Measuring Intergenerational Mobility

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In this talk, I wish to give some perspectives on the importance of developing more complex measures of intergenerational mobility.

equal access to advantage, where "advantage" is understood to include, but to be wider than, welfare. Under equal access to advantage, the fundamental distinction for an egalitarian is between choice and luck in the shaping of people's fates.

-Gerald Cohen, “On the Currency of Egalitarian Justice” Ethics 1989

Roemer’s (and others) ideas of responsibility-sensitive egalitarianism have this basis.

Why Measurement?

Equal access to advantage requires measurement.

One might say that the answer requires a complete structural model of intergenerational mobility.

Whether or not this is true, my goal today is to argue that basic measurements are an important next step.

To do this, I will discuss three dimensions of intergenerational mobility measurement where some progress has been made, but there is much to do.

No claim to be exhaustive and unsurprisingly focused on my own current research.

Background: Standard Mobility Measurement

$$y_{it+1}^{P} = \alpha + \beta y_{it}^{P} + \varepsilon_{it}.$$

y_{it}^{P} is permanent income, typically average of parental income over time. The coefficient β is the intergenerational elasticity of income/earnings (IGE) and the statistic conventionally used to measure intergenerational mobility.

My claim is that there are dimensions along which to enrich such a measure.

Dimension 1: Time

Why should parental influence be summarized by permanent income of parents? Work of Cunha and Heckman and others has established the importance of skill evolution in childhood and adolescence.

Key implication for measurement: family trajectories should matter.

Key Contributions

Siwei Cheng and Xi Song, "Linked Trajectories: Intergenerational Association of Intragenerational Trajectories," *American Sociological Review* 2019.

Geoffrey Wodtke, David Harding, and Felix Elwert, "Neighborhood Effects in Temporal Perspective: The Impact of Long-Term Exposure to Concentrated Disadvantage on High School Graduation" *American Sociological Review* 2011.

Geoffrey Wodtke, with Felix Elwert and David Harding, "Neighborhood Effect Heterogeneity by Family Income and Developmental Period," *American Journal of Sociology* 2016.

In “A Trajectories-Based Approach to Measuring Intergenerational Mobility,” Yoosoon Chang, SD, Seunghee Lee and Joon Park, a complementary answer is proposed using functional data analysis.

Basic Idea

Replace the standard IGE regression with

$$y^P_{it+1} = \alpha + \int_0^R \beta(r) y_{it}(r) dr + \varepsilon_{it}.$$

so that parental incomes across childhood and adolescence have distinct effects.

Key mathematical insight: random function $y_{it}(r)$ lives in space of random functions.

After demeaning, these random functions can be decomposed with a basis; this decomposition is known as the Karhunen-Loéve expansion

Basic Idea:

The variance covariance matrix function for $y_{it}(r)$ is associated with eigenfunctions $\nu_k(r)$ (deterministic functions) so that

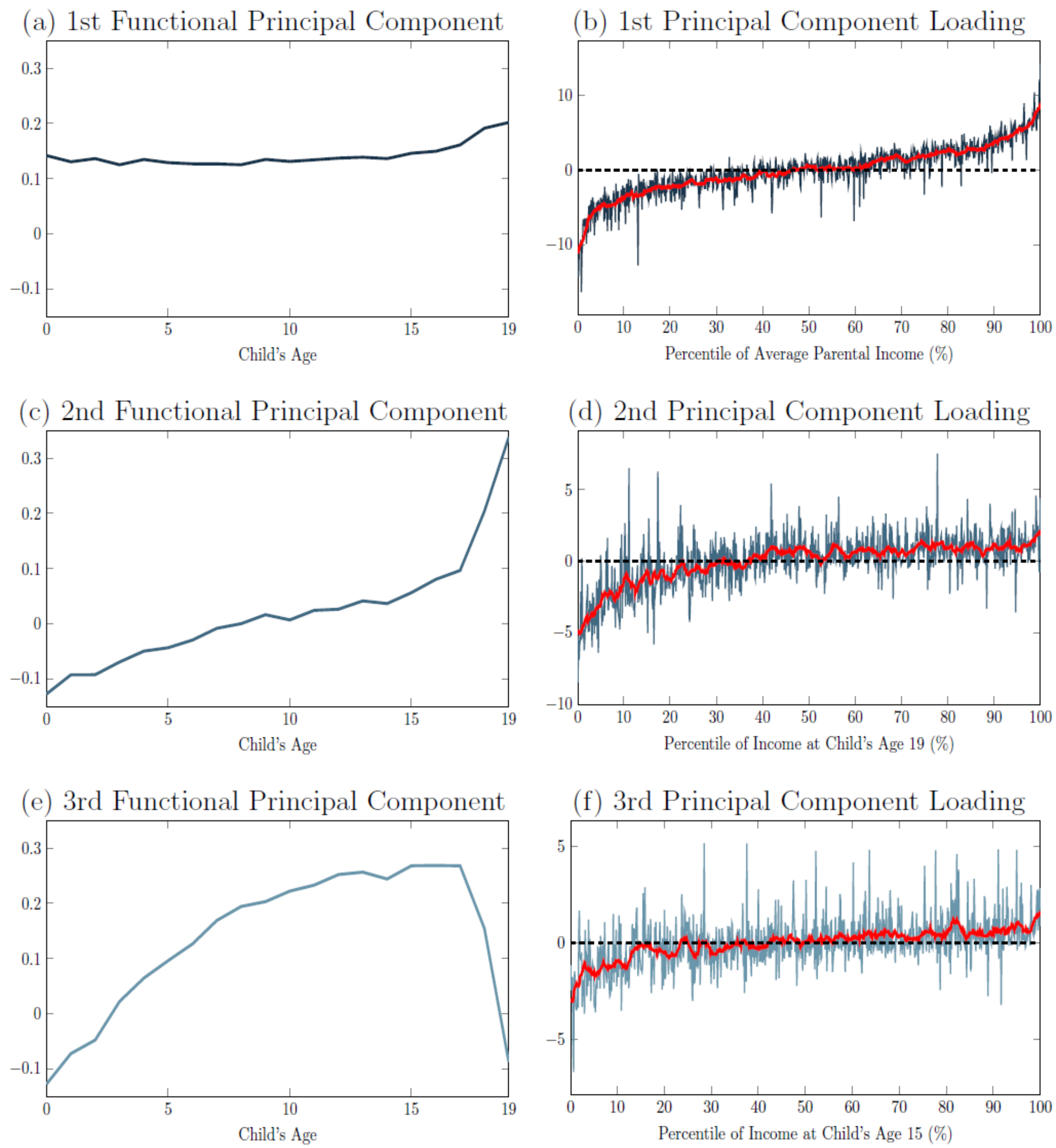
$$y_{it}(r) = \sum_{k=1}^{\infty} \xi_{ik} \nu_k \quad r$$

Chang and Park establish various optimality properties for this expansion in estimating functional regressions.

These include finite component approximation unbiasedness.

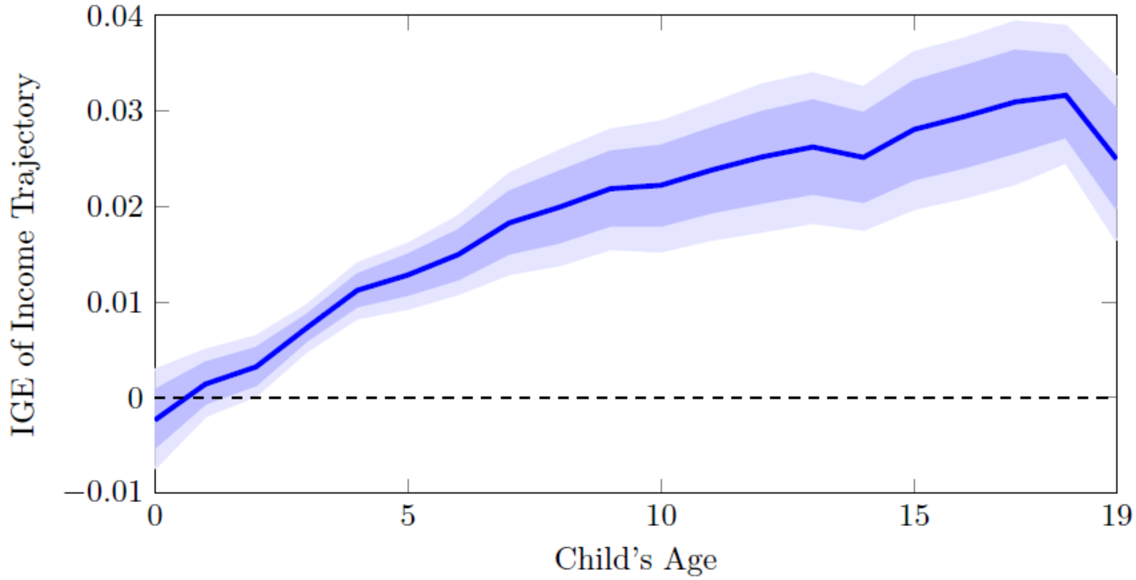
Cross-validation prediction methods lead to number of components being set at 3 for the PSID.

Figure 3: Functional Principal Components and Their Loadings



This approach produces an IGE curve $\beta(r)$

Figure 1: Age-Varying IGE of Income



Key substantive finding:

Marginal predictive value of income increases with age up to age 18.

Surprising, given current emphasis on early childhood investment. But logically consistent.

Conjecture: importance of schools and neighborhoods increases with age. Income is used for different “investments” at different ages.

How does our IGE curve relate to the standard intergenerational elasticity of income coefficient produced by a linear regression?

For the overall sample, if one were to estimate the conventional bivariate regression of permanent income onto permanent income, the estimated IGE is .57, similar to the estimate in Mazumder (2016).

The natural comparison to our function model involves the calculation of the effect of a unit increase in income for all r.

$$\int_0^R \beta(r)\,dr$$

The effect of such a change on expected future income change is the integral of the mobility curve. We estimate this integral to be .40.

This leads to the interesting result that, from the perspective of permanent income changes, the conventional linear regression *overstates* persistence across generations.

Put differently, our results show how the conventional regression produces misleading results when the underlying mobility process relies on features of parental income trajectories beyond the mean.

Conjectures on Explanations

Parental income as determinant of neighborhoods/schools. Link to Wodtke, Harding, and Elwert.

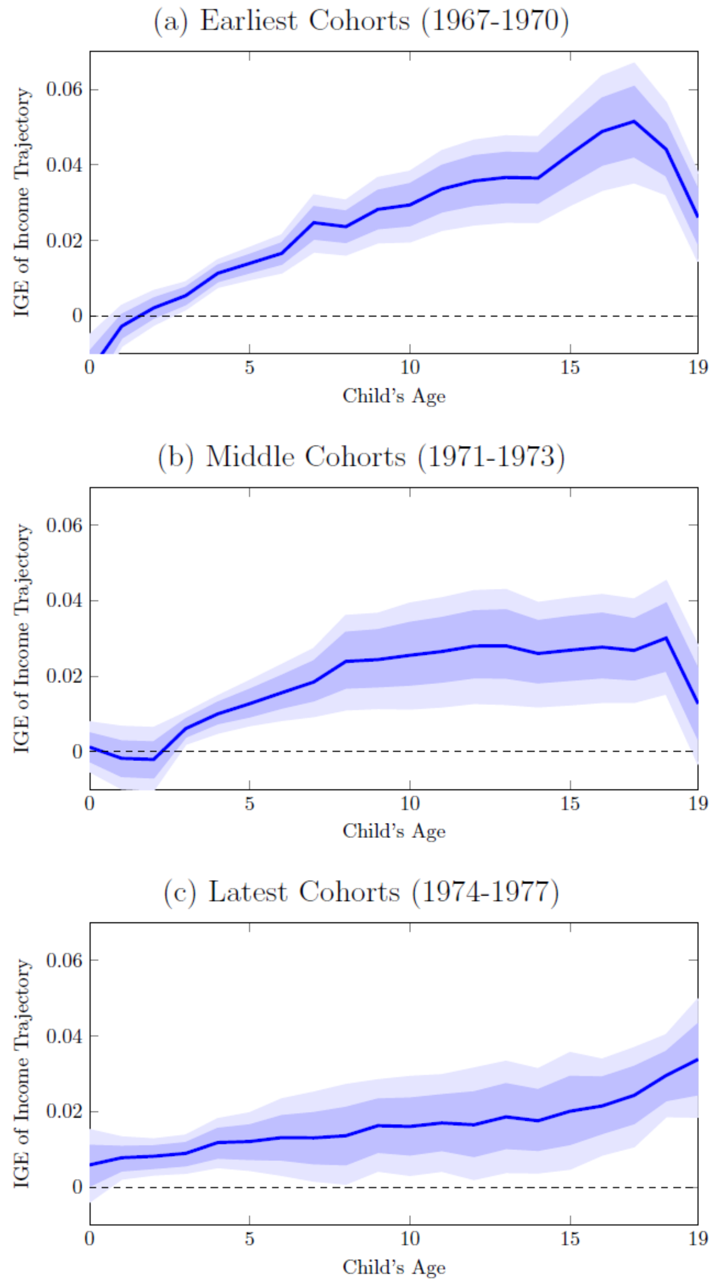
Transitory dynamics in parental income.

Results robust to Beveridge Nelson decomposition and IV.

But if transitory process is sufficiently nonstationary, then could explain results.

There are also some interesting new insights into evolution of mobility across time.

Figure 4: Age-Varying IGE of Income - Cohorts Groups



Observations

Mobility increases if one focuses on later years, decreases if one focuses on early years.

Last cohort does not exhibit age 18 effect.

Interactions/Nonlinearities

Here is a generalization of the baseline model

$$
\begin{aligned}
y_{it+1}^{P} = \\
\alpha + \int_0^R \beta(r) y_{it}(r) dr + \\
\int_0^R \int_0^R \Gamma(r,s) y_{it}(r) y_{ir}(s) ds dr + \varepsilon_{it}
\end{aligned}
$$

.

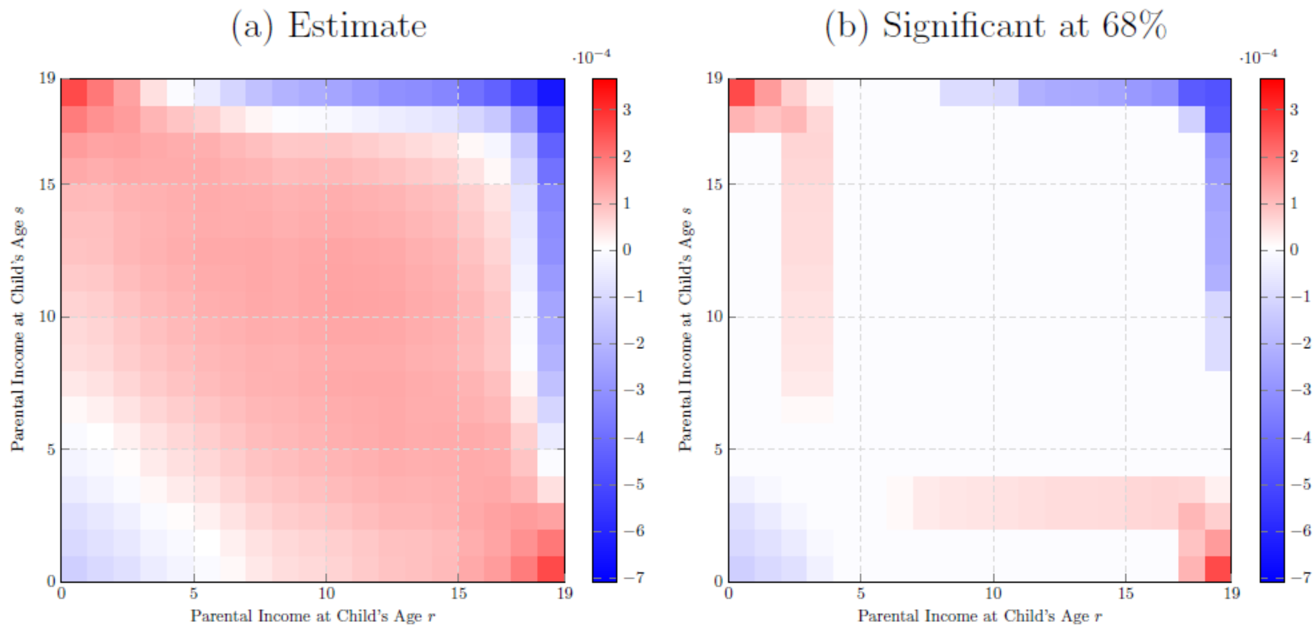
$\Gamma(r,s)$ captures interactions of incomes at different ages.

For $r \neq s$, positive values represent complementarity, negative value represent substitutability in terms of effects of incomes at different ages.

For $r = s$, the matrix captures nonlinearities.

This functional form is not ideal from perspective of theoretical models of poverty and affluence traps.

Figure 10: Quadratic Effect on Offspring's Income - Heatmap

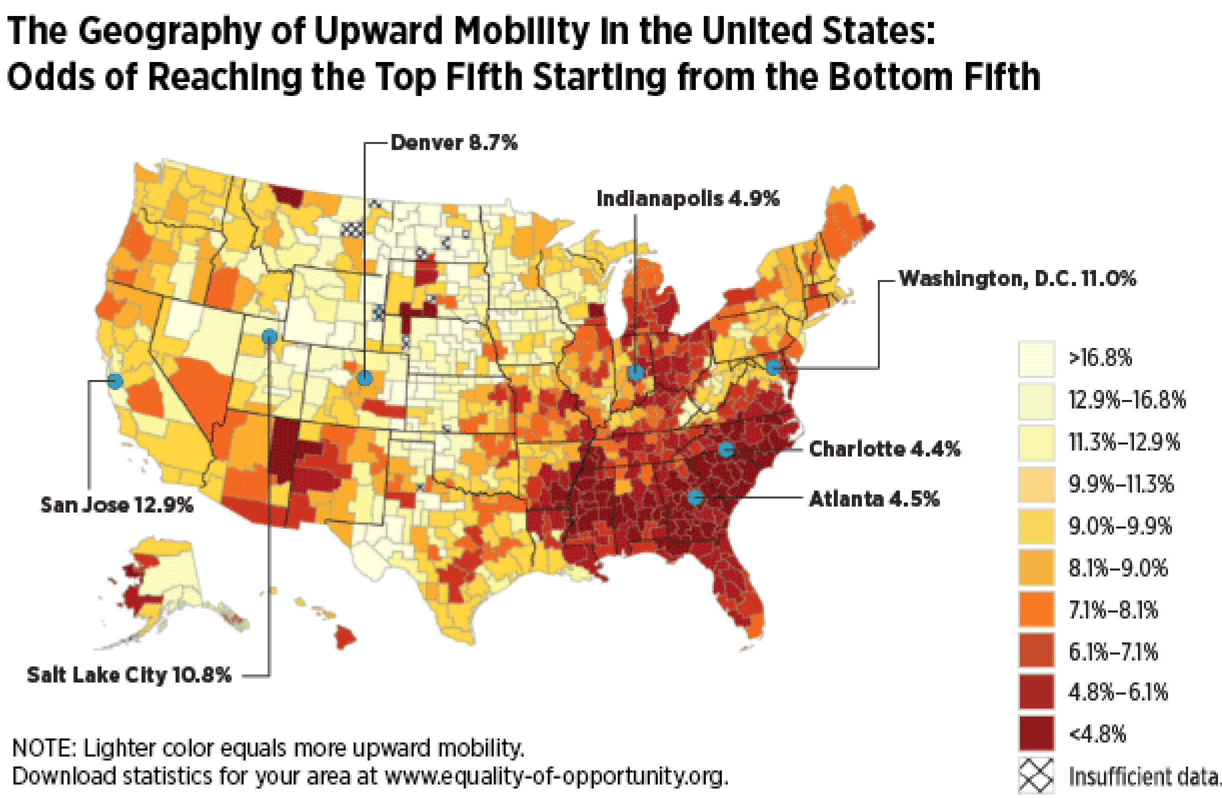


Red is complementarity, i.e. positive values for $\Gamma(r, s)$, blue is substitutability i.e. negative values

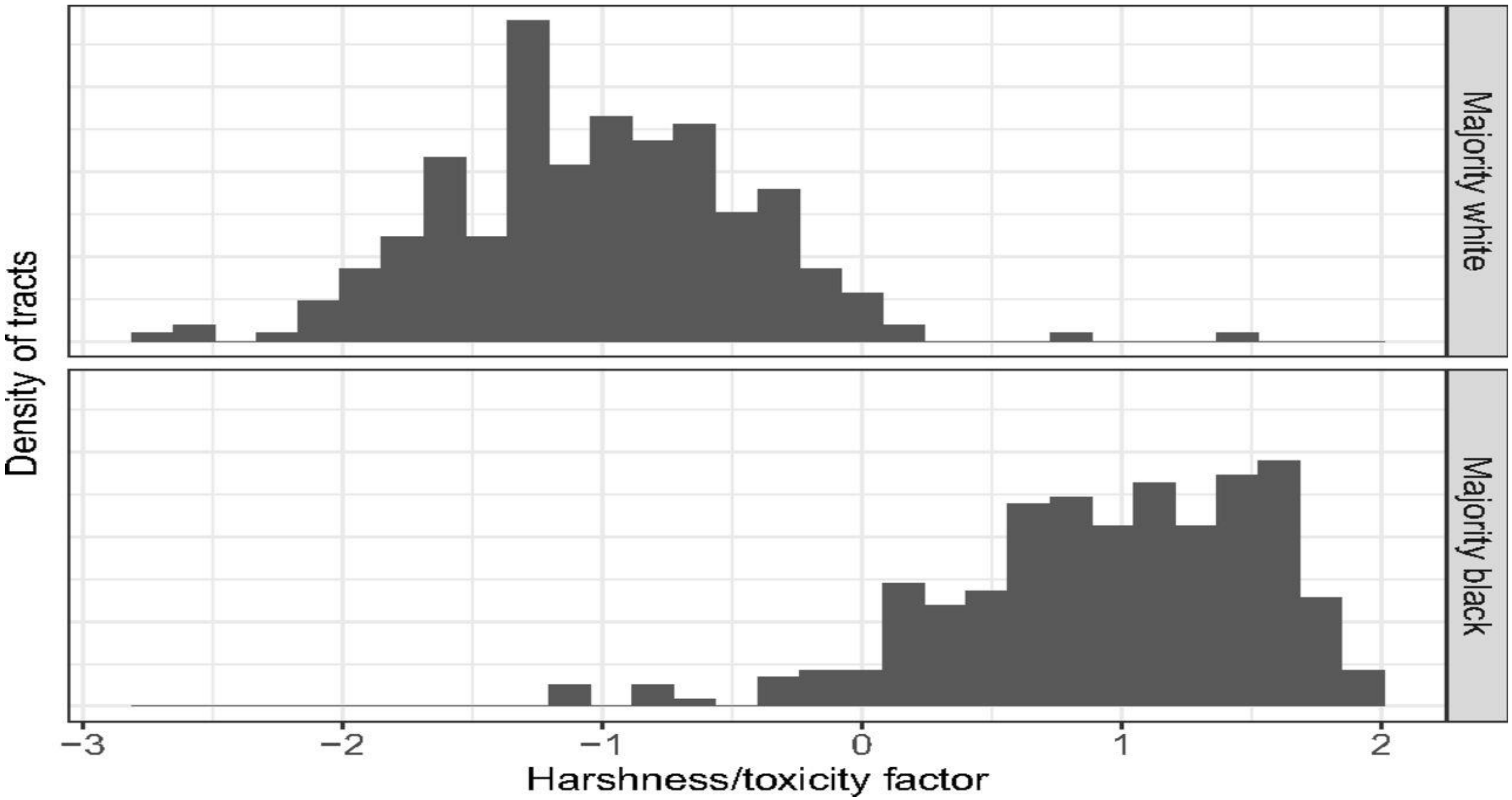
This might seem to conventional wisdom in sense that there is local substitutability and local complementarity.

However, this refers to predictive power of income rather than skills/human capital production function.

Next Step: Spatio-Temporal Integration



Raj Chetty et al

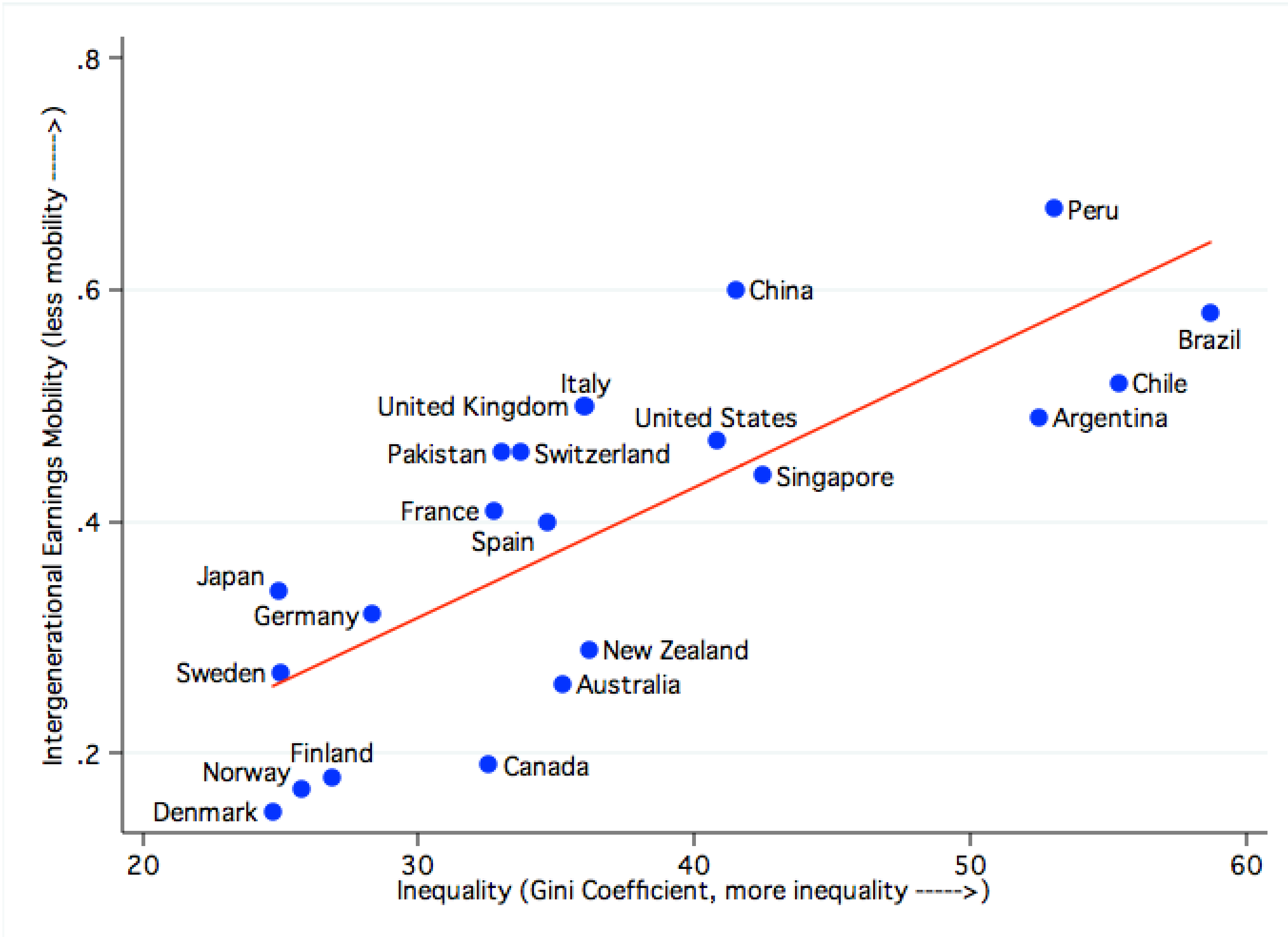


Robert Manduca and Robert Sampson

Preview of Coming Attractions

Functional Data methods can integrate time and space.

Dimension 2: Understanding Cross Sectional/Temporal Relationships: The Great Gatsby Curve



.8
.6
.4
.2
Intergenerational Earnings Mobility (less mobility ----->)
20
30
40
50
60
Inequality (Gini Coefficient, more inequality ----->)
Peru
China
Brazil
Chile
Argentina
Italy
United Kingdom
United States
Pakistan
Switzerland
Singapore
France
Spain
Japan
Germany
New Zealand
Sweden
Australia
Finland
Norway
Canada
Denmark

Miles Corak

Why is there a Measurement Issue?

Standard IGE generate GG mechanically.

$$\mathrm{var}(y_{it+1}) = \beta^2\,\mathrm{var}(y_{it}) + \mathrm{var}(\varepsilon_{it}),$$

but avoids the theoretically more natural mapping from inequality to intergenerational persistence. Here persistence begets mobility.

For relationship to move in other direction, need a nonlinear model

$$y_{it+1} = \alpha + \psi(x_{it})y_{it} + \eta_{it},$$

where x_{it} is some set of variables that are correlated with y_{it} and η_{it} is a martingale difference process.

This equation implies that the β produced in the linear projection, i.e standard regression can vary according to the distribution of y_{it}. This relationship is made explicit in White (1980)

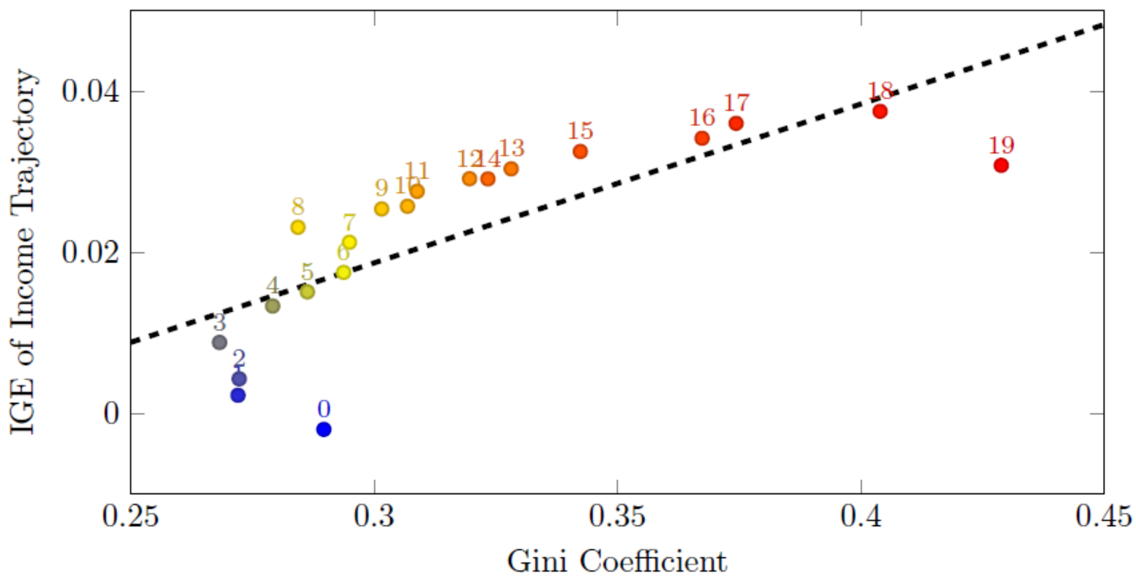
$$\beta_{linear} = \frac{\int y_{it}^2 \psi(x_{it}) \mu(x_{it}, y_{it}) dxdy}{\mathrm{var}(y_{it})},$$

where $\mu(x_{it}, y_{it})$ is the joint probability density over x_{it} and y_{it}. In this way, different variances of income may be associated with different β estimates.

Here CDLP, for example, gives two routes.

One is the quadratic model and the second is intrinsic in thinking about trajectories.

Figure 6: The Great Gatsby Curve across Child's Ages



Dimension 3:

Integrating Short Run and Long Run Mobility Measures

Which mobility statistics warrant attention?

Mobility literature divides between IGE and measure of persistence in Markov chain transition matrices (second largest eigenvalue).

Are these adequate?

Blume, Durlauf, Lukina (in progress)

Basic idea is to develop systematic measures of mobility that fully incorporate information in the Markov process.

Assume that there is a Markov chain representation for mobility P.

Assume there exists a unique invariant measure for the system μ^*

We argue that one should characterize mobility by dynamics of evolution of μ_o towards μ^*, Standard calculation of this type is the mixing rate which involves dynamics of

$$\sup_{\mu_0} \left\| \mu_o P^t - \mu^* \right\|$$

As t becomes large. This captures “worst case scenario of convergence rate”

We recommend calculation given initial conditions

$$AM(\mu_o, t) = \left\| \mu_o P^t - \mu^* \right\|$$

This curve gives dependence that “appropriately” depend on cross sectional distribution.

Applies to short as well as long time horizons

One important implication for intercountry comparisons: distinguish transition from cross section.

Comment: standard second eigenvalue measure reappears as mixing rate bound.

Collective memory does not equal individual memory. Bistochastic transition matrices as occur in rank analysis are standard example: population always at invariant measure.

So what is missing?

Exchange versus Structural Mobility

Exchange Mobility: Mobility at steady state

Structural Mobility: Mobility due to transition to steady state

These ideas have longstanding in sociology.

How to proceed?

For each state of the system j, calculate

$$IM(1_j, t) = \left\| 1_j P^t - \mu^* \right\|$$

and average across population

$$AIM(\mu_o, t) = \int \left\| 1_j P^t - \mu^* \right\| \mu_0$$

This gives a way of distinguishing exchange and structural mobility.

Exchange mobility may be interpreted via

$$EM(\mu_o, t) = AIM(\mu_o, t) - AM(\mu_o, t)$$

Other calculations

$$\sup_{\mu_0,\mu_o'} \left\| \mu_o P^t - \mu_o' P^t \right\| =$$
$$\sup_{1_i,1_j} \left\| 1_i P^t - 1_j P^t \right\|$$

Special case

$$\lim_{t\to\infty} \left\| 1_{\max} P^t - 1_{\min} P^t \right\| \neq 0$$

This is Durlauf (1996a,b) and Durlauf and Seshadri (2017) measure of traps.

Why Does this Matter?

How to think about mobility against background of dramatic growth?

Kristina Butaeva, Lien Chen, SD, Albert Park (in progress)

China and Russia exhibit more educational and occupational exchange mobility than the US even if they exhibit less mobility per se.

Structural Versus Exchange Mobility Father to Child

	Father to child			Younger cohort			Older cohort		
	China	Russia	US	China	Russia	US	China	Russia	US
				Education					
AM	0.74	0.19	0.16	0.66	0.19	0.10	0.71	0.20	0.23
EM	0.01	0.05	0.16	0.01	0.06	0.20	0.04	0.04	0.12
AIM=AM+EM	0.75	0.23	0.32	0.67	0.24	0.30	0.76	0.24	0.35
				Occupation					
AM	0.22	0.16	0.05	0.17	0.14	0.04	0.26	0.17	0.06
EM	0.03	0.10	0.18	0.02	0.08	0.23	0.07	0.12	0.14
AIM=AM+EM	0.25	0.26	0.23	0.20	0.22	0.27	0.33	0.29	0.20

Structural Versus Exchange Mobility: Mother To Child

	Mother to child			Younger cohort			Older cohort		
	China	Russia	US	China	Russia	US	China	Russia	US
				Education					
AM	0.92	0.16	0.10	0.78	0.13	0.05	1.07	0.21	0.18
EM	0.00	0.08	0.18	0.00	0.09	0.22	0.01	0.08	0.14
AIM=AM+EM	0.93	0.24	0.29	0.78	0.22	0.27	1.08	0.29	0.32
				Occupation					
AM	0.25	0.03	0.06	0.20	0.02	0.04	0.27	0.03	0.06
EM	0.03	0.22	0.14	0.04	0.20	0.18	0.10	0.24	0.13
AIM=AM+EM	0.28	0.25	0.20	0.24	0.21	0.23	0.36	0.27	0.19

SD, Gueyon Kim, Dohyeon Lee, and Xi Song, black-white mobility in US.

This work involves a new issue: Evolution of P.

For US, slavery, reconstruction, Jim Crow, post WW2, Civil Rights movement, and modern US are distinct regimes.

This type of approach needed more generally.

Bottom Line

Intergenerational mobility research one of the most exciting areas of contemporary social science, but much remains to be done in enriching measurement methods.