Constrained efficient borrowing with sovereign default risk*

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Abstract

Using a quantitative sovereign default model, we characterize constrained efficient borrowing by a Ramsey government that commits to income-history-contingent borrowing paths taking as given ex-post optimal future default decisions. The Ramsey government improves upon the Markov government because it internalizes the effects of borrowing decisions in period t on borrowing opportunities prior to t. We show the effect of borrowing decisions in t on utility flows prior to t can be encapsulated by two single dimensional variables. Relative to a Markov government, the Ramsey government distorts borrowing decisions more when bond prices are more sensitive to borrowing, and changes in bond prices have a larger effect on past utility. In a quantitative exercise, more than 80% of the default risk is eliminated by a Ramsey government, without decreasing borrowing. The Ramsey government also has a higher probability of completing a successful deleveraging (without defaulting), while smoothing out the fiscal consolidation.

Keywords: Sovereign Default, Long-term Debt, Time Inconsistency, Debt Dilution, Deleveraging, Austerity, Debt Management, Fiscal Rules

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1 Introduction

In response to the COVID-19 pandemic, most economies have implemented large fiscal stimulus programs that pushed public debt to their historical highest levels, as illustrated in Figure 1. These developments have brought the need to plan for future deleveraging strategies to the forefront of policy debates.[1] Fiscal rules that constrain authorities are often at the center of deleveraging debates and debt sustainability analyzes have been useful in gauging the magnitudes of fiscal consolidations needed to stabilize debt ratios.[2] Yet, there is little formal analysis regarding desirable debt paths during deleveraging. In this paper, we show how to compute constrained efficient borrowing paths for governments facing default risk. We thus provide a benchmark to inform the design of deleveraging plans for highly indebted countries, and against which to compare simpler policies to enhance fiscal discipline.

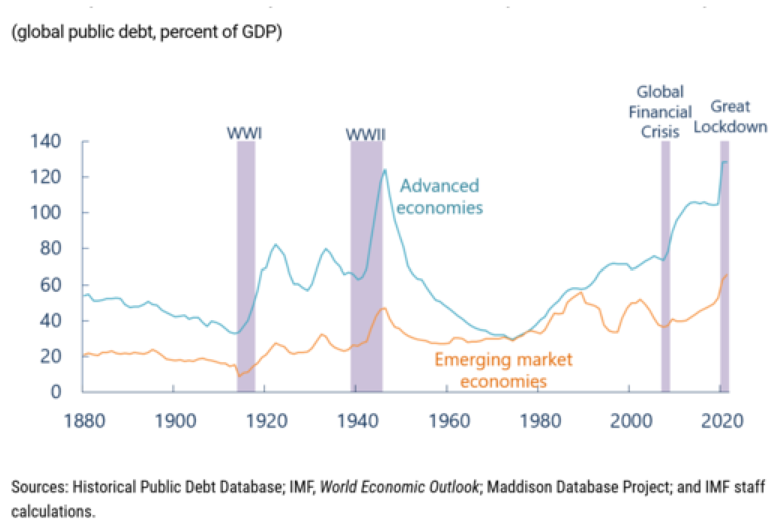

Figure 1: Update of World Economic Outlook, June 2020

Formally, we study a standard quantitative sovereign default framework à la Eaton and

[1] See, for instance, Reinhart and Rogoff (2020) and Gelpern et al. (2020). These debates are not novel. Abbas et al. (2011) study public debt cycles since 1880.

[2] As defined by the IMF (2017), "A fiscal rule is a long-lasting constraint on fiscal policy through numerical limits on budgetary aggregates." Hall and Sargent (2015) study the effectiveness of federal debt limits in the U.S. Poterba and Rueben (1999) study the effectiveness of fiscal rules in U.S. states and Thornton and Vasilakis (2017) study the effectiveness across countries. D'Erasmo et al. (2016) summarize work on debt sustainability.

Gersovitz (1981) with long-term debt. The Eaton-Gersovitz model has been widely used in studies of fiscal policy for countries with default risk. At the beginning of each period, the government first observes the aggregate endowment and the continuation value under defaulting, which are both stochastic. Then, the government decides whether to default on its debt. If the government repays, it can issue bonds priced by competitive risk-neutral foreign investors.

We solve the model using two equilibrium concepts. In the Markov Perfect Equilibrium (MPE), the government optimally chooses its default and borrowing actions each period taking as given future default and borrowing strategies. This is the equilibrium concept that the quantitative sovereign default literature has focused on.[3] In the Ramsey equilibrium, in period zero or in the first period after a default, the government commits to an income-history-contingent borrowing plan, taking as given ex-post optimal default decisions. We show in our quantitative exercise that the threat of permanently switching to the MPE after a deviation from the committed borrowing plan is enough to support the Ramsey equilibrium.

The "Ramsey government" improves upon the "Markov government" because the Ramsey government takes into account how borrowing decisions in period t affect the borrowing sets prior to t. If borrowing more in t raises default risk after t, it lowers the price of bonds issued prior to t, thus shrinking the borrowing sets in those periods. The Ramsey government takes these borrowing costs into account when choosing its borrowing plan, while the Markov government does not. By internalizing this intertemporal price effect, the Ramsey government implements the constrained efficient borrowing path.

Our contribution is to show the effect of borrowing decisions in period-t on welfare prior to t can be encapsulated by two single dimensional variables (which under conditions verified in our numerical implementation, can be collapsed into one variable). This result enables us to propose an algorithm to solve for the Ramsey government's borrowing plan.[4] Using the

[3] Aguiar and Amador (2020) show that in an Eaton-Gersovitz model with long-term debt, there may be multiple MPE equilibria. We rule out this possibility by focusing on the equilibrium that is the limit of the equilibrium of the finite-horizon economy.

[4] We cannot show the problem faced by the Ramsey government is convex, but by being able to compute the first-order derivative, we can numerically back out the shape of the Ramsey government's objective function. We use this information to verify numerically the optimal borrowing we find is indeed a global

optimality condition for the Ramsey government, we show it is optimal to distort the Markov government's borrowing decisions more in states where: i) bond prices are more sensitive to borrowing and ii) changes in bond prices have a larger effect on past welfare.

We impose discipline to our quantitative exercise by calibrating the MPE to match data from Mexico, a representative economy with sovereign risk and a standard reference in the literature. We also verify that the model's key testable implications for the relationship between the spread, income, and debt are aligned with the ones estimated for a sample of emerging economies. The overall match between the simulations for the Markov government and the data makes the model a plausible laboratory for the quantitative exercises we conduct in this paper. We measure the effects of commitment to future borrowing by comparing simulations between the Markov and Ramsey governments.

We find the welfare gain from permanently switching to an economy with a Ramsey government ranges from 0.3% to 0.7%, with larger gains for lower income.[5] The average spread is 0.5% in the economy with the Ramsey government and 3.3% in the economy with the Markov government. The Ramsey government achieves this significant reduction in default risk not by lowering average borrowing but by fanning out its borrowing: it reduces borrowing in later periods and in states where default risk is most affected by borrowing, and it expands borrowing in the remaining states. Since default risk is more sensitive to borrowing in low income states, the Ramsey government conducts a more procyclical fiscal policy than the Markov government. More dispersed borrowing even leads the Ramsey government to buy back debt in some low-income states because it internalizes that an increase in bond prices in period t may allow it to issue debt at better prices prior to t. In contrast, the Markov government never buys back debt because it does not benefit from the increase in bond prices implied by a buyback, as shown by Aguiar et al., 2019 and Bulow and Rogoff, 1988, 1991.

We also show that starting from a state with high debt, the Ramsey government has

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maximum.

[5]It should also be emphasized that our model underestimates the gains from committing to a long term borrowing plan. First, we are not considering the windfall gains of bondholders, who benefit from the increase in bond prices implied by the enhanced commitment. Second, the lower spreads implied by commitment are not reflected in higher aggregate income.

a higher probability of completing a successful deleveraging (without defaults) than the Markov government. In addition, the Ramsey government can afford to smooth out the initial adjustment during the deleveraging path by effectively reducing future default risk. Commitment to a simpler policy plan implemented through the optimal sequence of debt limits imposes harsher initial austerity and delivers 60% of the welfare gains achieved by the Ramsey government. Overall, these results are indicative of the quantitative importance of enhancing long-term fiscal discipline to reduce sovereign risk and to ensure the success of fiscal programs aiming at reducing debt levels.

We also find that fiscal discipline has a significant effect on the optimal duration of sovereign debt. If the Markov government could choose the optimal ex-ante debt duration (i.e., if it could choose the debt duration in the initial period and commit to that duration thereof), it would want to issue debt with a duration of 2.3 years.[6] The optimal ex-ante duration is much higher (over 30 years) for the Ramsey government.

1.1 Related literature

While some studies of inefficient borrowing with default risk have been able to characterize the constrained efficient allocation using tractable two-period models (see, for example, Bizer and DeMarzo, 1992 or Bolton and Jeanne, 2009), quantitative work has circumscribed to relatively simple policies that limit borrowing incentives: Hatchondo et al. (2016) and Chatterjee and Eyigungor (2015) consider alternative debt contracts, and Aguiar et al. (2020) and Hatchondo et al. (2015) consider simple borrowing constraints.[7] This paper computes the constrained efficient borrowing path in a quantitative sovereign default model with inefficient borrowing.

Aguiar et al. (2019) characterize efficient deleveraging in an Eaton-Gersovitz model without income uncertainty and with endogenous maturity. They show the MPE is constrained efficient and thus implements the borrowing plan a Ramsey government would choose–conditional on ex-post optimal defaults. This is so because a Markov government

[6]Our quantitative results are robust to assuming a debt duration of 2.3 years instead of the 4.8 years duration in our calibration.

[7]While Hatchondo et al. (2016) find allocations without debt dilution, the constraint efficient borrowing path features dilution.

has no incentive to actively trade long-term bonds and only rebalances its stock of one-period bonds during deleveraging. Relatedly, Aguiar and Amador (2019) shows that borrowing is constrained efficient in an Eaton-Gersovitz model with one-period bonds and income uncertainty. We consider an environment with income uncertainty and long-term bonds in which there is a hedging benefit of actively trading long-term bonds in the MPE.[8] Because of that, the MPE is not constrained efficient and there is a role for constraining borrowing by future governments.[9]

Mateos-Planas and Ríos-Rull (2016) derive a generalized Euler equation for borrowing with long-term bonds that isolates how the default and borrowing decisions in the next period affect the current bond price and current utility. They further show that when the government can commit to default and borrowing policy rules one period in advance, there is no difference between issuing one-period or long-term bonds. We study the case with commitment to borrowing in every future period but not to future default decisions. We show that issuing long-term bonds with commitment is different from issuing one-period bonds.

Adam and Grill (2017) study the effects of committing to the next-period default rule in an environment with one-period debt. Our focus on commitment to the future borrowing path without commitment to future defaults is motivated by discussions of how to design debt reduction programs and policies that weaken governments' incentives to borrow, either through fiscal rules or through changes in debt instruments (Chatterjee and Eyigungor, 2015; Hatchondo et al., 2016).[10] Hatchondo et al. (2015) show that, for empirically plausible values, commitment to repayment yields welfare gains orders of magnitude larger than commitment to restricting future borrowing. They also show that a rule which eliminates defaults could be too costly to enforce to be credible, which may explain why in practice fiscal rules do not

[8] Arellano and Ramanarayanan (2012) discuss the hedging benefits of issuing long-term debt.

[9] Unlike Aguiar et al. (2019), we assume an exogenous maturity structure. However, we do not see this as an important limitation for studying gains from committing to the constrained efficient borrowing plan. Hatchondo et al. (2016) shows the possibility for welfare enhancing policies that reduce borrowing in a model with endogenous maturity. We show that even for the Markov government's optimal ex-ante maturity, there are gains from constraining future borrowing.

[10] The enforcement for debt reduction programs can be partially, albeit imperfectly, provided by institutions like the IMF or Eurozone partners in the case of European countries, for example.

restrict defaulting (see IMF, 2017 for a description of existing fiscal rules).

Bianchi et al. (2019) study an Eaton-Gersovitz model with production, nominal rigidities and a fiscal sector richer than the one we consider. Their setup features a trade-off between the role for expansionary fiscal policies when nominal rigidities bind and the cost of expansionary fiscal policies in terms of higher default risk. They show that when the government can commit to the spending level one period in advance, an austerity program can be beneficial. We analyze long-lasting fiscal consolidations plans with commitment but abstract from the multiplier effects considered by Bianchi et al. (2019).

Nunes and Debortoli (2010, 2013) consider a time inconsistency problem generated by the turnover of policymakers with heterogenous preferences and they study how commitment to fiscal policies affect tax rates and debt levels. Jarred et al. (2017) study the optimal maturity with and without commitment in a model with domestic debt. The time-inconsistency problem in those environments is generated by the incentive to manipulate domestic consumption, and through that, the pricing kernel that pins down government's bond prices. Those papers consider closed economies with non-defaultable bonds. Our focus on an environment with default risk is motivated by the vast quantitative work that follows Aguiar and Gopinath (2006) and Arellano (2008), and studies aggregate dynamics and fiscal policy in emerging economies using the Eaton-Gersovitz model.

Lustig et al. (2008) study the optimal fiscal and monetary policies in a closed economy when the government issues non-contingent nominal bonds of different maturities. The government wants to mitigate fluctuations in tax rates driven by exogenous government expenditure shocks. Issuing bonds with different maturities is a useful hedge against those shocks. However, the government can only affect the real value of its bonds through changes in (distortionary) inflation or variations in the term structure. In our model, long-term debt presents hedging benefits because it moderates consumption falls when default risk increases and the government's borrowing set shrinks. Lustig et al. (2008) show that a recursive formulation for the Ramsey planner can be achieved with additional state variables that summarize the implicit promises embedded in the history of first order conditions up to period t. Our additional state variables fulfill a similar role, but instead increasing the spanning of the debt portfolio, ours correct for a time inconsistency problem in debt issuance.

Dovis (2019) characterizes the dynamically optimal contract between a risk-neutral lender and a risk averse sovereign that privately observes a local productivity shock. The constrained efficient allocation is sustained by the threat of permanent autarky. Net transfers between the lender and sovereign depend on the entire history of productivity realizations. Dovis (2019) provides a recursive structure of the problem by introducing promised utility to the sovereign as a state variable. He shows how the constrained efficient allocation can be implemented with non-contingent bonds of different maturities and mimics patterns of debt maturity composition, and the dynamics of output, consumption, imports, and exports during and after debt crises. We study a standard Eaton-Gersovitz model with outright defaults and perfect information and an exogenous maturity structure. In our paper, the additional state variables that render the problem recursive and capture how borrowing in t affects utility flows prior to t.

Our approach need not be the only one that can be used to compute the constrained efficient borrowing path. We study an optimization problem with forward-looking constraints, where actions in periods $t + s > t$ constrain the feasible set at t (in our case, borrowing at $t+s > t$ affect the price of bonds issued at t and thus the feasible set at t). The seminal work by Marcet and Marimon (2019) elaborates a recursive formulation for this class of problems that has been used in several applications. The applications closest to our work are introduced by Faraglia et al. (2016, 2019), who present recursive formulations for real models with bonds that mature M periods ahead. They study a closed economy in which bonds are priced by local consumers. The government can affect real bond prices by "manipulating" the consumption path through changes in tax rates. They show that using the formulation by Marcet and Marimon (2019) requires keeping track of M additional co-state variables. Faraglia et al. (2016) also study a setup with perpetuities, like the ones we assume, and show that a recursive formulation of the Ramsey problem requires a single additional state variable. As in our model, the state variable in Faraglia et al. (2016) captures the benefits before t of changing the real interest rate in t. We study a small open economy in which the government does not affect the risk-free interest rate but affects the default premium.

The rest of the article proceeds as follows. Section 2 introduces the environment. Section 3 characterizes the optimality conditions in an economy with a Markov government and in one with a Ramsey government. Section 4 discusses the calibration. Section 5 presents the quantitative results. Section 6 concludes.

2 The environment

We study an infinite-horizon small open economy that receives a stochastic endowment stream $\{y_t\}_{t=0}^{\infty}$ of a single tradable good. The endowment process y_t takes values in the set $\mathcal{Y} = \{y_1, ..., y_J\}$ and follows a Markov process with probabilities $Pr(y_{t+1} = y_j \mid y_t = y_i) > 0$ for all $i, j = 1, ..., J$.

Preferences over consumption streams are characterized by

$$E \sum_{t=0}^{\infty} \beta^t u(c_t),$$

where $\beta \in (0, 1)$, and u is defined over the non-negative reals and characterized by $u' > 0$ and $u'' < 0$.

If the government defaults, it writes off its entire debt obligations and, as in Aguiar et al. (2019), the economy receives a continuation value U_t. The continuation value under default follows a stochastic process with support $(-\infty, \infty)$ and may be correlated with the endowment. That is, after an endowment realization $y_t = y_j$, the random variable U_t is drawn from a probability distribution with a continuous p.d.f. f_j at all $U \in (-\infty, \infty)$ and c.d.f. F_j.[11] We further assume that

$$\int_x^{\infty} U f_j(U) dU < \infty \quad \text{for all } x \in (-\infty, \infty) \text{ and } j = 1, ..., J,$$

which implies finite expected continuation values under default.[12]

[11] The unbounded support for U allows for realizations above the value of autarky. However, we do not find this has a significant role in the simulations. For our benchmark calibration, U exceeds the value of autarky 0.003 percent of the time in the MPE and 0.01 percent of the time in the economy with a Ramsey government. In addition, that occurs for income realizations that are more than 4 standard deviations below the mean, for which either government almost never repays.

[12] The main role for the random variable U is to smooth out the Ramsey government's objective function, so we can exploit first-order conditions to compute the constrained efficient borrowing plan. The dependence of the probability distribution of U_t on the income realization in the period is used in the quantitative application, where we assume the expected continuation value under default coincides with the continuation value implied by stochastic exclusion from debt markets and an endowment loss (as usually assumed in the

As long as the government has not defaulted, it borrows by issuing long-term bonds as in Hatchondo and Martinez (2009). A bond issued at t pays a coupon stream $\delta\left\{(1-\delta)^{s-1}\right\}_{s=1}^{\infty}$ in period $t+s$, until the government defaults. Thus, the parameter $\delta \in [0,1]$ determines the exogenous debt duration. Bonds are priced by competitive risk-neutral lenders that discount future payoffs at the rate r.

Timing. At the beginning of period t, the government observes the endowment y_t and the continuation value under defaulting U_t, and chooses whether to default. If the government repays, it can issue bonds or save by buying back outstanding bonds. The government announces an issuance volume and is committed to this announcement. Bonds are sold at the price offered by risk-neutral competitive lenders. These standard timing assumptions rule out multiplicity a là Calvo (Lorenzoni and Werning, 2019 and Navarro et al., 2018). We also rule out self-fulfilling crises a là Cole-Kehoe (Conesa and Kehoe, 2017, Bocola and Dovis, 2019).

The government makes economic decisions on behalf of the small open economy and its objective is to maximize consumers' welfare. We study two economies depending on when the government chooses the number of bonds issued in each period: (i) the Markov government chooses its borrowing sequentially, and (ii) the Ramsey government chooses in period 0 the borrowing path contingent on future income histories.

2.1 Markov government

As long as it has not defaulted, the Markov government acting in period t chooses how many bonds to issue in t. The only payoff-relevant state variables at the borrowing stage are the income realization y_t and the number of bonds outstanding at the beginning of the period, which we denote by b_t. The continuation value under default U_t carries no additional information about the probability distributions of $\{U_{t+s}\}_{s=1}^{\infty}$ and thus is not payoff-relevant after the government has decided to repay. The government acting in t chooses its borrowing to maximize

$$E_t \sum_{s=t}^{\infty} \beta^{s-t} u\left(c_s\right).$$

quantitative default literature).

The government acting in t cannot commit to future default and borrowing decisions. Instead, it takes into account the strategies followed by future governments to evaluate how current borrowing decisions will affect the future consumption stream.

We use V to denote the continuation value under repayment, x' to denote the next-period value of variable x, $\pi_j(y) = Pr\left(y' = y_j \mid y\right)$, and q denotes the bond price function. It is optimal for the government to repay only when the realization of the continuation value under defaulting U is below the continuation value under repayment V. Given this, we compute the MPE by solving the following Bellman equation:

$$V(b,y) = \underset{b'}{Max}\left\{u(c) + \beta \sum_{j=1}^{J} \pi_j(y) \left[\overbrace{F_j\left(V(b',y_j)\right)}^{\substack{\text{Repayment prob} \\ \text{for } y'=y_j}} V(b',y_j) + \underbrace{\int_{V(b',y_j)}^{\infty} U\ f_j(U) dU}_{\substack{\text{Exp. cont. value under} \\ \text{default for } y'=y_j}} \right]\right\} \tag{1}$$

$$s.t. \quad c = y - \delta b + q(b',y)\left[b' - (1-\delta)b\right],$$

and

$$q(b',y) = \frac{1}{1+r} \sum_{j=1}^{J} \pi_j(y) F_j\left(V(b',y_j)\right)\left[\delta + (1-\delta) q(\hat{b}(b',y_j),y_j)\right]. \tag{2}$$

The budget constraint says that a government that has repaid in the current period, pays δ coupons per outstanding bonds (b) and issues $b' - (1-\delta)b$ new bonds at a price q. Competition in financial markets between risk-neutral lenders implies that lenders make zero expected profits: in states where the government repays in the next period, bondholders receive the coupon payment δ and can trade the claims to subsequent coupon payments (that add up to $1-\delta$) at prices $q(\hat{b}(b',y_j),y_j)$, which depend on the next-period income realization $y' = y_j$ and next-period borrowing $\hat{b}(b',y_j)$. The function $\hat{b}$ solves the optimization problem (1) for all b, y.[13]

[13]The government can accumulate assets by choosing $b' < 0$. We still use equation (2) to price bonds in those cases, which implies allowing the government to default on its assets. We follow this route to avoid a discontinuity in the bond price function at $b' = 0$, which would invalidate the general use of first-order conditions to compute the solution of the Ramsey government's problem. Given that we do not observe states with $b' < 0$ in the simulations of the economies with a Markov or a Ramsey government, the equilibrium will not be affected by enabling the government to save at the risk-free rate.

Appendix A provides a description of the Markov game and equilibrium definition that support the above Bellman equation. We use the same equilibrium concept used in Aguiar and Gopinath (2006), Arellano (2008), and the papers that followed them.

2.2 Ramsey government

In the Ramsey economy, the government acting in $t > 0$ does not control its borrowing. Instead, it implements the borrowing level prescribed in the plan chosen by the Ramsey government in period 0. On the other hand, the government acting in $t > 0$ decides whether to repay in t and, as in the MPE, it chooses to repay when that yields a higher continuation value than defaulting.

Let $y^t = \{y_0, ..., y_t\}$ denote the income history until period t and $\mathcal{Y}^t$ denote the set of all possible income histories until period t. In period 0, the Ramsey government chooses the borrowing path for every future period and income history. That is, at $t = 0$, the Ramsey government chooses the path $\vec{b} = \{b_{t+1}(y^t)\}_{t=0}^{\infty}$ for all $y^t = \{y_0, ..., y_t\} \in \mathcal{Y}^t$. This path will be implemented until there is a default.

The Ramsey government may want to condition borrowing in t on the realization of the continuation value under defaulting U_t in order to expand the repayment set, and thus increase bond prices prior to t. We rule out this possibility mainly for tractability reasons. If the Ramsey government could condition on (y^t, U_t), kinks would appear in the Ramsey government's objective. Suppose there is a cutoff $U_t^*(\vec{b}, y^t)$ at which the government acting in t would be indifferent between repaying and defaulting. The government could expand the repayment region with a borrowing rule $b_{t+1}(y^t, U_t)$ that increases the value or repayment V_t only for realizations of U_t over a range $(U_t^*(\vec{b}, y^t), U_t^*(\vec{b}, y^t) + \Delta)$. In this case, the trade-offs determining $b_{t+1}(y^t, U_t)$ would be different for $U_t \in (-\infty, U_t^*(\vec{b}, y^t))$ and $(U_t^*(\vec{b}, y^t), U_t^*(\vec{b}, y^t) + \Delta)$. We discuss this possibility in Appendix B. In addition, while income has a clear empirical counterpart (GDP) and borrowing rules that depend on GDP are observed in reality (consider for example escape clauses in some fiscal rules), it is unclear how to map the utility cost of defaulting to a verifiable variable that could affect borrowing rules.

The continuation value under repayment, V_t, is determined by the borrowing path chosen by the Ramsey government, namely

$$V_t(\vec{b}, y^t) = u(c_t(\vec{b}, y^t)) + \beta \sum_{j=1}^{J} \pi_j(y_t) \left[\; F_j(V_{t+1,j})V_{t+1,j} + \int_{V_{t+1,j}}^{\infty} U f_j(U) dU \; \right] \tag{3}$$

$$\text{where } V_{t+1,j} = V_{t+1}\left(\vec{b}, (y^t, y_j)\right), \text{ and}$$

$$c_t(\vec{b}, y^t) = y_t - \delta b_t(\vec{b}, y^{t-1}) + q_t(\vec{b}, y^t)\left[b_{t+1}(\vec{b}, y^t) - (1-\delta) b_t(\vec{b}, y^{t-1})\right], \tag{4}$$

where to simplify notation, we write the value of repayment at t as a function of the entire borrowing path $\vec{b}$, even though V_t only depends on the initial debt in t b_t and the borrowing path that follows after history y^t. The government acting in period t defaults whenever the realization of the continuation value under defaulting U_t is higher than $V_t(\vec{b}, y^t)$.

Investors observe the borrowing plan $\vec{b}$ and price bonds accordingly. The equilibrium bond price follows the recursion

$$q_t(\vec{b}, y^t) = \frac{\sum_{j=1}^{J} \pi_j(y_t) F_j\left(V_{t+1}\left(\vec{b}, (y^t, y_j)\right)\right)\left[\delta + (1-\delta) q_{t+1}\left(\vec{b}, (y^t, y_j)\right)\right]}{1+r} \tag{5}$$

for all $t = 0, 1, ...$

Let $\mathcal{U}$ denote the Ramsey government's objective function evaluated in the initial period. This function depends on consumption flows under repayment for histories (y^t, U^t) without a default and on the continuation value under default U_t for histories (y^t, U^t) where the government defaults in t. Formally,

$$\mathcal{U}(\vec{b}, y_0) = u(c_0) + \sum_{t=1}^{\infty} \beta^t \sum_{y^t \in \mathcal{Y}^t} Pr(y^t) \prod_{n=1}^{t-1} F_{I(n,y^t)}(V_n(\vec{b}, y^n)) \left[\begin{array}{l} F_{I(t,y^t)}(V_t(\vec{b}, y^t)) u(c_t(\vec{b}, y^t)) \\ + \int_{V_t(\vec{b}, y^t)}^{\infty} U \; f_{I(t,y^t)}(U) dU \end{array} \right], \tag{6}$$

where $I(n, y^t)$ denotes the income realization index in period $n < t$ for an income history y^t, $Pr(y^t)$ denotes the probability of observing an income history y^t given y_0, and y^n the sub-history of income realizations until period n given y^t. The government derives utility $u(c_t(\vec{b}, y^t))$ in period t after repaying and expects $\int_{V_t(\vec{b}, y^t)}^{\infty} U \; f_{I(t,y^t)}(U) dU$ after defaulting. Both scenarios are relevant only if the government repays in every period before t, which occurs with probability $\prod_{n=1}^{t-1} F_{I(n,y^t)}(V_n(\vec{b}, y^n))$.

The *Ramsey government's optimization problem* consists of

$$\underset{\vec{b}}{Max}\, \mathcal{U}(\vec{b}, y_0) \tag{7}$$
$$s.t. \quad (3),\ (4),\ \text{and } (5).$$

3 Optimality conditions

This section presents the optimality conditions for both the Markov and the Ramsey governments. For the Markov government, we do not establish that the objective function is differentiable. We only assume differentiability of q and V w.r.t. b' to illustrate the trade-off faced by each government. We do not rely on the optimality conditions to solve for the MPE numerically. In contrast, for the Ramsey government, we show that the objective function is differentiable.

3.1 Optimality condition for the Markov government

The next equation presents the standard optimality condition for the Markov government:

$$u'(c) \underbrace{\left[q(b', y) + \frac{\partial q(b', y)}{\partial b'} \iota \right]}_{\substack{\text{Marginal proceeds from} \\ \text{issuing an extra bond}}} = \beta \sum_{j=1}^{J} \pi_j(y) F_j(V_j') u'(c_j') \left[\delta + (1-\delta) q_j' \right], \tag{8}$$

where $V_j' = V(b', y_j)$, $q_j' = q\left(\hat{b}(b, y_j), y_j\right)$, $c_j' = \hat{c}(b, y_j)$, and we use $\iota = b' - (1-\delta)b$ to denote the number of bonds issued in the period. The above equation uses the envelope condition $\partial V(b, y)/\partial b = -u'(c)[\delta + (1-\delta) q(\hat{b}(b, y), y)]$.

The left-hand side of equation (8) represents the current marginal benefit from issuing an extra bond. The government collects $q(b', y) + \partial q(b', y)/\partial b' \iota$ additional units of the consumption good when it issues an extra bond, where the second term shows it is costly for the government to lower the current bond price, which lowers the proceeds the government obtains from issuing bonds. To measure the effect on welfare of issuing an extra bond, the marginal change in current consumption is weighted by the current consumption valuation $u'(c)$.

The right-hand side of equation (8) represents the cost of transferring more debt to future periods. In the states in which the government repays in the next period, it pays the coupon

δ and carries a stream $1-\delta$ of coupon obligations to future periods. The value of the latter is $q\left(\hat{b}(b,y_j),y_j\right)$ for $y'=y_j$.

3.2 Optimality condition for the Ramsey government

We first establish differentiability of the Ramsey government's objective (the proof of Proposition 1, which illustrates the role played by the shocks U_t in ensuing differentiability, is presented in Appendix C.1):

Proposition 1. *$\mathcal{U}$ is continuously differentiable w.r.t. $b_{t+1}(y^t)$ for all $t=0,1...$ and $y^t\in\mathcal{Y}^t$.*

Differentiability implies that the optimal borrowing path chosen by the Ramsey government satisfies the following condition for $b_{t+1}(\tilde{y}^t)$, for t=0,1,... and all $\tilde{y}^t\in\mathcal{Y}^t$ (to simplify notation, we spared the arguments of c_s, q_s, and V_s for $s\le t$):

$$\overbrace{\frac{\partial\left[u(c_0)+\sum_{k=1}^{t-1}\beta^k Pr(\tilde{y}^k)\prod_{n=1}^{k-1}F_{I(n)}(V_n)\left[F_{I(k,\tilde{y}^t)}(V_k)u(c_k)+\int_{V_k}^{\infty}U\ f_k(U)dU\right]\right]}{\partial b_{t+1}(\tilde{y}^t)}}^{\text{Effect of } b_{t+1}(\tilde{y}^t) \text{ on utility flows before } t \text{ along the income path } y_0,\tilde{y}_1,...\tilde{y}_t}+$$

$$\underbrace{\beta^t Pr(\tilde{y}^t)\prod_{n=1}^{t}F_{I(n,\tilde{y}^t)}(V_n)}_{\substack{\text{Prob of arriving at}\\ t \text{ without defaults}}}\times\underbrace{\begin{bmatrix}u'(c_t)\left[q_t+\frac{\partial q_t}{\partial b_{t+1}(\tilde{y}^t)}\iota_t\right]-\beta\sum_{j=1}^{J}\pi_j(\tilde{y}_t)\times\\ F_j(V_{t+1,j})u'(c_{t+1,j})\left[\delta+(1-\delta)q_{t+1,j}\right]\end{bmatrix}}_{\substack{\text{Effect of } b_{t+1}(\tilde{y}^t) \text{ on } V_t\text{: it resembles the optimality}\\ \text{condition for the Markov government}}}=0, \tag{9}$$

with $V_{t+1,j}=V_{t+1}(\vec{b},(\tilde{y}^t,y_j))$, $q_{t+1,j}=q_{t+1}(\vec{b},(\tilde{y}^t,\ y_j))$, $c_{t+1,j}=c_{t+1}(\vec{b},(\tilde{y}^t,y_j))$, and $\iota_t(\vec{b},\tilde{y}^t)=b_{t+1}(\vec{b},\tilde{y}^t)-(1-\delta)b_t(\vec{b},\tilde{y}^{t-1})$ denotes the number of bonds issued in period t after history $\tilde{y}^t$.

Comparing equations (8) and (9) illustrates the time inconsistency problem in the standard default model with long-term debt. The Ramsey government considers the effect that debt choices in period t ($b_{t+1}(\tilde{y}^t)$) have on utility flows prior to t. This is represented by the first line in equation (9). In contrast, as illustrated in equation (8), the Markov government acting in t only takes into account the effects of changing consumption in t and $t+1$. The second line in equation (9) shows that the Ramsey government also considers the same trade-off considered by the Markov government acting in t: borrowing more in t allows for more consumption in t at the expense of lowering consumption in $t+1$.

Sufficiency of the first-order condition. We do not show that $\mathcal{U}$ is concave and condition (9) is sufficient for finding the optimum. In the numerical application, we calculate $\partial\mathcal{U}/\partial b_{t+1}(\tilde{y}^t)$ for a range of $b_{t+1}(\tilde{y}^t)$ and approximate the shape of $\mathcal{U}(..., b_{t+1}(\tilde{y}^t), ...)$ over that range to verify we are finding a global maximum (Appendix D provides more details).

3.2.1 Recursive optimality condition for the Ramsey government

In this subsection we first show the cumulated effects of borrowing in t on utility flows prior to t can be condensed in two single dimensional state variables that follow recursive laws of motion. Second, we show that under a condition that we numerically verify in our quantitative application, those two variables can be condensed into one, which enables us to find the constrained efficient allocation by solving a recursion with only one additional state variable relative to the MPE.

Note first that the Ramsey government's commitment to borrow an extra unit in t after history $y^t = (y^{t-1}, y_i)$ affects (i) $q_t(\vec{b}, y^t)$ by changing the repayment probability in $t+1$ and (ii) the repayment probability in t by changing the value of repayment in t, $V_t(\vec{b}, y^t)$. Thus, the effect of $b_{t+1}(y^t)$ on the price of debt in period $t-1$ for history y^{t-1} can be written as function of the effect of $b_{t+1}(y^t)$ on q_t and V_t:

$$\frac{\partial q_{t-1}(\vec{b}, y^{t-1})}{\partial b_{t+1}(y^t)} = \overbrace{Pr(y_i \mid y_{t-1}) F_i\left(V_t(\vec{b}, y^t)\right)\left(\frac{1-\delta}{1+r}\right)\frac{\partial q_t(\vec{b}, y^t)}{\partial b_{t+1}(y^t)}}^{\text{Effect of changing the repayment probability in } t+1 \text{ on } q_{t-1}}$$

$$+ \underbrace{Pr(y_i \mid y_{t-1}) f_i\left(V_t(\vec{b}, y^t)\right)\left(\frac{\delta + (1-\delta)q_t(\vec{b}, y^t)}{1+r}\right)\frac{\partial V_t(\vec{b}, y^t)}{\partial b_{t+1}(y^t)}}_{\text{Effect of changing the repayment probability in } t \text{ on } q_{t-1}}. \tag{10}$$

More generally, $b_{t+1}(y^t)$ affects the price of debt in every previous period $t-n$ for subhistories y^{t-n} through $\frac{\partial q_t(\vec{b}, y^t)}{\partial b_{t+1}(y^t)}$ and $\frac{\partial V_t(\vec{b}, y^t)}{\partial b_{t+1}(y^t)}$. Proposition 2 shows how the effects of $b_{t+1}(y^t)$ on utility flows until $t-1$ can be expressed as a weighted sum of $\frac{\partial q_t(\vec{b}, y^t)}{\partial b_{t+1}(y^t)}$ and $\frac{\partial V_t(\vec{b}, y^t)}{\partial b_{t+1}(y^t)}$, and that the weights follow a recursive structure (the proof is in Appendix C.2).

Proposition 2. *The optimal borrowing plan for the Ramsey government $\vec{b}^*$ satisfies first order conditions*

$$h_t^q(\vec{b}^*, y^t)\frac{\partial q_t(\vec{b}^*, y^t)}{\partial b_{t+1}(y^t)} + h_t^V(\vec{b}^*, y^t)\frac{\partial V_t(\vec{b}, y^t)}{\partial b_{t+1}(y^t)} = 0 \tag{11}$$

for all t and $y^t = (y^{t-1}, y_i) \in \mathcal{Y}^t$, with

$$h_t^q = \frac{(1-\delta)F_i\left(V_t\right)}{1+r}\left[h_{t-1}^q + u'(c_{t-1})\iota_{t-1}h_{t-1}^V\right], \tag{12}$$

$$h_t^V = \frac{f_i\left(V_t\right)\left[\delta+(1-\delta)q_t\right]}{1+r}\left[h_{t-1}^q + u'(c_{t-1})\iota_{t-1}h_{t-1}^V\right] + \beta F_i\left(V_t\right)h_{t-1}^V, \tag{13}$$

with initial values $h_0^q(\vec{b}^*, y_0) = 0$ and $h_0^V(\vec{b}^*, y_0) = 1$, and $\iota_t(\vec{b}^*, y^t) = b_{t+1}^*(y^t) - (1-\delta)b_t^*(y^{t-1})$. To simplify notation, the arguments $(\vec{b}^*, y^t)$ and $(\vec{b}^*, y^{t-1})$ of variables in period t and $t-1$ are omitted in equations (12)-(13).

The term h_t^q factors how a change in q_t affects expected utility flows from period 0 until period $t-1$, while h_t^V factors how a change in V_t affects expected utility flows from period 0 until period $t-1$, plus the direct effect of changing V_t on $\mathcal{U}$. Equations (12)-(13) show that the laws of motion for those terms can be expressed as non-linear functions of variables in periods $t-1$ and t. This enables us to recast the optimality condition for the Ramsey government using a recursive structure.

Law of motions for h^q and h^V. When contracting more debt in t increases the default probability in $t+1$, it reduces q_t. A lower q_t reduces bond prices prior to period t. In equation (12), the direct effect of changing q_t on bond prices prior to $t-1$ is captured by h_{t-1}^q. In addition, if q_{t-1} changes, that affects c_{t-1} and thus the value of repaying in $t-1$. The effect of that on utility flows prior to $t-1$ is captured by h_{t-1}^V (and weighted by $u'(c_{t-1})\iota_{t-1}$).

Contracting more debt in t changes consumption in t and thus the continuation value V_t and the repayment probability in t. The change in the repayment probability in t affects bond prices and utility flows until $t-1$. As discussed in the previous paragraph, these effects are captured by $h_{t-1}^q + u'(c_{t-1})\iota_{t-1}h_{t-1}^V$, as presented in the first term of the law of motion (13). The second term in (13) arises because by changing V_t, contracting more debt in t also changes V_{t-1} and the repayment probability in $t-1$, and the effect of that on utility flows prior to $t-1$ is captured by h_{t-1}^V.

Equations (11)-(13) in Proposition 2 are possible because of the recursive structure of the bond price function (5), which in turn is the result of the assumption of geometrically declining coupon payments.

A formulation with only one state variable. As hinted by equation (11), what matters for the optimality condition is the relative weight on $\partial q_t(\vec{b}, y^t)/\partial b_{t+1}(y^t)$ and $\partial V(\vec{b}, y^t)/\partial b_{t+1}(y^t)$. If $h_t^V(\vec{b}, y^t) \neq 0$, we can define the variable

$$h_t(\vec{b}, y^t) = \frac{h_t^q(\vec{b}, y^t)}{h_t^V(\vec{b}, y^t)} + u'(c_t(\vec{b}, y^t))\iota_t(\vec{b}, y^t) \tag{14}$$

that encapsulates the relative effects of changing bond prices in period t on utility flows until period t. Armed with that auxiliary variable, the following lemma provides a simplified version of the optimality condition (the proof is in Appendix C.2).

Lemma 3. *If $h_t^V(\vec{b}, y^t) \neq 0$ for all $t = 0, 1, \ldots$ and $y^t = (y^{t-1}, y_i) \in \mathcal{Y}^t$, the optimal borrowing path for the Ramsey planner satisfies*

$$u'(c_t)q_t + \frac{\partial q_t}{\partial b_{t+1}(y^t)} h_t - \beta \sum_{j=1}^{J} \pi_j(y_i) F_j\left(V_{t+1,j}\right) u'(c_{t+1,j})[\delta + (1-\delta)q_{t+1,j}] = 0, \tag{15}$$

with the following law of motion for h:

$$h_t = \frac{F_i\left(V_t\right)(1-\delta)h_{t-1}}{f_i\left(V_t\right)\left[\delta + (1-\delta)q_t\right] h_{t-1} + \beta(1+r)F_i\left(V_t\right)} + u'(c_t)\iota_t. \tag{16}$$

The terms $X_{t+1,j}$ refer to functions $X_{t+1}(\vec{b}, (y^t, y_j))$ for $X = V, q, c$. We also omitted the arguments for variables in periods $t-1$ and t to simplify notation.

Equation (16) shows that the history variable h_t consists of a non-linear function of bond issuances until period t weighted by their corresponding consumption valuations. It represents the welfare cost of reducing the bond price in period t (h_t^q) relative to the one of changing V_t (h_t^V), as presented in equation (14). Those welfare costs consist of current and prior consumption sacrifices derived from lowering the prices at which debt is issued in periods $t-n$ with $n = 0, 1, \ldots t$.

Comparing the Markov and Ramsey optimality conditions. There are three differences between the Ramsey optimality condition (15) and the one for a Markov government (8). First, the Ramsey government weights the bond price derivative in t by h_t and the Markov government by $u'(c_t)\iota_t$. This means that if $h_{t-1} > 0$ (as it is always the case in the simulations) and thus $h_t > u'(c_t)\iota_t$, the bond price derivative is weighted by a higher number in the Ramsey optimality condition, indicating that the marginal cost of issuing debt is higher for the Ramsey government than for the Markov government. This captures the main difference between the two optimization problems: the Ramsey government internalizes the welfare effect of changing the proceeds from bond sales in all periods up to t, but the Markov government only internalizes the welfare effect of changing the proceeds from bond sales in t.

Second, the derivative $\frac{\partial q_t}{\partial b_{t+1}(y^t)}$ in the Ramsey condition computes the effect of changing the debt stock in $t+1$ alone, keeping the subsequent debt path after $t+1$ constant. This implies that changes in $b_{t+1}(y^t)$ only affect the default probability in t. In contrast, the derivative $\partial q/\partial b'$ in the Markov condition takes into account how changes in the debt stock in the next period (b') affect future debt paths. This implies that when the Markov government increases b_{t+1}, it affects all default probabilities after t.

Third, given that the path of future default probabilities are differentially affected, $q_{t+1,j}$ are different from their counterparts in the MPE. Since the debt path from period $t+2$ onwards is unaffected by changes in $b_{t+1}(y^t)$, no default probability after $t+1$ is affected, and thus the bond prices $q_{t+1,j}$ are invariant to changes in $b_{t+1}(y^t)$. In contrast, in the MPE, next-period bond prices change with b' because all future default probabilities are affected. This introduces a discrepancy in the marginal cost of increasing $b_{t+1}(y^t)$, which is $\beta \sum_{j=1}^{J} \pi_j(y) F_j\left(V_{t+1,j}\right) u'(c_{t+1,j})[\delta + (1-\delta)q_{t+1,j}]$ after using the envelope condition.

The above discussion means that if $h = 0$ (for instance, in period 0 or after a default), the trade-offs in the optimality conditions for the Ramsey and Markov government coincide: the derivative of the bond price is only weighted by the marginal utility and issuances in the current period. However, the debt choice need not coincide because $\partial q_t/\partial b_{t+1}(y^t)$ and $q_{t+1,j}$ are different from their counterparts in the MPE.

Comparison with one-period debt. Comparing equations (8) and (15) also shows that with one-period debt ($\delta = 1$), the incentives of the Ramsey and Markov governments coincide. If $\delta = 1$, $h_t = u'(c_t)b_{t+1}$ and h is no longer a relevant state variable.

Law of motion for h. Equation (16) shows that h_t is higher for states with a high repayment probability $F_i(V_t)$. The reason is that borrowing has a larger effect on prior bond prices after reaching a state with a high repayment probability, and it is thus optimal for the Ramsey government to borrow less after transiting through those states.

Equation (16) also shows that h_t decreases with respect to the density $f_i(V_t)$.[14] A higher value of the p.d.f. $f_i(V_t)$ means the repayment probability is more sensitive to increases in the continuation value V_t, inducing the Ramsey government to increase V_t by allowing for more borrowing in t and in subsequent periods (a lower h_t achieves that by lowering the marginal cost of borrowing in t).

Finally, equation (16) shows that for states with high marginal utility or with a high number of bonds issued, h_t takes a higher value. Recall that in the optimality condition, h_t represents the marginal cost of lowering the bond price. Intuitively, the Ramsey planner wants to increase the price at which it issues debt in states with high consumption valuation and/or a high debt issuance. A higher h_t achieves that by lowering borrowing in subsequent periods. Note also that h_t is higher when h_{t-1} is higher, and h_{t-1} is higher when previous states in the history feature high consumption valuation and/or high debt issuance. This effect from consumption valuations and debt issuance in previous periods is absent in the MPE.

Recursive representation with state variable h. Armed with Lemma 3, we find the constrained efficient borrowing plan by solving the recursion below, which requires keeping track of the history variable h in addition to the state variables used in the MPE. Formally, we find the repayment value V, bond price q, and policies $\left\{\hat{c}, \hat{b}, \hat{h}\right\}$ that satisfy the following:

[14] This assumes $h_t > 0$, which is always true in our simulations.

1) V solves

$$V(b,y_i,h) = u(c) + \beta\sum_j \pi_j(y_i)\left[F_j\left(V_j'\right)V_j' + \int_{V_j'} Uf_j(dU)\right] \tag{$\mathcal{R}$}$$

subject to the resource constraint:

$$c = y_i - \delta b + q(b',y_i,h')\iota, \text{ with } b' = (1-\delta)b + \iota; \tag{$\mathcal{R}_c$}$$

the necessary condition for an optimum:

$$u'(c)q(b',y_i,h') + \Delta_q(b',y_i,h')h' - \beta\sum_j \pi_j(y)F_j(V_j')u'(c_j')\left(\delta + (1-\delta)q_j'\right) = 0, \tag{$\mathcal{R}_{b'}$}$$

where the function $\Delta_q(b',y_i,h') = -\dfrac{\sum_j \pi_j(y_i)f_j(V_j')u'(c_j')\left(\delta+(1-\delta)q_j'\right)^2}{1+r}$

$V_j' = V(b',y_j,h'), c_j' = \hat{c}(b',y_j,h'), q_j' = q\left(\hat{b}(b',y_j,h'),y_j,\hat{h}(b',y_j,h')\right);$

and the law of motion for h :

$$h' = \frac{F_i(V(b,y_i,h))(1-\delta)h}{f_i(V(b,y_i,h))\left[\delta+(1-\delta)q(b',y_i,h')\right] + \beta(1+r)F_i(V(b,y_i,h))} + u'(c)\iota, \tag{$\mathcal{R}_{h'}$}$$

for all (b,y_i,h) given $\left\{q,\hat{c},\hat{b},\hat{h}\right\}$;

2) the policy functions $\left\{\hat{c},\hat{b},\hat{h}\right\}$ satisfy equations ($\mathcal{R}_c$), ($\mathcal{R}_{b'}$), and ($\mathcal{R}_{h'}$), for all (b,y_i,h) given $\{q,V\}$; and

3) the bond price q satisfies

$$q(b',y_i,h') = \frac{1}{1+r}\sum_j \pi_j(y_i)F_j(V(b',y_j,h'))\left[\delta + (1-\delta)q\left(\hat{b}(b',y_j,h'),y_j,\hat{h}(b',y_j,h')\right)\right]$$

for all (b',y_i,h') given $\left\{\hat{b},\hat{h},V\right\}$.

Equation ($\mathcal{R}_{h'}$) is based on the law of motion for the history variable in equation (16). The recursion above uses h to denote h_{t-1} in terms of equations (15)-(16). The advantage of this approach is that we only need to solve for one value of h'. If instead we had chosen h to represent h_t, we would have had to solve for J equations determining h_j' for each possible income realization in the next period.

The necessary condition for optimum in equation ($\mathcal{R}_{b'}$) is based on equation (15) and the bond price derivative function Δ_q is based on

$$\frac{\partial q_t(\vec{b},y^t)}{\partial b_{t+1}(y^t)} = \frac{-\sum_{j=1}^{J}\pi_j(y)f_j\left(V_{t+1,j}\right)u'(c_{t+1,j})[\delta+(1-\delta)q_{t+1,j}]}{1+r}.$$

The policy functions $\left\{\hat{c}, \hat{b}, \hat{h}\right\}$ are found by solving the system of equations ($\mathcal{R}_c$), ($\mathcal{R}_{b'}$), and ($\mathcal{R}_{h'}$), which requires solving for non-linear equations in (b', h'). We find there is a unique solution to both equations for our baseline parameterization. Furthermore, we verify that the sign of the left-hand side of equation ($\mathcal{R}_{b'}$) changes from positive to negative around the unique root, indicating we are finding a global maximum. Appendix D describes our numerically algorithm.

4 Calibration

We present a standard calibration such that the simulations of the MPE match data from Mexico. Mexico is a common reference in the default literature because its business cycle displays the same properties that are observed in other economies with sovereign default risk (Aguiar and Gopinath, 2007; Neumeyer and Perri, 2005). Unless otherwise specified, we use quarterly data from 1993 to 2018.

The utility function displays a constant coefficient of relative risk aversion, that is,

$$u(c) = \frac{c^{1-\gamma} - 1}{1-\gamma}, \text{ with } \gamma \neq 1.$$

The income process is a discretization of $log(y_t) = \rho log(y_{t-1}) + (1-\rho)\mu + \varepsilon_t$.

We endogenize the mean continuation utility of defaulting $V^D(y)$ by incorporating the standard assumptions on the cost of defaulting from the quantitative default literature: stochastic exclusion from debt markets and income losses. The continuation utility of defaulting U incorporates a gaussian shock to $V^D(y)$: $U \sim N(V^D(y), \sigma_U)$, where

$$V^D(y) = u(y(1 - d_0 - d_1 y)) + \beta \sum_j \pi_j(y) \left[\psi V(0, y_j, 0) + (1-\psi) V^D(y_j)\right], \tag{17}$$

and V denotes the continuation value under repayment in equations (1) or recursion ($\mathcal{R}$), for the Markov and Ramsey governments, respectively. The exclusion period is stochastic, with ψ denoting the probability of exiting this period. While the economy remains excluded, the government looses a proportion $d_0 + d_1 y$ of its income. As in Chatterjee and Eyigungor (2012), having two parameters in the cost of defaulting gives us the flexibility to match the levels of debt and spread in the data. Note that in equation (17), the history variable h resets to 0 after a default. This assumption rules out the possibility of the Ramsey government

Table 1: Parameter values.

Previous literature or estimated			Calibrated to match targets		
r	0.01	Standard	σ_U	0.1	Std spread = 2.4%
β	0.97	Standard	d_0	0.17	Avg debt = 44.2%
ρ	0.94	Mexico GDP	d_1	1.2	Avg spread = 3.3%
σ_ε	1.5%	Mexico GDP	γ	4.2	$\sigma(c)/\sigma(y) = 1.1$
μ	$-0.5\sigma_\varepsilon^2$	$E(y) = 1$			
ψ	0.083	E(excl. duration) = 3 years			
δ	0.035	Debt duration = 4.8 years			

manipulating the cost of defaulting with post-default borrowing promises.

Table 1 presents the values given to all parameters in the model. A period in the model refers to a quarter. The value of the risk-free rate and the domestic discount factor are standard in quantitative business cycle and sovereign default studies. Note that assuming $\beta(1+r) < 1$ gives the government incentives to borrow. The parameter values that govern the endowment process are chosen to mimic the cyclical component of the log-linearly detrended GDP in Mexico from 1980 to 2014. Setting $\delta = 0.035$ and targeting Mexico's level of sovereign spreads, implies an average debt duration in the simulations of 4.8 years, roughly the average duration of public debt in Mexico.[15]

Our modeling of the continuation utility of defaulting requires calibrating the value of an additional parameter relative to other quantitative papers: the volatility of the utility cost, σ_U. In order to calibrate this parameter, we choose to add as a target the volatility of the sovereign spread, a statistic of interest for our exercise that is strongly affected by σ_U (Appendix G.2). The targets for the mean and the standard deviation of the spread are 3.3% and 2.3%, respectively, and corresponds to the J.P. Morgan EMBI spread from the first quarter of 1994 to the first quarter of 2018. As in Bianchi et al. (2018), we make the domestic risk aversion part of the calibration. This is a key parameter determining the government's willingness to tolerate consumption fluctuations and thus the optimal cyclicality of fiscal

[15]We use the Macaulay definition of duration. The data for duration corresponds to the average Modified Duration for Mexican government bonds computed by J.P. Morgan between January 2002 and March 2018.

policy. Overall, we use the simulations to calibrate the value of four parameters: the values for the default cost d_0 and d_1 mainly determine the average debt and spread levels (Hatchondo and Martinez, 2017), σ_U mainly determines the spread volatility, and γ is determined mainly by the consumption-volatility target.

5 Quantitative results

Subsection 5.1 shows that the economy with a Ramsey government features a significantly lower spread but a higher average market value of debt claims, and that the welfare gain from permanently switching to an economy with a Ramsey government ranges from 0.3% to 0.7%. Subsection 5.2 shows that the Ramsey government reduces borrowing in low-income states where bond prices are more sensitive to borrowing, and expands borrowing in the remaining states. Subsection 5.3 shows the borrowing path chosen by the Ramsey government can be sustained with a trigger to the MPE. Subsection 5.4 shows that compared with a Markov government, the Ramsey government has a higher probability of completing a successful deleveraging (without default), even when smoothing out the initial adjustment. Subsection 5.5 discusses two robustness exercises that show: i) that our main quantitative findings do not depend on the assumed debt duration and that the optimal ex-ante debt duration is significantly higher for the Ramsey government than for the Markov government, and ii) that the role of shocks to the utility of defaulting.

5.1 Default risk and welfare

Table 2 reports long-run moments in the data and in the simulations. The table shows that the constrained efficient allocation features a significantly lower spread than the MPE. Compared with the Markov government, the Ramsey government eliminates more than 80% of both the average level and the volatility of the sovereign spread. While the mean debt level is lower with the Ramsey government, the mean market value of debt claims is higher, indicating that on average the Ramsey government does not borrow less (this is illustrated in the right panel of Figure 2). Our assumption of risk neutral lenders and competitive debt market implies that the market value of debt equals the present discounted value of net payments to lenders, so the market value of debt is a better gauge of the expected debt

Table 2: Data and model simulations

	Data	Markov	Ramsey	One-period
Mean debt (%)	44.2	44.0	39.5	26.6
Mean debt mkt. value (%)		37.7	38.2	26.4
Mean spread (in %)	3.3	3.3	0.5	0.2
Std dev spread	2.4	2.4	0.4	0.7
Corr(spread, y)	-0.4	-0.8	-0.8	-0.5
$\sigma(c)/\sigma(y)$	1.1	1.1	1.4	1.4

We simulate 1,000 samples of 500 periods (quarters) each, and then select samples of 88 periods without defaults and with the last default occurring at least 30 periods before the beginning of the sample. We report the average value of each moment. The debt level in the simulations is calculated as the present value of coupon payments discounted at the risk-free rate, i.e. $b\delta/(r+\delta)$. The market value of debt corresponds to $q \times b$. Debt ratios are computed using annualized income levels.

payments than the face value of debt.

Welfare gains. The left panel of Figure 2 presents welfare gains between 0.3% to 0.7% from permanently switching from the MPE to an economy where borrowing is decided by the Ramsey government. By correcting the time inconsistency problem, the Ramsey government can achieve a better allocation of resources across time and states. Given a $\beta(1+r) < 1$ parameterization (as in most of the quantitative literature on sovereign default), a better allocation is characterized by higher consumption in earlier periods, as illustrated in the right panel of Figure 2.

Table 3 present the sources of welfare gains following Aguiar et al. (2020) (details are presented in Appendix E. The decomposition of welfare gains is computed using consumption paths and does not incorporate the welfare effects of the shocks to the continuation value under defaulting, which are present when comparing value functions (first line). The first two lines of Table 3 show that shocks to the continuation value under defaulting do not play a significant role in welfare comparisons. The table illustrates that most of the welfare gains

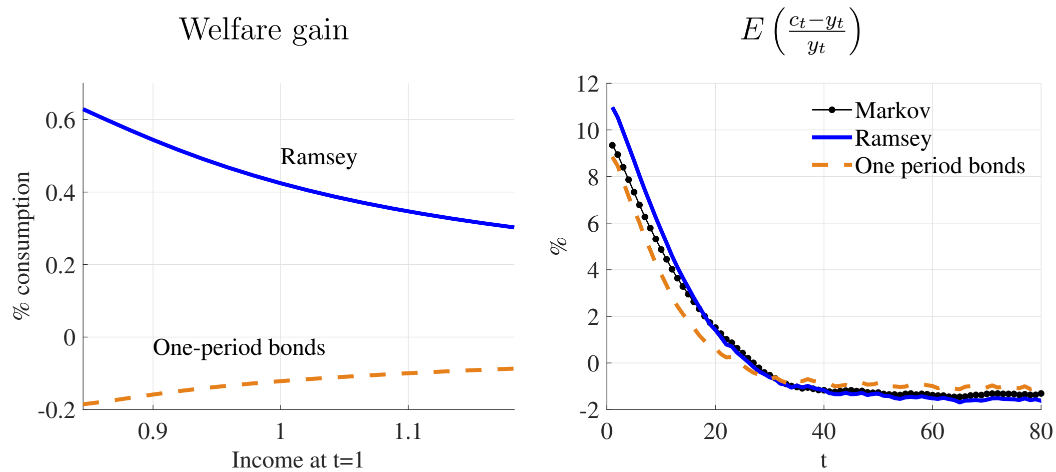

Figure 2: Welfare and net borrowing. The initial levels of debt and the history variable h are assume to be zero. Welfare gains are measured as the permanent proportional change in consumption that would leave consumers indifferent between staying in the economy with a Markov government and moving either to an economy with a Ramsey government or to an economy in which the government only issues one-period debt ($\delta = 1$ and all other parameter values as in the benchmark calibration). The right panel illustrates the average net transfer the economy receives from lenders in periods with repayment.

are accounted for by the lower default frequency with a Ramsey government, which reduces the deadweight costs of defaulting. The second contributor to welfare gains stems from a more front-loaded consumption profile. The more volatile consumption process implemented by a Ramsey government has a negative contribution to welfare.

If the government only issues one-period bonds, there is no time inconsistency in borrowing decisions (Aguiar and Amador, 2019). Table 2 shows that in this case default risk is lower than in our benchmark MPE with long-term debt, but Figure 2 shows welfare is also lower. This is, consumers are better off with a Markov government that issues debt with a duration of 4.8 years than with one that issues one-period debt. While one-period debt eliminates time inconsistency in borrowing decisions, it magnifies the exposure to shifts in borrowing opportunities. When the government issues only one-period bonds, it has to roll over its debt every period, and thus it is more vulnerable to adverse income shocks that contract its borrowing set. The optimal response to this source of risk is to issue less debt and reduce consumption frontloading (right panel of Figure 2).

Table 3: Decomposition of welfare gains

Welfare gain	$y_0 = E(y) - 2\sigma(y)$	$y_0 = E(y)$	$y_0 = E(y) + 2\sigma(y)$
From value functions	0.52	0.45	0.36
From consumption paths	0.55	0.47	0.37
From lowering income losses during default	0.38	0.33	0.26
From lowering consumption volatility	-0.12	-0.11	-0.10
From tilting consumption	0.30	0.25	0.21

Welfare gains from consumption paths are obtained using simulations of 25,000 samples of 750 periods each. All simulations start from an initial period with zero debt, $h_0 = 0$, and an initial continuation value under defaulting for which the government prefers to repay in $t = 0$ in both economies. The three bottom lines are computed using consumption paths.

5.2 Borrowing incentives and the cyclicality of fiscal policy

The top-left panel of Figure 3 shows that the borrowing policies of both the Markov and Ramsey governments are hump-shaped across income levels, with less issuances at high and low income realizations. Two opposing forces determine how equilibrium borrowing depends on the income level. On the one hand, as in models without default, borrowing is shaped by consumption-smoothing incentives. This implies that borrowing is lower when income is higher, as occurs for high income levels in the figure.

On the other hand, the main difference between the first-order conditions in default models (equations 8 and 11) and in models without default risk, is that in default models, borrowing may increase default risk and thus may lower the price of debt, weakening incentives to borrow. The top-right panel of Figure 3 shows that the effect of borrowing on the bond price is stronger for lower income levels.[16] If income is sufficiently low, this effect becomes dominant, and equilibrium borrowing is typically increasing with respect to income.

[16]The top-right panel of Figure 3 also shows that the derivative of the bond price is lower in the economy with a Markov government. In this economy, adding more debt in t has a persistent effect on the debt stock and thus raises the default probability also in subsequent periods. This is the case because unlike the Ramsey government, the Markov government acting in t does not control subsequent borrowing.

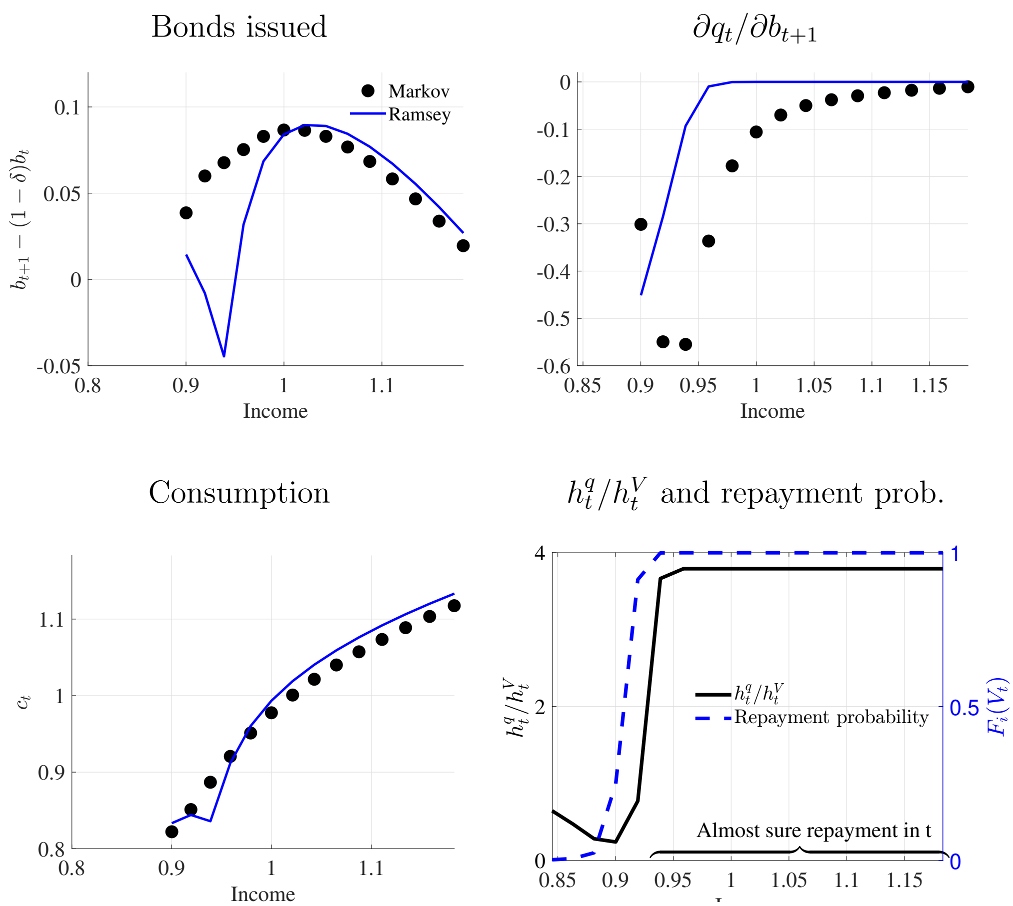

Figure 3: Cyclicality of fiscal policy. All functions plotted assume initial debt levels b_t equal to the mean debt in the simulations of each of the two economies, and history variable h_t equal to the average in the simulations of the economy with a Ramsey government. The range of income realizations included in the plots is such that the equilibrium repayment probability for $F_i(V_t)$ is above 5 percent in each of the two economies. Income takes values in a discrete set but to facilitate the comparison across economies, continuous lines are used to illustrate functions for the Ramsey government.

The top-left panel of Figure 3 also shows a jump in borrowing by the Ramsey government at the lowest income realizations. The optimality condition (11) shows that the Ramsey government weights the effect of the derivative of the bond price by h_t^q/h_t^V, which in turn depends on the repayment probability $F_i(V_t)$. Intuitively, the Ramsey government is less concerned about lowering the bond price for income realizations where repayment is unlikely. This occurs because lowering the bond price in those states has a small effect on bond prices and welfare in previous periods. The bottom-right panel of Figure 3 shows that for the lowest income realizations, $F_i(V_t)$ and thus h_t^q/h_t^V are low, lowering the cost of borrowing for the Ramsey government.

Table 2 shows that with either the Markov or the Ramsey government, consumption volatility is higher than the volatility of income. This indicates that the government tends to borrow less when income is lower. This is, governments prefer to conduct a procyclical fiscal policy.

Table 2 also shows that consumption volatility is higher with the Ramsey government, indicating that the Ramsey government prefers a more procyclical fiscal policy. This is further illustrated in the left panels of Figure 3. Lower debt issuances at lower income in the top-left panel lead to consumption being more sensitive to income (bottom-left panel).

The Ramsey government curbs borrowing more than the Markov government at moderately lower income levels because those are the states where both, bond prices are more sensitive to borrowing, and prior welfare is more sensitive to changes in bond prices. In effect, the top-right panel of Figure 3 shows that the derivative $\partial q_t/\partial b_{t+1}$ becomes significantly negative in the Ramsey economy for $y_t < 0.96$, while the bottom-right panel shows that h_t^q/h_t^V is significantly away from zero for $y_t > 0.92$. Thus, for $0.92 < y_t < 0.96$, it is optimal for the Ramsey government to curb borrowing more than the Markov government. Since the Ramsey government curbs borrowing more for relatively low levels of income, it implements a fiscal policy that is more procyclical than the one chosen by the Markov government.

In the economy with a Ramsey government, for income realizations above the mean, $\partial q_t/\partial b_{t+1} \simeq 0$ (top-right panel of Figure 3). The nearly zero value of this derivative reflects that, at high income realizations, the debt chosen by the Ramsey government commands almost no default risk for the next period and, given a gaussian distribution for U_{t+1}, default risk is insensitive to borrowing at the margin. Therefore, for income levels higher than the

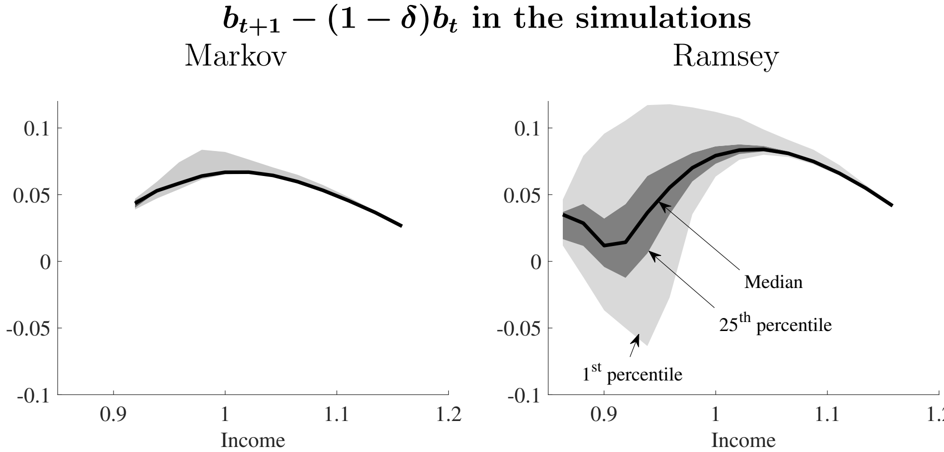

Figure 4: Debt issuance in the simulation periods used to compute the moments in Table 2.

mean, the Ramsey government does not have significant incentives to issue less debt than the Markov government.

Finally, the top-left panel of Figure 3 shows that in contrast to the Markov government, the Ramsey government may choose to buy back debt. While the Markov government does not benefit from the increase in bond prices implied by a buyback in the current period (Aguiar et al., 2019; Bulow and Rogoff, 1988, 1991), the Ramsey government may find it optimal because it internalizes the benefits that the increase in bond prices in t has on utility flows before t.

Higher borrowing dispersion with a Ramsey government. Figure 4 illustrates the distribution of bond issuances in the simulations. The borrowing path chosen by the Ramsey government entails a significant dispersion even after conditioning for income, underscoring the importance of a history variable for disciplining borrowing. The figure also shows that Ramsey borrowing is more dispersed mostly at low income realizations. As explained before, it is in low income states where the bond price is more sensitive to borrowing (top-right panel of Figure 3) and, thus, where dispersion in the history variable h generates borrowing dispersion. The higher borrowing dispersion at lower income levels with a Ramsey government is reflected in the higher consumption dispersion reported in Table 2. Finally, Figure 4 shows that the Ramsey government finds it optimal to repay at (low) income realizations at which the Markov government never repays. This enables the Ramsey government to borrow to buffer consumption drops over that income range.

Cross country evidence of $\partial q/\partial b'$. The policy prescription implied by the previous results is that implementing the constrained efficient allocation necessitates restricting debt issuances–a Markov government would choose–in states with a higher sensitivity of bond prices (spreads) to the debt level. Regardless of the government's type, in the model, the spread is more sensitive to the debt level in states with lower income levels, which (as in the data) coincide with higher spreads (see Table 2). This feature of the model seems consistent with the data. In a panel of 33 emerging economies, we find that the spread increases more with debt in years with high spread (at less than 1% statistical significance). The spread also increases more with debt in years with low growth, albeit at a weak statistical significance (less than 17%). Appendix F presents more details about these findings, showing that the MPE captures remarkably well the relationship between debt, spread, and income in the data. This makes the model a plausible laboratory for policy exercises.

5.3 Commitment to the borrowing plan chosen by the Ramsey government

Assume that if the government acting in t deviates from the borrowing plan designed by the Ramsey government in the initial period (or after a default), the government loses all credibility to future borrowing commitments. Namely, the economy permanently switches to the MPE. Figure 5 shows the distribution of welfare gains of those deviation across income realizations.[17] The figure shows that a trigger to the MPE suffices to enforce the Ramsey government's borrowing plan. The lowest welfare losses after permanently deviating to the MPE correspond to moderately lower income levels, at which the Ramsey governments constrains borrowing the most.

[17] Formally, let V^R (V^M) denote the value of repayment with a Ramsey (Markov) government, and $V^{D,R}$ ($V^{D,M}$) the expected value of defaulting with a Ramsey (Markov) government. Let the value of defaulting in t in the economy with a Ramsey government be denoted as $U_t^R = V^{D,R}(y_t) + \varepsilon_t^U$, and the value of defaulting in t in the economy with a Markov government be denoted as $U_t^M = V^{M,R}(y_t) + \varepsilon_t^U$. We assume the innovation to the value of defaulting ε_t^U does not depend on the type of government in office. We compute

$$\left[\frac{Max\left\{V^M(b_t, y_t), V^{D,M}(y_t) + \varepsilon_t^U\right\}}{V^R(b_t, y_t, h_t)}\right]^{\frac{1}{1-\sigma}} - 1$$

for simulation periods in the economy with the Ramsey government in which the government is not excluded and $V^R(b_t, y_t, h_t) \geq V^{D,R}(y_t) + \varepsilon_t^U$. The above calculation allows the government acting in t to choose the best deviation in the MPE, which may be defaulting.

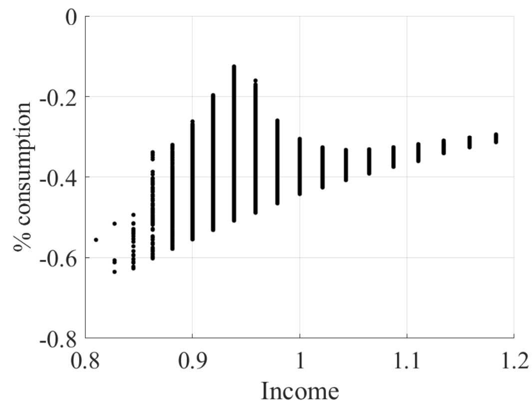

Figure 5: Welfare gains from permanently switching from the allocation chosen by the Ramsey government to the MPE. Gains are computed using all simulations periods with repayment in the economy with the Ramsey government.

5.4 Deleveraging

The COVID-19 outbreak has caused significant increases in budget deficits, sending public debt ratios to historically record levels, as illustrated in Figure 1. This episode will likely foster discussions about the best strategies to unwind those high debt levels. In this subsection we compute the debt path a Ramsey government would choose starting from a scenario with high debt, and compare it with two alternative debt paths: (i) the debt path chosen by an unconstrained Markov government, which allows us to quantify the importance of having full commitment to future borrowing, and (ii) the debt path chosen by a Markov government constrained by a sequence of debt limits, which allows us to measure the effectiveness of simple fiscal rules relative to the constrained efficient allocation.

We study simulations for the Markov and Ramsey governments for an initial state with average income, a debt-to-income ratio of 50%, and initial history $h_0 = 0$ for the Ramsey government. For the Markov government, this implies an initial spread of 6.6% and defaults during the first six years of the deleveraging process in 33% of the simulation samples. The Ramsey government achieves a significantly higher probability of a successful deleveraging, reducing the probability of default during the first six years of deleveraging to 22%. Furthermore, in none of the income paths for which the Ramsey government defaults, the Markov government finds it optimal to repay.

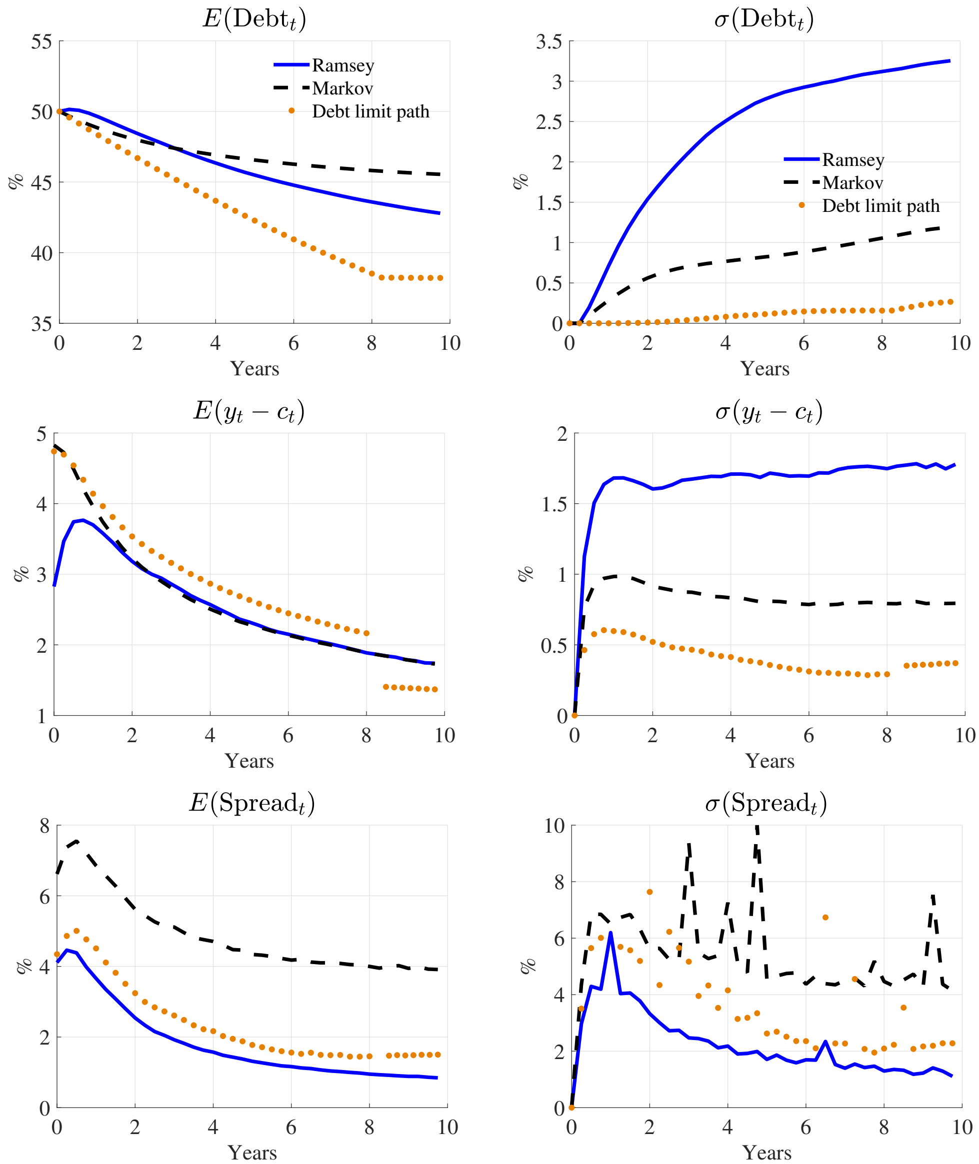

Figure 6: Deleveraging paths in 25,000 samples of 10 years each. The average and standard deviation in period t are computed using samples without defaults up to period t. All samples start with at debt ratio of 50%, mean income level and initial history $h_0 = 0$ for the Ramsey government. The debt and net transfer to lenders $(y_t - c_t)$ are expressed as a proportion of the unconditional mean income $E(y)$.

Figure 6 illustrates how the Ramsey government implements a more successful deleveraging even with weaker initial austerity. For the first year of deleveraging, compared with the Markov government, the Ramsey government chooses on average a slower deleveraging (top-left panel) and lowers net transfers to creditors ($y_t - c_t$; mid-left panel). This is a consequence of the relative impatience of domestic consumers. Since $\beta(1 + r) < 1$, earlier states are more valuable than later ones. Unlike in the case of zero debt (Figure 2), the Ramsey government cannot front-load consumption when it starts with high debt, yet it can moderate the adjustment in early periods.

The left panels of Figure 6 show that after the first year, the Ramsey government achieves a faster debt reduction with a similar level of net transfers in repayment states. This is a result of the higher repayment incentives in that economy: the higher prices at which the government issues debt (bottom-left panel of Figure 6) enables the government to collect the same proceeds from debt issuances with a lower number of bonds issued.[18] The bottom-left panel of Figure 6 shows that the lower probability of default with a Ramsey deleveraging path is reflected immediately by lower sovereign spreads.

Consistently with the discussion in the previous subsection, the right panels of Figure 6 illustrate that the conditionality imposed by the Ramsey government on its deleveraging path increases consumption and debt volatility compared to the economy with a Markov government.

Welfare is higher when the deleveraging plan is implemented by a Ramsey government. The welfare gain is equivalent to a permanent consumption increase of 0.44% relative to the MPE. In sum, our results indicate that the ability to commit to a long-term deleveraging plan allows for a smaller early consumption sacrifice while greatly increasing the probability of a successful deleveraging process that avoids default.[19]

[18]It should be noted that the Ramsey government repays in more states and thus on average transfers more resources to its creditors.

[19]Bi et al. (2013) show that expectations about future fiscal consolidations are an important determinant of the success of fiscal adjustments.

5.4.1 Deleveraging with debt limits

In this subsection, we compare the deleveraging plan chosen by a Ramsey government with the one chosen by a Markov government that is constrained by a sequence of debt limits. This exercise will enable us to measure the relative effectiveness of simpler deleveraging paths, as the ones implemented in practice. Formally, we solve the following problem

$$
\begin{aligned}
& V(b,y,t) = \underset{b'}{Max}\left\{u(c) + \beta \sum_{j=1}^{J} \pi_j(y)\left[F_j\left(V_j'\right)V_j' + \int_{V_j'}^{\infty} U\ f_j(U)dU\right]\right\} \\
& s.t. \quad c = y - \delta b + q(b', y, t)\left[b' - (1-\delta)b\right], \quad V_j' = V(b', y_j, t+1), \\
& b' \leq Max\left\{\bar{b}(t+1), (1-\delta)b\right\}, \qquad (18)
\end{aligned}
$$

where $\bar{b}(t+1)$ denotes the debt limit in period t and the constraint (18) says that the government is not forced to buy back bonds when $\bar{b}(t+1) < (1-\delta)b$.

For tractability reasons, we search over the following family of debt limit sequences. First, we assume $\bar{b}(0) = b_0$, where b_0 denotes the number of bonds outstanding at the beginning of the deleveraging process. Second, we assume the debt limit evolves according to $\bar{b}(t) - \bar{b}(t+1) = a_0 + a_1 t$. This formulation is flexible enough to allow for different adjustment paths. For instance, it allows for milder initial adjustments for $a_0 < 0$ and $a_1 > 0$, or faster initial adjustments for $a_0 > 0$ and $a_1 < 0$. Third, we assume that once the economy reaches period T, the debt limit becomes constant, i.e., $\bar{b}(t) = \bar{b}(\infty) \quad \forall t \geq T$. This enables us to introduce long-run discipline with only one parameter. We find the optimal sequence of debt limits by optimizing over $\left\{T, \bar{b}(\infty), a_1\right\}$.

The optimal sequence of debt limits features a transition of 8 years ($T = 32$), a final debt limit of 38.25%, with a slope $a_1 = -0.000235$ and $a_0 = 0.0223$. The top-left panel of Figure 6 presents the path of average debt, which in the case of the deleveraging process with debt limits almost coincide with the sequence of debt limits. This is because the debt limits are almost always binding in the simulations. Indeed, the top-right panel shows the debt volatility is close to zero during the deleveraging process. The top-left panel of Figure 6 shows that compared with the Ramsey deleveraging, the optimal sequence of debt limits imposes a much faster deleveraging. The Ramsey government imposes austerity in states where austerity is most effective at reducing default risk, while placing milder constraints in other states. A debt limit constrains borrowing in almost all states and thus imposes excessive austerity in states where it need not be necessary, driving the average debt down. The middle-left panel of Figure 6 shows that on average the deleveraging process with debt limits forces the government to transfers more net resources to lenders during the deleveraging period compared with the average net transfers chosen by the Ramsey government. The bottom-left panel of Figure 6 shows that the Ramsey government is more successful in reducing the spread than a Markov government with debt limits.

The inability to fine tune the states in which austerity is imposed leads to a lower welfare gain than with a Ramsey government. However, we find that simple constraints on future borrowing go a long way in terms of generating welfare gains. The optimal deleveraging with debt limits achieves a welfare gain of 0.26%, equivalent to 59% of the one achieved by a Ramsey government.

5.5 Robustness exercises

The inefficiency studied in the paper hinges on the government issuing long-term debt. While we assume an exogenous maturity structure, Arellano and Ramanarayanan (2012), Hatchondo et al. (2016), Dvorkin et al. (2020), and others show that with plausible calibrations of the government's incentives to hedge against changes in the borrowing cost, the Eaton-Gersovitz model can account for the debt maturities observed in the data. In Appendix G.1, we show that i) our main quantitative results do not change if we assume a debt duration equal to the optimal ex-ante value preferred by the Markov government (2.3 years vs. 4.8 years in our benchmark), and ii) once the Ramsey government corrects the time inconsistency in borrowing decisions, the optimal ex-ante debt duration is significantly higher than in our benchmark (more than 30 years).

The shock to the utility cost of defaulting enables us to exploit first-order conditions to solve for the Ramsey government's problem. The importance of the shock to the utility cost of defaulting is given by the volatility of U conditional on income (σ_U), which in our benchmark calibration is disciplined by the sovereign spread volatility.

We find that for the parameter values in Table 1, the shock to U plays a minor role in determining default decisions. For instance, the bottom-right panel of Figure 3 shows that for most income realizations, the government either repays or defaults almost surely regardless of the realization of U. This property is also confirmed in our simulations, where defaults are mostly driven by low income realizations. One can interpret shocks to U as encompassing default determinants that are not related to income. A higher σ_U would capture a higher role for those default determinants, which would make bond prices sensitive to borrowing even at high income realizations. Equation (11) indicates that in such scenarios, the Ramsey planner would also want to restrict borrowing at high income realizations. This is verified in Appendix G.2, where we show that higher values of σ_U moderate the additional procyclicality of fiscal policy chosen by the Ramsey government. Appendix G.2 also shows that the Ramsey government achieves significant welfare gains and significant reductions in sovereign default risk for different values of σ_U.

6 Conclusions

We solve a quantitative Eaton-Gersovitz sovereign default model with long-term debt in which a Ramsey government decides the entire borrowing plan taking as given ex-post optimal default decisions. The Ramsey government improves upon the Markov government because it takes into account how borrowing decisions in period t affect borrowing opportunities, and thus welfare, prior to t. Our contribution is to show that the effect of borrowing decisions in t on utility flows prior to t can be encapsulated by two single dimensional variables, and by one variable under conditions that we verify are satisfied in our quantitative exercise. This allows us to propose a tractable algorithm to find the constrained efficient borrowing policy. Relative to a Markov government, the Ramsey government distorts borrowing decisions more when i) bond prices are more sensitive to borrowing and ii) changes in bond prices have a more significant effect on past utility flows. For empirically plausible parameter values, more than 80% of the default risk is eliminated by a Ramsey government, without lowering average borrowing levels. An efficient reduction in default risk prescribes a higher volatility in borrowing and debt levels. The welfare gain of having a Ramsey government instead of a Markov government ranges from 0.3% to 0.7%, depending on initial income. The Ramsey government carries a more procyclical fiscal policy that includes debt buybacks in some states with low income. This higher procyclicality of fiscal policy is mitigated in economies where default determinants are less correlated with income.

Starting from a state with high debt, the Ramsey government has a higher probability of completing a successful deleveraging (without defaults) than the Markov government. In addition, compared with the Markov government, by effectively reducing future default risk, the Ramsey government can afford to smooth out the initial adjustment during the deleveraging path. The Markov government's commitment to a simple deleveraging plan consisting of a sequence of debt limits imposes harsher initial austerity and delivers 60% of the welfare gains achieved by the Ramsey government. These results are indicative of the quantitative importance of enhancing long-term fiscal discipline for the success of debt reduction programs.

Our methodology could be extended to study other aspects of debt management in which time inconsistency plays a role. Similar inefficiencies arise when governments issue debt in local and foreign currency, when they decide fiscal and monetary policy, when they issue debt with different maturities, and when they accumulate debt and assets (Bianchi et al., 2018). Extending our analysis to study and quantify these issues and inform the on the role of fiscal rules to regulate other debt management aspects is an interesting avenue for future research.

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Online Appendix

A Definition of the Markov Perfect Equilibrium

Let x' denote the next-period value of variable x, $\hat{b}$ denote the borrowing strategy followed by future governments (number of bonds outstanding at the end of the period), $\hat{d}$ denote the defaulting strategy followed by future governments, and V denote the continuation value under repayment when future governments follow the strategies $\hat{b}$ and $\hat{d}$. The function V is given by

$$V(b,y) = u(c) + \beta \sum_{j=1}^{J} \pi_j(y) \left\{ \int_{U'} \left[d'_j U' + (1-d'_j) V(b', y_j) \right] f(U') dU' \right\} \tag{19}$$

$$\begin{aligned} \text{where } & c = y - \delta b + q(b', y')[b' - (1-\delta)b], \\ & d'_j = \hat{d}(b', y_j, U'), \text{ and} \\ & b' = \hat{b}(b, y). \end{aligned}$$

In states where the government acting in the next period defaults ($\hat{d}(b', y_j, U') = 1$), it receives the continuation value U'.

Competition in financial markets between risk-neutral lenders implies a bond price function where lenders break even in expectation:

$$q(b', y) = \frac{\sum_{j=1}^{J} \pi_j(y) \left\{ \int_{U'} [1 - \hat{d}(b', y_j, U')] \left[\delta + (1-\delta) q(b''_j, y_j) \right] f(U') dU' \right\}}{1 + r}. \tag{20}$$

In states where the next-period government repays, bondholders receive the coupon payment δ and can trade the claims to subsequent coupon payments (that add up to $1-\delta$) at prices $q(b''_j, y_j)$, which depend on the next-period income realization $y' = y_j$ and next-period borrowing decisions $b''_j = \hat{b}(b', y_j)$.

Let $\mathcal{V}$ denote the maximum welfare a government can attain in the current period if it chooses to repay and expects future governments to follow strategies $\hat{b}$ and $\hat{d}$. Formally,

$$\mathcal{V}(b,y) = \underset{b'}{Max}\left\{u(c) + \beta \sum_{j=1}^{J} \pi_j(y) \left\{\int_{U'} \left[d'_j U' + (1-d'_j) V(b', y_j)\right] f(U') dU'\right\}\right\} \tag{21}$$

$$\text{s.t. } c = y - \delta b + q(b', y')[b' - (1-\delta)b], \text{ and}$$

$$d'_j = \hat{d}(b', y_j, U').$$

The problem above illustrates that the current government has a limited ability to affect future debt levels.

At the beginning of the period the government observes its continuation value under defaulting U and chooses its defaulting decision according to

$$d = \underset{d \in \{0,1\}}{Max} \left\{dU + (1-d)\mathcal{V}(b,y)\right\}. \tag{22}$$

A Markov Perfect Equilibrium is characterized by strategies $\hat{d}$ and $\hat{b}$ such that

(a) Given $\hat{d}$ and $\hat{b}$, the value function under repayment V satisfies the functional equation (19).

(b) Given $\hat{d}$ and $\hat{b}$, the bond price function q satisfies the functional equation (20).

(c) The function $\hat{d}$ solves problem (22) for all b, y, U.

(d) The function $\hat{b}$ solves problem (21) for all b, y.

B Ramsey government's choice set

Let $V_t(\cdot, b_{t+1}(y^t), \cdot)$ denote the repayment value for the Ramsey government in period t after an income history y^t. The assumption that the Ramsey government cannot condition its borrowing in t on U_t implies that V_t does not depend on U_t. Thus, the optimal repayment rule for the Ramsey government in t after income history y^t is to repay whenever $U_t \in (-\infty, U_t^*]$.

If, instead, the Ramsey government could condition its borrowing in t on U_t, it may find it optimal to do it as long as there is room to increase V_t. This is because by expanding the repayment set in t (after history y^t) the Ramsey government can increase the repayment

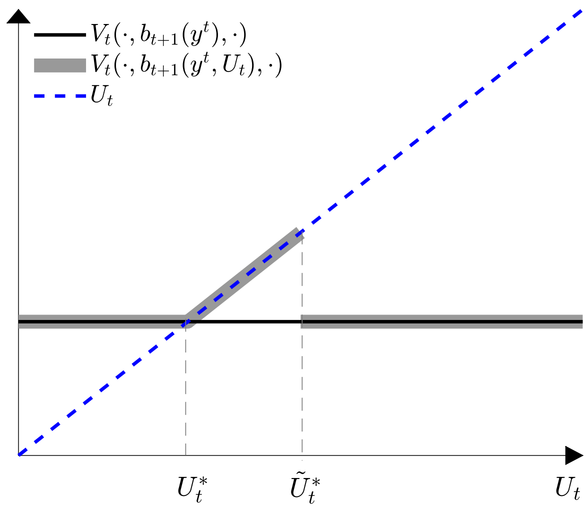

Figure 7: When the Ramsey government cannot condition the choice of b_{t+1} after history y^t on the realization of U_t, it repays when $U_t \in (-\infty, U_t^*]$. When the Ramsey government can condition b_{t+1} on U_t, it can expand the repayment region to $U_t \in (-\infty, \tilde{U}_t^*]$ by only distorting b_{t+1} over $[U_t^*, \tilde{U}_t^*]$.

probability in t, and therefore, bond prices prior to t. Figure 7 illustrates how distorting b_{t+1} for $U_t \in [U_t^*, \tilde{U}_t^*]$ could expand the repayment set. This can be achieved by a borrowing rule that increases V_t just enough to leave the government indifferent between repaying and defaulting over $[U_t^*, \tilde{U}_t^*]$ (we assume the government repays if indifferent).[20] Incorporating this possibility would significantly complicate the analysis and we rule it out.

C Proofs

C.1 Proof of proposition 1

Proof. Consider a particular history $\tilde{y}^t$. Changes in $b_{t+1}(\tilde{y}^t)$ only have effects in periods $s \leq t+1$. Given that the debt path after $t+1$ is not affected, neither is the consumption path after period $t+1$ and therefore the path of repayment probabilities after $t+1$.

Step 1: Differentiability in $t+1$.

If the economy starts with more debt in period $t+1$ and there is no change in the subsequent debt path, the only effect on V_{t+1} is due to the decrease in c_{t+1}. The following first-order derivative

$$\frac{\partial V_{t+1}(\vec{b}, (\tilde{y}^t, y_j))}{\partial b_{t+1}(\tilde{y}^t)} = -u'\left(c_{t+1}(\vec{b}, (\tilde{y}^t, y_j))\right)\left[\delta + (1-\delta) q_{t+1}(\vec{b}, (\tilde{y}^t, y_j)\right],$$

[20]More borrowing in t would also reduce the bond price in t. This deviation would be beneficial if the increase in the repayment set in t more than compensates the fall in q_t. Also, note that the borrowing rule for $U_t > \tilde{U}^*$ is not payoff relevant as long as it yields $V_t \leq \tilde{U}_t^*$.

exists and is continuous w.r.t. $b_{t+1}(\tilde{y}^t)$ because u' is continuous, and $c_{t+1}(\vec{b},(\tilde{y}^t,y_j))$ is continuous in $b_{t+1}(\tilde{y}^t)$. Moreover, the repayment probability in $t+1$, $F_j\left(V_{t+1}(\vec{b},(\tilde{y}^t,y_j))\right)$ is also continuously differentiable w.r.t. $b_{t+1}(\tilde{y}^t)$ because F_j is continuously differentiable.

Step 2: Differentiability of effects in t.

A higher amount of debt in $t+1$ affects the repayment probability in $t+1$ and thus

$$\frac{\partial q_t(\vec{b},\tilde{y}^t)}{\partial b_{t+1}(\tilde{y}^t)} = \frac{\sum_{j=1}^{J} \pi_j(\tilde{y}) f_j\left(V_{t+1}\left(\vec{b},(\tilde{y}^t,y_j)\right)\right) \frac{\partial V_{t+1}(\vec{b},(\tilde{y}^t,y_j))}{\partial b_{t+1}(\tilde{y}^t)} \left[\delta + (1-\delta) q_{t+1}\left(\vec{b},(\tilde{y}^t,y_j)\right)\right]}{1+r}$$

is continuously differentiable w.r.t. $b_{t+1}(\tilde{y}^t)$ because of *Step 1.* As a consequence,

$$\begin{aligned}\frac{\partial V_t(\vec{b},\tilde{y}^t)}{\partial b_{t+1}(\tilde{y}^t)} =& u'(c_t(\vec{b},\tilde{y}^t))\left[q_t(\vec{b},\tilde{y}^t) + \frac{\partial q_t(\vec{b},\tilde{y}^t)}{\partial b_{t+1}(\tilde{y}^t)}\left(b_{t+1}(\tilde{y}^{t-1}) - (1-\delta)b_t(\tilde{y}^t)\right)\right] \\ &+ \beta \sum_{j=1}^{J} \pi_j(\tilde{y}) F_j\left(V_{t+1}\left(\vec{b},(\tilde{y}^t,y_j)\right)\right) \frac{\partial V_{t+1}(\vec{b},(\tilde{y}^t,y_j))}{\partial b_{t+1}(\tilde{y}^t)}\end{aligned}$$

is also continuously differentiable w.r.t. $b_{t+1}(\tilde{y}^t)$ because u, q_t and V_{t+1} are continuously differentiable w.r.t. $b_{t+1}(\tilde{y}^t)$.

Step 3: Even though there is no change in the number of bonds issued in period $t-1$, the price at which those bonds are traded can change because the repayment probability and bond price in t for the income realization $y_t = \tilde{y}_t$ may change. The derivative

$$\frac{\partial q_{t-1}(\vec{b},\tilde{y}^{t-1})}{\partial b_{t+1}(\tilde{y}^t)} = \frac{Pr(\tilde{y}_t \mid \tilde{y}_{t-1})}{1+r}\left[\begin{array}{l} f_{I(t,\tilde{y}^t)}\left(V_t\left(\vec{b},\tilde{y}^t\right)\right) \frac{\partial V_t(\vec{b},\tilde{y}^t)}{\partial b_{t+1}(\tilde{y}^t)} \left[\delta + (1-\delta) q_t\left(\vec{b},\tilde{y}^t\right)\right] \\ +F_{I(t,\tilde{y}^t)}\left(V_t\left(\vec{b},\tilde{y}^t\right)\right)(1-\delta)\frac{\partial q_t(\vec{b},\tilde{y}^t)}{\partial b_{t+1}(\tilde{y}^t)} \end{array}\right]$$

exists and is continuous w.r.t. $b_{t+1}(\tilde{y}^t)$ because q_t and V_t are continuously differentiable w.r.t. $b_{t+1}(\tilde{y}^t)$. The term $I(t,\tilde{y}^t)$ denotes the income index realization in t for income history $\tilde{y}^t$. The derivative

$$\begin{aligned}\frac{\partial V_{t-1}(\vec{b},\tilde{y}^{t-1})}{\partial b_{t+1}(\tilde{y}^t)} =& u'(c_{t-1}(\vec{b},(\tilde{y}^{t-1})))\frac{\partial q_{t-1}(\vec{b},\tilde{y}^t)}{\partial b_{t+1}(\tilde{y}^t)}\left[b_t(\tilde{y}^{t-1}) - (1-\delta)b_{t-1}(\tilde{y}^{t-2})\right] \\ &+ \beta Pr(\tilde{y}_t \mid \tilde{y}_{t-1}) F_{I(t,\tilde{y}^t)}\left(V_t\left(\vec{b},\tilde{y}^t\right)\right)\frac{\partial V_t(\vec{b},(\tilde{y}^t,y_j))}{\partial b_{t+1}(\tilde{y}^t)}\end{aligned}$$

exists and is continuous because u, q_{t-1}, and V_t are continuously differentiable.

Step 4: The same logic applied in *Step 3* extends to periods $s = t-2, t-3, ..., 0$. □

C.2 Proof of Proposition 2

Proof. The proof is structured in four steps. Firstly, we show that the derivative of the Ramsey government's objective with respect to $b_{t+1}(\tilde{y}^t)$ can be expressed as a function of the derivatives of the repayment value in t and bond prices up to t. Secondly, we show that the derivatives of bond prices up to t can be written as functions of $\partial q_t/\partial b_{t+1}(\tilde{y}^t)$ and $\partial V_t/\partial b_{t+1}(\tilde{y}^t)$. Thirdly, we show how the coefficients that weight the current derivatives of the bond price and the expected utility under repayment follow a recursive structure. Finally, we combine the previous steps to establish the recursive structure of the necessary condition for the optimum.

Notation. To simplify notation, we drop the arguments $(\vec{b}, \tilde{y}^s)$ for variables evaluated in period $s \le t$ after income-history $\tilde{y}^s$, we use $I(s)$ to denote the income realization index in s, $I(s, \tilde{y}^t)$ for $s \le t$, and we use u_s' to denote $u'(c_s(\vec{b}, \tilde{y}^s))$. We use $V_{t+1,j}$, $q_{t+1,j}$ to denote $V_{t+1}(\vec{b}, (\tilde{y}^t, y_j))$, and $q_{t+1}(\vec{b}, (\tilde{y}^t, y_j))$, respectively.

First, note that the derivative of the Ramsey government's objective $\mathcal{U}$ w.r.t. $b_{t+1}(\tilde{y}^t)$, can be written as:

$$\frac{\partial \mathcal{U}(\vec{b}, y_0)}{\partial b_{t+1}(\tilde{y}^t)} = \frac{\partial V_0}{\partial b_{t+1}(\tilde{y}^t)} = \frac{\partial \left[u(c_0) + \beta Pr(\tilde{y}_1 \mid y_0)\left[F_{I(1)}(V_1)\, V_1 + \int_{V_1}^{\infty} U f_{I(1)}(U) dU\right]\right]}{\partial b_{t+1}(\tilde{y}^t)},$$

where we assume it is optimal to repay in the initial period. Since the continuation values under repayment and default are identical at $V_1(\vec{b}, \tilde{y}^1)$, changing the repayment probability $F_{I(1)}\left(V_1(\vec{b}, \tilde{y}^1)\right)$ does not affect the expected continuation value. Therefore,

$$\begin{aligned}
\frac{\partial \mathcal{U}(\vec{b}, y_0)}{\partial b_{t+1}(\tilde{y}^t)} &= u_0' \iota_0 \frac{\partial q_0}{\partial b_{t+1}(\tilde{y}^t)} + \beta Pr(\tilde{y}_1 \mid y_0) F_{I(1)}(V_1) \frac{\partial V_1}{\partial b_{t+1}(\tilde{y}^t)} \qquad (23)\\
&= u_0' \iota_0 \frac{\partial q_0}{\partial b_{t+1}(\tilde{y}^t)} + \beta Pr(\tilde{y}_1 \mid y_0) F_{I(1)}(V_1) \\
&\quad \times \left[u_1' \iota_1 \frac{\partial q_1}{\partial b_{t+1}(\tilde{y}^t)} + \beta Pr(\tilde{y}_2 \mid \tilde{y}_1) F_{I(2)}(V_2) \frac{\partial V_2}{\partial b_{t+1}(\tilde{y}^t)}\right],
\end{aligned}$$

where the second line substitutes $\partial V_1(\vec{b}, \tilde{y}^1)/\partial b_{t+1}(\tilde{y}^t)$. If we continue substituting away the derivatives $\frac{\partial V_{t-n}(\vec{b}, \tilde{y}^{t-n})}{\partial b_{t+1}(\tilde{y}^t)}$ for all $0 < n < t$, we obtain

$$\begin{aligned}
\frac{\partial \mathcal{U}(\vec{b}, y_0)}{\partial b_{t+1}(\tilde{y}^t)} = \sum_{n=1}^{t} \beta^{t-n} Pr(\tilde{y}^{t-n} \mid y_0) \prod_{m=1}^{t-n} F_{I(m)}(V_m)\, u_{t-n}' \iota_{t-n} \frac{\partial q_{t-n}}{\partial b_{t+1}} \\
+ \beta^t Pr(\tilde{y}^t \mid y_0) \prod_{m=1}^{t} F_{I(m)}(V_m) \frac{\partial V_t}{\partial b_{t+1}}, \qquad (24)
\end{aligned}$$

where sure repayment in the initial period implies $F(V_0) = 1$.

The following lemma states that the effect of bond issuances in t on bond prices in periods $t-n$ $(\frac{\partial q_{t-n}}{\partial b_{t+1}(\tilde{y}^t)})$ can be written as a function of the effect of bond issuances on the bond price in the issuance period $(\frac{\partial q_t}{\partial b_{t+1}(\tilde{y}^t)})$, and of the effect of bond issuances on the expected utility in the issuance period $(\frac{\partial V_t}{\partial b_{t+1}(\tilde{y}^t)})$. The latter affects the probability of a default in that period, which is not captured in the bond price for that period, but affects bond prices in previous periods.

Lemma 4. *The derivative of the bond price in $t-n$ w.r.t. $b_{t+1}(\tilde{y}^t)$ satisfies*

$$\frac{\partial q_{t-n}}{\partial b_{t+1}(\tilde{y}^t)} = Pr(\tilde{y}_{t-n+1}, ...\tilde{y}_t \mid \tilde{y}_{t-n}) \left[A^q_{t,t-n} \frac{\partial q_t}{\partial b_{t+1}(\tilde{y}^t)} + B^q_{t,t-n} \frac{\partial V_t}{\partial b_{t+1}(\tilde{y}^t)} \right]. \tag{25}$$

Proof. First, consider $n = 1$. The derivative

$$\frac{\partial q_{t-1}}{\partial b_{t+1}(\tilde{y}^t)} = \frac{Pr\left(\tilde{y}_t \mid \tilde{y}_{t-1}\right) \left[f_{I(t)}(V_t) \left[\delta + (1-\delta) q_t\right] \frac{\partial V_t}{\partial b_{t+1}(\tilde{y}^t)} + F_{I(t)}(V_t)(1-\delta) \frac{\partial q_t}{\partial b_{t+1}(\tilde{y}^t)} \right]}{1+r}, \tag{26}$$

indicating the bond price in $t-1$ may change because there is a change in the repayment probability in $t+1$ (captured by $\partial q_t / \partial b_{t+1}(\tilde{y})$) and/or because there is a change in the repayment probability in t (captured by $\partial V_t / \partial b_{t+1}(\tilde{y})$). Equation (26) implies equation (25) for

$$A^q_{t,t-1} = \frac{F_{I(t)}(V_t)(1-\delta)}{1+r} \quad \text{and} \quad B^q_{t,t-1} = \frac{f_{I(t)}(V_t)\left[\delta + (1-\delta) q_t\right]}{1+r}.$$

Second, consider $n = 2$. The derivative

$$\begin{aligned} \frac{\partial q_{t-2}}{\partial b_{t+1}(\tilde{y}^t)} = \frac{Pr\left(\tilde{y}_{t-1} \mid \tilde{y}_{t-2}\right)}{1+r} \Bigg[& f_{I(t-1)}(V_{t-1}) \frac{\partial V_{t-1}}{\partial b_{t+1}(\tilde{y}^t)} \left[\delta + (1-\delta) q_{t-1}\right] \\ & + F_{I(t-1)}(V_{t-1})(1-\delta) \frac{\partial q_{t-1}}{\partial b_{t+1}(\tilde{y}^t)} \Bigg], \end{aligned}$$

Using

$$\frac{\partial V_{t-1}}{\partial b_{t+1}(\tilde{y}^t)} = u'_{t-1} \iota_{t-1} \frac{\partial q_{t-1}}{\partial b_{t+1}(\tilde{y}^t)} + \beta Pr\left(\tilde{y}_t \mid \tilde{y}_{t-1}\right) F_{I(t)}(V_t) \frac{\partial V_t}{\partial b_{t+1}(\tilde{y}^t)}$$

and equation (26), $\partial q_{t-2} / \partial b_{t+1}(\tilde{y}^t)$ can be written as

$$\begin{aligned} \frac{\partial q_{t-2}}{\partial b_{t+1}(\tilde{y}^t)} &= \frac{Pr(\tilde{y}_{t-1}, \tilde{y}_t | \tilde{y}_{t-2})}{1+r} f_{I(t-1)}(V_{t-1}) \left[\delta + (1-\delta) q_{t-1}\right] \beta F_{I(t)}(V_t) \frac{\partial V_t}{\partial b_{t+1}(\tilde{y}^t)} \\ &+ \frac{Pr(\tilde{y}_{t-1}, \tilde{y}_t | \tilde{y}_{t-2})}{1+r} \left[f_{I(t-1)}(V_{t-1}) \left[\delta + (1-\delta) q_{t-1}\right] u'_{t-1} \iota_{t-1} + F_{I(t-1)}(V_{t-1})(1-\delta) \right] \\ &\times \left[A^q_{t,t-1} \frac{\partial q_t}{\partial b_{t+1}(\tilde{y}^t)} + B^q_{t,t-1} \frac{\partial V_t}{\partial b_{t+1}(\tilde{y}^t)} \right], \end{aligned}$$

implying that it can be recasted as in equation (25) for

$$A^q_{t,t-2} = \frac{\left[f_{I(t-1)}(V_{t-1}) \left[\delta + (1-\delta)q_{t-1}\right] u'\left(c_{t-1}\right) \iota_{t-1} + F_{I(t-1)}(V_{t-1})(1-\delta) \right]}{1+r} A^q_{t,t-1}$$

and

$$B^q_{t,t-2} = \frac{\left\{ \begin{array}{l} f_{I(t-1)}(V_{t-1}) \left[\delta + (1-\delta)q_{t-1}\right] \beta F_{I(t)}(V_t) + \\ \left[f_{I(t-1)}(V_{t-1}) \left[\delta + (1-\delta)q_{t-1}\right] u'\left(c_{t-1}\right) \iota_{t-1} F_{I(t-1)}(V_{t-1})(1-\delta) \right] B^q_{t,t-1} \end{array} \right\}}{1+r}.$$

For any other $n > 2$, we can recast $\partial q_{t-n}/\partial b_{t+1}(\tilde{y}^t)$ as in (25) after iterating on the bond price and repayment value functions as above. □

The next lemma shows how the coefficients that determine the effect of debt issuances in $t+1$ on the bond price in period $t-n$, $A^q_{t+1,t-n}$ and $B^q_{t+1,t-n}$, can be written as functions of $A^q_{t,t-n}$ and $B^q_{t,t-n}$.

Lemma 5. *Consider an income-history $\tilde{y}^{t-1}$ and continuation history $\tilde{y}^t = (\tilde{y}^{t-1}, y_i)$ with $y_i \in \mathcal{Y}$. For any $n > 1$, the coefficients are*

$$A^q_{t,t-n} = \left(A^q_{t-1,t-n} + B^q_{t-1,t-n} u'_{t-1} \iota_{t-1}\right) \frac{(1-\delta)F_i(V_t)}{1+r}, \tag{27}$$

$$B^q_{t,t-n} = \left(A^q_{t-1,t-n} + B^q_{t-1,t-n} u'_{t-1} \iota_{t-1}\right) \frac{f_i(V_t)\left[\delta + (1-\delta)q_t\right]}{1+r} + B^q_{t-1,t-n} \beta F_i(V_t). \tag{28}$$

For $n = 1$, the coefficients are

$$A^q_{t,t-1} = \frac{(1-\delta)F_i(V_t)}{1+r}, \tag{29}$$

$$B^q_{t,t-1} = \frac{f_i(V_t)\left[\delta + (1-\delta)q_t\right]}{1+r}. \tag{30}$$

Proof. Changes in borrowing in t affect q_{t-1} and V_{t-1}. Applying Lemma 4 to express the effects of borrowing in t on q_{t-n} through changes in q_{t-1} and V_{t-1}, implies

$$\frac{\partial q_{t-n}}{\partial b_{t+1}(\tilde{y}^t)} = Pr(\tilde{y}_{t-n+1}, ...\tilde{y}_t \mid \tilde{y}_{t-n}) \left[A^q_{t-1,t-n} \frac{\partial q_{t-1}}{\partial b_{t+1}(\tilde{y}^t)} + B^q_{t-1,t-n} \frac{\partial V_{t-1}}{\partial b_{t+1}(\tilde{y}^t)} \right]. \tag{31}$$

After using $\frac{\partial V_{t-1}}{\partial b_{t+1}(\tilde{y}^t)} = u'_{t-1} \iota_{t-1} \frac{\partial q_{t-1}}{\partial b_{t+1}(\tilde{y}^t)} + \beta Pr(y_i \mid \tilde{y}_{t-1}) F_i(V_t) \frac{\partial V_t}{\partial b_{t+1}(\tilde{y}^t)}$

to substitute $\frac{\partial V_{t-1}}{\partial b_{t+1}(\tilde{y}^t)}$ into equation (31), we obtain

$$\begin{aligned} \frac{\partial q_{t-n}}{\partial b_{t+1}(\tilde{y}^t)} = {} & Pr(\tilde{y}_{t-n+1}, ..., \tilde{y}_{t-1} \mid \tilde{y}_{t-n}) \left[A^q_{t-1,t-n} \frac{\partial q_{t-1}}{\partial b_{t+1}(\tilde{y}^t)} \right. \\ & \left. + B^q_{t-1,t-n} \left[u'_{t-1} \iota_{t-1} \frac{\partial q_{t-1}}{\partial b_{t+1}(\tilde{y}^t)} + \beta Pr(y_i \mid \tilde{y}_{t-1}) F_i(V_t) \frac{\partial V_t}{\partial b_{t+1}(\tilde{y}^t)} \right] \right], \end{aligned}$$

$$\begin{aligned} \frac{\partial q_{t-n}}{\partial b_{t+1}(\tilde{y}^t)} = {} & Pr(\tilde{y}_{t-n+1}, ..., \tilde{y}_{t-1} \mid \tilde{y}_{t-n}) \left[\left(A^q_{t-1,t-n} + B^q_{t-1,t-n} u'_{t-1} \iota_{t-1} \right) \frac{\partial q_{t-1}}{\partial b_{t+1}(\tilde{y}^t)} \right. \\ & \left. + B^q_{t-1,t-n} \beta Pr(y_i \mid \tilde{y}_t) F_i(V_t) \frac{\partial V_t}{\partial b_{t+1}(\tilde{y}^t)} \right]. \end{aligned} \tag{32}$$

Recall that

$$\frac{\partial q_{t-1}}{\partial b_{t+1}(\tilde{y}^t)} = \frac{Pr(y_i \mid \tilde{y}_{t-1})}{1+r} \left[f_i(V_t) \left[\delta + (1-\delta) q_t \right] \frac{\partial V_t}{\partial b_{t+1}(\tilde{y}^t)} + (1-\delta) F_i(V_t) \frac{\partial q_t}{\partial b_{t+1}(\tilde{y}^t)} \right].$$

After substituting the above equation into (32), we can recast $\frac{\partial q_{t-n}}{\partial b_{t+1}(\tilde{y}^t)}$ as a function of $\frac{\partial V_t}{\partial b_{t+1}(\tilde{y}^t)}$ and $\frac{\partial q_t}{\partial b_{t+1}(\tilde{y}^t)}$:

$$\begin{aligned} \frac{\partial q_{t-n}}{\partial b_{t+1}(\tilde{y}^t)} & = Pr(\tilde{y}_{t-n+1}, ..., \tilde{y}_{t-1} \mid \tilde{y}_{t-n}) \left[\left(A^q_{t-1,t-n} + B^q_{t-1,t-n} u'_{t-1} \iota_{t-1} \right) \tfrac{Pr(y_i \mid \tilde{y}_{t-1})}{1+r} \right. \\ & \times \left[f_i(V_t) \left[\delta + (1-\delta) q_t \right] \tfrac{\partial V_t}{\partial b_{t+1}(\tilde{y}^t)} + (1-\delta) F_i(V_t) \tfrac{\partial q_t}{\partial b_{t+1}(\tilde{y}^t)} \right] \\ & \left. + B^q_{t-1,t-n} \beta Pr(y_i \mid \tilde{y}_t) F_i(V_t) \tfrac{\partial V_t}{\partial b_{t+1}(\tilde{y}^t)} \right]. \end{aligned}$$

Since $Pr(\tilde{y}_{t-n+1},...,\tilde{y}_{t-1},y_i \mid \tilde{y}_{t-n}) = Pr(\tilde{y}_{t-n+1},...,\tilde{y}_{t-1} \mid \tilde{y}_{t-n}) Pr(y_i \mid \tilde{y}_{t-1})$,

$$\frac{\partial q_{t-n}}{\partial b_{t+1}(\tilde{y}^t)} = Pr(\tilde{y}_{t-n+1},...,y_i \mid \tilde{y}_{t-n}) \left\{ \left(A^q_{t-1,t-n} + B^q_{t-1,t-n} u'_{t-1}\iota_{t-1} \right) \frac{(1-\delta)F_i(V_t)}{1+r} \frac{\partial q_t}{\partial b_{t+1}(\tilde{y}^t)} \right.$$
$$\left. + \left[\left(A^q_{t-1,t-n} + B^q_{t-1,t-n} u'_{t-1}\iota_{t-1} \right) \frac{f_i(V_t)(\delta+(1-\delta)q_t)}{1+r} + B^q_{t-1,t-n}\beta F_i(V_t) \right] \frac{\partial V_t}{\partial b_{t+1}(\tilde{y}^t)} \right\}. \tag{33}$$

Equations (25) and (33) imply equations (27) and (28). Equations (29) and (30) follow from (27), (28), $A^q_{t,t} = 1$, and $B^q_{t,t} = 0$. $\square$

Necessary condition for optimum and law of motions for h^q and h^V. If we use equation (25) to substitute the derivatives $\partial q_{t-n}/\partial b_{t+1}(y^t)$ in the derivative (24), we can recast the derivative of the Ramsey government's objective as a function of the effect of bond issuances on the bond price in the issuance period ($\frac{\partial q_t}{\partial b_{t+1}(y^t)}$), and of the effect of bond issuances on the repayment value in the issuance period ($\frac{\partial V_t}{\partial b_{t+1}(y^t)}$). Using the expression for $\partial \mathcal{U}/\partial b_{t+1}(y^t)$ in (24) and the previous lemma,

$$\frac{\partial \mathcal{U}(\vec{b}, y_0)}{\partial b_{t+1}(\tilde{y}^t)} = Pr(\tilde{y}^t \mid y_0) \left[h^q_t \frac{\partial q_t}{\partial b_{t+1}(\tilde{y}^t)} + h^V_t \frac{\partial V_t}{\partial b_{t+1}(\tilde{y}^t)} \right] \tag{34}$$

for all t and $y^t \in \mathcal{Y}^t$, with

$$h^q_t = \sum_{n=1}^{t} \beta^{t-n} \prod_{m=1}^{t-n} F_{I(m)}(V_m)\, u'_{t-n}\iota_{t-n} A^q_{t,t-n}, \text{ and} \tag{35}$$
$$h^V_t = \sum_{n=1}^{t} \beta^{t-n} \prod_{m=1}^{t-n} F_{I(m)}(V_m) u'_{t-n}\iota_{t-n} B^q_{t,t-n} + \beta^t \prod_{m=1}^{t} F_{I(m)}(V_m). \tag{36}$$

The next lemma shows the welfare weights h^q_t, h^V_t can be written as functions of the weights h^q_{t-1} and h^V_{t-1} in period $t-1$.

Lemma 6. *Given an income history up to period t $\tilde{y}^t \in \mathcal{Y}^t$ with $y^t = y_i$*

$$h^q_t = \frac{(1-\delta)F_i(V_t)}{1+r} \left(h^q_{t-1} + u'_{t-1}\iota_{t-1} h^V_{t-1} \right), \tag{37}$$
$$h^V_t = \frac{f_i(V_t)\left[\delta + (1-\delta)q_t\right]}{1+r} \left(h^q_{t-1} + u'_t\iota_t h^V_{t-1} \right) + \beta F_i(V_t)\, h^V_{t-1}. \tag{38}$$

Proof. Using lemma 5 to substitute $A^q_{t,t-n}$ by $A^q_{t-1,t-n}$ and $B^q_{t-1,t-n}$ in equation (35), we obtain

$$
\begin{aligned}
h_t^q &= \sum_{n=1}^{t-1} \beta^{t-1-n} \prod_{m=1}^{t-1-n} F_{I(m)}(V_m) u'_{t-1-n} \iota_{t-1-n} \times \overbrace{A_{t,t-1-n}^q}^{\left(A_{t-1,t-1-n}^q + B_{t-1,t-1-n}^q u'_{t-1} \iota_{t-1}\right) \frac{(1-\delta)F_i(V_t)}{1+r}} \\
&+ \beta^{t-1} \prod_{m=1}^{t-1} F_{I(m)}(V_m) u'_{t-1} \iota_{t-1} \underbrace{A_{t,t-1}^q}_{\frac{(1-\delta)F_i(V_t)}{1+r}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{(1-\delta)F_i(V_t)}{1+r} \left[\sum_{n=1}^{t-1} \beta^{t-1-n} \prod_{m=1}^{t-1-n} F_{I(m)}(V_m) u'_{t-1-n} \iota_{t-1-n} A_{t-1,t-1-n}^q + \right. \\
&\left. u'_{t-1} \iota_{t-1} \left[\sum_{n=1}^{t-1} \beta^{t-1-n} \prod_{m=1}^{t-1-n} F_{I(m)}(V_m) u'_{t-1-n} \iota_{t-1-n} B_{t-1,t-1-n}^q + \beta^{t-1} \prod_{m=1}^{t-1} F_{I(m)}(V_m) \right] \right] \\
&= \frac{(1-\delta)F_i(V_t)}{1+r} \left[h_{t-1}^q + u'_{t-1} \iota_{t-1} h_{t-1}^V \right].
\end{aligned}
$$

Likewise, using lemma 5 to substitute $B_{t,t-n}^q$ by $A_{t-1,t-n}^q$ and $B_{t-1,t-n}^q$ in equation (36), we obtain

$$
\begin{aligned}
h_t^V &= \sum_{n=1}^{t} \beta^{t-n} \prod_{m=1}^{t-n} F_{I(m)}(V_m) u'_{t-n} \iota_{t-n} \left[A^q_{t-1,t-n} \frac{f_i(V_t)\left[\delta + (1-\delta)q_t\right]}{1+r} \right. \\
&\left. + B^q_{t-1,t-n} \left[u'_{t-1}\iota_{t-1} \frac{f_i(V_t)\left[\delta + (1-\delta)q_t\right]}{1+r} + \beta F_i(V_t) \right] \right] + \beta^t \prod_{m=1}^{t} F_{I(m)}(V_m) \\
&= \sum_{n=1}^{t-1} \beta^{t-1-n} \prod_{m=1}^{t-1-n} F_{I(m)}(V_m) u'_{t-1-n} \iota_{t-1-n} \left[A^q_{t-1,t-1-n} \frac{f_i(V_t)\left[\delta + (1-\delta)q_t\right]}{1+r} \right. \\
&\left. + B^q_{t-1,t-1-n} \left[u'_{t-1}\iota_{t-1} \frac{f_i(V_t)\left[\delta + (1-\delta)q_t\right]}{1+r} + \beta F_i(V_t) \right] \right] \\
&+ \beta^{t-1} \prod_{m=1}^{t-1} F_{I(m)}(V_m) u'_{t-1}\iota_{t-1} \frac{f_i(V_t)\left[\delta + (1-\delta)q_t\right]}{1+r} + \beta^t \prod_{m=1}^{t} F_{I(m)}(V_m) \\
&= \frac{f_i(V_t)\left[\delta + (1-\delta)q_t\right]}{1+r} \left[\sum_{n=1}^{t-1} \beta^{t-1-n} \prod_{m=1}^{t-n} F_{I(m)}(V_m) u'_{t-1-n}\iota_{t-1-n} A^q_{t-1,t-1-n} + \right. \\
&\left. u'_{t-1}\iota_{t-1} \left[\sum_{n=1}^{t-1} \beta^{t-1-n} \prod_{m=1}^{t-1-n} F_{I(m)}(V_m) u'_{t-1-n}\iota_{t-1-n} B^q_{t-1,t-1-n} + \beta^{t-1} \prod_{m=1}^{t-1} F_{I(m)}(V_m) \right] \right] \\
&+ \beta F_i(V_t) \left[\sum_{n=1}^{t-1} \beta^{t-1-n} \prod_{m=1}^{t-1-n} F_{I(m)}(V_m) u'_{t-1-n}\iota_{t-1-n} B^q_{t-1,t-1-n} + \beta^{t-1} \prod_{m=1}^{t-1} F_{I(m)}(V_m) \right] \\
&= \frac{f_i(V_t)\left[\delta + (1-\delta)q_t\right]}{1+r} \left(h^q_{t-1} + u'_{t-1}\iota_{t-1} h^V_{t-1} \right) + \beta F_i(V_t) h^V_{t-1},
\end{aligned}
$$

where the second equality uses $A^q_{t-1,t-1} = 1$ and $B^q_{t-1,t-1} = 0$. $\square$

Since $Pr(y^t \mid y_0) > 0$ for all $y^t \in \mathcal{Y}^t$, the borrowing plan $\vec{b}^*$ that solves the Ramsey government's problem must satisfy

$$
h_t^q(\vec{b}^*, y^t) \frac{\partial q_t(\vec{b}^*, y^t)}{\partial b_{t+1}(y^t)} + h_t^V(\vec{b}^*, y^t) \frac{\partial V_t(\vec{b}^*, y^t)}{\partial b_{t+1}(y^t)} = 0 \tag{39}
$$

for all t and $y^t \in \mathcal{Y}^t$. $\square$

C.3 Proof of Lemma 3

Proof. It follows from (14), the optimality condition (11), and

$$\begin{aligned}\frac{\partial V_t(\vec{b}, y^t)}{\partial b_{t+1}(y^t)} &= u'(c_t(\vec{b}, y^t)) \left[q_t(\vec{b}, y^t) + \frac{\partial q_t(\vec{b}, y^t)}{\partial b_{t+1}(y^t)} \iota_t(\vec{b}, y^{t-1}) \right] \\ &- \beta \sum_{j=1}^{J} \pi_j(y) F_j \left(V_{t+1} \left(\vec{b}, (y^t, y_j) \right) \right) u'(c_{t+1}(\vec{b}, y^{t+1}))[\delta + (1-\delta) q_{t+1}(\vec{b}, y^{t+1})].\end{aligned}$$

□

D Computation

We solve the MPE by iterating on the value and bond price functions as described in Hatchondo et al. (2010). We assume the initial iteration corresponds to the final period of the finite horizon model, implying that the equilibrium we find is the limit of the finite horizon game. We use 50 grid points for b and 21 grid points for y. We use cubic spline interpolation to evaluate V and q for debt levels in the grid $[-0.4, 4]$, and we verify those limits are never binding in the simulations. We do not extrapolate over b. We iterate until the maximum deviation across iterations for the functions V and q is below 10^{-6}.

We solve the Ramsey government's problem by solving the fixed point described in page 21 and assuming an initial iteration identical to the last period of the finite horizon model. We assume $h \in [\underline{h}, \bar{h}]$.[21] We use the same grids for b and y used for the MPE, and we interpolate linearly over h. We do not extrapolate over b or h. The two computational challenges relative to the standard default model are finding (h', b') that solve the non-linear equations ($\mathcal{R}_{b'}$)-($\mathcal{R}_{h'}$) and, more importantly, guaranteeing we are finding a global maximum.

D.1 Solving for h' given (b, y_i, h, b')

Each time we evaluate equations ($\mathcal{R}$)-($\mathcal{R}_{h'}$), we first solve for the value of h' that solves the law of motion ($\mathcal{R}_{h'}$) given the initial state (b, y, h) and debt choice b'. This implies solving

[21] We impose $\underline{h} = 0$ and $\bar{h} = 6$, and verify these bounds are never binding.

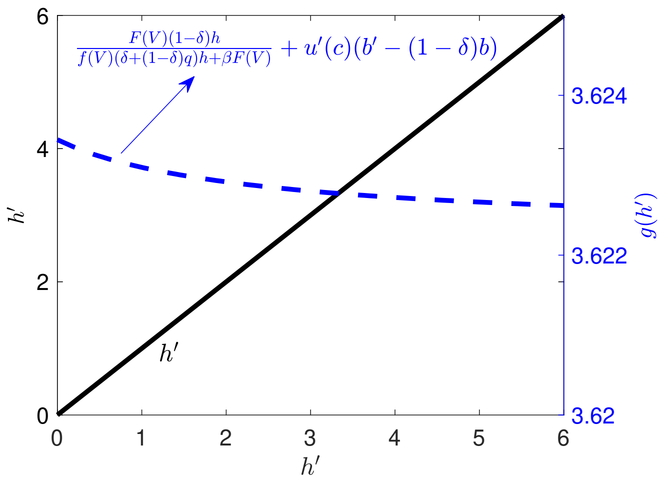

Figure 8: Finding the fixed point $h' = g(h')$. The figure assumes $(b,\, h)$ takes the average values in the simulations, a mean income y_i, and $b' = \hat{b}(b, h, y_i)$. The blue dashed line depicts $g(h')$, defined in equation (40). The left vertical axis corresponds to the 45 degrees line and the right axis to $g(h')$.

for the non-linear equation $h' - g(h') = 0$ where

$$g(h') = \frac{F_i(V)(1-\delta)h}{f_i(V)\left[\delta + (1-\delta)q\right] + \beta(1+r)F_i(V)} h + u'(c)\left[b' - (1-\delta)b\right], \tag{40}$$

with $V = V(b, y_i, h)$ and $q = q(b', y_i, h')$. Assuming that $h' - g(h')$ is increasing in h' (see below), we solve $h' - g(h')$ using the following steps:

1. If $\underline{h} - g(\underline{h}) > 0$, we impute $h' = \underline{h}$. We do this to avoid extrapolating in h.
2. If $\bar{h} - g(\bar{h}) < 0$, we impute $h' = \bar{h}$.
3. If 1. and 2. do not hold, we search over a grid $\{h_1, ...h_n\}$ starting from $h_1 = \underline{h}$ and find the lowest index i with $h_i - g(h_i) < 0$ and $h_{i+1} - g(h_{i+1}) > 0$. We then search for a root within the interval (h_i, h_{i+1}) using a bisection method.

Figure 8 presents a representative case where the root h'^{*} s.t. $h'^{*} - g(h'^{*}) = 0 \in (\underline{h}, \bar{h})$. The figure shows that the slope of g is significantly below the 45 degrees line, which validates our conjecture of an increasing $h' - g(h')$. We show below this property is more general than the case depicted in the figure.

Uniqueness of solution of $h' - g(h') = 0$. Once we have found a fixed point in $\left\{V, \hat{b}, \hat{h}, \hat{c}, q\right\}$, we verify that it satisfies the fixed point conditions when we use two alternative procedures to solve for $h' - g(h') = 0$. In the first procedure: a) we impute $h' = \underline{h}$ only when $h_i - g(h_i) > 0$ for all $i = 1, ..., n$; b) we impute $h' = \bar{h}$ only when $h_i - g(h_i) < 0$ for all $i = 1, ..., n$; and c) otherwise, we use a bisection method to find a root in $[\underline{h}, \bar{h}]$. We find that the fixed point we originally found also satisfies the fixed point conditions when using this alternative procedure. The second procedure is identical to the one described above but where the search for h' in step 3 starts from the highest h on the grid (instead of from the lowest h as in step 3). We also find this change makes no difference, indicating that the fixed point we find features a unique solution for $h' - g(h') = 0$ for all (b, y_i, h, b').

D.2 Finding the global maximum

We exploit the tractable formulation for $\partial\mathcal{U}/\partial b_{t+1}(y^t)$ to numerically verify the solution for equation ($\mathcal{R}_{b'}$) we find is a maximum. The borrowing plan chosen by the Ramsey government ($\vec{b}^*$) satisfies

$$\begin{aligned} \frac{\partial \mathcal{U}(\vec{b}^*, y_0)}{\partial b_{t+1}(y^t)} =& Pr(y^t \mid y_0) \left[[h_t - u'(c_t)\iota_t] \frac{\partial q_t}{\partial b_{t+1}(y^t)} + \frac{\partial V_t}{\partial b_{t+1}(y^t)} \right] \times \\ & \underbrace{\left[\frac{f_i\left(V_t\right)[\delta + (1-\delta)q_t]}{1+r} h_{t-1} + \beta F_i\left(V_t\right) \right] h^V_{t-1}}_{h^V_t} = 0. \end{aligned} \tag{41}$$

for all t and y^t. For simplicity, we omit the argument $(\vec{b}^*, y^t)$ in equation (41). The equation stems from taking $h^V_t(\vec{b}^*, y^t)$ as common factor in equation (11), and applying the law of motion (13) and $h_{t-1} = h^q_{t-1}/h^V_{t-1} + u'(c_{t-1})\iota_{t-1}$.

Based on equation (41), for each initial state (b, h, y_i), we define the function

$$\mathcal{O}(b') = \int_{\underline{b}}^{b'} \left\{ \begin{array}{l} \left[u'(c)q(b', y_i, h') - \frac{\sum_j \pi_j(y_i) f_j(V'_j) u'(c'_j)\left(\delta + (1-\delta)q'_j\right)^2}{1+r} h' \right. \\ \left. -\beta \sum_j \pi_j(y) F_j(V'_j) u'(c'_j) \left[\delta + (1-\delta)q'_j\right]\right] \\ \times \left[\frac{f_i(W(b,y_i,h,b'))[\delta + (1-\delta)q(b',y_i,h')]}{1+r} h + \beta F_i\left(W(b, y_i, h, b')\right)\right] \end{array} \right\} db', \tag{42}$$

with $V'_j = V(b', y_j, h')$, $q'_j = q(b', y_j, h')$, and where

$$W(b,h,y_i,b') = u\left(c\right) + \beta \sum_j \pi_j(y_i) \left[F_j\left(V_j'\right)V_j' + \int_{V_j'} U f_j(dU)\right]$$

denotes the continuation value after repaying and choosing b'. The value of h' used in equation (42) is the one that solves equation ($\mathcal{R}_{h'}$) given (b, y_i, h, b').

The function $\mathcal{O}$ approximates the shape of the Ramsey government's objective with respect to $b_{t+1} = b'$ after arriving at t with $(b_t, h_t, y) = (b, h, y_i)$.[22] It does so because the integrand in equation (42) is proportional to the derivative of the Ramsey government's objective. In terms of equation (41), the constant of proportionality is $Pr(y^t \mid y_0)h_{t-1}^V(\vec{B}, y^t)$, where $\vec{B} = \vec{b}^*$ for all components except for $b_{t+1}(y^t)$.[23] Equation (41) can be used to see how $\frac{\partial \mathcal{U}(\vec{B}, y_0)}{\partial b_{t+1}(y^t)}$ depends on $h_{t-1}^V(\vec{B}, y^{t-1})$, which depends on the choice of $b_{t+1}(y^t)$. This effect is absent in the computation of $\mathcal{O}$. For this reason, the function $\mathcal{O}$ is useful to identify local maxima but not the global maximum. We show below this is not a problem in our quantitative application.

We approximate $\mathcal{O}$ over a grid for b' and use its shape to verify if the candidate for the optimum is at the boundaries $\underline{b}$, $\bar{b}$ or if it is interior. In the first case, we do not extrapolate and assume b' is at one of the bounds. In the second case, we find $b'^* = \underset{b' \in \{b_1,,\ldots b_n\}}{Argmax}\ \mathcal{O}(b')$ and use a non-linear equation solver with initial guess b'^* to solve equation ($\mathcal{R}_{b'}$). Figure 9 depicts the shape of $\mathcal{O}$ for the average values of (b, h, y) in the simulations. The flat segment corresponds to b' choices at which the government buys back so many bonds that current consumption is too low to make repayment optimal for almost any possible realization of the continuation value of defaulting U.

Uniqueness of local maxima. The shape of $\mathcal{O}$ in Figure 9 resembles the ones we find for other states. In fact, once we have found the functions $\left\{V, \hat{b}, \hat{h}, \hat{c}, q\right\}$ that satisfy the

[22]The derivative used in $\mathcal{O}$ differs from the one in (11) because while in (11) we assume a one-time deviation in the debt path, in equation (42) we assume the government reoptimizes its future borrowing path after changing b_{t+1}. This is so because we are using $V_j' = V(b', h', y_j)$, $c_j' = \hat{c}(b', h', y_j)$, and $q_j' = q(\hat{b}(b', h', y_j), \hat{h}(b', h', y_j), y_j)$, which change with b'. We verify the fixed point satisfies the optimality condition with one-time deviations.

[23]We verify in our simulations that h_t^V always takes positive values, which implies that $\mathcal{O}$ increases (decreases) if and only if the Ramsey government's objective increases (decreases) in b_{t+1}.

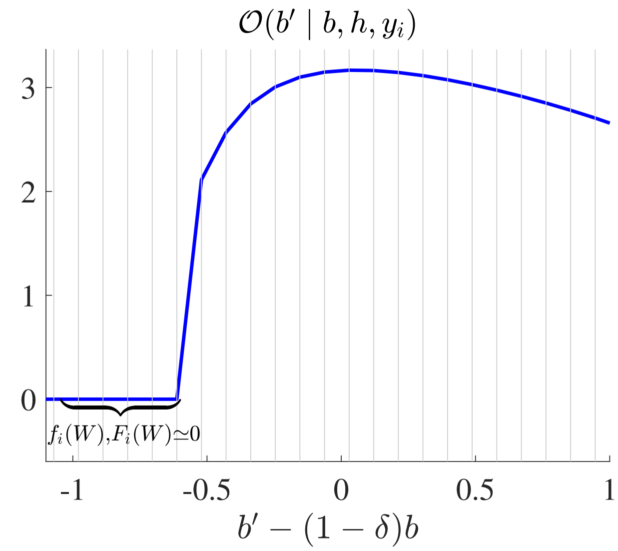

Figure 9: Approximated shape of the objective function of the Ramsey government. The figure assumes the average levels of b, h, and y in the simulations.

fixed point conditions on page 13, we verify that those conditions are also satisfied when we use two alternative procedures to compute b': i) choose the local maximum with the lowest b' and ii) choose the local maximum with the highest b'. That is, we find that both local maxima coincide, for all (b, h, y_i) on the grid, indicating that the solution we find is a global maximum.

Robustness without income uncertainty. To further illustrate how our algorithm finds the global maximum, we solve a version of the model without income uncertainty ($\sigma_\varepsilon = 0$, and all other parameter values as in the benchmark calibration). In this case, there is no hedging motive for issuing long-term debt, and it is optimal to issue one-period bonds (Aguiar et al., 2019), eliminating the time inconsistency problem and thus making the optimal solution for the Markov and Ramsey government coincide. We find that the optimal paths of consumption with one-period bonds, and the optimal paths of consumption implied by our algorithm (with long-term bonds and commitment to future borrowing) are almost identical.

E Decomposition of welfare gains

Let $\left\{\hat{c}^R, \hat{d}^R, \hat{b}^R\right\}$ denote the consumption, default and debt accumulation policy rules with a Ramsey government, and $\left\{\hat{c}^M, \hat{d}^M \ , \hat{b}^M\right\}$ the ones with a Markov government. Let

$$W^R(b_0, y_0, h_0, U_0) = \sum_{t=0}^{\infty} \beta^t \sum_{j=1}^{J} Pr(y_t = y_j \mid y_0) \int u\left(\hat{c}^R\left(b_t^R, y_j, h_t, U_t\right)\right) f_j^R(U_t) dU_t \tag{43}$$

denote the present expected discounted value of future utility flows in the economy with a Ramsey government starting from an initial state (b_0, y_0, h_0, U_0). The consumption rule $\hat{c}^R$ satisfies equation ($\mathcal{R}_c$) when the government repays and $\hat{c}^R(b, y, h) = y(1-d_0-d_1 y)$ when the government defaults. The paths for $\left\{b_t^R, h_t\right\}_{t=1}^{\infty}$ are computed using the debt accumulation policy $\hat{b}^R$ and equation (16). Similarly, let

$$W^M(b_0, y_0) = \sum_{t=0}^{\infty} \beta^t \sum_{j=1}^{J} Pr(y_t = y_j \mid y_0) \int u\left(\hat{c}^M\left(b_t^M, y_j, U_t\right)\right) f_j^M(U_t) dU_t \tag{44}$$

denote the present expected discounted value of future utility flows in the economy with a Markov government starting from an initial state (b_0, y_0, U_0). The p.d.f. for the continuation value under defaulting differ across the two economies because the value of being temporarily excluded differ. By using only consumption paths, the functions W^R, W^M do not incorporate the innovations to the continuation value under default $(U - V^D)$ into the welfare measure. We show below these innovations not play a critical role in accounting for welfare gains.

Define $c^{R,ND}$ $\hat{c}^{M,ND}$ as the consumption policy rules when we strip out the income costs of defaulting from the consumption path. Namely,

$$\hat{c}^{R,ND}(b, y, h, U) = \hat{d}^R(b, y, h, U)y + [1 - \hat{d}^R(b, y, h, U)]\hat{c}^R(b, y, h, U) \text{ and}$$
$$\hat{c}^{M,ND}(b, y, U) = \hat{d}^M(b, y, U)y + [1 - \hat{d}^M(b, y, U)]\hat{c}^M(b, y, U).$$

Let $\bar{c}_t^{R,ND}(b_0, y_0, h_0, U_0) = E\left[\hat{c}^{R,ND}(b_t^R, y_t, h_t, U_t) \mid b_0, y_0, h_0, U_0\right]$ denote the expected consumption in t without the income default cost and starting from an initial state (b_0, y_0, h_0, U_0) in the economy with a Ramsey government, and $\bar{c}_t^{M,ND}(b_0, y_0, U_0) = E\left[\hat{c}^{M,ND}(b_t^M, y_t, U_t) \mid b_0, y_0, U_0\right]$ denote the expected consumption in t without the income default cost and starting from an initial state (b_0, y_0, U_0) in the economy with a Markov government. Finally, let

$$\left\{W^{R,ND}, \bar{W}^{R,ND}, W^{M,ND}, \bar{W}^{M,ND}\right\}$$

denote the expected present discounted value of utility flows computed using the consumption policy rules $\left\{\hat{c}^{R,ND}, \bar{c}_t^{R,ND}, \hat{c}^{M,ND}, \bar{c}_t^{M,ND}\right\}$ for t = 0,1,....

The consumption-based welfare gain of being in an economy with a Ramsey government instead of one with a Markov government satisfies

$$\underbrace{\left(\frac{W^R}{W^M}\right)^{\frac{1}{1-\sigma}}}_{1+\lambda} = \underbrace{\left(\frac{W^R/W^{R,ND}}{W^M/W^{M,ND}}\right)^{\frac{1}{1-\sigma}}}_{1+\lambda_D} \times \underbrace{\left(\frac{W^{R,ND}/\bar{W}^{R,ND}}{W^{M,ND}/\bar{W}^{M,ND}}\right)^{\frac{1}{1-\sigma}}}_{1+\lambda_V} \times \underbrace{\left(\frac{\bar{W}^{R,ND}}{\bar{W}^{M,ND}}\right)^{\frac{1}{1-\sigma}}}_{1+\lambda_T}. \tag{45}$$

The first factor λ_D captures the role of income default costs and computes the percentage increase in consumption necessary to compensate consumers for the resources lost in default with a Markov government relative to resources lost in default with a Ramsey government. The second term λ_V represents the welfare benefit from shutting down consumption volatility (without the income cost of defaulting) in the economy with a Markov government relative to shutting down consumption volatility with a Ramsey government. Finally, λ_T captures the welfare benefit from consumption tilting, i.e., the percentage increase in consumption needed to compensate for the different average consumption profiles.[24]

F Cross-country evidence on the relation between debt and spreads

Akitoby and Stratmann (2008) and Jaramillo and Tejada (2011) document that fiscal variables and growth rates have statistically significant effects on sovereign spreads. We build on those papers to estimate how the relationship between the spread and public debt depends on income and spread levels. We use annual data from a sample of 33 emerging market countries ranging from 1994 to 2015. In order to facilitate the comparison of the model's

[24]Given that lenders are risk neutral $\sum_{t=0}^{\infty} \bar{c}^{R,ND}/(1+r)^t = \sum_{t=0}^{\infty} \bar{c}^{M,ND}/(1+r)^t$, meaning welfare differences are only driven by a different consumption allocation across time.

Table 4: Regressions using simulations of the MPE

Regression	(1)	(2)	(3)
Constant	-1.713	0.677	0.604
Public debt to GDP	0.167	0.106	0.109
Real GDP growth	-0.045	-0.0291	-0.025
Debt $\times$ I (growth <0)	0.000		
Debt $\times$ I (GDP $<$ Mean GDP)		0.011	
Debt $\times$ I (Spread $>$ Mean spread)			0.014
R^2	0.637	0.744	0.816

Annual variables are created out of the quarterly model simulations in the economy with a Markov government. The dependent variable is log(spread). The dummy variables I take a value of 1 when the condition in brackets is satisfied and 0 otherwise.

testable implications with the data, we subtract to GDP growth rates the average growth rate for each country (GDP is assumed to be stationary in the model). We remove country-year observations in which the country was in default according to the definition of Standard & Poor's.

The empirical strategy in Akitoby and Stratmann (2008), Jaramillo and Tejada (2011), and most of the references therein is based on Edwards (1984, 1986) and consists of regressing the logarithm of the spread on a set of explanatory variables. Table 4 shows the result of conducting that regression using the simulations of the economy with the Markov government (we assume that the data is generated by governments that lack commitment to future borrowing). All regressions show that more debt and lower growth rates are associated with a higher spread. Regression (2) shows that the spread is more sensitive to debt when aggregate income is below its long-run mean. Regression (3) shows the same result when the spread is above its mean.[25]

We contrast the testable implications presented in Table 4 with data by estimating a

[25] Regression (1) shows the relationship between spread and debt does not depend on the growth rate. This is an artifact of having a model with a stationary income process in which higher growth does not necessarily mean a good income state.

fixed effects panel regression with robust standard errors.[26] Namely, we estimate

$$log\,(\text{Spread})_{it} = \alpha + \beta X_{it} + \delta_i + \eta_{it}, \qquad i = 1, ..., N, \quad t = 1, ..., T,$$

where i denotes the country index, X_{it} is a vector of control variables for country specific and global macroeconomic factors; δ_i are country fixed effects; and η_{it} represents disturbances that are independent across countries and time.

The results are summarized in Table 5. As in Jaramillo and Tejada (2011), Akitoby and Stratmann (2008), and other studies, all the regressions in the table show that the spread increases with the debt level and decreases with GDP growth. The model is consistent with this and even implies a remarkably similar coefficient for the growth rate. The coefficient for debt is higher than the one in the data but the model abstracts from other determinants for borrowing and defaulting. Regression (2) shows that even though there seems to be evidence that the spread increases more with debt in years with low growth, the statistical significance is weak (17%). Regression (3) shows the spread increases more with debt in years with high spread. Regression (4) shows that the spread seems to increase more with government net borrowing but also at a low significance level (19%). Finally, regression (5) shows the spread increases more with net government borrowing in years with high spread.

[26] The robust variance matrix estimator in Wooldridge (2002, p. 152) is implemented with the option "hccme = 3 cluster" in SAS.

Table 5: Panel regressions

Regression	(1)	(2)	(3)	(4)	(5)
Public debt to GDP	0.020	0.019	0.004	0.020	0.017
	(0.004)	(0.004)	(0.002)	(0.004)	(0.003)
Real GDP growth	-0.042	-0.034	-0.031	-0.036	-0.036
	(0.006)	(0.009)	(0.004)	(0.007)	(0.006)
Reserves to GDP	-0.033	-0.033	-0.022	-0.033	-0.027
	(0.017)	(0.017)	(0.010)	(0.017)	(0.014)
Net gov borrowing to GDP	0.020	0.021	0.017	0.012	-0.023
	(0.014)	(0.015)	(0.009)	(0.015)	(0.017)
VIX	0.034	0.034	0.0194	0.035	0.028
	(0.003)	(0.003)	(0.004)	(0.003)	(0.004)
$\text{Debt}_{it} \times I$ $(\text{growth}_{it} < \text{mean}_i(\text{growth}))$		0.002			
		(0.001)			
$\text{Debt}_{it} \times I$ $(\text{spread}_{it} > \text{mean}_i(\text{spread}))$			0.013		
			(0.001)		
$\text{NGB}_{it} \times I$ $(\text{growth}_{it} < \text{mean}_i(\text{growth}))$				0.020 (0.016)	
$\text{NGB}_{it} \times I$ $(\text{spread}_{it} > \text{mean}_i(\text{spread}))$					0.102
					(0.016)
Observations	523	523	523	523	523
R-squared	0.77	0.77	0.87	0.77	0.82
Number of countries	33	33	33	33	33

The dummy variables $I(x) = 1$ when condition x is met and 0 otherwise. NGB stand for net government borrowing to GDP. Robust standard errors in parentheses.

Table 6: Simulations with a debt duration of 2.3 years

	Markov	Ramsey
Mean debt-to-income ratio (in %)	35.4	35.1
Mean debt market value (in %)	34.3	34.8
Mean spread (in %)	1.5	0.5
Std dev spread	1.8	0.6
$\sigma(c)/\sigma(y)$	1.3	1.4

G Robustness exercises

G.1 Optimal ex-ante debt duration

We calculate the optimal ex-ante duration by choosing the value of δ in an initial period with zero debt and income equal to the mean, and assuming the government commits to issuing bonds with that δ thereof. Welfare is measured relative to the economy with one-period bonds. The optimal ex-ante $\delta = 0.0963$, implying an average debt duration of 2.3 years, around half the value used in our parameterization. Hatchondo and Martinez (2013) quantify the optimal ex-ante duration and the endogenous duration in an Eaton-Gersovitz model calibrated to Mexico. They find the endogenous duration is higher than the optimal ex-ante duration, suggesting that if we allowed for an endogenous debt portfolio, the average duration would be above 2.3 years.

Table 6 shows that the Markov government issues less debt and at a lower spread for a debt duration of 2.3 years. The intuition is similar to the lower debt and spread discussed in the economy with one-period bonds in Section 5. However, default risk is still inefficiently high and a Ramsey government that issues bonds with the same coupon structure would choose a borrowing path with a lower default risk and higher average borrowing. The welfare gain from permanently switching to a Ramsey government in a period with no debt and mean income is 0.16% (it is 0.42% for our benchmark parameterization).

We could not find an interior solution for the ex-ante optimal debt duration for the Ramsey problem. We solved the Ramsey government's problem for debt durations of up to 30 years and found welfare always increases with duration over that interval. A debt duration of 30 years correspond to $\delta = -0.0016$, i.e., bond payments increase over time but at a lower pace than the risk-free interest rate.[27] Hatchondo et al. (2016) show that mitigating the government's time inconsistency problem with debt covenants would lead the Markov government to choose longer maturities. Once the inefficiency in the borrowing path is corrected, the Ramsey planner benefits from extending the debt duration to exploit the hedging benefits of long-term debt described by Arellano and Ramanarayanan (2012).

G.2 Shocks to the utility cost of defaulting

Table 7 presents simulation results for different values of σ_U, while keeping the benchmark values for all other parameters. The table shows that for both the Ramsey and the Markov governments, a higher σ_U implies lower debt levels and a higher mean and standard deviation of sovereign spreads (the latter being consistent with our calibration strategy). Intuitively, a higher σ_U increases the mass of states in which it is optimal to default. Lenders anticipate that and offer worse bond price schedules, and the government borrows less. By the same logic, a higher σ_U also increases the sensitivity of bond prices to debt at high income states. In this scenario, equation (11) implies the Ramsey government would also want to distort borrowing in those states, moderating the procyclicality of fiscal policy. This effect can be seen in column 5: for instance when $\sigma_U = 1$, the Ramsey government chooses an allocation with the same relative consumption volatility as the one chosen by the Markov government.

Despite the above differences in terms of debt and consumption volatility, Table 7 also shows that the Ramsey government achieves significant welfare gains and significant reductions in sovereign default risk for different values of σ_U. As expected, welfare gains decrease with σ_U. The lower the relative importance of income shocks as a default determinant, the lower the gain from conditioning borrowing on income histories.

[27] We rescale the sequence of coupon payments to allow for longer durations. Formally, we assume a bond issued at t pays a coupon $(r + \delta)(1 - \delta)^{n-1}$ in period $t + n$, for $n = 1, 2, ...$.

Table 7: Importance of the shock to the utility cost of defaulting

Std. dev. shock to V^D	Mean debt (%)	Mean spread (%)	Std dev spread	$\sigma(c)/\sigma(y)$	Welfare gain (%)
Markov					
0.08	44.3	3.0	2.2	1.1	
0.10	**44.3**	**3.3**	**2.4**	**1.1**	
0.25	40.7	3.9	2.8	1.1	
0.75	26.4	8.9	4.7	1.0	
1.00	22.5	12.1	5.2	1.0	
Ramsey					
0.08	40.4	0.5	0.4	1.4	0.43
0.10	**39.5**	**0.5**	**0.4**	**1.4**	**0.45**
0.25	34.5	0.9	0.6	1.3	0.41
0.75	17.8	4.4	2.1	1.1	0.17
1.00	13.8	8.4	3.5	1.0	0.13

Rows in bold typeface correspond to our baseline calibration.