STAGFLATION AND TOPSY-TURVY CAPITAL FLOWS*

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Abstract

Are unregulated capital flows excessive during a stagflation episode? We argue that they likely are, owing to a macroeconomic externality operating through the economy's supply side. Inflows raise domestic wages through a wealth effect on labor supply and cause unwelcome upward pressure on marginal costs in countries where monetary policy is trying to drive down costs to stabilize inflation. Yet, market forces are likely to generate such inflows. Optimal capital flow management instead requires net outflows, suggesting topsy-turvy capital flows following markup shocks.

Keywords: Stagflation, current account adjustment, macroeconomic externalities, stabilization policy, capital flow management

JEL Classifications: E32, E44, E52, F32, F41, F42

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1 Introduction

One of the most striking macroeconomic developments of the current recovery is the recent pickup in inflation, shown in the left panel of Figure 1 for G7 economies. To prevent inflation from settling at high levels, major central banks have engaged in their most aggressive tightening cycle in decades, as shown in the figure's right panel. This rapid increase in interest rates across advanced economies is raising the specter of a new "taper tantrum," by which global investors' search for yield leads to large capital outflows from emerging economies. Should we be concerned about these capital outflows, and the corresponding capital inflows into economies with rapidly tightening policy stances, being excessive? Are the rising odds of stagflation critical for this assessment? Over the past twenty years, a large body of research in macroeconomic theory has pointed to imperfections in financial, goods, and labor markets as possible causes of excessive capital flows (e.g., Bianchi 2011 and Schmitt-Grohe and Uribe 2016). Yet, perhaps because adverse supply shocks have been off policymakers' radar for much of this period, the literature has largely ignored the role of the trade-off between stabilizing inflation and promoting economic activity, which appears central in the current context. Our goal in this paper is to focus squarely on this issue.

We point to a previously undocumented macroeconomic externality associated with capital flows and operating through the economy's supply side: by propping up consumption, capital inflows shift up households' labor supply schedule, and as long as the trade elasticity is greater than the degree of home bias, they raise domestic firms' marginal costs.[1] When the economy operates at potential and inflation is perfectly stabilized, this externality does not cause any inefficiency. But when the economy operates below potential as a result of the central bank's attempt to fight off a markup shock (i.e., in a stagflation scenario), the rise in marginal costs worsens the policy trade-off: to stabilize inflation at a given level, the central bank needs to engineer a more severe recession. In this context, the macroeconomic externality generates first-order welfare effects and creates a wedge between the privately and socially optimal levels of external borrowing.

We formalize this insight in a simple two-country general equilibrium model with nominal rigidities, whose building blocks form the backbone of more complex dynamic stochastic general equilibrium models used by most central banks for policy analysis.

[1]This condition on the trade elasticity is weaker than the well-known Marshall-Lerner condition, which states that the sum of a country's export and import demand elasticities, commonly defined as the trade elasticity, is greater than one. When this condition is satisfied, an exchange rate depreciation improves the trade balance.

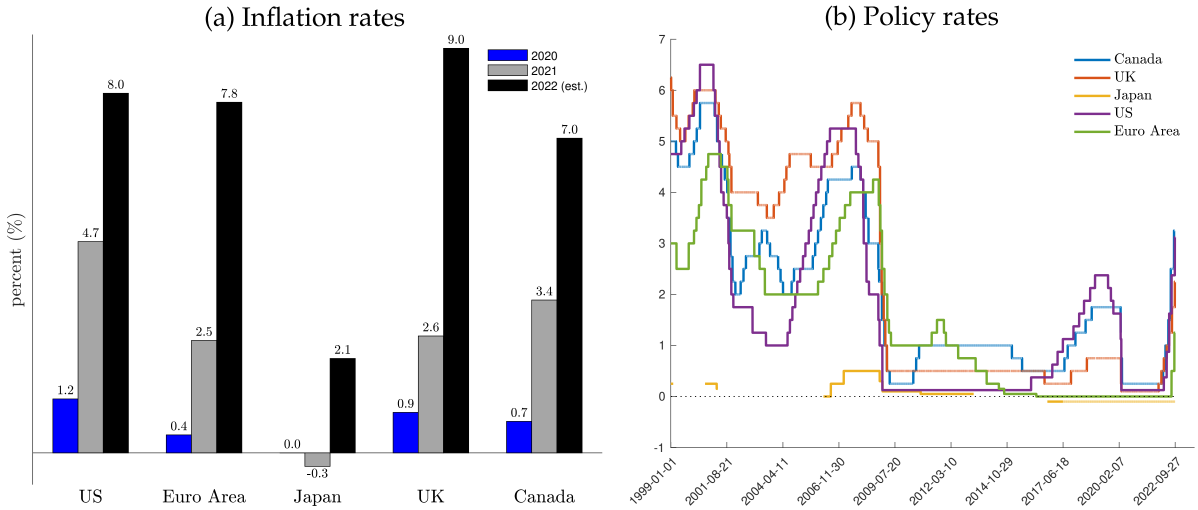

Figure 1: Inflation and policy rates in G7 countries.

Note: Left panel shows annual CPI inflation rates. Data from 2020 and 2021 are from the World Bank's Development Indicators, and 2022 data are from Bloomberg consensus forecasts. Right panel shows daily policy rates from the BIS. See Appendix A for details.

We find that under free capital mobility, capital tends to flow excessively toward the country where the degree of stringency of the output-inflation trade-off is the highest, owing to the externality described above.[2] Firms' marginal costs can be decomposed into a pure labor cost component, equal to the real wage measured in terms of the economy's consumption basket, and an adjustment term accounting for the economy's relative purchasing power. For a given output gap, a marginal increase in external borrowing raises domestic spending, shifting up the labor supply schedule and causing a rise in the equilibrium real wage. When the model features home bias in consumption, the increase in domestic spending also appreciates the terms of trade. In turn, the appreciation of the terms of trade raises the purchasing power of domestic firms and attenuates the rise in their marginal costs. As long as the trade elasticity is larger than the degree of home bias, the direct effect on labor costs outweighs the latter effect from the purchasing power, thereby leading to an overall increase in firms' marginal costs. Hence, in times when monetary policy adopts a particularly tight stance to limit domestic inflation, the upward pressure on domestic marginal costs caused by additional capital inflows makes monetary policy's job even harder. Either the central bank lets the rise in marginal costs translate into higher domestic inflation, or it is forced to depress economic activity further to achieve a given stabilization of inflation. Either way, the economy is worse off, and this adverse side

[2] In our model, such cross-country differences in the stringency of output-inflation trade-offs are the result of asymmetric markup shocks, but more generally, they could arise from any structural asymmetry across countries.

effect of external borrowing is not adequately signaled to domestic agents by its price in an unregulated market.

The externality we point to does not simply lead to inefficiencies at the margin. Indeed, it can be powerful enough to reverse the direction of capital flows in response to markup shocks. While a free capital mobility regime is likely to feature capital inflows into the region with the most depressed activity, an optimal capital flow management regime always prescribes outflows from that region. Our analysis hence suggests that ostensibly wrong price signals in international financial markets can lead to *tospy-turvy* capital flows during a stagflation episode.

Going beyond describing optimal capital flow management policy in target form and contrasting the behavior of the trade balance under alternative capital account regimes, we characterize the world economy's adjustment to an unanticipated markup shock. We show that in response to such a shock, apart from knife-edge cases, fluctuations in the cross-country differences in inflation and the output gap are smaller when capital flows are managed optimally than when capital flows freely across countries. Calibrating model parameters to values from the literature, we further find that countries experiencing a mean-reverting markup shock face fluctuations in output and inflation that are nearly 30% smaller when capital flows are managed optimally. Fluctuations in output and inflation in the rest of the world, which are an order of magnitude smaller, are meanwhile reduced by a factor of five.

Our externality resembles those stressed by two branches of the recent literature in financial, monetary and international economics. In the first one, elegantly formulated in general terms by Farhi and Werning (2016), privately optimal financial choices differ from socially optimal ones because of aggregate demand externalities in economies with nominal rigidities.[3] In the second one, summarized by Davila and Korinek (2017), pecuniary externalities generate inefficiencies in incomplete markets environments.[4] Our externality occurs in a setting of complete financial markets and aggregate demand imbalances, as aggregate demand externalities do, but works through prices, as like pecuniary externali-

[3]See also Farhi and Werning (2012, 2014, 2017), Korinek and Simsek (2016), Schmitt-Grohe and Uribe (2016), Acharya and Bengui (2018), Fornaro and Romei (2019) and Bianchi and Coulibaly (2021).

[4]For earlier articulations of these ideas in the information economics and general equilibrium literatures, see, e.g., Stiglitz (1982), Greenwald and Stiglitz (1986) and Geanakoplos and Polemarchakis (1986). In financial economics, see, e.g., Gromb and Vayanos (2002) and Lorenzoni (2008). In international macroeconomics, see Caballero and Krishnamurthy (2001), Korinek (2007, 2018), Bianchi (2011), Jeanne and Korinek (2010, 2019, 2020), Benigno, Chen, Otrok, Rebucci and Young (2013, 2016), Bengui (2014) and Bianchi and Mendoza (2018). Also, see Coulibaly (2020) and Ottonello (2021) for examples of studies combining pecuniary externalities that matter owing to financial frictions with aggregate demand externalities arising from nominal rigidities.

ties do. The fact that the externality we point to works through prices rather than quantity adjustments is most obvious in a special case of our model featuring no home bias in consumption. In this case, marginal costs respond to external borrowing only to the extent that the latter affects the real wage via a wealth effect on labor supply. Were we to prevent this price from adjusting by assuming fully rigid nominal wages, the externality would disappear, and the free capital mobility regime would become constrained efficient.[5]

The contrast between the macroeconomic externality we focus on and the aggregate demand externalities studied by Farhi and Werning (2016) and others goes beyond simple semantics. When aggregate demand externalities cause inefficiencies in contexts in which constraints on price adjustment and monetary policy prevent goods-specific labor wages from being closed, the general policy prescription is to incentivize agents to shift wealth toward states of nature in which their spending on goods whose provision is most depressed is relatively high. Boosting spending on these goods is something monetary policy would like to achieve but is unable to owing to constraints such as a fixed exchange rate (Farhi and Werning 2012, 2017, Schmitt-Grohe and Uribe 2016) or a zero lower bound (Farhi and Werning 2016, Korinek and Simsek 2016). This general principle does not apply in our context, in which it is usually optimal to tilt spending *away* from the country whose output gap is the most negative. Indeed, in our model, inefficiencies of financial decisions arise even when marginal propensities to consume (MPCs) are identical across agents. Instead of being designed to redirect spending toward relatively more depressed goods, in our setup financial market interventions are motivated by a desire to shift spending away from where it worsens the most unfavorable output-inflation trade-offs through supply-side channels. Our paper hence complements the aggregate demand externality literature by providing an insight specific to circumstances in which, as is the case at the current juncture, central banks may not be able to limit inflation without causing economic slowdowns.

Our paper relates to two further pieces of recent work. First, in contemporaneous and independent work, Cho, Kim and Kim (2021) show numerically that welfare under autarky might be higher than welfare under complete markets in a New Keynesian model with markup shocks. In this paper, in contrast, we formally identify the underlying externality and solve for the constrained efficient capital flow regime that accounts for it. Doing so enables us to analytically characterize cases in which capital flows in the wrong direction under free capital mobility and, consequently, rationalize why fully closing capital accounts

[5]We thank Anton Korinek and Ivan Werning for enlightening discussions on the distinction between our externality and both the aggregate demand externalities and pecuniary externalities emphasized in the literature.

may raise welfare in such cases. Second, Fornaro and Romei (2022) argue that national monetary policies may be excessively tight in response to shocks that reallocate demand toward tradable goods. Like us, they study the international ramifications of the ongoing recovery for policy. But they focus on monetary policy spillovers, while we emphasize an externality associated with private capital flows under cooperative monetary policy.

The remainder of the paper is organized as follows. Section 2 presents the model. Section 3 analyzes optimal monetary and capital flow management policy. Section 4 studies the interaction between capital flows and other macro variables in a stagflation episode. Section 5 discusses possible extensions of our analysis, and Section 6 concludes.

2 Model

The world is composed of two countries of equal size, Home and Foreign. In each country, households consume goods and supply labor, while firms hire labor to produce output. Variables pertaining to Foreign are denoted by asterisks.

2.1 Households

The Home country is populated by a continuum of households indexed by $h \in [0,1]$, each maximizing

$$\int_0^\infty e^{-\rho t}\left[\log C_t(h) - \frac{N_t(h)^{1+\phi}}{1+\phi}\right] dt, \tag{1}$$

where $C_t(h)$ is a consumption aggregate, $N_t(h)$ is labor supply, ϕ is the inverse Frisch elasticity of labor supply, and ρ is the discount rate.[6] The consumption aggregate $C_t(h)$ is defined as

$$C_t(h) \equiv \left[(1-\alpha)^{\frac{1}{\eta}}\left(C_{H,t}(h)\right)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}}\left(C_{F,t}(h)\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}},$$

where $C_{H,t}(h)$ and $C_{F,t}(h)$ are themselves CES aggregates over a continuum of goods produced respectively in Home and Foreign, with elasticity of substitution between varieties produced within a country equal to $\varepsilon > 1$. The elasticity of substitution between domestic and foreign goods is $\eta > 0$ and $\alpha \in (0, 1/2]$ is a home bias parameter capturing the degree of trade openness. When $\alpha = 1/2$, there is no home bias. In contrast, when $\alpha < 1/2$, households' preferences are biased toward domestically produced goods. Therefore, $1 - 2\alpha$

[6] Our exposition focuses on Home's representative household, but the environment faced by Foreign's representative household is symmetric.

summarizes the degree of home bias.

Households can trade two types of nominal bonds: an international bond, traded internationally, and a domestic bond, traded only domestically. Domestic bonds are denominated in domestic currency, while the international bond is (arbitrarily) denominated in Home's currency (without loss of generality given perfect foresight).

The Home household's budget constraint is given by

$$\dot{D}_t(h) + \dot{B}_t(h) = i_t D_t(h) + i_{B,t} B_t(h) + W_t(h) N_t(h) + \Pi_t \\ - \int_0^1 P_{H,t}(l)\, C_{H,t}(h,l) dl - \int_0^1 P_{F,t}(l) C_{F,t}(h,l) dl,$$

where $D_t(h)$ is domestic bond holdings, $B_t(h)$ is international bond holdings, i_t denotes the return on Home bonds, $i_{B,t}$ denotes the return on the international bond for Home households, $C_{H,t}(h,l)$ is its consumption of good l produced domestically, $C_{F,t}(h,l)$ its consumption of imported good l, $P_{H,t}(l)$ is the price of the good l produced domestically, $P_{F,t}(l)$ is the price of imported good l, and $W_t(h)$ is household h's nominal wage.

Each household h is a monopolistically competitive supplier of its labor services and faces a CES demand function of $N_t(h) = (W_t(h)/W_t)^{-\varepsilon_t^w} N_t$, where ε_t^w is the elasticity of substitution among labor varieties, which is the same across households but may vary over time, $W_t = \int_0^1 \left(W_t(h)^{1-\varepsilon_t^w} \right)^{1/(1-\varepsilon_t^w)}$ is the relevant (domestic) aggregate wage index, and N_t is aggregate employment. Wages are fully flexible and can be set at every instant. The household's optimal wage setting results in a wage markup over the marginal disutility of working per unit of consumption,

$$\frac{W_t(h)}{P_t} = \mu_t^w C_t(h) N_t(h)^{\phi}, \tag{2}$$

where $\mu_t^w \equiv \varepsilon_t^w / (\varepsilon_t^w - 1)$ is the gross wage markup. Variations in wage markups are the source of markup shocks that will give rise to a trade-off between stabilizing economic activity and inflation (see e.g., Clarida, Gali and Gertler 2002, Engel 2011 and Groll and Monacelli 2020).

In addition to their labor supply, households choose consumption and bond holdings to maximize utility. Because all Home households are identical, we can drop the index for the household, and the optimality conditions for domestic bond holdings and international bond holdings are given by

$$\dot{C}_t = (i_t - \pi_t - \rho)\, C_t, \tag{3}$$
$$\dot{C}_t = (i_{B,t} - \pi_t - \rho)\, C_t, \tag{4}$$

where $\pi_t \equiv \dot{P}_t / P_t$ is the Home consumer price index (CPI) inflation rate. Home's CPI follows from standard expenditure minimization:

$$P_t \equiv \left[(1-\alpha)\, (P_{H,t})^{1-\eta} + \alpha\, (P_{F,t})^{1-\eta} \right]^{1/(1-\eta)},$$

where $P_{H,t}$ is Home's producer price index (PPI) and $P_{F,t}$ is Home's price index of imported goods. Condition (3) is the Home household's Euler equation for the domestic bond, while (4) is the analogous Euler equation for the international bond.

Foreign households face an environment symmetric to that of Home households. Variables pertaining to Foreign households are indexed by asterisks. To accommodate possible deviations from perfect capital mobility, we allow for (tax-induced) return differentials across countries on the international bond. Specifically, we assume that the return on the international bond has two components: a component that is common across countries $\underline{i}_t$ and a country-specific component (τ_t for Home and τ_t^* for Foreign) that captures taxes on international financial transactions financed by lump-sum taxes raised locally. We then define τ_t^D as being related to the wedge between the return on the international bond faced by Home and Foreign households via

$$\tau_t^D \equiv \frac{i_{B,t} - i_{B,t}^*}{2} = \frac{\tau_t - \tau_t^*}{2}. \tag{5}$$

Under free capital mobility, we will have $\tau_t^D = 0$ for all $t \geq 0$. But we will also consider situations in which $\tau_t^D \neq 0$ as a result of different taxes across countries ($\tau_t \neq \tau_t^*$). Finally, we assume that countries have symmetric net foreign asset positions (i.e., equal to 0) at time 0. The optimality conditions of Foreign households are symmetric to (3)-(4) and given by

$$\dot{C}_t^* = (i_t^* - \pi_t^* - \rho)\, C_t^*, \tag{6}$$
$$\dot{C}_t^* = \left(i_{B,t}^* - \dot{e}_t - \pi_t^* - \rho\right) C_t^*, \tag{7}$$

where $\dot{e}_t$ denotes the depreciation rate of the nominal exchange rate E_t, defined as the Home currency price of the Foreign currency, and $\pi_t^* \equiv \dot{P}_t^* / P_t^*$ is the Foreign CPI inflation rate. The real exchange rate is defined as the ratio of the two countries' CPI expressed in a common currency, $Q_t \equiv E_t P_t^* / P_t$. Further, letting $P_{F,t}^*$ denote Foreign's PPI and $P_{H,t}^*$ Foreign's price index of imported good, the Home's terms of trade can be defined as $S_t = P_{F,t} / (E_t P_{H,t}^*)$.

Combining (3)-(7) with (5) leads to a distorted interest parity condition:

$$i_t = i_t^* + \dot{e}_t + 2\tau_t^D.$$

Under free capital mobility (i.e., when $\tau_t^D = 0$), standard interest parity holds. In contrast, when $\tau_t^D > 0$, the Home household faces a higher borrowing cost, while when $\tau_t^D < 0$, it is the Foreign household that faces a higher borrowing cost.

2.2 Firms

Technology. Home Firms, indexed by $l \in [0, 1]$, produce differentiated goods with a linear technology $Y_t(l) = N_t(l)$, where productivity is normalized to one for convenience and $N_t(l) \equiv \left(\int_0^1 N_t(h,l)^{(\varepsilon_t^w - 1)/\varepsilon_t^w} dh \right)^{\varepsilon_t^w / (\varepsilon_t^w - 1)}$ is a composite of domestic individual household labor. Variables are defined analogously in Foreign, where the production function is given by $Y_t(l) = N_t^*(l)$.

Price Setting. Firms operate under monopolistic competition and engage in infrequent price setting à la Calvo (1983). Prices are set in producers' currency, and the law of one price holds for each good. Each firm has an opportunity to reset its price when it receives a price-change signal, which itself follows a Poisson process with intensity $\rho_\delta \geq 0$. As a result, a fraction δ of firms receives a price-change signal per unit of time. These firms reset their price, $P_{H,t}^r(j)$, to maximize the expected discounted profits

$$\int_t^\infty \rho_\delta e^{-\rho_\delta (k-t)} \frac{\lambda_k}{\lambda_t} \left[P_{H,t}^r(j) - P_{H,k} MC_k \right] Y_{k|t} dk,$$

subject to the demand for their own good, $Y_{k|t} = \left(P_{H,t}^r / P_{H,k} \right)^{-\varepsilon} Y_k$, taking as given the paths of domestic output Y, of the domestic PPI P_H, and of the domestic real marginal cost MC. The real marginal cost is defined as $MC_k \equiv (1 - \tau^N) W_k / P_{H,k}$, where τ^N is a time-invariant labor subsidy.[7] The Home household's time k marginal utility of consumption

[7] As is standard in the New Keynesian literature, we will be assuming that this subsidy is set at the level that would be optimal in a steady state with flexible prices.

is denoted by λ_k, so that the ratio λ_k/λ_t is the firm's relevant discount factor between time t and time $k \geq t$. The pricing environment is symmetric in Foreign. In the limiting case of flexible prices (i.e. $\rho_\delta \to \infty$), firms are able to reset their prices continuously and optimal pricing setting reduces to a markup over marginal cost $P_{H,t} = \mu^p(1-\tau^N)W_t$, where $\mu^p = \varepsilon/(\varepsilon-1)$.

2.3 Equilibrium Dynamics

Given paths for interest rates and taxes on international financial transactions, an equilibrium is a constellation in which all households and firms optimize and markets clear.

International Intertemporal Sharing. Combining the Home and Foreign households' Euler equations for the international bonds yields an intertemporal sharing condition relating the ratio of marginal utility in both countries to the real exchange rate:

$$C_t = \Theta_t Q_t C_t^*, \tag{8}$$

where $\Theta_t \equiv \Theta_0 \exp\left[\int_0^t 2\tau_s^D ds\right]$ captures relative spending by Home consumers, with Θ_0 being a constant related to initial relative wealth positions.[8] Condition (8) indicates that under free capital mobility (i.e., in the absence of taxes on international financial transactions), Home and Foreign spending are related to each other through a time-invariant coefficient of proportionality Θ_0. Taxes on international financial transactions alter capital flows between the two countries and make relative spending Θ_t, sometimes referred to as *demand imbalance* in the literature, a time-varying object.

Output Determination. Market clearing for a good l produced in Home requires that the supply of the good equal the sum of the demand emanating from Home and Foreign:

$$Y_t(l) = \underbrace{(1-\alpha)\left(\frac{P_{H,t}(l)}{P_{H,t}}\right)^{-\varepsilon}\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t}_{C_{H,t}(l):\text{ Home demand for Home variety } l} + \underbrace{\alpha\left(\frac{P_{H,t}(l)}{P_{H,t}}\right)^{-\varepsilon}\left(\frac{P_{H,t}}{P_t^*}\right)^{-\eta} C_t^*}_{C_{H,t}^*(l):\text{ Foreign demand for Home variety } l}.$$

[8] In models featuring uncertainty and complete markets, this condition is often labeled as an international risk sharing condition.

At the level of Home's aggregate output, defined as $Y_t \equiv \left[\int_0^1 Y_t(l)^{(\varepsilon-1)/\varepsilon} dl\right]^{\varepsilon/(\varepsilon-1)}$, market clearing hence requires

$$Y_t = \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \left[(1-\alpha) C_t + \alpha Q_t^{\eta} C_t^*\right]. \tag{9}$$

Similarly, market clearing for the foreign goods requires

$$Y_t^* = \left(\frac{P_{F,t}^*}{P_t^*}\right)^{-\eta} \left[(1-\alpha) Q_t^{-\eta} C_t^* + \alpha C_t\right]. \tag{10}$$

Aggregate Home employment, defined as $N_t \equiv \int_0^1 N_t(l) dl$, relates to aggregate Home output according to $N_t = Y_t Z_t$, where $Z_t \equiv \int_0^1 (P_t(l)/P_t)^{-\varepsilon} dl$. An analogous relation holds between Foreign's employment and output.

2.4 Log-linearized Model and World-Difference Formulation

Following most of the literature, we focus on a first-order approximation of the equilibrium dynamics of the model around the non-distorted symmetric steady state. To ensure that the model's steady state is non-distorted, we assume that the time-invariant labor subsidy is set to $\tau^N = (\mu^p \mu^w - 1)/(\mu^p \mu^w)$ in both countries, so as to offset distortions form monopolistic competition.[9] Since the only shocks we consider are markup shocks, the efficient allocation is time-invariant and coincides with the non-distorted steady state allocation. Therefore, log deviations of variables from their steady state value can also be interpreted as gaps from the efficient allocation. Denoting such log deviations from steady state by hats on lower case letters, e.g., $\widehat{y}_t = y_t - \bar{y}$, and noting that our productivity normalization implies that output, employment, consumption and the terms of trade are all equal to one in steady state (see Appendix B.1), we have $\widehat{y}_t = y_t$. In what follows, we will therefore simply use lower case letters to denote gaps from the efficient allocation.

Log-linearized Model. A first-order approximation of the goods market-clearing conditions around the steady state yields the following log-linear expressions:

$$y_t = (1-\alpha)\left[c_t + \alpha\eta s_t\right] + \alpha\left[c_t^* + (1-\alpha)\eta s_t\right], \tag{11a}$$

$$y_t^* = (1-\alpha)\left[c_t^* - \alpha\eta s_t\right] + \alpha\left[c_t - (1-\alpha)\eta s_t\right], \tag{11b}$$

[9] The steady-state markup is denoted by μ^w, which is taken to be the same across the two countries: $\mu^w = \varepsilon^w/(\varepsilon^w - 1) = \varepsilon^{w*}/(\varepsilon^{w*} - 1) = \mu^{w*}$.

which indicate that output in each country depends on consumption in Home and Foreign, as well as on the terms of trade: a terms of trade improvement raises demand for Foreign output at the expense of demand for Home output, via the expenditure switching channel.

Taking logs on both sides of the intertemporal sharing condition (8), and taking into account the (first-order accurate) relationship between the real exchange rate and the terms of trade, $q_t = (1 - 2\alpha)s_t$, we obtain a log-linearized intertemporal sharing condition

$$c_t - c_t^* = \theta_t + (1 - 2\alpha)s_t. \tag{12}$$

Combining the intertemporal sharing condition (12) with the market-clearing conditions (11a) and (11b) yields an expression satisfied by the equilibrium terms of trade,

$$y_t - y_t^* = \omega s_t + (1 - 2\alpha)\theta_t, \tag{13}$$

where $\omega \equiv \eta - (\eta - 1)(1 - 2\alpha)^2 > 0$. This expression indicates that output is relatively higher in the country that has less favorable terms of trade or, in the presence of home bias in consumption, in the country benefiting from a positive demand imbalance. In the absence of home bias (i.e., when $\alpha = 1/2$), since the composition of consumption is identical across the two countries, demand imbalances do not translate into differences in output.

We define net exports in units of the Home good as $NX_t \equiv Y_t - P_t C_t / P_{H,t}$. Linearizing this relationship around the non-distorted steady state, and substituting the Home market clearing condition (11a) and the intertemporal smoothing condition (12), we obtain an expression for the trade balance,

$$nx_t = \frac{\omega - 1}{2} s_t - \alpha \theta_t, \tag{14}$$

where nx_t denotes Home's net exports as a share of Home's steady state output, $nx_t \equiv NX_t / \bar{Y}$. The expression indicates that the effect of an appreciated terms of trade on the trade balance depends on the relative importance of the elasticity of substitution across goods (η) and the intertemporal elasticity of substitution, which is set to one. Furthermore, all else equal, a positive demand imbalance ($\theta_t > 0$) deteriorates the trade balance.

Turning to the supply side of the economy, under our Calvo price setting assumption, up to a first order, the dynamics of PPI inflation are described by

$$\dot{\pi}_{H,t} = \rho\pi_{H,t} - \kappa mc_t, \tag{15a}$$
$$\dot{\pi}^*_{F,t} = \rho\pi^*_{F,t} - \kappa mc^*_t. \tag{15b}$$

where $\kappa \equiv \rho_\delta(\rho + \rho_\delta)$, and mc_t (resp. mc^*_t) denotes the log deviation of the real marginal cost from its steady state value. Using the linearized aggregate production functions, $y_t = n_t$ and $y^*_t = n^*_t$, and linearized labor supply equations, $w_t - p_t = \mu^w_t - \bar{\mu}^w + c_t + \phi n_t$ and $w^*_t - p^*_t = \mu^{w*}_t - \bar{\mu}^w + c^*_t + \phi n^*_t$, these are given by

$$mc_t = (1+\phi)y_t - \frac{\omega-1}{2}s_t + \alpha\theta_t + u_t, \tag{16a}$$
$$mc^*_t = (1+\phi)y^*_t + \frac{\omega-1}{2}s_t - \alpha\theta_t + u^*_t, \tag{16b}$$

where the markup shocks $u_t \equiv \mu^w_t - \bar{\mu}^w$ and $u^*_t \equiv \mu^{w*}_t - \bar{\mu}^w$ are deviations of wage markups from their steady state value. The real marginal cost (measured in units of the domestic good) depends positively on the marginal rate of substitution between consumption and leisure and negatively on the terms of trade.[10] However, since the equilibrium marginal rate of substitution itself depends ambiguously on the terms of trade (for given levels of output and the demand imbalance), the relationship between the terms of trade and the marginal cost is a priori ambiguous. Finally, for given levels of output and the terms of trade, a positive demand imbalance raises the marginal rate of substitution of a country's residents and thus increases the domestic firms' marginal cost.[11]

World and Difference Formulation. Before we turn to optimal policy, it is convenient to rewrite the dynamics of output and inflation in both regions in "world" and "difference" format. We respectively define the world output and the cross-country output differential as $y^W_t = (y_t + y^*_t)/2$ and $y^D_t = (y_t - y^*_t)/2$. Similarly, we define the world PPI inflation and cross-country PPI inflation differential as $\pi^W_t = (\pi_{H,t} + \pi^*_{F,t})/2$ and $\pi^D_t = (\pi_{H,t} - \pi^*_{F,t})/2$. Combining the expressions for PPI inflation dynamics (15a)-(15b) with the marginal cost expressions (16a)-(16b) yields New Keynesian Phillips curves (NKPC) for world and

[10] That is to say, an improvement in a country's terms of trade lowers its producers' marginal cost. A terms of trade improvement raises the price of the domestic good relative to that of the consumption basket. Noting that $p_t = p_{H,t} + \alpha s_t$, the labor supply equation implies that the real wage expressed in terms of the domestic good must be equal to $w_t - p_{H,t} = \phi n_t + c_t + \alpha s_t + u_t$, so that the real marginal cost is given by $mc_t = \phi n_t + c_t + \alpha s_t + u_t$.

[11] Note that for Home, improved terms of trade correspond to a lower s_t, while a positive demand imbalance corresponds to a higher θ_t. In contrast, for Foreign, improved terms of trade correspond to a higher s_t, while a positive demand imbalance corresponds to a lower θ_t.

differences:

$$\dot{\pi}_t^W = \rho \pi_t^W - \kappa(1+\phi) y_t^W - \kappa u_t^W, \tag{17}$$

$$\dot{\pi}_t^D = \rho \pi_t^D - \kappa \left[(1+\phi) y_t^D - \frac{\omega - 1}{2} s_t + \alpha \theta_t \right] - \kappa u_t^D. \tag{18}$$

The equilibrium terms of trade expression (13) can also be re-written as

$$2y_t^D = \omega s_t + (1-2\alpha)\theta_t. \tag{19}$$

3 Optimal Policy Analysis

We will now argue that when monetary policy faces an output-inflation trade-off, a free capital mobility regime is generically constrained inefficient because of a macroeconomic externality operating via firms' marginal costs. Under a condition weaker than the well-known Marshall-Lerner condition, capital inflows are inflationary. As a result, either inflows into the region experiencing the deepest policy-induced recession are excessive, or outflows from that region are insufficient. In addition, if, as is empirically plausible, the intratemporal elasticity of substitution between Home and Foreign goods is larger than one, capital flows in the wrong direction.

3.1 Welfare-Based Loss Function

To capture the various trade-offs to be resolved by optimal policies, we use a standard welfare-based loss function. To obtain this loss function, we take a second-order approximation of a symmetrically weighted average of households' utilities in Home and Foreign around the non-distorted steady state (see Appendix B.1). The instantaneous loss function is given by

$$\begin{aligned} L_t = & \left[(1+\phi)(y_t^W)^2 + \frac{\varepsilon}{\kappa}(\pi_t^W)^2 \right] + \left[(1+\phi)(y_t^D)^2 + \frac{\varepsilon}{\kappa}(\pi_t^D)^2 \right] \\ & + \alpha(1-\alpha)(1-\eta)\eta(s_t)^2 + \alpha(1-\alpha)\left[\theta_t - (\eta-1)(1-2\alpha)s_t\right]^2, \end{aligned} \tag{20}$$

where the output gap and inflation are again expressed in "world" and "difference" forms. The first two terms in (20) featuring squared output gaps and inflation reflect sticky price distortions familiar from the closed economy literature. The third and fourth terms, reflecting distortions specific to the open economy context, capture welfare losses stemming from an inefficient cross-country distribution of consumption potentially caused by two factors: the demand imbalance θ_t and the terms of trade gap s_t.[12]

Normative research in New Open Economy Macroeconomics (e.g., Benigno 2009) has traditionally studied how monetary policy should be set in a context where the demand imbalance term θ_t was equal to zero (i.e., under complete markets) or was endogenously responding to shocks and other macroeconomic variables (i.e., under incomplete markets). Our approach, in contrast, is to treat the demand imbalance θ_t as a policy variable and ask whether actively managing it may be desirable in a context where it could otherwise be left at zero.

3.2 Optimal Monetary Policy

The optimal monetary policy problem consists of choosing a path for the output gaps y_t^W, y_t^D, inflation π_t^W, π_t^D, and terms of trade s_t, to minimize the present value of the loss (20), subject to the NKPCs (17)-(18) and the equilibrium terms of trade expression (19).[13] We have the following characterization.

Proposition 1 (Optimal monetary policy)**.** *Optimal monetary policy is characterized by the following targeting rules:*

$$\dot{y}_t^W + \varepsilon \pi_t^W = 0 \tag{21}$$

$$\dot{y}_t^D + \varepsilon \pi_t^D = 0. \tag{22}$$

Proof. See Appendix B.2.1. □

This description of optimal cooperative monetary policy is analogous to that commonly encountered for complete markets open economy models with producer currency pricing (PCP) in the literature. Targeting rules (21) and (22) indicate that, in both "world" and "difference" terms, optimal policy strikes a balance between losses from inflation and losses from deviations of output from its efficient level. The two targeting rules can be combined to deliver targeting rules for each country that depend only on the domestic output gap and PPI inflation – that is, $\dot{y}_t + \varepsilon \pi_{H,t} = 0$ and $\dot{y}_t^* + \varepsilon \pi_{F,t}^* = 0$ – a feature referred to as *inward looking* monetary policy in the New Open Economy Macroeconomics literature. It

[12] This later factor, however, disappears in a widely studied special case featuring unit elasticities.

[13] Implicitly, in line with the literature, we assume that the policymaker has access to a date 0 transfer, so the optimal policy problem reflects efficiency rather than a mix of efficiency and redistributive considerations. For a formal statement of the optimal monetary policy problem, see Appendix B.2.1.

is worth stressing that this characterization does not rely on any particular assumption regarding the path of θ_t (other than it being exogenous, or chosen by policy). In particular, it holds under free capital mobility (i.e., $\theta_t = 0\ \forall t$), as well as under an optimally managed capital account regime to be derived below.

The targeting rules (21) and (22) lead us to one observation, summarized in the corollary below, which helps us narrow down the role played by capital flows in response to shocks.

Corollary 1 (Irrelevance of capital flow regime for world variables)**.** *The paths of the world output gap y_t^W and world inflation π_t^W are independent of the capital flow regime (i.e., of the path of θ_t).*

Proof. See Appendix B.2.2 □

This observation follows directly from combining the "world" NKPC (17) with the "world" monetary policy targeting rule (21) and means that the capital flow regime matters only for the determination of cross-country "difference" variables and the terms of trade.[14] Therefore, both from a positive and from a normative standpoint, an analysis of the role played by capital flows in the adjustment to shocks can legitimately center on the dynamics of cross-country difference variables y_t^D and π_t^D and external variables s_t and θ_t.

Discussion of Inward versus Outward Looking Monetary Policy When the path of the demand imbalance θ_t deviates from zero, asset markets are no longer complete, and the inward lookingness of monetary policy in (21)-(22) contrasts with the *outward looking* rules derived in studies assuming either other forms of market incompleteness (e.g., Corsetti, Dedola and Leduc 2010, 2018), or pricing to market (e.g., Engel 2011).[15] In these studies, the demand imbalance is an endogenous variable whose fluctuations depend on the interaction of shocks and other variables influenced by monetary policy, such as the cross-country difference in the output gap. As a result, monetary policy can manage distortions caused by market incompleteness or pricing to market, and generally chooses to do so, resulting in outward looking rules. In our case, in contrast, the demand imbalance is either exogenous or directly controlled by policy, so there is no scope for monetary policy to manage the market incompleteness distortion, hence the inward looking rules.

[14]See Groll and Monacelli (2020) for a similar result regarding the irrelevance of the exchange rate regime for the determination of "world" variables.

[15]The literature refers to outward looking monetary policy when targeting rules in open economy models also feature external variables, such as international relative prices or the demand imbalance term.

3.3 Optimal Capital Flow Management

To question the constrained efficiency of the free capital mobility regime, we make the demand imbalance θ_t a choice variable of the optimizing policymaker and ask under what circumstances θ_t is set to a value different from zero. The optimal policy problem now consists in choosing a path for the output gaps y_t^W, y_t^D, inflation π_t^W, π_t^D, terms of trade s_t *and* demand imbalance θ_t to minimize the present discounted value of the loss (20), subject to the NKPCs (17) and (18) and the equilibrium terms of trade relation (19).[16]

In addition to the targeting rules associated with monetary policy, (21) and (22), optimal policy now also pertains to an additional capital flow management margin. For future reference, it is convenient to define the trade elasticity as follows.

Definition 1 (Trade elasticity)**.** *The trade elasticity χ is defined as the sum of the absolute values of the price elasticity of imports and the price elasticity of exports, holding aggregate consumption constant. Formally,*

$$\chi \equiv \left.\frac{-\partial \log C_{F,t}}{\partial \log P_{F,t}/P_{H,t}}\right|_{C_t} + \left.\frac{-\partial \log C^*_{H,t}}{\partial P^*_{H,t}/P^*_{F,t}}\right|_{C^*_t} = 2(1-\alpha)\eta.$$

It is also useful to state a condition on parameters which will play an important role for our analysis.

Condition 1. *The trade elasticity is larger than the degree of home bias: $\chi \geq 1-2\alpha$.*

To our knowledge, this condition, which is weaker than the well-known Marshall-Lerner condition (stating that $\chi > 1$), has not received any attention in the literature. Yet, as we will now see, it happens to play a critical role for the normative properties of capital flows in the model.

Proposition 2 (Optimal capital flow management)**.** *The optimal capital flow regime is characterized by the targeting rule*

$$\theta_t = \frac{\chi - (1-2\alpha)}{\chi} 2y_t^D. \tag{23}$$

Proof. See Appendix B.2.3 □

This targeting rule embodies the paper's main insight. To the extent that shocks generating an output-inflation trade-off generally result in a non-zero cross-country difference

[16]See Appendix B.2 for a formal statement of the problem.

in output gaps, rule (23) indicates that the demand imbalance should always be set to zero only if Condition 1 holds with equality. Apart from this knife-edge case, the optimally managed demand imbalance is generally not zero. Therefore, the free capital mobility regime is generically constrained inefficient when monetary policy faces an output-inflation trade-off, and it is constrained efficient only under a very special parametric condition.

In favor of which country should policy want to tilt spending? Rule (23) suggests that the answer to this question depends on whether Condition 1 is met. When it is violated, optimal capital flow management generates a demand imbalance in favor of the country whose output is the most depressed (in short, the most depressed country), while when it is met with strict inequality, optimal capital flow management generates a demand imbalance in favor of the least depressed country. In this latter case, which appears to be the most relevant one empirically, the most depressed country experiences either excessive capital inflows or insufficient capital outflows.[17]

What is the source of this inefficiency? We next argue that it arises from a macroeconomic externality operating via firms' marginal costs.

3.4 Macroeconomic Externality via Firms' Marginal Costs

To nail down the inefficiencies at work in the free capital mobility regime, we find it useful to ask how a marginal deviation from the equilibrium external borrowing positions in that regime alters the constraints faced by monetary policy and hence aggregate outcomes.

Consider a marginal increase in borrowing by Home from Foreign at instant t (i.e., $\theta_t = \epsilon$ for some small $\epsilon > 0$, leaving $\theta_k = 0$ for all other $k \neq t$).[18] If we use (19) to substitute for the equilibrium terms of trade in the marginal cost expressions (16a)-(16b) and apply the envelope theorem, the change in the loss function induced by this perturbation is given by

$$\frac{dL_t}{d\theta_t} = \varphi_t^D \frac{\partial mc^D(y_t^D, \theta_t)}{\partial \theta_t}, \tag{24}$$

where φ_t^D is the co-state variable associated with the NKPC in differences (18). Equation (24) shows that the marginal increase in borrowing by Home from Foreign affects global

[17]Most calibrations of the the model place the trade elasticity above one, in which case Condition 1 is necessarily satisfied.

[18]For the sake of the argument, we assume that this increase in borrowing is compensated by a change in the date 0 implicit transfer. More generally, what matters for the externality to matter is that the balancing transaction occurs at a time when the government's multiplier on the NKPC (18) has a value different from the one at time t.

welfare losses via its effects on the cross-country "difference" in marginal costs. Now, observe that the cross-country difference in marginal costs can be decomposed into two components:

$$mc^D(y_t^D, \theta_t) = \underbrace{\left[\phi + \frac{1-2\alpha}{\omega}\right] y_t^D + \frac{\alpha\chi}{\omega}\theta_t + u_t^D}_{\text{difference in real wages}} + \underbrace{\frac{\alpha}{\omega}\left[2y_t^D - (1-2\alpha)\theta_t\right]}_{\text{difference in purchasing power}}. \tag{25}$$

The first component reflects cross-country differences in labor costs arising from a difference in the real wage (in terms of each country's consumption bundle). The second component reflects a cross-country difference in purchasing power arising from movements in the terms of trade. The marginal cost derivative in (24) is therefore given by

$$\frac{\partial mc^D(y_t^D, \theta_t)}{\partial \theta_t} = \frac{\alpha\chi}{\omega}\left[\underbrace{1}_{\text{real wage effect}} - \underbrace{\frac{1-2\alpha}{\chi}}_{\text{purchasing power effect}}\right], \tag{26}$$

where the two terms reflect the two aforementioned components of the cross-country difference in marginal costs. First, raising Home consumption and lowering Foreign consumption shifts up labor supply in Home while shifting it down in Foreign. In equilibrium, this leads to a rise in Home's real wage and a drop in Foreign's real wage, thereby raising the cross-country difference in marginal costs. Second, in the presence of home bias in preferences ($\alpha < 1/2$), the appreciation of the terms of trade induced by the increase in borrowing by Home from Foreign raises Home's purchasing power while decreasing Foreign's purchasing power. This lowers marginal costs in Home and raises them in Foreign, hence reducing the cross-country difference. The strength of this effect is proportional to the ratio of the degree of home bias $1-2\alpha$ to the trade elasticity χ. On the one hand, the stronger the home bias, the more changes in relative spending affect the relative price between Home and Foreign goods.[19] On the other hand, the higher the trade elasticity, the smaller are price movements associated with a given change in relative spending.

Condition 1 guarantees that the purchasing power effect is not strong enough to dominate the real wage effect. When it is met with strict inequality, the real wage effect dominates, and an increase in θ_t raises the cross-country difference in marginal costs. In contrast, when it is violated, the purchasing power effect dominates, and the increase in θ_t

[19] Without home bias ($\alpha = 1/2$), since households in both countries consume the exact same basket, changes in relative spending are not associated to any changes in relative prices.

lowers the cross-country difference in marginal costs.[20]

These effects of marginal changes in external borrowing work in general equilibrium as prices adjust in goods and labor markets. As a result, they are ignored by atomistic agents. Yet, when the output-inflation trade-off is more stringent in one of the two countries, – that is, when $\varphi_t^D \neq 0$ – a marginal increase in borrowing by Home from Foreign at instant t generates a first-order welfare effect by tightening or relaxing the constraint faced by the monetary authority, as indicated by (24).

We next argue that the externality just discussed is powerful enough to result in trade imbalances of opposite signs under optimally managed capital flows versus under free capital mobility.

3.5 Topsy-Turvy Capital Flows

Combining the targeting rule (23) with the equilibrium terms of trade expression (19), we obtain that the terms of trade are proportional to the cross-country difference in the output gap in the optimal CFM regime, $s_t = 2y_t^D/\chi$, albeit with a different coefficient than the one under free capital mobility, in which the relationship follows from (19) (with $\theta_t = 0$) and is given by $s_t = 2y_t^D/\omega$. Substituting these terms of trade expressions into the net export expression (14), we obtain a trade balance of

$$nx_t = -\frac{2\alpha}{\chi}\, y_t^D \tag{27}$$

under optimal CFM, while the trade balance under free capital mobility is given by

$$nx_t = \frac{\omega - 1}{\omega} y_t^D. \tag{28}$$

This points to qualitatively different patterns of trade imbalances under the two regimes, which we summarize in the following proposition.

Proposition 3 (Topsy-turvy capital flows)**.** *If $\eta > 1$, the most depressed country runs a trade surplus under optimal CFM, while it runs a trade deficit under free capital mobility.*

Proof. The proof follows directly from (27), (28), and the definitions of ω and χ. □

The proposition implies that in the presence of cross-country differences in the severity

[20]Note that the two effects exactly cancel out in the knife-edge cases in which Condition 1 is met with equality ($\chi = 1 - 2\alpha$), in which case the difference in marginal costs is independent of θ_t.

of (policy-induced) recessions, capital flows are *topsy-turvy* under free capital mobility in the empirically plausible case where $\eta > 1$. Hence, rather than simply causing capital flows to be excessive, the macroeconomic externality discussed in Section 3.4 can be strong enough to flip their direction.

Neoclassical and Keynesian Motives of Intertemporal Trade. To understand the essence of Proposition 2, we find it useful to decipher the various motives for intertemporal trade in the model. In the free capital mobility regime, these motives are purely neoclassical and have been well understood since at least Cole and Obstfeld (1991): a temporarily lower income in Home creates an incentive to borrow, but the terms of trade appreciation accompanying this lower income generates an incentive to save. When the intra-temporal elasticity is high (i.e., $\eta > 1$), terms of trade movements are muted, and the first effect dominates. When the intra-temporal elasticity is low (i.e., $\eta < 1$), terms of trade movements are strong, and the second effect dominates. And when the intra- and intertemporal elasticities are equal, the two effects neutralize each other.

In the optimal CFM regime, an additional Keynesian macroeconomic stabilization motive is also present. This motive calls for relaxing the output-inflation trade-off in the country where it is the least favorable. For the sake of illustrating the scope for topsy-turvy capital flows, consider the case of the Cole-Obstfeld parameter specification ($\eta = 1$). As we just argued, in this case the two neoclassical motives cancel out, and the result is zero trade imbalances under free capital mobility. Under optimal CFM, the Keynesian motive also generates a benefit from reducing consumption in the country with the lowest output, as Condition 1 is met with strict inequality.[21] That country thus experiences a trade surplus. For η slightly above one, the first of the two neoclassical effects is the strongest, generating a trade deficit by the country with the most negative output gap under free capital mobility. However, under optimal CFM the Keynesian effect still dominates to yield a trade surplus by that country. As η is raised further, the net neoclassical effect grows stronger in both regimes, but it happens to never overturn the Keynesian effect under optimal CFM.[22]

This topsy-turvy result is depicted in Figure 2. At a given point in time, we can define the most depressed country to be the country whose output is the lowest.[23] The figure

[21]When $\eta = 1$, we have $\chi = 2 - 2\alpha > 1 - 2\alpha$.

[22]In the limit where $\eta \to \infty$, as long as $\alpha > 0$, the trade balance becomes proportional to the difference in the output gap under free capital mobility, $nx_t = y_t^D$, but converges to zero in under optimal CFM, $nx_t = 0$.

[23]Since each country's output gap is proportional to the policymaker's co-state on the NKPCs, the most depressed country also happens to be the one for which relaxing the NKPC is the most valuable to the policymaker.

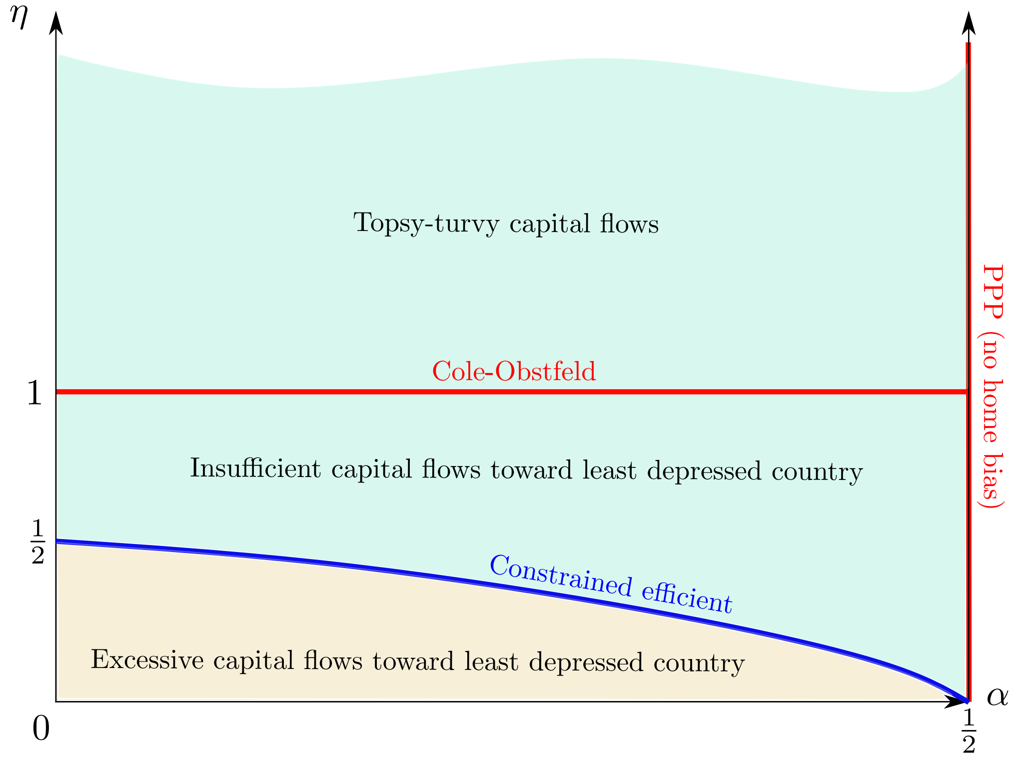

Figure 2: Characterization of distortions to capital flows in free capital mobility regime.

shows whether capital flows are topsy-turvy, excessive or insufficient as a function of the values of the parameters α and η. The red Cole-Obstfeld line depicts cases in which capital flows are zero under free capital mobility. Above this line, under free capital mobility, capital flows from the least depressed country, which runs a trade surplus, to the most depressed country, which runs a trade deficit. These flows go in the wrong direction, compared with those that would prevail under optimal CFM. Below the Cole-Obstfeld line, capital flows in the opposite direction, with the least depressed country running a trade deficit and the most depressed country running a surplus. Within this area, the blue concave curve depicts the knife-edge cases in which the free capital mobility regime is constrained efficient. In the area above the concave curve (but below the Cole-Obstfeld line), capital flows from the most to the least depressed country, but to an insufficient extent. In contrast, in the area under the concave curve, capital flows from the most to the least depressed country in an excessive way. The most plausible model calibrations place us in the area above the Cole-Obstfeld line, where capital flows are topsy-turvy (see Section 4.2 for an example).

3.6 Insights from Two Special Cases

We now briefly discuss two special cases of the model that reveal interesting insights.

No Home Bias and MPC Homogeneity. In the first case, which the early New Open Economy Macroeconomics literature has almost exclusively focused on (see, e.g., Clarida et al. 2002 or Benigno and Benigno 2003), home bias is abstracted from ($\alpha = 1/2$), and as a result, purchasing power parity (PPP) holds. By (12), the free capital mobility outcome features an international equalization of consumption at all times, despite possible divergences in economic activity. Yet, this regime is never constrained efficient, as Condition 1 is always met with strict inequality. According to the targeting rule (23), relative spending should be distorted away from the most depressed region, $\theta_t = 2y_t^D$. In addition, we have the following corollary to Proposition 2.

Corollary 2 (Topsy-turvy capital flows in the absence of home bias)**.** *If the Marshall-Lerner condition holds, capital flows are topsy-turvy.*

Without home bias, the condition of Proposition 2 reduces to the Marshall-Lerner condition. Hence, when this condition holds, the most depressed country runs a trade surplus under optimal CFM but runs a trade deficit under free capital mobility.

Since the no home bias case features identical MPCs on each good across countries (and over time), it clarifies that the intervention motive present in our model is conceptually distinct from the one emphasized by Farhi and Werning (2016) and others based on the idea of tilting spending in favor of agents with relatively higher MPCs on more depressed goods. In our context, capital flows alter the conditions faced by monetary policy through the economy's supply side rather than through the demand side. Since without home bias, the purchasing power effect in (26) is absent, capital flows only affect marginal costs via the real wage through a wealth effect on labor supply. Had we assumed fully rigid wages, changes in households' labor supply would not lead to changes in firms' marginal costs, and there would be no externality or inefficiency.

Cole-Obstfeld, Trade Imbalances and Spillovers. A second special case of interest, commonly referred to as the Cole and Obstfeld (1991) parametrization and popularized in the New Open Economy Macroeconomics literature by Corsetti and Pesenti (2001), features unitary elasticities ($\eta = 1$). This case is remarkable for at least two reasons: under free capital mobility, it features neither external imbalances nor any international spillovers from markup shocks or monetary policy. Indeed, in this regime net exports in (28) are always zero, and both output and inflation respond only to domestic shocks. Yet, we find that this regime is never constrained efficient either: Condition 1 is again always met with strict inequality, so according to the targeting rule (23), relative spending should be distorted away from the most depressed region, $\theta_t = y_t^D/(1-\alpha)$.

This special case hence illustrates that the macroeconomic externality we uncovered can generate inefficient intertemporal trade even in the absence of any external imbalances and thus that external imbalances can be larger under optimal CFM than under free capital mobility. This clarifies that capital mobility is not harmful *per se*. Instead, it is the extent of intertemporal trade that is inefficient because market prices do not accurately reflect the social value of external borrowing in the presence of an output-inflation trade-off.

Furthermore, since the free capital mobility regime is constrained inefficient even in the absence of any international spillovers from markup shocks or monetary policy, the inefficiency is clearly not related to the correction or internalization of such spillovers.

3.7 Decentralization with Taxes on Capital Flows

Several financial policies could be used by a global policymaker to implement the optimal CFM policy. One possible implementation is through taxes on capital flows. An explicit expression linking these taxes to the output path can be obtained from the targeting rule characterizing optimal capital flow management policy. From the intertemporal sharing condition (8), the tax differential satisfies $\tau_t^D = \dot{\theta}_t/2$. If we use the targeting rule (23), it can therefore be related to the growth rate of the cross-country difference in the output gap as

$$\tau_t^D = \left[1 - \frac{1-2\alpha}{\chi}\right] \dot{y}_t^D. \tag{29}$$

Hence, when Condition 1 is met with strict inequality, the tax is higher in the country with the fastest growing output gap, while when it is violated, the opposite is true. And since the free capital mobility regime is constrained efficient when Condition 1 holds with equality, the tax should naturally be set to zero in this case.

4 Stagflation Episodes and Capital Flows

To illustrate our results' implications for a stagflation episode, we now analyze the world economy's adjustment to an unanticipated temporary markup shock that gives rise to an output-inflation trade-off of unequal stringency in the two countries. First, in Section 4.1, we characterize the adjustment to a specific markup shock scenario without putting any constraints on parameter values. Then, in Section 4.2, we analyze the impulse response to a mean-reverting markup shock with parameters calibrated to standard values from the literature.

4.1 Characterization of a Stagflation Episode

For concreteness, suppose that Home is subject to an inflationary markup shock such that $u_t = 2\bar{u} > 0$ for some $\bar{u} > 0$ for $t \in [0, T)$ and $u_t = 0$ for $t \geq T$, while Foreign is not hit by any shock (i.e., $u_t^* = 0$ for $t \geq 0$). In terms of the "world" and "difference" shocks appearing in (17) and (18), we therefore have

$$u_t^W = u_t^D = \begin{cases} \bar{u} > 0 & \text{for} \quad t \in [0, T) \\ 0 & \text{for} \quad t \geq T. \end{cases} \tag{30}$$

As is well understood, monetary policy will not able to perfectly stabilize all variables under this scenario. Instead, it will trade off output gap and inflation distortions, as emphasized in Section 3.2. The main advantage of the step-function scenario in (30) is to allow for a sharp characterization of the adjustment under our two capital account regimes of interest.

4.1.1 Free Capital Mobility

In the free capital mobility regime, $\theta_t = 0 \, \forall t \geq 0$. Accounting for this fact when substituting the equilibrium terms of trade expression (19) into the NKPC in difference (18) yields a dynamic equation for the cross-country difference in inflation as a function of itself and the cross-country difference in the output gap:

$$\dot{\pi}_t^D = \rho \pi_t^D - \kappa \left(\frac{1}{\omega} + \phi \right) y_t^D - \kappa u_t^D. \tag{31}$$

Meanwhile, differentiating the targeting rule (22) with respect to time yields a dynamic equation for the cross-country difference in the output gap as a function of the cross-country difference in inflation:

$$\dot{y}_t^D = -\varepsilon \pi_t^D. \tag{32}$$

Equations (31) and (32) form a dynamical system in π_t^D and y_t^D whose solution encapsulates the dynamics of the cross-country block of the model; π_t^D is a jump variable, and although y_t^D could in principle jump, under the optimal plan it is predetermined at $y_0^D = 0$.[24] The system is thus saddle-path stable, and the solution can be conveniently represented in a phase diagram. The $\dot{y}_t^D = 0$ locus is described by $\pi_t^D = 0$, while the $\dot{\pi}_t^D = 0$ locus is described by $\rho\pi_t^D = \kappa\left(\frac{1}{\omega} + \phi\right) y_t^D + \kappa u_t^D$. Given our shock scenario, in the (y_t^D, π_t^D) space, the $\dot{y}_t^D = 0$ locus is therefore always a flat line at 0, while the $\dot{\pi}_t^D = 0$ locus is an upward sloping straight line with slope $\kappa\left(\frac{1}{\omega} + \phi\right)/\rho$ and intercept $\kappa\bar{u}/\rho > 0$ in the short-run (i.e., for $t \in [0, T)$) and intercept 0 in the long-run (i.e., for $t \geq T$).

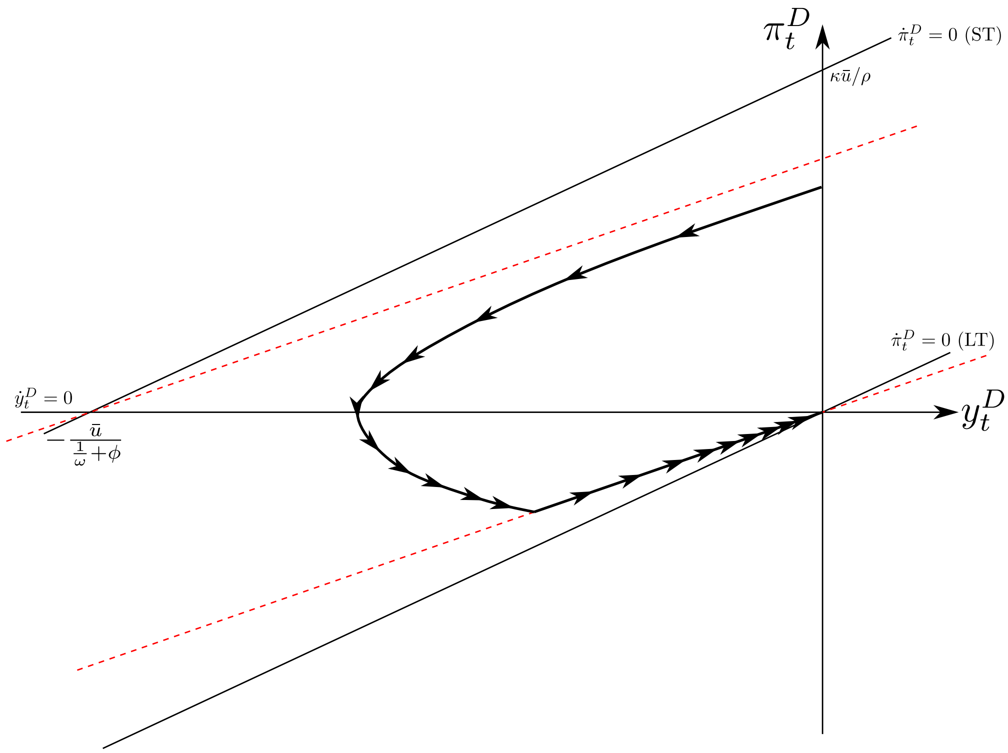

Figure 3: Output-inflation trade-off under free capital mobility.
Note: (ST) denotes short-term $\dot{\pi}_t^D = 0$ locus, (LT) denotes long-term $\dot{\pi}_t^D = 0$ locus.

The loci are represented in Figure 3, where y_t^D rises (diminishes) south (north) of the $\dot{y}_t^D = 0$ locus and π_t^D rises (diminishes) west (east) of the $\dot{\pi}_t^D = 0$ locus. The fictional saddle-path associated with the system being permanently governed by the short-term loci is represented by the upper dashed upward sloping line, while that associated with the system being permanently governed by the long-term loci is represented by the lower dashed upward sloping line. The actual saddle path is represented by the thick curve with arrows.

[24]The co-state variable φ_t^D is backward looking with an initial condition $\varphi_0^D = 0$, and both y_t^D and s_t are proportional to φ_t^D (see equations (B.21), (B.25) and (B.26) with $\theta_t = 0\ \forall t$).

The inflationary markup shock in Home naturally causes a cross-country difference in inflation on impact. But the initial jump in the inflation difference is limited by monetary policy's commitment to generate a more negative output gap in Home than in Foreign in the future, with the difference in the output gap displaying a hump shape. To support this path for the output gap differential, the terms of trade gap needs to follow a similar hump shape, indicating persistently (misaligned and) appreciated terms of trade throughout the episode. In line with our discussion of Section 3.5, several patterns regarding cross-border capital flows can arise. From (28), a hump-shaped trade deficit arises if $\eta > 1$, while a hump-shaped trade surplus arises if $\eta < 1$. When $\eta = 1$, trade remains balanced in response to the markup shock.

4.1.2 Optimal CFM

Under optimal CFM, the path of θ_t satisfies the targeting rule (23). Accounting for this fact when substituting the equilibrium terms of trade expression (19) into the NKPC in difference (18) again yields a dynamic equation for the cross-country difference in inflation as a function of itself and the cross-country difference in the output gap:

$$\dot{\pi}_t^D = \rho \pi_t^D - \kappa \left[\frac{1}{\omega} + \phi + \frac{2\alpha}{\chi\omega} \left[\chi - (1 - 2\alpha)\right]^2 \right] y_t^D - \kappa u_t^D, \tag{33}$$

where the last term in the large square bracket reflects the optimal management of the demand imbalance. This term is non-negative and equal to zero only in the knife-edge case in which $\chi = 1 - 2\alpha$. Equations (33) and (31) now form the dynamical system in π_t^D and y_t^D whose solution represents the dynamics of the cross-country block of the model. Again, π_t^D is a jump variable, and y_t^D is predetermined at $y_0^D = 0$ under the optimal plan. The system is again saddle-path stable and is represented with a phase diagram in Figure 4.

As is the case under free capital mobility, the $\dot{y}_t^D = 0$ locus is described by $\pi_t^D = 0$. But this time, the $\dot{\pi}_t^D = 0$ locus is described by $\rho \pi_t^D = \kappa \left[\frac{1}{\omega} + \phi + \frac{2\alpha}{\chi\omega} \left(\chi - (1 - 2\alpha)\right)^2 \right] y_t^D + \kappa u_t^D$. The only difference with the phase diagram of Figure 3 is that the $\dot{\pi}_t^D = 0$ locus now has a steeper slope of $\kappa \left[\frac{1}{\omega} + \phi + \frac{2\alpha}{\chi\omega} \left[\chi - (1 - 2\alpha)\right]^2 \right] / \rho$. This slope is strictly steeper, except when $\chi = (1 - 2\alpha)$, in which case the two phase diagrams coincide. The phase diagram shows that optimal CFM results in a more favorable trade-off between the stabilization of the cross-country difference in the output gap and that of the cross-country difference in domestic inflation, regardless of the direction of the inefficiency.

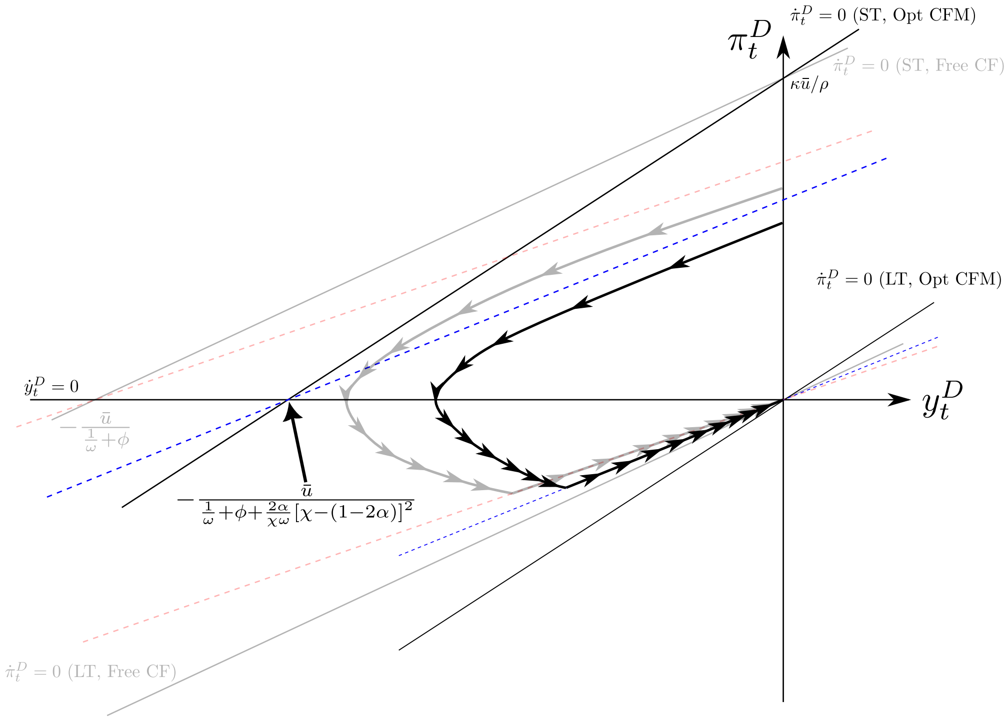

Figure 4: Output-inflation trade-off under optimal CFM.
Note: (ST) denotes short-term $\dot{\pi}_t^D = 0$ locus; (LT) denotes long-term $\dot{\pi}_t^D = 0$ locus.

As the path for the cross-country difference in the output gap again displays a hump-shape, (27) indicates that a hump-shaped trade surplus arises regardless of the value of the intratemporal elasticity η. As a result, in the empirically plausible case in which the intratemporal elasticity is above unity ($\eta > 1$), capital flows are topsy-turvy: throughout the stagflation episode, Home runs a trade deficit under free capital mobility, while it runs a trade surplus under optimal CFM. Furthermore, in the unit intratemporal elasticity case ($\eta = 1$), trade is balanced at all times under free capital mobility, while Home again runs a trade surplus under optimal CFM. This illustrates that the macroeconomic externality we point to can make external imbalances insufficiently volatile in response to shocks. This contrasts with most existing normative work on capital flows.

4.2 Quantitative Analysis

To further illustrate how the macroeconomic adjustments play out under both regimes, we now turn to presenting impulse response functions to an asymmetric markup shock in a calibrated version of the model. To do so, we draw heavily on the calibration used by Groll and Monacelli (2020) to study impulse responses to markup shocks. Rather than assuming a step function as in Section 4.1, we consider a more standard mean-reverting shock. In

Table 1: Calibration

Parameter	Description	Value/Target
ρ	Discount factor	0.04
α	Degree of trade openness	0.25
ε	Elasticity of substitution btw. differentiated goods	7.66
η	Elasticity of substitution btw. Home and Foreign goods	2
χ	Trade elasticity	3
ρ_δ	Probability of being able to reset price	$1 - 0.75^4$
ρ_μ	Persistence of Home markup shock	0.65

particular, we hit the economy with a markup shock of 10% that mean reverts at a rate of 0.42 per year, yielding an annual autocorrelation of 0.65 or, equivalently, a quarterly autocorrelation of 0.9.

The labor supply elasticity parameter ϕ is set to zero. The home bias parameter, α, is set to 0.25, which implies a degree of home bias of 0.5. The trade elasticity χ plays an important role for our results, as it determines the direction of the inefficiency and the scope for topsy-turvy capital flows, with high elasticities making topsy-turvy capital flows more likely. Simonovska and Waugh (2014) report a range of trade elasticity estimates from 2.69 to 4.47. We conservatively set χ near the lower bound of this range to $\chi = 3$, which implies an elasticity of substitution between domestic and foreign goods η of 2. Both the discount rate parameter, ρ, and the parameter for the probability of adjustment of nominal prices, ρ_δ, are set to standard values: $\rho = 0.04$ and $\rho_\delta = 1 - 0.75^4$. Finally, the elasticity of substitution among differentiated intermediate goods, ε, is set to 7.66, corresponding to a 15% net markup. All parameters are hence set to the same value as that in Groll and Monacelli (2020) (adjusting for our annual frequency), and we note that Condition 1 is met by a very wide margin ($\chi \gg 1 - 2\alpha$).

Figure 5 illustrates the difference in the response of macroeconomic variables to the markup shock under free capital mobility (solid lines) versus the response under optimal CFM (dashed lines). It is well understood that under free capital mobility, the efficient allocation cannot be achieved following markup shocks. To limit PPI inflation in Home following the shock, monetary policy engineers a recession in Home. This monetary policy response entails a terms of trade appreciation peaking at 9% and a positive spillover in Foreign, where the positive output gap nearly reaches 3%. Home runs a trade deficit of up to 3.5% of GDP. In the optimal CFM regime, opening a demand imbalance in favor of

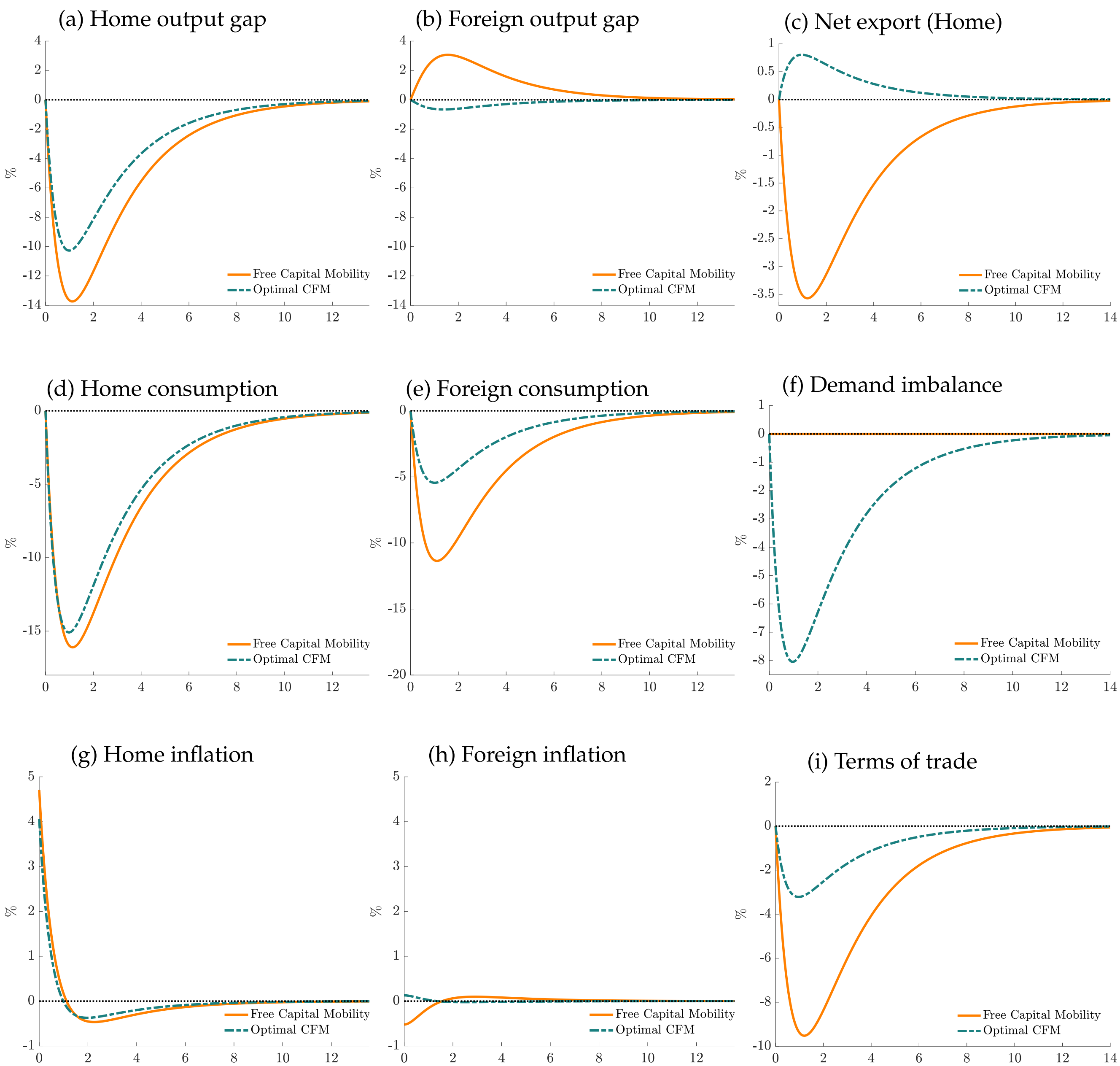

Figure 5: Impulse responses to a markup shock in Home.
Note: Solid lines represent free capital mobility, and dashed lines represent optimal CFM.

Foreign helps reduce the magnitude of the Home output gap and mitigate international relative price misalignment at the expense of distorting the intertemporal sharing condition. Despite this distortion, consumption appears to end up falling by less in both countries. Home runs a modest trade surplus rather than a sizable trade deficit, the Home output gap drops to -10% rather than to -13%, and the terms of trade appreciate by no more than 3%. The negative demand imbalance redirects demand toward Foreign, but the significantly smaller terms of trade appreciation results in a mildly negative Foreign output gap (rather than a positive one under free capital mobility). As a result, PPI inflation in Foreign is also more stable. Therefore, it is not a zero-sum game, and both countries achieve a superior stabilization of output and inflation under optimal CFM.

5 Discussion

In an effort to make our point as clear as possible, we have focused on the most basic specification of the open economy New Keynesian model. Enriching this basic model may create additional sources of externalities and inefficiencies, but as long as a wealth effect on labor supply is present and policy faces some output-inflation trade-off, our main insight regarding the poor functioning of international financial markets in a stagflation context can be expected to apply. In this section, we briefly discuss possible extensions of our analysis.

Additional Constraints on Monetary Policy. To streamline the implications of the output-inflation trade-off for the normative properties of a free capital mobility regime, we purposely abstracted from additional constraints on monetary policy. These include a lack of commitment to future policies (i.e., discretionary policy), a lack of international cooperation (i.e., non-cooperative policy setting), and a lack of monetary independence in the two countries (such as that resulting from a peg or a currency union). Such features would introduce extra constraints on stabilization policy, which capital inflows may contribute to loosen or tighten. Their presence would accordingly create distinct motives for financial market interventions, resulting in additional terms in the targeting rule for capital flow management (23).

Alternative Goods Pricing Specifications and Deviations from the Law of One Price. We assumed that export prices were sticky in producers' currency and that the law of one price held. Alternative assumptions regarding pricing currencies and deviations from the law of one price are known to place further constraints on monetary policy and accordingly give rise to more complex targeting rules than (21)-(22). Adopting these specifications would make other variables, such as the cross-country difference in consumer prices (also referred to as the average currency misalignment), relevant measures of the tightness of constraints on monetary policy, in addition to the output gap (see Engel, 2011). As a result, they would also yield additional terms in the targeting rule for capital flow management (23), without invalidating our main insight.

Non-cooperative Capital Flow Management. Our assumption that capital flow management policy is conducted cooperatively reflects our fundamental interest in understanding whether free capital flows help or hinder macroeconomic stabilization in a stagflation context *from the perspective of the world economy*. For the purposes of studying the related question of whether individual countries face incentives to actively manage capital flows in such a context, assuming non-cooperative policy-making may be more appropriate. In this case, capital flow management by individual countries would trade off the dynamic terms of trade manipulation motives stressed by Costinot, Lorenzoni and Werning (2014) with the macroeconomic stabilization motives we have emphasized here.

6 Conclusion

We point to a macroeconomic externality operating via firms' marginal costs in open economy models with nominal rigidities. For plausible values for the trade elasticity, this externality causes capital to flow in the wrong direction, following shocks that create an output-inflation trade-off: while an optimally managed capital account regime would require outflows from the regions where activity is the most depressed, a free capital mobility regime features capital inflows into such regions. Our results therefore cast doubt on the classical view that free capital mobility promotes macroeconomic adjustment, especially in a context of stagflation.

Our analysis has implications beyond open economy macroeconomics. Indeed, the insight that privately optimal financial decisions may worsen policy trade-offs via externalities operating on the economy's supply side ought to apply more generally to other heterogeneous agent, multi-sector macroeconomic models with nominal rigidities. Given the rising popularity of heterogeneous agent New Keynesian (HANK) models and current concerns about the possibility of stagflation, the study of such externalities appears to be a pressing issue for future research.

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APPENDIX TO "STAGFLATION AND TOPSY TURVY CAPITAL FLOWS"

A Data

This Appendix provides details on the sources and definitions of the data used in Figure 1.

Inflation (left panel of Figure 1) Data for 2020 and 2021 are annual CPI data from the World Bank's Development Indicators. Data for 2022 are from Bloomberg consensus forecasts. The Bloomberg tickers for the consensus CPI forecasts are as follows:

- For Canada, ECPICA 22 Index.
- For the United Kingdom, ECPIGB 22 Index.
- For Japan, ECPIJP 22 Index.
- For the United States, ECPIUS 22 Index.
- For the Euro area, ECPIR1 22 Index.

Policy rates (right panel of Figure 1) Policy rates are daily data from the BIS, available at https://www.bis.org/statistics/cbpol.htm.

- For Canada, the rate is the central bank target for the overnight rate.
- For the United Kingdom, the rate is the repo rate before August 3, 2006, and official bank rate from August 3, 2006 onwards.
- For Japan, the rate is the uncollateralized overnight call rate before April 2013 and the Japan cash rate (Complementary Deposit Facility Interest Rate) from April 2013 onward.
- For the United States, the rate is the midpoint of the Federal Reserve target rate.
- For the euro area, the rate is the main refinancing operations, minimum bid, before October 15, 2008, and official central bank liquidity providing, main refinancing operations, fixed rate, from October 15, 2008 onwards.

B Proofs

B.1 Derivation of the Loss Function

In this section, we write the objective function of the policy maker in terms of the squared output gap, squared inflation, squared terms of trade, and relative demand gap. The symmetrically weighted average of the period utility in the two countries is

$$v_t \equiv \frac{1}{2}\left[\log C_t - \frac{1}{1+\phi}(N_t)^{1+\phi}\right] + \frac{1}{2}\left[\log C_t^* - \frac{1}{1+\phi}(N_t^*)^{1+\phi}\right].$$

The loss relative to the efficient outcome is then $v_t - v^{max}$, where v^{max} is the maximized welfare, defined as welfare when C_t, C_t^*, N_t and N_t^* take on their efficient values. In what follows, we start by describing the efficient allocation and then turn to deriving a second-order approximation of the objective function.

Efficient Allocation. The socially optimal allocation solves the following static problem at each instant:

$$\max_{C_{H,t},C_{H,t}^*,C_{F,t},C_{F,t}^*,N_t,N_t^*} \frac{\eta}{\eta-1}\log\left[(1-\alpha)^{\frac{1}{\eta}}(C_{H,t})^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}}(C_{F,t})^{\frac{\eta-1}{\eta}}\right] - \frac{1}{1+\phi}(N_t)^{1+\phi}$$
$$+ \frac{\eta}{\eta-1}\log\left[(1-\alpha)^{\frac{1}{\eta}}(C_{F,t}^*)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}}(C_{H,t}^*)^{\frac{\eta-1}{\eta}}\right] - \frac{(N_t^*)^{1+\phi}}{1+\phi}$$

subject to

$$C_{H,t} + C_{H,t}^* = N_t, \tag{B.1}$$
$$C_{F,t} + C_{F,t}^* = N_t^*. \tag{B.2}$$

Let $\vartheta_{H,t}$ and $\vartheta_{F,t}$ denote the multipliers on (B.1) and (B.2). The first order conditions are

$$[C_{H,t}] :: \quad \vartheta_{H,t} = (1-\alpha)^{\frac{1}{\eta}}(C_{H,t})^{-\frac{1}{\eta}}(C_t)^{\frac{1}{\eta}-1} \tag{B.3a}$$
$$[C_{F,t}] :: \quad \vartheta_{F,t}^* = \alpha^{\frac{1}{\eta}}(C_{F,t})^{-\frac{1}{\eta}}(C_t)^{\frac{1}{\eta}-1} \tag{B.3b}$$
$$[C_{H,t}^*] :: \quad \vartheta_{H,t} = \alpha^{\frac{1}{\eta}}(C_{H,t}^*)^{-\frac{1}{\eta}}(C_t^*)^{\frac{1}{\eta}-1} \tag{B.4a}$$
$$[C_{F,t}^*] :: \quad \vartheta_{F,t}^* = (1-\alpha)^{\frac{1}{\eta}}(C_{F,t}^*)^{-\frac{1}{\eta}}(C_t^*)^{\frac{1}{\eta}-1} \tag{B.4b}$$
$$[N_t] :: \quad (N_t)^{\phi} = \vartheta_{H,t} \tag{B.5a}$$
$$[N_t^*] :: \quad (N_t^*)^{\phi} = \vartheta_{F,t}^*. \tag{B.5b}$$

Combining (B.3a) and (B.3b) after multiplying the first equation by $C_{H,t}$ and the second one by $C_{F,t}$, and proceeding similarly with (B.4a) and (B.4b), we arrive at

$$\vartheta_{H,t} C_{H,t} + \vartheta^*_{F,t} C_{F,t} = 1, \tag{B.6a}$$

$$\vartheta_{H,t} C^*_{H,t} + \vartheta^*_{F,t} C^*_{F,t} = 1. \tag{B.6b}$$

Substituting the resource constraints (B.1) and (B.2) into (B.5a) and (B.5b) yields $(N_t)^{1+\phi} + (N^*_t)^{1+\phi} = \vartheta_{H,t}(C_{H,t} + C^*_{H,t}) + \vartheta^*_{F,t}(C_{F,t} + C^*_{F,t})$, which, combined with (B.6a) and (B.6b), leads to

$$(N_t)^{1+\phi} + (N^*_t)^{1+\phi} = 2. \tag{B.7}$$

Using the resource constraints and symmetry, we arrive at

$$C^e_t = C^{*e}_t = N^e_t = N^{*e}_t = 1,$$

where variables with a superscript e denote efficient values. Finally, from the aggregate production functions, we have $Y^e_t = 1$ and $Y^{*e}_t = 1$. In logs, we therefore have

$$c^e_t = c^{*e}_t = n^e_t = n^{*e}_t = y^e_t = y^{*e}_t = 0. \tag{B.8}$$

Loss Function. The second-order approximation of the period utility around the non-distorted steady state (using $\bar{N}^{1+\phi} = 1$) is given by

$$v_t = -\frac{1}{1+\phi} + \frac{1}{2}\Big[(c_t + c^*_t) - (n_t + n^*_t) - \frac{1+\phi}{2}\left((n_t)^2 + (n^*_t)^2\right) + o\left(||u||^3\right)\Big], \tag{B.9}$$

where $+o\left(||u||^3\right)$ indicate the 3^{rd} and higher order terms left out. Note from (B.8) and (B.9) that $v^{max}_t = -\frac{1}{1+\phi}$. The period loss function is then

$$v_t - v^{max}_t = \frac{1}{2}\Big[(c_t + c^*_t) - (n_t + n^*_t) - \frac{1+\phi}{2}\left((n_t)^2 + (n^*_t)^2\right) + o\left(||u||^3\right)\Big] \tag{B.10}$$

We now use a second-order approximation of the aggregate demand equations and aggregate output to employment relation to substitute for c_t, c^*_t and n_t, n^*_t. First note that after substituting for the intertemporal sharing condition (8), the aggregate demand for Home goods can be rewritten as

$$Y_t = \left[(1-\alpha) + \alpha \left(S_t\right)^{1-\eta}\right]^{\frac{\eta}{1-\eta}} \left[(1-\alpha) + \alpha \Theta_t^{-1} Q_t^{\eta-1}\right] C_t.$$

Taking a second-order approximation around the non-distorted steady state, we get

$$\begin{aligned} y_t = c_t - \alpha\theta_t + \frac{\omega - (1-2\alpha)}{2} s_t + \frac{1}{2}\alpha(1-\alpha)(1-\eta)\eta(s_t)^2 \\ + \frac{1}{2}\alpha(1-\alpha)\left[\theta_t - (1-2\alpha)(\eta-1)s_t\right]^2 + o(||u||^3), \end{aligned} \tag{B.11}$$

where $\omega = \eta + (\eta-1)(1-2\alpha)^2$. Similarly, the aggregate demand for the Foreign good can be rewritten as

$$Y_t^* = \left[(1-\alpha) + \alpha \left(S_t\right)^{\eta-1}\right]^{\frac{\eta}{1-\eta}} \left[(1-\alpha) + \alpha \Theta_t Q_t^{1-\eta}\right] C_t^*,$$

and the second-order approximation is given by

$$\begin{aligned} y_t^* = c_t^* + \alpha\theta_t - \frac{\omega - (1-2\alpha)}{2} s_t + \frac{1}{2}\alpha(1-\alpha)(1-\eta)\eta(s_t)^2 \\ + \frac{1}{2}\alpha(1-\alpha)\left[\theta_t - (1-2\alpha)(\eta-1)s_t\right]^2 + o(||u||^3). \end{aligned} \tag{B.12}$$

We can combine (B.11) and (B.12) to obtain

$$\begin{aligned} c_t + c_t^* = y_t + y_t^* + \alpha(1-\alpha)(1-\eta)\eta(s_t)^2 \\ + \alpha(1-\alpha)\left[\theta_t - (1-2\alpha)(\eta-1)s_t\right]^2 + o(||u||^3). \end{aligned} \tag{B.13}$$

Using again (B.11) and (B.12), and after some algebraic manipulation, we get

$$\begin{aligned} (c_t)^2 + (c_t^*)^2 = (y_t)^2 + (y_t^*)^2 - 2\alpha(1-\alpha)\left(\eta\right)^2\left(s_t\right)^2 \\ + 2\alpha(1-\alpha)\left(\theta_t - (\eta-1)(1-2\alpha)s_t\right)^2 + o(||u||^3). \end{aligned} \tag{B.14}$$

Aggregate employment is given as $N_t = Y_t Z_t$, with $Z_t = \int_0^1 \left(P_{Ht(l)}/P_{Ht}\right)^{-\varepsilon} dl$. At the second-order approximation, $n_t = y_t + z_t + \frac{1}{2}y_t^2 + o(||u||^3)$, with $z_t = 0 + o(||u||^2)$. Thus, we have

$$n_t + n_t^* = y_t + y_t^* + \frac{1}{2}\left((y_t)^2 + (y_t^*)^2\right) + z_t + z_t^* + o(||u||^3) \tag{B.15}$$

$$(n_t)^2 + (n_t^*)^2 = (y_t)^2 + (y_t^*)^2 + o(||u||^3). \tag{B.16}$$

Plugging (B.13), (B.14), (B.15) and (B.16) into (B.10), we obtain the following second-order approximation of the period loss function:

$$v_t - v_t^{max} = \frac{1}{2}\Big[z_t + z_t^* + (1+\phi)(y_t)^2 + (1+\phi)(y_t^*)^2 + 2\alpha(1-\alpha)(1-\eta)\eta(s_t)^2 + 2\alpha(1-\alpha)\left(\theta_t - (\eta-1)(1-2\alpha)s_t\right)^2\Big] + o(||u||^3). \tag{B.17}$$

The objective of the policy maker is to minimize the loss function $\mathbb{L} = \int_0^\infty e^{-\rho t}(v_t - v_t^{max})dt$ where $v_t - v_t^{max}$ is given by (B.17). Then, using

$$\int_0^\infty e^{-\rho t} z_t dt = \int_0^\infty e^{-\rho t} \mathrm{var}_l \left(P_{H,t}(l)\right) dt = \frac{1}{\kappa}\int_0^\infty e^{-\rho t}\left(\pi_{H,t}\right)^2 dt,$$
$$\int_0^\infty e^{-\rho t} z_t^* dt = \int_0^\infty e^{-\rho t} \mathrm{var}_l \left(P_{F,t}^*(l)\right) dt = \frac{1}{\kappa}\int_0^\infty e^{-\rho t}\left(\pi_{F,t}^*\right)^2 dt,$$

and our definition of world and difference variables, we arrive at

$$\mathbb{L} = \frac{1}{2}\int_0^\infty e^{-\rho t}\Big[2\frac{\varepsilon}{\kappa}\left((\pi_t^W)^2 + (\pi_t^D)^2\right)^2 + 2(1+\phi)\left((y_t^W)^2 + (y_t^D)^2\right) + 2\alpha(1-\alpha)(1-\eta)\eta(s_t)^2 + 2\alpha(1-\alpha)\left(\theta_t - (\eta-1)(1-2\alpha)s_t\right)^2\Big], \tag{B.18}$$

which corresponds to (20).

B.2 Optimal policy problem

We divide the loss (20) by a factor 2, since we can equivalently minimize a linear transformation of the objection function of the global planner. The optimal monetary policy problem is given by

$$\min_{\{\pi^W,\pi^D,x^W,y^D,s\}} \frac{1}{2}\int_0^\infty e^{-\rho t}\Big[\frac{\varepsilon}{\kappa}\left((\pi_t^W)^2 + (\pi_t^D)^2\right) + (1+\phi)\left((y_t^W)^2 + (y_t^D)^2\right) + \alpha(1-\alpha)(1-\eta)\eta(s_t)^2 + \alpha(1-\alpha)\left(\theta_t - (\eta-1)(1-2\alpha)s_t\right)^2\Big]$$

subject to

$$\dot{\pi}_t^W = \rho\pi_t^W - \kappa(1+\phi)y_t^W - \kappa u_t^W, \tag{B.19}$$
$$\dot{\pi}_t^D = \rho\pi_t^D - \kappa(1+\phi)y_t^D + \kappa\frac{\omega-1}{2}s_t - \kappa\alpha\theta_t - \kappa u_t^D, \tag{B.20}$$
$$2y_t^D = \omega s_t + (1-2\alpha)\theta_t. \tag{B.21}$$

Letting φ_t^W, φ_t^D, be the co-state variables associated with (B.19) and (B.20), the first-order conditions are

$$
\begin{aligned}
\left[\pi_t^W\right] &:: \quad \dot{\varphi}_t^W = -\frac{\varepsilon}{\kappa}\pi_t^W, && \text{(B.22)}\\
\left[\pi_t^D\right] &:: \quad \dot{\varphi}_t^D = -\frac{\varepsilon}{\kappa}\pi_t^D, && \text{(B.23)}\\
\left[y_t^W\right] &:: \quad 0 = -(1+\phi)y_t^W + \kappa(1+\phi)\varphi_t^W, && \text{(B.24)}\\
\left[y_t^D\right] &:: \quad 0 = -(1+\phi)y_t^D + \kappa(1+\phi)\varphi_t^D - \Lambda_t, && \text{(B.25)}\\
[s_t] &:: \quad 0 = -(\omega-1)y_t^D + \kappa(\omega-1)\varphi_t^D - \omega\Lambda_t, && \text{(B.26)}\\
[\theta_t] &:: \quad 0 = -\alpha(1-\alpha)\theta_t + \frac{\omega-1}{4}(1-2\alpha)s_t + \kappa\alpha\sigma\varphi_t^D + \frac{1}{2}(1-2\alpha)\Lambda_t, && \text{(B.27)}
\end{aligned}
$$

together with the initial conditions $\varphi_0^j = 0$ and transversality conditions $\lim_{t\to\infty} e^{-\rho t}\varphi_t^j = 0$ for $j \in \{W, D\}$, and where Λ_t is the Lagrange multiplier on (B.21).

B.2.1 Proof of Proposition 1

Combining (B.25) and (B.26) we have $\Lambda_t = 0$. Substituting this back into (B.25), we obtain

$$
y_t^D - \kappa\varphi_t^D = 0. \tag{B.28}
$$

Differentiating (B.24) and (B.28) with respect to time and noting from (B.22) and (B.23) that $\kappa\dot{\varphi}_t^W = -\varepsilon\pi_t^W$ and $\kappa\dot{\varphi}_t^D = -\varepsilon\pi_t^D$, we obtain

$$
\begin{aligned}
\dot{y}_t^W + \varepsilon\pi_t^W &= 0, && \text{(B.29)}\\
\dot{y}_t^D + \varepsilon\pi_t^D &= 0.
\end{aligned}
$$

From (B.24), $y_t^W = \kappa\varphi_t^W$, and given that $\varphi_0^W = 0$, we have $y_0^W = 0$. From (B.28) and (B.21), we have $y_0^D = 0$ and $2y_0^D + \omega s_0 = 0$, which imply that $y_0^D = s_0 = 0$. Thus, integrating between 0 and t we arrive at

$$
\begin{aligned}
y_t^W + \varepsilon(p_t^W - p_0^W) &= 0, && \text{(B.30)}\\
y_t^D + \varepsilon(p_t^D - p_0^D) &= 0. && \text{(B.31)}
\end{aligned}
$$

B.2.2 Proof of Corollary 1

We consider the targeting rule (B.29) for world variables and differentiate this rule to obtain $\ddot{y}_t^W + \varepsilon \dot{\pi}_t^W = 0$. We then use (B.19), $\dot{\pi}_t^W = \rho \pi_t^W - \kappa(1+\phi) y_t^W - \kappa u_t^W$, to substitute for $\dot{\pi}_t^W$ and obtain

$$\ddot{y}_t^W - \rho \dot{y}_t^W - \varepsilon\kappa(1+\phi) y_t^W = \varepsilon\kappa u_t^W. \tag{B.32}$$

The characteristic polynomial of this equation has one negative eigenvalue $z_1 < 0$ and one positive eigenvalue $z_2 > 0$ where

$$z_1 = \frac{1}{2}\left(\rho - \sqrt{\rho^2 + 4\kappa\varepsilon(1+\phi)}\right) < 0 \quad \text{and} \quad z_2 = \frac{1}{2}\left(\rho + \sqrt{\rho^2 + 4\kappa\varepsilon(1+\phi)}\right) > 0.$$

The solution of this second-order differential equation takes the form

$$y_t^W = \vartheta_0 e^{z_1 t} + \vartheta_1 \int_0^t e^{z_1(t-s)} u_s^W ds + \vartheta_2 \int_t^\infty e^{z_2(t-s)} u_s^W ds. \tag{B.33}$$

Differentiating (B.33) and relating each term to (B.32), we obtain

$$\vartheta_1 = \vartheta_2 = -\frac{\varepsilon\kappa}{z_2 - z_1}.$$

Next, from (B.33) for $t = 0$, we get

$$\vartheta_0 = y_0^W + \frac{\varepsilon\kappa}{z_2 - z_1} \int_0^\infty e^{-z_2 s} u_s^W ds.$$

From the initial condition for the co-state variable $\varphi_0^W = 0$, the relation $y_t^W = \kappa \varphi_t^W$ implies that $y_t^W = 0$. The solution to the optimal monetary policy problem is thus

$$y_t^W = -\frac{\varepsilon\kappa}{z_2 - z_1}\left[e^{z_1 t}\int_0^t \left(e^{-z_1 s} - e^{-z_2 s}\right) u_s^W ds + \left(e^{z_2 t} - e^{z_1 t}\right)\int_t^\infty e^{-z_2 s} u_s^W ds\right]. \tag{B.34}$$

Using (B.29), the path for the world inflation under the optimal monetary policy satisfies

$$\pi_t^W = \frac{\kappa}{z_2 - z_1}\left[z_1 e^{z_1 t}\int_0^t \left(e^{-z_1 s} - e^{-z_2 s}\right) u_s^W ds + \left(z_2 e^{z_2 t} - z_1 e^{z_1 t}\right)\int_t^\infty e^{-z_2 s} u_s^W ds\right]. \tag{B.35}$$

From (B.34) and (B.35), it follows that the paths of the world variables y_t^W and π_t^W are independent of the path of θ_t.

B.2.3 Proof of Proposition 2

We start by combining (B.25) and (B.26) to obtain $\Lambda_t = 0$. Substituting it into the optimality condition (B.27), we arrive at

$$\begin{aligned} 2\alpha(1-\alpha)\theta_t &= (1-2\alpha)\frac{\omega-1}{2}s_t + 2\kappa\alpha\varphi_t^D \\ &= (1-2\alpha)\frac{\omega-1}{2}s_t + 2\alpha y_t^D, \end{aligned} \tag{B.36}$$

where the second equality uses (B.28). We then plug equation (B.21) into (B.36) to substitute for y_t^D. We get

$$\begin{aligned} 2\alpha(1-\alpha)\theta_t &= (1-2\alpha)\frac{\omega-1}{2}\left[\frac{2}{\omega}y_t^D - \left(\frac{1-2\alpha}{\omega}\right)\theta_t\right] + 2\alpha y_t^D \\ &= \frac{\omega-(1-2\alpha)}{\omega}y_t^D - (1-2\alpha)^2\frac{\omega-1}{2\omega}\theta_t. \end{aligned} \tag{B.37}$$

Rearranging the expression (B.37) leads to

$$\begin{aligned} \frac{1}{2\omega}\left[\omega-(1-2\alpha)^2\right]\theta_t &= \frac{\omega-(1-2\alpha)}{\omega}y_t^D \\ \frac{\alpha}{\omega}\chi\theta_t &= \frac{2\alpha}{\omega}\left[\chi-(1-2\alpha)\right]y_t^D, \end{aligned} \tag{B.38}$$

where we use $\omega = 2\alpha\chi - (1-2\alpha)^2$ to obtain the second equality (B.38). Finally, we simplify the above expression (B.38) and arrive at

$$\theta_t = \frac{\chi-(1-2\alpha)}{\chi}2y_t^D. \tag{B.39}$$