Does a Common Application Increase Access? Theory and Evidence from Boston's Charters*

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This version: November 2022

Abstract

Several US cities have recently adopted a common application to assign pupils to schools. In a common application, students can apply to multiple schools on a single form, and each school admits students without taking offers from other schools into account. A common application increases access by reducing application costs, but since each school receives more applications it can also decrease access. We develop a model of a common application system and compare it to a (1) decentralized system, where each application is costly, and a (2) ranked system, where applicants rank choices on a common application and offers are generated by a matching algorithm. Compared to the decentralized system, the common application yields more applications to each school. However, the screening benefits of costly applications in the decentralized system can outweigh increases in school access from lower application costs in the common application because of greater competition. We show that under independently drawn preferences, the distribution of ordinal applicant rankings from the decentralized system stochastically dominates the distribution from the common application. The ranked system improves upon the common application by minimizing issues caused by multiple offers, while still benefitting from lower application costs.

We then empirically examine the effects of switching from decentralized applications to a common application in Boston's charter sector. Demand estimation and counterfactual simulations suggest that the common application performs no better than either a decentralized application system or a ranked system on several dimensions. We estimate that fewer students receive their

*Our thanks to Cara Nickolaus, Marcela Ulloa, Sean Wang, and Hellary Zhang for research assistance and Eryn Heying and Jim Shen for valuable administrative support. We thank seminar participants at the MIT IO tea for helpful comments. This research would not be possible without the cooperation of Boston's charter schools and the Massachusetts Department of Elementary and Secondary Education (DESE), for which we're grateful. The research described here was carried out under data use agreements with Boston charter schools and MA DESE, but in no way reflects their views. Pathak is a co-founder of Avela Education.

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first choice under the common application than either decentralized or ranked systems. The common application does not appear to have differentially increased school access for disadvantaged student groups. We use our empirical model to examine how levels of competition and the extent of changing preferences between application and enrollment decisions affect comparisons across systems.

1 Introduction

Several US cities have recently adopted common application systems to assign pupils to K-12 choice schools. In a **common application** system, students apply to multiple schools on a single application, but do not report a ranking over their choices. Schools make offers to students without taking into account offers from other schools. Table 1 lists 12 large cities where a common application has been adopted since 2017. In most cases, only charter schools are part of the common application. Discussion from these locations focuses on three main rationales for adopting a common application: (1) lowering costs and logistical barriers for applicants, (2) increasing information about schools, and (3) equalizing opportunities to exercise choice for all families.[1]

Common application systems often replace decentralized systems. In a **decentralized** system, applicants must apply to each school by submitting an application to the school and schools make offers to students without taking into account offers from other schools. Often, applications must be physically submitted at the school building and an applicant must be present when the school makes offers of admission. This is common, for example, when a charter school runs a public admissions lottery. Relative to a decentralized system, a common application reduces application costs for students interested in multiple schools because a student interested in more than one school can simply fill out a single application form rather than separate applications for each school.

Several of the cities listed in Table 1 also considered a ranked system. In a **ranked** system, applicants use a common application form and submit a ranking over schools and offers of placement are given to students using a matching algorithm and each student is given at most one offer. Boston, for example, adopted a common application for charter schools in 2017, following more than a decade with a decentralized application system. During the debate over Boston's common charter application,

[1] These rationales were highlighted when new systems were announced. Oakland Enrolls, for example, described that "Oakland families no longer have to drive all over town from one charter school to another, fill out various papers applications, and keep track of different deadlines." Buffalo officials explained: "It's our hope that this platform increases access to and knowledge of the area's charter schools and simplifies the application process for families." Los Angeles staff highlighted that "A common enrollment system tends to level the playing field and get more equal opportunity to all families." Appendix A provides more details about these reforms.

several stakeholders argued that the city should adopt a ranked system unifying applications across all school sectors, following the model of Denver and New Orleans (Fox, 2016). The Boston Mayor convened a working group to consider a ranked system and the plan received strong support from several groups, but ultimately was not adopted (Globe, 2015).

In this paper, we develop a model to study the effects of a common application on student access to schools. The model captures the trade-off between reducing application barriers and increasing competition for schools. In the model, under the common application, a student's first application has cost c, but subsequent applications are costless. Under the decentralized application, each application to a school costs c. Under the ranked application, an application costs c and students express a ranking over choices. In all three cases, schools use independent admissions lotteries and offers are made to equilibrate supply and demand. Our main interest is using the model to compare the common application to these two alternatives.

The model generates several intuitive predictions: (1) each school receives (weakly) more applications in equilibrium with the common application than the decentralized application, (2) the equilibrium admissions rate under the common application is (weakly) lower than the equilibrium admissions rate under the decentralized application, and (3) the common application implements an assignment that is not pairwise stable with positive probability, but the ranked system's assignments are always pairwise stable. Our main theoretical result is that with independent symmetric preferences, the distribution of ordinal rankings for enrolled students under the decentralized system stochastically dominates the distribution of ordinal ranks under the common application.

Our model represents a frictionless comparison between assignment systems under specialized assumptions. These assumptions allow us to isolate fundamental trade-offs between the systems analytically. However, even with this frictionless environment, we do not have definitive comparisons about student welfare or increased school access across systems. Several examples from our model show why sharp comparisons are likely not possible without strong assumptions on costs or the distribution of student preferences. Moreover, the model does not incorporate changing preferences after applications and before enrollment, and abstracts away from dynamic market clearing, two features which are important in practice. We therefore develop an empirical model with several real-life frictions, taking advantage of unique data from Boston's charter sector, which moved from a decentralized system to a common application in the 2017-18 school year. We estimate student preferences for charter and traditional public schools together with school admissions rules to examine the decentralized, common, and ranked application systems. The model allows us to quantify magnitudes of forces highlighted by the theoretical model and incorporate some aspects that are not part of the theoretical model. We can also simulate the effects of a ranked system, which was nearly adopted by Boston's charter sector.

Using our empirical model, we find that fewer students receive their first choice under the common application (33%) than either a decentralized or ranked system (37% each). Boston's charter common application appears not to have differentially increased school access for disadvantaged student groups. Estimates from the demand model show that the common application does not significantly increase overall student welfare compared to a ranked system or the decentralized system. All three systems generate about 70% of the total possible utility from an assignment that maximizes total applicant utility, using cardinal utility estimates from the demand model.

To consider situations outside of Boston, we then use the empirical model to examine how levels of competition and the extent of changing preferences affect our comparison of one system over the other. The best-case scenario for the common application is in situations where students apply to a few schools and are admitted to almost all of them and when preferences change substantially between the application decision and the enrollment decision. The best-case scenario for a ranked system is when there is substantial competition for school spots. A decentralized system performs best when there is modest competition for school spots and there are not extreme changes between preferences between the application and enrollment decision.

This paper is related to several literatures. First, several studies examine the *Common App* used for U.S colleges and universities. Knight and Schiff (2022) document that the *Common App* increased the number of applications received by schools and reduces the yield on accepted students. We find the same pattern with the adoption of Boston's common charter application. Other studies of the *Common App* include Liu, Ehrenberg, and Mrdjenovic (2007), Smith (2013), and Klasik (2012). Fu (2014) develops a structural model of the college market, where tuition, applications, admissions and enrollment are determined as the outcome of a game. Second, our work also connects to the literature on "preference signaling" in matching markets, which also focuses on colleges and universities. Avery and Levin (2010) study how early admissions affects student applications and sorting. Chade, Lewis, and Smith (2014) examine frictional matching, motivated by college admissions. Che and Koh (2016) develop a model of college admissions when a college strategically admits students that are likely to be overlooked by competitors and consider an alternative institution based on a centralized matching algorithm. Grenet, He, and Kubler (2022) use data from Germany's university admissions system and demonstrate that applicants do not have full information about their own preferences and a dynamic mechanism, which facilitates preference discovery has desirable properties. Our empirical results on Boston's common application also highlight the importance of preference discovery since a benefit of the common application is that students can take advantage of the flexibility of multiple offers if their preferences change between the application and enrollment decision. Third, our work connects to a literature that focused on market clearing in student assignment. A non-exhaustive list of related papers includes Abdulkadiroğlu and Sönmez (2003), Abdulkadiroğlu, Agarwal, and Pathak (2017), and Chen and He (2021). Finally, since a common application is seen as a way to reduce application frictions, our work is related to literature which documents and examines ways to alleviate application barriers in the context of college admissions (see, e.g., Hoxby and Avery (2013), Hoxby and Turner (2015), and Pallais (2015)) and in accessing social programs (see, e.g., Deshpande and Li (2019)).

This paper is structured as follows. Section 2 compares decentralized application to the common application and Section 3 compares common application to the ranked application. Section 4 provides details on the Boston charter common application. Section 5 describes the structural model. Section 6 reports estimates and the counterfactual simulations. Section 7 uses the empirical model to examine situations other than Boston. The last section concludes.

2 Decentralized vs. Common Application

2.1 Model and Basic Properties

Suppose there are S schools and N students. Each school has capacity K. Student i has utility values $(u_{i1}, u_{i2}, \ldots, u_{iS})$ where $u_{ij} > 0$ represents student i's utility for attending school j. Each student's utility values $(u_{i1}, \ldots, u_{iS})$ are independent and identically distributed draws from a symmetric joint distribution F, so that all rank orders of preferences are equally likely for students.

We start by considering two application systems:

- **Decentralized Application**: Each application to a single school has known cost $c > 0$. Each student chooses to apply to any number of schools from 0 to S.
- **Common Application**: A student's first application has cost $c > 0$, but all subsequent applications are costless, so each student applies either to no schools or to all S schools.

We consider the Nash equilibrium of the game where applicants simultaneously decide which schools to apply to after observing their utility values, but not the utility values of other students. Our first result is about equilibrium existence:

Proposition 1 *There exists a unique pure-strategy equilibrium for the Common Application system and it yields the same admission probability at each school. There exists a symmetric equilibrium for the Decentralized Application system, but it is not necessarily unique.*

An increase in admissions probability at any school necessarily promotes applications. Thus, enrollment is broadly increasing in the admissions probability, which in turn results in a unique equilibrium with the Common Application.

By contrast, while an increase in admissions probability for one school promotes applications from students for whom that school is a first choice, it may discourage further applications to other schools from those same students. As a result, the total number of applications submitted need not be monotonic in the admission rates for the schools, which in turn can produce multiple equilibria. We illustrate the phenomenon of multiple equilibria in the following example.

Example 1 *Suppose that there are* $2N$ *students and two schools, each with capacity* $0.75N$*. Half of the students have utility values (8, 4) and the other half have utility values (4, 8).*

Under the Decentralized Application system, if each student submits a single application to her most preferred school, then the schools choose admission probability $p = 3/4$ to fill their classes. This is an equilibrium if (1) all students gain by submitting the first application, i.e. $(3/4) * 8 > c$, or $c < 6$ and (2) all students prefer to submit only one application, i.e. $(3/4)(1/4)4 < c$ or $c > 3/4$. By contrast, if each student applies to both schools, then the schools reduce their admission probabilities to $p' = 1/2$ in order to fill their classes. This is an equilibrium if each student's second application yields positive expected utility, i.e. $(1/2)(1/2)4 > c$, or $c < 1$. (Note that this condition is also sufficient for each student's first application to yield positive expected utility.)

In a pure-strategy equilibrium under the Common Application system, each student submits two applications and the schools choose admission probability $p' = 1/2$ to fill their classes. But now the equilibrium condition is that students prefer two applications at cost c to no applications, i.e. $(1/2)8 + (1/4)4 > c$ or $c < 5$.

Cost c	Decentralized Application	Common Application
$c < 3/4$	2 apps per student	2 apps per student
$3/4 \leq c \leq 1$	Multiple Equilibria	2 apps per student
$1 \leq c \leq 5$	1 app per student	2 apps per student

Comparison of Equilibria in Example 1

The table summarizes the equilibrium analysis above. One immediate observation is that there can be multiple pure-strategy equilibria of the Decentralized Application system with $3/4 < c < 1$. For the lowest values of c, students submit two applications under either admissions rule. In this case, the Common Application rule yields higher expected utility to students than the Decentralized Application rule, for the two rules achieve the same school assignments (in probability) but with lower application costs for the Common Application.

For higher values of c between 1 and 5, students submit one application given a Decentralized Application rule and two applications given the Common Application rule. In this case, application costs are the same in both systems, but students receive higher expected utility under the Decentralized Application rule because the Decentralized Application rule only assigns admitted students to first-choice schools whereas the Common Application rule assigns some admitted students to second-choice schools.

Propositions 2 and 3 below verify that the intuitions suggested by Example 1 hold for general symmetric distributions of utility values for students and for more than two schools.

Proposition 2 *Comparing the Common Application and (any) symmetric equilibrium of Decentralized Application, the Common Application equilibrium yields more applications to each school and each school admits a lower percentage of applicants.*

Proof. (Admissions Rate) If the admissions rate is the higher under Common Application than in the Decentralized Application system, then the same students who apply under Decentralized Application will all apply under the Common Application. In addition, some students who do not apply under the Decentralized Application will apply under Common Application. Then, more students will be admitted under the Common Application and more will enroll at each school. (The single exceptional case occurs if exactly the same students apply in both cases and all apply to all schools, in which case, both systems would yield the same outcome, but this does not occur with a continuous distribution of values.)

(Number of Applications) Suppose in equilibrium under Decentralized Application, there are n_k students who apply to k schools, so that we can summarize the number of applications by $(n_1, n_2, \ldots, n_S)$. In a symmetric equilibrium, these applications are distributed equally across the S schools. The number of students admitted to at least one school is then given by the sum:

$$pn_1+[p+p(1-p)]n_2\ldots+[p+(1-p)p+(1-p)^2p+\ldots+(1-p)^{S-1}p]n_S = \sum_{s=1}^{S}(\sum_{j=1}^{s} p(1-p)^{j-1})n_s.$$

Thus, a student's k^{th} application adds probability $(1-p)^{k-1}p$ of admission to at least one school, which is decreasing in k. For any given admission probability p, the minimum total applications to clear the market occurs if each student applies to only one school, while the maximum total applications to clear the market occurs if each student applies to all S schools. Further, both the minimum total number of applications and maximum total number of applications to clear the market are clearly decreasing in p.

The Common Application equilibrium has all students applying to S schools, so corresponds to the maximum number of applications for admission probability p_C, which in turn is greater than the maximum number of applications for the (larger) admission probability for the decentralized equilibrium. This proves the theorem. ■

2.2 Drawbacks of Common Application

Our next proposition shows that assignments of students to schools that result from a Decentralized Application equilibrium stochastically dominate (in student utility) the assignments of students to schools that result from a Common Application equilibrium. Intuitively, there are two distinct forces that are responsible for this result. First, the Common Application induces students to apply to a longer list of schools, thereby increasing the proportion of admissions to less preferred schools. Second, since the Common Application equilibrium has a lower probability of admission than an Decentralized Application equilibrium, a student who is admitted to a less preferred school is relatively unlikely to also be admitted to a more preferred school in a Common Application system. These two forces together yield a systematically less desirable assignment of students to schools with the Common Application.

To facilitate welfare comparisons, assume that when there are multiple equilibria for Decentralized Application, then the symmetric equilibrium with fewest applications is played. This is a Pareto improvement over any other symmetric equilibrium for the Decentralized Application rule. This refinement allows us to state the next result.

Proposition 3 *With independent symmetric preferences, the distribution of ordinal ranks for enrolled students under the Decentralized Application system stochastically dominates the distribution of ordinal ranks for enrolled students under the Common Application system.*

2.3 Heterogeneous Application Costs

One motivation for the Common Application is to enhance opportunities for low-income students and others who have limited access to skilled guidance. To allow for this possibility, we expand the model to allow for differential application costs for two types of students, where applicants of Type 1 ("Advantaged") have cost per application c_1 and applicants of Type 2 ("Disadvantaged") have cost per application $c_2 > c_1$. Under the Common Application system, we assume that Type 1 students can apply to all schools at cost c_1 and that Type 2 students can apply to all schools at cost c_2. We continue to assume that both types of students have identical and symmetric distributions of utility values for the schools.

Proposition 4 *With differential costs, if $c_1 = 0, c_2 > 0$ and Type 2 students do not all apply to both schools in the Decentralized Application equilibrium, then the Common Application unambiguously increases the number of Type 2 students who are assigned to a school.*

Proof. If $c_1 = 0$, then all Group 1 students apply to all schools under any admissions system. As in Proposition 2, the admission probability at each school in the Common Application must be lower than the admission probability at each school for the Decentralized Application. Otherwise, more students will apply and more students will be admitted to at least one school using the Common Application rule, thereby causing over-enrollment. But since all Type 1 students apply exhaustively in each system, they have lower probability of admission to at least one school in the Common Application equilibrium. Thus, to make up the deficit in enrollment and maintain the market clearing condition, a larger number of Type 2 students must enroll in the Common Application equilibrium than in the Decentralized Application equilibrium. ■

Proposition 4 is limited in two important ways. First, it does not consider application costs. Even if the assignments to Type 2's improve with the Common Application, there might be an even larger increase in costs for this group. Second, Proposition 4 pertains specifically to the number of seats assigned to Type 2 students and not the expected utility to Type 2 students of school assignments, either net of or excluding application costs. The assignments for Type 2 students might be worse with Common Application than with Decentralized Application. As a result, it is possible that Type 2 students could gain seats but lose expected utility from school assignments with the Common Application. (This is true for Type 1 in Example 2.)

Despite the natural intuition that the Common Application should level the playing field and thus be beneficial to Type 2 students, it could have the opposite effect, as shown by Example 2.

Example 2 *Adjust Example 1 as follows. Suppose that there are $2.5N$ students and two schools, each with capacity $0.75N$. There are $2N$ students of Type 1 with application cost c_1. Of these students, half*

(N students) have utility values (8, 2) and the other half (N students) have utility values (2, 8). There are $0.5N$ *students of Type 2 with application cost* $c_2 > c_1$*. Of these students, half (0.25N students) have utility values (8, 2) and the other half (0.25N students) have utility values (2, 8).*

Note that the conditions for Proposition 4 do not hold in this example. Under the Decentralized Application system, if all students submit a single application to their most preferred school, then each school receives $1.25N$ applications and each school can fill its class by choosing an admission probability of 0.6.

Under the Common Application system, if only students of Type 1 apply, $2N$ students apply to each school and, after accounting for the students who are admitted to both schools, each school would fill its class by choosing an admission probability of 1/2.

Given these admission probabilities, students achieve expected utility from school assignments of $0.6 * 8 = 4.8$ in the Decentralized Application system and expected utility from school assignments of $0.5 * 8 + 0.25 * 2 = 4.5$ with a Common Application rule. Thus, for any pair of values c_1 and c_2 satisfying $0.5 < c_1 < 4.5 < c_2 < 4.8$, these are the unique equilibrium outcomes for the two application rules.

In this example, the introduction of the Common Application induces students of Type 1 to apply to both schools rather than to a single school, and that choice in turn reduces the equilibrium admission probability and discourages Type 2 students from participating. Therefore, we have to add a caveat to the earlier intuition – the Common Application will be beneficial to disadvantaged types if advantaged types are already applying very extensively.

3 Common Application vs. Ranked Application

3.1 Ranked Application

Next, we consider a ranked application system. Under this system, students submit a rank ordering of schools. Schools use a single lottery to order students, and then the student-proposing deferred acceptance (DA) is run.[2] We no longer assume that all schools have the same capacity. Suppose now that school capacities may differ and denote school s's capacity as k_s.

Definition 1 *A Common Application equilibrium consists of admission probabilities* $(p_1, \dots, p_S)$ *such that expected enrollment at school* j *is* k_j *given independent admissions decisions.*

[2]When all schools share a common ranking over students, both student-proposing and school-proposing DA produce the same outcome.

This is the rule that would be chosen by a school that seeks to hit their enrollment target with symmetric loss function for over and under-enrollment.

Proposition 5 *There is a unique combination* $(p_{1C}, p_{2C}, \ldots, p_{SC})$ *of admission probabilities such that each school enrolls* K *students in expectation. In the limit as* N *and* K *become large, the school-proposing or student-proposing version of the DA algorithm match students to schools with the same probabilities as the Common Application if schools submit independent and randomly generated rankings of students.*

The mechanical intuition for the asymptotic equivalence of these rules is straightforward. In the school-proposing version of the DA algorithm, each school progresses down its rank order preference list, making offers to some number of most preferred candidates in each round in rote fashion. This process is analogous to a Common Application rule, as the students have implicitly submitted applications to all of the schools and the schools respond by making offers to students in stages as the algorithm progresses. The algorithm concludes with each school having enrolled its desired number of students, at which point, each school j will have admitted some proportion q_j of applicants. Since each school's rank order list in this context is generated at random, the admission probabilities $(q_1, q_2, \ldots, q_S)$ observed in a single implementation of the DA algorithm produce the same statistical distribution of assignments of students to schools as a Common Application rule with these same admission probabilities.

The proof of the equivalence of the student-proposing and school-proposing versions of the DA algorithms follows similar lines. In both the student-proposing and school-proposing versions of the algorithm, the schools set cutoffs in their ranking lists and then each school admits only those students who are above its cutoff. Unmatched students in the student-proposing version of the algorithm make proposals and are rejected by all schools. Each matched student makes proposals to schools on her preference lists until she reaches the school where she is accepted and assigned by the algorithm. Yet, though these students do not make proposals to all schools, they would receive the same assignments if they did make proposals to all schools, as these additional proposals would be to schools that they would not choose if admitted. We can therefore view the student-proposing version of the algorithm as another way of arriving at the assignment probabilities from the unique Common Application outcome.

Intuitively, the Ranked Application relies on revealed preference to determine assignments of students to schools, with the well-known result that it achieves an ex-post efficient matching. In particular, the first school to reach capacity in a Ranked Application only enrolls students who prefer it to all other schools. By contrast, under the Common Application, every school enrolls some students who were admitted only to that school and had no other options. With two schools, in fact, the ordinal distribution of matches from the Ranked Application stochastically dominates that from the Common Application. With more than two schools, the relative advantage of the Ranked Application is most easily expressed in terms of the stability of the assignment outcome.

3.2 Comparison in Large Markets

Our next two results assume large N and that admissions probabilities converge to expected values.

Proposition 6 *Each school offers admission to a smaller proportion of students with the Common Application than it does with Ranked Application.*

Proof. Order the schools so that in expectation, if $j < j'$ then School j reaches capacity before School j' with Ranked Admissions. With Ranked Application, every student who is admitted to School 1 is also admitted to all other schools. But with the Common Application, some students who are admitted to School 1 are not admitted to all other schools. Therefore the probability of enrolling at School 1 conditional on receiving an offer from School 1 is higher with the Common Application than with Ranked Application, which means in turn that School 1 fills its class with fewer offers using the Common Application than with Ranked Admissions.

Furthermore, with Ranked Application, every student admitted to School 2 is also admitted to Schools 3, 4, 5, ..., S, and some proportion are also admitted to School 1. School 1 admits a lower proportion of students with the Common Application than with Ranked Admissions, so for School 2 the probability of competing with School 1 is lower with the Common Application than with Ranked Application. Thus, the probability of enrolling at School 2 conditional on receiving an offer is higher with the Common Application than with Ranked Admissions, so once again, School 2 fills its class with fewer offers with the Common Application than with Ranked Application. A similar argument demonstrates in order that Schools 3, 4, ..., S, each make fewer offers in expectation with the Common Application than with Ranked Application. ■

Corollary 7 *A higher proportion of students are assigned to first-choice schools with Ranked Application than with the Common Application.*

This is immediate because the probability of admission to any particular (first-choice) school is higher with Ranked Application than with the Common Application. However, as shown in Example 3 in the Appendix, the ordinal distribution of assignment probabilities with Ranked Application does not necessarily stochastically dominate the ordinal distribution of assignment probabilities with the Common Application.

Proposition 8 *Given a set of applicants who do not all have the same set of preference orderings over schools, Ranked Application guarantees a pairwise stable assignment of students to schools, while there is positive probability that the Common Application implements an assignment of students to schools that is not pairwise stable.*

Proof. Comparing any two applicants with distinct set of preference orderings, suppose that student A is assigned to school s_A and student B is assigned to school s_B with Ranked Application. If student A prefers s_B to s_A, then by revealed preference, school s_B was not available when it was her turn to choose a school. So student B must had priority over A and had the option of choosing either s_A or s_B were available. By revealed preference, then both student A and student B prefer s_B to s_A and so cannot gain from pairwise trade.

By contrast, if students A and B have different preference orderings in the context of the Common Application rule, then there is some pair of schools s_A and s_B such that student A prefers s_A to s_B while student B prefers s_B to s_A. But there is positive probability that student A is admitted only to school B and that student B is admitted only to school A and thus positive probability that there would be a gain from pairwise trade for the students given their assignments in the Common Application. ■

4 Boston's Charter Common Application

The model developed in the previous section makes simplifying assumptions that allow for theoretical comparisons between assignment systems. Our analysis shows that the Decentralized Application exhibits a force towards positive selection in the choice of applications, which results in placing more applicants at preferred schools. However, students who submit multiple applications pay larger costs for doing so in Decentralized Application than with the Common Application system. This tradeoff between the better matches enacted with Decentralized Application and lower costs of applications with a Common Application system may either benefit or hurt students, and we do not believe that it is possible to produce a universal ranking of the systems in terms of student welfare without very strong assumptions. Moreover, admissions probabilities in the model are the consequence of static equilibration of demand and supply. In practice, applicant preferences may change in the period between application and enrollment decisions as students experience taste shocks due to changing information or circumstances. When a student turns down an admissions offers, schools often make a set of offers from their wait lists.

In this section, we develop an empirical model to quantify the differences between systems, taking advantage of unique data from Boston's charter sector, which moved from a decentralized admissions system to a common application system. Our empirical model is based on data from two different decision points for an applicant: application and enrollment. To accommodate dynamic aspects of admissions, the model allows for student preferences to change between the admissions and enrollment stage.

4.1 Background

In 2016, Boston charter schools announced an online common application for Fall 2017 enrollment known as the *Boston Public Charter School Application* (BPCSA). The BPCSA system replaced a decentralized application in which applicants would fill out a separate form for each school. This change was implemented partially in response to the failure of an effort to integrate the enrollment of Boston's charter and district schools (Vaznis, 2016). At the time of the application's launch, the BPCSA included all Commonwealth charters – those "overseen by the state education department and operate independently of Boston Public Schools" – with the exception of Bridge Boston (Kennedy, 2016).[3]

The main entry points for Boston charter schools are grades 5, 6, and 9. We analyze grade 5 throughout to focus on the charter sector and a fallback outside option of traditional public schools.[4] Our focus on grade 5 allows us to abstract away from choice within traditional public schools, where grade 6 is the main transition grade, and choice for exam schools, which admit students in grade 7.[5] For admissions starting at grade 5, our sample includes five schools for which we observe application data: Academy of the Pacific Rim, Boston Collegiate, Match, Neighborhood House, and Uncommon Schools/Roxbury Prep. We exclude additional schools (e.g. Excel and KIPP) due to incomplete application data. The BPSCSA website launched in October 2016 and the deadline to submit an application was February 2017. Each school obtained a list of applicants and separately conducted lotteries to generate offers of admission in March 2017.

Figure 1 shows the geographic distribution of schools for Grade 5 students in Boston in 2017, with

[3]Massachusetts Horace Mann charter schools, a hybrid model approved and part of the local district, were also not included in the BPCSA.

[4]For more background and related studies on Boston's charters, see Abdulkadiroğlu, Angrist, Dynarski, Kane, and Pathak (2011), Angrist, Cohodes, Dynarski, Pathak, and Walters (2016), Cohodes, Setren, and Walters (2021), and Walters (2018)

[5]Abdulkadiroğlu, Angrist, and Pathak (2014) provide additional detail and context on Boston exam school admissions.

charter schools shown as triangles and the charter schools that are in our sample shown as shaded triangles. The figure shows that Boston students have many choice options. Charters are concentrated in certain parts of the city, notably in the East Zone, which includes Dorchester, Mattapan, and Hyde Park.

4.2 Sample and Data

4.2.1 Sample Restrictions

The main data sources used are records from the Student Information Management System (SIMS) provided to us by the Massachusetts Department of Elementary and Secondary Education and individual charter school records. The SIMS data contain information on student demographics, such as race and subsidized lunch eligibility. Charter admissions records contain information on applicants and lottery offers. We match charter applicants to the SIMS file using name and date of birth, following the procedure described in Abdulkadiroğlu, Angrist, Dynarski, Kane, and Pathak (2011). We also use data from Boston Public Schools (BPS) to estimate a student's residential location; see Appendix F for more details on the data sources, location estimation, and computation of a student's distance to each school.

To build the sample, we start with all students who attended a Massachusetts school in fourth grade according to SIMS. For the time series analysis, we consider these students during the school years beginning in 2014-2016. For the structural analysis, we use students who were in fourth grade in the school year beginning in 2016. We then drop (1) students who are missing an ID number; (2) students who applied late to a charter school or applied but were deemed ineligible (due, for example, to being too young or withdrawing an application); and (3) students who applied to charter schools for multiple school grades within the same year.

For the structural analyses, we impose additional restrictions. First, we limit the sample to students who attended a school within Boston in fourth grade in the 2016-2017 school year other than one of the schools on the BPCSA for fifth grade. We impose this restriction since a large share of students remain at the school that they were already attending and are therefore not actively applying to new schools. Second, we limit the sample to students for whom we observe or can estimate a valid residential location, as described in Appendix F. Notably, our sample includes both applicants and non-applicants to schools listed on the BPCSA.

The final sample consists of 2,034 applicants across all three years for the time series analysis, and 5,138 individuals in 2017 for the structural analysis (701 applicants and 4,437 non-applicants).

4.3 Summary Statistics

Table 2 shows summary statistics for the sample used in the structural estimation. Student neighborhoods are determined based on the neighborhoods listed in the charter application files. Charter applicants are disproportionately located in the East Zone, consistent with the large concentration of charter schools in this zone. Similar to many other large urban school districts, Boston has a large fraction of Black and Hispanic students, and students who qualify for a subsidized school lunch. Compared to the overall pool of applicants, charter applicants are less likely to be Hispanic and more likely to be Black. This patterns contrasts with the patterns documented in Abdulkadiroğlu, Angrist, Dynarski, Kane, and Pathak (2011) where charter applicants were more likely to be Black and may reflect growing Hispanic enrollment in Boston. Applicants in the BPCSA tend to have similar baseline MCAS scores as non-applicants. Studies using application data from Boston's decentralized period tend to find that charter applicants have higher baseline MCAS scores than non-applicants (see, e.g., Walters (2018).). However, charter applicants tend to be more representative in cities where all applicants must rank schools (see, e.g., Abdulkadiroğlu, Angrist, Narita, and Pathak (2017)).

4.4 Motivating Descriptive Results

To motivate the structural estimation that follows, we use the time series patterns to check whether the theoretical predictions on applications and enrollment hold in the data. Specifically, in the model, the number of applications per applicant will increase and admissions rates will decrease as a result of moving from Decentralized Application to Common Application.

Figure 2 shows that applications per applicant increase and admissions rates decrease in 2017 – the first year of the Common Application – consistent with our theoretical model. The sample consists of grade 5 schools for years 2015-2017. We use a balanced panel of the three schools for which we have data in all of these years (APR, Match, and Uncommon). To account for the fact that our data on waitlist offers may be incomplete, we use initial offers rather than final offers as the measure of acceptances.

Panel (b) of Figure 2 shows time series of the unique number of applicants received by the Grade 5 schools. The previous theory is ambiguous about how the number of applications may change. Empirically, there is no obvious break from the previous pattern. That is, BPCSA appears not to have changed the number of students applying to a charter school. Panel (c) then shows the average number of applications received per BPCSA school. The slight increase in number of applications per school in 2017 does not appear to diverge from the trend between 2015 and 2016.

Finally, Panel (d) of Figure 2 shows the time series of applications per applicant separately by subsidized lunch status. Between 2016 and 2017, subsidized lunch applicants exhibited a slightly larger increase in the number of BPCSA schools that they applied to relative to non-subsidized lunch applicants. This is consistent with subsidized lunch applicants experiencing a larger reduction in the marginal costs required to apply to an additional charter school have the introduction of the BPCSA, although the small difference in magnitudes suggest that these marginal cost differences may also be small.

The patterns demonstrated in Figure 2 are only suggestive and cannot be solely attributed to the BPCSA because of other contemporaneous changes occurring during the roll out of BPCSA. For example, in November 2016, there was a statewide referendum on charter school expansion and this may have influenced parental preferences for charter schools. Therefore, we next turn to the structural model to evaluate counterfactual admissions systems holding fixed the demand patterns for a given year.

5 Structural Model and Estimation

5.1 Model Motivation and Setup

We develop a structural model of the BPCSA and use it to simulate decentralized and ranked systems for several reasons. First, to obtain analytical results, we have to abstract away from several important frictions in the theoretical model. Key abstractions ignore the possibility that: (1) preferences may evolve over time between the application and enrollment decisions and (2) waitlists and the aftermarket for each system do not result in exact market clearing. The structural model will allow us to examine whether some of the sharp theoretical predictions hold in practice with these real-life considerations. Second, some of our theoretical results are ambiguous and depend on the distribution of preferences and application costs. Estimates from the structural model will inform these parameters and will allow us to measure the welfare effects of different admissions systems. Finally, the structural model is also used for extrapolation. While Boston's charters switched from a decentralized system to the BPCSA, it only entertained the possibility of using a ranked system. We use the model to simulate the ranked admissions system.

Timing

The environment consists of two periods. In stage 1, potential applicants have an initial utility over each school. Between stages 1 and 2, schools make admissions decisions. In stage 2, students have an updated utility from each school, and use this utility to determine their enrollment based on their set of admitted schools. We follow Walters (2018) and include a second-stage shock to rationalize the fact that many students enroll in their outside option despite being admitted to a BPCSA school (see also Kapor, Neilson, and Zimmerman (2020)).

Application Decision: Common and Ranked Application

At the time of the application (Stage 1), utility for each BPCSA school $j \in \{1, ..., S\}$ for student i is given by:

$$u_{ij} = \beta_D D_{ij} + \delta_j + \epsilon_{ij},$$

where D_{ij} is the estimated driving distance in minutes, δ_j is a school fixed effect, and ϵ_{ij} is a Type 1 extreme value error term variance normalized to $\frac{\pi^2}{6}$. The outside option has a utility given by another Type 1 extreme value error term: $u_{i0} = \epsilon_{i0}$. There is separate no fixed effect for the outside option since we cannot separately identify all of the fixed effects; therefore each school fixed effect δ_j is relative to the outside option.

We assume that students do not take into account acceptance probabilities when applying to schools. This is a point of departure from the expected utility maximization model underlying the theoretical section and is made to justify the fact that students do not apply to every charter school. We also motivate the assumption based on evidence from other settings that families have mistaken beliefs about admissions probabilities (e.g., Kapor, Neilson, and Zimmerman (2020). We expect this phenomenon to be particularly relevant in during the first year of the BPCSA, where applicants will have to anticipate odds of admission in a completely new system. Moreover, the descriptive evidence from BPCSA suggests that the Boston's common charter app did not result in a large change in the set of applicants. Appendix E reports estimates for an alternate model in which applicants maximize expected utility over portfolios of applications.

We therefore instead impose the following assumptions:

- Submitting any application has a fixed cost C which does not vary across individuals and there is no additional marginal cost for each school that a student applies to under the common application. Students will apply using the common application if the utility from at least one school is greater than the outside cost plus the outside option, so

$$\max\{u_{i1}, ..., u_{iS}\} \geq C + u_{i0}$$

- If a student applies using the common application, she applies to all schools for which $u_{ij} \geq u_{i0}$ due to the assumption of zero marginal cost for applications beyond the first application.

We assume that the set of schools that students will list under ranked application is the exact same as the set of schools that they would apply to under the common application model. This implies that the fixed cost C is the same in each system, each system has zero marginal cost beyond the first school, and students use the same behavioral assumption under each system. However, under the common application model, students simply indicate the schools for which they are applying. Under the ranked application model, students rank these schools based on their u_{ij}, reporting truthful preferences.

Application Decision: Decentralized Applications

The key feature of the decentralized applications is that it becomes more costly to submit the applications beyond the initial application. We model this by assuming that in the first stage, identical to the common application model, individuals submit an application if and only if

$$\max\{u_{i1}, ..., u_{iN}\} \geq C + u_{i0}.$$

For each subsequent application, there is a marginal cost $\tilde{c}$, where we hypothesize (but do not impose) that $\tilde{c} < C$. Students then apply to all schools for which $u_{ij} \geq \tilde{c} + u_{i0}$.

In the decentralized application model, the set of students that will submit an application in this model is identical to the set that will submit an application in the common application model because the decision to apply only depends on the maximum school utility in the first stage and the fixed cost of an application, which remains the same. This feature is consistent with descriptive patterns above. Conditional on submitting an application, the number of applications per applicant will be lower in the decentralized application model than in the common application model. Finally, just like in the common application model, none of the costs C or $\tilde{c}$ are relevant for the preference ranking of schools or the enrollment decision after the first stage of deciding whether to submit an application.

Enrollment Decision

After application, a students may be accepted to a subset of the schools where they applied. In Stage 2, the utility for each school (including the outside option) is given by:

$$U_{ij} = u_{ij} + \xi_{ij},$$

where ξ_{ij} follows a Type 1 extreme value distribution, but with an estimated variance that differs from the Stage 1 error term. Specifically, ϵ_{ij} has standard deviation $\frac{\pi}{\sqrt{6}}$ while ξ_{ij} has standard deviation $\frac{\kappa\pi}{\sqrt{6}}$. The variable κ regulates the extent to which preferences change across stages. A student then attends the school (which may be the outside option) for which she's received an acceptance that has the highest U_{ij} value. By definition, all students are accepted at their outside option. The application cost C is no longer relevant in Stage 2, since it is a sunk cost.

For the common application and decentralization application models, students receive a fixed set of acceptances and then choose their best offer among these schools.

5.2 Likelihood and Estimation of Model Parameters

Among the set of applicants and non-applicants who are considered potential applicants, we estimate the parameters of the model above (school fixed effects, distance coefficients, the fixed cost C, and the variance of the second stage error) using simulated maximum likelihood (SML) using data from the BPCSA in 2017.

Define the following notation:

- A_i is the observed set of schools where student i applies. $A_i = \phi$ is the event where student i does not apply to any schools on the BPCSA (i.e. $\max\{u_{i1}, ..., u_{iN}\} < u_{i0} + C$).
- Z_i is the observed set of schools for which student i receives acceptances. This is determined in equilibrium based on school capacities.
- S_i is the observed school chosen by student i (which may be a BPCSA school or the outside option).
- Denote the set of parameters as $\theta = (\delta, \beta)$ and the set of observed individual characteristics (e.g., location) as X_i.

There are three cases for the likelihood terms for each student, depending on their observed applications and enrollments. We therefore consider the likelihood for each separately.

Case I: Non-Applicants. These are potential applicants who are not observed applying, so $A_i = \phi$. The likelihood of non-application is the probability that the outside option utility plus the application cost is greater than the utility from *all* of the common application model schools. The likelihood is given by:

$$\mathcal{L}_i = Pr(A_i = \phi) = \frac{1}{1 + \sum_j \exp(u_{ij} - C)}.$$

Case II: Applicants with No Acceptances. These are applicants who are not accepted to any school, so their contribution to the likelihood function is only given by the set of schools where they apply. Their contribution to the likelihood is:

$$\begin{aligned}\mathcal{L}_i &= Pr(A_i|\theta, X_i)\\ &= Pr(A_i|\theta, X_i, A_i \neq \phi)\ Pr(A_i \neq \phi|\theta, X_i).\end{aligned}$$

By the same logic as in Case I, we have $Pr(A_i \neq \phi) = 1 - \frac{1}{1+\sum_j \exp(u_{ij}-C)}$.

The term $Pr(A_i|\theta, X_i, A_i \neq \phi)$ can be estimated via simulation for each individual, by drawing a large number of error terms, keeping the simulations for which the individual would choose to apply, and then recording the empirical probability of the observed application set A_i among these simulations.

Case III: Applicants with Acceptances. These are applicants who apply to at least one Common Application school, and receive at least one acceptance, so that we observe a Stage 2 enrollment choice for these applicants.

Their (unconditional) likelihood is given by:

$$\begin{aligned}\mathcal{L}_i &= Pr(A_i, S_i|\theta, X_i)\\ &= Pr(A_i|\theta, X_i)\ Pr(Z_i|A_i, \theta, X_i)\ Pr(S_i|Z_i, A_i, \theta, X_i)\\ &= Pr(A_i|\theta, X_i)\ Pr(Z_i|A_i)\ Pr(S_i|Z_i, A_i, \theta, X_i).\end{aligned}$$

Conditional on the observed applications, the charter school acceptances do not depend on θ. Therefore, the θ that maximizes the *log* likelihood is equivalent to the θ that maximizes the following for each individual:

$$\ell_i = \log(Pr(A_i|\theta, X_i)) + \log(Pr(S_i|Z_i, A_i, \theta, X_i)).$$

The expression within the log in the second term is equal to:

$$\int Pr(S_i|Z_i,\theta,X_i,\epsilon_i)f(\epsilon_i|A_i,X_i,\theta)d\epsilon_i = \int \frac{Z_{ij}\exp(\frac{v_{ij}+\epsilon_{ij}}{\kappa})}{\exp(\frac{\epsilon_{i0}}{\kappa}) + \sum_{j'=1}^{J} Z_{ij'}\exp(\frac{v_{ij'}+\epsilon_{ij'}}{\kappa})} f(\epsilon_i|A_i,X_i,\theta)d\epsilon_i,$$

where $f(\epsilon_i|A_i, X_i, \theta)$ is the pdf of the ϵ_{ij} terms conditional on a given application decision, parameter estimates, and student characteristics. Note that we can simplify this conditional pdf as:

$$f(\epsilon_i|A_i,X_i,\theta) = \frac{Pr(A_i|\epsilon_i,X_i,\theta)f(\epsilon_i)}{Pr(A_i|X_i,\theta)}.$$

Since each error term is independent, $f(\epsilon_i)$ is simply the product of the unconditional pdfs of each extreme value error term. In addition, since the application decision is deterministically given by ϵ_i, X_i, θ, the quantity $Pr(A_i|\epsilon_i, X_i, \theta)$ is simply an indicator variable of whether an individual would have the observed application set. Finally, since $Pr(A_i|X_i, \theta)$ does not depend on ϵ_i, it can be taken out of the integral, and will cancel with the first term of ℓ_i. Putting this all together, the likelihood for these students is:

$$\ell_i = \log\Big(\int \frac{Z_{iS_i}\exp(\frac{v_{iS_i}+\epsilon_{iS_i}}{\kappa})}{\exp(\frac{\epsilon_{i0}}{\kappa}) + \sum_{j'=1}^{J} Z_{ij'}\exp(\frac{v_{ij'}+\epsilon_{ij'}}{\kappa})} \mathbb{1}_{(A_i|\epsilon_i,X_i,\theta)} f(\epsilon_i)d\epsilon_i\Big).$$

There are two computational steps to computing this likelihood. First, the integral within the log likelihood cannot be computed explicitly so is estimated via simulation. Second, the indicator variable within the integral is not computationally feasible for uncommon combinations of applications, so a logit smoother following Train (2009) combined with the analytical formula for common application combinations given the logit error term following Ophem, Stam, and Praag (1999) is used to avoid values of 0 and 1. The only way the indicator takes on a value of 0 now is if the ϵ_i and θ values imply that the individual will not submit a BPCSA at all. More details are in Appendix D.

5.3 Estimation of Decentralized Application Marginal Costs

We use the following procedure to estimate the marginal costs $\tilde{c}$ under a decentralized application:

1. From the estimated parameters of the common application model, generate simulated error terms which rationalize the observed application and enrollment decisions of each student in 2017.

2. Determine the empirical number of applications per applicant in 2016, the year before the BPCSA was introduced, when a decentralized application was still in effect. Table 6 shows that an

average of 1.6 applications per applicant was submitted; as expected, this is lower than the 2.65 applications per applicant observed under the BPCSA in 2017.

3. From the simulations in 2017, assume that if a decentralized application system were still in effect, the number of applications per applicant would have been the same as in 2016. Then find the value of $\tilde{c}$ such that, based on the simulated error terms and our model of a decentralized application, the average number of applications per applicant (across all simulations) matches the 2016 value.

5.4 Simulating the Application Systems

To evaluate each of the application systems, we first simulate the set of error terms that rationalize the given application and enrollment decisions for each student. We generate 100 unique simulations.

For the common application model, we use the observed acceptances and enrollment, and can determine the rank and utility associated with the school that each student accepted within each simulation.

For the ranked application model, we run the student-proposing deferred acceptance (DA) algorithm. Waitlists and an aftermarket are accounted for by using two rounds of the algorithm. School capacities are set to be exactly equal to the eventual enrollments under the common application model among the given set of applicants. School priorities for students are randomly generated by lotteries and are set to be identical across schools so that DA is efficient.

To complete the first round of DA, run DA given the setup described above. At this point, some students are assigned to a school and some students remain unassigned. We say that students are on the waitlist for a school if there are schools for which they originally applied and were not assigned under DA, but prefer to the school that they were assigned. DA is then run for a second round among individuals who did not receive their eventual first choice (either through DA assignment or if the outside option became their first choice) given the new waitlist rankings.[6]

For the decentralized application model, we assume that the capacity for each common application school among the estimation sample is identical to the observed final enrollment in 2017 among this group (as in the ranked simulation). Then we determine equilibrium acceptance rates in the decentralized application model using the following greedy algorithm:

[6] This is designed to mimic the waitlist structure used in the traditional Boston Public Schools match, in which "students will be on all waitlist for schools they ranked on their choice list until June 15, 2021" (`https://www.bostonpublicschools.org/Page/6487`). This does not perfectly map onto the system used, since "after June 15, 2021, students will automatically remain on one waitlist, their highest ranked school, unless a parent contacts a Welcome Center to specify otherwise."

- First, for each school, randomly generate a priority order among all potential applicants. This priority order may differ across schools.
- Each school then starts with accepting 0 students and incrementally accepts another student if they still have capacity.
- Then given the full set of acceptances, students attend the school where they have the highest utility in the second round.
- Schools stop increasing their acceptance rate when they either or at capacity or have an acceptance rate of 100%.

As with the two other assignment systems, the simulated utilities can be used to determine the rank and utility associated with the school that each student receives.

6 Structural Estimates

Table 3 shows the estimated demand parameters estimated via simulated maximum likelihood. Results are shown both for the full sample, as well as for a model in which fixed costs may differ by subsidized lunch status or minority status.

For the results within the full sample, the coefficient on distance (β) is negative. The negative fixed effect for each school reflects the fact that a very small share of the sample ends up applying to any of the common application model schools. In addition, estimated application costs and changes in preferences between the two stages are both large. The large estimated κ reflects, in part, the large share of students who enroll in the outside option. In our data, 50% of applicants enroll in the outside option, including 33% of the applicants who receive at least one acceptance to a BPCSA school. Only 60% of applicants who enroll in the outside option do so as a result of not receiving an acceptance to any BPCSA school. This implies that for the other applicants, there must be large changes in preferences to rationalize the large market share of the outside option; we estimate based on simulations that at the enrollment stage, 27% of those who enroll in the outside option would have it as the first choice school.

The other columns in each table show heterogeneous results by demographic groups, defined during Grade 4. Subsidized lunch students seem to have fixed larger fixed costs than non-subsidized lunch students, while fixed costs are similar for minorities and non-minorities. Based on the stark differences in application shares to each school by minority status in Panel C, when we perform counterfactuals with heterogeneous estimates by group, we focus only on subsidized lunch heterogeneity.

To interpret the implications of the large changes in preferences across stages implied by the large κ estimate, Table 4 shows the transition matrix across simulations of how school rankings change between the application stage (Stage 1) and the enrollment stage (Stage 2). The large variance of the error term in the second stage means that rankings shift drastically between stages. However, the fixed cost makes the first choice slightly more stable relative to the other choices. Table 5 shows, for each school rank in the second stage, the type of school at that rank. The simulations imply that at the enrollment stage, 59% of the time an applicant's first choice will be a school that she listed on the BPCSA (either the first choice at the time of application or another BPCSA school to which she applied), 26% of the time it will be a BPCSA school that was not listed, and 15% of the time it will be the outside option.

The inputs to estimating the decentralized system are shown in Table 6. We find that acceptance rates for all schools increase relative to the observed rates in the common application. On average, all enrollments in Column (3) match enrollments in Column (2) except for one school. This is because increasing this school's acceptance rate to 100% still does not reach the school's capacity due to the decrease in applications. Some observed acceptance rates in 2017 are much lower than expected. This could partially reflect the incomplete waitlist information or the substantial variation in acceptance rates by year. From this estimation procedure, we obtain an overall marginal cost estimate of $\tilde{c} \approx 1.07$. This is lower than C, as expected.[7] When implementing this procedure separately by subsidized lunch status, we estimate $\tilde{c} = 1.13$ for subsidized lunch students and $\tilde{c} = 0.87$ for non-subsidized lunch students. The larger marginal cost for subsidized lunch students is consistent with their larger increase in applications per applicant in 2017 relative to 2016.

6.1 Ranking and Welfare Comparisons

With the simulations for each application system in hand, we can compare the distribution of rankings of the schools that students enroll in under each. These results are shown in Table 7. In general, the decentralized system performs the best based on this metric, and is roughly similar to the ranked system. The magnitudes of differences between the systems are relatively small, but in proportional terms they are still substantial. For example, over 10% more students get their top choice under the decentralized system than the common application. Figure 3 shows the results of a "vote" between each pair of systems. Results for the votes are also pushed toward 50% given the large number of

[7] To benchmark these estimates, Fu (2014) estimates application costs for college applications, and finds that marginal application costs decrease rapidly in the number of applications, with the cost of the first application being quite large (1900 dollars for the first application and 900 dollars for the second application).

students who receive the same assignment under each system.

Note that these simulations require the assumption that preferences change to the same extent between stages regardless of the application system used. However, each application system may change the incentives to acquire information; in particular, having to rank all choices may incentivize students to learn more about their preferences prior to applying (Chen and He, 2021). As a result, we also report results for the "Ranked Full Information" case. Ranked Full Information corresponds to the limit case in which ranked induces students to learn their full preferences prior to applying (and therefore report their second-stage preferences).

We evaluate the magnitudes by benchmarking the total utility in each system relative to a first-best allocation and a lower-bound allocation. For the first-best, we consider the allocation that maximizes total utility, subject to school capacity constraints ("First-Best'), as well as the allocation that maximizes total utility, subject to both school capacity constraints and restricting students to be assigned to one of the schools that they list on the Common Application ("First-Best, Applied"). For the lower-bound allocations on utility, we consider a "Random" allocation, in which students are randomly assigned to schools subject to capacity, and a "No Choice" allocation in which every student receives their outside option. Figure 4 shows that relative to these benchmarks, differences between total utility in the application systems that we consider are fairly small. Nevertheless, if we assume that the marginal costs for applying in the decentralized system are primarily behavioral costs, the decentralized system and the full-information DA both perform better than the Common Application. The Common Application performs better than the baseline DA and the decentralized application if we assume that all of the marginal costs are "true costs" incurred by applicants.

6.2 Decentralized Applications with Heterogeneous Marginal Costs

It is possible ex ante that the previous results obscure heterogeneity on the basis of marginal costs if the common application reduces marginal costs for disadvantaged groups.

Our estimates suggest that applicants on subsidized lunch have a higher marginal (and fixed) cost for applications than those not on subsidized lunch. Therefore, we can consider subsidized lunch students as the "disadvantaged" group that would be more likely to benefit from the introduction of the common application.

Figure 5 carries out the thought experiment of comparing the common application and the decentralized application under different marginal costs for the subsidized lunch applicants (holding the marginal costs for the other applicants fixed). Panel (a) shows that at the observed marginal cost, approximately 38% of subsidized lunch applicants are expected to receive their first choice under a common application (relative to approximately 34% under the decentralized application). In fact, the marginal costs would have to be approximately four times higher than what we estimate in order for the common application to be beneficial for this group. Panel (b) shows the same exercise for students receiving their first or second choice. In this case as well, true marginal costs for subsidized lunch applicants would need to be larger than what we estimate in order for the common application to be beneficial.

The results in these figures suggest that the difference in marginal costs for disadvantaged groups makes only a minor difference in their average ability to obtain their preferred schools. This is particularly small relative to the difference between the decentralized system and the common application. Therefore, in this setting, it does not appear that the common application makes a large difference in helping disadvantaged groups. One potential explanation for these small differences is that the large share of applicants on subsidized lunch may mitigate the effects of having applicants with lower marginal costs, or the difference in marginal costs may be small enough to not matter substantially.

6.3 Testing Common Application vs. Decentralized Application Predictions

With the simulation results in hand, we can revisit the results from the theoretical model in Section 2. We highlight some of the results that hold mechanically due to the structure of the model.

First, Proposition 2 holds mechanically in our model due to the structure of the application rule, and the fact that in our model, students do not adjust application behavior based on admissions probabilities. This means that in our model, we will always have weakly more applications and weakly lower admissions rates under the Common Application than the Decentralized Application.

Next, the empirical results confirm the theoretical prediction from Proposition 3, despite the fact that our structural model no longer imposes an assumption of independent symmetric preferences. Table 6 shows that at each rank, more students cumulatively receive a school of that rank or better under the decentralized system than the Common Application. Therefore, the distribution of ordinal ranks for enrolled students under the decentralized system continues to stochastically dominate the distribution of ordinal ranks under the Common Application.

Finally, our structural model allows us to empirically evaluate comparisons for which the theory in Section 2 did not yield clear predictions. In Section 2, we were not able to say whether expected utility is higher or lower with the Common Application system than with the decentralized application rule. Our empirical results in Figure 4 suggest that this comparison depends crucially on whether application costs are considered part of expected utility. Neglecting application costs, the decentralized application results in higher expected utility than a Common Application, while a Common Application performs better if these application costs are "true" costs.

6.4 Testing Common Application vs. Ranked Application Predictions

We can similarly use the previous estimates to evaluate the theoretical claims comparing the common application and ranked application in Section 3. Whether or not these results hold once we impose the additional frictions from the structural model that were not present in the basic theory is an empirical question; for example, properties such as stability under a ranked system may not hold when preferences change over time. First, our empirical results support Corollary 7 by showing that a higher proportion of students are estimated to receive their first-choice schools with a Ranked system than with a Common Application in Table 7. In addition, our empirical results support Proposition 8; Table 8 reports that fewer students are eligible to trade under a Ranked Application model than under the Common Application.

7 Comparisons in Environments Other Than Boston

In this section, we examine the implications of our empirical model for environments other than Boston. We focus on three dimensions: the level of school oversubscription, the size of marginal costs, and the extent of preference changes over time.

Parameters are chosen for the two illustrative that follow to illustrate the implications of environments with either heavy or light oversubscription; these environments correspond to variation in applicant demand relative to the overall supply of school seats. To establish a distribution of students and driving times, we start with the sample of 701 applicants from the structural estimation sample and use the estimated driving times to schools. For simplicity, the model maintains these five schools and keeps admissions rates and fixed effects constant across all schools. These examples are set up as follows:

- **Heavy Oversubscription**: In this scenario, students apply to almost all schools and are admitted to approximately one school on average. This uses the parameters $\beta = -0.01$; $\delta_1 = \delta_2 = ... = \delta_5 = 2.5$; $C = 0.5$; and acceptance rates $\pi_1 = \pi_2 = ... = \pi_5 = 0.25$. When preferences do not change between the two stages ($\kappa = 0$), the average number of applications per applicant is 4.68 under a common application. This and other facts on application patterns for this example are summarized in Appendix Table C.1.

- **Light Oversubscription**: In this scenario, students apply to a few schools and are admitted to almost all of their choices. This uses the parameters $\beta = -0.08$; $\delta_1 = \delta_2 = ... = \delta_5 = 0.5$; $C = 0.5$; and acceptance rates $\pi_1 = \pi_2 = ... = \pi_5 = 0.75$. When $\kappa = 0$, the average number of applications per applicant is 2.34 under a common application. This and other facts on application patterns for this example are summarized in Appendix Table C.2.

Within each of these examples, we show cases for various levels of preference changes: no preference changes ($\kappa = 0$), low preference changes ($\kappa = 1$), and high preference changes ($\kappa = 4$). By construction, when $\kappa = 0$, the correlation between utilities u_{ij} and U_{ij} in the two stages is 1.00. When $\kappa = 1$, this correlation is 0.77 in the heavy oversubscription example and 0.76 in the light oversubscription example. When $\kappa = 4$, this correlation shrinks to 0.29 in the heavy oversubscription example and 0.28 in the light oversubscription sample. These examples then allow for comparison of each application system; since the performance of a decentralized application also depends on the extent of marginal costs, we show results for a low marginal cost $\tilde{c} = 0.5$ and a high marginal cost $\tilde{c} = 5.0$.

The light oversubscription scenario more closely mirrors our Boston scenario. Table 6 shows that the average number of applicants per applicant is 2.65 and acceptance rates across schools vary from 9% to 81% under Boston's Common Application. The heavy oversubscription environment therefore corresponds to scenarios where applicants apply to more schools and acceptance rates are lower.

Figures 6 and 7 show the performance of each application system (Ranked, Common Application, Ranked with Full Information, Decentralized Applications with low Marginal Cost, and Decentralized with high Marginal Cost) in each of these illustrative examples, in terms of the fraction of applicants who receive each choice of school, based on 10 simulations. In the heavy oversubscription environment (Figure 6), a Ranked system performs best because of its ability to sort through applicant choices. Ranked yields a higher share of applicants with their top choice regardless of the extent of preference changes or marginal costs. When preferences do not change (shown in panel (a)), Ranked and Decentralized with high Marginal Costs have many more applicants obtaining their top choice, compared to the Common Application. Intuitively, this can be thought of as the Common Application making it easy to apply to a lot of schools, and the low acceptance rates effectively randomly assigning each applicant to a school. Under moderate preference changes ($\kappa = 1$), the share who obtain their top choice drops in all mechanisms. This is because preferences used to make the assignments in the first stage now change in the second stage. Ranked and Decentralized with high marginal cost yield higher shares of applicants receiving their top choice, but Ranked now outperforms Decentralized. Under extreme preference changes ($\kappa = 4$), about half of applicants obtain their top choice under Ranked, which is higher than the two alternatives. Now, Common Application and Decentralized produce very similar aggregate outcomes.

In the light oversubscription environments, the Decentralized Application with high marginal costs performs best when preferences do not change and the Common Application performs best when preferences change. Panel (a) of Figure 7 shows that Decentralized results than a higher fraction receiving their top choice than the Common Application, as students in the decentralized system apply to schools that will remain their choice when they face the enrollment decision. Ranked has similar performance. The Common Application has a higher fraction receiving their top choice under extreme preference changes, shown in Panel (c). In these situations, the Common Application allows students to change their assignment in the second stage after getting offers from multiple schools, taking into account their updated preferences.

A summary of the application system that performs best in each of these illustrative examples can be summarized as follows:

	Heavy Oversubscription	Light Oversubscription
Few Preference Changes	Ranked	Decentralized / Ranked
Many Preference Changes	Ranked	Common Application

Notably, these examples suggest that the Common Application should perform best in light oversubscription environments with many preference changes, which closely resembles our estimates of Boston's environment. Our counterfactual simulations nevertheless show that the Common Application was suboptimal across the metrics that we consider; the intuition from these examples suggest that this may represent an upper bound on how well a Common Application could perform in other settings.

8 Conclusion

This paper develops a theoretical model of common applications for K-12 schools and compares it to a decentralized system and a ranked system. We then examine the effects of the 2017 Boston's charter common application system on school access. We report facts on application patterns under the common application compared to the decentralized system. Estimates of a structural model of the common application allow us to measure student welfare and simulate counterfactual scenarios. Finally, we use our estimated model from Boston to consider scenarios with different characteristics. We report on an empirical simulation varying the extent of oversubscription, preference changes, and application costs to measure when each system performs best.

Our results provide an important caution about the potential effects of a common application. Theoretically, a common application need not increase access to schools relative to either the decentralized system or ranked system. In the model, a common application lowers application costs but also increases congestion. But creating application frictions can facilitate sorting. The ranked system lowers application costs and also allows for sorting because it uses applicant preferences to determine allocations.

Our Boston case study provides support for many predictions of the model. The number of applications increase under a common application and admissions rates decrease. Boston's common application did not appear to change the characteristics of applicants, suggesting a limited role for equity effects of the system. We also find substantial changes between preferences submitted in the first stage and the preferences which rationalize enrollment decisions. This fact leads the common application to perform better than expected in a situation where applicants are certain about their preferences, as in our theoretical model. However, we estimate that the common application performs no better than either the decentralized application system or the ranked system on several dimensions. Our investigation of alternative environments shows that the common application performs best in environments with light oversubscription and with many preferences changes, which appears to be the case in Boston. Ranked systems perform best in environments with heavy oversubscription regardless of the extent of preference changes. Decentralized systems work well in light oversubscription environments with few preference changes, provided that application costs are low.

Our results show that creating frictions to market clearing may in fact increase efficiency because frictions can improved sorting. The performance of the decentralized system suggests that a switch to a common application did not significantly improve student welfare in Boston. At the same time, the ranked system does not outperform the common application in Boston. This is because the ranked system does not offer the same degree of flexibility as the common application when preferences change between the application stage and enrollment stage. This fact suggests that the sorting benefits of frictions in market clearing need to be weighed against the benefits of flexibility when preference discovery is important. Our investigation of alternative environments suggests that this this trade-off is important for understanding the welfare consequences of the three different assignment systems.

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Exhibits

Table 1: Examples of Metropolitan Areas with Common Application Systems

	(1)	(2)
Metro Area	For School Year	Name of Portal
Atlanta	2020-2021	Apply APS Charter
Baton Rouge	2018-2019	EnrollBR
Boston	2017-2018	Boston Charter Public School Application
Buffalo	2021-2022	Enroll Buffalo Charters
Kansas City	2019-2020	Show Me KC Schools
Houston	2018-2019	Apply Houston
Los Angeles	2019-2020	ApplyLA Charter Common Application
New York City	2010-2011	Common Online Charter School Application
Oakland	2017-2018	Oakland Enrolls
Philadelphia	2019-2020	Apply Philly Charters
Rhode Island	2020-2021	Enroll RI
Rochester	2017-2018	GoodschoolsRoc

Notes: This table shows a partial list of cities that use a single common application for charter schools, collected by the authors. The "For School Year" column indicates the first academic year for which the common application was implemented.

Table 2: Descriptive Statistics for Charter Applicant and Non-Applicant Samples

	(1)	(2)
	Charter Applicants	Non-Applicants
Observations	701	3591
	A: Demographics	
Special Education	0.20	0.24
Subsidized Lunch	0.76	0.79
Black, Non-Hispanic	0.41	0.28
Hispanic	0.41	0.48
	B: Geography	
Closest Charter Drive Time (minutes)	7.59	10.36
East Zone	0.49	0.36
North Zone	0.20	0.38
West Zone	0.31	0.25
	C: Grade 4 School	
Charter	0.08	0.04
Traditional Boston Public	0.92	0.96
	D: Grade 4 Test Scores	
Observations with Test Score	664	3355
Grade 4 MCAS Average Score	0.00	0.01

Notes: This table shows descriptive statistics separately for the samples of both applicants and non-applicants to Boston Public Charter School Application (BPCSA) Grade 5 charter schools in 2017. The mean is shown for each row, with the exception of rows that indicate the number of observations. The applicant sample consists of students with an estimated location who applied to at least one of the BPCSA charter schools in our sample. The non-applicant sample consists of students with an estimated location who did not apply to any of the BPCSA charter schools in our sample and attended a non-BPCSA school within Boston for Grade 4 in 2016-2017. All information is measured in Grade 4. Geographic zones in Panel B correspond to the zones shown in Figure 1. Test scores in Panel D are based on the Massachusetts Comprehensive Assessment System (MCAS) exam in Grade 4, and are normalized to have mean zero and a standard deviation of one among all Boston students during the given year; the score shown is the average of the standardized math and reading scores.

Table 3: Common Application Preference Parameter Estimates

	(1)	(2)	(3)	(4)	(5)
		Subsidized Lunch		**Black/Hispanic**	
Parameter	Full Sample	Subsidized	Not Subsidized	Black or Hispanic	Not Black or Hispanic
		A: Applicants			
Number of Applicants	701	536	165	569	132
Number of Non-Applicants	3591	2841	750	2758	833
Applications/Applicant (2016)	1.60	1.61	1.60	1.69	1.23
Applications/Applicant (2017)	2.65	2.74	2.34	2.80	1.97
		B: Parameter Estimates			
Drive Time Coefficient, 100*β	-3.40	-3.24		-2.86	
Fixed Cost, $C/\lvert\beta\rvert$	57.75	63.26	57.35	77.01	75.92
$\kappa/\lvert\beta\rvert$	115.38	111.30		129.42	
$\delta_1/\lvert\beta\rvert$	–10.01	–9.03		–17.57	
$\delta_2/\lvert\beta\rvert$	–16.79	–14.77		–14.06	
$\delta_3/\lvert\beta\rvert$	–16.86	–16.16		–15.55	
$\delta_4/\lvert\beta\rvert$	–25.54	–23.39		–33.72	
$\delta_5/\lvert\beta\rvert$	–29.84	–26.95		–28.80	
		C: Fraction of Applicants Applying			
School 1	0.51	0.52	0.48	0.55	0.33
School 2	0.62	0.58	0.74	0.55	0.89
School 3	0.56	0.61	0.39	0.64	0.23
School 4	0.46	0.47	0.41	0.48	0.36
School 5	0.50	0.56	0.32	0.58	0.17

Notes: This table shows the simulated maximum likelihood parameter estimates from the model of application decisions for Boston Grade 5 in 2017, estimated separately three times: once in the full sample (Column (1)), once with cost heterogeneity by subsidized lunch status (Columns (2) and (3)), and once with cost heterogeneity by racial minority status (Columns (4) and (5)). In models with cost heterogeneity, all other parameters are imposed to be equal for both groups within the model. Subsidized lunch status and racial minority status are defined during Grade 4. Panel B shows the parameter estimates, scaled by the minutes of drive time for all parameters other than the drive time coefficient itself (β). Panel C shows the share of applicants applying to each school on the Boston Public Charter School Applicant (BPCSA).

Table 4: Implied Ranking Between Application and Enrollment Decisions

	(1)	(2)	(3)	(4)	(5)	(6)
Stage 1	Stage 2 – First Choice	Stage 2 – Second Choice	Stage 2 – Third Choice	Stage 2 – Fourth Choice	Stage 2 – Fifth Choice	Stage 2 – Sixth Choice
First Choice	0.32	0.25	0.18	0.13	0.08	0.04
Second Choice	0.18	0.19	0.19	0.18	0.15	0.10
Third Choice	0.16	0.17	0.18	0.18	0.18	0.14
Fourth Choice	0.13	0.14	0.16	0.18	0.19	0.19
Fifth Choice	0.11	0.13	0.15	0.17	0.20	0.23
Sixth Choice	0.10	0.12	0.13	0.16	0.20	0.29

Notes: This table shows the transition matrix of implied school rankings between the application stage (Stage 1) and the enrollment stage (Stage 2), among the 701 students who apply to at least one school on the Boston Public Charter School Application (BPCSA) in 2017. Six schools are included for each applicant: the five BPCSA charter schools in our data, and the applicant's outside option. Implied rankings are generated by running 100 simulations of error terms that rationalize the observed application and enrollment decisions for each applicant.

Table 5: School Rankings at Time of Enrollment Decision

	(1)	(2)	(3)	(4)
Stage 2 School	Stage 1 First Choice	Other Applied BPCSA School	Non-Applied BPCSA School	Outside Option
First Choice	0.32	0.27	0.26	0.15
Second Choice	0.25	0.28	0.31	0.17
Third Choice	0.18	0.28	0.35	0.18
Fourth Choice	0.13	0.29	0.40	0.18
Fifth Choice	0.08	0.28	0.47	0.17
Sixth Choice	0.04	0.25	0.56	0.15

Notes: This table shows, for each implied ranking at the enrollment stage (Stage 2), the fraction of schools that are a part of each mutually exclusive category. The sample consists of the 701 students who apply to at least one school on the Boston Public Charter School Application (BPCSA) in 2017 for grade 5. There are six potential schools for each applicant: the five BPCSA charter schools in our data, and the applicant's outside option. Implied rankings are generated by running 100 simulations of error terms that raitonalize the observed application and enrollment decisions for each applicant.

Table 6: Comparison of Common and Decentralized Systems

	(1)	(2)	(3)
	2016	**2017**	
	Decentralized Observed	Common Application Observed	Decentralized Simulated
A: Applicants			
Applications per Applicant	1.60	2.65	1.61
Applicants in Sample	920	701	701
B: School Acceptance Rates			
School 1	52%	36%	60%
School 2	27%	16%	47%
School 3	31%	9%	20%
School 4	100%	12%	25%
School 5	100%	81%	100%
C: School Enrollments			
School 1	72	67	67
School 2	74	74	74
School 3	43	25	25
School 4	23	22	22
School 5	206	164	116

Notes: This table compares the observed decentralized system in Boston in 2016 for Grade 5 to the observed common and simulated decentralized systems in Boston in 2017 for grade 5. Columns (1) and (2) show observed outcomes within the estimation sample for 2016 and 2017, with the decentralized and common application systems, respectively. Column (3) shows results for simulated outcomes in 2017 using a decentralized system, with 100 simulations. Simulations in Column (3) set marginal costs for each application so that the average number of applications per applicant matches the value in Column (1), and school acceptance rates are set to match those in Column (2), unless the acceptance rate reaches 100 percent. Averages in Panels B and C are based on the averages across all 100 simulations, and are both based only on the applicants within the sample.

Table 7: Application System Comparisons: Distributions of Rankings

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Common		**Ranked**		**Decentralized**		**Ranked – Full Information**	
Final Ranking	Fraction	[90% CI]	Fraction	[90% CI]	Fraction	[90% CI]	Fraction	[90% CI]
1st Choice	0.335	[0.308, 0.361]	0.368	[0.343, 0.392]	0.369	[0.342, 0.397]	0.400	[0.371, 0.429]
1st-2nd Choice	0.575	[0.548, 0.603]	0.590	[0.563, 0.618]	0.612	[0.584, 0.640]	0.601	[0.574, 0.627]
1st-3rd Choice	0.747	[0.725, 0.769]	0.747	[0.723, 0.771]	0.772	[0.748, 0.796]	0.746	[0.722, 0.770]
1st-4th Choice	0.868	[0.850, 0.887]	0.863	[0.844, 0.882]	0.879	[0.859, 0.898]	0.856	[0.835, 0.877]
1st-5th Choice	0.951	[0.939, 0.964]	0.946	[0.934, 0.958]	0.952	[0.939, 0.964]	0.941	[0.927, 0.955]
1st-6th Choice	1.000	[1.000, 1.000]	1.000	[1.000, 1.000]	1.000	[1.000, 1.000]	1.000	[1.000, 1.000]

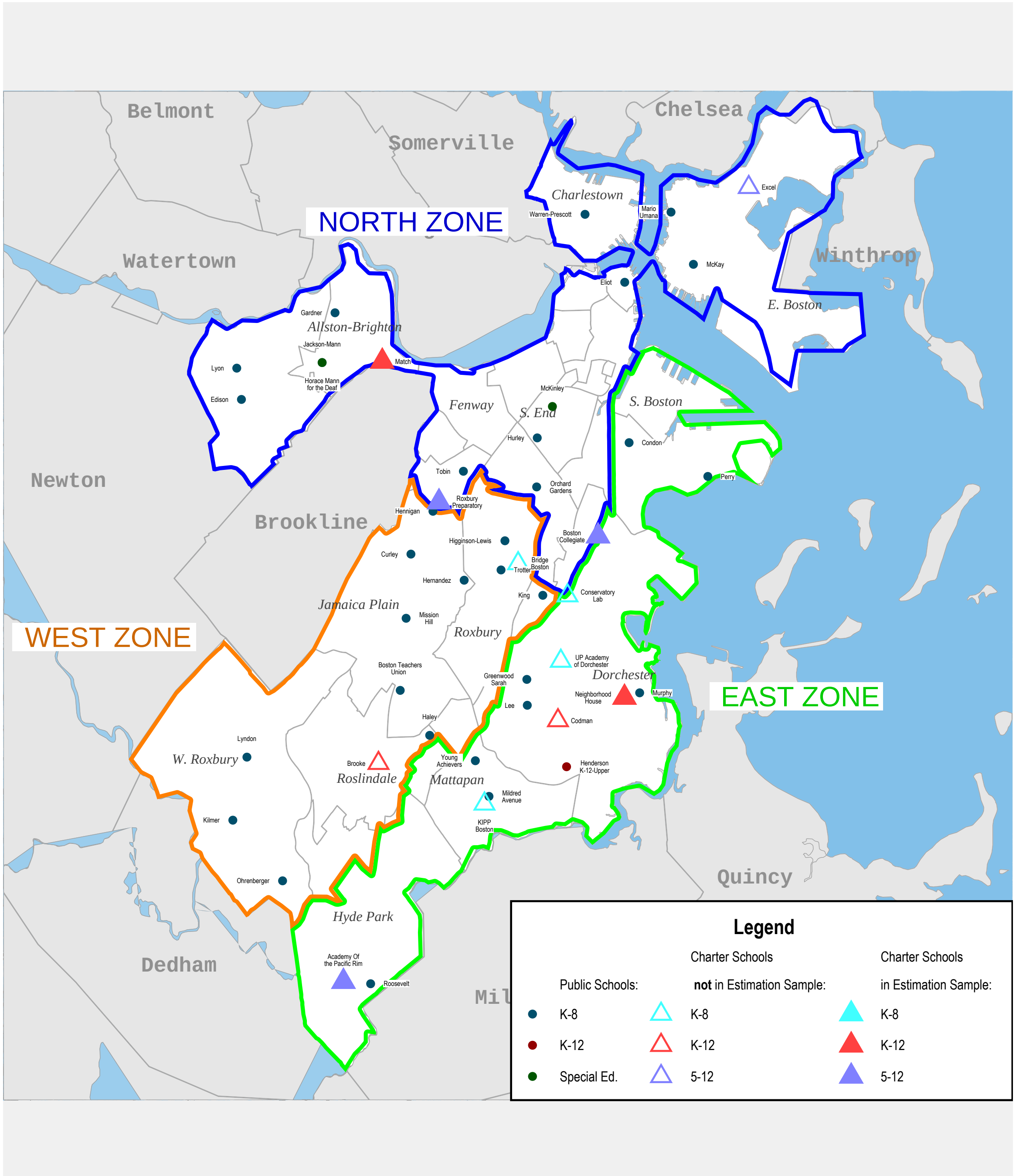
Notes: This table shows the fraction of the 701 applicants in the Boston Grade 5 sample in 2017 predicted to enroll in a school that is in the given position of their implied rankings at the end of the enrollment stage, using the estimated parameter values from demand model. Confidence intervals are based on the 5th and 95th percentiles of 100 simulations, and use variation in error terms across simulations while treating the original parameter estimates as fixed. Columns for Ranked applications assume that students are on the wait list for any school that they originally listed. The Ranked – Full Information column uses the implied school rankings at the enrollment stage as the input in the ranking.

Table 8: Students Eligible for Pairwise Trades by Application System

	(1)	(2)
Application System	Value	90% Confidence Interval
Common	20.7%	[18.5%, 23.1%]
Ranked	16.1%	[12.4%, 19.1%]
Decentralized	21.6%	[19.5%, 23.8%]

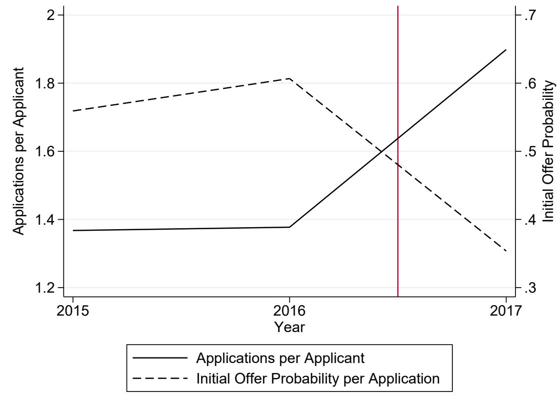
Notes: This table shows, for each application system across 100 simulations, the estimated fraction of the 701 Boston Public Charter School Application (BPCSA) applicants who would be eligible to trade placements with another student such that both students are better off, based on their simulated preferences during the enrollment stage. This represents an upper bound on trades that could occur, as an individual student could be involved in multiple potential trades. There are more potential trades under Common than Ranked in 99 percent of the simulations, more under Decentralized than Ranked in 100 of simulations, and more under Decentralized than Common in 68 percent of simulations.

Figure 1: Map of Schools and Geographic Zones in Boston

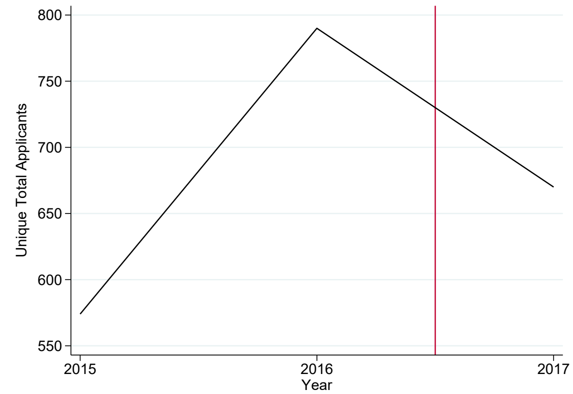


Notes: This figure shows the Grade 5 schools available to students during the 2017-2018 school year, as well as their placement in the three geographic zones corresponding to the zone configuration from the 1999-2013 Boston Public Schools choice system. Public schools are shown in circles, charter schools are in shown in triangles, and the charter schools in our structural estimation sample are shown in shaded triangles. Charter schools not in our estimation sample were either excluded due to incomplete data or because Grade 5 is not a primary entry year for the charter school. See the Data Appendix for the list of schools in this sample that are used in the time series analyses.

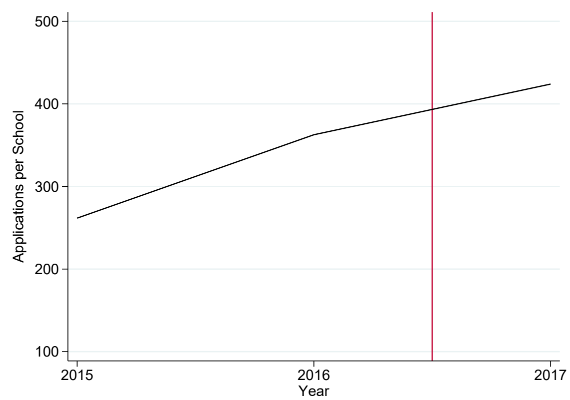
Figure 2: Time Series of Grade 5 Applicants and Applications



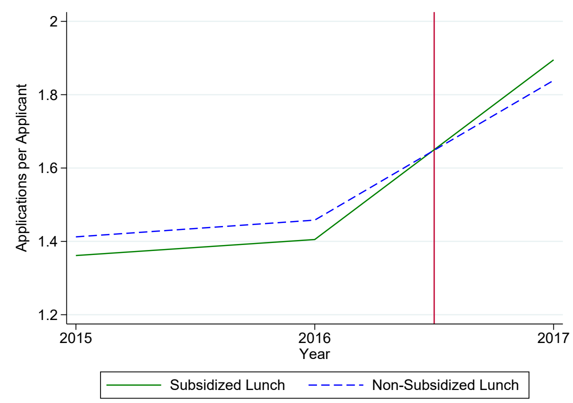
(a) Student-Side Demand



(b) Unique Total Applicants



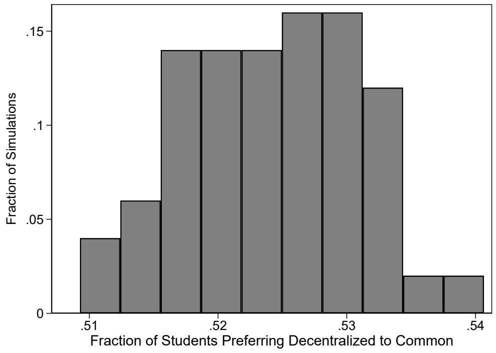
(c) Applications per School



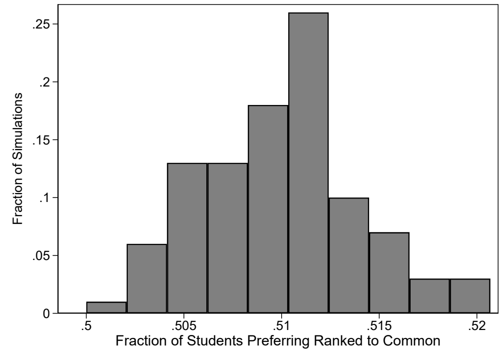
(d) Applications by Subsidized Lunch Status

Notes: These figures show application behavior and admissions outcomes from 2015-2017 for Boston Grade 5, among the three Grade 5 Boston Public Charter School Application (BPCSA) schools for which we have admissions and enrollment data across all three years. The sample size consists of $N = 2034$ applicants. Panel (a) shows, among applicants who apply to at least one BPCSA school, the time series of the average number of applications submitted, as well as the probability that each application results in an initial offer from the school. Panels (b) and (c) show, again among the three schools in the balanced panel, time series of the average number of applications received per BPCSA school as well as the total number of unique BPCSA applicants. Panel (d) shows, separately among subsidized lunch and non-subsidized lunch applicants to BPCSA schools, the time series of the average number of applications submitted. Subsidized lunch status is recorded in Grade 4.

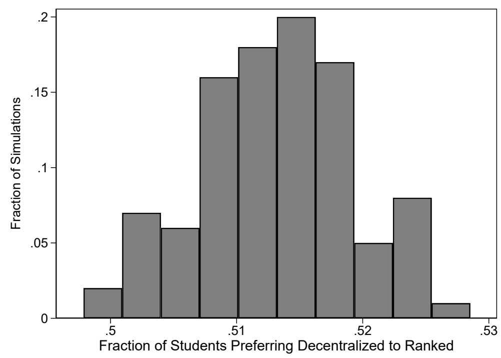
Figure 3: Pairwise Votes between Application Systems



(a) Decentralized vs. Common



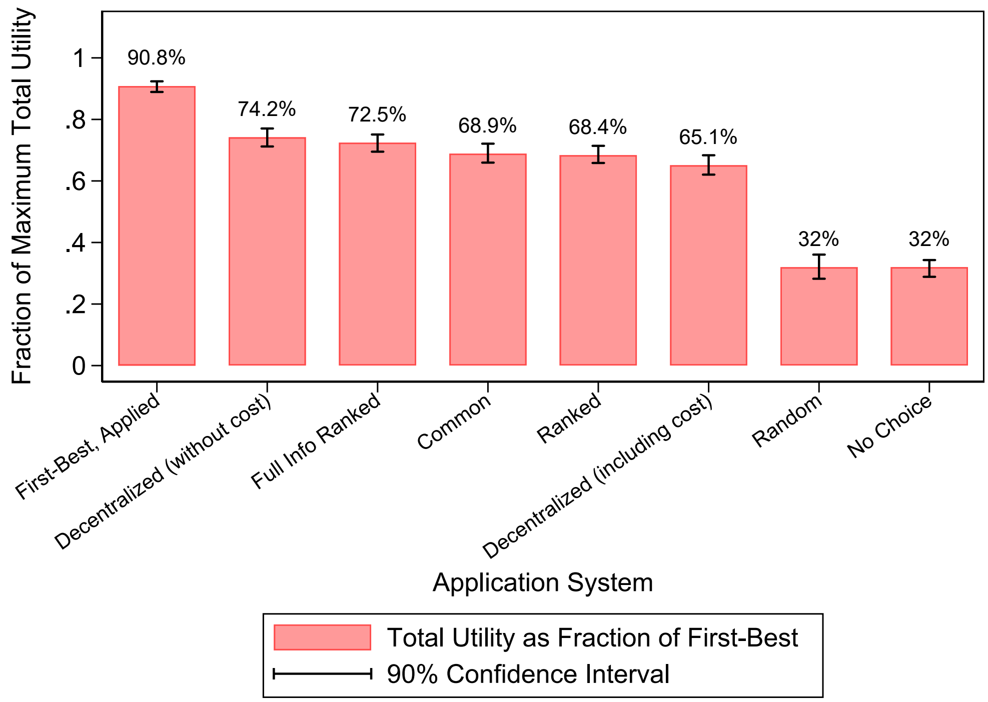
(b) Ranked vs. Common



(c) Decentralized vs. Ranked

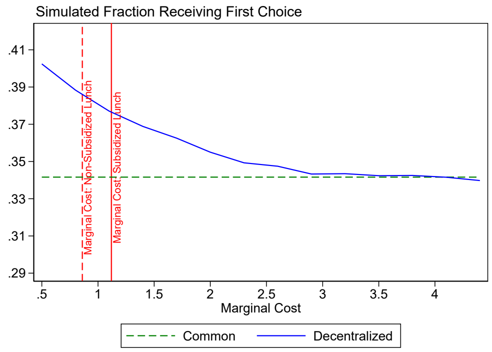
Notes: These histograms show – across 100 simulations of error terms – the fraction of the 701 applicants to a Boston Public Charter School Application (BPCSA) school in 2017 who would be predicted to prefer the outcomes from each application system based on the implied utility estimates. Utility calculations are drawn to rationalize the observed application and enrollment decisions for each applicant. The ranked application placement assumes that students are on the wait list for any school that they originally listed. The decentralized application placement only considers utility from the final placement, and does not subtract any marginal costs incurred during the application process from the total utility. Students who would be predicted to enroll in the same school under each of the two application systems cast half a vote for each system; this represents 63.4% of the students in panel (a), 67.4% of the students in panel (b), and 63.6% of the students in panel (c).

Figure 4: Comparison of Total Utility Across Application Systems

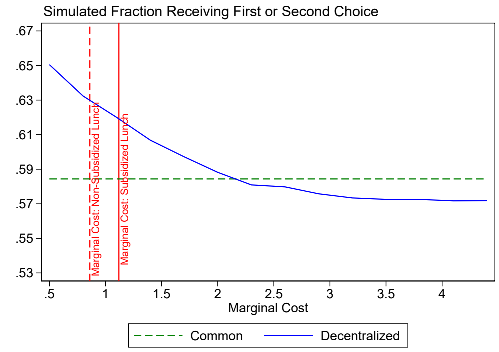


Notes: This figure shows, for Boston Grade 5 in 2017, the share of total utility under the first-best utility-maximizing allocation that is achieved by each alternate application system, among the 701 students who applied to at least one Boston Public Charter School Application (BPCSA) school. Utility calculations are based on 100 simulations of error terms that are drawn to rationalize the observed application and enrollment decisions for each applicant; confidence intervals are constructed based on variation across simulations, treating parameter estimates as fixed. The first-best allocation is the allocation that maximizes total utility, subject to capacity constraints at the BPCSA schools. Under the first-best allocation, 12.2% of applicants are placed in a school to which they are not observed submitting an application. The First-Best, Applied bar is the first-best subject to the constraint that students can only be placed in their outside option or in a school to which they applied. The Decentralized (including cost) bar subtracts out the marginal costs of applications beyond the first application, while the Decentralized (without cost) bar does not subtract these costs. The Random bar randomly places each applicant in a BPCSA school or the outside option, subject to capacity constraints at each BPCSA school. The No Choice bar places each applicant in their outside option, corresponding to an environment without BPCSA schools.

Figure 5: Common vs. Decentralized Application among Subsidized Lunch Applicants



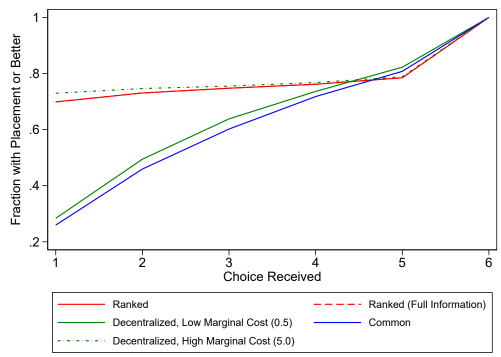
(a) 1st Choice



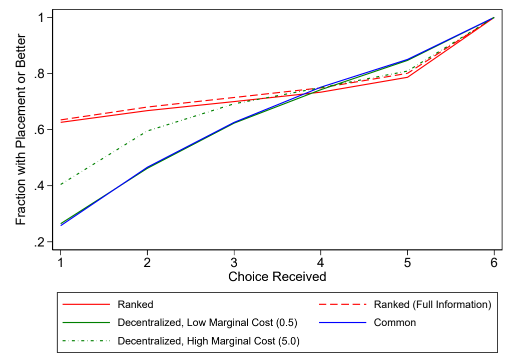
(b) 1st or 2nd Choice

Notes: These figures show, among the 536 subsidized lunch applicants to Boston Public Charter School Application (BPCSA) schools in 2017, the share of applicants estimated to receive their simulated first choice (panel (a)) and first or second choice (panel (b)). Estimates are shown for the common application system, as well as a decentralized application system in which the marginal costs for subsidized lunch applicants vary as shown on the x-axis while marginal costs for non-subsidized lunch applicants are fixed at the estimated value. Subsidized lunch status is measured as of Grade 4. Estimates are based on 100 simulations of error terms that are drawn to rationalize the observed application and enrollment decisions for each applicant.

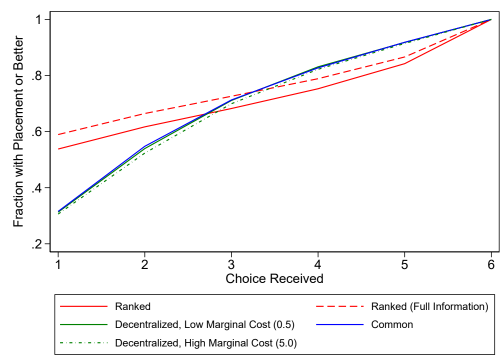
Figure 6: Example Distribution of School Enrollments – Heavy Oversubscription Environment



(a) No Preference Changes ($\kappa = 0$)



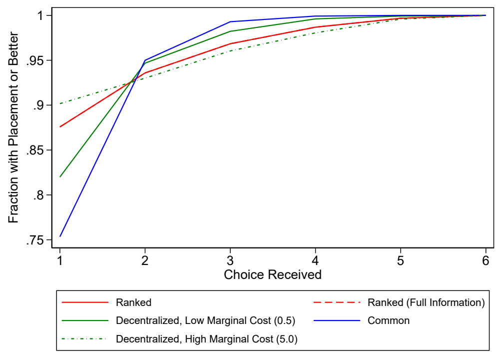
(b) Low Preference Changes ($\kappa = 1$)



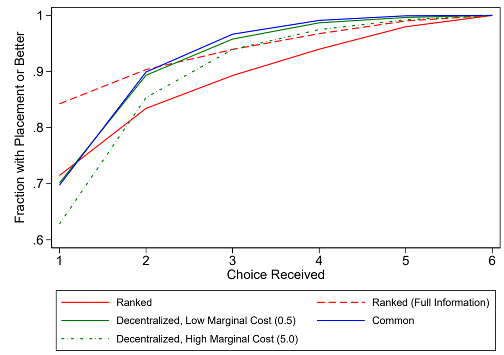
(c) High Preference Changes ($\kappa = 4$)

Notes: These figures show – for parameters calibrated to represent a heavy oversubscription environment – the distribution of school placements that students would receive under each application system. Each of the 10 simulations starts with the set of students and their driving times to the five schools from the structural estimation, and then imposes parameters $\beta = -0.01$; $\delta_1 = \delta_2 = ... = \delta_5 = 2.5$; $C = 0.5$; and acceptance rates $\pi_1 = \pi_2 = ...\pi_5 = 0.25$. As shown in Appendix Table C.1, this corresponds to each applicant applying to approximately 4.68 out of 5 schools on average. Each panel corresponds to different levels of changes in preferences between the application stage and the enrollment stage; the correlation between utilities u_{ij} and U_{ij} across the two stages equals 1.00 in panel (a), 0.77 in panel (b), and 0.29 in panel (c). In panel (a), the two ranked systems have the same distribution of school placements because student rankings are unchanged between the application and enrollment stage when $\kappa = 0$.

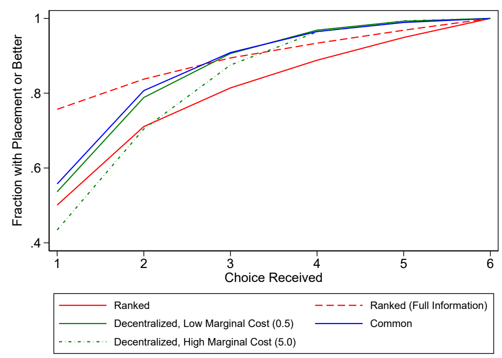
Figure 7: Example Distribution of School Enrollments – Light Oversubscription Environment



(a) No Preference Changes ($\kappa = 0$)



(b) Low Preference Changes ($\kappa = 1$)



(c) High Preference Changes ($\kappa = 4$)

Notes: These figures show – for parameters calibrated to represent a light oversubscription environment – the distribution of school placements that students would receive under each application system. Each of the 10 simulations starts with the set of students and their driving times to the five schools from the structural estimation, and then imposes parameters $\beta = -0.08$; $\delta_1 = \delta_2 = ... = \delta_5 = 0.5$; $C = 0.5$; and acceptance rates $\pi_1 = \pi_2 = ...\pi_5 = 0.75$. As shown in Appendix Table C.2, this corresponds to each applicant applying to approximately 2.34 out of 5 schools on average. Each panel corresponds to different levels of changes in preferences between the application stage and the enrollment stage; the correlation between utilities u_{ij} and U_{ij} across the two stages equals 1.00 in panel (a), 0.76 in panel (b), and 0.28 in panel (c). In panel (a), the two ranked systems have the same distribution of school placements because student rankings are unchanged between the application and enrollment stage when $\kappa = 0$.

Appendix A Theoretical Appendix

A.1 Existence and Uniqueness of Common Application Equilibrium

Suppose that there are S schools with capacities $K_1, K_2, \ldots, K_S$, where total capacity $K = K_1 + K_2 + \ldots + K_S < 1$. Define the enrollment function $e_j(p_1, \ldots, p_S)$ as the proportion of students who enroll in school j given these admission probabilities when all students apply to all schools.

Lemma 1 *Consider any two distinct vectors of probabilities $(p_1, \ldots, p_S)$ and $(q_1, \ldots, q_S)$ where $(1-p_1)(1-p_2)\ldots\ (1-p_S) = (1-q_1)(1-q_2)\ldots\ (1-q_S) = 1 - K$. Then for some school j, $e_j(p_1, \ldots, p_S) > e_j(q_1, \ldots, q_S)$ and also for some school $j\prime$, $e_{j'}(q_1, \ldots, q_S) > e_{j'}(p_1, \ldots, p_S)$.*

Proof. Divide the schools into three sets: S^1, where $p_j > q_j$ for each j, S^2 where $p_j < q_j$ for each j and S^3 where $p_j = q_j$ for each j. Sets S^1 and S^2 must each contain at least one school as otherwise $(1-p_1)(1-p_2)\ldots\ (1-p_S) < (1-q_1)(1-q_2)\ldots\ (1-q_S)$ (if S^1 is empty) or $(1-p_1)(1-p_2)\ldots\ (1-p_S) > (1-q_1)(1-q_2)\ldots\ (1-q_S)$ (if S^2 is empty) given that the two vectors are not identical. Now suppose that we start at $(p_1, \ldots, p_S)$ and adjust the admission probabilities from p_j to q_j one at a time starting with school 1 and continuing school by school through school S. (This procedure does not maintain the total capacity of enrollment at K at each step.). If $q_j > p_j$, then switching from $(q_1, \ldots, q_{j-1}, p_j, p_{j+1}, \ldots, p_S)$ to $(q_1, \ldots, q_{j-1}, q_j, p_{j+1}, \ldots, p_S)$ simply adds admissions offers to some students who were previously not admitted to school j. Students who are affected by this change either choose the same school under both systems or choose school j when they would not have done so in the original outcome. Since all admission probabilities are less than 1, a positive measure of students are admitted to none of the schools under original probabilities and are admitted only to school j given the adjusted probabilities. Thus, total enrollment in S^2 strictly increases and total enrollment in S^1 weakly decreases when we switch from $(q_1, \ldots, q_{j-1}, p_j, p_{j+1}, \ldots, p_S)$ to $(q_1, \ldots, q_{j-1}, q_j, p_{j+1}, \ldots, p_S)$. By a similar argument, total enrollment in S^2 weakly increases and total enrollment in S^1 strictly decreases given a switch from $(q_1, \ldots, q_{j-1}, p_j, p_{j+1}, \ldots, p_s)$ to $(q_1, \ldots, q_{j-1}, q_j, p_{j+1}, \ldots, p_S)$ when $q_j < p_j$. Combining these results, total enrollment is S^1 is strictly lower with $(q_1, q_2, \ldots, q_S)$ than with $(p_1, p_2, \ldots, p_S)$ and thus enrollment for at least one school in S^1 must be strictly lower with $(q_1, q_2, \ldots, q_S)$ while the reverse holds for S^2. ■

We now proceed to the proof, using the following induction on the number of unknown admission probabilities.

Part 1. Suppose that 2 (or 1) admission probabilities remain unknown.

Claim: If $S-2$ admission probabilities are fixed (and fewer than K students are admitted to those schools) then there is a unique choice of the last two probabilities such that

(1) K students enroll, i.e. $(1-p_1)(1-p_2)\dots(1-p_S) = 1-K$

(2) $e_{S-1}/e_S = k_{S-1}/k_S$.

If fewer than K students are admitted to schools 1 to $S-2$, then $(1-p_1)(1-p_2)\dots(1-p_{S-2}) > 1-K$. Define $\gamma = (1-p_1)(1-p_2)\dots(1-p_{S-2})$. Since $\gamma(1-p_{S-1}) < K$ and $\gamma(1-p_S) < K$, p_{S-1} and p_S are each bounded above by $1-(1-K)/\gamma$.

Solving (1) for p_S gives

$(1-p_{S-1})(1-p_S) = (1-K)/\gamma$

OR $p_S = [\gamma(1-p_S-1) - (1-K)]/\gamma(1-p_{S-1})]$. (3)

Thus, p_S is positive, continuous and strictly decreasing in p_{S-1} for $p_{S-1} < 1-(1-K)/\gamma$ and $p_S = 0$ at $p_{S-1} = 1-(1-K)/\gamma$. Given $p_1, p_2, \dots, p_{S-2}$ and $p_S = [\gamma(1-p_{S-1}) - (1-K)]/\gamma(1-p_{S-1})]$, the enrollment values e_{S-1} and e_S are continuous functions of p_{S-1} where e_{S-1} is strictly increasing and e_S is strictly decreasing in p_{S-1}. At the extreme values $p_{S-1} = 0$ and $p_{S-1} = (1-K)/\gamma$, one of the last two schools does not enroll any students: $e_{S-1}(p_{S-1} = 0) = 0$ and $e_S(p_{S-1} = (1-K)/\gamma) = 0$. Thus the ratio e_{S-1}/e_S is strictly decreasing in p_{S-1} and takes on all positive values as p_{S-1} ranges from 0 to $(1-K)/\gamma$. Therefore, there must be a unique choice p_{S-1} such that condition (2) holds.

More generally, this shows that for each $(p_1, \dots, p_{S-2})$ with $(1-p_1)(1-p_2)\dots(1-p_{S-2}) > 1-K$, there is a unique pair (p_{S-1}, p_S) such that (1) and (2) hold. Further, (p_{S-1}, p_S) varies continuously with $(p_1, \dots, p_{S-2})$ as a jump in (say) p_{S-1} would require a corresponding reduction in p_S with the result that at least one of the two schools would miss its enrollment target.

Part 2 (Induction Step). Suppose that these properties hold when J admission probabilities are fixed and $S-J$ admission probabilities are unknown. Show that the properties also hold when $J+1$ admission probabilities are fixed and $S-J-1$ admission probabilities are unknown.

Formally, suppose $J < S-2$ and that for each $(p_1, p_2, \dots, p_J)$ satisfying $(1-p_1)(1-p_2)\dots(1-p_J) > 1-K$, there exists a unique combination $(p_{J+1}, \dots, p_S)$ where $p_{J+1}, \dots, p_S$ are continuous in $(p_1, p_2, \dots, p_J)$ and

(1J) $(1-p_1)(1-p_2)\dots(1-p_S) = 1-K$;

(2J) For each pair (i,j) with $i, j > J$, the enrollment ratio $e_i/e_j = k_i/k_j$

We want to show that result holds when $J-1$ probabilities are known and $S-J+1$ are to be determined.

Fix $(p_1, p_2, \dots, p_{J-1})$ with $(1-p_1)(1-p_2)\dots(1-p_{J-1}) > 1-K$. Set $\lambda = (1-K)/[(1-p_1)(1-p_2)\dots(1-p_{J-1})]$.

If $p_J = \lambda$ then $(1-p_1)(1-p_2)\dots\ (1-p_J) = 1-K$, so $(1-p_1)(1-p_2)\dots\ (1-p_J) < 1-K$ if $p_J < \lambda$. By inductive hypothesis, for each p_J such that $0 < p_J < \lambda$, there exists a unique choice of probabilities $(p_{J+1}(p_J)\dots, p_S(p_J))$ such that $(1J)$ and $(2J)$ hold.

At p_J =0, enrollment $e_J = 0$ and enrollments $e_{J+1}, \dots, e_S > 0$, so $e_J/e_{J+1} = 0$. At $p_J = \lambda$, enrollment $e_J > 0$ and enrollments $e_{J+1}, \dots, e_S = 0$, so e_J/e_{J+1} becomes arbitrarily large in the limit at p_J approaches λ from below. Since $(p_{J+1}, \dots, p_S)$ vary continuously in p_J (holding $p_1, \dots, p_{J-1}$ fixed), the ratio e_J/e_{J+1} is continuous in p_J for $0 < pJ < \lambda$ and so there must exist some value p_J^* such that $e_J/e_{J+1} = k_J/k_{J+1}$ at p_J^*. By (2), then $e_J/e_{J+2} = k_J/k_{J+2}\ \dots\ e_J/e_S = k_J/k_S$ as desired.

Suppose that there are two different combinations of probabilities $(p_1, \dots, p_{J-1}, p_J, p_{J+1}, \dots, p_S)$ and $(p_1, \dots, p_{J-1}, q_J, q_{J+1}, \dots, q_S)$ satisfying (1). Then by the lemma, one school must have increased enrollment and antoher must have decreased enrollment as a result of the change in probaibliites, so it is not possible for both probabilities to sastify (2). By the lemma, there must be schools $j, j' > J$ such that

$e_j(p_1, \dots, p_{J-1}, p_J, p_{J+1}, \dots, p_S) > e_j(p_1, \dots, p_{J-1}, q_J, q_{J+1}, \dots, q_S)$ and

$e_{j'}(p_1, \dots, p_{J-1}, p_J, p_{J+1}, \dots, p_S) < e_{j'}(p_1, \dots, p_{J-1}, q_J, q_{J+1}, \dots, q_S)$ and

So the ratio of enrollments between schools j and $j\prime$ are different given the two different sets of admissions probabilities. This shows that there is (at most) a unique choice of probabilities for schools J to S that satisfy (1) and (2), where these probabilities must be continuous in $(p_1, p_2, ..., p_J)$ as a discontinuous jump up in any of $(p_{J+1}, ..., pS)$ would make it impossible to maintain (2J) for every pair (i, j) with $i, j > J$.

A.2 Proof of Proposition 6

Proof. Under the Decentralized Application system, with symmetric preferences and identical admission probabilities at each school, students apply in order to their most preferred schools, with sequentially increasing thresholds in utility values for each application. So, if m_k students apply to a k^{th} choice school in equilibrium, then $m_1 \geq m_2 \geq m_3 \dots \geq m_S$.

Step 1: Given the market clearing condition that K students enroll at each school and probability p of admission to each school, the number of students attending k^{th} choice schools or better is minimized by the Common Application pattern: $m_1 = m_2 = \dots\ = m_S$.

The minimum number of students attending k^{th} choice schools corresponds to the maximum number of students (given the market constraint) attending schools of $k+1^{st}$ choice or worse. For any combination of applications $(m_1, m_2, \dots, m_k; p)$ satisfying the market clearing constraint:

(1) If $m_l > m_{l+1}$ for any $l < k$, then we could maintain the market clearing constraint by increasing all of $(m_{l+1}, \dots\ m_S)$ while reducing each of $(m_1, m_2, \dots\ m_l)$. This change increases each of m_{k+1}, m_{k+2}, …, m_S, thus increasing the number of students attending $k + 1^{st}$ choice schools or worse and thereby reducing the number of students attending k^{th} choice schools or better. So the application pattern yielding the minimum number of students attending k^{th} choice schools or better sets $m_1 = m_2 = \dots\ = m_k$.

(2) Consider any application pattern $(m_1 = m, m_2 = m, \dots, m_k = m, m_{k+1}, \dots, m_S)$. If $m_l > m_{l+1}$ for any $l > k + 1$, then we could maintain the market clearing conditions by increasing each of $(m_{l+1}, m_{l+2}, \dots\ , m_S)$ and reducing each of $(m_1, m_2, \dots, m_{l+1})$. Since this change reduces each of $(m_1, m_2, \dots, m_k)$, it reduces the number of students attending k^{th} choice schools or better.

Combining points (1) and (2), the application pattern that yields the minimum number of students attending k^{th} choice schools or better is $m_1 = m_2 = \dots\ = m_S$, i.e. the Common Application pattern where every student applies to all S schools.

Step 2: Given the market clearing condition that K students enroll at each school, the Common Application pattern $m_1 = m_2 = \dots\ = m_S$ and probability p of admission to each school, the number of students attending k^{th} choice schools or better is increasing in p.

By symmetry, an equal number of students will be admitted to first choice schools, to second choice schools, … to S^{th} choice schools in a Common Application equilibrium. Denote this number as $A(p)$ as a function of the equilibrium admission probability p per application.

A student who is admitted to a k^{th} choice school will attend that school if not admitted to a preferred school – i.e. with probability $(1 - p)^{k-1}$. So the market clearing condition is

$$A(p) + A(p)(1 - p) + A(p)(1 - p)^2 + \dots\ + A(p)(1 - p)^{S-1} = K * S. \qquad (*)$$

Since each of the $(1 - p)$ terms is declining in p, $A(p)$ must be strictly increasing in p to maintain this market clearing condition. So a higher equilibrium probability of admission, means that a larger number of applicants must be admitted to each school (since this higher probability of admission increases the likelihood that an applicant will be admitted to multiple schools).

Compare the admission outcomes for two distinct values $p_1 > p_2$. Since $A(p)$ is increasing in p, a greater number of students will attend 1st choice schools when $p = p_1$ than when $p = p_2$. To maintain the market clearing condition, some terms in the sum must decline when p increases from p_1 to p_2. Since $p_1 > p_2$, we know that $(1 - p_2) > (1 - p_1)$, so $A(p_2)(1 - p_2)k - 1 > A(p_1)(1 - p_1)^{k-1}$ implies $A(p_2)(1 - p_2)^{n-1} > A(p_1)(1 - p_1)^{n-1}$ for all subsequent terms $n > k$. That is, the initial terms in the sequence $\{A(p_1), (1 - p_1)A(p_1) \dots, (1 - p_1)^{S-1}A(p_1)\}$, representing the number of students assigned to $1^{st}, 2^{nd}, \dots\ S^{th}$ choice schools given p_1, are greater than the initial terms in the corresponding sequence $\{A(p_2), (1-p_2)A(p_2)\ldots, (1-p_2)^{S-1}A(p_2)\}$, but the inequality is reversed for the final terms in those sequences.

Thus, the number of students attending 1st choice schools is greater for $p = p_1$ than for $p = p_2$. Beyond that point, the difference between the number of students attending kth choice schools for $p = p_1$ than for $p = p_2$ increases up to some point then gradually decreases until exactly the same number of students $(S * K)$ attend S^{th} choice schools or better with either p_1 or p_2. This proves that the number of students attending k^{th} choice schools or better with a Common Application equilibrium is increasing in the admission probability p.

Step 3: Combine Steps 1 and 2: In sum, Step 1 shows that the Common Application pattern achieves the lowest probability of attending a k^{th} choice school or better for a given probability of admission, while Step 2 shows that given the Common Application pattern, the probability of attending a k^{th} choice school or better increases with the probability of admission. Proposition 2 shows that in equilibrium the Common Application probability of admission is lower than the decentralized admission probability of admission. So, the Common Application pattern achieves the lowest probability of attending a k^{th} choice school or better given admission probability p_C and this is lower than the probability of attending a k^{th} choice school for any market clearing pattern of applications given admission probability $p > p_C$. This proves the result. ■

A.3 Example 3

Example 3 *Suppose that there are three schools S_1, S_2, and S_3, each wishing to enroll 1/3 of all students, that 80% of the students have strict preference ordering (S_1, S_2, S_3) and that 20% of the students have strict preference ordering (S_2, S_3, S_1) over the three schools. Denote the probabilities of receiving an offer to each school as p_{1R}, p_{2R}, p_{3R} for the Ranked Admission rule and as p_{1C}, p_{2C}, p_{3C} for the Common Application.*

Then with the Ranked Application rule, when there are a large number of students, the first students in line choose either School S_1 or School S_2, with School S_1 reaching capacity after making offers to $(1/3)/(0.8) = 5/12$ of the students. The next students in the line are offered the choice between Schools S_2 and S_3, and since all prefer School S_2, it reaches capacity when it has made offers to 2/3 of the students. Then School S_3 makes offers to the remaining students, who all enroll. The resulting offer probabilities are $p_{1R} = 5/12, p_{2R} = 2/3$ and $p_{3R} = 1$.

Similarly, with the Common Application, School 3 must make offers to all students to meet its enrollment target, so $p_{3C} = 1$. Then students with preference ordering (S_2, S_3, S_1) enroll at school S_2

with probability p_{2C} and at School S_3 with probability $1 - p_{2C}$. Students with preference ordering (S_1, S_2, S_3) enroll at school S_1 with probability p_{1C}, at School S_2 with probability $p_{2C}(1 - p_{1C})$ and otherwise enroll at school S_3. Solving in order for p_{1C} and p_{2C}, we have $0.8p_{1C} = 1/3$, or $p_{1C} = 5/12$ and $0.8\ p_{2C}\ (1 - p_{1C}) + 0.2p_{2C} = 1/3$, with solution $p_{2C} = 1/2$.

Under Ranked Application, all students with preferences (S_2, S_3, S_1) enroll at a first-choice or second-choice school while 2/3 of the students with preferences (S_1, S_2, S_3) do so. Thus, $0.8 * 2/3 + 0.2 * 1 = 11/15$ of students enroll at either a first-choice or second-choice school. Under the Common Application, all students with preferences (S_2, S_3, S_1) enroll at a first-choice or second-choice school while $(5/12 + 1/2 - 5/12 * 1/2) = 17/24$ of the students with preferences (S_1, S_2, S_3) do so. Thus, $0.8*17/24+0.2*1 = 23/30$ of students enroll at either a first-choice or second-choice school. Here, the probability of enrolling at a first-choice or second-choice school is higher with the Common Application than with the Ranked Application, so the ordinal distribution of outcomes with the Ranked Application does not stochastically dominate the ordinal distribution of outcomes with the Common Application.

Appendix B Common Applications in the Field

Table 1 describes the adoption of a common application in several U.S. cities since 2017. This appendix provides a summary of the anticipated effects of the authorities, organized into three categories. Below are excerpts from press releases or newspaper interviews with individuals involved in the reforms.

1) Ease of logistics

   - "Previously, parents had to research grade levels at each school and file separate applications, a process that could take hours." (Boston)
   - "The goal of EnrollBR is to simplify the process and make it easier for families to explore the choices available to them." (Baton Rouge)
   - "No more trying to find how to apply and where to apply, no more hassle filing out multiple forms, no more confusion for parents but definitely better access to a school choice." (Buffalo)
   - "Oakland families no longer have to drive all over town from one charter school to another, fill out various different paper applications and keep track of different deadlines. More importantly families have benefited from learning that they have choices when it comes to their child's education and that includes charter public schools." (Oakland)
   - "It's all about making it easier for families in Philadelphia to find a school that's best for their child... Running dozens of separate application processes only exacerbates the problem. Many say this fractured landscape disadvantages those without time or wherewithal to fill out multiple forms." (Philadelphia)
   - "Traditionally, most charter public schools have their own application and enrollment processes, meaning families must juggle separate applications, multiple websites, and different deadlines if they want to apply to more than one schools. The ApplyLA Charter Common Application seeks to simplify the process." (Los Angeles)
   - "We are partnered with the Rhode Island Department of Education to allow for an easier and more streamlined application process." (Rhode Island)
   - "SchoolMint is an application platform that allows parents to apply to multiple schools, and for multiple students with a single application. This will make applying for any charter school within the Atlanta Public School system easier and more transparent." (Atlanta)

- "Parents don't have to go to four different schools and fill out four separate applications... We really want to show the unity between charter schools and the ease of the application." (Kansas City)
- "The need for change, they said, became clear in January after the charters set up information booths at Dorchester Collegiate Charter School, a day after the state voted to shut it down. Charter staffs watched as families spent hours moving from one booth to another, filing out applications that requested mostly the same information as they agonized over their children's future." (Boston)

2) Increase information about options

- "A common application for Commonwealth charter schools will make it easier for families to enter their children in our enrollment lotteries, and provide them with more information about their educational choices." (Boston)
- "We think this is great for parents because it gives them the opportunity to explore multiple schools and make informed decisions about the best option for their children." (Baton Rouge)
- "It is our hope that this platform increases access to and knowledge of the area's charter schools and simplifies the application process for families." (Buffalo)
- "The promise of a marketplace of schools is also a promise that kids and parents can navigate that marketplace ... [Right now], there's no single place, time, or process for parents and kids to select and enroll in schools, so we're not really maximizing choice." (Detroit)
- "Our main purpose was to make it easier in a choice environment for parents to choose." (NYC)
- "The new process could help parents, who often rely on word of mouth for information, learn about schools they might not have considered." (Houston)
- "Houston parents have become accustomed to having a diversity of choices and this movement – this ApplyHouston.org – is really just about helping parents become better consumers to be able to better navigate the marketplace of school options [...] Before families had to keep track of different forms for different charter schools, which ... [is] big hassle."' (Houston)
- "I believe in school choice, and anything that helps families make wiser choices for their

children is a plus. (The website) gives a family the chance to see all the offerings and easily apply to the school that fits their preferences." (Rochester)

– "This will be great for families... [the online application] streamlines information for families and makes it easier for families to understand where there are seats across the charter sector." (Boston)

3) Leveling the playing field / equity

– "He [Peterson] said he looks at the common application process in Baton Rouge as a step toward greater equity. 'We need to do anything we can to remove barriers and obstacles for families,' Peterson said." (Baton Rouge)

– "It is important that we are creating a system that is fair and easy for parents to access, and allows schools to design programs and staff around the needs of their students." (Baton Rouge)

– "Some believe that a single application for both charter and district public schools would simplify enrollment for all parents, especially those without the time, money or privilege to navigate multiple public school options with different application deadlines and processes." (Oakland)

– "The initiative's backers say they're increasing access and helping parents. Given the dismal lottery odds at some city charter schools, they say, many families feel they have to apply to a bunch of charters to ensure they get in somewhere. Now more city parents can do that quickly, easily, and without the kind of extra legwork that can be difficult for working families to manage." (Philadelphia)

– "It helps enormously, because it simplifies the process, it simplifies their lives. It also creates a much more equitable system for all families. Educated parents who have the time have a tremendous advantage in getting their kids into the bets schools. They know how to bake brownies for the principal and get to know the principal. They know other ways to circumvent the rules and get their child into a good school. Then there are other parents who don't know those ways. They don't speak English, they may be working two jobs and will not be able to spend that time. A common enrollment system tends to level the playing field and get more equal opportunity to all families." (Los Angeles)

– "Michael Duffy, the head of the city's charter schools, said the city's goal was 'to widen the access for families' to charter schools. Duffy previously spearheaded a push to increase recruitment by charter schools, and said that the new common application should help charters reach out to groups of students, including those learning English, that charter recruiters often miss." (New York City)

– "For families, schools, and the District, the former system of charter school enrollment was disjointed and cumbersome with different processes and timelines across charter schools. This new more centralized system our office has developed in collaboration with the District's charter schools will allow for more equitable access to school choice options for families and make the whole process of charter school enrollment in APS more efficient and transparent." (Atlanta)

Appendix C Application Patterns Under Example Environments

Table C.1: Application Patterns in Heavy Oversubscription Environment Example

	(1)	(2)	(3)	(4)	(5)
		Decentralized		**Ranked**	
	Common	Low Marginal Cost (0.5)	High Marginal Cost (5.0)	Baseline	Full Information
No Preference Changes ($\kappa = 0$)					
Number of Applicants	680.00	680.00	680.00	680.00	680.00
Applications per Applicant	4.68	4.44	1.07		
Acceptance Rate	0.26	0.28	0.73		
Fraction Attending Outside	0.26	0.26	0.26	0.26	0.26
Low Preference Changes ($\kappa = 1$)					
Number of Applicants	676.70	676.70	676.70	676.70	676.70
Applications per Applicant	4.68	4.44	1.07		
Acceptance Rate	0.25	0.26	0.68		
Fraction Attending Outside	0.34	0.34	0.34	0.36	0.34
High Preference Changes ($\kappa = 4$)					
Number of Applicants	680.40	680.40	680.40	680.40	680.40
Applications per Applicant	4.69	4.45	1.07		
Acceptance Rate	0.25	0.26	0.70		
Fraction Attending Outside	0.48	0.48	0.48	0.54	0.48

Notes: This table shows the application patterns for the empirical example in which parameters are calibrated to represent a heavy oversubscription environment. Each value in the table corresponds to the average across 10 simulations. Each of the simulations starts with the set of students and their driving times to the five schools from the structural estimation, and then imposes parameters $\beta = -0.01$; fixed effects 2.5 for each school; fixed cost 0.5; and acceptance rates 0.25 for each school. Each panel corresponds to different levels of changes in preferences between the application stage and the enrollment stage; the correlation between utilities across the two stages equals 1.00 when $\kappa = 0$, 0.77 when $\kappa = 1$, and 0.29 when $\kappa = 4$. The Fraction Attending Outside row shows the share of students who would end up in a school that is not part of the common application.

Table C.2: Application Patterns in Light Oversubscription Environment Example

	(1)	(2)	(3)	(4)	(5)
		Decentralized		**Ranked**	
	Common	Low Marginal Cost (0.5)	High Marginal Cost (5.0)	Baseline	Full Information
No Preference Changes ($\kappa = 0$)					
Number of Applicants	412.00	412.00	412.00	412.00	412.00
Applications per Applicant	2.34	1.81	1.00		
Acceptance Rate	0.75	0.81	0.90		
Fraction Attending Outside	0.10	0.10	0.10	0.10	0.10
Low Preference Changes ($\kappa = 1$)					
Number of Applicants	415.20	415.20	415.20	415.20	415.20
Applications per Applicant	2.34	1.81	1.00		
Acceptance Rate	0.76	0.81	0.90		
Fraction Attending Outside	0.23	0.23	0.25	0.33	0.23
High Preference Changes ($\kappa = 4$)					
Number of Applicants	405.20	405.20	405.20	405.20	405.20
Applications per Applicant	2.36	1.83	1.00		
Acceptance Rate	0.76	0.86	0.96		
Fraction Attending Outside	0.36	0.37	0.40	0.52	0.36

Notes: This table shows the application patterns for the empirical example in which parameters are calibrated to represent a light oversubscription environment. Each value in the table corresponds to the average across 10 simulations. Each of the simulations starts with the set of students and their driving times to the five schools from the structural estimation, and then imposes parameters $\beta = -0.08$; fixed effects 0.5 for each school; fixed cost 0.5; and acceptance rates 0.75 for each school. Each panel corresponds to different levels of changes in preferences between the application stage and the enrollment stage; the correlation between utilities across the two stages equals 1.00 when $\kappa = 0$, 0.76 when $\kappa = 1$, and 0.28 when $\kappa = 4$. The Fraction Attending Outside row shows the share of students who would end up in a school that is not part of the common application.

Appendix D Estimation Computational Details

Implementation

Estimation of the baseline model uses a grid of 36 initial conditions of sets of parameter values. For each initial condition in this grid, we use the built-in Powell's Method solver from the `scipy.optimize` package in Python to solve for the solution, with convergence reached based on a tolerance of 10^{-6} in the objective function across subsequent iterations. Our estimation procedure also requires setting parameter values to address approximations in the likelihood function. To implement the logit smoother, we set $\lambda = 0.15$ The simulation of the application decision in Case II as described in Section 5.2 uses 1000 draws of the error terms, and the simulation of the integral in Case III uses 300 draws of the error terms.

Logit Smoother

Our approach to implementing the logit smoother in Case III of Section 5.2 closely follows Section 5.6.2 of Train (2009). In a "standard" discrete choice setting, in which we observe one choice from a set of options, the algorithm is described as follows:

1. For each of $s = 1, 2, ..., S$ simulations, draw a set of error terms $\epsilon_{ij}^{(s)}$ from a desired distribution for each student and school.

2. Using the drawn error terms and the deterministic utility estimates v_{ij}, construct for each simulation, student, and school: $u_{ij}^{(s)} = v_{ij} + \epsilon_{ij}^{(s)}$.

3. Suppose that each student was required to select exactly one school. Then given the error terms in each simulation, we should know deterministically the school chosen by each student. We can use a smooth approximation, however, by introducing an additional error term with a small variance. For small λ, the approximated probability that student i chooses school j is then $P_{ij} = \frac{1}{S} \sum_{s=1}^{S} \frac{\exp(u_{ij}^{(s)}/\lambda)}{\sum_{j=1}^{J} \exp(u_{ij}^{(s)}/\lambda)}$.

As λ approaches 0, the expression $\frac{\exp(u_{ij}^{(s)}/\lambda)}{\sum_{j=1}^{J} \exp(u_{ij}^{(s)}/\lambda)}$ approaches the indicator function.

Train (2009) also notes that this approximation is conceptually similar to adding a further extreme value error term with a low variance to the decision problem.

Our model differs from the standard discrete choice problem in that individuals may apply to multiple schools and do not rank these choices in the first stage. Nevertheless, the logic above extends directly to our setting. Consider a basic example in which there are three schools (A, B, C) and a student applies to A and B, but not C. Given a simulation with error terms ϵ_j for each school, the deterministic indicator function for this observed application would be $\mathbb{1}_{\{u_A, u_B > u_O > u_C\}}$, where u_j is the sum of the deterministic utility and its error term.

In contrast, the approximation with the logit smoother for small λ is given by the following the formula:

$$\Big(1 - \frac{\exp(\frac{u_O}{\lambda}) + \exp(\frac{u_C}{\lambda})}{\exp(\frac{u_A}{\lambda}) + \exp(\frac{u_O}{\lambda}) + \exp(\frac{u_C}{\lambda})} - \frac{\exp(\frac{u_O}{\lambda}) + \exp(\frac{u_C}{\lambda})}{\exp(\frac{u_B}{\lambda}) + \exp(\frac{u_O}{\lambda}) + \exp(\frac{u_C}{\lambda})} + \frac{\exp(\frac{u_O}{\lambda}) + \exp(\frac{u_C}{\lambda})}{\exp(\frac{u_A}{\lambda}) + \exp(\frac{u_B}{\lambda}) + \exp(\frac{u_O}{\lambda}) + \exp(\frac{u_C}{\lambda})}\Big)$$
$$\times \Big(\frac{\exp(\frac{u_O}{\lambda})}{\exp(\frac{u_O}{\lambda}) + \exp(\frac{u_C}{\lambda})}\Big)$$

Note that the first factor is analogous to the formula from Ophem, Stam, and Praag (1999) for $Pr(A, B \succ O, C)$ and the second factor is analogous to the standard discrete choice formula for $Pr(O \succ C)$.

Simulation Procedure

The integral within the likelihood function (for Case III) can be computed as follows:

1. For each of $s = 1, 2, ..., S$ simulations, draw a set of i.i.d. error terms $\epsilon_{ij}^{(s)}$ from the Gumbel distribution for each student and school. (Use this same set of simulated error terms in each iteration of the optimization procedure.)

2. Within each simulation, use the logit smoother to determine the probability that the resulting set of applications $A_i^{(s)}$ is consistent with the observed application decisions A_i. Denote this as P_i^s. This is a continuous approximation of the indicator variable function. This requires setting a value of $\lambda \in [0, 1]$, where lower values of λ are a better approximation of the indicator function.

3. Then approximate the integral within the likelihood function as:

$$\frac{1}{S}\sum_{i=1}^{s}\Big(\frac{Z_{iS_i}\exp(v_{iS_i} + \epsilon_{iS_i}^{(s)})}{\exp(\epsilon_{i0}^{(s)}) + \sum_{j'=1}^{J} Z_{ij'}\exp(v_{ij'} + \epsilon_{ij'}^{(s)})} P_i^{(s)}\Big)$$

Appendix E Alternate Application Model: Expected Utility

E.1 Model Setup

Notation

As an alternate model to the application model that we posit in our baseline estimation, we consider an expected utility model of application decisions, similar in spirit to Walters (2018). Notation is defined similar as in our baseline model setup; however, in contrast to that model in which students make a school-by-school application decision, the expected utility model requires making decisions over all potential portfolios of applications, including the possibility of applying to no schools. Specifically:

- $A_{ij} \in \{0, 1\}$ denotes the decision for individual i to apply to school j. A_i denotes the complete application portfolio in $\{0, 1\}^J$ of individual i, among all possible 2^J choices $a \in A_i$.

- $Z_{ij} \in \{0, 1\}$ denotes whether individual i was admitted to school j. Z_i denotes the complete set of admissions decisions in $\{0, 1\}^J$ of individual i, among all possible 2^J combinations of admissions decisions $z \in Z_i$. Not all admissions decisions are possible given an application set A_i, since being admitted to a school requires first applying to a school.

- S_i is the observed school chosen by student i (which may be a BPCSA school or the outside option).

- Denote the set of parameters as $\theta = (\delta, \beta)$ and the set of observed individual characteristics (e.g., location) as X_i.

Parameterization

Utility in each stage is parameterized identically to the baseline model. An expected utility model also requires imposing a non-monetary marginal cost on applications to BPCSA schools beyond the first school; otherwise – since option value is strictly increasing in the number of schools available because of the changing preferences between the two stages – every applicant would apply to all BPCSA schools.

We parameterize the total cost given a set of applications a as:

$$c(a) = (C - c)\mathbb{1}_{\{|a|>0\}} + c * |a|$$

where $|a|$ denotes the number of BPCSA schools applied to on the common application, C is the fixed cost for submitting the BPCSA application, and c is the marginal cost for applying to any additional school beyond the first school.

Enrollment and Application Decisions

Working backwards from enrollment decisions, the probability of enrollment given applications, acceptances, and known first-stage error terms ϵ_{ij} is identical to that from the behavioral model, given the extreme value Type I distribution properties:

$$Pr(S_i|Z_i, \theta, X_i, \epsilon_i) = \frac{Z_{ij}\exp(\frac{u_{ij}}{\epsilon_{ij}})}{\exp(\frac{u_{i0}}{\kappa}) + \sum_{j'=1}^{J} Z_{ij'}\exp(\frac{u_{ij'}}{\kappa})}$$

Assume that individual i is faced with a set of charter schools Z_i for which they have received acceptances. Define the set of options for student i as $\mathcal{O}(Z_i) = \{0\} \cup \{j : Z_{ij} = 1\}$.

Then prior to observing the Stage 2 shock ξ_{ij}, their expected utility relative to accepting the outside option is given by, due to properties of logit error terms:

$$\begin{aligned} w(Z_i) &= E\Big[\max_{j\in\mathcal{O}(Z_i)} U_{ij} - U_{i0}\Big] \\ &= \kappa * \log\Big(1 + \sum_{j=1}^{J} Z_{ij} * \exp(\frac{(u_{ij} - u_{i0})}{\kappa})\Big) \end{aligned}$$

Putting this together, assume that each applicant chooses to apply to the portfolio of schools that maximizes expected utility as of stage 2, net the application costs.

The optimal application decision is then:

$$A_i = \text{argmax}_{a\in\{0,1\}^J} \sum_{z\in\{0,1\}^J} [\hat{\pi}(Z_i|a)w(Z_i)] - c(a)$$

where $\hat{\pi}(Z_i|A_i)$ is the subjective probability of observing a given realization of acceptances given application decision a. There are many ways that students could form these subjective probabilities; we assume that students are backward looking and use the admissions probabilities from the previous year. This assumption plausibly seems more realistic than being able to anticipate how admissions probabilities will change during the first year of the common application's rollout.

Likelihood Functions and Estimation

In order to estimate this model via simulated maximum likelihood (SML), we first consider the likelihood of two separate cases, depending on whether or not we observe enrollment choices among BPCSA schools.

Case I – BPCSA Acceptances Not Observed:

The likelihood for this group can be computed as:

$$\begin{aligned}\mathcal{L}_i &= Pr(A_i|\theta, X_i)\\ &= Pr\Big(A_i \in \text{argmax}_{a\in\{0,1\}^J} \sum_{z\in\{0,1\}^J} [\hat{\pi}(Z_i|a)w(Z_i)] - c(a)\Big)\\ &\approx \int \Big(\frac{\exp\Big(\Big(\sum_{z\in\{0,1\}^J}[\hat{\pi}(Z_i|a)w(Z_i)] - c(a)\Big)/\lambda\Big)}{\sum_{a'}\exp\Big(\Big(\sum_{z\in\{0,1\}^J}[\hat{\pi}(Z_i|a')w(Z_i)] - c(a')\Big)/\lambda\Big)}\Big)f(\epsilon)d\epsilon\end{aligned}$$

where the last line uses a small value of λ as a logit smoother. In the case of non-applicants, note that the numerator within the integral in the last line always equals 1, since $c(\phi) = 0$ and $w(.)$ is defined as the payoffs relative to the outside option.

Case II – BPCSA Acceptances Observed:

When we not only observe application but also enrollment decisions, the likelihood is computed as:

$$\begin{aligned}\mathcal{L}_i &= Pr(A_i, S_i|\theta, X_i)\\ &= Pr(A_i|\theta, X_i) * Pr(Z_i|A_i) * Pr(S_i|Z_i, A_i, \theta, X_i)\end{aligned}$$

The $Pr(Z_i|A_i)$ term does not depend on the parameters that we are estimating so can be pulled out of the log likelihood. The log likelihood is then equal to:

$$\begin{aligned}\ell_i &= \log(Pr(A_i|\theta, X_i)) + \log\Big(\int Pr(S_i|Z_i, \theta, X_i, \epsilon)f(\epsilon|A_i, X_i, \theta)d\epsilon\Big)\\ &= \log\Big(\int Pr(S_i|Z_i, \theta, X_i, \epsilon)Pr(A_i|\epsilon_i, X_i, \theta)f(\epsilon)d\epsilon\Big)\\ &= \log\Big(\int Pr(S_i|Z_i, \theta, X_i, \epsilon)\mathbb{1}_{A_i|\epsilon_i, X_i, \theta}f(\epsilon)d\epsilon\Big)\\ &\approx \log\Big(\int \Big[\frac{Z_{ij}\exp(\frac{u_{ij}}{\kappa})}{\exp(\frac{u_{i0}}{\kappa}) + \sum_{j'=1}^{J} Z_{ij'}\exp(\frac{u_{ij'}}{\kappa})}\Big]\Big[\frac{\exp\Big(\Big(\sum_{z\in\{0,1\}^J}[\hat{\pi}(Z_i|a)w(Z_i)] - c(a)\Big)/\lambda\Big)}{\sum_{a'}\exp\Big(\Big(\sum_{z\in\{0,1\}^J}[\hat{\pi}(Z_i|a')w(Z_i)] - c(a')\Big)/\lambda\Big)}\Big]d\epsilon\Big)\end{aligned}$$

where the last line again applies a logit smoother via the λ parameter.

Combining the likelihood terms across all individuals in the sample, we again estimate the model via simulated maximum likelihood, using the same procedure as the baseline estimation. Here we use

$\lambda = 0.05$ and approximately integrals using 100 draws of error terms.

E.2 Estimation Results

Table E.1 below shows the resulting demand estimates from the alternate expected utility model. As expected, estimates of κ – the extent to which preferences change between the two stages – are now much smaller than in the baseline model. One intuition for this is that if applicants are admitted to a school but choose to attend the outside option, the baseline model infers that this is due to changes in preferences between the two stages, while the expected utility model allows for the possibility that the applicant applied to the school to preserve option value in case it became preferred to the outside option.

To interpret this difference in κ compared to the baseline model, Table E.2 shows the implied transition matrix of school rankings between the application and enrollment stage. Under this model, a student's first choice during the application stage remains their first choice at the time of enrollment over 80% of the time, in contrast to the baseline model in which it remains their first choice under 40% of the time.

Table E.1: Common Application Preference Parameter Estimates – Expected Utility Model

	(1)	(2)	(3)	(4)	(5)
		Subsidized Lunch		**Black/Hispanic**	
Parameter	Full Sample	Subsidized	Not Subsidized	Black or Hispanic	Not Black or Hispanic
			A: Applicants		
Number of Applicants	701	536	165	569	132
Number of Non-Applicants	3591	2841	750	2758	833
Applications/Applicant (2016)	1.60	1.61	1.60	1.69	1.23
Applications/Applicant (2017)	2.65	2.74	2.34	2.80	1.97
			B: Parameter Estimates		
Drive Time Coefficient, 100*β	–9.03	–8.08		–7.12	
Fixed Cost, $C/\lvert\beta\rvert$	3.69	4.65	3.86	5.00	4.42
Marginal Cost, $c/\lvert\beta\rvert$	0.28	0.28	0.55	0.17	0.97
$\kappa/\lvert\beta\rvert$	4.18	4.50		4.14	
$\delta_1/\lvert\beta\rvert$	–11.33	–11.81		–16.50	
$\delta_2/\lvert\beta\rvert$	–6.58	–5.75		–11.41	
$\delta_3/\lvert\beta\rvert$	–12.44	–10.28		–17.55	
$\delta_4/\lvert\beta\rvert$	–30.59	–32.13		–35.99	
$\delta_5/\lvert\beta\rvert$	–23.23	–25.3		–29.10	
			C: Fraction of Applicants Applying		
School 1	0.51	0.52	0.48	0.55	0.33
School 2	0.62	0.58	0.74	0.55	0.89
School 3	0.56	0.61	0.39	0.64	0.23
School 4	0.46	0.47	0.41	0.48	0.36
School 5	0.50	0.56	0.32	0.58	0.17

Notes: This table shows the simulated maximum liklelihood parameter estimates from the expected utility model of application decisions for Boston Grade in 2017, estimated separately three times: once in the full sample (Column (1)), once with cost heterogeneity by subsidized lunch status (Columns (2) and (3)), and once with cost heterogeneity by racial minority status (Columns (4) and (5)). In this model, subjective admissions probabilities for each school are set equal to their admissions rates during the previous year, such that applicants are backward-looking. In models with cost heterogeneity, all other parameters are imposed to be equal for both groups within the model. Subsidized lunch status and racial minority status are defined during Grade 4. Panel B shows the parameter estimates scaled by the minutes of drive time. Panel C shows the share of applicants applying to each school on the Boston Public Charter School Applicant (BPCSA).

Table E.2: Implied Ranking Between Application and Enrollment Decisions – Expected Utility Model

	(1)	(2)	(3)	(4)	(5)	(6)
Stage 1	Stage 2 – First Choice	Stage 2 – Second Choice	Stage 2 – Third Choice	Stage 2 – Fourth Choice	Stage 2 – Fifth Choice	Stage 2 – Sixth Choice
First Choice	0.83	0.15	0.02	0.00	0.00	0.00
Second Choice	0.13	0.68	0.16	0.03	0.00	0.00
Third Choice	0.03	0.13	0.63	0.18	0.03	0.00
Fourth Choice	0.01	0.03	0.15	0.61	0.17	0.02
Fifth Choice	0.00	0.01	0.04	0.15	0.66	0.14
Sixth Choice	0.00	0.00	0.01	0.03	0.13	0.83

Notes: This table shows, using the expected utility model of application decisions, the transition matrix of implied school rankings between the application stage (Stage 1) and the enrollment stage (Stage 2). Subjective probability for expected utility comes from empirical admissions rate sfor each school during the previous school year. The sample is the 701 students who apply to at least one school on the Boston Public Charter School Application (BPCSA) in 2017. Six schools are included for each applicant: the five BPCSA charter schools in our data, and the applicant's outside option. Implied rankings are generated by running 100 simulations of error terms that rationalize the observed application and enrollment decisions for each applicant.

Appendix F Data Appendix

Estimation of Location and Driving Time

Boston is split into 867 geocodes for the purposes of school assignment and transportation. A geocode is unit that is typically smaller than a census block group. For reference, in the 2020 Census Boston has 581 census block groups and 207 census tracts. We use data on a student's geocode to compute addresses as follows:

1. Check if the student also participated in a traditional BPS match during the same year. If so use this location. For grade 5 in 2017, 69% of students are assigned a location from this method.

2. Check if the student participated in a traditional BPS match during any of the previous 5 years. If so, use the location of the most recent year. For grade 5 in 2017, about 11% of students are assigned a location from this method.

3. If neither of the two methods above apply, infer student location based on the current school attended, using the closest geocode to the school address.For grade 5 in 2017, about 10% of students are assigned a location from this method.

Using this procedure, we assign 90% of students a location.

Once we determine student location, driving time (in minutes) between the center of the student geocode and the school address is used as the measure of distance, using the Google Maps API. For the Uncommon charter network, there are multiple school locations, so we compute the distance between each student and the closest location.

Figure F.1: Boston Charter Middle Schools

	Year Opened	Grade Span	Stuctural Analysis Sample	Reduced Form Sample	Reason Excluded
	(1)	(2)	(3)	(4)	(5)
Academy of the Pacific Rim	1997-98	5-12	Y	Y	
Boston Collegiate	1998-99	5-12	Y		
MATCH Middle School	2008-09	PK-12	Y	Y	
Neighborhood House	2016-17	PK-8	Y		
Uncommon Schools: Dorchester Campus	2012-13	5-8	Y	Y	
Uncommon Schools: Lucy Stone Campus	2011-12	5-8	Y	Y	
Uncommon Schools: Mission Hill Campus	1999-2000	5-8	Y	Y	
Boston Renaissance	1995-96	PK-6			Data not available
Bridge Boston	2011-12	PK-4 (6)			Grade 5 not an entry grade
Brooke East Boston	2012-13	K-8 (9)			Grade 5 not an entry grade
Brooke Mattapan	2011-12	K-8			Grade 5 not an entry grade
Codman Academy	2001-02	K-12			Grade 5 not an entry grade
Conservatory Lab	1999-2000	PK-8			Data not available
Excel Academy	2003-04	5-9 (11)			Data not available
KIPP Boston	2012-13	K-8			Data not available
UP Academy Dorchester	2013-14	PK-7 (8)			Grade 5 not an entry grade

Note: This table includes charter schools serving grade 5 in the study period (2015-2016 to 2017-2018 school years). Column (2) displays the grades served in the initial school year of our study period (2015-2016). If the grade span changed over the course of the study period, the grade number in parenthesis displays the highest grade served in the last year of the period.