Optimal Feedback in Contests

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with

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Motivation

- Contests can be an effective way to organize economic activity
  - Labor market (promotion) tournaments
  - Innovation contests
  - All-pay auctions
  - Legal & political battles
  - Athletic tournaments
- Contests are inherently dynamic, and designer may have informational advantage over participants about how they are doing mid-contest

This paper:

Characterizes optimal dynamic contests when the designer chooses *when* the contest ends, *how* a prize is allocated, and a real-time feedback policy

Applications

1. Promotion contests
   - A firm has an open VP slot and wants to promote one of its associates
   - It monitors efforts imperfectly, and is better informed than the associates themselves about their performance
   - How to design a contest to maximize the associates' efforts?
2. Innovation races
   - *2006 Netflix Prize*: $1M prize for an algorithm that predicts user film ratings with at least 10% better accuracy than Netflix' own algorithm
   - How to design the rules of contest?

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Related Literature

- Static tournaments / contests:
  - Lazear & Rosen ('81), Green & Stokey ('83), Nalebuff & Stiglitz ('83)
  - *Optimal prize allocation:* Moldovanu & Sela ('01), Drugov & Ryvkin ('18, '19), Olszewski & Siegel ('20)
  - *"Turning down the heat":* Fang, Noe & Strack ('18), Letina, Liu & Netzer ('20)
- Dynamic contests:
  - Taylor ('95), Benkert & Letina ('20)
- Feedback in contests:
  - *"Reveal intermediate progress?":* Yildirim ('05), Lizzeri, Meyer & Persico ('05), Aoyagi ('10), Ederer ('10), Goltsman & Mukherjee ('19)
  - *Contests for experimentation:* Halac, Kartik & Liu ('17)

Model (1/4): Players & Timing

- *Players:* A principal and $n \geq 2$ agents
- At $t = 0$, the principal designs a mechanism (contest) comprising
  i. a rule specifying *when* the mechanism will end,
  ii. a rule for allocating a \$1 prize, and
  iii. a real-time feedback policy
- At every $t > 0$, each agent
  - receives a message per the feedback policy, and
  - chooses to *work* or *shirk*; *i.e.*, $a_{i,t} \in \{0, 1\}$
- When mechanism ends, prize is awarded according to allocation rule

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Model (2/4): Effort, Signals & "Who observes what"

- Each agent's effort generates a binary signal: a Poisson "success"
  - Conditional on not having succeeded by t, an agent succeeds during $(t, t + dt)$ with probability $a_{i,t}dt$; *i.e.*, constant hazard rate of success
  - Each agent can succeed at most once (*extend to multiple successes later)
- **Who observes what:**
  - Principal observes successes but not efforts
  - Agents do not observe their rivals' successes
  - Ea. agent may or may not observe *own* success or do so probabilistically

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Model (3/4): Principal's Choice Variables

i. A **termination rule** is a stopping time w.r.t each agent's success time

- *e.g.*, mechanism may end at deadline, upon first success, randomly, etc

ii. A **prize allocation rule** specifies each agent's share of the prize q_i as a function of when each agent succeeds

- *e.g.*, prize may be awarded to first / second agent to succeed, split, etc

iii. A **feedback policy** specifies the message sent to each agent at every instant as a function of the agents' success times and past messages

- *e.g.*, Random feedback, private or public feedback, feedback about one's own or others' successes, feedback about feedback, etc
- $\mathcal{M}^{pronto}$: Keeps agents apprised of own success (but no other feedback)

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Model (4/4): Payoffs

- Given a contest, each agent's expected payoff is

$$u_{i,t} = \max_{a_{i,t} \in \{0,1\}} \mathbb{E}\left[q_i - c \int_0^{\tau} a_{i,t} dt \right],$$

where $c \in (1/n, 1)$.

- Principal designs a mechanism to maximize total effort

$$\max \mathbb{E}\left[\sum_{i=1}^{n} \int_0^{\tau} a_{i,t} dt \right]$$

$$\text{s.t. } \{a_{i,t}\} \text{ forms an equilibrium}$$

$$\sum_{i=1}^{n} q_i \leq 1. \qquad \text{(Budget Constraint)}$$

* Will argue that effort-maximizing contest also maximizes $\mathbb{E}[\#\text{successes}]$

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Roadmap

I. Sufficiency result for a mechanism to maximize total effort

II. Examples of effort-maximizing contests

III. Necessary conditions for optimality

IV. Effort-maximizing contest with shortest expected duration

V. *Extensions:* Multiple successes & Limited commitment

A Sufficiency Result

- Finding an optimal contest is hard because the choice variables are high-dimensional objects and can condition on the entire history.

Lemma 1. A contest is guaranteed to be optimal if in equilibrium:

i. The prize is awarded with probability 1

ii. Each agent earns zero rents

- The principal's objective can be written as

$$\mathbb{E}\left[\sum_{i=1}^{n}\int_0^{\tau} a_{i,t}\,dt\right] = \frac{1}{c}\left(\underbrace{\mathbb{E}\sum_i [q_i]}_{\text{Total Surplus} \leq 1} - \underbrace{\sum_i u_{i,0}}_{\text{Rents} \geq 0} \right) \leq \frac{1}{c}$$

- If a contest attains those bounds, it must be optimal!

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Example 1. Cyclical-Egalitarian Contest

- **Termination** τ^*. Runs in cycles of length T^* and is terminated at the end of the first cycle in which at least one agent has succeeded.
- **Egalitarian prize allocation**. Prize is shared equally among agents who have succeeded *irrespective of when they did so*.

Proposition 1.

- The contest with τ^*, *EGA*, and feedback policy $\mathcal{M}^{pronto}$ is optimal.
- In equilibrium, each agent works until they succeed and earns no rents

- Contest is optimal because it meets sufficiency conditions of Lemma:
  - T^* chosen such that marg. benefit of effort is equal to marg. cost
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Cyclical-Egalitarian Contest. Proof Sketch

- *Lemma 1:* Zero rents & prize awarded w.p 1 ⇒ Contest is optimal
  - Because contest ends only after an agent succeeds, 2^{nd} criterion is met
- Each agent's flow payoff can be expressed as

$$\underbrace{(1-p_t)}_{\Pr\{\text{no success by } t\}} \times \underbrace{a_t}_{\text{success rate}} \times \underbrace{R_t}_{\mathbb{E}[\text{prize}|\text{success at } t]} - \underbrace{c \times a_t}_{\text{cost of effort}}$$

- $\mathcal{M}^{pronto}$ implies that $p_t = 0$, and it jumps to 1 as soon as he succeeds
- Each agent's expected reward from success at t is:

$$R_t = \mathbb{E}\left[\frac{1}{1 + (\#\text{rivals who succeed by } T^*)}\right]$$

Can choose T^* such that $R_t = c$ so that working is *just* IC for each agent until he succeeds, and he earns zero rents.

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Example 2: Beeps Contest

- **Termination rule.** Conditional on at least *one* success, at T^* the contest ends w.p q, and from then onwards with rate r.
- **Prize allocation**. Prize shared equally among agents who succeeded prior to T^*. Otherwise, the first agent to succeed wins entire prize.

Proposition 2.

There exist $\{q, r\}$ such that this contest, coupled with $\mathcal{M}^{pronto}$ is optimal

- Before T^*, resembles a single cycle of the cyclical-egalitarian contest
- After T^*, termination rule keeps ea. unsuccessful agent's belief that nobody has succeeded constant at c. Flow payoff from working:

$$\underbrace{\Pr\{\text{no success yet}\}}_{=c} \times \underbrace{(\text{HR success})}_{=1} \times \underbrace{\mathbb{E}[\text{prize}]}_{=1} - c = 0$$

so ea. unsuccessful agent is just willing to work and earns no rents

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Example 3: Netflix-Style Contest

- **Termination.** The first agent to succeed triggers countdown T^c
- **Prize allocation**. First agent to succeed earns prize $\alpha/(\alpha + N)$, and each agent who succeeds during countdown earns $1/(\alpha + N)$

Proposition 3.

There exist $\{T^c, \alpha < 1\}$ s.t this contest, coupled with $\mathcal{M}^{pronto}$ is optimal

- If the first agent to succeed won the entire prize, he would earn rents
  - Can extract rents by extending contest & giving rivals another chance
- *Aim:* Expected reward from success $R_{i,t} = c$ for all i, t
  - During countdown agents know one agent has already succeeded, so must earn a bigger share of the prize than the first agent; hence $\alpha < 1$
- Resembles Netflix prize: first success triggered a 30-day countdown

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Why a contest instead of individual contracts

- *Definition:* "Contest" if effort creates a negative externality
  - *i.e.*, if an agent's payoff decreases in others' efforts or successes
- Suppose principal splits the prize and offers individual contracts
  - Because prize = \$1, the marginal benefits of effort $\sum_i R_{i,t} \leq \$1$
- Optimal contests have $R_{i,t} = c$ for all i, t, so $\sum_i R_{i,t} = cn > 1$
  - The advantage of a contest is that it allows pooling the agents' ICs
  - Prize not awarded to one agent can be used to incentivize another
  - This pooling is valuable whenever $c > 1/n$; *i.e.*, when prize is scarce
- *Remark:* If principal can meet \$1 budget constraint *in expectation*, then individual contracts suffice

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A Necessity Result

- *Obs.* Every optimal contest meets sufficiency conditions of Lemma 1

Proposition 4. *Every* optimal contest features:

i. Termination rule s.t. contest doesn't end until 1+ agents succeed

ii. $\mathcal{M}^{pronto}$ feedback

iii. Egalitarian prize structure; *i.e.*, $R_{i,t} = c$ whenever $a_{i,t} = 1$

- $\mathcal{M}^{pronto}$ ensures there is never asymmetric info btw principal & agent
  - Suppose on the eq'm path, there is an interval in which $p_{i,t} \in (0,1)$
  - IC requires $(1 - p_{i,t})R_{i,t} \geq c$, so $R_{i,t} > c$ during that interval
  - Agent could shirk until that interval so that $p_{i,t} = 0$ and earn rents
- Given $\mathcal{M}^{pronto}$, full rent extraction requires $R_{i,t} = c$ whenever an agent is supposed to be working

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Optimal Contests: Remarks

In *every* optimal contest:

i. Owing to $\mathcal{M}^{pronto}$, it is immaterial whether agents observe their successes directly, or do so probabilistically.
   - It *may* be important however that they don't observe *others'* successes

ii. Due to $\mathcal{M}^{pronto}$, an optimal contest maximizes total effort conditional on not having succeeded already. So it also maximizes $\mathbb{E}[\#\text{successes}]$

iii. Principal would be **no** better off with a more precise monitoring tech.
   - To extract all rents, monitoring tech. must generate no type-I errors

iv. Even if agents could succeed multiple times, because principal attains first-best payoff, wolog she can reward only the *first* success.

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Optimal Contests: Remarks

In *every* optimal contest:

i. Owing to $\mathcal{M}^{pronto}$, it is immaterial whether agents observe their successes directly, or do so probabilistically.
   - It *may* be important however that they don't observe *others'* successes

ii. Due to $\mathcal{M}^{pronto}$, an optimal contest maximizes total effort conditional on not having succeeded already. So it also maximizes $\mathbb{E}[\#\text{successes}]$

iii. Principal would be **no** better off with a more precise monitoring tech.
   - To extract all rents, monitoring tech. must generate no type-I errors

iv. Even if agents could succeed multiple times, because principal attains first-best payoff, wolog she can reward only the *first* success.

Minimum-duration, Effort-maximizing Contest

- Every effort-maximizing contest implements total effort $1/c$
- Here, we characterize the one with the shortest expected duration
  - *e.g.*, suppose the principal incurs a small cost p.u of time contest is *on*
- Fix an effort-maximizing contest, and define for each k,

$$T_k := \mathbb{E}[\text{time when } k \text{ agents are working}].$$

  - $T_k \leq 1/k$ because when k agents work, next success $\sim exp(1/k)$
  - Total effort $= \sum_k kT_k = 1/c$
  - Expected duration of contest $= \sum_k T_k$
- Roadmap:
  a. Suppose we can choose $T_1, \ldots, T_n$ directly $\Rightarrow$ Lower bound on duration
  b. Find a contest that achieves this lower bound

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A Lower Bound for Contest Duration

- Consider the following relaxed problem:

$$\min_{T_1,\dots,T_n} \sum_{k=1}^{n} T_k \quad \text{s.t.} \quad \sum_{k=1}^{n} kT_k = \frac{1}{c} \quad \text{and} \quad 0 \leq T_k \leq \frac{1}{k}.$$

- Define $K^* = \lfloor 1/c \rfloor$. The following is the unique solution:

$$\underline{T}_k = \begin{cases} 1/k & \text{if } k > n - K^* \\ (1/c - K^*)/(n - K^*) & \text{if } k = n - K^* \\ 0 & \text{if } k < n - K^*. \end{cases}$$

Lemma 2. Every effort-maximizing contest has $\mathbb{E}[\text{duration}] \geq \sum_k \underline{T}_k$

- W.p 1, contest must end *after* K^* but *before* $K^* + 1$ agents succeed
- None of the earlier examples satisfy this criterion!

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Second-Chance Contest

- **Termination rule.** After K^* agents succeed, contest is terminated upon the next success or countdown T^{sc} ends, whichever comes first
- **Prize allocation rule.**
  - If an agent succeeds during the countdown, he earns c
  - Remaining prize is shared equally among the first K^* successful agents

Proposition 5.

There exists a T^{sc} such that this contest coupled with $\mathcal{M}^{pronto}$ feedback has the smallest duration among effort-maximizing contests.

- Meets sufficiency conditions of Lemma 1 and lower bound of Lemma 2
- *Remark.* Remains optimal if agents also observe others' successes

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Extension I: Agents can succeed multiple times

- Assume principal wants to implement max. effort in shortest duration
- A second-chance contest with countdown duration $T^{sc} \in \{0, \infty\}$:
  - Ends w.p $(1 - 1/c + K^*)$ upon K^{*th} success; otherwise upon $K^* + 1^{st}$
  - Prize c for $K^* + 1^{st}$ success; remaining prize split among K^* successes
  - Agents are told when K^{*th} success occurs (other feedback immaterial)

Proposition 6.

This contest implements total effort $1/c$ and all agents work until the end

- Each agent's expected reward from a success is equal to its cost c
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Extension II: Limited Commitment

- Principal must credibly commit to feedback policy & termination rule
  - Ex-post, she has incentives to keep contest / agents "going"
- Suppose principal cannot credibly provide feedback and termination cannot condition on successes

Proposition 7. Assume agents *do not* observe their own successes.

- Optimal no-feedback contest ends at a deterministic deadline T, and the prize is shared equally among all agents who succeed by deadline.
- In equilibrium, all agents work continuously throughout $[0, T]$

- Over time, each agent believes it is ever more likely they have already succeeded, in which case continuing to exert effort is moot
- Egalitarian prize counteracts this by maximally backloading incentives

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Conclusions

- Contest design with endogenous feedback to maximize total effort
- Many contests are optimal. Every optimal one satisfies two criteria:
    - i. Agents are kept fully apprised of their own success
    - ii. Expected reward from success is constant
- Characterize the minimum-duration, effort-maximizing contest
    - Countdown is triggered once a pre-specified #agents succeed
    - Contest ends when countdown ends or another agent succeeds
    - Prize is shared (approximately) equally among successful agents
- *Broader agenda:* Information design in agency models
    - How to use information to provide incentives (under moral hazard)