A Simple Model of Group Conflict, Inequality and Stratification

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NBER Race and Stratification Conference
Boston, March 31, 2023

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- **Stratification Economics** (SE): discrimination = rational "defense" mechanism of the dominant group(s). Accordingly,
  - **Prejudicism** is a purposeful action aimed at maintaining the dominant group's dominance position.
- **Economics of discrimination** focuses on:
  - Taste for discrimination (Becker, 1957)
  - Statistical discrimination (Arrow, 1973; Phelps, 1972), but also
  - Identity-driven behavior (Akerlof and Kranton, 2000)
  - Unsuccessful behavior by marginalized group members (Fang & Loury, 2005)
- The difference with SE is that it presupposes purposeful **economic harm** perpetrated by some individuals onto others.

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- Formalize insights by Lewis (1985), Darity (2005), Chelwa, Hamilton and Stewart (2022).
- Keep the model as simple as possible.

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- Discriminatory effort $\rightarrow$ income & wealth inequality between groups; it is also **inefficient** from a societal standpoint.
- Yet, it **persists** because it is 'rational' for the dominant group, & the costly nature of anti-discriminatory measures & enforcement.

Key Elements of the Model

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  - D-group members can engage in discriminatory activities against members of the M-group in order to make them non-competitive in the market period.
  - D-individuals can free ride on discriminatory activity by other D-individuals.
  - But someone *must* discriminate, otherwise discrimination would not exist in equilibrium.

Individuals in the Marginalized Group

- There are $j = 1, \ldots, Q$ individuals in group M, choosing how much to invest $h_{j,M}$ in skill acquisition to earn income y_j^M in the market period.
- However, y_j^M can be reduced by the total discriminatory effort $d \in [0, 1]$ by the D-group. Thus, we postulate $y_j^M(h_{j,M}, d)$ & assume:
  1. $y_j^M(0, d) = 0$ (No free-lunch);
  2. $\partial y_j^M / \partial h_{j,M} > 0, \partial^2 y_j^M / \partial h_{j,M}^2 < 0$ (Monotonicity; strict concavity).
  3. $\partial y_j^M / \partial d < 0$ (Economic harm from discrimination).
- We assume:

$$y_j^M(h_{j,M}, d) = A h_{j,M}^{\alpha} (1 - d)^{1-\alpha} \, \alpha \in (0, 1), A \in (0, 1) \tag{1}$$

where A is a positive productivity parameter, restricted for model consistency.

Choice

- Individual j in group M chooses $h_{j,M}$ to maximize

$$y_j^M(h_{j,M}, d) - h_{j,M} \tag{2}$$

- **Reaction function**

$$h_M(d) = (\alpha A)^{\frac{1}{1-\alpha}}(1-d) \tag{3}$$

equal across all M-individuals.

Comments

- The intensity of "human capital" investment by a j individual *decreases* in the total discriminatory effort d by the dominant group.
- "Low educational attainment" by marginalized group members is due to discriminatory action against them.
- Market income for an M-individual is

$$\begin{aligned} y_j^M(d) &= \alpha^{\frac{\alpha}{1-\alpha}} A^{\frac{1}{1-\alpha}} (1-d) \\ &= y^M(d) \end{aligned} \tag{4}$$

  also symmetric across all $j \in M$ and linearly decreasing in d.

Individuals in the Dominant Group

- A D- group individual $i = 1, \ldots, N$ is not discriminated against.
- Thus, assuming away productivity differences between groups:

$$y_i^D = A h_{i,D}^{\alpha} \tag{5}$$

Choice & Free-riding

- A D-individual chooses $h_{i,D}$ and d_i to maximize the **difference** between their own market income and the income of a typical marginalized group individual.
- We assume that the cost of active discrimination is convex: $c(d_i) = d_i^2/2$.
- **Free-riding** issue: total discriminatory effort by the D-group is

$$d = \eta d_i + (1 - \eta) d_{-i};\ \eta \in (0, 1) \tag{6}$$

so that each of the $\{i, M\}$ individuals takes discriminatory effort by the other members of the same group as given.

Choice: Discrimination Effort

- The choice of skill investment and discriminatory effort are:

$$h_{i,D} = (\alpha A)^{\frac{1}{1-\alpha}} = h_D \,\forall i \quad (7)$$

$$d_i = \eta(1-\alpha)Ah_{j,M}^{\alpha}(1-d)^{-\alpha} \quad (8)$$

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  - Extent of skill investment by M-group;
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Equilibrium

An **equilibrium allocation** is defined as:

- A choice $h_{j,M}$ that max's market income for M-individuals given the PC and given d for all $j \in M$;
- A choice $\{h_{i,D}, d_i\}$ by $i \in D$ that max's difference in market incomes.

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Equilibrium investments in skill acquisition:

$$h^M = (\alpha A)^{\frac{1}{1-\alpha}} \left[1 - \eta \left(\frac{1-\alpha}{\alpha} \right) (\alpha A)^{\frac{1}{1-\alpha}} \right] \tag{10}$$

$$h^D = (\alpha A)^{\frac{1}{1-\alpha}} \tag{11}$$

Equilibrium Inequality

- Given the differences in human capital investment across the two groups, **racial income inequality** is obtained simply as the ratio:

$$\frac{y^D}{y^M} = \frac{1}{1-\eta\left(\frac{1-\alpha}{\alpha}\right)(\alpha A)^{\frac{1}{1-\alpha}}} > 1 \tag{12}$$

- Inequality would disappear if discriminatory effort had no effect on the D-group income $(\alpha = 1)$, or if $\eta = 0$ (complete free-riding by every individual i in the D-group).

Welfare

- A benevolent social planner chooses h_M, h_D, d to maximize the society's net average market income

$$W = \frac{1}{N+Q}\left[\sum_{j=1}^{Q}(y_j^M(h_{j,M}, d) - h_{j,M}) + \sum_{i=1}^{N}(y_i^D(h_{i,D}) - h_{i,D} - d_i)\right] \tag{13}$$

  taking into account that all the $j \in M$-individuals and the $i \in D$-individuals allocate the same amount of resources into skill acquisition (and discrimination activities).
- The SWF is **monotonically decreasing in discriminatory effort**:

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  taking into account that all the $j \in M$-individuals and the $i \in D$-individuals allocate the same amount of resources into skill acquisition (and discrimination activities).
- The SWF is **monotonically decreasing in discriminatory effort**:
- The **efficient allocation** involves $d_i^* = 0$ for all $i \in D$ $\rightarrow$ **is also egalitarian**.

Anti-Discrimination Policy

- A government could engage in anti-discrimination effort $\varepsilon \in [0,1]$ so that market income for an M-individual becomes

$$y^M(d;\varepsilon) = Ah_M^{\alpha}[1-d(1-\varepsilon)]^{1-\alpha}$$

  which eliminates the effects of discrimination when $\varepsilon = 1$.
- The reaction function and market income for an M-individual as a function of d and ε are now:

$$h^M(d;\varepsilon) = (\alpha A)^{\frac{1}{1-\alpha}}[1-d(1-\varepsilon)] \quad (14)$$

$$y^M(d;\varepsilon) = \alpha^{\frac{\alpha}{1-\alpha}} A^{\frac{1}{1-\alpha}}[1-d(1-\varepsilon)] \quad (15)$$

Policy (2)

- Consider D-individuals. It turns out that the difference is not the extent of discriminatory effort, but how effective the discriminatory effort will be:
- any amount $d_{i,M}$ will be scaled down by an amount ε because of anti-discriminatory policies.
- Thus, in equilbrium, the extent of market income inequality is

$$\frac{y^{E,D}}{y^{E,M}(\varepsilon)} = \frac{1}{1-(1-\varepsilon)\eta\left(\frac{1-\alpha}{\alpha}\right)(\alpha A)^{\frac{1}{1-\alpha}}} \tag{16}$$

and the egalitarian allocation is obtained when $\varepsilon = 1$.

Why isn't Discrimination Eliminated then?

- Suppose that the burden of proving to be a victim of discrimination falls upon the discriminated, and the cost of ensuring enforcement is convex $c(\varepsilon) = \frac{1}{2}\varepsilon^2$. A group-$M$ individual solves:

$$\max_{\{h_M, \varepsilon\}} A h_M^{\alpha} [1 - d(1-\varepsilon)]^{1-\alpha} - h_M - \frac{1}{2}\varepsilon^2 \tag{17}$$

- In equilibrium,

$$\varepsilon = \eta (1-\alpha)^2 \alpha^{\frac{2\alpha}{1-\alpha}} A^{\frac{2}{1-\alpha}} \propto d^2 < d \tag{18}$$

[Remember that $d \in (0,1)$]

- **Thus, discrimination will be lessened but never eliminated**.

Wealth Inequality & Stratification

- Through intergenerational altruism & bequests, income inequality reverberates into wealth inequality $\rightarrow$ stratify the society.
- We adapt the Galor-Zeira (1993) model to this setting.
- An individual in group $r = \{M, D\}$ earns market income y^r.
- Utility defined over **consumption** c_r and **bequests** b_r as follows:

$$u^r(c_r, b_r) = \beta \ln c_r + (1 - \beta) \ln b_r \tag{19}$$

- We need to consider the possibility of investing one's inheritance, earning **rate of return** $\rho > 0$.

Stratification (2)

- The PC's for group-M and group-D individuals are now:

$$w_j^M + y_{j,M} - h_{j,M} \geq w_j^M(1+\rho) \quad (20)$$

$$w_i^D + y_{i,D} - d_i - h_{i,D} \geq w_i^D(1+\rho) \quad (21)$$

- The chosen amount of **bequests** is a constant fraction of mkt income – the opportunity cost of interest on inherited wealth:

$$b_r = (1-\beta)(y^r - \rho w^r) \quad (22)$$

Stratification (3)

- Bequest left by the current generation = initial wealth of the following one $\rightarrow$ evolution of group r's wealth:

$$w^r_{+1} = (1-\beta)(y^r - \rho w^r) \tag{23}$$

Steady state:

$$w^r_{ss} = \frac{1-\beta}{1-(1+\beta)\rho} y^r \quad r = \{D, M\} \tag{24}$$

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- Thus,

$$\frac{w^D_{ss}}{w^M_{ss}} = \frac{y^D}{y^M} \tag{25}$$

Wealth inequality is proportional to income inequality.

Reparations

- A simple exercise shows that the amount of reparations needed to eliminate inequality in the baseline model with $\varepsilon = 0$ is

$$R = d = \eta \left(\frac{1-\alpha}{\alpha} \right) (\alpha A)^{\frac{1}{1-\alpha}} \tag{26}$$

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- This is way too simple!
  - Society has been stratified through several generations;
  - Intergenerational altruism can actually amplify wealth disparities;
  - Rates of return are different across racial groups.
- An infinite-horizon model will likely imply much higher wealth inequality between groups.

Conclusion

- Stratification Economics sees discrimination as a purposeful (costly) activity by dominant groups to maintain their status.
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- Stratification Economics sees discrimination as a purposeful (costly) activity by dominant groups to maintain their status.
- Even though some D-group members will not be actively engaged in discrimination, they will still benefit from it.
- **Someone must have discriminated**: $d_i > 0$ for at least one i.
- Discrimination is wasteful from a societal standpoint (not Pareto-efficient);
- Yet, it persists because anti-discrimination measures are costly to enforce, especially if the burden falls upon the discriminated.
- Intergenerational altruism provides the link from income inequality to wealth inequality.

Thank you!