Missing Financial Data

Svetlana Bryzgalova[1], Sven Lerner[2], Martin Lettau[3] and Markus Pelger[2]

[1]London Business School and CEPR [2]Stanford University [3]UC Berkeley, NBER, and CEPR

Missing financial data

Firm characteristics are crucial in asset pricing:

- Investment strategies (sorts, machine learning, panel models, etc.),
- Reduced-form, e.g. factors, and structural asset pricing models
- Test assets for models (e.g., double sorts),

Fundamental problem: Missing firm fundamentals

Key questions: Does missing data matter and how should we deal with it?

Current standard of dealing with missing data strongly biased

- Only fully observed data $\Rightarrow$ sample selection
- Ad-hoc imputation (cross-sectional average, past observations)

This paper:

1. Key facts on missing characteristics
2. Novel method to impute missing values
3. Implications for asset pricing

Broader impact: Methods and insights broadly applicable

- Missing fundamentals impact corporate finance and economics
- Growing importance due to new big data, ESG data, international data, etc.

Contribution of this paper I: Comprehensive empirical study

Stylized facts on missing fundamentals:

Fact #1: Missing data is prevalent:

- Almost all characteristics have missing observations
- Affects small and large, young and mature, profitable and distressed firms

Fact #2: Missingness particularly severe for multiple characteristics

- $> 70\%$ of the firms are missing some of the popular characteristics at any time
- 50% of market capitalization missing for fully observed panel

Fact #3: Data is not missing completely at random

- systematic patterns, clusters in time and characteristics
- more missingness for extreme realizations and smaller stocks

Fact #4: Returns depend on missingness

- Investment strategies more profitable with all imputed stocks
- Selection bias: missingness has price impact even on simple anomaly strategies.
- Imputation bias: ad-hoc imputation (median) severely distorts risk premia

$\Rightarrow$ widespread implications for the "multivariate challenge" in asset pricing,

Contribution of this paper II: Novel Data Imputation Method

Challenges of data imputation:

1. Requires good model for characteristics (avoid omitted variable bias)
2. Model has to be estimated on partially observed data (avoid selection bias)
   Characteristics are not missing completely at random!

Method: A cross-sectional and time-series factor model for characteristics

- Contemporaneous cross-sectional dependency (XS) explained by latent factors
- Persistence (TS) captured by a time-series model,
- Allows for general endogenous missing patterns:
  missingness can depend on time, stocks, characteristics and factor model

$\Rightarrow$ data-driven, transparent and simple-to-implement

Empirics:

- Comprehensive comparison of approaches to imputation,
- 40-50% reduction in the imputation error relative to existing benchmarks,
- TS and XS both matter, and depend on the characteristic and its missingness.

$\Rightarrow$ A reference dataset with imputed values for any follow-up work.

Missing Data: Stylized Facts

Data

Dataset:

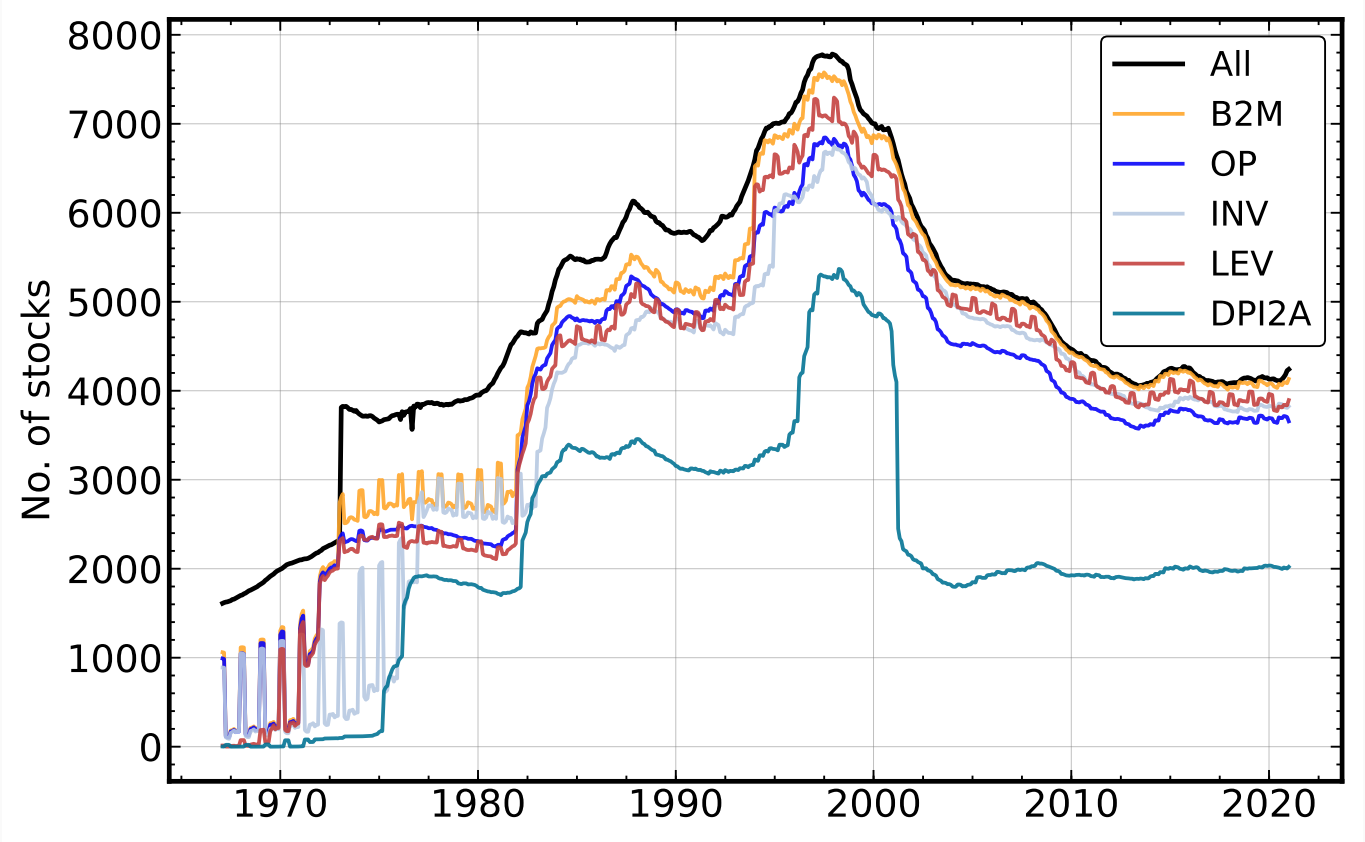
- Standard CRSP/Compustat universe + usual filters for outliers, exchanges, etc.
- Sample size: monthly returns 1967:07 – 2020:12
- 45 characteristics: value, investment, profitability, intangibles, past returns, trading frictions, etc.
- Characteristics raw values are converted into centered rank quantiles
- Characteristics are updated monthly or quarterly

Standard dataset for many modern asset pricing applications:

- the most popular characteristics, used individually and combined
- standard set of filters/transformations

Missing data: How big of a problem?

Even key firm characteristics are missing for many companies

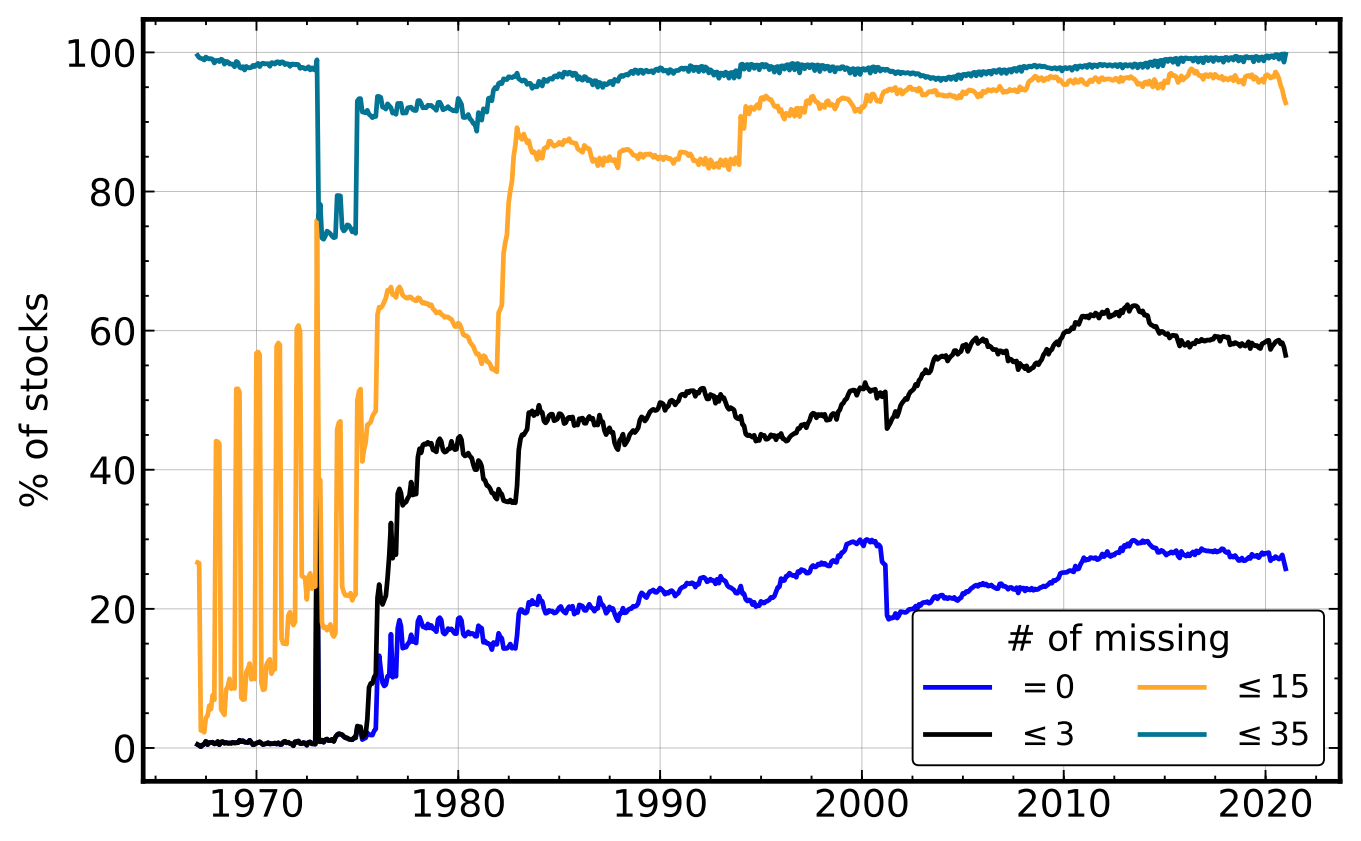


Firms with observed characteristic value

(value, gross profitability, investment, leverage, change in PPEI/assets)

- (Almost) any characteristic has missing observations
- The number of firms missing fundamentals is statistically and economically large
- Substantial cross-sectional and time variation

Elephant in the room: multiple characteristics

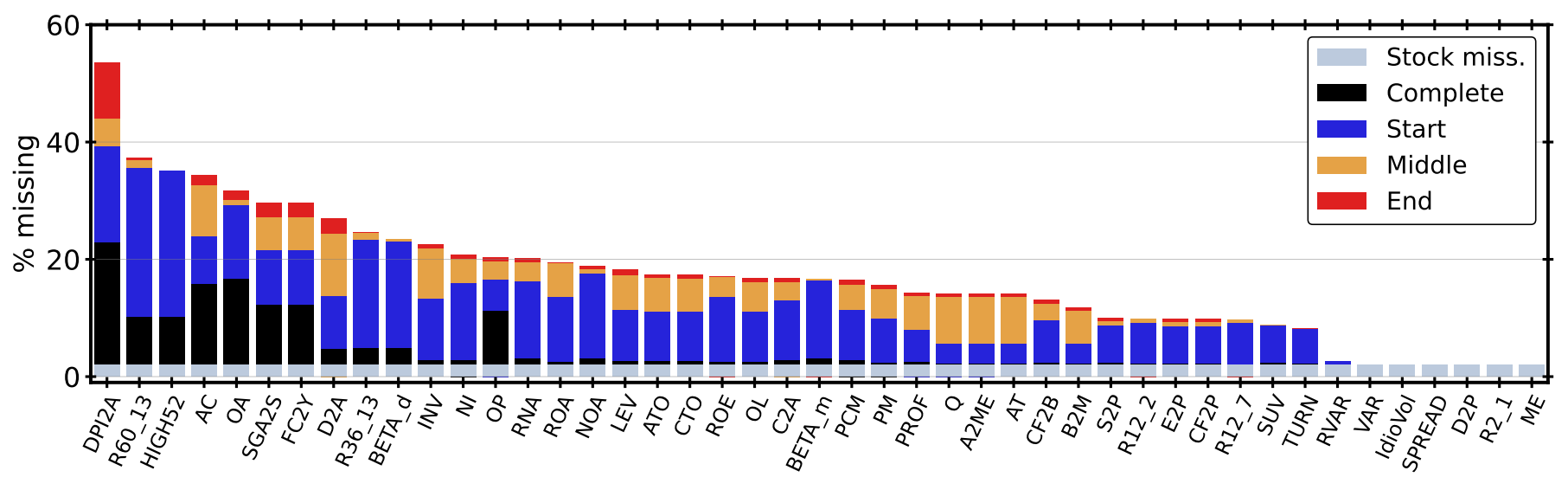


Percentage of firms missing some characteristics (from the list of 45)

- Missing data is a paramount problem whenever multiple characteristics are used
- $> 70\%$ of firms are missing at least some popular characteristics at any period
- Their total market cap is 48%

$\Rightarrow$ Using a fully observed panel of data may lead to massive sample selection: crucial for panel models, conditional factors, and machine learning.

When are characteristics missing?



Start = no previous observations

End = no further observations

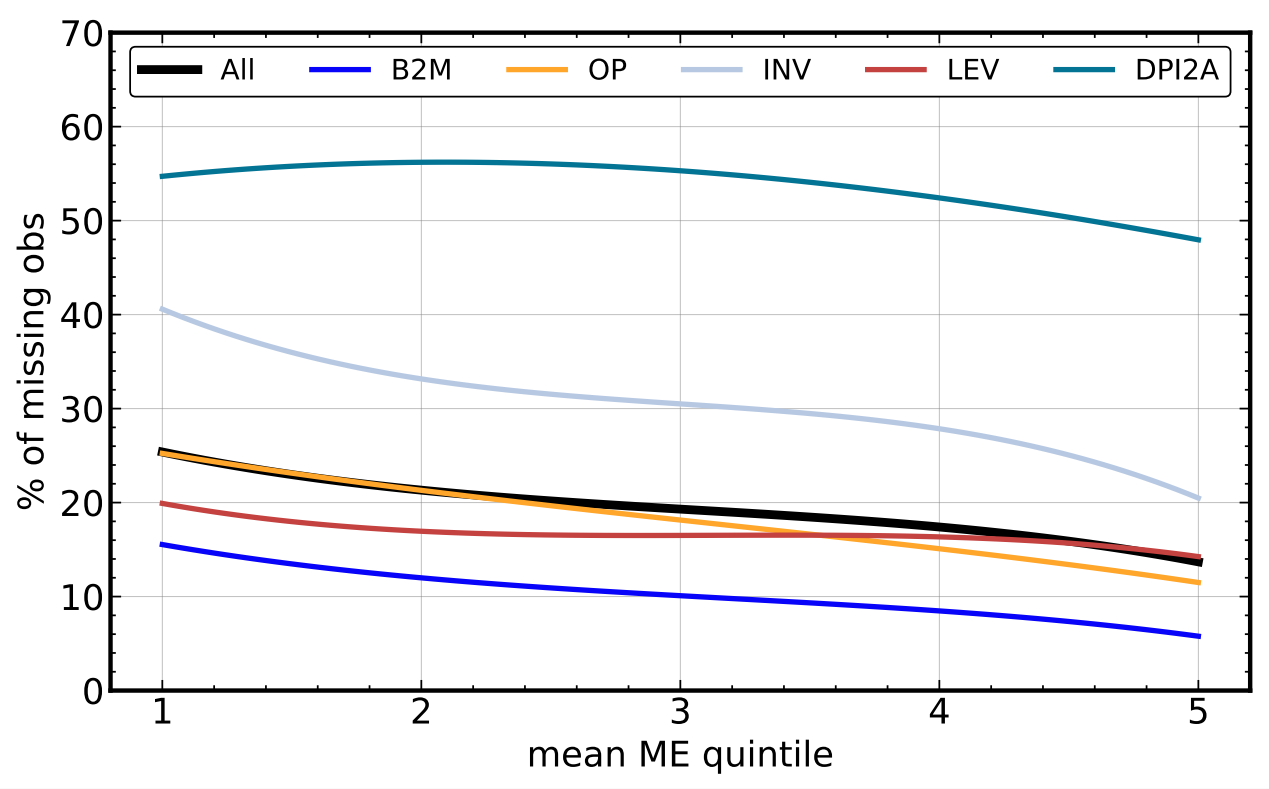
Middle = some previous and future observations

Complete = completely missing

- Some characteristics are mechanically missing for younger firms (e.g., LTrev)
- Many characteristics are missing after having been previously observed
- Some characteristics are missing at the end of the company's life
- Some are never observed

$\Rightarrow$ Imputation needs to allow for different information sets

Which stocks have missing observations?

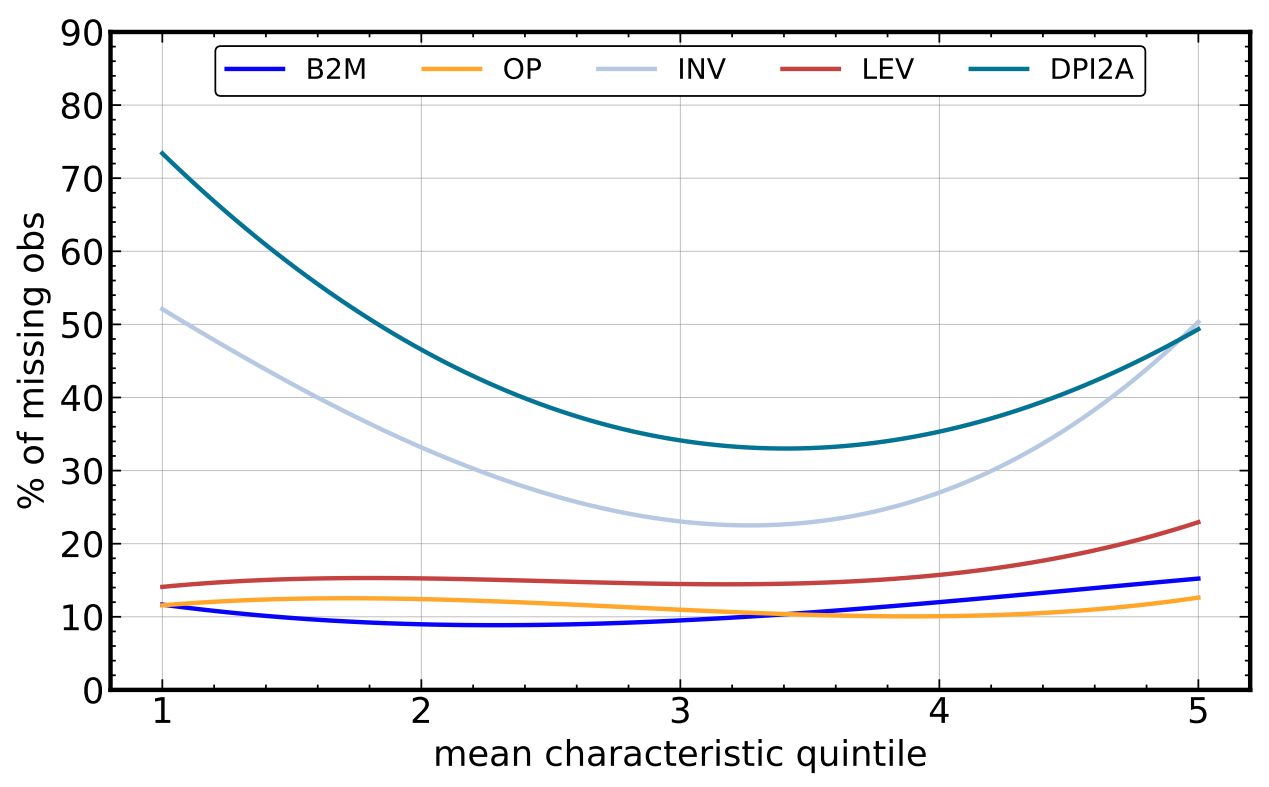


Percentage of missing characteristics by size quintiles

- Smaller companies have more missing observations
- Complex interactions of size and heterogeneous missingness

$\Rightarrow$ Firms with observed data are different $\Rightarrow$ selection bias

Which characteristic realizations are missing?

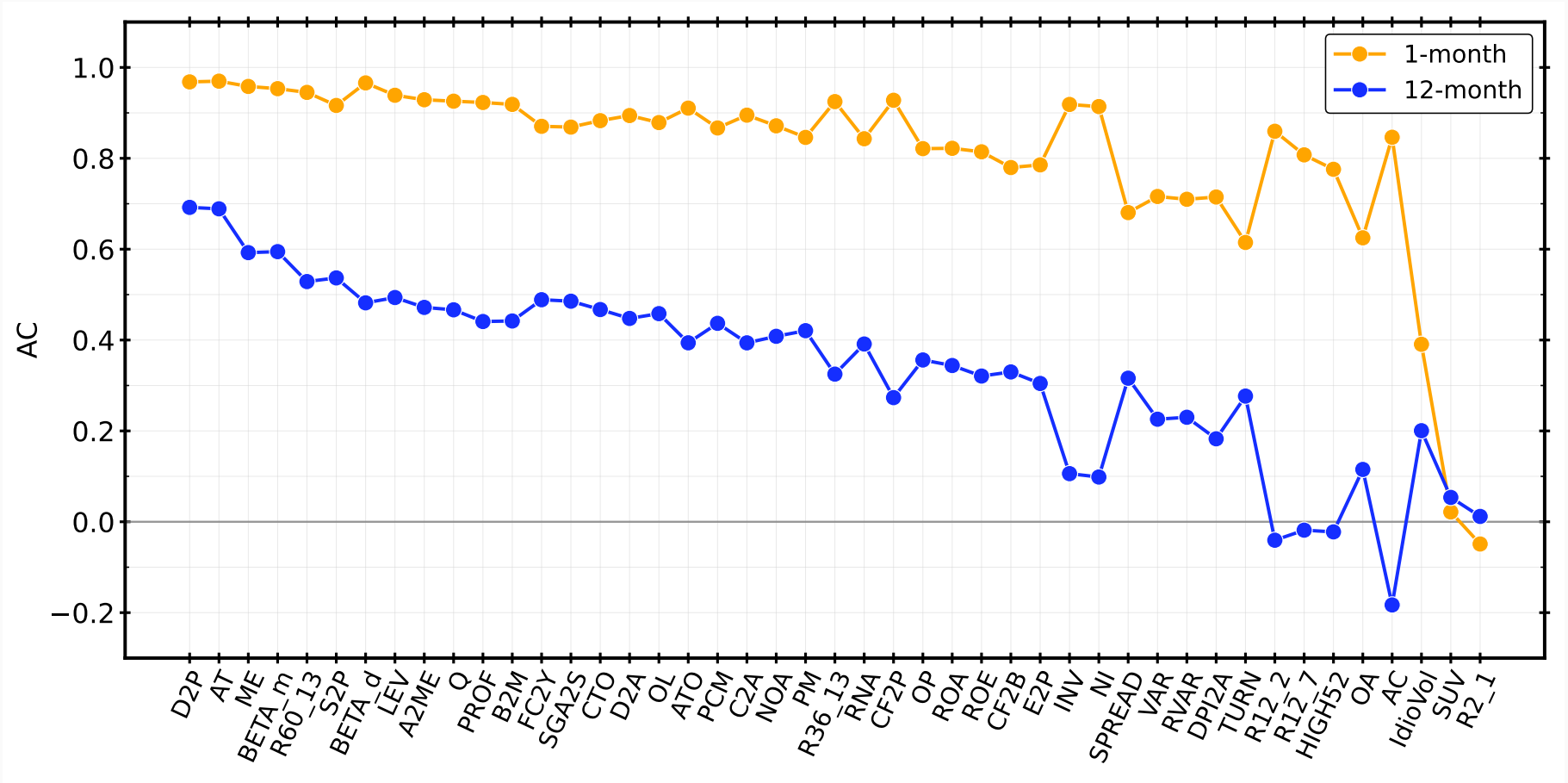


Percentage of missing characteristics by average characteristic quintiles

- More extreme realizations of characteristics are more likely to be unobserved
- U-shaped pattern generalizes to most characteristics
- Missingness depends on characteristic realization
- Endogenous missingness $\Rightarrow$ challenging statistical problem

Characteristics Dependency

Characteristics are persistent

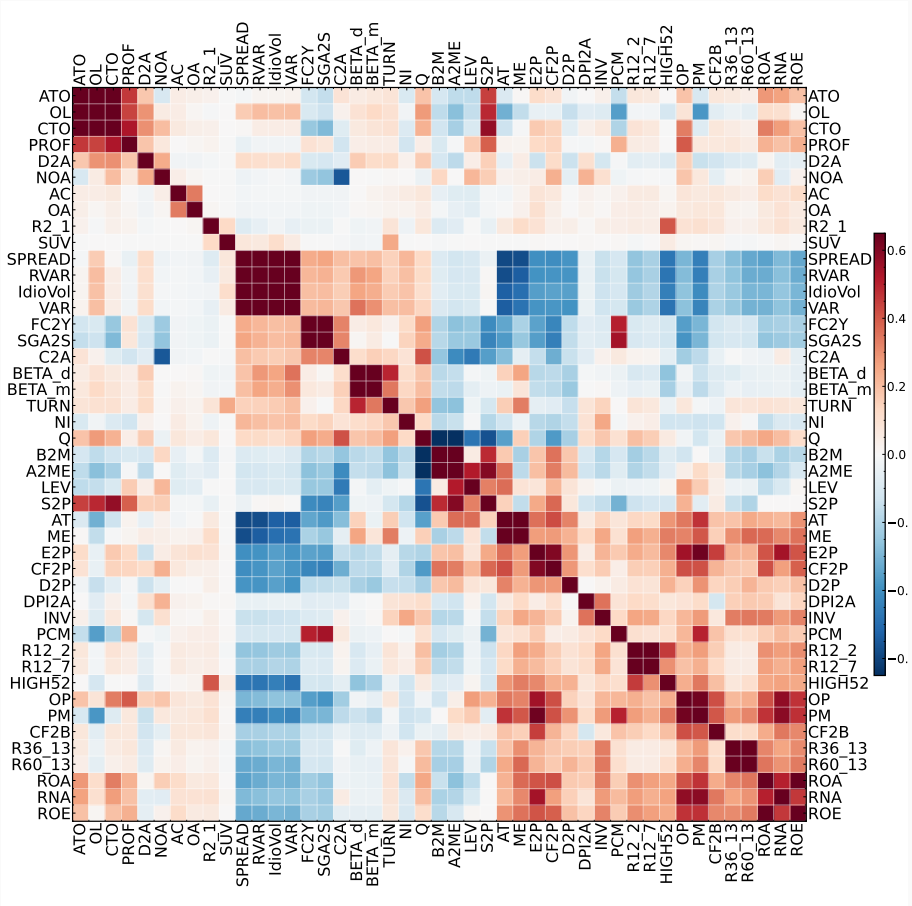


Average sample autocorrelation for each characteristic

- Many characteristics are very persistent
- Past (and future) values have information for missing values

$\Rightarrow$ Disregarding time dependency when imputing values might lead to a bias

Characteristics are cross-sectionally correlated



Pairwise correlations in characteristics, averaged over time and stocks

- Strong cross-sectional dependence
- Contemporaneous correlated characteristics have information for missing values
- Challenge: A model for the complex dependencies (avoid omitted variable bias)

We need a way of imputing characteristics using both cross-section and time-series.

Model

Model formulation

Characteristics form a 3-dimensional vector space:

$$C_{i,t,l} \qquad \text{with } i = 1, ..., N_t, t = 1, ..., T \text{ and } l = 1, ..., L$$

- Cross-sectional stock dimension $i = 1, ..., N_t$
- Time-series dimension $t = 1, ..., T$
- Different characteristics $l = 1, ..., L$

$\Rightarrow$ Goal: A low-dimensional model for cross-sectional and time-series dependency

Our baseline model uses centered rank quantiles:

- Stationarity in the cross-section and over time and deals with outliers
- Simple mapping between rank quantiles and raw values through empirical density
- Similar results in raw characteristic space after appropriate kernel transformation

Cross-sectional factor model

Approximate factor structure for $N_t \times L$ characteristic matrix C^t at time t:

$$C^t_{i,l} = F^t_i {\Lambda^t_l}^\top + e^t_{i,l} \qquad \text{with } i = 1, ..., N_t \text{ and } l = 1, ..., L.$$

- Allows for a separate factor model for each time t,
- K latent factors: $F^t \in \mathbb{R}^{N_t \times K}$ and $\Lambda^t \in \mathbb{R}^{L \times K}$,
- without missing values, estimate model with PCA applied to $C^t {C^t}^\top$.

A general approach to estimation (valid under general missing patterns):

1. Estimate F^t_i as the eigenvectors of the K largest eigenvalues of

$$\tilde{\Sigma}^{\text{XS},t}_{i,j} = \frac{1}{|Q^t_{i,j}|} \sum_{l \in Q^t_{i,j}} C^t_{i,l} C^t_{j,l},$$

   with $Q^t_{i,j}$ set of characteristics observed for stocks i and j at time t

2. Estimate loadings Λ^t_l from the characteristic regression:

$$\hat{\Lambda}^t_l = \left(\sum_{i=1}^{N_t} W^t_{i,l} \hat{F}^t_i {\hat{F}^t_i}^\top \right)^{-1} \left(\sum_{i=1}^{N_t} W^t_{i,l} \hat{F}^t_i C^t_{i,l} \right),$$

   where $W^t_{i,l} = 1$ if char. l is observed for stock i at time t and $W^t_{i,l} = 0$ o/w.

Asymptotic theory (including confidence intervals): Xiong and Pelger (2019).

Adding time-series information

Combine XS (cross-sectional) with TS (time-series) information:

- **B-XS-Model:** (backward-cross-sectional)

$$\hat{C}_{i,t}^{I,\text{B-XS}} = \beta^{I,\text{B-XS}\top} \begin{pmatrix} C_{i,t-1}^{I} & \hat{F}_{i,1}^{t} & \cdots & \hat{F}_{i,K}^{t} \end{pmatrix}$$

- Regression with stacked cross-sectional and time-series information in $X_i^{I,t}$:

$$\hat{\beta}^{I,t} = \left(\sum_{i=1}^{N_t} W_{i,I}^{t} X_i^{I,t} X_i^{I,t\top} \right)^{-1} \left(\sum_{i=1}^{N_t} W_{i,I}^{t} X_i^{I,t} C_{i,t}^{I} \right)$$

Method	Estimation
Backward-Forward-XS (BF-XS)	$\hat{C}_{i,t}^{\text{BF-XS}} = (\hat{\beta}^{\text{BF-XS}})^\top \begin{pmatrix} C_{i,t-1}^{I} & C_{i,t+1}^{I} & \hat{F}_{i,1}^{I} & \cdots & \hat{F}_{i,K}^{I} \end{pmatrix}$
Backward-XS (B-XS)	$\hat{C}_{i,t}^{\text{B-XS}} = (\hat{\beta}^{\text{B-XS}})^\top \begin{pmatrix} C_{i,t-1}^{I} & \hat{F}_{i,1}^{I} & \cdots & \hat{F}_{i,K}^{I} \end{pmatrix}$
Forward-XS (F-XS)	$\hat{C}_{i,t}^{\text{F-XS}} = (\hat{\beta}^{\text{F-XS}})^\top \begin{pmatrix} C_{i,t+1}^{I} & \hat{F}_{i,1}^{I} & \cdots & \hat{F}_{i,K}^{I} \end{pmatrix}$
Cross-sectional (XS)	$\hat{C}_{i,t}^{\text{XS}} = (\hat{\beta}^{\text{XS}})^\top \begin{pmatrix} \hat{F}_{i,1}^{I} & \cdots & \hat{F}_{i,K}^{I} \end{pmatrix}$
Time-series (B)	$\hat{C}_{i,t}^{\text{B}} = (\hat{\beta}^{\text{B}})^\top \begin{pmatrix} C_{i,t-1}^{I} \end{pmatrix}$
Previous value (PV)	$\hat{C}_{i,t}^{\text{PV}} = C_{i,t-1}^{I}$
Cross-sectional median	$\hat{C}_{i,t}^{\text{median}} = 0$

Different imputation methods sorted by the size of the information set

Imputing Characteristics

Evaluation

Metrics: RMSE (root mean squared errors) and R^2:

- $\text{RMSE} = \sqrt{\frac{1}{T\,L\,N_t}\sum_{t,l,i}\left(C_{i,t,l} - \hat{C}_{i,t,l}\right)^2}$
- $R^2 = 1 - \left(\sum_{t,l,i}\left(C_{i,t,l} - \hat{C}_{i,t,l}\right)^2\right) / \left(\sum_{t,l,i}\left(C_{i,t,l}\right)^2\right)$

Out-of-sample evaluation:

- OOS Block-missing: Masking 10% of characteristics in blocks of 1 year
- OOS Missing-at-random: Masking 10% of characteristics randomly
- OOS Logit: Masking with empirical distribution of missing data
- In-sample results on observed characteristics

Models:

- For each model: local (each month) and global (pooled) estimation.
  Local model avoids look-ahead bias but less efficient
- Current standard in the literature: Cross-sectional median or previous value

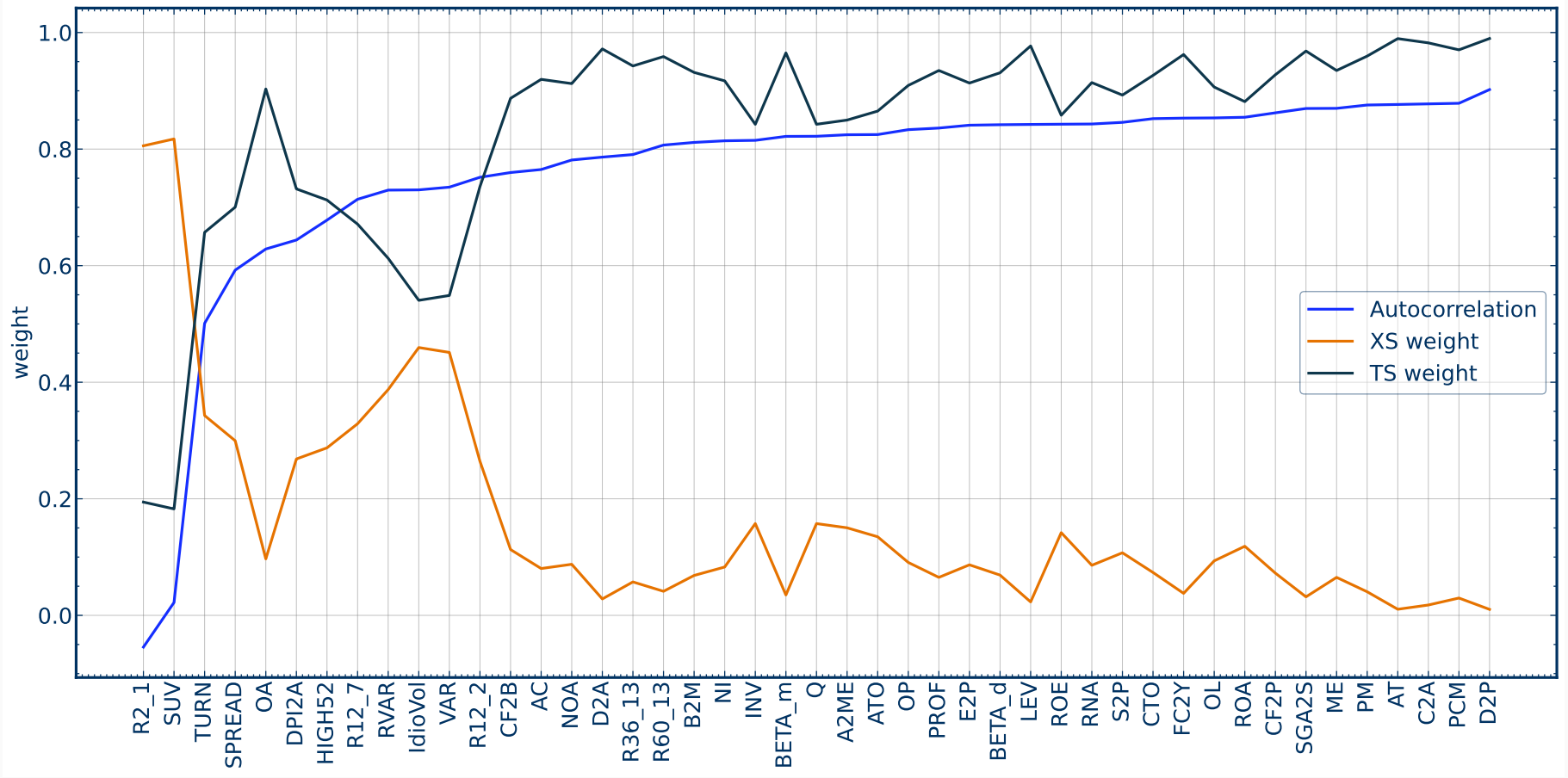
Aggregate results

Method	all characteristics	quarterly characteristics	monthly characteristics
global BF-XS	**0.83**	**0.94**	**0.77**
global F-XS	0.81	0.97	0.71
global B-XS	0.75	0.81	0.71
global XS	0.38	0.43	0.36
global B	0.74	0.79	0.71
local B-XS	**0.76**	**0.81**	**0.73**
local XS	0.37	0.38	0.35
local B	0.74	0.80	0.71
prev val	0.63	0.76	0.56
XS median	0.00	0.00	0.00
industry median	0.00	0.00	0.00

Out-of-sample R^2 relative to median for block-missing characteristics

- Baseline models:
  - local B-XS (no look-ahead-bias)
  - global BF-XS (full possible information)
- Current standard (cross-sectional median and last observed value) is the worst
- Similar results for logit masking
- Extensive evaluation for type of missingness (beginning, middle, end), different masking, extreme quantiles, size of companies, industry, over time, etc.

Information used for imputation



Relative importance of the TS and XS components of B-XS (L1 norm)

- Characteristics are sorted in ascending order based on their persistence
- Persistent characteristics put more weight on TS information
- Volatile characteristics put more on XS information

Asset Pricing Results

Two fundamental effects

Selection bias: Asset pricing results depend on which stocks are included

1. Portfolios based on observability of characteristics
2. Univariate portfolio sorts and factors
3. Asset pricing model (IPCA)

$\Rightarrow$ Subsamples of fully observed stocks lead to selection bias in asset pricing metrics

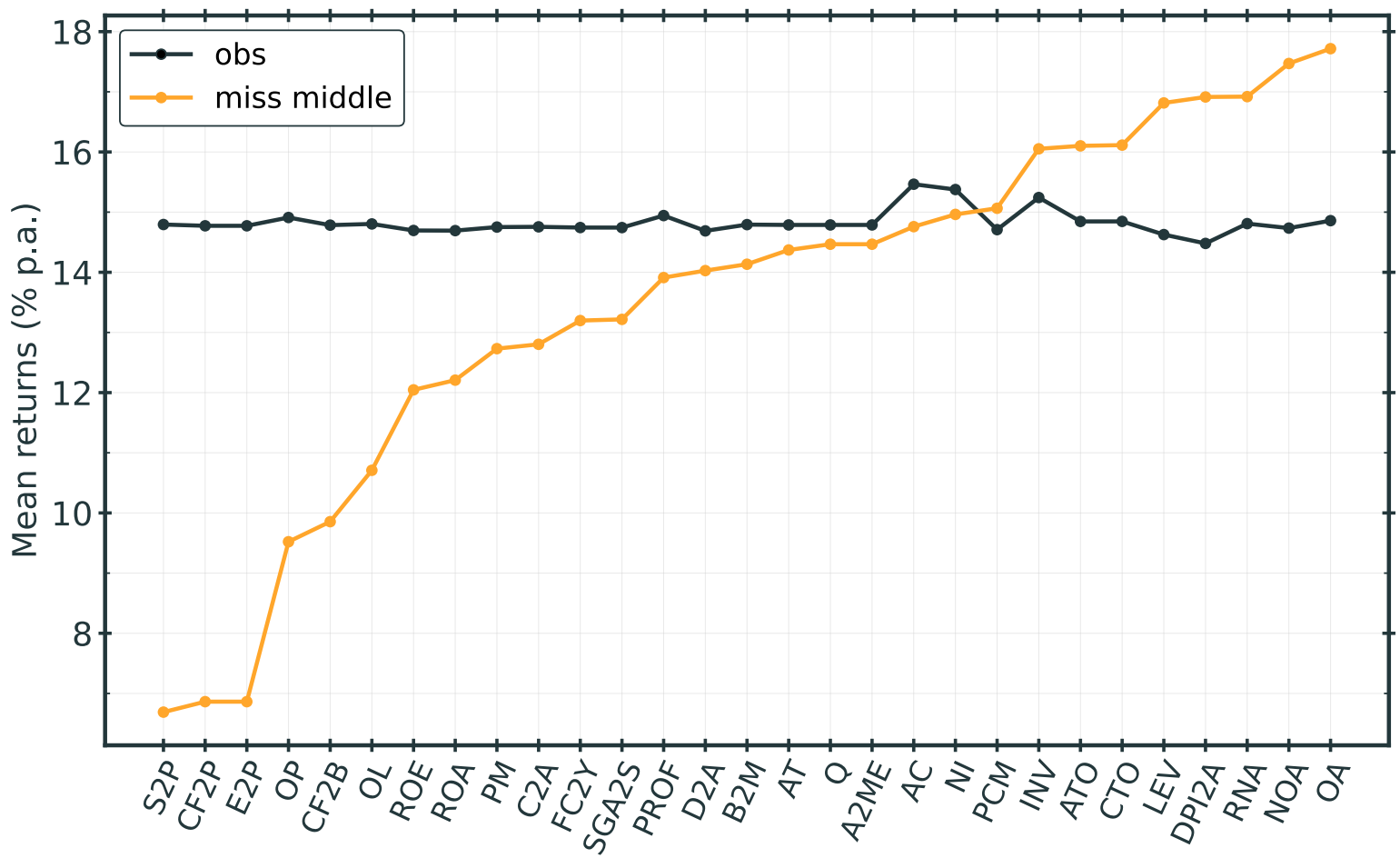
$\Rightarrow$ Out-of-sample investment substantially better with all stocks

Imputation bias: Asset pricing results depend on imputation method

- Mask observed values based on empirical observation pattern (logistic regression)
- Impute masked missing values with our local B-XS model or conventional median
- Cross-sectional regression on characteristics:
  - Risk premia for characteristic signals
  - Characteristic mimicking factor portfolio time-series

$\Rightarrow$ Uniformly and substantially larger errors in asset metrics for median imputation

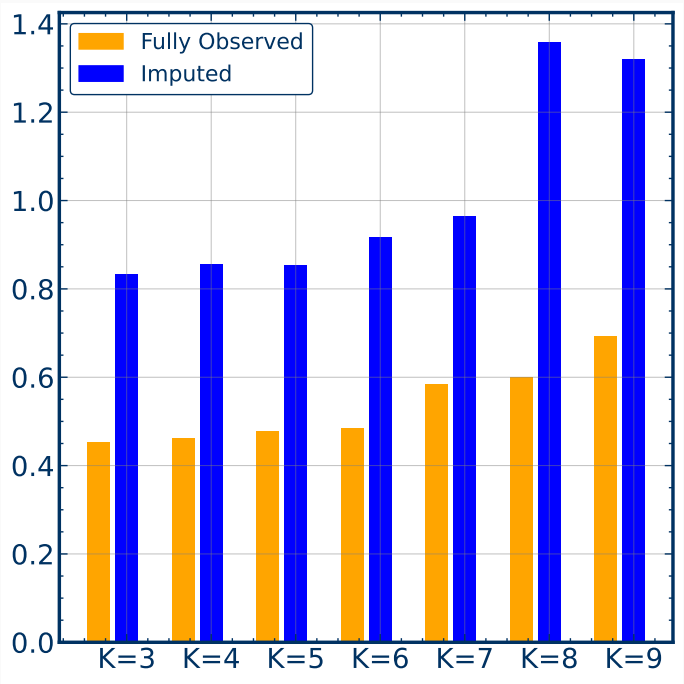
Missingness matters for simple portfolio strategies



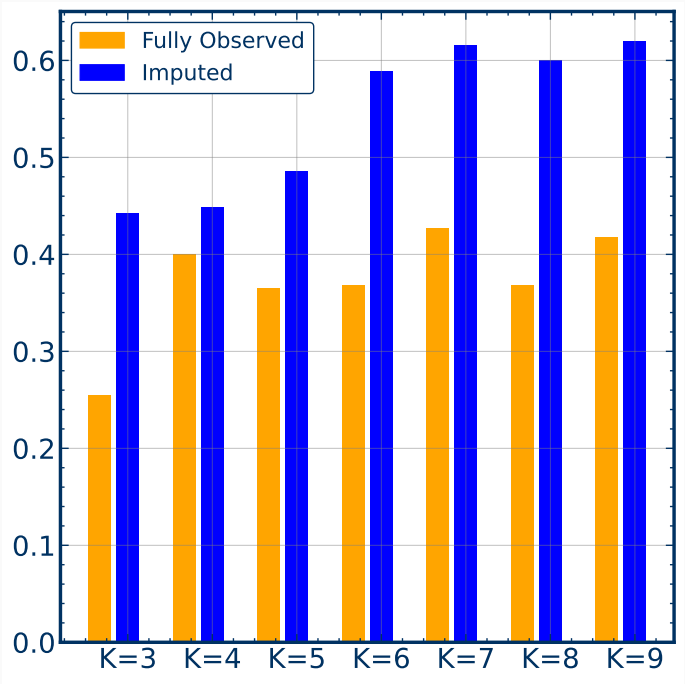
Returns on long-only portfolios that include/exclude particular characteristics

- Portfolios are formed by buying stocks with observed/missing characteristic value
- Significant difference in returns for many characteristics

Selection bias: Investment with IPCA factors



(a) In-sample Sharpe ratios



(b) Out-of-sample Sharpe ratios

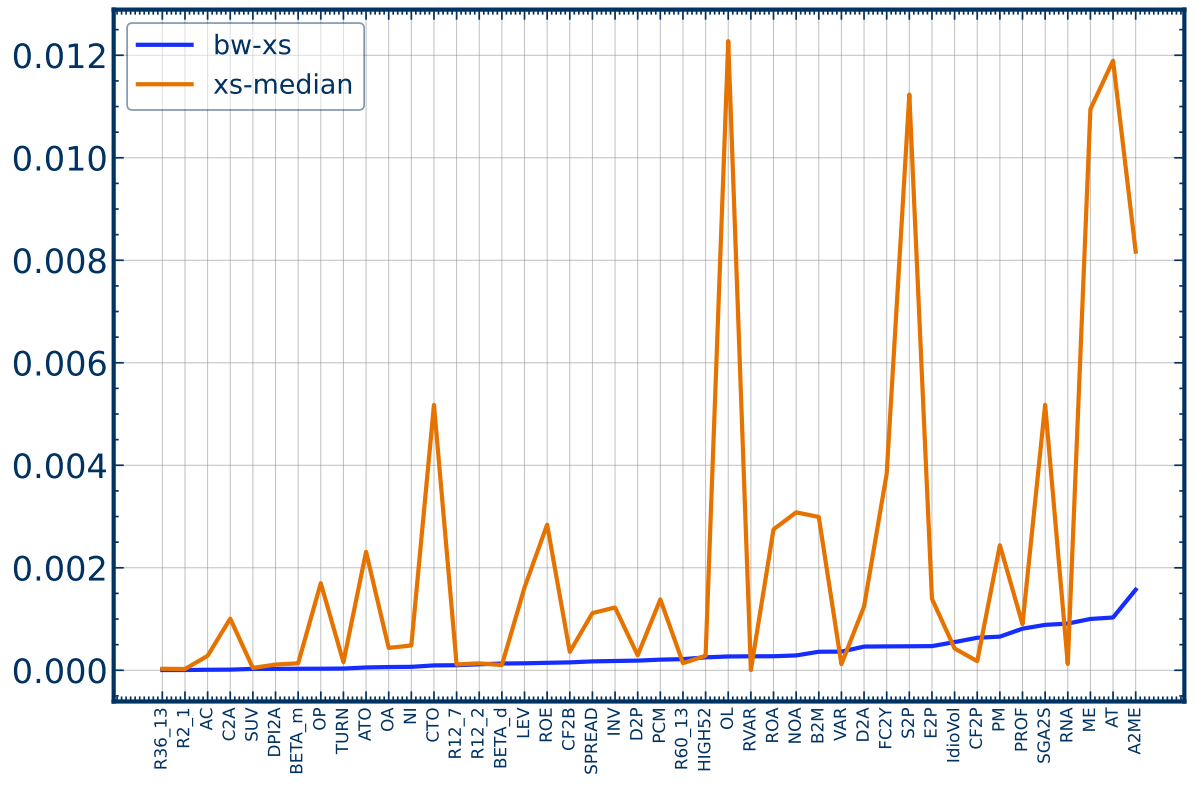
Sharpe ratios of mean-variance efficient combination for different number of factors

- Estimate conditional latent factor model with IPCA
  (Instrumented Principal Component Analysis by Kelly, Pruitt and Su (2019))
- Estimate on small subset of fully observed or large set of all imputed stocks

$\Rightarrow$ In- and out-of-sample Sharpe ratios substantially higher for all stocks

$\Rightarrow$ Investments with subset of fully observed stocks are suboptimal

Imputation Bias: Asset-Pricing with Different Imputation Methods

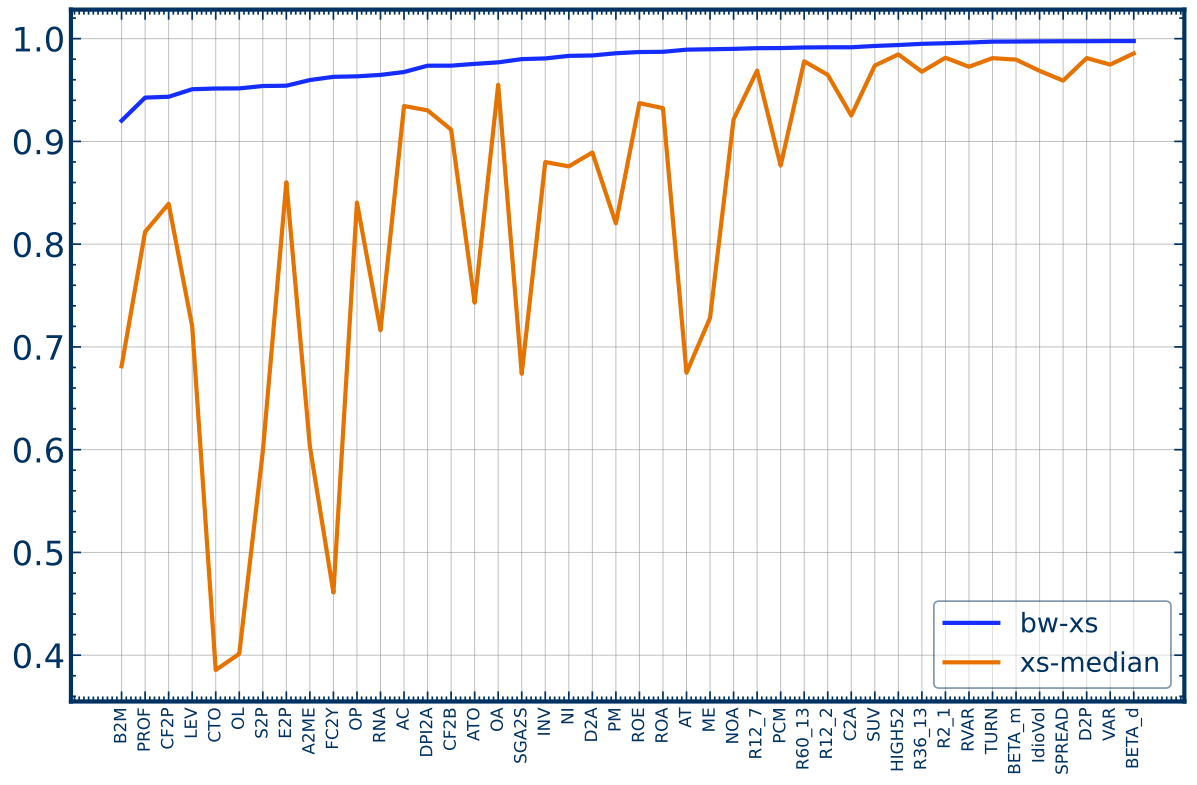


Absolute error in risk premium from cross-sectional regressions

- Comparison of B-XS and median imputed values relative to true observed values
- Mask values based on empirical pattern (logistic regression)
- Cross-sectional regression on characteristics: Compare risk premia and factor mimicking portfolios of imputation with observed data (truth)

$\Rightarrow$ Imputation bias: B-XS uniformly and substantially more accurate asset pricing

Imputation Bias: Asset-Pricing with Different Imputation Methods

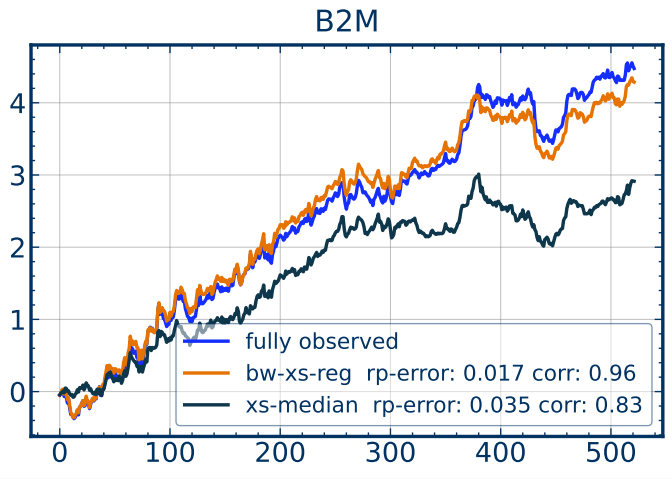


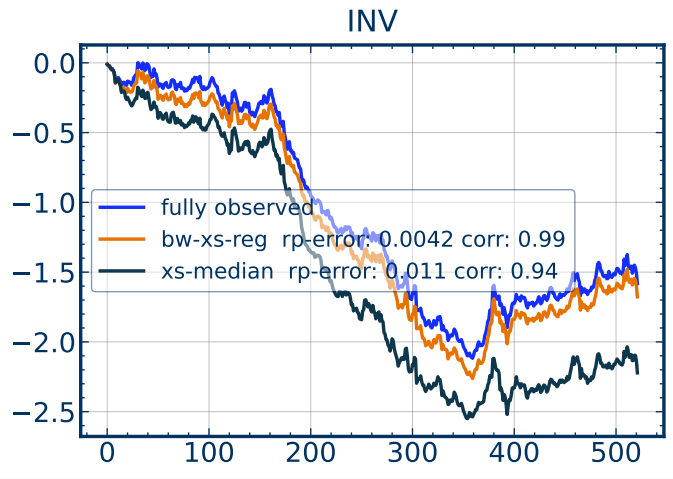
R^2 of factor mimicking portfolios from cross-sectional regressions

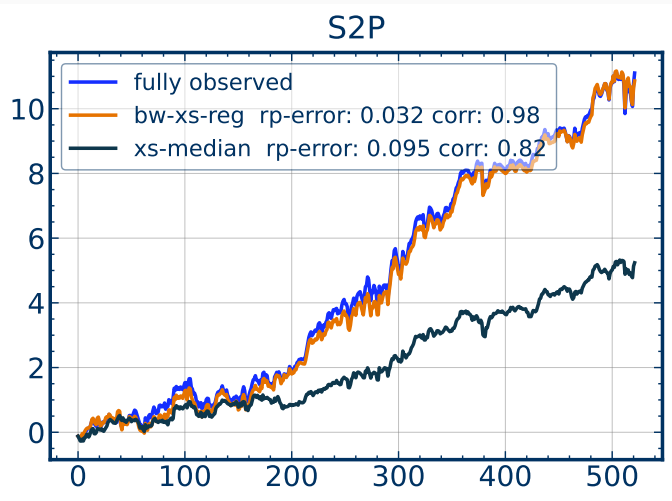
- Comparison of B-XS and median imputed values relative to true observed values
- Mask values based on empirical pattern (logistic regression)
- Cross-sectional regression on characteristics: Compare risk premia and factor mimicking portfolios of imputation with observed data (truth)

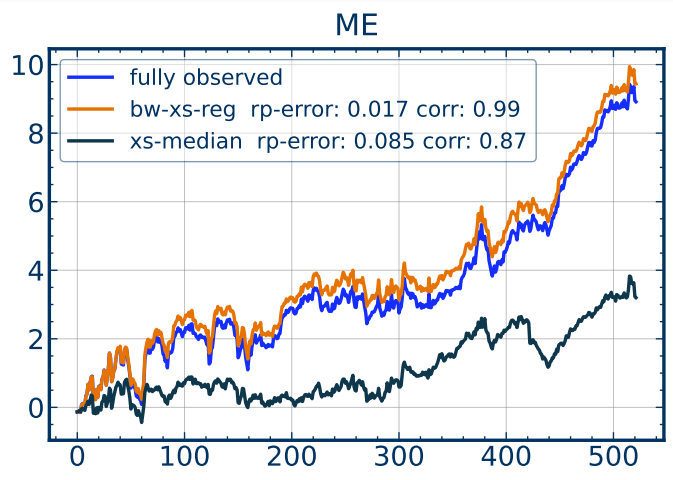
$\Rightarrow$ Imputation bias: B-XS uniformly and substantially more accurate asset pricing

Imputation Bias: Representative Examples









- Factor mimicking portfolio time-series for observed (true) and imputed data
- Risk premia equals mean, R^2 equals correlation with true time-series

$\Rightarrow$ Joint regression impacts even fully observed characteristics (e.g. size)

$\Rightarrow$ Extremely precise approximation of full time-series with B-XS

$\Rightarrow$ Substantial bias in time-series for median $\Rightarrow$ wrong mean, correlation, variance

Conclusion

Conclusion

A systematic study of missing data in characteristics:

- the problem is pervasive and affects even simple investment strategies,
- complex and endogenous patterns of missingness,
- simple solutions do not work.

A novel method to impute characteristic values:

- a parsimonious model for characteristic structure,
- time-series AND cross-sectional dependence,
- automatically captures a wide range of dependencies and missing patterns.

Outlook:

- a rising challenge in the presence of big data and machine learning,
- growing importance due to new large datasets, ESG data, international data, etc.
- numerous implications for asset pricing and corporate finance.

We will provide a publicly available dataset for researchers.

Appendix

Firm characteristics

Past Returns	Investment	Profitability	Intangibles	Value	Trading Frictions
Momentum	Investment	Operating profitability	Accrual	Book to Market Ratio	Size
Short-term Reversal	Net operating assets	Profitability	Operating accruals	Assets to market cap	Turnover
Long-term Reversal	Change in prop. to assets	Sales over assets	Operating leverage	Cash to assets	Idiosyncratic Volatility
Return 2-1	Net Share Issues	Capital turnover	Price to cost margin	Cash flow to book value	CAPM Beta
Return 12-2		Fixed costs to sales		Cashflow to price	Residual Variance
Return 36-13		Profit margin		Dividend to price	Total assets
		Return on net assets		Earnings to price	Market Beta
		Return on assets		Tobin's Q	Close to High
		Return on equity		Sales to price	Spread
		Expenses to sales		Leverage	Unexplained Volume
		Capital intensity			Variance

Literature (incomplete and partial list)

Missing financial data:

- GMM with missing data: Freyberger et al. (2021)
- Look-ahead-bias in imputation for out-of-sample investment: Blanchet et. al. (2022)
- Imputation for causal inference of publication effect: Xiong and Pelger (2022)

$\Rightarrow$ Different goal and complementary

Missing data in panel

- Latent factor models: Xiong and Pelger (2019), Bai and Ng (2021), Jin et al. (2021)
- Matrix completion: Athey et. al. (2018), Chen et al. (2019)
- Transfer learning with Target PCA: Duan, Pelger and Xiong (2022)

$\Rightarrow$ Only 2-D, challenge general missing patterns

Latent factor modeling in finance

- Unconditional: Connor et. al. (1988), Lettau and Pelger (2020a+b), Pelger (2019)
- Conditional: Kelly et. al. (2019), Pelger and Xiong (2021)

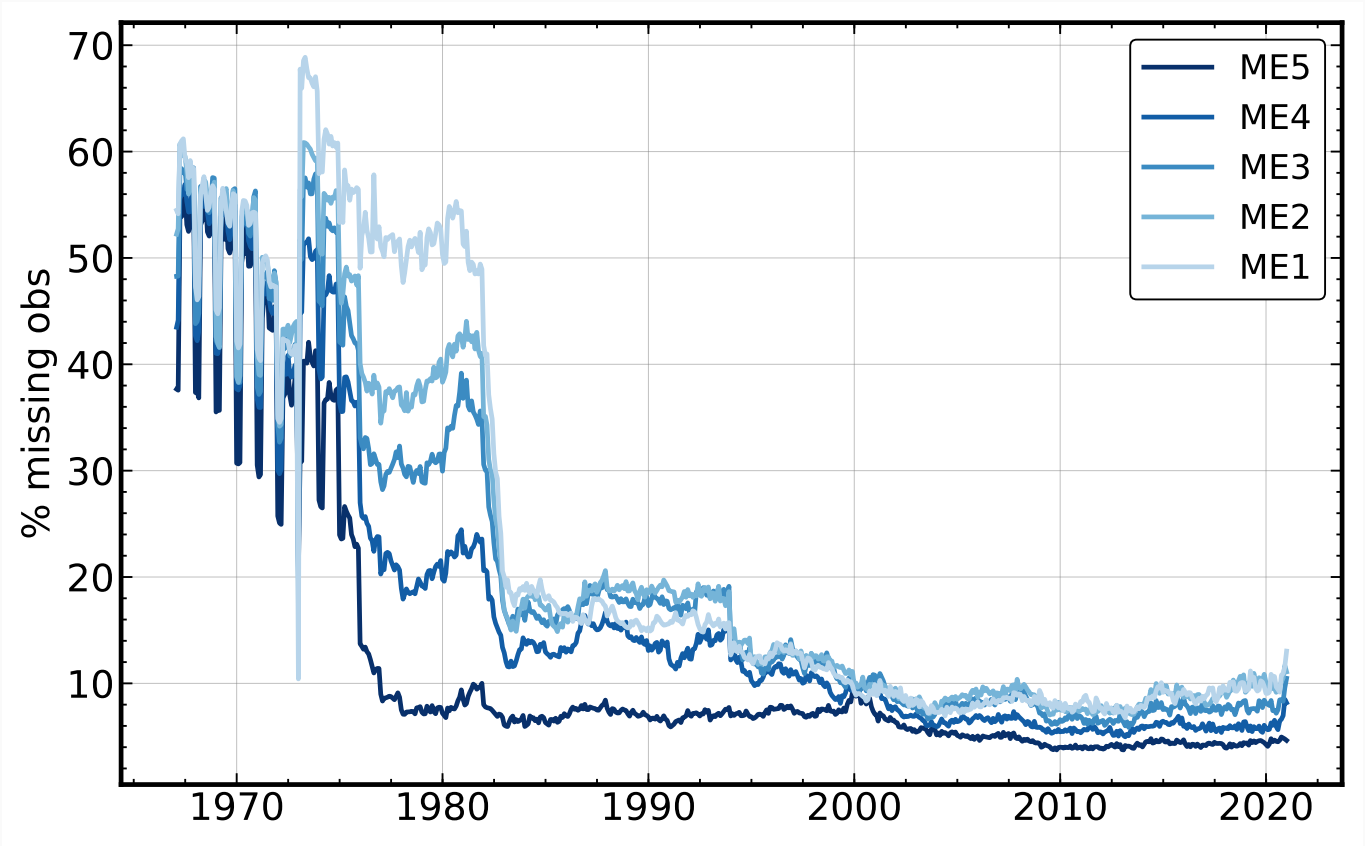
$\Rightarrow$ PCA type methods for fully observed panel of returns

Asset pricing with many characteristics

- Prediction: Freyberger et al. (2020), Gu et al. (2020), Kaniel et al. (2021)
- SDF modeling: Bryzgalova et al. (2019), Chen et al. (2019), Kozak et al. (2020)

$\Rightarrow$ Requires choices for missing data

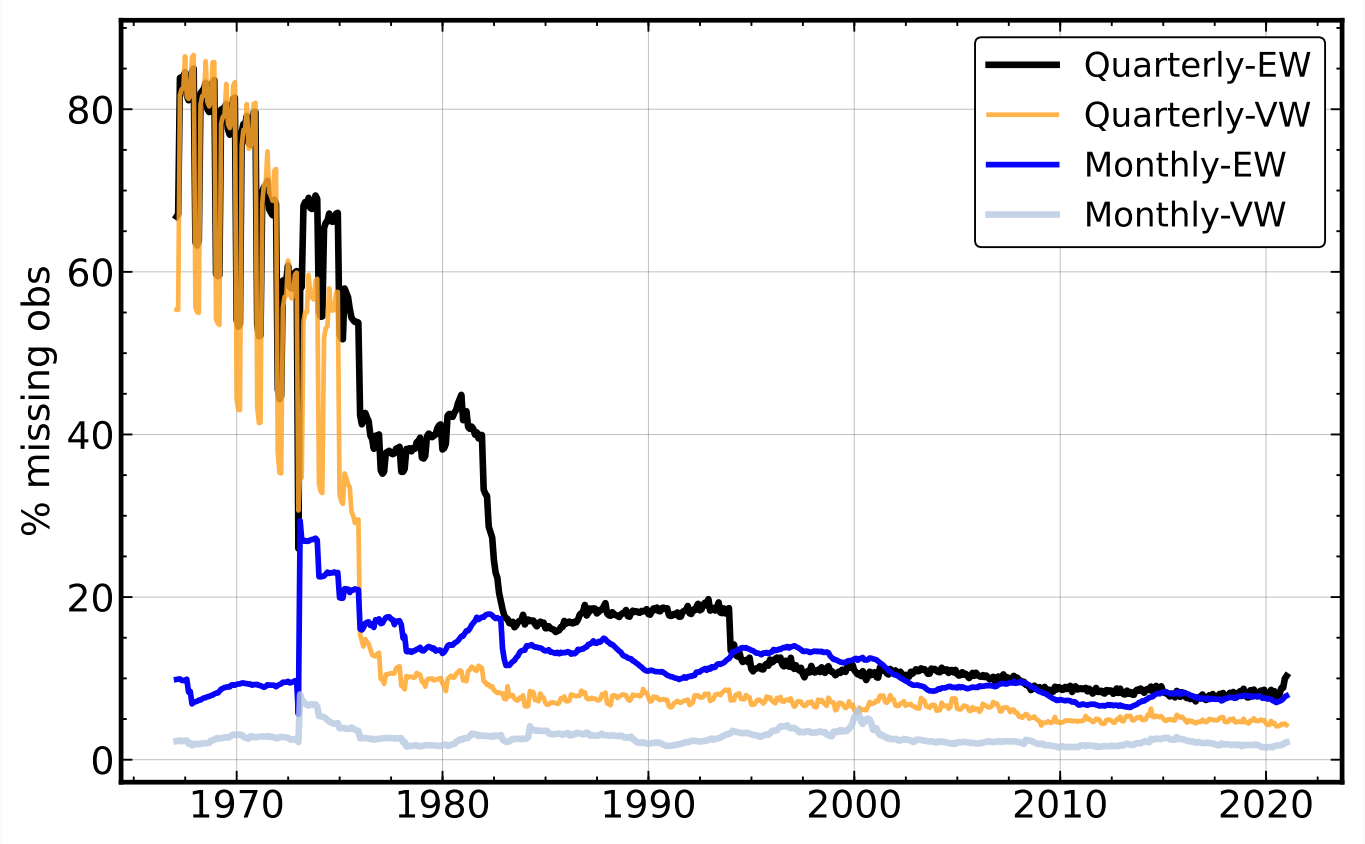
Missingness affects both small and large caps



Percentage of missing firm-month observations within quintiles

- Historically smaller companies used to have worse data coverage
- Last 20 years: similar patterns

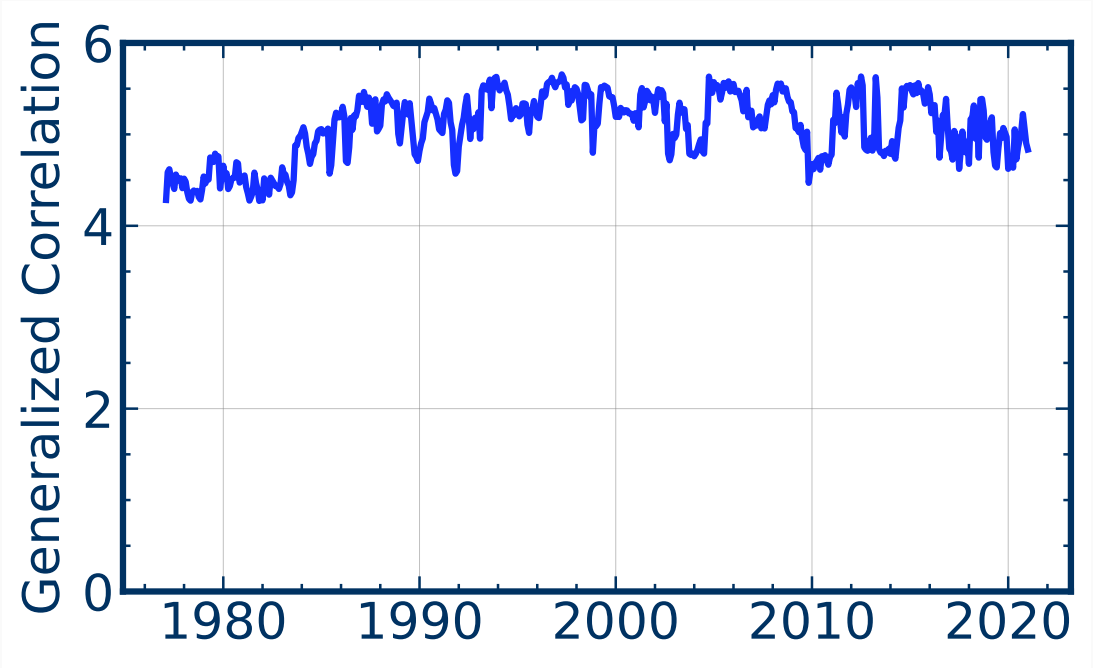
Missingness affects both quarterly and monthly characteristics



Percentage of missing quarterly and monthly updated characteristics

- Historically quarterly updated (usually accounting based) characteristics have more missing values than monthly updated (usually price based) characteristics
- Last 20 years: similar patterns

Generalized Correlation of Global and Local Factor Weights

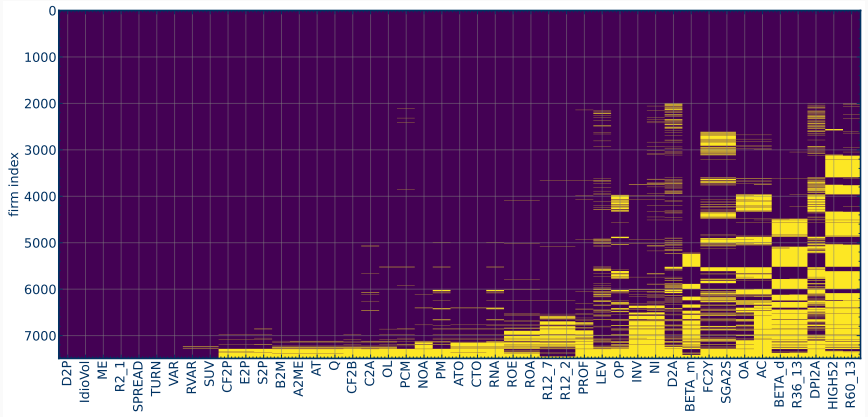


Generalized Correlation of Global and Local Factor Weights

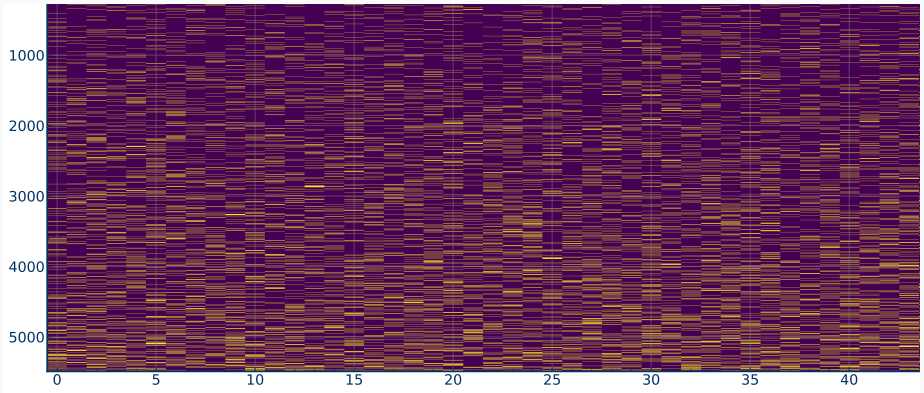
- Generalized correlation of constant global Λ with time-varying local Λ^t
- Six-factor model $\Rightarrow$ generalized correlation of 6 means the same span

$\Rightarrow$ Global and local loadings very close

Distribution of missingness



(a) Sample month: April 1981



(b) Simulated missing-at-random

Joint distribution of missing patterns (yellow missing)

- Missingness clusters in time and cross-section, and is heterogenous.
- Logistic regression to estimate $\mathbb{P}\left(W_{i,l}^t = 0\right)$:
  AUC (area under the curve) measure of fit (value of 1 optimal)
- Need characteristic fixed effects (heterogenity), observed characteristics (endogenity), past missingness (block missing)
- Characteristics are not missing at random.
- Selection bias for model estimated on observed data assuming missing-at-random
- Our approach allows for general missing patterns (different from most literature)

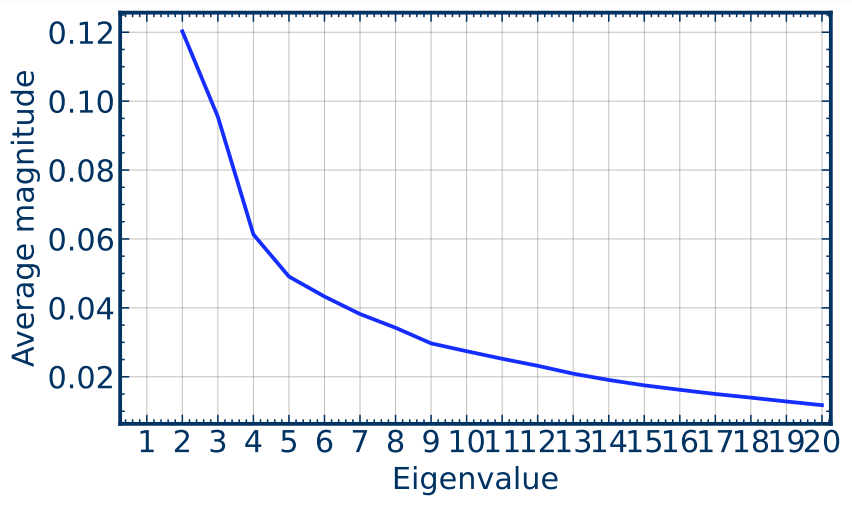
Distribution of missingness

D2P	IdioVol	ME	R2_1	SPREAD	TURN	VAR	FE	Last Val	Missing Gap	train AUC	test AUC
0.59*** [268.86]	0.63*** [28.28]	-0.44*** [-141.07]	0.04*** [18.04]	0.52*** [151.52]	0.27*** [118.95]	-0.82*** [-37.19]	F	F	F	0.55	0.52
							T	F	F	0.78	0.82
							T	5.37 [961.19]	F	0.92	0.96
							T	0.06 [137.87]	-4.74 [-279.65]	0.93	0.96
0.3*** [26.89]	-0.4*** [-3.3]	-0.65*** [-42.21]	0.07*** [7.09]	0.39*** [24.39]	-0.26*** [-24.38]	0.49*** [4.06]	T	0.06 [139.69]	-4.9 [-270.68]	**0.94**	**0.97**

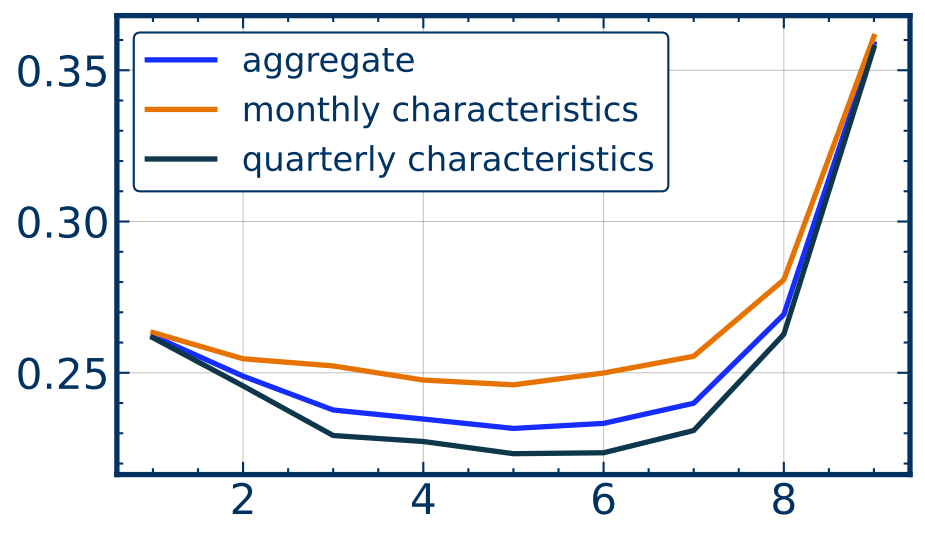
Logistic regressions explaining missingess

- Missingness clusters in time and cross-section, and is heterogenous.
- Logistic regression to estimate $\mathbb{P}\left(W_{i,l}^{t}=0\right)$:
  AUC (area under the curve) measure of fit (value of 1 optimal)
- Need characteristic fixed effects (heterogenity), observed characteristics (endogenity), past missingness (block missing)
- Characteristics are not missing at random.
- Selection bias for model estimated on observed data assuming missing-at-random
- Our approach allows for general missing patterns (different from most literature)

A factor model for characteristics



(a) Eigenvalues of $\tilde{\Sigma}_{l,p}^{XS,t}$ (time-averaged)



(b) Out-of-sample RMSE for different number of factors

- Strong factor structure in characteristics:
- $K = 6$ factors capture most of the cross-sectional variation (our baseline model).
- Extensive robustness results for different number of factors.
- Characteristic factors have economic interpretation.

Assumptions on missingness

Assumptions on probability of missingness $\mathbb{P}\left(W_{i,l}^t = 0\right) =: p_{i,l}^t$:

Dependence on time t:

- No assumption on temporal structure as different factor model for each t
- Examples: block-missing, mixed-frequency, dependence on prior missingness ...

Dependence on characteristic l:

- Characteristic specific heterogeneity
- Examples: Investment more likely to miss than B2M

Dependence on stock i:

- Extremely general dependence on features of stocks
- General time-varying and characteristic-specific function $p_{i,l}^t = f_{l,t}(F_i^t, S_i^t)$ of unknown stock-specific features $S_i^t \in \mathbb{R}^r$ and the stock-specific factors F_i^t
- Examples: small stocks or more extreme realizations more likely to miss

Identification restrictions:

- Missingness $W_{i,l}^t$ independent of loadings Λ_l^t and error $e_{i,l}^t$
- Same characteristic covariance matrix $\tilde{\Sigma}_{i,j}^{XS,t}$ on partially and fully observed data

$\Rightarrow$ Intuition: Identify "similar" stocks from observed data

Adding time-series information

Combine XS (cross-sectional) with TS (time-series) information:

- **B-XS-Model:** (backward-cross-sectional)

$$\hat{C}_{i,t}^{l,\text{B-XS}} = \beta^{l,\text{B-XS}\top} \begin{pmatrix} C_{i,t-1}^{l} & \hat{F}_{i,1}^{t} & \cdots & \hat{F}_{i,K}^{t} \end{pmatrix}$$

- **BF-XS-Model:** (backward-forward-cross-sectional)

$$\hat{C}_{i,t}^{l,\text{BF-XS}} = \beta^{l,\text{BF-XS}\top} \begin{pmatrix} C_{i,t-1}^{l} & C_{i,t+1}^{l} & \hat{F}_{i,1}^{t} & \cdots & \hat{F}_{i,K}^{t} \end{pmatrix}.$$

The framework includes several important special cases:

1. Time-series AR(1) model (B): $\beta^{l,\text{B-XS}} = \begin{pmatrix} \beta^{\text{B}} & 0 & \cdots & 0 \end{pmatrix}$.
2. Last observed value (PV): $\beta^{l,\text{B-XS}} = \begin{pmatrix} 1 & 0 & \cdots & 0 \end{pmatrix}$.
3. Cross-sectional median: $\beta^{l,\text{B-XS}} = \begin{pmatrix} 0 & 0 & \cdots & 0 \end{pmatrix}$ (ranks centered at 0).

For estimation, stack cross-sectional and time-series information in $X_i^{l,t}$ and run the following regression (averaged over observed stocks):

$$\hat{\beta}^{l,t} = \left(\sum_{i=1}^{N_t} W_{i,l}^{t} X_i^{l,t} X_i^{l,t\top} \right)^{-1} \left(\sum_{i=1}^{N_t} W_{i,l}^{t} X_i^{l,t} C_{i,t}^{l} \right)$$

Imputation models

Method	Estimation
Backward-Forward-XS (BF-XS)	$\hat{C}_{i,t}^{\text{BF-XS}} = (\hat{\beta}^{\text{BF-XS}})^\top \begin{pmatrix} C_{i,t-1}^I & C_{i,t+1}^I & \hat{F}_{i,1}^I & \cdots & \hat{F}_{i,K}^I \end{pmatrix}$
Backward-XS (B-XS)	$\hat{C}_{i,t}^{\text{B-XS}} = (\hat{\beta}^{\text{B-XS}})^\top \begin{pmatrix} C_{i,t-1}^I & \hat{F}_{i,1}^I & \cdots & \hat{F}_{i,K}^I \end{pmatrix}$
Forward-XS (F-XS)	$\hat{C}_{i,t}^{\text{F-XS}} = (\hat{\beta}^{\text{F-XS}})^\top \begin{pmatrix} C_{i,t+1}^I & \hat{F}_{i,1}^I & \cdots & \hat{F}_{i,K}^I \end{pmatrix}$
Cross-sectional (XS)	$\hat{C}_{i,t}^{\text{XS}} = (\hat{\beta}^{\text{XS}})^\top \begin{pmatrix} \hat{F}_{i,1}^I & \cdots & \hat{F}_{i,K}^I \end{pmatrix}$
Time-series (B)	$\hat{C}_{i,t}^{\text{B}} = (\hat{\beta}^{\text{B}})^\top \begin{pmatrix} C_{i,t-1}^I \end{pmatrix}$
Previous value (PV)	$\hat{C}_{i,t}^{\text{PV}} = C_{i,t-1}^I$
Cross-sectional median	$\hat{C}_{i,t}^{\text{median}} = 0$

Different imputation methods sorted by the size of the information set

- Current standard in the literature: Cross-sectional median or previous value
- The need for past/future information restricts available options for imputation
- For each model: local (each month) and global (pooled) estimation.
  Local model avoids look-ahead bias but less efficient

$\Rightarrow$ Different types of missing values might benefit from different methods

Global and local factor models

Global model assumes that factor composition Λ and β stays constant over time:

- Global model estimated with global (pooled) regression

$$\hat{\beta}^{l} = \left(\sum_{t=1}^{T} \sum_{i=1}^{N_t} \left(W_{i,l}^{t} X_{i}^{l,t} X_{i}^{l,t\top} \right) \right)^{-1} \left(\sum_{t=1}^{T} \sum_{i=1}^{N_t} \left(W_{i,l}^{t} X_{i}^{l,t} C_{i,t}^{l} \right) \right)$$

- Estimate rotation of global Λ from average characteristic covariance matrix

$$\tilde{\Sigma}_{l,p}^{\text{XS}} = \frac{1}{T} \sum_{t=1}^{T} \left(\frac{1}{|O_{l,p}^{t}|} \sum_{i \in O_{l,p}^{t}} C_{i,l}^{t} C_{i,p}^{t} \right)$$

- Local model estimates factor models and $\hat{\beta}^{l,t}$ for each t independently

Local vs. global tradeoff:

- Global estimation is more efficient (uses more information)
- Local estimation allows for time-variation (less bias) and avoids look-ahead bias

Aggregate results

Method	all characteristics	quarterly characteristics	monthly characteristics
global BF-XS	**0.10**	**0.08**	**0.13**
global F-XS	0.10	0.06	0.14
global B-XS	0.14	0.14	0.15
global XS	0.23	0.22	0.24
global B	0.15	0.15	0.15
local B-XS	**0.14**	**0.14**	**0.14**
local XS	0.23	0.23	0.24
local B	0.15	0.15	0.15
prev val	0.17	0.16	0.19
XS median	0.29	0.29	0.29
industry median	0.29	0.29	0.29

Out-of-sample RMSE for block-missing characteristics

- Baseline models:
  - local B-XS (no look-ahead-bias)
  - global BF-XS (full possible information)
- Current standard (cross-sectional median and last observed value) is the worst
- Similar results for logit masking
- Extensive evaluation for type of missingness (beginning, middle, end), different masking, extreme quantiles, size of companies, industry, over time, etc.

Aggregate results

	In-Sample			OOS MAR			OOS Block			OOS Logit		
Method	all	quarterly	monthly	all	quarterly	monthly	all	quarterly	monthly	all	quarterly	monthly
global BF-XS	**0.85**	**0.94**	**0.80**	**0.80**	**0.83**	**0.79**	**0.83**	**0.94**	**0.77**	**0.93**	**0.94**	**0.55**
global F-XS	0.85	0.98	0.77	0.75	0.77	0.74	0.81	0.97	0.71	0.49	0.74	0.06
global B-XS	0.78	0.81	0.77	0.76	0.79	0.74	0.75	0.81	0.71	0.87	0.87	0.48
global XS	0.57	0.61	0.54	0.42	0.47	0.39	0.38	0.43	0.36	0.23	0.35	0.11
global B	0.76	0.79	0.74	0.75	0.78	0.73	0.74	0.79	0.71	0.85	0.86	0.45
local B-XS	**0.79**	**0.82**	**0.78**	**0.77**	**0.80**	**0.75**	**0.76**	**0.81**	**0.73**	**0.87**	**0.87**	**0.49**
local XS	0.50	0.52	0.50	0.40	0.43	0.38	0.37	0.38	0.35	0.25	0.34	0.11
local B	0.76	0.80	0.74	0.75	0.78	0.73	0.74	0.80	0.71	0.85	0.86	0.45
prev	0.66	0.76	0.60	0.64	0.75	0.58	0.63	0.76	0.56	0.84	0.85	0.01
XS-median	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ind-median	0.00	0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	0.00	0.00	0.00

Out-of-sample explained variation R^2 relative to median

- Baseline models:
  - local B-XS (no look-ahead-bias)
  - global BF-XS (full possible information)
- Current standard (cross-sectional median and last observed value) is the worst

Imputation error for different types of missingness

	In-Sample			OOS MAR			OOS Block			OOS Logit		
Method	all	quarterly	monthly	all	quarterly	monthly	all	quarterly	monthly	all	quarterly	monthly
					Start of the sample							
global BF-XS	-	-	-	-	-	-	-	-	-	-	-	-
global F-XS	**0.10**	**0.05**	**0.16**	**0.17**	**0.17**	**0.18**	**0.12**	**0.07**	**0.17**	**0.22**	**0.20**	**0.26**
global B-XS	-	-	-	-	-	-	-	-	-	-	-	-
global XS	0.22	0.21	0.24	0.26	0.24	0.28	0.27	0.26	0.28	0.29	0.29	0.29
global B	-	-	-	-	-	-	-	-	-	-	-	-
local B-XS	-	-	-	-	-	-	-	-	-	-	-	-
local XS	**0.24**	**0.23**	**0.25**	**0.26**	**0.25**	**0.28**	**0.27**	**0.26**	**0.27**	**0.29**	**0.29**	**0.29**
local B	-	-	-	-	-	-	-	-	-	-	-	-
prev	-	-	-	-	-	-	-	-	-	-	-	-
XS-median	0.32	0.32	0.32	0.32	0.32	0.32	0.32	0.32	0.31	0.32	0.32	0.31
ind-median	0.32	0.32	0.32	0.32	0.32	0.32	0.32	0.32	0.31	0.32	0.32	0.31

In- and out-of-sample RMSE for different types of missing observations

- Bold indicates best local (lock-ahead-bias free) and global model
- Our baseline models dominate across all the missing patterns
- Availability of models depends on type of missingness

Imputation error for different types of missingness

	In-Sample			OOS MAR			OOS Block			OOS Logit		
Method	all	quarterly	monthly	all	quarterly	monthly	all	quarterly	monthly	all	quarterly	monthly
					Middle of the sample							
global BF-XS	**0.09**	**0.08**	**0.12**	**0.13**	**0.13**	**0.13**	**0.10**	**0.08**	**0.13**	**0.10**	**0.09**	**0.13**
global F-XS	0.09	0.06	0.13	0.14	0.15	0.14	0.1	0.06	0.14	0.13	0.12	0.15
global B-XS	0.13	0.14	0.13	0.14	0.14	0.14	0.14	0.14	0.15	0.13	0.12	0.15
global XS	0.19	0.18	0.21	0.22	0.21	0.23	0.22	0.21	0.24	0.22	0.21	0.24
global B	0.14	0.15	0.14	0.15	0.15	0.15	0.15	0.143	0.15	0.14	0.13	0.16
local B-XS	**0.13**	**0.14**	**0.13**	**0.14**	**0.14**	**0.14**	**0.14**	**0.14**	**0.14**	**0.13**	**0.12**	**0.15**
local XS	0.20	0.12	0.22	0.22	0.22	0.23	0.23	0.22	0.24	0.23	0.22	0.24
local B	0.14	0.15	0.14	0.15	0.15	0.15	0.15	0.15	0.15	0.14	0.13	0.16
prev	0.16	0.16	0.18	0.17	0.16	0.18	0.17	0.16	0.18	0.15	0.14	0.19
XS-median	0.29	0.29	0.29	0.29	0.29	0.29	0.29	0.29	0.29	0.29	0.29	0.29
ind-median	0.29	0.29	0.29	0.29	0.29	0.29	0.29	0.29	0.29	0.29	0.29	0.29

In- and out-of-sample RMSE for different types of missing observations

- Bold indicates best local (lock-ahead-bias free) and global model
- Our baseline models dominate across all the missing patterns
- Availability of models depends on type of missingness

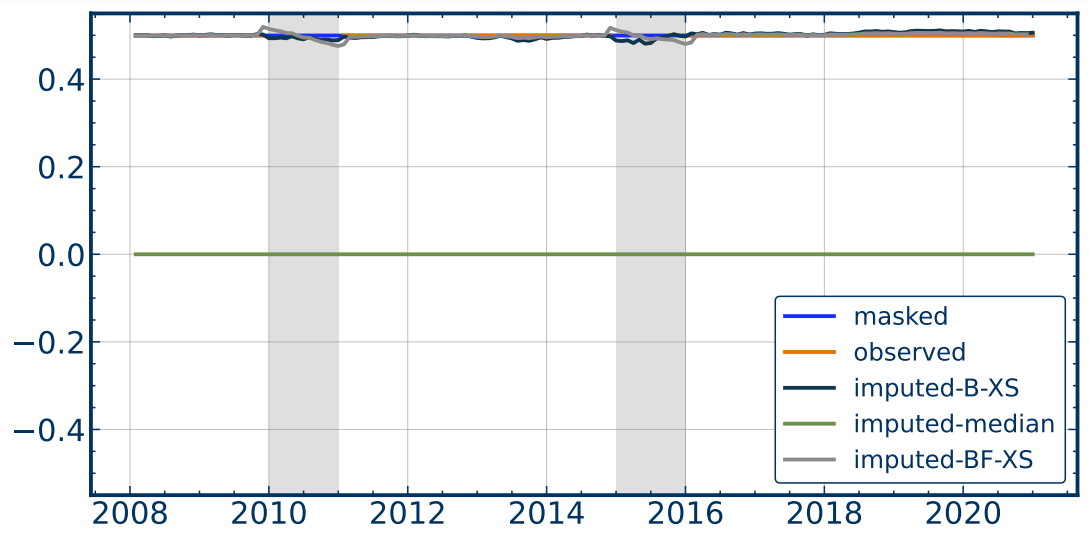
Imputation error for different types of missingness

	In-Sample			OOS MAR			OOS Block			OOS Logit		
Method	all	quarterly	monthly	all	quarterly	monthly	all	quarterly	monthly	all	quarterly	monthly
					End of the sample							
global BF-XS	-	-	-	-	-	-	-	-	-	-	-	-
global F-XS	-	-	-	-	-	-	-	-	-	-	-	-
global B-XS	**0.16**	**0.15**	**0.17**	**0.18**	**0.18**	**0.17**	**0.18**	**0.18**	**0.17**	**0.12**	**0.12**	**0.14**
global XS	0.23	0.23	0.24	0.26	0.25	0.267	0.27	0.27	0.27	0.25	0.25	0.27
global B	0.19	0.19	0.19	0.19	0.19	0.18	0.19	0.19	0.18	0.13	0.13	0.15
local B-XS	**0.16**	**0.15**	**0.17**	**0.18**	**0.18**	**0.17**	**0.18**	**0.18**	**0.17**	**0.12**	**0.12**	**0.14**
local XS	0.25	0.25	0.25	0.26	0.26	0.27	0.27	0.28	0.27	0.26	0.26	0.27
local B	0.19	0.19	0.19	0.18	0.19	0.18	0.19	0.19	0.18	0.13	0.13	0.15
prev	0.21	0.20	0.22	0.20	0.20	0.22	0.21	0.19	0.22	0.14	0.13	0.17
XS-median	0.35	0.37	0.34	0.34	0.33	0.33	0.35	0.37	0.33	0.32	0.32	0.33
ind-median	0.35	0.37	0.34	0.34	0.33	0.33	0.35	0.37	0.33	0.32	0.32	0.33

In- and out-of-sample RMSE for different types of missing observations

- Bold indicates best local (lock-ahead-bias free) and global model
- Our baseline models dominate across all the missing patterns
- Availability of models depends on type of missingness

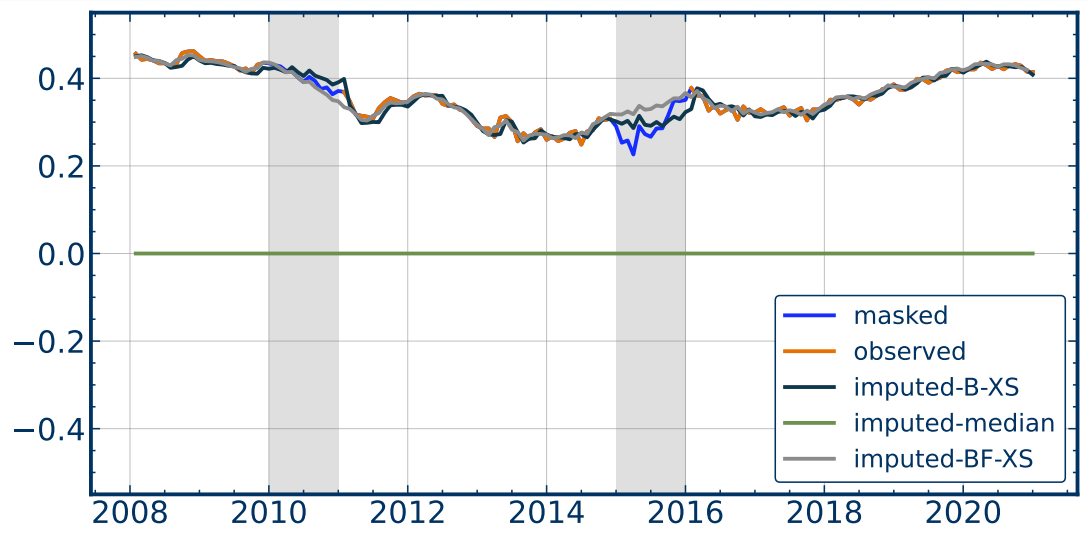
Illustration: Persistent characteristics



Size of Microsoft: Model-implied and observed time-series

- Size as representative persistent characteristic; other examples: AT, D2P, LEV
- Gray blocks: 1-year out-of-sample imputation
- B-XS and BF-XS extremely precise
- Time-series observation provides close to perfect prediction
- Median wrong level and dynamics

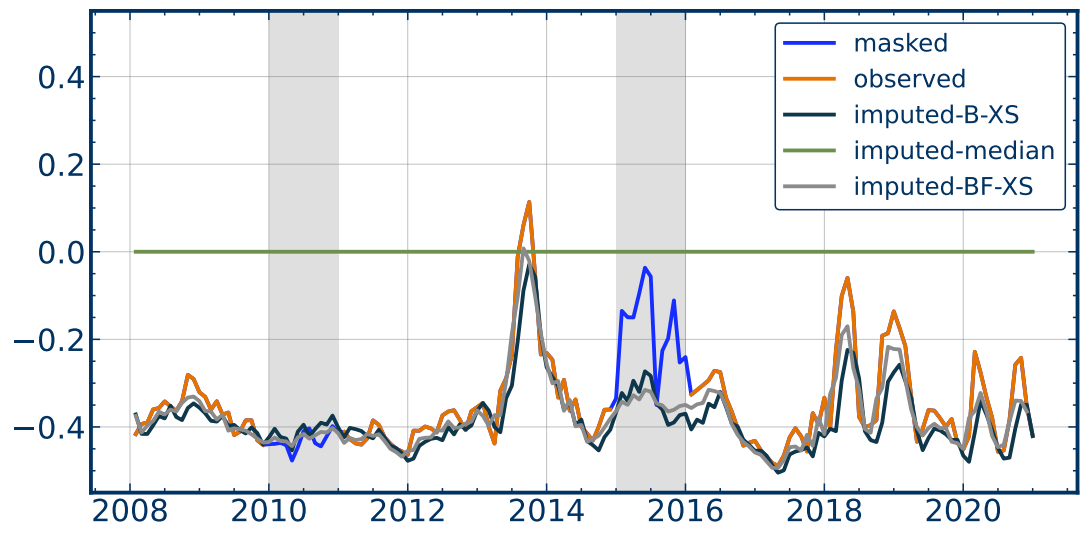
Illustration: Persistent and volatile characteristics



Operating Profitability of Microsoft: Model-implied and observed time-series

- Tobin's Q as representative persistent and volatile characteristic; other examples: B2M, E2P, INV, OP
- Gray blocks: 1-year out-of-sample imputation
- B-XS "anchors" at last observed value, dynamics from cross-section
- BF-XS connects endpoints, dynamics from cross-section
- Median wrong level and dynamics

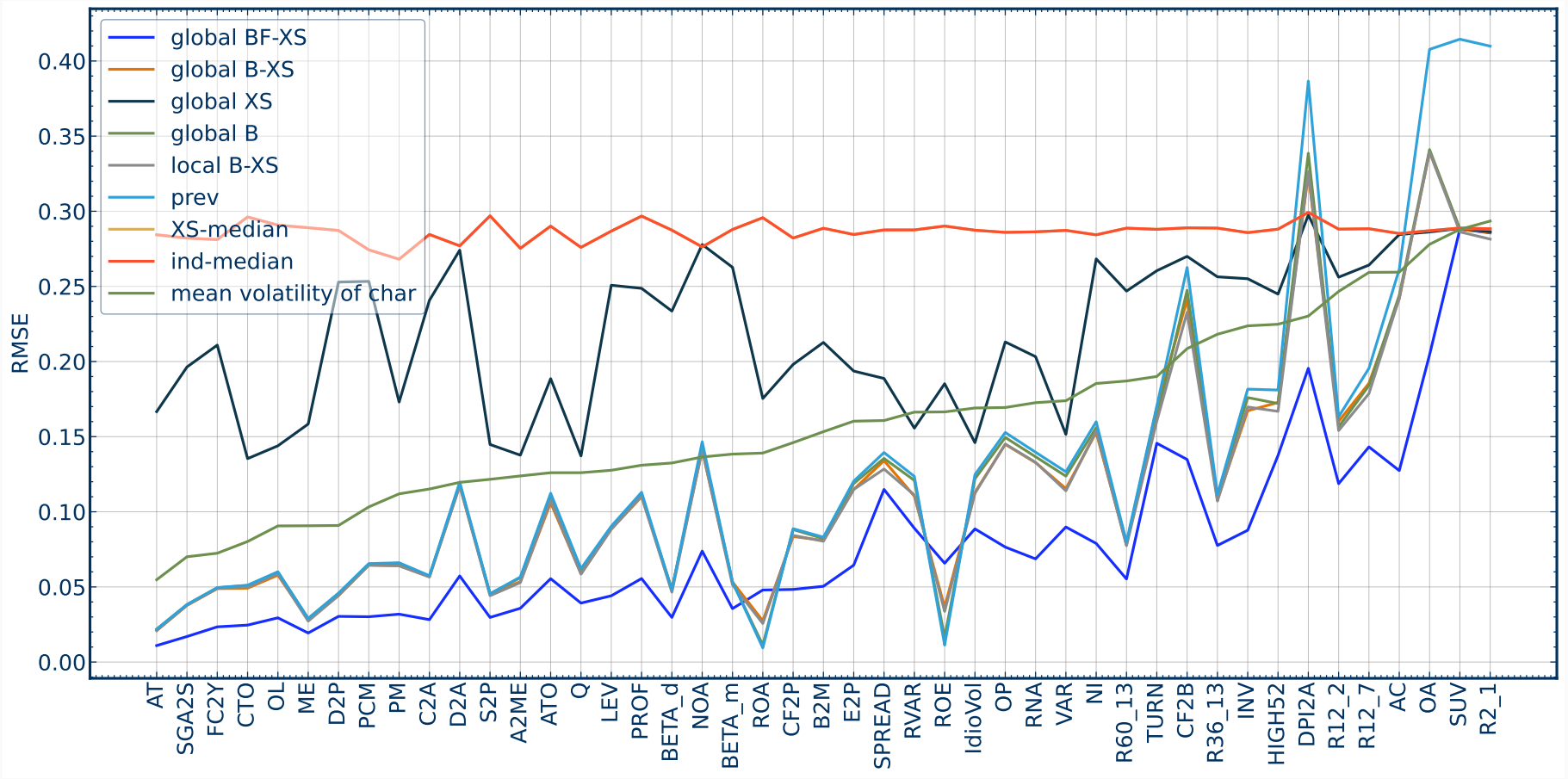
Illustration: Volatile characteristics



Variance of Microsoft: Model-implied and observed time-series

- VAR as representative volatile characteristic; other examples: R2_1, R12_2, SUV
- Gray blocks: 1-year out-of-sample imputation
- Dynamics driven by cross-sectional contemporaneous factors
- Median wrong level and dynamics

Comparison of imputation methods



Out-of-sample RMSE by imputation method across individual block-missing characteristics

- Characteristics are sorted in ascending order based on their volatility (black line)
- Imputation for persistent characteristics benefits from the TS data
- Imputation for more volatile characteristics relies more on XS information

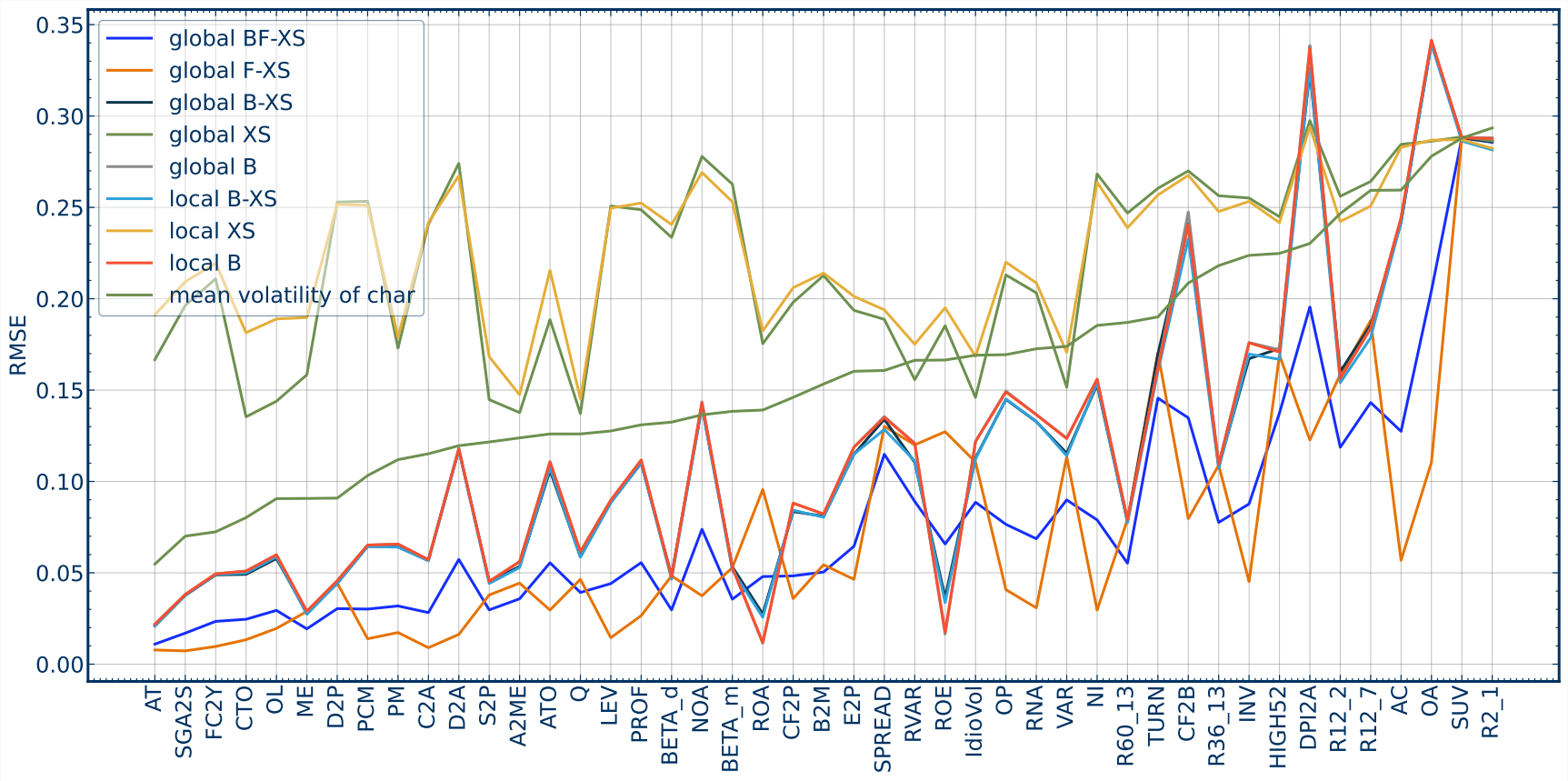
Aggregate results

	In-Sample			OOS MAR			OOS Block			OOS Logit		
Method	all	quarterly	monthly	all	quarterly	monthly	all	quarterly	monthly	all	quarterly	monthly
global BF-XS	**0.09**	**0.08**	**0.12**	**0.13**	**0.13**	**0.13**	**0.10**	**0.08**	**0.13**	**0.10**	**0.09**	**0.13**
global F-XS	0.09	0.06	0.13	0.15	0.15	0.14	0.10	0.06	0.14	0.18	0.16	0.23
global B-XS	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.15	0.13	0.12	0.15
global XS	0.19	0.18	0.21	0.22	0.21	0.24	0.23	0.22	0.24	0.25	0.24	0.27
global B	0.15	0.15	0.14	0.15	0.15	0.15	0.15	0.15	0.15	0.14	0.13	0.16
local B-XS	**0.14**	**0.14**	**0.13**	**0.14**	**0.14**	**0.14**	**0.14**	**0.14**	**0.14**	**0.13**	**0.12**	**0.15**
local XS	0.21	0.20	0.21	0.23	0.22	0.24	0.23	0.23	0.24	0.25	0.24	0.27
local B	0.15	0.15	0.14	0.15	0.15	0.15	0.15	0.15	0.15	0.14	0.13	0.16
prev	0.17	0.16	0.18	0.17	0.16	0.18	0.17	0.16	0.19	0.15	0.14	0.19
XS-median	0.29	0.29	0.29	0.29	0.29	0.29	0.29	0.29	0.29	0.30	0.30	0.31
ind-median	0.29	0.29	0.29	0.29	0.29	0.29	0.29	0.29	0.29	0.30	0.30	0.31

Out-of-sample RMSE for different imputation methods

- Baseline models:
  - local B-XS (no look-ahead-bias)
  - global BF-XS (full possible information)
- Current standard (cross-sectional median and last observed value) is the worst

Local vs global imputation



Out-of-sample RMSE by imputation method across individual block-missing characteristics

- Global models are slightly better
- Highly volatile characteristics benefit more from local models

Two fundamental effects

Selection bias: Asset pricing results depend on which stocks are included

- Portfolios based on observed or missing characteristics
- Univariate portfolios sorts with different stocks included:
  - Stocks that only have specific characteristic observed
  - Stocks that have multiple characteristics observed
  - Stocks that have all characteristics observed
- IPCA factors estimated on subset of fully observed or larger set of imputed data

$\Rightarrow$ Subsamples of fully observed stocks lead to selection bias in asset pricing metrics

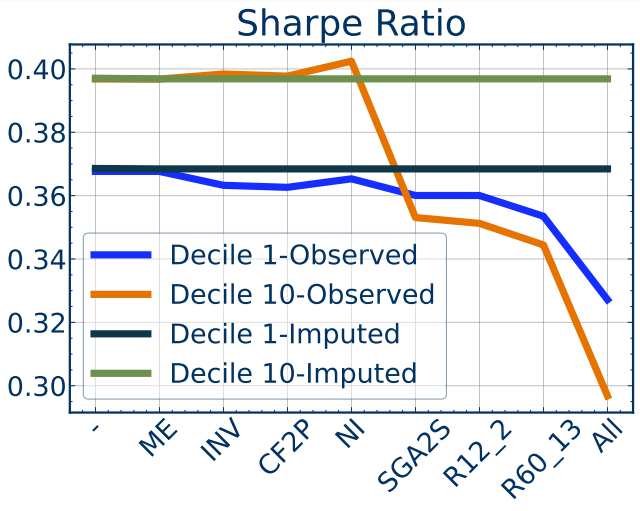
$\Rightarrow$ Out-of-sample investment substantially better with all stocks

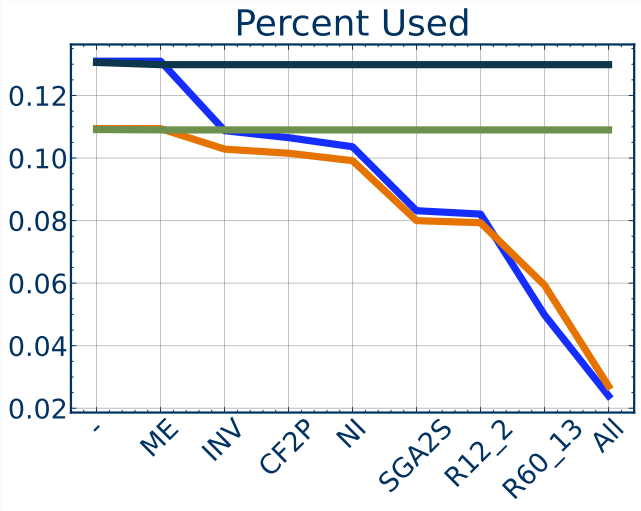
Imputation bias: Asset pricing results depend on imputation method

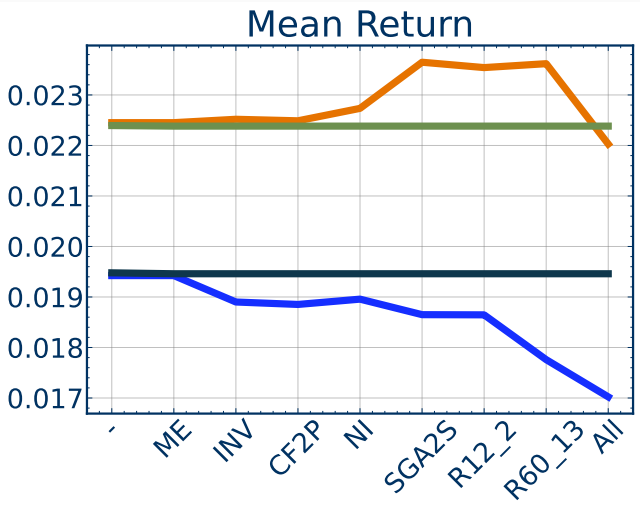
- Mask observed values based on empirical observation pattern (logistic regression)
- Impute masked missing values with our local B-XS model or conventional median
- Comparison of asset pricing metrics for observed and imputed data
- Cross-sectional regression on characteristics:
  - Risk premia for characteristic signals
  - Characteristic mimicking factor portfolio time-series

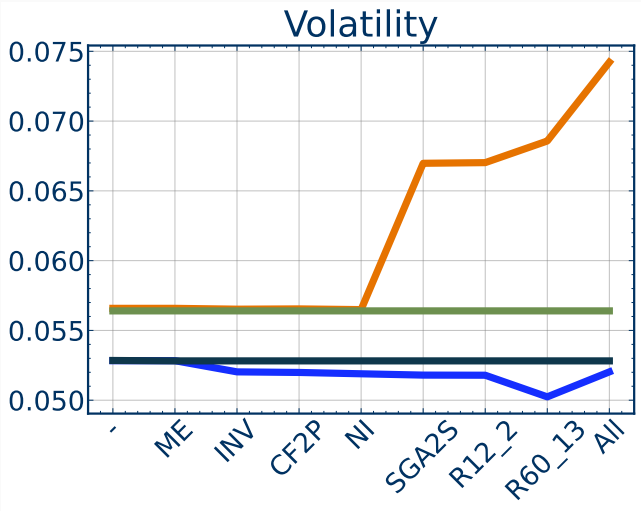
$\Rightarrow$ Uniformly and substantially larger errors in asset metrics for median imputation

Selection bias: Book-to-Market, conditional on other observables



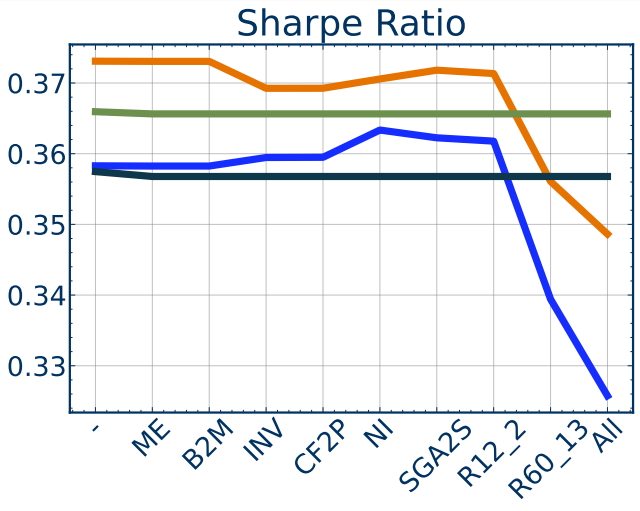


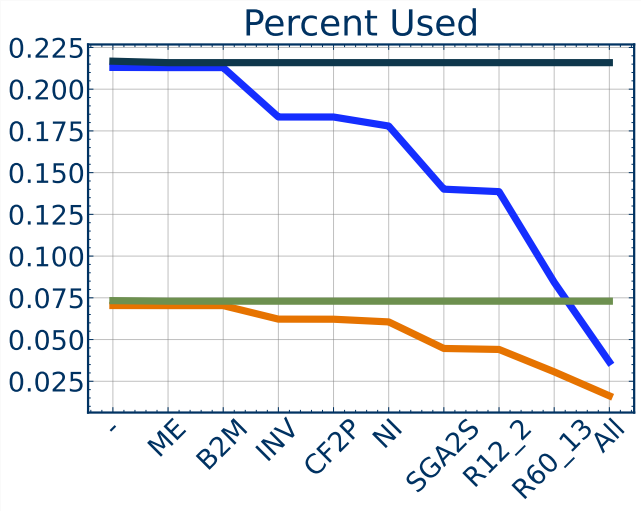


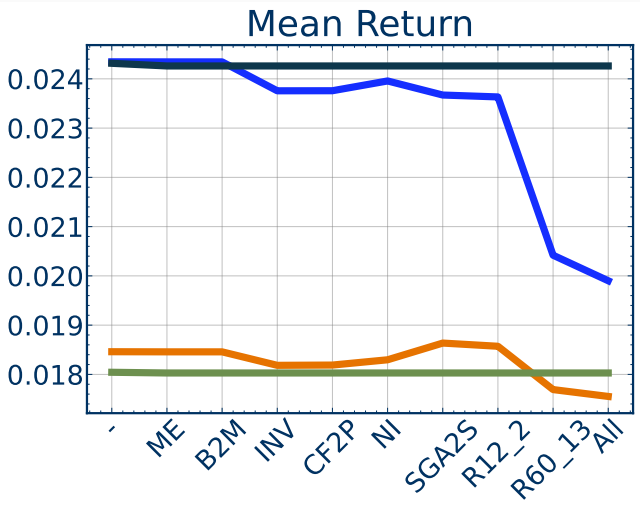


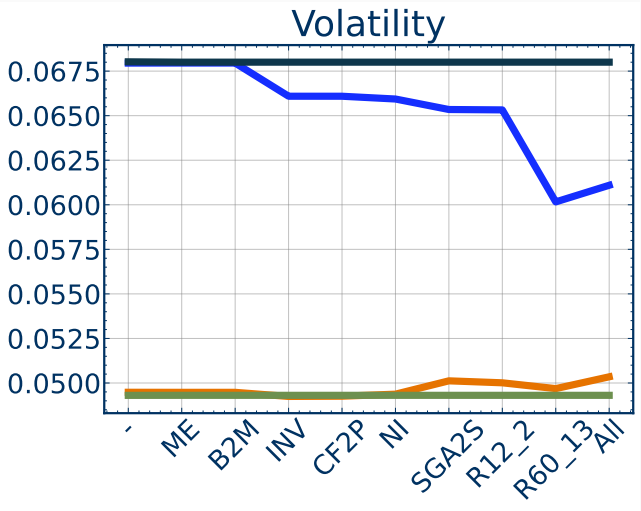
- Return on the value sorts, requiring additionally observed characteristics
- Missing information has a direct impact on the return of the simplest strategies
- Effect is larger when multiple signals are used

Selection bias: Operating Profitability, conditional on other observables





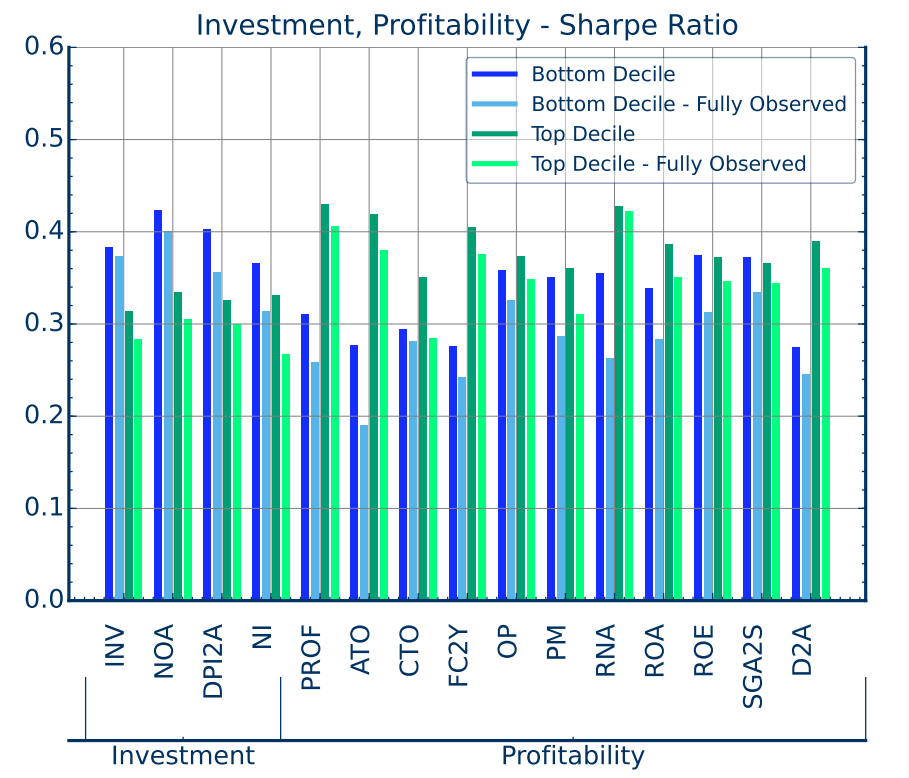




⇒ Missingness has a stronger impact when multiple characteristics are used

⇒ Implications for multiple sorts, machine learning, and the whole "multidimensional challenge" in asset pricing.

Univariate Portfolio Sorts with and without Missing Values

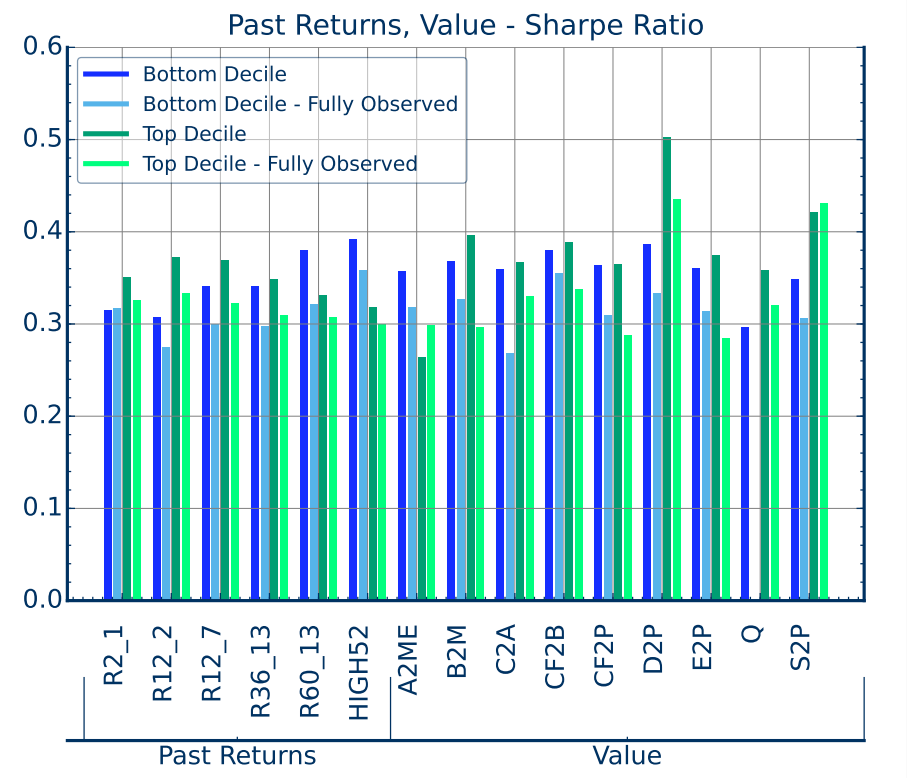


Sharpe ratios of top and bottom decile of sorted portfolios

- Sorts of stocks with observed single characteristic or all 45 characteristics
- Lower Sharpe ratios for fully observed subset
- Mean returns: complex interaction between characteristic and missingness
- Higher volatility: restricted sample has less diversification

$\Rightarrow$ Selection bias applies to all characteristic sorts

Univariate Portfolio Sorts with and without Missing Values

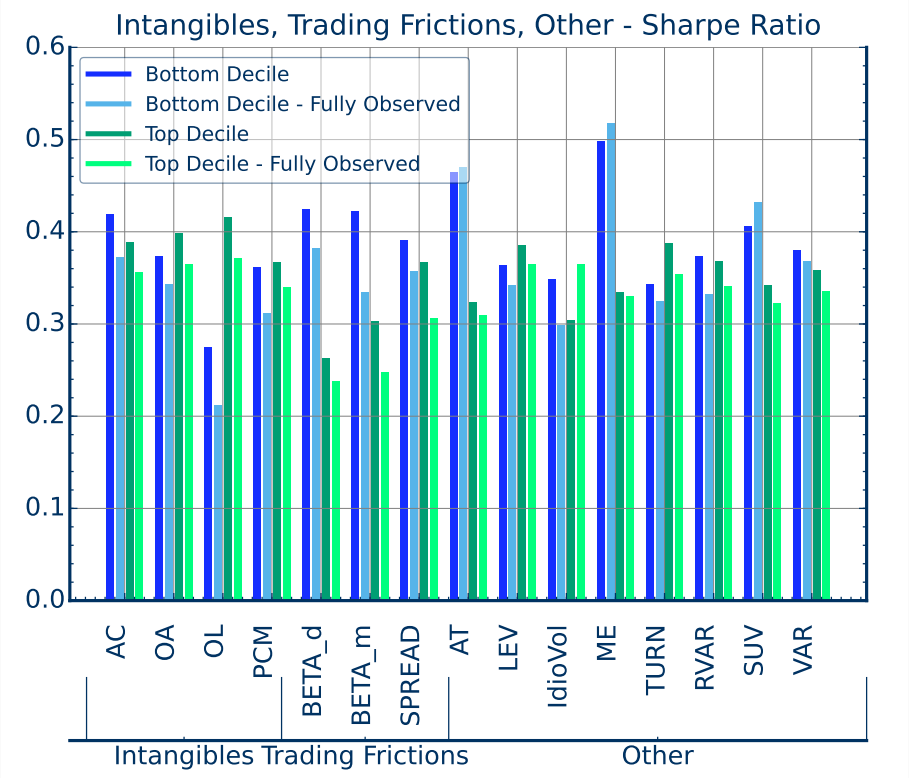


Sharpe ratios of top and bottom decile of sorted portfolios

- Sorts of stocks with observed single characteristic or all 45 characteristics
- Lower Sharpe ratios for fully observed subset
- Mean returns: complex interaction between characteristic and missingness
- Higher volatility: restricted sample has less diversification

$\Rightarrow$ Selection bias applies to all characteristic sorts

Univariate Portfolio Sorts with and without Missing Values



Sharpe ratios of top and bottom decile of sorted portfolios

- Sorts of stocks with observed single characteristic or all 45 characteristics
- Lower Sharpe ratios for fully observed subset
- Mean returns: complex interaction between characteristic and missingness
- Higher volatility: restricted sample has less diversification

$\Rightarrow$ Selection bias applies to all characteristic sorts

Imputation Error For Different Size Filters

estimation	evaluation	aggregate	quarterly	monthly
$<$ \$ 1 firms	$<$ \$ 1 firms	0.09	0.10	0.05
	$\geq$ \$ 1 firms	0.16	0.15	0.17
	all	0.16	0.15	0.17
$\geq$ \$ 1 firms	$<$ \$ 1 firms	0.26	0.30	0.24
	$\geq$ \$ 1 firms	0.14	0.14	0.14
	all	0.14	0.14	0.14
all	$<$ \$ 1 firms	0.26	0.30	0.24
	$\geq$ \$ 1 firms	0.14	0.14	0.14
	all	0.14	0.14	0.14

Imputation RMSE For Different Size Filters

$\Rightarrow$ Results are robust to size filters

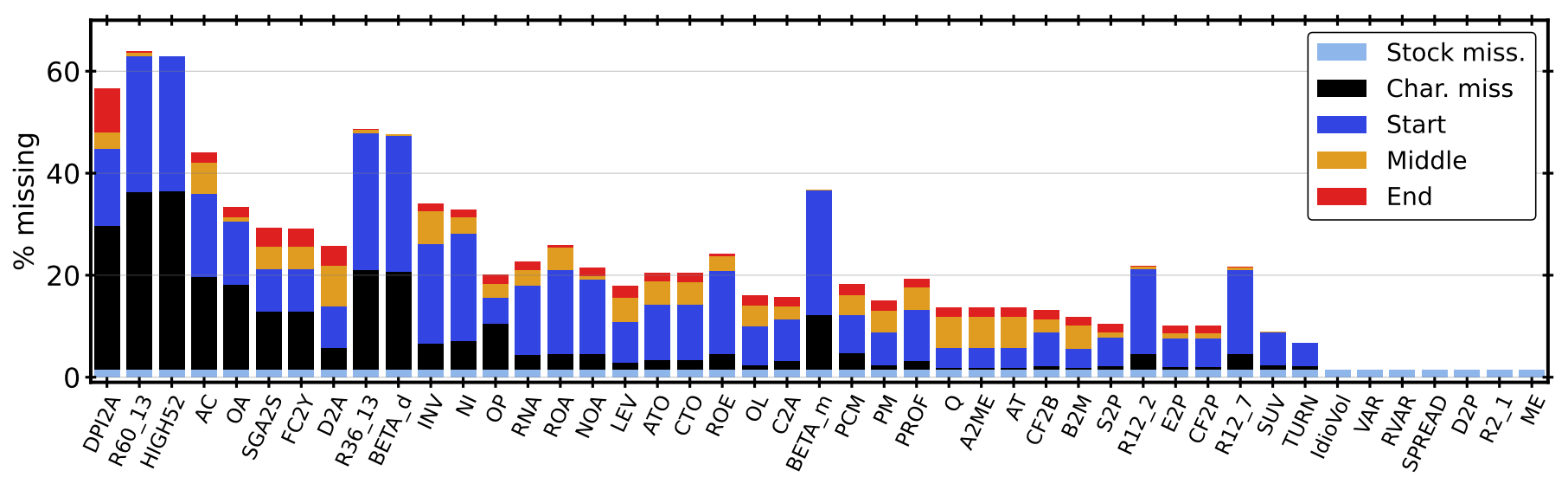
Imputation Results with and without Financial Firms

estimation	evaluation	aggregate	quarterly	monthly
financial firms	financial firms	0.14	0.13	0.14
	non financial firms	0.14	0.13	0.14
non financial firms	financial firms	0.14	0.14	0.14
	non financial firms	0.14	0.14	0.14

Imputation RMSE with and without financial firms

$\Rightarrow$ Results are robust to excluding financial firms

Pooled Mean across Stocks (Equally-weighted)



Start = no previous observations

End = no further observations

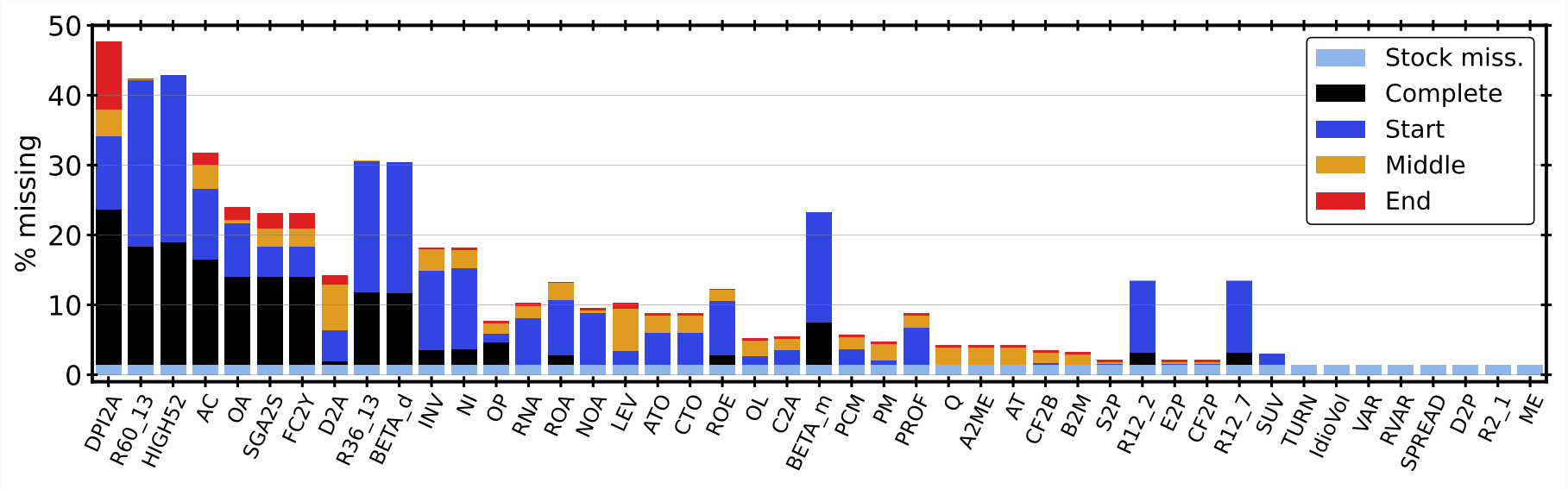
Middle = some previous and future observations

Complete = completely missing

- Some characteristics are mechanically missing for younger firms (e.g., LTrev)
- Many characteristics are missing after having been previously observed
- Some characteristics are missing at the end of the company's life
- Some are never observed

$\Rightarrow$ Imputation needs to allow for different information sets

Pooled Mean across Stocks (Value-weighted)



Start = no previous observations

End = no further observations

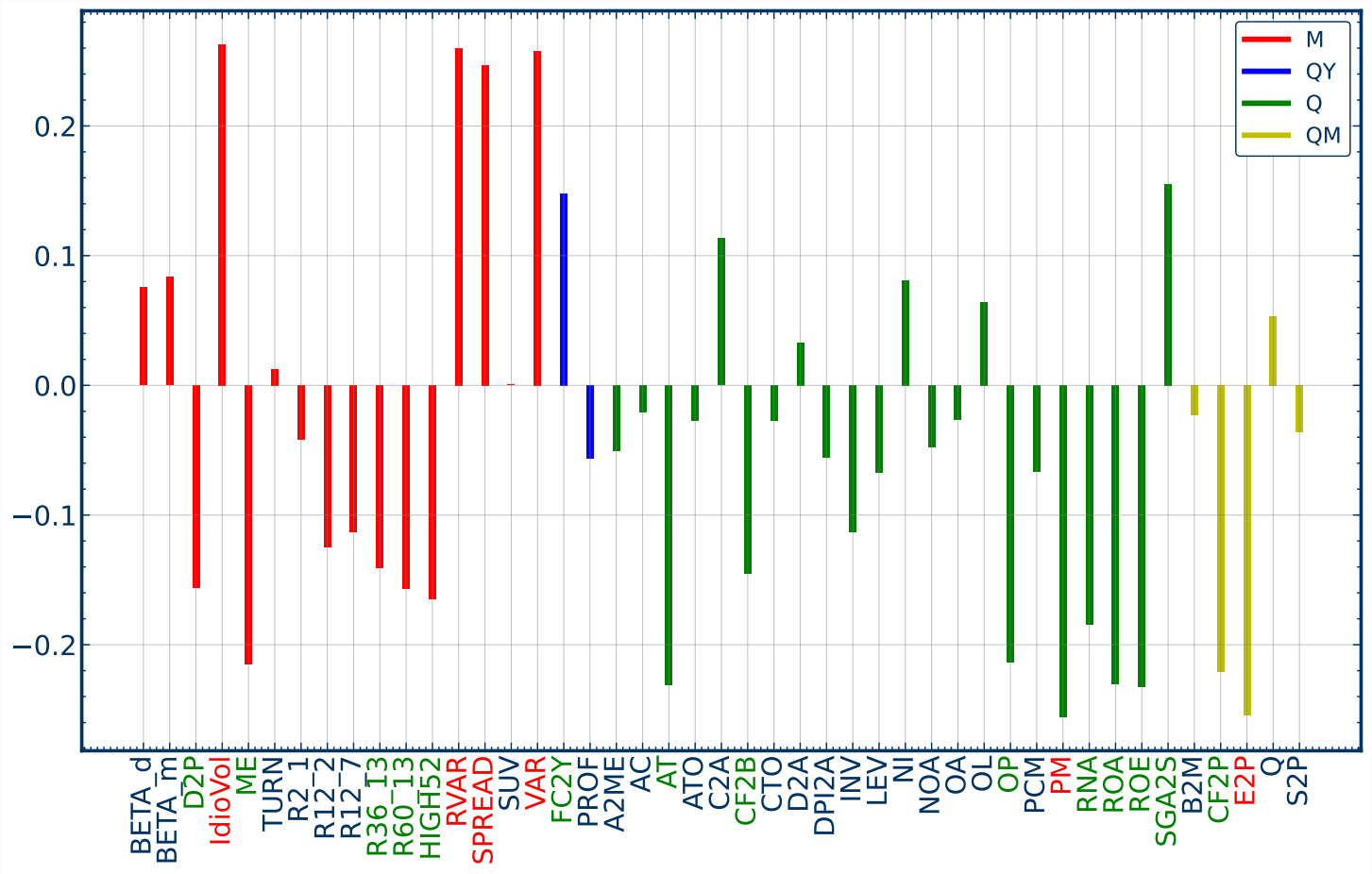
Middle = some previous and future observations

Complete = completely missing

- Some characteristics are mechanically missing for younger firms (e.g., LTrev)
- Many characteristics are missing after having been previously observed
- Some characteristics are missing at the end of the company's life
- Some are never observed

$\Rightarrow$ Imputation needs to allow for different information sets

Composition of Latent Factor 1

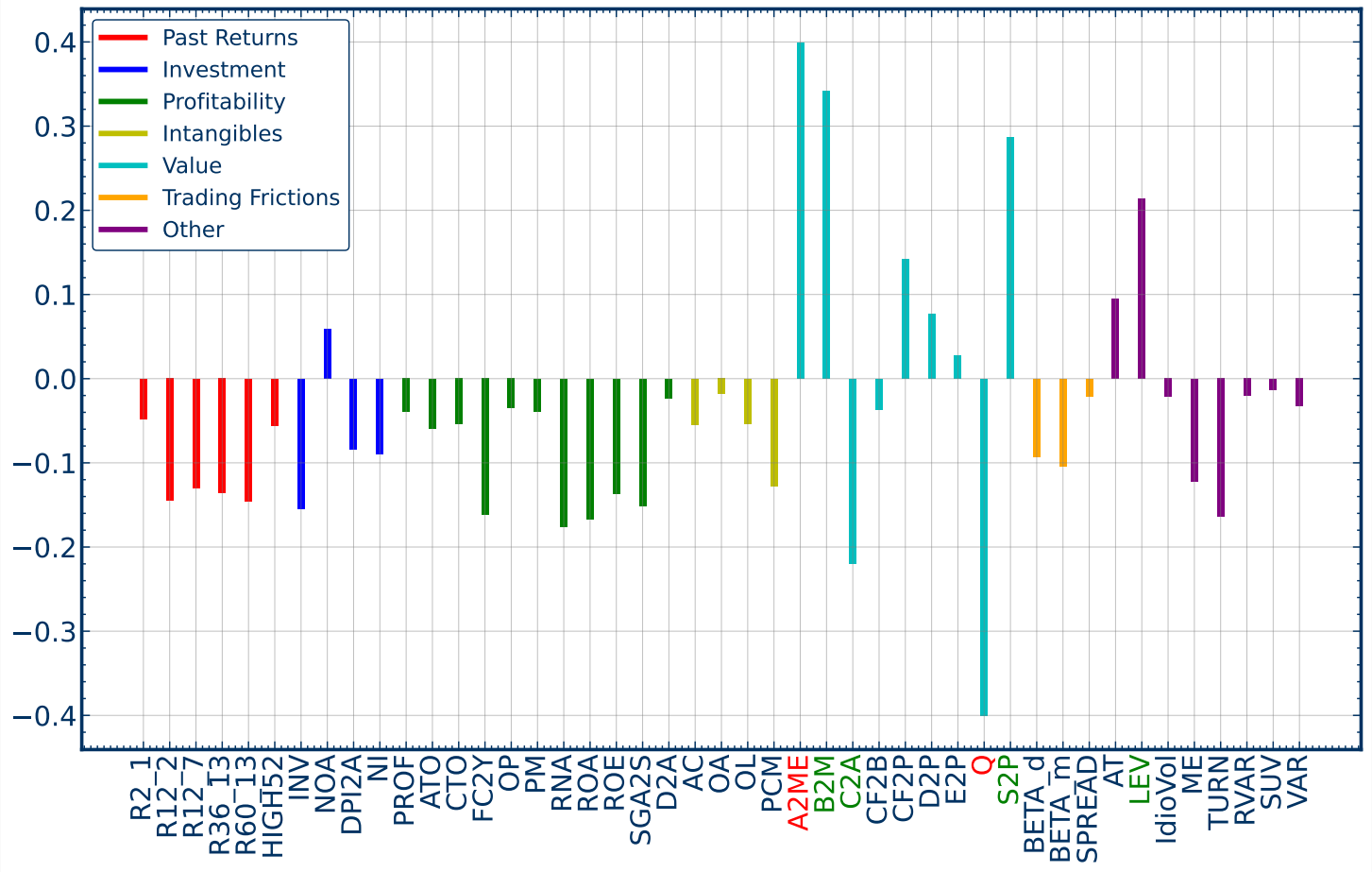


Composition of first latent factors grouped by frequency

- The loadings are colored by characteristic category

$\Rightarrow$ 1st factor = high volatility characteristics factor

Composition of Latent Factor 2

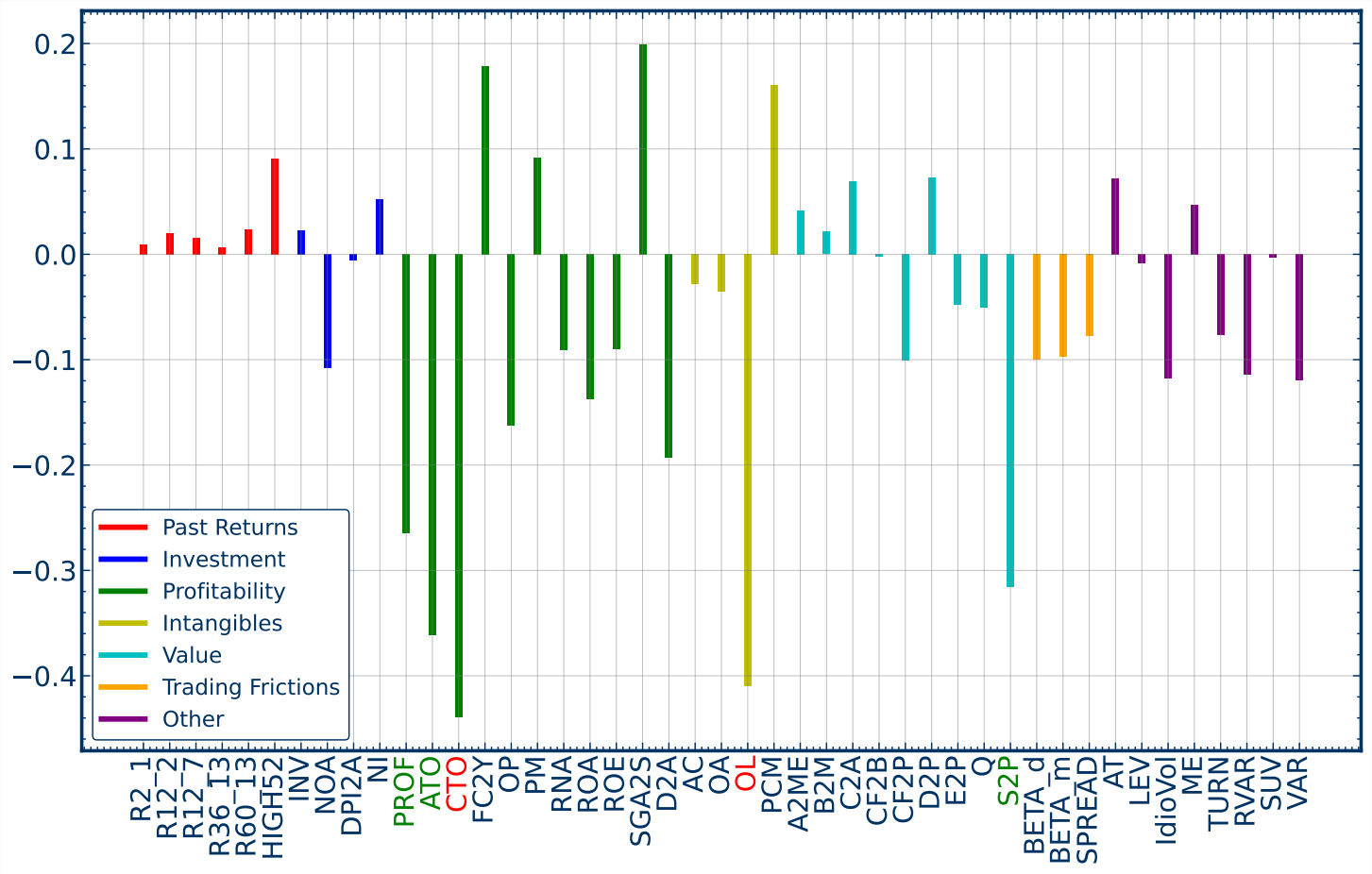


Composition of second latent factors by characteristic categories

- The loadings are colored by characteristic category

$\Rightarrow$ 2nd factor = value factor

Composition of Latent Factor 3

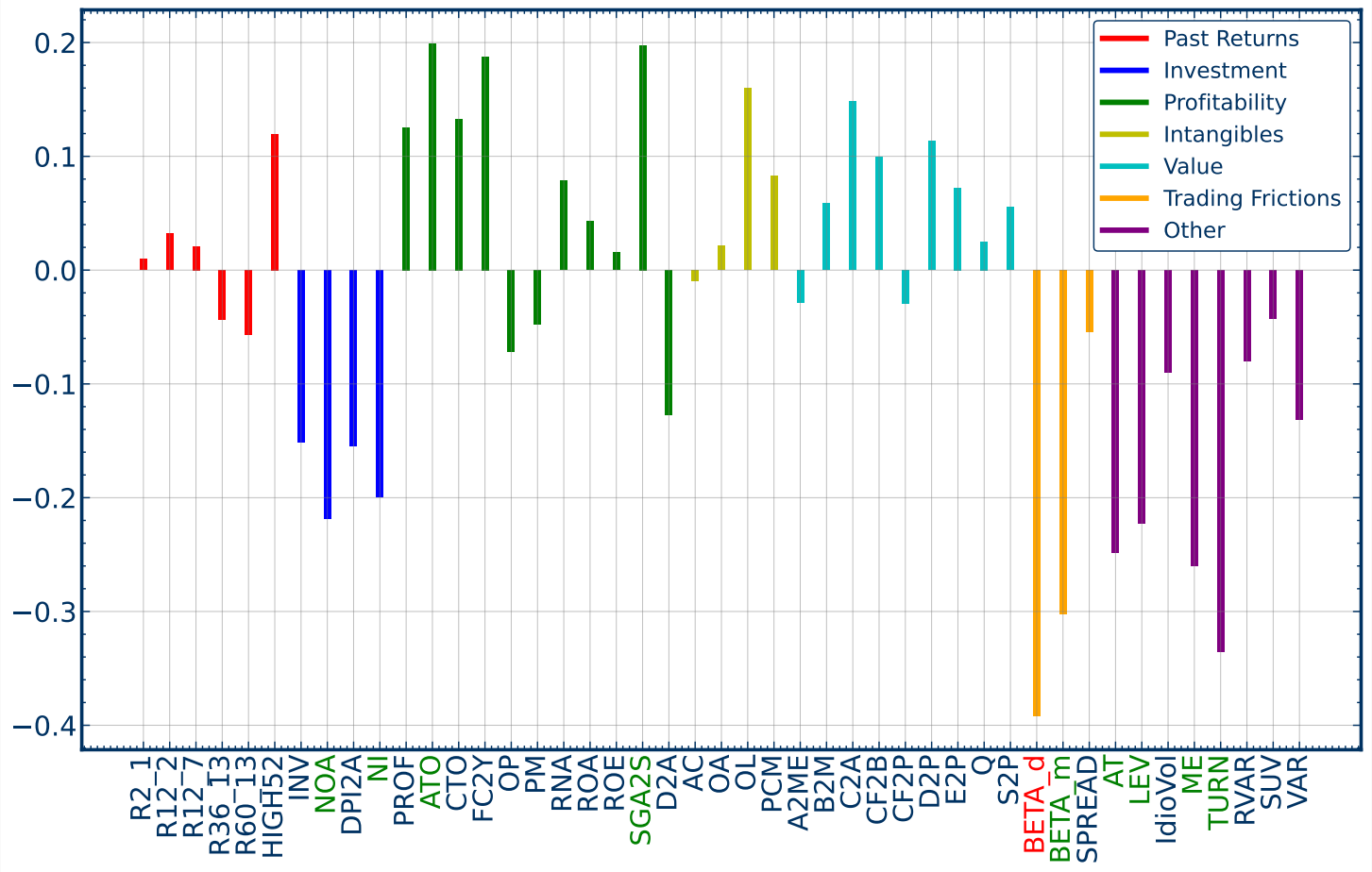


Composition of third latent factors by characteristic categories

- The loadings are colored by characteristic category

$\Rightarrow$ 3rd factor = profitability factor

Composition of Latent Factor 4

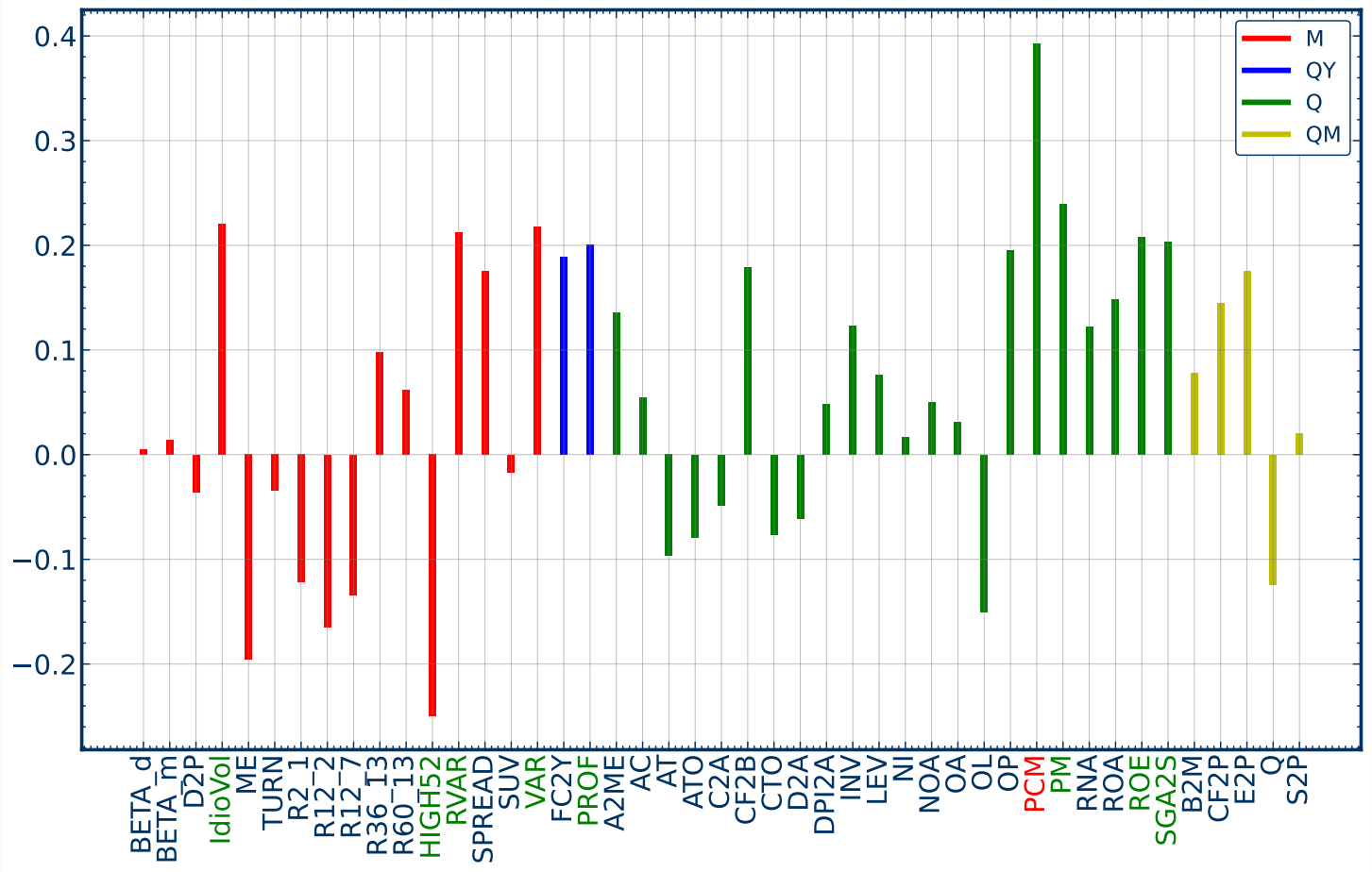


Composition of fourth latent factors by characteristic categories

- The loadings are colored by characteristic category

$\Rightarrow$ 4th factor = trading friction factor

Composition of Latent Factor 5

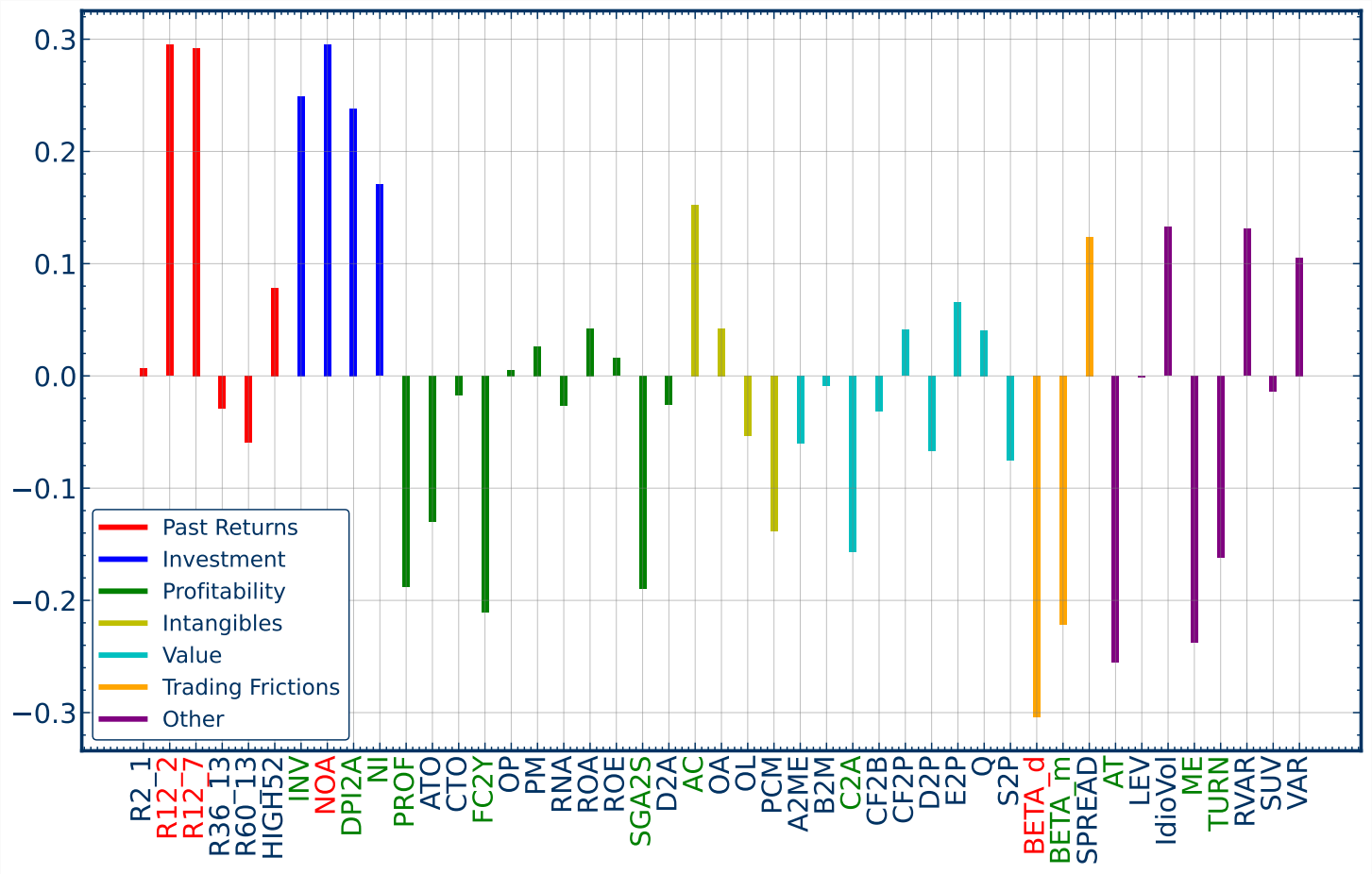


Composition of fifth latent factors grouped by frequency

- The loadings are colored by characteristic category

$\Rightarrow$ 5th factor = persistent characteristics factor

Composition of Latent Factor 6

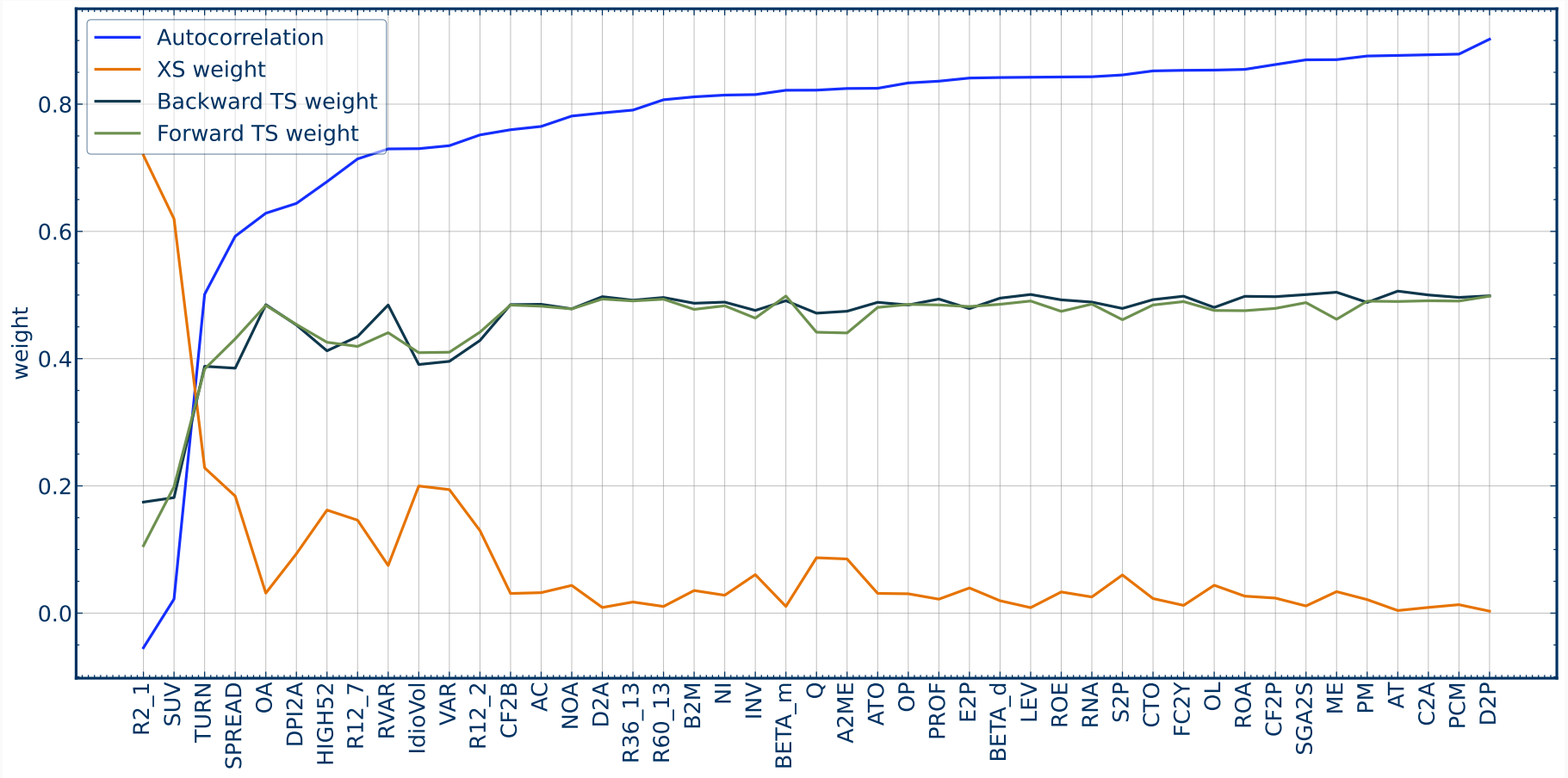


Composition of sixth latent factors by characteristic categories

- The loadings are colored by characteristic category

$\Rightarrow$ 6th factor = long past returns and investment

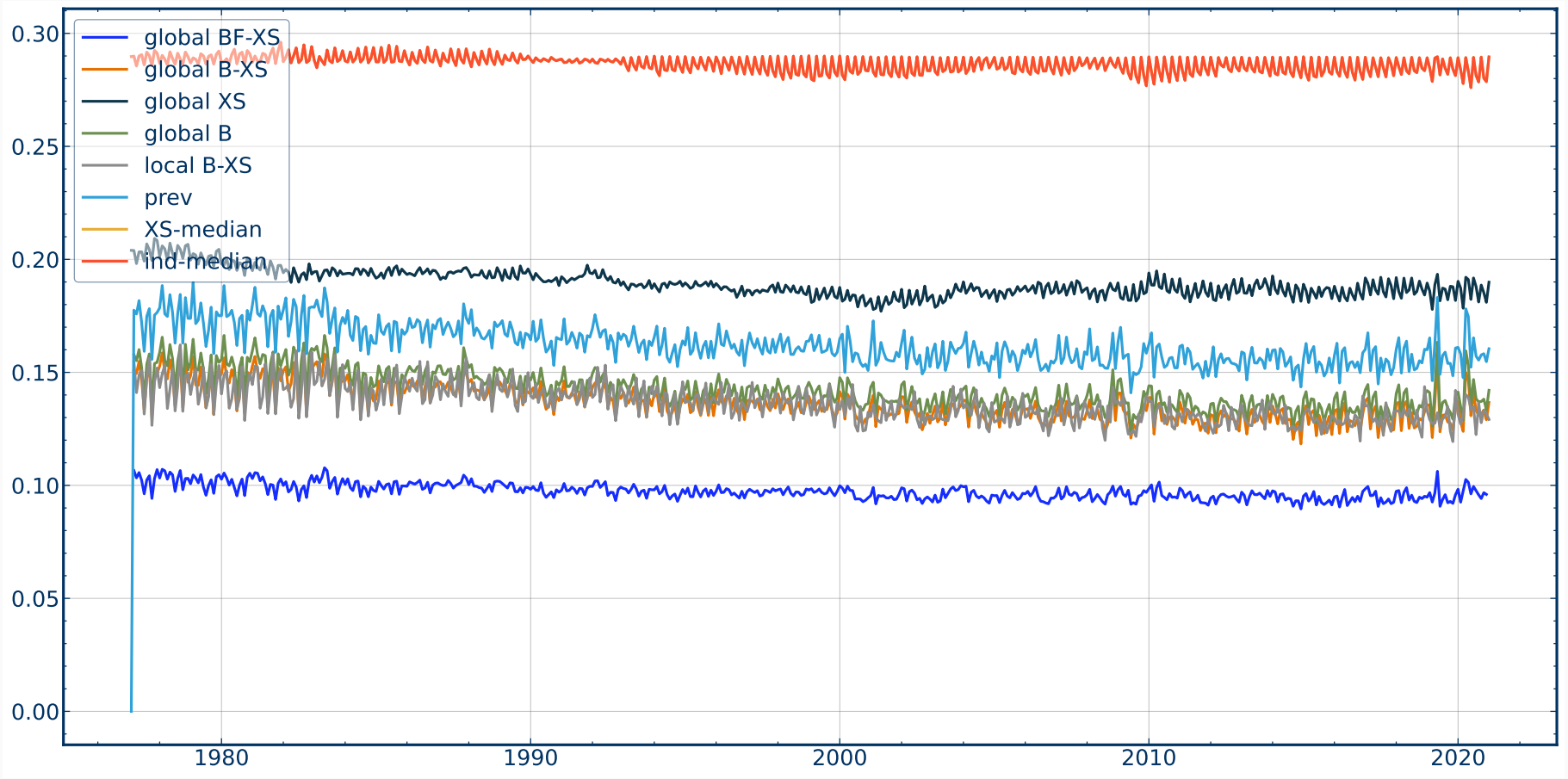
Information used for imputation



Relative importance of the TS and XS components (L1 norm) in BWFW-XS model

- Characteristics are sorted in ascending order based on their persistence
- Imputation for persistent characteristics benefits from the TS data
- Imputation for more volatile characteristics relies more on XS information

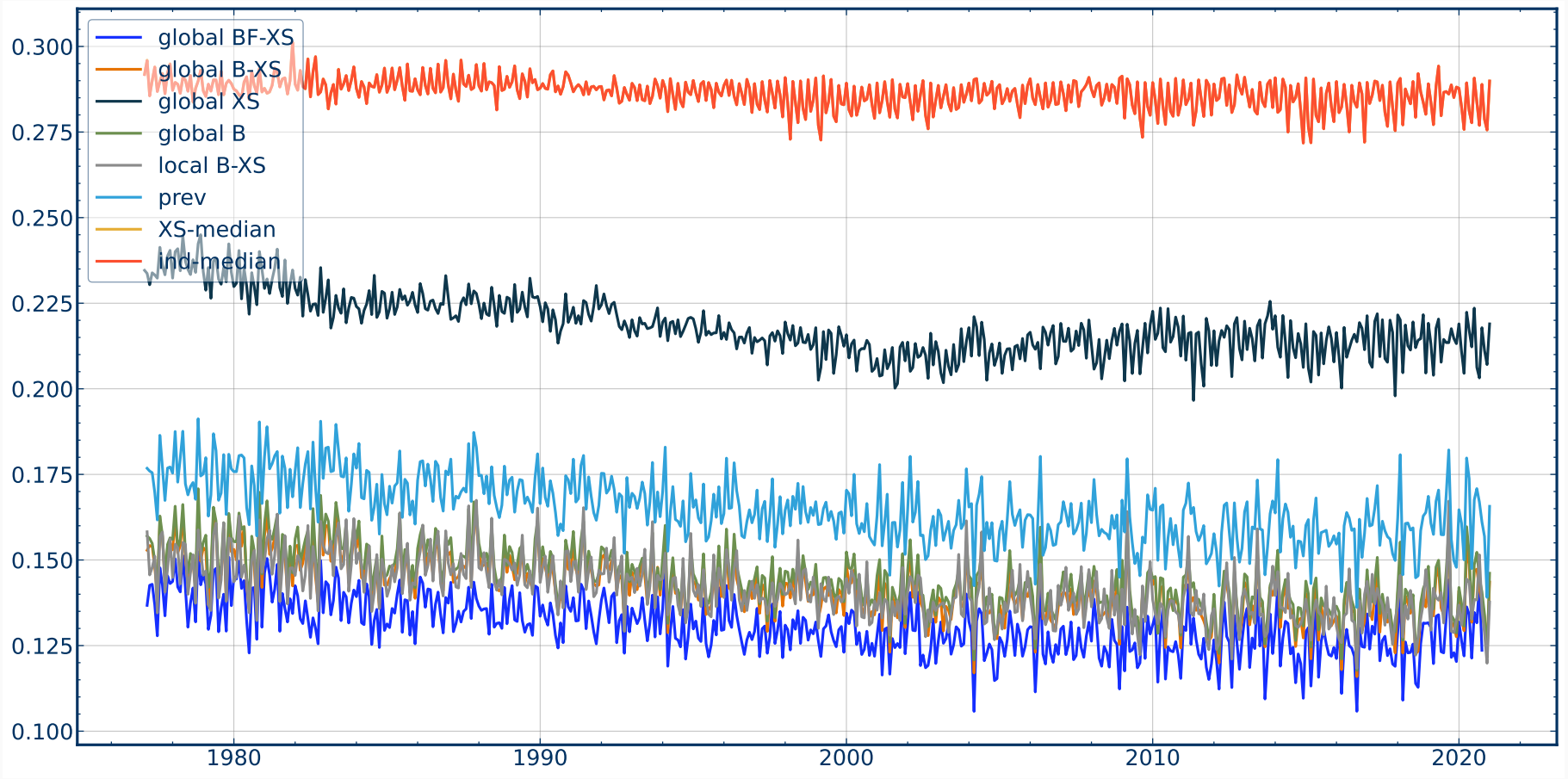
Comparison of imputation methods



In-sample RMSE by imputation method across individual characteristics

- Characteristics are sorted in ascending order based on their volatility (black line)
- Imputation for persistent characteristics benefits from the TS data
- Imputation for more volatile characteristics relies more on XS information

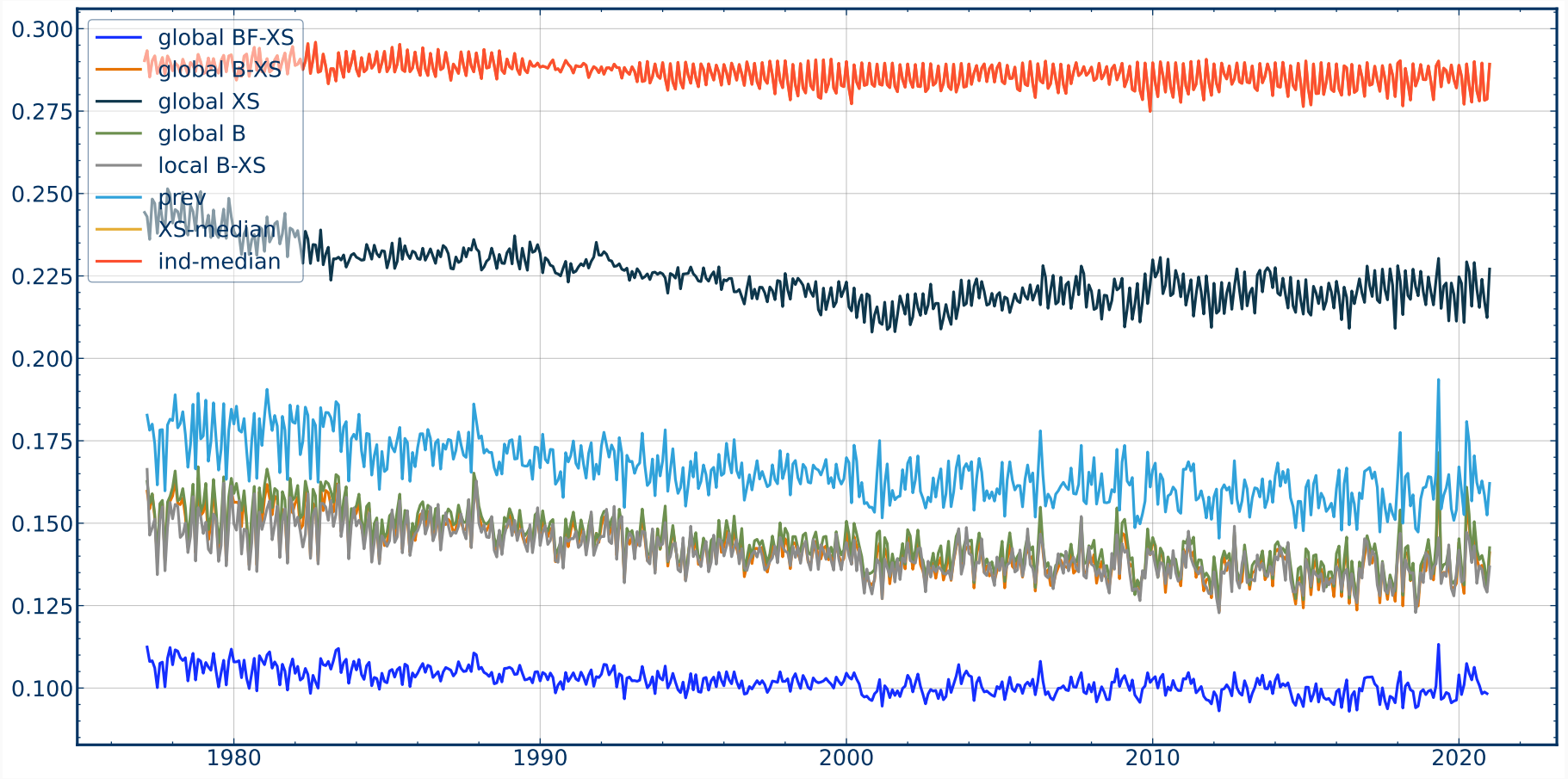
Comparison of imputation methods



Out-of-sample RMSE by imputation method across individual MAR masked characteristics

- Characteristics are sorted in ascending order based on their volatility (black line)
- Imputation for persistent characteristics benefits from the TS data
- Imputation for more volatile characteristics relies more on XS information

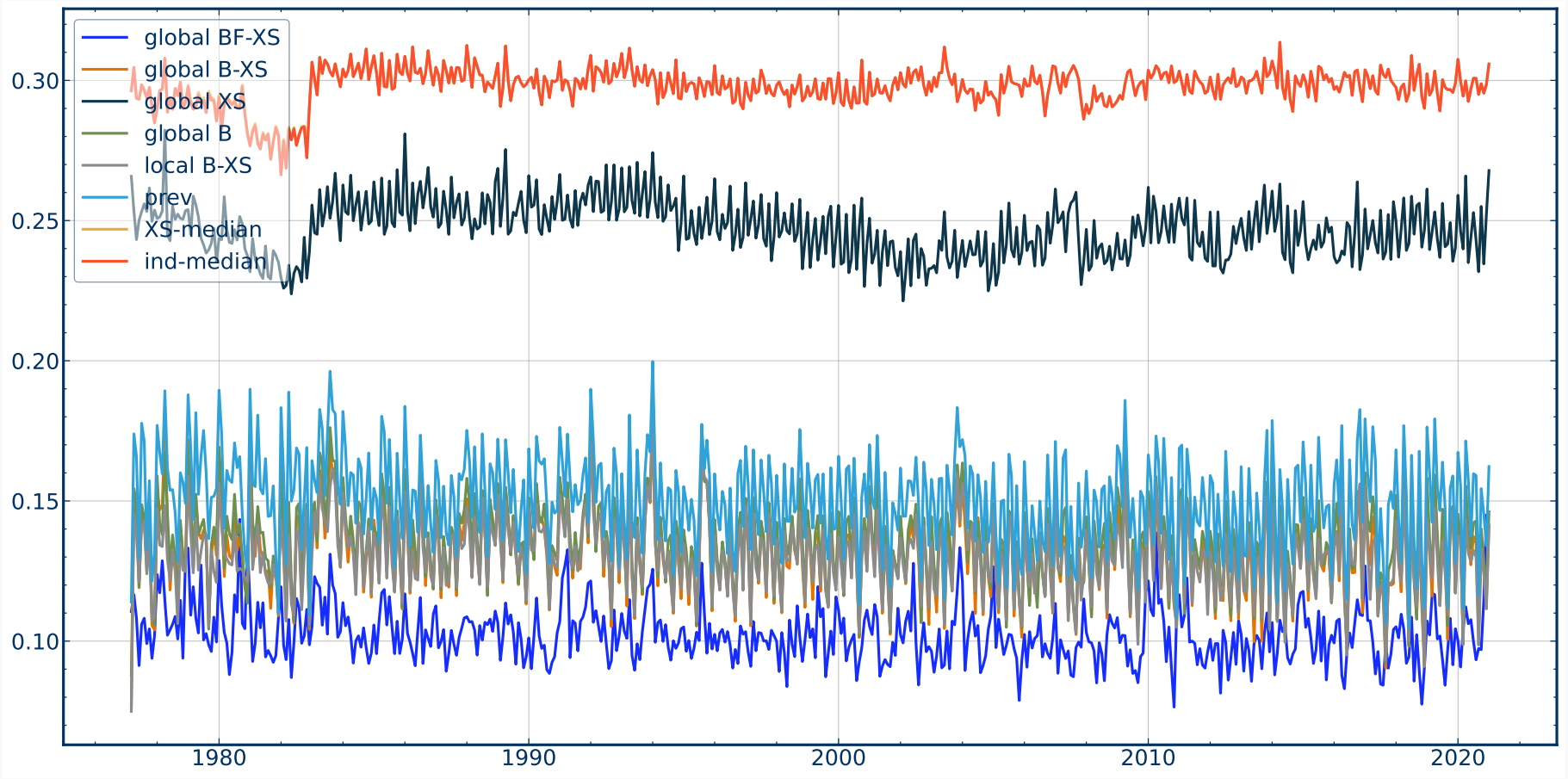
Comparison of imputation methods



Out-of-sample RMSE by imputation method across individual block-masked characteristics

- Characteristics are sorted in ascending order based on their volatility (black line)
- Imputation for persistent characteristics benefits from the TS data
- Imputation for more volatile characteristics relies more on XS information

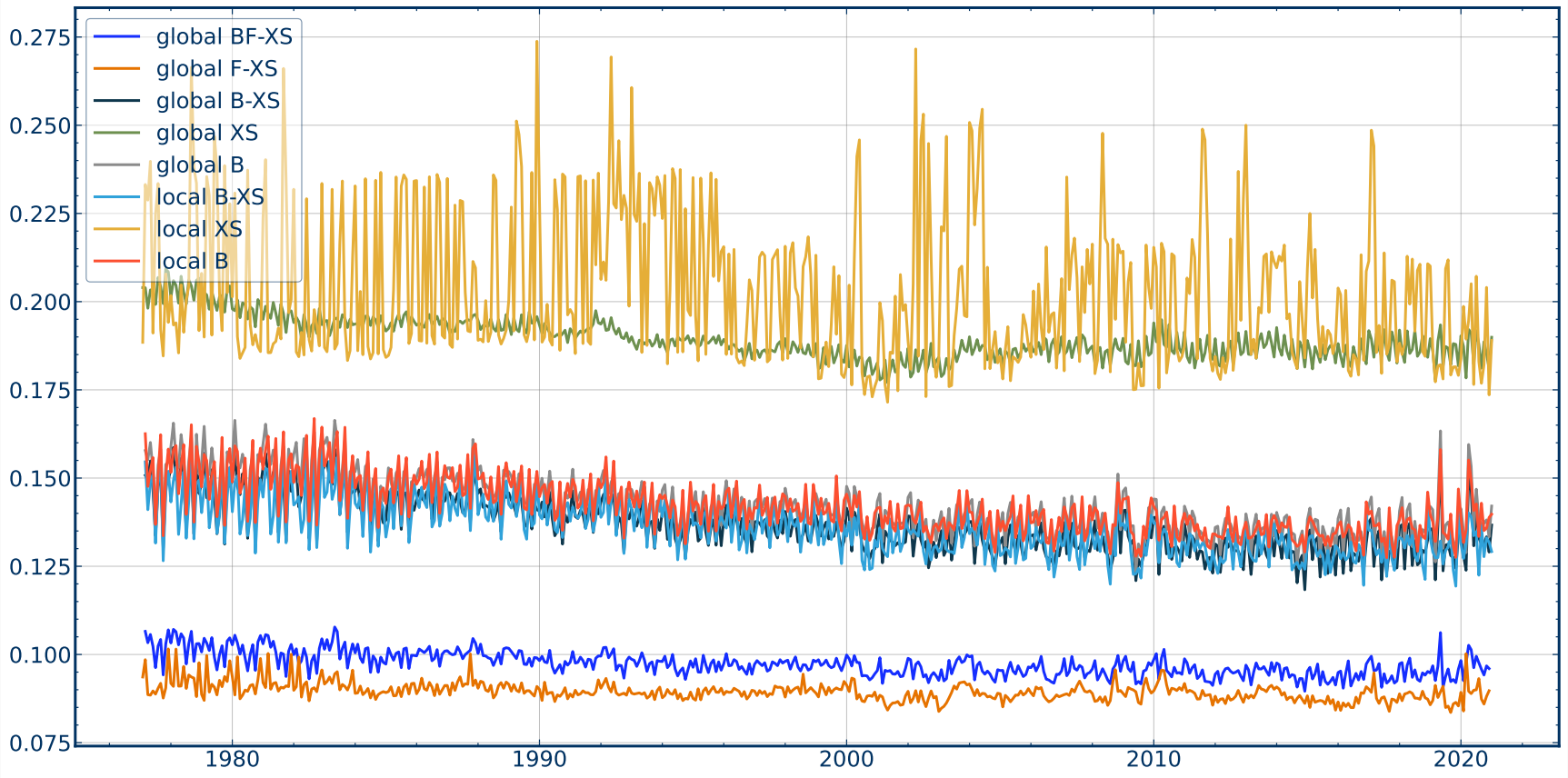
Comparison of imputation methods



Out-of-sample RMSE by imputation method across individual logit-masked characteristics

- Characteristics are sorted in ascending order based on their volatility (black line)
- Imputation for persistent characteristics benefits from the TS data
- Imputation for more volatile characteristics relies more on XS information

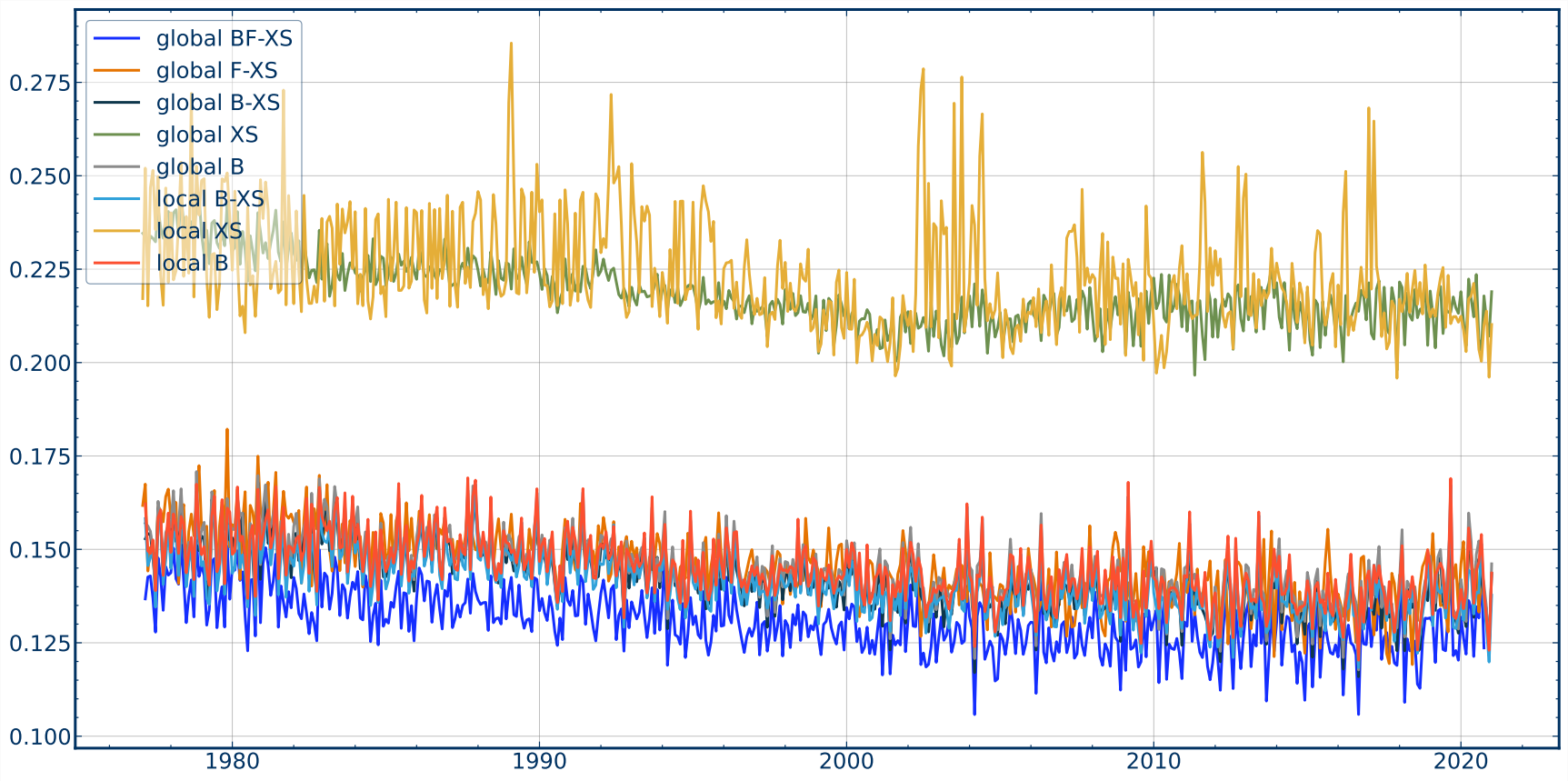
Local vs global imputation



In-sample RMSE by imputation method across individual characteristics

- Global models are slightly better
- Highly volatile characteristics benefit more from local models

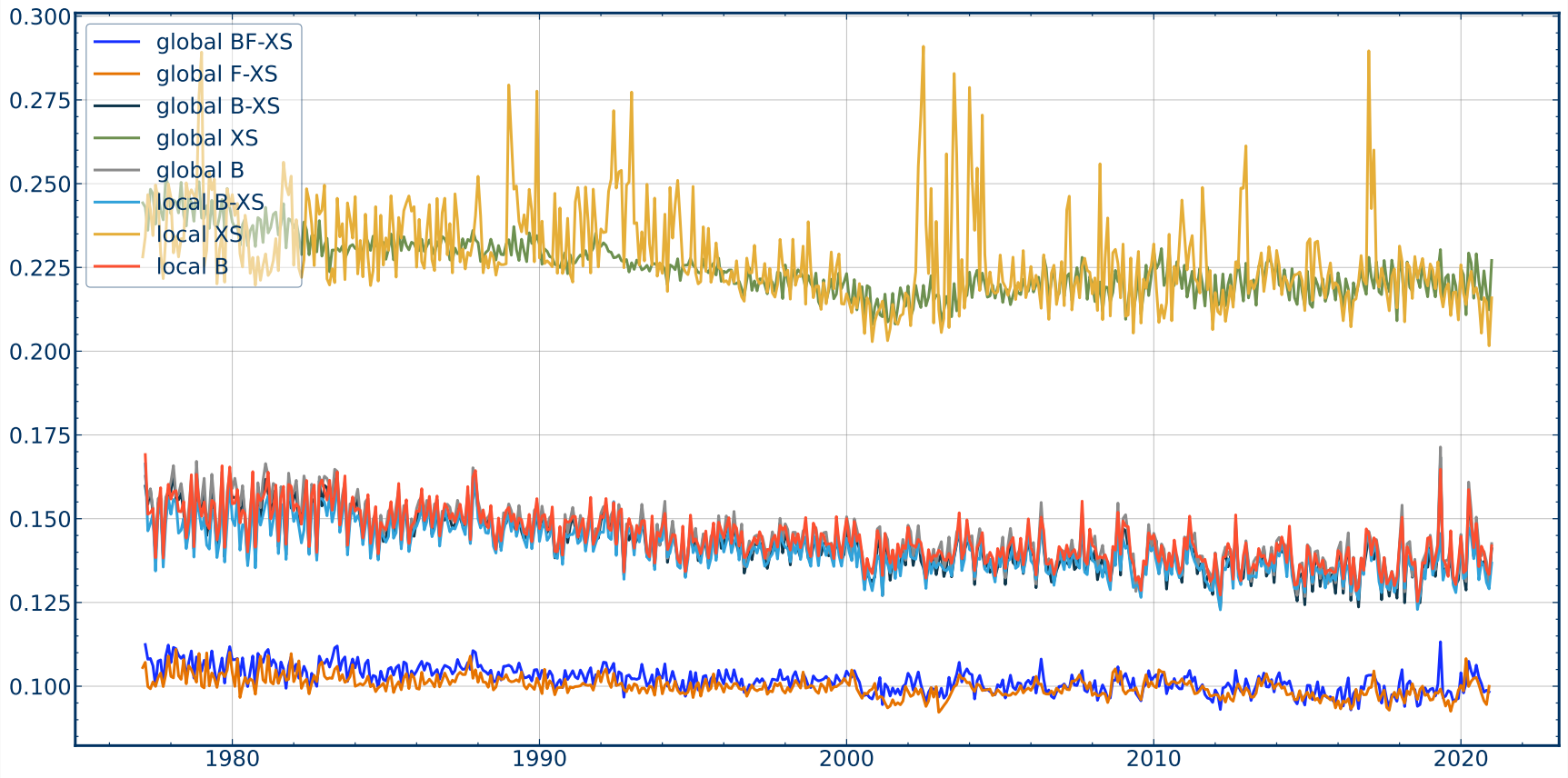
Local vs global imputation



Out-of-sample RMSE by imputation method across individual MAR characteristics

- Global models are slightly better
- Highly volatile characteristics benefit more from local models

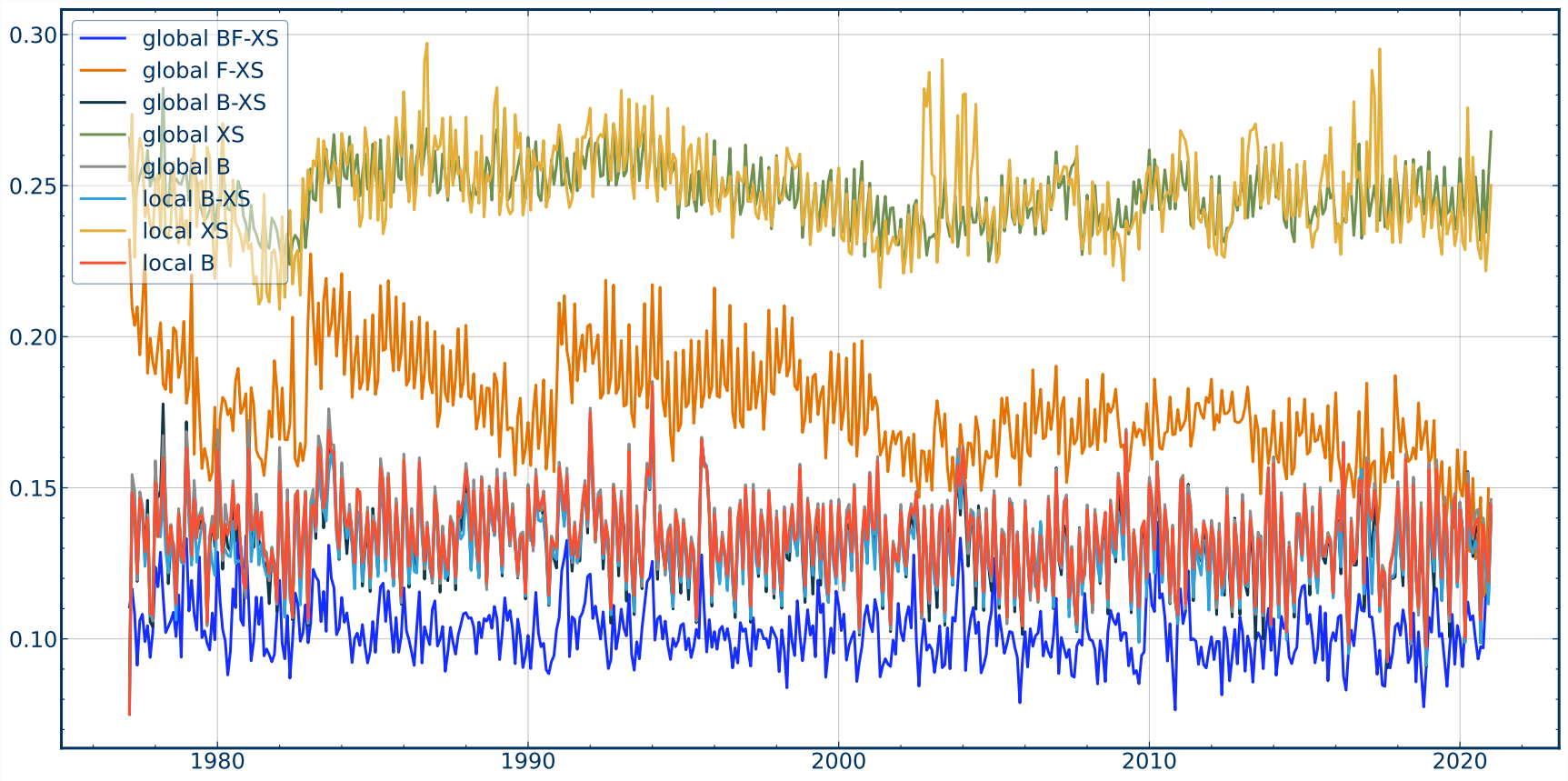
Local vs global imputation



Out-of-sample RMSE by imputation method across individual block-masked characteristics

- Global models are slightly better
- Highly volatile characteristics benefit more from local models

Local vs global imputation



Out-of-sample RMSE by imputation method across individual logit-masked characteristics

- Global models are slightly better
- Highly volatile characteristics benefit more from local models