Understanding Migration Responses to Local Shocks*

Kirill Borusyak†, Rafael Dix-Carneiro‡, and Brian K. Kovak§

November 14, 2022

Abstract

We examine how to interpret estimates from a commonly used migration regression relating changes in local population to exogenous local labor demand shocks. Using a simple model of local labor markets with mobility costs, we find that common conclusions drawn from migration regression estimates are likely to be substantially misleading. Intuitively, the conventional migration regression is misspecified due to the bilateral nature of location choices. Workers choose where to live based not only on the shock to their current location, but also on the shocks to potential alternative locations, which are omitted from the regression. Analytical results and simulations based on Brazilian data show that conventional migration regression estimates are inaccurate for the local population effects of either shocks to individual locations or all observed shocks taken together. These problems are particularly acute when workers face industry switching costs in addition to geographic mobility costs. A simple alternative approach leveraging the model's structure exhibits far better performance.

*We thank David Autor, Dave Choi, Gordon Hanson, Felix Koenig, Camille Landais, and participants at numerous seminars and conferences for helpful comments. Borusyak acknowledges financial support from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme (grant agreement Number 949995). All errors are our own.

†Department of Economics, University College London (k.borusyak@ucl.ac.uk)

‡Department of Economics, Duke University (rafael.dix.carneiro@duke.edu)

§Heinz College, Carnegie Mellon University (bkovak@cmu.edu)

1 Introduction

Workers' interregional migration responses are critical in determining the impacts of local economic shocks. The welfare effects of a given shock depend upon whether workers can smooth adverse labor market outcomes by migrating (Yagan, 2014). The speed of migration adjustment affects how long the impacts of local shocks persist (Topel, 1986; Blanchard and Katz, 1992), and a lack of migration response may help explain long-lasting economic effects of changes in local labor demand (Dix-Carneiro and Kovak, 2017, 2019; Autor et al., 2021). Mobility responses determine how local shocks influence interregional inequality (Topalova, 2010; Cadena and Kovak, 2016), and differences in mobility responses across demographic groups can drive between-group inequality as well (Bound and Holzer, 2000; Wozniak, 2010; Dix-Carneiro and Kovak, 2015). If migration frictions are particularly large, policy makers may consider programs to help workers relocate.[1]

A voluminous literature spanning subfields of economics studies migration responses to local shocks using what we refer to as the "conventional migration regression." This regression relates local population growth to observed labor demand shocks, as in the following regression specification:

$$\hat{L}_\ell = \alpha + \beta \hat{z}_\ell + \varepsilon_\ell, \tag{1}$$

where $\hat{L}_\ell$ is the proportional change in population in location ℓ, $\hat{z}_\ell$ is an observed local labor demand shock, and ε_ℓ is the error term. In many cases, researchers find substantial effects of the shocks on local economic outcomes such as wages and employment, but find estimates of β that are small and/or statistically indistinguishable from zero.[2] These results seem to imply a puzzle: despite substantial effects of employment shocks, workers do not appear to adjust by relocating.[3] Prior work has responded to these findings by inferring that migration costs are high, that workers are

[1] For example, the U.S. Trade Adjustment Assistance program includes funding for distant job search activities and an allowance for relocation costs (Hyman, 2018).

[2] Examples of papers finding substantial effects of local labor demand shocks on local economic outcomes but with estimates of β that are small and/or statistically indistinguishable from zero include Bound and Holzer (2000) (for those with a high-school degree or less); Topalova (2010); Autor et al. (2013); Dauth et al. (2014); Mian and Sufi (2014); Cadena and Kovak (2016); Dix-Carneiro and Kovak (2017); Dix-Carneiro and Kovak (2019); Yagan (2019); Autor et al. (2021) (except for those age 25-39); Choi et al. (2021); and Faber et al. (2021) (for Chinese import competition), among others. Additional papers estimating a version of (1) and finding substantial effects in at least some specifications include Black et al. (2005); Wozniak (2010); McCaig (2011); Bustos et al. (2016); Hakobyan and McLaren (2016); Bartik et al. (2019); Foote et al. (2019); Greenland et al. (2019); Albert and Monràs (2020); Boustan et al. (2020); Monràs (2020); Notowidigdo (2020); and Albert et al. (2021), among others.

[3] For example, Choi et al. (2021) echo a common sentiment in this literature: "This finding deepens the puzzle raised in recent papers that find no or limited migration response to large, negative local employment shocks."

generally unresponsive to changing local economic conditions, or that interregional migration can be ignored in quantitative models.

In this paper, we seek to resolve this puzzle by showing that most common interpretations of β in (1) can yield misleading conclusions. In particular, the estimate of β can be close to zero even when the observed shocks led to substantial spatial reallocation and workers are highly responsive to local economic conditions. These problems arise even when the observed shock is "exogenous," in the sense of being as good as random with respect to unobserved local labor demand and supply shocks.[4]

The core intuition emerging from our analysis is that the conventional migration regression is misspecified due to the bilateral nature of location choices. When workers consider economic conditions in deciding whether and where to move, their decisions depend upon both the shock to their current location and the shocks facing potential alternative locations. Since workers face different migration costs across origin-destination pairs, the most important potential alternatives differ across locations. By omitting the shocks facing these relevant alternative locations, the conventional migration regression in (1) is misspecified.

This misspecification leads to two problems. First, when the omitted shocks to relevant alternative locations are correlated with the shock to the current location, the conventional migration regression suffers from omitted-variable bias. For example, even if workers are very responsive to differences in labor demand conditions, there will be little incentive to migrate when workers' current and potential alternative locations face very similar labor demand shocks. In this case, the conventional migration regression estimate β will be close to zero even when the true migration elasticity with respect to local economic conditions is high. Second, the predicted values from the conventional regression fail to capture the true effects of the observed shocks on local populations because it omits variation in the outside options faced by potential migrants. Therefore, the conventional regression yields misleading conclusions about both worker mobility and the effects of observed shocks on local populations.

We analyze these interpretation problems in the context of a static model of local labor markets in which workers face mobility costs and have idiosyncratic preferences for living in different locations. The model is deliberately simple, focusing on costly migration while omitting features

[4]Equation (19) defines "as good as random" in this context.

such as housing markets or forward-looking behavior. The simplicity of the model allows us to derive an intuitive approximation for the effect of a given vector of local labor demand shocks on local population growth, capturing the intuition just described. The shocks' effect on a location's population depends on the direct shock to the location minus the migration-weighted average shock to other locations, which is a sufficient statistic for relevant spillovers.

We use this model to derive a novel decomposition for β in the conventional migration regression. We show that β increases in i) the ratio of migration and labor demand elasticities, ii) the baseline share of migrants in the national population, and iii) an "attenuation factor" that is below one when shocks are particularly positively correlated between regions with large migration flows. This decomposition allows us to contrast various common interpretations of β against true model-based responses to observed or counterfactual shocks. Examples include interpreting β as the effect of a unit shock to a single location or interpreting $\beta(\hat{z}_k - \hat{z}_\ell)$ as the difference in the effects of shocks on locations k and ℓ. We show that these and similar interpretations provide poor predictions for the true effects on local populations, particularly when the shocks used to estimate β are correlated across migrant-connected locations and thus attenuation is severe.[5]

In contrast, we find that these issues in population regressions do not imply similar problems when studying the effects of local shocks on local *wage* changes. When the conventional regression estimate of β is close to zero, wage regression estimates will experience minimal confounding from migration, irrespective of the mechanism driving the small estimate of β. Intuitively, because labor demand shocks have a direct effect on local wages and an indirect effect through migration spillovers, when $\beta \approx 0$ this indirect effect is approximately uncorrelated with the direct effect and therefore does not substantially bias the estimate of the local shock's effect on the local wage. So, even when population regressions yield misleading interpretations, associated wage analyses may still yield valid conclusions.

An extension of the model shows that the interpretation problems in the conventional migration regression are exacerbated when workers face mobility costs across both regions and industries. Many papers estimating (1) analyze local labor demand shocks with a shift-share structure in which typically $\hat{z}_\ell = \sum_n \frac{L^0_{\ell n}}{L^0_\ell} \hat{x}_n$, where $\hat{x}_n$ is a national labor demand shock facing industry n, and

[5] We show that β is correctly interpreted as the *difference* in the *average* effect of *all* shocks across locations facing higher vs. lower *direct* shocks. While correct, this interpretation is quite limited. Even when β is small, the shocks may have led to substantial spatial reallocation because of variation in shocks facing migrant-connected locations.

$L^0_{\ell n}/L^0_\ell$ is industry n's initial share of employment in location ℓ.[6] If industry switching costs are large, workers see diminished benefit to moving across locations because they primarily face the shock to their industry no matter where they choose to live. When labor demand shocks have an industry component, the presence of industry switching frictions thus reduces migration beyond what one would observe in a setting with regional frictions alone, further attenuating β. Moreover, with industry frictions, workers in different industries face different shocks and have different outside options, leading to complex spillovers across industries and regions, all of which is omitted by the conventional migration regression. As a result, our quantitative analysis finds that the conventional regression misses about half of the true population reallocation in response to national industry-level labor demand shocks. and generating intricate shock spillovers across locations and industries.

Given the failure of the conventional migration regression to provide accurate predictions of the effects of observed or counterfactual local labor demand shocks, we propose an alternative approach leveraging the model-based expression that relates the vector of local shocks to population changes in each location. Given observed labor demand shocks and data on the pre-shock migration flows between locations (or industry-location pairs in the full model), one can use nonlinear least squares (NLLS) to estimate the relevant parameter and predict the effects of observed or counterfactual labor demand shocks. As an alternative, we also show how, under an approximation to the model, one can simply control for a migration-weighted average of shocks to other locations (or location-industries) in the conventional migration regression. While these procedures are specific to our model, the same approach can be applied to extended models that include additional economic forces.

To assess the quantitative importance of our insights, we study the effects of simulated shocks in the context of real-world economic geography and migration patterns. We base our simulations on longitudinal administrative data covering formally employed workers in Brazil, which allow us to observe worker transitions across locations and industries. Due to data limitations, we use formal employment as a proxy for local population in this exercise. We simulate labor demand shocks following a variety of data-generating processes and generate model-implied local population

[6] Of the papers estimating conventional migration regressions listed in footnote 2, the following use shift-share labor demand shock measures: Topalova (2010); Wozniak (2010); McCaig (2011); Autor et al. (2013); Dauth et al. (2014); Cadena and Kovak (2016); Hakobyan and McLaren (2016); Dix-Carneiro and Kovak (2017); Dix-Carneiro and Kovak (2019); Greenland et al. (2019); Albert and Monràs (2020); Boustan et al. (2020); Notowidigdo (2020); Autor et al. (2021); Choi et al. (2021); and Faber et al. (2021).

changes when workers face migration frictions alone or both migration and industry frictions. The resulting population changes are then used to estimate the conventional migration regression, the results of which we compare to the true model-based effects of the simulated labor demand shocks.

The results of this simulation exercise confirm the practical quantitative importance of the concerns raised by the theoretical analysis. The regression-based estimates poorly predict the effects of the simulated shocks themselves, and estimates of the effects of shocks to individual locations are even less accurate. These problems are largest when analyzing industry-level shocks via shift-share regressions in the presence of location and industry frictions. When implementing simulations separately by skill level and assigning both groups the same migration elasticities, less-educated workers have substantially smaller estimates of β than more-educated workers when facing industry-level shocks. Differences in migration regression estimates across groups may therefore be uninformative about differences in true underlying responsiveness to economic conditions.

To assess the performance of our model-consistent NLLS procedure we introduce random variation from unobservable factors into the simulation procedure and vary the magnitude of this residual variation.[7] For all magnitudes of the unobserved component, the predictive performance of this NLLS procedure far outperforms that of the conventional migration regression, which often performs as poorly as an uninformative prediction of zero migration response. Our findings further highlight the importance of accounting for observed connections between industries in addition to locations and the value of leveraging data on employment changes at the location-industry level when studying the effects of industry shocks.

Our results inform the many literatures estimating the conventional migration regression in (1), which span development, international, labor, resource, and urban economics.[8] We emphasize interpretation challenges resulting from cross-location spillovers that violate the stable unit treatment value assumption (SUTVA). A small number of papers in this literature anticipated this spillover issue and introduced *ad-hoc* controls in an effort to address it. While studying the effects of Chinese import competition on U.S. local population changes, Greenland et al. (2019) control for the weighted average of shocks to other locations, with weights proportional to the squared inverse distance. In studying the effects of droughts on regional labor and capital reallocation in Brazil,

[7] We do not implement the NLLS procedure in the simulations without additional residual variation because it would fit perfectly by construction.

[8] See footnote 2 for references.

Albert et al. (2021) include a migration-weighted average of drought measures in other locations.[9] Our analysis refines and provides a theoretical rationalization for these approaches to dealing with cross-location spillovers.

Our findings have further implications for models of local labor markets. Since conventional migration regression estimates are not informative about underlying mobility parameters, the estimates should not be used to guide modeling choices. Specifically, estimates of β that are close to zero may simply reflect correlated shocks rather than a lack of mobility, so they do not justify omitting migration from models of local labor markets (e.g. Adão, 2016; Galle et al., forthcoming). Moreover, our approach elucidates how spatial migration networks lead to differences in the outside options facing workers in different locations. Studies that do not estimate the conventional regression but instead employ model-consistent specifications arising from models that assume away spillovers through migration networks likely face issues similar to those we examine in the context of conventional migration regressions (e.g. Kleven et al., 2013; Agrawal and Foremny, 2019; Notowidigdo, 2020).

Our simple model of costly migration follows the tradition of static gravity models of location choice used to analyze both migration and commuting (e.g. Ahlfeldt et al. (2015); Morten and Oliveira (2018); Monte et al. (2018); Amior and Manning (2019)). In these models, location choice probabilities take a logistic form across destinations, and bilateral costs of moving or commuting generate a network structure; most closely related in the migration context are the formulations by Tombe and Zhu (2019) and Fan (2019). On the labor demand side, the standard multi-sector Armington (1969) model that we employ has been used by Anderson and van Wincoop (2003), Head and Mayer (2014), and Adão et al. (2019), among many others.

Methodologically, our paper is related to Adão et al. (2021), who study how trade linkages generate interregional spillovers in the employment and wage effects of increased Chinese imports in the U.S. Like them, we provide a model-consistent reduced-form equation for the effects of a local labor demand shock in presence of spatial spillovers; our analysis focuses on spillovers from migration

[9] Spillovers via migration have also been considered contexts other than local population growth. When studying effects of trade liberalization on the non-agriculture share of local employment in China, Tian et al. (2021) include both the direct tariff shock facing a location and a migration-weighted average of tariff changes facing other locations. Imbert et al. (2021) control for an inverse-distance-weighted average of shocks to other locations in their study of how in-migration affects firms in China. Arthi et al. (2019) consider migration spillovers when studying the effects of the 19^{th} century cotton famine on local mortality rates in the UK.

while theirs focuses on interregional trade.[10] Yet, the papers' objectives are quite distinct, with Adão et al. (2021) seeking to estimate aggregate general equilibrium effects of a spatially heterogeneous shock to labor demand and our analysis informing a commonly used reduced-form approach that generally ignores potential spillovers.

The paper proceeds as follows. Section 2 describes the baseline model of local labor markets in which workers face costs of moving across locations, solves the model, and relates model-based expressions to observable quantities. Section 3 defines and characterizes the conventional migration regression coefficient and discusses problems when using the associated estimates to make within-sample or counterfactual predictions. It also describes the model-consistent estimation procedures. Section 4 presents the full model including frictions in switching both locations and industries. Section 5 presents the data and descriptive statistics on worker transitions across locations and industries. Section 6 presents the simulation-based quantitative results, and Section 7 concludes.

2 Simple Model of Mobility with Regional Frictions

2.1 Setup

This section presents a stylized model that yields clear and intuitive expressions for how local labor demand shocks affect local populations in the presence of costly mobility. Although this framework omits a number of potentially empirically relevant mechanisms, including housing, capital markets, and agglomeration economies, it highlights general issues with standard regression-based approaches to understanding how changes in local populations respond to local shocks. Here, we introduce a baseline version of the model in which workers face mobility costs across locations but are freely mobile across industries. Section 4 studies the full model with frictions in both dimensions.

Consider a small open economy consisting of multiple sectors indexed by $n \in \mathcal{N}$ produced in locations indexed by $\ell \in \mathscr{L}$. Products within a sector are differentiated by location of production, as in Armington (1969), Anderson and van Wincoop (2003), Head and Mayer (2014), and Adão et al. (2019), among many others. Product markets are frictionless, so the price of a sector-n good produced in location ℓ, $p_{\ell n}$, is constant across destinations. Homogeneous labor is the only input,

[10] The extended model of Adão et al. (2021) features migration, but because it lacks migration costs it does not allow for migration network linkages that vary by location.

and sector-location productivity is $\varphi_{\ell n}$. Output and labor markets are perfectly competitive, so $p_{\ell n} = w_\ell / \varphi_{\ell n}$, where w_ℓ is the wage in location ℓ.

Individuals make optimal choices over consumption bundles and their location of employment and face frictions in moving across locations. We assume that workers may migrate within the country of interest by choosing among locations $\ell \in \mathscr{L}$, but not internationally. Unless explicitly specified otherwise, sums over locations cover only places in the country of interest. Each of L workers inelastically supplies one unit of labor in their chosen location, so changes in employment are equivalent to changes in population. Utility from consumption is Cobb-Douglas across sectors and CES across location-specific varieties within sector:

$$U = \sum_n \eta_n \ln(Q_n), \quad \text{where } Q_n \equiv \left(\sum_{\ell \in \mathscr{L}_w} q_{\ell n}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \tag{2}$$

and $\sum_n \eta_n = 1$. Note that Q_n aggregates over location-specific varieties produced around the world ($\ell \in \mathscr{L}_w$), including those within the country of interest ($\mathscr{L} \subset \mathscr{L}_w$). Normalizing world aggregate expenditure to one, the aggregate optimal demand bundle satisfies

$$q_{\ell n} = p_{\ell n}^{-\sigma} P_n^{\sigma-1} \eta_n, \tag{3}$$

where $P_n \equiv \left(\sum_{\ell \in \mathscr{L}_w} p_{\ell n}^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$ is the CES exact price index for sector n, including varieties produced globally. The small-country assumption implies that P_n is set on the world market and is exogenous to developments in the country of interest.

The initial distribution of individuals across locations is taken as given, with L_o^0 workers in location o, and the indirect utility of choosing destination location ℓ for individual i initially living in o is[11]

$$V_{io\ell} = \ln\left(\frac{w_\ell}{P}\right) - \ln \tau_{o\ell} + \frac{1}{\theta} \epsilon_{i\ell}, \tag{4}$$

where $P \equiv (\Pi_n P_n^{\eta_n}) / (\Pi_n \eta_n^{\eta_n})$ is the Cobb-Douglas exact price index across sectors, $\ln \tau_{o\ell}$ is the utility cost of moving from o to ℓ, and $\epsilon_{i\ell}$ is a taste shock following a type-I extreme-value (standard Gumbel) distribution that is i.i.d. across individuals and locations. The parameter θ determines

[11] Exogenous local amenities can be incorporated without changing the analysis. Since we do not use data on wages, w_d can be reinterpreted as the amenity-adjusted wage.

the strength of location preferences relative to the log real wage, with smaller values of θ implying stronger location preference and larger frictions across locations.

2.2 Regional Labor Demand

Given perfectly competitive markets, $p_{\ell n} = w_\ell/\varphi_{\ell n}$, and revenue to sector-$n$ producers in location ℓ is

$$R_{\ell n} = \eta_n \left(\frac{w_\ell/\varphi_{\ell n}}{P_n}\right)^{1-\sigma}. \tag{5}$$

Perfect competition and regional trade balance imply that the regional wagebill equals total regional revenue across sectors:

$$w_\ell L_\ell = \sum_n R_{\ell n}. \tag{6}$$

Plugging in (5) and rearranging yields regional labor demand:

$$L_\ell = D_\ell w_\ell^{-\sigma}, \quad \text{where } D_\ell \equiv \sum_n \eta_n \left(\varphi_{\ell n} P_n\right)^{\sigma-1}. \tag{7}$$

The term D_ℓ is a regional demand shifter affected by changes in productivity $\varphi_{\ell n}$ or industry price indexes P_n. Equation (7) shows that σ is the labor demand elasticity, which is larger when product varieties across locations are more substitutable.

We consider an economic shock that affects this demand shifter across locations in the country of interest.[12] Let hats represent proportional changes relative to a no-shock counterfactual, and assume small changes so $\hat{x} \equiv dx/x = d\ln x$. The total derivative of (7) is then

$$\hat{L}_\ell = \hat{D}_\ell - \sigma \hat{w}_\ell. \tag{8}$$

This expression describes how the quantity of labor demanded in location ℓ responds to local wage

[12]For example, a change to the vector of sectoral price indexes yields a shift-share structure in $\hat{D}_\ell$. Differentiating the definition of D_ℓ in (7) yields

$$\hat{D}_\ell = (\sigma-1)\sum_n \frac{\eta_n(\varphi_{\ell n}P_n)^{\sigma-1}w_\ell^{-\sigma}}{D_\ell w_\ell^{-\sigma}}\hat{P}_n = (\sigma-1)\sum_n \frac{L_{\ell n}}{L_\ell}\hat{P}_n.$$

changes and local labor demand shocks, and can be represented in matrix notation as

$$\hat{\mathbf{L}} = \hat{\mathbf{D}} - \sigma \hat{\mathbf{w}}. \tag{9}$$

2.3 Regional Labor Supply

Given the assumptions in (4), the probability that an individual in location o chooses to live in location ℓ is given by $\pi_{o\ell}$, where

$$\pi_{o\ell} = \frac{\left(w_\ell / \tau_{o\ell}\right)^\theta}{\sum_d \left(w_d / \tau_{od}\right)^\theta}. \tag{10}$$

We refer to these location-choice probabilities as "out-migration shares." Equation (10) shows that θ is the migration elasticity reflecting how relative out-migration shares across destination locations respond to relative wage changes; a larger θ implies less variation in the idiosyncratic taste shocks and hence increased responsiveness to wage differences. Labor supply to location ℓ is then determined by these out-migration shares and the initial distribution of workers across locations, L_o^0:

$$L_\ell = \sum_o f_{o\ell} = \sum_o \pi_{o\ell} L_o^0, \tag{11}$$

where $f_{o\ell} \equiv \pi_{o\ell} L_o^0$ is the number of people initially in location o who choose location ℓ.

Now consider a labor demand shock leading to small changes in wages across (potentially all) locations, as in (8), while holding moving costs $\tau_{o\ell}$ fixed.[13] Totally differentiating (10) and (11) yields[14]

$$\hat{L}_\ell = \sum_o \gamma_{o\ell} \hat{\pi}_{o\ell} = \theta \left(\hat{w}_\ell - \sum_o \gamma_{o\ell} \sum_d \pi_{od} \hat{w}_d \right). \tag{12}$$

Here $\gamma_{o\ell} \equiv f_{o\ell}/L_\ell$ is the share of those in location ℓ who arrived from location o, which we refer to as the "in-migration share."[15] This expression describes how the quantity of labor supplied in location ℓ responds to wage changes across all destinations and can be written in matrix notation as

$$\hat{\mathbf{L}} = \theta \left(I - \Gamma' \Pi \right) \hat{\mathbf{w}}, \tag{13}$$

[13] Appendix A.1 generalizes our results to incorporate both labor demand shocks and labor supply shocks resulting from changes in $\tau_{o\ell}$. This does not affect the ensuing results.

[14] Note that, in a slight abuse of notation to avoid clutter, $\pi_{o\ell}$ in (11) is the out-migration share in the presence of labor demand shocks, while the version in (12) is the out-migration share in the absence of the labor demand shocks, such that $\hat{\pi}_{o\ell}$ is the proportional difference between these two sets of migration shares.

[15] The labor supply expression in (12) is similar to appendix equation (F.3) in Berkes et al. (2021).

where $\Gamma = (\gamma_{o\ell})$ is the matrix of in-migration shares and $\Pi = (\pi_{o\ell})$ is the matrix of out-migration shares.

2.4 Equilibrium Response to Shocks

Equating the labor demand and labor supply expressions in changes in (9) and (13) yields the equilibrium effects of regional labor demand shocks on regional population and wages:

$$\hat{\mathbf{L}} = \Omega \hat{\mathbf{D}}, \tag{14}$$

$$\hat{\mathbf{w}} = \frac{1}{\sigma}(I - \Omega)\hat{\mathbf{D}}, \tag{15}$$

where

$$\Omega \equiv I - \left[\left(1 + \frac{\theta}{\sigma}\right) I - \frac{\theta}{\sigma}\Gamma'\Pi \right]^{-1}. \tag{16}$$

Equation (14) shows the relationship between population changes and local labor demand shocks, i.e. the relationship whose empirical investigation we seek to inform. Two practical implications are immediately evident. First, the population change in a given location depends upon the shocks to *all* other locations. In fact, if all locations face the same shock so $\hat{D}_\ell = \hat{d}\ \forall \ell$, then population in each location remains unchanged.[16] Second, the importance of each shock in changing local population depends upon migrant connections between the focal location and other locations, as reflected in Γ and Π that enter Ω. This implies that Γ and Π are sufficient statistics for all relevant effects of the moving costs $(\tau_{o\ell})$ and sector-location productivity differences $(\varphi_{\ell n})$, so we will not need to estimate either as long as we can measure Γ and Π (as discussed in the next subsection).

To gain intuition for the ways in which these migrant connections influence the effects of shocks on connected regions, we consider an approximation in which mobility is low enough that the effects of indirect connections become unimportant. Specifically, write $\Gamma = I + \Delta\Gamma$ and $\Pi = I + \Delta\Pi$ and assume that $\Delta\Gamma'\Delta\Pi \approx \Delta\Gamma'\Delta\Gamma' \approx \Delta\Pi\Delta\Pi \approx \mathbf{0}$. Plugging this into (16) and then (14) yields[17]

$$\hat{L}_\ell \approx \frac{\theta}{\sigma}\left(\sum_o \gamma_{o\ell}(\hat{D}_\ell - \hat{D}_o) + \sum_d \pi_{\ell d}(\hat{D}_\ell - \hat{D}_d) \right). \tag{17}$$

[16] Specifically, we use that $\Gamma'\iota = \Pi\iota = \iota$, where ι is a column vector of ones. Thus, $\left[\left(1 + \frac{\theta}{\sigma}\right) I - \frac{\theta}{\sigma}\Gamma'\Pi\right]\iota = \iota$, $\left[\left(1 + \frac{\theta}{\sigma}\right) I - \frac{\theta}{\sigma}\Gamma'\Pi\right]^{-1}\iota = \iota$, $\Omega\iota = 0$, and finally $\Omega\iota\hat{d} = 0$ for any $\hat{d}$.

[17] Appendix A.2 derives (17).

The effect of a shock to location ℓ on the population in ℓ depends upon how the local shock compares to shocks facing all other locations, with more weight placed upon shocks in the location's typical migrant sources (captured by $\gamma_{o\ell}$) and its typical migrant destinations (captured by $\pi_{\ell d}$). This expression also shows that when the migration elasticity θ is larger, more individuals choose to migrate in response to a given vector of shocks. Similarly, when the local labor demand elasticity σ is smaller, a given vector of demand shifts leads to larger population changes.

2.5 Linking to Observables

The expressions relating population changes to labor demand shocks in (14) and (17) depend upon the ratio of elasticities θ/σ, the demand shocks $\hat{\mathbf{D}}$, and the migration share matrices Γ and Π, which refer to a counterfactual without labor demand shocks. While we keep θ/σ as an unknown parameter, in this subsection we consider how to link the labor demand shocks and counterfactual migration shares to observable quantities. Specifically, we address the possibility that the demand shocks may include observed and unobserved components and explain how we estimate the counterfactual migration shares using pre-shock observations.

We allow for unobserved labor demand shocks by assuming that we do not directly observe the overall labor demand shocks $\hat{\mathbf{D}} = (\hat{D}_\ell)$, but instead observe a demand shifter $\hat{\mathbf{z}} = (\hat{z}_\ell)$ such that

$$\hat{D}_\ell = \hat{z}_\ell + \zeta_{1\ell}, \tag{18}$$

where $\boldsymbol{\zeta}_1 = (\zeta_{1\ell})$ captures unobserved shocks to the local labor market. We assume that the observed demand shocks are as good as random with respect to unobserved shocks, such that

$$\mathbb{E}\left[\hat{z}_\ell \mid \boldsymbol{\zeta}_1\right] = \mu \quad \forall \ell, \tag{19}$$

where μ is a constant, while allowing the shocks to be mutually correlated (c.f. Borusyak et al. (2022), Assumption 1). In words, the shock to each location has the same mean conditional on all unobserved demand shocks. This assumption allows us to emphasize issues that arise even in the best-case empirical scenario, when the observed shocks are not confounded by unobservables.[18]

[18] As with any observational research design, the conventional migration regression will yield inconsistent estimates when the shocks are correlated with unobserved determinants of regional population growth. See, for example,

Given (18), we can rewrite (14) as

$$\hat{\mathbf{L}} = \Omega\hat{\mathbf{z}} + \boldsymbol{\zeta}_2, \tag{20}$$

where $\boldsymbol{\zeta}_2 \equiv \Omega\boldsymbol{\zeta}_1 = (\zeta_{2\ell})$ is a random error term. Similarly isolating the impacts of observed and unobserved shocks in (17) and rearranging terms, we arrive at an approximation for the population responses to shocks that will be particularly useful for our analytical results:

$$\hat{L}_\ell \approx \frac{2\theta}{\sigma}\frac{M_\ell}{L_\ell}\left(\hat{z}_\ell - \hat{z}_{-\ell}\right) + \zeta_{2\ell}, \qquad \text{where } \hat{z}_{-\ell} \equiv \sum_{k\neq\ell}\frac{F_{k\ell}}{M_\ell}\hat{z}_k, \tag{21}$$

$F_{k\ell} \equiv \frac{1}{2}(f_{k\ell} + f_{\ell k})$ is the average of migration flows between k and ℓ, and $M_\ell \equiv \sum_{k\neq\ell} F_{k\ell}$ is the average number of migrants to and from ℓ.[19] Equation (21) shows that local population responds to the gap between the local shock, $\hat{z}_\ell$, and the migration-weighted average of shocks to other locations, $\hat{z}_{-\ell}$. This response is stronger when the migration elasticity θ is larger and when the labor demand elasticity σ is smaller; it is also stronger in locations with a larger migration share, M_ℓ/L_ℓ.

To empirically operationalize the preceding two expressions, we must estimate the terms referring to the counterfactual setting without labor demand shocks: Ω in (20), which depends on Γ and Π, and M_ℓ, L_ℓ, and $F_{k\ell}$ in (21). We do so by assuming that the out-migration shares from the pre-shock period would have persisted in the absence of labor demand shocks.

To formalize this idea, define time $t = -1$, 0, and 1, such that the labor demand shocks arrive between $t = 0$ and $t = 1$, consistent with our earlier notation L^0_ℓ for the observed population before the arrival of the shock. Let $\Pi \equiv (\pi_{o\ell})$ be the matrix of counterfactual out-migration shares between $t = 0$ and 1 in the absence of labor demand shocks. We assume that these counterfactual out-migration shares equal those observed in the prior period, i.e. $\Pi = \Pi^0$ where $\Pi^0 \equiv (\pi^0_{o\ell})$ is the matrix of observed out-migration shares between $t = -1$ and 0. In other words, we presume that the existing migration patterns would continue absent observed and unobserved shocks at $t = 1$. This approach allows us to measure Π using pre-shock data while accounting for the possibility of persistent population trends across periods. Given these estimates and the initial population in each location, we calculate the remaining quantities in the no-shock counterfactual that appear in (20)

Greenland et al. (2019), who argue that the population regression in (Autor et al., 2013) was confounded by pre-existing trends in population growth.

[19] Appendix A.3 provides the derivation.

and (21), including Γ, M_ℓ, L_ℓ, and $F_{k\ell}$.[20] So, by assuming that the pre-shock out-migration shares would persist in the no-shock counterfactual, we can measure all relevant counterfactual quantities using observables.[21]

3 Conventional Regression and Model-Consistent Procedures

The model in the preceding section describes how local populations change in response to labor demand shocks across locations. In this section, we investigate how to interpret the results of conventional migration regressions when the data generating process follows the model just described.

3.1 The Conventional Migration Regression: Defining β

We consider a conventional approach to estimating the impact of labor demand shocks on local population, via Ordinary Least Squares (OLS) regression

$$\hat{L}_\ell = \alpha + \beta \hat{z}_\ell + \varepsilon_\ell, \tag{22}$$

with observations weighted by the pre-shock population L_ℓ^0. The corresponding OLS estimator of β is

$$\hat{\beta} = \frac{\sum_\ell L_\ell^0 \hat{L}_\ell \left(\hat{z}_\ell - \bar{z}\right)}{\sum_\ell L_\ell^0 \left(\hat{z}_\ell - \bar{z}\right)^2} \tag{23}$$

where $\bar{z} = \sum_\ell L_\ell^0 \hat{z}_\ell / \sum_\ell L_\ell^0$.[22]

We do not assume that equation (22) is correctly specified. Instead, we assume that the data on $\hat{L}_\ell$ are generated according to the model of Section 2: equation (20) or its approximation (21). We also maintain the assumption that the $(\hat{z}_\ell)$ are as-good-as-randomly assigned to regions with respect to all unobserved shocks to labor demand and supply, as in (19).

[20] We follow the definitions of these variables to calculate their values in the no-shock counterfactual: $f_{k\ell} = L_k^0 \pi_{k\ell}^0$, $F_{k\ell} = \frac{1}{2}(f_{k\ell} + f_{\ell k})$, $M_\ell = \sum_{k \neq \ell} F_{k\ell}$, $L_\ell = \sum_k f_{k\ell}$, and $\gamma_{k\ell} = f_{k\ell}/L_\ell$ with $\Gamma = (\gamma_{k\ell})$.

[21] Having linked the right-hand side of (20) and (21) to observables, we briefly discuss the left-hand side, which represents the difference in local population in $t = 1$ between counterfactuals with and without labor demand shocks. In place of this unobserved quantity, researchers employ the observed change in population between $t = 0$ and 1. As shown in Appendix A.4, the two are equivalent up to an additional error term representing population growth in the no-shock counterfactual. Because this additional term is orthogonal to the observed labor demand shocks under the exogeneity assumption (19), our identification and estimation procedures discussed below remain unaffected.

[22] Results similar to those derived below, albeit more cumbersome, can be obtained for unweighted regressions used in some studies.

Since the estimate $\hat{\beta}$ has random variation due to observed and unobserved shocks, we focus on the *estimand* of regression (22), which we define as

$$\beta = \frac{\mathbb{E}\left[\sum_{\ell} L_{\ell}^{0} \hat{L}_{\ell} \left(\hat{z}_{\ell} - \bar{z}\right)\right]}{\mathbb{E}\left[\sum_{\ell} L_{\ell}^{0} \left(\hat{z}_{\ell} - \bar{z}\right)^{2}\right]}. \tag{24}$$

This expression replaces the numerator and denominator of $\hat{\beta}$ with their expectations taken over observed and unobserved shocks (while viewing the no-shock equilibrium as fixed). This formulation has a number of benefits. First, it applies in finite samples of regions and thus allows us to avoid random sampling assumptions, which are inappropriate in typical applications including all regions of the country. Second, as we show in Appendix A.5, β approximates the probability limit of $\hat{\beta}$ in an asymptotic sequence of economies with growing numbers of regions, under appropriate regularity conditions. That is, in large samples where the impact of the unobserved shocks on $\hat{\beta}$ vanishes, we expect $\hat{\beta} \approx \beta$. Third, unlike $\mathbb{E}\left[\hat{\beta}\right]$, the formulation in (24) is analytically tractable, which allows for the convenient characterization presented in the following subsection.

3.2 Characterizing β

We now characterize β when the data were generated by the model described in Section 2. To make the results intuitively interpretable, we focus throughout on the low-mobility approximation to $\hat{L}_{\ell}$, shown in (21).

To understand our characterization, it is helpful to define some notation. Let $M \equiv \sum_{o \neq d} f_{od}$ be the national total number of migrants in the country of interest, so M/L is the migrant share of the population. We then define similar quantities that would prevail if the observed local populations reflected an economy with no migration costs, i.e. if $\tau_{od} = 1 \ \forall o, d$. Without migration costs, the probability of moving to any destination will equal the destination's share of national population, regardless of the origin.[23] Define $\widetilde{f}_{od} \equiv L_{o}^{0} \cdot \frac{L_d}{L}$ as the number of migrants from o to d and $\widetilde{M} \equiv \sum_{o \neq d} \widetilde{f}_{od}$ as the national total number of migrants in this costless-migration setting. The following theorem then characterizes β estimated with data generated by the model.

[23] When $\tau_{\ell d} = 1 \ \forall \ell, d$, (10) implies that the share of individuals choosing destination d does not depend on source ℓ: $\pi_{\ell d} = w_{d}^{\theta} / \sum_{d'} w_{d'}^{\theta} \equiv \pi_{d}$. In that case, $L_d = \sum_{\ell} \pi_{\ell d} L_{\ell}^{0} = \pi_{d} L$, so $\pi_d = L_d / L$, i.e. the probability of choosing d equals d's share of national population. This in turn implies that the flow of individuals from ℓ choosing d is $\pi_{\ell d} L_{\ell}^{0} = \pi_{d} L_{\ell}^{0} = L_{\ell}^{0} L_{d} / L$.

Theorem 1. *Suppose the data are generated by the low-mobility approximation to the baseline model, in which workers face mobility costs across locations but are freely mobile across industries (21), and the shocks are homoskedastic, i.e.* $\mathrm{Var}\left[\hat{z}_\ell\right]$ *is the same for all* ℓ*. Then*

$$\beta = \frac{2\theta}{\sigma} \cdot \frac{M/L}{\widetilde{M}/L} \cdot \frac{1-\rho}{1-\widetilde{\rho}}, \tag{25}$$

where $\rho \equiv \sum_{o \neq d} \frac{f_{od}}{M} \mathrm{Corr}\left[\hat{z}_o, \hat{z}_d\right]$ *is the average correlation between shocks to pairs of distinct regions weighted by migration flows, and* $\widetilde{\rho} = \sum_{o \neq d} \frac{\widetilde{f}_{od}}{\widetilde{M}} \mathrm{Corr}\left[\hat{z}_o, \hat{z}_d\right]$ *is a similar shock correlation weighted by the flows in the no-migration-cost scenario.*

Appendix A.6 proves this result, along with its generalization to heteroskedastic shocks.

Equation (25) shows that β can be viewed as a product of several factors with distinct economic meanings. First, β depends on the structural *elasticities*: as discussed above, true migration responses are larger when θ is large or σ is small, and β inherits those relationships. Second, β increases with the national migration *share* M/L (the term $\widetilde{M}/L$ is generally close to one and can therefore be ignored in practice).[24] Third, β is smaller when $\rho > \widetilde{\rho}$; that is, if shocks are particularly positively correlated between regions with large migration flows (i.e. flows that exceed those one would expect based solely on location sizes)—for instance, if migration is more likely at close geographic distances and shocks exhibit spatial correlation. We call the term $\frac{1-\rho}{1-\widetilde{\rho}}$ the *attenuation factor*: it reflects how the mutual correlation of shocks along the migration network attenuates the OLS coefficient. When shocks to migrant-connected locations are strongly correlated, individuals have minimal incentive to migrate, so we observe minimal migration response and estimate a relatively small (attenuated) value of β.

3.3 What Do We Learn from β?

The expression for β in Theorem 1 reveals how one can and cannot interpret estimates from conventional migration regressions. Here, we will consider various potential interpretations and compare them to correct interpretations in light of the model. To be precise about the causal effects of interest, we define $\hat{L}_\ell(\hat{\mathbf{z}}) \equiv \Omega\hat{\mathbf{z}} + \boldsymbol{\zeta}_2$, the model-predicted population growth in ℓ under shock vector

[24] Specifically, $\frac{\widetilde{M}}{L} = 1 - \sum_\ell \frac{\widetilde{f}_{\ell\ell}}{L} = 1 - \sum_\ell \left(\frac{L_\ell}{L}\right)^2$ equals one minus the Herfindahl index of population across regions. The Herfindahl index is small when the number of regions is sufficiently large and population is not too concentrated in a small number of them, as in our data.

$\hat{\mathbf{z}}$, as in (20).[25] $\hat{L}_\ell(\hat{\mathbf{z}})$ depends on the entire shock vector $\hat{\mathbf{z}}$, not just the direct shock to ℓ. To analyze counterfactual shocks, we denote a vector $\hat{\mathbf{z}}$ whose ℓ^{th} element has been changed from $\hat{z}_\ell$ to the scalar value a by $(\hat{\mathbf{z}} \overset{\ell}{\hookleftarrow} a) \equiv (\hat{z}_1, ..., \hat{z}_{\ell-1}, a, \hat{z}_{\ell+1}, ..., \hat{z}_R)$.

Effect of a shock to a single location. Researchers often use β to infer the causal effect of a labor demand shock to a single location on that location's population. A common approach interprets β as the effect of a unit labor demand shock to location ℓ on ℓ's population, implicitly fixing other locations' labor demand shocks at zero. Equation (21) shows that the true causal effect is $\hat{L}_\ell(\mathbf{0} \overset{\ell}{\hookleftarrow} 1) - \hat{L}_\ell(\mathbf{0}) \approx \frac{2\theta}{\sigma}\frac{M_\ell}{L_\ell}$. The estimate of β in (25) differs from this model-implied population growth in two important ways. First, it misses location-specific heterogeneity in the effect of a local shock; the model-implied population growth depends on the location-specific migration share $\frac{M_\ell}{L_\ell}$, while the regression estimate depends on the national migration share $\frac{M}{L}$ (recall $\widetilde{\frac{M}{L}} \approx 1$). As we will see in Section 5.2, local migration shares often vary substantially, so this heterogeneity can be important in practice.[26]

Second, the estimate of β includes the attenuation factor $\frac{1-\rho}{1-\widetilde{\rho}}$ resulting from the correlation in shocks among locations with strong migrant connections. Because of this attenuation factor, a researcher may estimate a small value of β that is close to zero even when a shock to any single location would actually lead to a substantial migration response. Consistent with this analysis, Section 6 shows that β substantially understates the true effect of a unit shock to a single location in a realistic empirical setting.

The same analysis and conclusions apply to other interpretations of β involving the effect of a shock to a single location. Examples include i) interpreting $\beta\hat{z}_\ell$ as the effect of the observed shock to ℓ, holding all other locations' shocks fixed at their observed values $(\hat{L}_\ell(\hat{\mathbf{z}}) - \hat{L}_\ell(\hat{\mathbf{z}} \overset{\ell}{\hookleftarrow} 0))$, ii) interpreting $\beta(\hat{z}_k - \hat{z}_\ell)$ as the difference in population growth caused by the difference in observed shocks across locations k and ℓ $(\hat{L}_\ell(\hat{\mathbf{z}} \overset{\ell}{\hookleftarrow} \hat{z}_k) - \hat{L}_\ell(\hat{\mathbf{z}}))$, and iii) interpreting $\beta(\hat{z}^{p75} - \hat{z}^{p25})$ as the effect of changing the shock to location ℓ from the 25^{th} to the 75^{th} percentile of the shock distribution $(\hat{L}_\ell(\hat{\mathbf{z}} \overset{\ell}{\hookleftarrow} \hat{z}^{p75}) - \hat{L}_\ell(\hat{\mathbf{z}} \overset{\ell}{\hookleftarrow} \hat{z}^{p25}))$. All of these cases suffer from the same problems as the unit shock discussed above: the regression-based prediction omits local heterogeneity in migration intensity

[25] Note that $\hat{L}_\ell(\hat{\mathbf{z}})$ includes the effect of unobserved shocks, so $\hat{L}_\ell(\mathbf{0}) = \zeta_{2\ell}$.

[26] This problem would not arise if the outcome variable of the regression was the net population change divided by the gross migration flow M_ℓ rather than divided by the overall population L_ℓ.

and suffers from attenuation resulting from correlated shocks across migrant-connected locations.[27]

Effect of shocks to all locations. The preceding discussion makes clear that the conventional migration regression does not capture the effects of shocks to individual locations. But one might still think it could capture the effect of the entire observed shock vector $\hat{\mathbf{z}}$ on population growth in each location ℓ, i.e. $\hat{L}_\ell(\hat{\mathbf{z}}) - \hat{L}_\ell(\mathbf{0}) \approx \frac{2\theta}{\sigma}\frac{M_\ell}{L_\ell}\left(\hat{z}_\ell - \hat{z}_{-\ell}\right)$.

It is important to note that the (population-weighted) average effect of any shock vector $\hat{\mathbf{z}}$ is zero by construction, since national population is assumed to be unaffected by the shock.[28] A meaningful interpretation of β must therefore involve comparisons across locations. To see why, consider using $\beta\hat{z}_\ell$ to predict the shock vector's effect on population growth in each location ℓ. In many empirical contexts, the shocks to all locations have the same sign, so $\beta\hat{z}_\ell$ would incorrectly imply either population growth in all locations or population declines in all locations. A more sensible approach attributes to the local shock only its effect relative to the mean, $\beta\left(\hat{z}_\ell - \bar{z}\right)$. This approach ensures that the predicted effects of the shocks average to zero across locations and is consistent with the inclusion of an intercept term α in the conventional regression (22). It also yields the best (mean squared error (MSE) minimizing) prediction of the expected population growth in location ℓ among linear functions of $(\hat{z}_\ell - \bar{z})$.[29]

However, using $\hat{z}_\ell - \bar{z}$ alone to predict local population growth is likely to yield very poor predictions. To see why, use the decomposition of β in (25) to write

$$\beta\left(\hat{z}_\ell - \bar{z}\right) = \frac{2\theta}{\sigma}\cdot\frac{M/L}{\widetilde{M/L}}\left(\left(\hat{z}_\ell - \bar{z}\right) - \frac{\rho - \widetilde{\rho}}{1-\widetilde{\rho}}\cdot\left(\hat{z}_\ell - \bar{z}\right)\right). \tag{26}$$

Compare this prediction to (21), the model's expected population growth in ℓ when facing the observed vector of demeaned shocks: $\frac{2\theta}{\sigma}\cdot\frac{M_\ell}{L_\ell}\left(\left(\hat{z}_\ell - \bar{z}\right) - \left(\hat{z}_{-\ell} - \bar{z}\right)\right)$. The estimate $\beta\left(\hat{z}_\ell - \bar{z}\right)$ omits two key sources of variation. As before, it omits heterogeneity based on the local migration intensity, $\frac{M_\ell}{L_\ell}$.

[27]In case i) the true effect is $\hat{L}_\ell(\hat{\mathbf{z}}) - \hat{L}_\ell(\hat{\mathbf{z}} \overset{\ell}{\hookleftarrow} 0) \approx \frac{2\theta}{\sigma}\frac{M_\ell}{L_\ell}\hat{z}_\ell$, while the regression-based prediction is $\beta\hat{z}_\ell = \frac{2\theta}{\sigma}\cdot\frac{M/L}{\widetilde{M/L}}\cdot\frac{1-\rho}{1-\widetilde{\rho}}\hat{z}_\ell$. In case ii) the true effect on location ℓ's population of changing ℓ's shock from $\hat{z}_\ell$ to $\hat{z}_k$, fixing other locations' shocks at their observed values is $\hat{L}_\ell(\hat{\mathbf{z}} \overset{\ell}{\hookleftarrow} \hat{z}_k) - \hat{L}_\ell(\hat{\mathbf{z}}) \approx \frac{2\theta}{\sigma}\frac{M_\ell}{L_\ell}(\hat{z}_k - \hat{z}_\ell)$, while the regression-based prediction is $\beta(\hat{z}_k - \hat{z}_\ell) = \frac{2\theta}{\sigma}\cdot\frac{M/L}{\widetilde{M/L}}\cdot\frac{1-\rho}{1-\widetilde{\rho}}(\hat{z}_k - \hat{z}_\ell)$. In case iii) the true effect is $\hat{L}_\ell(\hat{\mathbf{z}} \overset{\ell}{\hookleftarrow} \hat{z}^{p75}) - \hat{L}_\ell(\hat{\mathbf{z}} \overset{\ell}{\hookleftarrow} \hat{z}^{p25}) \approx \frac{2\theta}{\sigma}\frac{M_\ell}{L_\ell}(\hat{z}^{p75} - \hat{z}^{p25})$, while the regression-based prediction is $\beta(\hat{z}^{p75} - \hat{z}^{p25}) = \frac{2\theta}{\sigma}\cdot\frac{M/L}{\widetilde{M/L}}\cdot\frac{1-\rho}{1-\widetilde{\rho}}(\hat{z}^{p75} - \hat{z}^{p25})$.

[28]This assumption can be relaxed by allowing for population growth or international migration.

[29]See Appendix A.7 for a proof.

More importantly, it replaces the shocks to other migration-connected locations, $(\hat{z}_{-\ell} - \bar{z})$, with the scaled local shock relative to the mean $\frac{\rho-\widetilde{\rho}}{1-\rho}(\hat{z}_{\ell} - \bar{z})$. Shocks to other locations are important drivers of local population change according to the model and can differ substantially among locations facing similar direct shocks, but this information is omitted from (26). Because the fitted values from the population regression only use information on the direct shocks to the location, they cannot account for the extent to which other migrant sources and destinations faced similar or different shocks. As we will see in Section 5.2, the omission of these cross-location spillover effects can be quantitatively important.

Rather than attempting to use β to predict the effect of $\hat{\mathbf{z}}$ on one location's population growth, an alternative approach would interpret $\beta(\hat{z}_k - \hat{z}_{\ell})$ as the difference in the effects of all shocks between particular locations k and ℓ: $(\hat{L}_k(\hat{\mathbf{z}}) - \hat{L}_k(\mathbf{0})) - (\hat{L}_{\ell}(\hat{\mathbf{z}}) - \hat{L}_{\ell}(\mathbf{0}))$.[30] This interpretation suffers from similar problems to those already discussed: it omits heterogeneity in migration intensity and fails to account for the particular differences between migrant-connected shocks $\hat{z}_{-k}$ and $\hat{z}_{-\ell}$.[31]

Yet, there is a valid interpretation of β based on the *difference* in *average* effects of all shocks between locations facing different *direct* shocks. The most straightforward way to demonstrate this interpretation is to consider the following regression, which relates the effect of all shocks on population growth in location ℓ to location ℓ's direct shock:

$$\hat{L}_{\ell}(\hat{\mathbf{z}}) - \hat{L}_{\ell}(\mathbf{0}) = a + b\hat{z}_{\ell} + e_{\ell}. \tag{27}$$

Here b cannot be estimated directly, since the dependent variable is not observed. However, the only difference between the specification in (27) and the conventional migration regression in (22) is that the left side of (27) is $\hat{L}_{\ell}(\hat{\mathbf{z}}) - \hat{L}_{\ell}(\mathbf{0})$ rather than the observed change over time. As shown in Appendix A.4, the two are equivalent up to an additional error term representing population growth in the no-shock counterfactual plus the population effect of the unobserved shocks. Because this

[30]Note the distinction between this interpretation focusing on the effect of *all* shocks as opposed to interpretation ii) in footnote 27, which considers the effect of changing location $\ell's$ shock from $\hat{z}_{\ell}$ to $\hat{z}_k$, while holding all other locations' shocks fixed: $\hat{L}_{\ell}(\hat{\mathbf{z}} \overset{\ell}{\hookleftarrow} \hat{z}_k) - \hat{L}_{\ell}(\hat{\mathbf{z}})$.

[31]The difference in the effects of all shocks $\hat{\mathbf{z}}$ between locations k and ℓ is

$$\left(\hat{L}_k(\hat{\mathbf{z}}) - \hat{L}_k(\mathbf{0})\right) - \left(\hat{L}_{\ell}(\hat{\mathbf{z}}) - \hat{L}_{\ell}(\mathbf{0})\right) \approx \frac{2\theta}{\sigma}\left[\left(\frac{M_k}{L_k}\hat{z}_k - \frac{M_{\ell}}{L_{\ell}}\hat{z}_{\ell}\right) - \left(\frac{M_k}{L_k}\hat{z}_{-k} - \frac{M_{\ell}}{L_{\ell}}\hat{z}_{-\ell}\right)\right].$$

$\beta(\hat{z}_k - \hat{z}_{\ell})$ omits heterogeneity based on initial migration intensity and captures the differences in $\hat{z}_{-k}$ and $\hat{z}_{-\ell}$ using only $(\hat{z}_k - \hat{z}_{\ell})$.

additional term is orthogonal to the observed labor demand shocks under the exogeneity assumption (19), the β and b estimands are equal. We can therefore interpret an estimate of β as we would interpret b: the difference in the average effect of all shocks between places facing high vs. low direct shocks, scaled to be per-unit of the shock.[32] Importantly, even this limited interpretation only holds for the shocks used to estimate β. Counterfactual shocks will generally induce different correlation structures across migrant-connected regions that will not be captured by β estimated using the realized shocks.

To summarize, an estimate of β from the conventional migration regression in (22) does not yield accurate predictions regarding the causal effects of observed or counterfactual shocks because it ignores information on shocks to migrant-connected locations and omits heterogeneity based on local migration intensity. The estimate of β may be close to zero even when the observed shock led to substantial spatial reallocation, when workers are highly responsive to local economic conditions, and when a counterfactual shock to a single location would drive substantial migration. We learn little about interregional mobility costs or migration elasticities from an estimate of β and should therefore not use results from conventional migration regressions to justify model restrictions or to calibrate model parameters.

Confounding migration in wage regressions. Some studies estimate the conventional migration regression in (22) not to draw conclusions about the effects of shocks on local populations, but to assess whether interregional migration is likely to substantially affect estimates of the effect of local shocks on local economic outcomes such as wages. Appendix A.8 examines this approach in light of the model, finding that when the conventional regression estimate of β is close to zero, wage regression estimates will experience minimal confounding from migration, irrespective of the

[32] To formalize this point, split the sample of locations into those with above-average shocks, $\ell|\hat{z}_\ell \geq \overline{z}$, and those with below-average shocks, $\ell|\hat{z}_\ell < \overline{z}$. One can then express β in terms of the difference in the average effect of all shocks between these two groups of locations:

$$\beta = \Lambda \mathbb{E}\left[\sum_{\ell|\hat{z}_\ell \geq \overline{z}} v_\ell(\hat{L}_\ell(\mathbf{z}) - \hat{L}_\ell(\mathbf{0})) - \sum_{\ell|\hat{z}_\ell < \overline{z}} v_\ell(\hat{L}_\ell(\mathbf{z}) - \hat{L}_\ell(\mathbf{0}))\right],$$

where $v_\ell \equiv \frac{L_\ell^0|\hat{z}_\ell - \overline{z}|}{\mathbb{E}\left[\sum_{k|\hat{z}_k \geq \overline{z}} L_k^0|\hat{z}_k - \overline{z}|\right]}$ and $\Lambda \equiv \frac{\mathbb{E}\left[\sum_{\ell|\hat{z}_\ell \geq \overline{z}} L_\ell^0(\hat{z}_\ell - \overline{z})\right]}{\mathbb{E}\left[\sum_\ell L_\ell^0(\hat{z}_\ell - \bar{z})^2\right]}$. Each sum in this expression for β is the weighted-average of the effects of all shocks in the set of places with either above-average or below-average shocks, with weights given by v_ℓ, which add to one in each group of locations. β is the difference in these average effects scaled by Λ, which places β on a per-unit-of-$\hat{z}$ scale.

mechanism driving the small estimate of β. Intuitively, because labor demand shocks have a direct effect on local wages and an indirect effect through migration spillovers, when $\beta \approx 0$ this indirect effect is approximately uncorrelated with the direct effect and therefore does not substantially bias the estimate of the local shock's effect on the local wage.

3.4 Model-Consistent Specifications

We now describe alternative empirical approaches that integrate information on regional shocks and pre-shock migration patterns to estimate migration responses in a way that is consistent with the model.

The model-implied relationship between local population changes and the vector of local labor demand shocks is given by equation (20) above, rearranged slightly here.

$$\hat{\mathbf{L}} = \Omega(\theta/\sigma)\hat{\mathbf{z}} + \boldsymbol{\zeta}_2, \qquad \text{where } \Omega\left(\theta/\sigma\right) \equiv I - \left[I + \frac{\theta}{\sigma}\left(I - \Gamma'\Pi\right)\right]^{-1}. \tag{28}$$

Following Section 2.5, we observe the population change $\hat{\mathbf{L}}$, the migration matrices Γ and Π, and the shocks $\hat{\mathbf{z}}$, so we can estimate the parameter θ/σ using non-linear least squares (NLLS).[33] Formally, in Appendix A.10 we show that whenever $\hat{\mathbf{z}}$ has a non-degenerate variance-covariance matrix, θ/σ uniquely solves the NLLS problem[34]

$$\min_{\lambda} \mathbb{E}\left[\left(\hat{\mathbf{L}} - \Omega(\lambda)\hat{\mathbf{z}}\right)'\left(\hat{\mathbf{L}} - \Omega(\lambda)\hat{\mathbf{z}}\right)\right]. \tag{29}$$

Given the resulting estimate of θ/σ we can calculate shock-induced population changes in each location using the same relationship in (28), omitting the error term. This process can be used to predict internally valid effects of the observed shocks used to estimate θ/σ in (28) and externally valid effects of counterfactual shocks, under the model's assumptions.

An alternative approach relies on the low-mobility approximation to construct a control for the

[33] We recommend an intercept is included in equation (28) in practice to capture exogenous changes in national population (due to population growth or international migration) and to increase estimation efficiency.

[34] Borusyak and Hull (2021, Appendix D.5) show in a similar setting that a "recentering" adjustment is generally necessary for the appropriate moment condition to hold under the as-good-as-random assignment of the shocks, $\mathbb{E}\left[\hat{z}_\ell \mid \boldsymbol{\zeta}_1\right] = \mu$. We show in Appendix A.9 that this issue does not arise in our setting because NLLS estimates are invariant to μ: demeaning the shock has no implications for population changes. Recentering would be necessary, however, if the shocks were *conditionally* as-good-as-randomly assigned: e.g. higher in regions with a higher manufacturing employment share, as in many empirical settings (e.g. Autor et al. (2013)).

shocks to other locations, which can be included in the conventional migration regression. The relevant control is given by $\hat{z}_{-\ell}$ defined in (21), reflecting a migration-weighted average of shocks to locations other than ℓ.[35] While relying on the same information as the NLLS procedure just discussed, this approach maintains a specification that is linear in the parameters and can be estimated by OLS.

Two caveats should be kept in mind here. First, we assume that the researcher has available a set of shocks to local labor demand that are as good as randomly assigned (as in (19)) with which to estimate θ/σ. Any endogeneity in the shocks will lead to inconsistency in both the conventional migration regression and our proposed approaches. Second, our model is purposefully stylized in order to focus on the conventional migration regression's failure to account for shocks to other migrant-connected locations. The model does not include other mechanisms such as agglomeration economies, housing, capital markets, and forward-looking behavior. Instead, our relatively parsimonious model focuses on the cross-location spillover problem faced by conventional migration regressions.

4 Extension to Frictional Labor Mobility across Industries

The baseline model described in the preceding section allows for frictions in moving across locations, but assumes costless mobility across industries within a given location. Here, we examine the full model allowing for costly mobility across both industries and locations. We do so by applying the same analysis at the location-industry level. In particular, the indirect utility in (4) can be written identically, but with the subscript o referring to the worker's initial location-industry pair and ℓ referring to the location-industry pair that the worker might choose. It will be helpful to make this distinction explicit by defining the initial location and industry as o and p and the new location and industry as d and q. To clarify the distinction between prior and current location-industry information, we separate the subscripts with a comma. Workers face a moving cost $\tau_{op,dq}$ when switching from location-industry pair op to pair dq, and θ determines the responsiveness of labor supply to wage differences between one location-industry pair and another. Note that, for simplicity, this setup assumes that the same parameter θ applies to both the location and industry

[35] In Section 4, we generalize the model to include both location and industry switching costs and define a parallel linear specification capturing the effects of both shocks to ℓ and those in other locations in equation (31).

dimensions.[36]

The model extends directly to the location-industry context, and we again make use of the low-mobility approximation to derive an intuitive expression relating labor demand shocks to population changes at the location-industry level.[37] Applying (17) and (18) in that setting yields

$$\hat{L}_{\ell n} \approx \frac{\theta}{\sigma} \left(\sum_{o \in \mathscr{L}} \sum_{p \in \mathscr{N}} \gamma_{op,\ell n} (\hat{z}_{\ell n} - \hat{z}_{op}) + \sum_{d \in \mathscr{L}} \sum_{q \in \mathscr{N}} \pi_{\ell n,dq} (\hat{z}_{\ell n} - \hat{z}_{dq}) \right) + \zeta_{\ell n}, \tag{30}$$

where $\gamma_{op,\ell n} = f_{op,\ell n}/L_{\ell n}$ is the share of workers in location-industry cell ℓn who came from cell op and $\pi_{\ell n,dq} = f_{\ell n,dq}/L^0_{\ell n}$ is the share of workers in cell ℓn who went to cell dq (both in the no-shock counterfactual), the $\hat{z}_{\ell n}$ are observable labor demand shocks in cell ℓn, and $\boldsymbol{\zeta} = (\zeta_{\ell n})$ is an error term such that $\mathbb{E}[\hat{z}_{\ell n} \mid \boldsymbol{\zeta}] = \mu, \ \forall \ell, n$, as in (19).

This model allows us to interpret the conventional location-level migration regression when the data reflect a setting with frictions across both industries and locations. The conventional regression remains identical to equation (22), where we aggregate across industries to yield the location's average labor demand shock $\hat{z}_\ell \equiv \sum_n \frac{L^0_{\ell n}}{L^0_\ell} \hat{z}_{\ell n}$ and the location's change in population $\hat{L}_\ell = \sum_n \frac{L_{\ell n}}{L_\ell} \hat{L}_{\ell n}$. Applying the latter to (30) and rearranging yields

$$\hat{L}_\ell \approx \frac{2\theta}{\sigma} \frac{M_\ell}{L_\ell} (\hat{z}^{\text{mov}}_\ell - \hat{z}^{\text{mov}}_{-\ell}) + \zeta_\ell, \tag{31}$$

$$\text{where} \quad \hat{z}^{\text{mov}}_\ell \equiv \sum_n \frac{F_{-\ell,\ell n}}{M_\ell} \hat{z}_{\ell n}, \quad \hat{z}^{\text{mov}}_{-\ell} \equiv \sum_p \sum_{o \neq \ell} \frac{F_{op,\ell}}{M_\ell} \hat{z}_{op}, \tag{32}$$

and $\zeta_\ell = \sum_n \frac{L_{\ell n}}{L_n} \zeta_{\ell n}$ (see Appendix A.11 for the proof). In this expression, $F_{-\ell,\ell n} \equiv \sum_p \sum_{o \neq \ell} F_{op,\ell n}$ is the migration flow between location-industry ℓn and other locations (recall that $F_{op,\ell n} \equiv \frac{1}{2}(f_{op,\ell n} + f_{\ell n,op})$ is the average of flows in both directions), and $F_{op,\ell} \equiv \sum_n F_{op,\ell n}$ is the flow between location-industry op and location ℓ, all referring to the no-shock counterfactual. The terms $\hat{z}^{\text{mov}}_\ell$ and $\hat{z}^{\text{mov}}_{-\ell}$ reflect shocks facing industries in location ℓ and those in other locations, respectively. Both are weighted averages with weights depending on location ℓ's migrant connections to other locations, hence the superscripts referring to "movers."

[36]As discussed in footnote 45, available estimates suggest that θ is similar across locations and industries, supporting the restriction of a single parameter in both dimensions.

[37]Note that the low-mobility approximation may perform more poorly in the location-industry context, since there may be substantial mobility across industries within location. However, our quantitative investigation in Section 6 suggests this is not the case in our context.

Equation (31) shows that the local population change depends upon how local shocks compare to outside-option shocks to other locations, as in the baseline model. In fact, in the case of a purely regional shock, i.e. $\hat{z}_{\ell n} = \hat{x}_\ell \ \forall \ell, n$, the two models imply identical population responses under the low-mobility approximation, since $\hat{z}_\ell^{\text{mov}} = \hat{z}_\ell$ and $\hat{z}_{-\ell}^{\text{mov}} = \hat{z}_{-\ell}$. With regional shocks, (31) is therefore equivalent to (21). In other words, when the shock is purely regional, the presence of industry frictions does not alter migration behavior, and all of the problems of interpretation discussed in Section 3.3 apply to the model with location and industry frictions.

Now consider shocks that vary across industries. In a regional migration analysis, researchers typically incorporate industry-level shocks using a shift-share structure, in which the average shock facing workers in location ℓ is an employment-weighted average of industry shocks: $\hat{z}_\ell = \sum_n \frac{L_{\ell n}^0}{L_\ell^0} \hat{x}_n$. When switching industries is costless, as in the baseline model of Section 2, this shift-share measure captures the regional labor demand shock facing all workers in location ℓ, i.e. the shift-share measure *is* the regional labor demand shock when there are no industry switching frictions. In contrast, when industry switching frictions are present, as in the full model developed in this section, workers in the same location but in different industries experience different labor demand changes and have different incentives to migrate. We therefore find distinct migration behavior in models with and without industry frictions when workers face industry-level labor demand shocks.

We can see this distinction in (31) with purely industry-level shocks, i.e. $\hat{z}_{\ell n} = \hat{x}_n \ \forall \ell, n$. The term for shocks facing location ℓ becomes $\hat{z}_\ell^{\text{mov}} = \sum_n \frac{F_{-\ell, \ell n}}{M_\ell} \hat{x}_n$, which places more weight on shocks to local industries with stronger migrant connections to outside locations. These outside migrant connections are stronger for industries in which migration costs are relatively low, so a larger fraction of workers in these industries are close to indifferent between locations, and a positive (negative) shock to that industry will lead to more net in-migration (out-migration) among these marginal workers. Conversely, the outside-option term becomes $\hat{z}_{-\ell}^{\text{mov}} \equiv \sum_p \frac{F_{(-\ell)p, \ell}}{M_\ell} \hat{x}_p$, where $F_{(-\ell)p,\ell}$ is the migrant flow between all industries in location ℓ and industry p in other locations. This migrant connection is stronger when workers in ℓ can transition into industry p in other locations relatively easily, so a more positive (negative) shock to that industry will lead to more net out-migration from (net in-migration to) location ℓ.

To illustrate the implications of industry frictions for migration behavior, consider an extreme scenario in which workers cannot switch industries, formalized by $\tau_{op,dq} = \infty$ for all o, d, and $p \neq q$.

In that case, although regional shocks will induce migration, pure industry shocks will not. Because workers cannot change industries, those migrating to or from ℓ will have the same industry mix as those in ℓ. This means that $F_{-\ell,\ell n} = F_{(-\ell)n,\ell} = F_{(-\ell)n,\ell n}$, which implies that $\hat{z}_\ell^{\text{mov}} = \hat{z}_{-\ell}^{\text{mov}}$ and $\hat{L}_\ell = 0$ in (31). When workers cannot switch industries, they face the same industry-level shock irrespective of their location and thus have no incentive to migrate in response to pure industry shocks.

This extreme example illustrates the more general point that industry-switching frictions reduce migration responses when labor demand shocks have an national industry-specific component. Intuitively, if switching industries is difficult, workers see less benefit in moving to a location with a more favorably affected industry mix because they would still face much of their original industry's shock in the new location.

We can formalize this intuition by characterizing β, the conventional migration regression estimand in (24), in the context of the full model with location and industry mobility costs. Note that we lead with the case of heteroskedastic shocks because shift-share variables are inherently heteroskedastic.

Theorem 2. *Suppose the data are generated by the low-mobility approximation to the full model in which workers face mobility costs across both locations and industries (31). Then*

$$\beta = \frac{2\theta}{\sigma} \cdot \frac{M^v/L}{\widetilde{M}^v/L} \cdot \frac{\rho^{mov} - \rho}{1 - \widetilde{\rho}} \tag{33}$$

where $v_\ell \equiv \text{Var}\left[\hat{z}_\ell\right]$ *is the variance of the local shock to* ℓ*,* $M^v/L \equiv (\sum_\ell M_\ell v_\ell)/(\sum_\ell L_\ell v_\ell)$ *is the shock-variance weighted national average migration share, and* $\rho \equiv (\sum_\ell M_\ell v_\ell \rho_\ell)/(\sum_\ell M_\ell v_\ell)$ *for* $\rho_\ell \equiv \text{Cov}\left[\hat{z}_\ell, \hat{z}_{-\ell}^{mov}\right]/v_\ell$*. As in Theorem 1, tildes refer to the costless-migration setting, so* $\widetilde{M}^v/L \equiv (\sum_\ell \widetilde{M}_\ell v_\ell)/(\sum_\ell L_\ell v_\ell)$ *and* $\widetilde{\rho} \equiv (\sum_\ell \widetilde{M}_\ell v_\ell \widetilde{\rho}_\ell)/(\sum_\ell \widetilde{M}_\ell v_\ell)$ *for* $\widetilde{\rho}_\ell \equiv \text{Cov}\left[\hat{z}_\ell, \widetilde{z}_{-\ell}\right]/v_\ell$ *and* $\tilde{z}_{-\ell} \equiv \sum_{d \neq \ell} (\frac{1}{2}(\tilde{f}_{\ell d} + \tilde{f}_{d\ell})/\widetilde{M}_\ell)\hat{z}_d$*. Finally,* $\rho^{mov} \equiv (\sum_\ell M_\ell v_\ell \rho_\ell^{mov})/(\sum_\ell M_\ell v_\ell)$*, and* $\rho_\ell^{mov} \equiv \text{Cov}\left[\hat{z}_\ell^{mov}, \hat{z}_\ell\right]/v_\ell$*.*

The proof is given in Appendix A.11. As in the baseline model, β can be multiplicatively decomposed into the ratio of structural elasticities, a term depending on the national migration share (which differs from Theorem 1 in the baseline model only due to heteroskedasticity), and the attenuation factor $\frac{\rho^{\text{mov}} - \rho}{1 - \widetilde{\rho}}$. This attenuation factor captures two reasons for attenuation in the conventional migration regression estimate. As in the baseline model, ρ is larger when the shock to ℓ, $\hat{z}_\ell$, is more strongly correlated with the shocks to migrant-connected locations, $\hat{z}_{-\ell}^{\text{mov}}$. As this correlation grows, β exhibits more attenuation. In the presence of industry switching frictions, the conventional regression suffers from additional misspecification by using $\hat{z}_\ell$ as the direct shock measure rather than the model-consistent $\hat{z}_\ell^{\text{mov}}$. The more $\hat{z}_\ell^{\text{mov}}$ deviates from $\hat{z}_\ell$, the smaller is ρ^{mov} and the more β exhibits attenuation. Returning to the example of industry-level shocks with limited cross-industry mobility, workers moving between ℓ and other regions will have similar industry composition, and thus $\hat{z}_\ell^{\text{mov}}$ is similar to $\hat{z}_{-\ell}^{\text{mov}}$. This implies that $\rho^{\text{mov}} \approx \rho$, and attenuation is severe.

This discussion reveals how industry frictions undermine the intuitive appeal of measuring local labor demand shocks using shift-share measures. Only when industry switching frictions are absent does the industry shift-share represent a true regional labor demand shock facing all local workers. With industry frictions, workers in different industries face different shocks and have different outside options, leading to complex spillovers of the effects of shocks across industries and regions. Yet, all of these complexities are omitted by the conventional migration regression, exacerbating the biases examined in Section 3.3.

5 Data and Descriptive Statistics

To assess the practical importance of the conceptual problems with the conventional migration regression that we have discussed in Sections 2 through 4, we use data on observed worker transitions across locations and industries. This section describes the data source and presents descriptive statistics relevant to our quantitative analysis.

5.1 Data

The NLLS approach described in Section 3.4 and the model simulations presented in Section 6 require information on worker transitions across locations or across location-industry pairs, allowing us to calculate the transition matrices Γ and Π. We do so using administrative panel data that cover all formally employed workers in Brazil, allowing us to observe these detailed transitions across locations and industries. Subsequent revisions will present results using similar data in other countries.

We utilize data from the Relação Anual de Informações Sociais (RAIS) covering 1994 to 2000. This administrative dataset is a census of the Brazilian formal labor market that allows us to follow all formally employed workers across jobs in different industries and locations (De Negri et al., 2001; Saboia and Tolipan, 1985).[38] Locations in our analysis are based on the "microregion" definition of the Brazilian Statistical Agency (IBGE, 2002), which combines economically integrated contiguous municipalities (counties) with similar geographic and productive characteristics. We aggregate microregions slightly to ensure consistent boundaries over time, following Dix-Carneiro and Kovak (2017), which yields 486 time-consistent microregion locations in Brazil's 27 states.[39] Industries are based on the "Subsetor IBGE" classification the RAIS dataset uses to identify the industry associated with each employer. This classification identifies 25 distinct industries including 12 in manufacturing.

Our sample includes individuals age 18 to 64 with positive earnings in December of the relevant year.[40] We also drop individuals with missing or inconsistent information, including "other/ignored" industries, contradictory education levels across jobs in the same year, holding jobs in very distant geographic areas, etc. If a worker holds multiple jobs, we select the job with the highest December earnings and use it to assign the worker's location and industry of employment. When examining differences by skill level, we define skilled workers as those with a high school degree or more (34.5 percent of all workers) and unskilled workers as those without a high school degree.

The RAIS data allow us to observe formal employment in location-industry cells and transitions between them, but do not provide any information on non-employed or informally employed individuals. In our model, which does not include non-employment or informality, population and formal employment are indistinguishable. We therefore use formal employment as a proxy for population and use RAIS data to calculate yearly formally-employed worker flows between location-industry pairs, omitting transitions into or out of formal employment. Because we use detailed geographic

[38]Formality is defined as having a signed work card (*carteira assinada*), which registers the worker's contract with the Ministry of Labor and gives them the right to benefits and protections under the legal employment system. The restriction to formal workers is important in the Brazilian context, as informality rates measured in the Brazilian Census exceed 50% during our sample period (Dix-Carneiro and Kovak, 2019).

[39]As Dix-Carneiro and Kovak (2017) report using Census data, only 3.4 percent of individuals lived and worked in different microregions in the year 2000, so the microregion of employment is a quite accurate measure of both employment and residence location.

[40]RAIS reports earnings for December and average monthly earnings during employed months in the reference year. We use December earnings to ensure that our results are not influenced by seasonal variation or month-to-month inflation.

and industry information, workers may transition between 12,150 = 486 × 25 location-industry cells. To reduce noise in estimating transitions between small cells, we average the observed yearly flows between each location-industry cell from 1994 to 2000 and use these averages to calculate migration flows, employment levels, and employment changes both at the location and location-industry levels.[41]

5.2 Descriptive Evidence

We now investigate aspects of the transition matrices, Γ and Π, that are relevant to the population effects of local labor demand shocks. Section 3.3 shows that the conventional migration regression estimate β omits two important elements that drive the population effects of local labor demand shocks in the model: heterogeneity in local migration shares and differences in shocks to migrant-connected locations. This section therefore presents summary statistics on mobility rates and the concentration of migrant connections across locations and industries.

The conventional migration regression estimate β depends upon the national average mobility rate (M/L), which is shown in the first row of Table 1, column (1): 4.2 percent of workers migrate across locations (microregions) in a given year on average. The second row reports the probability of moving between the 27 Brazilian states (0.9 percent), showing that workers are much more likely to move across locations within states than across states, which reflects a lower likelihood of long-distance moves. Industry transitions (third row) are substantially more likely than migration events (although still rare, at 8.0 percent), and a small share of workers (1.3%) transitions between both locations and industries in a given year. Columns (2) and (3) show the same statistics for high-skilled and low-skilled workers separately, showing that geographic mobility rates are similar for the two groups, but that low-skilled workers are less likely to switch industries.

Underlying these national average mobility rates is substantial heterogeneity in migration shares across locations. Figure 1 shows a histogram of M_ℓ/L_ℓ, the share of individuals in ℓ who move to a different location in a typical year (weighted by the region's initial employment). As discussed in Section 3.3, this heterogeneity in baseline migration intensity drives heterogeneity in the true effect of local labor demand shocks on local population (as in (21)), but this heterogeneity is omitted from

[41] Specifically, define $f_{op,dq}^{t,t+1}$ as the observed the number of workers moving from cell op to cell dq between years t and $t+1$. Then calculate the average across years as $\overline{f}_{op,dq} \equiv \frac{1}{6}\sum_{t=1994}^{1999} f_{op,dq}^{t,t+1}$. These average flows are then used to measure the initial employment levels, local industry compositions, and transition matrices.

Table 1: Average Mobility Rates, %

Across	All Workers (1)	High-Skilled (2)	Low-Skilled (3)
Locations	4.2	4.2	4.3
States	0.9	0.8	1.0
Industries	8.0	8.3	7.5
Locations and Industries	1.3	1.3	1.2

Notes: Percent of individuals with yearly transitions between the cells defined in the first column, calculated from average yearly flows between location-industry cells in RAIS data from 1994 to 2000 as described in footnote 41. The last row measures the fraction of workers who change *both* location and industry in the same year. High-skilled is defined as having a high-school degree or more and low-skilled as less than high school.

the estimate of β in the conventional migration regression (as in (25)).[42]

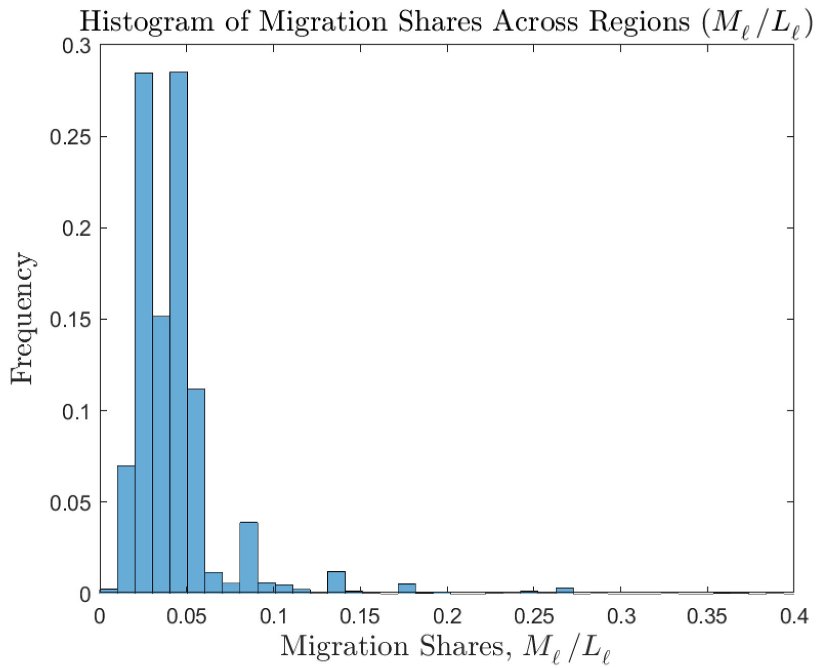
We next show that migration exhibits a network structure. The analyses in Sections 3.3 and 4 emphasized the role of spillovers between connected locations and location-industry cells in driving migration responses to labor demand shocks. These spillovers will be particularly important in practice if location or industry connections differ substantially for workers in different locations or industries, e.g. due to migration costs increasing in distance. To assess the degree to which these connections are concentrated, we calculate the Herfindahl–Hirschman index (HHI) measuring the average concentration of destinations and origins among those who migrated to or from a given region:

$$HHI_\ell = \sum_{k \neq \ell} \left(\frac{\frac{1}{2} \left(\pi_{\ell k} + \gamma_{k \ell} \right)}{1 - \frac{1}{2} \left(\pi_{\ell \ell} + \gamma_{\ell \ell} \right)} \right)^2 . \tag{34}$$

We also calculate a parallel expression for those who switched to or from industry n, HHI_n. The distributions of these concentration measures are shown in Figure 2. The average HHI_ℓ across locations is 0.165 (s.d. 0.174), which is equivalent to equally-weighted connections to only 6 other locations out of 485 possibilities. Industry transitions are more dispersed, with an average HHI_n across industries of 0.141 (s.d. 0.050), which is equivalent to equally-weighted connections to 7 other industries out of 24 possibilities. Figure 2 also documents the variation in these concentration measures across both locations and industries. The substantial concentration in connections and its heterogeneity across locations suggest that workers in different locations may face substantial differences in spillovers from connected locations and industries.

[42] We also find substantial heterogeneity across industries in the probability of switching industries, with rates ranging from close to zero up to 20%.

Figure 1: Geographic Mobility Rates Across Regions



Notes: Percent of individuals migrating across locations (microregions), calculated from average yearly flows between location-industry cells in RAIS data from 1994 to 2000 as described in footnote 41. The histogram weighted by each location's initial population.

6 Quantitative Assessment of Migration Regressions

In this section we quantitatively examine the predictive accuracy of the conventional migration regression in the context of Brazilian geography, industrial structure, and baseline migration patterns. We simulate migration behavior based on the baseline model with frictions across locations only and on the full model with frictions across both locations and industries. We consider population responses to labor demand shocks that follow a variety of data generating processes and compare these true responses to those inferred from conventional migration regression estimates. We further compare the conventional regression with the model-based NLLS procedure.

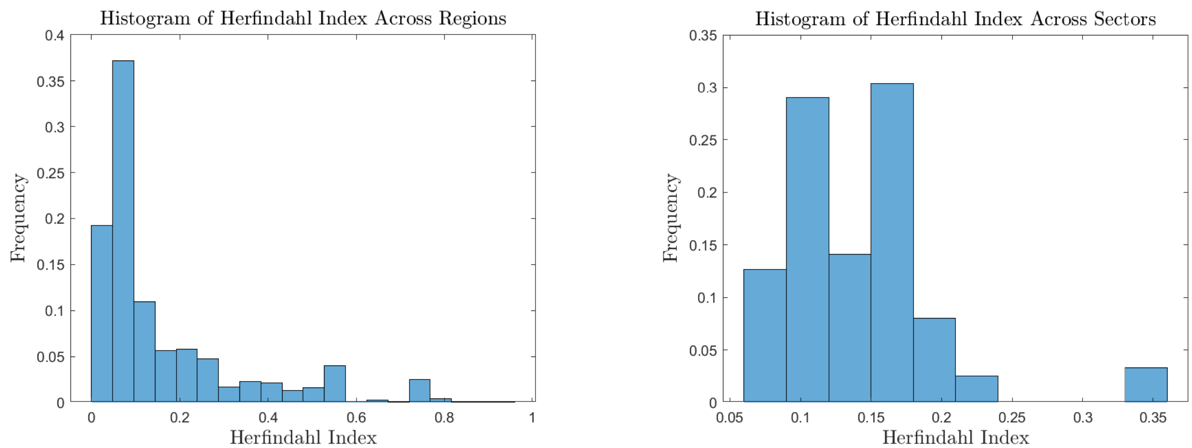
6.1 Simulation Procedure and Calibration

To generate model-simulated migration behavior, we must first specify the distribution of labor demand shocks and calibrate the relevant parameter ratio, θ/σ.

We consider three data-generating processes for shocks. The first set of shocks consists of independent standard normal draws across locations, i.e. $\hat{x}_\ell \sim N(0,1)$ and $\hat{z}_{\ell n} = \hat{x}_\ell$. We refer to

Figure 2: Concentration of Locations and Industries Among Switchers

Panel (a): Across Locations　　　　Panel (b): Across Industries



Notes: Panel (a) shows the distribution of HHI indexes measuring the concentration of sources and destinations for those switching locations, as defined in (34), weighted by the initial regional employment. Panel (b) similarly shows the distribution of HHI indexes across industries.

these shocks as "*iid* across locations." The second set of shocks introduces spatial correlation by assigning the same shock to all locations (microregions) in the same Brazilian state. The state-level shocks are independently and normally distributed, such that $\hat{x}_s \sim N(0, \sigma_2^2)$ and $\hat{z}_{\ell n} = \hat{x}_{s(\ell)}$, where $s(\ell)$ is the state containing location ℓ. We refer to these shocks as "*iid* across states." Finally, we generate independently and normally distributed shocks across industries, such that $\hat{x}_n \sim N(0, \sigma_3^2)$, and $\hat{z}_{\ell n} = \hat{x}_n$. We refer to these shocks as "*iid* across industries." For all three data generating processes, we calculate the regional shock $\hat{z}_\ell$ from the region-industry shocks $\hat{z}_{\ell n}$ using the weighted-average measure $\hat{z}_\ell = \sum_n \frac{L^0_{\ell n}}{L^0_\ell} \hat{z}_{\ell n}$. To ensure comparable magnitudes across the three types of shocks, we choose σ_2 and σ_3 so that the expected cross-location variance of shocks is equal to one.[43]

We draw on the literature to determine a realistic calibration of $\frac{\theta}{\sigma}$. Caliendo et al. (2019) estimate $\frac{1}{\theta} = 2.02$ across U.S. states (at the yearly level), so we set $\theta = 0.5$. In the absence of estimates of the Armington substitution elasticity across sub-national locations, we use standard estimates of the cross-country elasticity from Broda and Weinstein (2006) and Feenstra et al. (2018), both of which find $\sigma \approx 4$. We therefore set $\frac{\theta}{\sigma} = \frac{1}{8}$.

Given the relevant vector of shocks and calibrated value of θ/σ, we generate the change in employment based on $\hat{\mathbf{L}} = \Omega \hat{\mathbf{z}}$ (i.e. (20) without the error term, which we return to below).[44] Recall

[43]Specifically, $\sigma_2 = 1.076$ and $\sigma_3 = 5.595$ (when using flows for all workers regardless of skill).

[44]Note that we do not impose the low-mobility assumption in the simulation, since we use (20) rather than (21).

Table 2: Conventional Migration Regression Estimates

Shocks		Model: Location Frictions (1)	Model: Location-Industry Frictions (2)
iid across locations:	$100 \times \beta$	1.045	0.982
	R^2	0.577	0.608
iid across states:	$100 \times \beta$	0.254	0.247
	R^2	0.698	0.713
iid across industries:	$100 \times \beta$	0.875	0.283
	R^2	0.511	0.104

Notes: Average of regression estimates β and their associated R^2 across 500 simulations. See text for definitions of shock processes. Column "Location Frictions" simulates local employment changes using the model with cross-location frictions only (Section 2), while column "Location-Industry Frictions" simulates employment changes using the model with location-industry frictions (Section 4).

that Ω depends on the observed flow matrices Γ and Π, which we measure using the RAIS data (Section 5.1). To investigate the effects of industry switching frictions, we generate versions of $\hat{\mathbf{L}}$ from both the baseline model with regional frictions alone (Section 2) and the full version with industry-region frictions (Section 4), using the same structural parameters.[45]

We then estimate the conventional location-level migration regression using the model-implied employment changes and the associated regional shocks, and weighting by initial population, which yields an estimate of β and R^2 for a given simulation. We then repeat the entire process 500 times and report averages across the simulations, unless specified otherwise.

6.2 Initial Simulation Results

Table 2 presents the baseline simulation results. To interpret the magnitudes of the estimates in the model with location frictions only, reported in the first column, we rely on the characterization of β in Theorem 1. In our calibration, $\frac{2\theta}{\sigma} = \frac{1}{4}$. Table 1 reports the average mobility rate $\frac{M}{L} = 0.042$, and the mobility rate that would prevail without mobility frictions is $\frac{\widetilde{M}}{L} \approx 1$.[46] In the first row of Table 2, when shocks are *iid* across locations, there will be no attenuation, so $\rho \approx 0$ and $\frac{1-\rho}{1-\widetilde{\rho}} \approx 1$. Combining these, we expect $100 \times \beta \approx 1.05$, which is very similar to the realized estimate of 1.045

We do, however, maintain the assumption of small shocks, such that $\hat{x} \equiv dx/x = d\ln x$.

[45] Artuç et al. (2010) estimate $\frac{1}{\theta} = 1.88$ across 6 U.S. industries (at the yearly level), which supports the restriction in our model with location and industry frictions that θ is common to both dimensions.

[46] Footnote 24 shows that $\widetilde{M}/L$ equals one minus the Herfindahl index of population across regions.

in the model with only location frictions, confirming our theoretical predictions.[47]

The conventional migration regression performs poorly even with the simplest possible shock process (*iid* across locations) and in the absence of any unobserved labor demand or supply shocks. The average R^2 in these simulations is only 0.577, even though a perfect prediction would be possible using only the observed shocks. This poor predictive performance reflects the fact that β omits location-specific migration intensity $\frac{M_\ell}{L_\ell}$ and predicts the spillover effects from migrant-connected locations $(\hat{z}_{-\ell} - \overline{z})$ using only the direct shock facing the location $(\hat{z}_\ell - \overline{z})$, as discussed in Section 3.3.

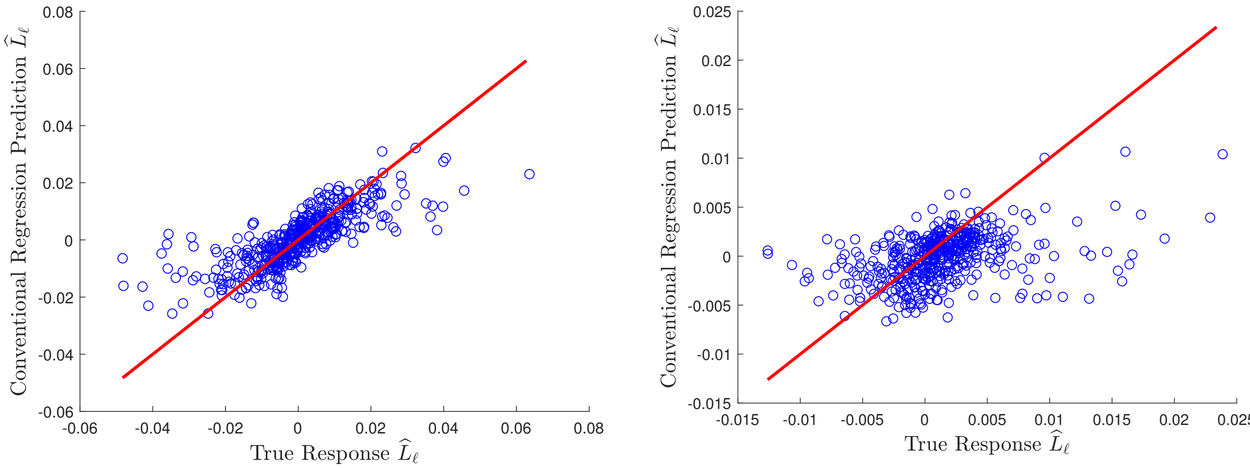
When shocks are correlated between migrant-connected locations, ρ in (25) is greater than zero and the estimate of β falls, biasing the estimates of population responses to counterfactual shocks. We can see this effect in the second row of Table 2, which presents results using *iid* shocks across states. The prevalence of intra-state migration (Table 1) induces substantial correlation between migrant-connected locations when facing state-level shocks. As a result, the β estimate falls sharply, with the implied attenuation factor declining from approximately 1 in the first row to 0.232 in the second row. Industry shift-share shocks in the third row similarly induce spatial correlation, but much less so than with state-level shocks, so the β estimate is only moderately below that for the *iid* location shocks.

While the model with location frictions alone yields straightforward intuition regarding the factors driving the conventional migration regression estimate of β , the model with both industry and location frictions is arguably more realistic and exhibits increased attenuation when shocks have an industry component. The simulation results from this full model appear in the second column of Table 2. When shocks are *iid* across locations, the regression estimate of 0.982 is very similar to the model with location frictions only, as is the associated R^2. This similarity reflects the finding in Section 4 that the two models are identical under the low-mobility approximation. However, when we introduce industry-level shocks in the bottom panel, the estimate is much smaller in the full model than in the baseline model, reflecting the additional attenuation induced by industry shocks in the presence of industry frictions. The regression also fits extremely poorly in this case, with

[47] The Table 1 coefficient could diverge from the prediction of Theorem 1 for two reasons, even with a very large number of simulation draws: the theorem uses the low-mobility approximation and characterizes the estimand of the regression, which is not the same as the estimator mean (because the ratio of expectations does not equal the expectation of the ratio in (23)). Neither of these differences turns out to be quantitatively important.

Figure 3: Conventional Regression Predictions vs. True Responses for Observed Shocks

Panel (a): *iid* shocks across locations Panel (b): *iid* shocks across industries



Notes: Scatter plots compare the predicted population response to the labor demand shocks against the true model-based response for a single representative simulation of the model with location-industry frictions (Section 4). Each point represents a location. The y-axis is the predicted population response based on the conventional migration regression estimate, β. Specifically, this prediction is $\beta(\hat{z}_\ell - \bar{z})$. The x-axis is the true model-based population change in the model with location-industry frictions. The left panel shows the result for *iid* shocks across locations, and the right panel shows the result for *iid* shocks across industries. 45-degree lines are shown in red. To maintain readability, we exclude the top and bottom 1% of regions based on the true model-based population change.

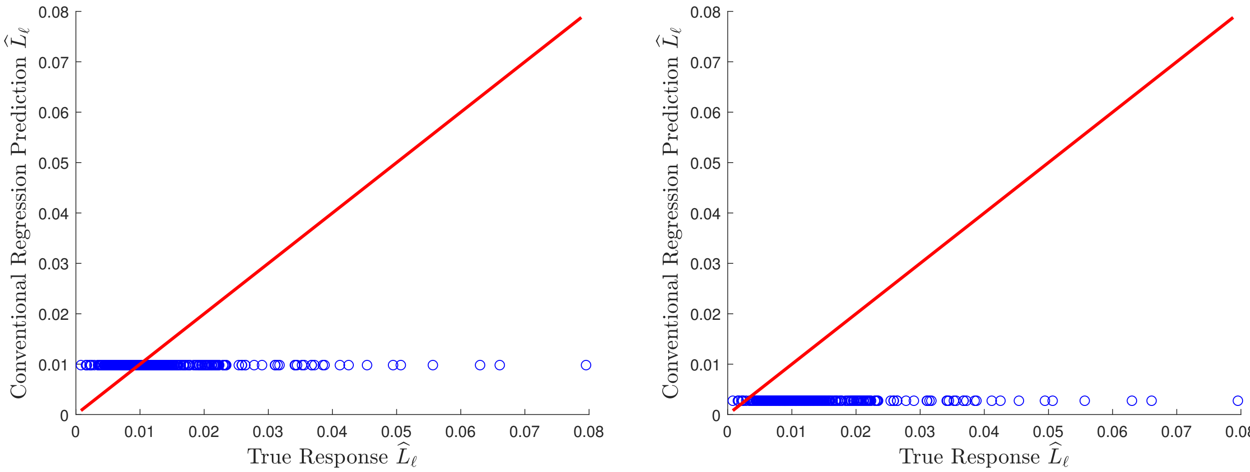
an R^2 value of only 10 percent, despite the fact that population growth depends solely upon the observed shocks.

Figure 3 visually presents the simulation results for a single representative simulation draw, emphasizing the conventional regression's poor performance in capturing the effects of observed shocks on local population growth. The y-axes measure predicted population changes based on the conventional migration regression estimates, $\beta(\hat{z}_\ell - \bar{z})$, and the x-axes measure the true population change in the full model with industry-region frictions. Each point represents a particular location. Panel (a) shows population changes when facing *iid* shocks across locations, while Panel (b) considers *iid* shocks across industries. Note the difference in x-axis scale between the two panels, showing that there is less true reallocation in Panel (b) with industry shocks. This difference confirms the theoretical conclusion from Section 4 that industry-switching frictions reduce true migration responses when labor demand shocks have an industry-specific component.

If the regression perfectly captured the effect of the shocks on local populations, all of the points would lie along the 45-degree line, shown in red. The fact that many points are distant from the line implies that the regression-based predictions are quite inaccurate in many cases, consistent with

Figure 4: Conventional Regression Predictions vs. True Responses for Counterfactual Unit Shocks

Panel (a): *iid* shocks across locations — Panel (b): *iid* shocks across industries



Notes: Scatter plots compare the predicted population response to a counterfactual unit shock to a single location against the true model-based response. Each point represents a location. The y-axis is the predicted population response based on the conventional migration regression estimate, β, averaged across simulations. The x-axis is the true model-based population change in the model with location-industry frictions (Section 4) when the relevant location faces a unit shock. The left panel shows the result when β is estimated using *iid* shocks across locations, and the right panel shows the result when β is estimated using *iid* shocks across industries. 45-degree lines are shown in red.

the low R^2 values in Table 2. This problem is substantially worse when facing industry-level shocks (in Panel (b)). The regression-based predictions are minimally related to the true effects. In this example, many regions experience substantial population changes as a result of the shocks despite the relatively small estimate of β. As a result, the regression-based predictions capture only half (50.7 percent) of the true population reallocation caused by the labor demand shocks (on average across simulations).[48]

Together, the results in Table 2 and Figure 3 confirm the patterns predicted in our theoretical analysis and reveal quantitatively substantial attenuation when shocks are correlated across migrant-connected locations. The low R^2 values also imply that the conventional regression yields quite poor predictions of the effects of the observed shocks used to estimate β even in a setting where migration is determined by the labor demand shocks alone, without additional noise. In other words, the conventional migration regression performs poorly in generating internally valid estimates of the effects of observed shocks.

[48]Formally, we define the true population reallocation index as $\frac{1}{2}\sum_\ell L_\ell \left|\hat{L}_\ell(\hat{\mathbf{z}}) - \hat{L}_\ell(\mathbf{0})\right|$ (where $\frac{1}{2}$ avoids double counting of population increases and decreases in different regions), while the regression-based predicted reallocation index is $\frac{1}{2}\sum_\ell L_\ell \left|\hat{\beta}(\hat{z}_\ell - \overline{z})\right|$. The ratio of the two reallocation indexes is 0.507 on average across simulations.

Figure 4 illustrates how the estimate of β cannot be interpreted as the effect of a counterfactual unit labor demand shock to a single location on that location's population either. The regression-based prediction for this counterfactual is $\beta \times 1$, so the counterfactual prediction is the constant β for all locations. The figure shows the true model-based population change when individually shocking each location and plots the responses against the estimate of β. Panel (a) considers the average β estimate using *iid* regional shocks in the model with location-industry frictions, where $100 \times \beta = 0.982$ (see Table 2). While the regression estimate based on *iid* regional shocks matches the mean response, it omits significant heterogeneity in the magnitude of the true counterfactual responses in each location. The situation is worse when β is estimated using a regional shift-share measure based on *iid* industry shocks in Panel (b), where $100 \times \beta = 0.283$. Not only does the estimate omit all heterogeneity, but it is subject to severe attenuation due to the shock correlation across locations and the industry-switching frictions (as seen in the bottom right panel of Table 2). As a result, the regression-based counterfactual estimate substantially understates the true effect in nearly every location.

6.3 Simulations by Skill Level

The simulation results thus far make clear that estimates from the conventional migration regression do not accurately reflect the effects of observed shocks on affected locations, nor can they be interpreted as the effect of counterfactual shocks to individual locations. We now investigate conventional migration estimates separately by skill level. Prior work has emphasized that less skilled workers' location choices are less responsive to local labor demand shocks than those of more highly skilled workers (Bound and Holzer, 2000; Cadena and Kovak, 2016). Our goal is to understand the extent to which the various problems identified above may explain these commonly-observed differences. For example, if labor demand shocks facing less skilled workers were more correlated between migrant-connected locations, their β estimates would be more attenuated than those for more skilled workers even if both groups of workers had similar mobility parameters.

To investigate this possibility, we implement the simulation procedure described in the preceding subsection separately for high-skilled and low-skilled workers.[49] We assign the same value of $\frac{\theta}{\sigma} = \frac{1}{8}$

[49]Specifically, we construct the mobility flows across locations and regions by skill level separately and redo the entire analysis. We do not consider a model in which both skill groups interact in the economy.

Table 3: Conventional Migration Regression Estimates – Comparison Across Skill Levels with Location-Industry Frictions

Shocks		All Workers (1)	High-Skilled (2)	Low-Skilled (3)
iid across locations:	$100 \times \beta$	0.982	1.014	0.956
	R^2	0.608	0.539	0.619
iid across states:	$100 \times \beta$	0.247	0.285	0.226
	R^2	0.713	0.703	0.669
iid across industries:	$100 \times \beta$	0.283	0.560	0.234
	R^2	0.104	0.139	0.110

Notes: Average of regression estimates β and R^2 across 500 simulations. See text for details of shock processes. All simulations are based on the model with location-industry frictions (Section 4). High-skilled workers are those with at least a high-school degree, and low-skilled workers are those with less than a high-school degree.

to both education groups so that both groups would be similarly responsive to shocks to one region at a time, given that their average mobility rates are also similar (as shown by Table 1). The identical mobility parameters and similar average mobility between the two groups implies that any substantial differences in conventional migration regression estimates across the two groups can only be explained by differences shock correlation or differences in the distribution of shocks faced by each group.

Table 3 shows the conventional migration estimates for all workers (replicating the second column of Table 2) and then by education group for the three shock processes in the model with location-industry frictions. The β estimates are quite similar across skill groups when shocks are *iid* across locations, reflecting our imposition of the same structural parameters on both groups and the groups' similar average migration rates. Both groups also experience roughly similar attenuation of β when facing spatially correlated state-level shocks. However, the two groups exhibit very different estimates when facing *iid* shocks across industries. The β estimate for low-skilled workers is less than half the magnitude of that for high-skilled workers. This difference suggests that low-skilled workers may appear to respond less to local industry shift-share shocks not because of fundamental differences in regional mobility but because they face different shift-share shocks or because they have larger costs of switching industries than high-skilled workers. These distinctions are essential for determining what kinds of policies will be most effective at addressing regional disparities in unemployment among low-skilled workers.

6.4 Introducing Noise

The preceding simulations determine the true population changes based on the model's structure and simulated labor demand shocks alone. In that setting, the model fits the data perfectly by construction, so we were unable to assess the quality of estimates based on the proposed NLLS procedure introduced in Section 3.4. Here, we add noise from unobserved changes in labor demand and supply to the baseline simulations of Section 6.2, to assess the relative performance of the conventional migration regression and the proposed NLLS procedure and to better mimic the data observed in a typical study of migration responses to local shocks. We assume the noise to be orthogonal to the observed labor demand shocks as in (19) to avoid introducing endogeneity bias.

We generate the true population changes at the location-industry level using the full model with location-industry frictions. For a shock vector $\hat{\mathbf{z}}$, we calculate the change in population as $\hat{\mathbf{L}}(\hat{\mathbf{z}}) - \hat{\mathbf{L}}(\mathbf{0}) = \Omega\hat{\mathbf{z}}$, which is the model-implied effect of shocks $\hat{\mathbf{z}}$ on the number of workers in each location-industry pair, and $\hat{\mathbf{L}}(\mathbf{0}) = \boldsymbol{\zeta}$, which represents unobserved shocks. Here Ω is computed using the true parameter $\frac{\theta}{\sigma} = \frac{1}{8}$, and we allow the unobserved shocks in $\boldsymbol{\zeta} = (\zeta_{\ell n})$ to consist of additive location, industry, and location-industry components, such that

$$\zeta_{\ell n} = \epsilon_\ell + \epsilon_n + \epsilon_{\ell n}.$$

To ensure a realistic scale for this noise term, we take the observed changes in employment in each location-industry cell and regress them on location fixed effects and industry fixed effects, which allows us to measure the location-level variation, industry-level variation, and residual location-industry variation, which we use to calibrate the variance of each noise component.[50]

We compare the in-sample fit of four methods for estimating the population effects of the simulated labor demand shocks. We first estimate β from the conventional regression (22) and, like before, take $\beta\left(\hat{z}_\ell - \bar{z}\right)$ as the predicted population growth in location ℓ. We then estimate θ/σ with two versions of the NLLS specification (28) based on the correctly specified model allowing

[50]From historical data, we estimate $\hat{L}_{\ell n} = \epsilon_\ell + \epsilon_n + \epsilon_{\ell n}$, where $\hat{L}_{\ell n}$ is the proportional change in the number of workers in location ℓ and industry n (averaged across years of our data), ϵ_ℓ are location fixed effects, ϵ_n are industry fixed effects, and $\epsilon_{\ell n}$ is the error term. The employment-weighted standard deviation of the estimated location terms is 0.0158, of the estimate industry terms is 0.0139, and of the estimated residual is 0.0512. We then simulate normally-distributed noise components as $\epsilon_\ell \sim \mathcal{N}(0, 0.0158^2)$, $\epsilon_n \sim \mathcal{N}(0, 0.0139^2)$, and $\epsilon_{\ell n} \sim \mathcal{N}(0, 0.0512^2)$. Finally, we demean the resulting overall noise term $\zeta_{\ell n}$ so that $\sum_{\ell,n} \frac{L_{\ell n}}{L}\zeta_{\ell n} = 0$ in each simulation.

for location-industry mobility frictions. First, we perform the estimation on location-industry observations, with the simulated $\hat{L}_{\ell n}$ as the dependent variable and the model's prediction for it as a function of $\hat{\mathbf{z}}$ and θ/σ on the right-hand side. The second version is analogous, except estimation is done with regional observations after aggregating across industries, in parallel to the conventional regression. That is, we use the simulated $\hat{L}_\ell$ as the dependent variable and the model's prediction for it, which aggregates predicted $\hat{L}_{\ell n}$ across industries for each region, on the right-hand side. Since both of these approaches are based on the correctly specified model, their prediction is only imperfect because of the noisy estimates of θ/σ. Finally, we consider NLLS estimation of (28) where $\Omega(\theta/\sigma)$ is constructed from the misspecified baseline model that does not allow industry mobility frictions, and again using $\hat{L}_\ell$ as the dependent variable. The advantage of this specification is that it is feasible without data on mobility across location-industry cells; it only requires pre-shock regional flows, which are much more readily available in many countries.

We implement this simulation and estimation procedure across $S = 500$ simulations and use the Root Mean Square Error (RMSE) as the measure of in-sample predictive quality for regional population changes $\hat{L}_\ell$. For a given simulation s, let $\hat{L}^{\text{true}}_{\ell;s} \equiv \hat{L}_\ell(\hat{\mathbf{z}}_s) - \hat{L}_\ell(\mathbf{0})$ and let $\hat{\boldsymbol{L}}^{\text{predicted}}_{\ell;s}$ be the predicted effect on local population growth using one of the four methods just described.

$$RMSE = \sqrt{\frac{1}{S}\sum_{s=1}^{S}\left[\sum_{\ell=1}^{R}\frac{L_\ell}{L}\left(\hat{L}^{\text{true}}_{\ell;s} - \hat{L}^{\text{predicted}}_{\ell;s}\right)^2\right]}, \tag{35}$$

which averages squared prediction errors both across locations (with population weights) and across the simulations. We rescale this RMSE measure relative to its value for the uninformed prediction, $\hat{L}^{\text{predicted}}_{\ell;s} = 0$, i.e. by the square root of the average cross-simulation variance of the component of population changes due to the shock:

$$RMSE_{\text{uninformative}} = \sqrt{\frac{1}{S}\sum_{s=1}^{S}\left[\sum_{\ell=1}^{R}\frac{L_\ell}{L}\left(\hat{L}^{\text{true}}_{\ell;s}\right)^2\right]}. \tag{36}$$

Table 4 reports the results for our four methods and for *iid* location and *iid* industry shock processes. The first row confirms that predictions by the conventional regression are very poor: it outperforms the uninformative prediction by less than 30% when the shocks are location-specific and performs 2% *worse* than the uninformative prediction for industry shocks. The second and

Table 4: RMSE of Conventional Regression and NLLS Predictions

Prediction method	RMSE, relative to uninformative prediction	
	iid location shocks (1)	*iid* industry shocks (2)
Conventional regression	0.719	1.020
NLLS, region-industry level	0.171	0.029
NLLS, regional level	0.250	0.339
NLLS, misspecified regional model	0.262	0.917

Notes: Each row corresponds to a method of predicting the effect of observed shocks on regional population, while each column defines the shock process; see the text for details. Each cells reports the ratio of the root mean squared error (RMSE) for a given prediction method to the RMSE from the uninformative prediction, computed via (35) and (36), respectively. All simulations are based on the model with location-industry frictions (Section 4).

third rows show that NLLS performs well when taking into account mobility frictions across both regions and industries. When estimated at the location-industry level, its RMSE is only 17% of that of the uninformative prediction with *iid* location shocks, and as little as 3% for *iid* industry shocks. For *iid* location shocks, estimation at the more aggregated regional level increases the estimation error only slightly, from 17% to 25% of the uninformative prediction RMSE. However, for *iid* industry shocks, data aggregation raises the RMSE from 3% to 34% — although still three times lower than with the conventional regression.[51] Finally, the fourth row of Table 4 reports the predictive quality of the misspecified model without industry frictions. For location shocks, the RMSE is almost identical to the previous row, at 26%. Indeed, as discussed before, the predictions of the baseline and full models would be identical if the low-mobility approximation was precise. However, with *iid* industry shocks the misspecified model provides very poor predictions, with a RMSE of 92% relative to the uninformative prediction and only slightly ahead of the conventional regression.

The absolute and, possibly, relative performance of different methods may depend on how much useful variation the observed shocks provide relative to the noise in the data — a parameter which may vary across empirical settings. To check robustness of our findings, we therefore repeat the Table 4 analysis varying the relative variance of $\hat{\mathbf{z}}$ and $\boldsymbol{\zeta}$, specifically by multiplying $\boldsymbol{\zeta}$ by a factor $\delta \in [0, 4]$, with $\delta = 1$ corresponding to the Table 4 results. Appendix Figure A1 shows that the

[51] This gap in performance can be linked to how much of the variation of the regional vs. region-industry levels the observed shocks capture. With *iid* industry shocks, 83.1% of the $\hat{L}_{\ell n}$ variation is due to the observed shocks, compared to only 10.0% for $\hat{L}_{\ell}$ (measured by the employment-weighted R^2 on average across simulations). This difference is much smaller, and in the opposite direction, for *iid* regional shocks: they on average explain 7.7% of the variation in $\hat{L}_{\ell n}$ but a higher 21.2% of the variation in $\hat{L}_{\ell}$.

results are robust.[52]

Taken together, these results yield two lessons. First, accounting for the spillovers via the model-consistent NLLS procedure allows one to capture the population responses of local labor demand shocks even in the presence of noise in the data, which the conventional regression is not capable of. Second, ignoring the industry dimension of the data — in either the unit of observations or the model employed — is relatively innocuous when the shocks are purely region-specific. However, when the shocks are industry-driven, it is crucial to take the structure of industry mobility into account, in using the appropriate model as well as estimating the structural parameter at the disaggregated location-industry level.

7 Conclusion

This paper has examined how to interpret conventional migration regression estimates relating changes in local population or employment to observed local labor demand shocks. These estimates often yield the seemingly puzzling finding that workers do not migrate in response to local labor demand shocks despite large effects on other outcomes, such as local wages. Using a simple model of local labor markets with mobility costs, we find that this puzzling conclusion may be unwarranted.

Analytical results and quantitative simulations show that the conventional migration regression provides inaccurate estimates of both the local population effects of shocks to individual locations and the within-sample effects of the entire vector of observed shocks (the distinction between which is often overlooked in the literature). These issues are particularly severe when the local shocks are constructed as shift-share aggregates of national industry shocks, which is common in the literature. These findings suggest that population may have responded substantially to the shocks studied in prior work, even when the associated conventional migration regression estimates were small.

Going forward, we recommend that researchers adopt methods that account for spillovers of shocks between connected locations, as in our proposed NLLS procedure. Yet, to implement that and related analyses, the data demands are significant. Our analysis using Brazilian data shows the substantial practical benefit of observing transitions between relatively detailed location-industry

[52] The only exception is NLLS estimated from *iid* industry shocks at the level of regions, which is sensitive to δ: it performs well for small δ (although uniformly worse than NLLS estimation by region-industry) but not for large amounts of noise.

cells to successfully capture the interrelationships in a large and diverse economy.

References

Adão, Rodrigo, "Worker Heterogeneity, Wage Inequality, and International Trade: Theory and Evidence from Brazil," *Working Paper*, 2016.

_ , Costas Arkolakis, and Federico Esposito, "General Equilibrium Effects in Space: Theory and Measurement," *Working Paper*, 2021.

Adão, Rodrigo, Michal Kolesár, and Eduardo Morales, "Shift-Share Designs: Theory and Inference," *The Quarterly Journal of Economics*, 08 2019, *134* (4), 1949–2010.

Agrawal, David R. and Dirk Foremny, "Relocation of the Rich: Migration in Response to Top Tax Rate Changes from Spanish Reforms," *The Review of Economics and Statistics*, 05 2019, *101* (2), 214–232.

Ahlfeldt, Gabriel M., Stephen J. Redding, Daniel M. Sturm, and Nikolaus Wolf, "The Economics of Density: Evidence From the Berlin Wall," *Econometrica*, 2015, *83* (6), 2127–2189.

Albert, Christoph and Joan Monràs, "The Regional Impact of Economic Shocks:Why Immigration is Different from Import Competition," *Working paper*, 2020.

_ , Paula Bustos, and Jacopo Ponticelli, "The effects of climate change on labor and capital reallocation," *Working Paper*, 2021.

Amior, Michael and Alan Manning, "Commuting, Migration and Local Joblessness," *CEP Discussion Paper No 1623*, 2019, (October), 1–45.

Anderson, James E. and Eric van Wincoop, "Gravity with Gravitas: A Solution to the Border Puzzle," *American Economic Review*, March 2003, *93* (1), 170–192.

Armington, Paul S., "A Theory of Demand for Products Distinguished by Place of Production," *IMF Staff Papers*, 1969, *16* (1), 159–178.

Arthi, V, B Beach, and W Hanlon, "Recessions, Mortality, and Migration Bias: Evidence from the Lancashire Cotton Famine," 2019, (1552692).

Artuç, Erhan, Shubham Chaudhuri, and John McLaren, "Trade shocks and labor adjustment: A structural empirical approach," *American Economic Review*, 2010, *100* (3), 1008–1045.

Autor, David, David Dorn, and Gordon Hanson, "On the Persistence of the China Shock," *NBER Working Paper*, 2021, (29401).

Autor, David H., David Dorn, and Gordon H. Hanson, "The China Syndrome: Local Labor Market Impacts of Import Competition in the United States," *American Economic Review*, 2013, *103* (6), 2121–2168.

Bartik, Alexander W., Janet Currie, Michael Greenstone, and Christopher R. Knittel, "The local economic and welfare consequences of hydraulic fracturing," *American Economic Journal: Applied Economics*, 2019, *11* (4), 105–155.

Berkes, Enrico, Ruben Gaetani, and Martí Mestieri, "Cities and Technological Waves," *CEPR Discussion Paper*, 2021, (DP16794).

Black, Dan, Terra McKinnish, and Seth Sanders, "The economic impact of the coal boom and bust," *Economic Journal*, 2005, *115* (503), 449–476.

Blanchard, Olivier Jean and Lawrence F. Katz, "Regional Evolutions," *Brookings Papers on Economic Activity*, 1992, (1), 1–75.

Borusyak, Kirill and Peter Hull, "Non-Random Exposure to Exogenous Shocks: Theory and Applications," *Mimeo*, 2021.

_ , _ , and Xavier Jaravel, "Quasi-Experimental Shift-Share Research Designs," *Review of Economic Studies*, 2022, *89* (1), 181–213.

Bound, John and Harry J. Holzer, "Demand shifts, population adjustments, and labor market outcomes during the 1980s," *Journal of Labor Economics*, 2000, *18* (1), 20–54.

Boustan, Leah, Matthew E. Kahn, Paul W. Rhode, and Maria Lucia Yanguas, "The Effect of Natural Disasters on Economics Activity in US Counties: A Century of Data," *Journal of Urban Economics*, 2020, *118* (103257).

Broda, Christian and David E. Weinstein, "Globalization and the Gains From Variety," *The Quarterly Journal of Economics*, 2006, *121* (2), 541–585.

Bustos, Paula, Bruno Caprettini, and Jacopo Ponticelli, "Agricultural Productivity and Structural Transformation: Evidence from Brazil," *American Economic Review*, June 2016, *106* (6), 1320–65.

Cadena, Brian C. and Brian K. Kovak, "Immigrants equilibrate local labor markets: Evidence from the great recession," *American Economic Journal: Applied Economics*, 2016, *8* (1), 257–290.

Caliendo, Lorenzo, Maximiliano Dvorkin, and Fernando Parro, "Trade and Labor Market Dynamics: General Equilibrium Analysis of the China Trade Shock," *Econometrica*, 2019, *87* (3), 741–835.

Choi, Jiwon, Ilyana Kuziemko, Ebonya Washington, and Gavin Wright, "Local Economic and Political Effects of Trade Deals: Evidence from NAFTA," *NBER Working Paper*, 2021, (29525).

Dauth, Wolfgang, Sebastian Findeisen, and Jens Suedekum, "The rise of the east and the far east: German labor markets and trade integration," *Journal of the European Economic Association*, 2014, *12* (6), 1643–1675.

De Negri, João Alberto, Paulo Furtado de Castro, Natalia Ribeiro de Souza, and Jorge Saba Arbache, "Mercado Formal de Trabalho: Comparação entre os Microdados da RAIS e da PNAD," *Texto Para Discussão*, 2001, (No. 840), 29.

Dix-Carneiro, Rafael and Brian K. Kovak, "Trade liberalization and the skill premium: A local labor markets approach," *American Economic Review: Papers & Proceedings*, 2015, *105* (5), 551–557.

_ and _ , "Trade Liberalization and Regional Dynamics," *American Economic Review*, 2017, *107* (10), 2908–2946.

_ and _ , "Margins of Labor Market Adjustment to Trade," *Journal of International Economics*, 2019, *117*, 125–142.

Faber, Marius, Andrés Sarto, and Marco Tabellini, "Local Shocks and Internal Migration: The Disparate Effects of Robots and Chinese Imports in the US," 2021, (14623).

Fan, Jingting, "Internal Geography, Labor Mobility, and the Distributional Impacts of Trade," *American Economic Journal: Macroeconomics*, July 2019, *11* (3), 252–88.

Feenstra, Robert, Philip Luck, Maurice Obsteld, and Katheryn Russ, "In Search of the Armington Elasticity," *Review of Economics and Statistics*, 2018, *100* (1), 135–150.

Foote, Andrew, Michel Grosz, and Ann Stevens, "Locate Your Nearest Exit: Mass Layoffs and Local Labor Market Response," *Industrial and Labor Relations Review*, 2019, *72* (1), 101–126.

Galle, Simon, Andrés Rodríguez-Clare, and Moises Yi, "Slicing the Pie: Quantifying the Aggregate and Distributional Effects of Trade," *Review of Economic Studies*, forthcoming.

Greenland, Andrew, John Lopresti, and Peter McHenry, "Import Competition and Internal Migration," *The Review of Economics and Statistics*, 2019, *101* (1), 44–59.

Hakobyan, Shushanik and John McLaren, "Looking for Local Labor Market Effects of NAFTA," *The Review of Economics and Statistics*, 10 2016, *98* (4), 728–741.

Head, Keith and Thierry Mayer, "Chapter 3 - Gravity Equations: Workhorse,Toolkit, and Cookbook," in Gita Gopinath, Elhanan Helpman, and Kenneth Rogoff, eds., *Handbook of International Economics*, Vol. 4 of *Handbook of International Economics*, Elsevier, 2014, pp. 131–195.

Hyman, Benjamin G., "Can Displaced Labor Be Retrained? Evidence from Quasi-Random Assignment to Trade Adjustment Assistance," *Working paper*, 2018.

IBGE, *Censo Demográfico 2000: Documentação das Microdados da Amostra*, Instituto Brasileiro de Geografia e Estatística, 2002.

Imbert, Clement, Marlon Seror, Yifan Zhang, and Yanos Zylberberg, "Migrants and Firms: Evidence from China," *working paper*, 2021.

Kleven, Henrik Jacobsen, Camille Landais, and Emmanuel Saez, "Taxation and International Migration of Superstars: Evidence from the European Football Market," *American Economic Review*, August 2013, *103* (5), 1892–1924.

McCaig, Brian, "Exporting out of poverty: Provincial poverty in Vietnam and U.S. market access," *Journal of International Economics*, 2011, *85* (1), 102–113.

Mian, Atif and Amir Sufi, "What Explains the 2007–2009 Drop in Employment?," *Econometrica*, 2014, *82* (6), 2197–2223.

Monràs, Joan, "Economic Shocks and Internal Migration," *Ssrn*, 2020.

Monte, Ferdinando, Stephen J. Redding, and Esteban Rossi-Hansberg, "Commuting, Migration, and Local Employment Elasticities," *American Economic Review*, 2018, *108* (12), 3855–3890.

Morten, Melanie and Jacqueline Oliveira, "The Effects of Roads on Trade and Migration: Evidence from a Planned Capital City," *NBER Working Paper*, 2018, (22158).

Notowidigdo, Matthew J., "The Incidence of Local Labor Demand Shocks," *Journal of Labor Economics*, 2020, *38* (3), 687–725.

Saboia, Joao L.M. and Ricardo M.L. Tolipan, "A Relação Anual de Informações Sociais (RAIS) e o mercado formal de trabalho no Brasil: uma nota," 1985.

Tian, Yuan, Junjie Xia, and Rudai Yang, "Trade-Induced Urbanization and the Making of Modern Agriculture," *working paper*, 2021.

Tombe, Trevor and Xiaodong Zhu, "Trade, Migration, and Productivity: A Quantitative Analysis of China," *American Economic Review*, May 2019, *109* (5), 1843–72.

Topalova, Petia, "Factor Immobility and Regional Impacts of Trade Liberalization : Evidence on Poverty from India," *American Economic Journal: Applied Economics*, 2010, *2* (4), 1–41.

Topel, Robert H., "Local labor markets," *Journal of Political Economy*, 1986, *94* (3), S111–S143.

Wozniak, Abigail, "Are College Graduates More Responsive to Distant Labor Market Opportunities?," *Journal of Human Resources*, 2010, *45* (4), 944–970.

Yagan, Danny, "Moving to Opportunity? Migratory Insurance over the Great Recession," *Working Paper*, 2014.

_ , "Employment Hysteresis from the Great Recession," *Journal of Political Economy*, 2019, *127* (5), 2505–2558.

A Theory Appendix

A.1 Incorporating Labor Supply Shocks

In this appendix we show that the expressions from Section 2 extend naturally when the moving costs $\tau_{\ell d}$, in addition to D_ℓ, are allowed to change, generating local labor supply shocks. In that case, (12) is replaced by

$$\begin{aligned}\hat{L}_d &= \sum_\ell \gamma_{\ell d} \hat{\pi}_{\ell d} \\ &= \theta \left(\hat{w}_d - \sum_\ell \gamma_{\ell d} \sum_{d'} \pi_{\ell d'} \hat{w}_{d'} \right) + \nu_d\end{aligned}$$

where the labor supply shock is defined as

$$\nu_d = -\theta \sum_\ell \gamma_{\ell d} \left(\hat{\tau}_{\ell d} - \sum_{d'} \pi_{\ell d'} \hat{\tau}_{\ell d'} \right).$$

The supply shock is high when moving costs to d decline, relative to the moving costs from the same origins to other destinations. In vector form, $\hat{\mathbf{L}} = \theta \left(I - \Gamma' \Pi \right) \hat{\mathbf{w}} + \boldsymbol{\nu}$ with $\boldsymbol{\nu} = \{\nu_d\}$. Solving the system of demand and supply (i.e. using (9)), this implies

$$\hat{\mathbf{L}} = \Omega \hat{\mathbf{D}} + (I - \Omega)\boldsymbol{\nu}.$$

Finally, (18) yields

$$\hat{\mathbf{L}} = \Omega \hat{\mathbf{z}} + \boldsymbol{\zeta}_2 + (I - \Omega)\boldsymbol{\nu},$$

which generates a second, supply-driven, error term. The results of the paper apply if the $\hat{\mathbf{z}}$ shocks are mean-independent from both unobserved labor demand and labor supply shocks.

A.2 Low-Mobility Approximation Derivation

Plugging $\Gamma = I + \Delta\Gamma$ and $\Pi = I + \Delta\Pi$ into (16) and assuming $\Delta\Gamma' \Delta\Pi \approx 0$ yields

$$\Omega \approx I - \left(I - \frac{\theta}{\sigma} \left(\Delta\Gamma' + \Delta\Pi \right) \right)^{-1}.$$

Let $A \equiv \frac{\theta}{\sigma} \left(\Delta\Gamma' + \Delta\Pi \right)$ such that $\Omega = I - (I - A)^{-1}$. It is straightforward to verify that $(I - A)^{-1} = I + A + A^2 + A^3 + \ldots$ and therefore $\Omega = -\left(A + A^2 + A^3 + \ldots \right)$. Moreover, under the low-mobility assumption, $A + A^2 + A^3 + \cdots \approx A$ since $\Delta\Gamma' \Delta\Pi \approx \Delta\Gamma' \Delta\Gamma' \approx \Delta\Pi \Delta\Pi \approx \mathbf{0}$.We thus have

$$\Omega \approx -A = \frac{\theta}{\sigma} \left(I - \Gamma' + I - \Pi \right)$$

and

$$\hat{\mathbf{L}} = \frac{\theta}{\sigma} \left(I - \Gamma' + I - \Pi \right) \hat{\mathbf{D}},$$

which is (17) in matrix notation.

A.3 Derivation of Equation (21)

From (17) we have:

$$\begin{aligned}
\hat{L}_\ell &\approx \frac{2\theta}{\sigma}\sum_{k\neq\ell}\frac{\gamma_{k\ell}+\pi_{\ell k}}{2}\left(\hat{D}_\ell-\hat{D}_k\right).\\
&= \frac{2\theta}{\sigma}\sum_{k\neq\ell}\frac{f_{k\ell}/L_\ell+f_{\ell k}/L_\ell^0}{2}\left(\hat{D}_\ell-\hat{D}_k\right)\\
&\approx \frac{2\theta}{\sigma}\sum_{k\neq\ell}\frac{F_{k\ell}}{L_\ell}\left(\hat{D}_\ell-\hat{D}_k\right)\\
&= \frac{2\theta}{\sigma}\frac{M_\ell}{L_\ell}\left(\hat{D}_\ell-\sum_{k\neq\ell}\frac{F_{k\ell}}{M_\ell}\hat{D}_k\right)\\
&= \frac{2\theta}{\sigma}\frac{M_\ell}{L_\ell}\left(\hat{z}_\ell+\zeta_{1\ell}-\sum_{k\neq\ell}\frac{F_{k\ell}}{M_\ell}\left(\hat{z}_k+\zeta_{1k}\right)\right)\\
&= \frac{2\theta}{\sigma}\frac{M_\ell}{L_\ell}\left(\hat{z}_\ell-\hat{z}_{-\ell}\right)+\zeta_{2\ell}.
\end{aligned}$$

where the first line cancels the terms for the same origin and destination, the third line uses that under the low-mobility approximation $L_\ell^0 \approx L_\ell$, and the other lines rearrange terms and use the definitions of $F_{k\ell}$, M_ℓ, and $\hat{z}_{-\ell}$.

A.4 Analyzing Observed Population Changes

Let $\mathbf{L}^0$ is the exogenously given vector of initial populations at $t=0$, and $\mathbf{L}^1(\hat{\mathbf{z}},\zeta_1)$ be the vector of populations at $t=1$ as a function of observed and unobserved labor demand shocks.[53] Then the observed population change between $t=0$ and $t=1$ is $\log\mathbf{L}^1(\hat{\mathbf{z}},\zeta_1)-\log\mathbf{L}^0$. We first can compare it to the population change relative to the no-shock counterfactual, as in footnote 21: by equation (20),

$$\log\mathbf{L}^1(\hat{\mathbf{z}},\zeta_1)-\log\mathbf{L}^1(\mathbf{0},\mathbf{0})\equiv\hat{\mathbf{L}}=\Omega\cdot\left(\hat{\mathbf{z}}+\zeta_1\right).$$

This counterfactual differs from the observed change because of the secular trend in population,

$$\log\mathbf{L}^1(\mathbf{0},\mathbf{0})-\log\mathbf{L}^0=\log\left(\Pi\mathbf{L}^0\right)-\log\mathbf{L}^0,$$

which is non-stochastic in our framework. Thus, it is orthogonal from $\hat{\mathbf{z}}$ by the as-good-as-randomness of the observed shocks.

The same argument applies to the causal impact of the observed shocks on population, $\log\mathbf{L}^1(\hat{\mathbf{z}},\zeta_1)-\log\mathbf{L}^1(\mathbf{0},\zeta_1)$, as in Section 3.3. It differs from the observed population change by

$$\log\mathbf{L}^1(\mathbf{0},\zeta_1)-\log\mathbf{L}^0=\log\left(\Pi\mathbf{L}^0\right)-\log\mathbf{L}^0+\Omega\zeta_1,$$

which captures both the secular trend and the effects of (small) unobserved labor demand shocks, and is again orthogonal to from $\hat{\mathbf{z}}$.

[53] Labor supply shocks which can be incorporated as in Appendix A.1

A.5 $\hat{\beta}$ Is Close to β in Large Samples

In this appendix we provide the regularity conditions under which both $\hat{\beta}$ and β converge to the same number in an asymptotic sequence of economies with a growing number of domestic regions R. Under those conditions $\hat{\beta}$ and β are expected to be similar to each other when the observed number of regions is large.

We consider a general sequence of data-generating processes, indexed by R, which jointly determine all relevant economic variables. We have:

Proposition A1. *Suppose* $\sum_\ell \frac{L_\ell^0}{L^0}\hat{L}_\ell\left(\hat{z}_\ell-\bar{z}\right)\xrightarrow{p} C$ *and* $\sum_\ell \frac{L_\ell^0}{L^0}\left(\hat{z}_\ell-\bar{z}\right)^2\xrightarrow{p} V>0$ *as* $R\to\infty$*, and both sequences are uniformly integrable. Then* $\hat{\beta}\xrightarrow{p}\frac{C}{V}$ *and* $\beta\to\frac{C}{V}$.

Proof. The first statement follows by the continuous mapping theorem. For the second statement, note that uniform integrability implies $\mathbb{E}\left[\sum_\ell \frac{L_\ell^0}{L^0}\hat{L}_\ell\left(\hat{z}_\ell-\bar{z}\right)\right]\to C$ and $\mathbb{E}\left[\sum_\ell \frac{L_\ell^0}{L^0}\left(\hat{z}_\ell-\bar{z}\right)^2\right]\to V$. Thus, $\beta\to\frac{C}{V}$, as required. □

A.6 Generalization and Proof of Theorem 1

We prove a generalization of Theorem 1 which applies beyond homoskedastic shocks:

Theorem A1. *Suppose the data are generated by the low-mobility approximation (21) to the baseline model. Then*

$$\beta=\frac{2\theta}{\sigma}\cdot\frac{M^v/L}{\widetilde{M}^v/L}\cdot\frac{1-\rho}{1-\tilde{\rho}}. \tag{37}$$

Here $\frac{M^v}{L}\equiv\frac{\sum_\ell M_\ell v_\ell}{\sum_\ell L_\ell v_\ell}$ *is the national average migration share weighted by shock variances* $v_\ell=\mathrm{Var}\left[\hat{z}_\ell\right]$*, and* $\rho=\frac{\sum_\ell M_\ell v_\ell\rho_\ell}{\sum_\ell M_\ell v_\ell}\in[-1,1]$ *for* $\rho_\ell=\frac{\mathrm{Cov}[\hat{z}_\ell,\hat{z}_{-\ell}]}{v_\ell}$*. Similarly,* $\frac{\widetilde{M}^v}{L}\equiv\frac{\sum_\ell \widetilde{M}_\ell v_\ell}{\sum_\ell L_\ell v_\ell}$, $\tilde{\rho}=\frac{\sum_\ell \widetilde{M}_\ell v_\ell\tilde{\rho}_\ell}{\sum_\ell \widetilde{M}_\ell v_\ell}\in[-1,1]$*, and* $\tilde{\rho}_\ell=\frac{\mathrm{Cov}[\hat{z}_\ell,\tilde{z}_{-\ell}]}{v_\ell}$ *for* $\widetilde{z}_{-\ell}=\sum_{d\neq\ell}\frac{(\widetilde{f}_{\ell d}^0+\widetilde{f}_{d\ell}^0)/2}{\widetilde{M}_\ell^0}\hat{z}_d$*. Moreover, when shocks are homoskedastic, i.e.* $v_\ell=v$ *for all* ℓ*, then* ρ *and* $\tilde{\rho}$ *coincide with the Theorem 1 definitions,* $M^v=M$*, and* $\widetilde{M}^v=\widetilde{M}$.

Proof. Given that regression (22) includes the constant and our interest is in the slope parameter, we assume without loss of generality that the shocks have zero expectation, $\mu=0$. Moreover, under the low-mobility approximation, $L_\ell\approx L_\ell^0$, and weighting the regression by the no-shock counterfactual employment levels L_ℓ is equivalent to weighting it by pre-shock employment L_ℓ^0. We therefore consider L_ℓ weights, which are more convenient for analytical results, and similarly redefine $\bar{z}=\sum_\ell\frac{L_\ell}{L}\hat{z}_\ell$.

We start with the numerator of β in (24), which can be represented as

$$\begin{aligned}\mathbb{E}\left[\sum_\ell L_\ell\hat{L}_\ell\left(\hat{z}_\ell-\bar{z}\right)\right]&=\mathbb{E}\left[\sum_\ell L_\ell\hat{L}_\ell\hat{z}_\ell\right]\\&=\mathbb{E}\left[\frac{2\theta}{\sigma}\sum_\ell M_\ell\hat{z}_\ell\left(\hat{z}_\ell-\hat{z}_{-\ell}\right)\right]\\&=\frac{2\theta}{\sigma}\sum_\ell M_\ell v_\ell\left(1-\rho_\ell\right).\end{aligned}\tag{38}$$

Here the first line used the fact that $\sum_\ell L_\ell\hat{L}_\ell\bar{z}=\bar{z}\sum_\ell L_\ell\hat{L}_\ell=\bar{z}\hat{L}=0$ because total national population is not allowed to change in the model. The second line plugged in (21) and used as-good-as-random assignment of $\hat{\mathbf{z}}$, and the last line used the definitions of v_ℓ and ρ_ℓ.

Now turn to the denominator of β. Since $\bar{z}$ is a weighted average of the shock to ℓ and other regions, specifically $\bar{z} = \frac{L_\ell}{L}\hat{z}_\ell + \left(1 - \frac{L_\ell}{L}\right)\widetilde{z}_{-\ell}$, we have

$$\hat{z}_\ell - \bar{z} = \left(1 - \frac{L_\ell}{L}\right)\left(\hat{z}_\ell - \widetilde{z}_{-\ell}\right). \tag{39}$$

Thus,

$$\begin{aligned}
\mathbb{E}\left[\sum_\ell L_\ell \left(\hat{z}_\ell - \bar{z}\right)^2\right] &= \mathbb{E}\left[\sum_\ell L_\ell \hat{z}_\ell \left(\hat{z}_\ell - \bar{z}\right)\right] \\
&= \mathbb{E}\left[\sum_\ell L_\ell \left(1 - \frac{L_\ell}{L}\right)\hat{z}_\ell \left(\hat{z}_\ell - \widetilde{z}_{-\ell}\right)\right] \\
&= \mathbb{E}\left[\sum_\ell \widetilde{M}_\ell \hat{z}_\ell \left(\hat{z}_\ell - \widetilde{z}_{-\ell}\right)\right] \\
&= \sum_\ell \widetilde{M}_\ell v_\ell \left(1 - \widetilde{\rho}_\ell\right).
\end{aligned} \tag{40}$$

Here the first line used $\sum_\ell L_\ell \bar{z}\left(\hat{z}_\ell - \bar{z}\right) = \bar{z}\sum_\ell L_\ell \left(\hat{z}_\ell - \bar{z}\right) = 0$ by definition of $\bar{z}$. The second line used (39). The third line used the definition of $\widetilde{M}_\ell$, and the final line used the definition of $\widetilde{\rho}_\ell$.

Combining (38) and (40) yields

$$\begin{aligned}
\beta &= \frac{2\theta}{\sigma}\frac{\sum_\ell M_\ell v_\ell \left(1 - \rho_\ell\right)}{\sum_\ell \widetilde{M}_\ell v_\ell \left(1 - \widetilde{\rho}_\ell\right)} \\
&= \frac{2\theta}{\sigma}\frac{\sum_\ell M_\ell v_\ell / \sum_\ell L_\ell v_\ell}{\sum_\ell \widetilde{M}_\ell v_\ell / \sum_\ell L_\ell v_\ell} \cdot \frac{\sum_\ell M_\ell v_\ell \left(1 - \rho_\ell\right) / \sum_\ell M_\ell v_\ell}{\sum_\ell \widetilde{M}_\ell v_\ell \left(1 - \widetilde{\rho}_\ell\right) / \sum_\ell \widetilde{M}_\ell v_\ell},
\end{aligned}$$

which implies (37).

To show that $\rho \in [-1, 1]$, we write

$$\rho = \frac{\sum_\ell M_\ell \mathrm{Cov}\left[\hat{z}_\ell, \hat{z}_{-\ell}\right]}{\sum_\ell M_\ell v_\ell} = \frac{\sum_{k \neq \ell} F_{k\ell} \mathrm{Cov}\left[\hat{z}_\ell, \hat{z}_k\right]}{\sum_\ell M_\ell v_\ell} = \frac{\sum_{k \neq \ell} f_{k\ell} \mathrm{Cov}\left[\hat{z}_\ell, \hat{z}_k\right]}{\sum_{k \neq \ell} f_{k\ell} \frac{\mathrm{Var}[\hat{z}_\ell] + \mathrm{Var}[\hat{z}_k]}{2}}.$$

We therefore have $\rho \leq 1$ because

$$\sum_{k \neq \ell} f_{k\ell} \frac{\mathrm{Var}\left[\hat{z}_\ell\right] + \mathrm{Var}\left[\hat{z}_k\right]}{2} - \sum_{k \neq \ell} f_{k\ell} \mathrm{Cov}\left[\hat{z}_\ell, \hat{z}_k\right] = \frac{1}{2}\sum_{k \neq \ell} f_{k\ell} \mathrm{Var}\left[\hat{z}_\ell - \hat{z}_k\right] \geq 0$$

and $\rho \geq -1$ because

$$\sum_{k \neq \ell} f_{k\ell} \frac{\mathrm{Var}\left[\hat{z}_\ell\right] + \mathrm{Var}\left[\hat{z}_k\right]}{2} + \sum_{k \neq \ell} f_{k\ell} \mathrm{Cov}\left[\hat{z}_\ell, \hat{z}_k\right] = \frac{1}{2}\sum_{k \neq \ell} f_{k\ell} \mathrm{Var}\left[\hat{z}_\ell + \hat{z}_k\right] \geq 0.$$

The argument for $\tilde{\rho} \in [-1, 1]$ follows similarly.

Finally, suppose shocks are homoskedastic; that is, $v_\ell = v$ for all ℓ. This immediately implies

$M^v/L = M/L$ and $\widetilde{M}^v/L = \widetilde{M}/L$. Moreover, the numerator of β equals

$$\begin{aligned}
\frac{2\theta}{\sigma} vM(1-\rho) &= \frac{2\theta}{\sigma}\sum_{\ell} v_{\ell} M_{\ell}(1-\rho_{\ell}) \\
&= \frac{2\theta}{\sigma}\sum_{\ell}\left(vM_{\ell} - \sum_{k\neq\ell} F_{k\ell}\text{Cov}\left[\hat{z}_k, \hat{z}_{\ell}\right]\right) \\
&= \frac{2\theta}{\sigma} v\sum_{\ell}\left(M_{\ell} - \sum_{k\neq\ell} f_{k\ell}\frac{\text{Cov}\left[\hat{z}_k, \hat{z}_{\ell}\right]}{v}\right) \\
&= \frac{2\theta}{\sigma} vM\left(1 - \sum_{k\neq\ell}\frac{f_{k\ell}}{M}\text{Corr}\left[\hat{z}_k, \hat{z}_{\ell}\right]\right),
\end{aligned}$$

such that the Theorem 1 formula for ρ applies. The denominator of β analogously justifies the Theorem 1 formula for $\tilde{\rho}$. □

A.7 Proof $\beta(\hat{z}_{\ell} - \bar{z})$ Yields MSE-Minimizing Predicted Population Growth

Proposition. *$\beta(z_{\ell} - \bar{z})$ is the best linear predictor of $\hat{L}_{\ell}$ among all $b(z_{\ell} - \bar{z})$, i.e. after demeaning the shocks, in the sense that it solves*

$$\min_b \mathbb{E}\left[\sum_{\ell} L_{\ell}\left(\hat{L}_{\ell} - b(z_{\ell} - \bar{z})\right)^2\right]. \tag{41}$$

Proof. The first order condition for (41) yields

$$\begin{aligned}
\mathbb{E}\left[\sum_{\ell} L_{\ell}\left(\hat{L}_{\ell} - b(z_{\ell} - \bar{z})\right)\cdot(z_{\ell} - \bar{z})\right] &= 0 \\
\mathbb{E}\left[\sum_{\ell} L_{\ell}\hat{L}_{\ell}(z_{\ell} - \bar{z})\right] &= b\cdot\mathbb{E}\left[\sum_{\ell} L_{\ell}(z_{\ell} - \bar{z})^2\right],
\end{aligned}$$

which implies $b = \beta$. □

A.8 Relationship Between $\beta \approx 0$ and Local Economic Outcomes

In this appendix we show in the context of our baseline model of Section 2 that the regression of local wage changes on the $\hat{\boldsymbol{z}}$ shock is approximately unbiased if and only if β from the migration regression is small, regardless of the reason why that it is small.[54]

By (8) and (18),

$$\hat{w}_{\ell} = \frac{1}{\sigma}\left(\hat{D}_{\ell} - \hat{L}_{\ell}\right) = \frac{1}{\sigma}\left(\hat{z}_{\ell} + \zeta_{1\ell} - \hat{L}_{\ell}\right).$$

Therefore, a conventional regression of $\hat{w}_{\ell}$ on the regional shock $\hat{z}_{\ell}$ would produce an estimand $\frac{1}{\sigma}(1-\beta)$. This estimand is close to $\frac{1}{\sigma}$ whenever $\beta \approx 0$, regardless of the causes, described in

[54] In a model extension (available by request) which allows for the choice of non-employment we establish a similar result for regressions in which the outcome variable is local employment change, which is now distinct from the change in local population.

Theorem 1. Note, however, that in some locations fitted values from the wage regression, $\frac{1}{\sigma}(1-\beta)\hat{z}_\ell$, may be far from the true effect, $\frac{1}{\sigma}\left(\hat{z}_\ell - \hat{L}_\ell\right)$, because migration responses $\hat{L}_\ell$ can be large in some regions. Moreover, the regression-based predicted effect of a counterfactual shock to a specific location ℓ, $\frac{1}{\sigma}(1-\beta)$, may differ substantially from the true effect of $\frac{1}{\sigma}\left(1 - \frac{2\theta}{\sigma}\frac{M_\ell}{L_\ell}\right)$, for locations with high migration shares.

A.9 Invariance to Recentering

In this appendix we show that recentering as in Borusyak and Hull (2021) is not needed for the NLLS problem (29) when the shocks are unconditionally as-good-as-randomly assigned to all regions, and how it can be performed with conditional as-good-as-random assignment.

Let $\Omega_\ell(\lambda)$ be the ℓth row of $\Omega(\lambda)$, such that $\hat{L}_\ell = \Omega_\ell(\lambda)\hat{\mathbf{z}} + \zeta_{2\ell}$. As we have shown in footnote 16, $\Omega(\lambda)\iota = 0$ (for any λ), and thus $\frac{\partial\Omega}{\partial\lambda}\iota = 0$ as well, where $\frac{\partial\Omega}{\partial\lambda}$ is a matrix of element-by-element derivatives of $\Omega(\lambda)$.

The first order condition corresponding to the NLLS problem (29) is

$$\mathbb{E}\left[\sum_\ell \left(\hat{L}_\ell - \Omega_\ell(\lambda)\hat{\mathbf{z}}\right) \cdot \frac{\partial\Omega_\ell}{\partial\lambda}\hat{\mathbf{z}}\right] = 0. \tag{42}$$

It holds at the true $\lambda = \frac{\theta}{\sigma}$ if and only if

$$\mathbb{E}\left[\sum_\ell \zeta_{2\ell} \cdot \frac{\partial\Omega_\ell}{\partial\lambda}\hat{\mathbf{z}}\right] = 0.$$

When $\mathbb{E}\left[\hat{z}_\ell \mid \boldsymbol{\zeta}_2\right] = \mu$ for all ℓ, i.e. in the unconditional as-good-as-random assignment case, this holds because, by the law of iterated expectations,

$$\mathbb{E}\left[\sum_\ell \zeta_{2\ell} \cdot \frac{\partial\Omega_\ell}{\partial\lambda}\hat{\mathbf{z}}\right] = \mathbb{E}\left[\sum_\ell \zeta_{2\ell} \cdot \frac{\partial\Omega_\ell}{\partial\lambda}\mu\iota\right] = 0.$$

This situation contrasts with *conditionally* as-good-as-random shocks, formalized by $\mathbb{E}\left[\hat{z}_\ell \mid \boldsymbol{\zeta}_2\right] = \widetilde{\mu}' q_\ell$, where q_ℓ is a vector of observables. This weaker assumption is appropriate, for instance, when $\hat{z}_\ell$ is a shift-share variable based on some shifters (e.g. the growth of import penetration from China) which only happen in manufacturing industries, with q_ℓ including the regional share of manufacturing employment in addition to the intercept (Borusyak et al., 2022). In that case the NLLS moment condition (42) need not hold, and its recentered version should be used (Borusyak and Hull, 2021):

$$\mathbb{E}\left[\sum_\ell \left(\hat{L}_\ell - \Omega_\ell(\lambda)\hat{\mathbf{z}}\right) \cdot \frac{\partial\Omega_\ell}{\partial\lambda}\left(\hat{\mathbf{z}} - Q\widetilde{\mu}\right)\right] = 0,$$

where matrix Q collects the q_ℓ, and $\widetilde{\mu}$ can be estimated in a zeroth step.

A.10 Non-Linear Least Squares Identification

In this appendix we prove that $\frac{\theta}{\sigma}$ uniquely solves the minimization problem (29) whenever the variance-covariance matrix of $\hat{\mathbf{z}}$ is non-degenerate (i.e., positive definite).

The NLLS objective function (in expectation) can be written as

$$\begin{aligned}
&\mathbb{E}\left[\left(\hat{\mathbf{L}}-\Omega\left(\lambda\right)\hat{\mathbf{z}}\right)'\left(\hat{\mathbf{L}}-\Omega\left(\lambda\right)\hat{\mathbf{z}}\right)\right] \\
&\qquad=\mathbb{E}\left[\left(\Omega\left(\frac{\theta}{\sigma}\right)\hat{\mathbf{z}}-\Omega\left(\lambda\right)\hat{\mathbf{z}}+\boldsymbol{\zeta}_2\right)'\left(\Omega\left(\frac{\theta}{\sigma}\right)\hat{\mathbf{z}}-\Omega\left(\lambda\right)\hat{\mathbf{z}}+\boldsymbol{\zeta}_2\right)\right] \\
&\qquad=\mathbb{E}\left[\left(\Omega\left(\frac{\theta}{\sigma}\right)\left(\hat{\mathbf{z}}-\mu\iota\right)-\Omega\left(\lambda\right)\left(\hat{\mathbf{z}}-\mu\iota\right)+\boldsymbol{\zeta}_2\right)'\left(\Omega\left(\frac{\theta}{\sigma}\right)\left(\hat{\mathbf{z}}-\mu\iota\right)-\Omega\left(\lambda\right)\left(\hat{\mathbf{z}}-\mu\iota\right)+\boldsymbol{\zeta}_2\right)\right] \\
&\qquad=\mathrm{Var}\left[\left(\Omega\left(\frac{\theta}{\sigma}\right)-\Omega\left(\lambda\right)\right)\hat{\mathbf{z}}\right]+\mathbb{E}\left[\boldsymbol{\zeta}_2'\boldsymbol{\zeta}_2\right], \qquad (43)
\end{aligned}$$

where the third line uses $\Omega\left(\lambda\right)\iota=0$ (see footnote 16 and Appendix A.9) and the last line follows since, under $\mathbb{E}\left[\hat{\mathbf{z}}-\mu\iota\mid\boldsymbol{\zeta}_2\right]=0$, the cross-terms

$$\mathbb{E}\left[\boldsymbol{\zeta}_2'\left(\Omega\left(\frac{\theta}{\sigma}\right)-\Omega\left(\lambda\right)\right)\left(\hat{\mathbf{z}}-\mu\iota\right)\right]=0$$

by the law of iterated expectations.

We now show that $\frac{\theta}{\sigma}$ uniquely minimizes (43). The second term in 43 does not depend on λ. The first term is zero when $\lambda=\frac{\theta}{\sigma}$ and positive whenever $\Omega\left(\lambda\right)\neq\Omega\left(\frac{\theta}{\sigma}\right)$ since the variance-covariance matrix of $\hat{\mathbf{z}}$ is non-degenerate. However, $\Omega\left(\lambda\right)=\Omega\left(\frac{\theta}{\sigma}\right)$ implies $\left(I+\lambda\left(I-\Gamma'\Pi\right)\right)^{-1}=\left(I+\frac{\theta}{\sigma}\left(I-\Gamma'\Pi\right)\right)^{-1}$ and in turn $\lambda\left(I-\Gamma'\Pi\right)=\frac{\theta}{\sigma}\left(I-\Gamma'\Pi\right)$, which is impossible for $\lambda\neq\frac{\theta}{\sigma}$ as long as there is any mobility in the no-shock equilibrium, i.e. $\Gamma'\Pi\neq I$.

A.11 Proofs for the Full Model

Here we prove equation (31) and Theorem 2. Analogously to the proofs for the baseline model in Appendices A.3 and A.6, we use the fact that under the low-mobility approximation, $L^0_{\ell n}\approx L_{\ell n}$, $L^0_\ell\approx L_\ell$, and that it is irrelevant whether regional observations in the regression and region-by-industry observations when defining $\hat{z}_\ell$ are weighted by the pre-shock or no-shock (counterfactual) employment levels.

From (30)

$$\begin{aligned}
\hat{L}_\ell&=\sum_{n\in\mathcal{N}}\frac{L_{\ell n}}{L_\ell}\hat{L}_{\ell n} \\
&\approx\frac{\theta}{\sigma}\sum_{n\in\mathcal{N}}\frac{L_{\ell n}}{L_\ell}\sum_{k\in\mathscr{L}}\sum_{p\in\mathcal{N}}\left(\frac{f_{kp,\ell n}}{L_{\ell n}}+\frac{f_{\ell n,kp}}{L^0_{\ell n}}\right)\left(\hat{z}_{\ell n}-\hat{z}_{kp}\right)+\zeta_\ell \\
&\approx\frac{2\theta}{\sigma}\sum_{n\in\mathcal{N}}\sum_{k\in\mathscr{L}}\sum_{p\in\mathcal{N}}\frac{F_{kp,\ell n}}{L_\ell}\left(\hat{z}_{\ell n}-\hat{z}_{kp}\right)+\zeta_\ell.
\end{aligned}$$

In this sum, the terms corresponding to $\ell=k$ add up to zero:

$$\sum_{n\in\mathcal{N}}\sum_{p\in\mathcal{N}}\frac{F_{\ell p,\ell n}}{L_\ell}\left(\hat{z}_{\ell n}-\hat{z}_{\ell p}\right)=\sum_{n}\frac{F_{\ell,\ell n}-F_{\ell n,\ell}}{L_\ell}\hat{z}_{\ell n}=0,$$

since $F_{\ell,\ell n} = F_{\ell n,\ell}$ for any ℓ and n by definition of average bilateral flows. Thus,

$$\begin{aligned}
\hat{L}_\ell &\approx \frac{2\theta}{\sigma} \sum_{k \neq \ell} \sum_{n \in \mathcal{N}} \sum_{p \in \mathcal{N}} \frac{F_{kp,\ell n}}{L_\ell} (\hat{z}_{\ell n} - \hat{z}_{kp}) + \zeta_\ell \\
&= \frac{2\theta}{\sigma} \left(\sum_{k \neq \ell} \sum_{n \in \mathcal{N}} \frac{F_{k,\ell n}}{L_\ell} \hat{z}_{\ell n} - \sum_{k \neq \ell} \sum_{p \in \mathcal{N}} \frac{F_{kp,\ell}}{L_\ell} \hat{z}_{kp} \right) + \zeta_\ell \\
&= \frac{2\theta}{\sigma} \frac{M_\ell}{L_\ell} \left(\hat{z}_\ell^{\text{mov}} - \hat{z}_{-\ell}^{\text{mov}} \right) + \zeta_\ell,
\end{aligned}$$

proving (31).

From (31), the proof of Theorem 2 follows similarly to the proof of Theorem A1. Specifically, the denominator of β is exactly the same:

$$\mathbb{E}\left[\sum_\ell L_\ell \left(\hat{z}_\ell - \bar{z} \right)^2 \right] = \sum_\ell \widetilde{M}_\ell v_\ell \left(1 - \widetilde{\rho}_\ell \right).$$

Assuming without loss of generality that $\mathbb{E}\left[\hat{z}_\ell \right] = \mu = 0$, the numerator can further be expressed as

$$\mathbb{E}\left[\sum_\ell L_\ell \hat{L}_\ell \hat{z}_\ell \right] = \mathbb{E}\left[\frac{2\theta}{\sigma} \sum_\ell M_\ell \hat{z}_\ell \left(\hat{z}_\ell^{\text{mov}} - \hat{z}_{-\ell}^{\text{mov}} \right) \right] = \frac{2\theta}{\sigma} \sum_\ell M_\ell v_\ell \left(\rho_\ell^{\text{mov}} - \rho_\ell \right).$$

Putting the two together,

$$\beta = \frac{2\theta}{\sigma} \frac{\sum_\ell M_\ell v_\ell \left(\rho_\ell^{\text{mov}} - \rho_\ell \right)}{\sum_\ell \widetilde{M}_\ell v_\ell \left(1 - \widetilde{\rho}_\ell \right)} = \frac{2\theta}{\sigma} \cdot \frac{M^v / L}{\widetilde{M}^v / L} \cdot \frac{\rho^{\text{mov}} - \rho}{1 - \widetilde{\rho}},$$

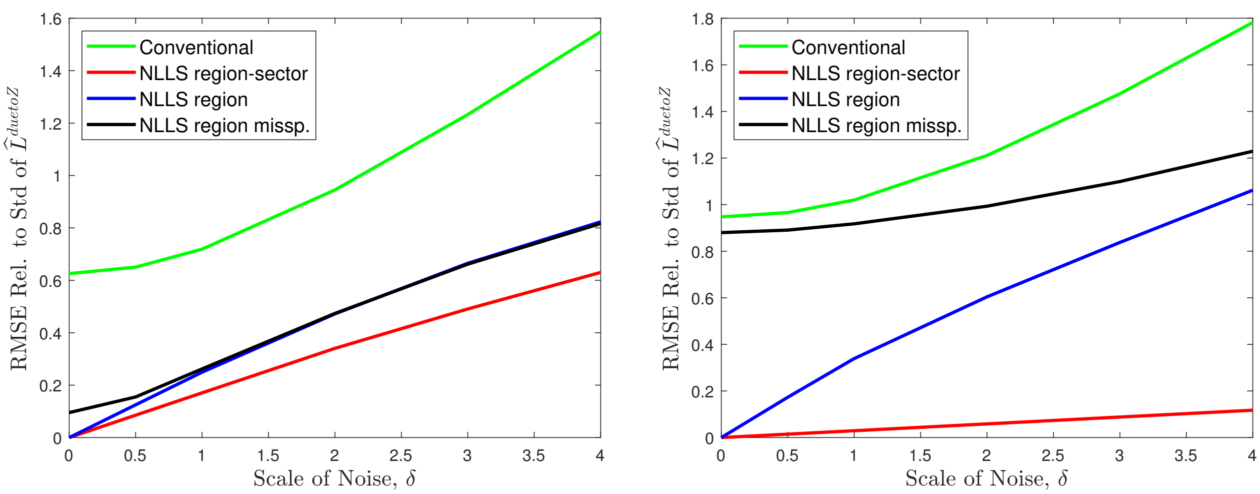
as required by the theorem.

Appendix Figures and Tables

Figure A1: Robustness of RMSE Measures to Noise Scaling

Panel (a): *iid* across locations

Panel (b): *iid* across industries



Notes: This figure repeats the analysis of Table 4 varying the noise scaling factor δ, which multiplies the error term $\boldsymbol{\zeta}$ in the simulations. Panel (a) reports the root mean-squared error (RMSE) of the four methods relative to the RMSE of the uninformative prediction for *iid* location shocks, while Panel (b) uses *iid* industry shocks. See notes to Table 4 for details.