Can Deficits Finance Themselves?*

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Abstract

We ask how fiscal deficits are financed in environments with two key features: (i) nominal rigidity and (ii) a violation of Ricardian equivalence due to finite lives or liquidity constraints. In such environments, fiscal deficits can contribute to their own financing by inducing a demand-driven Keynesian boom, raising both prices (which erodes the real debt burden) and real economic activity (which expands the tax base for given tax rates). We show that the extent of such self-financing increases with the delay in fiscal adjustment: pushing any tax hike further into the future helps generate a larger and more persistent boom, which in turn reduces the necessary tax hike. In fact, full self-financing is possible in the limit as fiscal adjustment is delayed more and more: the government can run a deficit now without having to raise tax rates ever in the future. We argue that a significant degree of self-financing is a robust feature of a large family of models with our two key features (e.g., HANK), as well as quantitatively potent and thus practically relevant.

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1 Introduction

Suppose the government runs a deficit today in order to stimulate aggregate demand. Suppose further that the steady-state rate of interest is positive—or more precisely, that it is higher than the steady-state growth rate of output—, so that there is no "free lunch" of the type considered in the recent "$r < g$" literature (e.g., Blanchard, 2019). What does the government have to do in order to make sure that public debt ultimately returns to back to baseline after the original policy change?

Perhaps the most obvious answer is *fiscal adjustment*, or *fiscal consolidation*: sooner or later, the government must adopt a package of tax hikes and/or spending cuts in order to pay down the accumulated debt. In this paper, we investigate a different margin—what we refer to as the *self-financing* of fiscal deficits. The basic idea is simple and familiar. Insofar as a deficit triggers a boom, it can contribute to its own financing in two complementary ways: by expanding the tax base, which in turn helps generate more tax revenue even without any tax hike; and by raising nominal prices, which helps reduce the real debt burden. The goal of this paper is to shed light on the theoretical properties and the quantitative relevance of this "self-financing" mechanism.[1]

We start by showing that *some* self-financing obtains robustly in environments that combine two key features: a failure of Ricardian equivalence due to finite horizons or liquidity constraints, so that debt and deficits can drive aggregate demand; and nominal rigidity, so that aggregate demand can in turn drive real economic activity. We next show that the degree of self-financing—the fraction of the initial deficit that pays for itself—depends crucially on the horizon of fiscal adjustment. In particular, pushing the tax hike further into the future helps raise both the magnitude and the persistence of the boom triggered by the initial deficit, thus contributing to more self-financing.

Pushing this logic to its limit, we show that even *full* self-financing is possible: as the fiscal adjustment is delayed more and more, the required tax hike becomes vanishingly small—that is, the deficit ultimately pays for itself. Importantly, this is true even in the absence of any monetary accommodation: the monetary authority can be "neutral"—in the sense of fixing the expected real rate—or even somewhat lean against the boom, yet our limiting self-financing result still obtains. Our contribution is completed by showing that a quantitatively meaningful degree of self-financing is consistent with realistic calibrations of the relevant parameters, notably: the extent of deviation from the permanent income hypothesis; the degree of nominal rigidity; and the delay in fiscal adjustment.

[1] The first of the two channels of self-financing—the expansion of the tax base—brings to mind the classical Keynesian cross and especially the paper by DeLong and Summers (2012). The second channel—the erosion of the real value of outstanding nominal public debt—is reminiscent of the classical Fiscal Theory of the Price Level. We expand on these relations in due course. For now, it suffices to note the following. First, unlike DeLong and Summers (2012), our analysis is grounded on an actual micro-founded model (rather than merely presenting reduced-form fiscal multiplier arithmetic). And second, unlike the FTPL, the equilibrium we study in this paper does not hinge on any sunspots or off-equilibrium threats to violate the government budget, and is fully consistent with the Taylor principle.

Environment. Our model is the same as that of the New Keynesian textbook, except for a key change in the demand block: we replace the representative, infinitely-lived consumer with overlapping generations of perpetual-youth consumers (OLG à la Blanchard, 1985). In any given period an existing consumer dies with probability $\omega \in (0, 1]$ and then gets replaced by a newborn consumer. When $\omega = 1$, our model reduces to the standard permanent-income representative-agent (PIH-RANK) framework. When instead $\omega < 1$, our model shares two key properties with state-of-the-art heterogeneous-agent (HANK) models: (i) consumers discount future tax hikes more heavily than in the PIH benchmark; and (ii) they have a larger short-run propensity to consume (MPC). As will become clear, our results on self-financing stimulus derive not from the OLG structure per se, but rather from these two more general and empirically relevant properties of consumer demand. The supply block remains the same as in the textbook New Keynesian model and boils down to a Phillips curve (NKPC).

The model is closed with a government, consisting of a fiscal and a monetary authority. The fiscal authority raises revenue through (i) a time-invariant proportional tax on income, at rate $\tau_y \in (0, 1)$; and (ii) additional lump-sum taxes, which can vary with public debt and deficits.[2] Our policy experiment is a date-0 increase in the fiscal deficit, paid out as a lump-sum transfer to all the households that are alive at date 0. A fiscal rule specifies when and how future lump-sum taxes will adjust so as to make sure that the fiscal authority remains solvent and that public debt ultimately returns back to its original level. That is, the government commits to hiking taxes in the future as needed, though potentially with a significant delay. Finally, the monetary authority sets the nominal rate of interest. Thanks to nominal rigidities, this allows the monetary authority to regulate the (expected) real interest rate. For the bulk of our analysis, we will consider the benchmark of a "neutral" monetary authority that neither accommodates fiscal deficits by reducing real rates nor leans against Keynesian booms by raising real rates. That is, we study the dynamics of aggregate spending, aggregate income, and government debt under a constant (expected) real interest rate. We later show how the possibility of full self-financing is robust to a more hawkish monetary authority, provided that expected real rates do not increase too much in response to the fiscal deficit.

Self-financing. We initialize the economy at its steady state and consider a positive innovation in the period-0 deficit. The question of interest is what fraction of this deficit can be self-financed, and how exactly such self-financing will be accomplished. When $\omega = 1$ (i.e., PIH-RANK), Ricardian equivalence holds and debt and deficits do not affect aggregate demand. As a result, no self-financing is possible.[3]

[2] Our baseline analysis assumes that distortionary taxes are time-invariant and that lump-sum taxes change over time, for analytical clarity. We show later that our limiting self-financing results do not depend on this assumption.

[3] To be precise, this statement is true as long as we rule out a class of sunspot equilibria where debt and deficits drive aggregate demand because, and only because, of a self-fulfilling prophecy. Such a self-fulfilling prophecy is the essence of the FTPL and can be ruled out by the Taylor principle and/or by appropriate perturbations of the information structure (Angeletos and Lian, 2023). The equilibrium we study in this paper does not rely on this or any other self-fulfilling

When $\omega < 1$, a higher deficit helps stimulate aggregate demand because it represents a net transfer from future generations to current generations (or, less literally, because it helps some consumers overcome liquidity constraints). Because of the nominal rigidity, the increase in aggregate demand in turn translates to an increase in real income and, via the NKPC, to an increase in the nominal price level. The government thus benefits from our two forms of self-financing: it collects more tax revenue even without any fiscal adjustment (as long as $\tau_y > 0$); and it enjoys a lower real debt burden (as long as prices are not fully rigid). Together, these effects mean that at least *some* of the deficit is self-financed. We let ν denote the fraction of the initial deficit that is self-financed via the aforementioned two channels, and we ask how ν varies with our assumptions on fiscal policy.

Our headline result is that the degree of self-financing ν is increasing in the horizon of fiscal adjustment, ultimately converging to *full* self-financing (i.e., $\nu = 1$). In other words, delaying the tax hike helps reduce the tax hike, with a limit of zero—i.e., no fiscal adjustment is needed. Intuitively, a longer delay in financing increases the impact increase in demand (because the future fiscal adjustment is discounted more) and leads to larger general equilibrium amplification via the "Intertemporal Keynesian Cross" (Auclert, Rognlie and Straub, 2018). As the fiscal adjustment is delayed more and more, debt returns to trend on its own, the required future tax hike vanishes, and the deficit pays for itself.

The economic intuition for this self-financing result can be understood through analogy with a simple two-period economy. In this economy the fiscal authority pays out a transfer to households at date $t = 0$, and then—if needed—increases taxes at $t = 1$ to return government debt to its steady state level. If date-0 prices are fully rigid, and if date-0 demand is independent of what happens at date 1 (including, in particular, future taxes), then the date-0 response of output to a one-unit transfer is just

$$y = \frac{\text{MPC}}{1 - \text{MPC} \times (1 - \tau_y)},$$

where MPC is the household marginal propensity to consume. Endogenous date-0 tax revenue is thus

$$\frac{\text{MPC}}{1 - \text{MPC} \times (1 - \tau_y)} \cdot \tau_y.$$

We see that, if $\text{MPC} \to 1$, then the endogenous tax revenue raised at date 0 would suffice to fully pay for the initial deficit, without any need to actually hike taxes at date 1. Our full dynamic economy echoes the intuition from this simple example. Consider first the direct increase in demand induced by the transfer. Discounting—i.e., the fact that $\omega < 1$—then implies the following: first, that the households that receive the initial transfer do not respond to the far-ahead future tax hike; and second, that they spend their transfer money relatively quickly. The transfer thus delivers a short-run demand increase with net present value close to 1—i.e., the short-run *cumulative* MPC approaches 1. This demand

prophecy, is robust to both the Taylor principle and the aforementioned perturbations, and is a direct extension of the New Keynesian model's "fundamental" solution from $\omega = 1$ to $\omega < 1$.

increase then leads to higher income which—again because $\omega < 1$—is also spent quite quickly, thus through the Keynesian cross delivering a front-loaded, short-lived boom with a multiplier approaching $\frac{1}{\tau_y}$. The boom thus endogenously raises enough tax revenue to stabilize debt *before* the promised future tax hike, exactly as in the two-period example. But then this tax hike vanishes, simply because government debt has already returned to trend. This is the essence of our self-financing result.

The above discussion has assumed perfectly rigid prices and thus emphasized the tax-base channel. Allowing for prices to be sticky (i.e., neither perfectly rigid nor perfectly flexible) changes very little—self-financing still obtains in the limit of delayed fiscal adjustment, just now with both the tax base channel and the inflation/debt erosion channel operative. In particular, the greater the degree of price flexibility, the larger the share of self-financing that obtains through a date-0 jump in prices.

Relation to FTPL. Our self-financing result shares with the Fiscal Theory of the Price Level (FTPL) the following flavor: deficits are financed not through outright fiscal adjustment but through equilibrium responses of prices (or quantities). We emphasize, however, that the mechanisms behind our result are actually very different from those behind the classical FTPL.

The typical formulation of the FTPL (see Sims, 1994; Leeper, 1991; Woodford, 1995; Bassetto, 2008; Cochrane, 2023) maintains the assumption of a representative, infinitely-lived, rational consumer, similarly to Barro (1974). In this environment, Ricardian equivalence can break—and self-financing can thus obtain—only by force of equilibrium selection: consumers are rational enough to understand that government debt is not net wealth; nevertheless, as long as the Taylor principle is violated, consumers can coordinate on an equilibrium in which prices (or output) adjust with public debt and deficits so as to achieve fiscal balance. In our environment, this mechanism is never at play. Instead, our self-financing result is grounded on a classical failure of Ricardian equivalence—one due to finite horizons or liquidity constraints. Unlike those of the FTPL, our conclusions are thus robust to both the kind of information perturbations considered in Angeletos and Lian (2023) and to a monetary authority that satisfies the Taylor principle, as discussed further below.

Generality and extensions. Our core result on the possibility of self-financing deficits is general, reflecting the simplicity and robustness of the underlying economic intuition.

Our first two extensions alter the assumptions on the conduct of policy. First, we show that nothing changes if the promised future tax hike is instead distortionary. The argument is trivial: under our assumptions, this tax hike will never materialize in equilibrium, and so our original construction of a self-financing equilibrium applies without change. Second, we consider what happens if the monetary authority moves away from our "neutral" benchmark, either accommodating or leaning against the deficit-induced boom. Intuitively, if the monetary authority accommodates (i.e., cuts real rates), then households are incentivized to front-load spending even further, so the Keynesian boom is even quicker, and convergence to the self-financing equilibrium is even faster. Conversely, if real rates are increased, then convergence is slower. In particular, we show that, if the monetary response is sufficiently aggressive in leaning against the boom, then convergence becomes so slow that a self-financing equilibrium ceases to exist. Equivalently, for such a monetary policy, equilibrium existence requires that fiscal adjustment is sufficiently quick.

Our second set of extensions generalizes the model environment. First, we show that the self-financing result continues to apply for aggregate demand blocks beyond the OLG structure. In particular, we provide a general set of sufficient conditions on MPCs out of current income, future income, and wealth that allow us to obtain our self-financing result. Echoing the discussion above, we again require "discounting"—ensuring that consumers spend their transfer receipts quickly and respond rather little to expectations of future taxes—and sufficiently front-loaded MPCs—ensuring a quick Keynesian boom. Second, we repeat our analysis in an environment with investment. The main insight here is that the "Keynesian cross"-type logic sketched above continues to apply, just now to total output less investment. And third, we show that our findings readily extend to fiscal stimulus in the form of government purchases (rather than transfers).

Quantitative relevance. As the final step in our analysis we ascertain the practical relevance of our self-financing result. Our theoretical analysis has revealed that the two model ingredients that will matter most are: (i) the deviation from permanent-income consumer behavior; and (ii) the delay in fiscal financing. Prior empirical work suggests that, even in classical "passive" fiscal regimes, fiscal adjustment tends to be very gradual (e.g., Galí, López-Salido and Vallés, 2007; Bianchi and Melosi, 2017; Auclert and Rognlie, 2018). On the consumer side, we enrich our baseline OLG model to also feature a margin of hand-to-mouth spenders; this extension is simple enough to fit into our generalized aggregate demand block mentioned above, yet rich enough to very closely approximate aggregate consumption behavior in HANK-type environments (Auclert, Rognlie and Straub, 2018; Wolf, 2021*a*). The model is disciplined using empirical evidence on household consumption in response to lump-sum income gains (Fagereng, Holm and Natvik, 2021), including at relatively far-out horizons.

We then use our quantitative model to study the effects of a one-off, deficit-financed lump-sum transfer to households—a policy reminiscent of the various rounds of "stimulus checks" seen recently in the U.S. Our headline finding is that, in equilibrium, such policies will indeed turn out to be meaningfully self-financed, inducing a large and persistent output boom and high inflation.[4] We note that those results are informative about the recent "excess savings" debate after deficit-driven stimulus checks: if households have finite horizons or face binding liquidity constraints, then our theory pre-

[4]For completeness, we show that our results extend with little change to a numerically solved HANK-type environment, confirming the findings of Auclert, Rognlie and Straub (2018) and Wolf (2021*a*).

dicts that such excess savings can induce long-lasting demand pressure. Depending on the extent of nominal rigidities this demand pressure will then feed through to either prices or quantities.

Literature. Our analysis relates and contributes to several strands of literature.

First, we add to prior work that considers the possibility of "self-financing" fiscal deficits. In the classical FTPL, deficits self-finance through a jump in the price level (Leeper, 1991; Woodford, 1995; Bassetto, 2002; Cochrane, 2005). In our theory, self-financing solely through a date-0 price-level jump occurs in the limit of vanishing pricing frictions. Relative to the conventional FTPL literature, however, our analysis sidesteps the controversies referenced above—we merely require finite *delays* in fiscal financing, and our results do not at all hinge on questions of equilibrium selection. Our New-Keynesian environment also allows us to emphasize a complementary, "real" source of self-financing: an output boom leading to an increase in the fiscal tax-and-transfer balance. The possibility of fiscal deficits paying for themselves in this way is discussed prominently in DeLong and Summers (2012). While those authors provide the arithmetic on fiscal multipliers necessary to generate such self-financing without a micro-founded model, our analysis characterizes explicit conditions on economic primitives required to actually see such self-financing.

Second, we contribute to recent work on the aggregate effects of stimulus check policies (Auclert, Rognlie and Straub, 2018; Wolf, 2021*a*,*b*). Relative to this prior work, our key contribution is to explicitly characterize the *limiting* effects of such stimulus checks as fiscal financing is delayed further and further. Our results are thus particularly relevant for the "excess savings" debate surrounding recent stimulus check policy experiments in the U.S.: those checks were sent out without any commitment to medium-term financing, so our theory predicts—qualitatively consistent with recent empirical evidence—protracted booms in inflation and output.

Third, we add to a fast-growing literature on the macroeconomic consequences of deviations from permanent-income consumer behavior (e.g., Galí, López-Salido and Vallés, 2007; Hagedorn, Manovskii and Mitman, 2019; Kaplan, Moll and Violante, 2018; Auclert, 2019; Bilbiie, 2020; Aggarwal et al., 2022; Mian, Straub and Sufi, 2022). It is well-known in that literature that fiscal deficits can induce demand-led booms, and thus have the potential to contribute to their own financing. Our contribution is to zero in on the question of fiscal financing and highlight that deficits in fact have the potential to fully finance themselves, with future tax adjustments becoming vanishingly small.

Outline. Sections 2 and 3 begin by presenting the model and characterizing its equilibrium. Section 4 then discusses our core self-financing result. Further extensions and the quantitative analysis follow in Sections 5 and 6, respectively. Finally Section 7 concludes, and supplementary results as well as proofs are relegated to several appendices.

2 Model

For our main analysis we consider a perpetual-youth, overlapping-generations version of the textbook New Keynesian model. Similarly to Del Negro, Giannoni and Patterson (2015), Farhi and Werning (2019), and Angeletos and Huo (2021), mortality risk (finite lives) is a convenient proxy for liquidity frictions: it breaks Ricardian Equivalence and lets fiscal policy—i.e., debt and deficits—affect aggregate demand. As will become clear, this departure from permanent-income consumer behavior is central to our results. We will show later how the insights obtained from our baseline model extend to more general aggregate demand structures, including those found in the HANK literature.[5]

Throughout, we study our economy's log-linearized dynamics in response to a surprise increase in fiscal deficits. We use uppercase variables to indicate levels; unless indicated otherwise, lowercase variables denote log-deviations from the economy's deterministic steady state. Time is discrete.

2.1 Households

We index households by $i = (i_1, i_2)$ where $i_1 \in \{0, 1, \ldots\}$ denotes their age and $i_2 \in [0, 1]$ their name. A household survives from one period to the next with probability $\omega \in (0, 1]$, so that $1 - \omega$ is the mortality rate. Whenever a household dies, it is replaced by a new household (with the same name i_2 but age reset to $i_1 = 0$). Households do not altruistically value the utility of the future households that replace them. Taking into account this mortality risk, the expected lifetime utility of any (alive) household i in period $t \in \{0, 1, \ldots\}$ is therefore given by

$$\mathbb{E}_t \left[\sum_{k=0}^{\infty} \left(\beta\omega\right)^k \left[u(C_{i,t+k}) - v(L_{i,t+k}) \right] \right], \tag{1}$$

where $C_{i,t+k}$ and $L_{i,t+k}$ denote household i's consumption and labor supply in period $t + k$ (conditional on survival), and preferences take the standard form $u(C) \equiv \frac{C^{1-1/\sigma}-1}{1-\frac{1}{\sigma}}$ and $v(L) = \chi \frac{L^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}$.

Households can save and borrow by trading an actuarially fair, risk-free nominal annuity. Conditional on survival, households therefore enjoy a nominal rate of return equal to I_t/ω, where I_t is the nominal rate on government bonds. Households furthermore receive labor income and dividend income, given respectively by $W_t L_{i,t}$ and $Q_{i,t}$ (in real terms). Households also pay taxes. This real tax payment $T_{i,t}$ depends on both on the individual's income and on aggregate fiscal conditions; namely, $T_{i,t} = \mathscr{T}(Y_{i,t}, Z_t)$, where $Y_{i,t} \equiv W_t L_{i,t} + Q_{i,t}$ is the household's total real income, Z_t captures aggregate conditions (including outstanding government debt), and $\mathscr{T}$ is a function describing tax policy (to be specified later). Finally, old households are obliged to make contributions to a "social fund" whose

[5] Also note that, consistent with Woodford (2003*b*) and Galí (2008), we will consider a "moneyless" limit. There is thus no seignorage revenue and the channel in Sargent and Wallace (1981) will not help finance deficits.

proceeds are distributed to the newborn households; the role of this fund is explained momentarily. All in all, the date-t budget constraint of household i is therefore given as

$$A_{i,t+1} = \underbrace{\frac{I_t}{\omega}}_{\text{annuity}} \left(A_{i,t} + P_t \cdot \big(\underbrace{W_t L_{i,t} + Q_{i,t}}_{Y_{i,t}} - C_{i,t} - T_{i,t} + S_{i,t}\big)\right), \tag{2}$$

where $A_{i,t}$ denotes i's nominal saving at the beginning of date t, P_t is the date-t price level, and $S_{i,t}$ is the transfer from or contribution to the fund, with $S_{i,t} = S^{\text{new}} > 0$ for newborns and $S_{i,t} = S^{\text{old}} < 0$ for old households (and where $(1-\omega)S^{\text{new}} + \omega S^{\text{old}} = 0$, ensuring that the social fund is balanced).

Compared to Blanchard (1985), the only novelty in our set-up is the social fund. We will set $S^{\text{new}} = D^{ss}$ (and therefore $S^{\text{old}} = -\frac{1-\omega}{\omega} D^{ss}$), where D^{ss} is the real steady-state value of public debt (and so in equilibrium also the real steady-state value of private wealth). The fund thus ensures that all cohorts, regardless of age, enjoy the same wealth and hence the same consumption in steady state. This in turn affords two simplifications. First, it simplifies aggregation when we log-linearize the model around its steady state, with every cohort equally weighted in aggregate demand. Second, it guarantees that the steady state of our model is invariant to both ω and the steady-state level of public debt, and hence is exactly the same as its RANK counterpart. The two models thus differ only in terms of how fiscal policy influences output gaps, cleanly isolating the mechanism that we are interested in.

It remains to specify how household income is determined. First, we assume that all households receive identical shares of aggregate dividends. Second, we also abstract from heterogeneity in labor supply. Specifically, we assume that labor supply is intermediated by labor unions. Those unions bargain on behalf of households, equalizing the (post-tax) real wage and the marginal rate of substitution between consumption and labor supply. Since all households work the same hours, it follows that all households receive the same labor income. Overall, the resulting household labor supply relation is exactly the same as in the textbook New Keynesian model.[6] Putting the pieces together, we conclude that $Y_{i,t} = Y_t$ and therefore also $T_{i,t} = T_t$,—all households receive identical post-tax income.

2.2 Firms

The production side of the economy is exactly the same as in the textbook New Keynesian model (e.g., Galí, 2008): there is a unit-mass continuum of monopolistically competitive retailers who set prices subject to a standard Calvo friction, hire labor from the aforementioned spot market, and pay out all their profits as dividends back to households. Combined with our assumptions on household labor supply this implies that the supply side of our economy reduces to a standard Phillips curve (NKPC).

[6] To be precise, this statement presumes that the tax distortion is time-invariant, which is indeed the case under our upcoming specification for taxes. Further details on the labor supply block of our economy are provided in Appendix A.1.

2.3 Policy

The government consists of two blocks: a fiscal authority issuing debt and setting taxes, and a monetary authority setting nominal interest rates.

Fiscal policy. We abstract from government spending and let B_t denote the total nominal public debt outstanding at the beginning of period t. We can then write the nominal flow budget constraint of the government as follows:

$$\frac{1}{I_t} B_{t+1} = B_t - P_t T_t. \tag{3}$$

where $T_t \equiv \int T_{i,t} di$ is the total real tax revenue at t. Letting $D_t \equiv B_t / P_t$ denote the *real* value of public debt, $\Pi_{t+1} \equiv P_{t+1}/P_t$ the realized inflation between t and $t+1$, and $R_t \equiv I_t / \mathbb{E}_t [\Pi_{t+1}]$ the (expected) real rate at t, we can rewrite the above as

$$D_{t+1} = R_t (D_t - T_t) \left(\frac{\mathbb{E}_t [\Pi_{t+1}]}{\Pi_{t+1}} \right).$$

We see that an inflation surprise eases fiscal space by eroding the real value of public debt.

We log-linearize around a steady state in which inflation is zero, real allocations are given by their flexible-price counterparts, and the real debt burden is constant at some arbitrary level D^{ss}. Thanks to the annuities (which offset the mortality risk) and the social fund (which makes sure that all cohorts enjoy identical wealth and consumption in steady state), the steady-state real rate is the same as in the analogous RANK model—that is, $R^{ss} = \frac{1}{\beta} > 1$.[7] Steady-state taxes then satisfy $T^{ss} = (1-\beta) D^{ss}$, where, as already noted, D^{ss} denotes the (arbitrary) steady-state level of public debt. We will focus on the empirically relevant scenario with $D^{ss} > 0$. Nonetheless, to accommodate $D^{ss} = 0$, we let $d_t \equiv (D_t - D^{ss}) / Y^{ss}$, $b_t \equiv (B_t - B^{ss}) / Y^{ss}$, and $t_t \equiv T_t / Y^{ss}$; that is, we measure fiscal variables in terms of absolute deviations (rather than log-deviations) from steady state, scaled by steady-state output. Rewriting (3) in real terms and linearizing, we obtain

$$d_{t+1} = \underbrace{\frac{1}{\beta}(d_t - t_t) + \frac{D^{ss}}{Y^{ss}} r_t}_{\text{expected debt burden tomorrow}} + \underbrace{\frac{D^{ss}}{Y^{ss}} (\mathbb{E}_t[\pi_{t+1}] - \pi_{t+1})}_{\text{debt erosion due to inflation surprise}} \tag{4}$$

where $r_t \equiv \log(R_t / R^{ss})$, $\pi_{t+1} \equiv \log(\Pi_{t+1}/\Pi_{ss})$, and Y^{ss} is the steady-state level of aggregate output. We will throughout impose a standard no-Ponzi condition on the fiscal authority. This will allow us to go back and forth between the sequence of flow budget constraints above and the corresponding intertemporal budget constraint.

It remains to specify a rule for how taxes adjust over time to balance the budget. Taxes in our economy consist of two components. First, there is a time-invariant distortionary tax $\tau_y \in [0,1)$ on

[7] Had we allowed for steady-state growth, this would translate to "$r > g$". Thus, and unlike the literature spurred by Blanchard (2019), we are in an environment where the real cost of government borrowing is positive.

household labor and dividend income (and thus total income). As a result, whenever there is a boom or a recession, tax revenue goes up and down "automatically," without the fiscal authority actually needing to adjust the tax code. Second, whenever public debt deviates from its steady state value (because of current or past shocks), the fiscal authority may adjust the aggregate tax bill for *given* aggregate income, for the sake of fiscal sustainability. We refer to the first component as "tax base effect" and to the second component as "fiscal adjustment." Much of our focus in the remainder of the paper is on the relative importance of these two financing margins. To this end we will study two particular examples of fiscal policy rules.

1. *Baseline fiscal policy.* Our baseline fiscal rule sets total taxes as follows:

$$T_{i,t} = \bar{T} + \tau_d (D_t + \mathscr{E}_t) + \tau_y Y_{i,t} - \mathscr{E}_t, \tag{5}$$

   where the "intercept" $\bar{T}$ is set to guarantee budget balance at steady state, $\mathscr{E}_t$ is a mean-zero and i.i.d. deficit shock, and $\tau_d \in [0,1)$ is a fixed fiscal adjustment parameter. Thanks to our simplifying assumption that all households receive the same income, we can drop the i index and rewrite (5), after (log-)linearization, as follows:

$$t_t = \underbrace{\tau_d \cdot (d_t + \epsilon_t)}_{\text{fiscal adjustment}} + \underbrace{\tau_y y_t}_{\text{tax base adjustment}} - \underbrace{\epsilon_t}_{\text{deficit shock}}, \tag{6}$$

   where $\epsilon_t \equiv \mathscr{E}_t / Y^{ss}$ and $y_t \equiv \ln(Y_t / Y^{ss})$. We note that (6) is a natural generalization of the fiscal rule found in Leeper (1991). As in that paper, the term $\tau_d \cdot (d_t + \epsilon_t)$ captures fiscal adjustment: the variation in current and future taxes induced by the exogenous deficit shock, holding the path of aggregate income constant. What is novel here is instead the term $\tau_y y_t$, capturing the tax base effect. τ_d and τ_y in this set-up will govern the relative importance of fiscal adjustment and the tax base as margins of financing: a lower τ_d means a smaller and slower fiscal adjustment, while a larger τ_y means a larger feedback from income to tax revenue.

2. *Alternative fiscal policy.* Our second fiscal rule is a time-dependent generalization of (5) in which τ_d and τ_y are allowed to vary with time. Written in date-0 sequence-space notation (see, e.g., Wolf, 2021*a*), this rule sets[8]

$$\tau_{d,t} = \begin{cases} 0 & t < H, \\ 1 & t \geq H, \end{cases} \qquad \text{and} \qquad \tau_{y,t} = \begin{cases} \tau_y & t < H, \\ 0 & t \geq H, \end{cases} \tag{7}$$

   This rule specifies that, after a deficit shock at date 0, there is no tax hike for $t < H$, with financing only coming from tax base and debt erosion effects. Then, for $t \geq H$, there is full fiscal

[8]The rule (7) should be interpreted as pertaining to the impulse responses induced by a date-0 deficit shock ϵ_0. See Wolf (2021*a*) for a discussion of the (notationally somewhat involved) mapping from policy rules in sequence-space to their state-space analogues.

adjustment ($t_t = d_t + \epsilon_t$), returning government debt to steady state at the start of $t = H+1$ and keeping it there forever after.

Note that, intuitively, we can capture a longer delay in fiscal adjustment through either a low τ_d (under the first rule) or through a high H (under the second rule). This suggests that the two rules are interchangeable for our main purposes—an intuition that turns out to be correct and that we will make precise in due course. Still, we like to work with *both* rules because each one serves different auxiliary purposes. On the one hand, (6) facilitates a tractable, recursive characterization of the equilibrium; a sharp contrast to earlier theoretical work; and a mapping between our theory and some relevant empirical work. On the other hand, (7) captures more directly the timing of fiscal adjustment; allows us to develop a sharper intuition behind our main result; and, last but not least, makes very clear that, for any finite H, our fiscal policy is "Ricardian" or "passive" in the sense of the FTPL literature. We will return to this point in Section 4.4, when we explain how the kind of self-financing documented in our paper is conceptually very different from that found in the FTPL literature.

Monetary policy. The monetary authority sets the rate of interest on nominal bonds according to the following rule:

$$I_t = R^{ss}\mathbb{E}_t\left[\frac{\Pi_{t+1}}{\Pi^{ss}}\right]\left(\frac{Y_t}{Y^{ss}}\right)^{\phi}, \tag{8}$$

for some $\phi \in \mathbb{R}$. This is equivalent to saying that the monetary authority implements the following relation between the (expected) real interest rate and real output:

$$r_t = \phi y_t. \tag{9}$$

In short, we parameterize monetary policy by the pro-cyclicality of the real interest rate. Since deficits will be shown to be expansionary in equilibrium (provided that $\omega < 1$, i.e., that Ricardian equivalence fails), ϕ also parameterizes the comovement between real rates and deficits. We can thus interpret $\phi < 0$ as an "accommodative" monetary authority that, in response to a positive deficit shock, lets real rates fall so as to increase fiscal space—the analogue of printing money in a cashless New Keynesian economy. Conversely, we can interpret $\phi > 0$ as a "hawkish" or "fiscally conservative" monetary authority that leans against any Keynesian boom (and any inflation) triggered by deficits.

For our main analysis, we let monetary policy be "neutral" in the sense that $\phi = 0$; that is, we keep the real rate fixed. This is the same baseline policy as in Woodford (2011) and allows us to cleanly isolate how the interaction of fiscal policy and private spending shapes the scope for fiscal self-financing, holding constant the government's cost of borrowing (and the private sector's rate of return). We will relax this restriction in Section 5.3.[9]

[9]One may also wonder how our specification of monetary policy relates to more standard Taylor rules, and how ϕ matters for equilibrium determinacy. As we make clear in due course (Section 5.3), the answers to both of these questions

3 Equilibrium

This section lays the groundwork for our self-financing result by characterizing the economy's equilibrium. We start by reducing the economy to a system of three equations: one for aggregate demand, one for aggregate supply, and one for the dynamics of public debt. We then characterize the unique bounded solution to this system. Throughout this section, we employ our baseline fiscal rule (6). Derivations for the alternative rule (7) are slightly different and relegated to Appendix A.3, though the economic essence is identical.

3.1 Aggregate demand

The consumption-savings problem of a household i is to choose sequences of consumption and asset holdings to maximize (1) subject to (2). Using the simplifying property that all households receive the same income (and pay the same taxes), we can express the (log-linearized) consumption function of household i in period t as follows:

$$c_{i,t} = (1-\beta\omega)\left(\tilde{a}_{i,t} + \mathbb{E}_t\left[\sum_{k=0}^{\infty}(\beta\omega)^k\left(y_{t+k} - t_{t+k}\right)\right]\right) - \gamma\mathbb{E}_t\left[\sum_{k=0}^{\infty}(\beta\omega)^k r_{t+k}\right], \tag{10}$$

where $\tilde{a}_{i,t}$ denotes the household's *real* financial wealth (inclusive of social fund payments) and $\gamma \equiv \sigma\beta\omega - (1-\beta\omega)\beta\frac{\tilde{A}^{ss}}{P^{ss}Y^{ss}}$ combines the intertemporal substitution and wealth effects of the real interest rate. Note that, when $\omega = 1$, then this demand function reduces to that of a standard permanent-income household: taxes are discounted at rate β, and the marginal propensity to consume (MPC) from both financial wealth and permanent income is $1-\beta$. Relative to this benchmark, $\omega < 1$ maps to (i) a higher MPC and (ii) more discounting of future income and future taxes. It is now well understood how these qualitative properties extend to richer, more realistic, HANK-type models (e.g., see Auclert, Rognlie and Straub, 2018; Farhi and Werning, 2019; Wolf, 2021*a*); as will become clear in due course, our results are driven by these more general qualitative properties of consumer demand, and not by the specific micro-foundations behind them.

Under our baseline monetary policy, $r_t = 0$ for all t, so the last term in (10) drops out. Aggregating across households, and using the fact that aggregate private financial wealth equals total government debt, we reach the following description of aggregate consumption:

$$c_t = (1-\beta\omega)\,d_t + (1-\beta\omega)\,\mathbb{E}_t\left[\sum_{k=0}^{\infty}(\beta\omega)^k\left(y_{t+k} - t_{t+k}\right)\right]. \tag{11}$$

Next, using (6) to express future taxes as functions of the current public debt and future output, and

are of little consequence for the lessons of our paper.

replacing c_t with y_t (market clearing), we arrive at the following representation of aggregate demand:

$$y_t = \mathscr{F}_1 \cdot (d_t + \epsilon_t) + \mathscr{F}_2 \cdot \mathbb{E}_t \left[(1-\beta\omega) \sum_{k=0}^{\infty} (\beta\omega)^k y_{t+k} \right], \tag{12}$$

where $\mathscr{F}_1 \equiv \frac{(1-\beta\omega)(1-\omega)(1-\tau_d)}{1-\omega(1-\tau_d)}$ and $\mathscr{F}_2 = 1 - \frac{(1-\omega)\tau_y}{1-\omega(1-\tau_d)}$. (12) is a key equation of this paper. The first term captures the direct (or "partial equilibrium") effect of fiscal deficits on aggregate demand. We see that $\mathscr{F}_1 > 0$ (i.e., deficits enter positively in aggregate demand) if and only if $\omega < 1$ (no Ricardian equivalence) and $\tau_d < 1$ (no *immediate* financing). Intuitively, $\tau_d < 1$ means that deficits today are financed at least in part with taxes in the future; as long as $\omega < 1$, this means that a deficit today translates to a positive real transfer from future cohorts to current cohorts, thus increasing aggregate demand. The second term then captures the general equilibrium feedback between aggregate demand and aggregate income—the "intertemporal Keynesian cross." Note in particular that $\mathscr{F}_2$ measures the "slope" of this Keynesian cross, in the following particular sense: if we raise expectations of future spending in all periods by 1, then current spending increases by $\mathscr{F}_2$.[10]

Note that, in the special case $\omega = 1$, (12) collapses to $y_t = \mathbb{E}_t \left[(1-\beta)\sum_{k=0}^{\infty} \beta^k y_{t+k}\right]$, with $\mathscr{F}_1 \equiv 0$ and $\mathscr{F}_2 = 1$; equivalently, written in standard Euler equation notation, this relation just becomes $y_t = \mathbb{E}_t[y_{t+1}]$. Debt and taxes thus drop out from (11), reflecting the fact that, with $\omega = 1$, Ricardian equivalence holds in the following partial equilibrium sense: holding constant behavior of other consumers (equivalently, the sequence of output), individual spending is invariant to fiscal policy.

3.2 Aggregate supply

By design, the supply side of our model is *exactly* the same as its RANK counterpart. In particular, labor supply is given by

$$\frac{1}{\varphi}\ell_t = w_t - \frac{1}{\sigma}c_t \tag{13}$$

Together with market clearing ($c_t = y_t$) and technology ($y_t = \ell_t$), this pins down the real wage as $w_t = \xi y_t$, where $\xi \equiv \frac{1}{\varphi} + \frac{1}{\sigma} > 0$. Firm optimality, on the other hand, pins down the optimal reset price a function of current and future real marginal costs. Following standard steps, we can then reduce the supply-side of the economy to the familiar NKPC:

$$\pi_t = \kappa y_t + \beta \mathbb{E}_t[\pi_{t+1}]. \tag{14}$$

where $\kappa \geq 0$ depends on ξ (the pro-cyclicality of real marginal costs) and θ (the Calvo reset probability). Since there is a one-to-one mapping between κ and θ, and since this mapping is invariant to both

[10] Also note that $\mathscr{F}_2 = 1$ when $\omega = 1$ or $\tau_y = 0$, but $\mathscr{F}_2 < 1$ as soon as $\omega < 1$ and $\tau_y > 0$. The combination of finite lives/liquidity constrains and proportional taxes thus attenuates the Keynesian feedback. This helps explain why our economy features a unique bounded equilibrium, as we discuss below and further in Appendix A.4.

fiscal and monetary policy, we henceforth treat κ as an exogenous parameter and (re)parameterize the degree of price flexibility by κ.

3.3 Law of motion for public debt

The remaining third equilibrium restriction comes from combining the government's flow budget constraint with the fiscal rule (6). This yields the following law of motion for public debt:

$$d_{t+1} = \beta^{-1}\big(d_t + \varepsilon_t - \underbrace{\tau_d \cdot (d_t + \epsilon_t)}_{\text{fiscal adjustment}} - \underbrace{\tau_y y_t}_{\text{tax base}}\big) - \underbrace{\frac{D^{ss}}{Y^{ss}}\left(\pi_{t+1} - \mathbb{E}_t\left[\pi_{t+1}\right]\right)}_{\text{debt erosion}}. \tag{15}$$

with initial condition[11]

$$d_0 = -\frac{D^{ss}}{Y^{ss}}\pi_0. \tag{16}$$

Finally, recall that the sequence of government flow budget constraints must be complemented with the usual no-Ponzi restriction $\lim_{k\to\infty}\mathbb{E}_t\left[\beta^k d_{t+k}\right] = 0$.

3.4 Equilibrium definition and characterization

A standard equilibrium definition combines (i) individual optimality for consumers, (ii) individual optimality for firms, (iii) market clearing, and (iv) budget balance for the government (together with the no-Ponzi constraint). The preceding analysis has log-linearized the model and has reduced the first three requirements to equations (12) and (14), and the last requirement to equation (15). These equations, like the log-linearization itself, make sense only insofar the economy remains in a neighborhood of the steady state. Accordingly, our notion of equilibrium is as follows.

Definition 1. An equilibrium is a stochastic path $\{y_t, \pi_t, d_t\}_{t=0}^{\infty}$ for output, inflation, and the real value of public debt that is bounded in the sense of Blanchard and Kahn (1980) and that satisfies aggregate demand (12), aggregate supply (14), and the law of motion for public debt (15), along with the initial condition (16) and the no-Ponzi game condition $\mathbb{E}_t\left[\lim_{k\to\infty}\beta^k d_{t+k}\right] = 0$.

We can now state our first main result.

Proposition 1. *Suppose that $\omega < 1$ and $\tau_y > 0$. There exists a unique (bounded) equilibrium. Along this equilibrium, real output and real public debt satisfy*

$$y_t = \chi\left(d_t + \epsilon_t\right) \qquad \textit{and} \qquad \mathbb{E}_t\left[d_{t+1}\right] = \rho_d\left(d_t + \epsilon_t\right), \tag{17}$$

[11] Note that $d_0 = \frac{1}{P^{ss}} b_0 - \frac{D^{ss}}{Y^{ss}}\left(\pi_0 - E_{-1}[\pi_0]\right)$. Since we start in steady state (i.e., $b_0 = E_{-1}[\pi_0] = 0$), we recover (16).

for some $\chi > 0$ *and* $\rho_d \in (0,1)$. *These coefficients solve the following fixed point problem:*

$$\chi = \frac{\mathscr{F}_1}{1 - \mathscr{F}_2 \frac{1-\beta\omega}{1-\beta\omega\rho_d}} \qquad \text{and} \qquad \rho_d = \beta^{-1}\left(1 - \tau_d - \tau_y \chi\right). \tag{18}$$

Finally, inflation satisfies $\pi_t = \frac{\kappa}{1-\beta\rho_d} y_t$.

(17) contains two relations. The first relation expresses the equilibrium level of output as a proportion χ of the private sector's real financial wealth (which itself equals d_t) and of the fiscal transfer (the deficit shock ϵ_t). Note that $\chi > 0$—i.e., deficits trigger booms. As emphasized previously, this is so because of the two key features of our model environment: the failure of Ricardian equivalence, which lets deficits stimulate aggregate demand; and the nominal rigidity, which lets aggregate demand drive output. The second relation gives the (expected) evolution of the real value of public debt, with ρ_d measuring the (expected) persistence of debt. But since y_t is proportional to d_t, we see that ρ_d here also measures the expected persistence of the Keynesian boom triggered by deficits.[12]

Condition (18) summarizes the fixed-point relation between χ and ρ_d—i.e., the two-way feedback between aggregate demand and fiscal conditions. On the one hand, as long as $\tau_y > 0$, higher aggregate demand contributes to higher output, higher tax revenue and thereby to lower public debt tomorrow. This feedback is reflected in the second part of condition (18), which pins down ρ_d as a function of χ and of the two fiscal-policy parameters (τ_d, τ_y). On the other hand, as long as $\omega < 1$, more delay in fiscal adjustment, or more persistence in public debt, will translate to a larger effective transfer from generations in the far future to generations in the present and the near future, thus stimulating aggregate demand both directly (the partial equilibrium effect) and indirectly (the general equilibrium Keynesian cross). This is reflected in the first part of condition (18), which pins down χ as a function of ρ_d and of the relevant aggregate-demand parameters $(\mathscr{F}_1, \mathscr{F}_2, \beta\omega)$. We emphasize that the feedback from deficits to aggregate demand is present only when $\omega < 1$, while the feedback from aggregate demand to tax revenue and thereby to public debt dynamics is present only when $\tau_y > 0$. The textbook model assumes away *both* feedbacks. We return to this point at the end of Section 4 and also in Appendix A.4, where we explain the conceptual differences between our analysis and the classical FTPL. For now, we wish to emphasize that the aforementioned two-way feedback is responsible *both* for the uniqueness of the equilibrium and for our upcoming self-financing results.[13]

[12] While this equilibrium characterization applies regardless of κ, we note that there is a subtlety as we move from $\kappa = 0$ to $\kappa > 0$. When prices are rigid, d_t is predetermined in the beginning of period t, and so the second part of (17) holds date-by-date, not just in expectation. When instead prices can move, then d_t is no longer predetermined—it depends on the concurrent price level, which itself responds to the shock ϵ_t (i.e., the debt erosion channel visible in (15)).

[13] The uniqueness of (bounded) equilibrium under our baseline policy rule (6), as stated in Proposition 1, holds true without further qualification. Under the alternative rule (7), instead, uniqueness requires any one of the following minor modifications: (i) a strengthening of the notion of boundedness to $\lim_{k\to\infty} \mathbb{E}_t\left[y_{t+k}\right] = 0$, which amounts to saying that expectations "at infinity" of the spending of others (beliefs of infinite order) are anchored to the steady state; (ii) the allowance of $\tau_y > 0$ also for $t > H$; or (iii) the reinterpretation of $\phi = 0$ as the limit of $\phi \to 0$ from above, which is basically

4 Self-financing of fiscal deficits

This section presents our headline result on the possibility of self-financing. We first in Section 4.1 use the intertemporal government budget constraint to provide a quantitative measure of the degree of self-financing, and also to decompose it into its two constituent sources: the expansion of the tax base (which operates whenever $\tau_y > 0$) and the erosion of the real debt burden (which operates whenever $\kappa > 0$). We then in Sections 4.2 and 4.3 show that *complete* self-financing is possible in the limit as fiscal adjustment is delayed further and further, and we explain the economics behind this key result. Finally, in Section 4.4, we characterize how the degree of self-financing depends on the slope of NKPC (κ) and the departure from Ricardian equivalence (as parameterized by $\omega < 1$).

4.1 Sources of fiscal financing

Iterating the debt relations (15) and (16) forward and using $\lim_{t\to\infty}\mathbb{E}_0\left[\beta^t d_t\right] = 0$ (since $\rho_d \in (0,1)$), we can re-write the government budget constraint in present-value form:

$$\underbrace{\epsilon_0}_{\text{deficit}} = \underbrace{\tau_d\left(\epsilon_0 + \sum_{k=0}^{\infty}\beta^k\mathbb{E}_0\left[d_k\right]\right)}_{\text{fiscal adjustment}} + \underbrace{\sum_{k=0}^{\infty}\tau_y\beta^k\mathbb{E}_0\left[y_k\right]}_{\text{self-financing via tax base}} + \underbrace{\frac{D^{ss}}{Y^{ss}}\left(\pi_0 - \mathbb{E}_{-1}\left[\pi_0\right]\right)}_{\text{self-financing via debt erosion}}. \tag{19}$$

The left-hand side of (19) is the initial deficit, while the right-hand side contains the three ways in which this deficit will be financed over time: the first term captures the adjustment in current and future taxes triggered by the initial deficit and any resulting accumulation of public debt; the second term collects the extra tax revenue generated by the deficit-driven boom; and the third term gives the erosion in the real debt burden caused by the deficit-driven inflation. Put differently, the first term captures the conventional notion of fiscal adjustment—the government actively adjusts its primary surplus to stabilize government debt—while the second and third terms reflect our two sources of self-financing. Finally, we note that a fourth source of financing—monetary accommodation—emerges if the monetary authority depresses real rates in response to deficits. In our main analysis, the assumption that $\phi = 0$ means that this channel is not operative.

We can now define the (overall) degree of self-financing as follows:

Definition 2. The degree of self-financing is the fraction of the initial deficit that is financed by an expansion in the tax base and/or an erosion in the real debt burden:

$$\nu \equiv \frac{\sum_{k=0}^{\infty}\tau_y\beta^k\mathbb{E}_0\left[y_k\right] + \frac{D^{ss}}{Y^{ss}}\pi_0}{\epsilon_0} \tag{20}$$

a (limit) Taylor principle. The sole role of any of these modifications is to remove a class of sunspot equilibria that are inherited from the standard New Keynesian model. See Appendix A.3 for details.

Note that this overall degree of self-financing can be decomposed into its two components as follows:

$$\nu \equiv \nu_y + \nu_p$$

where

$$\nu_y \equiv \frac{1}{\epsilon_0}\tau_y \sum_{k=0}^{\infty} \beta^k \mathbb{E}_0\left[y_k\right] \qquad \text{and} \qquad \nu_p \equiv \frac{1}{\epsilon_0}\frac{D^{ss}}{Y^{ss}}\pi_0 \tag{21}$$

measure, respectively, the tax-base and debt-erosion components of self-financing. Finally, because the NKPC implies that $\pi_0 = \kappa \sum_{k=0}^{\infty} \beta^k \mathbb{E}_0\left[y_k\right]$, it is immediate that

$$\nu_p = \frac{\kappa \frac{D^{ss}}{Y^{ss}}}{\tau_y}\nu_y. \tag{22}$$

By (22), once we understand the two-way feedback between real economic activity and fiscal deficits in the rigid-price benchmark ($\kappa = 0$), it will be straightforward to extend the analysis to the general case with $\kappa > 0$. In the rest of this section we therefore proceed as follows: we first state our headline result for the general case; we next expand on the economics behind it under the special case of $\kappa = 0$ (rigid prices); and we finally discuss the role of letting $\kappa > 0$.

Before proceeding to our main results we close with a brief remark on our two fiscal policy specifications (6) and (7). As previewed in Section 2.3, these two rules indeed turn out to be equivalent ways of thinking about the effects of delays in fiscal financing.

Lemma 1. *Suppose that $\omega < 1$ and $\tau_y > 0$. There exists a strictly decreasing mapping $\mathscr{T} : \mathbb{N} \to (0,1]$, with $\lim_{H\to\infty} \mathscr{T}(H) = 0$, such that the degree of self-financing generated by policy (7) is the same as that generated by policy (6) if and only if $\tau_d = \mathscr{T}(H)$.*

Importantly, this result establishes that "infinite financing delay" in the sense of $H \to \infty$ under the rule (7) maps to $\tau_d \to 0$ under the rule (6), and vice versa.

4.2 The self-financing result

We can now state our main theoretical result on the possibility of self-financing.

Theorem 1. *Suppose that $\omega < 1$ and $\tau_y > 0$, and that fiscal policy follows either our baseline rule (6) or the variant rule (7). The equilibrium degree of self-financing, ν, has the following properties:*

1. *It is increasing in the delay of fiscal adjustment, i.e., ν is decreasing in τ_d for the fiscal rule (6) and increasing in H for the fiscal rule (7)).*

2. *As fiscal adjustment is delayed further, the degree of self-financing converges to one; i.e., $\nu \to 1$ (from below) as $\tau_d \to 0$ (from above) or as $H \to \infty$. Furthermore, these two limits induce the same*

equilibrium paths $\{y_t, \pi_t, d_t\}_{t=0}^{\infty}$*, and in this common limit self-financing is sufficiently strong to return real government debt to steady state (i.e.,* $\lim_{k\to\infty} \mathbb{E}_t [d_{t+k}] \to 0$ *for the baseline rule* (6) *and* $\lim_{H\to\infty} \mathbb{E}_0 [d_H] \to 0$ *for the variant rule* (7)*).*

Theorem 1 is our core result. Its main implication is that the failure of Ricardian equivalence—here encapsulated in $\omega < 1$—opens the door for fiscal deficits to finance themselves. First, as fiscal adjustment is delayed, the initial fiscal deficit induces a larger (χ) and more persistent (ρ_d) Keynesian boom, thus increasing the share of self-financing through higher tax revenue and a larger date-0 price level jump. Second, the limit as $\tau_d \to 0$ or $H \to \infty$ is one of *complete* self-financing: the deficit-driven boom is large and fast enough to cover the cost of the initial fiscal outlay ϵ_0 and to make sure that public debt automatically returns back to steady state, without any fiscal adjustment.

A visual illustration. We provide a visual illustration of Theorem 1 in Figure 1.[14] The figure shows the effects of a deficit shock ϵ_0 under different assumptions about fiscal adjustment. The left and middle panels in the top half of the figure begin by showing impulse responses of output y_t and government debt d_t as a function of the fiscal adjustment parameter τ_d in our baseline fiscal rule (6). Consistently with Theorem 1, we see that smaller fiscal response coefficients correspond to larger impact output booms (i.e., larger χ) and more persistent deviations of output and government debt from steady state (i.e., larger ρ_d). This boom then contributes to financing of the initial deficit ϵ_0 through our two self-financing channels: a tax base expansion and a jump in the date-0 price level. The top right panel of the figure then reports this degree of self-financing ν—as well as the split into ν_y and ν_p—as a function of the fiscal adjustment parameter τ_d. We see that ν is decreasing in the strength of fiscal adjustment τ_d, i.e., increasing in the delay of fiscal adjustment. In particular, as τ_d declines towards zero, the degree of self-financing converges to one.

The bottom half of Figure 1 provides a different perspective on the same logic, using instead the alternative fiscal rule (7), in which taxes adjust after some (finite) horizon H to perfectly balance the budget. We see again that ν is increasing in the delay of fiscal adjustment, and in particular again converges to one as fiscal adjustment is delayed further and further ($H \to +\infty$). This not only offers a complementary interpretation of what "delay" means, but also proves the following important point: our self-financing result is consistent with fiscal policy being "Ricardian" or "passive" in the strong sense that it commits on bringing debt back to steady state at $t = H + 1$ regardless of the path that the economy has taken up to that point.

[14] We only emphasize *qualitative* features of our results here, so we do not in detail discuss the model parameterization; for the purposes of our analysis here, it suffices to note that we set $\omega \approx 0.75$—a meaningful departure from Ricardian equivalence. A more serious quantitative investigation of the likely importance of self-financing is relegated to Section 6.

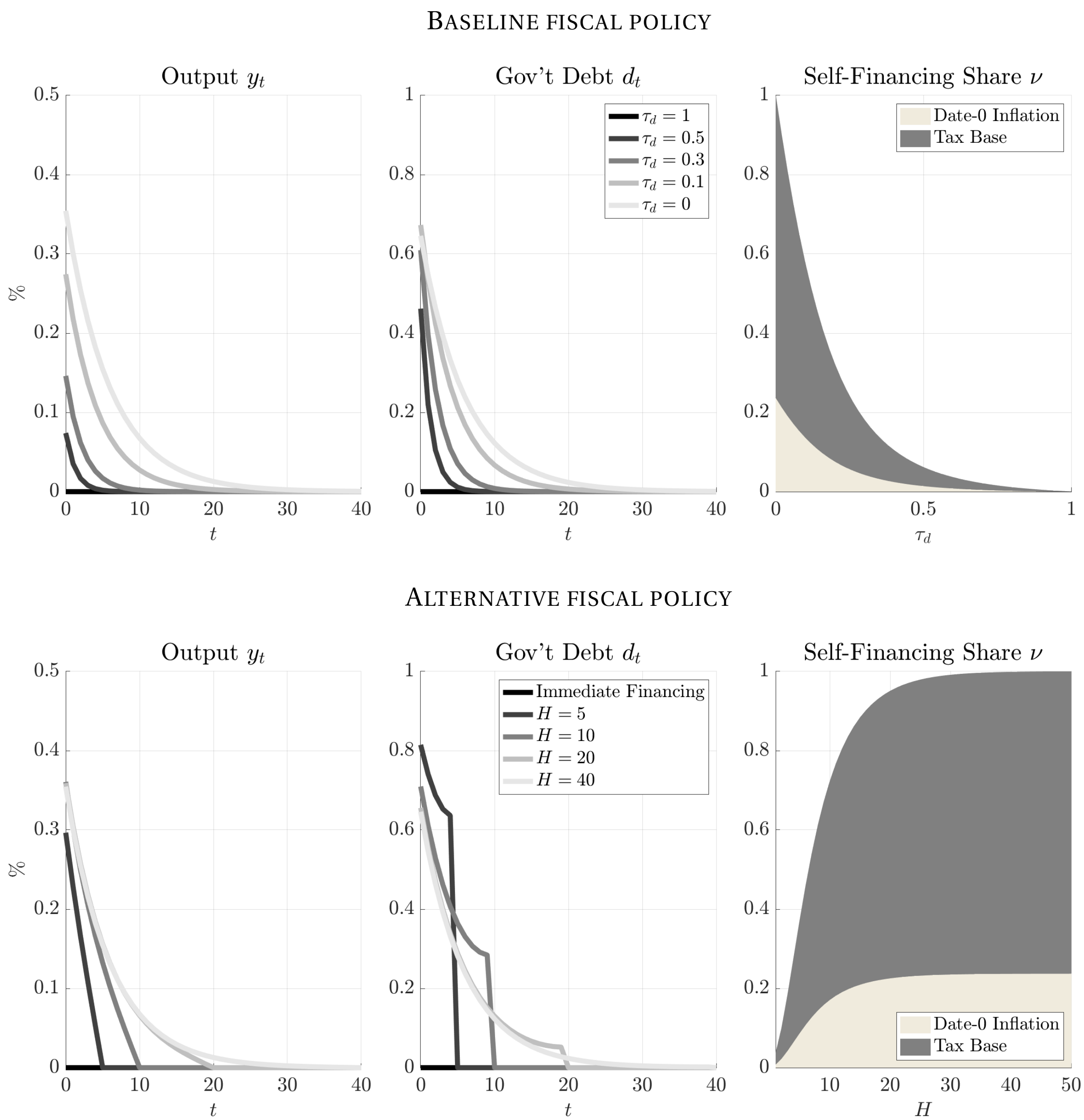

Figure 1: Top panel: impulse responses of output y_t, government debt d_t, and the self-financing share ν to the deficit shock ϵ_0 as a function of τ_d. Bottom panel: same as above, but as a function of H.

4.3 The economics behind the self-financing result

To understand the economics behind Theorem 1, we will in this section restrict attention to the special case of fully rigid prices (i.e., $\kappa = 0$). To maximize clarity, we will also focus on the fiscal rule (7). As anticipated before, this rule is pedagogically useful because it makes very clear what we mean by delay in fiscal adjustment, and furthermore delineates our result from the classical FTPL logic (as the fiscal rule is "Ricardian" in the sense of that literature). The policy experiment studied in this section is thus as follows: the fiscal authority pays out a lump-sum transfer to households at date 0 and promises to hike taxes at date H in order to return debt to its steady-state value at date $H+1$. The questions of interest are how this policy affects equilibrium outcomes, how large the required tax hike at H turns out to be, and what happens as we increase H.

Our analysis in this section proceeds in two steps. First, we discuss a simple example—an essentially static Keynesian cross. Second, we show how the intuition from this static example sheds light on the workings of our full intertemporal dynamic economy.

A simple static example. To build intuition it will prove instructive to consider a two-period economy in which consumption in the first period (date 0) is given by

$$c = \text{MPC} \cdot y_{\text{disp}}$$

where $\text{MPC} \in (0,1)$ is the marginal propensity to consume and

$$y_{\text{disp}} = (1-\tau_y)y + \epsilon$$

is disposable income, net of taxes and inclusive of a fiscal transfer ϵ. This set-up embeds the assumption that date-0 consumption is invariant to second-period (date-1) outcomes, allowing us to characterize the date-0 equilibrium without reference to what happens later. By imposing market clearing ($y = c$), we immediately see that the date-0 equilibrium level of income is given by

$$y = \frac{\text{MPC}}{1-(1-\tau_y)\text{MPC}} \times \epsilon \tag{23}$$

This equation is just the solution of the familiar, static Keynesian cross: MPC is the partial equilibrium effect of a unit transfer, $\frac{1}{1-(1-\tau_y)\text{MPC}}$ is the general equilibrium multiplier, and $(1-\tau_y)\text{MPC}$ is the slope of the Keynesian cross.

Consider now the government's budget constraint. Since the government hands out the transfer ϵ and collects taxes $\tau_y y$, the net deficit at the end of date 0 is $\epsilon - \tau_y y$. The amount of public debt inherited at date 1 is thus given by

$$\text{debt tomorrow} = R(\epsilon - \tau_y y), \tag{24}$$

where R is the real interest rate between the two periods. Plugging (23) into (24), we conclude that

$$\text{debt tomorrow} = R(1-\nu)\epsilon$$

where

$$\nu \equiv \frac{\tau_y y}{\epsilon} = \frac{\tau_y \text{MPC}}{1-(1-\tau_y)\text{MPC}} \tag{25}$$

is the degree of self-financing. Equation (25) reveals two important insights. First, we see that a higher MPC maps both to a larger partial equilibrium effect (numerator) and to a higher general equilibrium multiplier (denominator), and therefore overall to a larger degree of self-financing ν. Second, in the limit as MPC $\to 1$, the partial equilibrium effect converges to 1, the multiplier converges to $\frac{1}{\tau_y}$, and so there is full self-financing—i.e., $\nu \to 1$.

Back to the full model. To what extent is the simple static example informative about what is going on in our full dynamic economy? Full self-financing in the static model relies on two key properties: first, that the expected date-1 tax hike does not affect date-0 spending behavior; and second, that the date-0 transfer as well as the additional income it generates are fully spent at date 0 (MPC $\to 1$), thus generating enough tax revenue to stabilize debt *before* the promised date-1 tax hike. The core intuition for our self-financing result is that, as the financing delay H increases, our dynamic economy starts to emulate those two features of the simple static example.

To see why this is so, we begin by establishing two important properties of our consumption function (11).[15] This consumption function maps sequences of current and future (post-tax) income into sequences of current and future consumption. The first property concerns how consumption at date $t \geq 0$ responds to an anticipated future income change at some future date $t+\ell$, with $\ell \geq 0$. We write this response as $\mathcal{M}_{t,t+\ell}$—the $(t, t+\ell)$ element of the matrix of intertemporal MPCs studied by Auclert, Rognlie and Straub (2018). In our environment, as long as $\omega < 1$, a one-unit anticipated income change at date $t+\ell$ (in date-t present value terms) has a vanishing effect on date-t consumption as ℓ increases; i.e., we have that

$$\lim_{\ell\to\infty} \beta^{-\ell}\mathcal{M}_{t,t+\ell} = 0$$

The second property concerns how income changes at date $t \geq 0$ affect consumption at some future date-$t+\ell$, with $\ell \geq 0$. We write this response as $\mathcal{M}_{t+\ell,t}$—the $(t+\ell, t)$ element of the intertemporal MPC matrix. Analogously to the first property, as long as $\omega < 1$ and as ℓ increases, a one-unit income change at date t has a vanishing effect on consumer demand at date $t+\ell$; i.e., we have that

$$\lim_{\ell\to\infty} \mathcal{M}_{t+\ell,t} = 0$$

[15] See Lemma D.1 in Appendix D for a formalization of the discussion here.

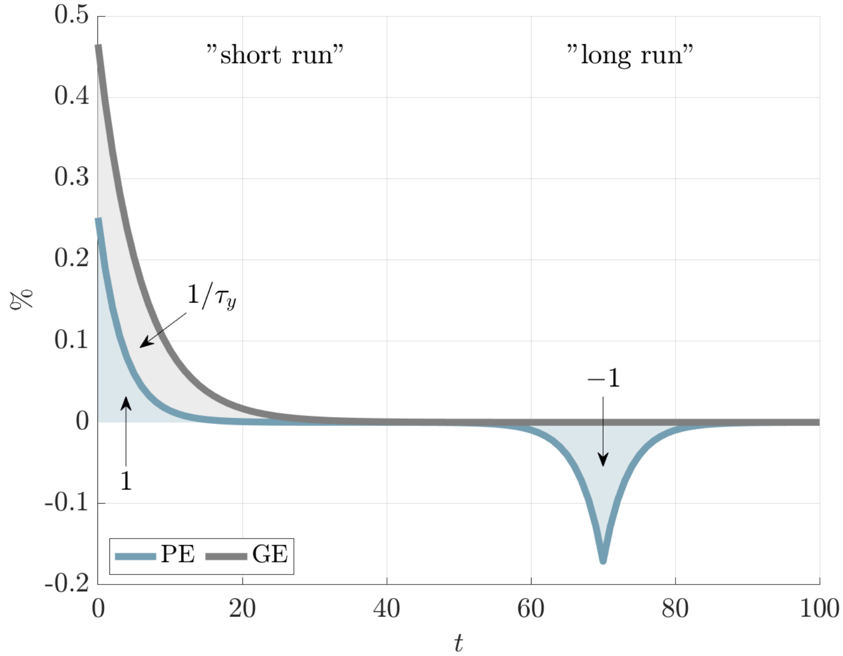

Figure 2: Direct, partial equilibrium demand effect (blue) and full general equilibrium output impulse response (grey) to a date-0 deficit shock with fiscal financing at date $H = 70$.

These two properties of the aggregate consumption function allow us to connect our full dynamic economy with the simple two-period example; a visual illustration to accompany the following discussion is furthermore provided in Figure 2. The figure shows two objects: first, the direct, partial equilibrium effect of the fiscal intervention (blue), defined as

$$\mathscr{M} \cdot \boldsymbol{t}^{PE}$$

where $\mathscr{M}$ is the full matrix of intertemporal MPCs, and the "partial equilibrium" tax-and-transfer vector $\boldsymbol{t}^{PE}$ equals -1 at date 0, β^{-H} at date H, and 0 otherwise. Second, the full general equilibrium impulse response of output to a transfer shock with delayed financing (grey).

Consider first the *partial equilibrium* demand effect. By the first property of consumer demand stated above, the date-0 cohort is essentially unaffected by the future announced tax hike; by the second property, it spends its lump-sum transfer receipt quickly—i.e., a cumulative MPC of 1 in the short-run. The cohorts born shortly after date-0 are also essentially unaffected by the future tax hike, so overall spending demand is back to trend after around 20 periods. It is only around $t = 60$ that expectations of the future tax hike at $H = 70$ start to depress demand. Our dynamic economy thus echoes the static example: the future tax hike does not affect "short-run" spending behavior (around $t = 0$), and the "short-run" cumulative MPC approaches 1.

Now turn to *general equilibrium* (grey). The initial increase in demand generates additional income; importantly, again by the second property of consumer demand established above, this income is spent quickly. Since the cumulative short-run MPC is 1, this delivers a front-loaded Keynesian multiplier of size $\frac{1}{\tau_y}$. Crucially, this Keynesian boom increases tax revenue (in terms of $t = 0$ present value) by $\tau_y \times \frac{1}{\tau_y} = 1$, thus returning government debt to trend far before date H (or "the long-run"). As a result, the subsequent tax hike at H—together with its negative effect on spending—vanishes, again echoing what happens with MPC = 1 at date 0 in the simple economy.

While the preceding discussion illuminated our limiting full self-financing result, the underlying intuition also readily connects with our monotonicity result—i.e., that the degree of self-financing is increasing in the fiscal delay H. As H is increased, the effect of the anticipated tax hike on short-run demand decreases, so the immediate partial equilibrium spending boom is increasing in H. Similarly, the larger H, the longer the general equilibrium Keynesian cross can play out before being moderated by the future tax hike. The size of the short-run boom thus is increasing in H, and by extension so is the endogenously raised tax revenue, decreasing the size of the subsequent tax hike. The exact same logic then also explains why the equilibrium boom becomes larger and more persistent for smaller τ_d under the Leeper (1991)-style fiscal rule (6). Either way, as fiscal adjustment is delayed further and further, the short-run Keynesian boom on its own becomes big enough to stabilize debt, with debt in the infinite-delay limit returning to steady state at rate $\lim_{\tau_d \to 0} \rho_d = \rho_d^{\text{full}} < 1$.

A comment on generality. We emphasize that the intuitions offered in this section transcend the particular OLG economy that underlies Theorem 1 as well as our graphical explorations in Figure 1. We will later make this claim precise in Section 5.1, where we generalize our self-financing result to much richer aggregate demand structures. This generalization will identify sufficient conditions in line with the intuition offered above—conditions that ensure (i) deficits inducing a partial equilibrium boom, and (ii) this boom in general equilibrium being large enough and fast enough to raise the required tax revenue prior to the promised future tax hike.

4.4 The roles of κ and ω

This section concludes our analysis by further investigating the role of two features of our environment: the strength of nominal rigidities κ and the distance from permanent-income behavior ω.

Partially sticky prices. The intuition offered in the previous subsection considered the special case of rigid prices ($\kappa = 0$), but extends with little change to the case of partially sticky prices. If we relax the assumption of rigid prices ($\kappa > 0$), then the Keynesian boom brings with it inflation, reducing the real value of public debt. Qualitatively, relative to the rigid-price baseline, moving to $\kappa > 0$ thus strictly *increases* the potency of self-financing. Furthermore, since the innovation in inflation is (via the NKPC) proportional to the innovation in the present discounted value of income, we can in fact readily compute the relative importance of the two forms of self-financing, as shown in equation (22).

Putting everything together, we have the following result.

Proposition 2. *Let $\omega < 1$, $\tau_y > 0$, $\kappa > 0$ and $\frac{D^{ss}}{Y^{ss}} > 0$, and consider either of our two fiscal policies.*

1. *The overall degree of self-financing, ν, increases with the degree of price flexibility κ, and the steady-state debt-to-GDP ratio, $\frac{D^{ss}}{Y^{ss}}$.*

2. *The relative contribution of the "real" channel (tax base) ν_y/ν, is increasing in τ_y, while that of the "nominal" channel (debt erosion), ν_p/ν, is increasing in κ and $\frac{D^{ss}}{Y^{ss}}$.*

To illustrate the effect of price flexibility, the top panel of Figure 3 repeats Figure 1 with a higher κ. We see that the degree of self-financing still increases with the delay in fiscal adjustment, again eventually converging to one. What changes is (i) the split between ν_p and ν_y—now the nominal channel has a relatively bigger contribution—and (ii) the level of ν for any given H—there is more self-financing because the period-0 jump in the price level is bigger when prices are more flexible. In particular, as $\kappa \to \infty$, *all* self-financing comes through the date-0 jump in prices.

Distance from permanent-income behavior. Finally we consider the importance of the deviation from permanent-income consumption behavior (i.e., the role of $\omega < 1$). For this purpose, we find it convenient to focus on our main fiscal policy specification (6), because this allows us to summarize the persistence of government debt in the single coefficient ρ_d.

Proposition 3. *Let $\omega < 1$ and $\tau_y > 0$ and consider the fiscal policy* (6).

1. *For any $\tau_d > 0$, a lower ω raises ν and decreases ρ_d; that is, a larger departure from the permanent-income benchmark yields both larger and faster self-financing.*

2. *Let $\rho_d^{full} \equiv \lim_{\tau_d \to 0} \rho_d$ be the persistence of government debt in the complete self-financing limit. $\rho_d^{full} < 1$ for any $\omega < 1$. But as the departure from the permanent-income benchmark vanishes ($\omega \to 1$), self-financing also vanishes ($\rho_d^{full} \to 1$).*

The intuition for the first part is straightforward: the smaller ω, the quicker and larger the Keynesian boom triggered by any deficit, and hence the larger ν and the smaller ρ_d. The second part, on the other hand, zeroes in on how ω matters for our self-financing limit. Provided that $\omega < 1$, Theorem 1 applies no matter how close ω is to 1. However, the closer ω is to 1, the smaller and less front-loaded the boom is, and hence the longer it takes for public debt to return to steady state.

The bottom panel of Figure 3 provides a visual illustration of what happens when ω is very close to 1—that is, we are very close to the permanent-income benchmark. Consistent with Theorem 1 the

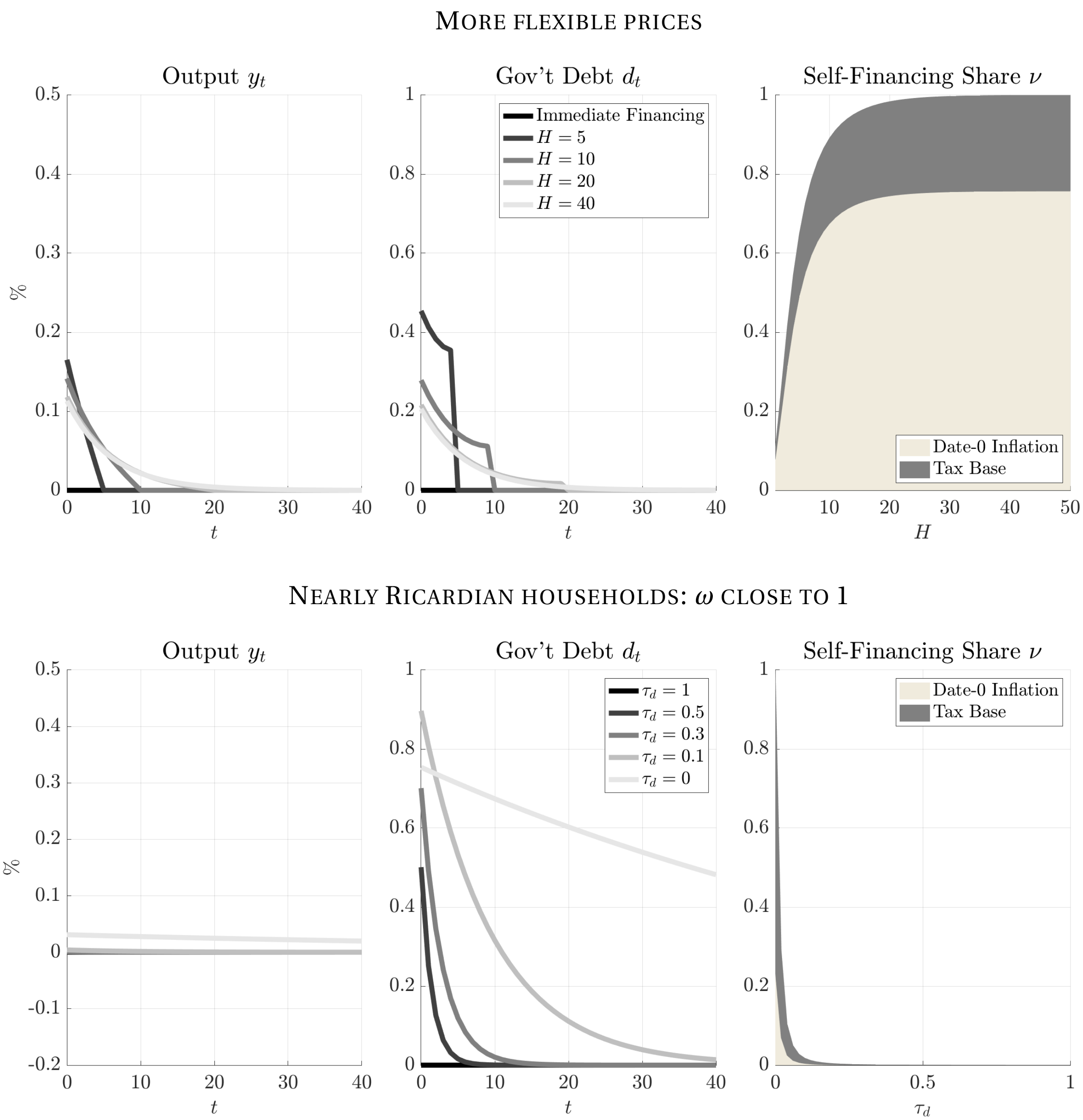

Figure 3: Top panel: impulse responses of output y_t, government debt d_t, and the total self-financing share ν to the deficit shock ϵ_0 as a function of H in an economy with relatively flexible prices. Bottom panel: same as above, but as a function of τ_d and with ω close to 1.

self-financing limit still exists; now, however, convergence to that limit is slow (in the sense that meaningful self-financing requires very small τ_d) and the limiting Keynesian boom is highly persistent.

To relate our analysis to the FTPL it is useful to briefly consider what happens in the permanent-income limit of $\omega = 1$. In that case, our aggregate demand relation reduces to

$$y_t = \mathbb{E}_t\left[y_{t+1}\right].$$

This equation has multiple bounded solutions, each corresponding to a different coordination equilibrium among consumers: if a permanent-income consumer expects everybody else to spend some amount z forever, then she expects her own income to be z forever, and she responds to this expectation by spending z forever, which closes the cycle. One of these solutions—$y_t = 0$ forever—is the one customarily selected by the Taylor principle. It is also the limit of our own equilibrium as $\omega \to 1^-$, for any given $\tau_d > 0$ or $H < \infty$. But if $\omega = 1$ and if $H = \infty$ (or $\tau_d = 0$), then a variant of the standard FTPL emerges: there is a (non-fundamental) equilibrium in which consumers coordinate on whatever level of spending is necessary for the government budget to hold. The conceptual difference between our theory and the conventional FTPL is now clear: breaking Ricardian equivalence on the consumer side allows us to replace the FTPL's assumption of a government that *never* adjusts the budget to one that does so with a *delay*. As a result, self-financing in our case requires neither an off-equilibrium threat to "blow up the budget" (Kocherlakota and Phelan, 1999), nor is it vulnerable to perturbations of social memory (Angeletos and Lian, 2023). We further elaborate on this discussion in Appendix A.4.

To summarize, our finding that, for a given fiscal policy, $\nu \to 0$ as $\omega \to 1$ suggests that the quantitative potency of our self-financing result critically depends on exactly how far the consumer block is from classical permanent-income behavior. We tackle this question in Section 6, where we quantify our result under empirically relevant assumptions on (i) delays in fiscal financing (τ_d and H) and (ii) household departures from permanent-income behavior (ω). Before doing so, however, we first in Section 5 further theoretically investigate the generality of our self-financing results.

5 Extensions

This section discusses five extensions of our self-financing result: the first two generalizing the model and the last three considering alternative specifications of fiscal and monetary policy.

First, in Section 5.1, we consider a much more general aggregate demand relation that nests—but goes materially beyond—our baseline OLG environment. Second, in Section 5.2, we discuss a model extension featuring investment. Third, in Section 5.3, we consider more general monetary policy rules, allowing the monetary authority to either accommodate or lean against the deficit-induced boom. Fourth, in Section 5.4, we allow for distortionary tax financing. Finally, in Section 5.5, we show that our results extend without any change to fiscal stimulus in the form of government purchases rather than lump-sum transfers.

5.1 A more general aggregate demand relation

We extend our self-financing result to much more general aggregate demand relations. The purpose of this extension is twofold. First, and more obviously, it illustrates the generality of our result. Second, our most general conditions offer additional important insights into the economics of self-financing.

Generalizing the demand block. Recall that, in our baseline OLG environment, aggregate consumer demand as a function of current wealth d_t as well as current and future income and taxes $y_t - t_t$ is

$$c_t = (1-\beta\omega)\left(d_t + \mathbb{E}_t\left[\sum_{k=0}^{\infty}(\beta\omega)^k\left(y_{t+k} - t_{t+k}\right)\right]\right). \tag{26}$$

This form of the aggregate demand relation embeds economically meaningful restrictions on consumer behavior: the MPC out of current income and wealth is the same (equal to $1-\beta\omega$), and MPCs out of the discounted presented value of future disposable income decline at a constant rate ω. We consider a generalized aggregate demand relationship that relaxes these constraints:

$$c_t = M_d \cdot d_t + M_y \cdot \left(\left(y_t - t_t\right) + \delta \cdot \mathbb{E}_t\left[\sum_{k=1}^{\infty}(\beta\omega)^k\left(y_{t+k} - t_{t+k}\right)\right]\right). \tag{27}$$

(27) allows for different MPCs out of current income ($M_y \in (0,1]$) and wealth ($M_d \leq M_y \in [0,1]$) as well as both constant and geometric discounting ($\delta, \omega \in [0,1]$) for future disposable income. We will give examples of several familiar models that are consistent with this more general structure.

A general sufficient condition for self-financing. We will now revisit our self-financing result in the baseline model of Section 2 but with the generalized demand relation (27) replacing our simpler OLG demand block. It turns out that self-financing obtains under two key restrictions on (27).

Assumption 1. *The aggregate consumption function features positive geometric discounting:* $\omega < 1$.

In words, MPCs out of future disposable income relative to MPCs out of current income and wealth decline strictly faster than the rate of interest β. Consistent with our discussion in Section 4.3, this is sufficient to ensure that far-ahead future tax hikes—i.e., tomorrow's tax hike in the analogy in Section 4.3—have vanishingly small effects on current aggregate demand, similar to the baseline OLG model. The fiscal deficit shock will thus lead to a demand boom around date 0.

Assumption 2. *Intertemporal MPCs are sufficiently front-loaded in the particular sense that*

$$M_d + \frac{1-\beta}{\tau_y}\left(1-\tau_y\right)M_y\left(1+\delta\sum_{k=1}^{\infty}(\beta\omega)^k\right) > \frac{1-\beta}{\tau_y}. \tag{28}$$

For (28) *to hold for all* $\tau_y \in (0,1]$*, the sufficient and necessary condition is*

$$M_d > 1-\beta \quad and \quad M_y\left(1+\delta\frac{\beta\omega}{1-\beta\omega}\right) \geq 1. \tag{29}$$

(28) is the condition required to ensure that the persistence of government debt ρ_d in the limiting self-financing equilibrium is strictly less than 1—i.e., that government debt will return to trend even as the future tax hike becomes vanishingly small. Intuitively, this requires the general equilibrium Keynesian boom to be sufficiently front-loaded, which in turn requires households to spend any income gains sufficiently quickly. If MPCs out of income and wealth are large enough—in the precise sense of (28)—then household spending is indeed sufficiently fast.[16]

Together, Assumptions 1 and 2 suffice to deliver a generalized self-financing result.

Theorem 2. *Consider the model of Section 2, with the consumer demand block taking the generalized form* (27)*. Suppose that Assumptions 1 and 2 hold.*

As fiscal adjustment is infinitely delayed there is complete self-financing, i.e., $\nu \to 1$ *(from below) as* $\tau_d \to 0$ *(from above) or as* $H \to \infty$*. Moreover, self-financing is sufficiently strong to return real government debt to steady state (i.e.,* $\lim_{k\to\infty}\mathbb{E}_t[d_{t+k}] \to 0$ *for the baseline rule* (6) *and* $\lim_{H\to\infty}\mathbb{E}_0[d_H] \to 0$ *for the variant rule* (7)*).*

Theorem 2 together with its underlying assumptions exactly echoes our intuitive discussion offered in Section 4.3. First, Assumption 1 guarantees that the future tax hike is discounted, so deficits will lead to a short-run boom. Second, Assumption 2 ensures that any additional income—both the initial transfer as well as all higher-order general equilibrium income gains—is spent sufficiently quickly to deliver a *short-lived* boom that raises the required revenue before the promised future tax hike becomes necessary. In the remainder of this section we will discuss examples of specific models of household demand that fit into the general form (27) and either satisfy or violate our two key conditions in Assumptions 1 and 2.

What environments are consistent with self-financing? Our generalized aggregate demand block (27) is consistent with many familiar models of household consumption-savings decisions.

Permanent-income consumers. The canonical permanent-income model readily fits into our generalized aggregate demand structure with $M_d = M_y = 1-\beta$ and $\delta = \omega = 1$ for all k. It is immediate that Assumptions 1 and 2 are violated. First, a deficit today together with (finitely) delayed financing does not induce a demand boom, simply because future tax hikes are not discounted further. Second, even

[16] Specifically, the first condition in (29) (i.e., that $M_d > 1-\beta$) corresponds to the second property of the consumption function ("front-loaded MPC") discussed in Section (4.3)—that is, we have that $\lim_{\ell\to\infty}\mathcal{M}_{t+\ell,t} = 0$. The second condition in (29) (i.e., that $M_y\left(1+\delta\frac{\beta\omega}{1-\beta\omega}\right) \geq 1$) guarantees that the general equilibrium Keynesian cross feedback is frontloaded enough to stabilize debt. In our baseline OLG economy both properties are ensured by $\omega < 1$.

if there was a general equilibrium boom, it would never be "quick", as permanent-income households postpone part of their spending into the infinite future, violating (28).

Liquidity constraints. Our generalized aggregate demand relation is evidently consistent with simple OLG models as a reduced-form representation of occasionally binding liquidity constraints (Farhi and Werning, 2019; Angeletos and Huo, 2021). More interestingly, since (27) disentangles the MPC out of wealth M_d and income M_y, it is also consistent with "hybrid" models that mix a margin of finite-life OLG households with classical spenders; i.e., a model with $M_y = \mu + (1-\mu)(1-\beta\omega)$, $M_d = (1-\beta\omega)$, $\omega < 1$ equal to the survival rate of the OLG households, and $\delta = \frac{(1-\mu)(1-\beta\omega)}{\mu+(1-\mu)(1-\beta\omega)}$, where $\mu \in (0,1)$ denotes the fraction of spenders. Such models have received attention in recent work because they tend to provide a relatively accurate approximation of aggregate demand in HANK-type models (see Auclert, Rognlie and Straub, 2018; Wolf, 2021*a*). It is straightforward to verify that, precisely because such hybrid models still feature "discounting" at rate $\omega < 1$, they also satisfy Assumptions 1 and 2, and thus deliver our self-financing result. A visual illustration is provided in the top panel of Figure 4.

Finally, it is instructive to briefly consider what happens in an alternative environment without any household discounting—the spender-saver model of Campbell and Mankiw (1989), with $M_y = \mu + (1-\mu)(1-\beta)$, $M_d = (1-\beta)$, $\omega = 1$, and $\delta = \frac{(1-\mu)(1-\beta)}{\mu+(1-\mu)(1-\beta)}$. Here it is straightforward to verify that both Assumptions 1 and 2 fail: the presence of permanent-income households means that (i) the effect of the future tax hike on date-0 consumption never vanishes, and (ii) the Keynesian boom is never fast enough to return the public debt back to trend. We see the result in the bottom panel of Figure 4: for any finite H, the self-financing share is zero, with the initial boom exactly offset (in net present value terms) by an increasingly large subsequent bust.[17] In our view this result is a feature and not a bug of our theory. Classical permanent-income savers correspond to an unrealistic infinite-horizon, infinite-liquidity, and infinite-rationality limit. In particular, the existence of a margin of these savers implies that the elasticity of household asset demand is infinite—a prediction of the model that is clearly at odds with data (Moll, Rachel and Restrepo, 2022). Finite horizons (as in our baseline model), wealth in the utility function of savers (Michaillat and Saez, 2021), binding household liquidity constraints (as in HANK), or certain kinds of behavioral frictions (as discussed further below) all break this unrealistic model feature and return us to our self-financing result.

Behavioral models. Our general demand block (27) also nests popular behavioral models of consumer spending behavior, including models with limited knowledge (Angeletos and Lian, 2018), limited rationality (Farhi and Werning, 2019; Vimercati, Eichenbaum and Guerreiro, 2021), or cognitive discounting (Gabaix, 2020). Behavioral frictions of this sort lead to additional discounting of future

[17] Similar conclusions apply when a margin of permanent-income households is added to our baseline OLG block. For large H, this model will feature a boom around 0 that looks very similar to our headline self-financing result. Around and after the financing horizon H, however, this economy enters a protracted slump, with permanent-income households consuming out of large accumulated savings, while consumption of the OLG margin is persistently depressed.

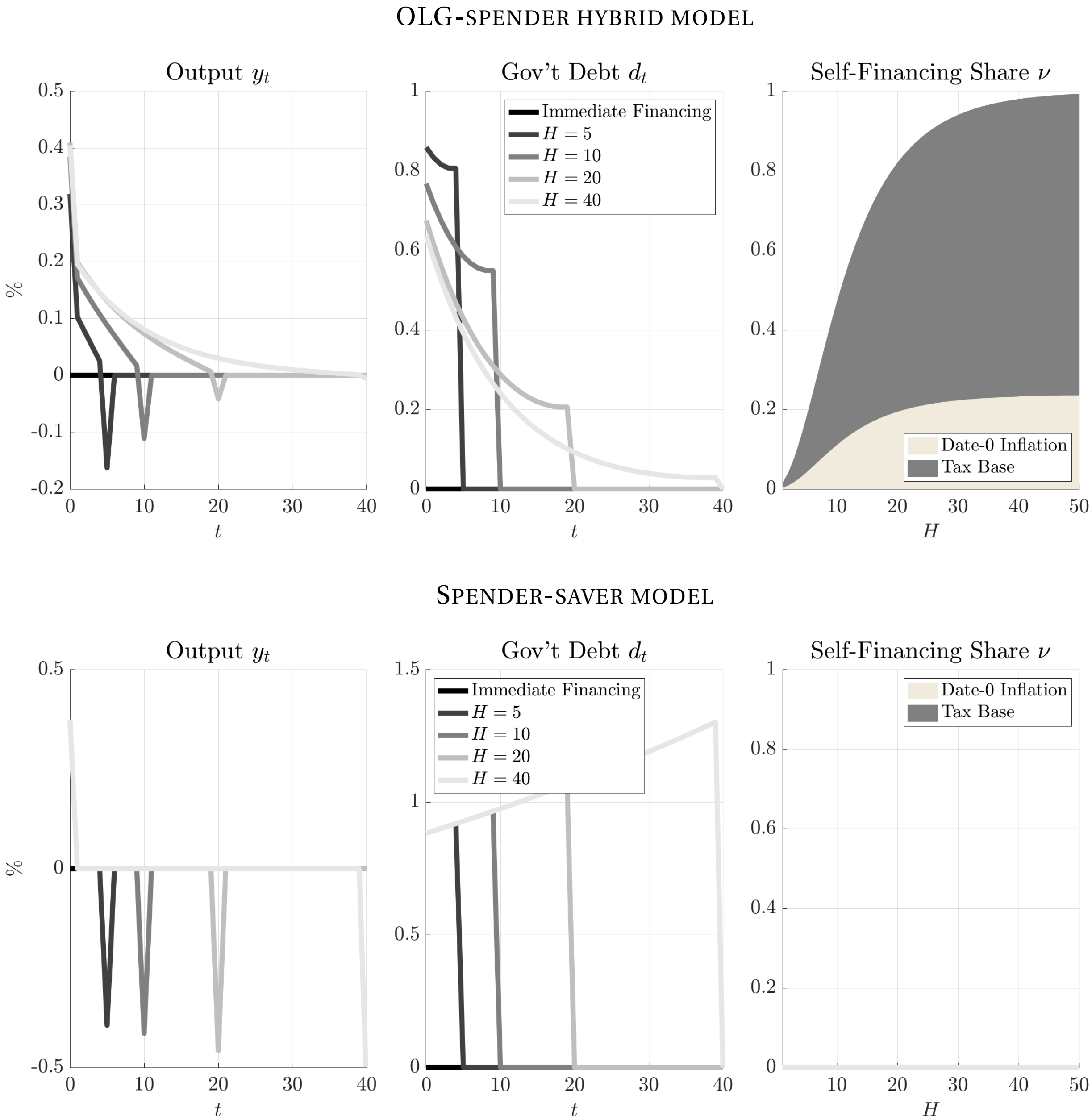

Figure 4: Top panel: impulse responses of output y_t, government debt d_t, and the total self-financing share ν to the deficit shock ϵ_0 as a function of H in an OLG-spender hybrid economy. Bottom panel: same as above, but in a spender-saver model.

income changes, thus lowering ω in our general relation (27). This has two interesting and partially offsetting effects. First, the initial partial equilibrium demand boom will be amplified: the future tax hike is discounted even further, and so the initial boom is larger. Second, the intertemporal Keynesian cross is weakened, as future gains in income do not feed back as strongly to today's consumer spending. We can see this in Assumption 2: a smaller ω decreases the left-hand side of (28), reflecting a Keynesian boom that is less front-loaded. We provide a quantitative investigation of these offsetting effects in Section 6.2 and Appendix C.5.

5.2 Investment

For our second extension we consider a substantially generalized model that also features firm investment. Our main result is that the self-financing logic extends almost without change to this case. We provide a sketch of the argument here, with details provided in Appendix B.2.

Model sketch. We enrich the supply side of our model. Firms now produce using labor and capital, owned and accumulated by the firm itself. As before firms are subject to standard nominal rigidities and pay out dividends to households. For our baseline results we assume that policy is specified exactly as in Section 2: the monetary authority fixes the expected real rate of interest while the fiscal authority taxes labor income and dividends, pays out transfers, and adjusts lump-sum taxes to balance the government budget as needed.

A generalized self-financing result. Our main insight is that the self-financing result of Theorem 1 extends with almost no change to this generalized environment. The logic of the argument is again seen easiest in the rigid-price case. Households consume out of labor plus dividend income less taxes, and the automatic fiscal tax adjustment τ_y is by assumption levied only on exactly this income. Furthermore, in equilibrium labor plus dividend income equals total consumption. The aggregate household demand relation (12) and the government debt equation (15) thus still constitute a bivariate system, just now in $\{c_t, d_t\}_{t=0}^{\infty}$ rather than $\{y_t, d_t\}_{t=0}^{\infty}$. Intuitively, with expected real interest rates fixed, the consumer and fiscal blocks of the model still induce a Keynesian cross relation, and so our equilibrium characterization in Section 4.2 applies completely unchanged, including in particular Theorem 1. Finally, partially sticky prices only somewhat complicate matters: the production block of the model now implies that the mapping from consumption as implied by the Keynesian cross into date-0 inflation is more complicated; conditional on this mapping, however, the same fixed-point logic as discussed in Section 4.2 applies, with inflation affecting the split into nominal and real self-financing, but without any effect on the overall self-financing limit.

5.3 Monetary accommodation

We now return to the environment of Section 2 but generalize our assumptions on policy. In this section we allow for more general monetary policy rules, taking the form of (9), restated here:

$$r_t = \phi y_t \tag{30}$$

Recall that our baseline monetary policy rule corresponds to the special case $\phi = 0$; we will now consider what happens if either $\phi < 0$ (i.e., monetary accommodation) or $\phi > 0$ (i.e., a monetary authority that leans against the deficit-induced boom). Intuitively, we would expect $\phi \neq 0$ to interact with the general equilibrium step underlying our self-financing intuition: $\phi < 0$ pulls spending forward in time and thus accelerates the deficit-driven boom even further, while $\phi > 0$ delays the boom and thus any tax base-related self-financing.[18] The remainder of this section formalizes these observations and in particular presents a generalized self-financing result. For completeness, we in Appendix B.3 provide a discussion of equilibrium determinacy, closely echoing and generalizing Leeper (1991).

On the possibility of self-financing equilibria Our main result is that self-financing continues to be possible in equilibrium if the monetary authority does not lean against the fiscal boom "too aggressively." We first state the formal result and then discuss its intuition.[19]

Theorem 3. *Consider our OLG-NK environment with $\omega < 1$ and $\tau_y > 1$, fiscal policy (6), and the monetary policy rule (30). There exists a $\bar{\phi} > 0$ such that:*

1. *If $\phi < \bar{\phi}$, complete self-financing is attained as the fiscal adjustment is infinitely delayed, i.e., $\nu \to 1$ and $\lim_{k\to\infty} \mathbb{E}_t[d_{t+k}] \to 0$ as $\tau_d \to 0$ (from above).*

2. *If $\phi > \bar{\phi}$, there is no bounded complete self-financing equilibrium as the fiscal adjustment is infinitely delayed ($\tau_d \to 0$ from above).*

[18] While we work with a "real" Taylor rule in (30) for expositional simplicity, we emphasize that our results here also hold for more traditional "nominal" Taylor rules of the form $i_t = \phi_\pi \pi_t$. In particular, the traditional "active" monetary policy $\phi_\pi > 1$ corresponds to $\phi > 0$ in (30), i.e., pro-cyclical expected real rates.

[19] With the more general monetary policy (9), the government budget constraint in (19) can be re-written as

$$\epsilon_0 + \frac{D^{ss}}{Y^{ss}} \sum_{k=0}^{+\infty} \beta^{k+1} E_0[r_k] = \tau_d \left(\epsilon_0 + \sum_{k=0}^{+\infty} \beta^k E_0[d_k] \right) + \sum_{k=0}^{+\infty} \tau_y \beta^k E_0[y_k] + \frac{D^{ss}}{Y^{ss}} (\pi_0 - \mathbb{E}_{-1}[\pi_0]),$$

where the new term $\frac{D^{ss}}{Y^{ss}} \sum_{k=0}^{+\infty} \beta^{k+1} E_0[r_k]$ captures how the time-varying interest rate in (9) changes the interest rate payments associated with the outstanding public debt. The share of self-financing (20) can then be re-written as

$$\nu \equiv \frac{\sum_{k=0}^{\infty} \tau_y \beta^k \mathbb{E}_0[y_k] + \frac{D^{ss}}{Y^{ss}} \pi_0}{\epsilon_0 + \frac{D^{ss}}{Y^{ss}} \sum_{k=0}^{+\infty} \beta^{k+1} E_0[r_k]} \tag{31}$$

The economic intuition underlying Theorem 3 is straightforward. Consider first the case of monetary accommodation (i.e., $\phi < 0$). As before, the fiscal deficit induces a demand boom. In response to this boom the monetary authority now cuts expected real rates, bringing household spending forward in time even further. The Keynesian boom is thus even quicker and debt is even less persistent—i.e., a smaller ρ_d than in our baseline self-financing equilibrium of Theorem 1. It follows that, with monetary accommodation, even less of a delay in the promised fiscal adjustment is needed to ensure material self-financing. Next consider a monetary authority leaning against the deficit-induced boom (i.e., $\phi > 0$). In that case, higher real rates result in households postponing spending, thus delaying the Keynesian boom—i.e., a larger ρ_d. The cutoff $\bar{\phi}$ is exactly the point where the monetary policy-induced delay exactly offsets the "quick" boom implied by finite household horizons, delivering $\rho_d(\bar{\phi}) = 1$. For any strictly more aggressive monetary policy the equilibrium explodes, and so no bounded self-financing equilibrium exists.

The previous discussion suggests a close connection between our results here and those for a more general aggregate demand relation presented in Section 5.1. Intuitively, both an aggressive monetary authority as well as low MPCs can result in consumer spending being sufficiently delayed to prevent fiscal revenue from being raised fast enough. This intuition indeed turns out to be correct: as we show in Appendix B.3, the cutoff $\bar{\phi}$ can equivalently be interpreted as being the value that delivers spending sufficiently postponed to violate our most general "GE" condition in Assumption 2.

5.4 Distortionary taxation

Our fourth model variant is one in which fiscal adjustment instead relies on distortionary taxation. The self-financing result turns out to extend with almost no change to this case; we only sketch the argument here, with details relegated to Appendix B.4.

Model sketch. We consider an extended environment in which the deficit shock ϵ_t is still paid out as a lump-sum transfer to households ("stimulus checks"), but then the distortionary tax τ_y adjusts over time to balance the government budget. Specifically, total taxes are now given as

$$T_{i,t} = \tau_{y,t} Y_{i,t} + \bar{T} - \mathscr{E}_t \tag{32}$$

where the distortionary tax rate $\tau_{y,t}$ is increasing in the fiscal deficit,

$$\tau_{y,t} = \tau_y + \tau_{d,t}(D_t - D^{ss}). \tag{33}$$

Note that the response coefficient $\tau_{d,t}$ is allowed to depend on time, nesting both kinds of fiscal rules considered in our original analysis. Relative to our baseline model, the *only* effect of this alternative fiscal financing rule is to introduce a time-varying wedge in the log-linearized aggregate NKPC:

$$\pi_t = \kappa y_t + \beta \mathbb{E}_t [\pi_{t+1}] + \zeta_t d_t \tag{34}$$

where ζ_t is a function of model primitives and such that $\zeta_t > 0$ if $\tau_{d,t} > 0$—intuitively, higher deficits map into higher distortionary taxes, introducing a labor wedge.

A generalized self-financing result. Our self-financing result extends without change. The logic echoes the proof of Theorem 1: as financing is delayed, the initial boom generates enough revenue to finance the deficit ϵ_0, so no tax adjustment is actually needed. It thus in particular does not matter whether this fiscal adjustment (which never happens) *would* have been distortionary or lump-sum.

5.5 Government spending

While we have so far considered lump-sum transfers to households as an example of a stimulative fiscal deficit policy, we emphasize that our results extend with almost no change to deficit-financed government purchases. We sketch our self-financing results for government purchases here, with details as well as a numerical illustration relegated to Appendix B.5.

Policy experiment. The only change relative to our baseline economy is that the government now also itself consumes some amount of the final good G_t. We assume that $G_t = \mathscr{G}_t$, where $\mathscr{G}_t$ is a stochastic spending shock. The linearized government budget constraint becomes

$$d_{t+1} = \frac{1}{\beta}\left[d_t - t_t + g_t + \beta \frac{D^{ss}}{Y^{ss}}(i_t - \pi_{t+1})\right] \tag{35}$$

and we consider linearized fiscal financing rules of the form

$$t_t = \tau_{d,t} \cdot (d_t + (1-\tau_y) g_t) + \tau_y y_t. \tag{36}$$

The presence of $(1-\tau_y)$ in the adjustment term ensures that $\tau_d = 1$ corresponds to immediate tax financing. Note that, since $\tau_{d,t}$ here is allowed to depend on time, this specification again nests both of our fiscal adjustment rules. Finally, the aggregate output market-clearing condition is replaced by

$$y_t = c_t + g_t \tag{37}$$

Self-financing government purchases. We obtain a result analogous to Theorem 1: as fiscal financing is delayed further and further, the tax adjustment vanishes. In particular, if prices are rigid, then the cumulative fiscal multiplier is again equal to $1/\tau_y$. The only difference is that, even for an immediately tax-financed spending increase, aggregate output responds, and thus the share of self-financing is not zero. This immediate tax-financed fiscal multiplier is in fact well-known to be equal to one (Woodford, 2011; Auclert, Rognlie and Straub, 2018), so the share of tax base self-financing is $\nu_y = \tau_y$.

6 Quantitative analysis

Having established the generality of our theory of self-financing deficits, we now ascertain its empirical relevance. Specifically, we ask: for empirically realistic delays in fiscal financing and for plausible departures from permanent-income behavior among households, how important is the self-financing margin likely to be? Our analysis proceeds in two steps. First, in Section 6.1, we first present and then calibrate a quantitatively relevant extension of our baseline model—a model rich enough to speak to empirical evidence on household consumption behavior, yet simple enough to be consistent with our general demand block studied in Section 5.1. Second, in Section 6.2, we use the model to compute the self-financing share ν under different assumptions on delays on fiscal financing.

6.1 Model and calibration strategy

Our analysis in Section 4 has revealed that the self-financing share is largely governed by two model features: household consumption behavior and the fiscal financing rule. In this section we review evidence on these two key ingredients and present a model that is consistent with that evidence.

Environment: spender-OLG hybrid. We begin with a sketch of the model environment. We consider a variant of our model in Section 2 with the baseline fiscal rule (6), but with one twist: we generalize the household block to consist of a mix of overlapping-generations households (as before) and a residual margin $\mu \in (0,1)$ of fully hand-to-mouth spenders, as already very briefly discussed in Section 5.1. Both groups receive labor and dividend income and pay taxes, but only the OLG block holds government bonds. Further details are presented in Appendix C.1.

A hybrid spender-OLG model is the ideal environment for our quantitative analysis, for two reasons. First, it remains simple enough to fit into our generalized aggregate demand relationship discussed in Section 5.1. As a result, we will later be able to invoke Theorem 2 to verify whether or not our calibrated model admits (limiting) full self-financing. Second, it is still rich enough to agree with quantitative HANK-type models on their implied intertemporal marginal propensities to consume (Auclert, Rognlie and Straub, 2018; Wolf, 2021*a*). Since the aggregate demand block of our economy affects the degree of self-financing only through those MPCs (see Appendix A.5 for the formal discussion), it follows that our model is quantitatively relevant in the particular sense of agreeing with state-of-the-art quantitative models of the household consumption-savings problem.[20]

[20] We note that the same is not true for our baseline OLG model. In that model, the coefficient ω governs both the average MPC as well as the time profile of how quickly income is spent. This restriction is inconsistent with empirical evidence on consumer spending behavior (see the discussion on calibration below). Allowing for spenders disentangles MPCs and dynamic spending profiles.

Parameter	Description	Value	Target	Value
Consumer spending				
μ	Share of HtM	0.11	Average MPC	0.22
ω	OLG survival rate	0.88	Short-run MPC slope	0.88
Fiscal adjustment				
τ_d	Tax feedback	$\{0.09, 0.04, 0.03\}$	Literature, see text	
Rest of the model				
σ	EIS	1	Standard	
β	Discount factor	0.99	Annual real rate	0.01
τ_y	Tax rate	0.3	Average Labor Tax	0.30
D^{ss}/Y^{ss}	Gov't debt	1.04	Liq. wealth holdings	1.04
κ	NKPC slope	0.1	Standard	

Table 1: Hybrid OLG-spender model, calibration.

Calibration. We discuss the model calibration in three steps: evidence on household consumption behavior to pin down the spender share μ and the OLG coefficient ω; evidence on fiscal adjustment to pin down the tax response coefficient τ_d; and all other parts of the model.

1. *Consumer spending behavior.* Empirical evidence on the household-level consumption response to income gains suggests two salient features of consumer spending behavior (Fagereng, Holm and Natvik, 2021). First, the average MPC out of income gains is elevated, with a standard quarterly value of around 0.22. Second, the income gain is spent gradually. Our baseline OLG environment of Section 2 provides a tight joint restriction on the level of the MPC and its dynamics: the level MPC (i.e., entry $\mathcal{M}_{0,0}$ of the matrix of MPCs) is $1-\beta\omega$, while the slope of the spending profile (i.e., the ratio of $\mathcal{M}_{\ell,0}$ to $\mathcal{M}_{\ell-1,0}$) is ω. This model-implied connection between level and slope is, unfortunately, inconsistent with the data; in particular, relative to the impact MPC, income in the data is subsequently spent much more quickly than predicted by the theoretical OLG-implied relationship. The spender-OLG hybrid model instead allows us to disentangle the level of the MPC and the spending slope in a way that is consistent with empirical evidence; in particular, we choose the spender share μ and the OLG coefficient ω to jointly match (i) impact and (ii) short-run (i.e., up to two years out) spending responses to lump-sum transfer receipt, as estimated by Fagereng, Holm and Natvik. A visual illustration of the implied intertemporal marginal propensities to consume is provided in Appendix C.1, and a discussion of several alternative calibration strategies will follow in Section 6.2.

   Current and future MPCs out of today's income gains—i.e., the estimand of Fagereng, Holm and

Natvik—is of course not all that matters for our theory; for our general equilibrium Keynesian cross, it is similarly important how anticipated income changes in the (far) future affect spending today (i.e., entries $\mathscr{M}_{t,t+\ell}$ for large ℓ in the matrix of MPCs). Given the lack of evidence on such responses to (far-away) income news shocks, our baseline exercise simply extrapolates these spending responses through the structure of the model. Encouragingly, as discussed in Wolf (2021*a*), our spender-OLG model extrapolates in a way very similar to state-of-the-art quantitative HANK-type models. Nevertheless, in Section 6.2 and in particular Appendix C.5, we also consider how results change with additional cognitive discounting among households, consistent with suggestive evidence from Ganong and Noel (2019).

2. *Delays in fiscal financing.* For our quantitative analysis we restrict attention to the fiscal rule (6), consistent with the seminal analysis of Leeper (1991). Prior work has estimated fiscal rules of this sort and thus in particular is informative about empirically relevant values of the speed of fiscal adjustment τ_d. For our quantitative analysis we consider three such studies (Galí, López-Salido and Vallés, 2007; Bianchi and Melosi, 2017; Auclert and Rognlie, 2018) with three implied values for τ_d, displayed in the middle part of Table 1. These three values imply half lives of government debt between two and ten years.[21]

3. *Rest of the model.* The remaining model parameters are set to standard values. First, we set $\sigma = 1$, giving log preferences. Since all quantitative exercises in this section fix the expected real rate of interest this parameter is in fact immaterial. Second, we set the discount factor to hit a steady-state real rate of interest of one per cent. Third, we set $\tau_y = 0.3$, in line with the discussion in DeLong and Summers (2012): for every dollar of additional output, we assume that the primary surplus automatically rises by 30 cents. Fourth, we set the slope of the NKPC to 0.1, a value at the large end of recent empirical estimates (e.g., Barnichon and Mesters, 2020) but arguably relevant for our application in Section 6.2—stimulus checks, as notably seen in the inflationary post-covid environment.

6.2 Are stimulus checks plausibly self-financed?

It is straightforward to verify numerically that our calibrated spender-OLG model satisfies the general sufficient conditions identified in Theorem 2 required for a limiting self-financing equilibrium to exist. It thus remains to ascertain how close we are to this limit for the empirically relevant fiscal rules discussed in Section 6.1. Results are displayed in Figure 5.

[21]We note that these half lives come from fiscal adjustment alone, without any equilibrium movements in prices and output. With those movements the half-life would be strictly smaller. Since empirical evidence arguably also contains these additional general equilibrium effects our quantitative analysis is conservative.

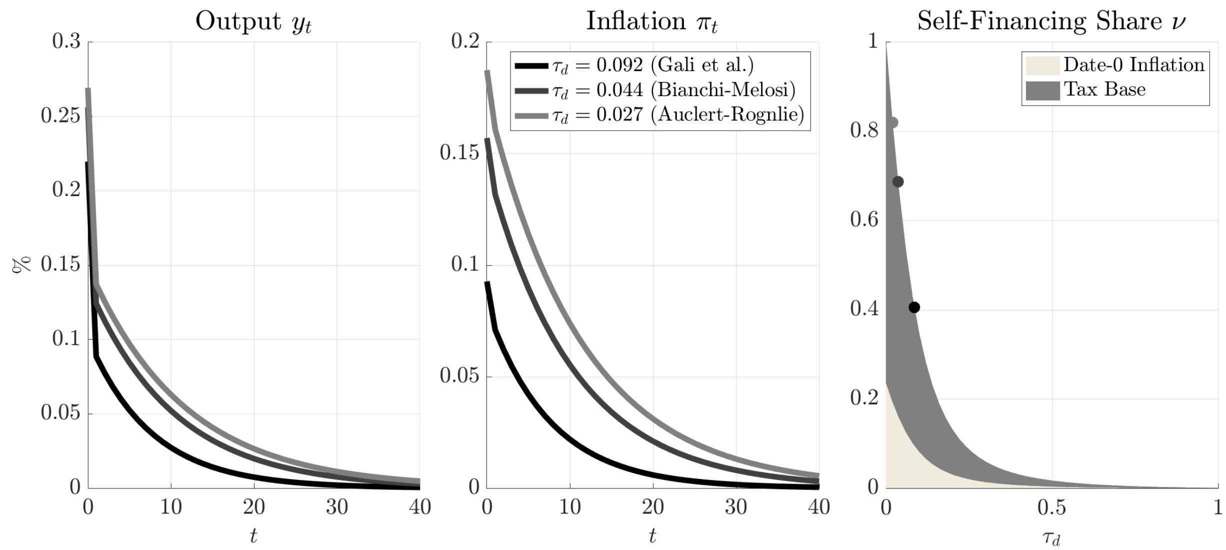

Figure 5: Impulse responses of output y_t, inflation π_t, and the total self-financing share ν to the deficit shock ϵ_0 as a function of τ_d. The left and middle panels show the impulse responses for the three values of τ_d discussed in Section 6.1. In the right panel these three points are marked with circles.

The figure shows output (left panel) and inflation (middle panel) impulse responses to a date-0 deficit shock for the three fiscal adjustment coefficients τ_d estimated in prior work (shades of grey); the right panel furthermore shows the degree of self-financing ν as a function of τ_d over the entire unit interval. As expected the share of self-financing ν is decreasing in τ_d and approaches one as $\tau_d \to 0$, consistent with Theorem 2. For the purposes of the quantitative analysis here, the key takeaway is that we are already quite close to this limit for the values of τ_d estimated in prior work, with ν around 0.4 - 0.8. We emphasize that these results are informative about the recent "excess savings" debate: if households violate Ricardian Equivalence because of reasonably long but finite horizons, then fiscal deficits without any quick tax offset are predicted to lead to a long-lived, inflationary boom.

As already remarked in Section 6.1, our calibration strategy relied on two standard but potentially material assumptions: first, we disciplined the slope of the spending profile (i.e., entries $\mathcal{M}_{\ell,0}$) from evidence on relatively *short-run* spending behavior (small ℓ); and second, we relied on model structure to extrapolate from responses to contemporaneous income changes to spending behavior following income news shocks (i.e., entries $\mathcal{M}_{t,t+\ell}$). The remainder of this section investigates how our conclusions would change with alternative calibration strategies and modeling assumptions.

Spending responses in the tails. As we emphasized throughout Sections 4 and 5.1, sufficiently front-loaded intertemporal MPCs are important for our self-financing results—they are what ensures a front-loaded Keynesian boom. We thus now consider alternative calibration strategies that try to more directly leverage information on spending responses to lump-sum transfer receipt in the *tails* (i.e.,

$\mathscr{M}_{\ell,0}$ for large ℓ). These tail responses are crucial to determine how fast cumulative MPCs converge to 1 and thus how front-loaded the Keynesian boom turns out to be.[22]

Results are reported in Figure 6. Here the three panels correspond to three different models—our baseline model of Section 6.1 (top panel) as well as two alternatives (middle and bottom panel). Our empirical targets, reported as the grey areas (corresponding to 95 per cent confidence intervals) in the three "cumulative MPC" panels, are again taken from Fagereng, Holm and Natvik (2021) (see their Figure 2). Now consider first the top panel. By construction, this model matches MPCs—and thus also cumulative MPCs—over the first couple of years after the income receipt. However, as revealed by the middle part of the panel, the model-implied cumulative MPC appears to converge to 1 somewhat too quickly, reflecting intertemporal MPCs that converge to 0 relatively fast, at rate ω. The middle panel—labeled "lower bound calibration"—instead disciplines the model parameters (μ,ω) by matching (i) the same impact MPC as before and then (ii) targeting the *lower bound* of the estimated confidence interval five years after the income receipt. This calibration strategy unsurprisingly delivers a larger ω (equal to 0.94), mapping into materially flatter intertemporal MPC profiles (left part), inconsistent with evidence on short-run spending responses (middle part). As expected, convergence to our self-financing limit is slowed, though even in this calibration the degree of self-financing remains rather substantial for our various fiscal adjustment rules. Finally, in the bottom panel, we consider an even further extended model—a model featuring spenders together with two types of OLG blocks, with heterogeneous ω_1 and ω_2—that is rich enough to provide a tight fit to the entire dynamic profile of cumulative MPCs, up to five years out. The left part of the bottom panel reveals that this model looks rather similar to our baseline calibration in the periods around income receipt, but then has somewhat flatter tail MPCs. Thus, as in the middle panel, the speed of convergence to the self-financing limit is slowed down, though the predicted degree of self-financing again remains meaningful, exactly as in our baseline environment.

Spending response to future income changes. All models discussed above are calibrated to be consistent with evidence on consumer spending behavior in response to income shocks *today*; the similarly important consumption behavior in response to income *news* shocks (i.e., $\mathscr{M}_{t,t+\ell}$), on the other hand, is extrapolated through the model structure. We here briefly discuss results from two alternative models that extrapolate somewhat differently, with details in Appendices C.4 and C.5.

First, in Appendix C.4, we consider a quantitative HANK model. Consistent with the results in Auclert, Rognlie and Straub (2018) and Wolf (2021*a*) we find that this model extrapolates MPCs across horizons in almost the same way as our reduced-form spender-OLG hybrid model. It is thus unsur-

[22]Yet another calibration strategy is presented in Appendix C.2. There we discipline the discounting coefficient ω through empirical evidence on long-run elasticities of household asset demand. Interestingly, this very different (and much less direct) approach suggests values of ω reasonably close to our baseline calibration strategy.

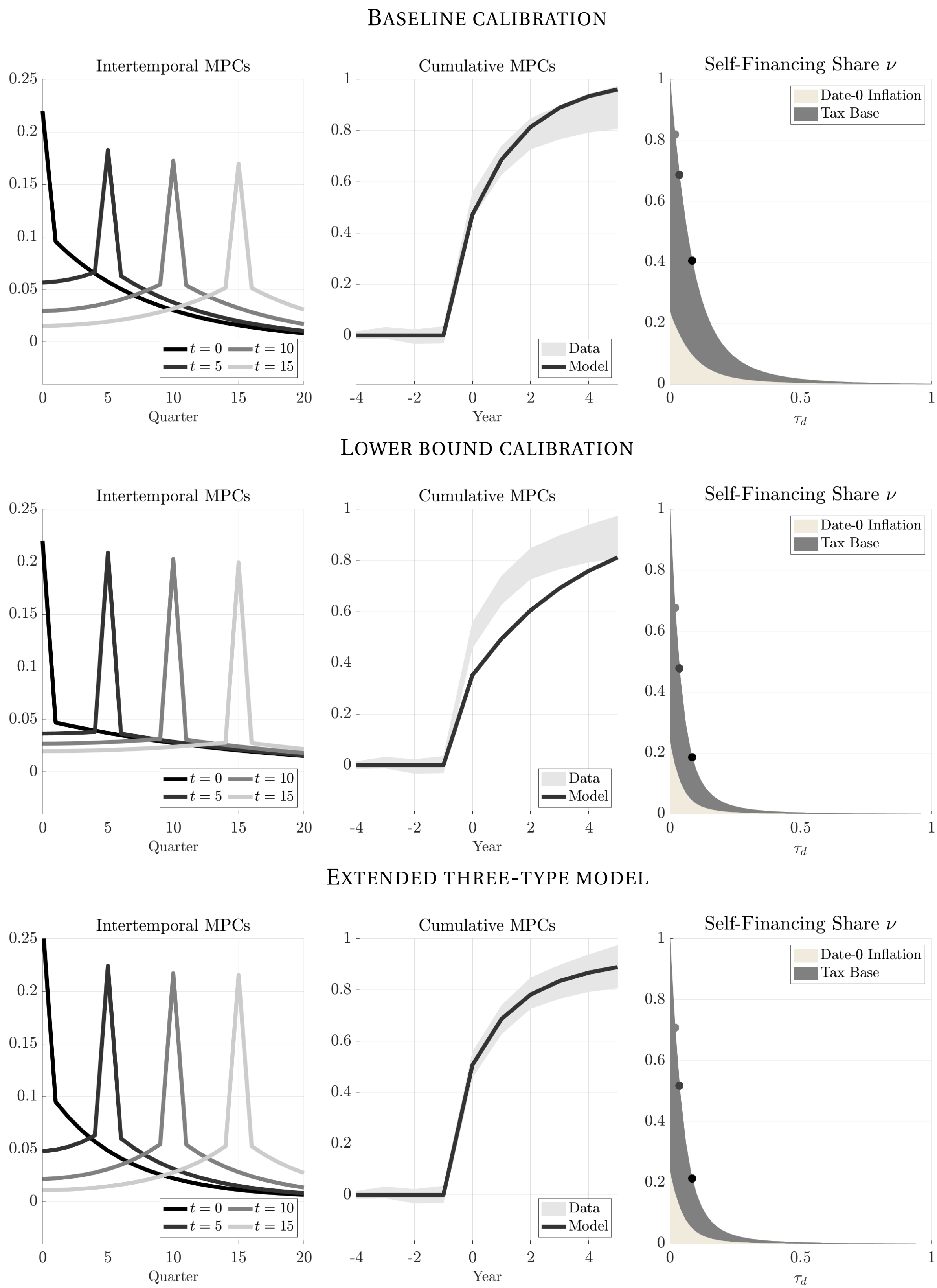

Figure 6: Top panel: iMPCs, cumulative MPCs, and self-financing share ν (as a function of τ_d in the baseline model. Middle and bottom panels: same as above, but for models calibrated to (also) match far-ahead tail MPCs. Parameter values are reported in Appendix C.2.

prising that our conclusions on self-financing are largely unchanged: self-financing equilibria still exist and the share of self-financing is large for empirically relevant values of τ_d.

Second, we consider a variant of our spender-OLG model with household cognitive discounting. As discussed in Section 5.1, we expect such behavioral discounting to introduce two offsetting effects on ν. On the one hand, the future tax hike is discounted by more, pushing up fiscal multipliers and thus the degree of self-financing. On the other hand, higher income in the future does not feed back to spending today, so it takes longer for the self-financing Keynesian boom to play out in equilibrium. Our model simulations confirm this intuition: we find that ν is higher than in our baseline model for intermediate values of τ_d, but converges to the full self-financing limit somewhat more gradually. Overall, however, the basic conclusions from our full-information rational-expectations analysis in Figure 5 extend with relatively little change.[23]

7 Conclusion

The central contribution of this paper is to clarify the conditions under which fiscal deficits can finance themselves. Our analysis applies to model environments with two empirically relevant features: (i) nominal rigidities and (ii) a violation of Ricardian equivalence due to finite lives or liquidity constraints. The headline result is that, as the fiscal authority delays the eventual tax adjustment further and further, this tax hike becomes smaller and smaller, with the deficit financed instead through a mix of higher output and inflation.

Our results have important conceptual as well as practical implications. *Conceptually*, they suggest that the spirit of the classical fiscal theory of the price level—that fiscal adjustment is not necessary to finance deficits—is more robust than commonly believed: it does not rely on the threat of a government violating its budget constraint, nor is it vulnerable to debates regarding equilibrium selection. Rather, it merely requires delays in fiscal financing together with empirically plausibly departures from permanent-income behavior. *Practically*, our theory predicts that deficit-financed stimulus will invariably induce a meaningful and persistent boom, with its effect on prices versus quantities governed by the degree of nominal rigidities and thus slack in the economy. These insights are particularly relevant for recent U.S. fiscal stimulus experiments.

We emphasize that our analysis has been entirely positive, not normative. We leave an investigation of the implications of our results for *optimal* fiscal and monetary policy to future work.

[23] We expect similar results in behavioral model variants that replace cognitive discounting with limited information and/or bounded rationality, as for example in Angeletos and Lian (2018), Farhi and Werning (2019), and Vimercati, Eichenbaum and Guerreiro (2021).

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Online Appendix for:
Can Deficits Finance Themselves?

This online appendix contains supplemental material for the article "Can Fiscal Stimulus Finance Itself?". We provide: (i) supplementary discussion of our baseline OLG model studied in Sections 2 - 4; (ii) details for the model extensions considered in Section 5; (iii) additional analysis and alternative results for our quantitative investigation in Section 6. The end of this appendix contains all proofs.

Any references to equations, figures, tables, assumptions, propositions, lemmas, or sections that are not preceded "A."—"D." refer to the main article.

A Supplementary model details

We here provide some additional discussion of our baseline structural model in Section 2. Appendix A.1 considers household labor supply, Appendix A.2 then combines this labor supply relationship with firm pricing decisions to derive our NKPC supply block (14), and Appendix A.3 explains how we characterize equilibria under the alternative fiscal policy rule (7). In Appendix A.4 we provide a detailed discussion of the permanent-income limit of our model. Finally, in Appendix A.5, we briefly interpret our self-financing results from a sequence-space perspective.

A.1 Labor supply

Unions equalize the post-tax real wage and the average marginal rate of substitution between labor supply and consumption. The aggregate optimal labor supply relation is thus

$$(1-\tau_y)W_t = \frac{\chi \int_0^1 L_{i,t}^{\frac{1}{\varphi}} di}{\int_0^1 C_{i,t}^{-1/\sigma} di} \tag{A.1}$$

Log-linearizing, we obtain (13).

A.2 Price-setting and the NKPC

Optimal firm pricing decisions as usual give inflation as a function of real marginal costs. With a standard constant-returns-to-scale, labor-only production function this gives (e.g., see the textbook derivations in Woodford, 2003*a*; Galí, 2008)

$$\pi_t = \tilde{\kappa} w_t + \beta \mathbb{E}_t [\pi_{t+1}], \tag{A.2}$$

where $\tilde{\kappa}$ is a function of firm-side primitives, including in particular the stickiness of prices. Combining (A.2) with (13) and imposing that $c_t = \ell_t = y_t$ we obtain

$$\pi_t = \underbrace{\tilde{\kappa}\left(\frac{1}{\varphi} + \frac{1}{\sigma}\right)}_{\equiv \kappa} y_t + \beta \mathbb{E}_t [\pi_{t+1}], \tag{A.3}$$

as required.

A.3 Equilibrium characterization with the alternative fiscal rule

We characterize the equilibrium in our OLG-NK environment with $\omega < 1$, $\tau_y > 0$, and the alternative fiscal rule (7) here. The aggregate demand relation (11) together with monetary policy (8) and market clearing $y_t = c_t$ lead to the following recursive aggregate demand curve[24]

$$y_t = \frac{\left(1-\beta\omega\right)(1-\omega)}{\beta\omega}\left(d_t - t_t\right) + \mathbb{E}_t\left[y_{t+1}\right], \tag{A.4}$$

where we use that $\mathbb{E}_t\left[d_{t+1}\right] = \frac{1}{\beta}\left(d_t - t_t\right)$ from (4).

We characterize the bounded equilibrium path through backward induction. Given the alternative fiscal rule (7), we know that, for $t \geq H$,

$$d_t - t_t = 0 \Longrightarrow y_t = \mathbb{E}_t\left[y_{t+1}\right].$$

We focus on the equilibrium with $y_t = 0$ for $t \geq H$. As discussed in Footnote 13, this equilibrium selection can be justified in three ways: strengthening the boundedness requirement in the equilibrium definition, allowing for $\tau_y > 0$ after H, or limiting to $\phi = 0$ from above. The sole role of any of these modifications is to remove a class of sunspot equilibria that are inherited from the standard New Keynesian model. Given this selection we use (A.4) to find the equilibrium path of $\left\{y_t, d_t\right\}_{t=0}^{H-1}$ through backward induction starting from

$$y_H = \chi_0\left(d_H + \epsilon_H\right) \quad \text{with} \quad \chi_0 = 0.$$

For $t \leq H-1$, substitute the alternative fiscal rule (7) into (A.4), giving

$$y_t = \frac{\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}}{1+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\tau_y}\left(d_t + \epsilon_t\right) + \frac{1}{1+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\tau_y}\mathbb{E}_t\left[y_{t+1}\right].$$

As a result, for $t \leq H-1$,

$$y_t = \chi_{H-t}\left(d_t + \epsilon_t\right) \quad \text{with} \quad \chi_{H-t} = \frac{\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}}{1+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\tau_y} + \frac{\frac{1}{\beta}\left(1-\tau_y\chi_{H-t}\right)}{1+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\tau_y}\chi_{H-t-1}. \tag{A.5}$$

Rearranging terms, we find the following recursive formula for the χs:

$$\chi_{H-t} = \frac{\frac{(1-\beta\omega)(1-\omega)}{\beta\omega} + \frac{\chi_{H-t-1}}{\beta}}{1+\left(\frac{(1-\beta\omega)(1-\omega)}{\beta\omega} + \frac{\chi_{H-t-1}}{\beta}\right)\tau_y} = g\left(\chi_{H-t-1}\right), \tag{A.6}$$

where

$$g\left(\chi\right) = \frac{\frac{(1-\beta\omega)(1-\omega)}{\beta\omega} + \frac{\chi}{\beta}}{1+\left(\frac{(1-\beta\omega)(1-\omega)}{\beta\omega} + \frac{\chi}{\beta}\right)\tau_y} \quad \text{and} \quad g'\left(\chi\right) = \frac{1}{\beta}\frac{1}{\left(1+\tau_y\left(\frac{(1-\beta\omega)(1-\omega)}{\beta\omega} + \frac{\chi}{\beta}\right)\right)^2} \geq 0 \quad \forall \chi \geq 0. \tag{A.7}$$

We thus know that

$$\chi_k \in (0, \frac{1}{\tau_y}) \quad \forall k \geq 1 \quad \text{and} \quad \chi_k \text{ increases in } k. \tag{A.8}$$

[24]Note that (12) does not apply here, since its derivation uses the baseline fiscal policy (6).

From (4) and (A.5), we also know that

$$\mathbb{E}_0[d_t] = \frac{1}{\beta^t}\Pi_{j=0}^{t-1}\left(1-\tau_y\chi_{H-j}\right)(d_0+\epsilon_0) \tag{A.9}$$

To further characterize the equilibrium we begin by considering an alternative economy with rigid prices ($\kappa = 0$) but otherwise identical to the baseline economy. Let ν' denote the self financing share in this alternative economy, i.e.,

$$\nu' \cdot \epsilon_0 = \nu'_y \cdot \epsilon_0 \equiv \sum_{k=0}^{\infty} \tau_y \beta^k \mathbb{E}_0[y_k],$$

In this economy, there is no $t = 0$ price level jump and so the real value of public outstanding at $t = 0$, $d_0 = b_0 = 0$ is pre-determined. From (A.5) and (A.9), we have that

$$\nu' = \sum_{t=0}^{H-1} \Pi_{j=0}^{t-1}\left(1-\tau_y\chi_{H-j}\right)\tau_y\chi_{H-t} = 1-\Pi_{j=0}^{H-1}\left(1-\tau_y\chi_{H-j}\right). \tag{A.10}$$

We can now return to the general case with $\kappa \geq 0$. From the NKPC (14) as well as the definitions in (20) – (21), we have that

$$\nu_p = \frac{\kappa\frac{D^{ss}}{Y^{ss}}}{\tau_y}\nu_y = \frac{\kappa\frac{D^{ss}}{Y^{ss}}}{\tau_y+\kappa\frac{D^{ss}}{Y^{ss}}}\nu \tag{A.11}$$

Finally, from the formula of d_0 (16), we know

$$d_0 = -\nu_p\epsilon_0 \quad \text{and} \quad \nu_y = \left(1-\nu_p\right)\nu^{'}.$$

Together, we have

$$\nu = \frac{\nu'}{\frac{\tau_y}{\tau_y+\kappa\frac{D^{ss}}{Y^{ss}}}+\frac{\kappa\frac{D^{ss}}{Y^{ss}}}{\tau_y+\kappa\frac{D^{ss}}{Y^{ss}}}\nu'} \quad \nu_y = \frac{\frac{\tau_y}{\tau_y+\kappa\frac{D^{ss}}{Y^{ss}}}\nu'}{\frac{\tau_y}{\tau_y+\kappa\frac{D^{ss}}{Y^{ss}}}+\frac{\kappa\frac{D^{ss}}{Y^{ss}}}{\tau_y+\kappa\frac{D^{ss}}{Y^{ss}}}\nu'}, \quad \text{and} \quad \nu_p = \frac{\frac{\kappa\frac{D^{ss}}{Y^{ss}}}{\tau_y+\kappa\frac{D^{ss}}{Y^{ss}}}\nu'}{\frac{\tau_y}{\tau_y+\kappa\frac{D^{ss}}{Y^{ss}}}+\frac{\kappa\frac{D^{ss}}{Y^{ss}}}{\tau_y+\kappa\frac{D^{ss}}{Y^{ss}}}\nu'}. \tag{A.12}$$

This completes our characterization of the equilibrium.

A.4 The PIH/RANK benchmark ($\omega = 1$)

This section provides a detailed comparison between our economy ($\omega < 1$) and its textbook, representative-agent counterpart ($\omega = 1$).

The aggregate demand block. Recall that, with $\omega = 1$, the aggregate demand block becomes

$$y_t = \mathbb{E}_t\left[(1-\beta)\sum_{k=0}^{\infty}\beta^k y_{t+k}\right] \tag{A.13}$$

or

$$y_t = \mathbb{E}_t[y_{t+1}]. \tag{A.14}$$

Although the Euler equation (A.14) is more familiar, the aggregate demand relation (A.13) is more insightful for our purposes, for the following reasons. First, it reveals the general equilibrium interactions hidden behind the representative consumer's Euler condition—i.e., the intertemporal Keynesian cross (IKC). Second, it translates this IKC to a dynamic game of strategic complementarity among the consumers: when a consumer expects others to spend more, she finds it optimal to spend more herself, because high spending by others means higher income for herself. Third, it makes clear why exactly this game admits multiple equilibria—because the cumulative marginal propensity to consume is one. And fourth, it reveals that fiscal policy does not enter the payoffs and the best responses of this game: holding constant aggregate spending (and hence also income), a consumer's optimal spending is invariant to fiscal policy. It follows that public debt and deficits can drive aggregate spending if and only if they serve as coordination devices (sunspots).

What are solutions of the aggregate demand block (A.13), studied in isolation? An obvious one is $y_t = 0$ for all t and states of nature, which means that the economy stays at steady state for ever. But $y_t = \bar{y}$ is also a solution, for arbitrary $\bar{y}$; this corresponds to consumers at every t spending $\bar{y}$ because, and only because, they expect all future consumers to do the same. Finally, there exist equilibria in which $y_t = \xi \sum_{k=0}^{t} \epsilon_{t-k}$, for arbitrary $\xi \in \mathbb{R}$. In any such equilibrium, consumers vary their spending with the current deficit by an amount equal to ξ because and only because they expect future consumers to keep responding to the current deficit in the same away, in perpetuity.

Feedback between consumers & fiscal policy in RANK. The above statements are valid characterizations of solutions to (A.13), without reference to fiscal policy. Conventional notions of equilibrium—including in particular our Definition 1—however also require that debt does not explode, thus putting the government budget constraint back into the picture. This in turn puts joint restrictions on fiscal policy and consumer behavior, despite the fact that consumers do not care for fiscal policy in the precise sense of the game described above.

What are these restrictions? For this it will prove instructive to consider first the case in which $\tau_y = 0$ and $\kappa = 0$. In this case, the law of motion for public debt reduces to

$$d_{t+1} = \beta^{-1}(1-\tau_d)(d_t + \epsilon_t).$$

This relation has two key properties. First, it is invariant to consumer behavior. And second, it has debt converge back to steady state if and only if $\tau_d > 1-\beta$. We thus arrive at the following conclusion.

Proposition A.1. *Suppose $\omega = 1$, $\tau_y = 0$, and $\kappa = 0$. Then:*

1. *If $\tau_d > 1-\beta$, there exist multiple self-fulfilling (and bounded) equilibria. In particular, $y_t = \bar{y} + \xi \sum_{k=0}^{t} \epsilon_{t-k}$ is an equilibrium for any $\bar{y} \in \mathbb{R}$. and any $\xi \in \mathbb{R}$.*

2. *If instead $\tau_d < 1-\beta$, there does not exist a (bounded) equilibrium.*

It is important to recognize that the non-existence of equilibrium in case (ii) is the byproduct of "bad" fiscal policy: when $\tau_d < 1-\beta$, there is no problem in finding a fixed point in the GE interaction among the consumers, but there is also no way for public debt to be sustainable (i.e., not to explode).

Now let us relax the restrictions $\tau_y = 0$ and/or $\kappa = 0$ (while maintaining $\omega = 1$). This does not change either condition (A.13) or our aforementioned logic about consumer behavior: fiscal policy continues not to enter the game among the consumers. But the opposite is no more true: consumer behavior now enters the government's budget via the tax base (when $\tau_y > 0$) and/or the real value of public debt (when $\kappa > 0$). This in turn modifies the second (but not the first) part of Proposition A.1.

Proposition A.2. *Suppose* $\omega = 1$ *and* $\max\{\tau_y, \kappa\} > 0$. *Then:*

1. *If* $\tau_d > 1-\beta$, *there exist multiple self-fulfilling (and bounded) equilibria.*
2. *If instead* $\tau_d < 1-\beta$, *there exists a unique bounded equilibrium.*

To understand the content of Proposition A.2, consider first the case with rigid prices ($\kappa = 0$). Had τ_y been zero, then $\tau_d < 1-\beta$ would have caused debt to explode regardless of how consumers behave. But now that $\tau_y > 0$, the following possibility is logically coherent (although, at least in our view, rather implausible): consumers coordinate on an equilibrium among them that generates just enough tax revenue to offset the adverse effects of deficit shocks and to make sure that debt does not explode. By direct analogy to the classical FTPL, we call this the "Fiscal Theory of Y" (FTY): in a world with rigid prices ($\kappa = 0$) and passive monetary policy ($\phi = 0$), consumers may coordinate on multiple self-fulling spending behaviors and hence to multiple self-fulfilling levels of aggregate income, but only one of them makes sure that debt does not explode when $\tau_d < 1-\beta$. Next, if we let $\tau_y = 0$ but $\kappa > 0$, then we recover the original Fiscal Theory of the Price Level (FTPL): the set of multiple self-fulling spending behaviors remains the same, but now there is a different unique selection among them that makes sure that debt does not explode when $\tau_d < 1-\beta$. And finally, a hybrid of these two extreme cases obtains (and a different equilibrium is selected) when both $\tau_y > 0$ and $\kappa > 0$.

We note that there is a long-standing theoretical debate about the plausibility of the FTPL. While this debate is *not* our paper's main theme or contribution, we hope that the above analysis helps make clear the following points: (i) the precise sense in which fiscal policy does not enter the Intertemporal Keynesian Cross (or the game among the consumers, when $\omega = 1$); (ii) the reason why this game admits multiple self-fulfilling equilibria; (iii) that the entirety of this multiplicity is consistent with non-explosive debt when $\tau_d > 1-\beta$; (iv) that no equilibrium is consistent with non-explosive debt when $\tau_d < 1-\beta$ and $\tau_y = \kappa = 0$; (v) that $\tau_y > 0$ or $\kappa > 0$ allows debt to be non-explosive when $\tau_d < 1-\beta$, but only insofar as consumers coordinate on a particular self-fulfilling equilibrium among the many of the aforementioned game; and (vi) that both the FTPL and the FTY translate to specific selections among the multiple self-fulfilling solutions of the IKC. Last but not least, the unique "fundamental" (or Markov Perfect) equilibrium of the aforementioned game is one where debt and deficits do not influence aggregate demand, and debt is sustainable along this equilibrium if and only if $\tau_d > 1-\beta$.

Returning to our environment. How does our environment fit into this picture? Once $\omega < 1$, fiscal policy becomes payoff-relevant in the game among the consumers. This then introduces a feedback from fiscal policy to aggregate demand, differently from the permanent-income baseline.

To see this most clearly, again suppose first that $\tau_y = \kappa = 0$, which means that consumer behavior does not enter the government budget. Similarly to the $\omega = 1$ case, this implies that public debt is non-explosive if and only if $\tau_d > 1-\beta$. Furthermore, it remains true that the game among consumers admits multiple equilibria. For instance, if there are no deficit shocks, then $y_t = \bar{y}$ is an equilibrium for any constant $\bar{y}$. Differently from the case above, however, the response of output to deficits is now uniquely pinned down.

Proposition A.3. *Suppose $\omega < 1$ but $\tau_y = \kappa = 0$. When $\tau_d < 1-\beta$, there does not exist an equilibrium. And when $\tau_d > 1-\beta$, there exist multiple bounded equilibria. The set of equilibria is then given by*

$$y_t = \bar{y} + \chi(d_t + \epsilon_t)$$

for arbitrary $\bar{y} \in \mathbb{R}$ and for a unique $\chi > 0$, given by

$$\chi = \frac{\mathscr{F}_1}{1 - \frac{(1-\beta\omega)}{1-\beta\omega(1-\tau_d)}}.$$

In short, the "intercept" or "level" remains indeterminate (because the dynamic strategic complementarity is still 1), but the response to debt and deficits is uniquely pinned down (because debt and deficits cease to be sunspots).

Finally return to the case where $\tau_y > 0$—i.e., our baseline case. Then and only then there is a *two-way* feedback between aggregate demand and the government budget. Furthermore, this two-way feedback translates to a reduction in the effective degree of strategic complementary. From

$$\mathscr{F}_2 = 1 - \frac{(1-\omega)\,\tau_y}{1-\omega\,(1-\tau_d)},$$

it is immediate that $\mathscr{F}_2 < 1$ if and only if *both* $\omega < 1$ and $\tau_y > 0$. This in turn helps explain why the multiplicity disappears when and only when *both* $\omega < 1$ and $\tau_y > 0$. On other hand, the presence of a feedback from output to the government budget constraint helps explain why a bounded equilibrium exists not only for $\tau_d > 1-\beta$ but also for $\tau_d < 1-\beta$.

Summary. From the preceding analysis we conclude that Proposition 1 combines the following subtle lessons: $\omega < 1$ alone explains why debt and deficits become fundamentals and feed into aggregate demand; $\tau_y > 0$ in turn explains how aggregate demand feeds into the government budget and can aid debt sustainability even when $\tau_d < 1-\beta$; and the combination of the two feedbacks finally helps explain why our economy features a unique bounded equilibrium, despite the apparent failure of the Taylor principle ($\phi = 0$).

A.5 A sequence-space perspective

All results in the main part of this paper are stated and proved using a standard state-space approach to equilibrium characterization. We can, however, develop some additional insights by instead adopting a sequence-space perspective. In the context of the paper as a whole the purpose of the sequence-space analysis in this section is twofold. First, by adopting this sequence-space perspective, we will be able to very easily substantiate a claim made in Section 6—that intertemporal MPCs fully characterize limiting self-financing equilibria. Second, we provide a different perspective on Assumption 2, re-phrasing it as a sufficient condition ensuring that intertemporal MPCs decay "sufficiently quickly."

Equilibrium. For the analysis in this section we again consider the baseline model of Sections 2 and 4, but substantially generalize the aggregate demand relation (11) to the following sequence-space relation:

$$\boldsymbol{c} = \mathscr{M} \times \left(\boldsymbol{y} - \boldsymbol{t}\right) + \mathscr{M}_i \times \boldsymbol{i} + \mathscr{M}_\pi \times \boldsymbol{\pi} \tag{A.15}$$

where boldface denotes sequences. Our objective in this section is to characterize self-financing equilibria, so we impose the baseline monetary policy rule (8), throughout consider the limiting case of $\tau_d = 0$, and for simplicity also assume that prices are rigid ($\kappa = 0$).[25] Imposing market-clearing as well as the monetary policy rule, the demand relation (A.15) becomes

$$\boldsymbol{y} = \mathscr{M} \times \left(\boldsymbol{y} - \boldsymbol{t}\right) \tag{A.16}$$

Now note that, under our assumptions on fiscal policy, taxes are given

$$\boldsymbol{t} = \tau_y \times \boldsymbol{y} - \boldsymbol{\epsilon} \tag{A.17}$$

where $\boldsymbol{\epsilon}$ denotes the fiscal policy stimulus. Combining (A.16) and (A.17), we find that output in the limiting self-financing equilibrium is completely characterized through the following system of dynamic equations:

$$\boldsymbol{y} = (1-\tau_y)\mathscr{M} \times \boldsymbol{y} + \mathscr{M} \times \boldsymbol{\epsilon} \tag{A.18}$$

(A.18) is a variant of the intertemporal Keynesian cross studied previously in Auclert, Rognlie and Straub (2018), but with a crucial difference: automatic tax financing is embedded in the tax revenue

[25] By an argument analogous to our discussion surrounding Theorem 1 the extension to the partially sticky price case is conceptually straightforward.

term $\tau_y \times \boldsymbol{y}$, rather than being specified directly as part of the policy intervention (here $\boldsymbol{\epsilon}$). This seemingly subtle distinction has important implications and in particular connects tightly with our self-financing results in Sections 4 and 5.1.

Discussion. The previous discussion first of all immediately substantiates the claim made in Section 6: for a large family of models (including in particular our spender-OLG hybrid), intertemporal MPCs fully characterize the dynamics of output in the limiting self-financing equilibrium. It remains to further characterize the solution of (A.18), allowing us to connect with the economic intuitions offered in Sections 4 and 5.1.

The remainder of the discussion here will leverage a crucial property of the intertemporal MPC matrix $\mathcal{M}$. Letting $\boldsymbol{r} \equiv (1, \frac{1}{1+\bar{r}}, \frac{1}{(1+\bar{r})^2}, \dots)$, we have that $\boldsymbol{r}' \cdot \mathcal{M}(\bullet, h) = \frac{1}{(1+\bar{r})^h}$—i.e., every dollar of income is spent at some point. It follows from this property that any solution $\boldsymbol{y}$ of (A.18) necessarily has net present value equal to $\frac{1}{\tau_y}$ times the net present value of the fiscal stimulus:

$$\boldsymbol{r}'\boldsymbol{y} = (1-\tau_y)\boldsymbol{r}'\mathcal{M} \times \boldsymbol{y} + \boldsymbol{r}'\mathcal{M} \times \boldsymbol{\epsilon}$$

and so from the properties of $\mathcal{M}$ we obtain that indeed

$$\tau_y \times \boldsymbol{r}'\boldsymbol{y} = \boldsymbol{r}'\boldsymbol{\epsilon}, \tag{A.19}$$

as claimed. Next we note that the solution of (A.18) takes the simple form

$$\boldsymbol{y} = \left[I - (1-\tau_y)\mathcal{M}\right]^{-1} \times \mathcal{M} \times \boldsymbol{\epsilon} \tag{A.20}$$

where for the purpose of the discussion here we simply assume that the stated inverse exists.[26] Our self-financing results in Theorems 1 and 2 concern the question of whether, as fiscal financing is gradually delayed further and further, we indeed converge to the general self-financing equilibrium characterized by (A.20). As discussed following Theorem 1, the condition required for such convergence to occur is that the Keynesian boom happens sufficiently quickly, raising all required revenue before fiscal adjustment is ever actually necessary. In (A.20), the "quickness" of the Keynesian boom is entirely governed by the properties of $\left[I - (1-\tau_y)\mathcal{M}\right]^{-1}$: if the off-diagonal entries of $\mathcal{M}$ decay to zero sufficiently quickly, then the same is true for the off-diagonal entries of $\left[I - (1-\tau_y)\mathcal{M}\right]^{-1}$ (e.g., see Bickel and Lindner, 2012). This then ensures that the solution $\boldsymbol{y}$ and thus the debt path $\boldsymbol{d}$ converge to zero, which in turn is precisely what is needed for self-financing to obtain as fiscal adjustment is delayed further and further. For our general aggregate demand relation (27), the condition stated in Assumption 2 is simply what is needed to ensure that indeed the off-diagonal entries of $\mathcal{M}$ and thus $\left[I - (1-\tau_y)\mathcal{M}\right]^{-1}$ decay to zero sufficiently quickly.

[26] Our analysis in the main text implies that, for standard models of the consumption-savings problem and if $\tau_y > 0$, then this inverse indeed exists.

B Details on model extensions

We elaborate on the various extensions considered in Section 5: the richer aggregate demand block in Appendix B.1; a model with investment in Appendix B.2; equilibria under more general monetary rules in Appendix B.3; fiscal financing through distortionary taxes in Appendix B.4; and fiscal stimulus in the form of government purchases in Appendix B.5.

B.1 A more general aggregate demand relation

In Section 5.1 we showed explicitly how several popular models of the household consumption-savings problem can be written in our general form (27). The only model for which we stated but did not prove such nesting was the model of cognitive discounting of Gabaix (2020).

Under cognitive discounting, a shock h periods in the future is additionally discounted by a factor of θ, with $\theta = 1$ corresponding to the standard full-information, rational-expectations model and $\theta = 0$ corresponding to myopic households. It is immediate that cognitive discounting added to our baseline OLG model gives the adjusted aggregate demand relation

$$c_t = (1-\beta\tilde{\omega})\left(d_t + \mathbb{E}_t\left[\sum_{k=0}^{\infty}(\beta\tilde{\omega}\theta)^k\left(y_{t+k} - t_{t+k}\right)\right]\right). \tag{B.1}$$

This fits into our demand structure (27) with $M_y = M_d = 1-\beta\tilde{\omega}$, $\delta = 1$ and $\omega = (\tilde{\omega}\theta)^k$. It is immediate that, for $\tilde{\omega} < 1$ and $\theta < 1$, Assumption 1 holds. Differently from the baseline OLG case, however, Assumption 2 does not hold automatically; plugging in to (28) and re-arranging we find that we need

$$\tau_y > \frac{\tilde{\omega}(1-\theta)}{1-\tilde{\omega}\theta}\frac{1-\beta}{1-\beta\tilde{\omega}} \tag{B.2}$$

This relation holds automatically for $\theta = 1$, but need not hold for $\theta < 1$; intuitively, as already discussed in the main text, $\theta < 1$ dampens demand spillovers from the future to the present and thus slows down the Keynesian boom. (B.2) is, however, a very mild condition: even for $\theta = 0$, as long as β is close to one and for values of $\tilde{\omega}$ as considered in Section 6, Assumption 2 holds even for small τ_y.

B.2 Investment

This section provides the missing details on the extension to models with investment discussed in Section 5.2. We begin by stating the (linearized) equations of the extended model before then characterizing its equilibrium.

Model equations The household block changes very little. Households still receive labor income and dividends; we now denote this total household income by e_t (which in equilibrium will be equal to total household consumption rather than total aggregate income). The linearized household demand relation is now

$$c_t = \left(1-\beta\omega\right)\left(d_t + \mathbb{E}_t\left[\sum_{k=0}^{\infty}\left(\beta\omega\right)^k\left(e_{t+k} - t_{t+k}\right)\right]\right) - \gamma\mathbb{E}_t\left[\sum_{k=0}^{\infty}(\beta\omega)^k r_{t+k}\right], \tag{B.3}$$

while labor supply still satisfies

$$\frac{1}{\varphi}\ell_t = w_t - \frac{1}{\sigma}c_t \tag{B.4}$$

The firm block on the other hand changes materially relative to our baseline model. Since this production side is entirely standard our discussion here will be brief and only present linearized optimality conditions; a detailed discussion of an almost identical model is offered in Wolf (2021*b*). The production sector consists of three parts: perfectly competitive intermediate goods producers who accumulate capital and hire labor on spot markets; monopolistically competitive retailers who purchase the intermediate good and costlessly differentiate it, subject to nominal rigidities; and a competitive final goods aggregator. Profits of the corporate sector as a whole are returned to households, subject to the time-invariant tax τ_y. The relevant equilibrium relations follow from the behavior of the intermediate goods producers and the retailers.

1. *Intermediate goods producers.* The production function takes a standard Cobb-Douglas form with capital share α, and capital depreciates at rate δ. We let p_t^I denote the real relative price of the intermediate good. Optimal labor demand gives the static relation

$$w_t = p_t^I + \alpha k_{t-1} - \alpha\ell_t \tag{B.5}$$

while optimal capital accumulation gives[27]

$$\frac{1}{\beta}\left(i_t - \mathbb{E}_t\left[\pi_{t+1}\right]\right) = \left(\frac{1}{\beta} - 1 + \delta\right) \times \mathbb{E}_t\left[p_{t+1}^I + (\alpha-1)k_t + (1-\alpha)\ell_{t+1}\right] \tag{B.6}$$

By our assumptions on the production function total output is given as

$$y_t = \alpha k_{t-1} + (1-\alpha)\ell_t \tag{B.7}$$

and finally investment x_t satisfies

$$x_t = \frac{1}{\delta}\left(k_t - (1-\delta)k_{t-1}\right) \tag{B.8}$$

2. *Retailers.* Optimal price-setting as usual relates real marginal costs—here the relative price of the intermediate good, p_t^I—to aggregate inflation:

$$\pi_t = \kappa p_t^I + \beta\mathbb{E}_t\left[\pi_{t+1}\right] \tag{B.9}$$

[27] Adjustment costs on the capital stock or investment flows would complicate this relation but not affect any of our subsequent arguments.

Aggregating dividend payments from intermediate goods producers and retailers, we obtain

$$Q_t = Y_t - W_t L_t - X_t \tag{B.10}$$

which implies that total household income E_t (in levels) is given as

$$E_t = W_t L_t + Q_t = Y_t - X_t \tag{B.11}$$

Aggregate output market-clearing dictates that

$$y_t = \frac{C^{ss}}{Y^{ss}} c_t + \frac{X^{ss}}{Y^{ss}} x_t \tag{B.12}$$

Finally we return to the government. The monetary rule (8) and the government budget constraint (4) are unchanged. The fiscal policy rules (6) or (7) are also unchanged up to the tax base revenue term: since the government taxes labor and dividend income, this term now equals $\tau_y \times E_t$.

Equilibrium characterization. Our key building block result is that we can reduce the equilibrium of this extended model to a system of equations almost as simple as that of our baseline model in Section 2. First, combining market-clearing and the policy rules with private-sector demand we obtain

$$c_t = \mathscr{F}_1 \cdot (d_t + \varepsilon_t) + \mathscr{F}_2 \cdot \mathbb{E}_t \left[(1 - \beta\omega) \sum_{k=0}^{\infty} (\beta\omega)^k c_{t+k} \right]. \tag{B.13}$$

Relative to our baseline model, the only change is that this equilibrium demand relationship is in aggregate consumption c_t rather than aggregate output y_t. We emphasize that this is possible precisely because the government taxes dividend and labor income, which as discussed above in equilibrium is equal to total consumption. Second, the law of motion for aggregate debt is now

$$d_{t+1} = \beta^{-1} \left(d_t + \varepsilon_t - \underbrace{\tau_d \cdot (d_t + \varepsilon_t)}_{\text{fiscal adjustment}} - \underbrace{\tau_y c_t}_{\text{tax base}} \right) - \underbrace{\frac{D^{ss}}{Y^{ss}} (\pi_{t+1} - \mathbb{E}_t [\pi_{t+1}])}_{\text{price level}}, \tag{B.14}$$

with real debt at date 0 given as

$$d_0 = b_0 - \frac{D^{ss}}{Y^{ss}} (\pi_0 - E_{-1} [\pi_0]) = -\frac{D^{ss}}{Y^{ss}} \pi_0. \tag{B.15}$$

Again, relative to the baseline model, the only change is that now it is aggregate consumption rather than aggregate output appearing in (B.14).

We note that (B.13) - (B.14) is a system in $\{c_t, d_t\}_{t=0}^{\infty}$ that depends on the rest of the economy—and so in particular the investment block—*only* through the presence of π_0. π_0 on the other hand can be obtained as a function of the consumption path $\{c_t\}_{t=0}^{\infty}$ by solving the system (B.4), (B.5), (B.6), (B.7), (B.8), (B.9) and (B.12) given consumption, and with the monetary policy rule (8) imposed. We write this function as

$$\pi_0 = \Pi_0 \left(\{c_t\}_{t=0}^{\infty} \right) \tag{B.16}$$

The equilibrium described by equations (B.13) - (B.16) is straightforward to characterize given our earlier analysis of the model without investment in Sections 2 and 4. We begin with the case of perfectly rigid prices ($\kappa = 0$), and for simplicity restrict attention to the limiting self-financing case ($\tau_d \to 0$ or $H \to \infty$). In that case $\pi_0 = 0$, so we can focus on the bivariate system (B.13) - (B.14) in $\{c_t, d_t\}_{t=0}^{\infty}$. Crucially, this system is *exactly* the same as that covered in Theorem 1, so the equilibrium characterization underlying that result applies unchanged, with c_t replacing y_t.

We now turn to the case of general κ. To this end let $c_{t,0}$ denote the solution of the rigid-price system, and furthermore let $p^I_{t,0}$ denote the corresponding equilibrium intermediate goods price obtained by solving the system (B.4), (B.5), (B.6), (B.7), (B.8), and (B.12) for p^I given $\{c_{t,0}\}_{t=0}^{\infty}$. Proceeding analogously to the proof of Theorem 1, we will now construct the equilibrium for general κ by simply scaling the $\kappa = 0$ equilibrium. To this end conjecture that equilibrium consumption satisfies $a \times c_{t,0}$, for some scalar a. It is then immediate that then we would also have $p^I_t = a \times p^I_{t,0}$. But then, from (B.9), we have that

$$\pi_0 = a \times \kappa \times \sum_{t=0}^{\infty} \beta^t p^I_{t,0} \tag{B.17}$$

Finally it follows from the government budget constraint that—again in our limiting self-financing equilibrium—we must have

$$\epsilon_0 = a \times \tau_y \times \sum_{t=0}^{\infty} \beta^t c_{t,0} + a \times \frac{D^{ss}}{Y^{ss}} \times \kappa \times \sum_{t=0}^{\infty} \beta^t p^I_{t,0}$$

Solving this equation for a we obtain consumption and thus inflation as well as government debt in the general sticky-price equilibrium. In particular we see that self-financing yet again obtains exactly as in our baseline economy. We summarize these observations in the following corollary.

Corollary B.1. *Consider the extended OLG-NK environment with investment. Fiscal adjustment is never necessary in equilibrium—that is, $\nu \to 1$—if the tax response is infinitely delayed, i.e., $\tau_d \to 0$ or $H \to \infty$. These two limits induce the same equilibrium paths $\{c_t, \pi_t, d_t\}_{t=0}^{\infty}$, and in this common limit self-financing is sufficient to return government debt to steady state (i.e., $\rho_d \in (0,1)$).*

B.3 Monetary accommodation

We provide two sets of supplementary results for our discussion of monetary policy feedback (i.e., $\phi \neq 0$). First, we investigate the model's determinacy regions, extending Leeper (1991). Second, we elaborate on the connection between monetary policy feedback and our analysis of more general aggregate demand relations.

Determinacy regions. We begin by providing a visual illustration of equilibrium determinacy—i.e., the famous Leeper (1991) regions—in our OLG model. The two panels in Figure B.1 show whether a

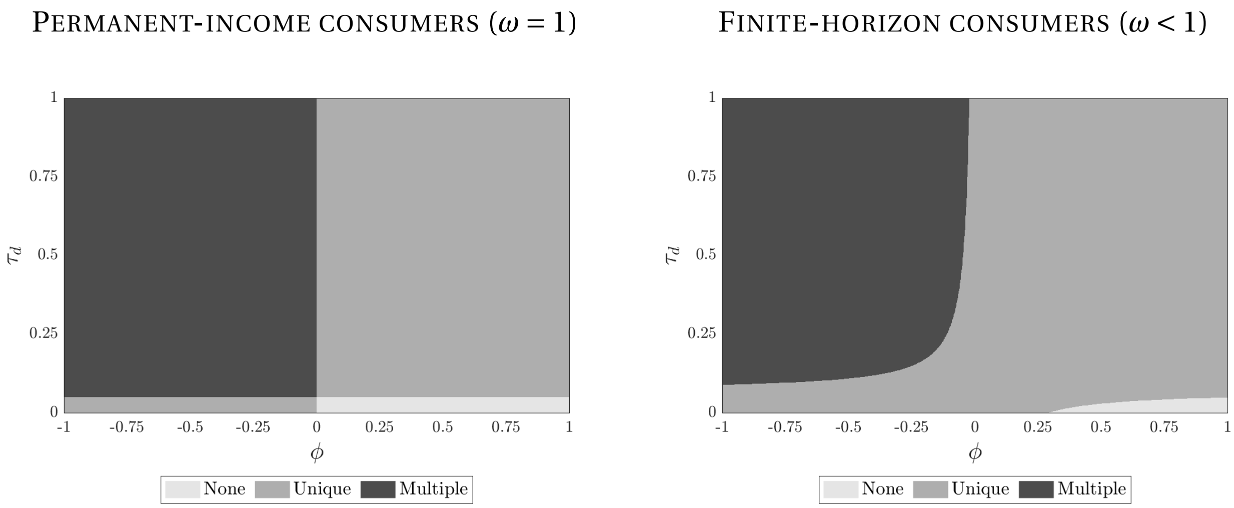

Figure B.1: Determinacy regions for $\omega = 1$ (left panel) and $\omega < 1$ (right panel) as a function of the fiscal and monetary policy rule coefficients τ_d (y-axis) and ϕ (x-axis).

bounded equilibrium (i.e., the standard solution concept of Blanchard and Kahn (1980)) exists and is unique under different assumptions on monetary policy (ϕ) and fiscal policy (τ_d in (6)), for a standard permanent-income model (left panel) and our OLG economy (right panel).

The figure reveals that the determinacy properties of the two economies are materially different. Results for the permanent-income model are well-known and require little explanation: equilibrium uniqueness requires that fiscal policy is passive ($\tau_d > 1 - \beta$) and monetary policy is active ($\phi > 0$), or vice-versa; if both rules are active then no bounded equilibrium exists, and if both are passive then there are multiple bounded equilibria. With discounting on the household side (i.e., $\omega < 1$), the regions of equilibrium determinacy look rather different. Perhaps most importantly, the benchmark monetary rule of $\phi = 0$ now induces determinate equilibria for any τ_d, consistent with Proposition 1. Intuitively, since deficits with $\omega < 1$ directly enter the aggregate demand relation, and since (with $\tau_y > 0$) output also directly affects the government budget, self-financing is now strong enough to pull debt as well as spending towards zero, even if interest rates do not provide any further Euler equation tilting. This automatic stabilization of government debt also shrinks the equilibrium non-existence region in the bottom right corner of the figure.

Connection between general monetary policy (9) and general aggregate demand (27). As discussed in the main text, the cutoff $\bar{\phi}$ plays the same role as Assumption 2—they both make sure that the deficit-driven Keynesian Boom is large enough to play out in finite time. In fact, when $\phi = \bar{\phi}$ or when (28) holds with equality, the expected persistence of real value of debt (ρ_d in (17)) becomes 1 when fiscal adjustment is infinitely delayed ($\tau_d \to 0$). This then prevents the existence of a bounded complete self-financing equilibrium.

To further elaborate on this connection we note that, when $\frac{D^{ss}}{Y^{ss}} = 0$, it is possible to re-write the OLG model's aggregate demand relation under a general monetary reaction in a way similar to the generalized aggregate demand relation (27) in Section 5.1. Specifically, aggregate demand in (11) together with monetary policy (30) and $\frac{D^{ss}}{Y^{ss}} = 0$ can be written as

$$c_t = (1-\beta\omega)\, d_t + (1-\beta\omega-\phi\sigma\beta\omega)\, E_t\left[\sum_{k=0}^{+\infty}(\beta\omega)^k\, y_{t+k}\right] - (1-\beta\omega)\, E_t\left[\sum_{k=0}^{+\infty}(\beta\omega)^k\, t_{t+k}\right], \tag{B.18}$$

which can be nested by a general aggregate demand equation:

$$c_t = M_y\left(y_t + E_t\left[\sum_{k=1}^{+\infty}(\beta\omega)^k y_{t+k}\right]\right) - M_t\left(t_t + E_t\left[\sum_{k=1}^{+\infty}(\beta\omega)^k t_{t+k}\right]\right) + M_d d_t, \tag{B.19}$$

with the only slight difference from (27) being that the MPC out of income M_y is now allowed to differ from the MPC out of taxes M_t. This difference however is immaterial: we can generalize the proof of Theorem 2 to get a more general version of Assumption 2, requiring that

$$M_d + \frac{1-\beta}{\tau_y}\left(M_y - \tau_y M_t\right)\left(1 + \sum_{k=1}^{+\infty}(\beta\omega)^k\right) > \frac{1-\beta}{\tau_y}. \tag{B.20}$$

(B.19) nests (B.18) with $M_y = 1-\beta\omega-\phi\sigma\beta\omega$ and $M_d = M_t = 1-\beta\omega$. We see that, when $\phi = \bar{\phi}$ in (D.31), (B.20) holds with equality. This formalizes our claim that the cutoff $\bar{\phi}$ indeed plays the same role as Assumption 2 in Section 5.1.

B.4 Distortionary taxation

We begin by showing how the equilibrium relations of the model change with time-varying distortionary taxes. We then discuss implications for our limiting self-financing equilibria.

Environment. With time-varying distortionary taxes the optimal labor supply relation becomes

$$(1-\tau_{y,t})W_t = \frac{\chi\int_0^1 L_{i,t}^{\frac{1}{\varphi}} di}{\int_0^1 C_{i,t}^{-1/\sigma} di} \tag{B.21}$$

Log-linearizing, we find that

$$w_t - \frac{1}{\sigma}c_t - \frac{1}{1-\tau_y}\widehat{\tau}_{y,t} = \frac{1}{\varphi}\ell_t \tag{B.22}$$

where $\widehat{\tau}_t \equiv \tau_{y,t} - \tau_y$. Next note that the firm optimal pricing relationship is still

$$\pi_t = \tilde{\kappa}\, w_t + \beta\mathbb{E}_t\left[\pi_{t+1}\right] \tag{B.23}$$

Combining (B.22), (B.23), and the adjusted fiscal rule (33) we obtain

$$\pi_t = \kappa y_t + \beta \mathbb{E}_t [\pi_{t+1}] + \underbrace{\frac{\tilde{\kappa}}{1-\tau_y} \tau_{d,t}}_{\equiv \zeta_t} \times d_t \tag{B.24}$$

All other equilibrium relations of the model are unaffected.

Self-financing result. Note that, as either $\tau_d \to 0$ or $H \to \infty$, we obtain that $\zeta_t \to 0$ for all t. It thus follows that the equilibrium characterization of Theorem 1 for the self-financing limit applies without change to the alternative economy in which fiscal adjustment is distortionary. Intuitively, since the adjustment is not necessary in equilibrium, it is immaterial whether the adjustment *would have been* distortionary or lump-sum.

B.5 Government spending

The government spending policy experiment was already described in Section 5.5. Since the analytics of the self-financing result are exactly analogous to our baseline "stimulus checks" case, we do not repeat those derivations here and instead just provide a visual illustration of the self-financing result.

We summarize our results in Figure B.2—the government spending analogue of Figure 1. We emphasize two main takeaways. First, as $\tau_d \to 0$ or $H \to \infty$, we indeed again converge to a self-financing limit. Second, even immediately tax-financed fiscal purchases have a positive spending multiplier, and thus the share of self-financing ν for $\tau_d = 1$ (top panel) and $H = 0$ (bottom panel) is already strictly positive.[28]

[28] As discussed in Section 5.5, if prices were rigid, then in this immediately tax-financed case $\nu = \nu_y = \tau_y$. With partially sticky prices the initial inflation further increases the degree of self-financing.

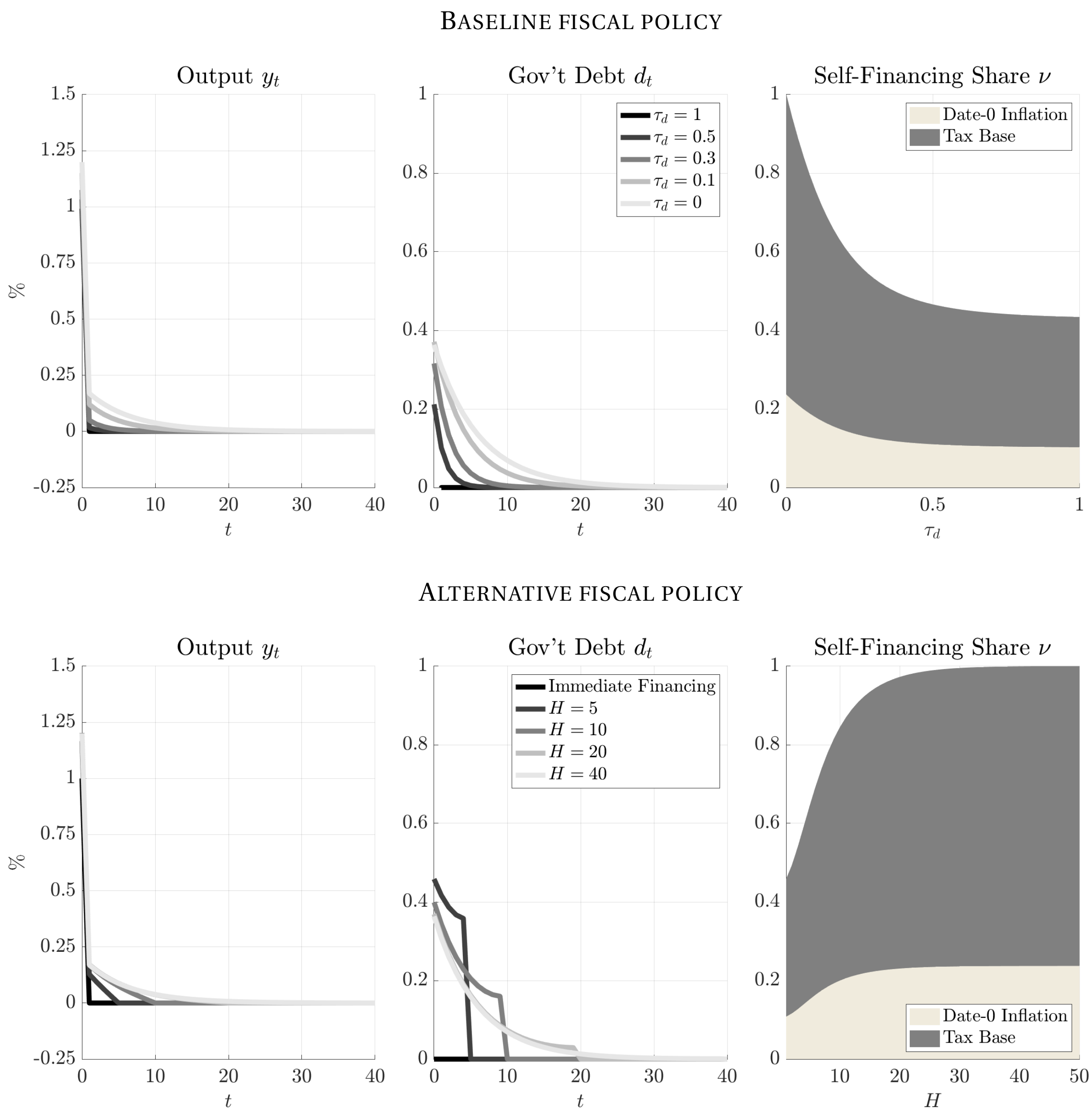

Figure B.2: Top panel: impulse responses of output y_t, government debt d_t, and the total self-financing share ν to the deficit shock ϵ_0 as a function of τ_d. Bottom panel: same as above, but as a function of H.

C Quantitative analysis

This section supplements our quantitative analysis in Section 6. We: provide some missing details on the specification of our spender-OLG hybrid model in Appendix C.1; discuss alternative calibration strategies in Appendix C.2; review empirical evidence on fiscal adjustment in Appendix C.3; and present detailed results from a HANK model and a model with cognitive discounting in Appendices C.4 and C.5, respectively.

C.1 Further details on the hybrid model

We first elaborate on the model environment and discuss in greater detail the model's implications for household consumption behavior, contrasting it in particular with the predictions of quantitative HANK-type models.

Model. The only change relative to our baseline model of Section 2 is that we generalize the household block to also feature a margin μ of spenders—that is, households who do not hold any assets and immediately spend any income they receive. The remaining fraction $1-\mu$ of households are exactly as described in Section 2.1. Both groups of households receive labor income as well as dividends and pay taxes, but only the OLG block holds government bonds.

We will make assumptions ensuring that both groups of households receive the same labor and dividend income, pay the same taxes (up to a between-group steady-state transfer), and have identical steady-state consumption. First, we assume that unions assign identical hours worked to both groups, and that dividends also accrue equally to both. Second, we assume that the government in lump-sum fashion redistributes between the two groups to ensure identical steady-state consumption; given that government bonds are held by the OLG block, this will generally require lump-sum transfers to spenders. Under those assumptions, it is first of all immediate that the supply block of the economy—notably (14)—is unchanged. Next, the demand block of the economy generalizes (26) as follows:

$$c_t = (1-\beta\omega)\cdot d_t + \left[\mu + (1-\mu)(1-\beta\omega)\right]\cdot\left((y_t - t_t) + (1-\mu)\mathbb{E}_t\left[\sum_{k=1}^{\infty}(\omega\beta)^k\left(y_{t+k}-t_{t+k}\right)\right]\right). \tag{C.1}$$

Replacing (26) by (C.1) is the only difference between our baseline OLG economy and the generalized hybrid model. Relative to (26), the key change in (C.1) is that we allow the MPC out of income to be larger than that out of wealth. As we discuss next, this minimal departure from our baseline OLG model is all that is needed to ensure (approximate) consistency with consumption-savings behavior even in quantitative HANK-type models.

Household consumption-savings behavior. By our discussion in Appendix A.5 we know that the role of the household consumption-savings decision in driving our self-financing result is fully gov-

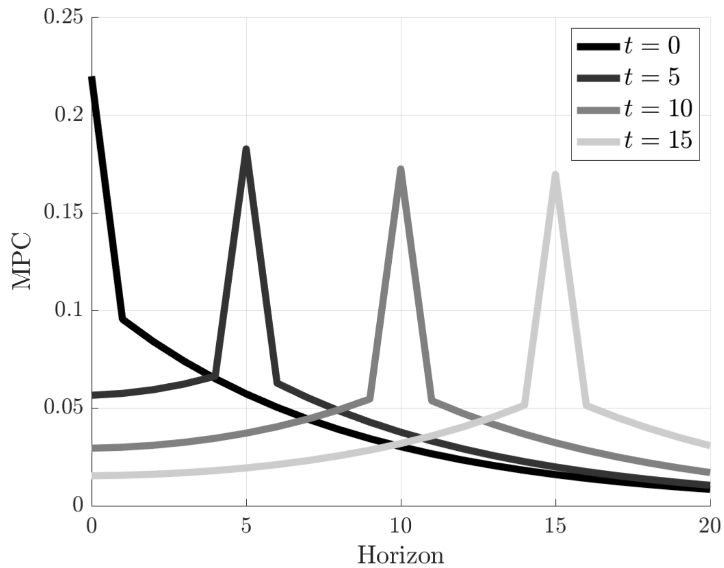

Figure C.1: Consumer spending responses to income gains at different dates (shades of grey) in our spender-OLG hybrid model. Model parameterization as in Table 1.

erned by the matrix of intertemporal marginal propensities to consume. Figure C.1 provides a visual illustration of this matrix in our spender-OLG hybrid model, as implied by the generalized demand block (C.1).

The figure plots the spending response over time to (anticipated) income gains at different dates. We emphasize two key takeaways. First, the response to a date-0 income gain—that is, the first column of $\mathscr{M}$—agrees closely with prior empirical evidence (Fagereng, Holm and Natvik, 2021), as already discussed in Section 6.1. Second, the higher-order columns are qualitatively and quantitatively similar to those implied by quantitative HANK-type models. This observation has been made previously in Auclert, Rognlie and Straub (2018) and Wolf (2021*a*). For our purposes, the important takeaway is that our analysis is indeed quantitatively relevant—as far as our question of self-financing is concerned, our model will have very similar predictions as richer quantitative HANK-type models. We further illustrate this observation in Appendix C.4.

C.2 Alternative calibration approaches for the household block

For our baseline analysis in Section 6 we discipline our model's departure from permanent-income behavior by requiring consistency with empirical evidence on the *level* and *slope* of (short-run) household consumption behavior lump-sum income receipt, as in Auclert, Rognlie and Straub (2018) and Wolf (2021*a*). We here instead discuss two different approaches: one based on farther-out spending responses, and one based on long-run price elasticities of household asset demand.

Calibration via tail MPCs. The calibration strategies were already discussed in Section 6.2: the "lower bound" hybrid model matches the impact MPC and the estimated lower bound of the five-year cumulative MPC, while the generalized three-type model is parameterized to match the five-year cumulative MPC path as well as possible, in a standard least-squares sense. For the "lower bound" calibration of the hybrid model we set $\mu = 0.17$ and $\omega = 0.94$; all other parameters are as in Table 1. For the three-type model, we set $\omega_1 = 0.97$ and $\omega_2 = 0.83$, with the fractions of the two groups equal to $\chi_1 = 0.22$ and $\chi_2 = 0.63$. The residual fraction $1 - \chi_1 - \chi_2$ are hand-to-mouth.

Calibration via asset supply elasticities. For this approach we combine evidence on level MPCs with long-run price elasticities of household asset demand. This calibration strategy is promising because models with permanent-income savers invariably imply a (counterfactual) infinite elasticity of household asset demand (e.g., Kaplan and Violante, 2018).

Our main building block result for this calibration approach is Proposition C.1. We there give the long-run elasticity of household asset demand as a function of model primitives.

Proposition C.1. *Consider the spender-OLG hybrid model. Let η denote the long-run interest rate elasticity of household asset demand—that is, the long-run response of asset demand to a permanent change in real interest rates. It is given as*

$$\eta \;=\; (1-\mu) \times \frac{\sigma}{1-\beta} \times \left(\frac{1}{1-\omega} - \frac{1}{1-\beta\omega}\right) \tag{C.2}$$

Empirical work suggests a range for η of around 1.25 to 35 (see Moll, Rachel and Restrepo, 2022). Setting $\beta = 0.99$, $\sigma = 1$, and requiring the model to generate an impact MPC of 22 per cent (all as in our baseline calibration), we find $\omega \in [0.21, 0.85]$. Our baseline calibration lies somewhat beyond the upper end of this range and is thus conservative.

C.3 Empirical evidence on fiscal adjustment

Notable prior work that has estimated fiscal financing rules and thus in particular the speed of fiscal adjustment in response to deficits includes Galí, López-Salido and Vallés (2007), Bianchi and Melosi (2017), and Auclert and Rognlie (2018).

1. Galí, López-Salido and Vallés (2007). The authors estimate impulse responses to an identified government spending shock and in particular report impulse responses of government debt. Setting $\tau_d = 0.09$ implies a deficit half-life comparable to their estimates.

2. Bianchi and Melosi (2017). The authors estimate a structural macroeconomic model with policy uncertainty in rich fiscal-monetary interactions. The model features both fiscal- as well as

monetary-led regimes; setting $\tau_d = 0.04$ implies a half-life of government debt of around five years, comparable to the simulations reported by the authors.

3. Auclert and Rognlie (2018). The authors estimate fiscal adjustments on data from a sample of OECD countries and find a half-life of deficits around 10 years. Translated to our notation this corresponds to around $\tau_d = 0.03$.

We emphasize that our calibration choices described above are conservative. We set τ_d so that, *even in the absence of self-financing fiscal feedback*, taxes adjust sufficiently quickly to deliver the desired debt half-life. Taking into account this self-financing fiscal feedback, the half-life of fiscal deficit shocks becomes even smaller.

C.4 A full HANK model

This section provides a sketch of the quantitative HANK model that we use to numerically illustrate the generality of our self-financing result. The discussion is brief because the household block of the model is essentially borrowed from Wolf (2021*a*).

Model sketch & calibration. The model economy is exactly as in Section 2, but with one twist: the OLG household block is replaced by a unit continuum of households $i \in [0,1]$ that face uninsurable income risk. Households have preferences

$$\mathbb{E}_t \left[\sum_{k=0}^{\infty} \beta^t \left[u(C_{i,t+k}) - v(L_{i,t+k}) \right] \right]$$

Households save and borrow (subject to a constraint) in a nominally risk-free bond, as in our baseline model. They receive labor and dividend income in proportion to their (stochastic) productivity, pay a proportional tax τ_y on that income, and finally pay additional lump-sum uniform taxes $\tilde{T}_t$. We can thus write the household budget constraint in real terms as

$$C_{i,t} + D_{i,t+1} = (1-\tau_y) e_{i,t} Y_t - \tilde{T}_t + \frac{I_{t-1}}{\Pi_t} D_{i,t}, \quad D_{i,t+1} \geq \underline{D}$$

Whenever possible we calibrate the model exactly as our baseline model. The remaining HANK-specific parameters are: the income risk process; the borrowing constraint; and the discount factor. The income risk process is taken from Kaplan, Moll and Violante (2018), just ported to discrete time as in Wolf (2021*b*). The borrowing constraint $\underline{D}$ is set to zero, and the discount factor β is backed out residually to clear the asset market. Finally, we need to make one more change relative to our baseline model: in the model set-up as described so far, tax revenue $\tau_y \times Y^{ss}$ would far exceed debt servicing costs, so the government would make a substantial uniform transfer, thus materially dampening household MPCs. We instead set the steady-state transfer share as in the data (following Kaplan, Moll

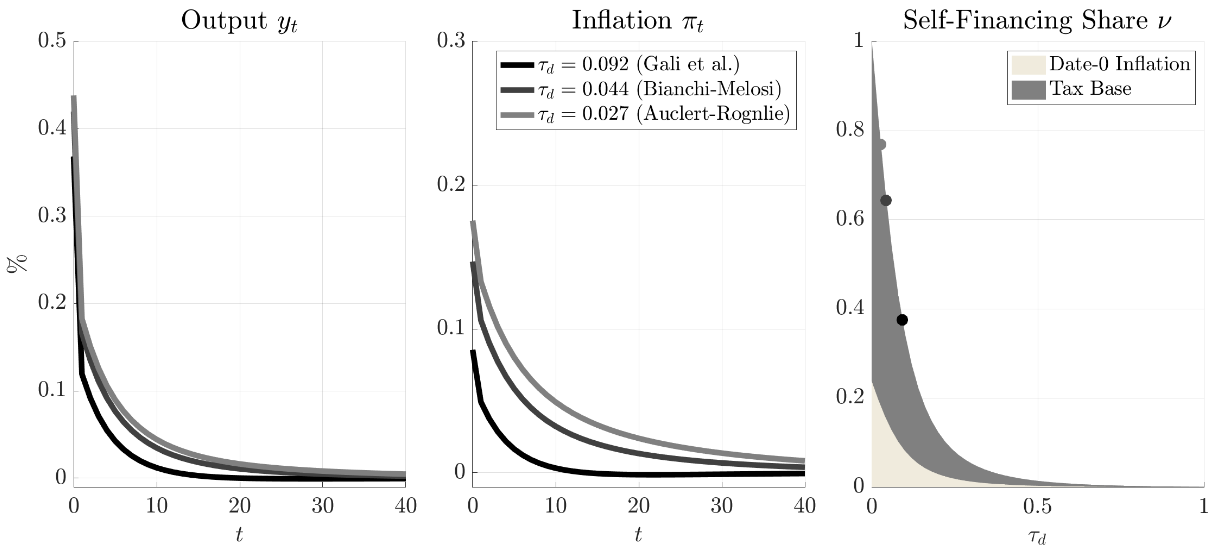

Figure C.2: Impulse responses of output y_t, inflation π_t, and the total self-financing share ν to the deficit shock ϵ_0 as a function of τ_d, for the quantitative HANK model. The left and middle panels show the impulse responses for the three particular values of τ_d discussed in Section 6.1. In the right panel these three points are marked with circles.

and Violante, 2018, which gives $\tilde{T}^{ss}/Y^{ss} = 0.06$), and then clear the government budget by additionally allowing for positive (and time-invariant) government purchases.

Results. We use the quantitative HANK model to revisit our numerical exercises in Section 6.2. Exactly as done there, we here compute the aggregate effects of one-off fiscal stimulus for different assumptions on the delay in fiscal financing. Results are reported in Figure C.2.

Our results closely echo those of Section 6.2. We emphasize two main takeaways. First, Figure C.2 is *qualitatively* very similar to Figure 5: output and inflation responses as well as the share of self-financing ν are all increasing in the delay in fiscal adjustment (i.e., decreasing in τ_d). Furthermore, as $\tau_d \to 0$, we again converge to a full self-financing limit. We have also verified numerically that, for each $\tau_d \in [0,1]$ considered in construction of Figure C.2, the constructed equilibrium is (locally) unique, again echoing our baseline analysis. Second, the two figures are also *quantitatively* similar: for our three values of τ_d taken from prior work, the impulse responses of output and inflation as well as the share of self-financing ν are very similar to the spender-OLG hybrid model. This conclusion confirms prior work arguing that, as far the dynamics of macroeconomic aggregates are concerned, spender-OLG hybrid models and fully specified HANK models look extremely similar (Auclert, Rognlie and Straub, 2018; Wolf, 2021*a*)

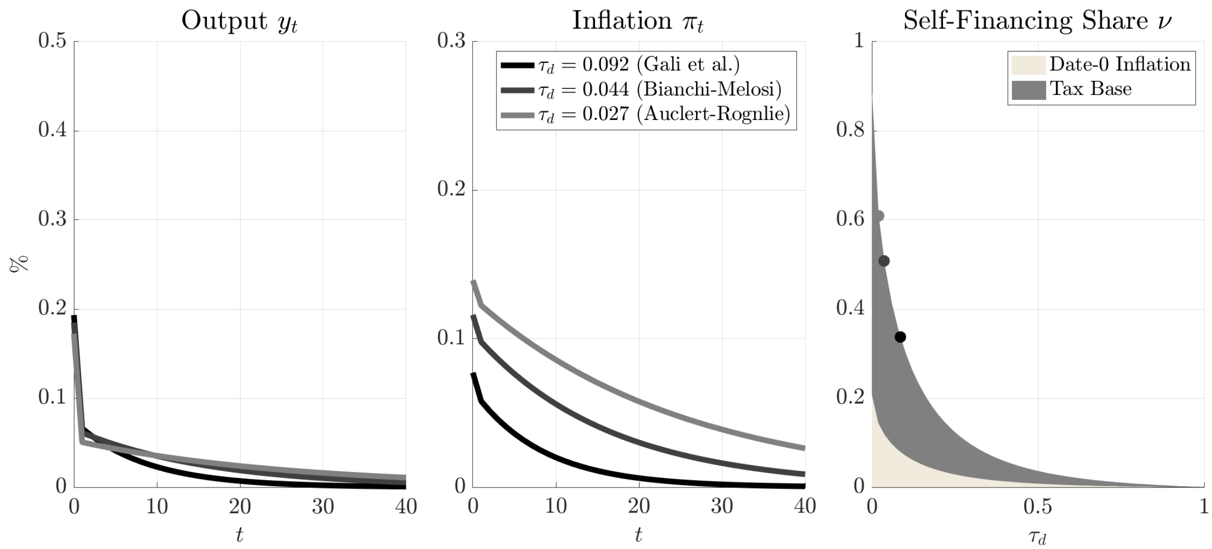

Figure C.3: Impulse responses of output y_t, inflation π_t, and the total self-financing share ν to the deficit shock ϵ_0 as a function of τ_d, with cognitive discounting. The left and middle panels show the impulse responses for the three particular values of τ_d discussed in Section 6.1. In the right panel these three points are marked with circles.

C.5 The effects of cognitive discounting

Figure C.3 repeats our analysis of Section 6.2 in a variant of our spender-OLG hybrid model with cognitive discounting. To illustrate the effects of discounting as clearly as possible we consider a rather significant degree of discounting ($\theta = 0.75$).

The figure illustrates the two effects described in Section 5.1. First, for τ_d close to one, the Keynesian boom and thus the share of self-financing ν are larger than in our baseline model. Intuitively, in this case, the strong discounting of the not-so-distant tax hike meaningfully amplifies the initial boom. Second, for τ_d close to one, the self-financing limit is approached more slowly, reflecting a weakening of the intertemporal Keynesian cross.

D Proofs and auxiliary lemmas

D.1 Proof of Proposition 1

Note that we restrict that $\omega \in (0,1)$, $\tau_y \in (0,1)$, and $\tau_d \in [0,1)$. We first write (12) recursively:

$$\begin{aligned} y_t - \mathscr{F}_1 \cdot (d_t + \epsilon_t) &= (1-\beta\omega)\mathscr{F}_2 \cdot y_t + \beta\omega \mathbb{E}_t \left[y_{t+1} - \mathscr{F}_1 \cdot (d_{t+1} + \epsilon_{t+1}) \right] \\ &= (1-\beta\omega)\mathscr{F}_2 \cdot y_t + \beta\omega \mathbb{E}_t \left[y_{t+1} - \mathscr{F}_1 \cdot \frac{1}{\beta} \left[(1-\tau_d)(d_t + \epsilon_t) - \tau_y y_t \right] \right]. \end{aligned}$$

After rearranging terms and using the formula of $\mathscr{F}_1$ and $\mathscr{F}_2$ (as stated after (12)), we have

$$\begin{aligned} y_t &= \frac{(1-\omega(1-\tau_d))\mathscr{F}_1}{1-\omega\tau_y\mathscr{F}_1 - (1-\beta\omega)\mathscr{F}_2}(d_t+\epsilon_t) + \frac{\beta\omega}{1-\omega\tau_y\mathscr{F}_1-(1-\beta\omega)\mathscr{F}_2}\mathbb{E}_t\left[y_{t+1}\right] \\ &= \frac{\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}(1-\tau_d)}{1+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\tau_y}(d_t+\varepsilon_t) + \frac{1}{1+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\tau_y}\mathbb{E}_t\left[y_{t+1}\right]. \end{aligned}$$

Applying period-t expectations $\mathbb{E}_t[\cdot]$ to (15), we have

$$\begin{pmatrix} \mathbb{E}_t[d_{t+1}] \\ \mathbb{E}_t[y_{t+1}] \end{pmatrix} = \begin{pmatrix} \frac{1-\tau_d}{\beta} & -\frac{\tau_y}{\beta} \\ -\frac{(1-\beta\omega)(1-\omega)(1-\tau_d)}{\beta\omega} & 1+\frac{(1-\beta\omega)(1-\omega)\tau_y}{\beta\omega} \end{pmatrix} \begin{pmatrix} d_t+\epsilon_t \\ y_t \end{pmatrix} \tag{D.1}$$

The two eigenvalues of the system are given by the solutions of

$$\lambda^2 - \lambda\left(\frac{1}{\beta}(1-\tau_d) + 1 + \frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right) + \frac{1}{\beta}(1-\tau_d) = 0,$$

with

$$\begin{aligned} \lambda_1 &= \frac{\left(\frac{1}{\beta}(1-\tau_d)+1+\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right) + \sqrt{\left(1-\frac{1}{\beta}(1-\tau_d)-\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right)^2 + 4\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)}}{2} \\ &> \frac{\left(\frac{1}{\beta}(1-\tau_d)+1+\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right) + \left|1-\frac{1}{\beta}(1-\tau_d)-\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right|}{2} \geq 1 \end{aligned} \tag{D.2}$$

and

$$\begin{aligned} \lambda_2 &= \frac{\left(\frac{1}{\beta}(1-\tau_d)+1+\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right) - \sqrt{\left(1-\frac{1}{\beta}(1-\tau_d)-\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right)^2 + 4\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)}}{2} \\ &< \frac{\left(\frac{1}{\beta}(1-\tau_d)+1+\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right) - \left|1-\frac{1}{\beta}(1-\tau_d)-\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right|}{2} \leq 1, \end{aligned} \tag{D.3}$$

with $\lambda_2 > 0$ too since $\lambda_1 = \lambda_2 = \frac{1}{\beta}(1-\tau_d) > 0$. Let $\left(1, \chi_2\right)'$ denote the eigenvector associated with λ_2,where

$$\lambda_2 = \frac{1}{\beta}\left(1-\tau_d-\tau_y\chi_2\right) \quad \text{and} \quad \chi_2 = \frac{\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}(1-\tau_d)}{1+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\tau_y - \lambda_2} > 0. \tag{D.4}$$

This means that any bounded equilibrium path $\{d_t, y_t\}_{t=0}^{+\infty}$ of (D.1)[29] takes the form of

$$y_t = \chi(d_t + \varepsilon_t) \quad \text{and} \quad \mathbb{E}_t[d_{t+1}] = \rho_d(d_t + \epsilon_t),$$

where χ and ρ_d are uniquely given by

$$\chi = \chi_2 > 0 \quad \text{and} \quad \rho_d = \lambda_2 \in (0,1). \tag{D.5}$$

In other words, the equilibrium takes the form of (17) and satisfies (18).[30]

To prove equilibrium uniqueness, note that the total amount of nominal public debt outstanding at the start of $t = 0$, $B_0 = B^{ss}$, is given. From (14) and (16), we know d_0 is uniquely pinned down by

$$d_0 = -\frac{D^{ss}}{Y^{ss}}\pi_0 = -\kappa\frac{D^{ss}}{Y^{ss}}\sum_{k=0}^{+\infty}\beta^k\mathbb{E}_0[y_k] = -\kappa\frac{D^{ss}}{Y^{ss}}\frac{\chi}{1-\beta\rho_d}(d_0+\epsilon_0) = -\frac{\kappa\frac{D^{ss}}{Y^{ss}}\frac{\chi}{1-\beta\rho_d}}{1+\kappa\frac{D^{ss}}{Y^{ss}}\frac{\chi}{1-\beta\rho_d}}\epsilon_0. \tag{D.6}$$

Similarly, for $t \geq 1$,

$$\begin{aligned} d_t - \mathbb{E}_{t-1}[d_t] &= -\frac{D^{ss}}{Y^{ss}}(\pi_t - \mathbb{E}_{t-1}[\pi_t]) = -\kappa\frac{D^{ss}}{Y^{ss}}\sum_{k=0}^{+\infty}\beta^k\left(\mathbb{E}_t[y_{t+k}] - \mathbb{E}_{t-1}[y_{t+k-1}]\right) \\ &= -\kappa\frac{D^{ss}}{Y^{ss}}\frac{\chi}{1-\beta\rho_d}(d_t - \mathbb{E}_{t-1}[d_t] + \epsilon_t) = -\frac{\kappa\frac{D^{ss}}{Y^{ss}}\frac{\chi}{1-\beta\rho_d}}{1+\kappa\frac{D^{ss}}{Y^{ss}}\frac{\chi}{1-\beta\rho_d}}\epsilon_t. \end{aligned}$$

This pins down the unique bounded equilibrium path of $\{\pi_t, d_t, y_t\}_{t=0}^{+\infty}$ with (14) and (17).

D.2 Proof of Lemma 1

The result follows directly from two facts in the proof of Proposition 1 and Theorem 1. First, under fiscal rule (6), ν decreases in τ_d, $\lim_{\tau_d \to 0^+}\nu \to 1$, and $\lim_{\tau_d \to 1^-}\nu \to 0$. Second, under fiscal rule (7), ν increases in H, $\nu = 0$ with $H = 0$, and $\lim_{H\to\infty}\nu \to 1$.

D.3 Proof of Theorem 1

We start with the case based on the baseline fiscal policy (6). Let ν' denote the self financing share in this alternative economy, which similarly to (21) is given as

$$\nu' \cdot \epsilon_0 = \nu'_y \cdot \epsilon_0 \equiv \sum_{k=0}^{\infty}\tau_y\beta^k\mathbb{E}_0[y_k],$$

Note that in this alternative economy all self-financing comes from tax base changes. In particular, there is no $t = 0$ price level jump and so the real value of public outstanding at $t = 0$, $d_0 = b_0 = 0$ is

[29] Boundedness means that $\lim_{k\to+\infty}\mathbb{E}_t[d_{t+k}]$ and $\lim_{k\to+\infty}\mathbb{E}_t[y_{t+k}]$ are bounded for any t, $d_t + \epsilon_t$, and y_t, similar to Blanchard and Kahn (1980).

[30] To see the first part of (18), combine (12) with (17).

pre-determined. From (17) and (21) we know that

$$\nu' = \tau_y \frac{\chi}{1-\beta\rho_d}. \tag{D.7}$$

Now, consider the general case with $\kappa \geq 0$. From NKPC (14) and the definitions in (20) – (21), we have

$$\nu_p = \frac{\kappa \frac{D^{ss}}{Y^{ss}}}{\tau_y} \nu_y = \frac{\kappa \frac{D^{ss}}{Y^{ss}}}{\tau_y + \kappa \frac{D^{ss}}{Y^{ss}}} \nu \tag{D.8}$$

From the formula of d_0 (16), we know

$$d_0 = -\nu_p \epsilon_0 \quad \text{and} \quad \nu_y = \left(1-\nu_p\right)\nu'.$$

Together, we have

$$\nu = \frac{\nu'}{\frac{\tau_y}{\tau_y+\kappa\frac{D^{ss}}{Y^{ss}}} + \frac{\kappa\frac{D^{ss}}{Y^{ss}}}{\tau_y+\kappa\frac{D^{ss}}{Y^{ss}}}\nu'} \quad \nu_y = \frac{\frac{\tau_y}{\tau_y+\kappa\frac{D^{ss}}{Y^{ss}}}\nu'}{\frac{\tau_y}{\tau_y+\kappa\frac{D^{ss}}{Y^{ss}}} + \frac{\kappa\frac{D^{ss}}{Y^{ss}}}{\tau_y+\kappa\frac{D^{ss}}{Y^{ss}}}\nu'}, \quad \text{and} \quad \nu_p = \frac{\frac{\kappa\frac{D^{ss}}{Y^{ss}}}{\tau_y+\kappa\frac{D^{ss}}{Y^{ss}}}\nu'}{\frac{\tau_y}{\tau_y+\kappa\frac{D^{ss}}{Y^{ss}}} + \frac{\kappa\frac{D^{ss}}{Y^{ss}}}{\tau_y+\kappa\frac{D^{ss}}{Y^{ss}}}\nu'}. \tag{D.9}$$

From (18), we know that

$$\frac{\chi}{1-\beta\rho_d} = \frac{\chi}{\tau_d + \tau_y\chi}. \tag{D.10}$$

From (D.3) and (D.5), we know

$$\rho_d = \lambda_2 = f(a,b) \equiv \frac{a+b+1-\sqrt{(a+b-1)^2+4b}}{2} \tag{D.11}$$

where $f(a,b)$, $a = \frac{1}{\beta}(1-\tau_d) > 0$, and $b = \frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega) > 0$. Since $\frac{\partial f}{\partial a} = \frac{1}{2} - \frac{(a+b-1)}{2\sqrt{(a+b-1)^2+4b}} > 0$,we know that ρ_d decreases with τ_d. From (D.4) and (D.5), we then know $\chi = \frac{\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}(1-\tau_d)}{1+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\tau_y - \rho_d}$ also decreases in τ_d. From (D.10), we know $\frac{\chi}{1-\beta\rho_d}$ decreases in τ_d. Finally, from (D.7) and (D.9), we know ν decreases in τ_d. This finishes the proof of Part 1.

For Part 2, from (D.3) and (18), we know that ρ_d and χ are continuous in $\tau_d \in [0,1)$, and

$$\rho_d^{\text{full}} \equiv \lim_{\tau_d \to 0^+} \rho_d = \frac{\left(\frac{1}{\beta}+1+\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right) - \sqrt{\left(1-\frac{1}{\beta}-\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right)^2 + 4\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)}}{2} < 1 \tag{D.12}$$

$$\chi^{\text{full}} \equiv \lim_{\tau_d \to 0^+} \chi = \frac{1-\beta\rho_d^{\text{full}}}{\tau_y} > 0 \tag{D.13}$$

From (D.10), we know $\lim_{\tau_d \to 0^+} \frac{\chi}{1-\beta\rho_d} = \frac{1}{\tau_y}$. From (D.7) and (D.9), we know $\lim_{\tau_d \to 0^+} \nu = 1$. Finally, $\lim_{k\to\infty} \mathbb{E}_t\left[d_{t+k}\right] \to 0$ follows directly from $\rho_d^{\text{full}} < 1$.

Now we turn to the alternative fiscal policy rule in (7), for which we use the equilibrium characterization in Appendix A.3. For the case with rigid prices ($\kappa = 0$), one can see from (A.10) that ν' increases in H, which proves Part I. For Part II and to find $\lim_{H\to\infty} \nu'$, first note that, from (A.8), $\{\chi_k\}_{k=0}^{\infty}$ is a bounded, increasing sequence. As a result, there exists $\chi^{\text{full,NM}}$ such that $\lim_{H\to\infty}\chi_k = \chi^{\text{full,NM}}$ and $\chi^{\text{full,NM}} = g\left(\chi^{\text{full,NM}}\right) \in \left(0, \frac{1}{\tau_y}\right)$. From (A.10), we know that $\lim_{H\to\infty}\nu' = 1$. From (A.9), we know that $\lim_{H\to\infty}\mathbb{E}_0\left[d_H\right] = 0$. From (D.4) and (D.5), we also know that $g\left(\chi^{\text{full}}\right) = \chi^{\text{full}}$ where χ^{full} defined in (D.13) parametrizes the output response in the complete self-financing limit ($\tau_d \to 0$) with the baseline fiscal rule (6). From the definition of $g(\cdot)$ in (A.7), we know that there is a unique $\chi > 0$ such that $g\left(\chi\right) = \chi$ when $\omega < 1$ and $\tau_d \in (0,1)$. As a result, $\chi^{\text{full,NM}} = \chi^{\text{full}} < \frac{1}{\tau_y}$ and $\lim_{H\to+\infty}\chi_k = \chi^{\text{full}}$. That is, these two limits ($\tau_d \to 0$ and $H \to 0$) share the same equilibrium path. Finally, for the general case with $\kappa \geq 0$, the desired result follows directly from the rigid price case together with (D.9).

D.4 Properties of Consumption Function

Lemma D.1. *Let $\mathscr{M}$ denote the the matrix of intertemporal MPCs corresponding to our consumption function* (11)*. Then, if and only if $\omega < 1$:*

1. *As ℓ increases, one unit anticipated income changes at date $t+\ell$ (in terms of present value at t) have a vanishing effect on consumer demand at date t:*

$$\lim_{\ell\to\infty}\beta^{-\ell}\mathscr{M}_{t,t+\ell} = 0$$

2. *As ℓ increases, one unit income changes at date t have a vanishing effect on consumer demand at date $t+\ell$:*

$$\lim_{\ell\to\infty}\mathscr{M}_{t+\ell,t} = 0$$

We prove the two parts of the lemma in turn. The proof leverages results on the properties of the intertemporal MPC matrix $\mathscr{M}$ in OLG models from Wolf (2021*a*).

1. The proof is by induction. First of all we have

$$\mathscr{M}_{0,\ell}\beta^{-\ell} = (1-\beta\omega)\omega^{\ell}$$

   Thus the claim holds for $t = 0$. Now suppose the claim holds for some $t-1$ (where $t \geq 1$), and consider horizon t. Here we have, for $\ell \geq 0$,

$$\mathscr{M}_{t,t+\ell}\beta^{-\ell} = -(1-\beta\omega)^2\beta^{t-1}\omega^{t+\ell+1} + \mathscr{M}_{t-1,t-1+\ell}\beta^{-(\ell-1)}\beta^{-1}$$

   As $\ell \to \infty$ the first term converges to zero since $\omega < 1$ while the second term converges to zero by the inductive assumption, completing the argument.

2. The proof is again by induction. Begin again with $t = 0$. Here we have

$$\mathscr{M}_{\ell,0} = (1-\beta\omega)\omega^{\ell}$$

and so the statement holds. Now suppose it holds for some $t-1$ (where $t \geq 1$), and consider horizon t. Here we have, for $\ell \geq 0$,

$$\mathcal{M}_{t+\ell,t} = -(1-\beta\omega)^2 \beta^{t-1} \omega^{2t+\ell-1} + \mathcal{M}_{t-1+\ell,t-1}$$

The first term converges to zero as $\ell \to \infty$, for any t. The second term furthermore also converges to zero (by the inductive hypothesis), completing the argument.

D.5 Proof of Proposition 2

For part 1 of the Proposition, we start with the case of the baseline fiscal policy (6). From (D.11), we know that ρ_d is independent of the degree of price flexibility κ and the steady-state debt-to-GDP ratio $\frac{D^{ss}}{Y^{ss}}$. From (18) and (D.7), we know that ν' is independent of the degree of price flexibility κ and the steady-state debt-to-GDP ratio $\frac{D^{ss}}{Y^{ss}}$. From (D.9), we know that ν is increasing in the degree of price flexibility κ and steady-state debt-to-GDP ratio $\frac{D^{ss}}{Y^{ss}}$.

Now we turn to part 1 with the alternative fiscal policy in (7), for which we use the equilibrium characterization in Appendix A.3. From (A.6) and (A.10), we know that ν' is independent of the degree of price flexibility κ and the steady-state debt-to-GDP ratio $\frac{D^{ss}}{Y^{ss}}$. From (A.12), we know that ν increases in the degree of price flexibility κ and the steady-state debt-to-GDP ratio $\frac{D^{ss}}{Y^{ss}}$.

Part 2 of the Proposition follows directly from (22), which holds for both fiscal policies and gives

$$\nu_y = \frac{\tau_y}{\tau_y + \kappa \frac{D^{ss}}{Y^{ss}}} \times \nu \quad \text{and} \quad \nu_p = \frac{\kappa \frac{D^{ss}}{Y^{ss}}}{\tau_y + \kappa \frac{D^{ss}}{Y^{ss}}} \times \nu.$$

D.6 Proof of Proposition 3

We start with the baseline fiscal policy (6). From (D.11), we know

$$\rho_d = \frac{a+b+1-\sqrt{(a+b+1)^2 - 4a}}{2} = \frac{2a}{a+b+1+\sqrt{(a+b+1)^2-4a}},$$

which decreases in $b = \frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)$. As a result, ρ_d increases in $\omega \in (0,1)$. From (18), we know χ decreases in ω. From, (D.7), (D.10) and (D.9), we know that ν decreases in ω. This proves Part I.

For Part II, from Theorem 1, we know that $\rho_d^{\text{full}} < 1$ for any $\omega < 1$. From (D.12), we know that $\lim_{\omega \to 1} \rho_d^{\text{full}} = \frac{\left(\frac{1}{\beta}+1\right) - \sqrt{\left(1-\frac{1}{\beta}\right)^2}}{2} = 1$.

D.7 Proof of Theorem 2

We start with the baseline fiscal policy (6). We focus on a bounded equilibrium similar to (17), taking the form of

$$y_t = \chi_d d_t + \chi_\epsilon \epsilon_t, \quad E_t[d_{t+1}] = \rho_d d_t + \rho_\epsilon \epsilon_t \quad \text{with} \quad \chi_d, \chi_\epsilon > 0, \ \rho_d \in (0,1). \tag{D.14}$$

For (D.14) to be an equilibrium, it needs to satisfy (15) and (27). For (D.14) to satisfy to the government budget (15), we need

$$\rho_d = \frac{1}{\beta}\left(1-\tau_d-\tau_y\chi_d\right) \quad \text{and} \quad \rho_\epsilon = \frac{1}{\beta}\left(1-\tau_d-\tau_y\chi_\epsilon\right). \tag{D.15}$$

For (D.14) to satisfy aggregate demand (27) (together with market clearing $c_t = y_t$), we need

$$\chi_d = M_d + M_y\left[\left(1-\tau_y\right)\chi_d - \tau_d\right]\left(1+\delta\sum_{k=1}^{+\infty}\left(\beta\omega\rho_d\right)^k\right) = \frac{M_d - \tau_d M_y\left(1+\frac{\delta\beta\omega\rho_d}{1-\beta\omega\rho_d}\right)}{1-M_y\left(1-\tau_y\right)\left(1+\frac{\delta\beta\omega\rho_d}{1-\beta\omega\rho_d}\right)}. \tag{D.16}$$

and

$$\chi_\epsilon = M_y\left(1+\left(\left(1-\tau_y\right)\chi_\epsilon - \tau_d + \sum_{k=1}^{+\infty}(\beta\omega)^k\rho_d^{k-1}\rho_\epsilon\left(\left(1-\tau_y\right)\chi_\epsilon-\tau_d\right)\right)\right). \tag{D.17}$$

(D.15) and (D.16) together mean that ρ_d needs to be the root of the following equation:

$$h\left(\rho_d;\tau_d\right) = \frac{1-\tau_d-\beta\rho_d}{\tau_y} - \frac{M_d-\tau_d M_y\left(1+\frac{\delta\beta\omega\rho_d}{1-\beta\omega\rho_d}\right)}{1-M_y\left(1-\tau_y\right)\left(1+\frac{\delta\beta\omega\rho_d}{1-\beta\omega\rho_d}\right)} = 0,$$

with $\lim_{\tau_d\to0^+} h(0;\tau_d) = \frac{1}{\tau_y} - \frac{M_d}{1-M_y(1-\tau_y)} \geq 0$ because $\tau_y > 0$ and $M_y \in [0,1]$ and $M_y \geq M_d$.

When Assumption 2 holds, we first show that there exists $\rho_d^{\text{full}} \in [0,1)$ such that $\lim_{\tau_d\to0^+} h\left(\rho_d^{\text{full}};\tau_d\right) = 0$. There are two cases. First, $M_y\left(1-\tau_y\right)\left(1+\frac{\delta\beta\omega}{1-\beta\omega}\right) > 1$. In this case, there exists $\bar{\rho} \in (0,1)$ such that

$$M_y\left(1-\tau_y\right)\left(1+\frac{\delta\beta\omega\bar{\rho}}{1-\beta\omega\bar{\rho}}\right) = 1,$$

and $\lim_{\tau_d\to0^+,\rho_d\to(\bar{\rho})^+} h\left(\rho_d;\tau_d\right) = -\infty$. As a result, there exists a unique $\rho_d^{\text{full}} \in [0,1)$ such that we have $\lim_{\tau_d\to0^+} h\left(\rho_d^{\text{full}};\tau_d\right) = 0$.

Second, $M_y\left(1-\tau_y\right)\left(1+\frac{\delta\beta\omega}{1-\beta\omega}\right) < 1$. In this case,

$$\lim_{\tau_d\to0^+} h(1;\tau_d) = \frac{1-\beta}{\tau_y} - \frac{M_d}{1-M_y\left(1-\tau_y\right)\left(1+\frac{\delta\beta\omega}{1-\beta\omega}\right)} < 0$$

from Assumption 2. As a result, there exists a unique $\rho_d^{\text{full}} \in (0,1)$ such that $\lim_{\tau_d\to0^+} h\left(\rho_d^{\text{full}};\tau_d\right) = 0$.

Since $h\left(\rho_d;\tau_d\right)$ is continuous, we know that, for each τ_d in a right neighborhood of 0, there exists $\rho_d(\tau_d) \in [0,1)$ such that $h\left(\rho_d(\tau_d);\tau_d\right) = 0$ and $\lim_{\tau_d\to0^+}\rho_d(\tau_d) = \rho_d^{\text{full}} \in [0,1)$. For each τ_d in this neighborhood, given $\rho_d(\tau_d)$, one can find $\rho_\epsilon(\tau_d)$ and $\chi_d(\tau_d), \chi_\epsilon(\tau_d) > 0$ from (D.15) – (D.17) to constitute a bounded equilibrium in the form of (D.14). Next note that from boundedness we have

$\lim_{\tau_d \to 0^+} \frac{\tau_d \left(\epsilon_0 + \sum_{k=0}^{\infty} \beta^k \mathbb{E}_0[d_k]\right)}{\epsilon_0} = 0$ and $\lim_{\tau_d \to 0^+} \nu = 1$, using also (19) and (20). Finally, $\lim_{k\to\infty} \mathbb{E}_t \left[d_{t+k}\right] \to 0$ follows directly from $\rho_d^{\text{full}} \in [0,1)$. This finishes the proof with the baseline fiscal policy (6).

We now turn to the case with (7). We first write the aggregate demand in (27) recursively

$$\begin{aligned} y_t &= \frac{M_d}{1-M_y} d_t - \frac{M_y}{1-M_y} t_t + \delta\beta\omega \frac{M_y}{1-M_y} \mathbb{E}_t \left[y_{t+1} - t_{t+1}\right] + \beta\omega \mathbb{E}_t \left[y_{t+1} - \frac{M_d}{1-M_y} d_{t+1} + \frac{M_y}{1-M_y} t_{t+1}\right] \\ &= \frac{M_d}{1-M_y} d_t - \frac{M_y}{1-M_y} t_t + \delta\beta\omega \frac{M_y}{1-M_y} \mathbb{E}_t \left[y_{t+1} - t_{t+1}\right] + \beta\omega \mathbb{E}_t \left[y_{t+1} + \frac{M_y}{1-M_y} t_{t+1}\right] - \omega \frac{M_d}{1-M_y} (d_t - t_t) \\ &= \frac{M_d (1-\omega)}{1-M_y} d_t - \frac{M_y - \omega M_d}{1-M_y} t_t + \beta\omega \left(\frac{1-(1-\delta) M_y}{1-M_y}\right) \mathbb{E}_t \left[y_{t+1}\right] + \beta\omega \frac{M_y}{1-M_y} (1-\delta) \mathbb{E}_t \left[t_{t+1}\right]. \end{aligned} \tag{D.18}$$

Without loss of generality (since we are working with linearized economy), we consider a one-time deficit shock ϵ_0, shut down all future deficit shocks (i.e., $\epsilon_t = 0$ for $t \geq 1$), and study the perfect foresight transitional dynamics after ϵ_0. From (7), we know that $t_t = d_t$ for all $t \geq H$. As a result, $d_{t+1} = 0$ for all $t \geq H$. Similar to the argument in Appendix A.3, we can then focus on the case that $y_t = d_t = 0$ for $t \geq H+1$. At $t = H$, from (D.18), we have

$$y_H = \frac{-\left(M_y - M_d\right)}{1-M_y} d_H = \chi_0 d_H \quad \text{with} \quad \chi_0 = \frac{-\left(M_y - M_d\right)}{1-M_y}. \tag{D.19}$$

Similar to the main analysis in Appendix A.3, we will now use (D.18) to find the equilibrium path of $\{y_t, d_t\}_{t=0}^{H-1}$ through backward induction. At $t = H-1$, from (7) and (D.18)

$$\begin{aligned} y_{H-1} &= \frac{\frac{M_d(1-\omega)}{1-M_y}}{1+\tau_y \frac{M_y-\omega M_d}{1-M_y}} d_{H-1} + \beta\omega \frac{\left(\frac{1-(1-\delta)M_y}{1-M_y}\right)\chi_0 + \frac{M_y(1-\delta)}{1-M_y}}{1+\tau_y \frac{M_y-\omega M_d}{1-M_y}} d_H \\ &= \frac{\frac{M_d(1-\omega)}{1-M_y}}{1+\tau_y \frac{M_y-\omega M_d}{1-M_y}} d_{H-1} + \omega \frac{\left(\frac{1-(1-\delta)M_y}{1-M_y}\right)\chi_0 + \frac{M_y(1-\delta)}{1-M_y}}{1+\tau_y \frac{M_y-\omega M_d}{1-M_y}} \left(d_{H-1} - \tau_y y_{H-1}\right) \\ y_{H-1} &= \frac{\frac{M_d(1-\omega)}{1-M_y} + \omega\left[\left(\frac{1-(1-\delta)M_y}{1-M_y}\right)\chi_0 + \frac{M_y(1-\delta)}{1-M_y}\right]}{1+\tau_y \frac{M_y-\omega M_d}{1-M_y} + \omega\tau_y \left(\frac{1-(1-\delta)M_y}{1-M_y}\right)\chi_0 + \omega \frac{M_y(1-\delta)}{1-M_y}} d_{H-1} \\ &= \chi_1 d_{H-1}, \end{aligned} \tag{D.20}$$

with

$$\begin{aligned} \chi_1 &= \frac{\frac{M_d(1-\omega)}{1-M_y} + \omega\left[\frac{-\delta M_y(1-M_d)+M_d(1-M_y)}{(1-M_y)^2}\right]}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y + \omega\tau_y\left[\frac{-\delta M_y(1-M_d)+M_d(1-M_y)}{(1-M_y)^2}\right]} \\ &= \frac{\frac{M_d(1-\omega)}{1-M_y} + \frac{\omega M_y}{1-M_y}\left(\frac{M_d}{M_y} - \delta\frac{1-M_d}{1-M_y}\right)}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y + \frac{\omega M_y}{1-M_y}\left(\frac{M_d}{M_y} - \delta\frac{1-M_d}{1-M_y}\right)\tau_y}. \end{aligned} \tag{D.21}$$

For $1 \le t \le H-2$, from (7) and (D.18),

$$
\begin{aligned}
y_t &= \frac{\frac{M_d(1-\omega)}{1-M_y}d_t}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y} + \beta\omega\frac{\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y}\mathbb{E}_t\left[y_{t+1}\right] \\
&= \frac{\frac{M_d(1-\omega)}{1-M_y}d_t}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y} + \omega\frac{\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y}\left(d_t-\tau_y y_t\right)\chi_{H-t-1} \\
&= \frac{\frac{M_d(1-\omega)}{1-M_y}+\omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\chi_{H-t-1}}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y+\omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\tau_y\chi_{H-t-1}}d_t \\
&= \chi_{H-t}d_t \text{ with } \chi_{H-t} = \frac{\frac{M_d(1-\omega)}{1-M_y}+\omega\left[\left(\frac{1-(1-\delta)M_y}{1-M_y}\right)\chi_{H-t-1}+\frac{M_y(1-\delta)}{1-M_y}\right]}{1+\tau_y\frac{M_y-\omega M_d}{1-M_y}+\omega\tau_y\left(\frac{1-(1-\delta)M_y}{1-M_y}\right)\chi_{H-t-1}+\frac{M_y(1-\delta)}{1-M_y}}
\end{aligned}
\tag{D.22}
$$

Finally, for $t=0$, from (7) and (D.18), we know

$$
\begin{aligned}
y_0 &= \frac{\frac{M_d(1-\omega)}{1-M_y}d_0+\frac{M_y-\omega M_d}{1-M_y}\epsilon_0}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y} + \beta\omega\frac{\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y}\mathbb{E}_0\left[y_1\right] \\
&= \frac{\frac{M_d(1-\omega)}{1-M_y}d_0+\frac{M_y-\omega M_d}{1-M_y}\epsilon_0}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y} + \omega\frac{\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y}\left(d_0+\epsilon_0-\tau_y y_0\right)\chi_{H-1} \\
&= \frac{\frac{M_d(1-\omega)}{1-M_y}+\omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\chi_{H-1}}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y+\omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\tau_y\chi_{H-1}}d_0 + \frac{\frac{M_y-\omega M_d}{1-M_y}+\omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\chi_{H-1}}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y+\omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\tau_y\chi_{H-1}}\epsilon_0 \\
&= \chi_H d_t + \chi_H^{\epsilon}\epsilon_0 \text{ with } \chi_H = \frac{\frac{M_d(1-\omega)}{1-M_y}+\omega\left[\left(\frac{1-(1-\delta)M_y}{1-M_y}\right)\chi_{H-t-1}+\frac{M_y(1-\delta)}{1-M_y}\right]}{1+\tau_y\frac{M_y-\omega M_d}{1-M_y}+\omega\tau_y\left(\frac{1-(1-\delta)M_y}{1-M_y}\right)\chi_{H-t-1}+\frac{M_y(1-\delta)}{1-M_y}},
\end{aligned}
\tag{D.23}
$$

and $\chi_H^{\epsilon} = \frac{\frac{M_d(1-\omega)}{1-M_y}+\omega\left[\left(\frac{1-(1-\delta)M_y}{1-M_y}\right)\chi_{H-t-1}+\frac{M_y(1-\delta)}{1-M_y}\right]}{1+\tau_y\frac{M_y-\omega M_d}{1-M_y}+\omega\tau_y\left(\frac{1-(1-\delta)M_y}{1-M_y}\right)\chi_{H-t-1}+\frac{M_y(1-\delta)}{1-M_y}}$. Define

$$
g(\chi) \equiv \frac{\frac{M_d(1-\omega)}{1-M_y}+\omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\chi}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y+\omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\tau_y\chi} = \frac{1}{\tau_y} - \frac{\frac{M_y-M_d}{1-M_y}+\frac{1}{\tau_y}}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y+\omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\tau_y\chi}. \tag{D.24}
$$

From (D.21), we know that $\chi_1 = g\left(\chi_0^{'}\right)$ with

$$
\chi_0^{'} = \frac{M_y}{1-(1-\tau_y)(1-\delta)M_y}\left(\frac{M_d}{M_y}-\delta\frac{1-M_d}{1-M_y}\right). \tag{D.25}
$$

From (D.22) and (D.23) we have $\chi_k = g\left(\chi_{k-1}\right)$ for all $k \in \{2,\cdots,H\}$. We first find the fixed point of $f(\chi)$:

$$\chi_{MSV}=\frac{\frac{M_d(1-\omega)}{1-M_y}+\omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\chi_{MSV}}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y+\omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\tau_y\chi_{MSV}},\tag{D.26}$$

which is equivalent to

$$\omega\left(\frac{1-\left(1-\tau_y\right)(1-\delta)M_y}{1-M_y}\right)\tau_y\chi_{MSV}^2+\chi_{MSV}\left(1+\frac{M_y-\omega M_d}{1-M_y}\tau_y-\omega\left(\frac{1-\left(1-\tau_y\right)(1-\delta)M_y}{1-M_y}\right)\right)-\frac{M_d(1-\omega)}{1-M_y}=0.\tag{D.27}$$

Let $\chi_{MSV,1}$ denote the smaller root and $\chi_{MSV,2}$ denote the larger root:

$$\chi_{MSV,1}=\frac{-\left(1+\frac{M_y-\omega M_d}{1-M_y}\tau_y-\omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\right)-\sqrt{\left(1+\frac{M_y-\omega M_d}{1-M_y}\tau_y-\omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\right)^2+4\frac{M_d(1-\omega)}{1-M_y}\omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\tau_y}}{2\omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\tau_y}$$

$$\chi_{MSV,2}=\frac{-\left(1+\frac{M_y-\omega M_d}{1-M_y}\tau_y-\omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\right)+\sqrt{\left(1+\frac{M_y-\omega M_d}{1-M_y}\tau_y-\omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\right)^2+4\frac{M_d(1-\omega)}{1-M_y}\omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\tau_y}}{2\omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\tau_y}.\tag{D.28}$$

If Assumption 1 holds ($\omega<1$), we know that $\chi_{MSV,1}\chi_{MSV,2}<0$ so $\chi_{MSV,1}<0$ and $\chi_{MSV,2}>0$. Note that $1+\frac{M_y-\omega M_d}{1-M_y}\tau_y+\omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\tau_y\chi_{MSV,1}>0$,[31] we have $g(\chi)>\chi$ if $\chi\in(\chi_{MSV,1},\chi_{MSV,2})$ and $g(\chi)<\chi$ if $\chi\in(\chi_{MSV,2},+\infty)$. From (D.24), we also know that $g(\chi)$ increases if $\chi\in[\chi_{MSV,1},+\infty]$.

Moreover, from (D.25), we know that $\chi_0^{'}\geq\chi_{MSV,1}$. To see this, define the left-hand side of (D.27) as

$$h(\chi)=\omega\left(\frac{1-\left(1-\tau_y\right)(1-\delta)M_y}{1-M_y}\right)\tau_y\chi^2+\left(1+\frac{M_y-\omega M_d}{1-M_y}\tau_y-\omega\left(\frac{1-\left(1-\tau_y\right)(1-\delta)M_y}{1-M_y}\right)\right)\chi-\frac{M_d(1-\omega)}{1-M_y}.$$

We have

$$\begin{aligned}
h\left(\chi_0^{'}\right)&=\left(\omega\frac{\tau_yM_y}{1-M_y}\left(\frac{M_d}{M_y}-\delta\frac{1-M_d}{1-M_y}\right)+1+\frac{M_y-\omega M_d}{1-M_y}\tau_y-\omega\left(\frac{1-\left(1-\tau_y\right)(1-\delta)M_y}{1-M_y}\right)\right)\chi_0^{'}-\frac{M_d(1-\omega)}{1-M_y}\\
&=\left(1+\frac{\tau_yM_y}{1-M_y}\left(1-\omega\delta\frac{1-M_d}{1-M_y}\right)\right)\chi_0^{'}-\omega\frac{M_y}{1-M_y}\left(\frac{M_d}{M_y}-\delta\frac{1-M_d}{1-M_y}\right)-\frac{M_d(1-\omega)}{1-M_y}\\
&=\frac{M_y\left(1+\frac{\tau_yM_y}{1-M_y}\left(1-\omega\delta\frac{1-M_d}{1-M_y}\right)\right)}{1-\left(1-\tau_y\right)(1-\delta)M_y}\left(\frac{M_d}{M_y}-\delta\frac{1-M_d}{1-M_y}\right)-\frac{M_d}{1-M_y}\left[1-\omega\delta\frac{M_y}{M_d}\frac{1-M_d}{1-M_y}\right]\\
&=\frac{\left(1-\left(1-\tau_y\right)M_y-\tau_yM_y\left(M_y+\omega\delta(1-M_d)\right)\right)}{1-\left(1-\tau_y\right)(1-\delta)M_y}\frac{M_d}{1-M_y}\left(1-\delta\frac{M_y}{M_d}\frac{1-M_d}{1-M_y}\right)-\frac{M_d}{1-M_y}\left[1-\omega\delta\frac{M_y}{M_d}\frac{1-M_d}{1-M_y}\right].
\end{aligned}$$

Since $\frac{1-(1-\tau_y)M_y-\tau_yM_y(M_y+\omega\delta(1-M_d))}{1-(1-\tau_y)(1-\delta)M_y}<1$ and $1-\delta\frac{M_y}{M_d}\frac{1-M_d}{1-M_y}<1-\omega\delta\frac{M_y}{M_d}\frac{1-M_d}{1-M_y}$,we know that $h\left(\chi_0^{'}\right)<0$ so $\chi_0^{'}\geq\chi_{MSV,1}$. The fact that $\chi_0^{'}\geq\chi_{MSV,1}$ together with the aforementioned property of $g(\chi)$ means that $\{\chi_k\}_{k=0}^{\infty}$ is a bounded, monotonic sequence converging to $\lim_{k\to+\infty}\chi_k=\chi_{MSV,2}>0$.

[31] If $1+\frac{M_y-\omega M_d}{1-M_y}\tau_y+\omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\tau_y\chi_{MSV,1}<0$, $\frac{M_d(1-\omega)}{1-M_y}+\omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\chi_{MSV,1}<0$, and $\chi_{MSV,1}>0$ from (D.26), a contradiction.

If Assumption 2 holds ($\omega < 1$), $\chi_{MSV,2} > \frac{1-\beta}{\tau_y}$. To see this, we have

$$
\begin{aligned}
h\left(\frac{1-\beta}{\tau_y}\right) &= \omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\frac{(1-\beta)^2}{\tau_y} + \left(1+\frac{M_y-\omega M_d}{1-M_y}\tau_y - \omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\right)\frac{1-\beta}{\tau_y} - \frac{M_d(1-\omega)}{1-M_y} \\
&= -\beta\omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\frac{(1-\beta)}{\tau_y} + \left(1+\frac{M_y}{1-M_y}\tau_y\right)\frac{1-\beta}{\tau_y} - \frac{(1-\beta\omega)M_d}{1-M_y} \\
&< -\beta\omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\frac{(1-\beta)}{\tau_y} + \left(1+\frac{M_y}{1-M_y}\tau_y - \frac{(1-\beta\omega)\left[1-(1-\tau_y)M_y\left(1+\delta\frac{\beta\omega}{1-\beta\omega}\right)\right]}{1-M_y}\right)\frac{1-\beta}{\tau_y} \\
&= -\beta\omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\frac{(1-\beta)}{\tau_y} + \beta\omega\left(\frac{1-(1-\tau_y)M_y(1-\delta)}{1-M_y}\right)\frac{1-\beta}{\tau_y} = 0.
\end{aligned}
$$

Similar to (A.9),

$$
\mathbb{E}_0[d_t] = \frac{1}{\beta^t}\Pi_{j=0}^{t-1}\left(1-\tau_y\chi_{H-j}\right)(d_0+\epsilon_0).
$$

Since $\lim_{k\to+\infty}\chi_k = \chi_{MSV,2} > \frac{1-\beta}{\tau_y}$, we know that $\lim_{H\to\infty}\mathbb{E}_0[d_H]\to 0$. This finishes the proof with the alternative fiscal policy (7).

D.8 Proof of Theorem 3

In this proof we restrict $\phi \in [-1/\sigma, \frac{\tau_y}{\beta\frac{D^{ss}}{Y^{ss}}}]$. The aggregate demand relation (11) together with monetary policy (30), market clearing $y_t = c_t$, and the government budget (4) lead to the following recursive aggregate demand relation:

$$
y_t = \frac{\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}}{1+\sigma\phi - \frac{(1-\beta\omega)(1-\omega)}{\omega}\phi\frac{D^{ss}}{Y^{ss}}}(d_t - t_t) + \frac{1}{1+\sigma\phi-\frac{(1-\beta\omega)(1-\omega)}{\omega}\phi\frac{D^{ss}}{Y^{ss}}}\mathbb{E}_t\left[y_{t+1}\right].
$$

Together with the baseline fiscal policy (6) we arrive at the following equation:

$$
y_t = \frac{\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}(1-\tau_d)}{1+\sigma\phi + \frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)}(d_t+\epsilon_t) + \frac{1}{1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)}\mathbb{E}_t\left[y_{t+1}\right].
$$

Applying the period-t expectation operator $\mathbb{E}_t[\cdot]$ to (15), we have

$$
\begin{pmatrix} \mathbb{E}_t[d_{t+1}] \\ \mathbb{E}_t\left[y_{t+1}\right] \end{pmatrix} = \begin{pmatrix} \frac{1-\tau_d}{\beta} & -\frac{\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}}{\beta} \\ -\frac{(1-\beta\omega)(1-\omega)(1-\tau_d)}{\beta\omega} & 1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)}{\beta\omega} \end{pmatrix}\begin{pmatrix} d_t+\epsilon_t \\ y_t \end{pmatrix} \tag{D.29}
$$

The two eigenvalues are given by the solutions of

$$
\lambda^2 - \lambda\left(\frac{1-\tau_d}{\beta} + 1 + \sigma\phi + \frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y - \beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right) + (1+\sigma\phi)\frac{1-\tau_d}{\beta} = 0. \tag{D.30}
$$

From $\phi \in [-1/\sigma, \frac{\tau_y}{\beta\frac{D^{ss}}{Y^{ss}}}]$ and $\tau_d \in [0,1]$, we know that $\lambda_1+\lambda_2 \geq 0$ and $\lambda_1\lambda_2 \geq 0$, so $\lambda_1 \geq 0$ and $\lambda_2 \geq 0$.

We first prove Part 1 of Theorem 3. That is, if

$$\phi < \bar{\phi} \equiv \frac{\frac{(1-\beta\omega)(1-\omega)}{\omega}\tau_y}{\sigma\left(1-\beta\right) + \frac{(1-\beta\omega)(1-\omega)}{\omega}\beta\frac{D^{ss}}{Y^{ss}}}, \tag{D.31}$$

complete self-financing is achieved as the fiscal adjustment is infinitely delayed. That is, there exists a bounded equilibrium of the form (17) with $\lim_{\tau_d\to 0^+}\rho_d \in (0,1)$ and $\lim_{\tau_d\to 0^+}\nu = 1$.

Since the eigenvalue of (D.30) is continuous in τ_d at 0, we have

$$\begin{aligned}\lim_{\tau_d\to 0^+}\lambda_1 &= \frac{\left(\frac{1}{\beta}+1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)+\sqrt{\left(\frac{1}{\beta}+1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)^2-4\frac{1+\sigma\phi}{\beta}}}{2}\\ &= \frac{\left(\frac{1}{\beta}+1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)+\sqrt{\left(1+\sigma\phi-\frac{1}{\beta}-\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)^2+4\left(1+\sigma\phi\right)\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)}}{2}\end{aligned}$$

and

$$\begin{aligned}\lim_{\tau_d\to 0^+}\lambda_2 &= \frac{\left(\frac{1}{\beta}+1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)-\sqrt{\left(\frac{1}{\beta}+1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)^2-4\frac{1+\sigma\phi}{\beta}}}{2}\\ &= \frac{\left(\frac{1}{\beta}+1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)-\sqrt{\left(1+\sigma\phi-\frac{1}{\beta}-\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)^2+4\left(1+\sigma\phi\right)\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)}}{2}.\end{aligned} \tag{D.32}$$

When $\phi \in [-1/\sigma, 0]$, from (D.32),

$$\lim_{\tau_d\to 0^+}\lambda_2 \le \frac{\left(\frac{1}{\beta}+1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)-\left|1+\sigma\phi-\frac{1}{\beta}-\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right|}{2} \le 1+\sigma\phi<1$$

When $\phi \in \left(0, \bar{\phi}\right)$, from (D.31), we have

$$\frac{\left(1-\beta\omega\right)\left(1-\omega\right)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right) > \sigma\phi\left(\frac{1}{\beta}-1\right).$$

Hence

$$\begin{aligned}\lim_{\tau_d\to 0^+}\lambda_2 &= \frac{2\frac{1+\sigma\phi}{\beta}}{\left(\frac{1}{\beta}+1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)+\sqrt{\left(\frac{1}{\beta}+1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)^2-4\frac{1+\sigma\phi}{\beta}}}\\ &< \frac{2\frac{1+\sigma\phi}{\beta}}{\frac{1}{\beta}+1+\frac{\sigma\phi}{\beta}+\sqrt{\left(1+\frac{1+\sigma\phi}{\beta}\right)^2-4\frac{1+\sigma\phi}{\beta}}} = \frac{2\frac{1+\sigma\phi}{\beta}}{1+\frac{1+\sigma\phi}{\beta}+\left|\frac{1+\sigma\phi}{\beta}-1\right|} = 1.\end{aligned} \tag{D.33}$$

Thus, as long as (D.31) holds, a bounded equilibrium in the form of (17) with $\rho_d = \lambda_2$ and $\chi = 1-\beta\rho_d > 0$ (from (18)) will be a solution of (D.29), with $\lim_{\tau_d\to 0^+}\rho_d < 1$. Given (17), one can then back out $\{\pi_t, d_t - \mathbb{E}_{t-1}[d_t]\}$ from (14) – (16). The fact that $\lim_{\tau_d\to 0^+}\nu \to 1$ follows directly from the definition of

ν and the fact that $\lim_{\tau_d \to 0^+}\left(\tau_d \frac{\epsilon_0 + \sum_{k=0}^{+\infty}\beta^k E_0[d_k]}{\epsilon_0}\right) = 0$ given $\lim_{\tau_d \to 0^+}\rho_d \in (0,1)$.

For Part 2 of Theorem 3, note that, when $\phi > \bar{\phi}$, we from (D.31) have

$$\frac{\left(1-\beta\omega\right)(1-\omega)}{\beta\omega}\left(\tau_y - \beta\phi\frac{D^{ss}}{Y^{ss}}\right) < \sigma\phi\left(\frac{1}{\beta}-1\right).$$

Hence, from (D.33),

$$\lim_{\tau_d \to 0^+}\lambda_2 > \frac{2\frac{1+\sigma\phi}{\beta}}{\frac{1}{\beta}+1+\frac{\sigma\phi}{\beta}+\sqrt{\left(1+\frac{1+\sigma\phi}{\beta}\right)^2 - 4\frac{1+\sigma\phi}{\beta}}} = 1.$$

As a result, there exists no bounded equilibrium if the fiscal adjustment is infinitely delayed (i.e., if $\tau_d \to 0$ from above).

D.9 Proof of Proposition C.1

We note that the proof heavily leverages results from Wolf (2021*a*). Following that paper, all arguments are established using sequence-space notation, with boldface denoting time paths.

The sequence of wealth holdings associated with an interest rate sequence $\boldsymbol{r}$ (both in deviation from steady state) is given as

$$\boldsymbol{d}(\boldsymbol{r}) = \mathcal{D}_r \times \boldsymbol{r}$$

where $\mathcal{D}_r$ is the sequence-space Jacobian of wealth holdings with respect to interest rates. The desired long-run elasticity η is the long-run response of asset holdings to a permanent change in interest rates; that is, it is given as the limit (if it exists) of the sequence $\boldsymbol{d}(\mathbf{1})$.

It follows from the aggregate household budget constraint that the savings matrix $\mathcal{D}_r$ and the analogous consumption matrix $\mathcal{M}_r$ are related as[32]

$$\mathcal{M}_r + \frac{1}{1+\bar{r}}\mathcal{D}_r = \begin{pmatrix} \mathbf{0}' \\ \mathcal{D}_r \end{pmatrix} \tag{D.34}$$

where $1+\bar{r} = \beta^{-1}$. Since by definition

$$\eta = \lim_{H\to\infty}\mathcal{D}_r(H,\bullet)\times\mathbf{1}$$

it follows from (D.34) that we have

$$\eta = \frac{1+\bar{r}}{\bar{r}}\lim_{H\to\infty}\mathcal{M}_r(H,\bullet)\times\mathbf{1} \tag{D.35}$$

It thus remains to characterize $\mathcal{M}_r$. For this we momentarily assume that there are no spenders ($\mu = 0$); the extension to the full spender-OLG model is straightforward and will come at the end. It follows

[32] Note that this construction removes income effects related to steady-state wealth holdings.

from the results in Wolf (2021*a*) that $\mathcal{M}_r$ has the following limiting properties:

$$\lim_{H\to\infty} \mathcal{M}_r(H,H) = -\sigma\beta\omega\frac{1-\omega}{1-\beta\omega^2}$$

$$\lim_{H\to\infty} \mathcal{M}_r(H,H-1) = \sigma\frac{\omega(1-\beta\omega)}{1-\beta\omega^2}$$

as well as

$$\lim_{H\to\infty} \frac{\mathcal{M}_r(H,H-s)}{\mathcal{M}_r(H,H-s+1)} = \omega\beta, \quad s\geq 2$$

$$\lim_{H\to\infty} \frac{\mathcal{M}_r(H,H+s)}{\mathcal{M}_r(H,H+s-1)} = \omega, \quad s\geq 1$$

Plugging those relations into (D.35) and simplifying, we find

$$\eta = \frac{1}{1-\beta}\sigma\left[\frac{1}{1-\theta}-\frac{1}{1-\beta\theta}\right] \tag{D.36}$$

Finally, if there is a margin of spenders, then the elasticity is simply scaled down to correspond to the margin of OLG households $(1-\mu)$, thus giving (C.2).