colonizer, common religion, common legal system, and regional trade agreements. These gravity variables are sourced from the CEPII.

I estimate a country's production cost (denoted as $\hat{c}_{i,t}$) based on the information from the PWT. Specifically, I compute a country's wage (w) as the ratio of its total labor compensation (output-side GDP ($rgdpo$) $\times$ share of labor compensation in GDP ($labsh$)) to total labor hours (number of employees (emp) $\times$ average hours per employee (avc)). Price of domestic absorption (pl_{da}) and price of capital services (pl_k) are used as the proxies for the price of intermediate inputs and capital rental fee respectively. Besides, I calibrate the share of intermediate input in production $\eta = .312$ based on Dekle et al. (2007) and the share of labor input $1-\mu$ as country-specific $labsh$ from the PWT. The production cost of ROW is calculated as the median cost across countries not included in table A.1.

The full estimating specification for all the country pairs in the sample follows

$$\ln(\frac{\pi_{ij,t}}{\pi_{jj,t}}) = \ln(T_{i,t}\hat{c}_{i,t}^{-\theta}) - \ln(T_{j,t}\hat{c}_{j,t}^{-\theta}) - \theta\hat{\tau}_{ij,t} + \gamma_{ij,t}, \tag{C.3}$$

The first two terms on the right $\ln(T_{i,t}\hat{c}_{i,t}^{-\theta})$ and $\ln(T_{j,t}\hat{c}_{j,t}^{-\theta})$ can be captured by the exporter and importer fixed effects respectively when running the estimation. $\hat{\tau}_{ij,t}$ represents the estimated bilateral trade costs as a linear combination of the gravity variables described above and $\gamma_{ij,t}$ stands for error terms. Exponentiating the importer fixed effects yields a term that combines country j's productivity and cost denoted as

$$\widehat{Tc}_{j,t} = T_{j,t}\hat{c}_{j,t}^{-\theta}. \tag{C.4}$$

If the US is the benchmark country whose productivity ($T_{US,t}$) is its TFP value from the PWT ($rtfpna$). Then other countries' Ricardian productivity can be calculated as

$$T_{j,t} = T_{US,t} \frac{\widehat{Tc}_{j,t}}{\widehat{Tc}_{US,t}} (\frac{\hat{c}_{j,t}}{\hat{c}_{US,t}})^{\theta}, \tag{C.5}$$

where trade elasticity $\theta = 4$ following Simonovska and Waugh (2014). After calculating countries' dynamic productivity $T_{j,t}$, I compute its mean value over time $\bar{T}$ and the cross-country covariance matrix Σ_T.