Constrained efficient borrowing with sovereign default risk

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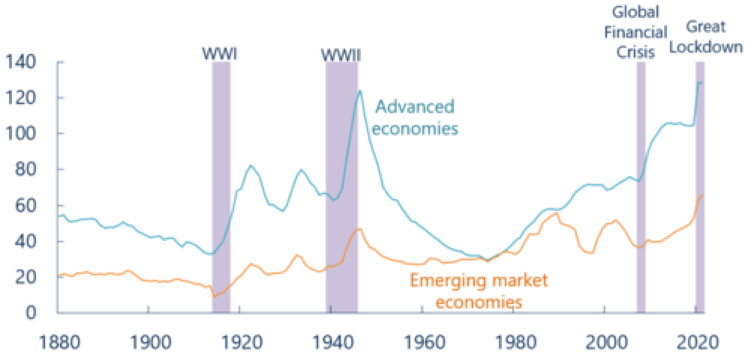
RECORD HIGH DEBT LEVELS

(global public debt, percent of GDP)

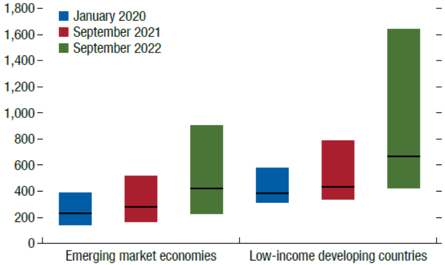


Sources: Historical Public Debt Database; IMF, *World Economic Outlook*; Maddison Database Project; and IMF staff calculations.

Recent increase in default risk

- Higher debt and lower growth ⇒ increase in sovereign spread yield.

Figure ES.5. Sovereign Spreads, by Income Group, 2020–22
(Basis points)

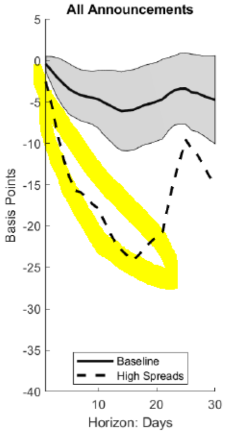


Source: JPMorgan Emerging Market Bond Index.
Note: Lines are median and shaded areas are interquartile ranges for a sample of 49 emerging market economies and 9 low-income developing countries.

Prompting discussions about how to lower debt in coming years.

SUCESS OF FISCAL CONSOLIDATIONS

- Sovereign yield spread declines after announcements of fiscal consolidation.
- Larger decline when accompanied by IMF programs. Stronger commitment to debt stabilization?



Source: David, Guajardo, and Yepez (IMF WP, 2019).

Fiscal rules lower the spread

- National governments: Thornton and Vasilakis (EI, 2017), Iara and Wolf (EJPE, 2014).
- US states: Eichengreen and Bayoumi (EER, 1994), Poterba and Rueben (1999, JUE 2001), Meng and Liu (2022).
- Evidence suggests that fiscal rules enhance commitment to lower future borrowing.

What do we do?

- Compute allocation with commitment to future borrowing paths (contingent on income paths).
- Quantify effects of commitment.
- Compare against simpler commitment devices: fiscal rules, IMF programs, fiscal consolidations.

Findings I

- Commitment to future borrowing:
  - reduces average sovereign spread by almost 80%.
  - can be enforced by reversal to equilibrium without commitment.
  - Optimal to curb borrowing more in periods
    - when the spread is more sensitive to borrowing,
    - after recessions, and
    - after significant debt expansions.
  - “Escape clauses”: more borrowing when close to defaulting.

FINDINGS I

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        - after significant debt expansions.
    - "Escape clauses": more borrowing when close to defaulting.

FINDINGS II

- Optimal debt reduction:
  - moderate fiscal adjustments early on,
  - rapid decrease in consumption in later periods,
- Commitment to future borrowing generates welfare gains of 0.45% (in consumption equivalent) vs. welfare gains of 0.25% with simple debt limit rules.

Environment

- Based on Aguiar, Amador, Hopenhayn, and Werning (Ecta, 2019).
- But with stochastic income: $y_t \in \{y_1, ..., y_n\}$, with $y_1 < y_2 < ... < y_n$, and $Pr(y_{t+1} = y_j \mid y_t = y_i) = \pi_{i,j} > 0$.
- Government issues long-term debt. A bond issued at t pays $\{\delta, \delta(1-\delta), \delta(1-\delta)^2, ...,\}$ at $t+1, t+2, t+3, ...$. Sum of all debt payments = 1 $\forall t$. ▸ Coupons δ is exogenous.
- Government can default on its debt. Zero recovery.
  - If the government defaults, it receives a continuation value U.
  - U has pdf f and cdf F, and $f > 0\ \forall U$.
- Lenders are foreign, atomistic, risk-neutral, and have a discount factor $= 1/R$.

Two commitment assumptions

1. **Without commitment to future borrowing or defaults**. Each period, a "Markov" government maximizes expected utility, taking as given policy rules of future (Markov) governments.
2. **With commitment to future borrowing (but without commitment to future defaults)**. Constrained efficient borrowing. "Ramsey planner" chooses income-path-contingent borrowing until it defaults.

Markov equilibrium

- Government decides each period whether to repay and how much to borrow (if it repays).
- The government defaults ($d_t = 1$) whenever $U \geq$ Value of repaying.
- Budget constraint:

$$c_t = y_t - \delta b_t + q_t i_t.$$

$b_t =$ Bonds outstanding.

$i_t =$ New bonds issued. Bellman

Equilibrium bond price

$$q_t = \frac{1}{R} E \left[(1 - d_{t+1}) \left(\delta + (1-\delta) q_{t+1} \right) \right]$$

q_{t+1} depends on future equilibrium (Markov) borrowing and defaulting policy rules.

Current Markov government can only manipulate future borrowing through the choice of b_{t+1}.

Constrained efficient borrowing

- Decided by a"Ramsey planner" that chooses borrowing path at $t = 0$, **taking as given future (ex-post optimal) default decisions.**
- Can be interpreted as the most sophisticated "debt stabilization plan" or fiscal rule.
  - Best case scenario for policy proposals aiming at disciplining sovereign borrowing.
- Assumption: Ramsey planner can condition its future borrowing on $y^t = \{y_1, ..., y_t\}$, not on U_t realizations.
  - Tractability.
  - Can interpret U_t as "policy noise" that cannot be used to condition fiscal policy.

Ramsey planner's objective

Need to solve for history dependent sequence $\vec{b} = \{b_{t+1}(y^t)\}_{t=1}^{\infty}\ \forall y^t$.

$$\underset{\vec{b}}{Max}\left\{ u(c_0) + E\sum_{t=1}^{\infty}\beta^{t-1}\underbrace{\prod_{j=2}^{t-1}(1-d_j)}_{\substack{\text{Pr. arriving at } t \\ \text{without defaults}}}\left[(1-d_t)u(c_t)+d_tU_t\right]\right\}$$

Optimality condition for Ramsey planner

- Takes into account how changes in $b_{t+1}(\tilde{y}^t)$ affects $c_{t-1}(\tilde{y}^{t-1})$, $c_{t-2}(\tilde{y}^{t-1})$,.... and default incentives at $t-1$, $t-2$,....

$$\beta^{t-1} \overbrace{Pr(\tilde{y}^t)\prod_{j=2}^{t} F(\mathcal{V}_j(\tilde{y}^j))}^{\substack{\text{Pr arriving at } \tilde{y}^t \\ \text{without prior defaults}}} \underbrace{\frac{\partial \mathcal{V}_t(\tilde{y}^t)}{\partial b_{t+1}(\tilde{y}^t)}}_{=0 \text{ in Markov}} + \overbrace{\left\{ \sum_{s=0}^{t-1} \beta^{s-1} Pr(\tilde{y^s}) \prod_{j=2}^{s} F(\mathcal{V}_j) u'(c_s) i_s \frac{\partial q_s}{\partial b_{t+1}(\tilde{y}^t)} \right\}}^{\text{Inefficiency in Markov: } \Delta b_{t+1} \text{ affects } c_{t-1}, c_{t-2}, \dots} = 0$$

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Tractable optimality condition for a Ramsey government

Proposition: the optimal borrowing plan for the Ramsey government satisfies

$$\underbrace{u'(c_t)\left(q_t + \frac{\partial q_t}{\partial b_{t+1}} i_t\right) - \beta E\left[(1-d_{t+1})u'(c_{t+1})(\delta + (1-\delta)q_{t+1})\right]}_{=\text{ 0 in Markov equilibrium}} + h_t \frac{\partial q_t}{\partial b_{t+1}} = 0$$

h_t = cumulative effects of Δq_t on welfare at $t-1, t-2, \ldots$.

When $h_t > 0$, Ramsey plan prescribes lower debt (b_{t+1}) than Markov equilibrium. Computation

Optimality condition for a Ramsey government

h_t has a recursive structure. Makes problem tractable.

$$h_t = \frac{(1-\delta)F\left(V_t\right)h_{t-1}}{f\left(V_t\right)\left(\delta + (1-\delta)q_t\right)h_{t-1} + \beta R \beta F\left(V_t\right)} + u'(c_{t-1})i_{t-1}$$

- h_t increases with $u'(c_{t-1})i_{t-1}$. Ramsey government reduces borrowing more after a period with more debt issuance (i_{t-1}) or with low consumption.
- h_t decreases with $f(V_t)$. Ramsey government borrows more when the repayment probability is sensitive to borrowing. Averting defaults at t improve q_{t-n}.

QUATITATIVE EVALUATION

- Preferences

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}$$

- Value of defaulting U_t: autarky and income drop for 3 years on average + ε_t

Calibration

1 period = 1 quarter.

β	0.97	Standard
R	1.01	Standard
ρ	0.94	Mexico GDP
σ_ε	1.5%	Mexico GDP
δ	0.035	Avg. debt duration = 4.8 years
ψ	0.083	E(exclusion duration) = 3 years
σ	4.2	$\sigma(c)/\sigma(y) = 1.1$
σ_{V^D}	0.1	Avg debt, avg. spread and spread volatility in Mexico
d_0	0.17	Avg debt, avg. spread and spread volatility in Mexico
d_1	1.2	Avg debt, avg. spread and spread volatility in Mexico

Simulations

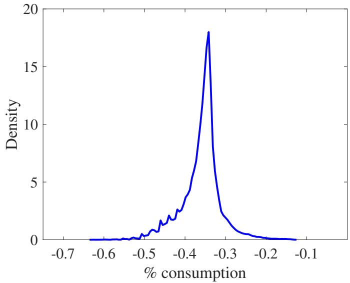
	Mexico	Markov	Ramsey	One-period bonds
Mean debt ratio	44.2	44.3	39.5	26.6
Average spread	3.3	3.3	0.5	0.2
Std spread	2.4	2.4	0.4	0.7
$\sigma(c)/\sigma(y)$	1.1	1.1	1.4	1.4

- No borrowing inefficiency when the government only issues short-term debt.
- But government cannot borrow as much as needed.

Enforcement of Ramsey debt plan

- Assumption: government loses "reputation" after one deviation ⇒ economy switches to Markov equilibrium forever.
- Government **never wants to deviate from Ramsey plan.**

Welfare gain from deviations from Ramsey plan

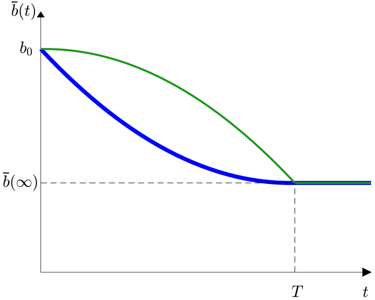


Optimal debt reduction path

- Start from initial state with "high" debt.
- Compute path chosen by a Ramsey planner with no prior commitments ($h = 0$).
- Finding: Ramsey planner reduces the debt faster in later periods but barely reduces it in the initial periods.
- Milder early sacrifices in consumption.
- Markov government defaults (within 10 years) in 41% of the transition paths. Ramsey planner defaults in 24% of the paths.

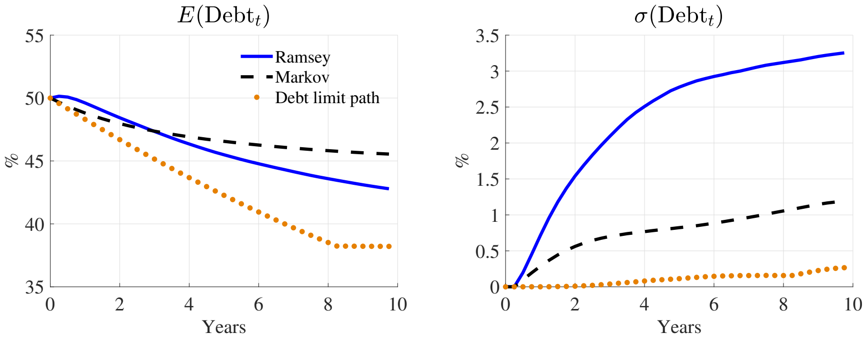
Debt reduction implemented with a simple fiscal rule (debt ceiling)

- Solve Markov problem with time-varying restriction $b' \leq \bar{b}(t)$.
- Time-varying debt limit depends on three parameters: transition length, long-run debt limit, and adjustment "speed".
- Examples:



Debt reduction paths

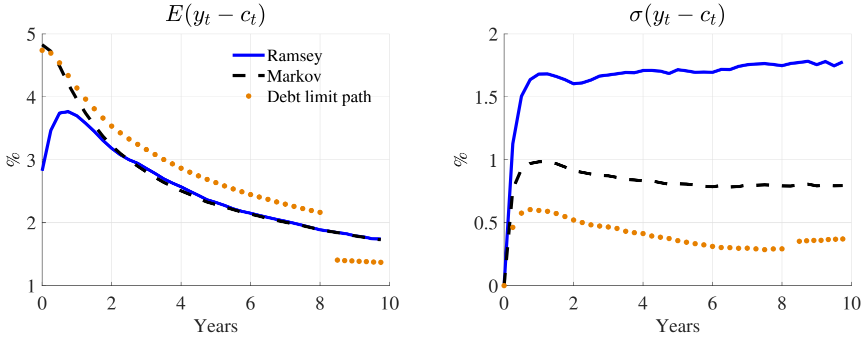
- Ramsey planner moderates debt reduction in early periods and adds more dispersion in later periods.
- Deleveraging with simple debt limits entail faster initial debt reduction.



Fiscal consolidation paths

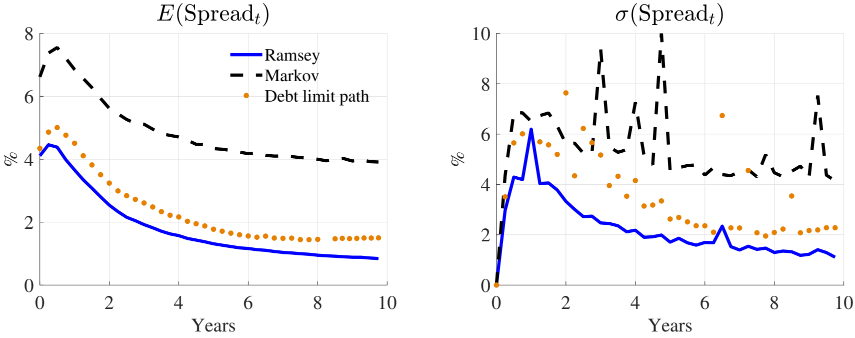
$y - c =$ net transfer to lenders.

Ramsey planner lowers debt with milder initial consumption sacrifice. But increases consumption dispersion over time.



SPREAD PATHS

- Immediate reduction in the spread when future fiscal discipline is imposed (Ramsey and Markov with debt limits).
- Ramsey government achieves larger spread reduction with milder initial consumption sacrifices.



Welfare

- A deleveraging process with an optimal sequence of debt limits achieve 60% of the welfare gains (0.44% of permanent consumption increase)

EX-ANTE OPTIMAL DEBT DURATION

- Government chooses δ at $t = 0$ before issuing any debt.
- In Markov: 2.3 years
- With commitment to borrowing paths: more than 30 years.

Comparative static w.r.t. shock to V^D

- More noise in continuation value under default:
  - reduces the debt, increases the spread and spread volatility.
  - reduces extra procyclicality of fiscal policy.

Std. dev. shock to V^D	Debt	Spread	Std dev spread	$\sigma(c)/\sigma(y)$	Welfare gain (%)
Markov					
0.07	44.3	3.0	2.2	1.1	
0.10	**44.3**	**3.3**	**2.4**	**1.1**	
0.25	40.7	3.9	2.8	1.1	
0.75	26.4	8.9	4.7	1.0	
1.00	22.5	12.1	5.2	1.0	
Ramsey					
0.075	40.4	0.5	0.4	1.4	0.43
0.10	**39.5**	**0.5**	**0.4**	**1.4**	**0.45**
0.25	34.5	0.9	0.6	1.3	0.41
0.75	17.8	4.4	2.1	1.1	0.17
1.00	13.8	8.4	3.5	1.0	0.13

Conclusions

- We show a way to compute the allocation with efficient borrowing in an Eaton-Gersovitz sovereign default model with long-term debt.
- Allocation with efficient borrowing:
  - reduces average sovereign spread and spread volatility by almost 80%.
  - Fiscal consolidations bundled with long-term commitments are more efficient in reducing debt with milder early sacrifices in consumption.
  - Simple debt limit rules attain 60% of the potential welfare gains.

Thanks!

Markov equilibrium

- Government decides each period whether to repay and how much to borrow (if it repays).
- The government defaults ($d_t = 1$) whenever $U \geq$ Value of repaying.
- Budget constraint:

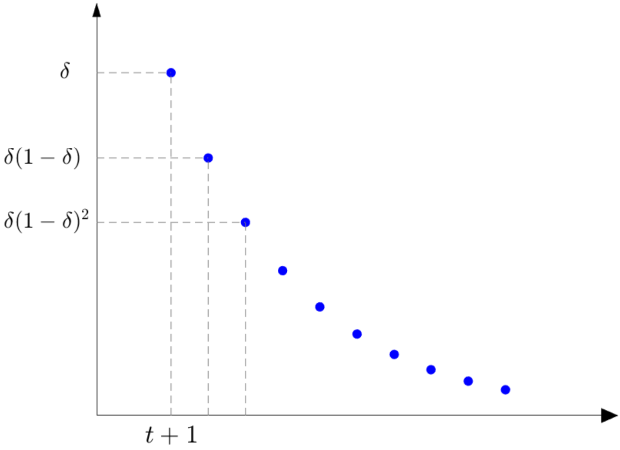
$$V(b,y) = \underset{b'}{Max} \left\{ u(c) + \beta E_{y',U'|y} \left[(1 - \hat{d}(b',y',U'))V(b',y') + \hat{d}(b',y',U')U' \right] \right\}$$

$$c_t = y_t - \delta b_t + q_t i_t.$$

$b_t =$ Bonds outstanding.

$i_t =$ New bonds issued. Markov

Profile of coupon payments



Setup

Bond price depends on future borrowing

$$q_{t-1} = \frac{1}{R} \sum Pr(y_t \mid y_{t-1}) F(V_t) \left(\delta + (1-\delta) q_t\right)$$

$$\frac{\partial q_{t-1}(\vec{b}, \tilde{y}^t)}{\partial b_{t+1}(\tilde{y}^t)} = \overbrace{Pr(\tilde{y}_t \mid \tilde{y}_{t-1}) F\left(V_t\right) \left(\frac{1-\delta}{R}\right) \frac{\partial q_t}{\partial b_{t+1}(\tilde{y}^t)}}^{\text{Effect of changing } q_t \text{ on } q_{t-1}}$$

$$+ \underbrace{Pr(\tilde{y}_t \mid \tilde{y}_{t-1}) f\left(V_t\right) \frac{\partial V_t}{\partial b_{t+1}(\tilde{y}^t)} \left(\frac{\delta + (1-\delta) q_t}{R}\right)}_{\text{Effect of changing the repayment probability in } t \text{ on } q_{t-1}}.$$

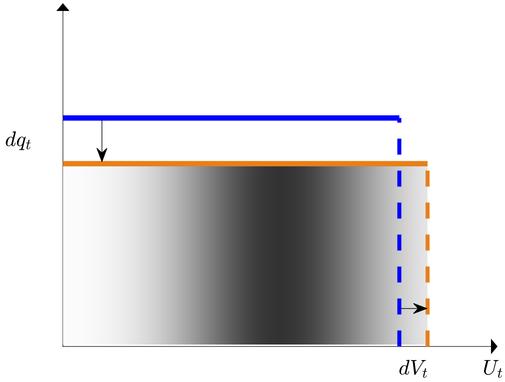
$$V_t = V_t(\vec{b}, \tilde{y}^t)$$

$$q_t = q_t(\vec{b}, \tilde{y}^t)$$

▸ Optimality condition

Change of q_{t-1} with $\uparrow b_{t+1}$

$$\frac{\partial q_{t-1}}{\partial b_{t+1}} = \underbrace{f(V_t)\frac{\partial V_t}{\partial b_{t+1}}}_{\text{Change in repay-ment prob. at } t} \frac{(\delta + (1-\delta)q_t)}{R} + \frac{F(V_t)(1-\delta)}{R}\frac{\partial q_t}{\partial b_{t+1}}$$



▸ Optimality condition

Optimality condition for Markov borrower

$$u'(c) \underbrace{\left(q + \frac{\partial q}{\partial b'} i\right)}_{\text{Mg revenue from } \Delta b'} + \sum_{y'} Pr(y' \mid y) \underbrace{F(V(b', y'))}_{\substack{\text{Repayment prob.} \\ \text{at } y_{t+1}=y'}} \frac{\partial V(b', y')}{\partial b'} = 0$$

$$u'(c) \left(q + \frac{\partial q}{\partial b'} i\right) - \sum_{y'} Pr(y' \mid y) F(V(b', y')) u'(c')(\delta + (1-\delta) q') = 0$$

$$\boldsymbol{\frac{\partial}{\partial b'}} \left\{ \boldsymbol{u(c) + \beta E_{y'|y} \left[F(V') u(c')(\delta + (1-\delta) q') \right]} \right\} = 0$$

$q' =$ equilibrium bond price next period when $y_{t+1} = y'$.

No gain from deviating from optimal debt path: gain from $\uparrow b'$ = expected loss from winding down $\uparrow b'$ at $t+1$. Markov borrower

Recursive formulation of Ramsey planer's problem

Optimality cond.

- Uses history variable h as an additional state.
- For a given state (b,h,y): find b' that satisfies Ramsey planner's foc.
- Continuation history h' depends on (b,h,y,b').

$$V(b,h,y) = u(c) + \beta \sum_{y'} Pr(y' \mid y) \left[F\left(V'(y')\right) V'(y') + \int_{V'(y')} Uf(U)dU \right]$$

$$s.t. \quad c = y_i - \delta b + qi \quad \text{with } i = b' - (1-\delta)b$$

$$u'(c)q - h' E_{y'|y} f\left(V'(y')\right) u'(c_j') \left(\delta + (1-\delta)q_j'\right)^2 / R - \beta E_{y'|y} F\left(V'(y')\right) u'(c_j') \left(\delta + (1-\delta)q_j'\right) = 0$$

$$h' = \left(\frac{F(V)(1-\delta)/R}{\frac{f(V)(\delta+(1-\delta)q)}{R} h + \beta F(V)} \right) h + u'(c)i$$

Recursive formulation

▸ Optimality cond.

$$V(b,h,y_i) = u(c) + \beta E_{y'|y}\left[F\left(V'(y')\right)V'(y') + \int_{V'(y')} Uf(U)dU\right]$$

$$s.t. \quad c = y_i - \delta b + qi \quad \text{with } i = b' - (1-\delta)b$$

$$u'(c)q - h'E_{y'|y}f\left(V'(y')\right)u'(c'_j)\left(\delta + (1-\delta)q'_j\right)^2/R - \beta E_{y'|y}F\left(V'(y')\right)u'(c'_j)\left(\delta + (1-\delta)q'_j\right) = 0$$

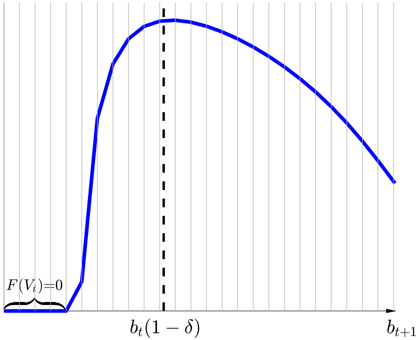
$$h' = \left(\frac{F(V)(1-\delta)/R}{\frac{f(V)(\delta+(1-\delta)q)}{R}h + \beta F(V)}\right)h + u'(c)i$$

$$q = q(b',h',y_i) = \frac{1}{R}\sum_{y'} Pr(y' \mid y)F\left(V'(y')\right)\left(\delta + (1-\delta)q'(y')\right)$$

$$V'(y') = V(b',h',y'), \quad q'(y') = q(\hat{b}(b',h',y'),h',y'), \quad c'(y') = \hat{c}(b',h',y')$$

Computing the Ramsey planners' problem

- We can compute the derivative of the Ramsey planner's objective $\partial \mathscr{U} / \partial b_{t+1}$ after arriving at t with (b_t, h_t, y_t).
- Approximate shape of $\mathscr{U}(b_{t+1})$ using $\partial \mathscr{U} / \partial b_{t+1}$
- Solve for $\partial \mathscr{U} / \partial b_{t+1} = 0$ around the maximum of the approximated function.



▸ Optimality Ramsey

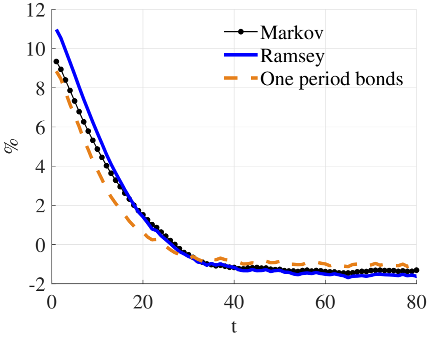
Sources of welfare gains

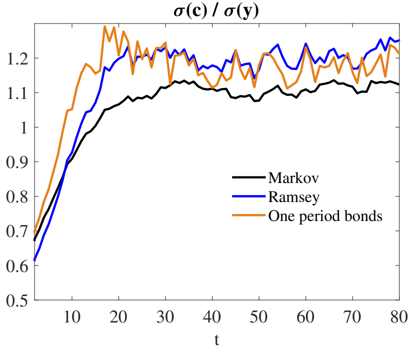
Starting from $y_0 = E(y)$, $b = 0$, and no prior commitments.

Total gain	0.47
From lowering deadweight cost of defaults	0.33
From tilting consumption	0.25
From lowering consumption volatility	-0.11

Sources of welfare gains

- Ramsey planner can increase initial consumption and reduce initial consumption volatility.
- One-period debt magnifies exposure to rollover risk $\Rightarrow$ less front-loading and at higher consumption volatility.





▸ Welfare gain

Run the following regression using a panel of cross-country data:

$$log\,(\text{Spread})_{it} = \alpha + \beta X_{it} + \delta_i + \eta_{it}, \qquad i = 1, ..., N, \quad t = 1, ..., T,$$

where

- α is a constant,
- X_{it} vector of control variables for country specific and global macroeconomic factors,
- δ_i country fixed effects,
- η_{it} disturbances that are independent across countries and time.

- Spread increases with debt and decreases with income growth.
- Spread increases more with debt when income is low (significance of 17%).
- Spread increases more with debt when spread is high.

Regression	Markov Model	Data	Markov Model	Data
Public debt to GDP	0.167	0.020 ***	0.106	0.019 ***
Real GDP growth	-0.045	-0.042 ***	-0.025	-0.034 ***
$\text{Debt}_{it} \times I\left(\Delta y_{it} < \text{mean}_i(\Delta y)\right)$	0.011	0.002		
$\text{Debt}_{it} \times I\left(\text{spread}_{it} > \text{mean}_i(\text{spread})\right)$			0.014	0.013 ***
Observations		523		523
R-squared		0.77		0.77
Number of countries		33		33

Table: The dummy variables $I(x) = 1$ when condition x is met and 0 otherwise.

▸ Slides

Density of b_t, h_t in the simulations of the economy with efficient borrowing.