The Coherence Side of Rationality

Rules of thumb, narrow bracketing, and managerial incoherence in corporate forecasts*

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Abstract

We develop a theory of forecast coherence in a firm production setting, which yields a normative ex ante benchmark of first-best coherent forecasts and statistical tests to detect incoherence ex post. Under the null, the forecasts—and the forecast errors—of output and inputs are "close" to one another. Using the Duke Survey of top executives of large US corporations, we reject the null of coherence for 55% of CFOs in our sample. In a positive version of our model, incoherence reflects intra-personal frictions in coordinating multiple forecasts, implying that some of the rules of thumb proposed by the managerial education literature to make contemporaneous forecasts may emerge as second-best optimal. Consistent with our model, we find that corporate performance correlates negatively with incoherence, being lowest for firms whose CFOs provide "narrow bracketing" forecasts—projecting past capital growth into the future while ignoring output and labor. We also find that the use of incoherent rules of thumb correlates negatively with corporate investment spending and positively with corporate leverage.

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I Introduction

Coherence refers to "the consistency of the elements of the person's judgment" (Hammond (2007), p. xvi). When forecasting many variables at the same time, coherence requires the forecaster to fully assess all the connections among the multiple variables under consideration, with the purpose of delivering rational forecasts. Coherence constitutes one of the two standards for evaluating the rationality of forecasts, together with accuracy (Hammond, 1990, 1996, 2000). While there are numerous theoretical and empirical analyses of forecast accuracy, centering on whether forecast errors are systematically predictable from information known at the time of the forecast (Tversky and Kahneman (1974); see Benjamin (2019) for a recent survey), coherence has received much less attention.[1] Our main contribution is to provide a theory of coherence in a firm production setting and to present the first evidence that top financial executives make severely incoherent forecasts of their own firm's output and input growth.

Incoherence may be very costly. Under a standard production technology, a firm that wishes to double its output will likely have to plan using a lot more of its inputs such as capital and labor, lest the desired output proves unattainable. Ignoring the technological constraint implies that the firm could end up using a suboptimal mix of capital and labor, thereby attaining a lower profit than it would have been otherwise possible with its production technology and budget constraint. More broadly, corporate planning and internal forecasting underlie all resource allocation and investment decisions inside the firm, and are still not well understood (Graham, 2022). The managerial education literature (e.g., Ruback (2004), Titman and Martin (2016), Welch (2017), Holthausen and Zmijewski (2020), Koller et al. (2020), and others) acknowledges the challenges of making detailed plans and provides a number of rules of thumb to help CFOs make rational forecasts. To the best of our knowledge, none of these rules of thumb have been assessed to date, be it theoretically or empirically.

[1] Tversky and Kahneman (1974) discuss coherence in the context of subjective expected utility and argue that to fully assess coherence it is not sufficient to elicit the individual's subjective probabilities, but one would also need to elicit "*the entire web of beliefs held by the individual*" (p. 1130). By contrast, we study coherence in the context of firms' production plans and thus in the context of observable, objective technological relationships among inputs and output.

One such rule of thumb, the "plain growth forecast" (e.g., Welch (2017), p. 593), is to forecast the growth rate of a firm's input (e.g., capital) by exclusively projecting that input's past growth rates into the future, thereby disregarding information about other inputs (e.g., labor). This is reminiscent of the "narrow-bracketing" behavior of decision makers who, facing multiple choices at the same time, make each choice in isolation. Narrow bracketing yields lower utility than the first best of broad bracketing (Thaler (1985); Read et al. (1999)). One mechanism underlying narrow bracketing is mental accounting (Thaler (1985); Tversky and Kahneman (1981)), whereby decision makers hold a separate mental account for each decision in isolation, as opposed to a single budget constraint for their total expenditures. For example, when considering two consumption goods—say food and gasoline—narrow bracketing implies allocating resources suboptimally by treating food consumption as independent of gasoline consumption, and vice versa. In principle, narrow bracketing could be even more problematic in a firm setting, for two reasons. First, corporate planning features not only a budget constraint but also a production technology. Second, top executives need to make very detailed and explicit forecasts (plans) for several years. As a result, narrow bracketing could lead to incoherent and suboptimal allocation of resources to future capital expenditures while ignoring labor costs, or vice versa.

We provide a theory of forecast coherence in a firm production setting, which yields several tests of forecast (in)coherence, and we present the first evidence that top financial executives make severely incoherent forecasts of their own firm's output and input growth. We implement our tests in a population of senior financial executives dealing with corporate forecasts and production decisions. We use the Duke Survey of top executives of large- and mid-size US corporations, who provide forecasts of annual growth rates of multiple firm-level balance-sheet variables simultaneously (e.g., Ben-David et al. (2013) and Graham (2022)). Besides being finance professionals, most of these forecasters are top financial executives and chief financial officers (CFOs) that are actively involved in setting the corporate investment and financing policies of their firms, which allows us to jointly assess corporate forecasts and corporate policies.

More specifically, we build a theoretical model of firm forecasts, which provides a normative benchmark of an ex ante coherent forecast that is first-best optimal. In a positive version of our model, we study the second-best optimal forecasting rules of agents who observe noisy signals about output and input prices and compare these second-best forecasting rules with the rules of thumb suggested by the managerial education literature. We then use our model to measure the extent to which financial executives are incoherent as well as the determinants of their incoherence. We examine empirically the extent to which executives' forecasts reflect the rules of thumb suggested by the managerial education literature, and whether the use of such rules of thumb is linked to more coherent forecasts. We also examine whether incoherence is related to CFOs' personal characteristics, to the intensity of corporate investment spending, to corporate debt policy—all corporate activities in which CFOs play a key role, over and above chief executive officers (CEOs); see Graham et al. (2015) and Malmendier et al. (2022). We find that the use of incoherent rules of thumb correlates negatively with corporate investment spending and positively with corporate leverage.

Using CFOs' forecasts of input and output variables over the period 2001-2018, we document a large cross-sectional dispersion in input forecasts conditional on an output forecast. More than one quarter of CFOs predict at the same time an increase in output (i.e., sales revenues) and a decrease in input (e.g., capital expenditures); about one third of CFOs predict at the same time a decrease in output and an increase in capital expenditures. Even more strikingly, more than 40% of CFOs make forecast errors in input growth that have the opposite sign of the forecast error in output growth. While suggestive of potential incoherence, these patterns are not conclusive.

Motivated by this evidence, we develop a theory of forecast coherence in a firm production setting. In our model, two inputs (capital and labor) combine to produce output according to a standard production technology. The optimal forecasts of inputs and output coherently reflect both the production technology and the budget constraint. In the first best, we establish that a forecaster who is asked to produce forecasts of growth rates of output and inputs should provide forecasts that are linked cross-sectionally by parameters reflecting the contribution of capital and labor to the firm's production technology and budget constraint. If the forecaster doesn't know these parameters, the forecaster can recover them by estimating regressions using realized data from a population of 'comparable' firms, for example, firms in the same industry as their own.

Our framework yields natural tests of forecast coherence across balance sheet variables. We find that most CFOs forecast a growth of output that is larger than the output growth implied by feeding into a general CES production function the same CFOs' forecasts of capital and labor input growth, thereby violating an inequality implied by our model. Under more stringent assumptions to account for uncertainty, we develop a test based on the idea that under the null hypothesis of coherence the forecast errors of output and inputs cannot be "too far" from one another. We find that for 55% of CFOs in our sample we reject the null hypothesis of coherence at the 95% confidence level.

Furthermore, our framework provides a benchmark for an ex ante optimal coherent forecast, which we use to evaluate the rules of thumb that the managerial literature has proposed to help managers make balance-sheet forecasts. We establish conditions under which these rules of thumb yield optimal coherent forecasts. We find that some (but not all) of these rules of thumb represent an optimal second-best forecast rule when CFOs observe noisy signals about the firm's production technology. In particular, narrow-bracketing forecasts projecting past capital expenditures into the future are second-best optimal in the limit in which the CFO observes infinitely noisy signals about the output and the other input (e.g., labor).

To assess our model empirically, we develop a continuous, ex ante measure of managerial incoherence given by the (absolute value of the) orthogonal distance between the actual forecast and the theoretically optimal coherent one. This distance measure is predetermined relative to corporate performance and corporate policies, and can thus be used to assess our model's predictions, unlike our test statistics that also make use of realizations. We validate our distance measure by showing that higher distance (in absolute value) from the ex ante coherent forecast predicts higher (absolute) values of our main test statistic. Consistent with our model, we find that (1) the narrow-bracketing rule of thumb is the most distant from the optimal coherent forecast among the managerial rules of thumb, and (2) corporate performance correlates negatively with managerial incoherence and is lowest for firms whose CFOs provide narrow-bracketing forecasts. We also show that the use of incoherent rules of thumb correlates negatively with corporate investment spending and positively with corporate leverage.

Finally, we examine corporate performance, investment, and debt around the date when the CFO takes office for the subset of CFOs who disclose their identity. We use hand-collected data to track these critical dates. We perform an event-study analysis around the dates the CFOs join their firms. Our results show that corporate performance (ROA) decreases in the years following the start date of an incoherent CFOs, and such decline is larger, the more incoherent the CFO. In the same spirit, corporate investment spending declines in the years following the start date of a narrow-bracketing CFO. Although these results are consistent with our model in which incoherence affects corporate performance and corporate policies, given our data, we are not able to conclude that these empirical relationships are causal. Our results indicate that incoherence is pervasive among top financial executives, and suggest that incoherence may come with the use of a suboptimal mix of capital and labor. To our knowledge, this is the first field evidence of incoherence and narrow bracketing in a setting featuring investment decisions and a production technology.

The paper proceeds as follows. Section II discusses the relevant literature. Section III describes the Duke Survey and presents some motivating empirical evidence. Section IV presents our model of optimal coherent firm forecasts and derives our statistical tests of coherence. Section V presents our empirical results. Section VI concludes.

II Related Literature

The idea that coherence is a pillar of rationality goes back at least to Aristotle.[2] Ethical and political philosophers have often seen incoherence in the context of human institutions such as moral and legal systems as concerning.[3] Rational choice theory typically argues

[2]See, e.g., Fogelin (2003)'s illustration using the law of noncontradiction.

[3]E.g., Raz (1994). Rawls (1999) describes the search for a 'reflective equilibrium' ensuring that one's beliefs, at multiple levels of abstraction, fit together as a sensible whole. Legal scholars (e.g., Dworkin (1986)) have expressed a similar aspiration to coherence. Sunstein et al. (2002) provide experimental evidence in the context of harms such as physical injury, commercial fraud, and ecological damage, that more severe punishments are often assigned to less serious misconducts, and attribute this pattern to individuals thinking in categories and having trouble translating outrage into a dollar scale.

that individuals are endowed with coherent systems of beliefs and preferences, and define coherence as the principal criterion of rationality (e.g., see Becker (1996), Posner (2014)). Sen (1993) formalizes the notion of "internal consistency" (i.e., coherence) in the context of individual decision making (e.g., demand theory, social choice, decision theory), and argues that there is no way of determining whether a choice function is coherent or not without referring to something external to choice behavior, such as objectives, values, or norms. Tversky and Kahneman (1974) similarly argue that to fully assess coherence one should not only elicit subjective probabilities, but also "the entire web of beliefs held by the individual" (p. 1130).

We study coherence in the context of production theory and firm plans. This way, we bypass the need to elicit elements external to the optimization process, such as beliefs, norms, or values. As long as economic agents (e.g., firms' executives) agree that firms' profits should be maximized, the features of the production technology, which are given in the short run, should constrain the process of firms' planning into making coherent forecasts. That is, if a firm wishes to double its output, it should plan to significantly increase its expenditures on capital and labor as well, otherwise the hoped-for output increase will be unattainable. In this precise sense, we construct a statistical test to determine whether the simultaneous forecasts of output and input are coherent with each other. In what follows, we discuss our contributions to behavioral economics with a specific focus on narrow bracketing, and to the literature on firms' survey expectations.

The Coherence and Accuracy Requirements of Rationality. The psychology literature has long recognized that rationality in probabilistic judgments (e.g., forecasts) involves both accuracy (sometimes called 'correspondence') and coherence.[4] This literature typically maintains that accuracy and coherence are separate properties of the same probabilistic judgement, but has struggled to provide a formal framework or direct evidence to assess such a claim. A number of experiments measure coherence in terms of adherence to the laws of probability theory and accuracy in terms of specific judgment tasks, and document that in their settings respondents' adherence to probability theory

[4] See, e.g., Hammond (1996), Gigerenzer et al. (1999), Gigerenzer and Todd (2000), Mandel (2005), Newell (2005), Gigerenzer and Gaissmaier (2011), Baron (2012), Lee and Zhang (2012), Wallin (2013).

is only weakly correlated with the same respondents' accuracy in some specific prediction task (see, e.g., Wright et al. (1994), Berg et al. (2022), Reyna and Lloyd (2006), Zhu et al. (2020), and Zhu et al. (2022)).[5]

More broadly, a large literature in behavioral economics has documented a number of biases in probabilistic judgment, including biases in beliefs about random sequences (e.g., gambler's fallacy and hot-hand fallacy), biases in beliefs about sample distributions (e.g., partition dependence and sample size neglect), biases in belief updating (e.g., violations of Bayes' theorem such as conservatism or base-rate neglect), and has presented a number of theories to understand these biases; e.g., see Tversky and Kahneman (1971), Rabin (2002), and Benjamin et al. (2016); see Benjamin (2019) for a recent review.

While Tversky and Kahneman (1974) have famously characterized this research program in terms of the "heuristics and biases" framework, according to which the use of heuristics generates systematic and predictable forecast inaccuracies (see also Gilovich et al. (2012); Thaler (2018)), more recently a number of authors have recognized that at least some of these results can also be cast in terms of the coherence-accuracy framework. For example, Hammond (1996), Tentori et al. (2013), Jönsson and Shogenji (2019), and others discuss how the conjunction fallacy,[6] first documented by Tversky and Kahneman (1983), can be understood as a violation of coherence with respect to probability laws. Similar arguments apply to the disjunction fallacy and violations of Bayes' theorem.

The focus in this literature has not been on providing a formal framework in which the concepts of accuracy and coherence are nested and can thus be both jointly assessed and disentangled; rather, the focus has been on establishing that inaccuracy comes from systematic cognitive biases, and chiefly among these, from the misconception of the laws

[5] A theoretical literature in philosophy develops axiomatic definitions of coherence using probability theory (see, e.g., Douven and Meijs (2007), Fitelson (2003), Glass (2002), Olsson (2002), Roche (2013), Schippers (2014), Schupbach (2011), Shogenji (1999)). A general theme in this literature is to define two propositions as coherent with each other if they are positively correlated, according to some suitably defined measure of correlation. In the data, however, a simple correlation among multiple forecasts may occur for a number of reasons other than coherence. Furthermore, none of these papers disentangle coherence from accuracy.

[6] That is, people wrongly believing that the conjunction of two events, A and B, has a higher probability than one of its constituents, say, A. In a famous experiment, respondents were first given a brief description of Linda, constructed to be representative of a feminist and not representative of a bank teller. Consistent with the conjunction fallacy, 85% of respondents indicated that it was more likely that Linda was both a feminist and a bank teller rather than a bank teller.

of probability theory. Therefore, in this literature documenting a form of incoherence such as the violation of probability laws (e.g., the law of total probability, or Bayes' theorem) typically serves the purpose of predicting future systematic inaccuracy, without aiming at disentangling coherence from accuracy.

Relative to this literature, our contribution is to provide a formal framework in a forecasting setting in which coherence and accuracy are defined with respect to the same forecasting task, which allows us to jointly assess forecast accuracy and coherence. We show that to do so one needs both theory and data. In terms of theory, an economic model provides a benchmark against which to judge coherence. This is similar to the use of probability theory for assessing coherence of probabilistic judgments; but crucially economic theory also allows us to nest forecast coherence and accuracy in a setting with optimizing agents. In terms of data, observing both forecasts and realizations—and thus forecast errors—allows us to jointly assess accuracy and coherence, and also to disentangle them. Accuracy is assessed by testing whether forecast errors of each variable are 'sufficiently' close to zero; coherence is assessed by testing whether forecast errors of different variables (inputs and output in our setting) are 'sufficiently' close to one another. In our data, we find that 12% of individuals are incoherent but accurate, and 13% of individuals are inaccurate but coherent.

Disentangling coherence and accuracy is crucial, because—at least under some conditions—coherence can be achieved ex ante. This is a similar intuition to the debiasing research program in psychology (e.g., Fischhoff (1982), Roy and Lerch (1996)),[7] with the difference that in our framework one can use economic theory and regression analysis to determine the appropriate ex ante rule of thumb to achieve coherence. In fact, the managerial education literature (e.g., Ruback (2004), Titman and Martin (2016), Welch (2017), Holthausen and Zmijewski (2020), Koller et al. (2020), and others) has recognized the challenge of making forecasts of many firm variables at the same time, and has provided a number of regression-based rules of thumb, without however relying on

[7]Debiasing strategies come in three forms: (i) modifying the presentation of a problem to elicit the appropriate mental procedure; (ii) training people to think correctly about a problem; and (iii) doing the calculations for people, so that they merely need to provide the inputs to the calculations (Roy and Lerch (1996); see also Fischhoff (1982)).

economic theory or empirical evidence to guide the choice among them. We show both in theory and in our data that not all rules of thumb are equivalent to one another. Quite to the contrary, while some rules of thumb do come close enough to the ex ante optimal coherent forecast, others provide severely incoherent forecasts. Most notably, the "plain growth forecast" (e.g., Welch (2017), p. 593) that we dub the narrow-bracketing rule of thumb provides severely incoherent forecasts.

Bracketing. Lab experiments and empirical research in psychology and behavioral economics show that the decision maker often narrowly brackets and makes each decision in isolation (e.g., Tversky and Kahneman (1981), Read et al. (1999), Kahneman and Lovallo (1993), Rabin and Weizsäcker (2009), Ellis and Freeman (2020)). To rationalize this evidence, Thaler (1985, 2018) and Heath and Soll (1996) argue that individuals hold a mental account of each decision (e.g., consumption or gambling) in isolation, and do not consider the interdependence among the various decisions brought about by the budget constraint and the marginal rates of substitution in the utility function.

Economic theories of narrow bracketing include Barberis et al. (2006) and Rabin and Weizsäcker (2009). These models study the monetary gambles and stock market participation decisions taken by narrow-bracketing agents, and show that narrow bracketing leads to stochastically dominated choices, including low stock market participation and the choice of stochastically dominated gambles. Related models of mental accounting in different contexts include Hastings and Shapiro (2013, 2018), in which consumers are allowed to deviate from an explicit mental budget subject to a cost. More recently, Lian (2021) provides a model of narrow thinking in which different decisions are based on different, non-nested information. In Lian (2021)'s model, the narrow thinker makes each decision with imperfect knowledge of other decisions and faces difficulties coordinating the multiple decisions, thereby endogenizing narrow bracketing.

Theories of narrow bracketing are also related to the rational inattention literature (e.g., Sims (2003), Mackowiak and Wiederholt (2009), Matějka and McKay (2015), Mackowiak et al. (2018)), which also uses noisy signals to capture the decision maker's inability to incorporate all relevant information when making each decision.

Recently, Kőszegi and Matějka (2020) microfound mental accounting by proposing a multiple-decision problem in which consumer's demand for multiple goods is based on the same, imperfect information. Related to rational inattention but using a deterministically imperfect perception of fundamentals rather than noisy signals, Gabaix (2014, 2019) develops a sparsity model in which, similar to the rational inattention approach, the sparse agent's multiple decisions are made based on the same, imprecise perception of the fundamental. Unlike the rational inattention literature, in recent models of narrow-bracketing agents make decisions and forecasts based on different, non-nested information, e.g., see Lian (2021).

We add to this literature by providing a model of narrow bracketing in a firm setting featuring a production technology, and by providing field evidence of narrow bracketing in a sample of senior financial executives and CFOs directly involved with corporate forecasts and production decisions. Similar to Lian (2021), our agent also makes decisions and forecasts based on different, non-nested information, which differentiates both us and Lian (2021) from the rational inattention literature, in which different decisions are made based on the same nested information. In our setting, there are no explicit mental budgets, which avoids the need to take a stand on where such mental budgets come from. Unlike Lian (2021), we study a production model where the interconnection among different decisions come from the budget constraint and the production technology, which delivers novel predictions about corporate forecasts and corporate performance. More broadly, while our paper shares with prior literature the common psychological underpinnings that individuals think in categories (e.g., Tversky and Kahneman (1991), Ellis and Masatlioglu (2022)), the extant literature has studied individual's choice over risky gambles (Rabin and Weizsäcker, 2009), stock market participation (Barberis et al., 2006), or consumption behavior (Lian, 2021). To our knowledge, we are the first to study narrow bracketing in the context of the firm's planning of resource allocation and production technology.

Survey Expectations of Firms. Our paper is related to the recent and growing empirical literature studying beliefs and forecasts of corporate top executives. Ben-David et al. (2013) and Boutros et al. (2020) show that top executives are miscalibrated, as they provide probability distributions of stock market returns that are too narrow, consistent with managerial overconfidence. Campello et al. (2010) develop a survey-based measure of financial constraints and find that constrained firms plan larger cuts in tech spending, employment, and capital spending. Campello et al. (2011, 2012) study how CFO's financial plans responded to the 2008-2009 financial crisis. Gennaioli et al. (2016) show that corporate investment plans as well as actual investment are explained by CFOs' expectations of earnings growth. We add to this literature by investigating the joint forecasts of different balance sheet items at the same time and showing that executives produce severely incoherent forecasts. We are able to directly control for the measures of overconfidence and optimism used in this literature and we show that, unlike overconfidence that predicts more aggressive corporate investment spending, incoherence correlates with reduced investment spending and increased leverage, consistent with the idea from psychology that incoherence and overconfidence are different traits.

Our paper is also related to the recent literature using survey data to analyze firm expectations about the macroeconomy and about own variables. This body of work has documented a wide heterogeneity in corporate forecasts in different settings, most notably, inflation, see D'Acunto et al. (2022) and Candia et al. (2022) for two recent surveys. Bloom et al. (2021) show that forecasting firms' own variables is potentially even harder than forecasting the aggregate economy. Graham (2022) documents that the revenue growth forecast is most important in terms of its consequences for the firm and its plans. Accordingly, Altig et al. (2022) show that firm-level growth expectations are highly predictive of realized growth rates. We confirm that firms make on average accurate sales growth forecasts also in our data, but we also show that expectations of other variables, for example capital expenditures, are much less predictive of realized growth rates, consistent with incoherence. Bachmann and Bayer (2013, 2014) find that the dispersion and volatility of expectations and expectation errors are countercyclical, and Bachmann et al. (2020) separately measure firms' subjective beliefs about risk and uncertainty. See Born et al. (2022) for a recent survey. We add to this literature by documenting heterogeneity in the extent to which corporate managers provide forecasts of multiple balance-sheet items at the same time.

Finally, our results imply that firms making incoherent plans leave money on the table by using a suboptimal mix of inputs. As a result, our paper also adds to the recent literature on behavioral firms making inefficient choices, e.g., see DellaVigna and Gentzkow (2019), Strulov-Shlain (2022), and DellaVigna (2018).

III Data and Motivating Evidence

A. Data

We use two main sources of data, one on CFO expectations and one on firm realizations. Our data on CFO expectations come from the Duke Survey led by John Graham and Campbell Harvey, which was launched in July 1996 and takes place on a quarterly basis.[8] Each quarter, the survey asks CFOs their views about the US economy and corporate policies, as well as their expectations of future firm performance and operational plans. Starting at the end of the 1990s, the CFO survey consistently asks respondents their expectations of the future twelve-month growth of key corporate variables, including revenues, capital expenditures, employment, and earnings, among others. In particular, our data comprises 72 quarterly surveys conducted between March 2001 and December 2018. In our data, we observe corporate forecasts as a single number per each item, which we interpret as the CFO's expected value and which corresponds to the firm's base case scenario. For many firms, the base case is the only scenario that gives rise to fleshed out forecasts in their internal planning process.[9] The original questions are presented to the CFOs as follows:

Relative to the previous 12 months, what will be your company's PERCENTAGE CHANGE during the next 12 months? (e.g., +3%, −2%, etc.) [Leave blank if not applicable] Revenues: _____; Capital spending: _____; R&D spending: _____; Technology spending: _____; Prices of your product: _____; Earnings: _____; Cash on balance sheet: _____; Number of domestic full-time employees: _____; Wage: _____; Dividends: _____. Advertising: _____. Share repurchases: _____.

[8]Historical surveys as well as aggregated responses can be accessed at https://cfosurvey.fuqua.duke.edu/.

[9]Firms that internally consider additional scenarios typically consider a downside scenario to plan for contingencies and an upside scenario to lay out stretch goals. However, these additional scenarios are often developed in less detail than the base case and do not necessarily lead to fleshed-out forecasts. See Graham (2022), p. 1997, for more details.

Figure 1 displays an actual screenshot of the above questions.

Each quarter, the study surveys between 2,000 and 3,000 financial officers. The usual response rate for the quarterly survey is 5% to 8%; most of the responses arrive within the first two days of the survey invitation date.

Our data on realizations comes from Compustat, which extracts the information from the Security and Exchange Commission (SEC) required public filing of financial statements. Compustat covers all publicly traded firms across all sectors of the U.S. economy since 1955. We exclude firms that have negative assets and we winsorize at the 1% level.

Table 1 reports summary statistics on CFO twelve-month ahead growth forecasts (Panel A) and on growth realizations in a matched Duke-Compustat sample (Panel B).[10] When matching Duke and Compustat data there are four sources of attrition: (1) due to privacy restrictions associated with these data, not all Duke respondents report their firm ID, so they cannot be matched to Compustat; (2) not all Duke respondents give forecasts about all variables in each survey; (3) not all variables elicited in the Duke survey have a precise counterpart in Compustat, namely, technology spending, outsourced employees, health spending, productivity, product prices, share repurchases; and (4) not all variables for which there is a precise counterpart in Compustat have full coverage, chiefly among those, wages are missing for about 90% of Compustat firms and R&D and advertising expenditures are also missing for a large fraction of Compustat firms.[11] Table A1 in the Appendix reports summary statistics on the full Compustat sample and on the matched Duke-Compustat sample. Firms with CFO expectations in the Duke data are typically larger than the median Compustat firm in terms of sales and assets. Firms with CFO expectations also appear to be more profitable and to pay more dividends than firms in the full Compustat sample, but are otherwise similar in terms of investment, market-to-

[10]Table A2 in the Appendix reports the same statistics in the full Compustat population.

[11]In our data, the matched Duke-Compustat sample mostly refers to the earlier part of the sample, until about 2011Q4; we will conduct most of our regression analysis in the pre-financial crisis period. Points (1) and (2) imply a potential selection problem. If anything, our respondents are positively selected among those more likely to give coherent and accurate forecasts of all variables, under the assumption that missing forecasts reflect lack of knowledge about the variables. Points (3) and (4) imply that our analysis of forecast errors needs to be limited to variables for which there is full coverage in both Duke and Compustat.

book ratio, and leverage. These patterns broadly concur with prior work using the Duke data (e.g., Ben-David et al. (2013)).

Comparing Panel A and B of Table 1 shows that CFOs are on average slightly more optimistic about output (i.e., revenues), although the medians of forecasts and realizations are quite close to one another, consistent with the observation in Graham (2022) that CFOs care about getting revenues forecasts right. Conversely, CFOs are on average more conservative about input (i.e., capital expenditures), with the distribution of capital expenditures realizations that is shifted to the right relative to the distribution of capital expenditures forecasts. However, these simple comparisons mask substantial cross-sectional heterogeneity. In the next subsection, we examine the joint distributions of output and input forecasts and forecast errors.

B. Motivating Evidence

Figure 2 shows the scatter plot of contemporaneous forecasts of output growth (i.e., revenues growth) and input growth (i.e., capital expenditures growth). Panel A refers to the whole Duke sample, whereas Panel B refers to the matched Duke-Compustat sample. According to the managerial education literature (e.g., Ruback (2004) and others), one would expect a strong positive association between output and input forecasts. Yet, both scatter plots in Panel A and B of Figure 2 show huge amounts of dispersion in the contemporaneous forecasts of output and capital. In particular, more than one quarter of CFOs predict at the same time an increase in output (i.e., sales revenues) and a decrease in input (e.g., capital expenditures); about one third of CFOs predict at the same time a decrease in output and an increase in capital expenditures. Furthermore, while the univariate regression coefficient of sales growth forecast on capital expenditures growth forecast in Panel A is positive (0.157) and strongly statistically significant as expected, there remains substantial unexplained variation (the R^2 is 4%). Similar patterns obtain in Panel B, where the regression coefficient is 0.171 and the R^2 is 8%.

While suggestive of incoherence, however, these patterns could still be reflecting rational coherent forecasts. Take for example the upper-left quadrant in which CFOs forecast higher sales but lower capital expenditures. These forecasts could make sense, for example, if the firm had a lot of accumulated inventory, so that over the following 12 months that firm could increase its sales while being able to accommodate a decrease in capital expenditures over the same horizon. Similarly, a firm could build up a lot of fixed capacity while at the same time not expecting to be able to increase its output in the following 12 months, explaining observations in the lower-right quadrant in which CFOs forecast higher capital expenditures and lower sales.

These observations suggest that a rational CFO would correctly anticipate these business needs, and thus the misaligned forecasts (higher sales and lower capital expenditures; or vice versa) would result in accurate ex post realizations. This line of reasoning suggests that it might be more informative of incoherence to examine the joint forecast errors in sales and capital expenditures, namely, the difference between realizations and forecasts. Figure 3 plots the forecast errors in capital expenditures against the forecast errors in output. Our sample size shrinks considerably, because for this exercise we need to match CFO forecasts to realizations in Compustat data. However, the pattern is very similar to that in Figure 2. While the univariate regression coefficient is positive (0.149) and strongly significant as expected, the R^2 is only 11%. Most important, 42% of observations still lie in the upper-left and lower-right quadrants, indicating that many CFOs make forecast errors that have the opposite sign to one another. Again, these patterns suggest pervasive incoherence across CFOs.[12]

While suggestive, these patterns of joint forecasts and joint forecast errors that are far off from each other are not conclusive of incoherence. We need a theoretical framework to formalize the notion of coherence and to derive statistical tests of (in)coherence. In the next section, we provide our theoretical framework.

IV Theoretical Framework

When preparing corporate plans, CFOs typically start from output by making a sales revenue forecast (aka top line forecast), and then proceed to make forecasts of all other

[12] In the body of the paper, we focus on the joint distributions of output and input forecasts and forecast errors to provide a close mapping with theory; in unreported tests we find similar empirical patterns in the joint distribution of forecasts and forecast errors of output and profit, and of input and profit.

balance sheet variables, including among others capital and labor expenditures (e.g., see Welch (2017), Graham (2022)), for a number of years. Therefore, the CFO faces a challenging multidimensional forecasting problem, which requires not only to make accurate forecasts of each and every one of the items under consideration, but also to make sure that these forecasts are coherent.

The managerial education literature acknowledges the challenges of making detailed corporate forecasts and provides a number of rules of thumb to guide managers in this process. A recent taxonomy is provided by Welch (2017)'s textbook (p. 593-594), who lists five possible heuristics:

(R1) A **plain growth** forecast, projecting into the future the past growth rates of each item (e.g., capital expenditures). In his teaching notes, Welch (2017) implements this rule by estimating the average of the two most recent past annual growth rates, and using this average as the predictor of future growth.

(R2) A pure **proportion of sales** forecast, forecasting each item as a fixed proportion of sales. Welch (2017) implements this rule by assigning each item the same growth rate as sales.

(R3) An **economies-of-scale** forecast, positing for each item both a fixed component and a variable component, the latter itself a proportion sales. Welch (2017) implements this rule by estimating a univariate mean linear regression of each balance sheet item's growth on contemporaneous sales growth using Compustat data. The estimated regression intercept is the fixed component and the estimated slope multiplied by the sales forecast is the variable component.

(R4) An **industry-based** forecast, drawing on information from other firms in the same industry. Welch (2017) implements this rule exactly as (R3), but using only data from other firms in the same industry rather than all Compustat data.

(R5) A **disaggregated** forecast, recognizing that each item may comove not only with sales but also with the other items. Welch (2017) implements this rule by conditioning on additional (relative to (R3)) contemporaneous items in the regressions using all Compustat data.

In his discussion of how to avoid potential mistakes in making cash flow forecasts, Ruback (2004) advocates using methods (R2) and (R3). Case studies developed at the Harvard Business School for teaching MBA students typically suggest a combination of methods (R1), (R2), and (R4), e.g., see Luehrman and Heilprin (2009), Stafford and Heilprin (2011), Cohen et al. (2008), among others. Koller et al. (2020) also advocate method (R2), as they claim that the "*net Property, Plant and Equipment should be forecast as a percentage of revenues*" (p. 286). Titman and Martin (2016), Chapter 2, describe a method akin to (R3). Holthausen and Zmijewski (2020) describe an elaborate process to come up with a forecast of capital expenditures that is akin to (R5).

Our motivating evidence in Section III shows that the forecasts and forecast errors of output and capital input of many CFOs of large US firms are far apart from each other, but stop short of allowing any conclusion about whether these patterns reflect actual incoherence. Furthermore, our evidence suggests that making coherent forecasts is indeed challenging, and raises the question of which of the above rules, if any, constitutes best practice. Indeed, the managerial education literature lacks a formal framework designed to offer guidance as to whether the above methods are one and the same and thus perfectly equivalent to one another, or they differ along key dimensions, and if so, which method is to be preferred and under which conditions.

Our objective in this section is threefold. First, we develop a formal theoretical framework, which, among other things, allows us to derive statistical tests of ex post incoherence. Second, we provide a benchmark normative framework to guide CFOs in developing ex ante rational, coherent corporate forecasts in the first best. We provide such a framework in Subsection *A*. Third, we develop a positive framework in Subsection *B*., describing how CFOs develop second-best optimal forecasts in the presence of noisy signals about the firm's technology. Our framework nests the above rules of thumb and shows conditions under which some of them emerge as second-best optimal.

A. A Benchmark Model of Optimal Corporate Forecasts

Consider a general class of production functions with constant elasticity of substitution (CES) between inputs,

$$\begin{aligned} y = f(x_1, x_2) &= \left(\frac{a}{a+b}x_1^{\xi} + \frac{b}{a+b}x_2^{\xi}\right)^{\frac{a+b}{\xi}} \\ \text{s.t. } p_1 x_1 + p_2 x_2 &\leq Z, \end{aligned}$$

where y is the output, x_1 and x_2 are input quantities (capital and labor), p_1 and p_2 are the input prices, the output price p_y is normalized to 1, Z is a real-valued budget constraints, $\nu \equiv a+b > 0$ are parameters governing the returns to scale (constant for $\nu = 1$, increasing for $\nu > 1$, and decreasing for $\nu < 1$), and the elasticity of substitution between x_1 and x_2 is $\chi = \frac{1}{1-\xi}$. We assume that factor-augmenting productivities are constant over time and we normalize them to one.[13] We also assume that the technological relationship is stable over time and not subject to aggregate shocks.[14] This formulation is very general (Moysan and Senouci, 2016) and it nests a number of widely used specifications as special cases.[15] Denote $\log p_i = \pi_i$, where $i = 1, 2$, and assume for now input prices are i.i.d., $\{\pi_{i,t}\}_{t\geq 1} \sim \mathcal{N}(0, \sigma_i^2)$, with $cov(\pi_1, \pi_2) = \rho_{1,2}$. Consider a forecaster who at time t issues a forecast F_t of the future realization of a generic variable, x_{t+1}, to minimize a quadratic loss function,

$$\min_{F_t} \mathbb{E}\left[(x_{t+1} - F_t)^2 | \Omega_t\right],$$

where Ω_t denotes the information set at time t and at the solution $F_t^* = \mathbb{E}[x_{t+1}|\Omega_t] \equiv \mathbb{E}_t[x_{t+1}]$.

[13]This is wlog because in our setting a TFP shock would be isomorphic, or observationally equivalent, to input price shocks in the same direction.

[14]This is plausible because we focus on cross-sectional differences in coherence across forecasters, and we implement most of our tests over 2001-2007 at the peak of the 'great moderation', a time when aggregate volatility was not a concern.

[15]For $\chi \to +\infty$ the inputs are perfect substitutes, and the production function is linear; for $\chi \to 0$ there is no substitution and the production function is Leontieff; and for $\chi = 1$ we have a Cobb-Douglas. The empirical literature suggests as plausible a range of $\chi \in (0.5, 1]$ (e.g., see Berndt (1976), Oberfield and Raval (2021)), implying $\xi \in (-1, 0]$.

A.1 Optimal Forecasts and Tests of Coherence

Proposition 1 (Inequality). *Forecast coherence requires that the output and inputs forecasts, $\mathbb{E}_t[y_{t+1}]$, $\mathbb{E}_t[x_{1,t+1}]$, and $\mathbb{E}_t[x_{2,t+1}]$, satisfy an inequality, whose direction depends on whether the CES production function is concave or convex. In particular, for $\xi \leq 1$ and $a+b \leq 1$, the CES function is concave, and thus forecast coherence requires*

$$\mathbb{E}_t[y_{t+1}] \leq f\left(\mathbb{E}_t[x_{1,t+1}], \mathbb{E}_t[x_{2,t+1}]\right) = \left(\frac{a}{a+b}\mathbb{E}_t[x_{1,t+1}]^{\xi} + \frac{b}{a+b}\mathbb{E}_t[x_{2,t+1}]^{\xi}\right)^{\frac{a+b}{\xi}}. \quad (1)$$

For $\xi \geq 1$ and $a+b \geq 1$, the CES function is convex, and thus coherence requires

$$\mathbb{E}_t[y_{t+1}] \geq f\left(\mathbb{E}_t[x_{1,t+1}], \mathbb{E}_t[x_{2,t+1}]\right) = \left(\frac{a}{a+b}\mathbb{E}_t[x_{1,t+1}]^{\xi} + \frac{b}{a+b}\mathbb{E}_t[x_{2,t+1}]^{\xi}\right)^{\frac{a+b}{\xi}}. \quad (2)$$

All Proofs are in the Appendix. Proposition 1 already provides a restriction on contemporaneous forecasts that a coherent forecaster must satisfy. Implementing this inequality restriction in the data requires observing $\mathbb{E}_t[y_{t+1}]$, $\mathbb{E}_t[x_{1,t+1}]$, and $\mathbb{E}_t[x_{2,t+1}]$ at the CFO level and requires knowledge of parameters a and b, where typically $a+b \leq 1$, and we can rely on the literature to obtain a range of plausible values for the elasticity of substitution, χ, for which it is often the case that $\chi \in (0.5, 1]$ (e.g., see Berndt (1976), Oberfield and Raval (2021)), so we can verify for how many CFOs the relevant inequality (1) of Proposition 1 is satisfied.

In general, the CES is a non-linear function of the inputs. Because the rules of thumb (R1)-(R5) are instead linear, and one of our objectives is to rationalize these rules of thumb, at least in a second best sense, in what follows we shall work with the limit case in which $\xi \to 0$ and the production function is thus Cobb-Douglas,

$$\lim_{\xi \to 0}\left(\frac{a}{a+b}x_1^{\xi} + \frac{b}{a+b}x_2^{\xi}\right)^{\frac{a+b}{\xi}} = x_1^a \cdot x_2^b.$$

Corollary 1 (Cobb-Douglas). *In the limit case in which $\xi \to 0$,*

$$\mathbb{E}_t \log[y_{t+1}] = a \cdot \mathbb{E}_t \log[x_{1,t+1}] + b \cdot \mathbb{E}_t \log[x_{2,t+1}].$$

Similarly,

$$\mathbb{E}_t \log\left[\frac{y_{t+1}}{y_t}\right] = a \cdot \mathbb{E}_t \log\left[\frac{x_{1,t+1}}{x_{1,t}}\right] + b \cdot \mathbb{E}_t \log\left[\frac{x_{2,t+1}}{x_{2,t}}\right].$$

The Cobb-Douglas production function is linear in logs, and as a result the coherence requirement holds with equality, both for forecasts expressed in levels and in growth rates. The Cobb-Douglas specification is also useful because it allows to construct a test statistic for coherence at the CFO level under the additional assumption that prices follow an AR(1) process, $\pi_{i,t+1} = \gamma_i \pi_{i,t} + \epsilon_{i,t+1}$, with $0 < \gamma_i < 1$ ($\gamma_i = 0$ denotes the i.i.d. case), where the error terms are i.i.d., normally distributed, and uncorrelated, namely, $\{\epsilon_{1,t}\}_{t\geq 1} \sim \mathcal{N}(0, \sigma_1^2)$, $\{\epsilon_{2,t}\}_{t\geq 1} \sim \mathcal{N}(0, \sigma_2^2)$, and $\{\epsilon_{1,t}\}_{t\geq 1} \perp \{\epsilon_{2,t}\}_{t\geq 1}$.

Proposition 2 (Test Statistics). *If $\xi \to 0$ and $\rho_{1,2} = 0$, under the null hypothesis of coherent forecasts it holds that*

$$\text{C1-stat} \equiv \frac{\frac{\mathbb{E}_t \log y_{t+1} - a\mathbb{E}_t \log x_{1,t+1}}{b} - \log \frac{b}{a+b} Z}{\gamma_2 \sigma_2} \sim \mathcal{N}(0,1) \tag{3}$$

and

$$\text{C2-stat} \equiv \frac{\text{FE}_t \log y_{t+1} - a\text{FE}_t \log x_{1,t+1}}{\sigma_2 b} \sim \mathcal{N}(0,1), \tag{4}$$

where $FE_t \log y_{t+1} = \log y_{t+1} - \mathbb{E}_t \log y_{t+1}$ and $FE_t \log x_{1,t+1} = \log x_{1,t+1} - \mathbb{E}_t \log x_{1,t+1}$.

Proposition 2 derives two test statistics at the individual CFO level. These statistics have an intuitive interpretation: under the null of coherence, the forecasts (3) and the forecast errors (4) of the output and of one input cannot be "too far" from each other. On the one hand, relative to Proposition 1, Proposition 2 does not require observing $\mathbb{E}_t[x_{2,t+1}]$ or realization $x_{2,t+1}$, so it can be implemented when $\mathbb{E}_t[x_{2,t+1}]$, $x_{2,t+1}$, or both are not observable. Furthermore, similar to Proposition 1, Proposition 2 requires knowledge of technology parameters a and b. On the other hand, Proposition 2 further requires knowledge of γ_2 or σ_2, as well as the more stringent assumptions that $\xi \to 0$ and that input prices follow the assumed processes.[16]

Comparing the C1 and C2 statistics is also instructive. On the one hand, computation of the C1-stat in (3) does not require observing realizations, so it can be potentially

[16] When some of these parameters need to be estimated, the test statistics will no longer be normally distributed. We further discuss implementation details in Subsection V.*A*.

implemented with expectations data only. On the other hand, the C1-stat requires information about the budget, Z, which is not only the cash and liquid securities from the firm's balance sheet but also the external resources that the firms in our sample can access from financial markets at short notice. While known to CFOs, this information is not easily observable by the econometrician for most firms. Second, the C1-stat does not allow to distinguish between forecast coherence and forecast accuracy.

Symmetrically, the C2-stat in (4) requires observing both forecasts and realizations; but it does not require observing the budget, Z, and it allows to distinguish between accuracy and coherence. In fact, forecast accuracy of the first input and of the output requires $\mathrm{FE}_t \log x_{1,t+1}$ and $\mathrm{FE}_t \log y_{t+1}$ to be not too far from zero in expectation (whereas forecast coherence requires the two forecast errors not to be too far apart from each other), formally $\frac{\mathrm{FE}_t \log x_{1,t+1}}{\sigma_1} \sim \mathcal{N}(0,1)$ and $\frac{\mathrm{FE}_t \log y_{t+1}}{\sigma_y} \sim \mathcal{N}(0,1)$.

Figure 4 depicts the theoretical connection between forecast accuracy and forecast coherence as implied by the C2-stat and shows that there are four conceptual areas. In the first area the forecaster is both accurate and coherent. This occurs when both $\frac{\mathrm{FE}_t \log x_{1,t+1}}{\sigma_1}$ and $\frac{\mathrm{FE}_t \log y_{t+1}}{\sigma_y}$ are close to zero and also close to each other. In the second area, either $\frac{\mathrm{FE}_t \log x_{1,t+1}}{\sigma_1}$ or $\frac{\mathrm{FE}_t \log y_{t+1}}{\sigma_y}$ or both are statistically different from zero but they are quite close to each other, so the forecaster is inaccurate but coherent. In the third area, both $\frac{\mathrm{FE}_t \log x_{1,t+1}}{\sigma_1}$ and $\frac{\mathrm{FE}_t \log y_{t+1}}{\sigma_y}$ are close to zero but they are sufficiently apart from each other, so the forecaster is accurate but incoherent. In the fourth area, either $\frac{\mathrm{FE}_t \log x_{1,t+1}}{\sigma_1}$ or $\frac{\mathrm{FE}_t \log y_{t+1}}{\sigma_y}$ or both are statistically different from zero and they are also far apart from each other, so the forecaster is both inaccurate and incoherent.

Figure 4 clarifies the theoretical connection between coherence and accuracy. Much research in psychology and elsewhere has been cast in terms of whether the accuracy or the coherence paradigm is the correct one, e.g., see Hammond (2007) and references therein. We highlight that these interpretations are incomplete or even misleading, because coherence and accuracy are two distinct but related concepts, and they are both necessary to understand individual forecasts. Furthermore, we underscore that one needs an analytical framework nesting both accuracy and coherence to distinguish between them and to understand their relationship.

A.2 Optimal Forecasts and Rules of Thumb

We now consider the forecasting problem in case the technological parameters a and b are unknown to the forecaster.

Proposition 3. *If parameters a and b are unknown, a forecaster can estimate them using a linear projection operator, with the forecasted variables in logs.*

Now, define the unconditional means $\mathbb{E}\log[y_{t+1}]=\mu_y$ and $\mathbb{E}\log[x_{i,t+1}]=\mu_i$, for $i = 1, 2$.

Corollary 2. *In a multivariate linear projection,* $\mathbb{E}_t \log[x_{1,t+1}] = \alpha + \beta_1 \cdot \mathbb{E}_t \log[y_{t+1}] + \beta_2 \cdot \mathbb{E}_t \log[x_{2,t+1}]$, *the parameters are*

$$\alpha = \mu_1 - \frac{1}{a}\mu_y + \frac{b}{a}\mu_2 = 0, \qquad \beta_1 = \frac{1}{a}, \qquad \beta_2 = -\frac{b}{a}.$$

In a multivariate linear projection with the variables in growth rates, $\mathbb{E}_t \log\left[\frac{x_{1,t+1}}{x_{1,t}}\right] = \alpha + \beta_1 \cdot \mathbb{E}_t \log\left[\frac{y_{t+1}}{y_t}\right] + \beta_2 \cdot \mathbb{E}_t \log\left[\frac{x_{2,t+1}}{x_{2,t}}\right]$, *the parameters are the same as above.*

Corollary 2 rationalizes how a rule of thumb akin to (R5) described above delivers the first-best optimal forecast. More precisely, Corollary 2 clarifies that one should implement (R5) to provide forecasts of capital growth using data on output growth and labor growth forecasts, and using parameters derived from a linear projection of the firm's input on the output and the other input. Note also that (R5), as well as (R1)-(R4), are defined in the managerial education literature as linear functions of growth rates (not in logs). Our analysis implies that such linear rules will be correct up to a first-order Taylor approximation. Importantly, Corollary 2 holds both in levels and in growth rates.

Corollary 3. *In a univariate linear projection,* $\mathbb{E}_t\log[x_{1,t+1}] = \alpha + \beta \cdot \mathbb{E}_t \log[y_{t+1}]$, *the parameters are*

$$\alpha = \mu_1 - \frac{a\sigma_1^2}{a^2\sigma_1^2 + b^2\sigma_1^2}\mu_y, \qquad \beta = \frac{a\sigma_1^2}{a^2\sigma_1^2 + b^2\sigma_1^2}.$$

In a univariate linear projection with the variables in growth rates, $\mathbb{E}_t\log\left[\frac{x_{1,t+1}}{x_{1,t}}\right] = \alpha + \beta \cdot \mathbb{E}_t \log\left[\frac{y_{t+1}}{y_t}\right]$, *the parameters are*

$$\alpha = 0, \qquad \beta = \frac{a\sigma_1^2}{a^2\sigma_1^2 + b^2\sigma_2^2}.$$

Corollary 3 shows that rule of thumb (R3), which makes use of information on the output and one input but neglects the other input, in general yields different forecasts from those of (R5), and thus (R3) in general yields incoherent forecasts. We now examine a special case in which (R3) yields coherent forecasts.

Corollary 4. *If $\rho_{1,2} = 1$ and $\sigma_1^2 = \sigma_2^2 = \sigma_{1,2} = \sigma^2$, then for a linear regression in growth rates, $\mathbb{E}_t\left[\frac{y_{t+1}}{y_t}\right] = \alpha + \beta \cdot \mathbb{E}_t\left[\frac{x_{i,t+1}}{x_{i,t}}\right] + e_{i,t+1}$, with $i = 1, 2$, we have $\alpha > 0 \iff 0 < \beta < 1 \iff \nu < 1$. The same is true for i.i.d. shocks, setting $\gamma_i = 0\ \forall i$.*

Corollary 4 shows that (R3) can be optimal under rather special circumstances, that is, when input prices are perfectly correlated and thus there is no added benefit from a multivariate rule like (R5) relative to the univariate rule (R3). Furthermore, Corollary 4 indicates that rule (R2), which amounts to setting $\mathbb{E}_t\left[\frac{x_{i,t+1}}{x_{i,t}}\right] = \mathbb{E}_t\left[\frac{y_{t+1}}{y_t}\right]$, is only optimal provided $\alpha = 0$ and $\beta = 1$, that is, under constant returns to scale $\nu = 1$, in which case (R2) is exactly equivalent to rule (R3). In the general case in which returns to scale are not constant, rule (R2) is suboptimal. More generally, whenever $\rho_{1,2} \in (-1, 1)$ and $\sigma_1^2 \neq \sigma_2^2$, rule (R3) is not going to yield an optimally coherent forecast and the forecaster would do better relying on information provided by all inputs and the output.

Finally, rule (R1) amounts to extrapolating past information of the input being forecasted, and disregarding information about the output and the other input. Because (R1) treats the forecast of each item in isolation, we interpret (R1) as an example of narrow bracketing. In general, (R1) amounts to setting the forecast of $x_{i,t+1}$ equal to the average of k past growth rates, $\log F_{i,t}^{R1} = \frac{1}{k}\sum_{j=1}^{k} \log \frac{x_{i,t+1-j}}{x_{i,t-j}}$. Welch (2017) advocates $k = 2$. We establish:

Corollary 5 (Losses Under Narrow Bracketing Forecasts). *Under (R1) and $k \to +\infty$, $\mathbb{E}_t\left[L_{t+1}^{R1}\right] = \mathbb{E}_t\left[L_{t+1}^{o}\right] + \left[(1-\gamma_i)\,\pi_{i,t}\right]^2 > \mathbb{E}_t\left[L_{t+1}^{o}\right]$ for $\gamma_i < 1$, where $\mathbb{E}_t\left[L_{t+1}^{R1}\right]$ and $\mathbb{E}_t\left[L_{t+1}^{o}\right]$ denote the expected losses under (R1) and the optimal forecast, respectively.*

Corollary 5 implies that using (R1) is optimal if and only if one uses a lot of data, $k \to +\infty$, and the correct autoregressive parameter is $\gamma_i = 1$, that is, if the input price follows a random walk, otherwise it is strictly inferior.

Our results so far establish that under optimal forecasts we can think of individuals as broad bracketers, in the sense that they compute forecasts taking into account the structure of the firm's problem and all available data on inputs, $x_{i,t}$, and output, y_t. Under narrow bracketing, individuals ignore the structure of the firm's problem and when forecasting growth of item x_i they examine data about x_i in isolation and ignore data on items $x_{\neg i}$ and y.

In reality, narrow bracketing could be a second-best optimal response to imperfect information. Furthermore, individuals may be producing forecasts between these two extremes of broad bracketing and narrow bracketing. They may be better informed about the price of input 1 and have difficulty accessing information about the price of input 2. For example, a CFO may be an expert on investment in physical assets, but not so on labor costs; another one may be an expert in innovative technology and R&D expenses, but not so in traditional technologies; and so on.

Following Lian (2021), in the next subsection we capture these possibilities by introducing noisy signals, and we recast the decision problem under narrow bracketing as multiple selves playing an incomplete information, common interest game.[17] With two inputs, capital and labor, the CFO "capital self" makes forecasts of capital expenditures growth by observing imprecise signals of output and labor growth. Conversely, the CFO "labor self" makes forecasts of labor expenditures growth by observing imprecise signals of output and capital growth. In the equilibrium of the game, since each self does not perfectly know other selves' signals (states of mind), each self's forecast is made with imperfect knowledge of other selves' forecasts. In this sense, narrow thinking introduces intra-personal frictions in coordinating multiple forecasts.

[17] In the multiple selves literature, multiple selves have conflicting interests (e.g., Marschak and Radner (1972), Piccione and Rubinstein (1997), and Benabou and Tirole (2002)); whereas in our paper and in Lian (2021) they have common interests. Despite common interests, since different selves do not share their information, they have difficulty in coordinating their decisions in response to shocks to the fundamentals.

B. A Model of Narrow Bracketing in Corporate Forecasts

We consider a forecaster self that makes a forecast for input 1, $F \log x_1$, to minimize a quadratic loss function,[18]

$$\min_{F \log x_1} \mathbb{E} \left(\log x_1 - F \log x_1 \right)^2$$

where for simplicity we drop the time subscript, t, because the problem is stationary, and as usual the firm's production technology is $y = x_1^a x_2^b$ and its budget constraint is $p_1 x_1 + p_2 x_2 = Z$. We assume that the forecaster observes two noisy signals, $\eta_y = \log y + \epsilon_y$ and $\eta_2 = \log x_2 + \epsilon_2$, where $\epsilon_y \sim \mathcal{N}\left(\mu_y, s_y^2\right)$ and $\epsilon_2 \sim \mathcal{N}\left(\mu_2, s_2^2\right)$.

Proposition 4. *The optimal forecast of input x_1 given signals η_y and η_2 is*

$$\mathbb{E}\left[\log x_1 | \eta_y, \eta_2\right] = \mu_1 + \beta_y \left(\eta_y - \mu_y\right) + \beta_2 \left(\eta_2 - \mu_2\right),$$

where

$$\beta_y = \frac{a\sigma_1^2}{a^2\sigma_1^2 + b^2\sigma_2^2 + s_y^2 - \frac{b^2\sigma_2^4}{\sigma_2^2 + s_2^2}}, \quad \beta_2 = \frac{ab\sigma_1^2\sigma_2^2}{b^2\sigma_2^4 - \left(\sigma_2^2 + s_2^2\right)\left(a^2\sigma_1^2 + b^2\sigma_2^2 + s_y^2\right)}.$$

Proposition 4 shows that, upon observing signals η_y and η_2, the optimal forecast of input x_1 is given by a linear projection of (the deviation of the signals from the prior means of) output y and input x_2. In such a linear projection, the constant term is the prior mean of x_1 and the slope coefficients are functions of the fundamental uncertainty and of the precision of the signals. Proposition 4 rationalizes rule of thumb (R5) as the optimal coherent forecast also in a second-best sense. Here, Proposition 4 clarifies that in a second-best world the accuracy of this linear projection will depend on the precision of the signals. Note that in our model the forecaster makes forecasts based on different, non-nested information, that is, in the sense of Blackwell, neither input i's signal is more informative than input $\neg i$'s signal nor input $\neg i$'s signal is more informative than input i's signal (see also Lian (2021)). Next, we examine a number of special cases.

Corollary 6 (Narrow Bracketing). *When $s_y^2, s_2^2 \to +\infty$, the optimal forecast is*

[18] This mirrors our empirical setting in which CFOs forecast capital (input 1) given information on the output and the other input (labor). The formulation is wlog as it can be cast in terms of generic inputs i and $\neg i$.

$\mathbb{E}\left[\log x_1 | \eta_y, \eta_2\right] = \mu_1$.

When both signals are infinitely noisy, the optimal forecast of input x_1 ignores the signals and instead extrapolates the prior mean μ_1 into the future. Corollary 6 rationalizes rule of thumb (R1) as an optimal forecast when the CFO observes infinitely noisy signals about the output and the other input.

Corollary 7 (Univariate Projections). *When* $s_2^2 \to +\infty$ *and* $0 < s_y^2 < +\infty$, *the optimal forecast is* $\mathbb{E}\left[\log x_1 | \eta_y, \eta_2\right] = \mu_1 + \beta_y (\eta_y - \mu_y)$, *where* $\beta_y = \frac{a\sigma_1^2}{a^2\sigma_1^2 + b^2\sigma_2^2 + s_y^2}$.

When only the signal of the other input is infinitely noisy and the signal of the output is noisy but informative, the optimal forecast of input x_1 is a univariate linear projection of (the deviation from the prior means of) the output. In such a linear projection, the constant term is still the prior mean of x_1 and the slope coefficient is a function of the fundamental uncertainty and of the precision of the signal. This corollary rationalizes rule of thumb (R3) as an optimal forecast when the CFO observes infinitely noisy signals about the other input. Note that in general β_y, $\beta_2 \neq 1$, implying that rule of thumb (R2) is in general not an optimal forecast, even in a second-best world.

Finally, note that rule of thumb (R4) can be thought of as a version of (R3) in which the linear projection is estimated for a subsample of firms in the same industry as the firm under consideration. On the one hand, using a smaller sample might hurt the performance of the linear projection, in which case (R3) may be superior to (R4). On the other hand, firms in an industry might differ from firms in other industries, for example because parameters a_j or b_j are specific to industry j, in which case (R4) may be superior to (R3). We will evaluate these possibilities empirically in the next section. For completeness, we provide the following corollary, stating that when both signals are infinitely precise, we fall back to the case of Corollary 2.

Corollary 8 (Precise Signals). *When* $s_y^2, s_2^2 \to 0$, *the optimal forecast is* $\mathbb{E}\left[\log x_1 | \eta_y, \eta_2\right] = \frac{1}{a}(\eta_y - b\eta_2)$.

To sum up, our theory yields a normative benchmark for an ex ante optimal coherent forecast, and a number of restrictions and tests to evaluate incoherence ex post. Our results imply that ex ante forecasts that differ from the normative benchmark are incoherent. The main empirical implication of our model is thus that expected corporate profits, $\mathbb{E}[\Psi] = \mathbb{E}[p_y y - p_1 x_1 - p_2 x_2]$, should decrease with the extent of incoherence. Furthermore, our positive framework formalizes a mechanism through which incoherence may arise as a result of narrow thinking, namely, intra-personal frictions in coordinating multiple forecasts. Our results deliver a pecking order of the rules of thumb proposed in the managerial education literature, and deliver a key additional empirical implication that the channel through which incoherence arises is by the use of certain rules of thumb, most notably, (R1) and (R2).[19] In the next section, we evaluate empirically these predictions.

V Empirical Analysis

In this section, we present our empirical analysis and our findings. In Subsection *A.*, we implement the tests of coherence derived above. In Subsection *B.*, we introduce a continuous measure of incoherence, we establish which rules of thumb are reflected in the CFOs forecasts, and how the use of such rules of thumb correlates with incoherence. In Subsection *C.*, we examine how incoherence and the rules of thumb correlate with corporate performance. In Subsection *D.*, we examine how the use of rules thumb correlate with corporate investment and debt policy. In Subsection *E.* we investigate how corporate behavior changes around the years in which CFOs take office.

A. Test Implementation and Results

We begin by implementing the inequality restriction developed in Proposition 1 under a very general CES specification. We view this inequality as imposing on the data as little restriction as possible to allow for an empirical investigation. We compute a and b using the universe of industries from the Bureau of Economic Analysis, and find that $a + b \leq 1$. Furthermore, the elasticity of substitution between capital and labor in the US economy, denoted with χ, is typically documented in the literature to be between

[19] Specifically, Proposition 3 says that the optimal ex ante coherent forecast is a specific version of (R5), because it implies the use of an optimal mix of inputs. Corollary 6 implies that (R1) is the most extreme—among those considered—deviation from (R5) when all signals are infinitely noisy. Corollary 7 implies that (R2) uses information about the output, but in a way that is not optimal. Corollary 7 further implies that (R3) and (R4) use optimally the information about the output but ignore the other input. As a result, (R5) should be the optimal coherent forecast rule; (R1) and (R2) should be the worst; (R3) and (R4) should be better than either (R1) or (R2), but not necessarily approximating (R5).

0.5 and 1 (e.g., see Berndt (1976), Oberfield and Raval (2021)), where $\chi = 1$ defines the Cobb-Douglas production function. Therefore, the CES function is weakly concave, and the relevant inequality is thus inequality (1) of Proposition 1. We account for heterogeneity by allowing a and b to vary by industry, and by presenting our results for three different values of the elasticity of substitution between capital and labor, $\chi = 0.5, 0.7, 0.9$. We implement our inequality restriction both in levels and in growth rates.[20]

Table 2 shows our results. Panel A shows that most CFOs' forecasts violate the inequality restriction of Proposition 1. In levels, almost all CFOs report joint forecasts of capital, labor, and output that jointly violate the inequality. However, as discussed just above, results in levels should be seen with caution, as they refer to a much smaller sample given the limitations of Compustat data on labor expenditures. In growth rates, about 73% of CFOs reports forecasts that violate the inequality. These results appear to be quite stable across different values of the elasticity of substitution between capital and labor. If anything, moving toward $\chi = 1$ and thus toward a Cobb-Douglas specification appears to give a slightly better shot at CFOs to report coherent forecasts. Panel B reports summary statistics of the difference between the left-hand side and the right-hand side of the inequality of Proposition 1.

The results in Table 2 show that the patterns of Figure 2 reflect a lot of incoherence. In particular, CFOs forecast a growth of output that is larger than the output growth implied by feeding into a CES production function the CFOs' forecasts of capital and labor input growth, thereby violating the inequality of Proposition 1. Panel B shows that the extent of these violations varies widely across CFOs. One possibility is that this variation is due to CFOs facing different conditions and amounts of uncertainty, which cannot be directly assessed based on the point forecasts in our data. To account for uncertainty, one thus needs to put more structure on the problem.

In particular, by assuming a Cobb-Douglas production function and by positing a distribution for the input price processes, we have developed two such tests in Proposition

[20] Note that (1) we observe CFO forecasts of growth rates, not of levels, and (2) while we observe the CFO forecast of the growth rate of labor expenditures, $\mathbb{E}_t \frac{[x_{2,t+1}]}{[x_{2,t}]}$, for a large sample, we observe few realizations in Compustat of $x_{2,t+1}$. Therefore, when we implement the inequality restriction in levels and we need to compute $\mathbb{E}_t [x_{2,t+1}] = x_{2,t} \cdot \mathbb{E}_t \frac{[x_{2,t+1}]}{[x_{2,t}]}$, we end with much fewer observations in levels than in growth rates.

2, the C1-stat in terms of forecasts and the C2-stat in terms of forecast errors. In our data, we can implement the C2-stat. We implement our C2-stat in (4) using the Duke data. To do so, we need to make a few remarks. First, the quantities $\mathrm{FE}_t \log x_{1,t+1}$ and $\mathrm{FE}_t \log y_{t+1}$ are directly observable in the data if and only if the forecasts are already elicited in logs; then, using the logs of realizations $x_{1,t+1}$ and y_{t+1} one can directly compute the forecast errors $\mathrm{FE}_t \log x_{1,t+1}$ and $\mathrm{FE}_t \log y_{t+1}$. However, if, as it is the case in the Duke Survey the forecasts are not elicited in logs and we only observe $\mathbb{E}_t x_{1,t+1}$ and $\mathbb{E}_t y_{t+1}$, one can use the transformation, written for a generic variable, x,

$$\mathbb{E}_t \log x_{t+1} = \log \mathbb{E}_t x_{t+1} - \frac{1}{2} V_t \log x_{t+1}, \tag{5}$$

where $V_t \log x_{t+1}$ is the conditional variance of $\log x$, and thus $V_t \log x_{i,t+1} = \sigma_i^2 = (1-\gamma_i^2)\, V \log x_{i,t+1}$ for $i = 1, 2$; $V_t \log y_{t+1} = a^2\sigma_1^2 + b^2\sigma_2^2$; and γ_i is the coefficient of an AR(1) regression of $\log x_{i,t}$ for $i = 1, 2$.

Therefore, for each CFO we observe four items, two forecasts and two realizations, and we estimate three parameters, a, b, and σ_2, from aggregate industry data from the Bureau of Economic Analysis and Bureau of Labor Statistics. As a result, our C2-stat is distributed according to a Student t distribution with $N - K = 4 - 3 = 1$ degree of freedom. Table 3 reports our results. Panel A of Table 3 shows that for 55.7% of CFOs in our sample we reject the null hypothesis of coherence at the 95% confidence level.[21] This striking result corroborates the view that our previous evidence in Figures 2 and 3 and Table 2 is inconsistent with coherence and indicates that a majority of CFOs in our sample provide incoherent forecasts of their firm output and capital input.

By contrast, Panel A of Table 3 further shows that CFOs are fairly accurate with respect to their output forecasts. In fact, we reject the null of accuracy for output forecasts at the 95% confidence level only for 27.2% of CFOs. CFOs are substantially less accurate with respect to their capital expenditures forecasts, as we reject the null of accuracy in capital expenditures forecasts for 47.9% of CFOs in our sample. These results are

[21] At the 99% confidence level, we reject the null of coherence for 7.7% of CFOs. The difference between the rejection regions at 95% and 99% confidence level reflects the distribution of the Student t with one degree of freedom.

consistent with the observations in Graham (2022) that top executives care the most about their output forecasts. When considering output and capital input forecasts together, we reject the null of accuracy for 57.0% of CFOs in our sample.

Panel B of Table 3 assesses coherence and accuracy together. It shows that 31.1% of CFOs in our sample are both coherent and accurate; 13.2% are coherent but inaccurate; 12.0% are accurate but incoherent; and the remaining 43.7% are both incoherent and inaccurate (all at the 95% confidence level).[22] The summary statistics of the cross-sectional distribution of our calculated C2-stat and of the forecast errors for output and capital input are shown in Panel C.

One concern with these results is the extent to which the computed C2-stat, as well as the accuracy statistics, are sensitive to the uncertainty coming from estimating the parameters a, b, and σ_2.[23] To address this concern, we perform a non-parametric bootstrap procedure, as follows. For each CFO, we generate 1,000 bootstrap repetitions of the C2-stat.[24] Using these 1,000 replications, we compute the fraction of cases out of 1,000 for which we reject the null of coherence at the 95% and 99% confidence levels. Hence, for each CFO and confidence level, the computed statistic is a number between 0 and 1, where 0 means that the null of coherence was never rejected across all 1,000 repetitions and 1 means that the null of coherence was rejected for all 1,000 repetitions.

In Figure 5, we plot the value of this statistic (on the vertical axis) against its empirical cumulative distribution function across CFOs (on the horizontal axis). The top plot refers to the calculation of the statistic at the 95% confidence level, while the bottom

[22]The figures at the 99% confidence level are 89.4%, 2.9%, 3.4%, and 4.3%, respectively.

[23]To be precise, the C2-stat in (4) depends on a, b, and σ_2 *directly*, where σ_2 is the square root of the conditional variance of the log of x_2 at $t+1$ given the information set at t. Furthermore, the C2-stat depends *indirectly* on σ_1 (defined analogously to σ_2) through the forecast error of the log of x_1, which depends on (5). Finally, the C2-stat further depends indirectly on σ_y through the forecast error of the log of y, which in turn depends on a, b, σ_1^2, and σ_2^2.

[24]For each of the 15 BEA industries and pair of consecutive years between 1987 and 2018, we resample observations with replacement 1,000 times (aka bootstrap replications) and, at each replication, we obtain an estimate of σ_1 based on the RSS of the regression of total capital compensation on its lag and obtain an estimate of σ_2 based on the residual sum of squares (RSS) of the regression of total labor compensation on its lag, using cluster bootstrap with 6 clusters corresponding to the following year windows: 1988-1992, 1993-1997, 1998-2002, 2003-2007, 2008-2012, and 2013-2018. We additionally generate bootstrap estimates of a and b as the capital and labor shares of total factor compensation. Endowed with bootstrap estimates of a, b, σ_1^2, and σ_2^2, we derive corresponding estimates for σ_y^2, for the forecast errors of the log of output (y) and of the log of capital input (x_1), and for the C2-stat.

plot refers to its calculation at the 99% confidence level. The graph on the top shows that for about 40% of CFOs the null of coherence is rejected in all bootstrap repetitions; for about 15% of CFOs the null of coherence is never rejected; and for the remaining 45% of CFOs the fraction of rejections across bootstrap repetitions is strictly between 0 and 1. Consistent with the results in Table 3, the proportion of CFOs for whom the null of coherence is rejected more than half of the times is approximately 55%.[25] We conclude that our results in Table 3 are robust to estimation uncertainty of the parameters.

B. Rules of Thumb Indicators and Measure of Incoherence

We begin by characterizing in our data the rules of thumb used by CFOs to produce corporate forecasts. We focus our exposition on forecasts about sales revenues (i.e., output) and capital expenditures because they have both a clear mapping with theory and we observe their realizations in Compustat, which allows computing forecast errors. In the Online Appendix we report data on other items.

We consider the five rules of thumb discussed in Section IV. Whenever the rules of thumb allow for multiple ways to be implemented, to tie our hands we follow the implementation of Welch (2017). Therefore, Rule (R1) uses the average growth of the past two years as the forecast. Rule (R2) amounts to using the same growth rate for sales revenues and capital expenditures; equivalently, rule (R2) amounts to set the joint forecasts of sales and capital expenditures growth using a univariate regression model, Sales Growth $= \alpha + \beta \cdot$ Capital Expenditures Growth $+ \varepsilon$, where $\alpha = 0$ and $\beta = 1$. Rule (R3) amounts to actually estimating the regression above in the population of Compustat firms.

Table 4 presents results of cross sectional regressions using 2000-2019 data of sales growth on the contemporaneous growth of selected balance sheet items. Column 2 of Table 4 shows that for the regression above, $\widehat{\alpha}$ =0.106 and $\widehat{\beta}$ =0.077, precisely estimated.

[25]The graph on the bottom of Figure 5 shows that for slightly less than 10% of CFOs the null of coherence is rejected in all bootstrap repetitions; for about 60% of CFOs the null of coherence is never rejected across all bootstrap repetitions; and for the remaining 30% of CFOs the fraction of rejections across bootstrap repetitions is strictly between 0 and 1. The proportion of CFOs for whom the null of coherence is rejected more than half of the times is slightly over 10%, that is, slightly higher but in the same ballpark as the proportion of rejections calculated in Table 3.

Rule (R4) amounts to estimating the same regression above by industry. We do so at the 1-digit SIC code level to make sure that each industry has enough observations to deliver meaningful estimates.[26]

Finally, Rule (R5) recognizes that the most sophisticated approach would use coefficients from a multivariate regression model, but does not fully specify which variables to include, possibly because no benchmark theory is available in the literature. Our model of Section IV indicates that the full multivariate model delivering the optimal rational coherent forecast should be as follows:

$$\text{Sales Growth} = \alpha + \beta \cdot \text{Capital Expenditures Growth} + \theta \cdot \text{Labor Costs Growth} + \varepsilon.$$

However, as mentioned above, Compustat provides only scant information about labor expenditures such as wages. Therefore, we replace Labor Costs Growth with Earnings Growth, because Earnings figures do contain information about labor costs, albeit indirectly. Column 7 of Table 4 shows that in this case $\widehat{\alpha}$ =0.106, $\widehat{\beta}$ =0.074, and $\widehat{\theta}$ =0.030, precisely estimated. We take these estimates as representing both Rule (R5) and as characterizing the rational coherent benchmark. Accordingly, we compute our ex ante measure of CFO incoherence as the orthogonal distance from the actual joint forecast of sales and capital expenditures and the optimal forecast computed under our implementation of Rule (R5).

Furthermore, for each CFO we establish to which rule of thumb, (R1)-(R5), the actual joint forecast of sales and capital expenditures is closest to, thereby establishing five CFO types. This approach enables us to assess the relative coherence of the four rules of thumb (R1)-(R4) relative to the rational coherent one (R5). We do so in two steps. First, for each CFO we establish the five orthogonal distances between the actual forecast and the ones implied by each of the rules of thumb (R1)-(R5). Second, for each CFO we compute the minimum distance among those five distances, and we assign a CFO type, $\tau = 1, .., 5$, based on which of the five rules of thumb is the closest to the actual forecast.

[26] SIC 1-digit codes roughly correspond to the following sectors: Agriculture, forestry, and fishing; Mining; Construction; Manufacturing; Transportation, communications, and public utilities; Wholesale trade; Retail trade; Finance, insurance, real estate; Services; and Public administration. SIC codes also allow a close mapping to the analogous classification of the Bureau of Economic Analysis.

Table 5 shows that of the 396 CFOs for which we observe their identity, the plurality, about 40%, use a rule of thumb that is closest to (R2), and 27% use exactly (R2). This is perhaps not surprising, because (R2) is a simple rule to implement, as it just requires to assign the same forecast to the two items under consideration. About 15% of CFOs use a rule of thumb that is closest to the rational coherent one, (R5); and 7.6% of CFOs use a rule of thumb that is closest to the narrow-bracketing one, (R1). Finally, about 11% use (R3) and 27% use (R4). These results underscore the large heterogeneity in the forecasting rules of thumb used by the CFOs, reflecting the fact that providing coherent forecasts is a challenging task and the managerial education literature has not achieved a consensus in recommending either rule of thumb.

Next, we compute our ex ante continuous measure of incoherence as the orthogonal distance between the actual forecast and (R5). We validate it by showing that it predicts our C2-stat, which contains information about realizations and forecast errors. Specifically, we estimate a univariate regression of the absolute value of our C2-stat on our ex ante measure of incoherence, and find

$$|\widehat{\text{C2-stat}}| = \underset{(0.022)}{0.229} + \underset{(0.197)}{0.629} \cdot \text{Incoherence}.$$

where standard errors are reported in parentheses under the point estimates. Our ex ante measure of incoherence strongly predicts the absolute value of the C2-stat.

Our model predicts a pecking order of rules of thumb, according to which (R5) is the first best optimal one (see Corollary 2), the narrow-bracketing rule (R1) should be the farthest from the optimal one (see Corollary 6, showing that the narrow-bracketing rule of thumb is second best optimal when the forecaster observes infinitely noisy signals about output and the other input), and the univariate rule of thumb (R3) should be somewhere in the middle (see Corollary 7, showing that the univariate rule of thumb is second best optimal when the forecaster observes an infinitely noisy signal about either the output or the other input, but an informative signal about the other input, or the output).

We evaluate these predictions by regressing our measure of incoherence on the dummy variables for the CFO type. Columns 1 through 4 of Table 6 present estimates from univariate regressions, and column 5 of Table 6 presents the full specification where (R5) is used as the reference group. Our estimates in column 5 show that, consistent with our theoretical model, the narrow-bracketing rule of thumb (R1) is the farthest away from (R5), followed by (R2). Both (R1) and (R2) deliver significantly different forecasts from (R5) and are the most distant from the optimal forecast (R5), implying the highest incoherence. By contrast, (R3) and (R4) deliver forecasts that are on average statistically indistinguishable from the rational coherent one.

Next, we explore how our measure of incoherence varies with CFOs personal characteristics such as age, gender, and whether the CFO has an MBA. We also report the Optimism and Miscalibration measures of Ben-David et al. (2013) to examine how incoherence is related to those. Panel A of Table 7 reports descriptive statistics. It shows that 45% of the CFOs in our sample has an MBA, 46% are between 41 and 50 years old, 40% are between 51 and 60 years old, and 9% are females. These figures are in line with those reported in prior work (e.g., Ben-David et al. (2013)).

Panel B of Table 7 reports our regression results. It shows that incoherence is lowest at the intermediate age range, 41-50, suggesting that experience may help CFOs form more coherent forecasts, but also that coherence declines at older ages. Incoherence is unrelated to optimism or miscalibration, consistent with the idea from psychology that incoherence and overconfidence are different traits. Perhaps the most interesting finding in Panel B is that having an MBA does not correlate with incoherence, likely reflecting the twin facts that some rules of thumb are quite simple and CFOs may come up with them on their own, and—most important—that there is no consensus in MBA textbooks on which of the different possible rules of thumb should be used. In fact, as we have just seen in Table 6, the rules of thumb perform very differently in terms of forecast coherence. We now turn to examine the correlation between incoherence, corporate performance, and corporate policies.

C. Incoherence, Rules of Thumb, and Firm Performance

We now examine the main prediction of our theoretical model that corporate performance should decrease with managerial incoherence, because incoherence implies the use of a suboptimal mix of inputs. We estimate the regression model,

$$\text{ROA}_{i,j,t} = \alpha + \lambda_j + \delta_t + \beta \cdot \text{Incoherence}_{i,j,t} + \theta \cdot X_{i,j,t} + \varepsilon_{i,j,t},$$

where i indexes the CFO-firm pair, j indexes the industry, t indexes time, the dependent variable $\text{ROA}_{i,j,t}$ is the percent return on the firm's assets, λ_j are industry fixed effects, δ_t are survey fixed effects, and $X_{i,j}$ includes firm-level controls—firm size, market-to-book, and dividends—and CFO-level variables such as miscalibration and optimism, measured at both short- and long-term horizons. Based on our model, we hypothesize $\beta < 0$. We also assume that the technological relationship is stable over time and not subject to aggregate shocks. Therefore, for this part of our empirical analysis we limit ourselves to the 2001-2007 period, to abstract from the impact of the financial crisis, which is arguably an aggregate shock and it has been documented to have an impact on managerial beliefs (e.g., Boutros et al. (2020)). Furthermore, the 2001-2007 period was the peak of the 'great moderation', a time when aggregate volatility was not a concern. We compute bootstrap standard errors following Cameron et al. (2008) and we cluster them at the firm level. Given the above and the fact that we have no source of exogenous variation in incoherence, the empirical results should simply be interpreted as correlations.

Table 8 reports our results. Column 1 indicates that a one standard deviation increase in incoherence (0.079 from Table 6) is associated with a 3-percent lower performance, as measured by ROA. The correlation is significant at the 5% level. Columns 2 and 3 show that the results are robust and quantitatively stable when we condition on measures of managerial miscalibration and optimism. Column 4 shows that the results are also stable when we include industry and survey fixed effects. Columns 5-8 report the same specifications when we add firm-level regressors. There is some attrition so sample size shrinks reflecting the availability of regressors, but the main result remains statistically significant and quite stable.

Next, we investigate the extent to which the previous results reflect the use of different rules of thumb. We estimate a specification similar to the one above, but instead of incoherence we include dummies for CFO types, corresponding to the use of rules of thumb (R1)-(R4), so our results will be interpreted relative to the corporate performance of firms whose CFOs use rule of thumb (R5). Our model predicts that performance should be lowest for CFOs using the narrow-bracketing rule of thumb (R1). Table 9 reports our results. Consistent with our model, Table 9 shows that in all specifications (R1) is associated with the lowest corporate performance, with an estimated coefficient that implies a 5%-to-6% lower ROA for firms whose CFO uses (R1) relative to firms whose CFO uses (R5). These are very large differences in economic terms. With respect to the other rules, Table 9 shows both (R2) and (R3) are associated with a 2%-3% lower ROA relative to (R5), whereas performance of firms whose CFOs use (R4) is indistinguishable from the performance of those firms whose CFOs use (R5). We conclude that, consistent with our model, corporate performance correlates negatively with incoherence and is lowest for firms whose CFOs use the narrow-bracketing rule of thumb.

D. Incoherence, Rules of Thumb, and Corporate Policies

We examine now the channels underlying the observed negative correlation between incoherence and performance, and between the narrow-bracketing rule of thumb and performance. According to our theoretical model, incoherence reflects the use of a suboptimal mix of inputs, which leads to lower earnings than it would otherwise be possible given the firm's technology and budget constraint. Given our results so far in Tables 2 and 3 and the patterns reported in Figure 2 Panel A and Panel B and Figure 3, we conjecture that one way through which the suboptimal mix of inputs may come up is by having a lower level of investment spending relative to the one needed to achieve the hoped-for growth in output and sales revenues. We investigate this conjecture by estimating the following regression model:

$$\mathrm{Y}_{i,j,t} = \alpha + \lambda_j + \delta_t + \beta \cdot \text{Rules of Thumb}_{i,j,t} + \theta \cdot X_{i,j,t} + \varepsilon_{i,j,t},$$

where Rules of $\text{Thumb}_{i,j,t}$ is a vector of binary indicators for the four rules (R1)-(R4), and the dependent variable $\mathrm{Y}_{i,j,t}$ in columns (1)-(3) of Table 10 is the ratio of capital expenditures divided by assets, and then in columns (4)-(6) of Table 10 the ratio of corporate long-term book debt divided by assets. Table 10 reports the results. Columns 1 and 2 of Table 10 show that both (R1) and (R2) are associated with 1.3%-1.6% lower levels of capital expenditures relative to (R5); the difference is larger in economic terms for (R1), whereas for (R2) it is also statistically significant. Columns 4 and 6 of Table 10 show also that (R1) and (R2) are associated with 4% and 9% higher leverage relative to (R5), and for (R2) the difference is statistically significant. Interestingly, our results are robust to conditioning for miscalibration and optimism and, consistent with Ben-David et al. (2013), we find that miscalibration and optimism are correlated with higher investment spending, underscoring the idea from psychology that incoherence and miscalibration are different phenomena. These results show that the most incoherent rules of thumb—the narrow-bracketing rule (R1) and rule (R2)—are associated with lower investment and higher leverage and suggest that, in line with our theory, managerial incoherence comes with suboptimal investment and financing policies.

E. Change in Performance and Policies when CFOs Take Office

Given the negative correlation between incoherence and corporate performance and given the correlations between the narrow-bracketing rules of thumb and corporate investment and financing policies, in this section we search for hints about the direction of causality. On the one hand, high incoherence might lead to lower investment and lower performance. Alternatively, lower investment levels might induce CFOs to be incoherent and forecast too high a growth in sales revenue. Related to the above, incoherent CFOs might be selected, or might self select to work in firms with low investment spending and poor performance.

To shed some light on the direction of causality, we exploit within-firm variation across time. In particular, we examine how corporate performance, investment, and leverage evolve in the years surrounding a CFO's hiring. We extract the dates when CFOs join firms from Execucomp and Boardex data, and we supplement this information by hand-collecting data from 10-K filings. A CFO is considered to take office in a firm when he or she first signs the financial reports. In addition, we match corporate performance, investment, leverage, and characteristics from Compustat for the year of taking office. The dependent variables in our regressions are the difference between the average ROA, corporate investment, and leverage in the two years following the CFO taking office and the average ROA, corporate investment, and leverage in the two years prior to the event.

Table 11 presents our results. Column 1 shows that corporate performance declines following the appointment of an incoherent CFO, and in particular, following the appointment of a CFO who uses a narrow-bracketing rule of thumb. The use of the narrow-bracketing rule of thumb is associated with a 2.2% lower investment intensity in the two years after that CFO takes office relative to the average investment intensity of 4.5 percentage points. On the other hand, we find no change in corporate leverage around the years an incoherent CFO takes office.

Although we cannot rule out reverse causality, our findings are consistent with CFO incoherence and the use of a narrow-bracketing rule of thumb leading to a decrease in corporate investment and a decrease in corporate performance.

VI Conclusion

We develop a theory of forecast coherence in a firm production setting, which yields a statistical test whereby under the null hypothesis of coherence the forecast errors of output and inputs are not "too far" from one another. Using the Duke Survey of top executives of large- and mid-size US corporations, we document that for 55% of CFOs in our sample we reject the null hypothesis of coherence.

Our baseline model provides a normative benchmark of an ex ante coherent forecast that is first-best optimal. In a positive version of our model in which agents observe noisy signals about output and inputs, we find that some of the rules of thumb suggested by the managerial education literature may emerge as second-best optimal forecasts. In particular, the narrow-bracketing rule of thumb–making forecasts about one input exclusively extrapolating past growth rates of that input–is the second-best optimal forecast if the agent observes infinitely noisy signals about output and the other input.

Consistent with our model we find that (1) the narrow-bracketing rule of thumb is the most distant from the ex ante benchmark of coherent forecast, and (2) corporate performance correlates negatively with managerial incoherence and is lowest for firms whose CFOs provide narrow-bracketing forecasts. We also provide evidence that the use of rules of thumb correlates negatively with corporate investment spending and positively with corporate leverage.

Much research in psychology and elsewhere has been cast in terms of whether the use of heuristics or rules of thumb to help forecasting is always necessarily good or bad, e.g., see Hammond (2007). We highlight that these interpretations are incomplete or even misleading, because heuristics per se can help individuals in challenging forecasting tasks, but not all heuristics are necessarily equally good or equivalent to one another. We highlight that one needs both theory and data to evaluate different heuristics with respect to their proposed goals.

Our results show that some rules of thumb do help; but at the same time, not all rules of thumb are equally good, and other rules of thumb are actually quite bad. We provide conditions under which some rules of thumb approximate the optimal forecast, and conditions under which they do not. Our results indicate that some rules of thumb lead to narrow-bracketing forecasting behavior and thus to severe incoherence, which correlates with low performance, suggesting that these rules of thumb should not belong in the toolkit to be provided to future corporate executives. Therefore, our results should inform the managerial education literature, suggesting a pecking order of the rules of thumb to be taught going forward.

Under the assumptions we have maintained in our paper, coherence can be achieved ex ante, unlike accuracy, and some rules of thumb can help achieve coherence. One such assumption is that the firm's production technology is stable over time. This is likely appropriate in our sample period, but of course it might not hold in general, and it will not hold when disruptive technological innovation occurs. At the same time, disruptive technological innovation will also threaten accuracy. In future work, it will be important to relax this assumption of a stable technology, and to establish more generally the promise of coherence as an overarching principle to help make better forecasts.

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Figure 1: **Survey Questions of Firm Forecasts**

4. Relative to the previous 12 months, what will be your company's PERCENTAGE CHANGE during the next 12 months? (e.g., +3%, -2%, etc.) [Leave blank if not applicable]	
% Prices of your products	% Technology spending
% Overtime	% Earnings
% Advertising/Marketing spending	% Revenues
% Number of employees	% Inventory
% Productivity (output per hour worked)	% M&A activity
% Wages/Salaries	% Capital spending
% Health care costs	% Dividends

Figure 2: **Contemporaneous Forecasts of Output and Capital**

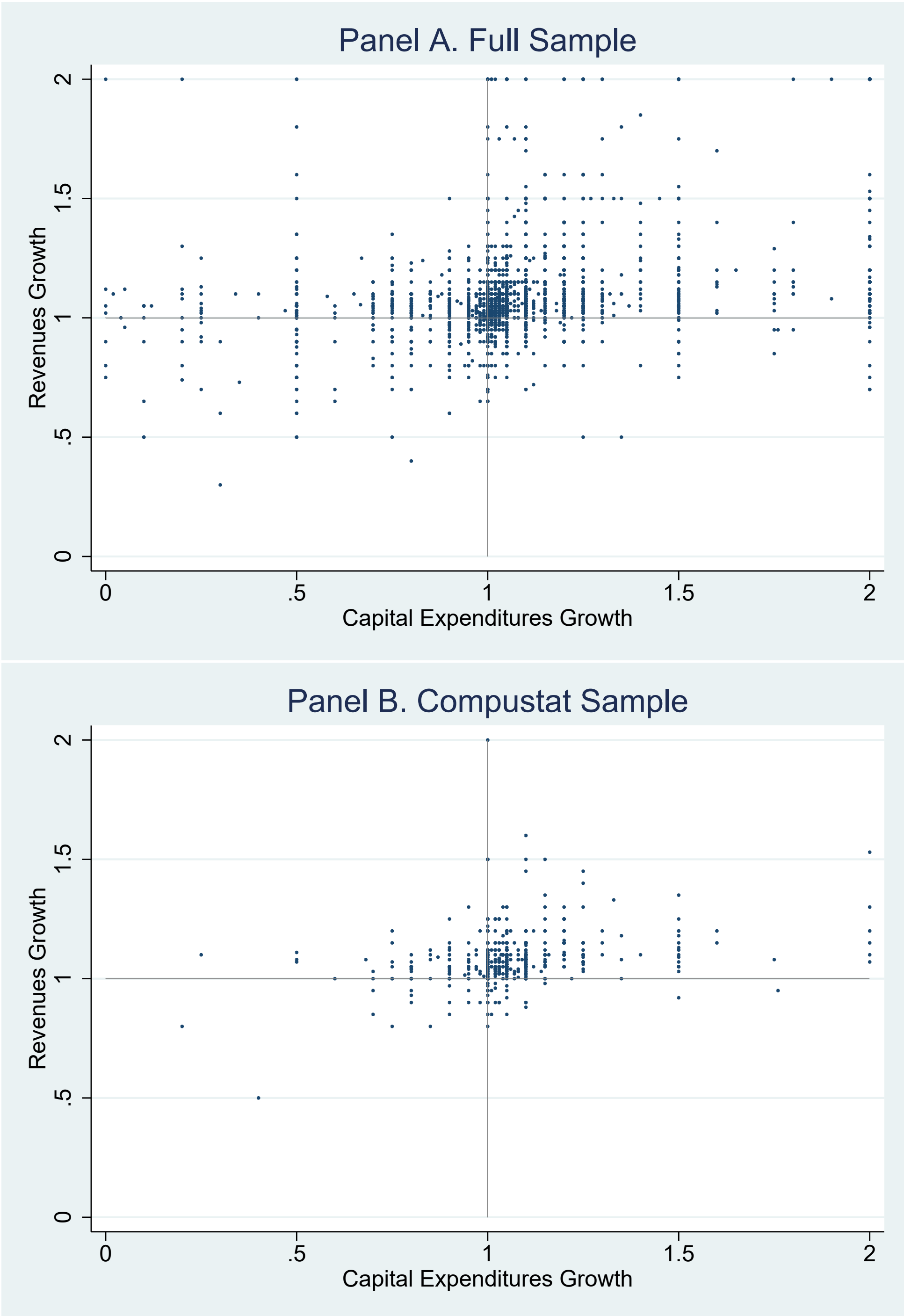

Figure 3: **Contemporaneous Forecasts Errors of Output and Capital**

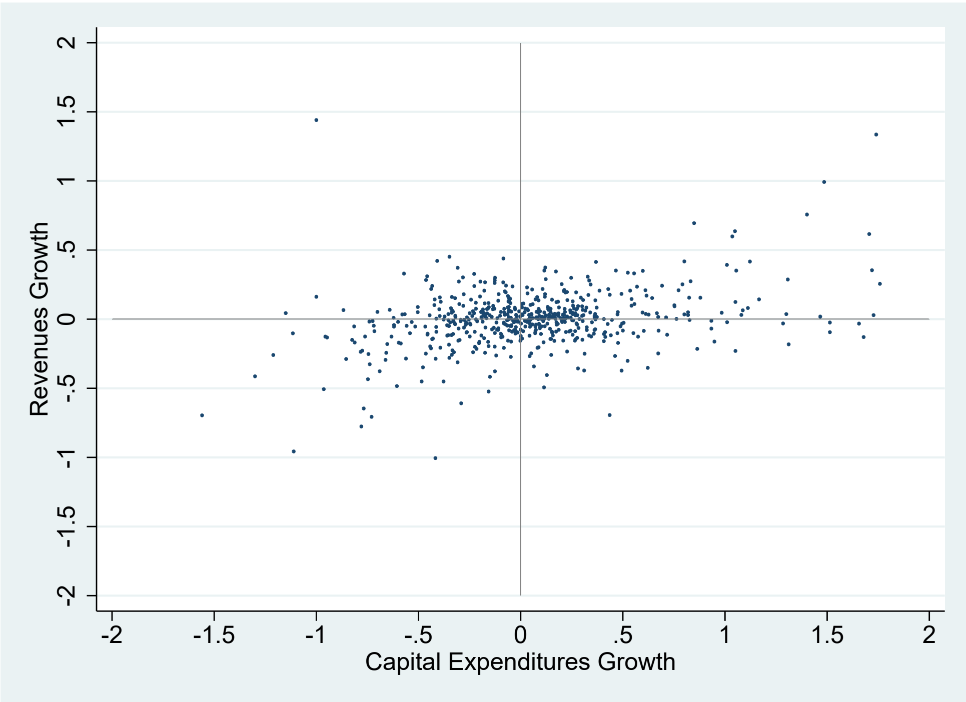

Figure 4: **(In)Coherence and (In)Accuracy Areas**

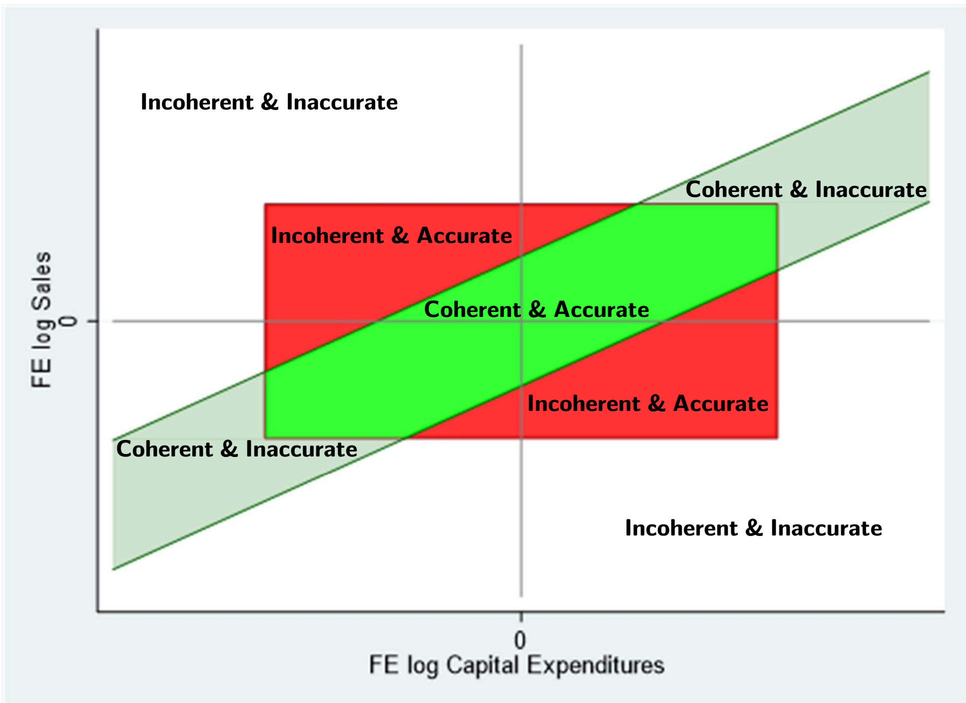

Figure 5: **Bootstrap of Coherence Test Statistic C2-stat**

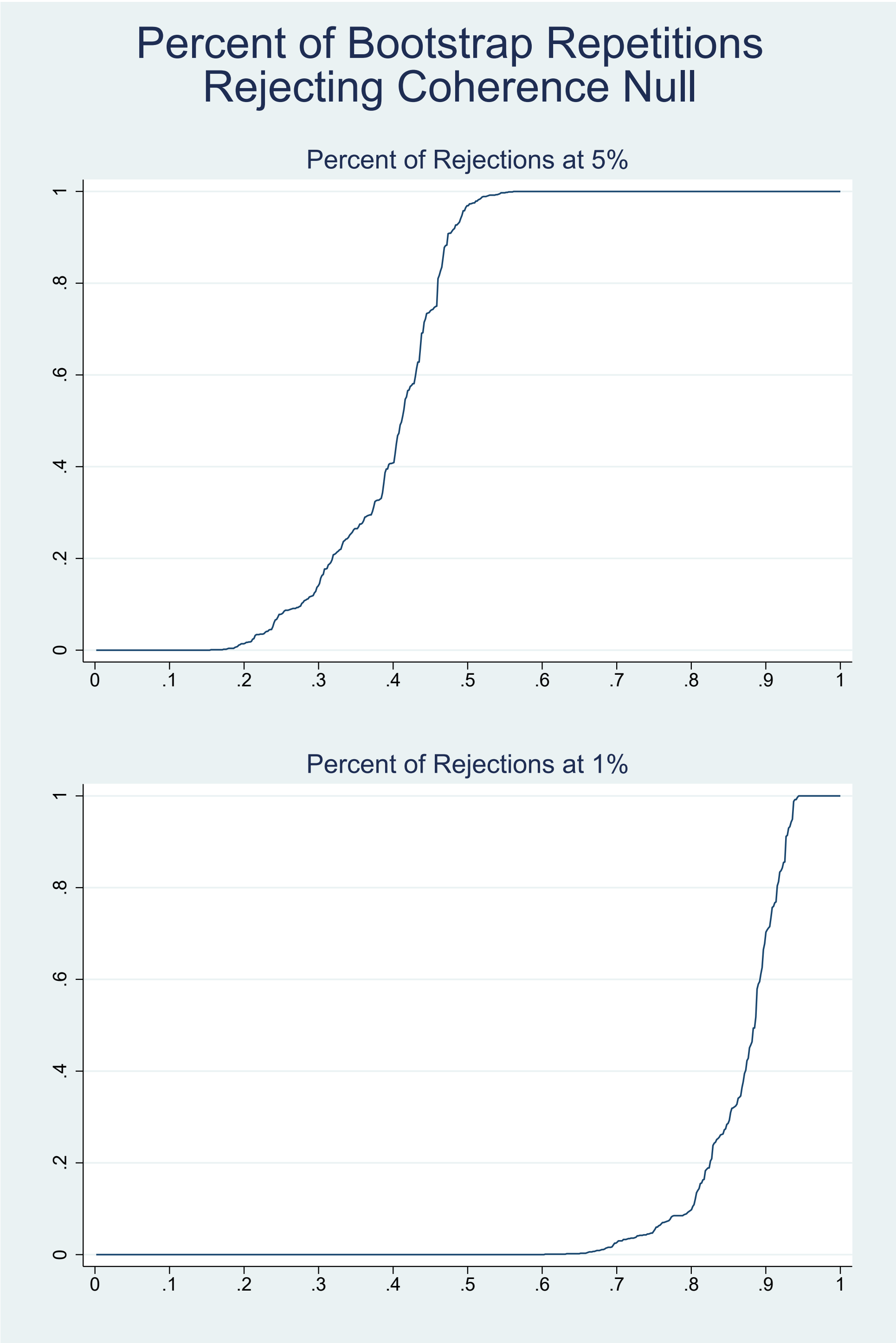

Table 1: **CFO Growth Forecasts and Realizations of Selected Balance Items**

Panel A – CFO Growth Forecasts (percent)

	Mean	Std. Dev.	Q10	Median	Q90	N Obs.
Expected Growth in Revenues and in Earnings						
Revenues	9.30	27.13	-5.00	5.00	20.00	14,490
Earnings	11.00	42.37	-10.00	5.00	30.00	25,472
Expected Growth in Capital-Related Expenditures						
Capital Expenditures	8.11	43.90	-15.00	3.00	25.00	25,305
R & D	4.51	21.65	0.00	0.00	15.00	8,325
Technology Spending	6.68	28.02	-5.00	3.00	20.00	22,404
Expected Growth in Labor-Related Costs						
Wages	3.90	12.41	0.00	3.00	7.00	27,472
Employees	3.95	30.16	-5.00	1.00	10.00	25,471
Outsourced Employees	3.74	21.19	0.00	0.00	10.00	10,990
Health Spending	8.59	11.65	1.00	8.00	15.00	25,064
Expected Growth in Productivity, Product Prices, and Advertising						
Productivity	3.91	9.38	0.00	3.00	10.00	18,197
Product Prices	2.08	8.22	-3.00	2.00	7.00	24,499
Advertising	4.75	21.83	-5.00	2.00	15.00	20,989
Expected Growth in Cash Holdings and Corporate Payout						
Cash	5.02	38.56	-20.00	0.00	20.00	16,876
Dividends	4.54	30.52	0.00	0.00	15.00	5,227
Share Repurchases	1.55	24.40	0.00	0.00	5.00	5,487

Panel B – Realizations, Matched Compustat-Duke Sample (percent)

	Mean	Std. Dev.	Q10	Median	Q90	N Obs.
Actual Growth in Revenues and in Earnings						
Revenues	6.80	21.32	-13.56	5.23	27.25	14,549
Earnings	-10.36	307.02	-161.71	2.36	124.59	14,580
Actual Growth in Capital-Related Expenditures						
Capital Expenditures	15.87	67.07	-42.59	3.96	75.70	13,770
R & D	7.09	29.57	-19.85	4.27	33.33	6,456
Technology Spending	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
Actual Growth in Labor-Related Costs						
Wages	7.02	14.65	-7.23	5.35	22.10	2,836
Employees	2.98	16.95	-11.88	1.19	17.71	14,359
Outsourced Employees	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
Health Spending	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
Actual Growth in Productivity, Product Prices, and Advertising						
Productivity	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
Product Prices	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
Advertising	8.03	42.14	-26.76	2.79	38.46	5,735
Actual Growth in Cash Holdings and Corporate Payout						
Cash	35.42	132.66	-46.26	5.76	113.78	14,520
Dividends	12.68	52.88	-12.22	6.15	38.44	8,762
Share Repurchases	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.

Table 2: **Violations of Coherence Inequality Restrictions**

Panel A – Inequality Test of Coherence

	$\chi = 0.5$	$\chi = 0.7$	$\chi = 0.9$
Inequality in Levels			
% Incoherent	100.00	100.00	99.07
% Coherent	0.00	0.00	0.93
% Total	100.00	100.00	100.00
N. Obs.	107	107	107
Inequality in Growth Rates			
% Incoherent	73.31	73.14	72.96
% Coherent	26.69	26.86	27.04
% Total	100.00	100.00	100.00
N. Obs.	577	577	577

Panel B – Summary stats of difference, LHS − RHS

	$\chi = 0.5$	$\chi = 0.7$	$\chi = 0.9$
Inequality in Levels			
Mean	15,419.59	15,201.16	15,033.78
Std. Dev.	28,574.83	28,233.56	27,990.89
Q10	213.3352	207.60	194.3495
Median	3,252.30	3228.92	3,205.96
Q90	39,737.28	39,505.75	39,360.82
N. Observations	107	107	107
Inequality in Growth Rates			
Mean	0.047	0.044	0.042
Std. Dev.	0.122	0.125	0.128
Q10	-0.061	-0.067	-0.067
Median	0.034	0.033	0.033
Q90	0.164	0.163	0.162
N. Observations	577	577	577

Note: χ denotes the elasticity of substitution between capital and labor.

Table 3: **The Coherence and Accuracy Sides of Rationality**

Panel A – Separate Assessment of Coherence and Accuracy (Percent of Rejections of Null)

Significance level α	Coherence Sales-CapEx	Accuracy Sales	Accuracy CapEx	Accuracy Both
	(1)	(2)	(3)	(4)
5%	55.7%	27.2%	47.9%	57.0%
1%	7.7%	1.8%	6.4%	7.1%

Panel B – Joint Assessment of Coherence and Accuracy

Significance level α	Coherent + Accurate	Coherent + Inaccurate	Incoherent + Accurate	Incoherent + Inaccurate
	(1)	(2)	(3)	(4)
5%	31.1%	13.2%	12.0%	43.7%
1%	89.4%	2.9%	3.4%	4.3%

Panel C – Test Statistics: Summary Statistics

	Mean	Std.Dev.	P05	Median	P95	N Obs.
C-statistic	-0.193	4.846	-8.335	-0.135	7.871	560
FE Sales	-0.538	19.07	-23.53	0.554	22.24	563
FE CapEx	-0.988	31.28	-54.18	1.186	41.20	560

Notes: In Panel B, Accuracy means both accurate; and inaccuracy means at least one inaccurate. Critical values are those of the t-student with one degree of freedom, +/-12.706 at the 5% and +/-63.657 at the 1%. Sales are the output. Capital Expenditures (CapEx) are input 1. Labor Expenditures are input 2 (unobserved).

Table 4: **Cross Sectional Regressions in Compustat Data for Rules of Thumb**

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Earnings growth	0.035***						0.030***	0.015***	0.014***
	(0.001)						(0.001)	(0.001)	(0.002)
Investment growth		0.077***					0.074***	0.068***	0.052***
		(0.002)					(0.002)	(0.002)	(0.004)
R & D growth			0.253***					0.210***	0.202***
			(0.007)					(0.007)	(0.011)
Advertising growth				0.139***					0.079***
				(0.005)					(0.005)
Cash growth					0.013***				0.018***
					(0.001)				(0.002)
Dividend growth						0.035***			0.003
						(0.002)			(0.005)
Constant	0.133***	0.106***	0.102***	0.096***	0.123***	0.128***	0.106***	0.085***	0.067***
	(0.001)	(0.001)	(0.002)	(0.002)	(0.001)	(0.001)	(0.001)	(0.002)	(0.003)
Adjusted R^2	0.027	0.081	0.132	0.128	0.012	0.004	0.102	0.190	0.284
N observations	103,738	100,441	40,711	34,182	103,363	100,247	100,040	40,315	15,358

Notes: The data is made of repeated cross sections. Standard errors corrected for heteroskedasticity and clustered by firm.

Table 5: **Minimum Distance of CapEx Forecasts from Rules of Thumb**

	All	R1	R2	R3	R4	R5
Mean	0.033	0.058	0.030	0.019	0.031	0.043
Std. Dev.	0.059	0.100	0.064	0.017	0.035	0.069
Frac. Zeros	0.106	0.000	0.268	0.000	0.000	0.000
P10	0.000	0.008	0.000	0.005	0.002	0.003
P25	0.007	0.015	0.000	0.006	0.007	0.008
P50	0.019	0.028	0.014	0.010	0.023	0.023
P75	0.036	0.064	0.035	0.028	0.048	0.043
P90	0.071	0.114	0.071	0.048	0.072	0.089
P95	0.106	0.143	0.106	0.048	0.100	0.140
N of Observations	396	30	157	43	107	59
Fraction	1.000	0.076	0.396	0.109	0.270	0.149

Notes: Cross-sectional analysis with 396 CFOs.

Table 6: **Incoherence and Rules of Thumb: Distance from Optimal Forecast**

	(1)	(2)	(3)	(4)	(5)
Rule 1 (CapEx)	0.081***				0.104***
	(0.014)				(0.016)
Rule 2 (CapEx)		0.039***			0.053***
		(0.008)			(0.011)
Rule 3 (CapEx)			-0.055***		-0.020
			(0.012)		(0.014)
Rule 4 (CapEx)				-0.027***	0.010
				(0.009)	(0.012)
Constant	0.066***	0.057***	0.079***	0.080***	0.043***
	(0.004)	(0.005)	(0.004)	(0.005)	(0.009)
Adjusted R^2	0.071	0.057	0.045	0.023	0.175
N observations	396	396	396	396	396

Summary Statistics of the dependent variable	
Mean	0.073
Std. Dev.	0.079
P10	0.012
Median	0.059
P90	0.139

Notes: *, **, *** denote two-tailed significance at the 10%, 5%, and 1% levels.

Table 7: **Incoherence and Personal CFO Characteristics**

Panel A – Summary statistics

	Mean	Std. Dev.	Q10	Median	Q90	N Obs.
CFO has MBA	0.452	0.498	0.000	0.000	1.000	396
Age 40-	0.063	0.244	0.000	0.000	0.000	396
Age 41-50	0.460	0.499	0.000	0.000	1.000	396
Age 51-60	0.402	0.491	0.000	0.000	1.000	396
Age 61+	0.076	0.265	0.000	0.000	0.000	396
Gender	0.088	0.284	0.000	0.000	0.000	396
Miscalibration ST	0.035	0.920	-1.166	0.329	0.985	360
Optimism ST	0.052	0.981	-0.918	-0.077	1.285	373
Miscalibration LT	0.039	0.979	-1.095	0.262	0.917	362
Optimism LT	0.033	1.088	-1.008	-0.078	1.077	374

Panel B – Incoherence and CFO characteristics

	(1)	(2)	(3)	(4)	(5)	(6)
CFO has MBA	0.004			0.005		
	(0.009)			(0.009)		
Age 40-		-0.011		-0.011		
		(0.022)		(0.022)		
Age 41-50		-0.026*		-0.027*		
		(0.015)		(0.016)		
Age 51-60		-0.024		-0.024		
		(0.017)		(0.017)		
Gender			-0.000	0.002		
			(0.011)	(0.010)		
Miscalibration ST					-0.012	
					(0.008)	
Optimism ST					-0.012	
					(0.007)	
Miscalibration LT						-0.005
						(0.004)
Optimism LT						0.001
						(0.004)
Constant	0.050**	0.078***	0.053***	0.075***	0.052*	0.046**
	(0.021)	(0.025)	(0.019)	(0.026)	(0.027)	(0.021)
Industry FE	Yes	Yes	Yes	Yes	Yes	Yes
Survey FE	Yes	Yes	Yes	Yes	Yes	Yes
Adjusted R^2	0.109	0.116	0.108	0.116	0.154	0.126
N of Observations	396	396	396	396	360	362

Notes: *, **, *** denote two-tailed significance at the 10%, 5%, and 1% levels, respectively. Robust standard errors are reported in parenthesis.

Table 8: **Incoherence and Corporate Performance (Return on Assets)**

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Incoherence	-0.377**	-0.378**	-0.360**	-0.396**	-0.399**	-0.386**	-0.317*	-0.307*
	(0.157)	(0.179)	(0.162)	(0.162)	(0.186)	(0.169)	(0.192)	(0.181)
Miscalibration ST		0.003			0.001		-0.001	
		(0.005)			(0.005)		(0.004)	
Optimism ST		0.000			0.000		0.001	
		(0.006)			(0.006)		(0.005)	
Miscalibration LT			0.004			0.002		0.001
			(0.005)			(0.005)		(0.005)
Optimism LT			0.008			0.007		0.009
			(0.006)			(0.006)		(0.006)
Firm size							0.009***	0.009**
							(0.003)	(0.003)
Market-to-Book							0.028**	0.027*
							(0.014)	(0.015)
Dividends							0.022*	0.023*
							(0.012)	(0.013)
Constant	0.069***	0.069***	0.068***	0.054***	0.056***	0.057***	-0.131***	-0.123***
	(0.011)	(0.011)	(0.011)	(0.014)	(0.020)	(0.019)	(0.047)	(0.0471)
Industry FE	No	No	No	Yes	Yes	Yes	Yes	Yes
Survey FE	No	No	No	Yes	Yes	Yes	Yes	Yes
Adjusted R^2	0.046	0.042	0.047	0.071	0.064	0.068	0.177	0.185
N of CFOs	311	282	284	311	282	284	263	265
N of Firms	277	252	254	277	252	254	235	237
N of Observations	468	423	428	468	423	428	396	401

Notes: *, **, *** denote two-tailed significance at the 10%, 5%, and 1% levels, respectively.
Standard errors are bootstrapped following Cameron, Gelbach, and Miller (2008) and clustered at the firm level.

Table 9: **Rules of Thumb and Corporate Performance (Return on Assets)**

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Rule 1 (CapEx)	-0.057**	-0.061**	-0.059**	-0.051**	-0.059**	-0.055**	-0.053**	-0.051**
	(0.022)	(0.025)	(0.024)	(0.023)	(0.025)	(0.025)	(0.026)	(0.025)
Rule 2 (CapEx)	-0.026*	-0.027*	-0.023	-0.023	-0.028*	-0.024	-0.034	-0.031
	(0.0138)	(0.015)	(0.015)	(0.015)	(0.017)	(0.016)	(0.021)	(0.019)
Rule 3 (CapEx)	-0.031*	-0.036*	-0.034*	-0.027	-0.037*	-0.034	-0.047**	-0.045**
	(0.017)	(0.019)	(0.019)	(0.019)	(0.020)	(0.021)	(0.023)	(0.022)
Rule 4 (CapEx)	-0.012	-0.010	-0.010	-0.008	-0.008	-0.007	-0.012	-0.011
	(0.012)	(0.014)	(0.014)	(0.013)	(0.014)	(0.015)	(0.015)	(0.015)
Miscalibration ST		0.001			-0.001		-0.002	
		(0.005)			(0.005)		(0.004)	
Optimism ST		0.001			0.000		0.001	
		(0.006)			(0.005)		(0.005)	
Miscalibration LT			0.003			0.002		0.001
			(0.006)			(0.005)		(0.004)
Optimism LT			0.007			0.006		0.008
			(0.006)			(0.006)		(0.005)
Firm size							0.010***	0.009***
							(0.004)	(0.004)
Market-to-Book							0.028**	0.028*
							(0.014)	(0.015)
Dividends							0.029**	0.030**
							(0.013)	(0.014)
Constant	0.065***	0.066***	0.064***	0.040***	0.045**	0.046*	-0.147***	-0.137***
	(0.011)	(0.012)	(0.013)	(0.015)	(0.019)	(0.028)	(0.046)	(0.050)
Industry FE	No	No	No	Yes	Yes	Yes	Yes	Yes
Survey FE	No	No	No	Yes	Yes	Yes	Yes	Yes
Adjusted R^2	0.014	0.014	0.019	0.033	0.031	0.034	0.165	0.170
N of CFOs	311	282	284	311	282	284	263	265
N of Firms	277	252	254	277	252	254	235	237
N of Observations	468	423	428	468	423	428	396	401

Notes: *, **, *** denote two-tailed significance at the 10%, 5%, and 1% levels, respectively.
Standard errors are bootstrapped following Cameron, Gelbach, and Miller (2008) and clustered at the firm level.

Table 10: **Rules of Thumb and Corporate Policies**

	Investment			**Leverage**		
	(1)	(2)	(3)	(4)	(5)	(6)
Rule 1 (CapEx)	-0.016	-0.014	-0.015	0.055	0.041	0.047
	(0.011)	(0.011)	(0.012)	(0.092)	(0.101)	(0.092)
Rule 2 (CapEx)	-0.013**	-0.015**	-0.012	0.093*	0.098	0.092*
	(0.006)	(0.007)	(0.008)	(0.053)	(0.060)	(0.053)
Rule 3 (CapEx)	-0.007	-0.011	-0.010	-0.023	-0.015	-0.027
	(0.008)	(0.010)	(0.010)	(0.073)	(0.091)	(0.084)
Rule 4 (CapEx)	-0.003	-0.003	-0.003	-0.004	0.005	0.001
	(0.007)	(0.008)	(0.008)	(0.045)	(0.050)	(0.046)
Miscalibration ST		0.001			0.012	
		(0.003)			(0.024)	
Optimism ST		0.002			-0.006	
		(0.003)			(0.019)	
Miscalibration LT			0.002			0.013
			(0.002)			(0.018)
Optimism LT			0.004*			-0.010
			(0.002)			(0.017)
Firm size	-0.002	-0.002	-0.002	0.012	0.011	0.012
	(0.002)	(0.002)	(0.002)	(0.017)	(0.019)	(0.018)
Market-to-Book	0.006**	0.006*	0.006*	-0.082***	-0.081***	-0.081***
	(0.003)	(0.003)	(0.003)	(0.020)	(0.021)	(0.022)
Dividends	0.000	0.004	0.004	0.037	0.044	0.043
	(0.009)	(0.008)	(0.008)	(0.072)	(0.076)	(0.080)
Constant	0.044*	0.043*	0.050**	0.568**	0.666***	0.620**
	(0.025)	(0.023)	(0.023)	(0.249)	(0.228)	(0.249)
Industry FE	Yes	Yes	Yes	Yes	Yes	Yes
Survey FE	Yes	Yes	Yes	Yes	Yes	Yes
Adjusted R^2	0.210	0.223	0.230	0.069	0.062	0.066
N of Observations	437	397	402	437	397	402

Notes: *, **, *** denote two-tailed significance at the 10%, 5%, and 1% levels, respectively. Standard errors are bootstrapped following Cameron, Gelbach, and Miller (2008) and clustered at the firm level.

Table 11: **Change in Performance and Corporate Policies when New CFOs Take Office**

	Change in ROA		**Change in Investment**		**Change in Leverage**	
	(1)	(2)	(3)	(4)	(5)	(6)
Incoherence	-1.633*		-0.049		-0.047	
	(0.989)		(0.045)		(1.115)	
Rule 1 (CapEx)		-0.274		-0.022*		-0.011
		(0.213)		(0.012)		(0.231)
Rule 2 (CapEx)		-0.000		-0.003		-0.201
		(0.036)		(0.008)		(0.199)
Rule 3 (CapEx)		-0.057		-0.008		-0.110
		(0.051)		(0.012)		(0.153)
Rule 4 (CapEx)		0.019		0.001		-0.070
		(0.048)		(0.009)		(0.118)
Firm characteristics	Yes	Yes	Yes	Yes	Yes	Yes
Industry FE	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Adjusted R^2	0.391	0.192	0.024	0.017	0.053	0.042
N of Observations	142	142	140	140	146	146

Notes: *, **, *** denote two-tailed significance at the 10%, 5%, and 1% levels, respectively. Standard errors are bootstrapped following Cameron, Gelbach, and Miller (2008) and clustered at the firm level.

Online Appendix with Supplementary Material for The Coherence Side of Rationality: Rules of thumb, narrow bracketing, and managerial incoherence in corporate forecasts

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Not for Publication

A Proofs

Proof of Proposition 1. Recall that for a concave function, f, it holds that $\mathbb{E}[f(x)] \leq f(\mathbb{E}[x])$. Assume $\xi \leq 1$ and start by assuming that $a+b=1$. The CES function f is homogeneous of degree one because, for a scalar λ, we have that

$$f(\lambda \mathbf{x}) = \left[a(\lambda x_1)^{\xi} + b(\lambda x_2)^{\xi}\right]^{\frac{1}{\xi}} = \lambda\left(a x_1^{\xi} + b x_2^{\xi}\right)^{\frac{1}{\xi}}.$$

Furthermore, note that f is also quasiconcave because it is a monotone transformation of a concave function. In fact,

$$f = g^{\frac{1}{\xi}},$$

and to see that g is concave, compute its Hessian, H_g,

$$H_g = \begin{bmatrix} \frac{\partial^2 g}{\partial x_1^2} & \frac{\partial^2 g}{\partial x_2 \partial x_1} \\ \frac{\partial^2 g}{\partial x_1 \partial x_2} & \frac{\partial^2 g}{\partial x_2^2} \end{bmatrix} = \begin{bmatrix} a\xi(1-\xi)x_1^{\xi-2} & 0 \\ 0 & b\xi(1-\xi)x_2^{\xi-2} \end{bmatrix}.$$

Since H_g is negative semi-definite, we can conclude that g is concave.

Now, let $a+b \leq 1$. We have that $f = \left(g^{\frac{1}{\xi}}\right)^{a+b}$, where g is concave, as shown above. Then, f is a concave increasing function of a concave function, from which we can conclude that f is concave, which proves the first part of the proposition. The second part of the proposition on convexity follows very similar arguments.
QED

Proof of Corollary 1. Here we prove the statement in growth rates (the one in levels follows similar steps). In a Cobb-Douglas for $\xi \to 0$, the solution for optimal input quantities are

$$x_1^* = \frac{Z}{p_1}\frac{a}{a+b}, \qquad x_2^* = \frac{Z}{p_2}\frac{b}{a+b}.$$

Therefore, for $i = 1,2$ we have

$$\begin{aligned} \log\left[\frac{x_{i,t+1}}{x_{i,t}}\right] &= \log\left[\frac{p_{i,t}}{p_{i,t+1}}\right] \\ \log\left[\frac{y_{t+1}}{y_t}\right] &= a \cdot \log\left[\frac{p_{1,t}}{p_{1,t+1}}\right] + b \cdot \log\left[\frac{p_{2,t}}{p_{2,t+1}}\right]. \end{aligned}$$

Putting these together, we obtain

$$\begin{aligned} \log\left[\frac{y_{t+1}}{y_t}\right] &= a \cdot \log\left[\frac{x_{1,t+1}}{x_{1,t}}\right] + b \cdot \log\left[\frac{x_{2,t+1}}{x_{2,t}}\right] \\ \mathbb{E}_t \log\left[\frac{y_{t+1}}{y_t}\right] &= a \cdot \mathbb{E}_t \log\left[\frac{x_{1,t+1}}{x_{1,t}}\right] + b \cdot \mathbb{E}_t \log\left[\frac{x_{2,t+1}}{x_{2,t}}\right]. \end{aligned}$$

QED

Proof of Proposition 2. To derive the test statistic, note that we have $\mathbb{E}_t \log y_{t+1} = a\mathbb{E}_t \log x_{1,t+1} + b\mathbb{E}_t \log x_{2,t+1}$ and $\log x_{2,t+1} = \log \frac{b}{a+b} Z - \pi_{2,t+1}$, implying that the CFO forecast for input 2 is

$$\mathbb{E}_t \log x_{2,t+1} = \log \frac{b}{a+b} Z - \gamma_2 \pi_{2,t}. \tag{6}$$

Note that (6) depends on the technological parameters, a and b, and budget, Z, which we assume are known to the CFO at the time of the forecast and are stable over time. From (6) we obtain that

$$\frac{\mathbb{E}_t \log x_{2,t+1} - \log \frac{b}{a+b} Z}{\gamma_2 \sigma_2} \sim \mathcal{N}(0,1).$$

We can then derive our test statistic C1-stat based on the joint forecasts of the first input and output by recalling that $\log y = a \log x_1 + b \log x_2$ as follows:

$$\text{C1-stat} \equiv \frac{\frac{\mathbb{E}_t \log y_{t+1} - a\mathbb{E}_t \log x_{1,t+1}}{b} - \log \frac{b}{a+b} Z}{\gamma_2 \sigma_2} \sim \mathcal{N}(0,1),$$

where the distribution here is obtained under the null hypothesis of coherent forecasts.

To derive our C2-stat, we start by defining the forecast error of a generic variable x forecasted at t and realized at $t+1$ as the difference between the realization and the forecast, $\mathrm{FE}_t x_{t+1} = x_{t+1} - \mathbb{E}_t x_{t+1}$. We then have that

$$\begin{aligned} \mathrm{FE}_t \log x_{2,t+1} &= \log x_{2,t+1} - \mathbb{E}_t \log x_{2,t+1} \\ &= \log \frac{b}{a+b} Z - \pi_{2,t+1} - \mathbb{E}_t \left[\log \frac{b}{a+b} Z - \pi_{2,t+1} \right] \\ &= -\mathrm{FE}_t \pi_{2,t+1} = -\epsilon_{2,t+1}. \end{aligned}$$

As a result, the forecast error of the log of the second input is the negative of the innovation of the second log-price process. It follows that

$$\frac{\mathrm{FE}_t \log x_{2,t+1}}{\sigma_2} \sim \mathcal{N}(0,1).$$

Noting that $\mathrm{FE}_t \log y_{t+1} = a\mathrm{FE}_t \log x_{1,t+1} + b\mathrm{FE}_t \log x_{2,t+1}$, we obtain our C2-stat,

$$\text{C2-stat} \equiv \frac{\mathrm{FE}_t \log y_{t+1} - a\mathrm{FE}_t \log x_{1,t+1}}{\sigma_2 b} \sim \mathcal{N}(0,1).$$

QED

Coherence Test Statistic: Multiple Inputs Case. Here we generalize our C2-stat to a multivariate case with N inputs. For this subsection, the production setting is

$$y = \prod_{i=1}^{N} x_i^{a_i}$$

$$\boldsymbol{p}'\boldsymbol{x} = Z,$$

where $\boldsymbol{p}$ and $\boldsymbol{x}$ are the column vectors of factor prices and quantities, respectively. As in the $N=2$ case we have a linear relationship between the logs of inputs and output,

$$\log y = \sum_{i=1}^{N} a_i \log x_i,$$

where the same equation holds for the forecast errors. Analogously to the bivariate case, we have that $\mathrm{FE}_t \log x_{1,t+1} = -\epsilon_{1,t+1}$ so that

$$\frac{\mathrm{FE}_t \log x_{1,t+1}}{\sigma_1} \sim \mathcal{N}(0,1).$$

Then, using the linear relationship between the logs of inputs and output we obtain our generalized C2-stat,

$$\frac{\mathrm{FE}_t \log y_{t+1} - \sum_{i=2}^{N} a_i \mathrm{FE}_t \log x_{i,t+1}}{\sigma_1 a_1} \sim \mathcal{N}(0,1).$$

Proof of Proposition 3. The Proof follows directly from the observation that in our setting the conditional expectation function is

$$\mathbb{E}\left[y|x_1,x_2\right] = \mathbb{E}\left[y\right] + \beta_1\left(x_1 - \mathbb{E}\left[x_1\right]\right) + \beta_2\left(x_2 - \mathbb{E}\left[x_2\right]\right),$$

where the parameters can be derived by the Frisch-Waugh-Lovell theorem, as

$$\beta_1 = \frac{cov\left(y,x_1\right) - \left(\frac{cov(x_1,x_2)cov(y,x_2)}{var(x_2)}\right)}{var\left(x_1\right) - \frac{cov(x_1,x_2)^2}{var(x_2)}},\ \ \beta_2 = \frac{cov\left(y,x_2\right) - \left(\frac{cov(x_1,x_2)cov(y,x_1)}{var(x_1)}\right)}{var\left(x_2\right) - \frac{cov(x_1,x_2)^2}{var(x_1)}},$$

and where $var\left(x_1\right)$, $var\left(x_2\right)$, $cov\left(y,x_1\right)$, and $cov\left(y,x_2\right)$ are functions of parameters a and b. QED

Proof of Corollary 2. In levels,

$$\mathbb{E}\left[\log x_1 | \log y, \log x_2\right] = \mu_1 + \beta_y\left(\log y - \mu_y\right) + \beta_2\left(\log x_2 - \mu_2\right),$$

where coefficients equal

$$\beta_y = \frac{cov\left(\log y, \log x_1\right) - \left(\frac{cov(\log y,\log x_2)cov(\log x_1,\log x_2)}{var(\log x_2)}\right)}{var\left(\log y\right) - \frac{cov(\log y,\log x_2)^2}{var(\log x_2)}},$$

$$\beta_2 = \frac{cov\left(\log x_2, \log x_1\right) - \left(\frac{cov(\log y,\log x_2)cov(\log x_1,\log y)}{var(\log y)}\right)}{var\left(\log x_2\right) - \frac{cov(\log y,\log x_2)^2}{var(\log y)}}.$$

In detail, we have:
$var\left(\log x_2\right) = \sigma_2^2$,
$var\left(\log y\right) = a^2\sigma_1^2 + b^2\sigma_2^2$,
$cov\left(\log x_2, \log x_1\right) = 0$,
$cov\left(\log y, \log x_1\right) = cov\left(a\log x_1 + b\log x_2, \log x_1\right) = a\sigma_1^2$,
$cov\left(\log y, \log x_2\right) = cov\left(a\log x_1 + b\log x_2, \log x_2\right) = b\sigma_2^2$.

Substituting yields

$$
\begin{aligned}
\mathbb{E}\left[\log x_1 | \log y, \log x_2\right] &= \mu_1 + \frac{a\sigma_1^2}{a^2\sigma_1^2 + b^2\sigma_2^2 - \frac{b^2\sigma_2^4}{\sigma_2^2}} \left(\log y - \mu_y\right) + \frac{-\left(\frac{b\sigma_2^2 a\sigma_1^2}{a^2\sigma_1^2+b^2\sigma_2^2}\right)}{\sigma_2^2 - \frac{b^2\sigma_2^4}{a^2\sigma_1^2+b^2\sigma_2^2}} \left(\log x_2 - \mu_2\right) \\
&= \mu_1 + \frac{1}{a}\left(\log y - \mu_y\right) - \frac{b\sigma_2^2 a\sigma_1^2}{\sigma_2^2\left(a^2\sigma_1^2 + b^2\sigma_2^2\right) - b^2\sigma_2^4}\left(\log x_2 - \mu_2\right) \\
&= \mu_1 + \frac{1}{a}\left(\log y - \mu_y\right) - \frac{b}{a}\left(\log x_2 - \mu_2\right).
\end{aligned}
$$

where $\mu_1 - \frac{1}{a}\mu_y + \frac{b}{a}\mu_2 = 0$ follows by Corollary 1. Proving the statement in growth rates follows similar steps.
QED

Proof of Corollary 3. In levels,

$$\mathbb{E}\left[\log x_1 | \log y\right] = \mu_1 + \beta_y\left(\log y - \mu_y\right),$$

where coefficients equal

$$\alpha = \mu_1 - \beta_y\mu_y, \qquad \beta_y = \frac{cov\left(\log y, \log x_1\right)}{var\left(\log y\right)}.$$

We have:
$cov\left(\log x_2, \log x_1\right) = 0$,
$cov\left(\log y, \log x_1\right) = cov\left(a\log x_1 + b\log x_2, \log x_1\right) = a\sigma_1^2$,
$var\left(\log y\right) = a^2\sigma_1^2 + b^2\sigma_2^2$.
Substituting yields

$$\alpha = \mu_1 - \frac{a\sigma_1^2}{a^2\sigma_1^2 + b^2\sigma_2^2}\mu_y, \qquad \beta_y = \frac{a\sigma_1^2}{a^2\sigma_1^2 + b^2\sigma_2^2},$$

and thus

$$\mathbb{E}\left[\log x_1 | \log y\right] = \mu_1 - \frac{a\sigma_1^2}{a^2\sigma_1^2 + b^2\sigma_2^2}\mu_y + \frac{a\sigma_1^2}{a^2\sigma_1^2 + b^2\sigma_2^2}\left(\log y - \mu_y\right).$$

Proving the statement in growth rates follows similar steps and intercept α is differenced away.
QED

Proof of Corollary 4. Consider the regression

$$\frac{y_{t+1}}{y_t} = \alpha + \beta\frac{x_{i,t+1}}{x_{i,t}} + e_{t+1}.$$

Denoting variable at optimum with superscripts *, we have

$$
\begin{aligned}
\beta &= \frac{cov\left(\frac{y_{t+1}}{y_t}, \frac{x^*_{i,t+1}}{x^*_{i,t}}\right)}{var\left(\frac{x^*_{i,t+1}}{x^*_{i,t}}\right)} = \frac{\mathbb{E}\left[\frac{y_{t+1}}{y_t}\cdot\frac{x^*_{i,t+1}}{x^*_{i,t}}\right] - \mathbb{E}\left[\frac{y_{t+1}}{y_t}\right]\cdot\mathbb{E}\left[\frac{x^*_{i,t+1}}{x^*_{i,t}}\right]}{\mathbb{E}\left[\left(\frac{x^*_{i,t+1}}{x^*_{i,t}}\right)^2\right] - \left(\mathbb{E}\left[\frac{x^*_{i,t+1}}{x^*_{i,t}}\right]\right)^2}, \\
\alpha &= \mathbb{E}\left[\frac{y_{t+1}}{y_t}\right] - \beta\mathbb{E}\left[\frac{x^*_{i,t+1}}{x^*_{i,t}}\right].
\end{aligned}
$$

Recall that

$$\left\{ \begin{pmatrix} \pi_{1,t} \\ \pi_{2,t} \end{pmatrix} \right\} \overset{iid}{\sim} \mathcal{N}_2 \left(\mathbf{0}, \begin{bmatrix} \sigma_1^2 & \sigma_{1,2} \\ \sigma_{1,2} & \sigma_2^2 \end{bmatrix} \right).$$

Under the assumption that $\sigma_1^2 = \sigma_2^2 = \sigma^2$, for every t we have that the Pearson correlation coefficient $\rho_{1,2} = 1$, from which it follows that $\pi_{1,t} = \pi_{2,t} + c$ almost surely, where c is a constant.[27] As a result, the setting can be recast as one in which prices are constant and the budget Z is stochastic because

$$\begin{aligned} \frac{x^*_{1,t+1}}{x^*_{1,t}} &= \frac{p_{1,t}}{p_{1,t+1}} \overset{\text{a.s.}}{=} \frac{p_{2,t}}{p_{2,t+1}} = \frac{x^*_{2,t+1}}{x^*_{2,t}} \\ \frac{y_{t+1}}{y_t} &= a\frac{p_{1,t}}{p_{1,t+1}} + b\frac{p_{2,t}}{p_{2,t+1}} \overset{\text{a.s.}}{=} (a+b)\frac{p_{1,t}}{p_{1,t+1}} \end{aligned}$$

and

$$\begin{aligned} &\left\{ \frac{x^*_{i,t+1}}{x^*_{i,t}} \right\} \overset{\text{iid}}{\sim} \mathcal{N}\left(0, 2\sigma^2\right), \quad i = 1, 2 \\ &\left\{ \frac{y_{t+1}}{y_t} \right\} \overset{\text{iid}}{\sim} \mathcal{N}\left(0, 2\left(a+b\right)^2\sigma^2\right). \end{aligned}$$

In fact, denote $z_t = \log(Z) \overset{\text{iid}}{\sim} \mathcal{N}\left(0, \sigma^2\right)$, and assume p_1 and p_2 constant for all t. We have

$$\begin{aligned} \frac{x^*_{1,t+1}}{x^*_{1,t}} &= \frac{x^*_{2,t+1}}{x^*_{2,t}} = e^{z_{t+1}-z_t} \\ \frac{y_{t+1}}{y_t} &= e^{(a+b)(z_{t+1}-z_t)} \end{aligned}$$

and

$$\begin{aligned} &\left\{ \frac{x^*_{i,t+1}}{x^*_{i,t}} \right\} \overset{\text{iid}}{\sim} \mathcal{N}\left(0, 2\sigma^2\right), \quad i = 1, 2 \\ &\left\{ \frac{y_{t+1}}{y_t} \right\} \overset{\text{iid}}{\sim} \mathcal{N}\left(0, 2\left(a+b\right)^2\sigma^2\right), \end{aligned}$$

as it was with stochastic prices. Therefore, for clarity from now on we drop the subscript i. Now, recalling that $\nu \equiv a + b$, that $cov\left(z_{t+1}, z_t\right) = \gamma\frac{\sigma^2}{1-\gamma^2}$, and that for any scalar, c, we have

$$\mathbb{E}\left[e^{c(z_{t+1}-z_t)}\right] = e^{\frac{1}{2}c^2 var(z_{t+1}-z_t)} = e^{\frac{1}{2}c^2\left(2\frac{\sigma^2}{1-\gamma^2} - 2\gamma\frac{\sigma^2}{1-\gamma^2}\right)} = e^{c^2\left(\frac{\sigma^2}{1+\gamma}\right)},$$

[27] To see this, suppose that X, Y are two random variables such that $\rho(X, Y) = 1$. Let $V = X - \mathbb{E}[X]$ and $W = Y - \mathbb{E}[Y]$. We have $\mathbb{E}\left[(V - W)^2\right] = var(X) + var(Y) - 2cov(X, Y) = 0$, so that $V \overset{\text{a.s.}}{=} W$, from which the result $\pi_{1,t} = \pi_{2,t} + c$ follows.

we obtain the expressions

$$
\begin{aligned}
\beta &= \frac{e^{\left[(\upsilon+1)^2\frac{\sigma^2}{1-\gamma}\right]}-e^{\left[\left(\upsilon^2+1\right)\frac{\sigma^2}{1-\gamma}\right]}}{e^{\left(4\frac{\sigma^2}{1-\gamma}\right)}-e^{\left(2\frac{\sigma^2}{1-\gamma}\right)}}=\frac{e^{\left[\left(\upsilon^2+2\upsilon\right)\frac{\sigma^2}{1-\gamma}\right]}-e^{\left[\upsilon^2\frac{\sigma^2}{1-\gamma}\right]}}{e^{\left(3\frac{\sigma^2}{1-\gamma}\right)}-e^{\left(\frac{\sigma^2}{1-\gamma}\right)}} \\
\alpha &= \frac{e^{\left[\left(\upsilon^2+2\right)\frac{\sigma^2}{1-\gamma}\right]}-e^{\left[\left(\upsilon^2+2\upsilon\right)\frac{\sigma^2}{1-\gamma}\right]}}{e^{\left(2\frac{\sigma^2}{1-\gamma}\right)}-1}.
\end{aligned}
$$

We can then directly verify the coimplications of the Corollary, that is,

$$
\begin{aligned}
\beta &< 1 \Longleftrightarrow e^{\left[\left(\upsilon^2+2\upsilon\right)\frac{\sigma^2}{1-\gamma}\right]}-e^{\left[\upsilon^2\frac{\sigma^2}{1-\gamma}\right]}<e^{\left(3\frac{\sigma^2}{1-\gamma}\right)}-e^{\left(\frac{\sigma^2}{1-\gamma}\right)} \Longleftrightarrow \upsilon<1 \\
\alpha &> 0 \Longleftrightarrow e^{\left[\left(\upsilon^2+2\right)\frac{\sigma^2}{1-\gamma}\right]}>e^{\left[\left(\upsilon^2+2\upsilon\right)\frac{\sigma^2}{1-\gamma}\right]} \Longleftrightarrow \upsilon<1,
\end{aligned}
$$

which also holds for i.i.d. shocks, that is, for $\gamma = 0$.
QED

Proof of Corollary 5. We have

$$\log\frac{x_{i,t+1}}{x_{i,t}}=\log\frac{1/p_{i,t+1}}{1/p_{i,t}}=\pi_{i,t}-\pi_{i,t+1}.$$

Denote $\log F_t^0$ the optimal forecast of log input $x_{i,t}$ growth. We have

$$\log F_t^o=\mathbb{E}_t\left[\log\frac{x_{i,t+1}}{x_{i,t}}\right]=(1-\gamma_i)\,\pi_{i,t}.$$

Under the optimal forecast, the forecast error will be minus the innovation of the log price shock,

$$\log\frac{x_{i,t+1}}{x_{i,t}}-\mathbb{E}_t\left[\log\frac{x_{i,t+1}}{x_{i,t}}\right]=-\epsilon_{i,t+1}|\Omega_t\ \sim\mathcal{N}\left(0,1\right),$$

so that the loss and the expected loss under the optimal forecast, L_{t+1}^o and $\mathbb{E}_t\left[L_{i,t+1}^o\right]$, are

$$
\begin{aligned}
L_{t+1}^o &= \epsilon_{i,t+1}^2=\sigma_i^2\frac{1}{\sigma_i^2}\epsilon_{i,t+1}^2 \\
\mathbb{E}_t\left[L_{i,t+1}^o\right] &= \sigma_i^2\mathbb{E}_t\left[\frac{1}{\sigma_i^2}\epsilon_{i,t+1}^2\right]=\sigma_i^2,
\end{aligned}
$$

where the last equality follows from $\epsilon_{i,t+1}^2=\sigma_i^2=\sigma_i^2\frac{1}{\sigma_i^2}\epsilon_{i,t+1}^2$, and $\frac{1}{\sigma_i^2}\epsilon_{i,t+1}^2|\Omega_t\ \sim\chi^2$ with mean 1. Under the narrow-bracketing rule (R1), $\log F_{i,t}^{R1}=\frac{1}{k}\sum_{j=1}^{k}\log\frac{x_{i,t+1-j}}{x_{i,t-j}}$, the forecast error in logs is

$$\log\frac{x_{i,t+1}}{x_{i,t}}-\log F_{i,t}^{R1}.$$

There are several possibilities. If $k=1$, $\log F_{i,t}^{R1}=\log\frac{x_{i,t}}{x_{i,t-1}}$, then the forecast error is $\log\frac{x_{i,t+1}}{x_{i,t}}-$

$\log \frac{x_{i,t}}{x_{i,t-1}} = -\left[\epsilon_{i,t+1} - (1-\gamma_i)\,\pi_{i,t} - (\pi_{i,t} - \pi_{i,t-1})\right]$, and

$$\mathbb{E}_t\left[L^{R1}_{i,t+1}\right] = \sigma_i^2\left(1 + \frac{\left[(1-\gamma_i)\,\pi_{i,t} + (\pi_{i,t} - \pi_{i,t-1})\right]^2}{\sigma_i^2}\right) = \mathbb{E}_t\left[L^{o}_{i,t+1}\right] + \left[(1-\gamma_i)\,\pi_{i,t} + (\pi_{i,t} - \pi_{i,t-1})\right]^2.$$

For a general k, one obtains

$$\mathbb{E}_t\left[L^{R1}_{i,t+1}\right] = \mathbb{E}_t\left[L^{o}_{i,t+1}\right] + \left[(1-\gamma_i)\,\pi_{i,t} + \frac{1}{k}\sum_{j=1}^{k}\left(\pi_{i,t+1-j} - \pi_{i,t-j}\right)\right]^2.$$

For $k \to \infty$,

$$\lim_{k\to\infty} \mathbb{E}_t\left[L^{R1}_{i,t+1}\right] = \mathbb{E}_t\left[L^{o}_{i,t+1}\right] + \left[(1-\gamma_i)\,\pi_{i,t}\right]^2.$$

QED

Proof of Proposition 4. The Proposition is stated in the text for the case of $\rho_{1,2} = 0$. Here we prove the Proposition for the general case with correlated prices, i.e., for a generic value of $\rho_{1,2} \in [0.1]$. We have that

$$\mathbb{E}\left[\log x_1 | \eta_y, \eta_2\right] = \mu_1 + \beta_y\left(\eta_y - \mu_y\right) + \beta_2\left(\eta_2 - \mu_2\right),$$

where coefficients equal

$$\beta_y = \frac{cov\left(\eta_y, \log x_1\right) - \left(\frac{cov(\eta_y,\eta_2)cov(\log x_1,\eta_2)}{var(\eta_2)}\right)}{var\left(\eta_y\right) - \frac{cov(\eta_y,\eta_2)^2}{var(\eta_2)}}, \ \ \beta_2 = \frac{cov\left(\eta_2, \log x_1\right) - \left(\frac{cov(\eta_y,\eta_2)cov(\log x_1,\eta_y)}{var(\eta_y)}\right)}{var\left(\eta_2\right) - \frac{cov(\eta_y,\eta_2)^2}{var(\eta_y)}}.$$

We have:
$cov\left(\eta_y, \log x_1\right) = cov\left(a\log x_1 + b\log x_2 + \epsilon_y, \log x_1\right) = a\sigma_1^2 + b\rho_{1,2}$,
$cov\left(\eta_2, \log x_1\right) = \rho_{1,2}$,
$cov\left(\eta_y, \eta_2\right) = cov\left(a\log x_1 + b\log x_2 + \epsilon_y, \log x_2 + \epsilon_2\right) = b\sigma_2^2 + a\rho_{1,2}$,
$var\left(\eta_y\right) = a^2\sigma_1^2 + b^2\sigma_2^2 + \sigma_y^2 + 2ab\rho_{1,2}$,
$var\left(\eta_2\right) = \sigma_2^2 + s_2^2$.
Substituting yields

$$\begin{aligned}
\beta_y &= \frac{a\sigma_1^2 + b\rho_{1,2} - \frac{\rho_{1,2}\left(a\rho_{1,2}+b\sigma_2^2\right)}{\sigma_2^2+s_2^2}}{a^2\sigma_1^2 + b^2\sigma_2^2 + s_y^2 + 2ab\rho_{1,2} - \frac{\left(a\rho_{1,2}+b\sigma_2^2\right)^2}{\sigma_2^2+s_2^2}} \\
\beta_2 &= \frac{\rho_{1,2} - \frac{\left(a\rho_{1,2}+b\sigma_2^2\right)\left(a\sigma_1^2+b\rho_{1,2}\right)}{a^2\sigma_1^2+b^2\sigma_2^2+s_y^2+2ab\rho_{1,2}}}{\sigma_2^2 + s_2^2 - \frac{\left(a\rho_{1,2}+b\sigma_2^2\right)^2}{a^2\sigma_1^2+b^2\sigma_2^2+s_y^2+2ab\rho_{1,2}}}
\end{aligned}$$

For $\rho_{1,2} = 0$, we obtain

$$\begin{aligned}
\beta_y &= \frac{a\sigma_1^2}{a^2\sigma_1^2 + b^2\sigma_2^2 + s_y^2 - \frac{b^2\sigma_2^4}{\sigma_2^2+s_2^2}}, \\
\beta_2 &= \frac{-\frac{\left(a\rho_{1,2}+b\sigma_2^2\right)\left(a\sigma_1^2\right)}{a^2\sigma_1^2+b^2\sigma_2^2+s_y^2}}{\sigma_2^2 + s_2^2 - \frac{b^2\sigma_2^4}{a^2\sigma_1^2+b^2\sigma_2^2+s_y^2}} = -\frac{ab\sigma_1^2\sigma_2^2}{a^2\sigma_1^2 + b^2\sigma_2^2 + s_y^2} \times \frac{a^2\sigma_1^2 + b^2\sigma_2^2 + s_y^2}{\left(\sigma_2^2 + s_2^2\right)\left(a^2\sigma_1^2 + b^2\sigma_2^2 + s_y^2\right) - b^2\sigma_2^4} \\
&= \frac{ab\sigma_1^2\sigma_2^2}{b^2\sigma_2^4 - \left(\sigma_2^2 + s_2^2\right)\left(a^2\sigma_1^2 + b^2\sigma_2^2 + s_y^2\right)}.
\end{aligned}$$

QED

Proof of Corollary 6.

$$\begin{aligned}
\lim_{s_y, s_2 \to +\infty} \beta_y &= \lim_{s_y, s_2 \to +\infty} \frac{a\sigma_1^2}{a^2\sigma_1^2 + b^2\sigma_2^2 + s_y^2 - \frac{b^2\sigma_2^4}{\sigma_2^2+s_2^2}} = 0, \\
\lim_{s_y, s_2 \to +\infty} \beta_2 &= \lim_{s_y, s_2 \to +\infty} \frac{ab\sigma_1^2\sigma_2^2}{b^2\sigma_2^4 - \left(\sigma_2^2 + s_2^2\right)\left(a^2\sigma_1^2 + b^2\sigma_2^2 + s_y^2\right)} = 0.
\end{aligned}$$

QED

Proof of Corollary 7.

$$\begin{aligned}
\lim_{s_2 \to +\infty} \beta_y &= \lim_{s_2 \to +\infty} \frac{a\sigma_1^2}{a^2\sigma_1^2 + b^2\sigma_2^2 + s_y^2 - \frac{b^2\sigma_2^4}{\sigma_2^2+s_2^2}} = \frac{a\sigma_1^2}{a^2\sigma_1^2 + b^2\sigma_2^2 + s_y^2}, \\
\lim_{s_2 \to +\infty} \beta_2 &= \lim_{s_2 \to +\infty} \frac{ab\sigma_1^2\sigma_2^2}{b^2\sigma_2^4 - \left(\sigma_2^2 + s_2^2\right)\left(a^2\sigma_1^2 + b^2\sigma_2^2 + s_y^2\right)} = 0.
\end{aligned}$$

QED

Proof of Corollary 8.

$$\begin{aligned}
\lim_{s_y, s_2 \to 0} \beta_y &= \lim_{s_y, s_2 \to 0} \frac{a\sigma_1^2}{a^2\sigma_1^2 + b^2\sigma_2^2 + s_y^2 - \frac{b^2\sigma_2^4}{\sigma_2^2+s_2^2}} = \frac{a\sigma_1^2}{a^2\sigma_1^2 + b^2\sigma_2^2 + - \frac{b^2\sigma_2^4}{\sigma_2^2}} = \frac{1}{a}, \\
\lim_{s_y, s_2 \to 0} \beta_2 &= \lim_{s_y, s_2 \to 0} \frac{ab\sigma_1^2\sigma_2^2}{b^2\sigma_2^4 - \left(\sigma_2^2 + s_2^2\right)\left(a^2\sigma_1^2 + b^2\sigma_2^2 + s_y^2\right)} = \frac{ab\sigma_1^2\sigma_2^2}{b^2\sigma_2^4 - \sigma_2^2\left(a^2\sigma_1^2 + b^2\sigma_2^2\right)} = -\frac{b}{a}.
\end{aligned}$$

QED

B Further Tables

Table A1: **Summary Statistics**

Panel A – Matched Duke-Compustat sample

	Mean	Std. Dev.	P05	Median	P95	N Obs.
Market-to-book	1.845	1.629	0.875	1.402	4.157	15,929
ROA	0.025	0.235	-0.154	0.037	0.167	16,591
Sales	10,617.34	28,482.18	53.66	2,043.96	49,545.00	17,799
Log(sales)	7.591	2.093	4.022	7.629	10.813	17,757
Assets	37,698.73	187,568.6	74.12	2,894.43	113,960.0	17,799
Log(assets)	7.993	2.221	4.306	7.971	11.644	17,799
Book Leverage	0.411	1.755	0.000	0.371	0.910	17,733
Capital Expenditure	0.045	0.058	0.001	0.031	0.134	16,200
R & D	0.060	0.202	0.000	0.025	0.211	9,043
Cash Flow	0.302	13.075	-1.127	0.413	3.005	17,010
Cash	4.895	62.328	0.014	0.563	15.188	17,269
Advertising	0.025	0.043	0.000	0.009	0.098	6,729
Dividends	0.117	1.420	0.000	0.060	0.349	17,391
Dividends (0/1)	0.607	0.488	0.000	1.000	1.000	17,391

Panel B – Compustat data

	Mean	Std. Dev.	P05	Median	P95	N Obs.
Market-to-book	1.843	2.526	0.708	1.297	4.489	105,769
ROA	0.006	0.279	-0.273	0.021	0.187	123,155
Sales	3,521.58	15,220.67	17.992	315.011	14,687.00	127,307
Log(sales)	5.911	2.062	2.890	5.753	9.595	127,307
Assets	12,626.69	98,056.55	24.147	597.555	30,241.99	140,894
Log(assets)	6.529	2.170	3.184	6.393	10.317	140,894
Book Leverage	0.408	22.120	0.000	0.362	0.996	139,264
Capital Expenditure	0.063	0.159	0.000	0.032	0.212	108,909
R & D	0.075	0.135	0.000	0.027	0.290	52,955
Cash Flow	0.050	0.261	-0.222	0.061	0.256	118,905
Cash	0.192	0.396	0.002	0.081	0.685	111,475
Advertising	0.034	0.103	0.000	0.009	0.131	39,813
Dividends	0.144	7.286	0.000	0.000	0.557	122,340
Dividends (0/1)	0.431	0.495	0.000	0.000	1.000	122,340

Table A2: **CFO Growth Forecasts and Realizations of Selected Balance Items**

Realizations in Compustat (percent)

	Mean	Std. Dev.	Q10	Median	Q90	N Obs.
Expected Growth in Revenues and in Earnings						
Revenues	13.38	35.96	-16.64	6.89	46.24	105,866
Earnings	-16.21	432.06	-207.66	-3.92	174.42	105,841
Expected Growth in Capital-Related Expenditures						
Capital Expenditures	35.71	132.99	-56.41	5.26	129.28	100,633
R & D	15.91	53.09	-25.24	6.75	57.64	40,715
Technology Spending	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
Expected Growth in Labor-Related Costs						
Wages	10.57	24.22	-9.57	6.94	31.96	29,491
Employees	6.18	25.30	-14.29	2.07	29.17	107,435
Outsourced Employees	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
Health Spending	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
Expected Growth in Productivity, Product Prices, and Advertising						
Productivity	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
Product Prices	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
Advertising	19.12	80.07	-37.35	4.39	71.33	34,251
Expected Growth in Cash Holdings and Corporate Payout						
Cash	76.55	308.36	-57.50	5.23	184.62	103,833
Dividends	18.74	97.74	-56.43	5.42	60.56	54,841
Share Repurchases	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.

Table A3: **Minimum Distance of Earnings Forecasts from Rules of Thumb**

	All	R1	R2	R3	R4	R5
Mean	0.026	0.045	0.021	0.028	0.032	0.031
Std. Dev.	0.033	0.054	0.028	0.046	0.035	0.030
Frac. Zeros	0.197	0.000	0.356	0.000	0.000	0.000
P10	0.000	0.002	0.000	0.002	0.005	0.004
P25	0.004	0.003	0.000	0.008	0.007	0.009
P50	0.014	0.027	0.014	0.010	0.017	0.015
P75	0.035	0.064	0.035	0.017	0.050	0.052
P90	0.068	0.101	0.057	0.099	0.071	0.073
P95	0.101	0.177	0.085	0.182	0.111	0.090
N of Observations	396	24	219	35	48	70
Fraction	1.000	0.061	0.553	0.088	0.121	0.177

Notes: Cross-sectional analysis with 396 CFOs.