Optimal Exchange Rate Policy*

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Abstract

We develop a general policy analysis framework that features nominal rigidities and financial frictions with endogenous PPP and UIP deviations. The goal of the optimal policy is to balance output gap stabilization and international risk sharing using a mix of monetary policy and FX interventions. The nominal exchange rate plays a dual role. First, it allows for the real exchange rate adjustments when prices are sticky, which are necessary to close the output gap. Monetary policy can eliminate the output gap, but this generally requires a volatile nominal exchange rate. Volatility in the nominal exchange rate, in turn, limits the extent of international risk sharing in the financial market with risk averse intermediaries. Optimal monetary policy closes the output gap, while optimal FX interventions eliminate UIP deviations. When the first-best real exchange rate is stable, both goals can be achieved by a fixed exchange rate policy — an open-economy divine coincidence. Generally, this is not the case, and the optimal policy requires a managed peg by means of a combination of monetary policy and FX interventions, without requiring the use of capital controls. We explore various constrained optimal policies, when either monetary policy or FX interventions are restricted, and characterize the possibility of central bank's income gains and losses from FX interventions.

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1 Introduction

What is the optimal exchange rate policy? Should exchange rates be optimally pegged, managed or allowed to freely float? What defines a freely floating exchange rate? Do open economies face a trilemma constraint in choosing between inflation and exchange rate stabilization? These are generally difficult questions, as the exchange rate is neither a policy instrument, nor a direct objective of the policy, but rather an endogenous general equilibrium variable with direct equilibrium links in both product and financial markets. At the same time, equilibrium exchange rate behavior features a variety of *puzzles* from the point of view of conventional business cycle models, which thus casts doubt on their exchange rate policy implications.

We address these questions by developing a general policy analysis framework with nominal rigidities and financial frictions that are both central for equilibrium exchange rate determination and result in an empirically realistic model of exchange rates. We extend the framework in Itskhoki and Mukhin (2021b), where we studied positive implications of a switch between floating and fixed exchange rate regimes, to allow for explicit policy analysis using both conventional monetary policy and foreign exchange interventions in the financial market. We show that this framework is easily amenable to normative analysis and characterize the optimal exchange rate policies implied by the model.

We focus on a problem of a small open economy with tradable and non-tradable goods with a segmented international financial market resulting in endogenous uncovered interest rate parity (UIP) deviations. With non-tradable goods, productivity shocks determine the value of the frictionless real exchange rate, or departures from purchasing power parity (PPP). Nominal rigidities is another — frictional — source of PPP violations. The presence of both endogenous PPP and UIP deviations is essential for the optimal exchange rate policy analysis, as exchange rates are key determinants of both violations.

The nominal exchange rate plays a dual role — in the goods and asset markets. First, it allows for expenditure switching and the real exchange rate adjustment when prices (or wages) are sticky, and in the absence of such nominal exchange rate movements, the economy features an output gap resulting in welfare losses. Monetary policy can eliminate the output gap, but this generally requires a volatile nominal exchange rate that accommodates fundamental macroeconomic shocks. Volatility in the nominal exchange rate, in turn, limits the extent of international risk sharing in the financial market, as international financial flows must be intermediated by risk-averse market makers who hold the nominal exchange rate risk. This also leads to welfare losses. Financial market interventions can shift this risk away from arbitrageurs, stabilizing the resulting equilibrium UIP deviations and improving the extent of international risk sharing. Thus, the goal of the optimal policy is to balance output gap stabilization and international risk sharing.

We begin our analysis by characterizing the optimal allocation, which ensures efficient level of production and optimal risk sharing in the tradable sector. We then show how an unconstrained joint use of monetary policy and FX interventions allows to implement the optimal allocation, with monetary policy eliminating the output gap and FX interventions eliminating the intermediation wedge and the resulting UIP deviation in the international financial market. Exchange rate stabilization is not a direct goal of a welfare maximizing policy. The resulting equilibrium generally features volatile nominal exchange rate and inflation targeting, with financial interventions targeting UIP deviations as their policy goal. Such policy mix allows the exchange rate to accommodate fundamental macroeconomic shocks, while it neutralizes the effects of currency demand shocks in the financial market on exchange rate volatility. This is the sense in which economies with a segmented financial market do not feature a trilemma constraint, as market segmentation offers the financial regulator an additional instrument to stabilize market volatility, even when monetary policy focuses exclusively on domestic inflation and output gap stabilization.

Implementing the optimal allocation in the goods and asset market generally requires an unconstrained use of both monetary and FX instruments. There exists, however, an important special case when addressing both frictions could be done with a nominal exchange rate peg by means of monetary policy alone. We refer to this case as divine coincidence in the open economy, by analogy with a closed-economy divine coincidence. Indeed, if the first-best real exchange rate that ensures efficient risk sharing is stable, then there is no tradeoff from the points of view of the goods and asset markets: A fixed nominal exchange rate is consistent with efficient expenditure switching under sticky prices in the goods market, as well as eliminates risk in the international financial market allowing for frictionless intermediation. Direct nominal exchange rate targeting is favored over inflation stabilization in this case as it guarantees a unique optimal equilibrium. While our analysis is consistent with the optimal currency areas logic, it identifies not only circumstances when the costs of a fixed exchange rate are low in the goods market, but also what the benefits of a fixed exchange rate are in the financial market.

Next, we explore circumstances where either monetary policy is constrained (e.g., due to the zero lower bound) or the financial interventions are constrained (e.g., due to non-negative requirement on central bank foreign reserves or value-at-risk constraints for the central bank portfolio). In this case, there are two independent policy goals — the output gap and the risk sharing wedge — and only one unconstrained policy instrument, thus making it generally impossible to replicate the optimal allocation. Fixing the exchange rate using the monetary policy tool is generally feasible, but is also generally suboptimal outside the case of divine coincidence. Similarly, targeting the output gap alone is also suboptimal, and monetary policy trades-off output gap and exchange rate stabilization (managed and crawling pegs) in the absence of FX interventions. Managed peg and dirty floats with monetary policy may emerge as the second best policy, even when divine coincidence is not satisfied, yet there are tight constraint on the balance sheet of the central bank making effective FX interventions infeasible. Using financial interventions to stabilize output gap is generally infeasible.

Lastly, we explore the monopoly power of the government in the international financial market and the ability of the central bank to earn rents without compromising the expenditure switching and risk sharing goals of optimal exchange rate policy. When the financial sector is offshore, the policymaker can compete with financial intermediaries for rents (international transfers) that emerge from exogenous shifts in currency demand. In the presence of an additional capital control instrument, it is possible to extract maximum rents without compromising the other objectives of the policy. In particular, FX interventions are used only part way in this case, without offsetting excess currency demand and eliminating the entire intermediation rent, while capital controls are used in addition to eliminate the effect of rents on the optimal risk sharing. This, however, requires a flexible state-contingent use of capital controls, which may be infeasible. Without capital controls, the policymaker can still use FX interventions to implement the frictionless allocation at no expected financial costs. This, however, is generally suboptimal as it fails to take advantage of trading off frictionless risk sharing for financial market rents from undersupplying currency reserves.

We finish our analysis with a number of extensions. In particular, we extend our small open economy model to global equilibrium with a continuum of small open economies, one of which issues a dominant funding currency that is used for international borrowing and lending against other national currencies. Unconstrained use of non-cooperative monetary policy and FX interventions eliminates all international risk sharing wedges and ensures efficient level of output in every country. When policies are constrained, however, international spillovers can no longer be internalized by non-cooperative policies. We characterize such spillovers that emerge in both dominant and non-dominant countries, and show how a cooperative policy of international FX interventions can address these spillovers. We also discuss costs and benefits associated with a global currency union and a gold standard.

Related literature We build on a vast literature studying the role of exchange rates in both goods and financial markets, as well as the optimal macroeconomic and financial policies in an open economy. The normative implications of the expenditure switching channel of monetary policy is the focus of Friedman (1953), Clarida, Galí, and Gertler (2000), Corsetti and Pesenti (2001a), Devereux and Engel (2003), Benigno and Benigno (2003), Goldberg and Tille (2009), **?** and the linear-quadratic representation of the planner's problem follows Clarida, Gali, and Gertler (1999), Gali and Monacelli (2005), Engel (2011), Corsetti, Dedola, and Leduc (2010).

Our model of frictional financial intermediation builds on Kouri (1983), Jeanne and Rose (2002), Alvarez, Atkeson, and Kehoe (2009), Gabaix and Maggiori (2015), Camanho, Hau, and Rey (2022), Itskhoki and Mukhin (2021a,b), Gourinchas, Ray, and Vayanos (2019), Greenwood, Hanson, Stein, and Sunderam (2020), Jiang, Krishnamurthy, and Lustig (2021), Bianchi, Bigio, and Engel (2021), Akinci, Kalemli-Özcan, and Queralto (2022), and the studied financial channel of monetary policy is related to the work of Obstfeld and Rogoff (2002), Benigno (2009), Rey (2013a), Fanelli (2017), Kekre and Lenel (2021), Hassan, Mertens, and Zhang (2021), Fornaro (2021).

The analysis is also related to the recent work on the costs and benefits of FX interventions by Jeanne (2012), Amador, Bianchi, Bocola, and Perri (2019), Cavallino (2019), Fanelli and Straub (2021) and the optimal capital controls by Jeanne and Korinek (2010), Bianchi (2011), Costinot, Lorenzoni, and Werning (2014), Farhi and Werning (2016, 2017), Schmitt-Grohé and Uribe (2016). Finally, we share with Basu, Boz, Gopinath, Roch, and Unsal (2020) the approach of studying multiple policy instruments in an open economy.

2 The Model of Exchange Rate Determination

We consider a small open economy with a tradable and a non-tradable sectors. In our baseline analysis, we make a number of strong assumptions that considerably simplify the policy problem, and we generalize our analysis in Section 7. In particular, we assume a separable log-linear utility of the households:

$$\mathbb{W}_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \big(\log C_t - (1-\gamma) L_t \big) \qquad \text{with} \qquad C_t = C_{Tt}^{\gamma} C_{Nt}^{1-\gamma},$$

who can borrow or lend using a one-period risk-free home-currency bond:

$$P_t C_t + \frac{B_t}{R_t} = B_{t-1} + W_t L_t + P_{Tt} Y_{Tt} + \Pi_t + T_t,$$

where R_t is the gross nominal interest rate.

The households own an exogenous stochastic endowment of the tradable good Y_{Tt}, which is homogenous and traded at a flexible international price that satisfies the law of one price:[1]

$$P_{Tt} = \mathcal{E}_t P_{Tt}^*,$$

where P_{Tt}^* is the international price of the tradable good and $\mathcal{E}_t$ is the nominal exchange rate (units of home currency for one unit of foreign currency; thus, an increase in $\mathcal{E}_t$ corresponds to a home depreciation). We assume a stable price level in the foreign country, $P_{Tt}^* = 1$, and therefore the home-currency tradable price tracks the nominal exchange rate, $P_{Tt} = \mathcal{E}_t$. Home net exports equal $NX_t = P_{Tt}(Y_{Tt} - C_{Tt}) = \mathcal{E}_t(Y_{Tt} - C_{Tt})$.

The non-tradable good is produced using labor subject to productivity shocks, $Y_{Nt} = A_t L_t$, and the firm profits are $\Pi_t = P_{Nt} Y_{Nt} - W_t L_t$. Household labor supply optimality requires $C_{Nt} = W_t / P_{Nt}$, and the market clearing requires $C_{Nt} = Y_{Nt}$. The competitive flexible price of non-tradables equals W_t/A_t, however, we assume that prices are permanently sticky at an exogenous level $P_{Nt} = 1$.[2] Equilibrium labor supply then equals $L_t = W_t/A_t$ and non-tradable output equals $C_{Nt} = Y_{Nt} = W_t$.

The policymaker uses wage inflation as monetary instrument. For simplicity, we assume the policymaker has direct control over nominal wages and chooses W_t in conducting monetary policy.[3] When monetary policy sets wages to peg non-tradable productivity, $W_t = A_t$, this results in the first best employment and output level, $L_t = 1$ and $Y_{Nt} = A_t$, i.e. zero output gap with a constant price $P_{Nt} = 1$.

[1] Homogenous tradable good together with log-linear utility make the terms of trade exogenous for a small open economy. This assumption combined with homogenous tradables in a small open economy eliminates all markup and terms of trade motives from policies that typically complicate the optimal policy analysis (see Corsetti and Pesenti 2001b, Benigno and Benigno 2003, Egorov and Mukhin 2021). In contrast, international risk sharing is not independent of the structure of the asset market, unlike in Cole and Obstfeld (1991), and our results generalize immediately for any utility separable across C_N, C_T and L.

[2] We focus on the fully sticky price case as a limiting benchmark which simplifies the analysis by avoiding an additional dynamic equation, yet maintains all the qualitative tradeoffs of a more general environment (see Section 7.1). By having price stickiness only in the non-tradable sector we avoid the need to choose between PCP, LCP and DCP frameworks (see Section 7.2); alternatively, we could focus on sticky wages.

[3] The first order condition for bond holdings implies $\beta R_t \mathbb{E}_t\{W_t/W_{t+1}\} = 1$, since $W_t = P_{Nt} C_{Nt}$. Therefore, an interest rate rule $R_t = \bar{R}_t \cdot (W_t/A_t)^{\phi}$ with a sufficiently large ϕ and $\bar{R}_t = (\beta \mathbb{E}_t\{A_t/A_{t+1}\})^{-1}$ implements $W_t = A_t$.

The total consumption expenditure is split between tradables and non-tradables, $P_tC_t = P_{Tt}C_{Tt} + P_{Nt}C_{Nt}$, such that $\gamma P_{Nt}C_{Nt} = (1-\gamma)P_{Tt}C_{Tt}$. Using the law of one price $P_{Tt} = \mathcal{E}_t$ and labor supply condition $C_{Nt} = W_t/P_{Nt}$, we obtain an equilibrium expenditure switching condition for the nominal exchange rate:

$$\frac{\gamma}{1-\gamma}\frac{C_{Nt}}{C_{Tt}} = \frac{P^*_{Tt}\mathcal{E}_t}{P_{Nt}} = \mathcal{E}_t. \tag{1}$$

Monetary policy W_t expands non-tradable consumption, $C_{Nt} = W_t$, and proportionally depreciates the nominal exchange rate, holding tradable consumption constant; in turn, holding constant monetary policy, greater tradable consumption appreciates the real exchange rate.

The real exchange rate, defined as $\mathcal{Q}_t = P^*_t\mathcal{E}_t/P_t = \mathcal{E}_t^{1-\gamma}$, where we take as given the foreign price level $P^*_t = 1$ and the home price level $P_t = P_{Tt}^{\gamma}P_{Nt}^{1-\gamma} = \mathcal{E}_t^{\gamma}$. With sticky non-tradable price, the real exchange rate tracks the nominal exchange rate. In the flexible price allocation, $\mathcal{Q}_t = \big(\frac{\gamma}{1-\gamma}\frac{A_t}{C_{Tt}}\big)^{1-\gamma}$, independently of the monetary policy W_t and hence the value of the nominal exchange rate $\mathcal{E}_t$.

Financial market Apart from households, three types of agents trade home and foreign currency bonds in the international financial market. Namely, these are the government, noise traders and arbitrageurs. The government holds a portfolio of (F_t, F^*_t) units of home- and foreign-currency bonds, respectively, with the value of the portfolio (government net foreign assets) given by $F_t/R_t + \mathcal{E}_tF^*_t/R^*_t$, where R^*_t is the gross nominal interest rate in foreign currency (dollar). Changes in F_t and F^*_t correspond to open market operations of the government.

Noise traders hold a zero capital portfolio (N_t, N^*_t) of the two bonds, such that $N_t/R_t + \mathcal{E}_tN^*_t/R^*_t = 0$, and N^*_t/R^*_t is an exogenous liquidity demand shock for foreign currency that is uncorrelated with macroeconomic fundamentals. A positive N^*_t/R^*_t means that noise traders short home-currency bonds to buy foreign-currency bonds, and vice versa. In turn, B_t is the fundamental demand of home households for the home-currency bond, which is shaped by the macroeconomic forces resulting in the equilibrium path of net exports. The choice of B_t is characterized by the household Euler equation:

$$\beta R_t\mathbb{E}_t\left\{\frac{C_{Tt}}{C_{Tt+1}}\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\right\} = 1, \tag{2}$$

where we used $P_{Tt} = \mathcal{E}_t$.

Finally, the arbitrageurs also hold a zero capital portfolio (D_t, D^*_t) such that $D_t/R_t + \mathcal{E}_tD^*_t/R^*_t = 0$, with a return on one foreign currency unit holding of such portfolio (i.e., $D^*_t = R^*_t$ and $D_t = -R_tD^*_t\mathcal{E}_t/R^*_t$) given by $\tilde{R}^*_{t+1} = R^*_t - R_t\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}$ in dollars. The income from this carry trade is given by $\pi^{D*}_{t+1} = D^*_t - D_t/\mathcal{E}_t = \tilde{R}^*_{t+1}\cdot\frac{D^*_t}{R^*_t}$ in foreign currency. Arbitrageurs choose their portfolio to maximize min-variance preferences over profits, $V_t(\pi^{D*}_{t+1}) = \mathbb{E}_t\left\{\Theta_{t+1}\pi^{D*}_{t+1}\right\} - \frac{\omega}{2}\mathrm{var}_t\big(\pi^{D*}_{t+1}\big)$, where $\Theta_{t+1} = \beta\frac{C_{Tt}}{C_{T,t+1}}$ is the stochastic discount factor of home households, and the second term in $V_t(\cdot)$ reflects the additional risk penalty of the arbitrageurs with ω being the risk aversion parameter. The optimal portfolio choice satisfies:

$$\frac{D^*_t}{R^*_t} = \frac{\mathbb{E}_t\big\{\Theta_{t+1}\tilde{R}^*_{t+1}\big\}}{\omega\sigma_t^2},$$

where $\sigma_t^2 \equiv \mathrm{var}_t(\tilde{R}^*_{t+1}) = R_t^2 \cdot \mathrm{var}_t(\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}})$ is a measure of the nominal exchange rate volatility.

The market clearing in the financial market requires that the home-currency bond positions of all four types of agents balance out:

$$B_t + N_t + D_t + F_t = 0.$$

The foreign-currency bond is in perfect elastic international supply at an exogenous interest rate R_t^*. The government budget constraint from operations in the financial market is given by:

$$\frac{F_t}{R_t} + \frac{\mathcal{E}_t F_t^*}{R_t^*} = F_{t-1} + \mathcal{E}_t F_{t-1}^* + \tau \mathcal{E}_t \pi_t^* - T_t, \qquad \pi_t^* = \tilde{R}_t^* \cdot \frac{N_{t-1}^* + D_{t-1}^*}{R_{t-1}^*},$$

where T_t is the lump-sum transfer to the home households and π_t^* is the combined income from the financial transactions of noise traders and arbitrageurs (in dollars), and $\tau \in [0,1]$ is the home country's ownership share of the financial sector.

3 The Policy Problem

Define the net foreign asset (NFA) position of the home country, B_t^* in foreign currency, which has the home-currency value:

$$\frac{\mathcal{E}_t B_t^*}{R_t^*} = \frac{B_t + F_t}{R_t} + \frac{\mathcal{E}_t F_t^*}{R_t^*},$$

that is the value of the combined position of the home households and the government. Using B_t^*, we introduce a sequences of lemmas that characterize the equilibrium conditions for the open economy.

Lemma 1 *The NFA of the home country equals the combined foreign-currency bond position in the financial market: $B_t^* = F_t^* + N_t^* + D_t^*$.*

Proof: Using the market clearing for home-currency bond, $B_t + N_t + D_t + F_t = 0$, and the zero capital portfolios of noise traders and arbitrageurs, we have $\frac{B_t + F_t}{R_t} - \frac{\mathcal{E}_t(N_t^* + D_t^*)}{R_t^*} = 0$. Then using the definition of NFA and rearranging yields $B_t^* = F_t^* + N_t^* + D_t^*$. ■

The NFA position allows to characterize concisely the home country budget constraint:

Lemma 2 *The combined home country budget constraint in foreign currency terms is given by:*

$$\frac{B_t^*}{R_t^*} - B_{t-1}^* = (Y_{Tt} - C_{Tt}) - (1-\tau)\tilde{R}_t^* \frac{B_{t-1}^* - F_{t-1}^*}{R_{t-1}^*}. \tag{3}$$

Proof: Combines household and government budget constraints and firm profits, and uses the definition of NFA. See Appendix A. ■

Note that $NX_t/\mathcal{E}_t = Y_{Tt} - C_{Tt}$ is the real (or foreign-currency) value of net exports. The last term in the budget constraint reflects the international transfer of financial-sector income (of arbitrageurs and noise traders) from the home country to the rest of the world, as in equilibrium $B_t^* - F_t^* = N_t^* + D_t^*$. When $\tau = 1$, that is financial sector is fully owned by domestic residents, there is no international transfer and the budget constraint is simply $B_t^*/R_t^* - B_{t-1}^* = Y_{Tt} - C_{Tt}$.

Finally, the equilibrium international risk sharing is characterized in:

Lemma 3 *The international risk sharing condition is given by:*

$$\beta R_t^* \mathbb{E}_t \frac{C_{Tt}}{C_{Tt+1}} = 1 + \omega\sigma_t^2 \frac{B_t^* - N_t^* - F_t^*}{R_t^*}, \qquad \textit{where} \quad \sigma_t^2 = R_t^2 \cdot \mathrm{var}_t\Big(\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\Big), \tag{4}$$

and $\omega\sigma_t^2 \frac{B_t^*-N_t^*-F_t^*}{R_t^*}$ *is the international risk sharing wedge.*

Proof: follows directly from the optimal portfolio of arbitrageurs, which we rewrite expanding the expressions for Θ_{t+1} and $\tilde{R}_{t+1}^*$ as:

$$\omega\sigma_t^2 \frac{D_t^*}{R_t^*} = \mathbb{E}_t\{\Theta_{t+1}\tilde{R}_{t+1}^*\} = \mathbb{E}_t\left\{\frac{\beta C_{Tt}}{C_{Tt+1}} \cdot \left[R_t^* - R_t \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\right]\right\}.$$

Subtracting the household Euler equation (2) and substituting for D_t^* from Lemma 1 finishes the proof. ■

In the absence or risk-sharing wedge, the international risk sharing condition reduces to the conventional Euler equation for the foreign-currency bond, $\beta R_t^* \mathbb{E}_t \frac{C_{Tt}}{C_{T,t+1}} = 1$, a property of the constrained optimal risk sharing in this economy. Combining international risk sharing (4) with the home household Euler equation (2) we obtain the modified UIP condition that holds in this economy:

$$\mathbb{E}_t\left\{\frac{C_{Tt}}{C_{Tt+1}}\left[R_t^* - R_t\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\right]\right\} = \omega\sigma_t^2 \frac{B_t^* - N_t^* - F_t^*}{\beta R_t^*}. \tag{5}$$

Therefore, the risk sharing wedge is also a UIP deviation from the perspective of home households, and it disappears in the limit of risk neutral arbitrageurs ($\omega \to 0$), in which case the international financial market converges to a frictionless single-bond incomplete market.

Equilibrium We can now define the equilibrium in this economy. Given the stochastic path of exogenous shocks $\{A_t, Y_{Tt}, R_t^*, N_t^*\}$, sticky non-tradable prices $P_{Nt} \equiv 1$, and the path of policies $\{R_t, F_t, F_t^*\}$, an equilibrium vector $\{C_{Nt}, C_{Tt}, B_t^*, \mathcal{E}_t\}$ and the implied $\{\sigma_t^2\}$ solve the dynamic system (1)–(4) with the initial condition B_{-1}^* and the transversality condition $\lim_{T\to\infty} B_T^* / \prod_{t=0}^T R_t^* = 0$.[4] Note that exogenous shocks include non-tradable productivity A_t, tradable endowment Y_{Tt}, foreign interest rate R_t^* and noise trader liquidity shocks for foreign vs home currency N_t^*, while the policy vector contains home-currency interest rate R_t and the central bank's portfolio of bonds (F_t, F_t^*). Note that Ricardian equivalence does not hold vis-à-vis foreign currency position F_t^*, as households cannot directly hold foreign currency bonds. Yet, the model features Ricardian equivalence for home-currency bonds, and a change in F_t crowds out private B_t and is not consequential for the equilibrium allocation.

3.1 Exact policy problem

In our baseline analysis, we focus on the Ramsey problem of welfare maximization with commitment for a given set of policy instruments. Given the equilibrium definition above, we can state the policy

[4] The other endogenous variables $\{W_t, L_t, Y_{Nt}, B_t, D_t^*\}$ are recovered from static equilibrium conditions. For example, from market clearing and labor supply $Y_{Nt} = C_{Nt} = W_t$ and from production function $L_t = Y_{Nt}/A_t$, while from Lemma 1 $D_t^* = B_t^* - F_t^* - N_t^*$, and household assets satisfy $\frac{B_t}{R_t} = \frac{\mathcal{E}_t(B_t^* - F_t^*)}{R_t^*} - \frac{F_t}{R_t}$.

problem as:

$$\max_{\{R_t, F_t^*, C_{Tt}, W_t, \mathcal{E}_t, B_t^*, \sigma_t^2\}_{t\geq 0}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\gamma \log C_{Tt} + (1-\gamma)\left(\log W_t - \frac{W_t}{A_t}\right)\right] \tag{6}$$

$$\begin{aligned}
\text{subject to} \qquad \frac{B_t^*}{R_t^*} - B_{t-1}^* &= (Y_{Tt} - C_{Tt}) - (1-\tau)\left[R_{t-1}^* - R_{t-1}\frac{\mathcal{E}_{t-1}}{\mathcal{E}_t}\right]\frac{B_{t-1}^* - F_{t-1}^*}{R_{t-1}^*}, \\
\beta R_t^* \mathbb{E}_t \frac{C_{Tt}}{C_{Tt+1}} &= 1 + \omega\sigma_t^2 \frac{B_t^* - N_t^* - F_t^*}{R_t^*}, \\
\beta R_t \mathbb{E}_t \left\{\frac{C_{Tt}}{C_{Tt+1}}\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\right\} &= 1, \\
\mathcal{E}_t &= \frac{\gamma}{1-\gamma}\frac{W_t}{C_{Tt}}, \\
\sigma_t^2 &= R_t^2 \cdot \mathrm{var}_t\Big(\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\Big),
\end{aligned}$$

given the stochastic path of exogenous variables $\{A_t, Y_{Tt}, R_t^*, N_t^*\}$ and subject to initial and transversality conditions on B_t^*. In writing the objective function we used the fact that under sticky prices, $P_{Nt} = 1$, we have $C_{Nt} = W_t$ and $L_t = W_t/A_t$. The constraints on the policy problem are: (1) the country budget constraint; (2) the international risk sharing condition; (3) the home-currency Euler equation; (4) the expenditure switching condition (which again $C_{Nt} = W_t$); and (5) the definition of the exchange rate volatility σ_t^2.

Two unusual properties of the optimal policy problem (6) are that the country budget constraint features an unconventional last term reflecting the international income transfer of the financial sector and that the international risk sharing condition features an endogenous risk sharing wedge that depends on the price of risk $\omega\sigma_t^2$ and quantity of risk $\frac{D_t^*}{R_t^*} = \frac{B_t^* - N_t^* - F_t^*}{R_t^*}$ held by the arbitrageurs. As a result, exchanger rate volatility directly enters the constraint of the policy problem, as it affects the equilibrium risk sharing wedge. In our baseline, we focus on the case without international transfers ($\tau = 1$), which considerably simplifies the analysis, and we study international transfers in Section 5.

Constrained optimum Focusing on the case without international transfer, $\tau = 1$, the constrained optimum problem maximizes (6) subject to the budget constraint $\frac{B_t^*}{R_t^*} - B_{t-1}^* = Y_{Tt} - C_{Tt}$ alone. Therefore, the constrained optimum allocation $\{\tilde{W}_t, \tilde{C}_{Tt}, \tilde{B}_t^*\}$ satisfies $\tilde{W}_t = A_t$ and the undistorted international risk sharing condition $\beta R_t^* \mathbb{E}_t \frac{C_{Tt}}{C_{Tt+1}} = 1$, which together with the budget constraint determines the path of $\{\tilde{C}_{Tt}, \tilde{B}_t^*\}$. Optimal $\tilde{C}_{Tt}$ is a function of shocks $\{Y_{Tt}, R_t^*\}$ and B_{-1}^*. The other variables can be recovered using static equilibrium conditions. In particular, $\tilde{C}_{Nt} = A_t$, $\tilde{L}_t = 1$, and $\tilde{\mathcal{E}}_t = \frac{\gamma}{1-\gamma}\frac{A_t}{\tilde{C}_{Tt}} = \tilde{\mathcal{Q}}_t^{1/(1-\gamma)}$, where $\tilde{\mathcal{Q}}$ is the first-best real exchange rate that ensures efficient expenditure switching in the goods market. Note that constrained optimum eliminates both the output gap in non-tradable consumption, i.e. $C_{Nt}/\tilde{C}_{Nt} = 1$, and the risk sharing wedge in tradable consumption, i.e. $C_{Tt}/\tilde{C}_{Tt} = 1$ — the two policy objectives.

3.2 Approximate policy problem

While the exact policy problem (6) is partially tractable (see Appendix A), we nonetheless consider a linear-quadratic approximation which allows us to obtain sharp characterization of the optimal policies. There are two challenges involved in the transition to a linear-quadratic environment. The first challenge relates to the quadratic approximation of the welfare function in an open economy, and in particular where the constrained optimal risk sharing is not full insurance, as the international financial market is incomplete and features risk free bonds only. Specifically, the optimal risk sharing corresponds to no UIP deviations in (5) rather than perfect consumption smoothing. The second challenge is associated with the risk sharing frictions that are proportional to second moments of the macro variables, namely the volatility of the nominal exchange rate σ_t^2 in (4). In our approximation, we must ensure that the risk sharing friction remains in the linear-quadratic environment to maintain the key tradeoff of the exact policy problem between output gap stabilization and risk sharing.

We denote with:

$$x_t \equiv \log(C_{Nt}/\tilde{C}_{Nt}) \qquad \text{and} \qquad z_t \equiv \log(C_{Tt}/\tilde{C}_{Tt})$$

the two wedges in our analysis, where $\{\tilde{C}_{Nt}, \tilde{C}_{Tt}\}$ is the constrained optimum allocation defined above. Recall that $C_{Nt} = W_t$ and $\tilde{C}_{Nt} = A_t$, and thus $x_t = w_t - a_t$ is our measure of output gap due to sticky prices in the non-tradable sector, where by convention small letters are log deviations of the corresponding variables (as clarified in Lemma 4 below). In turn, z_t is the measure of the risk sharing wedge, equal to the proportional gap between C_{Tt} and the constrained optimum $\tilde{C}_{Tt}$ which is defined by $\beta R_t^* \mathbb{E}_t\{\tilde{C}_{Tt}/\tilde{C}_{Tt+1}\} = 1$ and the budget constraint.

Lemma 4 *Without international transfers ($\tau = 1$), the equilibrium system (1)–(4) log-linearized around a non-stochastic equilibrium with $\bar{B}^* = \bar{N}^* = 0$, $\bar{R} = \bar{R}^* = 1/\beta$, and a finite non-zero $\omega\sigma_t^2$ is given by:*

$$\beta b_t^* = b_{t-1}^* - z_t, \tag{7}$$

$$\mathbb{E}_t \Delta z_{t+1} = -\bar{\omega}\sigma_t^2(\iota b_t^* - n_t^* - f_t^*), \tag{8}$$

$$\sigma_t^2 = \mathbb{E}_t e_{t+1}^2 - \big(\mathbb{E}_t e_{t+1}\big)^2, \tag{9}$$

$$e_t = \tilde{q}_t + x_t - z_t, \tag{10}$$

where $\mathbb{E}_t \Delta z_{t+1} = i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1}$ is the UIP deviation, $e_t = \log \mathcal{E}_t$ is the log nominal exchange rate, $i_t - i_t^* = \log(R_t/R_t^*)$, $\tilde{q}_t \equiv \tilde{c}_{Nt} - \tilde{c}_{Tt}$ is the first-best real exchange rate, $b_t^* \equiv (B_t^* - \tilde{B}_t^*)/\bar{Y}_T$, $n_t^* \equiv (N_t^* - \tilde{B}_t^*)/\bar{Y}_T$, $f_t^* \equiv F_t^*/\bar{Y}_T$, $\bar{\omega} \equiv \omega\bar{Y}_T/\beta$ and $\iota \in \{0, 1\}$ depending on the point of approximation.

The exogenous shocks in the linearized system are represented by two variables: (i) $\tilde{q}_t$, which reflects the evolution of non-tradable productivity A_t relative to tradable endowment Y_{Tt} and international interest rate R_t^* that shape the path of $\tilde{C}_{Tt}$; and (ii) n_t^* which reflects the foreign currency demand by noise traders N_t^* and households in the constrained optimum allocation $\tilde{B}_t^*$. Note that the path of $\tilde{B}_t^*$, like that of $\tilde{C}_{Tt}$, is shaped by exogenous shocks $\{Y_{Tt}, R_t^*\}$. The policy variables are the output gap x_t and the FX intervention f_t^*. Indeed, the output gap is directly determined by the path of wages W_t, which in turn can be induced by an interest rate rule R_t, and thus we treat x_t as the policy instrument. Finally, Lemma 4 emphasizes that the path of the risk sharing wedge z_t shapes the path of UIP deviations, with $i_t - i_t^* - \mathbb{E}_t\Delta e_{t+1} = \mathbb{E}_t\Delta z_{t+1}$.[5]

A distinctive feature of our approach and the key property of the linearized equilibrium system in Lemma 4 is that the second moment, namely the volatility of the nominal exchange rate σ_t^2, influences the first-order dynamics of the risk sharing wedge z_t, which in turn feeds back into the rest of the equilibrium system. The reason is that we take the approximation in a way that ensures that the risk premium approximated by $\bar{\omega}\sigma_t^2(\iota b_t^* - n_t^* - f_t^*)$ remains a first order object. Specifically, as shocks become small and $\tilde{\sigma}_t^2 = R_t^2 \cdot \mathrm{var}_t(\mathcal{E}_t/\mathcal{E}_{t+1}) \to 0$, we scale effective risk aversion of the financial sector ω to ensure that the sequence $\omega\tilde{\sigma}_t^2$ remains bounded away from zero by a constant (zero order term). We argue this provides a superior point of approximation for models that focus on the joint dynamics of macroeconomic variables and risk premia. Lastly, depending on the sequence of approximation, equation (8) features either $\iota = 1$ (baseline) or $\iota = 0$ (special case). The special case approximates the situation when macroeconomic demand for currency b_t^* is orders of magnitude smaller than financial (liquidity) demand for currency n_t^*, and disappears in relative terms in the limit.[6]

With Lemma 4, we can cast the policy problem as choosing the path of $\{x_t, z_t, e_t, b_t^*, f_t^*, \sigma_t^2\}$, where $\{x_t, f_t^*\}$ are policy instruments and $\{z_t, e_t, b_t^*, \sigma_t^2\}$ are endogenous variables solving the equilibrium system (7)–(10). The remaining element of the problem is the quadratic approximation to the welfare objective function around a constrained-optimal allocation with $x_t = z_t = 0$:

Lemma 5 *Without international transfers ($\tau = 1$), a second order approximation to the welfare objective (6) around a constrained optimal allocation $(\tilde{C}_{Tt}, \tilde{C}_{Nt}, \tilde{L}_t)$ is given by:*

$$\min \frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \big[\gamma z_t^2 + (1-\gamma)x_t^2\big]. \tag{11}$$

The difference of this approximation from a conventional approximation is that we do not use the first-best allocation with $C_{Tt} = const$, but rather a constrained optimal allocation with $C_{Tt} = \tilde{C}_{Tt}$ implied by $\beta R_t^* \mathbb{E}_t\{\tilde{C}_{Tt}/\tilde{C}_{T,t+1}\} = 1$. We prove Lemmas 4 and 5 in the appendix, and characterize the solution to the optimization problem (11) subject to (7)–(10) in Section 4.

Equilibrium dynamics We now impose some structural assumptions on the dynamics of shocks to solve the equilibrium system (7)–(10) in certain special cases which prove useful in future analysis. In particular, this allows to characterize the equilibrium exchange rate volatility σ_t^2. First, we characterize

[5] From the frictionless international Euler equation, we have $i_t^* = \log(R_t^*/\bar{R}^*) = \mathbb{E}_t\Delta\tilde{c}_{Tt+1}$. The home-currency Euler equation (2), in turn, implies $i_t = \log(R_t/\log\bar{R}) = \mathbb{E}_t\{\Delta c_{Tt+1} + \Delta e_{t+1}\}$. Subtracting one from the other yields the U|P result after using the definition of the risk sharing wedge $z_t = c_{Tt} - \tilde{c}_{Tt}$. Further note how the home-currency Euler equation, using labor supply $C_{Nt} = W_t$, also implies $i_t = \mathbb{E}_t\Delta c_{Nt+1} = \mathbb{E}_t\Delta w_{t+1}$, which shows the link between the path of the home interest rate and nominal wages, which in turn determine the output gap $x_t = w_t - a_t$. Shocks to R_t^* do not directly move R_t (chosen by monetary policy), and can instead be absorbed by the nominal exchange rate or UIP deviation.

[6] Formally, denoting with n and m the measures of symmetric noise traders and arbitrageurs respectively, the limit with $n, m \to \infty$ and $n/m = const$ results in $\iota = 0$, while the baseline with $n = m = 1$ features $\iota = 1$.

the dynamics of $\{\tilde{c}_{Tt}, \tilde{q}_t\}$ when y_{Tt} follows an AR(1) with persistence ρ. Assuming $\beta R_t^* \equiv 1$, we have:

$$\mathbb{E}_t \Delta\tilde{c}_{T,t+1} = 0,$$
$$\beta\tilde{b}_t^* = \tilde{b}_{t-1}^* + y_{Tt} - \tilde{c}_{Tt},$$

which result in the following solution:

$$\Delta\tilde{c}_{Tt} = \frac{1-\beta}{1-\beta\rho}(1-\rho L)y_{Tt} \sim \text{iid}, \tag{12}$$

which reduces to $\tilde{c}_{Tt} = y_{Tt}$ when y_{Tt} itself follows a random walk (i.e., $\rho = 1$).[7] The first-best real exchange rate is given by $\tilde{q}_t = a_t - \tilde{c}_{Tt}$, and thus in general follows an ARIMA(1,1,1). When a_t and y_{Tt} both follow random walks, $\tilde{q}_t$ is also a random walk with innovations reflecting non-tradable productivity growth relative to tradable endowment growth. Furthermore, $\tilde{q}_t = 0$ when a_t and y_{Tt} are perfectly comoving random walks so that the relative non-tradable and tradable outputs are stable.

Next, we consider the equilibrium path of $\{z_t, b_t^*\}$ when $n_t^* - f_t^*$ follows an AR(1) with persistence ρ. We conjecture and verify that $\sigma_t^2 = \sigma^2$. In this case, we can show that z_t satisfies:

$$z_t = (1-\beta\lambda_1)b_{t-1}^* - \frac{\beta\lambda_1\bar{\omega}\sigma^2}{1-\beta\rho\lambda_1}(n_t^* - f_t^*), \tag{13}$$

where $\lambda_1 \leq 1$ and $\lambda_2 \geq 1/\beta > 1$, such that $\lambda_1\lambda_2 = 1/\beta$, are the two roots of the equilibrium dynamic system, which in general depend on σ^2. When $\iota = 0$ in (8), we have $\lambda_1 = 1$ and $\lambda_2 = 1/\beta$ independently of the value of σ^2. Solving further for z_t, we can show that it follows an ARMA(2,1) process with autoregressive roots ρ and λ_1 and moving average root $1/\beta$.[8]

Equations (12)–(13) allow us to evaluate the resulting conditional volatility of the exchange rate:

$$\begin{aligned}\sigma^2 = \text{var}_t(e_{t+1}) = \text{var}_t\big(\tilde{q}_{t+1} + x_{t+1} - z_{t+1}\big) &= \text{var}_t\big(w_{t+1} - \tilde{c}_{T,t+1} - z_{t+1}\big) \\ &= \text{var}_t\left(\varepsilon_{t+1}^w - \frac{1-\beta}{1-\beta\rho}\varepsilon_{t+1}^y + \frac{\beta\lambda_1\bar{\omega}\sigma^2}{1-\beta\rho\lambda_1}(\varepsilon_{t+1}^n - \varepsilon_{t+1}^f)\right),\end{aligned}$$

where $(\varepsilon_{t+1}^y, \varepsilon_{t+1}^n)$ are exogenous innovations of tradable endowment and liquidity dollar demand shocks, respectively, and $(\varepsilon_{t+1}^w, \varepsilon_{t+1}^f)$ are innovations of monetary and FX policy, repsectively. Therefore, this equation characterizes a fixed point for σ^2, which is indeed constant as long as innovations $(\varepsilon_{t+1}^y, \varepsilon_{t+1}^n, \varepsilon_{t+1}^w, \varepsilon_{t+1}^f)$ have a constant covariance matrix Σ. In what follows, we consider various special cases in which the expression characterizing σ^2 simplifies and σ^2 can be solved explicitly. For example, when monetary policy stabilizes the output gap, $x_t = w_t - a_t = 0$, we have $\varepsilon_{t+1}^w = \varepsilon_{t+1}^a$, i.e. the innovation of non-tradable productivity.

[7] The implied solution for $\tilde{b}_t$ is an ARIMA(1,1,0) given by $\Delta\tilde{b}_t = \frac{1-\rho}{1-\beta\rho}y_t$, which reduces to $\tilde{b}_t = 0$ when $\rho = 1$.

[8] Specifically, $z_t = \lambda_1 z_{t-1} - \frac{\beta\lambda_1\bar{\omega}\sigma^2}{1-\beta\rho\lambda_1}(1-\beta^{-1}L)(n_t^* - f_t^*)$ and $b_t^* = \lambda_1 b_{t-1}^* + \frac{\lambda_1\bar{\omega}\sigma^2}{1-\beta\rho\lambda_1}(n_t^* - f_t^*)$, an AR(2). In the case with $\lambda_1 = 1$ (when $\iota = 0$), we have Δz_t follow an ARMA(1,1) and Δb_t^* an AR(1).

4 Optimal Policies

Lemmas 4 and 5 characterize the equilibrium system and the quadratic objective function respectively in the linearized environment, and we reproduce the policy problem here:

$$\min_{\{x_t,z_t,e_t,b_t^*,f_t^*,\sigma_t^2\}} \frac{1}{2}\mathbb{E}_0 \sum\nolimits_{t=0}^{\infty}\beta^t\big[\gamma z_t^2+(1-\gamma)x_t^2\big] \tag{14}$$

$$\begin{aligned}\text{subject to}\qquad \beta b_t^* &= b_{t-1}^*-z_t,\\ \mathbb{E}_t\Delta z_{t+1} &= -\bar{\omega}\sigma_t^2(\iota b_t^*-n_t^*-f_t^*),\\ \sigma_t^2 &= \mathbb{E}_t e_{t+1}^2-\big(\mathbb{E}_t e_{t+1}\big)^2 \quad\text{where}\quad e_t=\tilde{q}_t+x_t-z_t,\end{aligned}$$

given initial net foreign assets b_{-1}^* and the exogenous path of shocks $\{\tilde{q}_t, n_t^*\}$, where $\tilde{q}_t = a_t - \tilde{c}_{Tt}$ is the first best-real exchange rate and n_t^* is the aggregate demand for currency (including both fundamental and noise sources), as described above. We think of monetary policy as choosing directly the path of output gap x_t, while FX interventions f_t^* control the path of the risk sharing wedge z_t. The policies, thus, interact in determining the equilibrium volatility of the exchange rate, σ_t^2, which in turn feeds back into shaping the equilibrium risk sharing wedge without being directly a goal of the policy in itself. More specifically, the goal of the policy is to minimize the weighted average of the volatility (in the mean squared error sense) of the output gap and the risk sharing wedge, with the weight on the latter equal to the openness of the economy (share of tradables in consumption).

Exchange rate and the policy tradeoff Condition (10) emphasizes that the nominal exchange rate is an important equilibrium variable linking the goods market and the financial market. Specifically, the nominal exchange rate e_t can be decomposed into the first-best real exchange rate $\tilde{q}_t$, which ensures efficient expenditure switching in the goods market, the output gap in the goods market x_t, and the risk sharing wedge in the financial market z_t. The volatility of the nominal exchange is both necessary to accommodate the adjustment in the real exchange rate (when prices are sticky), but also results in the emergence of a risk sharing wedge in (8).

This emphasizes the tradeoff faced by the optimal exchange rate policy. If monetary policy fully stabilizes output gap, $x_t = 0$, then the nominal exchange rate must equal $e_t = \tilde{q}_t - z_t$, which in general results in $\sigma_t^2 > 0$ and a non-zero risk sharing wedge, $\mathbb{E}z_t^2 > 0$, unless it is stabilized by FX interventions f_t^*. Without FX interventions ($f_t^* \equiv 0$), optimal risk sharing with $z_t = 0$ can only be achieved with $\sigma_t^2 = 0$, which in turn requires $e_t = \tilde{q}_t + x_t = 0$, and thus in general a non-zero output gap, $\mathbb{E}x_t^2 > 0$. We now focus on the design of the optimal policies that arise from this tradeoff.

4.1 Constrained optimum implementation

The constrained optimum allocation features $x_t = z_t = 0$ for all t, as it is the global minimum of the loss function. This allocation can be always delivered by a combination of monetary and FX policies. Specifically, in addition to monetary policy that stabilized output gap, $x_t = 0$ (or $w_t = a_t$), the optimal FX interventions are $f_t^* = \iota b_t^* - n_t^* = -n_t^*$, since this policy ensures $z_t = 0$, and by consequence $b_t^* = 0$.[9] As a result, the risk sharing wedge is fully offset, and the optimal international risk sharing is restored independently of the currency demand shocks n_t^*. This solution is time consistent and its implementation requires no commitment.

Proposition 1 *If both policy instruments are available and unconstrained, the optimal policy fully stabilizes both wedges, the output gap $x_t = 0$ and the risk sharing wedge $z_t = 0$, by targeting home PPI inflation with monetary policy ($w_t = a_t$) and demand for currency with FX interventions ($f_t = \iota b_t^* - n_t^* = -n_t^*$). This solution is time consistent and its implementation requires no commitment.*

One notable feature of this result is that capital controls (see Section 5) are not needed for implementation, as FX interventions are sufficient to achieve the constrained optimum allocation when combined with optimal monetary policy. The second implication of this result is that FX interventions do not target exchange rate or ensure full exchange rate stabilization. The optimal policy ensures $x_t = z_t = 0$, which in turn implies that the nominal exchange rate equals the first-best real exchange rate:

$$e_t = \tilde{q}_t + x_t - z_t = \tilde{q}_t = a_t - \tilde{c}_{Tt}, \tag{15}$$

and generally $\sigma_t^2 = \mathrm{var}_t(\Delta e_{t+1}) > 0$. Instead, optimal FX interventions eliminate UIP deviations:

$$i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1} = \mathbb{E}_t \Delta z_{t+1} = 0, \tag{16}$$

where $i_t - i_t^* = \log(R_t/R_t^*)$ as defined in Lemma 4.

Optimal FX interventions offset currency demand shocks $n_t^* = (N_t^* - \tilde{B}_t^*)/\bar{Y}_T$ and allow the exchange rate to accommodate fundamental macroeconomic shocks $\{A_t, Y_{Tt}, R_t^*\}$ that drive the first-best real exchange rate $\tilde{q}_t$. Both liquidity N_t^* and macroeconomic $\tilde{B}_t^*$ currency demand shocks require intermediation in the financial market. To the extent this intermediation is frictional and results in distortions — namely, UIP deviations due to the exchange rate risk that would not be priced by the household SDF — FX interventions should step in to eliminate such UIP deviation.[10] In practice, this means providing FX liquidity to the market to offset currency demand shocks and alleviating the need for costly intermediation.

Implementation Optimal policies in Proposition 1 can be implemented using simple policy rule — a conventional Taylor interest rate rule targeting the output gap and a similar policy rule for FX interventions targeting ex ante UIP deviations $\mathbb{E}_t \Delta z_{t+1} = i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1}$, specifically $f_t^* = -\alpha \mathbb{E}_t \Delta z_{t+1}$. Substituting this policy rule into (8), we have $\mathbb{E}_t \Delta z_{t+1} = -\frac{\bar{\omega}\sigma_t^2}{1+\alpha\bar{\omega}\sigma_t^2}(\iota b_t^* - n_t^*) \to 0$ as $\alpha \to \infty$ and $f_t^* - (\iota b_t^* - n_t^*) \to 0$ in this limit. Simply put, FX interventions should lean against the wind intensively enough until the UIP wedge is entirely eliminated. Importantly, this does not require observing

[9] Note that the policy rule $f_t^* = -n_t^*$ also necessarily implies $z_t = b_t^* = 0$ as the unique solution, thus resulting in the same outcome. The reason is that the presence of b_t^* in (7) results in stable dynamics of $\{z_t, b_t^*\}$ and eliminates multiplicity.

[10] More generally, FX interventions should be used to eliminate (or minimize) rents in the currency market due to financial frictions (as e.g. in Gabaix and Maggiori 2015) or due to monopoly power of intermediaries. The portion of UIP deviations due to risk that is priced by the household SDF (e.g., default risk) should not be eliminated with FX interventions.

the shocks and distinguishing between macro-fundamental and non-fundamental sources of variation in the exchange rate.

The challenge with this implementation is that the UIP wedge is unobservable — neither $\mathbb{E}_t\Delta z_{t+1}$, nor $\mathbb{E}_t\Delta e_{t+1}$ is directly measurable in the data.[11] Interest rate differential $i_t - i_t^*$ and exchange rate e_t (or depreciation Δe_t) are, however, observable, and the policy rule can condition on this variables directly, e.g. $f_t^* = -\alpha e_t$.[12] Such policy rule stabilizes exchange rate which is not the goal of optimal policy, but may correspond to a second-best implementation if currency demand shocks n_t^* are the dominant source of exchange rate volatility. In contrast, if much of the exchange rate volatility is due to fundamental shocks $\tilde{q}_t$, then it is optimal not to respond to fluctuations in e_t, that is set $\alpha = 0$. Condition on the realization of the shocks, the policymaker can make a call whether to stabilize the exchange rate or not. This is akin to a challenge with the conventional Taylor rule, where the policymaker needs to make a judgement call whether output is high due to high productivity or due to opening output gap (or what is the natural rate of interest or NAIRU). Finally, an interesting property of an exchange rate rule $f_t^* = -\alpha e_t$ is that there is an upper bound on the value of α beyond which the policy becomes inconsistent with the budget constraint resulting in an explosive path for net foreign assets.[13] This, in particular, implies that a full credible exchange rate stabilization by means of an FX rule is infeasible.

Trilemma Does trilemma apply in this model? On one hand, full inflation (output gap) and UIP stabilization can be simultaneously achieved with the aid of FX interventions and without any capital controls. On the other hand, UIP stabilization generally does not result in a pegged exchange rate, as FX interventions only eliminate currency demand shocks and allow the exchange rate to accommodate macroeconomic shocks that require expenditure switching — as prescribed by optimal policy under sticky prices. Thus, exclusive inward focus of monetary policy on domestic inflation and output gap does not compromise the ability to achieve optimal exchange rate adjustment — a sense in which the trilemma constraint is relaxed. Furthermore, unconstrained FX interventions allow to manipulate the path of the exchange rate, provided $\sigma_t^2 > 0$ and subject to the country's intertemporal budget constraint, without compromising the ability of monetary policy to eliminate the output gap. However, if the goal of the policy were to fully stabilize the nominal exchange rate for some exogenous reason, this would result in $\sigma_t^2 = 0$ in (8), making the FX instrument irrelevant, and bringing back the classic trilemma constraint on monetary policy. Section 4.2 explores an important special case when this constraint is not binding.

We also note that domestic policy rate $i_t = \log(R_t/\bar{R})$ affects home output gap x_t and exchange rate e_t, but does not affect equilibrium capital flows. Indeed, home Euler equation (2) does not enter the policy problem (14), as international risk sharing — and thus the path of tradable consumption

[11] Even if $i_t - i_t^* - \mathbb{E}_t\Delta e_{t+1}$ were observed, one needs to make a call what portion of this UIP deviations corresponds to the intermediation wedge that needs to be eliminated by FX interventions and what portion is a fundamental risk premium that should not be eliminated.

[12] Alternatively, the policy can condition on ex post UIP realization, $i_{t-1} - i_{t-1}^* - \Delta e_{t+1}$, or on a noisy measure of the ex ante UIP, $\mathbb{E}_t\Delta z_{t+1} + u_t$ where u_t may be an expectational error. In both cases, first-best implementation is infeasible, and there is an internal optimum for the policy intensity α to avoid overreaction to noise in the measurement of the target.

[13] Intuitively, a very large α implements a nearly constant exchange rate, which generally is inconsistent with intertemporal budget constraint.

z_t and resulting international capital flows b_t^* — is shaped entirely by (4), a counterpart to (8). In other words, changes in home policy rate i_t are neutralized by changes in e_t and $\mathbb{E}_t \Delta e_{t+1}$, and do not affect equilibrium UIP deviations and the resulting international financial positions. As a result, while increasing home interest rate can curb exchange rate depreciation, if the goal of this policy is to stop capital outflows — then it is ineffective.[14]

4.2 Divine coincidence

We now study the optimal monetary policy when FX interventions are not available, that is $f_t^* \equiv 0$. We start with the case in which the constrained optimum is attainable with a single monetary policy instrument, and by analogy with the closed economy literature we label this case *divine coincidence.*

Proposition 2 *If the first-best real exchange rate is stable, $\tilde{q}_t = 0$, then monetary policy that fully stabilizes the nominal exchange rate, $e_t = 0$, ensures the first best allocation with $x_t = z_t = 0$, even in the absence of FX interventions $f_t^* = 0$. An exchange rate peg is superior to inflation targeting, as it rules out multiplicity of exchange rate equilibria.*

The first best solution (15) implies that the nominal exchange rate equals the first-best real exchange rate, $e_t = \tilde{q}_t$. Therefore, if $\tilde{q}_t = 0$, then $e_t = 0$ is part of the constrained optimal allocation, and this implies $\sigma_t^2 = 0$. In turn, with $\sigma_t^2 = 0$, equations (7)–(8) ensure $z_t = 0$ independently of the path of (b_t^*, n_t^*, f_t^*). Hence, if $e_t = 0$ is consistent with $x_t = z_t = 0$, then such policy ensures the optimal outcome as the unique equilibrium. Indeed, this is a "divine coincidence" as targeting one margin — a zero risk-sharing wedge $z_t = 0$ — simultaneously ensures an efficient real exchange rate and eliminates the output gap $x_t = 0$.[15]

This case provides a rationale for pegging the exchange rate. Moreover, in this case, a nominal exchange rate peg by means of monetary policy is not only efficient, but also effective, as it immediately eliminates the possibility of multiple equilibria. Consider the alternative policy of inflation targeting with $w_t = a_t$, which ensures $x_t = 0$ independently of z_t. Under divine coincidence, such policy is consistent with an equilibrium $z_t = e_t = \sigma_t^2 = 0$, however, this is not a unique equilibrium. Indeed, consider our example in Section 3.2, where n_t^* follows an AR(1). In this case, in light of $\tilde{q}_t = x_t = 0$, the solution for the nominal exchange rate is $e_t = -z_t = -(1-\beta\lambda_1)b_{t-1}^* + \frac{\beta\lambda_1\bar{\omega}\sigma^2}{1-\beta\rho\lambda_1}n_t^*$, which implies:

$$\sigma_t = \sigma = \frac{\beta\lambda_1\bar{\omega}\sigma^2}{1-\beta\rho\lambda_1}\mathrm{std}_t\big(\varepsilon_{t+1}^n\big).$$

This generally features two solutions — with $\sigma = 0$ and with $\sigma > 0$, the latter being a suboptimal outcome with $\mathbb{E}z_t^2 > 0$. Thus, under divine coincidence, exchange rate peg dominates inflation targeting, even though the outcome of exchange rate peg is stable inflation (*cf.* Marcet and Nicolini 2003).

[14]The use of monetary policy at $t+1$, however, can affect the financial market allocation at t by changing the conditional second moment of e_{t+1}, namely σ_t^2, as we study in Section 4.3.

[15]In Section 7.1 with Calvo staggered price changes, we show that the open economy divine coincidence nest the closed economy divine coincidence in the sense that $\tilde{q}_t = 0$ is an additional condition over and above the conventional conditions for divine coincidence in the closed economy (namely, the absence of cost-push shocks in the Phillips curve).

How does divine coincidence work? The nominal exchange rate has a dual role. On one hand, its movements ensure expenditure switching in the goods market, changing the relative price of domestic (home or non-tradable) and international (foreign or tradable) goods. In presence of sticky prices, without such exchange rate movements — and corresponding exchange rate volatility — the real exchange rate departs from its first-best level and, as a result, the goods market does not achieve the efficient allocation, as reflected in the output gap $x_t \neq 0$. At the same time, nominal exchange rate volatility, $\sigma_t^2 > 0$, results in UIP deviations for a given $(\iota b_t^* - n_t^* - f_t^*) \neq 0$, and thus departures from frictionless international risk sharing, $z_t \neq 0$. These deviations are increasing in the unpredictable exchange rate volatility, thus resulting in a conflict between the two policy objectives, or a policy tradeoff.

Divine coincidence is the situation when this policy tradeoff is absent, as it occurs when the first-best real exchange rate is stable $\tilde{q}_t = 0$, and thus $e_t = 0$ ensures both $x_t = 0$ and $z_t = 0$ — the latter due to $\sigma_t^2 = 0$, and the former as a coincidence due to $\tilde{q}_t = 0$. Note that in our baseline model, $\tilde{q}_t = a_t - \tilde{c}_{Tt}$ reflecting the Balassa-Samuelson forces in the model with non-tradables. In particular, if both non-tradable productivity a_t and tradable endowment y_{Tt} follow an identical random walk, and there are no international interest rate shocks ($\beta R_t^* \equiv 1$), then $\tilde{c}_{Tt} = y_{Tt} = a_t$, resulting in a divine coincidence $\tilde{q}_t = 0$. This is, of course, a knife-edge case which we do not expect to systematically hold in the data, yet it provides a key benchmark for our analysis and a stark illustration to the model's mechanism.

How special is the divine coincidence result? We explore its robustness below, where we show in particular that it does not rely on the specific model of the real exchange rate, namely the Balassa-Samuelson model with non-tradables and the law of one prices for tradables. What is crucial, however, is the model of the financial market in which ex post stable exchange rate, $e_{t+1} = 0$, implies ex ante certainty, namely $\sigma_t^2 = \mathrm{var}_t(e_{t+1}) = 0$, and it in turn guarantees that UIP holds and risk sharing is undistorted. This nests two assumptions. First, it requires that commitment to a peg is ex ante credible. Second, it relies on the structure of the model in which a fully stabilized exchange rate eliminates UIP deviations via the endogenous response of arbitrageurs, who are willing to take unbounded positions in the absence of exchange rate risk if UIP is violated. If either the peg is not credible, and there is a chance that $e_{t+1} \neq \mathbb{E}_t e_{t+1}$, or UIP deviations may coexist with $\sigma_t^2 = 0$, then divine coincidence result breaks down. To the extent a credible peg eliminates a large portion of UIP deviations, as the data seems to suggest, this result nonetheless may offer an accurate quantitative approximation, emphasizing robust economic forces at play.

Optimal currency areas The divine coincidence result also provides an important benchmark for common currency areas, which are optimal when the first-best real exchange rate between member countries is stable ($\tilde{q}_t = 0$). In particular, this is the case when member countries share correlated fundamental shocks (see Section 6), confirming the logic of Mundell (1961). What is new in our result is that it not only identifies the costs from lacking expenditure switching in suboptimal currency areas (resulting in $\mathbb{E}x_t^2 > 0$), but also emphasizes the benefits of the fixed exchange rate from the point of reduced financial volatility and improved risk sharing between member countries (resulting in $\mathbb{E}z_t^2 = 0$).

4.3 Optimal monetary policy

We now study the optimal monetary policy when FX interventions are constrained at some exogenous level f_t^*, which allows for no FX interventions $f_t^* \equiv 0$ as a special case, and divine coincidence does not hold, that is $\tilde{q}_t \neq 0$ and $\mathrm{var}_t(\tilde{q}_{t+1}) > 0$. Indeed, divine coincidence is unlikely to hold generally in the data and unconstrained FX interventions might be infeasible, and thus this case is arguably an important case of policy interest. The policy problem is still given by (14), yet taking the path of f_t^* as exogenously given and no longer a variable of optimization. We set up the Lagrangian for this problem and derive the optimality conditions in Appendix B.1, which yield the following result:

Proposition 3 *For a given exogenous path of FX interventions, Ramsey optimal monetary policy sets the path of output gap to satisfy* $\mathbb{E}_t x_{t+1} = 0$ *and*

$$(1-\gamma)x_{t+1} = -\gamma\bar{\omega}\mu_t(\iota b_t^* - n_t^* - f_t^*)\big(e_{t+1} - \mathbb{E}_t e_{t+1}\big), \tag{17}$$

where μ_t *is the Lagrange multiplier on the risk sharing constraint* (8) *such that* $\mu_t(\iota b_t^* - n_t^* - f_t^*) \geq 0$ *with strict inequality if and only if* $\bar{\omega}\sigma_t^2(\iota b_t^* - n_t^* - f_t^*) \neq 0$*. Without commitment, discretionary optimal monetary policy stabilizes output gap,* $x_{t+1} = 0$*.*

Proposition 3 has a number of implications. First, optimal monetary policy always stabilizes the expected output gap, $\mathbb{E}_t x_{t+1} = 0$, irrespective of the outcome for the risk sharing wedge z_t and $\mathbb{E}_t \Delta z_{t+1}$. Furthermore, when $\bar{\omega}\sigma_t^2(\iota b_t^* - n_t^* - f_t^*) = 0$ — that is period t UIP deviation is absent, $\mathbb{E}_t\Delta z_{t+1} = 0$ — monetary policy at $t+1$ eliminates output gap state-by-state, $x_{t+1} = 0$. Under these circumstances, the policymaker is confronted with no tradeoff, and this policy is time consistent.

Second, when $\bar{\omega}\sigma_t^2(\iota b_t^* - n_t^* - f_t^*) \neq 0$ and period t UIP deviations are present, $\mathbb{E}_t\Delta z_{t+1} \neq 0$, optimal monetary policy allows output gap x_{t+1} to vary state-by-state to reduce conditional exchange rate volatility at t, σ_t^2, that is to stabilize $e_{t+1} = \tilde{q}_{t+1} - z_{t+1} + x_{t+1}$ around $\mathbb{E}_t e_{t+1}$. In other words, monetary policy leans against the wind of surprise exchange rate pressure from $(\tilde{q}_{t+1} - \mathbb{E}_t\tilde{q}_{t+1})$ and $(z_{t+1} - \mathbb{E}_t z_{t+1})$.[16] This in particular implies that an unexpected depreciation, $e_{t+1} > \mathbb{E}_t e_{t+1}$, whether driven by goods market or financial market forces, requires a monetary tightening that results in an output gap, $x_{t+1} < 0$. This policy can be implemented as a Taylor interest rate rule which puts explicit (state contingent) weight on exchange rate surprises and raises interest rate i_t when $e_t > \mathbb{E}_{t-1}e_t$. This policy is not time consistent and requires commitment on the part of monetary authority. The only time consistent discretionary policy is output gap stabilization, $x_{t+1} = 0$, as departures from it alleviate only previous period's risk sharing wedge, $\mathbb{E}_t\Delta z_{t+1}$, and have no direct effect on current and future wedges.

Third, the intensity of departures from $x_{t+1} = 0$ increases in the relative welfare costs of period t UIP deviation as captures by $\frac{\gamma}{1-\gamma}\bar{\omega}\mu_t(\iota b_t^* - n_t^* - f_t^*)$, where $\gamma/(1-\gamma)$ is the relative consumption share of the tradable and non-tradable sectors. Periods with larger expected exchange rate volatility, σ_t^2,

[16] Rewriting $e_{t+1} = \tilde{q}_{t+1} - z_{t+1} + x_{t+1}$ in unexpected changes and using (17) results in:

$$e_{t+1} - \mathbb{E}_t e_{t+1} = \frac{(\tilde{q}_{t+1} - \mathbb{E}_t\tilde{q}_{t+1}) - (\Delta z_{t+1} - \mathbb{E}_t\Delta z_{t+1})}{1 + \frac{\gamma}{1-\gamma}\bar{\omega}\mu_t(\iota b_t^* - n_t^* - f_t^*)},$$

which shows how optimal policy dampens unexpected exchange rate volatility that comes from $\tilde{q}_{t+1}$ and z_{t+1} surprises.

and larger excess demand or supply of currency that requires intermediation, $|\iota b^*_t - n^*_t - f^*_t|$, call for a commitment to a stronger future response of monetary policy, x_{t+1}, to unexpected exchange rate movements, $e_{t+1} - \mathbb{E}_t e_{t+1}$. This allows to partially stabilize ex ante UIP deviations, $\mathbb{E}_t \Delta z_{t+1}$, and the risk sharing wedge z_t in periods when they are particularly large or costly for welfare. This suggests a state-contingent policy approach to financial market volatility, which can be ignored when it causes no spikes in risk premia (intermediation wedges), but should be smoothed out with monetary policy when such volatility distorts risk sharing and direct financial market (FX) interventions are limited.

Finally, optimal policy allows for any path of expected exchange rate changes, $\mathbb{E}_t \Delta e_{t+1} \neq 0$, which do not constrain monetary policy or result in risk sharing wedges. Indeed, if all exchange rate changes are expected, that is $\Delta e_{t+1} = \mathbb{E}_t \Delta e_{t+1}$ state by state, then (17) implies $x_{t+1} \equiv 0$ as well, and furthermore $\sigma_t^2 = \mathrm{var}_t(\Delta e_{t+1}) = 0$ in this case implies $\mathbb{E}_t \Delta z_{t+1} = 0$ from (8). The implication is that any medium-run exchange rate adjustment can be accommodated with expected exchange rate changes — that is, a managed *crawling peg* — without resulting in welfare costs in goods or financial markets. This, of course, can be only consistent with equilibrium if the first best real exchange rate itself features no unexpected surprise volatility, $\Delta \tilde{q}_{t+1} \equiv \mathbb{E}_t \Delta \tilde{q}_{t+1}$, otherwise a crawling peg will still result in ex post output gaps, $x_{t+1} \neq 0$. This is a restatement of the divine coincidence result of Proposition 2, in its stronger form (weaker requirement on $\tilde{q}_{t+1}$). Even outside this case, crawling pegs, "floating bands" and "dirty floats" (Ilzetzki, Reinhart, and Rogoff 2019) can offer second best compromises between stabilizing output gap and risk sharing wedges, akin to the one suggested by our optimality condition (17).

4.4 Optimal FX interventions

Consider next the opposite situation when monetary policy is constrained and the planner can only choose FX interventions. No divine coincidence with one instrument closing the two gaps emerges in this case. This is because FX interventions have only indirect effect on output gap choosing consumption of tradables, while the path of x_t is determined by the response of monetary policy to movements in z_t (and/or e_t). Under the zero lower bound (ZLB) — a particularly relevant constraint on monetary policy for many developed countries — the path of x_t is determined by the Euler equation $\mathbb{E}_t\big\{\Delta x_{t+1} + \Delta \tilde{c}_{Nt+1}\big\} = 0$ and is independent from z_t. As a result, FX interventions cannot close the output gap and optimally focus on closing the risk-sharing wedge $z_t = 0$. This contrasts with the case of a currency union where a fixed nominal exchange rate $e_t = \tilde{q}_t + x_t - z_t = 0$ induces an endogenous response of output gap x_t to consumption of tradables z_t (Farhi and Werning 2017). Yet, as discussed above, FX policy becomes irrelevant in this limit when carry trade is risk free and government interventions crowd out positions of arbitrageurs without any affect on risk premia or allocations.[17]

Another important difference from monetary policy is that for any given path of x_t, FX interventions are time consistent, so that the optimal discretionary policy still closes UIP deviations and implements $z_t = 0$. This does not mean, however, that commitment on behalf of the planner provides no additional benefits. The gains from commitment arise when FX interventions are subject to occa-

[17]This illustrates an important limitation of FX interventions relative to capital controls in eliminating aggregate demand externality (Farhi and Werning 2016).

sionally binding constraints. For example, countries might be unable to take negative reserve positions in foreign currency at the world interest rate and therefore, can only choose $f_t^* \geq 0$. Similarly, taking large FX positions can lead to government losses which would require a bailout by households compromising central bank independence. This means a planner might be subject to an additional value-at-risk constraint that puts an upper bound on FX interventions, e.g. $\sigma_t^2 \cdot f_t^{*2} \leq \bar{\alpha}$ where the left hand side is the conditional volatility of the value of central bank's portfolio.

If either of these constraints binds, the FX instrument cannot be used to fully offset liquidity shock n_t^* opening a risk-sharing wedge. In this case, the commitment technology allows the planner to improve the allocation by offering *forward guidance* about the future path of f_t^*. This can be done in two ways. First, similarly to conventional monetary guidance, the planner can exploit the fact that z_t is a forward-looking variable and depends on its own future expectation, $\mathbb{E}_t z_{t+1}$ as a result of consumption smoothing. For example, consider a capital outflow shock $n_t^* > 0$ that depresses the present consumption of tradable goods z_t. Even if reserves f_t^* are subject to a non-negative constraint, a planner can promise to tolerate future capital inflows $n_{t+j}^* < 0$, which stimulate z_{t+j} and consequently z_t (*cf.* Werning 2011). Second, and differently from conventional forward guidance, future FX interventions can be used to smooth out the variation of z_{t+1} around the same mean $\mathbb{E}_t z_{t+1}$, thus reducing conditional volatility of the exchange rate σ_t^2 and encouraging arbitrageurs to offset the present distortionary shock n_t^*. Importantly, the country's budget constraint puts limits on how far the government can manipulate z_t and the exchange rate e_t. If poorly designed, forward guidance risks ending with large imbalances and a currency run.[18]

5 International Transfers and Capital Controls

The analysis above shows that two instruments — nominal interest rate and FX interventions — are sufficient to implement the constrained optimal allocation. In this section, we complement conventional monetary policy and quantitative interventions in the FX market with capital controls. The goal is twofold. First, we discuss whether macroprudential policy can substitute for other instruments when the latter are constrained. Second, we show that under certain conditions, all three types of instruments are necessary to support the optimal allocation.

To this end, assume that the planner sets state-contingent agent-specific taxes on holding assets with net income transferred lump-sum to households. In particular, let τ_t^h denote a tax on household positions in local bonds and let τ_t^a and τ_t^{a*} denote respectively taxes on home and foreign bonds held by financial agents — arbitrageurs and noise traders. To keep the problem interesting, we restrict the set of available instruments by excluding taxes on foreign households, which would allow the planner

[18] Formally, we can rewrite the risk sharing condition (8) as:

$$z_t = \mathbb{E}_t z_{t+1} + \bar{\omega}\sigma_t^2(\iota b_t^* - n_t^* - f_t^*), \qquad \text{where} \qquad \sigma_t^2 = \mathbb{E}_t\{(\tilde{q}_{t+1} - \mathbb{E}_t\tilde{q}_{t+1}) - (z_{t+1} - \mathbb{E}_t z_{t+1})\}^2,$$

and we assumed for simplicity that $x_{t+1} = 0$ is ensured by monetary policy. When f_t^* is constrained, forward guidance using f_{t+j}^* for $j \geq 1$ can impact both $\mathbb{E}_t z_{t+1}$ and σ_t^2 via $(z_{t+1} - \mathbb{E}_t z_{t+1})$ on the right hand side, thus affecting z_t. Country budget constraint, $\beta b_t^* - b_{t-1}^* = -z_t$ with $\lim_{j\to\infty} \beta^j b_{t+j}^* = 0$, imposes a limit on how much future FX interventions can affect the path of z_{t+j} and, in particular, contemporaneous z_t.

to directly extract surplus from foreigners' positions that are unlikely to be in its jurisdiction. For the same reasons, we do not allow for discriminatory taxes on noise traders.

To see how the equilibrium system changes, notice that capital controls only affect intertemporal decisions, and static conditions remain unchanged, including the expenditure switching condition for the nominal exchange rate (1). The household intertemporal consumption smoothing depends on net returns on home bonds:

$$\frac{\beta R_t}{1+\tau_t^h}\mathbb{E}_t\frac{C_{Nt}}{C_{Nt+1}}=1,$$

and for any given path of τ_t^h, monetary policy R_t can still implement any demand for non-tradable goods and effectively controls the output gap x_t.

Solving portfolio problem of arbitrageurs and combining it with the market clearing for assets and the household Euler equation, we get a modified international risk-sharing condition:

$$\beta R_t^*\mathbb{E}_t\frac{C_{Tt}}{C_{Tt+1}}=\frac{(1+\tau_t^h)(1+\tau_t^{a*})}{1+\tau_t^a}+\omega\sigma_t^2\frac{B_t^*-N_t^*-F_t^*}{R_t^*}. \tag{18}$$

This equation clarifies two important properties of capital controls in the economy. From theoretical perspective, taxes and quantity interventions in FX markets are largely isomorphic and can be used interchangeably to implement the optimal risk sharing. In particular, any of the three taxes is sufficient to offset distortionary effects of liquidity shocks N_t^*. At the same time, the use of capital controls might be complicated in practice. First, the optimal risk sharing requires state-contingent instruments and cannot be implemented with slow-moving taxes. Second, the planner might be unable to distinguish different types of agents and impose agent-specific capital controls. As the expression above makes clear, setting a uniform tax on home bonds for all agents $\tau_t^h=\tau_t^a$ is isomorphic to a change in local interest rate and cannot be used to eliminate the UIP deviations. Finally, imposing a tax on holdings of foreign assets is complicated by the fact that some of these agents are foreigners and are outside of home jurisdiction. In short, the optimal risk sharing requires complicated capital controls that have to be state-contingent as well as agent- and asset-specific and can be challenging to implement in practice (Rebucci and Ma 2020).

With these caveats in mind, we proceed under the assumption that both arbitrageurs and noise traders are foreigners and the only form of capital controls used by the planner is a tax on their positions in home bonds τ_t^a. Using the optimal portfolio choice of arbitrageurs, the country's budget constraint (3) can be expressed as

$$\frac{B_t^*}{R_t^*}=B_{t-1}^*+(Y_{Tt}-C_{Tt})-\left[\frac{\mathbb{E}_{t-1}\Theta_t\tilde{R}_t^{*a}}{\omega\sigma_{t-1}^2}+\frac{N_{t-1}^*}{R_{t-1}^*}\right]\tilde{R}_t^{*a},$$

where $\tilde{R}_{t+1}^{*a}=R_t^*-\frac{R_t}{1+\tau_t^a}\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}$ are net returns on carry trade after paying taxes. The term in bracket is demand of foreign arbitrageurs and noise traders, which is multiplied by the ex-post return $\tilde{R}_t^{*a}$ to obtain a net transfer to the rest of the world. For example, a depreciation of the exchange rate generates a positive valuation effect when foreign traders have long positions in home bonds. While it is easy to generate such transfers from inelastic noise traders, the arbitrageurs invest in assets with a higher expected returns and make positive profits in expectation, which puts a limit on how much rents the planner can extract from foreigners.

Following the same approach as in Section 3.2, we next take the second-order approximation around the optimal allocation with the maximum extraction of foreign rents. The extended policy problem can be written as:[19]

$$\min_{\{x_t,z_t,b_t^*,f_t^*,\psi_t,\tau_t^a,\sigma_t^2\}} \frac{1}{2}\mathbb{E}_0 \sum\nolimits_{t=0}^{\infty}\beta^t \left[(1-\gamma)x_t^2+\gamma z_t^2+2\beta\gamma\left(\frac{1}{\bar\omega\sigma_t^2}\psi_t-n_t^*\right)\psi_t\right] \tag{19}$$

$$\begin{aligned} \text{s.t.} \qquad \mathbb{E}_t\Delta z_{t+1} &= \psi_t+\tau_t^a, \\ \psi_t &= -\bar\omega\sigma_t^2(\iota b_t^*-n_t^*-f_t^*), \\ \beta b_t^* &= b_{t-1}^*-z_t, \\ \sigma_t^2 &= \mathrm{var}_t\big(\tilde q_{t+1}+x_{t+1}-z_{t+1}\big), \end{aligned}$$

where $\mathbb{E}_t\Delta z_{t+1} = i_t - i_t^* - \Delta\mathbb{E}_t e_{t+1} = \psi_t + \tau_t^a$ is the UIP deviation evaluated from the point of view of households (i.e., the risk sharing wedge), while $\psi_t = i_t - i_t^* - \Delta\mathbb{E}_t e_{t+1} - \tau_t^a$ is the after-tax expected carry trade return for arbitrageurs. Two main differences from the benchmark problem (14) stand out. First, in addition to output gap and the risk-sharing wedge, the objective function also includes transfers from foreign traders. Interestingly, to the second-order approximation, these rents depend only on expected returns, while any variation in ex-post valuation effects is of a higher order.[20] As a result, the expression for transfers is largely isomorphic to the one in a deterministic case with CIP deviations replaced by UIP deviations (*cf.* Fanelli and Straub 2021). Second, capital controls τ_t^a provide an additional degree of freedom to the planner breaking a tight link between consumption of tradables z_t and returns on carry trade τ_t.

Given the three motives in the loss function, three instruments are required in general case to achieve the optimal allocation. As before, the interest rate policy controls the aggregate demand and closes the output gap $x_t = 0$. In contrast to the baseline model, however, FX interventions are reserved to extract rents rather than to close the risk-sharing wedge. Transfers are a quadratic function of ψ_t and attain the maximum value when $\psi_t = \frac{\bar\omega\sigma_t^2}{2}n_t^*$, which in turn, requires that FX interventions partially satisfy demand of noise traders $f_t^* = \iota b_t^* - \frac{1}{2}n_t^*$. Finally, the efficient risk sharing $z_t = 0$ (and consequently $b_t^* = 0$) still requires closing the UIP wedge (from the household perspective), but it is now implemented using capital controls, $\tau_t^a = -\psi_t$.[21] Neither policy instrument explicitly targets the exchange rate and its dynamics is still determined by $\tilde q_t$.

[19]The log-linearized version of (18), when $\tau_t^h = \tau_t^{a*} = 0$, is given by $\mathbb{E}_t\Delta z_{t+1} = \tau_t^a - \bar\omega\sigma_t^2(\iota b_t^* - n_t^* - f_t^*)$, where as before $z_t = \log(C_{Tt}/\tilde C_{Tt})$, and we denote $\psi_t = -\bar\omega\sigma_t^2(\iota b_t^* - n_t^* - f_t^*)$ as it is the term that emerges in the loss function (19).

[20]This implies that given the structure of international asset markets and the order of approximation, the planner does not aim to use state-contingent valuation effects to "complete the markets" (*cf.* Fanelli 2017).

[21]Consider, for example, a response to a liquidity demand for foreign currency (dollar), $n_t^* > 0$, that is noise traders borrow in home currency to invest in dollars. The planner responds with a capital control tax on borrowing in home currency, $\tau_t^a < 0$, which in turn is pocketed by intermediaries who extend (part of the) home currency lending, $\psi_t = -\tau_t^a > 0$, while UIP still holds from the perspective of households, $i_t - i_t^* - \mathbb{E}_t\Delta e_{t+1} = 0$. To maximize rents, the government optimally satisfies half of the currency demand of the noise traders, $f_t^* = -n_t^*/2$, which results in $\psi_t = \bar\omega\sigma_t^2 n_t^*/2$ and rents $\bar\omega\sigma_t^2(n_t^*)^2/4$.

Proposition 4 *Assume that arbitrageurs and noise traders are foreign agents. Then implementing the constrained optimal allocation requires closing the output gap with monetary policy $x_t = 0$, partially offsetting demand of noise traders with FX interventions $f_t^* = -n_t^*/2$, and closing the UIP deviations and the risk-sharing wedge $z_t = 0$ with capital controls, $\tau_t^a = -\bar{\omega}\sigma_t^2 n_t^*/2$.*

While fairly simple, this optimal policy has a few interesting properties. First, it follows that whenever $n_t^* \neq 0$, a country can generate a positive transfer from the rest of the world by exploiting the monopoly power in the home currency market.[22] The optimal FX interventions always lean against the wind, but offset only a part of the liquidity demand of noise traders leaving the rest to be absorbed by arbitrageurs to ensure positive equilibrium rents. Echoing the recent experience of Switzerland, this result implies that a positive demand for home currency should be addressed by issuing reserves and accumulating assets in foreign currency.

Second, there remains a one-to-one mapping between policy instruments and optimal targets. In particular, FX interventions still address noise trader demand, however, now it is no longer optimal to fully offset it, as it would then eliminate all rents (that is, $f_t^* = -n_t^*$ results in $\psi_t = 0$). Optimal FX interventions partially offset noise trader demand, leaving an opportunity for positive rents, $\psi_t \neq 0$. However, if left unaddressed by capital controls, this would result in a risk sharing wedge. Therefore, capital controls are optimally used to close the risk sharing wedge, $\tau_t^a = -\psi_t$, without eliminating equilibrium rents. As a result, UIP (optimal risk sharing) holds from the perspective of households ($i_t - i_t^* - \mathbb{E}_t\Delta e_{t+1} = 0$), while arbitrageurs receive equilibrium rents in proportion with $\psi_t = -\tau_t^a$.[23] Note that capital controls cannot substitute for FX interventions, as they do not change demand for domestic currency and, therefore, cannot be used to collect rents from foreign traders: Similarly to interest rate shocks R_t, any changes in capital controls τ_t^a are absorbed by expected depreciation $\mathbb{E}_t\Delta e_{t+1}$ and do not affect net carry trade returns ψ_t, which are pinned by the balance of supply and demand in the currency market and determine, in turn, the size of the international transfer.

Lastly, if for the reasons discussed above, capital controls are not available to the planner $\tau_t^a = 0$, closing the risks sharing wedge (UIP deviation) with FX interventions is still feasible and associated with no additional costs as the net transfer is equal to zero when $\psi_t = 0$. At the same time, the optimal policy in this case balances between closing the risk-sharing wedge and collecting international rents and. That is, unlike in Proposition 1, the optimal policy does not fully offset the noise trader demand, leaving a non-zero equilibrium UIP deviation.

Does the divine coincidence still hold in a model with transfers? Consider again the case with a stable optimal real exchange rate $\tilde{q}_t = 0$ and no FX interventions $f_t^* = 0$ or capital controls $\tau_t^a = 0$. It follows from the equilibrium system that a nominal peg ensures the optimal risk sharing $z_t = 0$ and closes the output gap $x_t = 0$. At the same time, international income loss can be evaluated using the

[22] Most of the existing literature focuses on the case of $n_t^* = 0$ and foreign arbitrageurs when any FX interventions result in negative rents (Jeanne 2012, Amador, Bianchi, Bocola, and Perri 2019, Fanelli and Straub 2021).

[23] Recall that arbitrageur's carry trade return is $i_t - i_t^* - \mathbb{E}_t\Delta e_{t+1} - \tau_t^a = \psi_t$, while households cannot take carry trade positions, yet nonetheless their equilibrium risk sharing is still governed by $\mathbb{E}_t\Delta z_{t+1} = i_t - i_t^* - \mathbb{E}_t\Delta e_{t+1} = \psi_t + \tau_t^a$.

equation for expected returns ψ_t:

$$2\gamma\bar{\omega}\sigma_t^2(\iota b_t^* - f_t^*)(\iota b_t^* - n_t^* - f_t^*)$$

and are equal to zero given $\sigma_t^2 = 0$. Thus, for any value of n_t^*, closing the two gaps with a nominal peg comes at no extra cost in terms of international income loss, yet requires leaving out potential rents in the currency market. Such rents may offer a reason to abandon a nominal peg even when the conditions for divine coincidence are satisfied.

6 International Cooperation

So far, we focused on the optimal policy in a small open economy that takes as given global economic conditions. This section studies the international dimensions of monetary policy, the spillovers across countries, and a classical question about international cooperation.

To this end, consider a world comprised a continuum of small open economies index by $i \in [0, 1]$, each one isomorphic to a country in the baseline model, and country $i = 0$ (the US) issues the global funding currency (the dollar), and we denote this country with $*$. There is a global market for the tradable good and a non-tradable sector in each economy. The law of one price still holds for tradables, and now we write it in logs as $p_{Tit} = p_{Tt}^* + e_{it}$ for all $i \in (0, 1]$ with e_{it} denoting the country i nominal exchange rate against the dollar. We allow for $p_{Tt}^* \neq 0$ and $\pi_{Tt}^* = \Delta p_{Tt}^*$ to denote the US tradable inflation. The expenditure switching condition (1) in this case can be written as

$$e_{it} = \tilde{q}_{it} - p_{Tt}^* + x_{it} - z_{it}, \tag{20}$$

with the wedges x_{it} and z_{it} and the first-best real exchange rate $\tilde{q}_{it} = \tilde{c}_{Nit} - \tilde{c}_{Tit}$ still defined as before.

We make two assumptions about the structure of asset markets. First, only nominal dollar bonds are available for international risk sharing, which as we will see, generates an asymmetry between the US and other economies. Second, for each currency there is a separate market, in which agents can trade it against dollars. This segmentation of currency markets is in line with the fact that the dollar accounts for 88% of the global FX market turnover, but it is not crucial for our results which remain largely unchanged if one assumes that arbitrageurs can invest simultaneously in a portfolio of currencies. For simplicity, we assume local financial markets to exclude the redistributive motive in the national policies (see discussed in Section 5). Appendix B.2 provides detailed derivations.

The equilibrium conditions for a given economy are the same as in the baseline model. Instead, the main difference is that the international interest rate, $i_t^* \equiv \log R_t^* - \log \tilde{R}_t^*$, is endogenous and is shaped by the dollar inflation and the global market clearing condition for tradables:

$$\int_0^1 c_{Tit}\mathrm{d}i = \int_0^1 y_{Tit}\mathrm{d}i \equiv y_{Tt},$$

where y_{Tt} is the global tradable output endowment. We denote $\tilde{r}_t^* = \log \tilde{R}_t^*$ the world interest rate that obtains in a global constrained optimum allocation with zero tradable inflation ($\pi^*_{Tt} = 0$), that is:

$$\tilde{r}^*_t = \mathbb{E}_t \Delta y_{Tt+1} = \mathbb{E}_t \Delta \tilde{c}_{Tit+1} \qquad \text{for all } i \in [0,1],$$

where $\{\tilde{c}_{Tit}\}_i$ now corresponds to the allocation with constrained optimum risk sharing among $i \in (0,1]$ rather than to a small open economy constrained optimum which takes R^*_t as given.

With this, we log-linearize the international risk-sharing condition (the analog to (4) which now also features P^*_{Tt}/P^*_{Tt+1} inside the expectation) to obtain:

$$\mathbb{E}_t \Delta z_{it+1} = i^*_t - \mathbb{E}_t \pi^*_{Tt+1} + \psi_{it}, \qquad \text{where} \quad \psi_{it} \equiv -\bar{\omega}_i \sigma^2_{it}(\iota b^*_{it} - n^*_{it} - f^*_{it}), \tag{21}$$

and as before $\sigma^2_{it} = \text{var}_t(\Delta e_{it})$ and $\beta b^*_{it} - b^*_{it-1} = -z_{it}$. We now use a short-hand ψ_{it} for currency i UIP wedge equal to the product of the unit price of risk of currency i, $\bar{\omega}_i \sigma^2_{it}$, and excess demand for the dollar relative to currency i that needs to be absorbed by the intermediaries, $n^*_{it} + f^*_{it} - \iota b^*_{it}$.

Tradable market clearing with global tradable endowment y_{Tt} ensures that $\int_0^1 z_{it} \mathrm{d}i = 0$, since market clearing must hold for both frictional $\{c_{Tit}\}_i$ and optimal $\{\tilde{c}_{Tit}\}_i$ allocations (and $z_{it} \equiv c_{Tit} - \tilde{c}_{Tit}$). Therefore, we can solve for the equilibrium interest rate deviation by integrating (21):

$$i^*_t - \mathbb{E}_t \pi^*_{Tt+1} = -\bar{\psi}_t, \qquad \text{where} \quad \bar{\psi}_t \equiv \int_0^1 \psi_{it} \mathrm{d}i = -\int_0^1 \bar{\omega}_i \sigma^2_{it}(\iota b^*_{it} - n^*_{it} - f^*_{it}) \mathrm{d}i. \tag{22}$$

Therefore, global excess demand for the dollar, $\bar{\psi}_t > 0$, creates a force that depresses the global dollar real interest rate, $i^*_t - \mathbb{E}_t \pi^*_{Tt+1}$.[24] Finally, substituting (22) back into (21) yields $\mathbb{E}_t \Delta z_{it+1} = (\psi_{it} - \bar{\psi}_t)$, which generalizes condition (8) in a small open economy with an endogenous i^*_t.

National policymakers take i^*_t as given and their problems (14) remain unchanged. However, a global planner — or a cooperative policy problem for all $i \in (0,1]$ jointly — internalizes the endogeneity of i^*_t and its international spillovers. Such a planner solves:

$$\min_{\{\{x_{it}, z_{it}, b^*_{it}, f^*_{it}, \sigma^2_{it}\}_i, i^*_t\}_t} \quad \frac{1}{2}\, \mathbb{E} \sum_{t=0}^{\infty} \beta^t \int_0^1 \big[\gamma z^2_{it} + (1-\gamma) x^2_{it}\big] \mathrm{d}i,$$

subject to (21)–(22), as well as (7) and (9) with e_{it} now defined in (20), and taking the path of dollar inflation $\pi^*_{Tt} = \Delta p^*_{Tt}$ as given. One interpretation is that the US monetary policy sets i^*_t to implement a specific path of inflation π^*_{Tt} taking as given the condition for the global (natural) real interest rate (22).

The first thing to note about this problem is that the optimal non-cooperative policies from Proposition 1 translate into a globally optimal outcome: that is, the Nash equilibrium played by the national policymakers results in zero output gap and optimal risk sharing between all economies.[25] Elimination of UIP deviations with privately optimal FX interventions country-by-country, $\psi_{it} = 0$ for all $i \in (0,1]$,

[24] A correlated excess demand for dollar relative to other currencies that results in $\bar{\psi}_t > 0$ creates correlated UIP premia on non-dollar currencies, which in turn result in $z_{it} = c_{Tit} - \tilde{c}_{Tit} < 0$ on average across $i \in (0,1]$. This creates an excess global supply of the tradable good — *global savings glut* — which depresses the global real interest rate.

[25] This result contrasts with the inefficient non-cooperative equilibrium in Fanelli and Straub (2021) with countries participating in a 'rat race' of reserve accumulation.

also eliminates the pressure on the global real interest rate beyond its efficient level $\tilde{r}_t^* = \mathbb{E}_t \Delta y_{Tt+1}$. Indeed, there are no externalities when countries choose consumption of tradables subject to intertemporal budget constraint. Although international asset markets are incomplete, the fact that there is only one tradable good implies that there is no pecuniary externality as in Geanakoplos and Polemarchakis (1986) and Greenwald and Stiglitz (1986). Similarly, there is no aggregate demand externality in the risk sharing as long as the monetary policy closes the output gap (Farhi and Werning 2016).

On the other hand, when FX policies of a subset of countries are constrained and shocks have a correlated component resulting in $\bar{\psi}_t \neq 0$, this creates negative international spillovers that are not internalized by national policymakers that take $i_t^* = 0$ as given. The unconstrained national policymakers use FX interventions according to Proposition 1 to target $\psi_{it} = 0$, while the optimal cooperative policy prescription for such countries is to target $\psi_{it} = \bar{\psi}_t$ to ensure $\mathbb{E}_t \Delta z_{it+1} = 0$ in (21). This cooperative policy eliminates the risk sharing wedge between the group of unconstrained and constrained countries $i \in (0, 1]$. Intuitively, a correlated global demand shock for dollars, $\bar{n}_t^* = \int_0^1 n_{it}^* \mathrm{d}i > 0$, if not offset with FX interventions in a subset of countries results in $\bar{\psi}_t > 0$ and depresses the world dollar interest rate $i_t^* < 0$ in (22). Without taking the endogeneity of i_t^* into account, this creates a wedge in the path of tradable consumption that is depressed in the constrained economies (due to ψ_{it} shocks) and expands in unconstrained economies (due to lower i_t^*). A cooperative policy aims to close this gap by under-reacting to the ψ_{it} shock in unconstrained economies to curb capital inflows, emphasizing the complementarity in the use of FX interventions across countries.[26]

Fixing the exchange rate to the dollar $\sigma_{it}^2 = \mathrm{var}_t(\Delta e_{it+1}) = 0$, if done by all countries $i \in (0, 1]$, eliminate risk sharing wedges across countries by ensuring $\psi_{it} = \bar{\psi}_t = 0$ for all $i \in (0, 1]$. The reason why the peg to the dollar has such an effect is not a particular form of currency market segmentation, but rather the assumption that the dollar is the international funding currency, i.e. the dollar bond is the internationally traded asset. This explains the central role of the bilateral exchange rates against the dollar and that pegging other bilateral or weighted exchange rates is suboptimal and can potentially exacerbate risk sharing wedges by increasing σ_{it}^2. Thus, taking as given the dominance of the dollar in international borrowing and lending (Maggiori, Neiman, and Schreger 2020), the model explains why most countries in the world — including the ones with weak trade linkages to the US — use the dollar as an anchor currency in their monetary and FX policies (Ilzetzki, Reinhart, and Rogoff 2019).[27]

The divine coincidence of Proposition 2, however, may fail for two reasons. First, a peg to the dollar by an individual country ensures $\psi_{it} = 0$, but not $\psi_{it} = \bar{\psi}_t$, and thus fails to eliminate capital flow externalities among $i \in (0, 1]$ when some countries are constrained and $\bar{\psi}_t \neq 0$, as discussed above. In addition to this, there are highly asymmetric spillover effects of US monetary policy via tradable inflation π_{Tt}^* that affects Δe_{it} according to (20). This is inconsequential under the optimal policy that ensures $x_{it} = z_{it} = 0$ country-by-country, as Δe_{it} simply accommodates shocks to π_{Tt}^*. In contrast, the

[26] This policy is cooperatively optimal for $i \in (0, 1]$ and does not take into account risk sharing with the US $i = 0$, which is assumed infinitesimal. With large US, the optimal policy for $(0, 1]$ must weigh in the risk sharing wedge with the US, which makes $|\psi_{it}| < |\bar{\psi}_t|$ optimal for unconstrained economies, and may give rise to substitutability in the use of FX interventions across ubconstrained and constrained economies.

[27] Similarly to Hassan, Mertens, and Zhang (2021), the goal of the peg in our model is to eliminate the UIP deviation, but the anchor status of the dollar is due to the structure of financial markets, not the size of the US economy.

peg to the dollar, or even a partial peg in (17), imports the US monetary policy stance and results in the output gap x_{it} trailing US inflation π^*_{Tt}. For example, consider a tightening of US monetary policy that leads to an appreciation of the dollar ($\Delta e_{it} > 0$) and lowers the price of tradables ($\pi^*_{Tt} < 0$). By leaning against the wind and also raising interest rates, other countries partially stabilizes their exchange rates against the dollar at the expense of a negative output gap $x_{it} < 0$. Thus, despite a zero mass of the US economy, all countries import its monetary stance giving rise to the global monetary cycle (Rey 2013b, Egorov and Mukhin 2021).

Proposition 5 *Cooperation is not required when all countries follow unconstrained privately optimal monetary and FX policies. Under constrained policies, global dollar demand shocks and US monetary policy shocks result in international spillovers.*

Lastly, we briefly comment on the optimal US policy and an alternative of a global gold standard. With constrained policies, an inward looking US policy fails to internalize the spillovers associated with global dollar demand shocks and the global monetary cycle induced by its monetary policy. While the latter requires some compromise between output gaps in the US and in the rest of the world, the former can be accommodated with a supply of dollar liquidity, e.g. in the form of currency swap lines between the Federal Reserve and monetary authorities in the rest of the world to eliminate wedges in international risk sharing. As an alternative, consider a global financial system dominated by gold. Equilibrium risk sharing conditions remain the same as before, except that the relevant source of risk σ^2_{it} is now the volatility of exchange rates against gold. In this case, the gold price of tradables P^*_{Tt} is pinned down by the market clearing condition for gold, which does not depend directly on any monetary policy, yet depends on the supply of gold reserves around the world. This leads to less asymmetric spillovers than under a US-centric financial markets, however at the cost of potentially greater volatility in P^*_{Tt}.

7 Extensions

The baseline model makes several stark assumptions to get a sharp characterization of the optimal policy. This section relaxes some of them — in particular, allowing for sluggish price adjustment, expenditure switching in tradables, and alternative structures of UIP deviations — to evaluate robustness of our main results. Detailed derivations are relegated to Appendix B.3.

7.1 Adjusting prices

The assumption of fully rigid prices in our baseline analysis provides emphasis to our main focus on the trade-off between output gap and international risk sharing, yet is admittedly very stark, and in particular removes domestic inflation as a policy consideration. We now generalize our results to an environment with staggered price adjustment. In particular, we assume that there is a continuum of varieties of non-tradable goods with an elasticity of substitution equal $\varepsilon > 1$ that are produced by monopolistic competitors. Firms are subject to a Calvo (1983) friction and update prices with probability $1 - \lambda$. We allow for markup shocks ν_t and assume that a constant production subsidy is used to eliminate the steady state markup wedge. The resulting planner's problem is largely isomorphic to the baseline (14), but features both inflation π_{Nt} and output gap x_t in the objective function:

$$\begin{aligned}\min_{\{x_t,\pi_{Nt},z_t,b_t^*,f_t^*,\sigma_t^2\}} \quad & \frac{1}{2}\,\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\big[\gamma z_t^2+(1-\gamma)(x_t^2+\alpha\pi_{Nt}^2)\big],\\ \text{subject to} \quad & \mathbb{E}_t\Delta z_{t+1}=-\bar{\omega}\sigma_t^2(\iota b_t^*-n_t^*-f_t^*),\\ & \beta b_t^*=b_{t-1}^*-z_t,\\ & \sigma_t^2=\mathrm{var}_t\big(\tilde{q}_{t+1}-z_{t+1}+x_{t+1}+\pi_{Nt+1}\big),\\ & \pi_{Nt}=\kappa x_t+\beta\mathbb{E}_t\pi_{Nt+1}+\nu_t,\end{aligned}$$

where $\alpha\equiv\varepsilon/\lambda$ is the relative weight on welfare losses from inflation and the set of constraints now additionally features a standard NKPC with $\kappa\equiv\frac{(1-\lambda)(1-\beta\lambda)}{\lambda}$.

The first thing to notice is that the results about the first-best policies remain largely unchanged. When two policy instruments are available, the FX interventions $f_t^*=\iota b_t^*-n_t^*$ eliminate the risk-sharing wedge and the interest rates implements the optimal path of inflation π_{Nt} and output gap x_t, as in the closed economy, generalizing Proposition 1. Similarly, by adopting an exchange rate peg, monetary policy on its own can implement the optimal allocation with $z_t=x_t=\pi_{Nt}=0$, if firms' markups are constant, $\nu_t=0$, and the first-best real exchange rate is stable, $\tilde{q}_t=0$. Hence, open economy divine coincidence requires that closed economy divine coincidence is satisfied, that is there is no conflict between output gap and inflation stabilization, and additionally that a fixed exchange rate does not interfere with efficient expenditure switching. This generalizes Proposition 2.

This isomorphism to the baseline model extends further and applies also to the second-best policies. To see this, notice that the only interaction between the two sectors comes from the nominal exchange rate via expenditure switching (1), which results in the $x_{t+1}+\pi_{Nt+1}$ term in the definition of σ_t^2 in the constraint set. This implies that the planner's problem can be broken into two sequential steps: first, solve for the optimal path $\{x_t,\pi_{Nt}\}$ given shocks to aggregate demand $m_t\equiv x_t+\pi_{Nt}$, and second, solve for the optimal trade-off between risk sharing z_t and domestic conditions summarized by m_t. The latter problem is the same as in the baseline model, except that the output losses x_t^2 are replaced with the overall welfare losses due to output gap and inflation from suboptimal monetary response to markup innovations. This implies that results about the second-best policies, including the optimal partial peg (17), extend to the setup with adjusting prices.[28]

7.2 Terms of trade

Another important limitation of the baseline model are constant terms of trade and no expenditure switching in exports. Following the previous normative open-economy literature (Galí and Monacelli 2005, Devereux and Engel 2003, Benigno and Benigno 2003), this extension replaces tradables and non-tradables with a home good consumed locally C_{Ht} and exported abroad C_{Ht}^* and with an imported

[28]Interestingly, in contrast to the prescriptions of the standard New-Keynesian model (Galí 2008), the optimal Ramsey policy does not target the long-run price level, and shocks in both sectors have permanent effects on price levels.

foreign good C_{Ft}. We keep the assumption of log-linear preferences with $C_t = C_{Ht}^{1-\gamma} C_{Ft}^{\gamma}$, linear technology, and CES demand for exports:

$$A_t L_t = C_{Ht} + C_{Ht}^*, \qquad C_{Ht}^* = \gamma P_{Ht}^{*-\varepsilon} C_t^*,$$

where P_{Ht}^* is the export price in foreign currency, $\varepsilon > 1$ is the elasticity of foreign demand, and C_t^* is the global demand shifter. For simplicity, all prices are fully sticky in the currency of invoicing. We assume that domestic prices are set in local currency and consider two alternatives for export prices: producer currency (PCP) with rich terms-of-trade dynamics and dollar pricing, which provides a better description of the current international price system (Gopinath, Boz, Casas, Díez, Gourinchas, and Plagborg-Møller 2020).

Producer currency pricing When export prices are sticky in the currency of exporter, the monetary policy can generate expenditure switching in the market of destination and simultaneously close the output gap in domestic and export sectors. As a result, the loss function can be written in terms of the total output gap x_t and the deviations of imports from the optimal level z_t:

$$\begin{aligned} \min_{\{x_t, z_t, b_t^*, f_t^*, \sigma_t^2\}} \quad & \frac{1}{2}\,\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\kappa z_t^2 + x_t^2\right] \\ \text{s.t.} \quad & \mathbb{E}_t \Delta z_{t+1} = -\bar{\omega}\sigma_t^2(\iota b_t^* - n_t^* - f_t^*), \\ & \beta b_t^* = b_{t-1}^* + \frac{\varepsilon - 1}{\varepsilon} x_t - z_t, \\ & \sigma_t^2 = \operatorname{var}_t\big(\tilde{q}_{t+1} + x_{t+1} - (1-\bar{\gamma}) z_{t+1}\big), \end{aligned}$$

where $\kappa \equiv \frac{\varepsilon^2 \gamma}{\varepsilon - \gamma}$ and $\bar{\gamma} \equiv \frac{\gamma(\varepsilon-1)}{\varepsilon-\gamma}$ is the steady-state share of exports in total output. The only substantial difference from the baseline problem (14) is that the monetary policy affects exports via expenditure switching channel and therefore, x_t appears in the country's budget constraint with a multiplier that depends on the elasticity of substitution ε. This additional channel does not change the main results about the first-best policies. When two instruments are available, the planner can implement efficient allocation by closing the output gap $x_t = 0$ with interest rate policy and eliminating the risk-sharing wedge with the FX interventions $f_t^* = \iota b_t^* - n_t^*$. Moreover, the divine coincidence still holds when efficient real exchange rate is constant: by stabilizaing the nominal exchange rate, monetary policy alone can close both wedges $x_t = z_t = 0$. The condition that $\tilde{q}_t = 0$ is satisfied when local productivity shocks move one-to-one with global demand shocks $a_t = c_t^*$ and both shocks follow a random walk.

Moving to the second-best policies, because of the effect of monetary policy on country's exports, a nominal peg $\sigma_t^2 = 0$ is no longer sufficient to implement $z_t = 0$. However, for any given path of x_t, it is still optimal to close the UIP deviations — either using the FX interventions or by stabilizing the nominal exchange rate. In particular, a partial peg remains optimal when FX interventions are not available. While the monetary policy can also stimulate exports to increase country's imports, the effect is relatively weak because of consumption smoothing and is not very useful to offset financial shocks.

Dominant currency pricing When export prices are sticky in foreign currency, the law of one price does not hold creating an additional gap in the planner's problem:

$$\begin{aligned}
\min_{\{x_t,z_t,b_t^*,f_t^*,\sigma_t^2\}} \quad & \frac{1}{2}\,\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\big[\gamma z_t^2+(1-\gamma)x_t^2+\gamma(\varepsilon-1)\tilde{q}_t^2\big] \\
\text{s.t.} \quad & \mathbb{E}_t\Delta z_{t+1} = -\bar{\omega}\sigma_t^2(\iota b_t^* - n_t^* - f_t^*), \\
& \beta b_t^* = b_{t-1}^* - (\varepsilon-1)\tilde{q}_t - z_t, \\
& \sigma_t^2 = \mathrm{var}_t\big(\tilde{q}_{t+1} + x_{t+1} - z_{t+1}\big),
\end{aligned}$$

where x_t is the output gap in domestic sector and z_t is the deviation of consumption of foreign goods from the optimal level. Because the export prices do not respond to shocks, the deviations from the optimal exports, $(1-\varepsilon)\tilde{q}_t$, fluctuate together with the optimal level of the real exchange rate. As a result, the exports are exogenous to monetary policy and neither interest rates nor FX interventions can close the output gap in the export sector (see Egorov and Mukhin 2021). Moreover, the suboptimal exports imply that it is impossible to achieve the efficient level of imports $z_t = 0$. Yet, when two policy instruments are available, the optimal targets are the same as in the baseline model: the monetary policy closes domestic output gap $x_t = 0$ and the FX interventions offset financial shocks $f_t^* = \iota b_t^* - n_t^*$.

Interestingly, the divine coincidence from the baseline model is still valid under DCP: if the real exchange rate is stable, the monetary policy alone can implement the first-best allocation. Indeed, if $\tilde{q}_t = 0$, then there is no need for export prices to adjust and exports are efficient. Pegging a nominal exchange rate encourages arbitrageurs and eliminates the risk-sharing wedge, while simultaneously closing the output gap. Away from this knife-edge case, a partial peg balances x_t and z_t.

7.3 Financial shocks

While the analysis above focuses on noise trader shocks as the main source of volatility in financial markets, the previous literature suggests that other shocks may play an important role as well. To study robustness of the optimal policy, we augment the model with three additional financial shocks. The first one is the expectation error of arbitrageurs ξ_t in the spirit of Gourinchas and Tornell (2004), which implies that the subjective beliefs are given by $\tilde{\mathbb{E}}_t\Delta e_{t+1} = \mathbb{E}_t\Delta e_{t+1} - \xi_t$. Second, following Brunnermeier, Nagel, and Pedersen (2009) and Gabaix and Maggiori (2015), we allow for risk-appetite shocks to ω_t. Finally, assume that there is a time-varying probability of default modelled as a shock δ_t to the returns on the home currency bond. Because this latter shock applies for both households and arbitrageurs, it is absorbed by the equilibrium interest rate and does not directly affect UIP deviations. Instead, it creates an additional source of carry trade risk. Combining these pieces together, the new risk-sharing condition is:

$$\mathbb{E}_t\Delta z_{t+1} = \xi_t - \bar{\omega}_t\sigma_t^2(\iota b_t^* - n_t^* - f_t^*), \qquad \sigma_t^2 = \mathrm{var}_t(e_{t+1} + \delta_{t+1}).$$

The rest of the equilibrium system and the objective function remain the same as in the baseline model.

It follows that the first-best policy remains largely unchanged in the presence of additional shocks. In particular, it remains optimal to target UIP deviations with FX interventions and offset both demand shocks of noise traders and expectational errors of arbitrageurs, $f_t^* = \iota b_t^* - n_t^* - \xi_t/\bar{\omega}_t\sigma_t^2$, aiming to implement $\mathbb{E}_t \Delta z_{t+1} = 0$. In contrast, the divine coincidence result holds with respect to the risk-appetite shocks ω_t, but does not apply more generally as stabilizing the nominal exchange rate is no longer sufficient to eliminate the UIP wedge in the presence of ξ_t and δ_t shocks. Nonetheless, an exchange rate peg still eliminates a part of the UIP wedge associated with the noise trader shocks.

8 Conclusion

This paper studies optimal exchange rate policy in an open economy with frictional goods and asset markets. In contrast to the previous normative literature, we use a framework that is consistent with the major exchange rate puzzles, including the change in macroeconomic dynamics after a switch from a peg to a float associated with the end of the Bretton-Woods system. The model is tractable and allows for an intuitive linear-quadratic approximation of the planner's problem, yet rich enough to accommodate interesting policy trade-offs and multiple policy instruments.

We show that the constrained optimum can be implemented with monetary policy closing the output gap under sticky prices in goods markets and FX interventions targeting UIP deviations due to intermediary frictions in asset markets. In addition, when foreign agents participate in financial intermediation, the government can collect monopoly rents in home currency markets and the optimal mix of policy tools includes capital controls. The open-economy divine coincidence holds when the first-best real exchange rate is constant and allows closing the two wedges with one monetary instrument by pegging the nominal exchange rate. More generally, when FX interventions are subject to additional constraints, the planner can use a crawling peg and/or FX forward guidance to mitigate financial distortions. International cooperation is not required when countries follow the unconstrained privately optimal policies, but helps mitigate international spillovers from global liquidity shocks and US monetary shocks under constrained policies.

Appendix

A Derivations and Proofs

Lemma 2 (country budget constraint) Substitute firm profits $\Pi_t = P_{Nt}Y_{Nt} - W_tL_t$ and household consumption expenditure $P_tC_t = P_{Nt}C_{Nt} + P_{Tt}C_{Tt}$ into the household budget constraint and use market clearing $C_{Nt} = Y_{Nt}$ to obtain:

$$\frac{B_t}{R_t} - B_{t-1} = NX_t + T_t,$$

where $NX_t = P_{Tt}Y_{Tt} - P_{Tt}C_{Tt} = \mathcal{E}_t(Y_{Tt} - C_{Tt})$. Next combine the household and government budget constraints to obtain:

$$\frac{B_t + F_t}{R_t} + \frac{\mathcal{E}_t F_t^*}{R_t^*} - B_{t-1} - F_{t-1} - \mathcal{E}_t F_{t-1}^* = NX_t + \tau \mathcal{E}_t \pi_t^*.$$

Define B_t^* such that $\frac{B_t^*}{R_t^*} = \frac{F_t^*}{R_t^*} + \frac{B_t+F_t}{\mathcal{E}_t R_t}$ and use the market clearing $B_t + D_t + N_t + F_t = 0$ and Lemma 1 that $B_t^* = D_t^* + N_t^* + F_t^*$ to rewrite:

$$\frac{\mathcal{E}_t B_t^*}{R_t^*} - \mathcal{E}_t B_{t-1}^* + \mathcal{E}_t(D_{t-1}^* + N_{t-1}^*) + (D_{t-1} + N_{t-1}) = NX_t + \tau \mathcal{E}_t \pi_t^*.$$

Finally, recall that $\pi_t^* = \tilde{R}_t^* \frac{D_{t-1}^* + N_{t-1}^*}{R_{t-1}^*} = \left[1 - \frac{R_{t-1}}{R_{t-1}^*}\frac{\mathcal{E}_{t-1}}{\mathcal{E}_t}\right](D_{t-1}^* + N_{t-1}^*)$. Subtract $\mathcal{E}_t \pi_t^*$ on both sides of the budget of the budget constraint to obtain:

$$\frac{\mathcal{E}_t B_t^*}{R_t^*} - \mathcal{E}_t B_{t-1}^* + \underbrace{(D_{t-1} + N_{t-1}) + \frac{R_{t-1}}{R_{t-1}^*}\mathcal{E}_{t-1}(D_{t-1}^* + N_{t-1}^*)}_{=0 \text{ as zero capital portfolio at } t-1} = NX_t - (1-\tau)\tilde{R}_t^* \frac{\mathcal{E}_t(D_{t-1}^* + N_{t-1}^*)}{R_{t-1}^*},$$

Divide through by $\mathcal{E}_t$, use the fact that $NX_t/\mathcal{E}_t = Y_{Tt} - C_{Tt}$, and Lemma 1 that $D_{t-1}^* + N_{t-1}^* = B_{t-1}^* - F_{t-1}^*$ to rewrite:

$$\frac{B_t^*}{R_t^*} - B_{t-1}^* = (Y_{Tt} - C_{Tt}) - (1-\tau)\tilde{R}_t^* \frac{\mathcal{E}_t(B_{t-1}^* - F_{t-1}^*)}{R_{t-1}^*},$$

completing the proof of the lemma. ■

Exact optimal policy (constrained optimum) The planner solves in this case:

$$
\begin{aligned}
\mathbb{W}_0 = \max_{\{C_{Tt},B_t^*,\mathcal{E}_t,R_t,W_t,F_t^*,\sigma_t^2\}} &\mathbb{E}_0 \sum_{t=0}^{\infty}\beta^t\left[\gamma\log C_{Tt} + (1-\gamma)\left(\log W_t - \frac{W_t}{A_t}\right)\right],\\
\text{subject to}\quad & \frac{B_t^*}{R_t^*} - B_{t-1}^* = Y_{Tt} - C_{Tt},\\
\beta R_t^*\mathbb{E}_t\frac{C_{Tt}}{C_{T,t+1}} &= 1+\omega\sigma_t^2\frac{B_t^*-N_t^*-F_t^*}{R_t^*}, \qquad \sigma_t^2 = R_t^2\cdot\mathrm{var}_t\Big(\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\Big),\\
\beta R_t\mathbb{E}_t\left\{\frac{C_{Tt}}{C_{T,t+1}}\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\right\} &= 1, \qquad \mathcal{E}_t = \frac{\gamma}{1-\gamma}\frac{W_t}{C_{Tt}}.
\end{aligned}
$$

The Lagrange multipliers on all constraints, but the budget constraint must be zero, and thus the problem is equivalent to maximizing the objective with respect to $\{C_{Tt}, B_t^*, W_t\}$ subject to the budget constraint only. First, note that Lagrange multipliers on the two constraints in the third line must be zero: since F_t^* enters only one constraint, and σ_t^2 enters only one other constraint, and neither enter the objective, F_t^* can be chosen to relax both constraints (ensure zero multipliers). Second, dropping these constraints, optimization over R_t and $\mathcal{E}_t$, which are featured only in the two of the remaining three constraints and not in the objective, ensures zero Lagrange multiplier on those constraints as well.

Solving the remaining problem, as stated in the proposition, results in the solution $\{\tilde{C}_{Tt}, \tilde{B}_t^*, \tilde{W}_t\}$ with $\tilde{W}_t = A_t$ and $\{\tilde{C}_{Tt}, \tilde{B}_t^*\}$ the unique solution of:

$$
\beta R_t^*\mathbb{E}_t\{C_{Tt}/C_{T,t+1}\} = 1 \qquad \text{and} \qquad \frac{B_t^*}{R_t^*} - B_{t-1}^* = Y_{Tt} - C_{Tt}.
$$

Using the remaining constraints of the problem, we back out $\{\tilde{\mathcal{E}}_t, \tilde{R}_t, \tilde{F}_t^*, \tilde{\sigma}_t^2\}$, and in particular we have $\tilde{F}_t^* = \tilde{B}_t^* - N_t^*$ and $\tilde{\mathcal{E}}_t = \frac{\gamma}{1-\gamma}\frac{A_t}{\tilde{C}_{Tt}}$. This is the exact counterpart to Proposition 1.

B Linear-Quadratic Policy Problem

B.1 Optimal monetary policy without FX interventions

Consider policy problem (14) with $f_t^* \equiv 0$:

$$
\begin{aligned}
\mathcal{L} = \sum_{t,s^t}\beta^t\pi(s^t)\bigg[&\frac{1}{2}\big(\gamma z_t^2 + (1-\gamma)x_t^2\big) + \lambda_t\big(\beta b_t^* - b_{t-1}^* - z_t\big)\\
&+\frac{\beta\gamma}{2}\mu_t\left(\sum_{s_{t+1}|s^t}\pi(s_{t+1}|s^t)\Delta z_{t+1} + \bar{\omega}\sigma_t^2(\iota b_t^* - n_t^*)\right)\\
&+\frac{\beta}{2}\nu_t\left(\sum_{s_{t+1}|s^t}\pi(s_{t+1}|s^t)(\tilde{q}_{t+1}+x_{t+1}-z_{t+1})^2 - \Big(\sum_{s_{t+1}|s^t}\pi(s_{t+1}|s^t)(\tilde{q}_{t+1}+x_{t+1}-z_{t+1})\Big)^2 - \sigma_t^2\right)\bigg]
\end{aligned}
$$

with the optimality conditions for the choice of x_t and σ_t^2 given by:

$$(1-\gamma)x_t + \nu_{t-1}(e_t - \mathbb{E}_{t-1}e_t) = 0,$$
$$\frac{\beta\gamma}{2}\mu_t\bar{\omega}(\iota b_t^* - n_t^*) - \frac{\beta}{2}\nu_t = 0.$$

Note that we introduced γ in front of μ_t constraint to weight it proportionally to the role of z_t^2 in the objective function. The optimality conditions with respect to b_t^* and z_t determine the optimal dynamics of μ_t without affecting the qualitative insights for the choice of x_t. From the two optimality conditions above, we have $\mathbb{E}_t x_{t+1} = 0$. This follows from the fact that ν_t is inside the t-information set. We also have:

$$(1-\gamma)x_{t+1} = -\gamma\bar{\omega}\mu_t(\iota b_t^* - n_t^*)\big(e_{t+1} - \mathbb{E}_t e_{t+1}\big),$$

where $\gamma\bar{\omega}\mu_t(\iota b_t^* - n_t^*) = \nu_t \geq 0$. Indeed, ν_t measures the impact of extra variance σ_t^2 on the objective function, and it is non-negative, and positive whenever risk sharing constraint is binding.

B.2 Derivations for Section 6

Generalize the equilibrium conditions from the baseline model to include the price of tradables. The household optimality condition for goods

$$\frac{\gamma}{1-\gamma}\frac{C_{Nit}}{C_{Tit}} = \frac{\mathcal{E}_{it}P_{Tt}}{P_{Nit}}$$

implies that the nominal exchange rate is given by

$$e_{it} = c_{Nit} - c_{Tit} - p_{Tt} = \tilde{q}_{it} - p_{Tt} + x_{it} - z_{it}.$$

Linearizing the budget constraint

$$\frac{B_{it}^*}{R_t^*} = B_{it-1}^* + P_{Tt}(Y_{Tit} - C_{Tit})$$

around the zero steady-state positions, we get an expression without valuation effects:

$$\beta b_{it}^* = b_{it-1}^* - z_{it}.$$

The household Euler equation combined with the optimal portfolio choice of arbitrageurs implies that the risk-sharing condition is given by

$$\beta R_t^* \mathbb{E}_t \frac{C_{Tit}}{C_{Tit+1}}\frac{P_{Tt}}{P_{Tt+1}} = 1 + \frac{\omega\sigma_{it}^2}{R_t^*}\left(B_{it}^* - N_{it}^* - F_{it}\right).$$

The equilibrium in the U.S. is described by the same conditions, except that $\mathcal{E}_{it} = 1$ and $\sigma_{it}^2 = 0$. Linearize this condition and integrate it across countries using the fact that

$$\int C_{Tit}\mathrm{d}i = \int Y_{Tit}\mathrm{d}i \equiv Y_{Tt}$$

to express the log *real* interest rate as follows

$$r_t^* = \mathbb{E}_t \Delta y_{Tt+1} - \bar{\omega} \int \sigma_{it}^2 (b_{it}^* - n_{it}^* - f_{it}^*) \mathrm{d}i.$$

Substituting this expression back into the risk-sharing condition of an individual economy, we get

$$\mathbb{E}_t \Delta z_{it+1} = \int \bar{n}_{jt}^* \mathrm{d}j - \bar{n}_{it}^*, \quad \bar{n}_{it}^* \equiv \bar{\omega} \sigma_{it}^2 (\iota b_{it}^* - n_{it}^* - f_{it}^*).$$

The derivation of the global planner's objective function follows the same steps as above. In particular, consider a relaxed problem with the constraints that bind in the steady state:

$$\begin{aligned} \max \quad & \mathbb{E} \sum_{t=0}^{\infty} \beta^t \int \left[\gamma \log C_{Tit} + (1-\gamma) \left(\log C_{Nit} - \frac{C_{Nit}}{A_{it}} \right) \right] \mathrm{d}i \\ \text{s.t.} \quad & \frac{B_{it}^*}{R_t^*} = B_{it-1}^* + Y_{Tit} - C_{Tit}, \qquad \int B_{it}^* \mathrm{d}i = 0. \end{aligned}$$

The former constraint limits the number of assets available to share the risk between countries, while the second constraint is equivalent to the resource constraint $\int C_{Tit} \mathrm{d}i = \int Y_{Tit} \mathrm{d}i$. Writing the Lagrangian and taking the second-order approximation around the efficient allocation, we get quadratic loss function: $\frac{1}{2} \ \mathbb{E} \ \sum_{t=0}^{\infty} \beta^t \int \left[\gamma z_{it}^2 + (1-\gamma) x_{it}^2 \right] \mathrm{d}i$. While the output gap is defined the same way as in a non-cooperative case, the risk-sharing wedge is now defined relative to the globally efficient benchmark $\tilde{c}_{it}$ that satisfies $\mathbb{E}_t \Delta \tilde{c}_{it+1} = \mathbb{E}_t \Delta y_{Tt+1}$ and the country's budget constraint.

Combining all pieces together, the global planner's problem can be written as

$$\begin{aligned} \min_{\{x_{it}, z_{it}, b_{it}^*, f_{it}^*, \sigma_{it}^2, p_{Tt}\}} \quad & \frac{1}{2} \ \mathbb{E} \ \sum_{t=0}^{\infty} \beta^t \int \left[\gamma z_{it}^2 + (1-\gamma) x_{it}^2 \right] \mathrm{d}i \\ \text{s.t.} \quad & \mathbb{E}_t \Delta z_{it+1} = \int \bar{n}_{jt}^* \mathrm{d}j - \bar{n}_{it}^*, \quad \bar{n}_{it}^* \equiv \bar{\omega} \sigma_{it}^2 (\iota b_{it}^* - n_{it}^* - f_{it}^*), \\ & \beta b_{it}^* = b_{it-1}^* - z_{it}, \\ & \sigma_{it}^2 = \mathrm{var}_t \big(\tilde{q}_{it} - p_{Tt} + x_{it} - z_{it} \big), \end{aligned}$$

The first-best solution is then to close the output gap $x_{it} = 0$ with monetary policy and to eliminate the risk-sharing wedge $f_{it}^* = \iota b_{it}^* - n_{it}^*$ with the FX instruments. It follows that $r_t^* = \mathbb{E}_t \Delta y_{Tt+1}$ and the efficient consumption of tradables for a given economy is also globally efficient.

Turning next to the second-best policy, the non-cooperative planner takes the world interest rate as given and accommodates global shocks. Instead, the cooperative planner aims to close the risk-sharing wedge by setting $\bar{n}_{it}^* = \int \bar{n}_{jt}^* \mathrm{d}j$, i.e. if FX interventions are constrained in other economies and $\int \bar{n}_{jt}^* \mathrm{d}j \neq 0$, it is optimal to deviate from $\bar{n}_{it}^* = 0$. In this sense, there are strategic complementarities in FX interventions across countries under the optimal cooperative policy. Finally, according to the Fischer equation $r_t^* = i_t^* - \mathbb{E}_t \Delta p_{Tt+1}$, U.S. nominal interest rate i_t^* affects the price of tradabales and the bilateral exchange rates against the dollar e_{it}. Therefore, a partial peg requires depressing the output x_{it} in response to tightening of U.S. policy that lowers p_{Tt}.

B.3 Proofs for Section 7

B.3.1 Adjusting prices

The derivation of the NKPC and the loss function in the presence of inflation follows the standard steps. Using the property of the model that monetary policy affects exchange rates only via σ_t^2, the planner's problem can be partitioned in two steps. The first one solves for the optimal trade-off between output gap and inflation. Because of the certainty equivalence and only first-period innovations affecting σ_t^2, it is sufficient to focus on the following problem:

$$\begin{aligned} \min_{\{x_t,\pi_{Nt}\}} \quad & \frac{1}{2}\sum_{t=0}^{\infty}\beta^t(x_t^2+\alpha\pi_{Nt}^2) \\ \text{s.t.} \quad & \pi_{Nt}=\kappa x_t+\beta\pi_{Nt+1}+\nu_t, \\ & x_0+\pi_{N0}=m_t. \end{aligned}$$

Taking the first-order conditions, we get

$$\beta^t x_t=\kappa\lambda_t+\mu_t,$$

$$\beta^t\alpha\pi_{Nt}=-\lambda_t+\lambda_{t-1}\beta+\mu_t,$$

where $\mu_t=0$ for $t>0$ and $\lambda_{-1}=0$. It follows that the optimality conditions are

$$\alpha\kappa\pi_{Nt}=-x_t+x_{t-1}$$

for $t\geq 1$ and

$$\alpha\kappa\pi_{Nt}=-x_t+(1+\kappa)\mu_t,$$

for $t=0$. Substitute the optimality condition into the NKPC, so that dynamics for $t>0$ is given by

$$\beta x_{t+1}-\left(1+\beta+\alpha\kappa^2\right)x_t+x_{t-1}=\alpha\kappa\nu_t.$$

This difference equation has two roots $\lambda_1>1$ and $\lambda_2<1$

$$\lambda_{1,2}=\frac{1}{2\beta}\left[1+\beta+\alpha\kappa^2\pm\sqrt{(1+\beta+\alpha\kappa^2)^2-4\beta}\right],$$

and assuming for simplicity that ν_t follows an AR(1) process, we get

$$x_t=\lambda_2x_{t-1}-\frac{\alpha\kappa}{\beta}\frac{1}{\lambda_1-\rho}\nu_t.$$

This means that one initial condition x_0 is required. At the same time, the NKPC for the first period together with the initial condition imply that

$$\alpha\kappa(m_t-x_0)=\alpha\kappa^2x_0-\beta\Delta x_1+\alpha\kappa\varepsilon_{\nu 0}.$$

Substitute in expression for x_1 and solve for

$$x_0 = \frac{\alpha\kappa}{\alpha\kappa^2 + \alpha\kappa + \beta - \beta\lambda_2}\left[m_t - \frac{\lambda_1}{\lambda_1 - \rho}\varepsilon_{\nu 0}\right].$$

Substituting this result into equation for x_t, we get

$$x_t = k^x_{xt} m_t - k^x_{\nu t}\varepsilon_{\nu 0},$$

$$\pi_{Nt} = k^\pi_{xt} m_t - k^\pi_{\nu t}\varepsilon_{\nu 0}$$

for some coefficients k. Substitute this back into the objective function:

$$\sum_{t=0}^{\infty}\beta^t(x_t^2 + \alpha\pi_{Nt}^2) = \sum_{t=0}^{\infty}\beta^t\Big[(k^x_{xt}m_t - k^x_{\nu t}\varepsilon_{\nu 0})^2 + \alpha(k^\pi_{xt}m_t - k^\pi_{\nu t}\varepsilon_{\nu 0})^2\Big]$$

$$= \mathcal{K}_x m_t^2 + \mathcal{K}_\nu \varepsilon_{\nu 0}^2 + \mathcal{K}_{x\nu} m_t \varepsilon_{\nu 0} = k_1(m_t - k_2\varepsilon_{\nu 0})^2 + k_3\varepsilon_{\nu 0}^2.$$

Substitute solution from the first step keeping in mind that it holds for every innovation $\varepsilon_{\nu 0}$ to get the second-stage problem, which is largely isomorphic to the baseline model:

$$\begin{aligned}
\min_{\{z_t, m_t, b_t^*, f_t^*, \sigma_t^2\}} \quad & \frac{1}{2}\,\mathbb{E}\sum_{t=0}^{\infty}\beta^t\Big[\gamma z_t^2 + (1-\gamma)k_1(m_t - k_2\varepsilon_{\nu t})^2\Big] \\
\text{s.t.} \quad & \mathbb{E}_t\Delta z_{t+1} = -\bar{\omega}\sigma_t^2\big(\iota b_t^* - n_t^* - f_t^*\big), \\
& \beta b_t^* = b_{t-1}^* - z_t, \\
& \sigma_t^2 = \mathrm{var}_t\left(\tilde{q}_{t+1} - z_{t+1} + m_{t+1}\right).
\end{aligned}$$

Going back to the policy in the non-tradable sector, consider whether the price level converges to the initial level in the long run. The optimal policy implements $\alpha\kappa\pi_{Nt} = -\Delta x_t$ for $t \geq 1$, just as in a closed economy. However, in the latter case, this condition holds also for $t = 0$ (under timeless perspective), which implies that $\alpha\kappa p_{Nt} = -x_t$ in all periods and given that x_t is stationary, the price level converges in the long run to the initial level. In contrast, in our model $\alpha\kappa p_{Nt} = -x_t + (x_0 + \alpha\kappa\pi_0)$ and given that $x_t \to 0$ in the long run, we get $p_{Nt} \to \frac{1}{\alpha\kappa}x_0 + \pi_0$, which is generically not equal zero.

B.3.2 Terms of trade

To derive the loss function, follow the same steps as in the baseline model. Write down the Lagrangian of the relaxed problem without nominal or financial frictions:

$$\begin{aligned}
\mathcal{L} = \mathbb{E}\sum_{t=0}^{\infty}\beta^t\Bigg\{&(1-\gamma)\log C_{Ht} + \gamma\log C_{Ft} - L_t \\
&+ \lambda_t\Big(A_t L_t - C_{Ht} - \gamma P_{Ht}^{*-\varepsilon}C_t^*\Big) + \mu_t\left[B_{t-1}^* + \gamma P_{Ht}^{*1-\varepsilon}C_t^* - C_{Ft} - \frac{B_t^*}{R_t^*}\right]\Bigg\}.
\end{aligned}$$

Notice that the planner is allowed to set optimal price in foreign market and, in equilibrium, charges a constant markup $\frac{\varepsilon}{\varepsilon-1}$ over domestic price for the same goods. Take the first-order conditions and solve for the steady-state values of the Lagrange multipliers: $\lambda = 1/A$, $\mu = \left(\frac{\varepsilon}{\varepsilon-1}\frac{C^*}{A}\right)^{\frac{\varepsilon-1}{\varepsilon}}\frac{1}{C^*}$. Using these values and expression (**??**), derive quadratic loss function:

$$\mathcal{L} \propto \frac{1}{2}\mathbb{E}\sum_{t=0}^{\infty}\beta^t\Big\{(1-\gamma)c_{Ht}^2 + \gamma c_{Ft}^2 + \gamma(\varepsilon-1)p_{Ht}^{*2}\Big\}, \tag{A1}$$

where as before, the small letters denote the deviations from the first-best allocation.

PCP When sticky in producer currency, the export price in the currency of destination is equal

$$P_{Ht}^* = \frac{\varepsilon}{\varepsilon-1}\frac{P_{Ht}}{\mathcal{E}_t} = \frac{\varepsilon}{\varepsilon-1}\frac{1-\gamma}{\gamma}\frac{C_{Ft}}{C_{Ht}},$$

where the latter equality follows from household demand for goods (**??**). It follows that

$$p_{Ht}^* = c_{Ft} - c_{Ht}$$

and it is sufficient to close two gaps in the loss function (A1) to implement efficient allocation. Linearizing the market clearing condition, we get

$$l_t = (1-\bar{\gamma})c_{Ht} - \bar{\gamma}\varepsilon p_{Ht}^*,$$

where $\bar{\gamma} \equiv \frac{\gamma(\varepsilon-1)}{\varepsilon-\gamma}$ is the steady-state share of exports in total output. The last two equations can be solved to express c_{Ht} and p_{Ht}^* in terms of the normalized output gap $x_t \equiv \frac{1}{1+\bar{\gamma}(\varepsilon-1)}l_t$ and the risk-sharing gap $z_t \equiv \frac{1}{1+\bar{\gamma}(\varepsilon-1)}c_{Ft}$:

$$c_{Ht} = \varepsilon\bar{\gamma}z_t + x_t, \qquad p_{Ht}^* = (1-\bar{\gamma})z_t - x_t.$$

Substitute these expressions into the loss function to obtain $\frac{1}{2}\mathbb{E}\sum_{t=0}^{\infty}\beta^t\Big[\kappa z_t^2 + x_t^2\Big]$, where $\kappa \equiv \frac{\varepsilon^2\gamma}{\varepsilon-\gamma}$.

Linearizing the budget constraint and substituting in expression for p_{Ht}^*, we get

$$\beta b_t^* = b_{t-1}^* + \frac{\varepsilon-1}{\varepsilon}x_t - z_t,$$

where $b_t^* \equiv \frac{B_t^*-\tilde{B}_t^*}{\varepsilon C_F}$. Normalizing noise trader shocks N_t^* and FX interventions F_t^* by $\frac{1}{\varepsilon C_F}$, we get the risk-sharing condition

$$\mathbb{E}_t\Delta z_{t+1} = -\bar{\omega}\sigma^2\Big(\iota b_t^* - n_t^* - f_t^*\Big),$$

where $\bar{\omega} \equiv \frac{\omega\varepsilon C_F}{\beta(1+\bar{\gamma}(\varepsilon-1))}$. As before, the nominal exchange rate is given by

$$e_t = (c_{Ht} + \tilde{c}_{Ht}) - (c_{Ft} + \tilde{c}_{Ft}) = \tilde{q}_t + x_t - (1-\bar{\gamma})z_t.$$

Combining these conditions, we get the planner's problem:

$$\begin{aligned}
\min_{\{x_t,z_t,b_t^*,f_t^*,\sigma_t^2\}} \quad & \frac{1}{2}\,\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\big[\kappa z_t^2+x_t^2\big] \\
\text{s.t.} \quad & \mathbb{E}_t\Delta z_{t+1}=-\bar{\omega}\sigma_t^2(\iota b_t^*-n_t^*-f_t^*), \\
& \beta b_t^* = b_{t-1}^*+\frac{\varepsilon-1}{\varepsilon}x_t-z_t, \\
& \sigma_t^2=\operatorname{var}_t\big(\tilde{q}_{t+1}+x_{t+1}-(1-\bar{\gamma})z_{t+1}\big).
\end{aligned}$$

Because x_t drops from the budget constraint in the first-best allocation, the latter can be implemented under the same conditions as in the baseline model. A sufficient condition for $\tilde{q}_t=0$ is that $r_t^*=0$ and $a_t=c_t^*$ follow a random walk. Indeed, in this case $\tilde{c}_{Ft}$ is also a random walk and moves one-to-one with a_t, which given $\tilde{c}_{Ht}=a_t$ implies that $\tilde{q}_t=\tilde{c}_{Ht}-\tilde{c}_{Ft}=0$.

DCP The dollar pricing implies that P_{Ht}^* is fixed and therefore,

$$p_{Ht}^*=-\tilde{p}_{Ht}^*=\tilde{c}_{Ht}-\tilde{c}_{Ft}=\tilde{q}_t.$$

Define output gap as deviations from the optimal production of locally consumed goods $x_t=c_{Ht}$ and the risk-sharing wedge as the deviation from the optimal consumption of foreign goods $z_t=c_{Ft}$ and write the loss function (A1) as $\frac{1}{2}\mathbb{E}\sum_{t=0}^{\infty}\beta^t\Big[(1-\gamma)x_t^2+\gamma z_t^2+\gamma(\varepsilon-1)\tilde{q}_t^2\Big]$. The first-order approximation to the budget constraint is

$$\beta b_t^*=b_{t-1}^*-(\varepsilon-1)\tilde{q}_t-z_t,$$

where $b_t^*\equiv\frac{B_t^*-\tilde{B}_t^*}{C_F}$. Intuitively, when the first-best real exchange rate depreciates, the export price become too high reducing exports relative to the efficient allocation. Normalizing N_t^* and F_t^* by C_F and defining $\bar{\omega}\equiv\omega C_F/\beta$, the planner's problem can be written as

$$\begin{aligned}
\min_{\{x_t,z_t,b_t^*,f_t^*,\sigma_t^2\}} \quad & \frac{1}{2}\,\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\big[\gamma z_t^2+(1-\gamma)x_t^2+\gamma(\varepsilon-1)\tilde{q}_t^2\big] \\
\text{s.t.} \quad & \mathbb{E}_t\Delta z_{t+1}=-\bar{\omega}\sigma_t^2(\iota b_t^*-n_t^*-f_t^*), \\
& \beta b_t^*=b_{t-1}^*-(\varepsilon-1)\tilde{q}_t-z_t, \\
& \sigma_t^2=\operatorname{var}_t\big(\tilde{q}_{t+1}+x_{t+1}-z_{t+1}\big).
\end{aligned}$$

It follows that when $\tilde{q}_t=0$, the first-best allocation with zero losses and $x_t=z_t=0$ is implementable with monetary policy that pegs the nominal exchange rate $\sigma_t^2=0$. When two policy instruments are available, the risk-sharing condition is not binding and the problem reduces to minimizing the losses subject to the intertemporal budget constraint:

$$\begin{aligned} \min_{\{x_t, z_t\}} \quad & \frac{1}{2}\, \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \big[\gamma z_t^2 + (1-\gamma) x_t^2\big] \\ \text{s.t.} \quad & \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \big[z_t + (\varepsilon - 1)\tilde{q}_t\big] = 0. \end{aligned}$$

Denote the Lagrange multiplier on the budget constraint with μ and take the first-order conditions:

$$\beta^t \gamma z_t = \beta^t \mu, \qquad \beta^t (1-\gamma) x_t = 0.$$

Therefore, the monetary policy closes the output gap $x_t = 0$ and the FX interventions close the UIP gap $\mathbb{E}_t \Delta z_{t+1} = 0$ by setting $f_t^* = \iota b_t^* - n_t^*$.

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