A Simple Model of Group Conflict, Inequality and Stratification

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Stratification Economics vs. Economics of Discrimination

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  - **Prejudicisim** is a purposeful action aimed at maintaining the dominant group’s dominance position.

- **Economics of discrimination** focuses on:
  - Taste for discrimination (Becker, 1957)
  - Statistical discrimination (Arrow, 1973; Phelps, 1972), but also
  - Identity-driven behavior (Akerlof and Kranton, 2000)
  - Unsuccessful behavior by marginalized group members (Fang & Loury, 2005)

- The difference with SE is that it presupposes purposeful **economic harm** perpetrated by some individuals onto others.
Contribution

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- Keep the model as simple as possible.
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- Marginalized group members have limited ability to counter discriminatory effort:
  - Clear power imbalance.
- Discriminatory effort → income & wealth inequality between groups; it is also inefficient from a societal standpoint.
- Yet, it persists because it is ‘rational’ for the dominant group, & the costly nature of anti-discriminatory measures & enforcement.
Key Elements of the Model

- The model builds on ideas advanced verbally by Lewis (1985).
- Two groups: a dominant group ($D$) and a marginalized group ($M$).
- Each group member lives for two periods:
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  - $D$-group members can engage in discriminatory activities against members of the $M$-group in order to make them non-competitive in the market period.
  - $D$-individuals can free ride on discriminatory activity by other $D$-individuals.
  - But someone *must* discriminate, otherwise discrimination would not exist in equilibrium.
Individuals in the Marginalized Group

There are $j = 1, \ldots, Q$ individuals in group $M$, choosing how much to invest $h_{j,M}$ in skill acquisition to earn income $y_{j}^{M}$ in the market period.

However, $y_{j}^{M}$ can be reduced by the total discriminatory effort $d \in [0, 1]$ by the $D$-group. Thus, we postulate $y_{j}^{M}(h_{j,M}, d)$ & assume:

1. $y_{j}^{M}(0, d) = 0$ (No free-lunch);
2. $\partial y_{j}^{M} / \partial h_{j,M} > 0$, $\partial^{2} y_{j}^{M} / \partial h_{j,M}^{2} < 0$ (Monotonicity; strict concavity).
3. $\partial y_{j}^{M} / \partial d < 0$ (Economic harm from discrimination).

We assume:

$$y_{j}^{M}(h_{j,M}, d) = Ah_{j,M}^{\alpha}(1 - d)^{1-\alpha} \quad \alpha \in (0, 1), A \in (0, 1)$$  \hspace{1cm} (1)$$

where $A$ is a positive productivity parameter, restricted for model consistency.
Choice

- Individual $j$ in group $M$ chooses $h_{j,M}$ to maximize
  \[ y^M_j (h_{j,M}, d) - h_{j,M} \]  

- **Reaction function**
  \[ h_M(d) = (\alpha A)^{\frac{1}{1-\alpha}} (1 - d) \] (3)

  equal across all $M$-individuals.
The intensity of “human capital” investment by a $j$ individual decreases in the total discriminatory effort $d$ by the dominant group.

“Low educational attainment” by marginalized group members is due to discriminatory action against them.

Market income for an $M$-individual is

$$y_j^M(d) = \alpha \frac{\alpha}{1-\alpha} A \frac{1}{1-\alpha} (1 - d) = y^M(d)$$

(4)

also symmetric across all $j \in M$ and linearly decreasing in $d$. 
Individuals in the Dominant Group

- A $D$-group individual $i = 1, \ldots, N$ is not discriminated against.
- Thus, assuming away productivity differences between groups:

$$y_i^D = Ah_{i,D}^\alpha$$  \hspace{1cm} (5)
Choice & Free-riding

- A $D$-individual chooses $h_{i,D}$ and $d_i$ to maximize the difference between their own market income and the income of a typical marginalized group individual.

- We assume that the cost of active discrimination is convex: $c(d_i) = d_i^2/2$.

- **Free-riding** issue: total discriminatory effort by the $D$-group is

$$d = \eta d_i + (1 - \eta) d_{-i}; \quad \eta \in (0, 1)$$

so that each of the $\{i, M\}$ individuals takes discriminatory effort by the other members of the same group as given.
Choice: Discrimination Effort

- The choice of skill investment and discriminatory effort are:

\[
h_{i,D} = (\alpha A)^{\frac{1}{1-\alpha}} = h_D \forall i
\]

\[
d_i = \eta(1 - \alpha)Ah_{j,M}^\alpha(1 - d)^{-\alpha}
\]

- Extent of discrimination increases in
  - Extent of skill investment by \(M\)-group;
  - Productivity \(A\): as \(M\)-group members become more productive, efforts to make them non-competitive will intensify.
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  - Extent of skill investment by \( M \)-group;
  - Productivity \( A \): as \( M \)-group members become more productive, efforts to make them non-competitive will intensify.
An equilibrium allocation is defined as:

- A choice \( h_{j,M} \) that max’s market income for \( M \)-individuals given the PC and given \( d \) for all \( j \in M \);
- A choice \( \{h_{i,D}, d_i\} \) by \( i \in D \) that max’s difference in market incomes.

Equilibrium discriminatory effort is symmetric and equal to

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\]

Equilibrium investments in skill acquisition:

\[
\begin{align*}
    h^M &= (\alpha A)^{\frac{1}{1-\alpha}} \left[ 1 - \eta \left( \frac{1 - \alpha}{\alpha} \right) (\alpha A)^{\frac{1}{1-\alpha}} \right] \tag{10} \\
    h^D &= (\alpha A)^{\frac{1}{1-\alpha}} \tag{11}
\end{align*}
\]
Equilibrium Inequality

- Given the differences in human capital investment across the two groups, **racial income inequality** is obtained simply as the ratio:

\[
\frac{y_D}{y_M} = \frac{1}{1 - \eta \left(\frac{1-\alpha}{\alpha}\right)(\alpha A)^{\frac{1}{1-\alpha}}} > 1
\]  

(12)

- Inequality would disappear if discriminatory effort had no effect on the $D$-group income ($\alpha = 1$), or if $\eta = 0$ (complete free-riding by every individual $i$ in the $D$-group).
Welfare

- A benevolent social planner chooses \( h_M, h_D, d \) to maximize the society’s net average market income

\[
W = \frac{1}{N + Q} \left[ \sum_{j=1}^{Q} (y_j^M (h_j, M, d) - h_j, M) + \sum_{i=1}^{N} (y_i^D (h_i, D) - h_i, D - d_i) \right]
\]

(13)

taking into account that all the \( j \in M \)-individuals and the \( i \in D \)-individuals allocate the same amount of resources into skill acquisition (and discrimination activities).

- The SWF is **monotonically decreasing in discriminatory effort**:
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- The SWF is **monotonically decreasing in discriminatory effort**:
- The **efficient allocation** involves $d^*_i = 0$ for all $i \in D \rightarrow$ is also egalitarian.
Anti-Discrimination Policy

- A government could engage in anti-discrimination effort $\varepsilon \in [0, 1]$ so that market income for an $M$-individual becomes

$$y^M(d; \varepsilon) = Ah^\alpha_M[1 - d(1 - \varepsilon)]^{1-\alpha}$$

which eliminates the effects of discrimination when $\varepsilon = 1$.

- The reaction function and market income for an $M$-individual as a function of $d$ and $\varepsilon$ are now:

$$h^M(d; \varepsilon) = (\alpha A)^{1-\alpha} [1 - d(1 - \varepsilon)]$$ (14)

$$y^M(d; \varepsilon) = \alpha^{1-\alpha} A^{1-\alpha} [1 - d(1 - \varepsilon)]$$ (15)
Consider $D$-individuals. It turns out that the difference is not the extent of discriminatory effort, but how effective the discriminatory effort will be:

- any amount $d_{i,M}$ will be scaled down by an amount $\varepsilon$ because of anti-discriminatory policies.

Thus, in equilibrium, the extent of market income inequality is

$$\frac{y^{E,D}}{y^{E,M}(\varepsilon)} = \frac{1}{1 - (1 - \varepsilon)\eta \left(\frac{1-\alpha}{\alpha}\right)(\alpha A)^{\frac{1}{1-\alpha}}} \tag{16}$$

and the egalitarian allocation is obtained when $\varepsilon = 1$. 
Why isn’t Discrimination Eliminated then?

Suppose that the burden of proving to be a victim of discrimination falls upon the discriminated, and the cost of ensuring enforcement is convex $c(\varepsilon) = \frac{1}{2} \varepsilon^2$. A group-$M$ individual solves:

$$\max \{ h_M, \varepsilon \} \quad A h_M^\alpha [1 - d(1 - \varepsilon)]^{1-\alpha} - h_M - \frac{1}{2} \varepsilon^2$$

In equilibrium,

$$\varepsilon = \eta(1 - \alpha)^2 \alpha^{\frac{2\alpha}{1-\alpha}} A^{\frac{2}{1-\alpha}} \propto d^2 < d$$

[Remember that $d \in (0, 1)$]

Thus, discrimination will be lessened but never eliminated.
Wealth Inequality & Stratification

- Through intergenerational altruism & bequests, income inequality reverberates into wealth inequality → stratify the society.
- We adapt the Galor-Zeira (1993) model to this setting.
- An individual in group \( r = \{ M, D \} \) earns market income \( y^r \).
- Utility defined over consumption \( c_r \) and bequests \( b_r \) as follows:

  \[
  u^r(c_r, b_r) = \beta \ln c_r + (1 - \beta) \ln b_r \tag{19}
  \]

- We need to consider the possibility of investing one’s inheritance, earning rate of return \( \rho > 0 \).
Stratification (2)

The PC’s for group-M and group-D individuals are now:

\[ w_j^M + y_{j,M} - h_{j,M} \geq w_j^M (1 + \rho) \quad (20) \]
\[ w_i^D + y_{i,D} - d_i - h_{i,D} \geq w_i^D (1 + \rho) \quad (21) \]

The chosen amount of bequests is a constant fraction of mkt income — the opportunity cost of interest on inherited wealth:

\[ b_r = (1 - \beta)(y^r - \rho w^r) \quad (22) \]
Wealth Inequality

Stratification (3)

- Bequest left by the current generation = initial wealth of the following one → evolution of group $r$’s wealth:

$$w_{r+1}^r = (1 - \beta)(y^r - \rho w^r) \quad (23)$$

Steady state:

$$w_{ss}^r = \frac{1 - \beta}{1 - (1 + \beta)\rho} y^r \quad r = \{D, M\} \quad (24)$$

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- Thus,

$$\frac{w_{ss}^D}{w_{ss}^M} = \frac{y^D}{y^M}$$ \hspace{1cm} (25)

Wealth inequality is proportional to income inequality.
A simple exercise shows that the amount of reparations needed to eliminate inequality in the baseline model with $\varepsilon = 0$ is

$$R = d = \eta \left( \frac{1 - \alpha}{\alpha} \right) (\alpha A)^{\frac{1}{1-\alpha}}$$

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This is way too simple!

- Society has been stratified through several generations;
- Intergenerational altruism can actually amplify wealth disparities;
- Rates of return are different across racial groups.

An infinite-horizon model will likely imply much higher wealth inequality between groups.
Stratification Economics sees discrimination as a purposeful (costly) activity by dominant groups to maintain their status.

Even though some $D$-group members will not be actively engaged in discrimination, they will still benefit from it.

Someone must have discriminated: $d_i > 0$ for at least one $i$. Discrimination is wasteful from a societal standpoint (not Pareto-efficient); Yet, it persists because anti-discrimination measures are costly to enforce, especially if the burden falls upon the discriminated. Intergenerational altruism provides the link from income inequality to wealth inequality.
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Thank you!